Computer Algebra Independent Integration Tests

Summer 2024

3-Logarithms/176-3.8

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**728**]. This is test number [176].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	94.23 (686)	5.77 (42)
Rubi	87.91 (640)	12.09 (88)
Fricas	60.58 (441)	39.42 (287)
Maple	58.52 (426)	41.48 (302)
Giac	53.71 (391)	46.29 (337)
Reduce	53.43 (389)	46.57 (339)
Maxima	51.24 (373)	48.76 (355)
Mupad	45.05 (328)	54.95 (400)
Sympy	29.81 (217)	70.19 (511)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

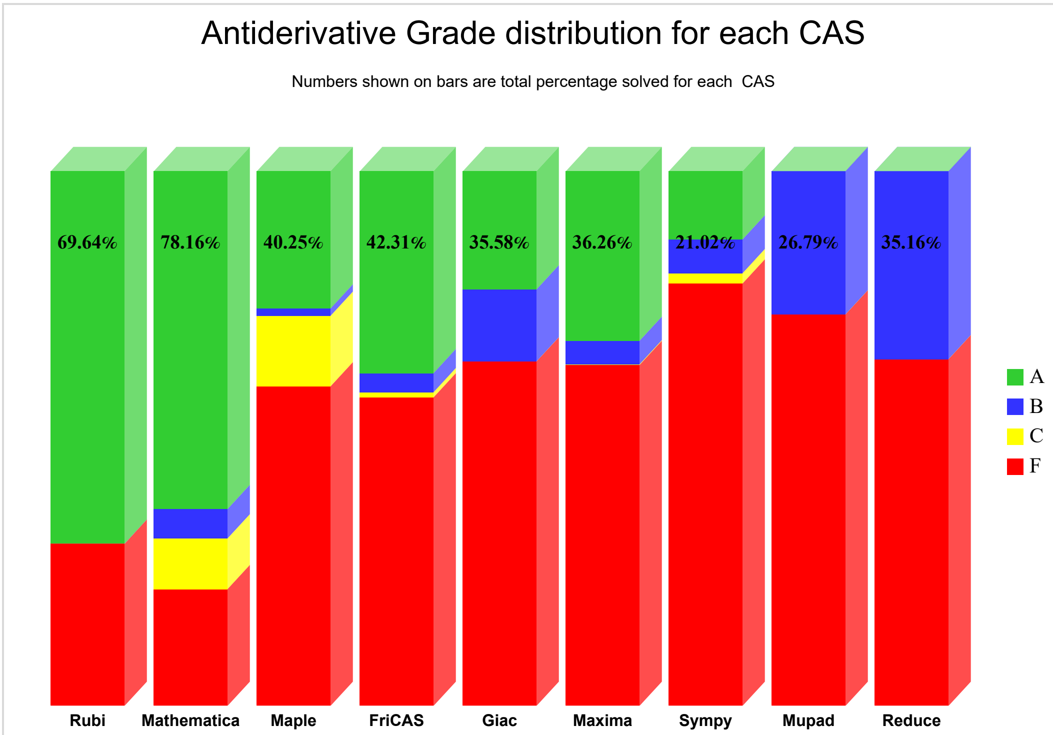
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

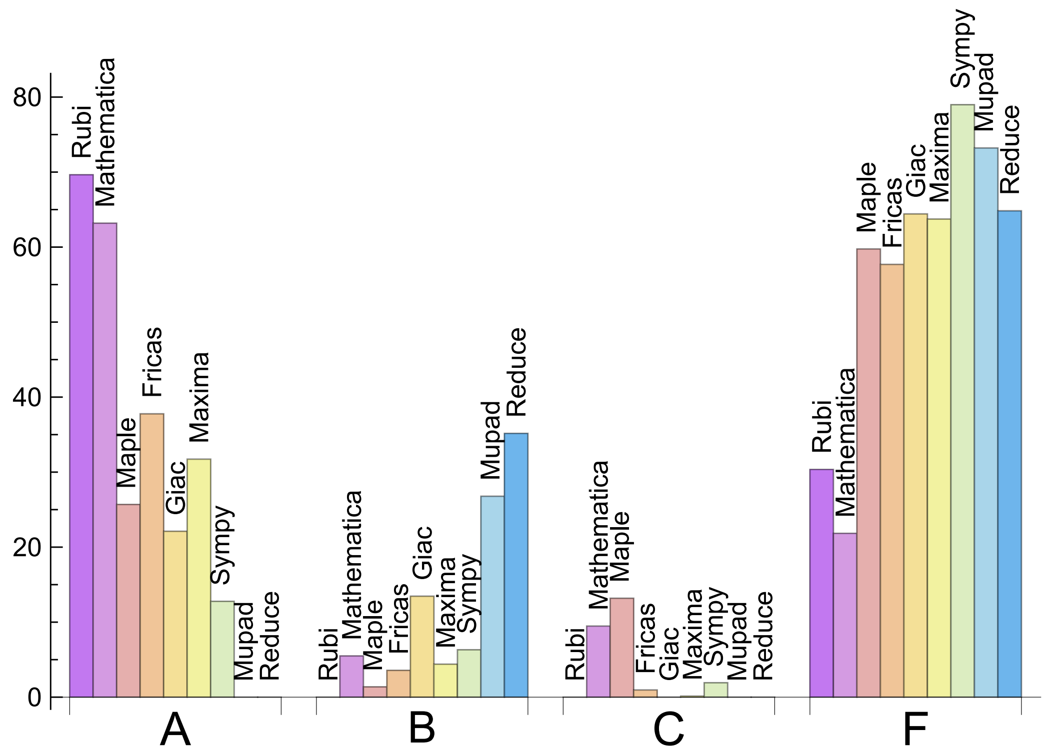
System	% A grade	% B grade	% C grade	% F grade
Rubi	69.643	0.000	0.000	30.357
Mathematica	63.187	5.495	9.478	21.841
Fricas	37.775	3.571	0.962	57.692
Maxima	31.731	4.396	0.137	63.736
Maple	25.687	1.374	13.187	59.753
Giac	22.115	13.462	0.000	64.423
Sympy	12.775	6.319	1.923	78.984
Mupad	0.000	26.786	0.000	73.214
Reduce	0.000	35.165	0.000	64.835

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	42	100.00	0.00	0.00
Rubi	88	100.00	0.00	0.00
Fricas	287	100.00	0.00	0.00
Maple	302	100.00	0.00	0.00
Maxima	355	77.75	0.00	22.25
Giac	337	97.33	0.00	2.67
Reduce	339	100.00	0.00	0.00
Mupad	400	0.00	100.00	0.00
Sympy	511	49.32	46.97	3.72

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.11
Fricas	0.16
Reduce	0.19
Giac	0.29
Mathematica	0.93
Rubi	0.99
Maple	2.15
Sympy	20.43
Mupad	22.87

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	130.22	1.04	47.00	1.00
Maxima	152.74	1.65	98.00	1.00
Sympy	187.72	2.04	83.00	1.08
Maple	193.19	1.44	77.00	1.00
Giac	220.53	1.52	96.00	1.11
Rubi	225.04	1.00	116.00	1.00
Mathematica	251.62	3.24	114.00	1.00
Fricas	280.41	1.69	84.00	1.14
Reduce	293.81	4.93	115.00	1.19

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

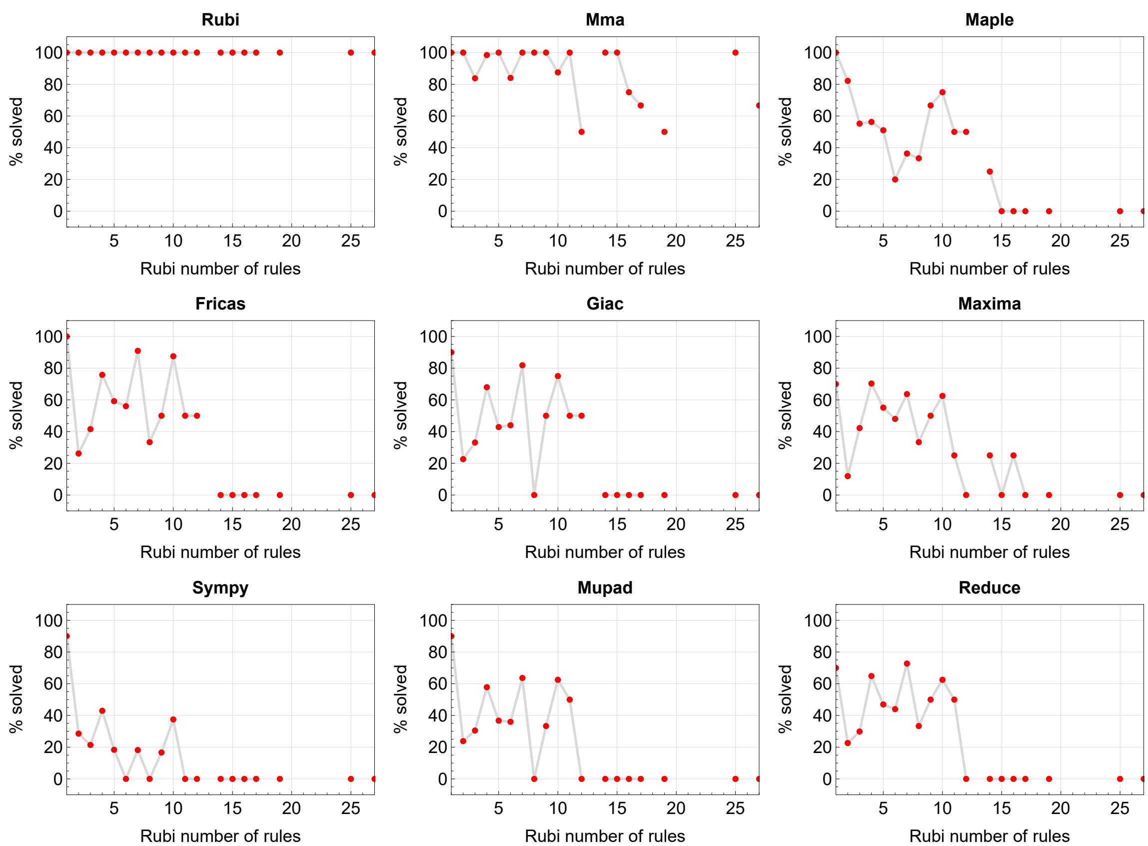


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

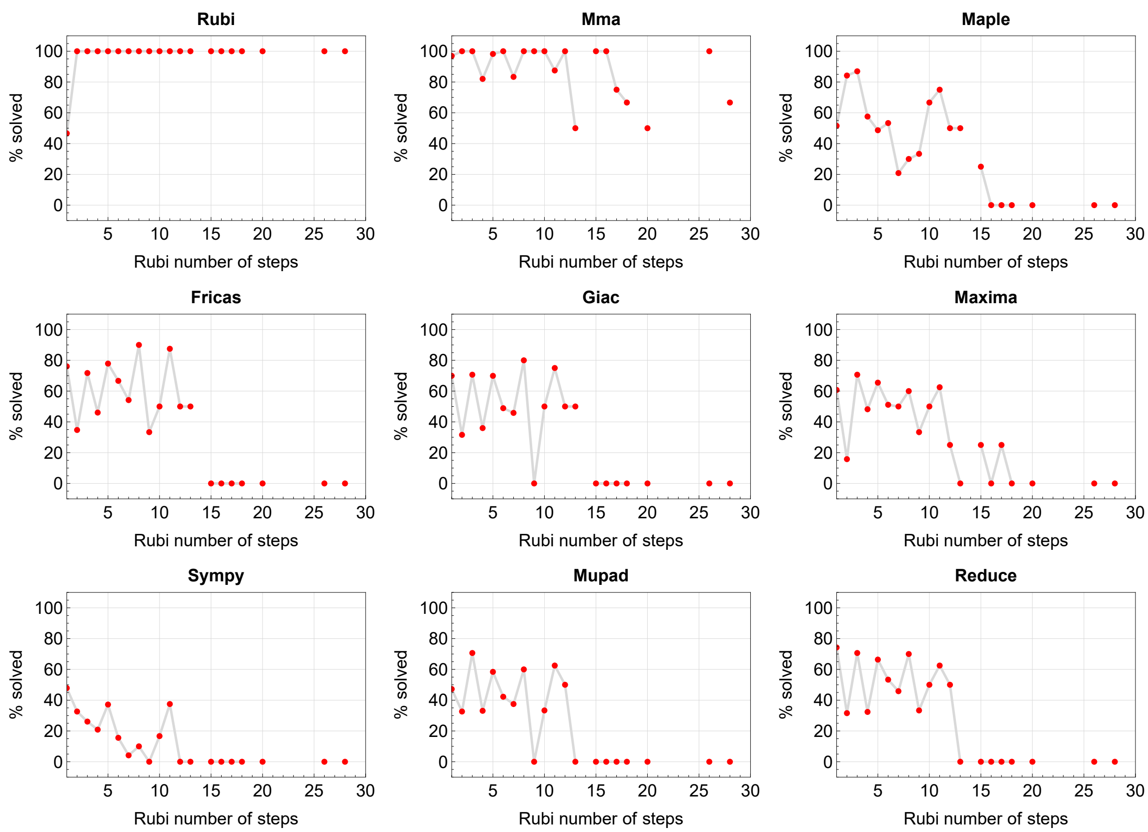


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

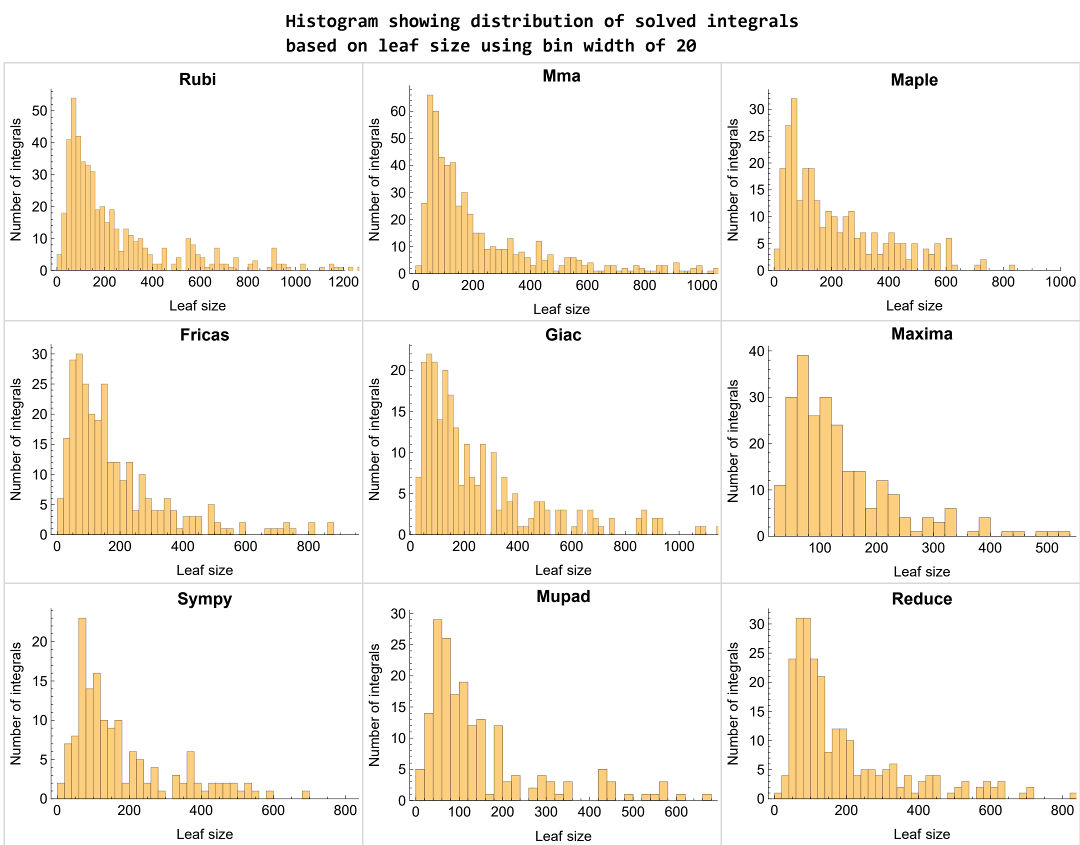


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

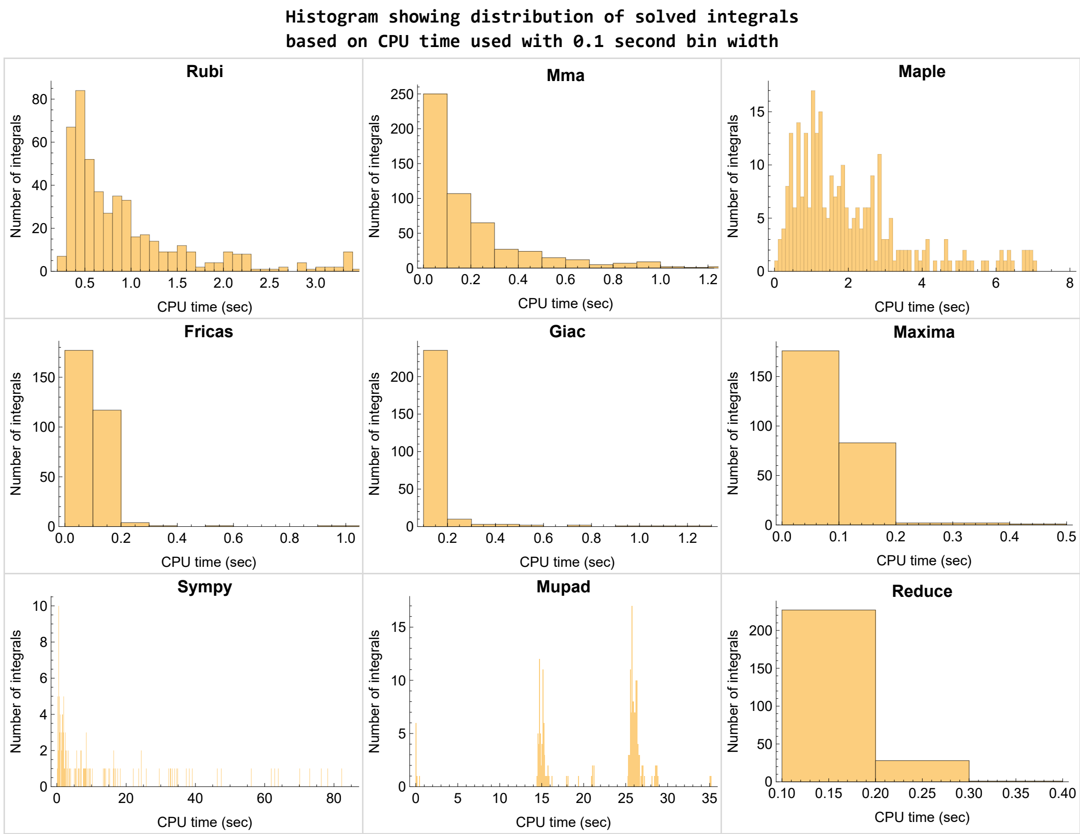


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

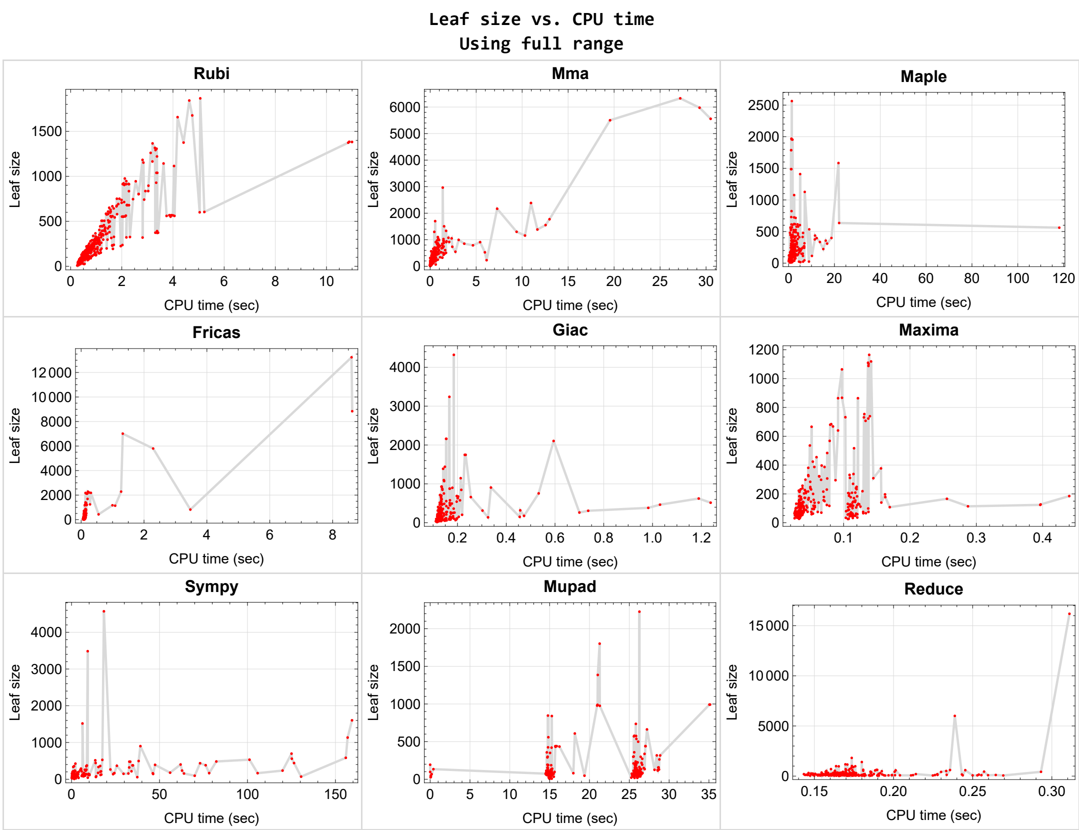


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 381, 382, 383, 384, 385, 386, 387, 388, 398, 485, 486, 487, 488, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 722, 727, 728}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {98, 99, 100, 101, 158, 159, 277, 298, 299, 485, 486, 487, 488, 528, 530, 531}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{77, 82, 83, 96, 97, 163, 168, 292, 408, 409, 411, 412, 413, 414, 418, 419, 420, 429, 430, 431, 432, 434, 435, 436, 437, 438, 450, 451, 452, 453, 454, 455, 460, 461, 462, 471, 472, 473, 474, 475, 483, 484, 498, 499, 500, 501, 502, 503, 504, 505, 516, 517, 518, 519, 520, 524, 525, 526\}$

Mathematica $\{131, 134, 158, 159, 168, 272, 292, 353, 355, 358, 376, 656, 657\}$

Maple $\{67, 73, 81, 82, 83, 84, 85, 87, 88, 89, 90, 102, 103, 109, 110, 116, 117, 132, 133, 135, 136, 137, 138, 139, 140, 148, 149, 150, 166, 173, 174, 175, 238, 239, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 271, 273, 274, 291, 293, 294, 295, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 619, 620, 624\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
```

```
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

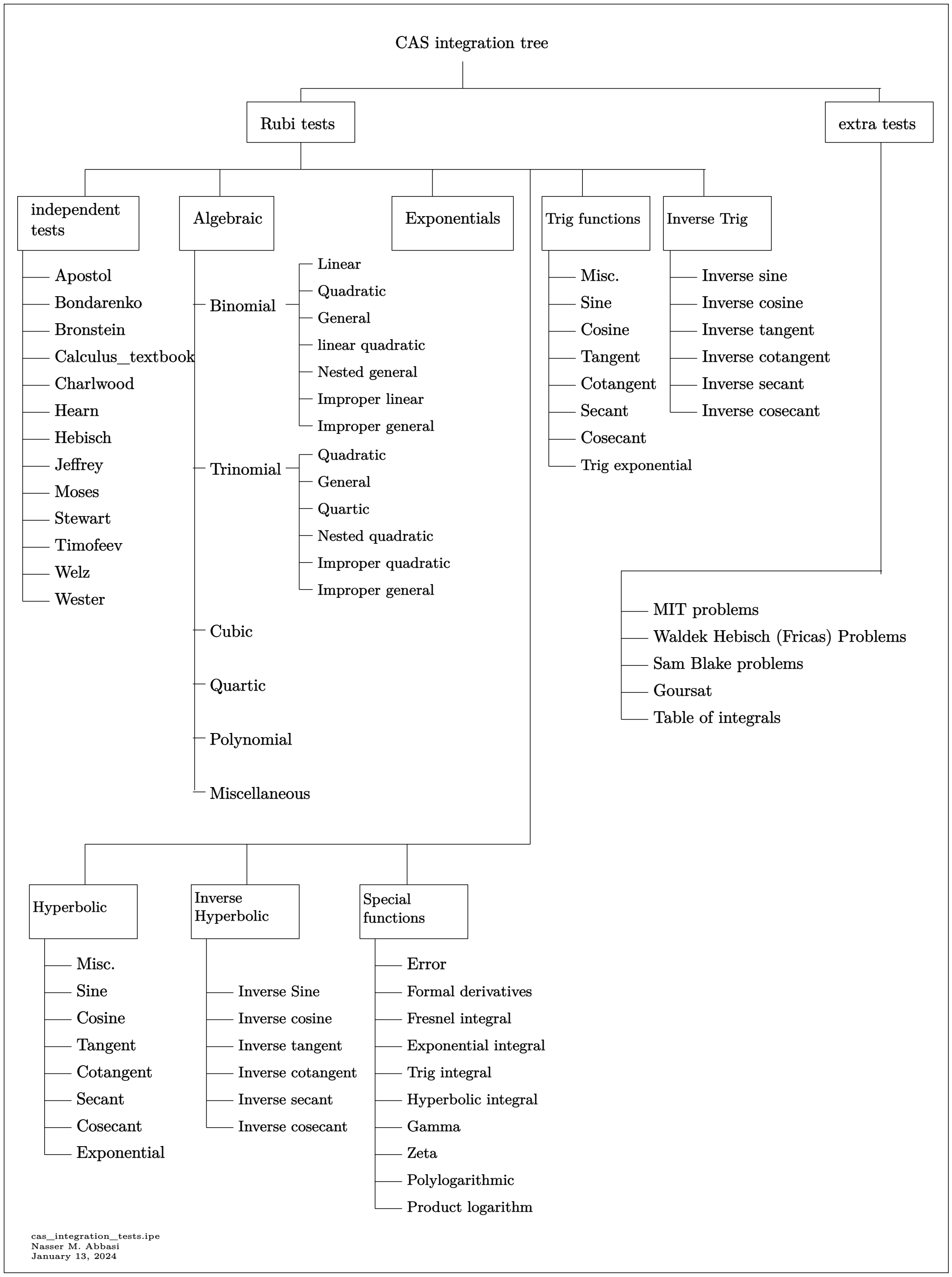
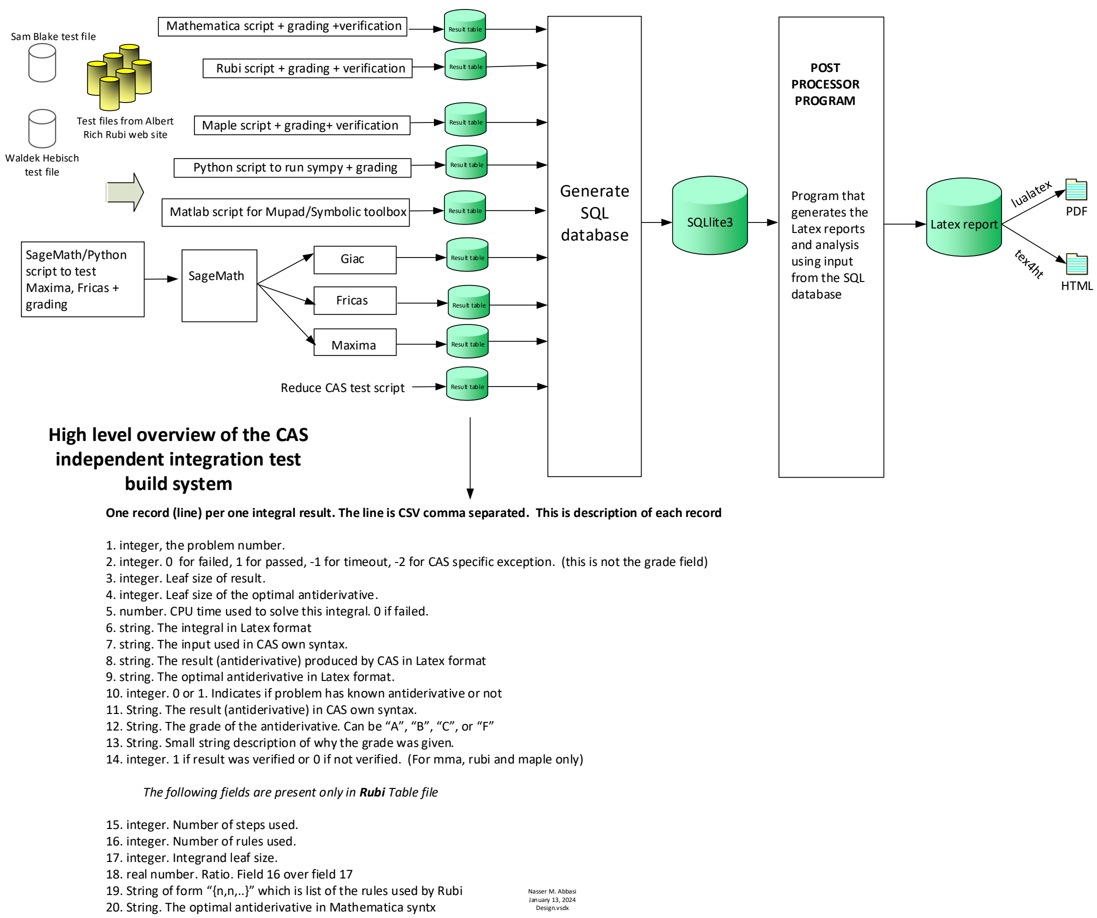


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 715, 716, 717, 718, 719, 720, 721, 723, 724, 725, 726 }

B grade { }

C grade { }

F normal fail { 497, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 15, 16, 18, 19, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 97, 102, 103, 109, 110, 116, 117, 124, 126, 128, 129, 130, 132, 134, 135, 136, 138, 139, 140, 148, 149, 150, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 257, 258, 261, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 375, 378, 379, 380, 389, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 469, 471, 472, 474, 475, 476, 477, 478, 481, 482, 484, 485, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 507, 508, 510, 512, 513, 514, 515, 522, 523, 527, 534, 535, 541, 547, 548, 549, 559, 585, 586, 587, 606, 607, 608, 611, 612, 613, 616, 617, 618, 619, 624, 629, 630, 631, 633, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 725, 726 }

B grade { 45, 80, 94, 95, 96, 98, 99, 101, 158, 159, 174, 175, 277, 298, 299, 376, 390, 411, 418, 419, 432, 453, 460, 473, 483, 486, 487, 488, 500, 516, 517, 528, 530, 531, 620, 621, 622, 623, 723, 724 }

C grade { 9, 11, 14, 17, 20, 23, 24, 36, 38, 89, 90, 100, 131, 133, 137, 191, 192, 193, 196, 197, 233, 234, 238, 239, 247, 254, 255, 256, 259, 260, 266, 267, 321, 322, 335, 336, 337, 347, 433, 434, 435, 468, 470, 479, 480, 501, 502, 509, 511, 518, 519, 520, 521, 524, 609, 610, 614, 615, 628, 632, 634, 643, 656, 657, 658, 670, 671, 683, 721 }

F normal fail { 123, 125, 127, 370, 373, 374, 377, 436, 437, 438, 503, 504, 505, 525, 526, 532, 533, 538, 539, 540, 553, 554, 555, 556, 557, 558, 562, 563, 564, 565, 568, 569, 575, 576, 591, 592, 593, 635, 696, 697, 709, 710 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 77, 78, 79, 91, 92, 93, 123, 124, 125, 126, 127, 128, 129, 130, 169, 172, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 268, 269, 270, 288, 289, 290, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 358, 359, 378, 379, 380, 389, 390, 392, 393, 395, 396, 397, 403, 424, 426, 445, 466, 492, 636, 637, 638, 639, 649, 650, 651, 652, 717, 718, 719, 726 }

B grade { 6, 41, 170, 171, 176, 313, 391, 394, 511, 720 }

C grade { 19, 67, 73, 81, 82, 83, 84, 85, 87, 88, 89, 90, 102, 103, 109, 110, 116, 117, 132, 133, 135, 136, 137, 138, 139, 140, 148, 149, 150, 166, 173, 174, 175, 195, 233, 234, 235, 236, 237, 238, 239, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 271, 273, 274, 291, 293, 294, 295, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 619, 620, 624, 716, 721 }

F normal fail { 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 74, 75, 76, 80, 86, 94, 95, 96, 97, 131, 134, 160, 163, 164, 165, 167, 168, 206, 207, 208, 209, 210, 212, 213, 214, 215, 264, 265, 266, 267, 272, 292, 353, 354, 355, 356, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 621, 622, 623, 628, 629, 630, 631, 632, 633, 634, 635, 640, 641, 642, 643, 644, 645, 646, 647, 648, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 723, 724, 725 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 67, 68, 69, 73, 77, 78, 79, 91, 92, 93, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 148, 149, 150, 163, 164, 165, 167, 168, 169, 172, 173, 177, 178, 179, 181, 184, 185, 186, 187, 189, 194, 198, 199, 200, 202, 268, 269, 270, 288, 289, 290, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 378, 379, 380, 389, 390, 391, 395, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 464, 465, 466, 468, 469, 470, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 507, 508, 509, 510, 513, 514, 515, 519, 520, 527, 619, 620, 623, 624, 636, 637, 638, 639, 645, 646, 647, 648, 649, 650, 651, 652, 659, 660, 661, 662, 663, 664, 665, 666, 672, 673, 674, 675, 676, 677, 678, 679, 685, 686, 687, 688, 689, 690, 691, 692, 698, 699, 700, 701, 702, 703, 704, 705, 711, 712, 713, 714, 717, 718, 719, 726 }

B grade { 170, 171, 176, 182, 183, 190, 203, 204, 392, 417, 439, 506, 511, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 621, 622, 720 }

C grade { 191, 192, 193, 196, 197, 716, 721 }

F normal fail { 6, 19, 31, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 70, 71, 72, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 166, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 370, 371, 372, 373, 374, 375, 376, 377, 393, 394, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 628, 629, 630, 631, 632, 633, 634, 635, 640, 641, 642, 643, 644, 653, 654, 655, 656, 657, 658, 667, 668, 669, 670, 671, 680, 681, 682, 683, 684, 693, 694, 695, 696, 697, 706, 707, 708, 709, 710, 715, 723, 724, 725 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 68, 69, 77, 78, 79, 80, 81, 82, 83, 91, 92, 93, 95, 96, 97, 129, 130, 132, 163, 164, 165, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 223, 224, 225, 243, 244, 245, 246, 264, 310, 311, 312, 315, 316, 317, 323, 324, 325, 329, 330, 331, 341, 342, 350, 351, 352, 358, 371, 372, 375, 376, 393, 396, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 465, 469, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 510, 514, 519, 520, 527, 636, 637, 638, 639, 645, 646, 647, 648, 649, 650, 651, 652, 659, 660, 661, 662, 663, 664, 665, 666, 676, 677, 678, 679, 685, 689, 690, 691, 692, 702, 703, 704, 705, 711, 718, 719, 726 }

B grade { 6, 19, 31, 41, 45, 50, 94, 180, 222, 340, 378, 379, 390, 391, 394, 404, 425, 439, 446, 467, 493, 512, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615 }

C grade { 263 }

F normal fail { 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 70, 71, 72, 73, 74, 75, 76, 84, 85, 86, 87, 88, 89, 90, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 131, 138, 139, 140, 148, 149, 150, 160, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 188, 195, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 265, 266, 267, 271, 291, 313, 314, 326, 327, 328, 338, 339, 345, 348, 349, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 373, 374, 377, 380, 389, 392, 395, 397, 399, 411, 412, 413, 414, 418, 419, 420, 429, 430, 431, 432, 436, 437, 438, 453, 454, 455, 460, 461, 462, 473, 474, 475, 483, 484, 497, 498, 499, 500, 503, 504, 505, 516, 517, 518, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 630, 631, 632, 633, 634, 635, 640, 641, 642, 643, 644, 653, 654, 655, 656, 657, 658, 667, 668, 669, 670, 671, 672, 673, 674, 675, 680, 681, 682, 683, 684, 686, 687, 688, 693, 694, 695, 696, 697, 698, 699, 700, 701, 706, 707, 708, 709, 710, 712, 713, 714, 715, 716, 721, 723, 724, 725 }

F(-1) timedout fail { }

F(-2) exception fail { 100, 133, 134, 135, 136, 137, 268, 269, 270, 272, 273, 274, 276, 277, 279, 288, 289, 290, 292, 293, 294, 295, 297, 298, 299, 301, 318, 319, 320, 321, 322, 332, 333, 334, 335, 336, 337, 343, 344, 346, 347, 353, 354, 355, 356, 381, 382, 383, 384, 385, 386, 387, 388, 464, 466, 468, 470, 476, 477, 478, 479, 480, 485, 486, 487, 488, 509, 511, 513, 515, 521, 522, 523, 528, 529, 530, 531, 717, 720 }

Giac

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 36, 37, 38, 39, 40, 42, 43, 44, 54, 77, 78, 79, 92, 93, 102, 103, 110, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 150, 178, 179, 181, 182, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 202, 268, 269, 270, 288, 289, 290, 312, 318, 319, 320, 321, 322, 332, 333, 334, 335, 336, 337, 350, 403, 410, 421, 423, 424, 433, 434, 452, 464, 465, 466, 468, 469, 470, 472, 482, 489, 490, 491, 492, 494, 495, 496, 501, 508, 509, 510, 511, 513, 514, 515, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 636, 637, 638, 639, 645, 646, 647, 648, 649, 650, 651, 652, 659, 660, 661, 662, 663, 664, 665, 666, 676, 677, 678, 679, 689, 690, 691, 692, 702, 703, 704, 705, 716, 717, 718 }

B grade { 10, 12, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 45, 46, 47, 48, 49, 51, 52, 53, 91, 109, 116, 117, 148, 149, 176, 177, 183, 198, 199, 200, 203, 204, 310, 311, 315, 316, 317, 323, 324, 325, 329, 330, 331, 390, 393, 400, 401, 402, 405, 406, 407, 408, 409, 415, 416, 417, 422, 426, 427, 428, 435, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 456, 457, 458, 459, 463, 471, 481, 502, 506, 507, 698, 699, 700, 701, 711, 712, 713, 714, 719, 720, 721, 726 }

C grade { }

F normal fail { 6, 19, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 389, 391, 392, 394, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 630, 631, 632, 633, 634, 635, 640, 641, 642, 643, 644, 653, 654, 655, 656, 657, 658, 667, 668, 669, 670, 671, 680, 681, 682, 683, 684, 693, 694, 695, 696, 697, 706, 707, 708, 709, 710, 715, 723, 724, 725 }

F(-1) timedout fail { }

F(-2) exception fail { 211, 672, 673, 674, 675, 685, 686, 687, 688 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 77, 78, 79, 91, 92, 93, 124, 126, 128, 129, 130, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 202, 203, 204, 268, 269, 270, 288, 289, 290, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 379, 390, 391, 393, 394, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 465, 466, 469, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 510, 511, 514, 519, 520, 527, 639, 652, 716, 717, 718, 719, 720, 721, 726 }

C grade { }

F normal fail { }

F(-1) timedout fail { 6, 19, 31, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 125, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 380, 389, 392, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 464, 467, 468, 470, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 509, 512, 513, 515, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 723, 724, 725 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 4, 8, 10, 12, 13, 16, 17, 18, 20, 22, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 37, 39, 43, 46, 47, 48, 49, 56, 58, 59, 77, 78, 79, 91, 92, 93, 124, 126, 128, 129, 130, 160, 178, 179, 191, 192, 193, 194, 200, 268, 289, 290, 311, 312, 332, 400, 401, 402, 403, 421, 422, 423, 424, 442, 443, 444, 445, 465, 466, 490, 491, 492, 511, 636, 637, 638, 639, 649, 650, 651, 652, 659, 663, 664, 665, 666, 689, 690, 691, 692, 726 }

B grade { 3, 5, 7, 9, 11, 36, 38, 40, 42, 44, 51, 52, 54, 176, 177, 181, 182, 183, 184, 185, 186, 187, 198, 199, 202, 203, 269, 270, 315, 318, 319, 320, 321, 322, 325, 333, 334, 335, 336, 405, 406, 426, 447, 717, 718, 719 }

C grade { 45, 70, 71, 72, 74, 75, 76, 169, 170, 171, 212, 213, 214, 215 }

F normal fail { 6, 19, 31, 41, 50, 57, 61, 62, 63, 64, 65, 66, 67, 73, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 125, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 180, 188, 201, 205, 209, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 262, 263, 264, 271, 273, 274, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 343, 344, 345, 346, 347, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 390, 391, 392, 393, 394, 395, 396, 397, 399, 404, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 425, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 446, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 462, 467, 472, 473, 477, 483, 493, 498, 499, 500, 501, 503, 504, 505, 506, 522, 535, 559, 620, 622, 624, 628, 629, 630, 631, 632, 633, 634, 635, 640, 641, 642, 643, 644, 645, 646, 647, 653, 654, 655, 656, 657, 658, 660, 661, 662, 667, 668, 669, 670, 671, 672, 673, 674, 675, 679, 680, 681, 683, 684, 685, 686, 693, 694, 695, 696, 697, 698, 699, 700, 701, 704, 705, 706, 707, 709, 710, 711, 712, 715, 723, 724, 725 }

F(-1) timedout fail { 14, 15, 21, 23, 24, 53, 55, 60, 190, 195, 196, 197, 204, 206, 207, 210, 211, 218, 224, 231, 232, 233, 234, 235, 236, 237, 238, 239, 245, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 265, 266, 267, 272, 276, 279, 287, 288, 291, 292, 296, 297, 300, 301, 305, 309, 310, 316, 317, 323, 324, 329, 330, 331, 337, 341, 342, 348, 349, 350, 351, 352, 353, 354, 355, 356, 370, 371, 373, 374, 375, 376, 377, 380, 381, 384, 385, 388, 407, 427, 428, 435, 441, 448, 449, 456, 463, 464, 468, 469, 470, 471, 474, 475, 476, 478, 479, 480, 481, 482, 484, 485, 487, 488, 489, 494, 495, 496, 497, 502, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 609, 610, 614, 615, 616, 617, 618, 621, 626, 627, 648, 676, 677, 678, 682, 687, 688, 702, 703, 708, 713, 714, 716, 720, 721 }

F(-2) exception fail { 68, 69, 189, 208, 372, 378, 379, 382, 383, 387, 389, 606, 607, 608, 611, 612, 613, 619, 623 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 68, 69, 77, 78, 79, 91, 92, 93, 129, 130, 163, 164, 165, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 202, 203, 204, 268, 269, 270, 288, 289, 290, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 464, 465, 466, 468, 469, 470, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 509, 510, 511, 513, 514, 515, 519, 520, 527, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 636, 637, 638, 639, 645, 646, 647, 648, 649, 650, 651, 652, 659, 660, 661, 662, 663, 664, 665, 666, 672, 673, 674, 675, 676, 677, 678, 679, 685, 686, 687, 688, 689, 690, 691, 692, 698, 699, 700, 701, 702, 703, 704, 705, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 726 }

C grade { }

F normal fail { 6, 19, 31, 41, 45, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 160, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 630, 631, 632, 633, 634, 635, 640, 641, 642, 643, 644, 653, 654, 655, 656, 657, 658, 667, 668, 669, 670, 671, 680, 681, 682, 683, 684, 693, 694, 695, 696, 697, 706, 707, 708, 709, 710, 715, 723, 724, 725 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	81	74	71	72	188	156	71	75	62
N.S.	1	1.01	0.92	0.89	0.90	2.35	1.95	0.89	0.94	0.78
time (sec)	N/A	0.368	0.048	1.009	0.111	0.089	32.310	0.121	0.170	26.401

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	65	59	57	55	57	65	97	60	51
N.S.	1	1.10	1.00	0.97	0.93	0.97	1.10	1.64	1.02	0.86
time (sec)	N/A	0.394	0.020	0.553	0.035	0.088	0.831	0.125	0.190	26.289

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	62	60	59	152	141	59	61	50
N.S.	1	1.03	0.94	0.91	0.89	2.30	2.14	0.89	0.92	0.76
time (sec)	N/A	0.370	0.028	0.610	0.121	0.097	7.887	0.134	0.269	26.282

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	38	34	37	44	40	51	43	44	39
N.S.	1	1.09	0.97	1.06	1.26	1.14	1.46	1.23	1.26	1.11
time (sec)	N/A	0.337	0.012	0.812	0.033	0.071	0.287	0.121	0.159	26.345

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	45	46	45	107	100	41	44	37
N.S.	1	1.11	1.00	1.02	1.00	2.38	2.22	0.91	0.98	0.82
time (sec)	N/A	0.301	0.015	0.442	0.113	0.077	2.020	0.121	0.167	26.385

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	43	110	80	0	0	0	50	0
N.S.	1	0.95	0.98	2.50	1.82	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.398	0.010	0.493	0.037	0.000	0.000	0.000	0.202	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	36	105	258	40	43	36
N.S.	1	1.00	1.00	0.84	0.82	2.39	5.86	0.91	0.98	0.82
time (sec)	N/A	0.282	0.012	0.431	0.121	0.082	7.537	0.125	0.180	26.175

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	47	45	42	44	43	65	58	51	41
N.S.	1	1.24	1.18	1.11	1.16	1.13	1.71	1.53	1.34	1.08
time (sec)	N/A	0.353	0.006	0.435	0.028	0.092	0.932	0.124	0.174	26.351

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	52	49	135	496	58	57	46
N.S.	1	1.00	0.82	0.87	0.82	2.25	8.27	0.97	0.95	0.77
time (sec)	N/A	0.320	0.006	0.829	0.115	0.103	38.137	0.122	0.165	26.395

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	62	56	54	54	58	83	132	64	56
N.S.	1	0.97	0.88	0.84	0.84	0.91	1.30	2.06	1.00	0.88
time (sec)	N/A	0.410	0.043	0.622	0.038	0.088	2.551	0.125	0.176	26.375

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	49	61	62	170	583	71	71	61
N.S.	1	1.04	0.66	0.82	0.84	2.30	7.88	0.96	0.96	0.82
time (sec)	N/A	0.337	0.006	1.292	0.114	0.095	155.888	0.122	0.250	26.215

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	76	75	66	69	71	97	191	77	68
N.S.	1	0.97	0.96	0.85	0.88	0.91	1.24	2.45	0.99	0.87
time (sec)	N/A	0.427	0.035	1.040	0.032	0.078	7.013	0.121	0.175	26.275

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	65	59	57	55	57	65	97	60	51
N.S.	1	1.10	1.00	0.97	0.93	0.97	1.10	1.64	1.02	0.86
time (sec)	N/A	0.399	0.018	1.210	0.029	0.074	2.435	0.121	0.174	26.229

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	162	69	139	147	161	0	162	116	157
N.S.	1	1.02	0.43	0.87	0.92	1.01	0.00	1.02	0.73	0.99
time (sec)	N/A	0.540	0.007	1.214	0.115	0.076	0.000	0.135	0.178	28.684

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	158	147	136	144	144	0	160	103	129
N.S.	1	1.01	0.94	0.87	0.92	0.92	0.00	1.02	0.66	0.82
time (sec)	N/A	0.494	0.068	0.984	0.117	0.090	0.000	0.121	0.181	28.734

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	38	34	37	44	40	51	43	44	39
N.S.	1	1.09	0.97	1.06	1.26	1.14	1.46	1.23	1.26	1.11
time (sec)	N/A	0.341	0.012	0.829	0.030	0.083	0.661	0.119	0.174	26.257

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	159	53	128	131	150	178	150	97	121
N.S.	1	1.08	0.36	0.87	0.89	1.02	1.21	1.02	0.66	0.82
time (sec)	N/A	0.567	0.006	0.741	0.122	0.083	56.025	0.131	0.171	28.551

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	142	129	122	125	110	165	143	91	134
N.S.	1	1.07	0.97	0.92	0.94	0.83	1.24	1.08	0.68	1.01
time (sec)	N/A	0.531	0.044	0.513	0.112	0.093	24.320	0.126	0.168	26.933

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	43	55	80	0	0	0	50	0
N.S.	1	0.95	0.98	1.25	1.82	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.390	0.011	0.694	0.032	0.000	0.000	0.000	0.314	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	138	47	113	119	126	165	137	88	149
N.S.	1	1.04	0.35	0.85	0.89	0.95	1.24	1.03	0.66	1.12
time (sec)	N/A	0.523	0.006	0.650	0.109	0.082	105.770	0.127	0.201	26.817

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	137	134	113	120	150	0	138	95	115
N.S.	1	0.99	0.96	0.81	0.86	1.08	0.00	0.99	0.68	0.83
time (sec)	N/A	0.510	0.037	0.762	0.109	0.103	0.000	0.131	0.175	28.582

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	47	45	42	44	43	65	58	51	41
N.S.	1	1.04	1.00	0.93	0.98	0.96	1.44	1.29	1.13	0.91
time (sec)	N/A	0.344	0.006	0.631	0.033	0.076	1.896	0.121	0.191	26.196

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	157	49	128	127	138	0	153	113	125
N.S.	1	1.04	0.32	0.85	0.84	0.91	0.00	1.01	0.75	0.83
time (sec)	N/A	0.555	0.006	1.243	0.110	0.111	0.000	0.128	0.180	28.168

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	154	49	128	128	172	0	149	112	156
N.S.	1	1.02	0.32	0.85	0.85	1.14	0.00	0.99	0.74	1.03
time (sec)	N/A	0.543	0.006	1.701	0.123	0.083	0.000	0.122	0.176	28.635

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	62	56	54	54	58	83	132	64	56
N.S.	1	0.97	0.88	0.84	0.84	0.91	1.30	2.06	1.00	0.88
time (sec)	N/A	0.384	0.042	1.376	0.028	0.091	7.055	0.121	0.190	26.165

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	83	85	74	74	89	100	308	99	77
N.S.	1	0.93	0.96	0.83	0.83	1.00	1.12	3.46	1.11	0.87
time (sec)	N/A	0.414	0.053	0.665	0.026	0.080	2.042	0.132	0.166	26.262

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	70	74	63	64	77	87	257	87	65
N.S.	1	0.93	0.99	0.84	0.85	1.03	1.16	3.43	1.16	0.87
time (sec)	N/A	0.393	0.035	0.325	0.036	0.082	1.207	0.126	0.260	26.118

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	57	62	52	51	64	73	210	74	53
N.S.	1	0.93	1.02	0.85	0.84	1.05	1.20	3.44	1.21	0.87
time (sec)	N/A	0.374	0.030	0.280	0.027	0.081	0.736	0.125	0.197	26.099

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	44	40	41	40	50	60	152	61	41
N.S.	1	0.94	0.85	0.87	0.85	1.06	1.28	3.23	1.30	0.87
time (sec)	N/A	0.347	0.022	0.266	0.035	0.082	0.504	0.123	0.181	26.246

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	28	27	33	36	96	45	27
N.S.	1	1.00	1.37	1.04	1.00	1.22	1.33	3.56	1.67	1.00
time (sec)	N/A	0.283	0.005	0.095	0.033	0.092	0.263	0.122	0.172	0.056

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	67	83	0	0	152	21	0
N.S.	1	1.00	1.02	1.68	2.08	0.00	0.00	3.80	0.52	0.00
time (sec)	N/A	0.392	0.005	0.272	0.034	0.000	0.000	0.205	0.185	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	36	30	37	50	36	37	63	48	40
N.S.	1	1.20	1.00	1.23	1.67	1.20	1.23	2.10	1.60	1.33
time (sec)	N/A	0.346	0.007	0.536	0.034	0.074	0.438	0.126	0.163	26.129

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	59	63	63	55	61	150	63	53
N.S.	1	1.03	1.00	1.07	1.07	0.93	1.03	2.54	1.07	0.90
time (sec)	N/A	0.394	0.018	0.303	0.034	0.075	0.644	0.124	0.182	25.982

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	73	75	74	66	75	234	76	65
N.S.	1	1.01	1.00	1.03	1.01	0.90	1.03	3.21	1.04	0.89
time (sec)	N/A	0.431	0.021	0.355	0.036	0.071	1.028	0.127	0.180	25.765

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	85	85	79	88	317	88	78
N.S.	1	1.00	1.00	0.98	0.98	0.91	1.01	3.64	1.01	0.90
time (sec)	N/A	0.453	0.026	0.435	0.032	0.068	1.619	0.128	0.185	25.998

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	68	49	60	59	178	148	75	73	56
N.S.	1	0.94	0.68	0.83	0.82	2.47	2.06	1.04	1.01	0.78
time (sec)	N/A	0.412	0.010	1.039	0.110	0.086	29.553	0.128	0.253	25.645

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	48	56	46	44	56	66	59	71	45
N.S.	1	0.94	1.10	0.90	0.86	1.10	1.29	1.16	1.39	0.88
time (sec)	N/A	0.375	0.024	0.441	0.034	0.080	1.217	0.128	0.186	25.503

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	47	49	48	141	133	63	58	44
N.S.	1	0.98	0.81	0.84	0.83	2.43	2.29	1.09	1.00	0.76
time (sec)	N/A	0.327	0.006	0.463	0.118	0.085	10.134	0.120	0.172	25.545

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	34	33	42	53	47	57	33
N.S.	1	1.00	1.22	0.92	0.89	1.14	1.43	1.27	1.54	0.89
time (sec)	N/A	0.301	0.006	0.317	0.034	0.077	0.591	0.125	0.195	25.532

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	43	34	33	107	95	42	46	33
N.S.	1	1.00	1.05	0.83	0.80	2.61	2.32	1.02	1.12	0.80
time (sec)	N/A	0.298	0.011	0.172	0.114	0.080	3.288	0.133	0.170	0.090

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	126	89	0	0	0	25	0
N.S.	1	1.00	1.02	2.86	2.02	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.383	0.006	0.263	0.034	0.000	0.000	0.000	0.178	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	56	52	52	49	119	97	54	54	42
N.S.	1	1.12	1.04	1.04	0.98	2.38	1.94	1.08	1.08	0.84
time (sec)	N/A	0.329	0.017	0.352	0.116	0.084	7.725	0.125	0.171	25.510

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	38	34	37	54	41	53	57	59	47
N.S.	1	1.09	0.97	1.06	1.54	1.17	1.51	1.63	1.69	1.34
time (sec)	N/A	0.340	0.011	0.645	0.034	0.088	0.825	0.121	0.185	19.375

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	77	70	61	62	154	138	73	70	55
N.S.	1	1.13	1.03	0.90	0.91	2.26	2.03	1.07	1.03	0.81
time (sec)	N/A	0.357	0.030	0.648	0.110	0.080	23.635	0.129	0.211	15.129

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	34	9	35	11	8	110	14	8
N.S.	1	1.00	4.25	1.12	4.38	1.38	1.00	13.75	1.75	1.00
time (sec)	N/A	0.259	0.007	0.594	0.032	0.070	1.534	0.154	0.232	14.964

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	151	133	121	120	129	146	339	131	121
N.S.	1	0.99	0.87	0.79	0.78	0.84	0.95	2.22	0.86	0.79
time (sec)	N/A	0.537	0.150	0.848	0.034	0.086	9.657	0.127	0.154	15.114

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	112	99	98	105	119	255	105	97
N.S.	1	1.00	0.91	0.80	0.80	0.85	0.97	2.07	0.85	0.79
time (sec)	N/A	0.491	0.059	0.687	0.034	0.089	2.827	0.123	0.153	15.051

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	95	88	77	76	80	92	171	79	73
N.S.	1	1.02	0.95	0.83	0.82	0.86	0.99	1.84	0.85	0.78
time (sec)	N/A	0.461	0.043	0.652	0.030	0.082	1.000	0.117	0.181	15.116

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	58	53	52	50	51	61	97	53	47
N.S.	1	1.09	1.00	0.98	0.94	0.96	1.15	1.83	1.00	0.89
time (sec)	N/A	0.360	0.032	0.388	0.034	0.103	0.503	0.113	0.178	0.076

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	58	79	0	0	0	90	0
N.S.	1	1.00	1.02	1.26	1.72	0.00	0.00	0.00	1.96	0.00
time (sec)	N/A	0.397	0.006	0.636	0.035	0.000	0.000	0.000	0.173	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	68	55	54	53	55	352	132	57	49
N.S.	1	1.08	0.87	0.86	0.84	0.87	5.59	2.10	0.90	0.78
time (sec)	N/A	0.402	0.045	0.425	0.034	0.088	8.425	0.128	0.176	15.247

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	98	90	77	76	84	435	232	85	72
N.S.	1	0.98	0.90	0.77	0.76	0.84	4.35	2.32	0.85	0.72
time (sec)	N/A	0.446	0.051	0.427	0.028	0.086	73.060	0.126	0.199	15.498

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	114	99	98	109	0	324	111	97
N.S.	1	0.97	0.88	0.76	0.75	0.84	0.00	2.49	0.85	0.75
time (sec)	N/A	0.477	0.068	0.434	0.036	0.103	0.000	0.125	0.265	15.620

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	37	33	32	31	28	156	31	31	33
N.S.	1	1.16	1.03	1.00	0.97	0.88	4.88	0.97	0.97	1.03
time (sec)	N/A	0.339	0.013	0.395	0.032	0.082	0.249	0.114	0.171	15.521

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	0	0	145	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	1.79	0.00
time (sec)	N/A	0.391	0.033	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	377	0	145	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	4.65	0.00	1.79	0.00
time (sec)	N/A	0.364	0.032	0.000	0.000	0.000	34.916	0.000	0.182	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	56	0	0	0	0	0	185	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	2.68	0.00
time (sec)	N/A	0.337	0.032	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	231	0	95	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	3.45	0.00	1.42	0.00
time (sec)	N/A	0.384	0.022	0.000	0.000	0.000	8.013	0.000	0.189	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	85	76	0	0	0	366	0	29	0
N.S.	1	1.04	0.93	0.00	0.00	0.00	4.46	0.00	0.35	0.00
time (sec)	N/A	0.415	0.035	0.000	0.000	0.000	25.767	0.000	0.168	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	87	76	0	0	0	0	0	29	0
N.S.	1	1.02	0.89	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.440	0.038	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	93	76	0	0	0	0	0	550	0
N.S.	1	1.12	0.92	0.00	0.00	0.00	0.00	0.00	6.63	0.00
time (sec)	N/A	0.383	0.041	0.000	0.000	0.000	0.000	0.000	0.269	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	70	77	0	0	0	0	0	238	0
N.S.	1	0.88	0.96	0.00	0.00	0.00	0.00	0.00	2.98	0.00
time (sec)	N/A	0.407	0.047	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	0	0	0	0	0	147	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.401	0.045	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	106	92	0	115	112	0	0	89	0
N.S.	1	0.75	0.65	0.00	0.82	0.79	0.00	0.00	0.63	0.00
time (sec)	N/A	0.473	0.086	0.000	0.038	0.098	0.000	0.000	0.194	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	91	74	0	95	92	0	0	75	0
N.S.	1	0.81	0.66	0.00	0.85	0.82	0.00	0.00	0.67	0.00
time (sec)	N/A	0.441	0.048	0.000	0.040	0.112	0.000	0.000	0.182	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	67	48	0	70	57	0	0	52	0
N.S.	1	0.97	0.70	0.00	1.01	0.83	0.00	0.00	0.75	0.00
time (sec)	N/A	0.404	0.037	0.000	0.038	0.096	0.000	0.000	0.178	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	45	46	182	0	63	0	0	58	0
N.S.	1	0.90	0.92	3.64	0.00	1.26	0.00	0.00	1.16	0.00
time (sec)	N/A	0.409	0.013	2.223	0.000	0.073	0.000	0.000	0.181	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	69	57	0	71	75	0	0	60	0
N.S.	1	0.86	0.71	0.00	0.89	0.94	0.00	0.00	0.75	0.00
time (sec)	N/A	0.362	0.022	0.000	0.039	0.101	0.000	0.000	0.169	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	88	76	0	99	104	0	0	83	0
N.S.	1	0.73	0.63	0.00	0.82	0.87	0.00	0.00	0.69	0.00
time (sec)	N/A	0.435	0.051	0.000	0.040	0.088	0.000	0.000	0.186	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	128	0	45	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	1.97	0.00	0.69	0.00
time (sec)	N/A	0.346	0.038	0.000	0.000	0.000	6.190	0.000	0.230	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	128	0	43	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	1.97	0.00	0.66	0.00
time (sec)	N/A	0.342	0.041	0.000	0.000	0.000	3.004	0.000	0.231	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	76	0	35	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	1.41	0.00	0.65	0.00
time (sec)	N/A	0.325	0.034	0.000	0.000	0.000	1.532	0.000	0.175	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	43	170	0	60	0	0	55	0
N.S.	1	0.95	0.98	3.86	0.00	1.36	0.00	0.00	1.25	0.00
time (sec)	N/A	0.401	0.001	1.978	0.000	0.110	0.000	0.000	0.186	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	59	0	0	0	73	0	47	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	1.11	0.00	0.71	0.00
time (sec)	N/A	0.351	0.038	0.000	0.000	0.000	3.512	0.000	0.211	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	0	0	0	78	0	50	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	1.08	0.00	0.69	0.00
time (sec)	N/A	0.355	0.032	0.000	0.000	0.000	6.680	0.000	0.164	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	0	0	0	78	0	50	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	1.11	0.00	0.71	0.00
time (sec)	N/A	0.350	0.032	0.000	0.000	0.000	14.004	0.000	0.179	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	170	127	190	145	189	182	370	165	126
N.S.	1	0.79	0.59	0.88	0.67	0.88	0.85	1.72	0.77	0.59
time (sec)	N/A	0.714	0.108	1.218	0.039	0.105	3.236	0.130	0.193	14.773

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	148	105	151	120	148	139	216	126	100
N.S.	1	1.02	0.72	1.04	0.83	1.02	0.96	1.49	0.87	0.69
time (sec)	N/A	0.588	0.064	1.748	0.038	0.084	1.311	0.118	0.181	14.778

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	64	63	105	97	96	90	96	85	70
N.S.	1	1.05	1.03	1.72	1.59	1.57	1.48	1.57	1.39	1.15
time (sec)	N/A	0.421	0.011	1.105	0.035	0.074	0.476	0.120	0.166	14.780

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	163	0	118	0	0	0	52	0
N.S.	1	1.06	2.26	0.00	1.64	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.685	0.154	0.000	0.055	0.000	0.000	0.000	0.164	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	93	481	118	0	0	0	109	0
N.S.	1	0.99	1.16	6.01	1.48	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.574	0.038	0.595	0.044	0.000	0.000	0.000	0.189	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	129	122	141	554	142	0	0	0	118	0
N.S.	1	0.95	1.09	4.29	1.10	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.986	0.084	0.921	0.040	0.000	0.000	0.000	0.242	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	193	198	190	607	173	0	0	0	154	0
N.S.	1	1.03	0.98	3.15	0.90	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	1.407	0.092	1.813	0.041	0.000	0.000	0.000	0.206	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	336	320	248	612	0	0	0	0	179	0
N.S.	1	0.95	0.74	1.82	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.086	0.192	1.237	0.000	0.000	0.000	0.000	0.157	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	294	281	223	565	0	0	0	0	139	0
N.S.	1	0.96	0.76	1.92	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.951	0.142	0.824	0.000	0.000	0.000	0.000	0.164	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	235	193	0	0	0	0	0	96	0
N.S.	1	0.99	0.81	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.817	0.102	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	190	192	173	446	0	0	0	0	98	0
N.S.	1	1.01	0.91	2.35	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	1.050	0.067	0.792	0.000	0.000	0.000	0.000	0.168	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	254	238	207	522	0	0	0	0	118	0
N.S.	1	0.94	0.81	2.06	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.870	0.107	1.019	0.000	0.000	0.000	0.000	0.180	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	296	273	294	568	0	0	0	0	134	0
N.S.	1	0.92	0.99	1.92	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.925	0.206	1.476	0.000	0.000	0.000	0.000	0.167	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	338	314	353	619	0	0	0	0	148	0
N.S.	1	0.93	1.04	1.83	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	1.012	0.243	2.418	0.000	0.000	0.000	0.000	0.178	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	331	178	289	239	359	289	662	264	187
N.S.	1	0.99	0.53	0.87	0.72	1.07	0.87	1.98	0.79	0.56
time (sec)	N/A	0.978	0.216	1.963	0.038	0.096	5.362	0.132	0.174	14.799

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	146	223	203	275	223	385	198	144
N.S.	1	1.00	0.69	1.06	0.96	1.30	1.06	1.82	0.94	0.68
time (sec)	N/A	0.678	0.132	15.150	0.040	0.105	2.087	0.125	0.167	14.758

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	90	87	151	164	176	143	169	128	103
N.S.	1	0.97	0.94	1.62	1.76	1.89	1.54	1.82	1.38	1.11
time (sec)	N/A	0.478	0.017	1.650	0.036	0.069	0.786	0.117	0.162	15.030

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	109	279	0	217	0	0	0	52	0
N.S.	1	1.03	2.63	0.00	2.05	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.809	0.227	0.000	0.046	0.000	0.000	0.000	0.171	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	113	302	0	202	0	0	0	172	0
N.S.	1	0.95	2.54	0.00	1.70	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.838	0.418	0.000	0.108	0.000	0.000	0.000	0.171	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	193	477	0	270	0	0	0	121	0
N.S.	1	0.88	2.18	0.00	1.23	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	1.653	0.466	0.000	0.114	0.000	0.000	0.000	0.182	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	352	319	571	0	338	0	0	0	212	0
N.S.	1	0.91	1.62	0.00	0.96	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	2.812	0.564	0.000	0.115	0.000	0.000	0.000	0.179	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	909	18	123	20	17	20	221	20
N.S.	1	1.00	50.50	1.00	6.83	1.11	0.94	1.11	12.28	1.11
time (sec)	N/A	1.654	5.430	0.060	0.903	0.085	3.708	0.143	0.158	25.159

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	789	14	111	16	14	16	150	16
N.S.	1	1.00	56.36	1.00	7.93	1.14	1.00	1.14	10.71	1.14
time (sec)	N/A	1.095	4.652	0.049	0.690	0.087	1.791	0.132	0.180	25.002

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	C	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	505	18	0	20	17	20	157	20
N.S.	1	1.00	28.06	1.00	0.00	1.11	0.94	1.11	8.72	1.11
time (sec)	N/A	0.417	1.721	0.057	0.000	0.145	2.795	0.143	0.158	25.061

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	851	18	117	20	17	20	181	20
N.S.	1	1.00	47.28	1.00	6.50	1.11	0.94	1.11	10.06	1.11
time (sec)	N/A	1.004	3.710	0.058	0.733	0.095	4.391	0.147	0.178	25.203

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	107	106	96	547	0	68	0	69	20	0
N.S.	1	0.99	0.90	5.11	0.00	0.64	0.00	0.64	0.19	0.00
time (sec)	N/A	0.578	0.150	1.000	0.000	0.089	0.000	0.121	0.179	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	272	0	29	0	31	70	0
N.S.	1	1.00	1.00	5.33	0.00	0.57	0.00	0.61	1.37	0.00
time (sec)	N/A	0.456	0.081	2.883	0.000	0.071	0.000	0.119	0.188	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	60	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	3.33	1.11
time (sec)	N/A	0.297	0.220	0.026	0.097	0.100	1.809	0.113	0.184	25.136

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11	1.11
time (sec)	N/A	0.290	0.378	0.026	0.104	0.095	4.266	0.112	0.170	25.447

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11	1.11
time (sec)	N/A	0.285	0.300	0.054	0.074	0.066	1.577	0.112	0.163	24.980

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14	1.14
time (sec)	N/A	0.263	0.014	0.049	0.115	0.067	1.087	0.114	0.195	25.077

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11	1.11
time (sec)	N/A	0.286	0.433	0.044	0.069	0.096	3.170	0.118	0.165	25.117

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	138	193	157	1487	0	141	0	313	20	0
N.S.	1	1.40	1.14	10.78	0.00	1.02	0.00	2.27	0.14	0.00
time (sec)	N/A	0.968	0.120	1.063	0.000	0.068	0.000	0.119	0.173	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	83	79	97	421	0	78	0	141	88	0
N.S.	1	0.95	1.17	5.07	0.00	0.94	0.00	1.70	1.06	0.00
time (sec)	N/A	0.486	0.049	2.825	0.000	0.066	0.000	0.118	0.172	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	69	20	17	20	78	20
N.S.	1	1.00	1.11	1.00	3.83	1.11	0.94	1.11	4.33	1.11
time (sec)	N/A	0.283	0.253	0.026	0.081	0.093	3.050	0.118	0.191	25.122

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	78	20	19	20	20	20
N.S.	1	1.00	1.11	1.00	4.33	1.11	1.06	1.11	1.11	1.11
time (sec)	N/A	0.285	1.347	0.026	0.073	0.093	5.473	0.133	0.188	25.165

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	66	20	17	20	20	20
N.S.	1	1.00	1.11	1.00	3.67	1.11	0.94	1.11	1.11	1.11
time (sec)	N/A	0.278	0.371	0.043	0.069	0.069	2.458	0.114	0.193	25.081

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	73	16	15	16	16	16
N.S.	1	1.00	1.14	1.00	5.21	1.14	1.07	1.14	1.14	1.14
time (sec)	N/A	0.256	0.274	0.046	0.063	0.070	1.902	0.110	0.254	25.101

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	79	20	19	20	20	20
N.S.	1	1.00	1.11	1.00	4.39	1.11	1.06	1.11	1.11	1.11
time (sec)	N/A	0.278	1.114	0.049	0.071	0.075	4.264	0.119	0.186	25.605

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	204	315	185	1969	0	270	0	874	20	0
N.S.	1	1.54	0.91	9.65	0.00	1.32	0.00	4.28	0.10	0.00
time (sec)	N/A	1.588	0.169	1.218	0.000	0.085	0.000	0.138	0.190	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	115	113	716	0	157	0	406	18	0
N.S.	1	1.01	0.99	6.28	0.00	1.38	0.00	3.56	0.16	0.00
time (sec)	N/A	0.576	0.061	3.071	0.000	0.077	0.000	0.120	0.167	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	161	20	17	20	80	20
N.S.	1	1.00	1.11	1.00	8.94	1.11	0.94	1.11	4.44	1.11
time (sec)	N/A	0.279	0.406	0.026	0.072	0.086	4.217	0.127	0.167	25.898

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	184	20	19	20	20	20
N.S.	1	1.00	1.11	1.00	10.22	1.11	1.06	1.11	1.11	1.11
time (sec)	N/A	0.277	3.042	0.046	0.072	0.071	7.676	0.121	0.170	25.809

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	180	20	17	20	20	20
N.S.	1	1.00	1.11	1.00	10.00	1.11	0.94	1.11	1.11	1.11
time (sec)	N/A	0.269	0.526	0.004	0.078	0.075	4.116	0.118	0.201	25.719

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	164	16	15	16	16	16
N.S.	1	1.00	1.14	1.00	11.71	1.14	1.07	1.14	1.14	1.14
time (sec)	N/A	0.254	0.369	0.051	0.065	0.066	2.660	0.119	0.180	25.715

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	187	20	19	20	20	20
N.S.	1	1.00	1.11	1.00	10.39	1.11	1.06	1.11	1.11	1.11
time (sec)	N/A	0.274	2.026	0.049	0.069	0.071	5.822	0.115	0.171	25.852

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	44	0	43	0	54	0	44	19	0
N.S.	1	0.98	0.00	0.96	0.00	1.20	0.00	0.98	0.42	0.00
time (sec)	N/A	0.450	0.000	2.430	0.000	0.064	0.000	0.124	0.162	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	0	19	27	20	63	18
N.S.	1	1.00	1.00	1.15	0.00	0.95	1.35	1.00	3.15	0.90
time (sec)	N/A	0.346	0.018	0.768	0.000	0.085	0.696	0.117	0.187	26.011

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	92	0	74	0	99	0	100	19	0
N.S.	1	1.30	0.00	1.04	0.00	1.39	0.00	1.41	0.27	0.00
time (sec)	N/A	0.685	0.000	2.947	0.000	0.074	0.000	0.127	0.170	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	43	43	48	0	55	49	47	80	46
N.S.	1	0.91	0.91	1.02	0.00	1.17	1.04	1.00	1.70	0.98
time (sec)	N/A	0.392	0.024	0.731	0.000	0.061	0.724	0.117	0.176	25.666

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	165	0	105	0	142	0	151	19	0
N.S.	1	1.30	0.00	0.83	0.00	1.12	0.00	1.19	0.15	0.00
time (sec)	N/A	1.050	0.000	2.848	0.000	0.069	0.000	0.129	0.176	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	71	55	70	0	79	70	71	17	74
N.S.	1	0.97	0.75	0.96	0.00	1.08	0.96	0.97	0.23	1.01
time (sec)	N/A	0.453	0.030	0.743	0.000	0.065	0.747	0.121	0.183	25.742

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	148	105	151	120	148	136	216	126	100
N.S.	1	0.99	0.70	1.01	0.80	0.99	0.91	1.44	0.84	0.67
time (sec)	N/A	0.594	0.077	6.193	0.037	0.089	3.734	0.123	0.170	25.590

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	63	101	97	96	100	96	85	71
N.S.	1	0.97	0.95	1.53	1.47	1.45	1.52	1.45	1.29	1.08
time (sec)	N/A	0.429	0.013	1.317	0.035	0.069	1.033	0.121	0.169	25.561

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	76	2965	0	0	0	0	0	52	0
N.S.	1	0.99	38.51	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.691	1.369	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	86	79	99	411	118	0	0	0	109	0
N.S.	1	0.92	1.15	4.78	1.37	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.534	0.033	1.157	0.033	0.000	0.000	0.000	0.208	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1294	1307	1041	1957	0	0	0	0	165	0
N.S.	1	1.01	0.80	1.51	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	3.359	0.880	1.354	0.000	0.000	0.000	0.000	0.176	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1304	1316	1101	0	0	0	0	0	148	0
N.S.	1	1.01	0.84	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	3.295	0.835	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1137	1153	972	1787	0	0	0	0	154	0
N.S.	1	1.01	0.85	1.57	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	2.845	0.753	1.096	0.000	0.000	0.000	0.000	0.181	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1170	1183	766	1787	0	0	0	0	163	0
N.S.	1	1.01	0.65	1.53	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	2.811	0.950	1.162	0.000	0.000	0.000	0.000	0.204	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1328	1292	912	1954	0	0	0	0	183	0
N.S.	1	0.97	0.69	1.47	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	3.318	1.756	1.702	0.000	0.000	0.000	0.000	0.197	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	164	160	146	823	0	116	0	108	20	0
N.S.	1	0.98	0.89	5.02	0.00	0.71	0.00	0.66	0.12	0.00
time (sec)	N/A	0.709	0.242	1.290	0.000	0.073	0.000	0.132	0.186	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	107	106	96	547	0	68	0	69	20	0
N.S.	1	0.99	0.90	5.11	0.00	0.64	0.00	0.64	0.19	0.00
time (sec)	N/A	0.583	0.124	1.120	0.000	0.076	0.000	0.128	0.230	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	272	0	29	0	31	70	0
N.S.	1	1.00	1.00	5.33	0.00	0.57	0.00	0.61	1.37	0.00
time (sec)	N/A	0.454	0.073	2.914	0.000	0.082	0.000	0.116	0.272	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	60	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	3.33	1.11
time (sec)	N/A	0.288	0.210	0.025	0.100	0.076	8.891	0.113	0.209	25.357

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11	1.11
time (sec)	N/A	0.283	0.366	0.023	0.100	0.086	32.415	0.113	0.186	25.521

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11	1.11
time (sec)	N/A	0.285	0.282	0.046	0.090	0.067	10.135	0.122	0.216	25.485

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.273	0.263	0.049	0.090	0.066	4.797	0.117	0.182	25.641

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14	1.14
time (sec)	N/A	0.264	0.015	0.049	0.102	0.074	4.795	0.128	0.197	25.621

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11	1.11
time (sec)	N/A	0.295	0.410	0.050	0.092	0.089	17.853	0.111	0.173	25.605

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11	1.11
time (sec)	N/A	0.289	0.425	0.052	0.095	0.093	23.843	0.119	0.194	25.550

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	195	309	290	2564	0	211	0	487	20	0
N.S.	1	1.58	1.49	13.15	0.00	1.08	0.00	2.50	0.10	0.00
time (sec)	N/A	1.104	0.194	1.424	0.000	0.073	0.000	0.146	0.188	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	193	157	1487	0	141	0	313	20	0
N.S.	1	1.37	1.11	10.55	0.00	1.00	0.00	2.22	0.14	0.00
time (sec)	N/A	0.978	0.111	1.234	0.000	0.069	0.000	0.125	0.174	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	83	79	97	421	0	78	0	141	88	0
N.S.	1	0.95	1.17	5.07	0.00	0.94	0.00	1.70	1.06	0.00
time (sec)	N/A	0.515	0.048	3.156	0.000	0.086	0.000	0.123	0.214	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	69	20	17	20	78	20
N.S.	1	1.00	1.11	1.00	3.83	1.11	0.94	1.11	4.33	1.11
time (sec)	N/A	0.282	0.242	0.026	0.112	0.083	16.041	0.115	0.204	25.731

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	78	20	19	20	20	20
N.S.	1	1.00	1.11	1.00	4.33	1.11	1.06	1.11	1.11	1.11
time (sec)	N/A	0.288	1.314	0.026	0.113	0.075	38.405	0.125	0.238	25.587

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	66	20	17	20	20	20
N.S.	1	1.00	1.11	1.00	3.67	1.11	0.94	1.11	1.11	1.11
time (sec)	N/A	0.284	0.341	0.004	0.113	0.093	14.732	0.111	0.198	25.537

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	74	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	4.62	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.272	0.460	0.025	0.104	0.095	8.728	0.117	0.192	25.485

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	77	16	15	16	16	16
N.S.	1	1.00	1.14	1.00	5.50	1.14	1.07	1.14	1.14	1.14
time (sec)	N/A	0.256	0.290	0.052	0.107	0.075	8.920	0.120	0.186	25.495

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	79	20	19	20	20	20
N.S.	1	1.00	1.11	1.00	4.39	1.11	1.06	1.11	1.11	1.11
time (sec)	N/A	0.277	1.094	0.052	0.110	0.080	21.661	0.115	0.190	25.463

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	80	20	19	20	20	20
N.S.	1	1.00	1.11	1.00	4.44	1.11	1.06	1.11	1.11	1.11
time (sec)	N/A	0.279	1.106	0.004	0.104	0.085	29.381	0.112	0.191	25.509

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	994	20	171	22	19	22	1504	22
N.S.	1	1.00	49.70	1.00	8.55	1.10	0.95	1.10	75.20	1.10
time (sec)	N/A	0.531	3.101	0.067	0.246	0.082	109.178	0.251	0.173	25.471

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	466	20	126	22	19	22	602	22
N.S.	1	1.00	23.30	1.00	6.30	1.10	0.95	1.10	30.10	1.10
time (sec)	N/A	0.500	0.903	0.063	0.214	0.154	58.003	0.183	0.183	25.436

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	377	0	145	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	4.65	0.00	1.79	0.00
time (sec)	N/A	0.367	0.026	0.000	0.000	0.000	34.607	0.000	0.160	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	24	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.20	1.10
time (sec)	N/A	0.299	0.837	0.054	0.116	0.091	12.390	0.114	0.164	25.546

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	97	22	19	22	24	22
N.S.	1	1.00	1.10	1.00	4.85	1.10	0.95	1.10	1.20	1.10
time (sec)	N/A	0.292	1.716	0.030	0.124	0.097	30.968	0.115	0.157	25.773

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	196	171	0	239	266	0	0	186	0
N.S.	1	0.53	0.46	0.00	0.64	0.72	0.00	0.00	0.50	0.00
time (sec)	N/A	0.878	0.278	0.000	0.050	0.118	0.000	0.000	0.169	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	164	140	0	200	204	0	0	143	0
N.S.	1	0.64	0.55	0.00	0.78	0.80	0.00	0.00	0.56	0.00
time (sec)	N/A	0.721	0.221	0.000	0.054	0.110	0.000	0.000	0.167	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	78	74	0	146	121	0	0	93	0
N.S.	1	0.77	0.73	0.00	1.45	1.20	0.00	0.00	0.92	0.00
time (sec)	N/A	0.529	0.029	0.000	0.051	0.076	0.000	0.000	0.167	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	88	79	168	614	0	0	0	0	60	0
N.S.	1	0.90	1.91	6.98	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.707	0.163	3.916	0.000	0.000	0.000	0.000	0.157	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	95	150	0	0	197	0	0	125	0
N.S.	1	0.77	1.21	0.00	0.00	1.59	0.00	0.00	1.01	0.00
time (sec)	N/A	0.641	0.107	0.000	0.000	0.087	0.000	0.000	0.168	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	146	288	0	0	279	0	0	164	0
N.S.	1	0.73	1.44	0.00	0.00	1.40	0.00	0.00	0.82	0.00
time (sec)	N/A	1.095	0.399	0.000	0.000	0.135	0.000	0.000	0.157	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	12	14	0	40	0
N.S.	1	1.00	1.00	1.08	0.00	0.92	1.08	0.00	3.08	0.00
time (sec)	N/A	0.275	0.005	1.552	0.000	0.100	1.484	0.000	0.161	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	24	21	40	0	40	100	0	42	0
N.S.	1	1.14	1.00	1.90	0.00	1.90	4.76	0.00	2.00	0.00
time (sec)	N/A	0.387	0.006	1.546	0.000	0.115	1.638	0.000	0.164	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	24	21	41	0	41	100	0	44	0
N.S.	1	1.14	1.00	1.95	0.00	1.95	4.76	0.00	2.10	0.00
time (sec)	N/A	0.367	0.007	1.547	0.000	0.096	1.631	0.000	0.175	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	38	39	38	0	54	0	0	49	0
N.S.	1	0.93	0.95	0.93	0.00	1.32	0.00	0.00	1.20	0.00
time (sec)	N/A	0.396	0.016	2.508	0.000	0.109	0.000	0.000	0.167	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	43	170	0	60	0	0	55	0
N.S.	1	0.95	0.98	3.86	0.00	1.36	0.00	0.00	1.25	0.00
time (sec)	N/A	0.409	0.006	0.656	0.000	0.077	0.000	0.000	0.160	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	79	76	164	578	0	0	0	0	57	0
N.S.	1	0.96	2.08	7.32	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.690	0.063	4.165	0.000	0.000	0.000	0.000	0.160	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	113	109	270	1409	0	0	0	0	57	0
N.S.	1	0.96	2.39	12.47	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.810	0.181	5.042	0.000	0.000	0.000	0.000	0.188	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	131	185	284	214	269	335	573	296	208
N.S.	1	0.94	1.32	2.03	1.53	1.92	2.39	4.09	2.11	1.49
time (sec)	N/A	0.500	0.227	1.719	0.034	0.087	0.974	0.133	0.166	25.766

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	107	121	184	136	172	202	316	188	131
N.S.	1	0.96	1.08	1.64	1.21	1.54	1.80	2.82	1.68	1.17
time (sec)	N/A	0.442	0.118	0.891	0.032	0.105	0.564	0.121	0.161	25.730

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	78	77	74	91	105	136	98	68
N.S.	1	0.99	0.93	0.92	0.88	1.08	1.25	1.62	1.17	0.81
time (sec)	N/A	0.406	0.052	0.656	0.028	0.067	0.354	0.125	0.154	25.711

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	29	24	32	35	32	36	39	35	29
N.S.	1	1.21	1.00	1.33	1.46	1.33	1.50	1.62	1.46	1.21
time (sec)	N/A	0.300	0.009	0.415	0.030	0.075	0.141	0.111	0.181	0.073

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	98	118	0	0	0	97	0
N.S.	1	1.00	0.98	1.69	2.03	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.471	0.007	2.895	0.033	0.000	0.000	0.000	0.167	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	65	62	66	65	80	236	81	115	70
N.S.	1	0.96	0.91	0.97	0.96	1.18	3.47	1.19	1.69	1.03
time (sec)	N/A	0.331	0.057	1.309	0.026	0.089	1.841	0.120	0.175	26.545

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	90	80	88	120	236	1518	185	429	96
N.S.	1	0.86	0.76	0.84	1.14	2.25	14.46	1.76	4.09	0.91
time (sec)	N/A	0.437	0.085	1.586	0.032	0.100	6.210	0.120	0.168	26.100

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	116	105	111	232	443	4571	365	898	145
N.S.	1	0.87	0.79	0.83	1.74	3.33	34.37	2.74	6.75	1.09
time (sec)	N/A	0.488	0.121	2.233	0.035	0.090	18.396	0.124	0.175	26.335

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	189	249	244	177	498	527	218	254	222
N.S.	1	1.06	1.40	1.37	0.99	2.80	2.96	1.22	1.43	1.25
time (sec)	N/A	0.818	0.837	2.020	0.111	0.119	17.511	0.133	0.165	26.022

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	151	211	187	131	320	374	152	178	263
N.S.	1	1.07	1.50	1.33	0.93	2.27	2.65	1.08	1.26	1.87
time (sec)	N/A	0.711	0.541	1.391	0.106	0.120	8.718	0.135	0.181	28.778

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	112	83	78	80	198	199	85	90	81
N.S.	1	1.13	0.84	0.79	0.81	2.00	2.01	0.86	0.91	0.82
time (sec)	N/A	0.481	0.030	0.815	0.104	0.090	4.050	0.130	0.181	26.662

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	45	46	45	107	100	41	44	37
N.S.	1	1.11	1.00	1.02	1.00	2.38	2.22	0.91	0.98	0.82
time (sec)	N/A	0.296	0.011	0.125	0.109	0.074	2.008	0.117	0.171	25.953

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	213	201	196	0	0	0	0	108	0
N.S.	1	1.06	1.00	0.98	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.817	0.095	1.679	0.000	0.000	0.000	0.000	0.200	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	108	137	99	108	261	0	121	179	337
N.S.	1	0.91	1.15	0.83	0.91	2.19	0.00	1.02	1.50	2.83
time (sec)	N/A	0.436	0.075	1.965	0.107	0.102	0.000	0.120	0.161	26.914

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	181	217	147	206	744	0	278	600	272
N.S.	1	1.04	1.25	0.84	1.18	4.28	0.00	1.60	3.45	1.56
time (sec)	N/A	0.657	0.608	2.566	0.107	0.140	0.000	0.129	0.174	26.628

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	327	286	398	332	8840	265	381	435	536
N.S.	1	1.02	0.89	1.24	1.04	27.62	0.83	1.19	1.36	1.68
time (sec)	N/A	1.692	0.532	2.692	0.109	8.621	22.087	0.162	0.201	25.895

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	262	231	336	249	5799	173	274	290	358
N.S.	1	1.05	0.92	1.34	1.00	23.20	0.69	1.10	1.16	1.43
time (sec)	N/A	1.198	0.327	1.688	0.120	2.290	15.085	0.154	0.192	25.620

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	242	204	247	187	2284	112	212	207	210
N.S.	1	1.06	0.89	1.08	0.82	9.97	0.49	0.93	0.90	0.92
time (sec)	N/A	0.916	0.078	1.105	0.108	1.272	10.203	0.154	0.188	25.531

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	142	129	122	125	110	165	143	91	134
N.S.	1	1.07	0.97	0.92	0.94	0.83	1.24	1.08	0.68	1.01
time (sec)	N/A	0.559	0.028	0.165	0.109	0.083	24.322	0.128	0.170	0.407

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	322	313	101	0	0	0	0	110	0
N.S.	1	1.05	1.02	0.33	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.190	0.183	1.879	0.000	0.000	0.000	0.000	0.220	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	296	254	277	311	7010	0	363	6006	736
N.S.	1	1.01	0.87	0.95	1.07	24.01	0.00	1.24	20.57	2.52
time (sec)	N/A	1.276	0.353	2.214	0.110	1.324	0.000	0.184	0.239	25.826

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	379	325	347	517	13236	0	660	16189	2227
N.S.	1	0.97	0.83	0.89	1.32	33.85	0.00	1.69	41.40	5.70
time (sec)	N/A	1.518	0.631	3.302	0.116	8.600	0.000	0.255	0.311	26.273

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	135	114	244	166	239	355	847	339	184
N.S.	1	0.97	0.82	1.76	1.19	1.72	2.55	6.09	2.44	1.32
time (sec)	N/A	0.617	0.151	0.967	0.036	0.134	1.443	0.138	0.172	25.791

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	100	86	168	102	153	216	490	218	111
N.S.	1	0.98	0.84	1.65	1.00	1.50	2.12	4.80	2.14	1.09
time (sec)	N/A	0.501	0.085	0.688	0.028	0.080	0.875	0.132	0.191	25.724

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	85	65	55	80	112	233	115	57
N.S.	1	1.00	1.09	0.83	0.71	1.03	1.44	2.99	1.47	0.73
time (sec)	N/A	0.443	0.031	0.348	0.030	0.076	0.574	0.132	0.173	25.664

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	116	114	137	159	0	0	0	25	0
N.S.	1	1.03	1.01	1.21	1.41	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.681	0.034	2.894	0.055	0.000	0.000	0.000	0.172	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	85	81	86	85	148	425	145	113	85
N.S.	1	1.05	1.00	1.06	1.05	1.83	5.25	1.79	1.40	1.05
time (sec)	N/A	0.486	0.072	1.204	0.027	0.134	2.205	0.129	0.171	25.796

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	123	113	120	160	428	3485	470	489	217
N.S.	1	0.97	0.89	0.94	1.26	3.37	27.44	3.70	3.85	1.71
time (sec)	N/A	0.570	0.191	1.616	0.033	0.555	9.184	0.136	0.193	26.535

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	170	164	165	299	818	0	1097	984	662
N.S.	1	0.97	0.94	0.94	1.71	4.67	0.00	6.27	5.62	3.78
time (sec)	N/A	0.660	0.276	2.628	0.042	3.474	0.000	0.144	0.176	27.220

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	111	115	114	82	0	0	0	20	0
N.S.	1	1.06	1.10	1.09	0.78	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.689	0.064	6.255	0.035	0.000	0.000	0.000	0.168	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	301	296	239	0	0	0	0	0	736	0
N.S.	1	0.98	0.79	0.00	0.00	0.00	0.00	0.00	2.45	0.00
time (sec)	N/A	1.442	0.748	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	206	176	0	0	0	0	0	730	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	3.56	0.00
time (sec)	N/A	0.700	0.267	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	0	0	743	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	8.35	0.00
time (sec)	N/A	0.380	0.062	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	123	0	0	0	0	0	453	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	3.36	0.00
time (sec)	N/A	0.481	0.083	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	257	258	211	0	0	0	0	0	207	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.992	0.499	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	100	22	0	0	800	22
N.S.	1	1.00	1.10	1.00	5.00	1.10	0.00	0.00	40.00	1.10
time (sec)	N/A	0.287	0.566	0.601	0.145	0.080	0.000	0.000	0.206	25.778

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	230	224	0	0	0	515	0	205	0
N.S.	1	0.98	0.96	0.00	0.00	0.00	2.20	0.00	0.88	0.00
time (sec)	N/A	0.868	0.490	0.000	0.000	0.000	13.487	0.000	0.171	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	182	178	0	0	0	360	0	143	0
N.S.	1	1.01	0.98	0.00	0.00	0.00	1.99	0.00	0.79	0.00
time (sec)	N/A	0.739	0.262	0.000	0.000	0.000	8.413	0.000	0.174	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	136	130	0	0	0	214	0	83	0
N.S.	1	1.03	0.98	0.00	0.00	0.00	1.62	0.00	0.63	0.00
time (sec)	N/A	0.644	0.126	0.000	0.000	0.000	5.737	0.000	0.166	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	76	0	35	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	1.41	0.00	0.65	0.00
time (sec)	N/A	0.322	0.026	0.000	0.000	0.000	1.563	0.000	0.186	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	123	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	6.15	1.10
time (sec)	N/A	0.288	2.493	0.553	0.144	0.073	3.269	0.129	0.187	25.580

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	105	33	19	22	169	22
N.S.	1	1.00	1.10	1.00	5.25	1.65	0.95	1.10	8.45	1.10
time (sec)	N/A	0.283	0.552	0.602	0.174	0.078	45.541	0.148	0.180	25.665

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	174	44	0	22	401	22
N.S.	1	1.00	1.10	1.00	8.70	2.20	0.00	1.10	20.05	1.10
time (sec)	N/A	0.281	0.587	0.565	0.176	0.095	0.000	0.153	0.179	25.740

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	183	297	0	0	0	0	329	0
N.S.	1	1.00	0.73	1.19	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.847	0.199	2.846	0.000	0.000	0.000	0.000	0.174	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	127	213	0	0	0	0	231	0
N.S.	1	1.00	0.80	1.34	0.00	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.659	0.100	2.666	0.000	0.000	0.000	0.000	0.185	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	79	156	0	0	0	0	149	0
N.S.	1	1.00	0.87	1.71	0.00	0.00	0.00	0.00	1.64	0.00
time (sec)	N/A	0.497	0.043	2.639	0.000	0.000	0.000	0.000	0.163	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	98	118	0	0	0	97	0
N.S.	1	1.00	0.98	1.69	2.03	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.450	0.001	2.512	0.037	0.000	0.000	0.000	0.168	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	156	123	0	0	0	24	0
N.S.	1	1.00	1.01	1.61	1.27	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.539	0.026	1.027	0.075	0.000	0.000	0.000	0.170	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	147	201	156	0	0	0	26	0
N.S.	1	1.00	1.01	1.38	1.07	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.658	0.045	1.211	0.070	0.000	0.000	0.000	0.166	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	194	260	216	0	0	0	26	0
N.S.	1	1.00	0.85	1.15	0.95	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.807	0.188	1.492	0.069	0.000	0.000	0.000	0.171	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	370	412	0	0	0	0	326	0
N.S.	1	1.00	0.94	1.05	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	1.227	0.356	2.109	0.000	0.000	0.000	0.000	0.167	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	310	348	0	0	0	0	228	0
N.S.	1	1.00	0.99	1.11	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	1.001	0.147	1.773	0.000	0.000	0.000	0.000	0.170	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	238	263	0	0	0	0	169	0
N.S.	1	1.00	0.93	1.03	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.854	0.130	1.694	0.000	0.000	0.000	0.000	0.166	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	213	201	196	0	0	0	0	108	0
N.S.	1	1.06	1.00	0.98	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.744	0.001	1.468	0.000	0.000	0.000	0.000	0.167	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	230	311	0	0	0	0	26	0
N.S.	1	1.00	0.93	1.26	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.948	0.096	1.104	0.000	0.000	0.000	0.000	0.165	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	282	354	0	0	0	0	77	0
N.S.	1	1.00	0.92	1.16	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.018	0.234	1.178	0.000	0.000	0.000	0.000	0.179	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	347	410	0	0	0	0	196	0
N.S.	1	1.00	0.94	1.11	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.169	0.217	1.750	0.000	0.000	0.000	0.000	0.178	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	581	302	0	0	0	0	458	0
N.S.	1	1.00	0.84	0.44	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	1.864	0.467	2.299	0.000	0.000	0.000	0.000	0.209	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	643	504	250	0	0	0	0	372	0
N.S.	1	1.00	0.78	0.39	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	1.684	0.449	2.040	0.000	0.000	0.000	0.000	0.195	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	430	206	0	0	0	0	225	0
N.S.	1	1.00	0.94	0.45	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	1.302	0.325	1.796	0.000	0.000	0.000	0.000	0.215	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	322	313	101	0	0	0	0	110	0
N.S.	1	1.05	1.02	0.33	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.984	0.002	1.533	0.000	0.000	0.000	0.000	0.202	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	327	170	0	0	0	0	26	0
N.S.	1	1.00	0.93	0.48	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.177	0.143	1.375	0.000	0.000	0.000	0.000	0.201	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	395	292	0	0	0	0	127	0
N.S.	1	1.00	0.77	0.57	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.465	0.171	1.556	0.000	0.000	0.000	0.000	0.216	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	542	421	0	0	0	0	245	0
N.S.	1	1.00	0.80	0.62	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.801	0.498	2.405	0.000	0.000	0.000	0.000	0.256	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	308	301	0	0	0	0	28	0
N.S.	1	1.00	1.04	1.01	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.007	0.163	3.100	0.000	0.000	0.000	0.000	0.191	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	229	240	0	0	0	0	28	0
N.S.	1	1.00	1.05	1.10	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.849	0.099	2.827	0.000	0.000	0.000	0.000	0.178	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	165	188	0	0	0	0	26	0
N.S.	1	1.00	1.09	1.25	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.732	0.052	2.629	0.000	0.000	0.000	0.000	0.207	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	116	114	137	159	0	0	0	25	0
N.S.	1	1.03	1.01	1.21	1.41	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.685	0.001	2.693	0.035	0.000	0.000	0.000	0.212	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	161	209	179	0	0	0	131	0
N.S.	1	1.00	1.01	1.31	1.13	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.810	0.051	1.180	0.070	0.000	0.000	0.000	0.205	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	200	264	230	0	0	0	181	0
N.S.	1	1.00	1.01	1.33	1.16	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.891	0.083	1.286	0.075	0.000	0.000	0.000	0.176	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	263	345	307	0	0	0	271	0
N.S.	1	1.00	0.92	1.20	1.07	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	1.090	0.252	1.619	0.075	0.000	0.000	0.000	0.203	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	404	411	0	0	0	0	32	0
N.S.	1	1.00	0.96	0.98	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.455	0.278	2.435	0.000	0.000	0.000	0.000	0.190	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	349	362	0	0	0	0	32	0
N.S.	1	1.00	0.99	1.03	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.191	0.196	2.169	0.000	0.000	0.000	0.000	0.182	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	271	296	0	0	0	0	30	0
N.S.	1	1.00	0.93	1.02	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.048	0.175	1.914	0.000	0.000	0.000	0.000	0.212	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	249	242	234	0	0	0	0	29	0
N.S.	1	1.03	1.00	0.97	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.004	0.070	2.016	0.000	0.000	0.000	0.000	0.178	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	289	362	0	0	0	0	33	0
N.S.	1	1.00	1.01	1.26	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.128	0.100	1.506	0.000	0.000	0.000	0.000	0.208	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	327	418	0	0	0	0	35	0
N.S.	1	1.00	0.92	1.17	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.252	0.227	1.648	0.000	0.000	0.000	0.000	0.178	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	384	488	0	0	0	0	211	0
N.S.	1	1.00	0.93	1.18	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	1.340	0.300	2.078	0.000	0.000	0.000	0.000	0.226	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	714	714	536	301	0	0	0	0	32	0
N.S.	1	1.00	0.75	0.42	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.091	0.364	3.093	0.000	0.000	0.000	0.000	0.210	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	483	269	0	0	0	0	32	0
N.S.	1	1.00	0.73	0.40	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.791	0.250	2.603	0.000	0.000	0.000	0.000	0.198	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	403	238	0	0	0	0	30	0
N.S.	1	1.00	0.83	0.49	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.490	0.228	2.531	0.000	0.000	0.000	0.000	0.193	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	344	354	350	142	0	0	0	0	29	0
N.S.	1	1.03	1.02	0.41	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.183	0.169	2.268	0.000	0.000	0.000	0.000	0.188	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	395	220	0	0	0	0	33	0
N.S.	1	1.00	1.02	0.57	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.356	0.176	1.883	0.000	0.000	0.000	0.000	0.198	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	463	345	0	0	0	0	35	0
N.S.	1	1.00	0.83	0.62	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.637	0.199	2.326	0.000	0.000	0.000	0.000	0.210	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	737	737	535	489	0	0	0	0	35	0
N.S.	1	1.00	0.73	0.66	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.991	0.259	3.190	0.000	0.000	0.000	0.000	0.201	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	749	706	867	272	0	0	0	0	24	0
N.S.	1	0.94	1.16	0.36	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.995	0.841	1.906	0.000	0.000	0.000	0.000	0.199	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	533	506	564	449	0	0	0	0	24	0
N.S.	1	0.95	1.06	0.84	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.318	0.411	1.770	0.000	0.000	0.000	0.000	0.171	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	178	392	309	0	0	0	22	0
N.S.	1	1.00	0.78	1.71	1.35	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.790	0.132	1.271	0.145	0.000	0.000	0.000	0.206	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	360	326	373	0	377	0	0	0	27	0
N.S.	1	0.91	1.04	0.00	1.05	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.006	0.263	0.000	0.156	0.000	0.000	0.000	0.181	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	597	557	706	0	0	0	0	0	31	0
N.S.	1	0.93	1.18	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.563	0.481	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	541	543	422	0	0	0	0	0	23	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.722	0.307	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	561	563	912	0	0	0	0	0	24	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.159	0.605	0.000	0.000	0.000	0.000	0.000	200.018	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	215	258	0	596	697	268	329	298
N.S.	1	1.00	0.64	0.76	0.00	1.76	2.06	0.79	0.97	0.88
time (sec)	N/A	0.880	0.257	3.895	0.000	0.102	125.043	0.129	0.159	26.428

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	151	167	0	404	478	174	212	193
N.S.	1	1.00	0.68	0.76	0.00	1.83	2.16	0.79	0.96	0.87
time (sec)	N/A	0.641	0.122	1.813	0.000	0.118	32.846	0.121	0.164	26.618

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	89	0	220	260	94	113	97
N.S.	1	1.00	1.00	0.76	0.00	1.88	2.22	0.80	0.97	0.83
time (sec)	N/A	0.464	0.045	0.882	0.000	0.095	8.270	0.124	0.146	25.950

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	533	506	564	449	0	0	0	0	24	0
N.S.	1	0.95	1.06	0.84	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.242	0.121	0.915	0.000	0.000	0.000	0.000	0.166	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	877	0	0	0	0	0	35	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.907	1.471	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	945	945	435	1077	0	0	0	0	478	0
N.S.	1	1.00	0.46	1.14	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	2.545	0.538	2.423	0.000	0.000	0.000	0.000	0.179	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	281	729	0	0	0	0	252	0
N.S.	1	1.00	0.51	1.33	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	1.591	0.295	1.151	0.000	0.000	0.000	0.000	0.166	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.08	1.08
time (sec)	N/A	0.313	3.866	0.625	0.847	0.105	10.155	0.124	0.148	25.978

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	37	0	26	37	26
N.S.	1	1.00	1.08	1.00	0.00	1.54	0.00	1.08	1.54	1.08
time (sec)	N/A	0.308	10.962	0.681	0.000	0.079	0.000	0.125	0.175	25.692

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	1772	22	0	24	20	24	397	24
N.S.	1	1.00	80.55	1.00	0.00	1.09	0.91	1.09	18.05	1.09
time (sec)	N/A	2.542	12.979	0.082	0.000	0.072	10.183	0.182	0.159	25.744

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.08	1.08
time (sec)	N/A	0.313	6.563	0.575	1.751	0.135	14.270	0.146	0.154	25.613

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	37	0	26	37	26
N.S.	1	1.00	1.08	1.00	0.00	1.54	0.00	1.08	1.54	1.08
time (sec)	N/A	0.314	10.948	1.063	0.000	0.087	0.000	0.148	0.183	25.457

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	35	20	26	69	26
N.S.	1	1.00	1.08	1.00	1.08	1.46	0.83	1.08	2.88	1.08
time (sec)	N/A	0.308	0.426	0.266	0.130	0.097	8.438	0.125	0.158	25.527

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	19	24	41	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.86	1.09	1.86	1.09
time (sec)	N/A	0.289	0.252	0.092	0.134	0.109	4.498	0.118	0.154	25.573

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	36	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.50	1.08
time (sec)	N/A	0.313	0.748	0.983	0.095	0.091	15.427	0.112	0.159	25.539

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	37	22	26	59	26
N.S.	1	1.00	1.08	1.00	1.08	1.54	0.92	1.08	2.46	1.08
time (sec)	N/A	0.309	1.829	0.935	0.101	0.088	167.597	0.116	0.159	25.559

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	148	35	22	26	69	26
N.S.	1	1.00	1.08	1.00	6.17	1.46	0.92	1.08	2.88	1.08
time (sec)	N/A	0.309	0.924	0.092	0.164	0.118	13.084	0.118	0.149	25.466

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	101	24	20	24	41	24
N.S.	1	1.00	1.09	1.00	4.59	1.09	0.91	1.09	1.86	1.09
time (sec)	N/A	0.282	0.488	0.089	0.154	0.072	7.664	0.118	0.155	25.618

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	157	26	22	26	40	26
N.S.	1	1.00	1.08	1.00	6.54	1.08	0.92	1.08	1.67	1.08
time (sec)	N/A	0.303	3.349	0.091	0.108	0.084	22.363	0.124	0.150	25.558

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	214	37	0	26	65	26
N.S.	1	1.00	1.08	1.00	8.92	1.54	0.00	1.08	2.71	1.08
time (sec)	N/A	0.303	6.530	0.007	0.110	0.080	0.000	0.136	0.160	25.436

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	258	309	0	708	0	325	356	316
N.S.	1	1.00	0.70	0.84	0.00	1.93	0.00	0.89	0.97	0.86
time (sec)	N/A	0.970	0.257	16.825	0.000	0.090	0.000	0.137	0.155	28.876

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	178	195	0	454	440	204	228	317
N.S.	1	1.00	0.77	0.84	0.00	1.97	1.90	0.88	0.99	1.37
time (sec)	N/A	0.686	0.208	5.223	0.000	0.090	126.468	0.131	0.151	28.454

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	104	0	250	214	98	112	94
N.S.	1	1.00	1.00	0.95	0.00	2.27	1.95	0.89	1.02	0.85
time (sec)	N/A	0.484	0.051	1.865	0.000	0.080	16.777	0.124	0.145	26.495

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1165	1165	990	577	0	0	0	0	24	0
N.S.	1	1.00	0.85	0.50	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	3.197	0.967	3.458	0.000	0.000	0.000	0.000	0.166	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1865	1867	2168	0	0	0	0	0	884	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	5.066	7.284	0.000	0.000	0.000	0.000	0.000	0.288	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1221	1221	780	1584	0	0	0	0	819	0
N.S.	1	1.00	0.64	1.30	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	3.375	1.334	21.773	0.000	0.000	0.000	0.000	0.161	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	835	835	475	1127	0	0	0	0	513	0
N.S.	1	1.00	0.57	1.35	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	2.278	0.617	7.054	0.000	0.000	0.000	0.000	0.193	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	398	724	0	0	0	0	239	0
N.S.	1	1.00	1.01	1.83	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.268	0.143	2.378	0.000	0.000	0.000	0.000	0.259	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.303	22.033	2.203	0.866	0.104	0.000	0.131	0.157	25.823

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	37	0	26	2374	26
N.S.	1	1.00	1.08	1.00	0.00	1.54	0.00	1.08	98.92	1.08
time (sec)	N/A	0.306	31.521	6.498	0.000	0.090	0.000	0.139	0.495	25.949

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	2385	24	0	35	22	26	812	26
N.S.	1	1.00	99.38	1.00	0.00	1.46	0.92	1.08	33.83	1.08
time (sec)	N/A	4.239	10.961	1.742	0.000	0.094	33.140	0.200	0.158	26.184

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	1051	22	0	24	20	24	374	24
N.S.	1	1.00	47.77	1.00	0.00	1.09	0.91	1.09	17.00	1.09
time (sec)	N/A	1.683	2.339	2.313	0.000	0.109	10.116	0.152	0.160	25.958

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.313	32.223	2.149	1.290	0.069	0.000	0.156	0.155	25.913

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	37	0	26	3753	26
N.S.	1	1.00	1.08	1.00	0.00	1.54	0.00	1.08	156.38	1.08
time (sec)	N/A	0.305	36.293	1.848	0.000	0.100	0.000	0.161	0.727	25.845

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	35	20	26	69	26
N.S.	1	1.00	1.08	1.00	1.08	1.46	0.83	1.08	2.88	1.08
time (sec)	N/A	0.310	0.283	0.265	0.136	0.072	14.146	0.120	0.154	25.953

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	19	24	41	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.86	1.09	1.86	1.09
time (sec)	N/A	0.291	0.227	0.083	0.134	0.079	5.816	0.117	0.145	25.985

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	36	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.50	1.08
time (sec)	N/A	0.312	8.631	0.899	0.101	0.085	103.270	0.117	0.157	25.947

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	37	0	26	59	26
N.S.	1	1.00	1.08	1.00	1.08	1.54	0.00	1.08	2.46	1.08
time (sec)	N/A	0.308	11.939	0.908	0.108	0.078	0.000	0.120	0.180	26.144

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	152	35	22	26	69	26
N.S.	1	1.00	1.08	1.00	6.33	1.46	0.92	1.08	2.88	1.08
time (sec)	N/A	0.304	0.660	0.091	0.155	0.093	21.703	0.120	0.156	26.128

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	104	24	20	24	41	24
N.S.	1	1.00	1.09	1.00	4.73	1.09	0.91	1.09	1.86	1.09
time (sec)	N/A	0.284	0.386	0.006	0.157	0.084	8.762	0.114	0.164	26.082

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	159	26	22	26	40	26
N.S.	1	1.00	1.08	1.00	6.62	1.08	0.92	1.08	1.67	1.08
time (sec)	N/A	0.309	10.743	0.007	0.113	0.072	140.391	0.124	0.156	26.024

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	215	37	0	26	65	26
N.S.	1	1.00	1.08	1.00	8.96	1.54	0.00	1.08	2.71	1.08
time (sec)	N/A	0.307	12.824	0.089	0.112	0.081	0.000	0.130	0.251	25.958

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	146	170	140	132	152	0	399	164	127
N.S.	1	1.03	1.20	0.99	0.93	1.07	0.00	2.81	1.15	0.89
time (sec)	N/A	0.658	0.056	3.322	0.032	0.080	0.000	0.131	0.165	26.029

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	124	140	117	108	128	156	269	138	103
N.S.	1	1.04	1.18	0.98	0.91	1.08	1.31	2.26	1.16	0.87
time (sec)	N/A	0.596	0.035	2.252	0.032	0.073	63.909	0.130	0.144	25.942

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	97	98	127	99	99	126	143	112	78
N.S.	1	1.03	1.04	1.35	1.05	1.05	1.34	1.52	1.19	0.83
time (sec)	N/A	0.462	0.036	1.990	0.028	0.075	16.937	0.121	0.154	25.948

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	75	80	157	0	0	0	0	105	0
N.S.	1	0.91	0.98	1.91	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.497	0.025	0.813	0.000	0.000	0.000	0.000	0.143	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	89	92	155	0	0	0	0	181	0
N.S.	1	0.96	0.99	1.67	0.00	0.00	0.00	0.00	1.95	0.00
time (sec)	N/A	0.525	0.032	1.015	0.000	0.000	0.000	0.000	0.147	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	100	105	88	77	97	167	209	120	85
N.S.	1	1.08	1.13	0.95	0.83	1.04	1.80	2.25	1.29	0.91
time (sec)	N/A	0.597	0.044	1.157	0.036	0.090	78.228	0.125	0.172	26.088

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	118	134	108	104	129	0	316	149	113
N.S.	1	0.94	1.07	0.86	0.83	1.03	0.00	2.53	1.19	0.90
time (sec)	N/A	0.582	0.069	1.809	0.030	0.092	0.000	0.122	0.254	26.322

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	143	162	131	132	155	0	379	175	134
N.S.	1	0.97	1.09	0.89	0.89	1.05	0.00	2.56	1.18	0.91
time (sec)	N/A	0.638	0.108	3.219	0.029	0.086	0.000	0.126	0.171	26.476

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	118	118	0	300	320	121	142	126
N.S.	1	1.00	0.77	0.77	0.00	1.95	2.08	0.79	0.92	0.82
time (sec)	N/A	0.531	0.065	2.533	0.000	0.106	32.765	0.121	0.145	26.382

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	89	0	220	260	94	113	97
N.S.	1	1.00	1.00	0.76	0.00	1.88	2.22	0.80	0.97	0.83
time (sec)	N/A	0.446	0.031	0.881	0.000	0.092	8.479	0.128	0.144	0.002

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	93	62	73	0	199	204	68	100	83
N.S.	1	1.29	0.86	1.01	0.00	2.76	2.83	0.94	1.39	1.15
time (sec)	N/A	0.431	0.055	1.454	0.000	0.124	16.414	0.120	0.153	26.372

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	96	77	0	191	901	88	105	65
N.S.	1	1.00	0.89	0.71	0.00	1.77	8.34	0.81	0.97	0.60
time (sec)	N/A	0.471	0.043	1.211	0.000	0.125	39.139	0.119	0.160	26.506

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	101	96	0	259	1134	117	132	88
N.S.	1	1.00	0.72	0.69	0.00	1.85	8.10	0.84	0.94	0.63
time (sec)	N/A	0.496	0.012	2.404	0.000	0.093	156.924	0.123	0.158	26.358

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	250	205	253	223	262	0	753	296	224
N.S.	1	1.00	0.82	1.01	0.89	1.04	0.00	3.00	1.18	0.89
time (sec)	N/A	0.933	0.197	6.712	0.032	0.086	0.000	0.144	0.173	26.045

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	209	257	211	185	224	0	544	252	184
N.S.	1	1.00	1.22	1.00	0.88	1.07	0.00	2.59	1.20	0.88
time (sec)	N/A	0.838	0.091	4.329	0.034	0.111	0.000	0.134	0.149	25.773

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	123	135	206	152	180	235	340	208	142
N.S.	1	0.99	1.09	1.66	1.23	1.45	1.90	2.74	1.68	1.15
time (sec)	N/A	0.547	0.114	3.073	0.033	0.075	62.820	0.128	0.148	25.860

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	152	121	209	0	0	0	0	194	0
N.S.	1	0.99	0.79	1.37	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.661	0.089	1.568	0.000	0.000	0.000	0.000	0.152	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	134	133	211	0	0	0	0	250	0
N.S.	1	0.99	0.99	1.56	0.00	0.00	0.00	0.00	1.85	0.00
time (sec)	N/A	0.645	0.082	2.741	0.000	0.000	0.000	0.000	0.152	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	174	152	208	0	0	0	0	206	0
N.S.	1	1.01	0.88	1.21	0.00	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.726	0.122	2.839	0.000	0.000	0.000	0.000	0.149	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	134	141	159	137	183	0	464	225	151
N.S.	1	1.03	1.08	1.22	1.05	1.41	0.00	3.57	1.73	1.16
time (sec)	N/A	0.663	0.126	2.193	0.036	0.083	0.000	0.137	0.161	25.783

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	208	184	193	183	230	0	604	271	190
N.S.	1	0.96	0.85	0.89	0.85	1.06	0.00	2.80	1.25	0.88
time (sec)	N/A	0.831	0.169	3.527	0.033	0.112	0.000	0.135	0.146	25.795

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	244	215	230	223	268	0	699	315	225
N.S.	1	0.96	0.85	0.91	0.88	1.06	0.00	2.76	1.25	0.89
time (sec)	N/A	0.912	0.201	5.785	0.034	0.096	0.000	0.134	0.146	25.835

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	188	211	0	492	559	215	259	235
N.S.	1	1.00	0.68	0.76	0.00	1.77	2.01	0.77	0.93	0.85
time (sec)	N/A	0.774	0.155	5.363	0.000	0.090	125.207	0.127	0.148	25.796

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	151	167	0	404	478	174	212	193
N.S.	1	1.00	0.68	0.76	0.00	1.83	2.16	0.79	0.96	0.87
time (sec)	N/A	0.619	0.066	2.328	0.000	0.092	33.368	0.122	0.166	0.002

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	112	127	0	366	400	136	194	180
N.S.	1	1.00	0.63	0.71	0.00	2.06	2.25	0.76	1.09	1.01
time (sec)	N/A	0.589	0.149	3.434	0.000	0.089	61.974	0.127	0.228	25.712

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	113	125	0	350	381	135	188	108
N.S.	1	1.00	0.67	0.74	0.00	2.07	2.25	0.80	1.11	0.64
time (sec)	N/A	0.591	0.134	3.800	0.000	0.116	76.240	0.123	0.156	25.806

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	156	134	0	351	1603	163	194	115
N.S.	1	1.00	0.78	0.67	0.00	1.76	8.02	0.82	0.97	0.58
time (sec)	N/A	0.638	0.071	3.660	0.000	0.118	159.420	0.130	0.143	25.835

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	161	167	0	429	0	207	239	149
N.S.	1	1.00	0.64	0.66	0.00	1.70	0.00	0.82	0.95	0.59
time (sec)	N/A	0.726	0.036	4.610	0.000	0.112	0.000	0.132	0.169	25.709

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	188	181	143	410	0	0	0	0	261	0
N.S.	1	0.96	0.76	2.18	0.00	0.00	0.00	0.00	1.39	0.00
time (sec)	N/A	0.775	0.116	4.696	0.000	0.000	0.000	0.000	0.172	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	112	107	91	358	0	0	0	0	173	0
N.S.	1	0.96	0.81	3.20	0.00	0.00	0.00	0.00	1.54	0.00
time (sec)	N/A	0.612	0.048	2.876	0.000	0.000	0.000	0.000	0.160	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	64	301	138	0	0	0	113	0
N.S.	1	0.97	0.91	4.30	1.97	0.00	0.00	0.00	1.61	0.00
time (sec)	N/A	0.554	0.011	3.599	0.034	0.000	0.000	0.000	0.183	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	119	113	92	420	140	0	0	0	26	0
N.S.	1	0.95	0.77	3.53	1.18	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.647	0.044	2.387	0.157	0.000	0.000	0.000	0.149	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	176	168	168	471	178	0	0	0	28	0
N.S.	1	0.95	0.95	2.68	1.01	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.755	0.066	4.079	0.163	0.000	0.000	0.000	0.159	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	667	667	697	616	0	0	0	0	148	0
N.S.	1	1.00	1.04	0.92	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.632	0.676	2.835	0.000	0.000	0.000	0.000	0.160	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	619	525	0	0	0	0	75	0
N.S.	1	1.00	1.06	0.90	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.384	0.413	2.193	0.000	0.000	0.000	0.000	0.164	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	533	506	564	449	0	0	0	0	24	0
N.S.	1	0.95	1.06	0.84	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.207	0.110	1.569	0.000	0.000	0.000	0.000	0.159	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	613	526	0	0	0	0	28	0
N.S.	1	1.00	1.06	0.91	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.390	0.417	3.103	0.000	0.000	0.000	0.000	0.181	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	670	602	0	0	0	0	28	0
N.S.	1	1.00	1.03	0.92	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.508	0.572	4.935	0.000	0.000	0.000	0.000	0.177	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	199	191	166	442	0	0	0	0	1132	0
N.S.	1	0.96	0.83	2.22	0.00	0.00	0.00	0.00	5.69	0.00
time (sec)	N/A	0.815	0.236	4.123	0.000	0.000	0.000	0.000	0.295	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	155	145	131	381	0	0	0	0	897	0
N.S.	1	0.94	0.85	2.46	0.00	0.00	0.00	0.00	5.79	0.00
time (sec)	N/A	0.682	0.088	4.010	0.000	0.000	0.000	0.000	0.326	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	77	73	75	74	91	0	126	140	80
N.S.	1	0.93	0.88	0.90	0.89	1.10	0.00	1.52	1.69	0.96
time (sec)	N/A	0.417	0.054	2.986	0.028	0.118	0.000	0.127	0.161	17.977

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	201	187	170	492	197	0	0	0	37	0
N.S.	1	0.93	0.85	2.45	0.98	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.812	0.106	2.876	0.162	0.000	0.000	0.000	0.186	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	251	248	224	535	295	0	0	0	39	0
N.S.	1	0.99	0.89	2.13	1.18	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.922	0.171	9.009	0.088	0.000	0.000	0.000	0.191	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	802	802	915	0	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.656	2.070	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	850	0	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.439	1.330	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	877	0	0	0	0	0	35	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.807	0.718	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	803	803	939	0	0	0	0	0	39	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.662	1.780	0.000	0.000	0.000	0.000	0.000	0.284	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	128	264	0	0	0	0	24	0
N.S.	1	1.03	0.79	1.62	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.923	0.057	2.510	0.000	0.000	0.000	0.000	0.145	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	222	159	194	208	0	0	0	20	0
N.S.	1	0.93	0.67	0.81	0.87	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.920	0.136	1.866	0.113	0.000	0.000	0.000	0.166	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	245	335	282	0	0	0	0	20	0
N.S.	1	1.13	1.54	1.30	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.897	0.148	1.057	0.000	0.000	0.000	0.000	0.179	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	144	127	118	267	0	148	0	0	153	0
N.S.	1	0.88	0.82	1.85	0.00	1.03	0.00	0.00	1.06	0.00
time (sec)	N/A	0.621	0.208	6.833	0.000	0.115	0.000	0.000	0.162	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	124	110	100	249	0	133	0	0	136	0
N.S.	1	0.89	0.81	2.01	0.00	1.07	0.00	0.00	1.10	0.00
time (sec)	N/A	0.581	0.143	6.438	0.000	0.122	0.000	0.000	0.170	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	83	75	68	223	0	100	0	0	110	0
N.S.	1	0.90	0.82	2.69	0.00	1.20	0.00	0.00	1.33	0.00
time (sec)	N/A	0.519	0.064	5.127	0.000	0.092	0.000	0.000	0.183	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	97	91	90	242	0	114	0	0	189	0
N.S.	1	0.94	0.93	2.49	0.00	1.18	0.00	0.00	1.95	0.00
time (sec)	N/A	0.580	0.098	6.838	0.000	0.099	0.000	0.000	0.180	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	126	117	104	267	0	150	0	0	197	0
N.S.	1	0.93	0.83	2.12	0.00	1.19	0.00	0.00	1.56	0.00
time (sec)	N/A	0.637	0.210	6.708	0.000	0.084	0.000	0.000	0.167	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	327	286	209	436	0	291	0	0	319	0
N.S.	1	0.87	0.64	1.33	0.00	0.89	0.00	0.00	0.98	0.00
time (sec)	N/A	0.955	0.318	11.540	0.000	0.097	0.000	0.000	0.155	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	254	222	171	375	0	242	0	0	263	0
N.S.	1	0.87	0.67	1.48	0.00	0.95	0.00	0.00	1.04	0.00
time (sec)	N/A	0.782	0.271	11.435	0.000	0.110	0.000	0.000	0.157	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	176	156	124	314	0	192	0	0	203	0
N.S.	1	0.89	0.70	1.78	0.00	1.09	0.00	0.00	1.15	0.00
time (sec)	N/A	0.661	0.180	6.934	0.000	0.087	0.000	0.000	0.158	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	193	180	160	334	0	210	0	0	275	0
N.S.	1	0.93	0.83	1.73	0.00	1.09	0.00	0.00	1.42	0.00
time (sec)	N/A	0.782	0.325	13.674	0.000	0.116	0.000	0.000	0.158	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	257	235	188	396	0	265	0	0	341	0
N.S.	1	0.91	0.73	1.54	0.00	1.03	0.00	0.00	1.33	0.00
time (sec)	N/A	0.897	0.573	12.272	0.000	0.095	0.000	0.000	0.221	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	266	252	0	478	0	0	0	0	29	0
N.S.	1	0.95	0.00	1.80	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.078	0.000	6.312	0.000	0.000	0.000	0.000	0.228	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	121	113	92	323	154	0	0	0	27	0
N.S.	1	0.93	0.76	2.67	1.27	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.688	0.094	3.611	0.118	0.000	0.000	0.000	0.162	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	64	243	112	0	0	0	146	0
N.S.	1	0.97	0.91	3.47	1.60	0.00	0.00	0.00	2.09	0.00
time (sec)	N/A	0.667	0.029	6.094	0.125	0.000	0.000	0.000	0.167	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	221	213	0	398	0	0	0	0	140	0
N.S.	1	0.96	0.00	1.80	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	1.050	0.000	18.708	0.000	0.000	0.000	0.000	0.194	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	419	393	0	635	0	0	0	0	43	0
N.S.	1	0.94	0.00	1.52	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.342	0.000	22.052	0.000	0.000	0.000	0.000	0.160	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	204	187	171	414	233	0	0	0	41	0
N.S.	1	0.92	0.84	2.03	1.14	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.836	0.176	6.136	0.120	0.000	0.000	0.000	0.157	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	156	145	433	356	209	0	0	0	1267	0
N.S.	1	0.93	2.78	2.28	1.34	0.00	0.00	0.00	8.12	0.00
time (sec)	N/A	0.781	1.462	16.210	0.118	0.000	0.000	0.000	0.195	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD	TBD
size	377	357	0	561	0	0	0	0	0	0
N.S.	1	0.95	0.00	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.379	0.000	118.098	0.000	0.000	0.000	0.000	0.222	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	26	26	23	106	25	0	0	233	0
N.S.	1	1.04	1.04	0.92	4.24	1.00	0.00	0.00	9.32	0.00
time (sec)	N/A	0.543	0.084	4.539	0.119	0.112	0.000	0.000	0.168	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	26	26	23	109	25	0	0	233	21
N.S.	1	1.04	1.04	0.92	4.36	1.00	0.00	0.00	9.32	0.84
time (sec)	N/A	0.497	0.004	6.309	0.040	0.100	0.000	0.000	0.165	14.889

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	27	34	24	0	30	0	0	252	0
N.S.	1	1.04	1.31	0.92	0.00	1.15	0.00	0.00	9.69	0.00
time (sec)	N/A	0.537	0.087	8.892	0.000	0.075	0.000	0.000	0.167	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	0	42	0	31	496	31
N.S.	1	1.00	1.07	1.00	0.00	1.45	0.00	1.07	17.10	1.07
time (sec)	N/A	0.408	0.404	0.319	0.000	0.079	0.000	3.833	0.181	14.745

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	0	40	0	29	492	29
N.S.	1	1.00	1.07	1.00	0.00	1.48	0.00	1.07	18.22	1.07
time (sec)	N/A	0.395	0.281	0.319	0.000	0.096	0.000	2.675	0.178	14.748

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	0	47	0	31	514	31
N.S.	1	1.00	1.07	1.00	0.00	1.62	0.00	1.07	17.72	1.07
time (sec)	N/A	0.456	0.464	0.631	0.000	0.106	0.000	2.493	0.189	14.766

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	31	0	49	0	33	532	33
N.S.	1	1.00	1.07	1.07	0.00	1.69	0.00	1.14	18.34	1.14
time (sec)	N/A	0.459	0.415	0.547	0.000	0.098	0.000	2.464	0.226	14.747

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	0	31	0	31	31	31
N.S.	1	1.00	1.07	1.00	0.00	1.07	0.00	1.07	1.07	1.07
time (sec)	N/A	0.406	1.487	0.872	0.000	0.087	0.000	2.537	0.264	15.843

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	0	29	22	29	29	29
N.S.	1	1.00	1.07	1.00	0.00	1.07	0.81	1.07	1.07	1.07
time (sec)	N/A	0.397	1.254	1.012	0.000	0.120	8.911	2.511	0.183	15.696

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	0	32	0	31	285	31
N.S.	1	1.00	1.07	1.00	0.00	1.10	0.00	1.07	9.83	1.07
time (sec)	N/A	0.528	1.598	1.463	0.000	0.103	0.000	2.515	0.187	15.172

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	31	0	36	0	33	279	33
N.S.	1	1.00	1.07	1.07	0.00	1.24	0.00	1.14	9.62	1.14
time (sec)	N/A	0.793	0.334	1.429	0.000	0.076	0.000	2.537	0.200	15.254

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	88	75	66	0	76	0	0	16	0
N.S.	1	1.28	1.09	0.96	0.00	1.10	0.00	0.00	0.23	0.00
time (sec)	N/A	0.735	0.069	2.611	0.000	0.076	0.000	0.000	0.151	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	12	34	9	59	11	0	68	14	8
N.S.	1	1.50	4.25	1.12	7.38	1.38	0.00	8.50	1.75	1.00
time (sec)	N/A	0.281	0.006	1.117	0.030	0.089	0.000	0.146	0.166	15.267

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	76	69	15	0	0	16	10
N.S.	1	1.00	1.00	6.33	5.75	1.25	0.00	0.00	1.33	0.83
time (sec)	N/A	0.305	0.006	0.549	0.028	0.064	0.000	0.000	0.154	15.135

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	61	0	0	18	0
N.S.	1	1.00	1.00	1.07	0.00	4.36	0.00	0.00	1.29	0.00
time (sec)	N/A	0.311	0.008	5.154	0.000	0.089	0.000	0.000	0.154	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	36	34	67	0	0	204	16	37
N.S.	1	1.00	1.03	0.97	1.91	0.00	0.00	5.83	0.46	1.06
time (sec)	N/A	0.450	0.007	2.138	0.029	0.000	0.000	0.219	0.165	15.110

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	108	77	0	0	0	18	33
N.S.	1	1.00	1.03	2.77	1.97	0.00	0.00	0.00	0.46	0.85
time (sec)	N/A	0.428	0.007	0.474	0.032	0.000	0.000	0.000	0.154	15.149

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	43	44	44	0	67	0	0	20	0
N.S.	1	0.91	0.94	0.94	0.00	1.43	0.00	0.00	0.43	0.00
time (sec)	N/A	0.441	0.021	6.906	0.000	0.076	0.000	0.000	0.154	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	111	115	114	124	0	0	0	20	0
N.S.	1	1.06	1.10	1.09	1.18	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.733	0.057	10.180	0.033	0.000	0.000	0.000	0.162	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	240	228	230	0	0	0	0	22	0
N.S.	1	1.06	1.00	1.01	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.136	0.125	4.626	0.000	0.000	0.000	0.000	0.164	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	14	24	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.64	1.09	1.09	1.09
time (sec)	N/A	0.327	0.562	1.090	0.099	0.072	24.900	0.136	0.158	15.048

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	0	0	0	0	0	166	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	1.80	0.00
time (sec)	N/A	0.412	0.055	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	156	157	0	128	148	155	348	150	137
N.S.	1	0.94	0.95	0.00	0.77	0.89	0.93	2.10	0.90	0.83
time (sec)	N/A	0.572	0.141	0.000	0.034	0.138	6.947	0.124	0.158	15.233

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	128	129	0	106	122	128	264	122	111
N.S.	1	0.96	0.96	0.00	0.79	0.91	0.96	1.97	0.91	0.83
time (sec)	N/A	0.520	0.101	0.000	0.035	0.100	2.521	0.126	0.164	15.155

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	100	101	0	84	95	100	180	94	85
N.S.	1	0.98	0.99	0.00	0.82	0.93	0.98	1.76	0.92	0.83
time (sec)	N/A	0.472	0.073	0.000	0.034	0.126	1.219	0.126	0.161	15.175

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	53	57	65	66	102	64	52
N.S.	1	1.00	1.00	0.88	0.95	1.08	1.10	1.70	1.07	0.87
time (sec)	N/A	0.336	0.032	0.995	0.032	0.075	0.501	0.130	0.161	15.098

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	108	0	0	0	99	0
N.S.	1	1.00	1.04	0.00	2.12	0.00	0.00	0.00	1.94	0.00
time (sec)	N/A	0.420	0.006	0.000	0.170	0.000	0.000	0.000	0.176	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	73	69	0	61	65	442	144	67	58
N.S.	1	1.04	0.99	0.00	0.87	0.93	6.31	2.06	0.96	0.83
time (sec)	N/A	0.418	0.049	0.000	0.032	0.110	13.690	0.120	0.163	15.328

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	103	100	0	84	97	532	249	97	83
N.S.	1	0.94	0.92	0.00	0.77	0.89	4.88	2.28	0.89	0.76
time (sec)	N/A	0.459	0.043	0.000	0.035	0.099	101.143	0.129	0.212	15.166

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	131	128	0	106	124	0	343	125	110
N.S.	1	0.93	0.91	0.00	0.75	0.88	0.00	2.43	0.89	0.78
time (sec)	N/A	0.520	0.133	0.000	0.035	0.094	0.000	0.127	0.228	15.269

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	480	298	311	0	324	487	0	930	439	434
N.S.	1	0.62	0.65	0.00	0.68	1.01	0.00	1.94	0.91	0.90
time (sec)	N/A	0.966	0.395	0.000	0.047	0.127	0.000	0.134	0.157	16.235

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	224	223	0	257	357	0	624	319	420
N.S.	1	0.65	0.65	0.00	0.75	1.04	0.00	1.82	0.93	1.23
time (sec)	N/A	0.880	0.214	0.000	0.038	0.095	0.000	0.134	0.175	15.240

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	202	150	0	179	225	0	340	198	186
N.S.	1	1.04	0.77	0.00	0.92	1.15	0.00	1.74	1.02	0.95
time (sec)	N/A	0.693	0.089	0.000	0.036	0.137	0.000	0.131	0.182	15.194

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	90	195	0	0	0	0	0	209	0
N.S.	1	0.97	2.10	0.00	0.00	0.00	0.00	0.00	2.25	0.00
time (sec)	N/A	0.781	0.145	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	143	189	0	0	0	0	0	237	0
N.S.	1	0.92	1.22	0.00	0.00	0.00	0.00	0.00	1.53	0.00
time (sec)	N/A	1.118	0.186	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	325	353	0	0	0	0	0	357	0
N.S.	1	1.11	1.20	0.00	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	2.319	0.361	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	408	560	437	0	0	0	0	0	477	0
N.S.	1	1.37	1.07	0.00	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	4.058	0.449	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	913	577	0	666	1197	0	2160	973	976
N.S.	1	1.01	0.64	0.00	0.73	1.32	0.00	2.38	1.07	1.08
time (sec)	N/A	2.281	0.508	0.000	0.052	0.173	0.000	0.153	0.165	21.300

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	595	598	433	0	536	861	0	1440	693	840
N.S.	1	1.01	0.73	0.00	0.90	1.45	0.00	2.42	1.16	1.41
time (sec)	N/A	1.545	0.321	0.000	0.049	0.141	0.000	0.148	0.170	15.276

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	284	289	241	0	381	527	0	714	417	350
N.S.	1	1.02	0.85	0.00	1.34	1.86	0.00	2.51	1.47	1.23
time (sec)	N/A	0.826	0.216	0.000	0.047	0.100	0.000	0.128	0.230	15.053

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	128	333	0	0	0	0	0	318	0
N.S.	1	0.95	2.47	0.00	0.00	0.00	0.00	0.00	2.36	0.00
time (sec)	N/A	0.968	0.177	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	224	536	0	0	0	0	0	477	0
N.S.	1	0.85	2.04	0.00	0.00	0.00	0.00	0.00	1.81	0.00
time (sec)	N/A	1.958	0.838	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	573	604	841	0	0	0	0	0	774	0
N.S.	1	1.05	1.47	0.00	0.00	0.00	0.00	0.00	1.35	0.00
time (sec)	N/A	5.227	1.315	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	157	155	0	118	179	143	266	164	140
N.S.	1	0.92	0.91	0.00	0.69	1.05	0.84	1.56	0.96	0.82
time (sec)	N/A	0.568	0.142	0.000	0.036	0.129	46.372	0.172	0.157	15.318

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	129	127	0	96	153	116	232	136	106
N.S.	1	0.93	0.91	0.00	0.69	1.10	0.83	1.67	0.98	0.76
time (sec)	N/A	0.531	0.100	0.000	0.035	0.091	16.459	0.173	0.156	15.118

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	101	99	0	74	126	88	81	108	86
N.S.	1	0.94	0.93	0.00	0.69	1.18	0.82	0.76	1.01	0.80
time (sec)	N/A	0.470	0.066	0.000	0.033	0.126	5.764	0.171	0.162	15.312

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	63	86	48	90	56	52	76	44
N.S.	1	1.00	1.19	1.62	0.91	1.70	1.06	0.98	1.43	0.83
time (sec)	N/A	0.331	0.035	1.040	0.036	0.092	2.422	0.148	0.171	14.869

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	124	0	0	0	31	0
N.S.	1	1.00	1.04	0.00	2.43	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.424	0.006	0.000	0.396	0.000	0.000	0.000	0.162	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	74	68	63	75	70	391	116	78	60
N.S.	1	1.14	1.05	0.97	1.15	1.08	6.02	1.78	1.20	0.92
time (sec)	N/A	0.426	0.040	1.182	0.061	0.089	47.490	0.133	0.244	14.604

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	102	104	0	95	96	0	237	105	87
N.S.	1	0.98	1.00	0.00	0.91	0.92	0.00	2.28	1.01	0.84
time (sec)	N/A	0.483	0.070	0.000	0.055	0.105	0.000	0.144	0.199	14.562

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	130	132	0	117	123	0	357	133	113
N.S.	1	0.96	0.97	0.00	0.86	0.90	0.00	2.62	0.98	0.83
time (sec)	N/A	0.575	0.091	0.000	0.079	0.085	0.000	0.140	0.165	14.598

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	404	552	432	0	0	0	0	0	509	0
N.S.	1	1.37	1.07	0.00	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	3.906	0.400	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	321	321	0	0	0	0	0	377	0
N.S.	1	1.11	1.11	0.00	0.00	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	2.180	0.235	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	151	172	0	0	0	0	0	248	0
N.S.	1	0.99	1.13	0.00	0.00	0.00	0.00	0.00	1.63	0.00
time (sec)	N/A	1.179	0.123	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	90	386	0	0	0	0	0	65	0
N.S.	1	0.97	4.15	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.782	0.365	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	202	298	0	248	235	0	278	248	193
N.S.	1	1.04	1.53	0.00	1.27	1.21	0.00	1.43	1.27	0.99
time (sec)	N/A	0.723	0.306	0.000	0.050	0.089	0.000	0.157	0.162	14.714

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	341	228	440	0	321	361	0	578	376	424
N.S.	1	0.67	1.29	0.00	0.94	1.06	0.00	1.70	1.10	1.24
time (sec)	N/A	0.879	0.329	0.000	0.063	0.108	0.000	0.169	0.186	14.802

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F(-1)	B	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	480	302	545	0	387	490	0	877	510	440
N.S.	1	0.63	1.14	0.00	0.81	1.02	0.00	1.83	1.06	0.92
time (sec)	N/A	0.981	0.566	0.000	0.071	0.096	0.000	0.168	0.233	15.716

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	569	600	0	0	0	0	0	0	1023	0
N.S.	1	1.05	0.00	0.00	0.00	0.00	0.00	0.00	1.80	0.00
time (sec)	N/A	5.043	0.000	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	236	0	0	0	0	0	0	682	0
N.S.	1	0.91	0.00	0.00	0.00	0.00	0.00	0.00	2.62	0.00
time (sec)	N/A	2.025	0.000	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	128	0	0	0	0	0	0	99	0
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.942	0.000	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	289	558	0	568	541	0	543	521	357
N.S.	1	1.01	1.96	0.00	1.99	1.90	0.00	1.91	1.83	1.25
time (sec)	N/A	0.880	0.638	0.000	0.079	0.091	0.000	0.197	0.158	14.646

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	595	598	766	0	732	869	0	1147	831	846
N.S.	1	1.01	1.29	0.00	1.23	1.46	0.00	1.93	1.40	1.42
time (sec)	N/A	1.553	1.068	0.000	0.103	0.139	0.000	0.213	0.173	14.789

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	913	950	0	864	1203	0	1747	1153	989
N.S.	1	1.01	1.05	0.00	0.95	1.33	0.00	1.93	1.27	1.09
time (sec)	N/A	2.172	1.477	0.000	0.092	0.182	0.000	0.234	0.171	21.013

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	216	218	0	172	201	216	516	201	189
N.S.	1	0.92	0.93	0.00	0.74	0.86	0.92	2.21	0.86	0.81
time (sec)	N/A	0.668	0.245	0.000	0.047	0.117	34.074	0.127	0.214	14.815

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	173	174	0	140	161	173	390	162	150
N.S.	1	0.94	0.94	0.00	0.76	0.87	0.94	2.11	0.88	0.81
time (sec)	N/A	0.588	0.168	0.000	0.062	0.088	6.973	0.130	0.243	14.774

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	130	132	0	106	122	131	264	123	111
N.S.	1	0.96	0.97	0.00	0.78	0.90	0.96	1.94	0.90	0.82
time (sec)	N/A	0.522	0.105	0.000	0.040	0.091	2.027	0.131	0.158	14.594

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	66	70	77	82	132	80	65
N.S.	1	1.00	1.00	0.86	0.91	1.00	1.06	1.71	1.04	0.84
time (sec)	N/A	0.365	0.049	0.806	0.066	0.105	0.766	0.126	0.160	14.533

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	166	0	0	0	60	0
N.S.	1	1.00	1.04	0.00	3.25	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.425	0.006	0.000	0.256	0.000	0.000	0.000	0.190	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	85	0	75	81	483	207	86	74
N.S.	1	1.02	0.98	0.00	0.86	0.93	5.55	2.38	0.99	0.85
time (sec)	N/A	0.452	0.038	0.000	0.051	0.130	82.297	0.129	0.163	14.689

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	133	131	0	106	125	0	343	127	109
N.S.	1	0.93	0.92	0.00	0.74	0.87	0.00	2.40	0.89	0.76
time (sec)	N/A	0.511	0.135	0.000	0.045	0.104	0.000	0.131	0.170	14.776

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	175	172	0	139	163	0	489	166	154
N.S.	1	0.91	0.90	0.00	0.72	0.85	0.00	2.55	0.86	0.80
time (sec)	N/A	0.586	0.225	0.000	0.056	0.111	0.000	0.135	0.164	14.724

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	680	404	427	0	424	674	0	1389	621	608
N.S.	1	0.59	0.63	0.00	0.62	0.99	0.00	2.04	0.91	0.89
time (sec)	N/A	1.122	0.628	0.000	0.054	0.141	0.000	0.141	0.190	18.162

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	480	302	317	0	323	484	0	930	447	431
N.S.	1	0.63	0.66	0.00	0.67	1.01	0.00	1.94	0.93	0.90
time (sec)	N/A	0.949	0.374	0.000	0.063	0.128	0.000	0.135	0.257	15.739

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	190	197	0	217	287	0	465	267	290
N.S.	1	0.71	0.74	0.00	0.81	1.07	0.00	1.74	1.00	1.09
time (sec)	N/A	0.843	0.136	0.000	0.043	0.133	0.000	0.127	0.171	14.717

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	90	195	0	0	0	0	0	114	0
N.S.	1	0.97	2.10	0.00	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.773	0.133	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	230	273	0	0	0	0	0	255	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	1.625	0.251	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	405	566	443	0	0	0	0	0	433	0
N.S.	1	1.40	1.09	0.00	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	3.995	0.498	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1835	1843	1025	0	1064	2183	0	4320	1816	1802
N.S.	1	1.00	0.56	0.00	0.58	1.19	0.00	2.35	0.99	0.98
time (sec)	N/A	4.637	1.447	0.000	0.097	0.311	0.000	0.185	0.174	21.279

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1357	1366	808	0	867	1688	0	3240	1405	1386
N.S.	1	1.01	0.60	0.00	0.64	1.24	0.00	2.39	1.04	1.02
time (sec)	N/A	3.202	0.932	0.000	0.098	0.204	0.000	0.167	0.180	21.042

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	913	589	0	668	1190	0	2160	994	979
N.S.	1	1.01	0.65	0.00	0.74	1.31	0.00	2.38	1.10	1.08
time (sec)	N/A	2.154	0.531	0.000	0.084	0.165	0.000	0.154	0.164	20.963

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	438	444	362	0	455	690	0	1072	575	558
N.S.	1	1.01	0.83	0.00	1.04	1.58	0.00	2.45	1.31	1.27
time (sec)	N/A	1.156	0.240	0.000	0.059	0.112	0.000	0.145	0.201	14.838

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	128	333	0	0	0	0	0	178	0
N.S.	1	0.95	2.47	0.00	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.933	0.204	0.000	0.000	0.000	0.000	0.000	0.318	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	439	371	733	0	0	0	0	0	493	0
N.S.	1	0.85	1.67	0.00	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	3.390	0.960	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	765	1382	1074	0	0	0	0	0	908	0
N.S.	1	1.81	1.40	0.00	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	10.997	1.988	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	134	135	0	108	129	0	254	125	113
N.S.	1	0.97	0.98	0.00	0.78	0.93	0.00	1.84	0.91	0.82
time (sec)	N/A	0.538	0.119	0.000	0.057	0.113	0.000	0.182	0.178	14.586

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	124	128	0	0	337	0	115	117	0
N.S.	1	0.95	0.98	0.00	0.00	2.59	0.00	0.88	0.90	0.00
time (sec)	N/A	0.467	0.163	0.000	0.000	0.103	0.000	0.153	0.164	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	91	94	0	76	83	95	82	86	74
N.S.	1	1.02	1.06	0.00	0.85	0.93	1.07	0.92	0.97	0.83
time (sec)	N/A	0.453	0.033	0.000	0.046	0.086	70.008	0.166	0.165	14.494

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	62	0	231	70	72	72	56
N.S.	1	1.00	1.00	0.86	0.00	3.21	0.97	1.00	1.00	0.78
time (sec)	N/A	0.351	0.031	0.815	0.000	0.128	1.887	0.141	0.162	14.642

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	55	0	114	0	0	0	60	0
N.S.	1	0.96	1.00	0.00	2.07	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	0.421	0.015	0.000	0.287	0.000	0.000	0.000	0.168	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	59	0	0	208	0	64	65	0
N.S.	1	1.01	0.87	0.00	0.00	3.06	0.00	0.94	0.96	0.00
time (sec)	N/A	0.360	0.021	0.000	0.000	0.096	0.000	0.147	0.180	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	91	93	0	77	85	0	95	90	74
N.S.	1	0.97	0.99	0.00	0.82	0.90	0.00	1.01	0.96	0.79
time (sec)	N/A	0.450	0.038	0.000	0.040	0.099	0.000	0.151	0.255	14.911

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	128	65	0	0	313	0	100	109	0
N.S.	1	1.04	0.53	0.00	0.00	2.54	0.00	0.81	0.89	0.00
time (sec)	N/A	0.425	0.018	0.000	0.000	0.101	0.000	0.150	0.157	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	482	308	323	0	330	508	0	905	453	440
N.S.	1	0.64	0.67	0.00	0.68	1.05	0.00	1.88	0.94	0.91
time (sec)	N/A	0.967	0.400	0.000	0.047	0.166	0.000	0.337	0.170	15.880

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	198	239	0	231	304	0	313	279	299
N.S.	1	0.72	0.87	0.00	0.84	1.11	0.00	1.14	1.01	1.09
time (sec)	N/A	0.823	0.168	0.000	0.051	0.122	0.000	0.303	0.170	14.751

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	92	199	0	0	0	0	0	117	0
N.S.	1	0.97	2.09	0.00	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.786	0.142	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	232	264	0	0	0	0	0	265	0
N.S.	1	0.97	1.11	0.00	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	1.636	0.343	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	412	568	449	0	0	0	0	0	439	0
N.S.	1	1.38	1.09	0.00	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	3.935	0.530	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	547	498	438	0	0	0	0	0	420	0
N.S.	1	0.91	0.80	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	1.532	0.462	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	364	349	319	0	0	0	0	0	236	0
N.S.	1	0.96	0.88	0.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	1.118	0.275	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	289	247	0	0	0	0	0	193	0
N.S.	1	0.97	0.83	0.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	1.028	0.180	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	476	431	473	0	0	0	0	0	296	0
N.S.	1	0.91	0.99	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	1.350	0.633	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	640	579	678	0	0	0	0	0	389	0
N.S.	1	0.90	1.06	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	1.815	1.245	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	913	915	593	0	680	1241	0	2104	1006	992
N.S.	1	1.00	0.65	0.00	0.74	1.36	0.00	2.30	1.10	1.09
time (sec)	N/A	2.265	0.960	0.000	0.080	0.283	0.000	0.594	0.174	35.140

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	446	428	0	484	720	0	755	595	575
N.S.	1	0.99	0.95	0.00	1.08	1.60	0.00	1.68	1.33	1.28
time (sec)	N/A	1.209	0.434	0.000	0.075	0.154	0.000	0.533	0.170	25.852

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	130	339	0	0	0	0	0	178	0
N.S.	1	0.94	2.44	0.00	0.00	0.00	0.00	0.00	1.28	0.00
time (sec)	N/A	0.977	0.223	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	451	373	764	0	0	0	0	0	511	0
N.S.	1	0.83	1.69	0.00	0.00	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	3.419	0.854	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	1552	22	0	75	0	24	929	24
N.S.	1	1.00	64.67	0.92	0.00	3.12	0.00	1.00	38.71	1.00
time (sec)	N/A	4.214	12.551	0.152	0.000	0.092	0.000	0.257	0.197	25.258

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	1299	18	0	62	19	20	504	20
N.S.	1	1.00	64.95	0.90	0.00	3.10	0.95	1.00	25.20	1.00
time (sec)	N/A	1.899	9.400	0.108	0.000	0.124	61.681	0.266	0.185	25.532

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	1158	22	0	66	0	24	393	24
N.S.	1	1.00	48.25	0.92	0.00	2.75	0.00	1.00	16.38	1.00
time (sec)	N/A	1.198	10.312	0.154	0.000	0.097	0.000	0.241	0.182	25.394

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	1385	22	0	66	0	24	564	24
N.S.	1	1.00	57.71	0.92	0.00	2.75	0.00	1.00	23.50	1.00
time (sec)	N/A	2.926	11.662	0.152	0.000	0.094	0.000	0.244	0.206	25.482

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	217	214	0	162	232	0	169	216	191
N.S.	1	0.91	0.90	0.00	0.68	0.97	0.00	0.71	0.90	0.80
time (sec)	N/A	0.654	0.279	0.000	0.047	0.135	0.000	0.182	0.155	25.723

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	173	171	0	128	192	162	134	177	153
N.S.	1	0.91	0.90	0.00	0.67	1.01	0.85	0.71	0.93	0.81
time (sec)	N/A	0.561	0.134	0.000	0.038	0.093	46.209	0.175	0.163	25.905

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	131	128	0	96	153	119	103	138	112
N.S.	1	0.93	0.91	0.00	0.68	1.09	0.84	0.73	0.98	0.79
time (sec)	N/A	0.496	0.099	0.000	0.053	0.131	9.535	0.166	0.165	26.155

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	79	108	59	107	73	63	96	59
N.S.	1	1.00	1.13	1.54	0.84	1.53	1.04	0.90	1.37	0.84
time (sec)	N/A	0.360	0.055	0.869	0.052	0.093	2.160	0.145	0.153	25.466

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	185	0	0	0	32	0
N.S.	1	1.00	1.04	0.00	3.63	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.426	0.007	0.000	0.440	0.000	0.000	0.000	0.176	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	89	85	0	86	107	0	96	95	73
N.S.	1	1.09	1.04	0.00	1.05	1.30	0.00	1.17	1.16	0.89
time (sec)	N/A	0.463	0.040	0.000	0.040	0.103	0.000	0.172	0.153	25.916

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	132	133	0	117	165	0	127	134	113
N.S.	1	0.96	0.96	0.00	0.85	1.20	0.00	0.92	0.97	0.82
time (sec)	N/A	0.533	0.103	0.000	0.057	0.089	0.000	0.182	0.152	25.527

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	175	177	0	150	213	0	160	173	152
N.S.	1	0.94	0.95	0.00	0.80	1.14	0.00	0.86	0.93	0.81
time (sec)	N/A	0.597	0.167	0.000	0.071	0.130	0.000	0.170	0.163	25.716

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	572	0	603	0	0	0	0	0	715	0
N.S.	1	0.00	1.05	0.00	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.000	0.638	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	400	560	437	0	0	0	0	0	520	0
N.S.	1	1.40	1.09	0.00	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	3.750	0.355	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	238	241	0	0	0	0	0	324	0
N.S.	1	1.05	1.06	0.00	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	1.688	0.193	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	90	389	0	0	0	0	0	67	0
N.S.	1	0.97	4.18	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.769	0.239	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F	A	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	196	374	0	284	357	0	428	323	299
N.S.	1	0.73	1.39	0.00	1.06	1.33	0.00	1.59	1.20	1.11
time (sec)	N/A	0.865	0.391	0.000	0.050	0.107	0.000	0.154	0.158	25.496

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F(-1)	B	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	479	306	551	0	387	597	0	877	518	439
N.S.	1	0.64	1.15	0.00	0.81	1.25	0.00	1.83	1.08	0.92
time (sec)	N/A	0.976	0.594	0.000	0.056	0.117	0.000	0.169	0.163	27.047

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	759	1374	0	0	0	0	0	0	1132	0
N.S.	1	1.81	0.00	0.00	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	10.850	0.000	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	436	391	0	0	0	0	0	0	664	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	3.379	0.000	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	128	0	0	0	0	0	0	102	0
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.997	0.000	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	438	444	666	0	640	814	0	846	700	570
N.S.	1	1.01	1.52	0.00	1.46	1.86	0.00	1.93	1.60	1.30
time (sec)	N/A	1.237	0.839	0.000	0.092	0.159	0.000	0.216	0.160	25.579

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	913	962	0	864	1404	0	1747	1174	992
N.S.	1	1.01	1.06	0.00	0.95	1.55	0.00	1.93	1.29	1.09
time (sec)	N/A	2.274	1.443	0.000	0.122	0.138	0.000	0.230	0.170	35.060

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	135	137	0	98	160	0	105	140	112
N.S.	1	0.94	0.96	0.00	0.69	1.12	0.00	0.73	0.98	0.78
time (sec)	N/A	0.530	0.126	0.000	0.055	0.099	0.000	0.185	0.153	25.588

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	111	65	0	0	399	0	103	114	0
N.S.	1	0.92	0.54	0.00	0.00	3.30	0.00	0.85	0.94	0.00
time (sec)	N/A	0.468	0.019	0.000	0.000	0.140	0.000	0.197	0.154	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	91	93	0	63	113	0	69	100	73
N.S.	1	0.97	0.99	0.00	0.67	1.20	0.00	0.73	1.06	0.78
time (sec)	N/A	0.442	0.029	0.000	0.054	0.099	0.000	0.192	0.156	25.446

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	154	0	279	61	60	68	51
N.S.	1	1.00	0.82	2.37	0.00	4.29	0.94	0.92	1.05	0.78
time (sec)	N/A	0.339	0.017	1.154	0.000	0.132	13.836	0.155	0.170	25.370

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	55	0	127	0	0	0	32	0
N.S.	1	0.96	1.00	0.00	2.31	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.452	0.014	0.000	0.397	0.000	0.000	0.000	0.292	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	86	80	0	0	235	0	78	80	0
N.S.	1	1.12	1.04	0.00	0.00	3.05	0.00	1.01	1.04	0.00
time (sec)	N/A	0.401	0.054	0.000	0.000	0.110	0.000	0.194	0.154	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	91	94	0	88	84	0	105	97	74
N.S.	1	1.02	1.06	0.00	0.99	0.94	0.00	1.18	1.09	0.83
time (sec)	N/A	0.452	0.039	0.000	0.043	0.088	0.000	0.197	0.160	25.320

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	145	129	0	0	339	0	111	123	0
N.S.	1	1.10	0.98	0.00	0.00	2.57	0.00	0.84	0.93	0.00
time (sec)	N/A	0.463	0.180	0.000	0.000	0.157	0.000	0.198	0.160	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	412	568	830	0	0	0	0	0	471	0
N.S.	1	1.38	2.01	0.00	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	3.860	0.755	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	232	542	0	0	0	0	0	276	0
N.S.	1	0.97	2.27	0.00	0.00	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	1.623	0.466	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	92	1701	0	0	0	0	0	67	0
N.S.	1	0.97	17.91	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.750	0.544	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F(-1)	F	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	276	198	691	0	298	307	0	0	329	302
N.S.	1	0.72	2.50	0.00	1.08	1.11	0.00	0.00	1.19	1.09
time (sec)	N/A	0.860	0.629	0.000	0.066	0.109	0.000	0.000	0.162	25.554

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	F(-1)	F	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	482	308	988	0	397	513	0	0	524	440
N.S.	1	0.64	2.05	0.00	0.82	1.06	0.00	0.00	1.09	0.91
time (sec)	N/A	0.965	0.912	0.000	0.067	0.095	0.000	0.000	0.163	27.019

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	490	441	735	0	0	0	0	0	429	0
N.S.	1	0.90	1.50	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	1.507	2.406	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	295	523	0	0	0	0	0	228	0
N.S.	1	0.95	1.69	0.00	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.967	1.157	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	361	347	609	0	0	0	0	0	160	0
N.S.	1	0.96	1.69	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	1.261	1.627	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	773	1384	5557	0	0	0	0	0	971	0
N.S.	1	1.79	7.19	0.00	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	10.888	30.498	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	451	373	0	0	0	0	0	0	454	0
N.S.	1	0.83	0.00	0.00	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	3.311	0.000	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	130	0	0	0	0	0	0	102	0
N.S.	1	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.924	0.000	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	446	692	0	684	725	0	0	712	578
N.S.	1	0.99	1.54	0.00	1.52	1.61	0.00	0.00	1.59	1.29
time (sec)	N/A	1.252	1.357	0.000	0.081	0.098	0.000	0.000	0.172	25.626

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	5975	22	0	93	0	24	935	24
N.S.	1	1.00	248.96	0.92	0.00	3.88	0.00	1.00	38.96	1.00
time (sec)	N/A	3.152	29.295	0.175	0.000	0.096	0.000	0.524	0.203	25.265

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	80	0	20	451	20
N.S.	1	1.00	1.10	0.90	0.00	4.00	0.00	1.00	22.55	1.00
time (sec)	N/A	1.126	6.265	0.129	0.000	0.115	0.000	0.224	0.196	25.209

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	5502	22	0	84	0	24	245	24
N.S.	1	1.00	229.25	0.92	0.00	3.50	0.00	1.00	10.21	1.00
time (sec)	N/A	2.236	19.560	0.175	0.000	0.081	0.000	0.529	0.178	25.233

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	6328	22	0	84	0	24	302	24
N.S.	1	1.00	263.67	0.92	0.00	3.50	0.00	1.00	12.58	1.00
time (sec)	N/A	4.666	27.193	0.175	0.000	0.086	0.000	0.543	0.181	22.116

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	730	741	0	0	0	0	0	0	0	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.873	0.000	0.000	0.000	0.000	0.000	0.000	0.293	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	551	552	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.070	0.000	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	360	364	229	0	0	0	0	0	0	0
N.S.	1	1.01	0.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.431	6.137	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	178	130	0	0	0	0	0	1228	0
N.S.	1	1.02	0.75	0.00	0.00	0.00	0.00	0.00	7.06	0.00
time (sec)	N/A	0.803	0.064	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	224	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	10.18	1.00
time (sec)	N/A	0.367	0.173	0.093	0.140	0.094	0.000	0.155	0.307	25.749

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	188	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	8.55	1.00
time (sec)	N/A	0.376	0.159	0.089	0.148	0.101	0.000	0.166	0.190	25.469

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	896	0	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.042	0.000	0.000	0.000	0.000	0.000	0.000	0.345	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	677	679	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.200	0.000	0.000	0.000	0.000	0.000	0.000	0.286	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	445	442	0	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.532	0.000	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	216	182	0	0	0	0	0	0	0
N.S.	1	1.01	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.865	0.075	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	33	0	24	292	24
N.S.	1	1.00	1.08	0.92	1.00	1.38	0.00	1.00	12.17	1.00
time (sec)	N/A	0.388	0.178	0.095	0.152	0.086	0.000	1.154	0.181	25.492

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	33	0	24	257	24
N.S.	1	1.00	1.08	0.92	1.00	1.38	0.00	1.00	10.71	1.00
time (sec)	N/A	0.384	0.179	0.095	0.154	0.090	0.000	1.185	0.207	25.598

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	26	0	20	25	20
N.S.	1	1.00	1.10	0.90	1.00	1.30	0.00	1.00	1.25	1.00
time (sec)	N/A	0.371	1.261	0.142	0.168	0.111	0.000	0.195	0.163	25.468

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	24	0	18	23	18
N.S.	1	1.00	1.11	0.89	1.00	1.33	0.00	1.00	1.28	1.00
time (sec)	N/A	0.340	0.051	0.096	0.182	0.093	0.000	0.155	0.157	25.383

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	27	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.23	1.00
time (sec)	N/A	0.379	0.481	0.017	0.181	0.086	0.000	0.185	0.160	25.392

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	178	131	0	0	0	0	0	0	0
N.S.	1	1.02	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.821	0.331	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	325	0	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.067	1.318	0.000	0.000	0.000	0.000	0.000	0.270	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	926	929	525	0	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.344	5.952	0.000	0.000	0.000	0.000	0.000	0.512	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	35	0	22	34	22
N.S.	1	1.00	1.09	0.91	1.00	1.59	0.00	1.00	1.55	1.00
time (sec)	N/A	0.374	0.388	0.189	0.175	0.119	0.000	1.825	0.155	25.779

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	33	0	20	32	20
N.S.	1	1.00	1.10	0.90	1.00	1.65	0.00	1.00	1.60	1.00
time (sec)	N/A	0.348	0.053	0.098	0.177	0.097	0.000	1.041	0.154	25.644

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	37	0	24	36	24
N.S.	1	1.00	1.08	0.92	1.00	1.54	0.00	1.00	1.50	1.00
time (sec)	N/A	0.397	0.242	0.100	0.175	0.084	0.000	1.873	0.185	25.532

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	676	679	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.283	0.000	0.000	0.000	0.000	0.000	0.000	0.350	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1143	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.632	0.000	0.000	0.000	0.000	0.000	0.000	0.541	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1121	1115	0	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.041	0.000	0.000	0.000	0.000	0.000	0.000	0.480	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	831	836	0	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.033	0.000	0.000	0.000	0.000	0.000	0.000	0.444	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	553	552	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.030	0.000	0.000	0.000	0.000	0.000	0.000	0.316	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	269	174	0	0	0	0	0	0	0
N.S.	1	1.01	0.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.127	0.036	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	130	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	5.91	1.00
time (sec)	N/A	0.377	0.159	0.092	0.144	0.091	0.000	0.160	0.264	25.331

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	532	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	24.18	1.00
time (sec)	N/A	0.373	0.155	0.093	0.148	0.097	0.000	0.165	0.197	25.431

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1363	1375	0	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.414	0.000	0.000	0.000	0.000	0.000	0.000	0.771	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1035	1039	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.352	0.000	0.000	0.000	0.000	0.000	0.000	0.489	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	679	0	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.231	0.000	0.000	0.000	0.000	0.000	0.000	0.326	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	344	0	0	0	0	0	0	0	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.253	0.000	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	193	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	8.04	1.00
time (sec)	N/A	0.381	0.190	0.093	0.165	0.113	0.000	1.728	0.285	25.506

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	773	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	32.21	1.00
time (sec)	N/A	0.383	0.180	0.092	0.159	0.120	0.000	1.646	0.239	25.320

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	557	554	0	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.127	0.000	0.000	0.000	0.000	0.000	0.000	0.306	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	271	0	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.114	0.000	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	133	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	6.05	1.00
time (sec)	N/A	0.385	0.197	0.094	0.167	0.096	0.000	0.174	0.228	25.368

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	533	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	24.23	1.00
time (sec)	N/A	0.385	0.211	0.090	0.156	0.101	0.000	0.178	0.604	25.373

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	1475	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	67.05	1.00
time (sec)	N/A	0.388	0.201	0.092	0.157	0.119	0.000	0.184	0.326	25.216

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	19	0	18	578	18
N.S.	1	1.00	1.11	0.89	1.00	1.06	0.00	1.00	32.11	1.00
time (sec)	N/A	0.356	0.045	0.092	0.156	0.118	0.000	0.154	0.224	25.236

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	100	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	4.55	1.00
time (sec)	N/A	0.377	0.170	0.092	0.150	0.106	0.000	0.172	0.371	25.235

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	678	681	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.202	0.000	0.000	0.000	0.000	0.000	0.000	0.379	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	346	0	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.211	0.000	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	193	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	8.04	1.00
time (sec)	N/A	0.389	0.248	0.095	0.155	0.106	0.000	1.978	0.187	25.396

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	773	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	32.21	1.00
time (sec)	N/A	0.396	0.323	0.093	0.158	0.097	0.000	1.956	0.730	25.500

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	2087	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	86.96	1.00
time (sec)	N/A	0.398	0.241	0.092	0.154	0.149	0.000	1.957	0.437	25.350

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	31	0	20	830	20
N.S.	1	1.00	1.10	0.90	1.00	1.55	0.00	1.00	41.50	1.00
time (sec)	N/A	0.349	0.054	0.092	0.152	0.146	0.000	1.099	0.294	25.219

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	148	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	6.17	1.00
time (sec)	N/A	0.389	0.200	0.091	0.152	0.118	0.000	1.965	0.450	25.341

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	26	0	20	25	20
N.S.	1	1.00	1.10	0.90	1.00	1.30	0.00	1.00	1.25	1.00
time (sec)	N/A	0.362	1.628	0.144	0.168	0.125	0.000	0.188	0.184	25.770

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	24	0	18	23	18
N.S.	1	1.00	1.11	0.89	1.00	1.33	0.00	1.00	1.28	1.00
time (sec)	N/A	0.344	0.052	0.095	0.166	0.097	0.000	0.158	0.165	25.476

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	27	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.23	1.00
time (sec)	N/A	0.375	0.407	0.100	0.175	0.137	0.000	0.196	0.170	25.413

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	269	175	0	0	0	0	0	0	0
N.S.	1	1.01	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.050	0.433	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	554	552	325	0	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.008	1.326	0.000	0.000	0.000	0.000	0.000	0.365	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	832	836	502	0	0	0	0	0	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.921	1.431	0.000	0.000	0.000	0.000	0.000	0.466	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	38	0	22	37	22
N.S.	1	1.00	1.09	0.91	1.00	1.73	0.00	1.00	1.68	1.00
time (sec)	N/A	0.370	0.401	0.189	0.174	0.111	0.000	1.095	0.169	25.434

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	36	0	20	35	20
N.S.	1	1.00	1.10	0.90	1.00	1.80	0.00	1.00	1.75	1.00
time (sec)	N/A	0.367	0.058	0.098	0.173	0.123	0.000	0.628	0.186	25.416

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	40	0	24	39	24
N.S.	1	1.00	1.08	0.92	1.00	1.67	0.00	1.00	1.62	1.00
time (sec)	N/A	0.395	0.249	0.099	0.178	0.131	0.000	1.080	0.161	25.331

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	342	344	0	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.277	0.000	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	673	679	0	0	0	0	0	0	0	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.206	0.000	0.000	0.000	0.000	0.000	0.000	0.561	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1036	1039	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.388	0.000	0.000	0.000	0.000	0.000	0.000	0.644	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	27	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.23	1.00
time (sec)	N/A	0.376	0.403	0.141	0.174	0.164	0.000	0.243	0.160	25.374

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	27	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.23	1.00
time (sec)	N/A	0.376	0.227	0.095	0.177	0.095	0.000	0.239	0.178	25.126

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	26	0	20	25	20
N.S.	1	1.00	1.10	0.90	1.00	1.30	0.00	1.00	1.25	1.00
time (sec)	N/A	0.368	0.268	0.095	0.176	0.129	0.000	0.233	0.156	25.265

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	24	0	18	23	18
N.S.	1	1.00	1.11	0.89	1.00	1.33	0.00	1.00	1.28	1.00
time (sec)	N/A	0.348	0.052	0.094	0.170	0.108	0.000	0.187	0.162	25.192

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	27	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.23	1.00
time (sec)	N/A	0.373	0.222	0.102	0.175	0.151	0.000	0.243	0.165	25.254

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	116	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	5.27	1.00
time (sec)	N/A	0.373	0.188	0.096	0.179	0.104	0.000	0.254	0.199	25.247

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	42	0	24	39	24
N.S.	1	1.00	1.08	0.92	1.00	1.75	0.00	1.00	1.62	1.00
time (sec)	N/A	0.382	0.478	0.190	0.180	0.108	0.000	1.731	0.191	25.449

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	42	0	24	39	24
N.S.	1	1.00	1.08	0.92	1.00	1.75	0.00	1.00	1.62	1.00
time (sec)	N/A	0.380	0.272	0.096	0.185	0.138	0.000	1.754	0.260	25.257

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	40	0	22	37	22
N.S.	1	1.00	1.09	0.91	1.00	1.82	0.00	1.00	1.68	1.00
time (sec)	N/A	0.368	0.315	0.096	0.172	0.097	0.000	1.785	0.169	25.361

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	38	0	20	35	20
N.S.	1	1.00	1.10	0.90	1.00	1.90	0.00	1.00	1.75	1.00
time (sec)	N/A	0.348	0.060	0.096	0.172	0.169	0.000	0.959	0.169	25.466

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	42	0	24	39	24
N.S.	1	1.00	1.08	0.92	1.00	1.75	0.00	1.00	1.62	1.00
time (sec)	N/A	0.380	0.249	0.100	0.182	0.108	0.000	1.746	0.179	25.941

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	42	0	24	164	24
N.S.	1	1.00	1.08	0.92	1.00	1.75	0.00	1.00	6.83	1.00
time (sec)	N/A	0.389	0.222	0.095	0.181	0.092	0.000	1.729	0.223	25.863

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	474	609	373	0	754	1196	0	556	360	0
N.S.	1	1.28	0.79	0.00	1.59	2.52	0.00	1.17	0.76	0.00
time (sec)	N/A	1.566	0.394	0.000	0.131	0.121	0.000	0.154	0.164	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	581	332	0	731	1162	0	448	368	0
N.S.	1	1.29	0.74	0.00	1.63	2.59	0.00	1.00	0.82	0.00
time (sec)	N/A	1.466	0.504	0.000	0.131	0.117	0.000	0.170	0.172	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	431	575	271	0	707	1236	0	455	365	0
N.S.	1	1.33	0.63	0.00	1.64	2.87	0.00	1.06	0.85	0.00
time (sec)	N/A	1.413	0.454	0.000	0.133	0.130	0.000	0.168	0.163	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	460	608	235	0	723	1348	0	494	400	0
N.S.	1	1.32	0.51	0.00	1.57	2.93	0.00	1.07	0.87	0.00
time (sec)	N/A	1.463	0.257	0.000	0.137	0.160	0.000	0.177	0.175	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	481	629	100	0	739	1369	0	507	410	0
N.S.	1	1.31	0.21	0.00	1.54	2.85	0.00	1.05	0.85	0.00
time (sec)	N/A	1.560	0.078	0.000	0.140	0.149	0.000	0.168	0.175	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	760	976	628	0	1165	2178	0	900	606	0
N.S.	1	1.28	0.83	0.00	1.53	2.87	0.00	1.18	0.80	0.00
time (sec)	N/A	2.118	0.911	0.000	0.139	0.139	0.000	0.179	0.236	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	709	922	478	0	1119	2118	0	577	590	0
N.S.	1	1.30	0.67	0.00	1.58	2.99	0.00	0.81	0.83	0.00
time (sec)	N/A	2.068	0.922	0.000	0.141	0.167	0.000	0.210	0.245	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F(-2)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	692	904	527	0	1102	2112	0	631	595	0
N.S.	1	1.31	0.76	0.00	1.59	3.05	0.00	0.91	0.86	0.00
time (sec)	N/A	2.095	1.206	0.000	0.138	0.192	0.000	0.201	0.172	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	693	913	340	0	1088	2205	0	675	623	0
N.S.	1	1.32	0.49	0.00	1.57	3.18	0.00	0.97	0.90	0.00
time (sec)	N/A	2.033	1.077	0.000	0.137	0.257	0.000	0.203	0.189	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	723	947	294	0	1110	2283	0	688	634	0
N.S.	1	1.31	0.41	0.00	1.54	3.16	0.00	0.95	0.88	0.00
time (sec)	N/A	2.177	0.287	0.000	0.137	0.216	0.000	0.197	0.181	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1601	1677	1506	0	0	0	0	0	354	0
N.S.	1	1.05	0.94	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	4.754	1.515	0.000	0.000	0.000	0.000	0.000	0.340	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1361	1261	1297	0	0	0	0	0	64	0
N.S.	1	0.93	0.95	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	3.127	0.450	0.000	0.000	0.000	0.000	0.000	0.310	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1580	1658	1336	0	0	0	0	0	421	0
N.S.	1	1.05	0.85	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	4.180	1.733	0.000	0.000	0.000	0.000	0.000	0.478	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	148	0	35	0	0	20	0
N.S.	1	1.00	1.00	4.48	0.00	1.06	0.00	0.00	0.61	0.00
time (sec)	N/A	0.401	0.016	5.630	0.000	0.108	0.000	0.000	0.145	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	220	496	0	105	0	0	26	0
N.S.	1	1.00	2.93	6.61	0.00	1.40	0.00	0.00	0.35	0.00
time (sec)	N/A	0.665	0.285	2.611	0.000	0.083	0.000	0.000	0.146	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	179	659	0	0	417	0	0	46	0
N.S.	1	1.11	4.09	0.00	0.00	2.59	0.00	0.00	0.29	0.00
time (sec)	N/A	1.165	0.335	0.000	0.000	0.142	0.000	0.000	0.181	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	149	456	0	0	281	0	0	46	0
N.S.	1	1.13	3.45	0.00	0.00	2.13	0.00	0.00	0.35	0.00
time (sec)	N/A	0.959	0.280	0.000	0.000	0.128	0.000	0.000	0.152	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	118	265	0	0	161	0	0	44	0
N.S.	1	1.16	2.60	0.00	0.00	1.58	0.00	0.00	0.43	0.00
time (sec)	N/A	0.858	0.250	0.000	0.000	0.090	0.000	0.000	0.151	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	47	49	180	0	69	0	0	65	0
N.S.	1	0.96	1.00	3.67	0.00	1.41	0.00	0.00	1.33	0.00
time (sec)	N/A	0.428	0.016	4.779	0.000	0.097	0.000	0.000	0.162	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	44	30	24	30	43	30
N.S.	1	1.00	1.07	1.00	1.57	1.07	0.86	1.07	1.54	1.07
time (sec)	N/A	0.713	0.658	0.128	0.194	0.090	46.857	0.125	0.154	25.944

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	91	30	0	30	89	30
N.S.	1	1.00	1.07	1.00	3.25	1.07	0.00	1.07	3.18	1.07
time (sec)	N/A	0.523	3.402	0.128	0.140	0.117	0.000	0.123	0.221	26.014

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	238	30	0	30	79	30
N.S.	1	1.00	1.07	1.00	8.50	1.07	0.00	1.07	2.82	1.07
time (sec)	N/A	0.507	13.935	0.129	0.151	0.094	0.000	0.132	0.228	26.116

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	113	0	0	0	0	0	17	0
N.S.	1	0.00	1.55	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.000	0.263	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	135	0	0	0	0	0	19	0
N.S.	1	0.00	1.80	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.000	0.236	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	192	0	0	0	0	0	21	0
N.S.	1	0.00	1.79	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.522	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	0	118	0	0	0	0	0	19	0
N.S.	1	0.00	1.12	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.235	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	136	0	0	0	0	0	19	0
N.S.	1	0.00	1.60	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.205	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	0	113	0	0	0	0	0	17	0
N.S.	1	0.00	1.36	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.195	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	0	120	0	0	0	0	0	20	0
N.S.	1	0.00	0.78	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.329	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	0	0	0	0	0	0	0	22	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	0	62	76	118	55	70	128	76	0
N.S.	1	0.00	0.42	0.52	0.81	0.38	0.48	0.88	0.52	0.00
time (sec)	N/A	0.000	0.062	1.031	0.114	0.084	130.483	0.120	0.204	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	49	66	90	45	56	92	58	0
N.S.	1	0.00	0.44	0.59	0.81	0.41	0.50	0.83	0.52	0.00
time (sec)	N/A	0.000	0.047	1.012	0.108	0.101	37.370	0.123	0.160	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	38	52	61	33	42	54	40	0
N.S.	1	0.00	0.52	0.71	0.84	0.45	0.58	0.74	0.55	0.00
time (sec)	N/A	0.000	0.035	1.037	0.108	0.092	9.243	0.124	0.147	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	23	32	31	21	24	31	21	21
N.S.	1	0.00	0.66	0.91	0.89	0.60	0.69	0.89	0.60	0.60
time (sec)	N/A	0.000	0.011	1.876	0.026	0.083	2.478	0.112	0.166	25.250

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	0	85	0	0	0	0	0	24	0
N.S.	1	0.00	1.60	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.000	0.036	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	0	144	0	0	0	0	0	24	0
N.S.	1	0.00	1.45	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.000	0.098	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	0	185	0	0	0	0	0	24	0
N.S.	1	0.00	1.29	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.000	0.146	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	374	0	193	0	0	0	0	0	24	0
N.S.	1	0.00	0.52	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	0.412	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	192	0	0	0	0	0	21	0
N.S.	1	0.00	1.79	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.137	0.000	0.000	0.000	0.000	0.000	0.148	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	29	0	27	43	0	47	60	0
N.S.	1	0.00	1.00	0.00	0.93	1.48	0.00	1.62	2.07	0.00
time (sec)	N/A	0.000	0.035	0.000	0.108	0.111	0.000	0.131	0.225	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	0	47	0	57	60	0	119	129	0
N.S.	1	0.00	0.61	0.00	0.74	0.78	0.00	1.55	1.68	0.00
time (sec)	N/A	0.000	0.057	0.000	0.106	0.089	0.000	0.131	0.234	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	0	58	0	86	70	0	195	195	0
N.S.	1	0.00	0.48	0.00	0.71	0.58	0.00	1.61	1.61	0.00
time (sec)	N/A	0.000	0.064	0.000	0.106	0.080	0.000	0.134	0.172	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F(-1)	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	0	67	0	114	80	0	267	261	0
N.S.	1	0.00	0.41	0.00	0.69	0.48	0.00	1.62	1.58	0.00
time (sec)	N/A	0.000	0.076	0.000	0.111	0.114	0.000	0.137	0.157	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	0	60	62	99	53	139	84	74	0
N.S.	1	0.00	0.47	0.48	0.77	0.41	1.08	0.65	0.57	0.00
time (sec)	N/A	0.000	0.053	1.016	0.137	0.075	8.350	0.124	0.175	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	50	52	75	43	102	66	56	0
N.S.	1	0.00	0.51	0.53	0.77	0.44	1.04	0.67	0.57	0.00
time (sec)	N/A	0.000	0.039	1.023	0.129	0.070	2.806	0.120	0.150	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	0	36	38	50	31	65	46	38	0
N.S.	1	0.00	0.56	0.59	0.78	0.48	1.02	0.72	0.59	0.00
time (sec)	N/A	0.000	0.030	1.008	0.112	0.076	0.902	0.113	0.164	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	20	27	26	18	24	26	17	18
N.S.	1	0.00	0.67	0.90	0.87	0.60	0.80	0.87	0.57	0.60
time (sec)	N/A	0.000	0.009	0.940	0.106	0.104	0.297	0.119	0.166	25.662

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	0	105	0	0	0	0	0	22	0
N.S.	1	0.00	1.81	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	0	159	0	0	0	0	0	22	0
N.S.	1	0.00	1.57	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.101	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	0	195	0	0	0	0	0	22	0
N.S.	1	0.00	1.38	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.000	0.168	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	0	238	0	0	0	0	0	22	0
N.S.	1	0.00	0.85	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.623	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	189	0	0	0	0	0	22	0
N.S.	1	0.00	0.97	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	0.427	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	136	0	0	0	0	0	19	0
N.S.	1	0.00	1.60	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.000	0.050	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	36	0	34	36	37	57	56	0
N.S.	1	0.00	1.00	0.00	0.94	1.00	1.03	1.58	1.56	0.00
time (sec)	N/A	0.000	0.038	0.000	0.103	0.077	3.993	0.134	0.190	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	64	0	66	60	0	94	100	0
N.S.	1	0.00	0.79	0.00	0.81	0.74	0.00	1.16	1.23	0.00
time (sec)	N/A	0.000	0.045	0.000	0.112	0.077	0.000	0.138	0.160	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	0	78	0	101	70	0	141	130	0
N.S.	1	0.00	0.64	0.00	0.83	0.58	0.00	1.17	1.07	0.00
time (sec)	N/A	0.000	0.063	0.000	0.113	0.078	0.000	0.138	0.170	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	0	87	0	140	80	0	186	160	0
N.S.	1	0.00	0.54	0.00	0.87	0.50	0.00	1.16	0.99	0.00
time (sec)	N/A	0.000	0.069	0.000	0.113	0.080	0.000	0.145	0.182	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	0	115	0	206	154	333	225	318	0
N.S.	1	0.00	0.52	0.00	0.92	0.69	1.49	1.01	1.43	0.00
time (sec)	N/A	0.000	0.133	0.000	0.043	0.103	15.285	0.123	0.164	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	91	0	161	116	250	175	236	0
N.S.	1	0.00	0.54	0.00	0.96	0.69	1.49	1.04	1.40	0.00
time (sec)	N/A	0.000	0.093	0.000	0.047	0.090	4.995	0.124	0.162	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	0	64	0	112	75	167	125	154	0
N.S.	1	0.00	0.58	0.00	1.01	0.68	1.50	1.13	1.39	0.00
time (sec)	N/A	0.000	0.075	0.000	0.038	0.094	1.599	0.125	0.161	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	0	37	0	60	37	78	76	74	0
N.S.	1	0.00	0.70	0.00	1.13	0.70	1.47	1.43	1.40	0.00
time (sec)	N/A	0.000	0.019	0.000	0.034	0.087	0.571	0.120	0.155	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	0	99	0	0	0	0	0	76	0
N.S.	1	0.00	1.15	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.000	0.092	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	0	260	0	0	0	0	0	103	0
N.S.	1	0.00	1.74	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.000	0.162	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	0	347	0	0	0	0	0	123	0
N.S.	1	0.00	1.61	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.000	0.236	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	0	548	0	0	0	0	0	77	0
N.S.	1	0.00	1.43	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	2.746	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	0	373	0	0	0	0	0	57	0
N.S.	1	0.00	2.14	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.000	0.564	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	F(-2)	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	0	59	0	0	142	0	0	185	0
N.S.	1	0.00	0.98	0.00	0.00	2.37	0.00	0.00	3.08	0.00
time (sec)	N/A	0.000	0.139	0.000	0.000	0.098	0.000	0.000	0.160	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	F(-2)	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	81	0	0	219	0	0	307	0
N.S.	1	0.00	0.68	0.00	0.00	1.84	0.00	0.00	2.58	0.00
time (sec)	N/A	0.000	0.188	0.000	0.000	0.102	0.000	0.000	0.186	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	F(-2)	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	0	107	0	0	297	0	0	426	0
N.S.	1	0.00	0.59	0.00	0.00	1.63	0.00	0.00	2.34	0.00
time (sec)	N/A	0.000	0.224	0.000	0.000	0.101	0.000	0.000	0.293	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	F(-2)	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	0	130	0	0	373	0	0	536	0
N.S.	1	0.00	0.53	0.00	0.00	1.52	0.00	0.00	2.19	0.00
time (sec)	N/A	0.000	0.255	0.000	0.000	0.112	0.000	0.000	0.181	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F(-1)	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	235	0	117	0	219	156	0	314	207	0
N.S.	1	0.00	0.50	0.00	0.93	0.66	0.00	1.34	0.88	0.00
time (sec)	N/A	0.000	0.154	0.000	0.128	0.081	0.000	0.127	0.149	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F(-1)	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	0	93	0	171	118	0	230	153	0
N.S.	1	0.00	0.53	0.00	0.97	0.67	0.00	1.30	0.86	0.00
time (sec)	N/A	0.000	0.118	0.000	0.116	0.105	0.000	0.123	0.162	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F(-1)	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	0	66	0	119	77	0	134	99	0
N.S.	1	0.00	0.56	0.00	1.02	0.66	0.00	1.15	0.85	0.00
time (sec)	N/A	0.000	0.089	0.000	0.113	0.113	0.000	0.122	0.162	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	40	0	66	40	0	83	50	0
N.S.	1	0.00	0.70	0.00	1.16	0.70	0.00	1.46	0.88	0.00
time (sec)	N/A	0.000	0.021	0.000	0.134	0.092	0.000	0.122	0.154	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	0	104	0	0	0	0	0	49	0
N.S.	1	0.00	1.16	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.000	0.108	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	0	273	0	0	0	0	0	73	0
N.S.	1	0.00	1.75	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.000	0.185	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	0	364	0	0	0	0	0	92	0
N.S.	1	0.00	1.61	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.000	0.287	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	568	0	477	0	0	0	0	0	70	0
N.S.	1	0.00	0.84	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	1.589	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	0	378	0	0	0	0	0	48	0
N.S.	1	0.00	1.97	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.000	0.490	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	F(-2)	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	0	61	0	63	139	0	0	90	0
N.S.	1	0.00	0.98	0.00	1.02	2.24	0.00	0.00	1.45	0.00
time (sec)	N/A	0.000	0.145	0.000	0.119	0.114	0.000	0.000	0.163	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	F(-2)	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	0	84	0	0	214	0	0	188	0
N.S.	1	0.00	0.68	0.00	0.00	1.73	0.00	0.00	1.52	0.00
time (sec)	N/A	0.000	0.215	0.000	0.000	0.095	0.000	0.000	0.171	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	0	108	0	0	292	0	0	293	0
N.S.	1	0.00	0.57	0.00	0.00	1.54	0.00	0.00	1.54	0.00
time (sec)	N/A	0.000	0.267	0.000	0.000	0.107	0.000	0.000	0.170	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	0	132	0	0	368	0	0	395	0
N.S.	1	0.00	0.52	0.00	0.00	1.44	0.00	0.00	1.54	0.00
time (sec)	N/A	0.000	0.349	0.000	0.000	0.129	0.000	0.000	0.168	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	0	139	0	227	182	360	253	347	0
N.S.	1	0.00	0.59	0.00	0.96	0.77	1.53	1.07	1.47	0.00
time (sec)	N/A	0.000	0.174	0.000	0.042	0.087	16.288	0.128	0.167	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	0	106	0	173	132	267	191	254	0
N.S.	1	0.00	0.61	0.00	0.99	0.75	1.53	1.09	1.45	0.00
time (sec)	N/A	0.000	0.111	0.000	0.042	0.085	5.423	0.124	0.159	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	0	69	0	115	80	173	129	161	0
N.S.	1	0.00	0.61	0.00	1.02	0.71	1.53	1.14	1.42	0.00
time (sec)	N/A	0.000	0.078	0.000	0.040	0.100	1.692	0.121	0.149	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	0	37	0	60	37	80	65	76	0
N.S.	1	0.00	0.70	0.00	1.13	0.70	1.51	1.23	1.43	0.00
time (sec)	N/A	0.000	0.022	0.000	0.038	0.077	0.531	0.117	0.157	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	0	120	0	0	0	0	0	92	0
N.S.	1	0.00	1.19	0.00	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.000	0.114	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	0	310	0	0	0	0	0	122	0
N.S.	1	0.00	1.73	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.000	0.211	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	0	412	0	0	0	0	0	145	0
N.S.	1	0.00	1.64	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.000	0.305	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	406	0	0	0	0	0	0	0	80	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	0	0	0	0	0	0	0	59	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	B	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	0	81	0	0	139	0	152	195	0
N.S.	1	0.00	1.16	0.00	0.00	1.99	0.00	2.17	2.79	0.00
time (sec)	N/A	0.000	0.147	0.000	0.000	0.088	0.000	0.455	0.168	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	B	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	0	111	0	0	229	0	263	322	0
N.S.	1	0.00	0.80	0.00	0.00	1.65	0.00	1.89	2.32	0.00
time (sec)	N/A	0.000	0.222	0.000	0.000	0.119	0.000	0.700	0.165	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	B	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	0	147	0	0	328	0	383	448	0
N.S.	1	0.00	0.71	0.00	0.00	1.58	0.00	1.84	2.15	0.00
time (sec)	N/A	0.000	0.236	0.000	0.000	0.118	0.000	0.983	0.185	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	B	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	0	182	0	0	429	0	516	568	0
N.S.	1	0.00	0.66	0.00	0.00	1.55	0.00	1.86	2.05	0.00
time (sec)	N/A	0.000	0.285	0.000	0.000	0.126	0.000	1.238	0.188	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F(-1)	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	0	141	0	240	184	0	358	188	0
N.S.	1	0.00	0.57	0.00	0.97	0.74	0.00	1.44	0.76	0.00
time (sec)	N/A	0.000	0.170	0.000	0.119	0.118	0.000	0.130	0.163	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F(-1)	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	0	108	0	183	134	0	254	135	0
N.S.	1	0.00	0.58	0.00	0.99	0.72	0.00	1.37	0.73	0.00
time (sec)	N/A	0.000	0.119	0.000	0.120	0.086	0.000	0.127	0.154	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	0	72	0	122	81	0	138	82	0
N.S.	1	0.00	0.61	0.00	1.03	0.68	0.00	1.16	0.69	0.00
time (sec)	N/A	0.000	0.090	0.000	0.120	0.086	0.000	0.128	0.168	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	A	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	0	40	0	66	40	0	71	40	0
N.S.	1	0.00	0.70	0.00	1.16	0.70	0.00	1.25	0.70	0.00
time (sec)	N/A	0.000	0.023	0.000	0.130	0.094	0.000	0.128	0.186	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	0	125	0	0	0	0	0	58	0
N.S.	1	0.00	1.19	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	0	323	0	0	0	0	0	85	0
N.S.	1	0.00	1.75	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.000	0.208	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	0	429	0	0	0	0	0	107	0
N.S.	1	0.00	1.64	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.000	0.437	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	613	0	0	0	0	0	0	0	73	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	0	0	0	0	0	0	0	51	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	A	A	F	B	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	71	0	75	156	0	178	101	0
N.S.	1	0.00	0.97	0.00	1.03	2.14	0.00	2.44	1.38	0.00
time (sec)	N/A	0.000	0.163	0.000	0.137	0.142	0.000	0.473	0.166	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	B	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	0	100	0	0	242	0	308	209	0
N.S.	1	0.00	0.69	0.00	0.00	1.67	0.00	2.12	1.44	0.00
time (sec)	N/A	0.000	0.226	0.000	0.000	0.103	0.000	0.736	0.171	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	0	134	0	0	344	0	465	326	0
N.S.	1	0.00	0.62	0.00	0.00	1.59	0.00	2.14	1.50	0.00
time (sec)	N/A	0.000	0.292	0.000	0.000	0.128	0.000	1.031	0.166	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F(-1)	B	B	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	0	171	0	0	444	0	623	443	0
N.S.	1	0.00	0.59	0.00	0.00	1.54	0.00	2.16	1.53	0.00
time (sec)	N/A	0.000	0.325	0.000	0.000	0.123	0.000	1.189	0.170	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	74	65	0	0	0	0	0	82	0
N.S.	1	0.97	0.86	0.00	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.366	0.031	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	174	147	132	0	1134	0	261	205	192
N.S.	1	1.03	0.87	0.78	0.00	6.71	0.00	1.54	1.21	1.14
time (sec)	N/A	0.628	0.124	3.126	0.000	1.083	0.000	0.188	0.168	25.873

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	70	63	105	0	206	235	92	98	82
N.S.	1	1.11	1.00	1.67	0.00	3.27	3.73	1.46	1.56	1.30
time (sec)	N/A	0.359	0.044	1.812	0.000	0.097	119.943	0.123	0.155	25.671

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	47	47	54	46	80	64	65	46
N.S.	1	1.26	1.34	1.34	1.54	1.31	2.29	1.83	1.86	1.31
time (sec)	N/A	0.336	0.039	1.298	0.027	0.075	0.293	0.120	0.152	25.702

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	43	56	65	65	65	102	171	95	67
N.S.	1	0.96	1.24	1.44	1.44	1.44	2.27	3.80	2.11	1.49
time (sec)	N/A	0.340	0.059	0.705	0.034	0.095	0.524	0.129	0.149	0.192

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	79	111	0	287	0	139	132	163
N.S.	1	0.97	1.34	1.88	0.00	4.86	0.00	2.36	2.24	2.76
time (sec)	N/A	0.361	0.153	1.882	0.000	0.092	0.000	0.325	0.152	26.263

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	158	66	138	0	1169	0	319	304	499
N.S.	1	0.96	0.40	0.84	0.00	7.08	0.00	1.93	1.84	3.02
time (sec)	N/A	0.597	0.604	4.129	0.000	0.996	0.000	0.457	0.176	26.116

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	30	19	24	251	24
N.S.	1	1.00	1.09	1.00	1.09	1.36	0.86	1.09	11.41	1.09
time (sec)	N/A	0.279	0.508	0.152	1.135	0.086	15.630	0.208	0.188	26.289

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	196	743	0	0	0	0	0	433	0
N.S.	1	0.89	3.36	0.00	0.00	0.00	0.00	0.00	1.96	0.00
time (sec)	N/A	1.206	1.466	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	150	441	0	0	0	0	0	325	0
N.S.	1	0.89	2.62	0.00	0.00	0.00	0.00	0.00	1.93	0.00
time (sec)	N/A	0.975	0.751	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	103	219	0	0	0	0	0	213	0
N.S.	1	0.90	1.90	0.00	0.00	0.00	0.00	0.00	1.85	0.00
time (sec)	N/A	0.596	0.262	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	70	77	70	76	107	176	104	61
N.S.	1	1.00	1.40	1.54	1.40	1.52	2.14	3.52	2.08	1.22
time (sec)	N/A	0.342	0.055	0.431	0.034	0.082	0.516	0.127	0.226	25.383

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	30	19	24	661	24
N.S.	1	1.00	1.09	1.00	1.09	1.36	0.86	1.09	30.05	1.09
time (sec)	N/A	0.272	0.490	0.136	0.175	0.074	0.635	0.788	0.186	26.007

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	148	62	20	24	2449	24
N.S.	1	1.00	1.09	1.00	6.73	2.82	0.91	1.09	111.32	1.09
time (sec)	N/A	0.281	0.658	0.154	0.235	0.099	3.491	0.265	0.222	25.814

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [503] had the largest ratio of [1.22727000000000008]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.01	16	0.188
2	A	5	4	1.10	16	0.250
3	A	3	3	1.03	16	0.188
4	A	4	3	1.09	14	0.214
5	A	3	3	1.11	12	0.250
6	A	4	3	0.95	16	0.188
7	A	2	2	1.00	16	0.125
8	A	6	5	1.24	16	0.312
9	A	3	3	1.00	16	0.188
10	A	5	4	0.97	16	0.250
11	A	4	4	1.04	16	0.250
12	A	5	4	0.97	16	0.250
13	A	5	4	1.10	16	0.250
14	A	3	3	1.02	16	0.188
15	A	3	3	1.01	16	0.188
16	A	4	3	1.09	16	0.188
17	A	11	10	1.08	14	0.714
18	A	11	10	1.07	12	0.833
19	A	4	3	0.95	16	0.188
20	A	10	9	1.04	16	0.562
21	A	10	9	0.99	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	5	1.04	16	0.312
23	A	11	10	1.04	16	0.625
24	A	11	10	1.02	16	0.625
25	A	5	4	0.97	16	0.250
26	A	4	4	0.93	16	0.250
27	A	4	4	0.93	16	0.250
28	A	4	4	0.93	16	0.250
29	A	4	4	0.94	14	0.286
30	A	3	3	1.00	12	0.250
31	A	4	3	1.00	16	0.188
32	A	4	3	1.20	16	0.188
33	A	5	4	1.03	16	0.250
34	A	5	4	1.01	16	0.250
35	A	5	4	1.00	16	0.250
36	A	4	4	0.94	16	0.250
37	A	6	5	0.94	16	0.312
38	A	4	4	0.98	16	0.250
39	A	3	3	1.00	14	0.214
40	A	3	3	1.00	12	0.250
41	A	4	3	1.00	16	0.188
42	A	4	4	1.12	16	0.250
43	A	4	3	1.09	16	0.188
44	A	5	5	1.13	16	0.312
45	A	1	1	1.00	12	0.083
46	A	5	4	0.99	18	0.222
47	A	5	4	1.00	18	0.222
48	A	5	4	1.02	16	0.250
49	A	5	4	1.09	14	0.286
50	A	4	3	1.00	18	0.167
51	A	5	4	1.08	18	0.222
52	A	5	4	0.98	18	0.222
53	A	5	4	0.97	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	3	1.16	16	0.188
55	A	3	3	1.00	18	0.167
56	A	3	3	1.00	18	0.167
57	A	2	2	1.00	16	0.125
58	A	5	4	1.00	18	0.222
59	A	5	4	1.04	18	0.222
60	A	5	4	1.02	18	0.222
61	A	5	4	1.12	20	0.200
62	A	6	5	0.88	20	0.250
63	A	3	3	1.00	18	0.167
64	A	6	5	0.75	22	0.227
65	A	6	5	0.81	22	0.227
66	A	6	5	0.97	20	0.250
67	A	5	4	0.90	19	0.211
68	A	7	6	0.86	22	0.273
69	A	6	5	0.73	22	0.227
70	A	2	2	1.00	16	0.125
71	A	2	2	1.00	14	0.143
72	A	2	2	1.00	12	0.167
73	A	4	3	0.95	16	0.188
74	A	2	2	1.00	16	0.125
75	A	2	2	1.00	16	0.125
76	A	2	2	1.00	16	0.125
77	A	8	7	0.79	18	0.389
78	A	4	3	1.02	18	0.167
79	A	5	4	1.05	16	0.250
80	A	6	5	1.06	18	0.278
81	A	5	4	0.99	18	0.222
82	A	10	9	0.95	18	0.500
83	A	15	14	1.03	18	0.778
84	A	3	3	0.95	18	0.167
85	A	3	3	0.96	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	0.99	14	0.214
87	A	10	9	1.01	18	0.500
88	A	3	3	0.94	18	0.167
89	A	3	3	0.92	18	0.167
90	A	3	3	0.93	18	0.167
91	A	4	3	0.99	18	0.167
92	A	4	3	1.00	18	0.167
93	A	6	5	0.97	16	0.312
94	A	7	6	1.03	18	0.333
95	A	7	6	0.95	18	0.333
96	A	12	11	0.88	18	0.611
97	A	17	16	0.91	18	0.889
98	N/A	3	0	1.00	18	0.000
99	N/A	3	0	1.00	14	0.000
100	N/A	2	0	1.00	18	0.000
101	N/A	3	0	1.00	18	0.000
102	A	4	3	0.99	18	0.167
103	A	5	4	1.00	16	0.250
104	N/A	1	0	1.00	18	0.000
105	N/A	1	0	1.00	18	0.000
106	N/A	1	0	1.00	18	0.000
107	N/A	1	0	1.00	14	0.000
108	N/A	1	0	1.00	18	0.000
109	A	8	7	1.40	18	0.389
110	A	6	5	0.95	16	0.312
111	N/A	1	0	1.00	18	0.000
112	N/A	1	0	1.00	18	0.000
113	N/A	1	0	1.00	18	0.000
114	N/A	1	0	1.00	14	0.000
115	N/A	1	0	1.00	18	0.000
116	A	13	12	1.54	18	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
117	A	7	6	1.01	16	0.375
118	N/A	1	0	1.00	18	0.000
119	N/A	1	0	1.00	18	0.000
120	N/A	1	0	1.00	18	0.000
121	N/A	1	0	1.00	14	0.000
122	N/A	1	0	1.00	18	0.000
123	A	4	3	0.98	16	0.188
124	A	4	3	1.00	14	0.214
125	A	7	6	1.30	16	0.375
126	A	5	4	0.91	14	0.286
127	A	11	10	1.30	16	0.625
128	A	6	5	0.97	14	0.357
129	A	4	3	0.99	18	0.167
130	A	5	4	0.97	18	0.222
131	A	6	5	0.99	18	0.278
132	A	5	4	0.92	18	0.222
133	A	3	3	1.01	16	0.188
134	A	3	3	1.01	14	0.214
135	A	3	3	1.01	18	0.167
136	A	3	3	1.01	18	0.167
137	A	3	3	0.97	18	0.167
138	A	4	3	0.98	18	0.167
139	A	4	3	0.99	18	0.167
140	A	5	4	1.00	18	0.222
141	N/A	1	0	1.00	18	0.000
142	N/A	1	0	1.00	18	0.000
143	N/A	1	0	1.00	18	0.000
144	N/A	1	0	1.00	16	0.000
145	N/A	1	0	1.00	14	0.000
146	N/A	1	0	1.00	18	0.000
147	N/A	1	0	1.00	18	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
148	A	5	4	1.58	18	0.222
149	A	8	7	1.37	18	0.389
150	A	6	5	0.95	18	0.278
151	N/A	1	0	1.00	18	0.000
152	N/A	1	0	1.00	18	0.000
153	N/A	1	0	1.00	18	0.000
154	N/A	1	0	1.00	16	0.000
155	N/A	1	0	1.00	14	0.000
156	N/A	1	0	1.00	18	0.000
157	N/A	1	0	1.00	18	0.000
158	N/A	2	0	1.00	20	0.000
159	N/A	2	0	1.00	20	0.000
160	A	3	3	1.00	18	0.167
161	N/A	1	0	1.00	20	0.000
162	N/A	1	0	1.00	20	0.000
163	A	9	8	0.53	24	0.333
164	A	5	4	0.64	24	0.167
165	A	6	5	0.77	22	0.227
166	A	7	6	0.90	21	0.286
167	A	6	5	0.77	24	0.208
168	A	11	10	0.73	24	0.417
169	A	1	1	1.00	12	0.083
170	A	4	3	1.14	12	0.250
171	A	4	3	1.14	14	0.214
172	A	4	3	0.93	14	0.214
173	A	4	3	0.95	16	0.188
174	A	6	5	0.96	18	0.278
175	A	7	6	0.96	18	0.333
176	A	3	3	0.94	18	0.167
177	A	3	3	0.96	18	0.167
178	A	3	3	0.99	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
179	A	3	2	1.21	10	0.200
180	A	4	3	1.00	18	0.167
181	A	3	3	0.96	18	0.167
182	A	3	3	0.86	18	0.167
183	A	3	3	0.87	18	0.167
184	A	4	4	1.06	20	0.200
185	A	4	4	1.07	20	0.200
186	A	4	4	1.13	18	0.222
187	A	3	3	1.11	12	0.250
188	A	3	3	1.06	20	0.150
189	A	6	6	0.91	20	0.300
190	A	5	5	1.04	20	0.250
191	A	7	7	1.02	20	0.350
192	A	5	5	1.05	20	0.250
193	A	3	3	1.06	18	0.167
194	A	11	10	1.07	12	0.833
195	A	3	3	1.05	20	0.150
196	A	3	3	1.01	20	0.150
197	A	3	3	0.97	20	0.150
198	A	4	4	0.97	20	0.200
199	A	4	4	0.98	20	0.200
200	A	4	4	1.00	18	0.222
201	A	4	4	1.03	20	0.200
202	A	4	4	1.05	20	0.200
203	A	4	4	0.97	20	0.200
204	A	4	4	0.97	20	0.200
205	A	4	4	1.06	16	0.250
206	A	3	3	0.98	20	0.150
207	A	3	3	1.00	20	0.150
208	A	2	2	1.00	18	0.111
209	A	5	5	1.00	20	0.250
210	A	4	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
211	N/A	1	0	1.00	20	0.000
212	A	3	3	0.98	20	0.150
213	A	3	3	1.01	20	0.150
214	A	3	3	1.03	18	0.167
215	A	2	2	1.00	12	0.167
216	N/A	1	0	1.00	20	0.000
217	N/A	1	0	1.00	20	0.000
218	N/A	1	0	1.00	20	0.000
219	A	2	2	1.00	21	0.095
220	A	2	2	1.00	21	0.095
221	A	2	2	1.00	19	0.105
222	A	4	3	1.00	18	0.167
223	A	2	2	1.00	21	0.095
224	A	2	2	1.00	21	0.095
225	A	2	2	1.00	21	0.095
226	A	2	2	1.00	23	0.087
227	A	2	2	1.00	23	0.087
228	A	2	2	1.00	21	0.095
229	A	3	3	1.06	20	0.150
230	A	2	2	1.00	23	0.087
231	A	2	2	1.00	23	0.087
232	A	2	2	1.00	23	0.087
233	A	2	2	1.00	23	0.087
234	A	2	2	1.00	23	0.087
235	A	2	2	1.00	21	0.095
236	A	3	3	1.05	20	0.150
237	A	2	2	1.00	23	0.087
238	A	2	2	1.00	23	0.087
239	A	2	2	1.00	23	0.087
240	A	2	2	1.00	23	0.087
241	A	2	2	1.00	23	0.087
242	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
243	A	4	4	1.03	20	0.200
244	A	2	2	1.00	23	0.087
245	A	2	2	1.00	23	0.087
246	A	2	2	1.00	23	0.087
247	A	2	2	1.00	23	0.087
248	A	2	2	1.00	23	0.087
249	A	2	2	1.00	21	0.095
250	A	4	4	1.03	20	0.200
251	A	2	2	1.00	23	0.087
252	A	2	2	1.00	23	0.087
253	A	2	2	1.00	23	0.087
254	A	2	2	1.00	23	0.087
255	A	2	2	1.00	23	0.087
256	A	2	2	1.00	21	0.095
257	A	4	4	1.03	20	0.200
258	A	2	2	1.00	23	0.087
259	A	2	2	1.00	23	0.087
260	A	2	2	1.00	23	0.087
261	A	4	4	0.94	22	0.182
262	A	4	4	0.95	22	0.182
263	A	2	2	1.00	20	0.100
264	A	5	5	0.91	22	0.227
265	A	5	5	0.93	22	0.227
266	A	4	3	1.00	24	0.125
267	A	6	5	1.00	24	0.208
268	A	2	2	1.00	22	0.091
269	A	2	2	1.00	22	0.091
270	A	2	2	1.00	20	0.100
271	A	4	4	0.95	22	0.182
272	A	2	2	1.00	22	0.091
273	A	2	2	1.00	24	0.083
274	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
275	N/A	1	0	1.00	24	0.000
276	N/A	1	0	1.00	24	0.000
277	N/A	2	0	1.00	22	0.000
278	N/A	1	0	1.00	24	0.000
279	N/A	1	0	1.00	24	0.000
280	N/A	1	0	1.00	24	0.000
281	N/A	1	0	1.00	22	0.000
282	N/A	1	0	1.00	24	0.000
283	N/A	1	0	1.00	24	0.000
284	N/A	1	0	1.00	24	0.000
285	N/A	1	0	1.00	22	0.000
286	N/A	1	0	1.00	24	0.000
287	N/A	1	0	1.00	24	0.000
288	A	2	2	1.00	22	0.091
289	A	2	2	1.00	22	0.091
290	A	2	2	1.00	20	0.100
291	A	2	2	1.00	22	0.091
292	A	2	2	1.00	22	0.091
293	A	2	2	1.00	24	0.083
294	A	2	2	1.00	24	0.083
295	A	2	2	1.00	22	0.091
296	N/A	1	0	1.00	24	0.000
297	N/A	1	0	1.00	24	0.000
298	N/A	2	0	1.00	24	0.000
299	N/A	2	0	1.00	22	0.000
300	N/A	1	0	1.00	24	0.000
301	N/A	1	0	1.00	24	0.000
302	N/A	1	0	1.00	24	0.000
303	N/A	1	0	1.00	22	0.000
304	N/A	1	0	1.00	24	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	N/A	1	0	1.00	24	0.000
306	N/A	1	0	1.00	24	0.000
307	N/A	1	0	1.00	22	0.000
308	N/A	1	0	1.00	24	0.000
309	N/A	1	0	1.00	24	0.000
310	A	6	5	1.03	23	0.217
311	A	6	5	1.04	23	0.217
312	A	5	4	1.03	21	0.190
313	A	4	3	0.91	23	0.130
314	A	4	3	0.96	23	0.130
315	A	6	5	1.08	23	0.217
316	A	6	5	0.94	23	0.217
317	A	6	5	0.97	23	0.217
318	A	2	2	1.00	23	0.087
319	A	2	2	1.00	20	0.100
320	A	2	2	1.29	23	0.087
321	A	2	2	1.00	23	0.087
322	A	2	2	1.00	23	0.087
323	A	6	5	1.00	25	0.200
324	A	6	5	1.00	25	0.200
325	A	5	4	0.99	23	0.174
326	A	4	3	0.99	25	0.120
327	A	4	3	0.99	25	0.120
328	A	4	3	1.01	25	0.120
329	A	6	5	1.03	25	0.200
330	A	6	5	0.96	25	0.200
331	A	6	5	0.96	25	0.200
332	A	2	2	1.00	25	0.080
333	A	2	2	1.00	22	0.091
334	A	2	2	1.00	25	0.080
335	A	2	2	1.00	25	0.080
336	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
337	A	2	2	1.00	25	0.080
338	A	4	3	0.96	25	0.120
339	A	4	3	0.96	25	0.120
340	A	5	4	0.97	23	0.174
341	A	4	3	0.95	25	0.120
342	A	4	3	0.95	25	0.120
343	A	2	2	1.00	25	0.080
344	A	2	2	1.00	25	0.080
345	A	4	4	0.95	22	0.182
346	A	2	2	1.00	25	0.080
347	A	2	2	1.00	25	0.080
348	A	4	3	0.96	25	0.120
349	A	4	3	0.94	25	0.120
350	A	5	4	0.93	23	0.174
351	A	4	3	0.93	25	0.120
352	A	4	3	0.99	25	0.120
353	A	2	2	1.00	25	0.080
354	A	2	2	1.00	25	0.080
355	A	2	2	1.00	22	0.091
356	A	2	2	1.00	25	0.080
357	A	9	8	1.03	22	0.364
358	A	4	4	0.93	18	0.222
359	A	3	3	1.13	18	0.167
360	A	4	3	0.88	25	0.120
361	A	4	3	0.89	25	0.120
362	A	4	3	0.90	23	0.130
363	A	5	4	0.94	25	0.160
364	A	5	4	0.93	25	0.160
365	A	4	3	0.87	27	0.111
366	A	4	3	0.87	27	0.111
367	A	4	3	0.89	25	0.120
368	A	5	4	0.93	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
369	A	5	4	0.91	27	0.148
370	A	4	3	0.95	27	0.111
371	A	4	3	0.93	25	0.120
372	A	6	5	0.97	27	0.185
373	A	5	4	0.96	27	0.148
374	A	4	3	0.94	27	0.111
375	A	4	3	0.92	25	0.120
376	A	5	4	0.93	27	0.148
377	A	5	4	0.95	27	0.148
378	A	6	5	1.04	33	0.152
379	A	5	4	1.04	29	0.138
380	A	5	4	1.04	33	0.121
381	N/A	1	0	1.00	29	0.000
382	N/A	1	0	1.00	27	0.000
383	N/A	2	0	1.00	29	0.000
384	N/A	2	0	1.00	29	0.000
385	N/A	1	0	1.00	29	0.000
386	N/A	1	0	1.00	27	0.000
387	N/A	4	0	1.00	29	0.000
388	N/A	5	0	1.00	29	0.000
389	A	4	4	1.28	14	0.286
390	A	1	1	1.50	12	0.083
391	A	2	2	1.00	14	0.143
392	A	2	2	1.00	16	0.125
393	A	5	4	1.00	14	0.286
394	A	5	4	1.00	16	0.250
395	A	5	4	0.91	18	0.222
396	A	5	5	1.06	18	0.278
397	A	5	5	1.06	20	0.250
398	N/A	2	0	1.00	22	0.000
399	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
400	A	5	4	0.94	22	0.182
401	A	5	4	0.96	22	0.182
402	A	5	4	0.98	20	0.200
403	A	1	1	1.00	18	0.056
404	A	4	3	1.00	22	0.136
405	A	5	4	1.04	22	0.182
406	A	5	4	0.94	22	0.182
407	A	5	4	0.93	22	0.182
408	A	7	6	0.62	24	0.250
409	A	7	6	0.65	22	0.273
410	A	4	3	1.04	20	0.150
411	A	6	5	0.97	24	0.208
412	A	10	9	0.92	24	0.375
413	A	18	17	1.11	24	0.708
414	A	26	25	1.37	24	1.042
415	A	4	3	1.01	24	0.125
416	A	4	3	1.01	22	0.136
417	A	4	3	1.02	20	0.150
418	A	7	6	0.95	24	0.250
419	A	12	11	0.85	24	0.458
420	A	20	19	1.05	24	0.792
421	A	5	4	0.92	22	0.182
422	A	5	4	0.93	22	0.182
423	A	5	4	0.94	20	0.200
424	A	1	1	1.00	18	0.056
425	A	4	3	1.00	22	0.136
426	A	5	4	1.14	22	0.182
427	A	5	4	0.98	22	0.182
428	A	5	4	0.96	22	0.182
429	A	26	25	1.37	24	1.042
430	A	18	17	1.11	22	0.773
431	A	11	10	0.99	20	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	6	5	0.97	24	0.208
433	A	4	3	1.04	24	0.125
434	A	7	6	0.67	24	0.250
435	A	7	6	0.63	24	0.250
436	A	20	19	1.05	22	0.864
437	A	13	12	0.91	20	0.600
438	A	7	6	0.95	24	0.250
439	A	4	3	1.01	24	0.125
440	A	4	3	1.01	24	0.125
441	A	4	3	1.01	24	0.125
442	A	5	4	0.92	22	0.182
443	A	5	4	0.94	22	0.182
444	A	5	4	0.96	20	0.200
445	A	1	1	1.00	18	0.056
446	A	4	3	1.00	22	0.136
447	A	5	4	1.02	22	0.182
448	A	5	4	0.93	22	0.182
449	A	5	4	0.91	22	0.182
450	A	8	7	0.59	24	0.292
451	A	7	6	0.63	22	0.273
452	A	8	7	0.71	20	0.350
453	A	6	5	0.97	24	0.208
454	A	15	14	1.00	24	0.583
455	A	26	25	1.40	24	1.042
456	A	4	3	1.00	24	0.125
457	A	4	3	1.01	24	0.125
458	A	4	3	1.01	22	0.136
459	A	4	3	1.01	20	0.150
460	A	7	6	0.95	24	0.250
461	A	17	16	0.85	24	0.667
462	A	28	27	1.81	24	1.125
463	A	5	4	0.97	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
464	A	5	4	0.95	22	0.182
465	A	5	4	1.02	20	0.200
466	A	1	1	1.00	18	0.056
467	A	4	3	0.96	22	0.136
468	A	5	4	1.01	22	0.182
469	A	5	4	0.97	22	0.182
470	A	8	7	1.04	22	0.318
471	A	7	6	0.64	24	0.250
472	A	8	7	0.72	22	0.318
473	A	6	5	0.97	24	0.208
474	A	15	14	0.97	24	0.583
475	A	26	25	1.38	24	1.042
476	A	5	4	0.91	24	0.167
477	A	5	4	0.96	20	0.200
478	A	5	4	0.97	24	0.167
479	A	5	4	0.91	24	0.167
480	A	5	4	0.90	24	0.167
481	A	4	3	1.00	24	0.125
482	A	4	3	0.99	22	0.136
483	A	7	6	0.94	24	0.250
484	A	17	16	0.83	24	0.667
485	N/A	5	0	1.00	24	0.000
486	N/A	5	0	1.00	20	0.000
487	N/A	5	0	1.00	24	0.000
488	N/A	5	0	1.00	24	0.000
489	A	5	4	0.91	22	0.182
490	A	5	4	0.91	22	0.182
491	A	5	4	0.93	20	0.200
492	A	1	1	1.00	18	0.056
493	A	4	3	1.00	22	0.136
494	A	5	4	1.09	22	0.182
495	A	5	4	0.96	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
496	A	5	4	0.94	22	0.182
497	F	0	0	N/A	0.000	N/A
498	A	26	25	1.40	22	1.136
499	A	16	15	1.05	20	0.750
500	A	6	5	0.97	24	0.208
501	A	8	7	0.73	24	0.292
502	A	7	6	0.64	24	0.250
503	A	28	27	1.81	22	1.227
504	A	18	17	0.90	20	0.850
505	A	7	6	0.95	24	0.250
506	A	4	3	1.01	24	0.125
507	A	4	3	1.01	24	0.125
508	A	5	4	0.94	22	0.182
509	A	6	5	0.92	22	0.227
510	A	5	4	0.97	20	0.200
511	A	1	1	1.00	18	0.056
512	A	4	3	0.96	22	0.136
513	A	7	6	1.12	22	0.273
514	A	5	4	1.02	22	0.182
515	A	10	9	1.10	22	0.409
516	A	26	25	1.38	24	1.042
517	A	15	14	0.97	22	0.636
518	A	6	5	0.97	24	0.208
519	A	8	7	0.72	24	0.292
520	A	7	6	0.64	24	0.250
521	A	6	5	0.90	24	0.208
522	A	5	4	0.95	20	0.200
523	A	6	5	0.96	24	0.208
524	A	28	27	1.79	24	1.125
525	A	17	16	0.83	22	0.727
526	A	7	6	0.94	24	0.250
527	A	4	3	0.99	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
528	N/A	6	0	1.00	24	0.000
529	N/A	5	0	1.00	20	0.000
530	N/A	6	0	1.00	24	0.000
531	N/A	6	0	1.00	24	0.000
532	A	4	3	1.02	22	0.136
533	A	4	3	1.00	22	0.136
534	A	4	3	1.01	20	0.150
535	A	4	3	1.02	18	0.167
536	N/A	3	0	1.00	22	0.000
537	N/A	3	0	1.00	22	0.000
538	A	4	3	0.99	24	0.125
539	A	4	3	1.00	24	0.125
540	A	4	3	0.99	22	0.136
541	A	4	3	1.01	20	0.150
542	N/A	3	0	1.00	24	0.000
543	N/A	3	0	1.00	24	0.000
544	N/A	3	0	1.00	20	0.000
545	N/A	3	0	1.00	18	0.000
546	N/A	3	0	1.00	22	0.000
547	A	4	3	1.02	22	0.136
548	A	4	3	1.00	22	0.136
549	A	4	3	1.00	22	0.136
550	N/A	3	0	1.00	22	0.000
551	N/A	3	0	1.00	20	0.000
552	N/A	3	0	1.00	24	0.000
553	A	4	3	1.00	24	0.125
554	A	4	3	1.00	24	0.125
555	A	4	3	1.00	24	0.125
556	A	4	3	0.99	22	0.136
557	A	4	3	1.01	22	0.136
558	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
559	A	4	3	1.01	18	0.167
560	N/A	3	0	1.00	22	0.000
561	N/A	3	0	1.00	22	0.000
562	A	4	3	1.01	24	0.125
563	A	4	3	1.00	24	0.125
564	A	4	3	1.01	22	0.136
565	A	4	3	1.02	20	0.150
566	N/A	3	0	1.00	24	0.000
567	N/A	3	0	1.00	24	0.000
568	A	4	3	0.99	22	0.136
569	A	4	3	0.99	20	0.150
570	N/A	3	0	1.00	22	0.000
571	N/A	3	0	1.00	22	0.000
572	N/A	3	0	1.00	22	0.000
573	N/A	3	0	1.00	18	0.000
574	N/A	3	0	1.00	22	0.000
575	A	4	3	1.00	24	0.125
576	A	4	3	0.99	22	0.136
577	N/A	3	0	1.00	24	0.000
578	N/A	3	0	1.00	24	0.000
579	N/A	3	0	1.00	24	0.000
580	N/A	3	0	1.00	20	0.000
581	N/A	3	0	1.00	24	0.000
582	N/A	3	0	1.00	20	0.000
583	N/A	3	0	1.00	18	0.000
584	N/A	3	0	1.00	22	0.000
585	A	4	3	1.01	22	0.136
586	A	4	3	1.00	22	0.136
587	A	4	3	1.00	22	0.136
588	N/A	3	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
589	N/A	3	0	1.00	20	0.000
590	N/A	3	0	1.00	24	0.000
591	A	4	3	1.01	24	0.125
592	A	4	3	1.01	24	0.125
593	A	4	3	1.00	24	0.125
594	N/A	3	0	1.00	22	0.000
595	N/A	3	0	1.00	22	0.000
596	N/A	3	0	1.00	20	0.000
597	N/A	3	0	1.00	18	0.000
598	N/A	3	0	1.00	22	0.000
599	N/A	3	0	1.00	22	0.000
600	N/A	3	0	1.00	24	0.000
601	N/A	3	0	1.00	24	0.000
602	N/A	3	0	1.00	22	0.000
603	N/A	3	0	1.00	20	0.000
604	N/A	3	0	1.00	24	0.000
605	N/A	3	0	1.00	24	0.000
606	A	5	4	1.28	29	0.138
607	A	5	4	1.29	29	0.138
608	A	5	4	1.33	29	0.138
609	A	5	4	1.32	29	0.138
610	A	5	4	1.31	29	0.138
611	A	5	4	1.28	31	0.129
612	A	5	4	1.30	31	0.129
613	A	5	4	1.31	31	0.129
614	A	5	4	1.32	31	0.129
615	A	5	4	1.31	31	0.129
616	A	5	4	1.05	31	0.129
617	A	7	6	0.93	31	0.194
618	A	5	4	1.05	31	0.129
619	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	3	3	1.00	23	0.130
621	A	6	6	1.11	28	0.214
622	A	5	5	1.13	28	0.179
623	A	4	4	1.16	26	0.154
624	A	4	3	0.96	20	0.150
625	N/A	2	0	1.00	28	0.000
626	N/A	2	0	1.00	28	0.000
627	N/A	2	0	1.00	28	0.000
628	F	0	0	N/A	0.000	N/A
629	F	0	0	N/A	0.000	N/A
630	F	0	0	N/A	0.000	N/A
631	F	0	0	N/A	0.000	N/A
632	F	0	0	N/A	0.000	N/A
633	F	0	0	N/A	0.000	N/A
634	F	0	0	N/A	0.000	N/A
635	F	0	0	N/A	0.000	N/A
636	F	0	0	N/A	0.000	N/A
637	F	0	0	N/A	0.000	N/A
638	F	0	0	N/A	0.000	N/A
639	F	0	0	N/A	0.000	N/A
640	F	0	0	N/A	0.000	N/A
641	F	0	0	N/A	0.000	N/A
642	F	0	0	N/A	0.000	N/A
643	F	0	0	N/A	0.000	N/A
644	F	0	0	N/A	0.000	N/A
645	F	0	0	N/A	0.000	N/A
646	F	0	0	N/A	0.000	N/A
647	F	0	0	N/A	0.000	N/A
648	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
649	F	0	0	N/A	0.000	N/A
650	F	0	0	N/A	0.000	N/A
651	F	0	0	N/A	0.000	N/A
652	F	0	0	N/A	0.000	N/A
653	F	0	0	N/A	0.000	N/A
654	F	0	0	N/A	0.000	N/A
655	F	0	0	N/A	0.000	N/A
656	F	0	0	N/A	0.000	N/A
657	F	0	0	N/A	0.000	N/A
658	F	0	0	N/A	0.000	N/A
659	F	0	0	N/A	0.000	N/A
660	F	0	0	N/A	0.000	N/A
661	F	0	0	N/A	0.000	N/A
662	F	0	0	N/A	0.000	N/A
663	F	0	0	N/A	0.000	N/A
664	F	0	0	N/A	0.000	N/A
665	F	0	0	N/A	0.000	N/A
666	F	0	0	N/A	0.000	N/A
667	F	0	0	N/A	0.000	N/A
668	F	0	0	N/A	0.000	N/A
669	F	0	0	N/A	0.000	N/A
670	F	0	0	N/A	0.000	N/A
671	F	0	0	N/A	0.000	N/A
672	F	0	0	N/A	0.000	N/A
673	F	0	0	N/A	0.000	N/A
674	F	0	0	N/A	0.000	N/A
675	F	0	0	N/A	0.000	N/A
676	F	0	0	N/A	0.000	N/A
677	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
678	F	0	0	N/A	0.000	N/A
679	F	0	0	N/A	0.000	N/A
680	F	0	0	N/A	0.000	N/A
681	F	0	0	N/A	0.000	N/A
682	F	0	0	N/A	0.000	N/A
683	F	0	0	N/A	0.000	N/A
684	F	0	0	N/A	0.000	N/A
685	F	0	0	N/A	0.000	N/A
686	F	0	0	N/A	0.000	N/A
687	F	0	0	N/A	0.000	N/A
688	F	0	0	N/A	0.000	N/A
689	F	0	0	N/A	0.000	N/A
690	F	0	0	N/A	0.000	N/A
691	F	0	0	N/A	0.000	N/A
692	F	0	0	N/A	0.000	N/A
693	F	0	0	N/A	0.000	N/A
694	F	0	0	N/A	0.000	N/A
695	F	0	0	N/A	0.000	N/A
696	F	0	0	N/A	0.000	N/A
697	F	0	0	N/A	0.000	N/A
698	F	0	0	N/A	0.000	N/A
699	F	0	0	N/A	0.000	N/A
700	F	0	0	N/A	0.000	N/A
701	F	0	0	N/A	0.000	N/A
702	F	0	0	N/A	0.000	N/A
703	F	0	0	N/A	0.000	N/A
704	F	0	0	N/A	0.000	N/A
705	F	0	0	N/A	0.000	N/A
706	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
707	F	0	0	N/A	0.000	N/A
708	F	0	0	N/A	0.000	N/A
709	F	0	0	N/A	0.000	N/A
710	F	0	0	N/A	0.000	N/A
711	F	0	0	N/A	0.000	N/A
712	F	0	0	N/A	0.000	N/A
713	F	0	0	N/A	0.000	N/A
714	F	0	0	N/A	0.000	N/A
715	A	4	3	0.97	16	0.188
716	A	12	11	1.03	16	0.688
717	A	5	4	1.11	16	0.250
718	A	4	3	1.26	14	0.214
719	A	5	4	0.96	16	0.250
720	A	5	4	0.97	16	0.250
721	A	12	11	0.96	16	0.688
722	N/A	1	0	1.00	22	0.000
723	A	9	8	0.89	22	0.364
724	A	8	7	0.89	22	0.318
725	A	6	5	0.90	22	0.227
726	A	1	1	1.00	20	0.050
727	N/A	1	0	1.00	22	0.000
728	N/A	1	0	1.00	22	0.000

CHAPTER 3

Listing of integrals

3.1 $\int x^4 \log \left(c\left(a+bx^2\right)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right) \, dx = -\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{2a^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)$$

output
```
-2/5*a^2*p*x/b^2+2/15*a*p*x^3/b-2/25*p*x^5+2/5*a^(5/2)*p*arctan(b^(1/2)*x/
a^(1/2))/b^(5/2)+1/5*x^5*ln(c*(b*x^2+a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{1}{75}\left(-\frac{30a^2px}{b^2}+\frac{10apx^3}{b}-6px^5+\frac{30a^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+15x^5\log\left(c\left(a+bx^2\right)^p\right)\right)$$

input
```
Integrate[x^4*Log[c*(a + b*x^2)^p],x]
```

output
```
((-30*a^2*p*x)/b^2 + (10*a*p*x^3)/b - 6*p*x^5 + (30*a^(5/2)*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/b^(5/2) + 15*x^5*Log[c*(a + b*x^2)^p])/75
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{5}x^5 \log \left(c\left(a+bx^2\right)^p\right)-\frac{2}{5}bp\int \frac{x^6}{bx^2+a}dx$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{5}x^5 \log \left(c\left(a+bx^2\right)^p\right)-\frac{2}{5}bp\int \left(\frac{x^4}{b}-\frac{ax^2}{b^2}-\frac{a^3}{b^3\left(bx^2+a\right)}+\frac{a^2}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5 \log \left(c\left(a+bx^2\right)^p\right)-\frac{2}{5}bp\left(-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}\right)$$

input
```
Int[x^4*Log[c*(a + b*x^2)^p],x]
```

output
```
(-2*b*p*((a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/b^(7/2)))/5 + (x^5*Log[c*(a + b*x^2)^p])/5
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

method	result
parts	$\frac{x^{5} \ln\left(c \left(b \,x^{2}+a \right)^{p}\right)}{5}-\frac{2 p b \left(\frac{\frac{1}{5} b^{2} x^{5}-\frac{1}{3} a b \,x^{3}+a^{2} x}{b^{3}}-\frac{a^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{b^{3} \sqrt{a b}}\right)}{5}$
risch	$\frac{x^{5} \ln\left(\left(b \,x^{2}+a \right)^{p}\right)}{5}+\frac{i \pi \,x^{5} \operatorname{csgn}\left(i \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \left(b \,x^{2}+a \right)^{p}\right)^{2}}{10}-\frac{i \pi \,x^{5} \operatorname{csgn}\left(i \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \right)}{10}-\frac{i \pi \,x^{5}}{}$

input
```
int(x^4*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*ln(c*(b*x^2+a)^p)-2/5*p*b*(1/b^3*(1/5*b^2*x^5-1/3*a*b*x^3+a^2*x)-a
^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.35

$$\int x^4\log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\left[\frac{15\,b^2px^5\log\left(bx^2+a\right)-6\,b^2px^5+15\,b^2x^5\log\left(c\right)+10\,abpx^3+15\,a^2p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\,a\right.}{75\,b^2}$$

input
```
integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
[1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 + 15*b^2*x^5*log(c) + 10*
a*b*p*x^3 + 15*a^2*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2
+ a)) - 30*a^2*p*x)/b^2, 1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 +
 15*b^2*x^5*log(c) + 10*a*b*p*x^3 + 30*a^2*p*sqrt(a/b)*arctan(b*x*sqrt(a/b
)/a) - 30*a^2*p*x)/b^2]
```

Sympy [A] (verification not implemented)

Time = 32.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.95

$$\int x^4\log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\begin{cases}\frac{x^5\log\left(0^pc\right)}{5} & \text{for } a=0\wedge b=0\\ \frac{x^5\log\left(a^pc\right)}{5} & \text{for } b=0\\ -\frac{2px^5}{25}+\frac{x^5\log\left(c\left(bx^2\right)^p\right)}{5} & \text{for } a=0\\ \frac{2a^3p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{5b^3\sqrt{-\frac{a}{b}}}-\frac{a^3\log\left(c\left(a+bx^2\right)^p\right)}{5b^3\sqrt{-\frac{a}{b}}}-\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{x^5\log\left(c\left(a+bx^2\right)^p\right)}{5} & \text{otherwise}\end{cases}$$

input
```
integrate(x**4*ln(c*(b*x**2+a)**p),x)
```

output
```
Piecewise((x**5*log(0**p*c)/5, Eq(a, 0) & Eq(b, 0)), (x**5*log(a**p*c)/5,
Eq(b, 0)), (-2*p*x**5/25 + x**5*log(c*(b*x**2)**p)/5, Eq(a, 0)), (2*a**3*p
*log(x - sqrt(-a/b))/(5*b**3*sqrt(-a/b)) - a**3*log(c*(a + b*x**2)**p)/(5*
b**3*sqrt(-a/b)) - 2*a**2*p*x/(5*b**2) + 2*a*p*x**3/(15*b) - 2*p*x**5/25 +
 x**5*log(c*(a + b*x**2)**p)/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{5}\,x^5 \log \left(\left(bx^2+a\right)^p c\right) + \frac{2}{75}\,bp\left(\frac{15\,a^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{3\,b^2x^5 - 5\,abx^3 + 15\,a^2x}{b^3}\right)$$

input
```
integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
1/5*x^5*log((b*x^2 + a)^p*c) + 2/75*b*p*(15*a^3*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*b^3) - (3*b^2*x^5 - 5*a*b*x^3 + 15*a^2*x)/b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{5}\,px^5 \log \left(bx^2+a\right) - \frac{1}{25}\,(2\,p - 5\,\log\left(c\right))x^5 + \frac{2\,apx^3}{15\,b} + \frac{2\,a^3p \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{5\,\sqrt{ab}b^2} - \frac{2\,a^2px}{5\,b^2}$$

input
```
integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
1/5*p*x^5*log(b*x^2 + a) - 1/25*(2*p - 5*log(c))*x^5 + 2/15*a*p*x^3/b + 2/
5*a^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 2/5*a^2*p*x/b^2
```

Mupad [B] (verification not implemented)

Time = 26.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{x^5 \, \ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{5} - \frac{2\,p\,x^5}{25} + \frac{2\,a^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5\,b^{5/2}} + \frac{2\,a\,p\,x^3}{15\,b} - \frac{2\,a^2\,p\,x}{5\,b^2}$$

input
```
int(x^4*log(c*(a + b*x^2)^p),x)
```

output
```
(x^5*log(c*(a + b*x^2)^p))/5 - (2*p*x^5)/25 + (2*a^(5/2)*p*atan((b^(1/2)*x
)/a^(1/2)))/(5*b^(5/2)) + (2*a*p*x^3)/(15*b) - (2*a^2*p*x)/(5*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^2p + 15\log\left(\left(b\,x^2+a\right)^p c\right) b^3x^5 - 30a^2bpx + 10a\,b^2p\,x^3 - 6b^3p\,x^5}{75b^3}$$

input
```
int(x^4*log(c*(b*x^2+a)^p),x)
```

output
```
(30*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*p + 15*log((a + b*x
**2)**p*c)*b**3*x**5 - 30*a**2*b*p*x + 10*a*b**2*p*x**3 - 6*b**3*p*x**5)/(
75*b**3)
```

3.2 $\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{apx^2}{4b}-\frac{px^4}{8}-\frac{a^2p\log \left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4\log \left(c\left(a+bx^2\right)^p\right)$$

output
```
1/4*a*p*x^2/b-1/8*p*x^4-1/4*a^2*p*ln(b*x^2+a)/b^2+1/4*x^4*ln(c*(b*x^2+a)^p
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{apx^2}{4b}-\frac{px^4}{8}-\frac{a^2p\log \left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4\log \left(c\left(a+bx^2\right)^p\right)$$

input
```
Integrate[x^3*Log[c*(a + b*x^2)^p],x]
```

output
```
(a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*
(a + b*x^2)^p])/4
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int x^2 \log \left(c\left(bx^2+a\right)^p\right)dx^2$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4 \log \left(c\left(a+bx^2\right)^p\right)-\frac{1}{2}bp\int \frac{x^4}{bx^2+a}dx^2\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4 \log \left(c\left(a+bx^2\right)^p\right)-\frac{1}{2}bp\int \left(\frac{a^2}{b^2\left(bx^2+a\right)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4 \log \left(c\left(a+bx^2\right)^p\right)-\frac{1}{2}bp\left(\frac{a^2\log \left(a+bx^2\right)}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{2b}\right)\right)$$

input
```
Int[x^3*Log[c*(a + b*x^2)^p],x]
```

output
```
(-1/2*(b*p*(-((a*x^2)/b^2) + x^4/(2*b) + (a^2*Log[a + b*x^2])/b^3)) + (x^4
*Log[c*(a + b*x^2)^p])/2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
parts	$\frac{x^4 \ln\left(c\left(b\,x^2+a\right)^p\right)}{4}-\frac{pb\left(-\frac{-\frac{1}{2}b\,x^4+a\,x^2}{2b^2}+\frac{a^2\ln\left(b\,x^2+a\right)}{2b^3}\right)}{2}$	57
parallelrisch	$-\frac{-2x^4\ln\left(c\left(b\,x^2+a\right)^p\right)b^2+b^2p\,x^4-2abp\,x^2+2\ln\left(b\,x^2+a\right)a^2p+2a^2p}{8b^2}$	63
risch	Expression too large to display	1190

input
```
int(x^3*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*ln(c*(b*x^2+a)^p)-1/2*p*b*(-1/2/b^2*(-1/2*b*x^4+a*x^2)+1/2*a^2/b^3
*ln(b*x^2+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx = -\frac{b^2 p x^4 - 2\, b^2 x^4 \log \left(c\right) - 2\, a b p x^2 - 2\, \left(b^2 p x^4 - a^2 p\right) \log \left(b x^2 + a\right)}{8\, b^2}$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
-1/8*(b^2*p*x^4 - 2*b^2*x^4*log(c) - 2*a*b*p*x^2 - 2*(b^2*p*x^4 - a^2*p)*l
og(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \begin{cases} -\frac{a^2 \log \left(c\left(a+bx^2\right)^p\right)}{4b^2} + \frac{apx^2}{4b} - \frac{px^4}{8} + \frac{x^4 \log \left(c\left(a+bx^2\right)^p\right)}{4} & \text{for}\: b \neq 0 \\ \frac{x^4 \log \left(a^p c\right)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*ln(c*(b*x**2+a)**p),x)
```

output
```
Piecewise((-a**2*log(c*(a + b*x**2)**p)/(4*b**2) + a*p*x**2/(4*b) - p*x**4
/8 + x**4*log(c*(a + b*x**2)**p)/4, Ne(b, 0)), (x**4*log(a**p*c)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^3 \log \left(c(a+bx^2)^p\right)\,dx = \frac{1}{4}\,x^4 \log \left((bx^2+a)^p c\right) - \frac{1}{8}\,bp\left(\frac{2\,a^2 \log \left(bx^2+a\right)}{b^3} + \frac{bx^4 - 2\,ax^2}{b^2}\right)$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
1/4*x^4*log((b*x^2 + a)^p*c) - 1/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4
- 2*a*x^2)/b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.64

$$\int x^3 \log \left(c(a+bx^2)^p\right)\,dx = \frac{2\,(bx^2+a)^2 p \log \left(bx^2+a\right) - (bx^2+a)^2 p + 2\,(bx^2+a)^2 \log \left(c\right)}{8\,b^2} + \frac{(bx^2 - (bx^2+a)\log \left(bx^2+a\right) + a)ap - (bx^2+a)a \log \left(c\right)}{2\,b^2}$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
1/8*(2*(b*x^2 + a)^2*p*log(b*x^2 + a) - (b*x^2 + a)^2*p + 2*(b*x^2 + a)^2*
log(c))/b^2 + 1/2*((b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a*p - (b*x^2 +
 a)*a*log(c))/b^2
```

Mupad [B] (verification not implemented)

Time = 26.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{x^4 \, \ln \left(c \left(b \, x^2+a\right)^p\right)}{4} - \frac{p \, x^4}{8} - \frac{a^2 \, p \, \ln \left(b \, x^2+a\right)}{4 \, b^2} + \frac{a \, p \, x^2}{4 \, b}$$

input
```
int(x^3*log(c*(a + b*x^2)^p),x)
```

output
```
(x^4*log(c*(a + b*x^2)^p))/4 - (p*x^4)/8 - (a^2*p*log(a + b*x^2))/(4*b^2)
+ (a*p*x^2)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{-2\log\left(\left(b\,x^2+a\right)^p c\right) a^2+2\log\left(\left(b\,x^2+a\right)^p c\right) b^2x^4+2abp\,x^2-b^2p\,x^4}{8b^2}$$

input
```
int(x^3*log(c*(b*x^2+a)^p),x)
```

output
```
( - 2*log((a + b*x**2)**p*c)*a**2 + 2*log((a + b*x**2)**p*c)*b**2*x**4 + 2
*a*b*p*x**2 - b**2*p*x**4)/(8*b**2)
```

3.3 $\int x^2 \log \left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int x^2 \log \left(c(a+bx^2)^p\right) \, dx = \frac{2apx}{3b} - \frac{2px^3}{9} - \frac{2a^{3/2}p \arctan \left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} + \frac{1}{3}x^3 \log \left(c(a+bx^2)^p\right)$$

output
```
2/3*a*p*x/b-2/9*p*x^3-2/3*a^(3/2)*p*arctan(b^(1/2)*x/a^(1/2))/b^(3/2)+1/3*
x^3*ln(c*(b*x^2+a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int x^2 \log \left(c(a+bx^2)^p\right) \, dx = \frac{1}{9}\left(\frac{6apx}{b} - 2px^3 - \frac{6a^{3/2}p \arctan \left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} + 3x^3 \log \left(c(a+bx^2)^p\right)\right)$$

input
```
Integrate[x^2*Log[c*(a + b*x^2)^p],x]
```

output
```
((6*a*p*x)/b - 2*p*x^3 - (6*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
 + 3*x^3*Log[c*(a + b*x^2)^p])/9
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2905, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(c\left(a+bx^2\right)^p\right) dx$$

$$\Big\downarrow \; 2905$$

$$\frac{1}{3}x^3 \log \left(c\left(a+bx^2\right)^p\right) - \frac{2}{3}bp\int \frac{x^4}{bx^2+a}dx$$

$$\Big\downarrow \; 254$$

$$\frac{1}{3}x^3 \log \left(c\left(a+bx^2\right)^p\right) - \frac{2}{3}bp\int \left(\frac{a^2}{b^2\left(bx^2+a\right)} - \frac{a}{b^2} + \frac{x^2}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}x^3 \log \left(c\left(a+bx^2\right)^p\right) - \frac{2}{3}bp\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}\right)$$

input
```
Int[x^2*Log[c*(a + b*x^2)^p],x]
```

output
```
(-2*b*p*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
b^(5/2)))/3 + (x^3*Log[c*(a + b*x^2)^p])/3
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

method	result
parts	$\frac{x^{3} \ln\left(c \left(b \,x^{2}+a \right)^{p}\right)}{3}-\frac{2 p b \left(-\frac{-\frac{1}{3} b \,x^{3}+a x}{b^{2}}+\frac{a^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{b^{2} \sqrt{a b}}\right)}{3}$
risch	$\frac{x^{3} \ln\left(\left(b \,x^{2}+a \right)^{p}\right)}{3}+\frac{i \pi \,x^{3} \operatorname{csgn}\left(i \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \left(b \,x^{2}+a \right)^{p}\right)^{2}}{6}-\frac{i \pi \,x^{3} \operatorname{csgn}\left(i \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \right)}{6}-i \pi \,x^{3}$

input
```
int(x^2*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(c*(b*x^2+a)^p)-2/3*p*b*(-1/b^2*(-1/3*b*x^3+a*x)+a^2/b^2/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.30

$$\int x^2 \log \left(c \left(a + bx^2\right)^p\right) dx$$
$$= \left[\frac{3\,bpx^3 \log\left(bx^2 + a\right) - 2\,bpx^3 + 3\,bx^3 \log\left(c\right) + 3\,ap\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 6\,apx}{9\,b}, \frac{3\,bpx^3 \log\left(bx^2 +\right.}{}\right.$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
[1/9*(3*b*p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) + 3*a*p*sqrt(-
a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*a*p*x)/b, 1/9*(3*
b*p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) - 6*a*p*sqrt(a/b)*arct
an(b*x*sqrt(a/b)/a) + 6*a*p*x)/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(63) = 126.

Time = 7.89 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.14

$$\int x^2 \log \left(c \left(a + bx^2\right)^p\right) dx$$
$$= \begin{cases} \frac{x^3 \log\left(0^p c\right)}{3} & \text{for } a = 0 \wedge b = 0 \\ \frac{x^3 \log\left(a^p c\right)}{3} & \text{for } b = 0 \\ -\frac{2px^3}{9} + \frac{x^3 \log\left(c\left(bx^2\right)^p\right)}{3} & \text{for } a = 0 \\ -\frac{2a^2 p \log\left(x - \sqrt{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}} + \frac{a^2 \log\left(c\left(a + bx^2\right)^p\right)}{3b^2\sqrt{-\frac{a}{b}}} + \frac{2apx}{3b} - \frac{2px^3}{9} + \frac{x^3 \log\left(c\left(a + bx^2\right)^p\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*ln(c*(b*x**2+a)**p),x)
```

output
```
Piecewise((x**3*log(0**p*c)/3, Eq(a, 0) & Eq(b, 0)), (x**3*log(a**p*c)/3,
Eq(b, 0)), (-2*p*x**3/9 + x**3*log(c*(b*x**2)**p)/3, Eq(a, 0)), (-2*a**2*p
*log(x - sqrt(-a/b))/(3*b**2*sqrt(-a/b)) + a**2*log(c*(a + b*x**2)**p)/(3*
b**2*sqrt(-a/b)) + 2*a*p*x/(3*b) - 2*p*x**3/9 + x**3*log(c*(a + b*x**2)**p
)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int x^2 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{3}\,x^3\log\left(\left(bx^2+a\right)^p c\right) - \frac{2}{9}\,bp\left(\frac{3\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bx^3-3\,ax}{b^2}\right)$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
1/3*x^3*log((b*x^2 + a)^p*c) - 2/9*b*p*(3*a^2*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b^2) + (b*x^3 - 3*a*x)/b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int x^2 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{3}\,px^3\log\left(bx^2+a\right) - \frac{1}{9}\,(2\,p-3\,\log\left(c\right))x^3 - \frac{2\,a^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b} + \frac{2\,apx}{3\,b}$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
1/3*p*x^3*log(b*x^2 + a) - 1/9*(2*p - 3*log(c))*x^3 - 2/3*a^2*p*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b) + 2/3*a*p*x/b
```

Mupad [B] (verification not implemented)

Time = 26.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int x^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{x^3 \ln \left(c\left(b\,x^2+a\right)^p\right)}{3} - \frac{2\,p\,x^3}{9} - \frac{2\,a^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\,b^{3/2}} + \frac{2\,a\,p\,x}{3\,b}$$

input
```
int(x^2*log(c*(a + b*x^2)^p),x)
```

output
```
(x^3*log(c*(a + b*x^2)^p))/3 - (2*p*x^3)/9 - (2*a^(3/2)*p*atan((b^(1/2)*x)
/a^(1/2)))/(3*b^(3/2)) + (2*a*p*x)/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int x^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{-6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ap + 3\log\left(\left(b\,x^2+a\right)^p c\right)b^2x^3 + 6abpx - 2b^2p\,x^3}{9b^2}$$

input
```
int(x^2*log(c*(b*x^2+a)^p),x)
```

output
```
( - 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*p + 3*log((a + b*x**
2)**p*c)*b**2*x**3 + 6*a*b*p*x - 2*b**2*p*x**3)/(9*b**2)
```

3.4 $\int x \log\left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int x \log\left(c(a+bx^2)^p\right) \, dx = -\frac{px^2}{2} + \frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2b}$$

output
```
-1/2*p*x^2+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int x \log\left(c(a+bx^2)^p\right) \, dx = \frac{1}{2}\left(-px^2 + \frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}\right)$$

input
```
Integrate[x*Log[c*(a + b*x^2)^p],x]
```

output
```
(-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b)/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(c \left(a+b x^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2} \int \log \left(c \left(b x^2+a\right)^p\right)dx^2$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log \left(c \left(b x^2+a\right)^p\right)d\left(b x^2+a\right)}{2 b}$$

$$\Big\downarrow \text{2732}$$

$$\frac{\left(a+b x^2\right) \log \left(c \left(a+b x^2\right)^p\right)-p \left(a+b x^2\right)}{2 b}$$

input
```
Int[x*Log[c*(a + b*x^2)^p],x]
```

output
```
(-(p*(a + b*x^2)) + (a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*b)
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)\left(b\,x^{2}+a\right)-\left(b\,x^{2}+a\right)p}{2b}$
default	$\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)\left(b\,x^{2}+a\right)-\left(b\,x^{2}+a\right)p}{2b}$
norman	$-\frac{p\,x^{2}}{2}+\frac{x^{2}\ln\left(c\,{\mathrm e}^{p\ln\left(b\,x^{2}+a\right)}\right)}{2}+\frac{pa\ln\left(b\,x^{2}+a\right)}{2b}$
parts	$\frac{x^{2}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{2}-pb\left(\frac{x^{2}}{2b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2b^{2}}\right)$
parallelrisch	$\frac{x^{2}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)bp-x^{2}b\,p^{2}+\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)ap+a\,p^{2}}{2pb}$
risch	$\frac{x^{2}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{2}+\frac{i\pi\,x^{2}\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}}{4}-\frac{i\pi\,x^{2}\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}}{4}$

input
```
int(x*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(ln(c*(b*x^2+a)^p)*(b*x^2+a)-(b*x^2+a)*p)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int x\log\left(c\left(a+bx^2\right)^p\right)\,dx=-\frac{bpx^2-bx^2\log\left(c\right)-\left(bpx^2+ap\right)\log\left(bx^2+a\right)}{2\,b}$$

input
```
integrate(x*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
-1/2*(b*p*x^2 - b*x^2*log(c) - (b*p*x^2 + a*p)*log(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int x\log\left(c(a+bx^2)^p\right)\,dx=\begin{cases}\frac{a\log\left(c(a+bx^2)^p\right)}{2b}-\frac{px^2}{2}+\frac{x^2\log\left(c(a+bx^2)^p\right)}{2} & \text{for } b\neq 0\\ \frac{x^2\log(a^pc)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*ln(c*(b*x**2+a)**p),x)
```

output
```
Piecewise((a*log(c*(a + b*x**2)**p)/(2*b) - p*x**2/2 + x**2*log(c*(a + b*x
**2)**p)/2, Ne(b, 0)), (x**2*log(a**p*c)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int x\log\left(c(a+bx^2)^p\right)\,dx=-\frac{1}{2}\,bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)+\frac{1}{2}\,x^2\log\left(\left(bx^2+a\right)^pc\right)$$

input
```
integrate(x*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
-1/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2) + 1/2*x^2*log((b*x^2 + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int x \log \left(c \left(a+b x^2\right)^p\right) \, dx=-\frac{(b x^2-(b x^2+a) \log (b x^2+a)+a) p-(b x^2+a) \log (c)}{2 b}$$

input
```
integrate(x*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
-1/2*((b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p - (b*x^2 + a)*log(c))/b
```

Mupad [B] (verification not implemented)

Time = 26.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int x \log \left(c \left(a+b x^2\right)^p\right) \, dx=\frac{x^2 \ln \left(c \left(b x^2+a\right)^p\right)}{2}-\frac{p x^2}{2}+\frac{a p \ln (b x^2+a)}{2 b}$$

input
```
int(x*log(c*(a + b*x^2)^p),x)
```

output
```
(x^2*log(c*(a + b*x^2)^p))/2 - (p*x^2)/2 + (a*p*log(a + b*x^2))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int x \log \left(c \left(a+b x^2\right)^p\right) \, dx=\frac{\log \left(\left(b x^2+a\right)^p c\right) a+\log \left(\left(b x^2+a\right)^p c\right) b x^2-b p x^2}{2 b}$$

input
```
int(x*log(c*(b*x^2+a)^p),x)
```

output
```
(log((a + b*x**2)**p*c)*a + log((a + b*x**2)**p*c)*b*x**2 - b*p*x**2)/(2*b
)
```

3.5 $\int \log \left(c\left(a+b x^2\right)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \log \left(c\left(a+b x^2\right)^p\right) \, dx=-2 p x+\frac{2 \sqrt{a} p \arctan \left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}+x \log \left(c\left(a+b x^2\right)^p\right)$$

output
```
-2*p*x+2*a^(1/2)*p*arctan(b^(1/2)*x/a^(1/2))/b^(1/2)+x*ln(c*(b*x^2+a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(a+b x^2\right)^p\right) \, dx=-2 p x+\frac{2 \sqrt{a} p \arctan \left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}+x \log \left(c\left(a+b x^2\right)^p\right)$$

input
```
Integrate[Log[c*(a + b*x^2)^p],x]
```

output
```
-2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + 
b*x^2)^p]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2898, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c\left(a+bx^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{2898}$$

$$x \log \left(c\left(a+bx^2\right)^p\right)-2bp\int \frac{x^2}{bx^2+a}dx$$

$$\Big\downarrow \text{262}$$

$$x \log \left(c\left(a+bx^2\right)^p\right)-2bp\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)$$

$$\Big\downarrow \text{218}$$

$$x \log \left(c\left(a+bx^2\right)^p\right)-2bp\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)$$

input
```
Int[Log[c*(a + b*x^2)^p],x]
```

output
```
-2*b*p*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)) + x*Log[c*(a
+ b*x^2)^p]
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result
default	$x \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)-2 p b \left(\frac{x}{b}-\frac{a \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{b \sqrt{a b}}\right)$
parts	$x \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)-2 p b \left(\frac{x}{b}-\frac{a \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{b \sqrt{a b}}\right)$
risch	$x \ln \left(\left(b \,x^{2}+a \right)^{p}\right)+\frac{i \operatorname{csgn}\left(i c \left(b \,x^{2}+a \right)^{p}\right)^{2} \operatorname{csgn}\left(i \left(b \,x^{2}+a \right)^{p}\right) \pi x}{2}-\frac{i \pi x \,\operatorname{csgn}\left(i \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \right)}{2}-$

input
```
int(ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(b*x^2+a)^p)-2*p*b*(x/b-1/b*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int \log \left(c\left(a+bx^2\right)^p\right)\,dx = \left[px\log\left(bx^2+a\right)+p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) - 2\,px + x\log\left(c\right), px\log\left(bx^2+a\right) + 2\,p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 2\,px + x\log\left(c\right)\right]$$

input
```
integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
[p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) - 2*p*x + x*log(c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 2.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \log \left(c\left(a+bx^2\right)^p\right)\,dx = \begin{cases} x\log\left(0^pc\right) & \text{for}\ a=0 \wedge b=0 \\ x\log\left(a^pc\right) & \text{for}\ b=0 \\ -2px + x\log\left(c\left(bx^2\right)^p\right) & \text{for}\ a=0 \\ \frac{2ap\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b\sqrt{-\frac{a}{b}}} - 2px + x\log\left(c\left(a+bx^2\right)^p\right) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x**2+a)**p),x)
```

output
```
Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (x*log(a**p*c), Eq(b, 0)),
 (-2*p*x + x*log(c*(b*x**2)**p), Eq(a, 0)), (2*a*p*log(x - sqrt(-a/b))/(b*
sqrt(-a/b)) - a*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) - 2*p*x + x*log(c*(a
 + b*x**2)**p), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(a+bx^2\right)^p\right)\,dx = 2\,bp\left(\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}-\frac{x}{b}\right)+x\log\left(\left(bx^2+a\right)^pc\right)$$

input
```
integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
2*b*p*(a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - x/b) + x*log((b*x^2 + a)^p*
c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \log \left(c\left(a+bx^2\right)^p\right)\,dx = px\log\left(bx^2+a\right)+\frac{2\,ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-(2\,p-\log(c))x$$

input
```
integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c)
)*x
```

Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \log \left(c\left(a+bx^2\right)^p\right)\,dx=x\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)-2\,p\,x+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
int(log(c*(a + b*x^2)^p),x)
```

output
```
x*log(c*(a + b*x^2)^p) - 2*p*x + (2*a^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/b
^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \log \left(c\left(a+bx^2\right)^p\right)\,dx=\frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)p+\log\left(\left(b\,x^2+a\right)^p c\right)bx-2bpx}{b}$$

input
```
int(log(c*(b*x^2+a)^p),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*p + log((a + b*x**2)**p*c
)*b*x - 2*b*p*x)/b
```

3.6 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx=\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}p\,\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right)$$

output
```
1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)+1/2*p*polylog(2,1+b*x^2/a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx=\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+p\,\text{PolyLog}\left(2,\frac{a+bx^2}{a}\right)\right)$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/x,x]
```

output
```
(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, (a + b*x^2)/a])/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^2\right)^p\right)}{x} \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{\log \left(c\left(bx^2+a\right)^p\right)}{x^2}dx^2$$

$$\Big\downarrow \text{2841}$$

$$\frac{1}{2}\left(\log \left(-\frac{bx^2}{a}\right)\log \left(c\left(a+bx^2\right)^p\right)-bp\int \frac{\log \left(-\frac{bx^2}{a}\right)}{bx^2+a}dx^2\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}\left(\log \left(-\frac{bx^2}{a}\right)\log \left(c\left(a+bx^2\right)^p\right)+p\,\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\right)$$

input
```
Int[Log[c*(a + b*x^2)^p]/x,x]
```

output
```
(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 + (b*x^2)/a])/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(40) = 80$.

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.50

method	result
parts	$\ln\left(c(b\,x^2+a)^p\right)\ln\left(x\right)-2pb\left(\frac{\ln(x)\left(\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2b}+\frac{\operatorname{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2b}\right)$
risch	$\ln\left((b\,x^2+a)^p\right)\ln\left(x\right)-p\ln\left(x\right)\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)-p\ln\left(x\right)\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)-p\operatorname{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)-$

input
```
int(ln(c*(b*x^2+a)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(c*(b*x^2+a)^p)*ln(x)-2*p*b*(1/2*ln(x)*(ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1
/2))+ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b+1/2*(dilog((-b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))+dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b)
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{x}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\begin{aligned}
&\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx \\
&= \frac{1}{2}bp\left(\frac{2\log\left(bx^2+a\right)\log\left(x\right)}{b} - \frac{2\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)}{b}\right) \\
&\quad - p\log\left(bx^2+a\right)\log\left(x\right) + \log\left(\left(bx^2+a\right)^p c\right)\log\left(x\right)
\end{aligned}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="maxima")
```

output
```
1/2*b*p*(2*log(b*x^2 + a)*log(x)/b - (2*log(b*x^2/a + 1)*log(x) + dilog(-b
*x^2/a))/b) - p*log(b*x^2 + a)*log(x) + log((b*x^2 + a)^p*c)*log(x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx=\int \frac{\log\left((bx^2+a)^p c\right)}{x}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx=\int \frac{\ln\left(c\,(b\,x^2+a)^p\right)}{x}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)/x,x)
```

output
```
int(log(c*(a + b*x^2)^p)/x, x)
```

Reduce [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx=\frac{4\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^3+ax}dx\right)ap+\log\left((b\,x^2+a)^p\,c\right)^2}{4p}$$

input
```
int(log(c*(b*x^2+a)^p)/x,x)
```

output
```
(4*int(log((a + b*x**2)**p*c)/(a*x + b*x**3),x)*a*p + log((a + b*x**2)**p*
c)**2)/(4*p)
```

3.7 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2} \, dx = \frac{2\sqrt{b}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\log \left(c(a+bx^2)^p\right)}{x}$$

output
```
2*b^(1/2)*p*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)-ln(c*(b*x^2+a)^p)/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2} \, dx = \frac{2\sqrt{b}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\log \left(c(a+bx^2)^p\right)}{x}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/x^2,x]
```

output
```
(2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2905, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$$

$$\Big\downarrow \text{ 2905}$$

$$2bp\int \frac{1}{bx^2+a}dx-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}$$

input
```
Int[Log[c*(a + b*x^2)^p]/x^2,x]
```

output
```
(2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{\ln(c(b\,x^2+a)^p)}{x}+\frac{2pb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$
risch	$-\frac{\ln((b\,x^2+a)^p)}{x}-\frac{i\pi a\ \mathrm{csgn}(i(b\,x^2+a)^p)\mathrm{csgn}(ic(b\,x^2+a)^p)^2-i\pi a\ \mathrm{csgn}(i(b\,x^2+a)^p)\ \mathrm{csgn}(ic(b\,x^2+a)^p)\ \mathrm{csgn}(ic)-i\pi a\mathrm{csgn}(ic(b}{2}$

input
```
int(ln(c*(b*x^2+a)^p)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(b*x^2+a)^p)/x+2*p*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.39

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx$$
$$=\left[\frac{px\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-p\log\left(bx^2+a\right)-\log\left(c\right)}{x},\frac{2\,px\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-p\log\left(bx^2+a\right)-\mathrm{l}}{x}\right.$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="fricas")
```

output
```
[(p*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - p*log(b
*x^2 + a) - log(c))/x, (2*p*x*sqrt(b/a)*arctan(x*sqrt(b/a)) - p*log(b*x^2
+ a) - log(c))/x]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(39) = 78$.

Time = 7.54 (sec) , antiderivative size = 258, normalized size of antiderivative = 5.86

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\,dx$$
$$= \begin{cases} -\frac{\log (0^p c)}{x} \\ -\frac{\log (a^p c)}{x} \\ -\frac{2p}{x} - \frac{\log \left(c(bx^2)^p\right)}{x} \\ -\frac{\log (0^p c)}{x} \\ -\frac{a^2 \log \left(c(a+bx^2)^p\right)}{a^2x+abx^3} - \frac{2apx\sqrt{-\frac{a}{b}}\log \left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{a^2x}{b}+ax^3} - \frac{ax^2 \log \left(c(a+bx^2)^p\right)}{\frac{a^2x}{b}+ax^3} + \frac{ax\sqrt{-\frac{a}{b}}\log \left(c(a+bx^2)^p\right)}{\frac{a^2x}{b}+ax^3} - \frac{2bpx^3\sqrt{-\frac{a}{b}}\log \left(x-\sqrt{-\frac{a}{b}}\right.}{\frac{a^2x}{b}+ax^3} \end{cases}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x**2,x)
```

output
```
Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/x, Eq(b, 0)
), (-2*p/x - log(c*(b*x**2)**p)/x, Eq(a, 0)), (-log(0**p*c)/x, Eq(a, -b*x*
*2)), (-a**2*log(c*(a + b*x**2)**p)/(a**2*x + a*b*x**3) - 2*a*p*x*sqrt(-a/
b)*log(x - sqrt(-a/b))/(a**2*x/b + a*x**3) - a*x**2*log(c*(a + b*x**2)**p)
/(a**2*x/b + a*x**3) + a*x*sqrt(-a/b)*log(c*(a + b*x**2)**p)/(a**2*x/b + a
*x**3) - 2*b*p*x**3*sqrt(-a/b)*log(x - sqrt(-a/b))/(a**2*x/b + a*x**3) + b
*x**3*sqrt(-a/b)*log(c*(a + b*x**2)**p)/(a**2*x/b + a*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\,bp\arctan \left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{\log \left((bx^2+a)^p c\right)}{x}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="maxima")
```

output
```
2*b*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - log((b*x^2 + a)^p*c)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\,bp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{p\log\left(bx^2+a\right)}{x} - \frac{\log\left(c\right)}{x}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="giac")
```

output
```
2*b*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - p*log(b*x^2 + a)/x - log(c)/x
```

Mupad [B] (verification not implemented)

Time = 26.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\,\sqrt{b}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{x}$$

input
```
int(log(c*(a + b*x^2)^p)/x^2,x)
```

output
```
(2*b^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/a^(1/2) - log(c*(a + b*x^2)^p)/x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)px - \log\left(\left(b\,x^2+a\right)^p c\right)a}{ax}$$

input
```
int(log(c*(b*x^2+a)^p)/x^2,x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*p*x - log((a + b*x**2)**p
*c)*a)/(a*x)
```

3.8 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3} dx = \frac{bp\log(x)}{a} - \frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2ax^2}$$

output
```
b*p*ln(x)/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3} dx = \frac{bp\log(x)}{a} - \frac{bp\log(a+bx^2)}{2a} - \frac{\log\left(c(a+bx^2)^p\right)}{2x^2}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/x^3,x]
```

output
```
(b*p*Log[x])/a - (b*p*Log[a + b*x^2])/(2*a) - Log[c*(a + b*x^2)^p]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2904, 2842, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^3} \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2} \int \frac{\log \left(c(bx^2+a)^p\right)}{x^4} dx^2$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2} \left(bp \int \frac{1}{x^2 (bx^2+a)} dx^2 - \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2} \left(bp \left(\frac{\int \frac{1}{x^2} dx^2}{a} - \frac{b \int \frac{1}{bx^2+a} dx^2}{a}\right) - \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2} \left(bp \left(\frac{\log (x^2)}{a} - \frac{b \int \frac{1}{bx^2+a} dx^2}{a}\right) - \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2} \left(bp \left(\frac{\log (x^2)}{a} - \frac{\log (a+bx^2)}{a}\right) - \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\right)$$

input
```
Int[Log[c*(a + b*x^2)^p]/x^3,x]
```

output
```
(b*p*(Log[x^2]/a - Log[a + b*x^2]/a) - Log[c*(a + b*x^2)^p]/x^2)/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{2x^{2}}+pb\left(-\frac{\ln\left(b\,x^{2}+a\right)}{2a}+\frac{\ln\left(x\right)}{a}\right)$
parallelrisch	$\frac{2p^{2}b\ln\left(x\right)x^{2}-x^{2}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)bp-\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)ap}{2x^{2}ap}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{2x^{2}}-\frac{i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}-i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\right)-i\pi a\mathrm{csg}}{4a\,x^{2}}$

input
```
int(ln(c*(b*x^2+a)^p)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(b*x^2+a)^p)/x^2+p*b*(-1/2/a*ln(b*x^2+a)+ln(x)/a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{2\,bpx^2\log\left(x\right)-\left(bpx^2+ap\right)\log\left(bx^2+a\right)-a\log\left(c\right)}{2\,ax^2}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b*p*x^2*log(x) - (b*p*x^2 + a*p)*log(b*x^2 + a) - a*log(c))/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx = \begin{cases} -\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}+\frac{bp\log\left(x\right)}{a}-\frac{b\log\left(c(a+bx^2)^p\right)}{2a} & \text{for } a\neq 0 \\ -\frac{p}{2x^2}-\frac{\log\left(c(bx^2)^p\right)}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x**3,x)
```

output
```
Piecewise((-log(c*(a + b*x**2)**p)/(2*x**2) + b*p*log(x)/a - b*log(c*(a +
b*x**2)**p)/(2*a), Ne(a, 0)), (-p/(2*x**2) - log(c*(b*x**2)**p)/(2*x**2),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx = -\frac{1}{2}\,bp\left(\frac{\log\left(bx^2+a\right)}{a} - \frac{\log\left(x^2\right)}{a}\right) - \frac{\log\left(\left(bx^2+a\right)^p c\right)}{2\,x^2}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="maxima")
```

output
```
-1/2*b*p*(log(b*x^2 + a)/a - log(x^2)/a) - 1/2*log((b*x^2 + a)^p*c)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx = -\frac{\frac{b^2p\log(bx^2+a)}{a} - \frac{b^2p\log(bx^2)}{a} + \frac{bp\log(bx^2+a)}{x^2} + \frac{b\log(c)}{x^2}}{2\,b}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="giac")
```

output
```
-1/2*(b^2*p*log(b*x^2 + a)/a - b^2*p*log(b*x^2)/a + b*p*log(b*x^2 + a)/x^2
 + b*log(c)/x^2)/b
```

Mupad [B] (verification not implemented)

Time = 26.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{b\,p\,\ln(x)}{a} - \frac{b\,p\,\ln\left(b\,x^2+a\right)}{2\,a} - \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{2\,x^2}$$

input
```
int(log(c*(a + b*x^2)^p)/x^3,x)
```

output
```
(b*p*log(x))/a - (b*p*log(a + b*x^2))/(2*a) - log(c*(a + b*x^2)^p)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{-\log\left((b\,x^2+a)^p\,c\right)a-\log\left((b\,x^2+a)^p\,c\right)b\,x^2+2\log(x)\,bp\,x^2}{2a\,x^2}$$

input
```
int(log(c*(b*x^2+a)^p)/x^3,x)
```

output
```
( - log((a + b*x**2)**p*c)*a - log((a + b*x**2)**p*c)*b*x**2 + 2*log(x)*b*
p*x**2)/(2*a*x**2)
```

3.9 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4} dx = -\frac{2bp}{3ax} - \frac{2b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{\log\left(c(a+bx^2)^p\right)}{3x^3}$$

output
```
-2/3*b*p/a/x-2/3*b^(3/2)*p*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)-1/3*ln(c*(b*x
^2+a)^p)/x^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4} dx = -\frac{2bp\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax} - \frac{\log\left(c(a+bx^2)^p\right)}{3x^3}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/x^4,x]
```

output
```
(-2*b*p*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)])/(3*a*x) - Log[c*(a
+ b*x^2)^p]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2905, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^2\right)^p\right)}{x^4} \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{2}{3} bp \int \frac{1}{x^2 \left(bx^2+a\right)}dx-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{3x^3}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{3} bp \left(-\frac{b \int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{3x^3}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2}{3} bp \left(-\frac{\sqrt{b} \arctan \left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{3x^3}$$

input
```
Int[Log[c*(a + b*x^2)^p]/x^4,x]
```

output
```
(2*b*p*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/3 - L
og[c*(a + b*x^2)^p]/(3*x^3)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{3x^{3}}+\frac{2pb\left(-\frac{1}{ax}-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}\right)}{3}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{3x^{3}}-\frac{i\pi a\,\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}-i\pi a\,\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\right)-i\pi a\operatorname{csgn}\left(ic\left(b\right.\right.}{}$

input
```
int(ln(c*(b*x^2+a)^p)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(c*(b*x^2+a)^p)/x^3+2/3*p*b*(-1/a/x-1/a*b/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.25

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx = \left[\frac{bpx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,bpx^2-ap\log\left(bx^2+a\right)-a\log\left(c\right)}{3\,ax^3}, -\frac{2\,bpx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,bpx^2+ap\log\left(bx^2+a\right)+a\log\left(c\right)}{3\,ax^3}\right]$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="fricas")
```

output
```
[1/3*(b*p*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) -
 2*b*p*x^2 - a*p*log(b*x^2 + a) - a*log(c))/(a*x^3), -1/3*(2*b*p*x^3*sqrt(
b/a)*arctan(x*sqrt(b/a)) + 2*b*p*x^2 + a*p*log(b*x^2 + a) + a*log(c))/(a*x
^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(56) = 112$.

Time = 38.14 (sec) , antiderivative size = 496, normalized size of antiderivative = 8.27

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx = \begin{cases} -\frac{\log\left(0^p c\right)}{3x^3} \\ -\frac{\log\left(a^p c\right)}{3x^3} \\ -\frac{2p}{9x^3}-\frac{\log\left(c\left(bx^2\right)^p\right)}{3x^3} \\ -\frac{\log\left(0^p c\right)}{3x^3} \\ -\frac{a^2\sqrt{-\frac{a}{b}}\log\left(c(a+bx^2)^p\right)}{3a^2x^3\sqrt{-\frac{a}{b}}+3abx^5\sqrt{-\frac{a}{b}}}-\frac{2apx^3\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}}-\frac{2apx^2\sqrt{-\frac{a}{b}}}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}}+\frac{ax^3\log\left(c(a+bx^2)^p\right)}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}}-\frac{ax^2\sqrt{-\frac{a}{b}}\log\left(c(a+\cdots\right.}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\cdots} \end{cases}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x**4,x)
```

output
```
Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/(3*x
**3), Eq(b, 0)), (-2*p/(9*x**3) - log(c*(b*x**2)**p)/(3*x**3), Eq(a, 0)),
(-log(0**p*c)/(3*x**3), Eq(a, -b*x**2)), (-a**2*sqrt(-a/b)*log(c*(a + b*x*
*2)**p)/(3*a**2*x**3*sqrt(-a/b) + 3*a*b*x**5*sqrt(-a/b)) - 2*a*p*x**3*log(
x - sqrt(-a/b))/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)) - 2*a*p*x
**2*sqrt(-a/b)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)) + a*x**3*l
og(c*(a + b*x**2)**p)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)) - a
*x**2*sqrt(-a/b)*log(c*(a + b*x**2)**p)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x*
*5*sqrt(-a/b)) - 2*b*p*x**5*log(x - sqrt(-a/b))/(3*a**2*x**3*sqrt(-a/b)/b
+ 3*a*x**5*sqrt(-a/b)) - 2*b*p*x**4*sqrt(-a/b)/(3*a**2*x**3*sqrt(-a/b)/b +
 3*a*x**5*sqrt(-a/b)) + b*x**5*log(c*(a + b*x**2)**p)/(3*a**2*x**3*sqrt(-a
/b)/b + 3*a*x**5*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^4}\,dx = -\frac{2}{3}\,bp\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{1}{ax}\right)-\frac{\log\left((bx^2+a)^pc\right)}{3\,x^3}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="maxima")
```

output
```
-2/3*b*p*(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x)) - 1/3*log((b*x^
2 + a)^p*c)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^4}\,dx = -\frac{2\,b^2 p \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a} - \frac{p\log\left(bx^2+a\right)}{3\,x^3} - \frac{2\,bpx^2 + a\log\left(c\right)}{3\,ax^3}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="giac")
```

output
```
-2/3*b^2*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/3*p*log(b*x^2 + a)/x^3
- 1/3*(2*b*p*x^2 + a*log(c))/(a*x^3)
```

Mupad [B] (verification not implemented)

Time = 26.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^4}\,dx = -\frac{\ln\left(c\,(b\,x^2+a)^p\right)}{3\,x^3} - \frac{2\,b^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\,a^{3/2}} - \frac{2\,b\,p}{3\,a\,x}$$

input
```
int(log(c*(a + b*x^2)^p)/x^4,x)
```

output
```
- log(c*(a + b*x^2)^p)/(3*x^3) - (2*b^(3/2)*p*atan((b^(1/2)*x)/a^(1/2)))/(
3*a^(3/2)) - (2*b*p)/(3*a*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^4}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)bp\,x^3 - \log\left(\left(b\,x^2+a\right)^p c\right)a^2 - 2abp\,x^2}{3a^2x^3}$$

input
```
int(log(c*(b*x^2+a)^p)/x^4,x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*p*x**3 - log((a + b*
x**2)**p*c)*a**2 - 2*a*b*p*x**2)/(3*a**2*x**3)
```

3.10 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 64

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx=-\frac{bp}{4ax^2}-\frac{b^2p\log(x)}{2a^2}+\frac{b^2p\log\left(a+bx^2\right)}{4a^2}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}$$

output
```
-1/4*b*p/a/x^2-1/2*b^2*p*ln(x)/a^2+1/4*b^2*p*ln(b*x^2+a)/a^2-1/4*ln(c*(b*x
^2+a)^p)/x^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx=\frac{1}{4}bp\left(-\frac{1}{ax^2}-\frac{2b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{a^2}\right)-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/x^5,x]
```

output
```
(b*p*(-(1/(a*x^2)) - (2*b*Log[x])/a^2 + (b*Log[a + b*x^2])/a^2))/4 - Log[c
*(a + b*x^2)^p]/(4*x^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^2\right)^p\right)}{x^5} \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{\log \left(c\left(bx^2+a\right)^p\right)}{x^6}dx^2$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\left(\frac{1}{2}bp\int \frac{1}{x^4\left(bx^2+a\right)}dx^2-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\left(\frac{1}{2}bp\int \left(\frac{b^2}{a^2\left(bx^2+a\right)}-\frac{b}{a^2x^2}+\frac{1}{ax^4}\right)dx^2-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}bp\left(-\frac{b\log \left(x^2\right)}{a^2}+\frac{b\log \left(a+bx^2\right)}{a^2}-\frac{1}{ax^2}\right)-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

input
```
Int[Log[c*(a + b*x^2)^p]/x^5,x]
```

output
```
((b*p*(-(1/(a*x^2)) - (b*Log[x^2])/a^2 + (b*Log[a + b*x^2])/a^2))/2 - Log[
c*(a + b*x^2)^p]/(2*x^4))/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{4x^{4}}+\frac{pb\left(\frac{b\ln\left(b\,x^{2}+a\right)}{2a^{2}}-\frac{1}{2a\,x^{2}}-\frac{b\ln\left(x\right)}{a^{2}}\right)}{2}$
parallelrisch	$-\frac{2b^{2}p^{2}\ln\left(x\right)x^{4}-x^{4}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)b^{2}p-b^{2}p^{2}x^{4}+ab\,p^{2}x^{2}+\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)a^{2}p}{4x^{4}a^{2}p}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{4x^{4}}-\frac{4b^{2}p\ln\left(x\right)x^{4}-2b^{2}p\ln\left(-b\,x^{2}-a\right)x^{4}+i\pi\,a^{2}\,\mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}-i\pi\,a^{2}\,\mathrm{csgn}\left(i\left(b\,x^{2}+a\right)\right.}{}$

input
```
int(ln(c*(b*x^2+a)^p)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(c*(b*x^2+a)^p)/x^4+1/2*p*b*(1/2*b/a^2*ln(b*x^2+a)-1/2/a/x^2-1/a^2*
b*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{2\,b^2px^4\log(x) + abpx^2 + a^2\log(c) - (b^2px^4 - a^2p)\log(bx^2+a)}{4\,a^2x^4}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*p*x^4*log(x) + a*b*p*x^2 + a^2*log(c) - (b^2*p*x^4 - a^2*p)*lo
g(b*x^2 + a))/(a^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = \begin{cases} -\frac{\log\left(c(a+bx^2)^p\right)}{4x^4} - \frac{bp}{4ax^2} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2\log\left(c(a+bx^2)^p\right)}{4a^2} & \text{for } a \neq 0 \\ -\frac{p}{8x^4} - \frac{\log\left(c(bx^2)^p\right)}{4x^4} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x**5,x)
```

output
```
Piecewise((-log(c*(a + b*x**2)**p)/(4*x**4) - b*p/(4*a*x**2) - b**2*p*log(
x)/(2*a**2) + b**2*log(c*(a + b*x**2)**p)/(4*a**2), Ne(a, 0)), (-p/(8*x**4
) - log(c*(b*x**2)**p)/(4*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = \frac{1}{4}\,bp\left(\frac{b\log\left(bx^2+a\right)}{a^2} - \frac{b\log\left(x^2\right)}{a^2} - \frac{1}{ax^2}\right) - \frac{\log\left(\left(bx^2+a\right)^p c\right)}{4\,x^4}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="maxima")
```

output
```
1/4*b*p*(b*log(b*x^2 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2)) - 1/4*log((b*x
^2 + a)^p*c)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(56) = 112.

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.06

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{\frac{b^3p\log(bx^2+a)}{(bx^2+a)^2-2\,(bx^2+a)a+a^2} - \frac{b^3p\log(bx^2+a)}{a^2} + \frac{b^3p\log(bx^2)}{a^2} + \frac{(bx^2+a)b^3p-ab^3p+ab^3\log(c)}{(bx^2+a)^2a-2\,(bx^2+a)a^2+a^3}}{4\,b}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="giac")
```

output
```
-1/4*(b^3*p*log(b*x^2 + a)/((b*x^2 + a)^2 - 2*(b*x^2 + a)*a + a^2) - b^3*p
*log(b*x^2 + a)/a^2 + b^3*p*log(b*x^2)/a^2 + ((b*x^2 + a)*b^3*p - a*b^3*p
+ a*b^3*log(c))/((b*x^2 + a)^2*a - 2*(b*x^2 + a)*a^2 + a^3))/b
```

Mupad [B] (verification not implemented)

Time = 26.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^5}\,dx = \frac{b^2\,p\,\ln\left(b\,x^2+a\right)}{4\,a^2} - \frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{4\,x^4} - \frac{b^2\,p\,\ln\left(x\right)}{2\,a^2} - \frac{b\,p}{4\,a\,x^2}$$

input
```
int(log(c*(a + b*x^2)^p)/x^5,x)
```

output
```
(b^2*p*log(a + b*x^2))/(4*a^2) - log(c*(a + b*x^2)^p)/(4*x^4) - (b^2*p*log
(x))/(2*a^2) - (b*p)/(4*a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^5}\,dx = \frac{-\log\left(\left(b\,x^2+a\right)^p c\right)a^2+\log\left(\left(b\,x^2+a\right)^p c\right)b^2x^4-2\log(x)\,b^2p\,x^4-abp\,x^2}{4a^2x^4}$$

input
```
int(log(c*(b*x^2+a)^p)/x^5,x)
```

output
```
( - log((a + b*x**2)**p*c)*a**2 + log((a + b*x**2)**p*c)*b**2*x**4 - 2*log
(x)*b**2*p*x**4 - a*b*p*x**2)/(4*a**2*x**4)
```

3.11 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6} dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6} dx = -\frac{2bp}{15ax^3} + \frac{2b^2p}{5a^2x} + \frac{2b^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}} - \frac{\log\left(c(a+bx^2)^p\right)}{5x^5}$$

output
```
-2/15*b*p/a/x^3+2/5*b^2*p/a^2/x+2/5*b^(5/2)*p*arctan(b^(1/2)*x/a^(1/2))/a^
(5/2)-1/5*ln(c*(b*x^2+a)^p)/x^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6} dx = -\frac{2bp\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{bx^2}{a}\right)}{15ax^3} - \frac{\log\left(c(a+bx^2)^p\right)}{5x^5}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/x^6,x]
```

output
```
(-2*b*p*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/a)])/(15*a*x^3) - Log[c
*(a + b*x^2)^p]/(5*x^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2905, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^6}dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{2}{5}bp\int \frac{1}{x^4\left(bx^2+a\right)}dx-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{5}bp\left(-\frac{b\int \frac{1}{x^2(bx^2+a)}dx}{a}-\frac{1}{3ax^3}\right)-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{5}bp\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

$$\Big\downarrow \text{218}$$

$$\frac{2}{5}bp\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

input
```
Int[Log[c*(a + b*x^2)^p]/x^6,x]
```

output
```
(2*b*p*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/a^(3/2)))/a))/5 - Log[c*(a + b*x^2)^p]/(5*x^5)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{5x^{5}}+\frac{2pb\left(\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}-\frac{1}{3a\,x^{3}}+\frac{b}{a^{2}x}\right)}{5}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{5x^{5}}-\frac{-6\left(\sum_{\_R=\mathrm{RootOf}\left(a^{5}\_Z^{2}+b^{5}p^{2}\right)}\_R\ln\left(\left(3\_R^{2}a^{5}+2b^{5}p^{2}\right)x-a^{3}b^{2}p\_R\right)\right)a^{2}x^{5}+3i\pi\,a^{2}\,\mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)}{}$

input
```
int(ln(c*(b*x^2+a)^p)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*ln(c*(b*x^2+a)^p)/x^5+2/5*p*b*(b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))-1/3/a/x^3+b/a^2/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.30

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6}\,dx$$
$$= \left[\frac{3\,b^2px^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,b^2px^4-2\,abpx^2-3\,a^2p\log\left(bx^2+a\right)-3\,a^2\log\left(c\right)}{15\,a^2x^5}, \frac{6\,b^2px^5\sqrt{\frac{b}{a}}\ldots}{\ldots}\right.$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="fricas")
```

output
```
[1/15*(3*b^2*p*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
a)) + 6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(
a^2*x^5), 1/15*(6*b^2*p*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 6*b^2*p*x^4 -
2*a*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(a^2*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(70) = 140$.

Time = 155.89 (sec) , antiderivative size = 583, normalized size of antiderivative = 7.88

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6}\,dx$$
$$= \begin{cases} -\frac{\log(0^pc)}{5x^5} \\ -\frac{\log(a^pc)}{5x^5} \\ -\frac{2p}{25x^5}-\frac{\log\left(c(bx^2)^p\right)}{5x^5} \\ -\frac{\log(0^pc)}{5x^5} \\ -\frac{3a^3\sqrt{-\frac{a}{b}}\log\left(c(a+bx^2)^p\right)}{15a^3x^5\sqrt{-\frac{a}{b}}+15a^2bx^7\sqrt{-\frac{a}{b}}}-\frac{2a^2px^2\sqrt{-\frac{a}{b}}}{\frac{15a^3x^5\sqrt{-\frac{a}{b}}}{b}+15a^2x^7\sqrt{-\frac{a}{b}}}-\frac{3a^2x^2\sqrt{-\frac{a}{b}}\log\left(c(a+bx^2)^p\right)}{\frac{15a^3x^5\sqrt{-\frac{a}{b}}}{b}+15a^2x^7\sqrt{-\frac{a}{b}}}+\frac{6abpx^5\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{15a^3x^5\sqrt{-\frac{a}{b}}}{b}+15a^2x^7\sqrt{-\frac{a}{b}}}+\ldots \end{cases}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x**6,x)
```

output
```
Piecewise((-log(0**p*c)/(5*x**5), Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/(5*x
**5), Eq(b, 0)), (-2*p/(25*x**5) - log(c*(b*x**2)**p)/(5*x**5), Eq(a, 0)),
 (-log(0**p*c)/(5*x**5), Eq(a, -b*x**2)), (-3*a**3*sqrt(-a/b)*log(c*(a + b
*x**2)**p)/(15*a**3*x**5*sqrt(-a/b) + 15*a**2*b*x**7*sqrt(-a/b)) - 2*a**2*
p*x**2*sqrt(-a/b)/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) -
3*a**2*x**2*sqrt(-a/b)*log(c*(a + b*x**2)**p)/(15*a**3*x**5*sqrt(-a/b)/b +
 15*a**2*x**7*sqrt(-a/b)) + 6*a*b*p*x**5*log(x - sqrt(-a/b))/(15*a**3*x**5
*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) + 4*a*b*p*x**4*sqrt(-a/b)/(15*a**
3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) - 3*a*b*x**5*log(c*(a + b*x
**2)**p)/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) + 6*b**2*p*
x**7*log(x - sqrt(-a/b))/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a
/b)) + 6*b**2*p*x**6*sqrt(-a/b)/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*
sqrt(-a/b)) - 3*b**2*x**7*log(c*(a + b*x**2)**p)/(15*a**3*x**5*sqrt(-a/b)/
b + 15*a**2*x**7*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6}\,dx = \frac{2}{15}\,bp\left(\frac{3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{3\,bx^2-a}{a^2x^3}\right)-\frac{\log\left((bx^2+a)^pc\right)}{5\,x^5}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="maxima")
```

output
```
2/15*b*p*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/(a^2
*x^3)) - 1/5*log((b*x^2 + a)^p*c)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6}\,dx = \frac{2\,b^3p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{5\,\sqrt{ab}a^2}-\frac{p\log\left(bx^2+a\right)}{5\,x^5} + \frac{6\,b^2px^4-2\,abpx^2-3\,a^2\log\left(c\right)}{15\,a^2x^5}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="giac")
```

output
```
2/5*b^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/5*p*log(b*x^2 + a)/x^5
 + 1/15*(6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*log(c))/(a^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 26.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^6}\,dx = \frac{2\,b^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5\,a^{5/2}} - \frac{\frac{2\,b\,p}{3\,a}-\frac{2\,b^2\,p\,x^2}{a^2}}{5\,x^3} - \frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{5\,x^5}$$

input
```
int(log(c*(a + b*x^2)^p)/x^6,x)
```

output
```
(2*b^(5/2)*p*atan((b^(1/2)*x)/a^(1/2)))/(5*a^(5/2)) - ((2*b*p)/(3*a) - (2*
b^2*p*x^2)/a^2)/(5*x^3) - log(c*(a + b*x^2)^p)/(5*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^6}\,dx = \frac{6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2p\,x^5 - 3\log\left(\left(b\,x^2+a\right)^p c\right)a^3 - 2a^2bp\,x^2 + 6a\,b^2p\,x^4}{15a^3x^5}$$

input
```
int(log(c*(b*x^2+a)^p)/x^6,x)
```

output
```
(6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*p*x**5 - 3*log((a +
b*x**2)**p*c)*a**3 - 2*a**2*b*p*x**2 + 6*a*b**2*p*x**4)/(15*a**3*x**5)
```

3.12 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log(x)}{3a^3}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}-\frac{\log\left(c(a+bx^2)^p\right)}{6x^6}$$

output
```
-1/12*b*p/a/x^4+1/6*b^2*p/a^2/x^2+1/3*b^3*p*ln(x)/a^3-1/6*b^3*p*ln(b*x^2+a
)/a^3-1/6*ln(c*(b*x^2+a)^p)/x^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = \frac{1}{3}bp\left(-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}\right)-\frac{\log\left(c(a+bx^2)^p\right)}{6x^6}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/x^7,x]
```

output
```
(b*p*(-1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x
^2])/(2*a^3)))/3 - Log[c*(a + b*x^2)^p]/(6*x^6)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^2\right)^p\right)}{x^7} \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2} \int \frac{\log \left(c\left(bx^2+a\right)^p\right)}{x^8}dx^2$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2} \left(\frac{1}{3} bp \int \frac{1}{x^6 \left(bx^2+a\right)}dx^2-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \left(\frac{1}{3} bp \int \left(-\frac{b^3}{a^3 \left(bx^2+a\right)}+\frac{b^2}{a^3 x^2}-\frac{b}{a^2 x^4}+\frac{1}{a x^6}\right)dx^2-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{1}{3} bp \left(\frac{b^2 \log \left(x^2\right)}{a^3}-\frac{b^2 \log \left(a+bx^2\right)}{a^3}+\frac{b}{a^2 x^2}-\frac{1}{2 a x^4}\right)-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

input
```
Int[Log[c*(a + b*x^2)^p]/x^7,x]
```

output
```
((b*p*(-1/2*1/(a*x^4) + b/(a^2*x^2) + (b^2*Log[x^2])/a^3 - (b^2*Log[a + b*
x^2])/a^3))/3 - Log[c*(a + b*x^2)^p]/(3*x^6))/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{6x^6}+\frac{pb\left(-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^3}-\frac{1}{4a\,x^4}+\frac{b^2\ln(x)}{a^3}+\frac{b}{2a^2x^2}\right)}{3}$
parallelrisch	$\frac{4b^3p^2\ln(x)x^6-2x^6\ln\left(c\left(b\,x^2+a\right)^p\right)b^3p-2x^6b^3p^2+2x^4a\,b^2p^2-x^2a^2b\,p^2-2\ln\left(c\left(b\,x^2+a\right)^p\right)a^3p}{12x^6p\,a^3}$
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{6x^6}-\frac{2b^3p\ln\left(b\,x^2+a\right)x^6-4b^3p\ln(x)x^6+i\pi\,a^3\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2-i\pi\,a^3\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right.}{}$

input
```
int(ln(c*(b*x^2+a)^p)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(c*(b*x^2+a)^p)/x^6+1/3*p*b*(-1/2*b^2/a^3*ln(b*x^2+a)-1/4/a/x^4+b^2
/a^3*ln(x)+1/2*b/a^2/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^7} dx = \frac{4 b^3px^6 \log (x) + 2 ab^2px^4 - a^2bpx^2 - 2 a^3 \log (c) - 2 (b^3px^6 + a^3p) \log (bx^2 + a)}{12 a^3x^6}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="fricas")
```

output
```
1/12*(4*b^3*p*x^6*log(x) + 2*a*b^2*p*x^4 - a^2*b*p*x^2 - 2*a^3*log(c) - 2*
(b^3*p*x^6 + a^3*p)*log(b*x^2 + a))/(a^3*x^6)
```

Sympy [A] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^7} dx = \begin{cases} -\frac{\log \left(c(a+bx^2)^p\right)}{6x^6} - \frac{bp}{12ax^4} + \frac{b^2p}{6a^2x^2} + \frac{b^3p\log (x)}{3a^3} - \frac{b^3 \log \left(c(a+bx^2)^p\right)}{6a^3} & \text{for } a \neq 0 \\ -\frac{p}{18x^6} - \frac{\log \left(c(bx^2)^p\right)}{6x^6} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x**7,x)
```

output
```
Piecewise((-log(c*(a + b*x**2)**p)/(6*x**6) - b*p/(12*a*x**4) + b**2*p/(6*
a**2*x**2) + b**3*p*log(x)/(3*a**3) - b**3*log(c*(a + b*x**2)**p)/(6*a**3)
, Ne(a, 0)), (-p/(18*x**6) - log(c*(b*x**2)**p)/(6*x**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{1}{12}\,bp\left(\frac{2\,b^2\log\left(bx^2+a\right)}{a^3}-\frac{2\,b^2\log\left(x^2\right)}{a^3}-\frac{2\,bx^2-a}{a^2x^4}\right) - \frac{\log\left(\left(bx^2+a\right)^p c\right)}{6\,x^6}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="maxima")
```

output
```
-1/12*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 - a)/(
a^2*x^4)) - 1/6*log((b*x^2 + a)^p*c)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(68) = 136.

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.45

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{\frac{2\,b^4p\log\left(bx^2+a\right)}{\left(bx^2+a\right)^3-3\left(bx^2+a\right)^2a+3\left(bx^2+a\right)a^2-a^3}+\frac{2\,b^4p\log\left(bx^2+a\right)}{a^3}-\frac{2\,b^4p\log\left(bx^2\right)}{a^3}-\frac{2\left(bx^2+a\right)^2b^4p-5\left(bx^2+a\right)ab^4p+3\,a^2b^4p-2\,a^2b^4\log}{\left(bx^2+a\right)^3a^2-3\left(bx^2+a\right)^2a^3+3\left(bx^2+a\right)a^4-a^5}}{12\,b}$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="giac")
```

output
```
-1/12*(2*b^4*p*log(b*x^2 + a)/((b*x^2 + a)^3 - 3*(b*x^2 + a)^2*a + 3*(b*x^
2 + a)*a^2 - a^3) + 2*b^4*p*log(b*x^2 + a)/a^3 - 2*b^4*p*log(b*x^2)/a^3 -
(2*(b*x^2 + a)^2*b^4*p - 5*(b*x^2 + a)*a*b^4*p + 3*a^2*b^4*p - 2*a^2*b^4*l
og(c))/((b*x^2 + a)^3*a^2 - 3*(b*x^2 + a)^2*a^3 + 3*(b*x^2 + a)*a^4 - a^5)
)/b
```

Mupad [B] (verification not implemented)

Time = 26.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = \frac{b^2\,p}{6\,a^2\,x^2} - \frac{b^3\,p\,\ln\left(b\,x^2+a\right)}{6\,a^3} - \frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{6\,x^6} + \frac{b^3\,p\,\ln\left(x\right)}{3\,a^3} - \frac{b\,p}{12\,a\,x^4}$$

input
```
int(log(c*(a + b*x^2)^p)/x^7,x)
```

output
```
(b^2*p)/(6*a^2*x^2) - (b^3*p*log(a + b*x^2))/(6*a^3) - log(c*(a + b*x^2)^p
)/(6*x^6) + (b^3*p*log(x))/(3*a^3) - (b*p)/(12*a*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = \frac{-2\log\left(\left(b\,x^2+a\right)^p c\right)a^3 - 2\log\left(\left(b\,x^2+a\right)^p c\right)b^3x^6 + 4\log(x)\,b^3p\,x^6 - a^2bp\,x^2 + 2a\,b^2p\,x^4}{12a^3x^6}$$

input
```
int(log(c*(b*x^2+a)^p)/x^7,x)
```

output
```
( - 2*log((a + b*x**2)**p*c)*a**3 - 2*log((a + b*x**2)**p*c)*b**3*x**6 + 4
*log(x)*b**3*p*x**6 - a**2*b*p*x**2 + 2*a*b**2*p*x**4)/(12*a**3*x**6)
```

3.13 $\int x^5 \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int x^5 \log \left(c(a+bx^3)^p\right) \, dx = \frac{apx^3}{6b} - \frac{px^6}{12} - \frac{a^2 p \log(a+bx^3)}{6b^2} + \frac{1}{6}x^6 \log \left(c(a+bx^3)^p\right)$$

output
```
1/6*a*p*x^3/b-1/12*p*x^6-1/6*a^2*p*ln(b*x^3+a)/b^2+1/6*x^6*ln(c*(b*x^3+a)^
p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^5 \log \left(c(a+bx^3)^p\right) \, dx = \frac{apx^3}{6b} - \frac{px^6}{12} - \frac{a^2 p \log(a+bx^3)}{6b^2} + \frac{1}{6}x^6 \log \left(c(a+bx^3)^p\right)$$

input
```
Integrate[x^5*Log[c*(a + b*x^3)^p],x]
```

output
```
(a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c
*(a + b*x^3)^p])/6
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{3}\int x^3 \log \left(c\left(bx^3+a\right)^p\right)dx^3$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6 \log \left(c\left(a+bx^3\right)^p\right)-\frac{1}{2}bp\int \frac{x^6}{bx^3+a}dx^3\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6 \log \left(c\left(a+bx^3\right)^p\right)-\frac{1}{2}bp\int \left(\frac{x^3}{b}+\frac{a^2}{b^2\left(bx^3+a\right)}-\frac{a}{b^2}\right)dx^3\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6 \log \left(c\left(a+bx^3\right)^p\right)-\frac{1}{2}bp\left(\frac{a^2\log \left(a+bx^3\right)}{b^3}-\frac{ax^3}{b^2}+\frac{x^6}{2b}\right)\right)$$

input
```
Int[x^5*Log[c*(a + b*x^3)^p],x]
```

output
```
(-1/2*(b*p*(-((a*x^3)/b^2) + x^6/(2*b) + (a^2*Log[a + b*x^3])/b^3)) + (x^6
*Log[c*(a + b*x^3)^p])/2)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
parts	$\frac{x^{6} \ln\left(c \left(b \,x^{3}+a \right)^{p}\right)}{6}-\frac{p b \left(-\frac{-\frac{1}{2} b \,x^{6}+a \,x^{3}}{3 b^{2}}+\frac{a^{2} \ln\left(b \,x^{3}+a \right)}{3 b^{3}}\right)}{2}$	57
parallelrisch	$-\frac{-2 x^{6} \ln\left(c \left(b \,x^{3}+a \right)^{p}\right) b^{2}+x^{6} b^{2} p -2 a b p \,x^{3}+2 \ln\left(b \,x^{3}+a \right) a^{2} p +2 a^{2} p}{12 b^{2}}$	63
risch	Expression too large to display	1190

input
```
int(x^5*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*ln(c*(b*x^3+a)^p)-1/2*p*b*(-1/3/b^2*(-1/2*b*x^6+a*x^3)+1/3*a^2/b^3
*ln(b*x^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$= -\frac{b^2px^6 - 2\,b^2x^6\log\left(c\right) - 2\,abpx^3 - 2\,\left(b^2px^6 - a^2p\right)\log\left(bx^3+a\right)}{12\,b^2}$$

input
```
integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
-1/12*(b^2*p*x^6 - 2*b^2*x^6*log(c) - 2*a*b*p*x^3 - 2*(b^2*p*x^6 - a^2*p)*
log(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \begin{cases} -\frac{a^2\log\left(c\left(a+bx^3\right)^p\right)}{6b^2} + \frac{apx^3}{6b} - \frac{px^6}{12} + \frac{x^6\log\left(c\left(a+bx^3\right)^p\right)}{6} & \text{for}\ b \neq 0 \\ \frac{x^6\log\left(a^pc\right)}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*ln(c*(b*x**3+a)**p),x)
```

output
```
Piecewise((-a**2*log(c*(a + b*x**3)**p)/(6*b**2) + a*p*x**3/(6*b) - p*x**6
/12 + x**6*log(c*(a + b*x**3)**p)/6, Ne(b, 0)), (x**6*log(a**p*c)/6, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right) dx = \frac{1}{6} x^6 \log \left(\left(bx^3+a\right)^p c\right) - \frac{1}{12} bp\left(\frac{2 a^2 \log \left(bx^3+a\right)}{b^3} + \frac{bx^6-2 ax^3}{b^2}\right)$$

input
```
integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
1/6*x^6*log((b*x^3 + a)^p*c) - 1/12*b*p*(2*a^2*log(b*x^3 + a)/b^3 + (b*x^6
 - 2*a*x^3)/b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.64

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right) dx = \frac{2 \left(bx^3+a\right)^2 p \log \left(bx^3+a\right) - \left(bx^3+a\right)^2 p + 2 \left(bx^3+a\right)^2 \log \left(c\right)}{12 b^2} + \frac{(bx^3 - (bx^3+a) \log \left(bx^3+a\right) + a)ap - (bx^3+a)a \log \left(c\right)}{3 b^2}$$

input
```
integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
1/12*(2*(b*x^3 + a)^2*p*log(b*x^3 + a) - (b*x^3 + a)^2*p + 2*(b*x^3 + a)^2
*log(c))/b^2 + 1/3*((b*x^3 - (b*x^3 + a)*log(b*x^3 + a) + a)*a*p - (b*x^3
+ a)*a*log(c))/b^2
```

Mupad [B] (verification not implemented)

Time = 26.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right)\,dx = \frac{x^6\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{6}-\frac{p\,x^6}{12}-\frac{a^2\,p\,\ln\left(b\,x^3+a\right)}{6\,b^2}+\frac{a\,p\,x^3}{6\,b}$$

input
```
int(x^5*log(c*(a + b*x^3)^p),x)
```

output
```
(x^6*log(c*(a + b*x^3)^p))/6 - (p*x^6)/12 - (a^2*p*log(a + b*x^3))/(6*b^2)
 + (a*p*x^3)/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right)\,dx = \frac{-2\log\left(\left(b\,x^3+a\right)^p c\right) a^2+2\log\left(\left(b\,x^3+a\right)^p c\right) b^2x^6+2abp\,x^3-b^2p\,x^6}{12b^2}$$

input
```
int(x^5*log(c*(b*x^3+a)^p),x)
```

output
```
( - 2*log((a + b*x**3)**p*c)*a**2 + 2*log((a + b*x**3)**p*c)*b**2*x**6 + 2
*a*b*p*x**3 - b**2*p*x**6)/(12*b**2)
```

3.14 $\int x^4 \log\left(c(a+bx^3)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 159

$$\int x^4 \log\left(c(a+bx^3)^p\right)\,dx = \frac{3apx^2}{10b} - \frac{3px^5}{25} + \frac{\sqrt{3}a^{5/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5b^{5/3}} + \frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}} - \frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10b^{5/3}} + \frac{1}{5}x^5\log\left(c(a+bx^3)^p\right)$$

output
```
3/10*a*p*x^2/b-3/25*p*x^5+1/5*3^(1/2)*a^(5/3)*p*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)*3^(1/2)/a^(1/3))/b^(5/3)+1/5*a^(5/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(5/3
)-1/10*a^(5/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)+1/5*x^5
*ln(c*(b*x^3+a)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \frac{3apx^2}{10b} - \frac{3px^5}{25} - \frac{3apx^2 \text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)}{10b} + \frac{1}{5}x^5 \log \left(c\left(a+bx^3\right)^p\right)$$

input
```
Integrate[x^4*Log[c*(a + b*x^3)^p],x]
```

output
```
(3*a*p*x^2)/(10*b) - (3*p*x^5)/25 - (3*a*p*x^2*Hypergeometric2F1[2/3, 1, 5
/3, -((b*x^3)/a)])/(10*b) + (x^5*Log[c*(a + b*x^3)^p])/5
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2905, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{5}x^5 \log \left(c\left(a+bx^3\right)^p\right) - \frac{3}{5}bp\int \frac{x^7}{bx^3+a}dx$$

$$\Big\downarrow \text{ 831}$$

$$\frac{1}{5}x^5 \log \left(c\left(a+bx^3\right)^p\right) - \frac{3}{5}bp\int \left(\frac{x^4}{b}+\frac{a^2x}{b^2\left(bx^3+a\right)}-\frac{ax}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5\log\left(c(a+bx^3)^p\right)-\frac{3}{5}bp\left(-\frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}-\frac{ax^2}{2b^2}+\frac{x^5}{5b}\right)$$

input
```
Int[x^4*Log[c*(a + b*x^3)^p],x]
```

output
```
(-3*b*p*(-1/2*(a*x^2)/b^2 + x^5/(5*b) - (a^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) - (a^(5/3)*Log[a^(1/3) + b^(1/
3)*x])/(3*b^(8/3)) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(6*b^(8/3))))/5 + (x^5*Log[c*(a + b*x^3)^p])/5
```

Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

method	result
parts	$\frac{x^5 \ln(c(b x^3+a)^p)}{5} - \frac{3pb\left(-\frac{-\frac{1}{5}b x^5+\frac{1}{2}a x^2}{b^2}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a^2}{b^2}\right)}{5}$
risch	$\frac{x^5 \ln((b x^3+a)^p)}{5}+\frac{i\pi x^5 \operatorname{csgn}(i(b x^3+a)^p)\operatorname{csgn}(ic(b x^3+a)^p)^2}{10}-\frac{i\pi x^5 \operatorname{csgn}(i(b x^3+a)^p)\operatorname{csgn}(ic(b x^3+a)^p)\operatorname{csgn}(ic)}{10}-\frac{i\pi x^5}{}$

input
```
int(x^4*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*ln(c*(b*x^3+a)^p)-3/5*p*b*(-1/b^2*(-1/5*b*x^5+1/2*a*x^2)+(-1/3/b/(
1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*
x+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x-1)))*a^2/b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right)\,dx$$
$$= \frac{10\,bpx^5\log\left(bx^3+a\right) - 6\,bpx^5 + 10\,bx^5\log\left(c\right) + 15\,apx^2 - 10\,\sqrt{3}ap\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right) - 5}{50\,b}$$

input
```
integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
1/50*(10*b*p*x^5*log(b*x^3 + a) - 6*b*p*x^5 + 10*b*x^5*log(c) + 15*a*p*x^2
 - 10*sqrt(3)*a*p*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3
) - sqrt(3)*a)/a) - 5*a*p*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3)
+ a*(a^2/b^2)^(1/3)) + 10*a*p*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3))
)/b
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right)\,dx = \text{Timed out}$$

input
```
integrate(x**4*ln(c*(b*x**3+a)**p),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int x^4 \log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{1}{5}\,x^5\log\left(\left(bx^3+a\right)^p c\right) - \frac{1}{50}\,bp\left(\frac{10\,\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{10\,a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + 3$$

input
```
integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
1/5*x^5*log((b*x^3 + a)^p*c) - 1/50*b*p*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 5*a^2*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 10*a^2*log(x + (a/b)^(1/3))/(
b^3*(a/b)^(1/3)) + 3*(2*b*x^5 - 5*a*x^2)/b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.02

$$\int x^4 \log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{1}{10}\,a^2b^4p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^5} + \frac{2\,\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^7} - \frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\right.}{ab^7} + \frac{1}{5}\,px^5\log\left(bx^3+a\right) - \frac{1}{25}\left(3\,p-5\,\log\left(c\right)\right)x^5 + \frac{3\,apx^2}{10\,b}$$

input
```
integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
1/10*a^2*b^4*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) + 2*sqrt
(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(
a*b^7) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^7))
+ 1/5*p*x^5*log(b*x^3 + a) - 1/25*(3*p - 5*log(c))*x^5 + 3/10*a*p*x^2/b
```

Mupad [B] (verification not implemented)

Time = 28.68 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right)\,dx = \frac{x^5\,\ln\left(c\,\left(b\,x^3+a\right)^p\right)}{5} - \frac{3\,p\,x^5}{25} + \frac{a^{5/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{5\,b^{5/3}} + \frac{3\,a\,p\,x^2}{10\,b} + \frac{a^{5/3}\,p\,\ln\left(\frac{9\,a^4\,p^2\,x}{25\,b}+\frac{9\,a^{13/3}\,p^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{25\,b^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{5\,b^{5/3}} - \frac{a^{5/3}\,p\,\ln\left(\frac{9\,a^4\,p^2\,x}{25\,b}+\frac{9\,a^{13/3}\,p^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{25\,b^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{5\,b^{5/3}}$$

input
```
int(x^4*log(c*(a + b*x^3)^p),x)
```

output
```
(x^5*log(c*(a + b*x^3)^p))/5 - (3*p*x^5)/25 + (a^(5/3)*p*log(b^(1/3)*x + a
^(1/3)))/(5*b^(5/3)) + (3*a*p*x^2)/(10*b) + (a^(5/3)*p*log((9*a^4*p^2*x)/(
25*b) + (9*a^(13/3)*p^2*((3^(1/2)*1i)/2 - 1/2)^2)/(25*b^(4/3)))*((3^(1/2)*
1i)/2 - 1/2))/(5*b^(5/3)) - (a^(5/3)*p*log((9*a^4*p^2*x)/(25*b) + (9*a^(13
/3)*p^2*((3^(1/2)*1i)/2 + 1/2)^2)/(25*b^(4/3)))*((3^(1/2)*1i)/2 + 1/2))/(5
*b^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\int x^4 \log \left(c \left(a+b x^3\right)^p\right) \, dx$$

$$= \frac{10\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) a^2 p + 10b^{\frac{5}{3}} a^{\frac{1}{3}} \log\left(\left(b\, x^3+a\right)^p c\right) x^5 + 15 b^{\frac{2}{3}} a^{\frac{4}{3}} p\, x^2 - 6 b^{\frac{5}{3}} a^{\frac{1}{3}} p\, x^5 + 15 \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right)}{50 b^{\frac{5}{3}} a^{\frac{1}{3}}}$$

input
```
int(x^4*log(c*(b*x^3+a)^p),x)
```

output
```
(10*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*p + 10
*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*b*x**5 + 15*b**(2/3)*a**(1/3)*a*
p*x**2 - 6*b**(2/3)*a**(1/3)*b*p*x**5 + 15*log(a**(1/3) + b**(1/3)*x)*a**2
*p - 5*log((a + b*x**3)**p*c)*a**2)/(50*b**(2/3)*a**(1/3)*b)
```

3.15 $\int x^3 \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 157

$$\int x^3 \log \left(c(a+bx^3)^p\right) \, dx = \frac{3apx}{4b} - \frac{3px^4}{16} + \frac{\sqrt{3}a^{4/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}} - \frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}} + \frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}} + \frac{1}{4}x^4\log\left(c(a+bx^3)^p\right)$$

output
```
3/4*a*p*x/b-3/16*p*x^4+1/4*3^(1/2)*a^(4/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)*3^(1/2)/a^(1/3))/b^(4/3)-1/4*a^(4/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)+1
/8*a^(4/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)+1/4*x^4*ln(
c*(b*x^3+a)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int x^3 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$

$$= \frac{12a\sqrt[3]{b}px - 3b^{4/3}px^4 + 4\sqrt{3}a^{4/3}p\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 4a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + 2a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\right.}{16b^{4/3}}$$

input
```
Integrate[x^3*Log[c*(a + b*x^3)^p],x]
```

output
```
(12*a*b^(1/3)*p*x - 3*b^(4/3)*p*x^4 + 4*Sqrt[3]*a^(4/3)*p*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*a^(4/3)*p*Log[a^(1/3) + b^(1/3)*x] + 2*a^(
4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*b^(4/3)*x^4*Log[
c*(a + b*x^3)^p])/(16*b^(4/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2905, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{4}x^4 \log \left(c\left(a+bx^3\right)^p\right) - \frac{3}{4}bp\int \frac{x^6}{bx^3+a}dx$$

$$\Big\downarrow \text{ 831}$$

$$\frac{1}{4}x^4 \log \left(c\left(a+bx^3\right)^p\right) - \frac{3}{4}bp\int \left(\frac{x^3}{b}+\frac{a^2}{b^2\left(bx^3+a\right)}-\frac{a}{b^2}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{1}{4}x^4\log\left(c(a+bx^3)^p\right)-\frac{3}{4}bp\left(-\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}}-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}}-\frac{ax}{b^2}+\frac{x^4}{4b}\right)$$

input
```
Int[x^3*Log[c*(a + b*x^3)^p],x]
```

output
```
(-3*b*p*(-((a*x)/b^2) + x^4/(4*b) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*x
])/(3*b^(7/3)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
(6*b^(7/3))))/4 + (x^4*Log[c*(a + b*x^3)^p])/4
```

Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

method	result
parts	$\frac{x^4\ln\left(c\left(b\,x^3+a\right)^p\right)}{4}-\frac{3pb\left(-\frac{-\frac{1}{4}b\,x^4+ax}{b^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a^2}{b^2}\right)}{4}$
risch	$\frac{x^4\ln\left(\left(b\,x^3+a\right)^p\right)}{4}+\frac{i\pi\,x^4\,\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{8}-\frac{i\pi\,x^4\,\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\,\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)\,\mathrm{csgn}\left(ic\right)}{8}-\frac{i\pi\,x^4}{}$

input
```
int(x^3*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*ln(c*(b*x^3+a)^p)-3/4*p*b*(-1/b^2*(-1/4*b*x^4+a*x)+(1/3/b/(1/b*a)^
(2/3)*ln(x+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*
a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*
x-1)))*a^2/b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int x^3 \log\left(c(a+bx^3)^p\right)\,dx$$

$$= \frac{4\,bpx^4\log\left(bx^3+a\right) - 3\,bpx^4 + 4\,bx^4\log\left(c\right) + 4\,\sqrt{3}ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3\,a}\right) - 2\,ap\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log}{16\,b}$$

input
```
integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
1/16*(4*b*p*x^4*log(b*x^3 + a) - 3*b*p*x^4 + 4*b*x^4*log(c) + 4*sqrt(3)*a*
p*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) - 2*
a*p*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 4*a*p*(-a/b)^(
1/3)*log(x - (-a/b)^(1/3)) + 12*a*p*x)/b
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \log\left(c(a+bx^3)^p\right)\,dx = \text{Timed out}$$

input
```
integrate(x**3*ln(c*(b*x**3+a)**p),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int x^3 \log \left(c \left(a+b x^3\right)^p\right) dx = \frac{1}{4} x^4 \log \left(\left(b x^3+a\right)^p c\right) - \frac{1}{16} b p \left(\frac{4 \sqrt{3} a^2 \arctan \left(\frac{\sqrt{3} \left(2 x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3 \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{2 a^2 \log \left(x^2-x \left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^3 \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{4 a^2 \log \left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3 \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{3 (b \cdots}{\cdots}\right.$$

input
```
integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
1/4*x^4*log((b*x^3 + a)^p*c) - 1/16*b*p*(4*sqrt(3)*a^2*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 2*a^2*log(x^2 - x*(a/
b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 4*a^2*log(x + (a/b)^(1/3))/(b^
3*(a/b)^(2/3)) + 3*(b*x^4 - 4*a*x)/b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02

$$\int x^3 \log \left(c \left(a+b x^3\right)^p\right) dx = \frac{1}{8} a^2 b^3 p \left(\frac{2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log \left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a b^4} - \frac{2 \sqrt{3} (-a b^2)^{\frac{1}{3}} \arctan \left(\frac{\sqrt{3} \left(2 x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a b^5} - \frac{(-a b^2)^{\frac{1}{3}} \log \left(x^2+x \left(\cdots\right.\right.}{a b^5} \cdots\right. + \frac{1}{4} p x^4 \log \left(b x^3+a\right) - \frac{1}{16} (3 p - 4 \log (c)) x^4 + \frac{3 a p x}{4 b}$$

input
```
integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
1/8*a^2*b^3*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 2*sqrt(
3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^5) - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5)) +
 1/4*p*x^4*log(b*x^3 + a) - 1/16*(3*p - 4*log(c))*x^4 + 3/4*a*p*x/b
```

Mupad [B] (verification not implemented)

Time = 28.73 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int x^3\log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{x^4\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{4}-\frac{3\,p\,x^4}{16}+\frac{3\,a\,p\,x}{4\,b}-\frac{a^{4/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{4\,b^{4/3}}+\frac{a^{4/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,b^{4/3}}-\frac{a^{4/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,b^{4/3}}$$

input
```
int(x^3*log(c*(a + b*x^3)^p),x)
```

output
```
(x^4*log(c*(a + b*x^3)^p))/4 - (3*p*x^4)/16 + (3*a*p*x)/(4*b) - (a^(4/3)*p
*log(b^(1/3)*x + a^(1/3)))/(4*b^(4/3)) + (a^(4/3)*p*log(2*b^(1/3)*x - 3^(1
/2)*a^(1/3)*1i - a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(4*b^(4/3)) - (a^(4/3)*p
*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(
4*b^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

$$\int x^3\log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)p-6a^{\frac{4}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)p+2a^{\frac{4}{3}}\log\left(\left(b\,x^3+a\right)^p c\right)+4b^{\frac{4}{3}}\log\left(\left(b\,x^3+a\right)^p c\right)x^4+12\cdots}{16b^{\frac{4}{3}}}$$

input
```
int(x^3*log(c*(b*x^3+a)^p),x)
```

output
```
(4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*p
 - 6*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*p + 2*a**(1/3)*log((a + b*x**3)
**p*c)*a + 4*b**(1/3)*log((a + b*x**3)**p*c)*b*x**4 + 12*b**(1/3)*a*p*x -
3*b**(1/3)*b*p*x**4)/(16*b**(1/3)*b)
```

3.16 $\int x^2 \log \left(c\left(a+b x^3\right)^p\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int x^2 \log \left(c\left(a+b x^3\right)^p\right) dx=-\frac{p x^3}{3}+\frac{\left(a+b x^3\right) \log \left(c\left(a+b x^3\right)^p\right)}{3 b}$$

output
```
-1/3*p*x^3+1/3*(b*x^3+a)*ln(c*(b*x^3+a)^p)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int x^2 \log \left(c\left(a+b x^3\right)^p\right) dx=\frac{1}{3}\left(-p x^3+\frac{\left(a+b x^3\right) \log \left(c\left(a+b x^3\right)^p\right)}{b}\right)$$

input
```
Integrate[x^2*Log[c*(a + b*x^3)^p],x]
```

output
```
(-(p*x^3) + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/b)/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log\left(c(a+bx^3)^p\right) dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{3}\int \log\left(c(bx^3+a)^p\right) dx^3$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log\left(c(bx^3+a)^p\right) d(bx^3+a)}{3b}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(a+bx^3)\log\left(c(a+bx^3)^p\right) - p(a+bx^3)}{3b}$$

input
```
Int[x^2*Log[c*(a + b*x^3)^p],x]
```

output
```
(-(p*(a + b*x^3)) + (a + b*x^3)*Log[c*(a + b*x^3)^p])/(3*b)
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)\left(b\,x^{3}+a\right)-\left(b\,x^{3}+a\right)p}{3b}$
default	$\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)\left(b\,x^{3}+a\right)-\left(b\,x^{3}+a\right)p}{3b}$
parts	$\frac{x^{3}\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{3}-pb\left(\frac{x^{3}}{3b}-\frac{a\ln\left(b\,x^{3}+a\right)}{3b^{2}}\right)$
parallelrisch	$\frac{x^{3}\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)bp-x^{3}b\,p^{2}+\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)ap+a\,p^{2}}{3pb}$
risch	$\frac{x^{3}\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{3}+\frac{i\pi\,x^{3}\,\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}}{6}-\frac{i\pi\,x^{3}\,\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}}{6}$

input
```
int(x^2*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(ln(c*(b*x^3+a)^p)*(b*x^3+a)-(b*x^3+a)*p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int x^{2}\log\left(c\left(a+bx^{3}\right)^{p}\right)\,dx=-\frac{bpx^{3}-bx^{3}\log\left(c\right)-\left(bpx^{3}+ap\right)\log\left(bx^{3}+a\right)}{3\,b}$$

input
```
integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
-1/3*(b*p*x^3 - b*x^3*log(c) - (b*p*x^3 + a*p)*log(b*x^3 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int x^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \begin{cases} \frac{a \log \left(c(a+bx^3)^p\right)}{3b} - \frac{px^3}{3} + \frac{x^3 \log \left(c(a+bx^3)^p\right)}{3} & \text{for } b \neq 0 \\ \frac{x^3 \log \left(a^p c\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*ln(c*(b*x**3+a)**p),x)
```

output
```
Piecewise((a*log(c*(a + b*x**3)**p)/(3*b) - p*x**3/3 + x**3*log(c*(a + b*x
**3)**p)/3, Ne(b, 0)), (x**3*log(a**p*c)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int x^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \frac{1}{3} x^3 \log \left(\left(bx^3+a\right)^p c\right) - \frac{1}{3} \left(\frac{x^3}{b} - \frac{a \log \left(bx^3+a\right)}{b^2}\right) bp$$

input
```
integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
1/3*x^3*log((b*x^3 + a)^p*c) - 1/3*(x^3/b - a*log(b*x^3 + a)/b^2)*b*p
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int x^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx = -\frac{\left(bx^3 - \left(bx^3+a\right) \log \left(bx^3+a\right) + a\right)p - \left(bx^3+a\right) \log \left(c\right)}{3\,b}$$

input
```
integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
-1/3*((b*x^3 - (b*x^3 + a)*log(b*x^3 + a) + a)*p - (b*x^3 + a)*log(c))/b
```

Mupad [B] (verification not implemented)

Time = 26.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int x^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \frac{x^3 \ln \left(c \left(b \, x^3+a\right)^p\right)}{3}-\frac{p \, x^3}{3}+\frac{a \, p \, \ln \left(b \, x^3+a\right)}{3 \, b}$$

input
```
int(x^2*log(c*(a + b*x^3)^p),x)
```

output
```
(x^3*log(c*(a + b*x^3)^p))/3 - (p*x^3)/3 + (a*p*log(a + b*x^3))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int x^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \frac{\log\left(\left(b \, x^3+a\right)^p c\right) a+\log\left(\left(b \, x^3+a\right)^p c\right) b \, x^3-bp \, x^3}{3b}$$

input
```
int(x^2*log(c*(b*x^3+a)^p),x)
```

output
```
(log((a + b*x**3)**p*c)*a + log((a + b*x**3)**p*c)*b*x**3 - b*p*x**3)/(3*b
)
```

3.17 $\int x \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 147

$$\int x \log \left(c(a+bx^3)^p\right) \, dx = -\frac{3px^2}{4} - \frac{\sqrt{3}a^{2/3}p \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} - \frac{a^{2/3}p \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} + \frac{a^{2/3}p \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}} + \frac{1}{2}x^2 \log\left(c(a+bx^3)^p\right)$$

output
```
-3/4*p*x^2-1/2*3^(1/2)*a^(2/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/
a^(1/3))/b^(2/3)-1/2*a^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)+1/4*a^(2/3)*p
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)+1/2*x^2*ln(c*(b*x^3+a)^
p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.36

$$\int x \log \left(c\left(a+b x^3\right)^p\right) \, dx=-\frac{3 p x^2}{4}+\frac{3}{4} p x^2 \text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{b x^3}{a}\right)+\frac{1}{2} x^2 \log \left(c\left(a+b x^3\right)^p\right)$$

input
```
Integrate[x*Log[c*(a + b*x^3)^p],x]
```

output
```
(-3*p*x^2)/4 + (3*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 +
(x^2*Log[c*(a + b*x^3)^p])/2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2905, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(c\left(a+b x^3\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{2} x^2 \log \left(c\left(a+b x^3\right)^p\right)-\frac{3}{2} b p \int \frac{x^4}{b x^3+a}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{2} x^2 \log \left(c\left(a+b x^3\right)^p\right)-\frac{3}{2} b p \left(\frac{x^2}{2 b}-\frac{a \int \frac{x}{b x^3+a}dx}{b}\right)$$

$$\Big\downarrow \text{ 821}$$

$$\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{3}{2}bp\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)$$

$\Big\downarrow$ 16

$$\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{3}{2}bp\left(\frac{x^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)$$

$\Big\downarrow$ 1142

$$\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{3}{2}bp\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{3}{2}bp\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{3}{2}bp\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{3}{2}bp\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)$$

$\downarrow$ 217

$$\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{3}{2}bp\left(\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)$$

$\downarrow$ 1103

$$\frac{1}{2}x^2\log\left(c(a+bx^3)^p\right)-\frac{3}{2}bp\left(\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)$$

input
```
Int[x*Log[c*(a + b*x^3)^p],x]
```

output
```
(-3*b*p*(x^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))
))/b))/2 + (x^2*Log[c*(a + b*x^3)^p])/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

method	result
parts	$\frac{x^{2}\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{2}-\frac{3pb\left(\frac{x^{2}}{2b}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}\right)}{2}$
risch	$\frac{x^{2}\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{2}+\frac{i\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)x^{2}\pi}{4}-\frac{i\pi\,x^{2}\,\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\right)}{4}-\frac{i\pi\,x^{2}}{}$

input
```
int(x*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(b*x^3+a)^p)-3/2*p*b*(1/2*x^2/b-(-1/3/b/(1/b*a)^(1/3)*ln(x+(1
/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3
*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))/b*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

$$\int x\log\left(c(a+bx^3)^p\right)\,dx=\frac{1}{2}px^2\log\left(bx^3+a\right)-\frac{3}{4}px^2+\frac{1}{2}x^2\log\left(c\right)+\frac{1}{2}\sqrt{3}p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-\frac{1}{4}p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+\frac{1}{2}p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)$$

input
```
integrate(x*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
1/2*p*x^2*log(b*x^3 + a) - 3/4*p*x^2 + 1/2*x^2*log(c) + 1/2*sqrt(3)*p*(-a^
2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) -
1/4*p*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/
3)) + 1/2*p*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3))
```

Sympy [A] (verification not implemented)

Time = 56.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.21

$$\int x\log\left(c(a+bx^3)^p\right)\,dx=\begin{cases}\frac{x^2\log\left(0^pc\right)}{2}\\ \frac{x^2\log\left(a^pc\right)}{2}\\ -\frac{3px^2}{4}+\frac{x^2\log\left(c(bx^3)^p\right)}{2}\\ -\frac{3px^2}{4}+\frac{3p\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{4}-\frac{\sqrt{3}p\left(-\frac{a}{b}\right)^{\frac{2}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{2}+\frac{x^2\log\left(c(a+bx^3)^p\right)}{2}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(c\right.}{2}\end{cases}$$

input
```
integrate(x*ln(c*(b*x**3+a)**p),x)
```

output
```
Piecewise((x**2*log(0**p*c)/2, Eq(a, 0) & Eq(b, 0)), (x**2*log(a**p*c)/2,
Eq(b, 0)), (-3*p*x**2/4 + x**2*log(c*(b*x**3)**p)/2, Eq(a, 0)), (-3*p*x**2
/4 + 3*p*(-a/b)**(2/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/4
 - sqrt(3)*p*(-a/b)**(2/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt(3)/3)
/2 + x**2*log(c*(a + b*x**3)**p)/2 - (-a/b)**(2/3)*log(c*(a + b*x**3)**p)/
2, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int x\log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{4}\,bp\left(\frac{3\,x^2}{b}-\frac{2\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) + \frac{1}{2}\,x^2\log\left(\left(bx^3+a\right)^pc\right)$$

input
```
integrate(x*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
-1/4*b*p*(3*x^2/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(b^2*(a/b)^(1/3)) - a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^
2*(a/b)^(1/3)) + 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))) + 1/2*x^2*log
((b*x^3 + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

$$\int x \log \left(c \left(a+b x^3\right)^p\right) \, dx = -\frac{1}{4}\, ab^2 p \left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log \left(\left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right| \right)}{ab^2} + \frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}} \arctan \left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{ab^4} - \frac{(-ab^2)^{\frac{2}{3}} \log \left(x^2 + x \right.}{ab} \right.$$

$$+\frac{1}{2}\, px^2 \log \left(bx^3 + a\right) - \frac{1}{4}\,(3\,p - 2\,\log \left(c\right))x^2$$

input
```
integrate(x*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
-1/4*a*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2*sqrt(3
)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*
b^4) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)) +
1/2*p*x^2*log(b*x^3 + a) - 1/4*(3*p - 2*log(c))*x^2
```

Mupad [B] (verification not implemented)

Time = 28.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int x \log \left(c \left(a+b x^3\right)^p\right) \, dx = \frac{x^2\,\ln \left(c\,{\left(b\,x^3+a\right)}^p\right)}{2} - \frac{3\,p\,x^2}{4} - \frac{a^{2/3}\,p\,\ln \left(b^{1/3}\,x + a^{1/3}\right)}{2\,b^{2/3}} - \frac{a^{2/3}\,p\,\ln \left(4\,b^{1/3}\,x - 2\,a^{1/3} - \sqrt{3}\,a^{1/3}\,2{}\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,b^{2/3}} + \frac{a^{2/3}\,p\,\ln \left(4\,b^{1/3}\,x - 2\,a^{1/3} + \sqrt{3}\,a^{1/3}\,2{}\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,b^{2/3}}$$

input
```
int(x*log(c*(a + b*x^3)^p),x)
```

output
```
(x^2*log(c*(a + b*x^3)^p))/2 - (3*p*x^2)/4 - (a^(2/3)*p*log(b^(1/3)*x + a^
(1/3)))/(2*b^(2/3)) - (a^(2/3)*p*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*
a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(2/3)) + (a^(2/3)*p*log(3^(1/2)*a^(1
/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int x\log \left(c(a+bx^3)^p\right)\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)ap+2b^{\frac{2}{3}}a^{\frac{1}{3}}\log\left(\left(b\,x^3+a\right)^p c\right)x^2-3b^{\frac{2}{3}}a^{\frac{1}{3}}p\,x^2-3\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)ap+\log\left(\left(b\,x^3+\right.\right.}{4b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(x*log(c*(b*x^3+a)^p),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*p + 2*b
**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*x**2 - 3*b**(2/3)*a**(1/3)*p*x**2
- 3*log(a**(1/3) + b**(1/3)*x)*a*p + log((a + b*x**3)**p*c)*a)/(4*b**(2/3)
*a**(1/3))
```

3.18 $\int \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \log \left(c\left(a+bx^3\right)^p\right) \, dx = -3px - \frac{\sqrt{3}\sqrt[3]{a}p \arctan \left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}p \log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}p \log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x \log \left(c\left(a+bx^3\right)^p\right)$$

output
```
-3*p*x-3^(1/2)*a^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))
/b^(1/3)+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-1/2*a^(1/3)*p*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)+x*ln(c*(b*x^3+a)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \log \left(c\left(a+bx^3\right)^p\right) \, dx = -3px - \frac{\sqrt{3}\sqrt[3]{a}p \arctan \left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}p \log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}p \log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x \log \left(c\left(a+bx^3\right)^p\right)$$

input
```
Integrate[Log[c*(a + b*x^3)^p],x]
```

output
```
-3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b
^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)
^p]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {2898, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c\left(a+bx^3\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2898}$$

$$x \log \left(c\left(a+bx^3\right)^p\right)-3bp\int \frac{x^3}{bx^3+a}dx$$

$$\Big\downarrow \text{ 843}$$

$$x \log \left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^3+a}dx}{b}\right)$$

$$\Big\downarrow \text{ 750}$$

$$x \log \left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)$$

$$\Big\downarrow \text{ 16}$$

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\downarrow$ 1142

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\downarrow$ 25

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\downarrow$ 27

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\Big\downarrow$ 1082

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\Big\downarrow$ 217

$$3bp\left(x\log\left(c\left(a+bx^3\right)^p\right)-\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$$\Big\downarrow \text{1103}$$

$$3bp\left(x\log\left(c\left(a+bx^3\right)^p\right)-\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

input

```
Int[Log[c*(a + b*x^3)^p],x]
```

output
```
-3*b*p*(x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[
3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b) + x*Log[c*(a
 + b*x^3)^p]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

method	result
default	$x \ln\left(c\left(b\,x^{3}+a\right)^{p}\right)-3pb\left(\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}\right)$
parts	$x \ln\left(c\left(b\,x^{3}+a\right)^{p}\right)-3pb\left(\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}\right)$
risch	$x \ln\left(\left(b\,x^{3}+a\right)^{p}\right)+\frac{i\operatorname{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}\operatorname{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\pi x}{2}-\frac{i\pi x\,\operatorname{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}\left(ic\right)}{2}-\frac{i\pi}{}$

input
```
int(ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(b*x^3+a)^p)-3*p*b*(x/b-(1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/
6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))/b*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \log \left(c\left(a+bx^3\right)^p\right)\,dx = px\log\left(bx^3+a\right)+\sqrt{3}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right) - \frac{1}{2}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-3\,px+x\log\left(c\right)$$

input
```
integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)
^(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)
^(2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

Sympy [A] (verification not implemented)

Time = 24.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = \begin{cases} x\log\left(0^p c\right) \\ -3px+x\log\left(c\left(bx^3\right)^p\right) \\ x\log\left(a^p c\right) \\ -3px+x\log\left(c\left(a+bx^3\right)^p\right)-\frac{3bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\sqrt{3}bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a}+\frac{b\left(-\frac{a}{b}\right)}{} \end{cases}$$

input
```
integrate(ln(c*(b*x**3+a)**p),x)
```

output
```
Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (-3*p*x + x*log(c*(b*x**3)
**p), Eq(a, 0)), (x*log(a**p*c), Eq(b, 0)), (-3*p*x + x*log(c*(a + b*x**3)
**p) - 3*b*p*(-a/b)**(4/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3
))/(2*a) - sqrt(3)*b*p*(-a/b)**(4/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) +
sqrt(3)/3)/a + b*(-a/b)**(4/3)*log(c*(a + b*x**3)**p)/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \log \left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{2}\,bp\left(\frac{6\,x}{b}-\frac{2\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) + x\log\left(\left(bx^3+a\right)^pc\right)$$

input
```
integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
-1/2*b*p*(6*x/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(b^2*(a/b)^(2/3)) + a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(2/3)) - 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + x*log((b*x^3
+ a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{2}\,abp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab} - \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\right.}{ab^2}\right) + px\log\left(bx^3+a\right) - (3\,p-\log\left(c\right))x$$

input
```
integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
-1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2)
 - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*
log(b*x^3 + a) - (3*p - log(c))*x
```

Mupad [B] (verification not implemented)

Time = 26.93 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \log\left(c\left(a+bx^3\right)^p\right)\,dx &= x\,\ln\left(c\left(b\,x^3+a\right)^p\right) - 3\,p\,x - \frac{(-a)^{1/3}\,p\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)}{b^{1/3}} \\ &\quad + \frac{(-a)^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}-\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{1/3}} \\ &\quad - \frac{(-a)^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{1/3}}\end{aligned}$$

input
```
int(log(c*(a + b*x^3)^p),x)
```

output
```
x*log(c*(a + b*x^3)^p) - 3*p*x - ((-a)^(1/3)*p*log((-a)^(4/3) + a*b^(1/3)*
x))/b^(1/3) + ((-a)^(1/3)*p*log(2*a*b^(1/3)*x - 3^(1/2)*(-a)^(4/3)*1i - (-
a)^(4/3))*((3^(1/2)*1i)/2 + 1/2))/b^(1/3) - ((-a)^(1/3)*p*log(3^(1/2)*(-a)
^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2))/b^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.68

$$\int \log \left(c(a+bx^3)^p\right)\,dx$$
$$= \frac{-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)p+3a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)p-a^{\frac{1}{3}}\log\left((b\,x^3+a)^p\,c\right)+2b^{\frac{1}{3}}\log\left((b\,x^3+a)^p\,c\right)x-6b^{\frac{1}{3}}}{2b^{\frac{1}{3}}}$$

input
```
int(log(c*(b*x^3+a)^p),x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
p + 3*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*p - a**(1/3)*log((a + b*x**3)**p
*c) + 2*b**(1/3)*log((a + b*x**3)**p*c)*x - 6*b**(1/3)*p*x)/(2*b**(1/3))
```

3.19 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}\,dx=\frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{3}p\,\text{PolyLog}\left(2,1+\frac{bx^3}{a}\right)$$

output
```
1/3*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)+1/3*p*polylog(2,1+b*x^3/a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}\,dx=\frac{1}{3}\left(\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)+p\,\text{PolyLog}\left(2,\frac{a+bx^3}{a}\right)\right)$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/x,x]
```

output
```
(Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p] + p*PolyLog[2, (a + b*x^3)/a])/3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{x} \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{3} \int \frac{\log \left(c\left(bx^3+a\right)^p\right)}{x^3}dx^3$$

$$\Big\downarrow \text{2841}$$

$$\frac{1}{3} \left(\log \left(-\frac{bx^3}{a}\right) \log \left(c\left(a+bx^3\right)^p\right)-bp \int \frac{\log \left(-\frac{bx^3}{a}\right)}{bx^3+a}dx^3\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{3} \left(\log \left(-\frac{bx^3}{a}\right) \log \left(c\left(a+bx^3\right)^p\right)+p\,\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)\right)$$

input
```
Int[Log[c*(a + b*x^3)^p]/x,x]
```

output
```
(Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p] + p*PolyLog[2, 1 + (b*x^3)/a])/3
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.69 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

method	result
parts	$\ln\left(c(b\,x^3+a)^p\right)\ln\left(x\right)-p\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3+a\right)}\left(\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)$
risch	$\ln\left((b\,x^3+a)^p\right)\ln\left(x\right)-p\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3+a\right)}\left(\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)+\left(\frac{i\pi}{}\right.$

input
```
int(ln(c*(b*x^3+a)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(c*(b*x^3+a)^p)*ln(x)-p*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1
=RootOf(_Z^3*b+a))
```

Fricas [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x}\,dx = \int \frac{\log \left((bx^3+a)^p c\right)}{x}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="fricas")
```

output
```
integral(log((b*x^3 + a)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x}\,dx = \int \frac{\log \left(c(a+bx^3)^p\right)}{x}\,dx$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x,x)
```

output
```
Integral(log(c*(a + b*x**3)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x}\,dx = \frac{1}{3}\,bp\left(\frac{3\,\log \left(bx^3+a\right)\log \left(x\right)}{b} - \frac{3\,\log \left(\frac{bx^3}{a}+1\right)\log \left(x\right) + \mathrm{Li}_2\left(-\frac{bx^3}{a}\right)}{b}\right) - p\log \left(bx^3+a\right)\log \left(x\right) + \log \left((bx^3+a)^p c\right)\log \left(x\right)$$

input
```
integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="maxima")
```

output
```
1/3*b*p*(3*log(b*x^3 + a)*log(x)/b - (3*log(b*x^3/a + 1)*log(x) + dilog(-b
*x^3/a))/b) - p*log(b*x^3 + a)*log(x) + log((b*x^3 + a)^p*c)*log(x)
```

Giac [F]

$$\int \frac{\log(c(a+bx^3)^p)}{x}\,dx=\int \frac{\log((bx^3+a)^pc)}{x}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="giac")
```

output
```
integrate(log((b*x^3 + a)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx^3)^p)}{x}\,dx=\int \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{x}\,dx$$

input
```
int(log(c*(a + b*x^3)^p)/x,x)
```

output
```
int(log(c*(a + b*x^3)^p)/x, x)
```

Reduce [F]

$$\int \frac{\log(c(a+bx^3)^p)}{x}\,dx=\frac{6\left(\int \frac{\log\left((b\,x^3+a)^p c\right)}{b\,x^4+ax}dx\right)ap+\log\left((b\,x^3+a)^p\,c\right)^2}{6p}$$

input
```
int(log(c*(b*x^3+a)^p)/x,x)
```

output
```
(6*int(log((a + b*x**3)**p*c)/(a*x + b*x**4),x)*a*p + log((a + b*x**3)**p*
c)**2)/(6*p)
```

3.20 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2} dx = -\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}} - \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}} - \frac{\log\left(c(a+bx^3)^p\right)}{x}$$

output
```
-3^(1/2)*b^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))/a^(1/
3)-b^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)+1/2*b^(1/3)*p*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)-ln(c*(b*x^3+a)^p)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2} dx = \frac{3bpx^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a} - \frac{\log\left(c(a+bx^3)^p\right)}{x}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/x^2,x]
```

output
```
(3*b*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/(2*a) - Log[c*(a
+ b*x^3)^p]/x
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2905, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{x^2} \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$3bp\int \frac{x}{bx^3+a}dx-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{x}$$

$$\Big\downarrow \text{ 821}$$

$$3bp\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{x}$$

$$\Big\downarrow \text{ 16}$$

$$3bp\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{x}$$

$$\Big\downarrow \text{ 1142}$$

$$3bp\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}$$

$$\Big\downarrow \; 25$$

$$3bp\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}$$

$$\Big\downarrow \; 27$$

$$3bp\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}$$

$$\Big\downarrow \; 1082$$

$$3bp\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}$$

$$\Big\downarrow \; 217$$

$$3bp\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}$$

$$\Big\downarrow \text{1103}$$

$$3bp\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}$$

input
```
Int[Log[c*(a + b*x^3)^p]/x^2,x]
```

output
```
3*b*p*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))) - Log[c*(a + b*x
^3)^p]/x
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{x}+3pb\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{x}-\frac{i\pi\,\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}-i\pi\,\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\right)-i\pi\mathrm{csgn}\left(ic\left(b\,x^{3}-\right.\right.}{}$

input
```
int(ln(c*(b*x^3+a)^p)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(b*x^3+a)^p)/x+3*p*b*(-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6/b
/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx$$
$$=\frac{2\sqrt{3}px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+2\,px\left(-\frac{b}{a}\right)}{2\,x}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*p*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sq
rt(3)) - p*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) +
 2*p*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) - 2*p*log(b*x^3 + a) - 2*log
(c))/x
```

Sympy [A] (verification not implemented)

Time = 105.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx = \begin{cases} -\frac{\log(0^p c)}{x} \\ -\frac{3p}{x} - \frac{\log\left(c(bx^3)^p\right)}{x} \\ -\frac{\log(a^p c)}{x} \\ -\frac{\log\left(c(a+bx^3)^p\right)}{x} + \frac{3bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a} - \frac{\sqrt{3}bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a} - \frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(c(a+bx^3)^p\right)}{a} \end{cases}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x**2,x)
```

output
```
Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-3*p/x - log(c*(b*x**3)*
*p)/x, Eq(a, 0)), (-log(a**p*c)/x, Eq(b, 0)), (-log(c*(a + b*x**3)**p)/x +
 3*b*p*(-a/b)**(2/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*
a) - sqrt(3)*b*p*(-a/b)**(2/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt(3
)/3)/a - b*(-a/b)**(2/3)*log(c*(a + b*x**3)**p)/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{\log(c(a+bx^3)^p)}{x^2}\,dx = \frac{1}{2}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) - \frac{\log\left((bx^3+a)^pc\right)}{x}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*
(a/b)^(1/3)) + log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 2*
log(x + (a/b)^(1/3))/(b*(a/b)^(1/3))) - log((b*x^3 + a)^p*c)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03

$$\int \frac{\log(c(a+bx^3)^p)}{x^2}\,dx = -\frac{bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\,p\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{p\log(bx^3+a)}{x} + \frac{(-ab^2)^{\frac{2}{3}}\,p\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,ab} - \frac{\log(c)}{x}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="giac")
```

output
```
-b*p*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*p*
arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - p*log(b*x^3
+ a)/x + 1/2*(-a*b^2)^(2/3)*p*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*
b) - log(c)/x
```

Mupad [B] (verification not implemented)

Time = 26.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^2}\,dx = \frac{(-b)^{1/3}\,p\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)}{a^{1/3}}-\frac{\ln\left(c\,(b\,x^3+a)^p\right)}{x} + \frac{(-b)^{1/3}\,p\,\ln\left(9\,b^3\,p^2\,x+9\,a^{1/3}\,(-b)^{8/3}\,p^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}} - \frac{(-b)^{1/3}\,p\,\ln\left(9\,b^3\,p^2\,x+9\,a^{1/3}\,(-b)^{8/3}\,p^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}}$$

input
```
int(log(c*(a + b*x^3)^p)/x^2,x)
```

output
```
((-b)^(1/3)*p*log(a^(1/3)*(-b)^(8/3) + b^3*x))/a^(1/3) - log(c*(a + b*x^3)
^p)/x + ((-b)^(1/3)*p*log(9*b^3*p^2*x + 9*a^(1/3)*(-b)^(8/3)*p^2*((3^(1/2)
*1i)/2 - 1/2)^2)*((3^(1/2)*1i)/2 - 1/2))/a^(1/3) - ((-b)^(1/3)*p*log(9*b^3
*p^2*x + 9*a^(1/3)*(-b)^(8/3)*p^2*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/
2 + 1/2))/a^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^2}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bpx-2b^{\frac{2}{3}}a^{\frac{1}{3}}\log\left((b\,x^3+a)^p\,c\right)-3\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)bpx+\log\left((b\,x^3+a)^p\,c\right)bx}{2b^{\frac{2}{3}}a^{\frac{1}{3}}x}$$

input
```
int(log(c*(b*x^3+a)^p)/x^2,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*p*x - 2
*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c) - 3*log(a**(1/3) + b**(1/3)*x)*b
*p*x + log((a + b*x**3)**p*c)*b*x)/(2*b**(2/3)*a**(1/3)*x)
```

3.21 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\,dx = -\frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}} + \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}} - \frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

output
```
-1/2*3^(1/2)*b^(2/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))/a
^(2/3)+1/2*b^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)-1/4*b^(2/3)*p*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)-1/2*ln(c*(b*x^3+a)^p)/x^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\,dx = -\frac{2\sqrt{3}b^{2/3}px^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2b^{2/3}px^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + b^{2/3}px^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}x^2}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/x^3,x]
```

output
```
-1/4*(2*Sqrt[3]*b^(2/3)*p*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
- 2*b^(2/3)*p*x^2*Log[a^(1/3) + b^(1/3)*x] + b^(2/3)*p*x^2*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a^(2/3)*Log[c*(a + b*x^3)^p])/(a^(2/3)
*x^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2905, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{x^3}dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{3}{2}bp\int \frac{1}{bx^3+a}dx-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{3}{2}bp\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3}{2}bp\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{3}{2}bp\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{2}bp\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{2}bp\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{3}{2}bp\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

$$\Big\downarrow \; 217$$

$$\frac{3}{2}bp\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

$\Big\downarrow$ 1103

$$\frac{3}{2}bp\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2x^2}$$

input
```
Int[Log[c*(a + b*x^3)^p]/x^3,x]
```

output
```
(3*b*p*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[
(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/2 - Log[c*(a + b*x^3)^p]/
(2*x^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{2x^{2}}+\frac{3pb\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{2}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{2x^{2}}-\frac{i\pi\,\operatorname{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}-i\pi\,\operatorname{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}\left(ic\right)-i\pi\operatorname{csgn}\left(ic\left(b\,x^{3}\right.\right.}{}$

input
```
int(ln(c*(b*x^3+a)^p)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(b*x^3+a)^p)/x^2+3/2*p*b*(1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3)
)-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^
(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3}\,dx$$
$$= \frac{2\sqrt{3}px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2px^2\left(\frac{b^2}{a^2}\right)}{4x^2}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*p*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(
2/3) - sqrt(3)*b)/b) - p*x^2*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)
^(1/3) + a^2*(b^2/a^2)^(2/3)) + 2*p*x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a
^2)^(1/3)) - 2*p*log(b*x^3 + a) - 2*log(c))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx = \frac{1}{4}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) - \frac{\log\left((bx^3+a)^p c\right)}{2\,x^2}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="maxima")
```

output
```
1/4*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*
(a/b)^(2/3)) - log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 2*
log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))) - 1/2*log((b*x^3 + a)^p*c)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx = -\frac{1}{4}\,bp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x(\cdot\right.}{ab}\right. - \frac{p\log\left(bx^3+a\right)}{2\,x^2} - \frac{\log\left(c\right)}{2\,x^2}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="giac")
```

output
```
-1/4*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)
^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b
^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)) - 1/2*p*log(b*x^
3 + a)/x^2 - 1/2*log(c)/x^2
```

Mupad [B] (verification not implemented)

Time = 28.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3}\,dx = \frac{b^{2/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{2\,a^{2/3}} - \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{2\,x^2} - \frac{b^{2/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,a^{2/3}} + \frac{b^{2/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,a^{2/3}}$$

input
```
int(log(c*(a + b*x^3)^p)/x^3,x)
```

output
```
(b^(2/3)*p*log(b^(1/3)*x + a^(1/3)))/(2*a^(2/3)) - log(c*(a + b*x^3)^p)/(2
*x^2) - (b^(2/3)*p*log(2*b^(1/3)*x - 3^(1/2)*a^(1/3)*1i - a^(1/3))*((3^(1/
2)*1i)/2 + 1/2))/(2*a^(2/3)) + (b^(2/3)*p*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/
3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*a^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bp\,x^2 - 2b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left((b\,x^3+a)^p\,c\right) + 3\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)bp\,x^2 - \log\left((b\,x^3+a)^p\,c\right)b\,x^2}{4b^{\frac{1}{3}}a^{\frac{2}{3}}x^2}$$

input
```
int(log(c*(b*x^3+a)^p)/x^3,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*p*x**2
- 2*b**(1/3)*a**(2/3)*log((a + b*x**3)**p*c) + 3*log(a**(1/3) + b**(1/3)*x
)*b*p*x**2 - log((a + b*x**3)**p*c)*b*x**2)/(4*b**(1/3)*a**(2/3)*x**2)
```

3.22 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4}\,dx = \frac{bp\log(x)}{a} - \frac{bp\log\left(a+bx^3\right)}{3a} - \frac{\log\left(c(a+bx^3)^p\right)}{3x^3}$$

output
```
b*p*ln(x)/a-1/3*b*p*ln(b*x^3+a)/a-1/3*ln(c*(b*x^3+a)^p)/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4}\,dx = \frac{bp\log(x)}{a} - \frac{bp\log\left(a+bx^3\right)}{3a} - \frac{\log\left(c(a+bx^3)^p\right)}{3x^3}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/x^4,x]
```

output
```
(b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2904, 2842, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^4}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{3}\int \frac{\log\left(c\left(bx^3+a\right)^p\right)}{x^6}dx^3$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{3}\left(bp\int \frac{1}{x^3\left(bx^3+a\right)}dx^3-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{3}\left(bp\left(\frac{\int \frac{1}{x^3}dx^3}{a}-\frac{b\int \frac{1}{bx^3+a}dx^3}{a}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{3}\left(bp\left(\frac{\log\left(x^3\right)}{a}-\frac{b\int \frac{1}{bx^3+a}dx^3}{a}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(bp\left(\frac{\log\left(x^3\right)}{a}-\frac{\log\left(a+bx^3\right)}{a}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3}\right)$$

input
```
Int[Log[c*(a + b*x^3)^p]/x^4,x]
```

output
```
(b*p*(Log[x^3]/a - Log[a + b*x^3]/a) - Log[c*(a + b*x^3)^p]/x^3)/3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{3x^{3}}+pb\left(\frac{\ln\left(x\right)}{a}-\frac{\ln\left(b\,x^{3}+a\right)}{3a}\right)$
parallelrisch	$\frac{3p^{2}b\ln\left(x\right)x^{3}-x^{3}\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)bp-\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)ap}{3x^{3}ap}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{3x^{3}}-\frac{-6bp\ln\left(x\right)x^{3}+2bp\ln\left(b\,x^{3}+a\right)x^{3}+i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}-i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right) \mathrm{cs}}{6a\,x^{3}}$

input
```
int(ln(c*(b*x^3+a)^p)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(c*(b*x^3+a)^p)/x^3+p*b*(ln(x)/a-1/3/a*ln(b*x^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4}\,dx = \frac{3\,bpx^3\log\left(x\right)-\left(bpx^3+ap\right)\log\left(bx^3+a\right)-a\log\left(c\right)}{3\,ax^3}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*b*p*x^3*log(x) - (b*p*x^3 + a*p)*log(b*x^3 + a) - a*log(c))/(a*x^3)
```

Sympy [A] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4}\,dx = \begin{cases} -\frac{\log\left(c(a+bx^3)^p\right)}{3x^3}+\frac{bp\log\left(x\right)}{a}-\frac{b\log\left(c(a+bx^3)^p\right)}{3a} & \text{for } a \neq 0 \\ -\frac{p}{3x^3}-\frac{\log\left(c(bx^3)^p\right)}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x**4,x)
```

output
```
Piecewise((-log(c*(a + b*x**3)**p)/(3*x**3) + b*p*log(x)/a - b*log(c*(a +
b*x**3)**p)/(3*a), Ne(a, 0)), (-p/(3*x**3) - log(c*(b*x**3)**p)/(3*x**3),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^4}\,dx = -\frac{1}{3}\,bp\left(\frac{\log\left(bx^3+a\right)}{a}-\frac{\log\left(x^3\right)}{a}\right)-\frac{\log\left(\left(bx^3+a\right)^p c\right)}{3\,x^3}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="maxima")
```

output
```
-1/3*b*p*(log(b*x^3 + a)/a - log(x^3)/a) - 1/3*log((b*x^3 + a)^p*c)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^4}\,dx = -\frac{\frac{b^2p\log\left(bx^3+a\right)}{a}-\frac{b^2p\log\left(bx^3\right)}{a}+\frac{bp\log\left(bx^3+a\right)}{x^3}+\frac{b\log\left(c\right)}{x^3}}{3\,b}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="giac")
```

output
```
-1/3*(b^2*p*log(b*x^3 + a)/a - b^2*p*log(b*x^3)/a + b*p*log(b*x^3 + a)/x^3
 + b*log(c)/x^3)/b
```

Mupad [B] (verification not implemented)

Time = 26.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^4}\,dx = \frac{b\,p\,\ln\left(x\right)}{a}-\frac{b\,p\,\ln\left(b\,x^3+a\right)}{3\,a}-\frac{\ln\left(c\,\left(b\,x^3+a\right)^p\right)}{3\,x^3}$$

input
```
int(log(c*(a + b*x^3)^p)/x^4,x)
```

output
```
(b*p*log(x))/a - (b*p*log(a + b*x^3))/(3*a) - log(c*(a + b*x^3)^p)/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4}\,dx = \frac{-\log\left((b\,x^3+a)^p\,c\right)a-\log\left((b\,x^3+a)^p\,c\right)b\,x^3+3\log(x)\,bp\,x^3}{3a\,x^3}$$

input
```
int(log(c*(b*x^3+a)^p)/x^4,x)
```

output
```
( - log((a + b*x**3)**p*c)*a - log((a + b*x**3)**p*c)*b*x**3 + 3*log(x)*b*
p*x**3)/(3*a*x**3)
```

3.23 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 151

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx=-\frac{3bp}{4ax}+\frac{\sqrt{3}b^{4/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

output
```
-3/4*b*p/a/x+1/4*3^(1/2)*b^(4/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2
)/a^(1/3))/a^(4/3)+1/4*b^(4/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)-1/8*b^(4/3)
*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)-1/4*ln(c*(b*x^3+a)^p)
/x^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.32

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx=-\frac{3bp\,\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},-\frac{bx^3}{a}\right)}{4ax}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/x^5,x]
```

output
```
(-3*b*p*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x^3)/a)])/(4*a*x) - Log[c*(a
+ b*x^3)^p]/(4*x^4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2905, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{x^5} \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{3}{4}bp\int \frac{1}{x^2\left(bx^3+a\right)}dx-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{3}{4}bp\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{3}{4}bp\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3}{4}bp\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \ 1142$$

$$\frac{3}{4}bp\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \ 25$$

$$\frac{3}{4}bp\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{4}bp\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \ 1082$$

$$\frac{3}{4}bp\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \ 217$$

$$\frac{3}{4}bp\left(-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \; 1103$$

$$\frac{3}{4}bp\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

input
```
Int[Log[c*(a + b*x^3)^p]/x^5,x]
```

output
```
(3*b*p*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))
))/a))/4 - Log[c*(a + b*x^3)^p]/(4*x^4)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{\ln(c(b x^3+a)^p)}{4x^4}+\frac{3pb\left(-\frac{1}{ax}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}\right)}{4}$
risch	$-\frac{\ln((b x^3+a)^p)}{4x^4}-\frac{i\pi a\ \mathrm{csgn}(i(b x^3+a)^p)\mathrm{csgn}(ic(b x^3+a)^p)^2-i\pi a\ \mathrm{csgn}(i(b x^3+a)^p)\ \mathrm{csgn}(ic(b x^3+a)^p)\ \mathrm{csgn}(ic)-i\pi a\mathrm{csgn}(ic(b}{}$

input

```
int(ln(c*(b*x^3+a)^p)/x^5,x,method=_RETURNVERBOSE)
```

output

```
-1/4*ln(c*(b*x^3+a)^p)/x^4+3/4*p*b*(-1/a/x-(-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b
*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*3
^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))/a*b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx =$$

$$-\frac{2\sqrt{3}bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{lo}}{8\,ax^4}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(3)*b*p*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3
*sqrt(3)) + b*p*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3
)) - 2*b*p*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6*b*p*x^3 + 2*a*p*lo
g(b*x^3 + a) + 2*a*log(c))/(a*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx = -\frac{1}{8}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{6}{ax}\right)-\frac{\log\left((bx^3+a)^pc\right)}{4\,x^4}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="maxima")
```

output
```
-1/8*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a
*(a/b)^(1/3)) + log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) - 2
*log(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) + 6/(a*x)) - 1/4*log((b*x^3 + a)^p*c
)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx$$
$$=\frac{1}{8}b^2p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b^2}-\frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x(-\right.}{a^2b^2}\right.$$
$$-\frac{p\log\left(bx^3+a\right)}{4\,x^4}-\frac{3\,bpx^3+a\log\left(c\right)}{4\,ax^4}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="giac")
```

output
```
1/8*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2)
- (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)) - 1/4
*p*log(b*x^3 + a)/x^4 - 1/4*(3*b*p*x^3 + a*log(c))/(a*x^4)
```

Mupad [B] (verification not implemented)

Time = 28.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx=\frac{b^{4/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{4\,a^{4/3}}-\frac{\ln\left(c\,\left(b\,x^3+a\right)^p\right)}{4\,x^4}-\frac{3\,b\,p}{4\,a\,x}$$
$$+\frac{b^{4/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,a^{4/3}}$$
$$-\frac{b^{4/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,a^{4/3}}$$

input
```
int(log(c*(a + b*x^3)^p)/x^5,x)
```

output
```
(b^(4/3)*p*log(b^(1/3)*x + a^(1/3)))/(4*a^(4/3)) - log(c*(a + b*x^3)^p)/(4
*x^4) - (3*b*p)/(4*a*x) + (b^(4/3)*p*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i
- 2*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(4*a^(4/3)) - (b^(4/3)*p*log(3^(1/2)*
a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(4*a^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^5}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^2 p\,x^4-2b^{\frac{2}{3}}a^{\frac{4}{3}}\log\left(\left(b\,x^3+a\right)^p c\right)-6b^{\frac{5}{3}}a^{\frac{1}{3}}p\,x^3+3\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right) b^2 p\,x^4-\log\left(\left(b\,x^3\right.\right.}{8b^{\frac{2}{3}}a^{\frac{4}{3}}x^4}$$

input
```
int(log(c*(b*x^3+a)^p)/x^5,x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*p*x**4
- 2*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*a - 6*b**(2/3)*a**(1/3)*b*p*x
**3 + 3*log(a**(1/3) + b**(1/3)*x)*b**2*p*x**4 - log((a + b*x**3)**p*c)*b*
*2*x**4)/(8*b**(2/3)*a**(1/3)*a*x**4)
```

3.24 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 151

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = -\frac{3bp}{10ax^2}+\frac{\sqrt{3}b^{5/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}} - \frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}} + \frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10a^{5/3}} - \frac{\log\left(c(a+bx^3)^p\right)}{5x^5}$$

output
```
-3/10*b*p/a/x^2+1/5*3^(1/2)*b^(5/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(
1/2)/a^(1/3))/a^(5/3)-1/5*b^(5/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)+1/10*b^(
5/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)-1/5*ln(c*(b*x^3+a
)^p)/x^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.32

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = -\frac{3bp\,\text{Hypergeometric2F1}\left(-\frac{2}{3},1,\frac{1}{3},-\frac{bx^3}{a}\right)}{10ax^2} - \frac{\log\left(c(a+bx^3)^p\right)}{5x^5}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/x^6,x]
```

output
```
(-3*b*p*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x^3)/a)])/(10*a*x^2) - Log[c*
(a + b*x^3)^p]/(5*x^5)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2905, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{3}{5}bp\int \frac{1}{x^3\,(bx^3+a)}dx - \frac{\log\left(c(a+bx^3)^p\right)}{5x^5}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{3}{5}bp\left(-\frac{b\int \frac{1}{bx^3+a}dx}{a} - \frac{1}{2ax^2}\right) - \frac{\log\left(c(a+bx^3)^p\right)}{5x^5}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{3}{5}bp\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{5}bp\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{5}bp\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{5}bp\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{5}bp\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}$$

$$\Big\downarrow \; 1082$$

$$\frac{3}{5}bp\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}$$

$\downarrow$ 217

$$\frac{3}{5}bp\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}$$

$\downarrow$ 1103

$$\frac{3}{5}bp\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{5x^5}$$

input
```
Int[Log[c*(a + b*x^3)^p]/x^6,x]
```

output
```
(3*b*p*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3))
+ (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/
5 - Log[c*(a + b*x^3)^p]/(5*x^5)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{5x^{5}}+\frac{3pb\left(-\frac{1}{2a\,x^{2}}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}\right)}{5}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{5x^{5}}-\frac{-2\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{3}+b^{5}p^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{5}-3b^{5}p^{3}\right)x-a^{2}b^{3}p^{2}\_R\right)\right)a\,x^{5}+i\pi a\,\operatorname{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)}{}$

input
```
int(ln(c*(b*x^3+a)^p)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*ln(c*(b*x^3+a)^p)/x^5+3/5*p*b*(-1/2/a/x^2-(1/3/b/(1/b*a)^(2/3)*ln(x+(
1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/
3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))/a*b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.14

$$\int \frac{\log(c(a+bx^3)^p)}{x^6}\,dx = \frac{2\sqrt{3}bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+}{10\,ax^5}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="fricas")
```

output
```
1/10*(2*sqrt(3)*b*p*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a
^2)^(2/3) - sqrt(3)*b)/b) - b*p*x^5*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(
-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*b*p*x^5*(-b^2/a^2)^(1/3)*log(b
*x - a*(-b^2/a^2)^(1/3)) - 3*b*p*x^3 - 2*a*p*log(b*x^3 + a) - 2*a*log(c))/
(a*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx^3)^p)}{x^6}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = -\frac{1}{10}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{3}{ax^2}\right) - \frac{\log\left((bx^3+a)^p c\right)}{5\,x^5}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="maxima")
```

output
```
-1/10*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(
a*(a/b)^(2/3)) - log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) +
2*log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) + 3/(a*x^2)) - 1/5*log((b*x^3 + a)^
p*c)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = \frac{b^2p\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{5\,a^2} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\,bp\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{5\,a^2} - \frac{(-ab^2)^{\frac{1}{3}}\,bp\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{10\,a^2} - \frac{p\log\left(bx^3+a\right)}{5\,x^5} - \frac{3\,bpx^3+2\,a\log\left(c\right)}{10\,ax^5}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="giac")
```

output
```
1/5*b^2*p*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 1/5*sqrt(3)*(-a*b^
2)^(1/3)*b*p*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1
/10*(-a*b^2)^(1/3)*b*p*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 1/5*
p*log(b*x^3 + a)/x^5 - 1/10*(3*b*p*x^3 + 2*a*log(c))/(a*x^5)
```

Mupad [B] (verification not implemented)

Time = 28.64 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = \frac{(-b)^{5/3}\,p\,\ln\left(a^{1/3}\,(-b)^{11/3}-b^4\,x\right)}{5\,a^{5/3}}-\frac{\ln\left(c\,(b\,x^3+a)^p\right)}{5\,x^5}-\frac{3\,b\,p}{10\,a\,x^2}+\frac{(-b)^{5/3}\,p\,\ln\left(225\,a^2\,b^4\,p\,x-225\,a^{7/3}\,(-b)^{11/3}\,p\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{5\,a^{5/3}}-\frac{(-b)^{5/3}\,p\,\ln\left(225\,a^2\,b^4\,p\,x+225\,a^{7/3}\,(-b)^{11/3}\,p\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{5\,a^{5/3}}$$

input
```
int(log(c*(a + b*x^3)^p)/x^6,x)
```

output
```
((-b)^(5/3)*p*log(a^(1/3)*(-b)^(11/3) - b^4*x))/(5*a^(5/3)) - log(c*(a + b
*x^3)^p)/(5*x^5) - (3*b*p)/(10*a*x^2) + ((-b)^(5/3)*p*log(225*a^2*b^4*p*x
- 225*a^(7/3)*(-b)^(11/3)*p*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2)
)/(5*a^(5/3)) - ((-b)^(5/3)*p*log(225*a^2*b^4*p*x + 225*a^(7/3)*(-b)^(11/3
)*p*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(5*a^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2p\,x^5-2b^{\frac{1}{3}}a^{\frac{5}{3}}\log\left((b\,x^3+a)^p\,c\right)-3b^{\frac{4}{3}}a^{\frac{2}{3}}p\,x^3-3\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b^2p\,x^5+\log\left((b\,x^3\right.}{10b^{\frac{1}{3}}a^{\frac{5}{3}}x^5}$$

input
```
int(log(c*(b*x^3+a)^p)/x^6,x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*p*x**5
- 2*b**(1/3)*a**(2/3)*log((a + b*x**3)**p*c)*a - 3*b**(1/3)*a**(2/3)*b*p*x
**3 - 3*log(a**(1/3) + b**(1/3)*x)*b**2*p*x**5 + log((a + b*x**3)**p*c)*b*
*2*x**5)/(10*b**(1/3)*a**(2/3)*a*x**5)
```

3.25 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^7} dx$

Optimal result

Integrand size = 16, antiderivative size = 64

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7} dx = -\frac{bp}{6ax^3} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log(a+bx^3)}{6a^2} - \frac{\log\left(c(a+bx^3)^p\right)}{6x^6}$$

output
```
-1/6*b*p/a/x^3-1/2*b^2*p*ln(x)/a^2+1/6*b^2*p*ln(b*x^3+a)/a^2-1/6*ln(c*(b*x
^3+a)^p)/x^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7} dx = \frac{1}{6}bp\left(-\frac{1}{ax^3} - \frac{3b\log(x)}{a^2} + \frac{b\log(a+bx^3)}{a^2}\right) - \frac{\log\left(c(a+bx^3)^p\right)}{6x^6}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/x^7,x]
```

output
```
(b*p*(-(1/(a*x^3)) - (3*b*Log[x])/a^2 + (b*Log[a + b*x^3])/a^2))/6 - Log[c
*(a + b*x^3)^p]/(6*x^6)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{x^7} \, dx$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{3} \int \frac{\log \left(c\left(bx^3+a\right)^p\right)}{x^9}dx^3$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{1}{3} \left(\frac{1}{2} bp \int \frac{1}{x^6 \left(bx^3+a\right)}dx^3-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{2x^6}\right)$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{3} \left(\frac{1}{2} bp \int \left(\frac{b^2}{a^2 \left(bx^3+a\right)}-\frac{b}{a^2x^3}+\frac{1}{ax^6}\right)dx^3-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{2x^6}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\frac{1}{2} bp \left(-\frac{b \log \left(x^3\right)}{a^2}+\frac{b \log \left(a+bx^3\right)}{a^2}-\frac{1}{ax^3}\right)-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{2x^6}\right)$$

input
```
Int[Log[c*(a + b*x^3)^p]/x^7,x]
```

output
```
((b*p*(-(1/(a*x^3)) - (b*Log[x^3])/a^2 + (b*Log[a + b*x^3])/a^2))/2 - Log[
c*(a + b*x^3)^p]/(2*x^6))/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{6x^{6}}+\frac{pb\left(-\frac{1}{3a\,x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(b\,x^{3}+a\right)}{3a^{2}}\right)}{2}$
parallelrisch	$-\frac{3b^{2}p^{2}\ln\left(x\right)x^{6}-x^{6}\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)b^{2}p-b^{2}p^{2}x^{6}+ab\,p^{2}x^{3}+\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)a^{2}p}{6x^{6}a^{2}p}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{6x^{6}}-\frac{-2b^{2}p\ln\left(-b\,x^{3}-a\right)x^{6}+6b^{2}p\ln\left(x\right)x^{6}+i\pi\,a^{2}\operatorname{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}-i\pi\,a^{2}\operatorname{csgn}\left(i\left(b\,x^{3}+\right.\right.}{}$

input
```
int(ln(c*(b*x^3+a)^p)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(c*(b*x^3+a)^p)/x^6+1/2*p*b*(-1/3/a/x^3-1/a^2*b*ln(x)+1/3*b/a^2*ln(
b*x^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^7}\,dx = -\frac{3\,b^2px^6\log(x)+abpx^3+a^2\log(c)-(b^2px^6-a^2p)\log(bx^3+a)}{6\,a^2x^6}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="fricas")
```

output
```
-1/6*(3*b^2*p*x^6*log(x) + a*b*p*x^3 + a^2*log(c) - (b^2*p*x^6 - a^2*p)*lo
g(b*x^3 + a))/(a^2*x^6)
```

Sympy [A] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^7}\,dx = \begin{cases} -\frac{\log\left(c(a+bx^3)^p\right)}{6x^6}-\frac{bp}{6ax^3}-\frac{b^2p\log(x)}{2a^2}+\frac{b^2\log\left(c(a+bx^3)^p\right)}{6a^2} & \text{for } a\neq 0 \\ -\frac{p}{12x^6}-\frac{\log\left(c(bx^3)^p\right)}{6x^6} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x**7,x)
```

output
```
Piecewise((-log(c*(a + b*x**3)**p)/(6*x**6) - b*p/(6*a*x**3) - b**2*p*log(
x)/(2*a**2) + b**2*log(c*(a + b*x**3)**p)/(6*a**2), Ne(a, 0)), (-p/(12*x**
6) - log(c*(b*x**3)**p)/(6*x**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx = \frac{1}{6}\,bp\left(\frac{b\log\left(bx^3+a\right)}{a^2} - \frac{b\log\left(x^3\right)}{a^2} - \frac{1}{ax^3}\right) - \frac{\log\left(\left(bx^3+a\right)^p c\right)}{6\,x^6}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="maxima")
```

output
```
1/6*b*p*(b*log(b*x^3 + a)/a^2 - b*log(x^3)/a^2 - 1/(a*x^3)) - 1/6*log((b*x
^3 + a)^p*c)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(56) = 112.

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.06

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx = -\frac{\frac{b^3p\log\left(bx^3+a\right)}{\left(bx^3+a\right)^2-2\left(bx^3+a\right)a+a^2} - \frac{b^3p\log\left(bx^3+a\right)}{a^2} + \frac{b^3p\log\left(bx^3\right)}{a^2} + \frac{\left(bx^3+a\right)b^3p-ab^3p+ab^3\log\left(c\right)}{\left(bx^3+a\right)^2a-2\left(bx^3+a\right)a^2+a^3}}{6\,b}$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="giac")
```

output
```
-1/6*(b^3*p*log(b*x^3 + a)/((b*x^3 + a)^2 - 2*(b*x^3 + a)*a + a^2) - b^3*p
*log(b*x^3 + a)/a^2 + b^3*p*log(b*x^3)/a^2 + ((b*x^3 + a)*b^3*p - a*b^3*p
+ a*b^3*log(c))/((b*x^3 + a)^2*a - 2*(b*x^3 + a)*a^2 + a^3))/b
```

Mupad [B] (verification not implemented)

Time = 26.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx = \frac{b^2\,p\,\ln\left(b\,x^3+a\right)}{6\,a^2} - \frac{\ln\left(c\left(b\,x^3+a\right)^p\right)}{6\,x^6} - \frac{b^2\,p\,\ln\left(x\right)}{2\,a^2} - \frac{b\,p}{6\,a\,x^3}$$

input
```
int(log(c*(a + b*x^3)^p)/x^7,x)
```

output
```
(b^2*p*log(a + b*x^3))/(6*a^2) - log(c*(a + b*x^3)^p)/(6*x^6) - (b^2*p*log
(x))/(2*a^2) - (b*p)/(6*a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx = \frac{-\log\left(\left(b\,x^3+a\right)^p c\right)a^2 + \log\left(\left(b\,x^3+a\right)^p c\right)b^2x^6 - 3\log(x)\,b^2p\,x^6 - abp\,x^3}{6a^2x^6}$$

input
```
int(log(c*(b*x^3+a)^p)/x^7,x)
```

output
```
( - log((a + b*x**3)**p*c)*a**2 + log((a + b*x**3)**p*c)*b**2*x**6 - 3*log
(x)*b**2*p*x**6 - a*b*p*x**3)/(6*a**2*x**6)
```

3.26 $\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = -\frac{b^4px}{5a^4}+\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}+\frac{bpx^4}{20a}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^5p\log(b+ax)}{5a^5}$$

output
```
-1/5*b^4*p*x/a^4+1/10*b^3*p*x^2/a^3-1/15*b^2*p*x^3/a^2+1/20*b*p*x^4/a+1/5*
x^5*ln(c*(a+b/x)^p)+1/5*b^5*p*ln(a*x+b)/a^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{abpx(-12b^3+6ab^2x-4a^2bx^2+3a^3x^3)+12b^5p\log\left(a+\frac{b}{x}\right)+12a^5x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+12b^5p\log(x)}{60a^5}$$

input
```
Integrate[x^4*Log[c*(a + b/x)^p],x]
```

output
```
(a*b*p*x*(-12*b^3 + 6*a*b^2*x - 4*a^2*b*x^2 + 3*a^3*x^3) + 12*b^5*p*Log[a
+ b/x] + 12*a^5*x^5*Log[c*(a + b/x)^p] + 12*b^5*p*Log[x])/(60*a^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2905, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{5} b p \int \frac{x^3}{a+\frac{b}{x}}dx+\frac{1}{5} x^5 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 795}$$

$$\frac{1}{5} b p \int \frac{x^4}{b+a x}dx+\frac{1}{5} x^5 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{5} b p \int \left(\frac{b^4}{a^4 (b+a x)}-\frac{b^3}{a^4}+\frac{x b^2}{a^3}-\frac{x^2 b}{a^2}+\frac{x^3}{a}\right)dx+\frac{1}{5} x^5 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5} b p \left(\frac{b^4 \log (a x+b)}{a^5}-\frac{b^3 x}{a^4}+\frac{b^2 x^2}{2 a^3}-\frac{b x^3}{3 a^2}+\frac{x^4}{4 a}\right)+\frac{1}{5} x^5 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

input
```
Int[x^4*Log[c*(a + b/x)^p],x]
```

output
```
(x^5*Log[c*(a + b/x)^p])/5 + (b*p*(-((b^3*x)/a^4) + (b^2*x^2)/(2*a^3) - (b
*x^3)/(3*a^2) + x^4/(4*a) + (b^4*Log[b + a*x])/a^5))/5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

method	result
parts	$\frac{x^{5} \ln\left(c \left(a+\frac{b}{x}\right)^{p}\right)}{5}+\frac{p b \left(\frac{\frac{1}{4} a^{3} x^{4}-\frac{1}{3} a^{2} b \,x^{3}+\frac{1}{2} a \,b^{2} x^{2}-b^{3} x}{a^{4}}+\frac{b^{4} \ln\left(a x +b \right)}{a^{5}}\right)}{5}$
parallelrisch	$-\frac{-12 x^{5} \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) a^{5} p -3 x^{4} a^{4} b \,p^{2}+4 x^{3} a^{3} b^{2} p^{2}-6 x^{2} a^{2} b^{3} p^{2}-12 \ln\left(x \right) b^{5} p^{2}+12 x a \,b^{4} p^{2}-12 \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) b^{5} p -12 b^{5} p^{2}}{60 a^{5} p}$

input
```
int(x^4*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*ln(c*(a+b/x)^p)+1/5*p*b*(1/a^4*(1/4*a^3*x^4-1/3*a^2*b*x^3+1/2*a*b^
2*x^2-b^3*x)+b^4/a^5*ln(a*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{12\,a^5px^5\log\left(\frac{ax+b}{x}\right)+12\,a^5x^5\log(c)+3\,a^4bpx^4-4\,a^3b^2px^3+6\,a^2b^3px^2-12\,ab^4px+12\,b^5p\log(ax+b)}{60\,a^5}$$

input
```
integrate(x^4*log(c*(a+b/x)^p),x, algorithm="fricas")
```

output
```
1/60*(12*a^5*p*x^5*log((a*x + b)/x) + 12*a^5*x^5*log(c) + 3*a^4*b*p*x^4 -
4*a^3*b^2*p*x^3 + 6*a^2*b^3*p*x^2 - 12*a*b^4*p*x + 12*b^5*p*log(a*x + b))/
a^5
```

Sympy [A] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \begin{cases} \frac{x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{5}+\frac{bpx^4}{20a}-\frac{b^2px^3}{15a^2}+\frac{b^3px^2}{10a^3}-\frac{b^4px}{5a^4}+\frac{b^5p\log(ax+b)}{5a^5} & \text{for}\ a\neq 0 \\ \frac{px^5}{25}+\frac{x^5\log\left(c\left(\frac{b}{x}\right)^p\right)}{5} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*ln(c*(a+b/x)**p),x)
```

output
```
Piecewise((x**5*log(c*(a + b/x)**p)/5 + b*p*x**4/(20*a) - b**2*p*x**3/(15*
a**2) + b**3*p*x**2/(10*a**3) - b**4*p*x/(5*a**4) + b**5*p*log(a*x + b)/(5
*a**5), Ne(a, 0)), (p*x**5/25 + x**5*log(c*(b/x)**p)/5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{1}{5}x^5\log\left(\left(a+\frac{b}{x}\right)^p c\right) + \frac{1}{60}bp\left(\frac{12\,b^4\log\left(ax+b\right)}{a^5}+\frac{3\,a^3x^4-4\,a^2bx^3+6\,ab^2x^2-12\,b^3x}{a^4}\right)$$

input
```
integrate(x^4*log(c*(a+b/x)^p),x, algorithm="maxima")
```

output
```
1/5*x^5*log((a + b/x)^p*c) + 1/60*b*p*(12*b^4*log(a*x + b)/a^5 + (3*a^3*x^
4 - 4*a^2*b*x^3 + 6*a*b^2*x^2 - 12*b^3*x)/a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(77) = 154.

Time = 0.13 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.46

$$\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = -\frac{\frac{12\,b^6p\log\left(\frac{ax+b}{x}\right)}{a^5-\frac{5\,(ax+b)a^4}{x}+\frac{10\,(ax+b)^2a^3}{x^2}-\frac{10\,(ax+b)^3a^2}{x^3}+\frac{5\,(ax+b)^4a}{x^4}-\frac{(ax+b)^5}{x^5}}+\frac{12\,b^6p\log\left(-a+\frac{ax+b}{x}\right)}{a^5}-\frac{12\,b^6p\log\left(\frac{ax+b}{x}\right)}{a^5}-\frac{25\,a^4b^6p-12\,a^4b^6\cdots}{a^9-\frac{5\,(a\cdots}{\cdots}}}{60\,b}$$

input
```
integrate(x^4*log(c*(a+b/x)^p),x, algorithm="giac")
```

output
```
-1/60*(12*b^6*p*log((a*x + b)/x)/(a^5 - 5*(a*x + b)*a^4/x + 10*(a*x + b)^2
*a^3/x^2 - 10*(a*x + b)^3*a^2/x^3 + 5*(a*x + b)^4*a/x^4 - (a*x + b)^5/x^5)
 + 12*b^6*p*log(-a + (a*x + b)/x)/a^5 - 12*b^6*p*log((a*x + b)/x)/a^5 - (2
5*a^4*b^6*p - 12*a^4*b^6*log(c) - 77*(a*x + b)*a^3*b^6*p/x + 94*(a*x + b)^
2*a^2*b^6*p/x^2 - 54*(a*x + b)^3*a*b^6*p/x^3 + 12*(a*x + b)^4*b^6*p/x^4)/(
a^9 - 5*(a*x + b)*a^8/x + 10*(a*x + b)^2*a^7/x^2 - 10*(a*x + b)^3*a^6/x^3
+ 5*(a*x + b)^4*a^5/x^4 - (a*x + b)^5*a^4/x^5))/b
```

Mupad [B] (verification not implemented)

Time = 26.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{x^5\,\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{5} - \frac{b^2\,p\,x^3}{15\,a^2} + \frac{b^3\,p\,x^2}{10\,a^3} + \frac{b^5\,p\,\ln\left(b+a\,x\right)}{5\,a^5} + \frac{b\,p\,x^4}{20\,a} - \frac{b^4\,p\,x}{5\,a^4}$$

input
```
int(x^4*log(c*(a + b/x)^p),x)
```

output
```
(x^5*log(c*(a + b/x)^p))/5 - (b^2*p*x^3)/(15*a^2) + (b^3*p*x^2)/(10*a^3) +
 (b^5*p*log(b + a*x))/(5*a^5) + (b*p*x^4)/(20*a) - (b^4*p*x)/(5*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

$$\int x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{12\log\left(\frac{(ax+b)^p c}{x^p}\right)a^5x^5 + 12\log\left(\frac{(ax+b)^p c}{x^p}\right)b^5 + 12\log(x)\,b^5p + 3a^4bp\,x^4 - 4a^3b^2p\,x^3 + 6a^2b^3p\,x^2 - 12a\,b^4px}{60a^5}$$

input
```
int(x^4*log(c*(a+b/x)^p),x)
```

output
```
(12*log(((a*x + b)**p*c)/x**p)*a**5*x**5 + 12*log(((a*x + b)**p*c)/x**p)*b
**5 + 12*log(x)*b**5*p + 3*a**4*b*p*x**4 - 4*a**3*b**2*p*x**3 + 6*a**2*b**
3*p*x**2 - 12*a*b**4*p*x)/(60*a**5)
```

3.27 $\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{b^3px}{4a^3} - \frac{b^2px^2}{8a^2} + \frac{bpx^3}{12a} + \frac{1}{4}x^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right) - \frac{b^4p\log(b+ax)}{4a^4}$$

output
```
1/4*b^3*p*x/a^3-1/8*b^2*p*x^2/a^2+1/12*b*p*x^3/a+1/4*x^4*ln(c*(a+b/x)^p)-1
/4*b^4*p*ln(a*x+b)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{abpx(6b^2-3abx+2a^2x^2) - 6b^4p\log \left(a+\frac{b}{x}\right) + 6a^4x^4\log \left(c\left(a+\frac{b}{x}\right)^p\right) - 6b^4p\log(x)}{24a^4}$$

input
```
Integrate[x^3*Log[c*(a + b/x)^p],x]
```

output
```
(a*b*p*x*(6*b^2 - 3*a*b*x + 2*a^2*x^2) - 6*b^4*p*Log[a + b/x] + 6*a^4*x^4*
Log[c*(a + b/x)^p] - 6*b^4*p*Log[x])/(24*a^4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2905, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{4} b p \int \frac{x^2}{a+\frac{b}{x}}dx+\frac{1}{4} x^4 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 795}$$

$$\frac{1}{4} b p \int \frac{x^3}{b+a x}dx+\frac{1}{4} x^4 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{4} b p \int \left(-\frac{b^3}{a^3 (b+a x)}+\frac{b^2}{a^3}-\frac{x b}{a^2}+\frac{x^2}{a}\right)dx+\frac{1}{4} x^4 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} b p \left(-\frac{b^3 \log (a x+b)}{a^4}+\frac{b^2 x}{a^3}-\frac{b x^2}{2 a^2}+\frac{x^3}{3 a}\right)+\frac{1}{4} x^4 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

input
```
Int[x^3*Log[c*(a + b/x)^p],x]
```

output
```
(x^4*Log[c*(a + b/x)^p])/4 + (b*p*((b^2*x)/a^3 - (b*x^2)/(2*a^2) + x^3/(3*
a) - (b^3*Log[b + a*x])/a^4))/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

method	result	size
parts	$\frac{x^{4} \ln\left(c \left(a+\frac{b}{x}\right)^{p}\right)}{4}+\frac{p b \left(\frac{\frac{1}{3} a^{2} x^{3}-\frac{1}{2} a b \,x^{2}+b^{2} x}{a^{3}}-\frac{b^{3} \ln\left(a x +b \right)}{a^{4}}\right)}{4}$	63
parallelrisch	$-\frac{-6 x^{4} \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) a^{4} p -2 x^{3} a^{3} b \,p^{2}+3 x^{2} a^{2} b^{2} p^{2}+6 \ln\left(x \right) b^{4} p^{2}-6 x a \,b^{3} p^{2}+6 \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) b^{4} p +6 b^{4} p^{2}}{24 a^{4} p}$	107

input
```
int(x^3*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*ln(c*(a+b/x)^p)+1/4*p*b*(1/a^3*(1/3*a^2*x^3-1/2*a*b*x^2+b^2*x)-b^3
/a^4*ln(a*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{6 a^4 p x^4 \log\left(\frac{ax+b}{x}\right) + 6 a^4 x^4 \log(c) + 2 a^3 b p x^3 - 3 a^2 b^2 p x^2 + 6 a b^3 p x - 6 b^4 p \log(ax+b)}{24 a^4}$$

input
```
integrate(x^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

output
```
1/24*(6*a^4*p*x^4*log((a*x + b)/x) + 6*a^4*x^4*log(c) + 2*a^3*b*p*x^3 - 3*
a^2*b^2*p*x^2 + 6*a*b^3*p*x - 6*b^4*p*log(a*x + b))/a^4
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \begin{cases} \frac{x^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4} + \frac{bpx^3}{12a} - \frac{b^2px^2}{8a^2} + \frac{b^3px}{4a^3} - \frac{b^4p\log(ax+b)}{4a^4} & \text{for } a \neq 0 \\ \frac{px^4}{16} + \frac{x^4\log\left(c\left(\frac{b}{x}\right)^p\right)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*ln(c*(a+b/x)**p),x)
```

output
```
Piecewise((x**4*log(c*(a + b/x)**p)/4 + b*p*x**3/(12*a) - b**2*p*x**2/(8*a
**2) + b**3*p*x/(4*a**3) - b**4*p*log(a*x + b)/(4*a**4), Ne(a, 0)), (p*x**
4/16 + x**4*log(c*(b/x)**p)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{1}{4}\,x^4 \log \left(\left(a+\frac{b}{x}\right)^p c\right) - \frac{1}{24}\,bp\left(\frac{6\,b^3 \log \left(ax+b\right)}{a^4} - \frac{2\,a^2x^3 - 3\,abx^2 + 6\,b^2x}{a^3}\right)$$

input
```
integrate(x^3*log(c*(a+b/x)^p),x, algorithm="maxima")
```

output
```
1/4*x^4*log((a + b/x)^p*c) - 1/24*b*p*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3
 - 3*a*b*x^2 + 6*b^2*x)/a^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.43

$$\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{\frac{6\,b^5 p \log\left(\frac{ax+b}{x}\right)}{a^4 - \frac{4\,(ax+b)a^3}{x} + \frac{6\,(ax+b)^2a^2}{x^2} - \frac{4\,(ax+b)^3a}{x^3} + \frac{(ax+b)^4}{x^4}} + \frac{6\,b^5 p \log\left(-a+\frac{ax+b}{x}\right)}{a^4} - \frac{6\,b^5 p \log\left(\frac{ax+b}{x}\right)}{a^4} - \frac{11\,a^3b^5p - 6\,a^3b^5\log(c) - \frac{26\,(ax+b)a^2b^5p}{x}}{a^7 - \frac{4\,(ax+b)a^6}{x} + \frac{6\,(ax+b)^2a^5}{x^2} - }}{24\,b}$$

input
```
integrate(x^3*log(c*(a+b/x)^p),x, algorithm="giac")
```

output
```
1/24*(6*b^5*p*log((a*x + b)/x)/(a^4 - 4*(a*x + b)*a^3/x + 6*(a*x + b)^2*a^
2/x^2 - 4*(a*x + b)^3*a/x^3 + (a*x + b)^4/x^4) + 6*b^5*p*log(-a + (a*x + b
)/x)/a^4 - 6*b^5*p*log((a*x + b)/x)/a^4 - (11*a^3*b^5*p - 6*a^3*b^5*log(c)
 - 26*(a*x + b)*a^2*b^5*p/x + 21*(a*x + b)^2*a*b^5*p/x^2 - 6*(a*x + b)^3*b
^5*p/x^3)/(a^7 - 4*(a*x + b)*a^6/x + 6*(a*x + b)^2*a^5/x^2 - 4*(a*x + b)^3
*a^4/x^3 + (a*x + b)^4*a^3/x^4))/b
```

Mupad [B] (verification not implemented)

Time = 26.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx = \frac{x^4 \ln \left(c \left(a+\frac{b}{x}\right)^p\right)}{4} - \frac{b^2 \, p \, x^2}{8 \, a^2} - \frac{b^4 \, p \, \ln \left(b+a \, x\right)}{4 \, a^4} + \frac{b \, p \, x^3}{12 \, a} + \frac{b^3 \, p \, x}{4 \, a^3}$$

input
```
int(x^3*log(c*(a + b/x)^p),x)
```

output
```
(x^4*log(c*(a + b/x)^p))/4 - (b^2*p*x^2)/(8*a^2) - (b^4*p*log(b + a*x))/(4
*a^4) + (b*p*x^3)/(12*a) + (b^3*p*x)/(4*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx = \frac{6 \log \left(\frac{(ax+b)^p c}{x^p}\right) a^4 x^4 - 6 \log \left(\frac{(ax+b)^p c}{x^p}\right) b^4 - 6 \log (x) \, b^4 p + 2 a^3 b p \, x^3 - 3 a^2 b^2 p \, x^2 + 6 a \, b^3 p x}{24 a^4}$$

input
```
int(x^3*log(c*(a+b/x)^p),x)
```

output
```
(6*log(((a*x + b)**p*c)/x**p)*a**4*x**4 - 6*log(((a*x + b)**p*c)/x**p)*b**
4 - 6*log(x)*b**4*p + 2*a**3*b*p*x**3 - 3*a**2*b**2*p*x**2 + 6*a*b**3*p*x)
/(24*a**4)
```

3.28 $\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = -\frac{b^2px}{3a^2}+\frac{bpx^2}{6a}+\frac{1}{3}x^3\log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^3p\log(b+ax)}{3a^3}$$

output
```
-1/3*b^2*p*x/a^2+1/6*b*p*x^2/a+1/3*x^3*ln(c*(a+b/x)^p)+1/3*b^3*p*ln(a*x+b)
/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{abpx(-2b+ax)+2b^3p\log\left(a+\frac{b}{x}\right)+2a^3x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+2b^3p\log(x)}{6a^3}$$

input
```
Integrate[x^2*Log[c*(a + b/x)^p],x]
```

output
```
(a*b*p*x*(-2*b + a*x) + 2*b^3*p*Log[a + b/x] + 2*a^3*x^3*Log[c*(a + b/x)^p
] + 2*b^3*p*Log[x])/(6*a^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2905, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{3} b p \int \frac{x}{a+\frac{b}{x}}dx+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 795}$$

$$\frac{1}{3} b p \int \frac{x^2}{b+a x}dx+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3} b p \int \left(\frac{b^2}{a^2 (b+a x)}-\frac{b}{a^2}+\frac{x}{a}\right)dx+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} b p \left(\frac{b^2 \log (a x+b)}{a^3}-\frac{b x}{a^2}+\frac{x^2}{2 a}\right)+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right)$$

input
```
Int[x^2*Log[c*(a + b/x)^p],x]
```

output
```
(x^3*Log[c*(a + b/x)^p])/3 + (b*p*(-((b*x)/a^2) + x^2/(2*a) + (b^2*Log[b +
 a*x])/a^3))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

method	result	size
parts	$\frac{x^{3} \ln\left(c \left(a+\frac{b}{x}\right)^{p}\right)}{3}+\frac{p b \left(\frac{\frac{1}{2} a\, x^{2}-b x}{a^{2}}+\frac{b^{2} \ln \left(a x +b \right)}{a^{3}}\right)}{3}$	52
parallelrisch	$-\frac{-2 x^{3} \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) a^{3} p -x^{2} a^{2} b \,p^{2}-2 \ln \left(x \right) b^{3} p^{2}+2 x a \,b^{2} p^{2}-2 \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) b^{3} p -2 b^{3} p^{2}}{6 a^{3} p}$	93

input
```
int(x^2*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(c*(a+b/x)^p)+1/3*p*b*(1/a^2*(1/2*a*x^2-b*x)+b^2/a^3*ln(a*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{2\,a^3px^3\log\left(\frac{ax+b}{x}\right)+2\,a^3x^3\log(c)+a^2bpx^2-2\,ab^2px+2\,b^3p\log(ax+b)}{6\,a^3}$$

input
```
integrate(x^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

output
```
1/6*(2*a^3*p*x^3*log((a*x + b)/x) + 2*a^3*x^3*log(c) + a^2*b*p*x^2 - 2*a*b
^2*p*x + 2*b^3*p*log(a*x + b))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \begin{cases} \frac{x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3}+\frac{bpx^2}{6a}-\frac{b^2px}{3a^2}+\frac{b^3p\log(ax+b)}{3a^3} & \text{for } a\neq 0 \\ \frac{px^3}{9}+\frac{x^3\log\left(c\left(\frac{b}{x}\right)^p\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*ln(c*(a+b/x)**p),x)
```

output
```
Piecewise((x**3*log(c*(a + b/x)**p)/3 + b*p*x**2/(6*a) - b**2*p*x/(3*a**2)
 + b**3*p*log(a*x + b)/(3*a**3), Ne(a, 0)), (p*x**3/9 + x**3*log(c*(b/x)**
p)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{1}{3}\,x^3\log\left(\left(a+\frac{b}{x}\right)^p c\right) + \frac{1}{6}\,bp\left(\frac{2\,b^2\log\left(ax+b\right)}{a^3}+\frac{ax^2-2\,bx}{a^2}\right)$$

input
```
integrate(x^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

output
```
1/3*x^3*log((a + b/x)^p*c) + 1/6*b*p*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*
b*x)/a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(53) = 106.

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.44

$$\int x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = -\frac{\frac{2\,b^4p\log\left(\frac{ax+b}{x}\right)}{a^3-\frac{3\,(ax+b)a^2}{x}+\frac{3\,(ax+b)^2a}{x^2}-\frac{(ax+b)^3}{x^3}}+\frac{2\,b^4p\log\left(-a+\frac{ax+b}{x}\right)}{a^3}-\frac{2\,b^4p\log\left(\frac{ax+b}{x}\right)}{a^3}-\frac{3\,a^2b^4p-2\,a^2b^4\log(c)-\frac{5\,(ax+b)ab^4p}{x}+\frac{2\,(ax+b)^2b^4}{x^2}}{a^5-\frac{3\,(ax+b)a^4}{x}+\frac{3\,(ax+b)^2a^3}{x^2}-\frac{(ax+b)^3a^2}{x^3}}}{6\,b}$$

input
```
integrate(x^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

output
```
-1/6*(2*b^4*p*log((a*x + b)/x)/(a^3 - 3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/
x^2 - (a*x + b)^3/x^3) + 2*b^4*p*log(-a + (a*x + b)/x)/a^3 - 2*b^4*p*log((
a*x + b)/x)/a^3 - (3*a^2*b^4*p - 2*a^2*b^4*log(c) - 5*(a*x + b)*a*b^4*p/x
+ 2*(a*x + b)^2*b^4*p/x^2)/(a^5 - 3*(a*x + b)*a^4/x + 3*(a*x + b)^2*a^3/x^
2 - (a*x + b)^3*a^2/x^3))/b
```

Mupad [B] (verification not implemented)

Time = 26.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{x^3\,\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{3}+\frac{b^3\,p\,\ln\left(b+a\,x\right)}{3\,a^3}+\frac{b\,p\,x^2}{6\,a}-\frac{b^2\,p\,x}{3\,a^2}$$

input
```
int(x^2*log(c*(a + b/x)^p),x)
```

output
```
(x^3*log(c*(a + b/x)^p))/3 + (b^3*p*log(b + a*x))/(3*a^3) + (b*p*x^2)/(6*a
) - (b^2*p*x)/(3*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

$$\int x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{2\log\left(\frac{(ax+b)^p c}{x^p}\right)a^3x^3+2\log\left(\frac{(ax+b)^p c}{x^p}\right)b^3+2\log(x)\,b^3p+a^2bp\,x^2-2a\,b^2px}{6a^3}$$

input
```
int(x^2*log(c*(a+b/x)^p),x)
```

output
```
(2*log(((a*x + b)**p*c)/x**p)*a**3*x**3 + 2*log(((a*x + b)**p*c)/x**p)*b**
3 + 2*log(x)*b**3*p + a**2*b*p*x**2 - 2*a*b**2*p*x)/(6*a**3)
```

3.29 $\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx=\frac{bpx}{2a}+\frac{1}{2}x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^2p\log(b+ax)}{2a^2}$$

output
```
1/2*b*p*x/a+1/2*x^2*ln(c*(a+b/x)^p)-1/2*b^2*p*ln(a*x+b)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx=\frac{1}{2}\left(x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp(ax-b\log(b+ax))}{a^2}\right)$$

input
```
Integrate[x*Log[c*(a + b/x)^p],x]
```

output
```
(x^2*Log[c*(a + b/x)^p] + (b*p*(a*x - b*Log[b + a*x]))/a^2)/2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2905, 772, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{2}bp\int \frac{1}{a+\frac{b}{x}}dx+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 772}$$

$$\frac{1}{2}bp\int \frac{x}{b+ax}dx+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}bp\int \left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)dx+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}bp\left(\frac{x}{a}-\frac{b\log(ax+b)}{a^2}\right)+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)$$

input
```
Int[x*Log[c*(a + b/x)^p],x]
```

output
```
(x^2*Log[c*(a + b/x)^p])/2 + (b*p*(x/a - (b*Log[b + a*x])/a^2))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result	size
parts	$\frac{x^2 \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{2}+\frac{pb\left(\frac{x}{a}-\frac{b\ln(ax+b)}{a^2}\right)}{2}$	41
parallelrisch	$-\frac{-x^2\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)a^2p+\ln(x)b^2p^2-xab\,p^2+\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)b^2p+b^2p^2}{2a^2p}$	76

input
```
int(x*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(a+b/x)^p)+1/2*p*b*(x/a-1/a^2*b*ln(a*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int x \log \left(c \left(a+\frac{b}{x}\right)^p\right) dx = \frac{a^2 p x^2 \log \left(\frac{ax+b}{x}\right) + a^2 x^2 \log (c) + abpx - b^2 p \log (ax+b)}{2\, a^2}$$

input
```
integrate(x*log(c*(a+b/x)^p),x, algorithm="fricas")
```

output
```
1/2*(a^2*p*x^2*log((a*x + b)/x) + a^2*x^2*log(c) + a*b*p*x - b^2*p*log(a*x
 + b))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int x \log \left(c \left(a+\frac{b}{x}\right)^p\right) dx = \begin{cases} \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2} + \frac{bpx}{2a} - \frac{b^2 p \log (ax+b)}{2a^2} & \text{for}\ a \neq 0 \\ \frac{px^2}{4} + \frac{x^2 \log \left(c\left(\frac{b}{x}\right)^p\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*ln(c*(a+b/x)**p),x)
```

output
```
Piecewise((x**2*log(c*(a + b/x)**p)/2 + b*p*x/(2*a) - b**2*p*log(a*x + b)/
(2*a**2), Ne(a, 0)), (p*x**2/4 + x**2*log(c*(b/x)**p)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x \log \left(c \left(a+\frac{b}{x}\right)^p\right) dx = \frac{1}{2}\, bp\left(\frac{x}{a} - \frac{b \log (ax+b)}{a^2}\right) + \frac{1}{2}\, x^2 \log \left(\left(a+\frac{b}{x}\right)^p c\right)$$

input
```
integrate(x*log(c*(a+b/x)^p),x, algorithm="maxima")
```

output
```
1/2*b*p*(x/a - b*log(a*x + b)/a^2) + 1/2*x^2*log((a + b/x)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.23

$$\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}}+\frac{b^3p\log\left(-a+\frac{ax+b}{x}\right)}{a^2}-\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2}-\frac{ab^3p-ab^3\log(c)-\frac{(ax+b)b^3p}{x}}{a^3-\frac{2\,(ax+b)a^2}{x}+\frac{(ax+b)^2a}{x^2}}}{2\,b}$$

input
```
integrate(x*log(c*(a+b/x)^p),x, algorithm="giac")
```

output
```
1/2*(b^3*p*log((a*x + b)/x)/(a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + b^
3*p*log(-a + (a*x + b)/x)/a^2 - b^3*p*log((a*x + b)/x)/a^2 - (a*b^3*p - a*
b^3*log(c) - (a*x + b)*b^3*p/x)/(a^3 - 2*(a*x + b)*a^2/x + (a*x + b)^2*a/x
^2))/b
```

Mupad [B] (verification not implemented)

Time = 26.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{x^2\,\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{2}+\frac{b\,p\,x}{2\,a}-\frac{b^2\,p\,\ln\left(b+a\,x\right)}{2\,a^2}$$

input
```
int(x*log(c*(a + b/x)^p),x)
```

output
```
(x^2*log(c*(a + b/x)^p))/2 + (b*p*x)/(2*a) - (b^2*p*log(b + a*x))/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right) a^2 x^2 - \log\left(\frac{(ax+b)^p c}{x^p}\right) b^2 - \log(x) b^2 p + abpx}{2a^2}$$

input
```
int(x*log(c*(a+b/x)^p),x)
```

output
```
(log(((a*x + b)**p*c)/x**p)*a**2*x**2 - log(((a*x + b)**p*c)/x**p)*b**2 -
log(x)*b**2*p + a*b*p*x)/(2*a**2)
```

3.30 $\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx=x \log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (b+a x)}{a}$$

output
```
x*ln(c*(a+b/x)^p)+b*p*ln(a*x+b)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx=\frac{b p \log \left(a+\frac{b}{x}\right)}{a}+x \log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b p \log (x)}{a}$$

input
```
Integrate[Log[c*(a + b/x)^p],x]
```

output
```
(b*p*Log[a + b/x])/a + x*Log[c*(a + b/x)^p] + (b*p*Log[x])/a
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2898, 795, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

$$\Big\downarrow \ 2898$$

$$bp\int \frac{1}{\left(a+\frac{b}{x}\right)x}dx+x\log \left(c\left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \ 795$$

$$bp\int \frac{1}{b+ax}dx+x\log \left(c\left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \ 16$$

$$x\log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\log(ax+b)}{a}$$

input
```
Int[Log[c*(a + b/x)^p],x]
```

output
```
x*Log[c*(a + b/x)^p] + (b*p*Log[b + a*x])/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
parts	$x \ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)+\frac{bp\ln(ax+b)}{a}$	28
default	$x \ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)+\frac{bp\ln(ax+b)}{a}$	30
parallelrisch	$-\frac{-\ln(x)b^{2}p^{2}-x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)abp-\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{2}p}{abp}$	63

input
```
int(ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(a+b/x)^p)+b*p*ln(a*x+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \log\left(c\left(a+\frac{b}{x}\right)^{p}\right)\,dx=\frac{apx\log\left(\frac{ax+b}{x}\right)+bp\log\left(ax+b\right)+ax\log\left(c\right)}{a}$$

input
```
integrate(log(c*(a+b/x)^p),x, algorithm="fricas")
```

output
```
(a*p*x*log((a*x + b)/x) + b*p*log(a*x + b) + a*x*log(c))/a
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \begin{cases} x\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{bp\log(ax+b)}{a} & \text{for } a \neq 0 \\ px + x\log\left(c\left(\frac{b}{x}\right)^p\right) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(a+b/x)**p),x)
```

output
```
Piecewise((x*log(c*(a + b/x)**p) + b*p*log(a*x + b)/a, Ne(a, 0)), (p*x + x
*log(c*(b/x)**p), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = x\log\left(\left(a+\frac{b}{x}\right)^p c\right) + \frac{bp\log(ax+b)}{a}$$

input
```
integrate(log(c*(a+b/x)^p),x, algorithm="maxima")
```

output
```
x*log((a + b/x)^p*c) + b*p*log(a*x + b)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.56

$$\int \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = -\frac{\frac{b^2p\log\left(-a+\frac{ax+b}{x}\right)}{a} + \frac{b^2p\log\left(\frac{ax+b}{x}\right)}{a-\frac{ax+b}{x}} - \frac{b^2p\log\left(\frac{ax+b}{x}\right)}{a} + \frac{b^2\log(c)}{a-\frac{ax+b}{x}}}{b}$$

input
```
integrate(log(c*(a+b/x)^p),x, algorithm="giac")
```

output
```
-(b^2*p*log(-a + (a*x + b)/x)/a + b^2*p*log((a*x + b)/x)/(a - (a*x + b)/x)
 - b^2*p*log((a*x + b)/x)/a + b^2*log(c)/(a - (a*x + b)/x))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = x \ln \left(c \left(a+\frac{b}{x}\right)^p\right)+\frac{b\,p\,\ln\left(b+a\,x\right)}{a}$$

input
```
int(log(c*(a + b/x)^p),x)
```

output
```
x*log(c*(a + b/x)^p) + (b*p*log(b + a*x))/a
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right) ax+\log\left(\frac{(ax+b)^p c}{x^p}\right) b+\log(x)\, bp}{a}$$

input
```
int(log(c*(a+b/x)^p),x)
```

output
```
(log(((a*x + b)**p*c)/x**p)*a*x + log(((a*x + b)**p*c)/x**p)*b + log(x)*b*
p)/a
```

3.31 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right) - p\,\text{PolyLog}\left(2,1+\frac{b}{ax}\right)$$

output
```
-ln(c*(a+b/x)^p)*ln(-b/a/x)-p*polylog(2,1+b/a/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right) - p\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x}}{a}\right)$$

input
```
Integrate[Log[c*(a + b/x)^p]/x,x]
```

output
```
-(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, (a + b/x)/a]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c(a+\frac{b}{x})^p\right)}{x}\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$-\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 2841}$$

$$bp\int \frac{\log\left(-\frac{b}{ax}\right)}{a+\frac{b}{x}}d\frac{1}{x}-\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)$$

$$\Big\downarrow \text{ 2752}$$

$$\log\left(-\frac{b}{ax}\right)\left(-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right)-p\,\text{PolyLog}\left(2,\frac{b}{ax}+1\right)$$

input
```
Int[Log[c*(a + b/x)^p]/x,x]
```

output
```
-(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, 1 + b/(a*x)]
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

method	result	size
parts	$\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln\left(x\right)+pb\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{ax+b}{b}\right)}{a}+\frac{\ln(x)\ln\left(\frac{ax+b}{b}\right)}{a}\right)a}{b}+\frac{\ln(x)^{2}}{2b}\right)$	67

input
```
int(ln(c*(a+b/x)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(c*(a+b/x)^p)*ln(x)+p*b*(-(dilog((a*x+b)/b)/a+ln(x)*ln((a*x+b)/b)/a)/b*a
+1/2*ln(x)^2/b)
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{x}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/x,x, algorithm="fricas")
```

output
```
integral(log(c*((a*x + b)/x)^p)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x}\,dx=\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x}\,dx$$

input
```
integrate(ln(c*(a+b/x)**p)/x,x)
```

output
```
Integral(log(c*(a + b/x)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x}\,dx\\ &=\frac{1}{2}bp\left(\frac{2\log\left(a+\frac{b}{x}\right)\log\left(x\right)}{b}+\frac{\log\left(x\right)^{2}}{b}-\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log\left(x\right)+\mathrm{Li}_{2}\left(-\frac{ax}{b}\right)\right)}{b}\right)\\ &\quad -p\log\left(a+\frac{b}{x}\right)\log\left(x\right)+\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)\log\left(x\right)\end{aligned}$$

input
```
integrate(log(c*(a+b/x)^p)/x,x, algorithm="maxima")
```

output
```
1/2*b*p*(2*log(a + b/x)*log(x)/b + log(x)^2/b - 2*(log(a*x/b + 1)*log(x) +
 dilog(-a*x/b))/b) - p*log(a + b/x)*log(x) + log((a + b/x)^p*c)*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(39) = 78$.

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.80

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$$
$$= -\frac{\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}}+\frac{b^3p\log\left(-a+\frac{ax+b}{x}\right)}{a^2}-\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2}-\frac{ab^3p-ab^3\log(c)-\frac{(ax+b)b^3p}{x}}{a^3-\frac{2\,(ax+b)a^2}{x}+\frac{(ax+b)^2a}{x^2}}}{2\,b^2}$$

input
```
integrate(log(c*(a+b/x)^p)/x,x, algorithm="giac")
```

output
```
-1/2*(b^3*p*log((a*x + b)/x)/(a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + b
^3*p*log(-a + (a*x + b)/x)/a^2 - b^3*p*log((a*x + b)/x)/a^2 - (a*b^3*p - a
*b^3*log(c) - (a*x + b)*b^3*p/x)/(a^3 - 2*(a*x + b)*a^2/x + (a*x + b)^2*a/
x^2))/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$$

input
```
int(log(c*(a + b/x)^p)/x,x)
```

output
```
int(log(c*(a + b/x)^p)/x, x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = \int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)}{x}dx$$

input
```
int(log(c*(a+b/x)^p)/x,x)
```

output
```
int(log(((a*x + b)**p*c)/x**p)/x,x)
```

3.32 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \frac{p}{x} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

output
```
p/x-(a+b/x)*ln(c*(a+b/x)^p)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \frac{p}{x} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

input
```
Integrate[Log[c*(a + b/x)^p]/x^2,x]
```

output
```
p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}dx$$

$$\Big\downarrow \text{2904}$$

$$-\int \log\left(c\left(a+\frac{b}{x}\right)^p\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2836}$$

$$-\frac{\int \log\left(c\left(a+\frac{b}{x}\right)^p\right)d\left(a+\frac{b}{x}\right)}{b}$$

$$\Big\downarrow \text{2732}$$

$$-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)-p\left(a+\frac{b}{x}\right)}{b}$$

input
```
Int[Log[c*(a + b/x)^p]/x^2,x]
```

output
```
-((-(p*(a + b/x)) + (a + b/x)*Log[c*(a + b/x)^p])/b)
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$-\frac{\left(a+\frac{b}{x}\right)\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)-\left(a+\frac{b}{x}\right)p}{b}$	37
default	$-\frac{\left(a+\frac{b}{x}\right)\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)-\left(a+\frac{b}{x}\right)p}{b}$	37
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x}-pb\left(\frac{a\ln(ax+b)}{b^{2}}-\frac{1}{bx}-\frac{a\ln(x)}{b^{2}}\right)$	51
parallelrisch	$-\frac{x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{2}p+\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)abp-ab\,p^{2}}{xabp}$	61

input
```
int(ln(c*(a+b/x)^p)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b*((a+b/x)*ln(c*(a+b/x)^p)-(a+b/x)*p)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \frac{bp - b\log(c) - (apx + bp)\log\left(\frac{ax+b}{x}\right)}{bx}$$

input
```
integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="fricas")
```

output
```
(b*p - b*log(c) - (a*p*x + b*p)*log((a*x + b)/x))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \begin{cases} -\frac{a\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b} + \frac{p}{x} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x} & \text{for } b \neq 0 \\ -\frac{\log(a^p c)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(a+b/x)**p)/x**2,x)
```

output
```
Piecewise((-a*log(c*(a + b/x)**p)/b + p/x - log(c*(a + b/x)**p)/x, Ne(b, 0
)), (-log(a**p*c)/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = -bp\left(\frac{a\log(ax+b)}{b^2} - \frac{a\log(x)}{b^2} - \frac{1}{bx}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{x}$$

input
```
integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="maxima")
```

output
```
-b*p*(a*log(a*x + b)/b^2 - a*log(x)/b^2 - 1/(b*x)) - log((a + b/x)^p*c)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.10

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = -\frac{\frac{(ax+b)p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{x} - \frac{(ax+b)p}{x} + \frac{(ax+b)\log(c)}{x}}{b}$$

input
```
integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="giac")
```

output
```
-((a*x + b)*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/x - (a*x + b)*p/x + (a*x
 + b)*log(c)/x)/b
```

Mupad [B] (verification not implemented)

Time = 26.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \frac{p}{x} - \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x} - \frac{2\,a\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b}$$

input
```
int(log(c*(a + b/x)^p)/x^2,x)
```

output
```
p/x - log(c*(a + b/x)^p)/x - (2*a*p*atanh((2*a*x)/b + 1))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \frac{-\log\left(\frac{(ax+b)^p c}{x^p}\right)ax - \log\left(\frac{(ax+b)^p c}{x^p}\right)b + bp}{bx}$$

input
```
int(log(c*(a+b/x)^p)/x^2,x)
```

output
```
( - log(((a*x + b)**p*c)/x**p)*a*x - log(((a*x + b)**p*c)/x**p)*b + b*p)/(
b*x)
```

3.33 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3} dx = \frac{p}{4x^2} - \frac{ap}{2bx} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

output
```
1/4*p/x^2-1/2*a*p/b/x+1/2*a^2*p*ln(a+b/x)/b^2-1/2*ln(c*(a+b/x)^p)/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3} dx = \frac{p}{4x^2} - \frac{ap}{2bx} + \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

input
```
Integrate[Log[c*(a + b/x)^p]/x^3,x]
```

output
```
p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)
^p]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3} \, dx$$

$$\Big\downarrow \text{ 2904}$$

$$-\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{1}{2}bp\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}d\frac{1}{x}-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}bp\int \left(\frac{a^2}{b^2\left(a+\frac{b}{x}\right)}-\frac{a}{b^2}+\frac{1}{bx}\right)d\frac{1}{x}-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}bp\left(\frac{a^2\log \left(a+\frac{b}{x}\right)}{b^3}-\frac{a}{b^2x}+\frac{1}{2bx^2}\right)-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

input
```
Int[Log[c*(a + b/x)^p]/x^3,x]
```

output
```
(b*p*(1/(2*b*x^2) - a/(b^2*x) + (a^2*Log[a + b/x])/b^3))/2 - Log[c*(a + b/
x)^p]/(2*x^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{2x^{2}}-\frac{pb\left(-\frac{a^{2}\ln(ax+b)}{b^{3}}-\frac{1}{2b\,x^{2}}+\frac{a^{2}\ln(x)}{b^{3}}+\frac{a}{b^{2}x}\right)}{2}$	63
parallelrisch	$-\frac{2\ln(x)x^{2}a^{2}p-2\ln(ax+b)x^{2}a^{2}p-2x^{2}a^{2}p+2apxb+2\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{2}-b^{2}p}{4x^{2}b^{2}}$	76

input
```
int(ln(c*(a+b/x)^p)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(a+b/x)^p)/x^2-1/2*p*b*(-1/b^3*a^2*ln(a*x+b)-1/2/b/x^2+1/b^3*a^2
*ln(x)+1/b^2*a/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = -\frac{2\,abpx - b^2p + 2\,b^2\log\left(c\right) - 2\left(a^2px^2 - b^2p\right)\log\left(\frac{ax+b}{x}\right)}{4\,b^2x^2}$$

input
```
integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*b*p*x - b^2*p + 2*b^2*log(c) - 2*(a^2*p*x^2 - b^2*p)*log((a*x +
b)/x))/(b^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = \begin{cases} \frac{a^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2b^2} - \frac{ap}{2bx} + \frac{p}{4x^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2} & \text{for}\ b \neq 0 \\ -\frac{\log\left(a^pc\right)}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(a+b/x)**p)/x**3,x)
```

output
```
Piecewise((a**2*log(c*(a + b/x)**p)/(2*b**2) - a*p/(2*b*x) + p/(4*x**2) -
log(c*(a + b/x)**p)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = \frac{1}{4}\,bp\left(\frac{2\,a^2\log\left(ax+b\right)}{b^3} - \frac{2\,a^2\log\left(x\right)}{b^3} - \frac{2\,ax-b}{b^2x^2}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{2\,x^2}$$

input
```
integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="maxima")
```

output
```
1/4*b*p*(2*a^2*log(a*x + b)/b^3 - 2*a^2*log(x)/b^3 - (2*a*x - b)/(b^2*x^2)
) - 1/2*log((a + b/x)^p*c)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(51) = 102$.

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.54

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx$$
$$= \frac{\frac{4\,(ax+b)ap\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{bx} - \frac{4\,(ax+b)ap}{bx} - \frac{2\,(ax+b)^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{bx^2} + \frac{4\,(ax+b)a\log(c)}{bx} + \frac{(ax+b)^2p}{bx^2} - \frac{2\,(ax+b)^2\,\mathrm{l}}{bx^2}}{4\,b}$$

input
```
integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="giac")
```

output
```
1/4*(4*(a*x + b)*a*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b*x) - 4*(a*x +
b)*a*p/(b*x) - 2*(a*x + b)^2*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b*x^2)
 + 4*(a*x + b)*a*log(c)/(b*x) + (a*x + b)^2*p/(b*x^2) - 2*(a*x + b)^2*log(
c)/(b*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 25.98 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = \frac{\frac{p}{2}-\frac{a\,p\,x}{b}}{2\,x^2} - \frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{2\,x^2} + \frac{a^2\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^2}$$

input
```
int(log(c*(a + b/x)^p)/x^3,x)
```

output
```
(p/2 - (a*p*x)/b)/(2*x^2) - log(c*(a + b/x)^p)/(2*x^2) + (a^2*p*atanh((2*a
*x)/b + 1))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = \frac{2\log\left(\frac{(ax+b)^p c}{x^p}\right)a^2x^2 - 2\log\left(\frac{(ax+b)^p c}{x^p}\right)b^2 - 2abpx + b^2p}{4b^2x^2}$$

input
```
int(log(c*(a+b/x)^p)/x^3,x)
```

output
```
(2*log(((a*x + b)**p*c)/x**p)*a**2*x**2 - 2*log(((a*x + b)**p*c)/x**p)*b**
2 - 2*a*b*p*x + b**2*p)/(4*b**2*x**2)
```

3.34 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx=\frac{p}{9x^3}-\frac{ap}{6bx^2}+\frac{a^2p}{3b^2x}-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

output
```
1/9*p/x^3-1/6*a*p/b/x^2+1/3*a^2*p/b^2/x-1/3*a^3*p*ln(a+b/x)/b^3-1/3*ln(c*(
a+b/x)^p)/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx=\frac{p}{9x^3}-\frac{ap}{6bx^2}+\frac{a^2p}{3b^2x}-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

input
```
Integrate[Log[c*(a + b/x)^p]/x^4,x]
```

output
```
p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*
b^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4} \, dx$$

$$\Big\downarrow \text{ 2904}$$

$$-\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{1}{3}bp\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}d\frac{1}{x}-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}bp\int \left(-\frac{a^3}{b^3\left(a+\frac{b}{x}\right)}+\frac{a^2}{b^3}-\frac{a}{b^2x}+\frac{1}{bx^2}\right)d\frac{1}{x}-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}bp\left(-\frac{a^3\log \left(a+\frac{b}{x}\right)}{b^4}+\frac{a^2}{b^3x}-\frac{a}{2b^2x^2}+\frac{1}{3bx^3}\right)-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

input
```
Int[Log[c*(a + b/x)^p]/x^4,x]
```

output
```
(b*p*(1/(3*b*x^3) - a/(2*b^2*x^2) + a^2/(b^3*x) - (a^3*Log[a + b/x])/b^4))
/3 - Log[c*(a + b/x)^p]/(3*x^3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{3x^{3}}-\frac{pb\left(\frac{a^{3}\ln\left(ax+b\right)}{b^{4}}-\frac{1}{3b\,x^{3}}-\frac{a^{2}}{b^{3}x}-\frac{a^{3}\ln\left(x\right)}{b^{4}}+\frac{a}{2b^{2}x^{2}}\right)}{3}$	75
parallelrisch	$-\frac{6x^{3}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{3}p+6x^{3}a^{3}p^{2}-6x^{2}a^{2}b\,p^{2}+3xa\,b^{2}p^{2}+6\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{3}p-2b^{3}p^{2}}{18x^{3}b^{3}p}$	97

input
```
int(ln(c*(a+b/x)^p)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(c*(a+b/x)^p)/x^3-1/3*p*b*(1/b^4*a^3*ln(a*x+b)-1/3/b/x^3-1/b^3*a^2/
x-1/b^4*a^3*ln(x)+1/2/b^2*a/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = \frac{6\,a^2bpx^2-3\,ab^2px+2\,b^3p-6\,b^3\log\left(c\right)-6\,\left(a^3px^3+b^3p\right)\log\left(\frac{ax+b}{x}\right)}{18\,b^3x^3}$$

input
```
integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="fricas")
```

output
```
1/18*(6*a^2*b*p*x^2 - 3*a*b^2*p*x + 2*b^3*p - 6*b^3*log(c) - 6*(a^3*p*x^3
+ b^3*p)*log((a*x + b)/x))/(b^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = \begin{cases} -\frac{a^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3b^3}+\frac{a^2p}{3b^2x}-\frac{ap}{6bx^2}+\frac{p}{9x^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} & \text{for } b\neq 0 \\ -\frac{\log\left(a^pc\right)}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(a+b/x)**p)/x**4,x)
```

output
```
Piecewise((-a**3*log(c*(a + b/x)**p)/(3*b**3) + a**2*p/(3*b**2*x) - a*p/(6
*b*x**2) + p/(9*x**3) - log(c*(a + b/x)**p)/(3*x**3), Ne(b, 0)), (-log(a**
p*c)/(3*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx$$
$$=-\frac{1}{18}\,bp\left(\frac{6\,a^3\log\left(ax+b\right)}{b^4}-\frac{6\,a^3\log\left(x\right)}{b^4}-\frac{6\,a^2x^2-3\,abx+2\,b^2}{b^3x^3}\right)-\frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{3\,x^3}$$

input
```
integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="maxima")
```

output
```
-1/18*b*p*(6*a^3*log(a*x + b)/b^4 - 6*a^3*log(x)/b^4 - (6*a^2*x^2 - 3*a*b*
x + 2*b^2)/(b^3*x^3)) - 1/3*log((a + b/x)^p*c)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(63) = 126.

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.21

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx =$$
$$-\frac{\frac{18\,(ax+b)a^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^2x}-\frac{18\,(ax+b)a^2p}{b^2x}-\frac{18\,(ax+b)^2ap\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^2x^2}+\frac{18\,(ax+b)a^2\log(c)}{b^2x}+\frac{9\,(ax+b)^2ap}{b^2x^2}}{18\,b}$$

input
```
integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="giac")
```

output
```
-1/18*(18*(a*x + b)*a^2*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^2*x) - 18
*(a*x + b)*a^2*p/(b^2*x) - 18*(a*x + b)^2*a*p*log(-b*(a/b - (a*x + b)/(b*x
)) + a)/(b^2*x^2) + 18*(a*x + b)*a^2*log(c)/(b^2*x) + 9*(a*x + b)^2*a*p/(b
^2*x^2) + 6*(a*x + b)^3*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^2*x^3) -
18*(a*x + b)^2*a*log(c)/(b^2*x^2) - 2*(a*x + b)^3*p/(b^2*x^3) + 6*(a*x + b
)^3*log(c)/(b^2*x^3))/b
```

Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = \frac{\frac{p}{3}+\frac{a^2\,p\,x^2}{b^2}-\frac{a\,p\,x}{2\,b}}{3\,x^3}-\frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{3\,x^3}-\frac{2\,a^3\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{3\,b^3}$$

input
```
int(log(c*(a + b/x)^p)/x^4,x)
```

output
```
(p/3 + (a^2*p*x^2)/b^2 - (a*p*x)/(2*b))/(3*x^3) - log(c*(a + b/x)^p)/(3*x^
3) - (2*a^3*p*atanh((2*a*x)/b + 1))/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = \frac{-6\log\left(\frac{(ax+b)^p c}{x^p}\right)a^3x^3-6\log\left(\frac{(ax+b)^p c}{x^p}\right)b^3+6a^2bp\,x^2-3a\,b^2px+2b^3p}{18b^3x^3}$$

input
```
int(log(c*(a+b/x)^p)/x^4,x)
```

output
```
( - 6*log(((a*x + b)**p*c)/x**p)*a**3*x**3 - 6*log(((a*x + b)**p*c)/x**p)*
b**3 + 6*a**2*b*p*x**2 - 3*a*b**2*p*x + 2*b**3*p)/(18*b**3*x**3)
```

3.35 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx=\frac{p}{16x^4}-\frac{ap}{12bx^3}+\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

output
```
1/16*p/x^4-1/12*a*p/b/x^3+1/8*a^2*p/b^2/x^2-1/4*a^3*p/b^3/x+1/4*a^4*p*ln(a
+b/x)/b^4-1/4*ln(c*(a+b/x)^p)/x^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx=\frac{p}{16x^4}-\frac{ap}{12bx^3}+\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

input
```
Integrate[Log[c*(a + b/x)^p]/x^5,x]
```

output
```
p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) +
(a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}dx$$

$$\Big\downarrow \text{2904}$$

$$-\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{4}bp\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}d\frac{1}{x}-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}bp\int \left(\frac{a^4}{b^4\left(a+\frac{b}{x}\right)}-\frac{a^3}{b^4}+\frac{a^2}{b^3x}-\frac{a}{b^2x^2}+\frac{1}{bx^3}\right)d\frac{1}{x}-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}bp\left(\frac{a^4\log \left(a+\frac{b}{x}\right)}{b^5}-\frac{a^3}{b^4x}+\frac{a^2}{2b^3x^2}-\frac{a}{3b^2x^3}+\frac{1}{4bx^4}\right)-\frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

input
```
Int[Log[c*(a + b/x)^p]/x^5,x]
```

output
```
(b*p*(1/(4*b*x^4) - a/(3*b^2*x^3) + a^2/(2*b^3*x^2) - a^3/(b^4*x) + (a^4*L
og[a + b/x])/b^5))/4 - Log[c*(a + b/x)^p]/(4*x^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{4x^{4}}-\frac{pb\left(-\frac{a^{4}\ln(ax+b)}{b^{5}}-\frac{1}{4b\,x^{4}}+\frac{a^{4}\ln(x)}{b^{5}}-\frac{a^{2}}{2b^{3}x^{2}}+\frac{a^{3}}{b^{4}x}+\frac{a}{3b^{2}x^{3}}\right)}{4}$	85
parallelrisch	$-\frac{12\ln(x)x^{4}a^{4}p-12\ln(ax+b)x^{4}a^{4}p-12x^{4}a^{4}p+12x^{3}a^{3}bp-6x^{2}a^{2}b^{2}p+4xa\,b^{3}p+12\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{4}-3b^{4}p}{48x^{4}b^{4}}$	100

input
```
int(ln(c*(a+b/x)^p)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(c*(a+b/x)^p)/x^4-1/4*p*b*(-1/b^5*a^4*ln(a*x+b)-1/4/b/x^4+1/b^5*a^4
*ln(x)-1/2/b^3*a^2/x^2+1/b^4*a^3/x+1/3/b^2*a/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5} \, dx = -\frac{12 \, a^3bpx^3 - 6 \, a^2b^2px^2 + 4 \, ab^3px - 3 \, b^4p + 12 \, b^4 \log \left(c\right) - 12 \, \left(a^4px^4 - b^4p\right) \log \left(\frac{ax+b}{x}\right)}{48 \, b^4x^4}$$

input
```
integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="fricas")
```

output
```
-1/48*(12*a^3*b*p*x^3 - 6*a^2*b^2*p*x^2 + 4*a*b^3*p*x - 3*b^4*p + 12*b^4*l
og(c) - 12*(a^4*p*x^4 - b^4*p)*log((a*x + b)/x))/(b^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5} \, dx = \begin{cases} \frac{a^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{4b^4} - \frac{a^3p}{4b^3x} + \frac{a^2p}{8b^2x^2} - \frac{ap}{12bx^3} + \frac{p}{16x^4} - \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4} & \text{for } b \neq 0 \\ -\frac{\log \left(a^pc\right)}{4x^4} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(a+b/x)**p)/x**5,x)
```

output
```
Piecewise((a**4*log(c*(a + b/x)**p)/(4*b**4) - a**3*p/(4*b**3*x) + a**2*p/
(8*b**2*x**2) - a*p/(12*b*x**3) + p/(16*x**4) - log(c*(a + b/x)**p)/(4*x**
4), Ne(b, 0)), (-log(a**p*c)/(4*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx = \frac{1}{48}\,bp\left(\frac{12\,a^4\log\left(ax+b\right)}{b^5}-\frac{12\,a^4\log\left(x\right)}{b^5}-\frac{12\,a^3x^3-6\,a^2bx^2+4\,ab^2x-3\,b^3}{b^4x^4}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{4\,x^4}$$

input

```
integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="maxima")
```

output

```
1/48*b*p*(12*a^4*log(a*x + b)/b^5 - 12*a^4*log(x)/b^5 - (12*a^3*x^3 - 6*a^
2*b*x^2 + 4*a*b^2*x - 3*b^3)/(b^4*x^4)) - 1/4*log((a + b/x)^p*c)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(75) = 150$.

Time = 0.13 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.64

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx = \frac{\frac{48\,(ax+b)a^3p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x}-\frac{48\,(ax+b)a^3p}{b^3x}-\frac{72\,(ax+b)^2a^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x^2}+\frac{48\,(ax+b)a^3\log(c)}{b^3x}+\frac{36\,(ax+b)^2a^2p}{b^3x^2}}{}$$

input

```
integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="giac")
```

output

```
1/48*(48*(a*x + b)*a^3*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x) - 48*
(a*x + b)*a^3*p/(b^3*x) - 72*(a*x + b)^2*a^2*p*log(-b*(a/b - (a*x + b)/(b*
x)) + a)/(b^3*x^2) + 48*(a*x + b)*a^3*log(c)/(b^3*x) + 36*(a*x + b)^2*a^2*
p/(b^3*x^2) + 48*(a*x + b)^3*a*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*
x^3) - 72*(a*x + b)^2*a^2*log(c)/(b^3*x^2) - 16*(a*x + b)^3*a*p/(b^3*x^3)
 - 12*(a*x + b)^4*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x^4) + 48*(a*x
 + b)^3*a*log(c)/(b^3*x^3) + 3*(a*x + b)^4*p/(b^3*x^4) - 12*(a*x + b)^4*lo
g(c)/(b^3*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 26.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5} \, dx = \frac{\frac{p}{4}+\frac{a^2\,p\,x^2}{2\,b^2}-\frac{a^3\,p\,x^3}{b^3}-\frac{a\,p\,x}{3\,b}}{4\,x^4}-\frac{\ln \left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{4\,x^4}+\frac{a^4\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{2\,b^4}$$

input
```
int(log(c*(a + b/x)^p)/x^5,x)
```

output
```
(p/4 + (a^2*p*x^2)/(2*b^2) - (a^3*p*x^3)/b^3 - (a*p*x)/(3*b))/(4*x^4) - lo
g(c*(a + b/x)^p)/(4*x^4) + (a^4*p*atanh((2*a*x)/b + 1))/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5} \, dx = \frac{12 \log \left(\frac{(ax+b)^p c}{x^p}\right) a^4 x^4-12 \log \left(\frac{(ax+b)^p c}{x^p}\right) b^4-12 a^3 b p\, x^3+6 a^2 b^2 p\, x^2-4 a\, b^3 p x+3 b^4 p}{48 b^4 x^4}$$

input
```
int(log(c*(a+b/x)^p)/x^5,x)
```

output
```
(12*log(((a*x + b)**p*c)/x**p)*a**4*x**4 - 12*log(((a*x + b)**p*c)/x**p)*b
**4 - 12*a**3*b*p*x**3 + 6*a**2*b**2*p*x**2 - 4*a*b**3*p*x + 3*b**4*p)/(48
*b**4*x**4)
```

3.36 $\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = -\frac{2b^2px}{5a^2}+\frac{2bpx^3}{15a}+\frac{2b^{5/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{5a^{5/2}} + \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

output
```
-2/5*b^2*p*x/a^2+2/15*b*p*x^3/a+2/5*b^(5/2)*p*arctan(a^(1/2)*x/b^(1/2))/a^
(5/2)+1/5*x^5*ln(c*(a+b/x^2)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \frac{2bpx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{b}{ax^2}\right)}{15a} + \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

input
```
Integrate[x^4*Log[c*(a + b/x^2)^p],x]
```

output
```
(2*b*p*x^3*Hypergeometric2F1[-3/2, 1, -1/2, -(b/(a*x^2))])/(15*a) + (x^5*L
og[c*(a + b/x^2)^p])/5
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2905, 795, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{2}{5}bp\int \frac{x^2}{a+\frac{b}{x^2}}dx+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{ 795}$$

$$\frac{2}{5}bp\int \frac{x^4}{ax^2+b}dx+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{2}{5}bp\int \left(\frac{b^2}{a^2\left(ax^2+b\right)}-\frac{b}{a^2}+\frac{x^2}{a}\right)dx+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}bp\left(\frac{b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{5/2}}-\frac{bx}{a^2}+\frac{x^3}{3a}\right)+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

input
```
Int[x^4*Log[c*(a + b/x^2)^p],x]
```

output
```
(2*b*p*(-((b*x)/a^2) + x^3/(3*a) + (b^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a
^(5/2)))/5 + (x^5*Log[c*(a + b/x^2)^p])/5
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

method	result	size
parts	$\frac{x^{5} \ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)}{5}+\frac{2 p b \left(\frac{\frac{1}{3} a \,x^{3}-b x}{a^{2}}+\frac{b^{2} \arctan\left(\frac{a x}{\sqrt{a b}}\right)}{a^{2} \sqrt{a b}}\right)}{5}$	60

input
```
int(x^4*ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*ln(c*(a+b/x^2)^p)+2/5*p*b*(1/a^2*(1/3*a*x^3-b*x)+b^2/a^2/(a*b)^(1/
2)*arctan(a*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.47

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx$$
$$= \left[\frac{3\,a^2px^5\log\left(\frac{ax^2+b}{x^2}\right)+3\,a^2x^5\log\left(c\right)+2\,abpx^3+3\,b^2p\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)-6\,b^2px}{15\,a^2},\ \frac{3\,a^2px^5\log\left(\right.}{}\right.$$

input
```
integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

output
```
[1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*a*b*p*x^3 +
 3*b^2*p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) - 6*b^
2*p*x)/a^2, 1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*
a*b*p*x^3 + 6*b^2*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) - 6*b^2*p*x)/a^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(70) = 140.

Time = 29.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.06

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx$$
$$= \begin{cases} \frac{x^5\log\left(0^pc\right)}{5} & \text{for}\ a=0\wedge b=0 \\ \frac{2px^5}{25}+\frac{x^5\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{5} & \text{for}\ a=0 \\ \frac{x^5\log\left(a^pc\right)}{5} & \text{for}\ b=0 \\ \frac{x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{5}+\frac{2bpx^3}{15a}-\frac{2b^2px}{5a^2}+\frac{b^3p\log\left(x-\sqrt{-\frac{b}{a}}\right)}{5a^3\sqrt{-\frac{b}{a}}}-\frac{b^3p\log\left(x+\sqrt{-\frac{b}{a}}\right)}{5a^3\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*ln(c*(a+b/x**2)**p),x)
```

output
```
Piecewise((x**5*log(0**p*c)/5, Eq(a, 0) & Eq(b, 0)), (2*p*x**5/25 + x**5*l
og(c*(b/x**2)**p)/5, Eq(a, 0)), (x**5*log(a**p*c)/5, Eq(b, 0)), (x**5*log(
c*(a + b/x**2)**p)/5 + 2*b*p*x**3/(15*a) - 2*b**2*p*x/(5*a**2) + b**3*p*lo
g(x - sqrt(-b/a))/(5*a**3*sqrt(-b/a)) - b**3*p*log(x + sqrt(-b/a))/(5*a**3
*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{5} x^5 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right) + \frac{2}{15} bp\left(\frac{3 b^2 \arctan \left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{ax^3-3 bx}{a^2}\right)$$

input
```
integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

output
```
1/5*x^5*log((a + b/x^2)^p*c) + 2/15*b*p*(3*b^2*arctan(a*x/sqrt(a*b))/(sqrt
(a*b)*a^2) + (a*x^3 - 3*b*x)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{5} px^5 \log \left(ax^2+b\right) - \frac{1}{5} px^5 \log \left(x^2\right) + \frac{1}{5} x^5 \log \left(c\right) + \frac{2 bpx^3}{15 a} + \frac{2 b^3 p \arctan \left(\frac{ax}{\sqrt{ab}}\right)}{5 \sqrt{ab}a^2} - \frac{2 b^2 px}{5 a^2}$$

input
```
integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

output
```
1/5*p*x^5*log(a*x^2 + b) - 1/5*p*x^5*log(x^2) + 1/5*x^5*log(c) + 2/15*b*p*
x^3/a + 2/5*b^3*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 2/5*b^2*p*x/a^2
```

Mupad [B] (verification not implemented)

Time = 25.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^4 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx = \frac{x^5 \ln \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{5} + \frac{2\, b^{5/2}\, p \operatorname{atan}\left(\frac{\sqrt{a}\, x}{\sqrt{b}}\right)}{5\, a^{5/2}} + \frac{2\, b\, p\, x^3}{15\, a} - \frac{2\, b^2\, p\, x}{5\, a^2}$$

input
```
int(x^4*log(c*(a + b/x^2)^p),x)
```

output
```
(x^5*log(c*(a + b/x^2)^p))/5 + (2*b^(5/2)*p*atan((a^(1/2)*x)/b^(1/2)))/(5*
a^(5/2)) + (2*b*p*x^3)/(15*a) - (2*b^2*p*x)/(5*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int x^4 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx = \frac{6\sqrt{b}\,\sqrt{a}\, atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right) b^2 p + 3 \log\left(\frac{\left(a\, x^2+b\right)^p c}{x^{2p}}\right) a^3 x^5 + 2a^2 bp\, x^3 - 6a\, b^2 px}{15a^3}$$

input
```
int(x^4*log(c*(a+b/x^2)^p),x)
```

output
```
(6*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b**2*p + 3*log(((a*x**2 +
 b)**p*c)/x**(2*p))*a**3*x**5 + 2*a**2*b*p*x**3 - 6*a*b**2*p*x)/(15*a**3)
```

3.37 $\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{bpx^2}{4a}+\frac{1}{4}x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{b^2p\log(b+ax^2)}{4a^2}$$

output
```
1/4*b*p*x^2/a+1/4*x^4*ln(c*(a+b/x^2)^p)-1/4*b^2*p*ln(a*x^2+b)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{4}x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{1}{4}bp\left(\frac{x^2}{a}-\frac{b\log\left(a+\frac{b}{x^2}\right)}{a^2}-\frac{2b\log(x)}{a^2}\right)$$

input
```
Integrate[x^3*Log[c*(a + b/x^2)^p],x]
```

output
```
(x^4*Log[c*(a + b/x^2)^p])/4 + (b*p*(x^2/a - (b*Log[a + b/x^2])/a^2 - (2*b
*Log[x])/a^2))/4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2905, 795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{2}bp\int \frac{x}{a+\frac{b}{x^2}}dx+\frac{1}{4}x^4\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{795}$$

$$\frac{1}{2}bp\int \frac{x^3}{ax^2+b}dx+\frac{1}{4}x^4\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}bp\int \frac{x^2}{ax^2+b}dx^2+\frac{1}{4}x^4\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}bp\int \left(\frac{1}{a}-\frac{b}{a\left(ax^2+b\right)}\right)dx^2+\frac{1}{4}x^4\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}bp\left(\frac{x^2}{a}-\frac{b\log \left(ax^2+b\right)}{a^2}\right)+\frac{1}{4}x^4\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

input
```
Int[x^3*Log[c*(a + b/x^2)^p],x]
```

output
```
(x^4*Log[c*(a + b/x^2)^p])/4 + (b*p*(x^2/a - (b*Log[b + a*x^2])/a^2))/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
parts	$\frac{x^4 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{4}+\frac{pb\left(\frac{x^2}{2a}-\frac{b\ln\left(a\,x^2+b\right)}{2a^2}\right)}{2}$	46
parallelrisch	$-\frac{-x^4\ln\left(c\left(\frac{a\,x^2+b}{x^2}\right)^p\right)a^2p-ab\,p^2x^2+2\ln\left(x\right)b^2p^2+\ln\left(c\left(\frac{a\,x^2+b}{x^2}\right)^p\right)b^2p+b^2p^2}{4a^2p}$	83

input
```
int(x^3*ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*ln(c*(a+b/x^2)^p)+1/2*p*b*(1/2*x^2/a-1/2/a^2*b*ln(a*x^2+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{a^2 p x^4 \log \left(\frac{a x^2+b}{x^2}\right)+a^2 x^4 \log (c)+a b p x^2-b^2 p \log \left(a x^2+b\right)}{4 a^2}$$

input
```
integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

output
```
1/4*(a^2*p*x^4*log((a*x^2 + b)/x^2) + a^2*x^4*log(c) + a*b*p*x^2 - b^2*p*l
og(a*x^2 + b))/a^2
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \begin{cases} \frac{x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{4}+\frac{b p x^2}{4 a}-\frac{b^2 p \log \left(a x^2+b\right)}{4 a^2} & \text{for } a \neq 0 \\ \frac{p x^4}{8}+\frac{x^4 \log \left(c\left(\frac{b}{x^2}\right)^p\right)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*ln(c*(a+b/x**2)**p),x)
```

output
```
Piecewise((x**4*log(c*(a + b/x**2)**p)/4 + b*p*x**2/(4*a) - b**2*p*log(a*x
**2 + b)/(4*a**2), Ne(a, 0)), (p*x**4/8 + x**4*log(c*(b/x**2)**p)/4, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{4} x^4 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right) + \frac{1}{4} bp\left(\frac{x^2}{a} - \frac{b\log \left(ax^2+b\right)}{a^2}\right)$$

input
```
integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

output
```
1/4*x^4*log((a + b/x^2)^p*c) + 1/4*b*p*(x^2/a - b*log(a*x^2 + b)/a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{4} px^4 \log \left(ax^2+b\right) - \frac{1}{4} px^4 \log \left(x^2\right) + \frac{1}{4} x^4 \log \left(c\right) + \frac{bpx^2}{4\,a} - \frac{b^2 p \log \left(ax^2+b\right)}{4\,a^2}$$

input
```
integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

output
```
1/4*p*x^4*log(a*x^2 + b) - 1/4*p*x^4*log(x^2) + 1/4*x^4*log(c) + 1/4*b*p*x
^2/a - 1/4*b^2*p*log(a*x^2 + b)/a^2
```

Mupad [B] (verification not implemented)

Time = 25.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{x^4\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{4} - \frac{b^2\,p\,\ln\left(a\,x^2+b\right)}{4\,a^2} + \frac{b\,p\,x^2}{4\,a}$$

input
```
int(x^3*log(c*(a + b/x^2)^p),x)
```

output
```
(x^4*log(c*(a + b/x^2)^p))/4 - (b^2*p*log(b + a*x^2))/(4*a^2) + (b*p*x^2)/
(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right) a^2 x^4 - \log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right) b^2 - 2\log(x)\, b^2 p + abp\, x^2}{4a^2}$$

input
```
int(x^3*log(c*(a+b/x^2)^p),x)
```

output
```
(log(((a*x**2 + b)**p*c)/x**(2*p))*a**2*x**4 - log(((a*x**2 + b)**p*c)/x**
(2*p))*b**2 - 2*log(x)*b**2*p + a*b*p*x**2)/(4*a**2)
```

3.38 $\int x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{2bpx}{3a} - \frac{2b^{3/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}} + \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

output
```
2/3*b*p*x/a-2/3*b^(3/2)*p*arctan(a^(1/2)*x/b^(1/2))/a^(3/2)+1/3*x^3*ln(c*(
a+b/x^2)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{2bpx \operatorname{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b}{ax^2}\right)}{3a} + \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

input
```
Integrate[x^2*Log[c*(a + b/x^2)^p],x]
```

output
```
(2*b*p*x*Hypergeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))])/(3*a) + (x^3*Log[c
*(a + b/x^2)^p])/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2905, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{2}{3} b p \int \frac{1}{a+\frac{b}{x^2}}dx+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{ 772}$$

$$\frac{2}{3} b p \int \frac{x^2}{a x^2+b}dx+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{3} b p \left(\frac{x}{a}-\frac{b \int \frac{1}{a x^2+b}dx}{a}\right)+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2}{3} b p \left(\frac{x}{a}-\frac{\sqrt{b} \arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{a^{3/2}}\right)+\frac{1}{3} x^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)$$

input
```
Int[x^2*Log[c*(a + b/x^2)^p],x]
```

output
```
(2*b*p*(x/a - (Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(3/2)))/3 + (x^3*Log
[c*(a + b/x^2)^p])/3
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

method	result	size
parts	$\frac{x^{3} \ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)}{3}+\frac{2 p b \left(\frac{x}{a}-\frac{b \arctan\left(\frac{a x}{\sqrt{a b}}\right)}{a \sqrt{a b}}\right)}{3}$	49

input
```
int(x^2*ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(c*(a+b/x^2)^p)+2/3*p*b*(x/a-1/a*b/(a*b)^(1/2)*arctan(a*x/(a*b)^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \left[\frac{apx^3 \log \left(\frac{ax^2+b}{x^2}\right) + ax^3 \log(c) + bp\sqrt{-\frac{b}{a}} \log \left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right) + 2\,bpx}{3\,a}, \frac{apx^3 \log \left(\frac{ax^2+b}{x^2}\right) + ax^3 \log(c)\right.$$

input
```
integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

output
```
[1/3*(a*p*x^3*log((a*x^2 + b)/x^2) + a*x^3*log(c) + b*p*sqrt(-b/a)*log((a*
x^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) + 2*b*p*x)/a, 1/3*(a*p*x^3*log((a
*x^2 + b)/x^2) + a*x^3*log(c) - 2*b*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) +
2*b*p*x)/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(54) = 108$.

Time = 10.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.29

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \begin{cases} \frac{x^3 \log(0^p c)}{3} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{2px^3}{9} + \frac{x^3 \log\left(c\left(\frac{b}{x^2}\right)^p\right)}{3} & \text{for}\ a = 0 \\ \frac{x^3 \log(a^p c)}{3} & \text{for}\ b = 0 \\ \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3} + \frac{2bpx}{3a} - \frac{b^2 p \log\left(x-\sqrt{-\frac{b}{a}}\right)}{3a^2\sqrt{-\frac{b}{a}}} + \frac{b^2 p \log\left(x+\sqrt{-\frac{b}{a}}\right)}{3a^2\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*ln(c*(a+b/x**2)**p),x)
```

output
```
Piecewise((x**3*log(0**p*c)/3, Eq(a, 0) & Eq(b, 0)), (2*p*x**3/9 + x**3*lo
g(c*(b/x**2)**p)/3, Eq(a, 0)), (x**3*log(a**p*c)/3, Eq(b, 0)), (x**3*log(c
*(a + b/x**2)**p)/3 + 2*b*p*x/(3*a) - b**2*p*log(x - sqrt(-b/a))/(3*a**2*s
qrt(-b/a)) + b**2*p*log(x + sqrt(-b/a))/(3*a**2*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{3}x^3 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right) - \frac{2}{3}bp\left(\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{x}{a}\right)$$

input
```
integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

output
```
1/3*x^3*log((a + b/x^2)^p*c) - 2/3*b*p*(b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)
*a) - x/a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{3}px^3 \log \left(ax^2+b\right) - \frac{1}{3}px^3 \log \left(x^2\right) + \frac{1}{3}x^3 \log (c) - \frac{2\,b^2p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{3\sqrt{ab}a} + \frac{2\,bpx}{3\,a}$$

input
```
integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

output
```
1/3*p*x^3*log(a*x^2 + b) - 1/3*p*x^3*log(x^2) + 1/3*x^3*log(c) - 2/3*b^2*p
*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) + 2/3*b*p*x/a
```

Mupad [B] (verification not implemented)

Time = 25.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \frac{x^3\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{3}-\frac{2\,b^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3\,a^{3/2}}+\frac{2\,b\,p\,x}{3\,a}$$

input
```
int(x^2*log(c*(a + b/x^2)^p),x)
```

output
```
(x^3*log(c*(a + b/x^2)^p))/3 - (2*b^(3/2)*p*atan((a^(1/2)*x)/b^(1/2)))/(3*
a^(3/2)) + (2*b*p*x)/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)bp+\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)a^2x^3+2abpx}{3a^2}$$

input
```
int(x^2*log(c*(a+b/x^2)^p),x)
```

output
```
( - 2*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b*p + log(((a*x**2 + b
)**p*c)/x**(2*p))*a**2*x**3 + 2*a*b*p*x)/(3*a**2)
```

3.39 $\int x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{2}x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{bp\log(b+ax^2)}{2a}$$

output
```
1/2*x^2*ln(c*(a+b/x^2)^p)+1/2*b*p*ln(a*x^2+b)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{bp\log\left(a+\frac{b}{x^2}\right)}{2a} + \frac{1}{2}x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{bp\log(x)}{a}$$

input
```
Integrate[x*Log[c*(a + b/x^2)^p],x]
```

output
```
(b*p*Log[a + b/x^2])/(2*a) + (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[x])/a
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2905, 795, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right) \, dx$$

$$\Big\downarrow \text{2905}$$

$$bp \int \frac{1}{\left(a+\frac{b}{x^2}\right) x}dx+\frac{1}{2} x^2 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{795}$$

$$bp \int \frac{x}{a x^2+b}dx+\frac{1}{2} x^2 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{2} x^2 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)+\frac{b p \log \left(a x^2+b\right)}{2 a}$$

input
```
Int[x*Log[c*(a + b/x^2)^p],x]
```

output
```
(x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[b + a*x^2])/(2*a)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
parts	$\frac{x^{2} \ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)}{2}+\frac{b p \ln\left(a \,x^{2}+b \right)}{2 a}$	34
parallelrisch	$-\frac{-x^{2} \ln\left(c \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{p}\right) a b p -2 \ln\left(x \right) b^{2} p^{2}-\ln\left(c \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{p}\right) b^{2} p}{2 a b p}$	69

input
```
int(x*ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(a+b/x^2)^p)+1/2*b*p*ln(a*x^2+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{apx^2\log\left(\frac{ax^2+b}{x^2}\right)+ax^2\log\left(c\right)+bp\log\left(ax^2+b\right)}{2\,a}$$

input
```
integrate(x*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

output
```
1/2*(a*p*x^2*log((a*x^2 + b)/x^2) + a*x^2*log(c) + b*p*log(a*x^2 + b))/a
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\begin{cases}\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2}+\frac{bp\log\left(ax^2+b\right)}{2a} & \text{for } a\neq 0\\ \frac{px^2}{2}+\frac{x^2\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*ln(c*(a+b/x**2)**p),x)
```

output
```
Piecewise((x**2*log(c*(a + b/x**2)**p)/2 + b*p*log(a*x**2 + b)/(2*a), Ne(a
, 0)), (p*x**2/2 + x**2*log(c*(b/x**2)**p)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{1}{2}\,x^2\log\left(\left(a+\frac{b}{x^2}\right)^pc\right)+\frac{bp\log\left(ax^2+b\right)}{2\,a}$$

input
```
integrate(x*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

output
```
1/2*x^2*log((a + b/x^2)^p*c) + 1/2*b*p*log(a*x^2 + b)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{1}{2}\,px^2\log\left(ax^2+b\right)-\frac{1}{2}\,px^2\log\left(x^2\right)+\frac{1}{2}\,x^2\log\left(c\right)+\frac{bp\log\left(ax^2+b\right)}{2\,a}$$

input
```
integrate(x*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

output
```
1/2*p*x^2*log(a*x^2 + b) - 1/2*p*x^2*log(x^2) + 1/2*x^2*log(c) + 1/2*b*p*l
og(a*x^2 + b)/a
```

Mupad [B] (verification not implemented)

Time = 25.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx = \frac{x^2 \ln \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{2} + \frac{b\,p\,\ln \left(a\,x^2+b\right)}{2\,a}$$

input
```
int(x*log(c*(a + b/x^2)^p),x)
```

output
```
(x^2*log(c*(a + b/x^2)^p))/2 + (b*p*log(b + a*x^2))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx = \frac{\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right) a\,x^2 + \log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right) b + 2\log(x)\,bp}{2a}$$

input
```
int(x*log(c*(a+b/x^2)^p),x)
```

output
```
(log(((a*x**2 + b)**p*c)/x**(2*p))*a*x**2 + log(((a*x**2 + b)**p*c)/x**(2*
p))*b + 2*log(x)*b*p)/(2*a)
```

3.40 $\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 41

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}}+x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

output
```
2*b^(1/2)*p*arctan(a^(1/2)*x/b^(1/2))/a^(1/2)+x*ln(c*(a+b/x^2)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=-\frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{a}}+x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

input
```
Integrate[Log[c*(a + b/x^2)^p],x]
```

output
```
(-2*Sqrt[b]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + x*Log[c*(a + b/x^2)^p
]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2898, 795, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

$$\Big\downarrow \text{2898}$$

$$2bp\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}dx+x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{795}$$

$$2bp\int \frac{1}{ax^2+b}dx+x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}}+x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

input
```
Int[Log[c*(a + b/x^2)^p],x]
```

output
```
(2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
parts	$x \ln \left(c {\left(a+\frac{b}{x^{2}}\right)}^{p}\right)+\frac{2 p b \arctan \left(\frac{a x}{\sqrt{a b}}\right)}{\sqrt{a b}}$	34
default	$x \ln \left(c {\left(\frac{a\,x^{2}+b}{x^{2}}\right)}^{p}\right)+\frac{2 p b \arctan \left(\frac{a x}{\sqrt{a b}}\right)}{\sqrt{a b}}$	38

input
```
int(ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(a+b/x^2)^p)+2*p*b/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.61

$$\int \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)\, dx = \left[px \log \left(\frac{ax^2+b}{x^2}\right)+p\sqrt{-\frac{b}{a}}\log \left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+x\log \left(c\right), px \log \left(\frac{ax^2+b}{x^2}\right)+2\,p\sqrt{\frac{b}{a}}\arctan \left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)+x\log \left(c\right)\right]$$

input
```
integrate(log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

output
```
[p*x*log((a*x^2 + b)/x^2) + p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b
)/(a*x^2 + b)) + x*log(c), p*x*log((a*x^2 + b)/x^2) + 2*p*sqrt(b/a)*arctan
(a*x*sqrt(b/a)/b) + x*log(c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(39) = 78$.

Time = 3.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.32

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \begin{cases} x\log\left(0^p c\right) & \text{for } a=0 \wedge b=0 \\ 2px + x\log\left(c\left(\frac{b}{x^2}\right)^p\right) & \text{for } a=0 \\ x\log\left(a^p c\right) & \text{for } b=0 \\ x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{bp\log\left(x-\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} - \frac{bp\log\left(x+\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(a+b/x**2)**p),x)
```

output
```
Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (2*p*x + x*log(c*(b/x**2)*
*p), Eq(a, 0)), (x*log(a**p*c), Eq(b, 0)), (x*log(c*(a + b/x**2)**p) + b*p
*log(x - sqrt(-b/a))/(a*sqrt(-b/a)) - b*p*log(x + sqrt(-b/a))/(a*sqrt(-b/a
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \frac{2\,bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}} + x\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)$$

input
```
integrate(log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

output
```
2*b*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) + x*log((a + b/x^2)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = px\log\left(ax^2+b\right)-px\log\left(x^2\right)+\frac{2\,bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}+x\log\left(c\right)$$

input
```
integrate(log(c*(a+b/x^2)^p),x, algorithm="giac")
```

output
```
p*x*log(a*x^2 + b) - p*x*log(x^2) + 2*b*p*arctan(a*x/sqrt(a*b))/sqrt(a*b)
+ x*log(c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = x\,\ln\left(c\,\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2\,\sqrt{b}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
int(log(c*(a + b/x^2)^p),x)
```

output
```
x*log(c*(a + b/x^2)^p) + (2*b^(1/2)*p*atan((a^(1/2)*x)/b^(1/2)))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)p+\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)ax}{a}$$

input
```
int(log(c*(a+b/x^2)^p),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*p + log(((a*x**2 + b)**p*
c)/x**(2*p))*a*x)/a
```

3.41 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx=-\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)-\frac{1}{2}p\,\mathrm{PolyLog}\left(2,1+\frac{b}{ax^2}\right)$$

output
```
-1/2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)-1/2*p*polylog(2,1+b/a/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx=-\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)-\frac{1}{2}p\,\mathrm{PolyLog}\left(2,\frac{a+\frac{b}{x^2}}{a}\right)$$

input
```
Integrate[Log[c*(a + b/x^2)^p]/x,x]
```

output
```
-1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))]) - (p*PolyLog[2, (a + b/x^2)/
a])/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+\frac{b}{x^2})^p\right)}{x} dx$$

$$\Big\downarrow \ 2904$$

$$-\frac{1}{2}\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) d\frac{1}{x^2}$$

$$\Big\downarrow \ 2841$$

$$\frac{1}{2}\left(bp\int \frac{\log (-\frac{b}{ax^2})}{a+\frac{b}{x^2}}d\frac{1}{x^2}-\log \left(-\frac{b}{ax^2}\right)\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\right)$$

$$\Big\downarrow \ 2752$$

$$\frac{1}{2}\left(\log \left(-\frac{b}{ax^2}\right)\left(-\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\right)-p\,\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)\right)$$

input
```
Int[Log[c*(a + b/x^2)^p]/x,x]
```

output
```
(-(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))]) - p*PolyLog[2, 1 + b/(a*x^2)])/
2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(40) = 80$.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.86

method	result
parts	$\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln\left(x\right)+2pb\left(-\frac{\left(\frac{\ln(x)\left(\ln\left(\frac{-ax+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{ax+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{-ax+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{ax+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2a}\right)a}{b}\right.$

input
```
int(ln(c*(a+b/x^2)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(c*(a+b/x^2)^p)*ln(x)+2*p*b*(-(1/2*ln(x)*(ln((-a*x+(-a*b)^(1/2))/(-a*b)^
(1/2))+ln((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a+1/2*(dilog((-a*x+(-a*b)^(1/2
))/(-a*b)^(1/2))+dilog((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a)/b*a+1/2*ln(x)^
2/b)
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{x}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="fricas")
```

output
```
integral(log(c*((a*x^2 + b)/x^2)^p)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = \int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$$

input
```
integrate(ln(c*(a+b/x**2)**p)/x,x)
```

output
```
Integral(log(c*(a + b/x**2)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.02

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = \frac{1}{2}bp\left(\frac{2\log\left(a+\frac{b}{x^2}\right)\log(x)}{b}+\frac{2\log(x)^2}{b}-\frac{2\log\left(\frac{ax^2}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax^2}{b}\right)}{b}\right) - p\log\left(a+\frac{b}{x^2}\right)\log(x)+\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)\log(x)$$

input
```
integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="maxima")
```

output
```
1/2*b*p*(2*log(a + b/x^2)*log(x)/b + 2*log(x)^2/b - (2*log(a*x^2/b + 1)*lo
g(x) + dilog(-a*x^2/b))/b) - p*log(a + b/x^2)*log(x) + log((a + b/x^2)^p*c
)*log(x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{x}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="giac")
```

output
```
integrate(log((a + b/x^2)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$$

input
```
int(log(c*(a + b/x^2)^p)/x,x)
```

output
```
int(log(c*(a + b/x^2)^p)/x, x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = \int \frac{\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)}{x}dx$$

input
```
int(log(c*(a+b/x^2)^p)/x,x)
```

output
```
int(log(((a*x**2 + b)**p*c)/x**(2*p))/x,x)
```

3.42 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 50

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \frac{2p}{x}+\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

output
```
2*p/x+2*a^(1/2)*p*arctan(a^(1/2)*x/b^(1/2))/b^(1/2)-ln(c*(a+b/x^2)^p)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \frac{2p}{x}-\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{b}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

input
```
Integrate[Log[c*(a + b/x^2)^p]/x^2,x]
```

output
```
(2*p)/x - (2*Sqrt[a]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b] - Log[c*(a + b
/x^2)^p]/x
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2905, 795, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2} \, dx$$

$$\Big\downarrow \; 2905$$

$$-2bp\int \frac{1}{\left(a+\frac{b}{x^2}\right) x^4}dx-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

$$\Big\downarrow \; 795$$

$$-2bp\int \frac{1}{x^2 \left(a x^2+b\right)}dx-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

$$\Big\downarrow \; 264$$

$$-2bp\left(-\frac{a\int \frac{1}{a x^2+b}dx}{b}-\frac{1}{b x}\right)-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

$$\Big\downarrow \; 218$$

$$-2bp\left(-\frac{\sqrt{a} \arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{b x}\right)-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

input
```
Int[Log[c*(a + b/x^2)^p]/x^2,x]
```

output
```
-2*b*p*(-(1/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(3/2)) - Log[
c*(a + b/x^2)^p]/x
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)}{x}-2pb\left(-\frac{1}{bx}-\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{b\sqrt{ab}}\right)$	52

input
```
int(ln(c*(a+b/x^2)^p)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(a+b/x^2)^p)/x-2*p*b*(-1/b/x-1/b*a/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.38

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \left[\frac{px\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-p\log\left(\frac{ax^2+b}{x^2}\right)+2\,p-\log\left(c\right)}{x}, \frac{2\,px\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)-p\log\left(\frac{ax^2+b}{x^2}\right)-}{x}\right.$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="fricas")
```

output
```
[(p*x*sqrt(-a/b)*log((a*x^2 + 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) - p*log((
a*x^2 + b)/x^2) + 2*p - log(c))/x, (2*p*x*sqrt(a/b)*arctan(x*sqrt(a/b)) -
p*log((a*x^2 + b)/x^2) + 2*p - log(c))/x]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(44) = 88$.

Time = 7.73 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.94

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \begin{cases} -\frac{\log\left(0^p c\right)}{x} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2p}{x} - \frac{\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{x} & \text{for}\: a = 0 \\ -\frac{\log\left(a^p c\right)}{x} & \text{for}\: b = 0 \\ \frac{p\log\left(x-\sqrt{-\frac{b}{a}}\right)}{\sqrt{-\frac{b}{a}}} - \frac{p\log\left(x+\sqrt{-\frac{b}{a}}\right)}{\sqrt{-\frac{b}{a}}} + \frac{2p}{x} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(a+b/x**2)**p)/x**2,x)
```

output
```
Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (2*p/x - log(c*(b/x**2)**
p)/x, Eq(a, 0)), (-log(a**p*c)/x, Eq(b, 0)), (p*log(x - sqrt(-b/a))/sqrt(-
b/a) - p*log(x + sqrt(-b/a))/sqrt(-b/a) + 2*p/x - log(c*(a + b/x**2)**p)/x
, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = 2\,bp\left(\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{1}{bx}\right)-\frac{\log\left(\left(a+\frac{b}{x^2}\right)^pc\right)}{x}$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="maxima")
```

output
```
2*b*p*(a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/(b*x)) - log((a + b/x^2)^
p*c)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \frac{2\,ap\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{p\log\left(ax^2+b\right)}{x}+\frac{p\log\left(x^2\right)}{x}+\frac{2\,p-\log\left(c\right)}{x}$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="giac")
```

output
```
2*a*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) - p*log(a*x^2 + b)/x + p*log(x^2)/x
+ (2*p - log(c))/x
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx=\frac{2\,p}{x}-\frac{\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{x}+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

input
```
int(log(c*(a + b/x^2)^p)/x^2,x)
```

output
```
(2*p)/x - log(c*(a + b/x^2)^p)/x + (2*a^(1/2)*p*atan((a^(1/2)*x)/b^(1/2)))
/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx=\frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)px-\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)b+2bp}{bx}$$

input
```
int(log(c*(a+b/x^2)^p)/x^2,x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*p*x - log(((a*x**2 + b)**
p*c)/x**(2*p))*b + 2*b*p)/(b*x)
```

3.43 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx=\frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

output
```
1/2*p/x^2-1/2*(a+b/x^2)*ln(c*(a+b/x^2)^p)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx=\frac{1}{2}\left(\frac{p}{x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{b}\right)$$

input
```
Integrate[Log[c*(a + b/x^2)^p]/x^3,x]
```

output
```
(p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/b)/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c(a+\frac{b}{x^2})^p\right)}{x^3}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{1}{2}\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2836}$$

$$-\frac{\int \log\left(c(a+\frac{b}{x^2})^p\right)d(a+\frac{b}{x^2})}{2b}$$

$$\Big\downarrow \text{2732}$$

$$-\frac{(a+\frac{b}{x^2})\log\left(c(a+\frac{b}{x^2})^p\right)-p(a+\frac{b}{x^2})}{2b}$$

input
```
Int[Log[c*(a + b/x^2)^p]/x^3,x]
```

output
```
-1/2*(-(p*(a + b/x^2)) + (a + b/x^2)*Log[c*(a + b/x^2)^p])/b
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\left(a+\frac{b}{x^{2}}\right) \ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)-\left(a+\frac{b}{x^{2}}\right) p}{2 b}$	37
default	$-\frac{\left(a+\frac{b}{x^{2}}\right) \ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)-\left(a+\frac{b}{x^{2}}\right) p}{2 b}$	37
parts	$-\frac{\ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)}{2 x^{2}}-p b \left(\frac{a \ln\left(a \,x^{2}+b \right)}{2 b^{2}}-\frac{1}{2 b \,x^{2}}-\frac{a \ln\left(x \right)}{b^{2}}\right)$	54
parallelrisch	$-\frac{x^{2} \ln\left(c \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{p}\right) a^{2} p+\ln\left(c \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{p}\right) a b p-a b \,p^{2}}{2 x^{2} a p b}$	67

input
```
int(ln(c*(a+b/x^2)^p)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*((a+b/x^2)*ln(c*(a+b/x^2)^p)-(a+b/x^2)*p)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx = \frac{bp - b\log\left(c\right) - \left(apx^2+bp\right)\log\left(\frac{ax^2+b}{x^2}\right)}{2\,bx^2}$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="fricas")
```

output
```
1/2*(b*p - b*log(c) - (a*p*x^2 + b*p)*log((a*x^2 + b)/x^2))/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx = \begin{cases} -\frac{a\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b} + \frac{p}{2x^2} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2x^2} & \text{for}\ b \neq 0 \\ -\frac{\log\left(a^p c\right)}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(a+b/x**2)**p)/x**3,x)
```

output
```
Piecewise((-a*log(c*(a + b/x**2)**p)/(2*b) + p/(2*x**2) - log(c*(a + b/x**
2)**p)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx = -\frac{1}{2}\,bp\left(\frac{a\log\left(ax^2+b\right)}{b^2} - \frac{a\log\left(x^2\right)}{b^2} - \frac{1}{bx^2}\right) - \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{2\,x^2}$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="maxima")
```

output
```
-1/2*b*p*(a*log(a*x^2 + b)/b^2 - a*log(x^2)/b^2 - 1/(b*x^2)) - 1/2*log((a
+ b/x^2)^p*c)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx = -\frac{p\left(\frac{(ax^2+b)\log\left(\frac{ax^2+b}{x^2}\right)}{x^2}-\frac{ax^2+b}{x^2}\right)+\frac{(ax^2+b)\log(c)}{x^2}}{2\,b}$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="giac")
```

output
```
-1/2*(p*((a*x^2 + b)*log((a*x^2 + b)/x^2)/x^2 - (a*x^2 + b)/x^2) + (a*x^2
+ b)*log(c)/x^2)/b
```

Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx = \frac{p}{2\,x^2}-\frac{\ln\left(c\,\left(a+\frac{b}{x^2}\right)^p\right)}{2\,x^2}-\frac{a\,p\,\ln\left(a\,x^2+b\right)}{2\,b}+\frac{a\,p\,\ln\left(x\right)}{b}$$

input
```
int(log(c*(a + b/x^2)^p)/x^3,x)
```

output
```
p/(2*x^2) - log(c*(a + b/x^2)^p)/(2*x^2) - (a*p*log(b + a*x^2))/(2*b) + (a
*p*log(x))/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx = \frac{-\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)a\,x^2-\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)b+bp}{2b\,x^2}$$

input
```
int(log(c*(a+b/x^2)^p)/x^3,x)
```

output
```
( - log(((a*x**2 + b)**p*c)/x**(2*p))*a*x**2 - log(((a*x**2 + b)**p*c)/x**
(2*p))*b + b*p)/(2*b*x**2)
```

3.44 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 68

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx=\frac{2p}{9x^3}-\frac{2ap}{3bx}-\frac{2a^{3/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}$$

output
```
2/9*p/x^3-2/3*a*p/b/x-2/3*a^(3/2)*p*arctan(a^(1/2)*x/b^(1/2))/b^(3/2)-1/3*
ln(c*(a+b/x^2)^p)/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx=\frac{2p}{9x^3}-\frac{2ap}{3bx}+\frac{2a^{3/2}p\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{3b^{3/2}}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}$$

input
```
Integrate[Log[c*(a + b/x^2)^p]/x^4,x]
```

output
```
(2*p)/(9*x^3) - (2*a*p)/(3*b*x) + (2*a^(3/2)*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)]
)/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2905, 795, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4} \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$-\frac{2}{3} b p \int \frac{1}{\left(a+\frac{b}{x^2}\right) x^6}dx-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3 x^3}$$

$$\Big\downarrow \text{ 795}$$

$$-\frac{2}{3} b p \int \frac{1}{x^4 \left(a x^2+b\right)}dx-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3 x^3}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{2}{3} b p \left(-\frac{a \int \frac{1}{x^2 \left(a x^2+b\right)}dx}{b}-\frac{1}{3 b x^3}\right)-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3 x^3}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{2}{3} b p \left(-\frac{a \left(-\frac{a \int \frac{1}{a x^2+b}dx}{b}-\frac{1}{b x}\right)}{b}-\frac{1}{3 b x^3}\right)-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3 x^3}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{2}{3} b p \left(-\frac{a \left(-\frac{\sqrt{a} \arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{b x}\right)}{b}-\frac{1}{3 b x^3}\right)-\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3 x^3}$$

input
```
Int[Log[c*(a + b/x^2)^p]/x^4,x]
```

output
```
(-2*b*p*(-1/3*1/(b*x^3) - (a*(-(1/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqr
t[b]])/b^(3/2)))/b))/3 - Log[c*(a + b/x^2)^p]/(3*x^3)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)}{3x^{3}}-\frac{2pb\left(\frac{a^{2}\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{b^{2}\sqrt{ab}}-\frac{1}{3b\,x^{3}}+\frac{a}{b^{2}x}\right)}{3}$	61

input
```
int(ln(c*(a+b/x^2)^p)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(c*(a+b/x^2)^p)/x^3-2/3*p*b*(1/b^2*a^2/(a*b)^(1/2)*arctan(a*x/(a*b)
^(1/2))-1/3/b/x^3+1/b^2*a/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.26

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx$$
$$= \left[\frac{3\,apx^3\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-6\,apx^2-3\,bp\log\left(\frac{ax^2+b}{x^2}\right)+2\,bp-3\,b\log\left(c\right)}{9\,bx^3},\right.$$
$$\left.-\frac{6\,apx^3\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+6\,apx^2+3\,bp\log\left(\frac{ax^2+b}{x^2}\right)-2\,bp+3\,b\log\left(c\right)}{9\,bx^3}\right]$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="fricas")
```

output
```
[1/9*(3*a*p*x^3*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b))
 - 6*a*p*x^2 - 3*b*p*log((a*x^2 + b)/x^2) + 2*b*p - 3*b*log(c))/(b*x^3), -
1/9*(6*a*p*x^3*sqrt(a/b)*arctan(x*sqrt(a/b)) + 6*a*p*x^2 + 3*b*p*log((a*x^
2 + b)/x^2) - 2*b*p + 3*b*log(c))/(b*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(63) = 126$.

Time = 23.63 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.03

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = \begin{cases} -\frac{\log(0^pc)}{3x^3} & \text{for } a=0\wedge b=0 \\ \frac{2p}{9x^3}-\frac{\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{3x^3} & \text{for } a=0 \\ -\frac{\log(a^pc)}{3x^3} & \text{for } b=0 \\ -\frac{ap\log\left(x-\sqrt{-\frac{b}{a}}\right)}{3b\sqrt{-\frac{b}{a}}}+\frac{ap\log\left(x+\sqrt{-\frac{b}{a}}\right)}{3b\sqrt{-\frac{b}{a}}}-\frac{2ap}{3bx}+\frac{2p}{9x^3}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(a+b/x**2)**p)/x**4,x)
```

output
```
Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (2*p/(9*x**3) - lo
g(c*(b/x**2)**p)/(3*x**3), Eq(a, 0)), (-log(a**p*c)/(3*x**3), Eq(b, 0)), (
-a*p*log(x - sqrt(-b/a))/(3*b*sqrt(-b/a)) + a*p*log(x + sqrt(-b/a))/(3*b*s
qrt(-b/a)) - 2*a*p/(3*b*x) + 2*p/(9*x**3) - log(c*(a + b/x**2)**p)/(3*x**3
), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = -\frac{2}{9}\,bp\left(\frac{3\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{3\,ax^2-b}{b^2x^3}\right)-\frac{\log\left(\left(a+\frac{b}{x^2}\right)^pc\right)}{3\,x^3}$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="maxima")
```

output
```
-2/9*b*p*(3*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + (3*a*x^2 - b)/(b^2
*x^3)) - 1/3*log((a + b/x^2)^p*c)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = -\frac{2\,a^2p\,\mathrm{arctan}\left(\frac{ax}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b} - \frac{p\log\left(ax^2+b\right)}{3\,x^3} + \frac{p\log\left(x^2\right)}{3\,x^3} - \frac{6\,apx^2-2\,bp+3\,b\log\left(c\right)}{9\,bx^3}$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="giac")
```

output
```
-2/3*a^2*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/3*p*log(a*x^2 + b)/x^3
+ 1/3*p*log(x^2)/x^3 - 1/9*(6*a*p*x^2 - 2*b*p + 3*b*log(c))/(b*x^3)
```

Mupad [B] (verification not implemented)

Time = 15.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = \frac{\frac{2\,p}{3}-\frac{2\,a\,p\,x^2}{b}}{3\,x^3} - \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3\,x^3} - \frac{2\,a^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3\,b^{3/2}}$$

input
```
int(log(c*(a + b/x^2)^p)/x^4,x)
```

output
```
((2*p)/3 - (2*a*p*x^2)/b)/(3*x^3) - log(c*(a + b/x^2)^p)/(3*x^3) - (2*a^(3
/2)*p*atan((a^(1/2)*x)/b^(1/2)))/(3*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = \frac{-6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)ap\,x^3-3\log\left(\frac{\left(a\,x^2+b\right)^p c}{x^{2p}}\right)b^2-6abp\,x^2+2b^2p}{9b^2x^3}$$

input
```
int(log(c*(a+b/x^2)^p)/x^4,x)
```

output
```
( - 6*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*p*x**3 - 3*log(((a*x
**2 + b)**p*c)/x**(2*p))*b**2 - 6*a*b*p*x**2 + 2*b**2*p)/(9*b**2*x**3)
```

3.45 $\int \frac{\log\left(1+\frac{b}{x}\right)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x} dx = \text{PolyLog}\left(2,-\frac{b}{x}\right)$$

output
```
polylog(2,-b/x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x} dx = -\log\left(-\frac{b}{x}\right)\log\left(\frac{b+x}{x}\right) - \text{PolyLog}\left(2,-\frac{-b-x}{x}\right)$$

input
```
Integrate[Log[1 + b/x]/x,x]
```

output
```
-(Log[-(b/x)]*Log[(b + x)/x]) - PolyLog[2, -((-b - x)/x)]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{b}{x}+1\right)}{x} \, dx$$

$$\Big\downarrow \; 2838$$

$$\text{PolyLog}\left(2,-\frac{b}{x}\right)$$

input
```
Int[Log[1 + b/x]/x,x]
```

output
```
PolyLog[2, -(b/x)]
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\operatorname{dilog}\left(1+\frac{b}{x}\right)$	9
default	$\operatorname{dilog}\left(1+\frac{b}{x}\right)$	9
risch	$\operatorname{dilog}\left(1+\frac{b}{x}\right)$	9
parts	$\ln\left(1+\frac{b}{x}\right)\ln\left(x\right)+b\left(-\frac{\operatorname{dilog}\left(\frac{x+b}{b}\right)+\ln\left(x\right)\ln\left(\frac{x+b}{b}\right)}{b}+\frac{\ln\left(x\right)^{2}}{2b}\right)$	50

input
```
int(ln(1+b/x)/x,x,method=_RETURNVERBOSE)
```

output
```
dilog(1+b/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = \mathrm{Li}_2\left(-\frac{b+x}{x}+1\right)$$

input
```
integrate(log(1+b/x)/x,x, algorithm="fricas")
```

output
```
dilog(-(b + x)/x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = \mathrm{Li}_2\left(\frac{be^{i\pi}}{x}\right)$$

input
```
integrate(ln(1+b/x)/x,x)
```

output
```
polylog(2, b*exp_polar(I*pi)/x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 4.38

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = \log\left(b+x\right)\log\left(x\right) - \frac{1}{2}\log\left(x\right)^2 - \log\left(x\right)\log\left(\frac{x}{b}+1\right) - \mathrm{Li}_2\left(-\frac{x}{b}\right)$$

input
```
integrate(log(1+b/x)/x,x, algorithm="maxima")
```

output
```
log(b + x)*log(x) - 1/2*log(x)^2 - log(x)*log(x/b + 1) - dilog(-x/b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(7) = 14$.

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 13.75

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx$$
$$= -\frac{b^3\left(\frac{1}{\frac{b+x}{x}-1} - \log\left(\frac{|b+x|}{|x|}\right) + \log\left(\left|\frac{b+x}{x}-1\right|\right)\right) + \frac{b^3\log\left(-b\left(\frac{\left(b-\frac{1}{\frac{1}{b}-\frac{b+x}{bx}}\right)\left(\frac{1}{b}-\frac{b+x}{bx}\right)}{b}+\frac{1}{b}\right)+1\right)}{\left(\frac{b+x}{x}-1\right)^2}}{2\,b^2}$$

input
```
integrate(log(1+b/x)/x,x, algorithm="giac")
```

output
```
-1/2*(b^3*(1/((b + x)/x - 1) - log(abs(b + x)/abs(x)) + log(abs((b + x)/x
- 1))) + b^3*log(-b*((b - 1/(1/b - (b + x)/(b*x)))*(1/b - (b + x)/(b*x))/b
 + 1/b) + 1)/((b + x)/x - 1)^2)/b^2
```

Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(1+\frac{b}{x}\right)}{x} \, dx = \operatorname{polylog}\left(2,-\frac{b}{x}\right)$$

input
```
int(log(b/x + 1)/x,x)
```

output
```
polylog(2, -b/x)
```

Reduce [F]

$$\int \frac{\log \left(1+\frac{b}{x}\right)}{x} \, dx = \int \frac{\log \left(\frac{b+x}{x}\right)}{x} dx$$

input
```
int(log(1+b/x)/x,x)
```

output
```
int(log((b + x)/x)/x,x)
```

3.46 $\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 153

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{a^7p\sqrt{x}}{4b^7} - \frac{a^6px}{8b^6} + \frac{a^5px^{3/2}}{12b^5} - \frac{a^4px^2}{16b^4} + \frac{a^3px^{5/2}}{20b^3} - \frac{a^2px^3}{24b^2} + \frac{apx^{7/2}}{28b} - \frac{px^4}{32} - \frac{a^8p\log\left(a+b\sqrt{x}\right)}{4b^8} + \frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

output
```
1/4*a^7*p*x^(1/2)/b^7-1/8*a^6*p*x/b^6+1/12*a^5*p*x^(3/2)/b^5-1/16*a^4*p*x^
2/b^4+1/20*a^3*p*x^(5/2)/b^3-1/24*a^2*p*x^3/b^2+1/28*a*p*x^(7/2)/b-1/32*p*
x^4-1/4*a^8*p*ln(a+b*x^(1/2))/b^8+1/4*x^4*ln(c*(a+b*x^(1/2))^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = -\frac{p\left(-840a^7b\sqrt{x}+420a^6b^2x-280a^5b^3x^{3/2}+210a^4b^4x^2-168a^3b^5x^{5/2}+140a^2b^6x^3-120ab^7x^{7/2}+105\right.}{3360b^8} + \frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

input
```
Integrate[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

output
```
-1/3360*(p*(-840*a^7*b*Sqrt[x] + 420*a^6*b^2*x - 280*a^5*b^3*x^(3/2) + 210
*a^4*b^4*x^2 - 168*a^3*b^5*x^(5/2) + 140*a^2*b^6*x^3 - 120*a*b^7*x^(7/2) +
 105*b^8*x^4 + 840*a^8*Log[a + b*Sqrt[x]]))/b^8 + (x^4*Log[c*(a + b*Sqrt[x
])^p])/4
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int x^{7/2} \log \left(c\left(a+b\sqrt{x}\right)^p\right) d\sqrt{x}$$

$$\Big\downarrow \text{ 2842}$$

$$2\left(\frac{1}{8}x^4 \log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{8}bp\int \frac{x^4}{a+b\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 49}$$

$$2\left(\frac{1}{8}x^4 \log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{8}bp\int \left(\frac{a^8}{b^8\left(a+b\sqrt{x}\right)}-\frac{a^7}{b^8}+\frac{\sqrt{x}a^6}{b^7}-\frac{xa^5}{b^6}+\frac{x^{3/2}a^4}{b^5}-\frac{x^2a^3}{b^4}+\frac{x^{5/2}a^2}{b^3}-\frac{x^3a}{b^2}+\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{1}{8}x^4 \log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{8}bp\left(\frac{a^8\log\left(a+b\sqrt{x}\right)}{b^9}-\frac{a^7\sqrt{x}}{b^8}+\frac{a^6x}{2b^7}-\frac{a^5x^{3/2}}{3b^6}+\frac{a^4x^2}{4b^5}-\frac{a^3x^{5/2}}{5b^4}+\frac{a^2x^3}{6b^3}-\frac{ax^{7/2}}{7b^2}\right.\right.$$

input
```
Int[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

output
```
2*(-1/8*(b*p*(-((a^7*Sqrt[x])/b^8) + (a^6*x)/(2*b^7) - (a^5*x^(3/2))/(3*b^
6) + (a^4*x^2)/(4*b^5) - (a^3*x^(5/2))/(5*b^4) + (a^2*x^3)/(6*b^3) - (a*x^
(7/2))/(7*b^2) + x^4/(8*b) + (a^8*Log[a + b*Sqrt[x]])/b^9)) + (x^4*Log[c*(
a + b*Sqrt[x])^p])/8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

method	result	size
parts	$\frac{x^{4} \ln\left(c \left(a+b \sqrt{x}\right)^{p}\right)}{4}-\frac{p b \left(-\frac{2 \left(-\frac{x^{4} b^{7}}{8}+\frac{a \,x^{\frac{7}{2}} b^{6}}{7}-\frac{a^{2} x^{3} b^{5}}{6}+\frac{a^{3} x^{\frac{5}{2}} b^{4}}{5}-\frac{x^{2} a^{4} b^{3}}{4}+\frac{a^{5} x^{\frac{3}{2}} b^{2}}{3}-\frac{a^{6} b x}{2}+a^{7} \sqrt{x}\right)}{b^{8}}+\frac{2 a^{8} \ln\left(a+b \sqrt{x}\right)}{b^{9}}\right)}{8}$	121

input

```
int(x^3*ln(c*(a+b*x^(1/2))^p),x,method=_RETURNVERBOSE)
```

output

```
1/4*x^4*ln(c*(a+b*x^(1/2))^p)-1/8*p*b*(-2/b^8*(-1/8*x^4*b^7+1/7*a*x^(7/2)*
b^6-1/6*a^2*x^3*b^5+1/5*a^3*x^(5/2)*b^4-1/4*x^2*a^4*b^3+1/3*a^5*x^(3/2)*b^
2-1/2*a^6*b*x+a^7*x^(1/2))+2*a^8/b^9*ln(a+b*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx =$$
$$-\frac{105\,b^8 p x^4 - 840\,b^8 x^4 \log\left(c\right) + 140\,a^2 b^6 p x^3 + 210\,a^4 b^4 p x^2 + 420\,a^6 b^2 p x - 840\,\left(b^8 p x^4 - a^8 p\right) \log\left(b\sqrt{x} + \right.}{3360\,b^8}$$

input

```
integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

output

```
-1/3360*(105*b^8*p*x^4 - 840*b^8*x^4*log(c) + 140*a^2*b^6*p*x^3 + 210*a^4*
b^4*p*x^2 + 420*a^6*b^2*p*x - 840*(b^8*p*x^4 - a^8*p)*log(b*sqrt(x) + a) -
 8*(15*a*b^7*p*x^3 + 21*a^3*b^5*p*x^2 + 35*a^5*b^3*p*x + 105*a^7*b*p)*sqrt
(x))/b^8
```

Sympy [A] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\, dx = -\frac{bp\left(\frac{2a^8\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^8} - \frac{2a^7\sqrt{x}}{b^8} + \frac{a^6x}{b^7} - \frac{2a^5x^{\frac{3}{2}}}{3b^6} + \frac{a^4x^2}{2b^5} - \frac{2a^3x^{\frac{5}{2}}}{5b^4} + \frac{a^2x^3}{3b^3} - \frac{2ax^{\frac{7}{2}}}{7b^2} + \frac{x^4}{4b}\right)}{8} + \frac{x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{4}$$

input
```
integrate(x**3*ln(c*(a+b*x**(1/2))**p),x)
```

output
```
-b*p*(2*a**8*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True)
)/b**8 - 2*a**7*sqrt(x)/b**8 + a**6*x/b**7 - 2*a**5*x**(3/2)/(3*b**6) + a*
*4*x**2/(2*b**5) - 2*a**3*x**(5/2)/(5*b**4) + a**2*x**3/(3*b**3) - 2*a*x**
(7/2)/(7*b**2) + x**4/(4*b))/8 + x**4*log(c*(a + b*sqrt(x))**p)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\, dx = \frac{1}{4}\,x^4\log\left(\left(b\sqrt{x}+a\right)^pc\right) - \frac{1}{3360}\,bp\left(\frac{840\,a^8\log\left(b\sqrt{x}+a\right)}{b^9} + \frac{105\,b^7x^4 - 120\,ab^6x^{\frac{7}{2}} + 140\,a^2b^5x^3 - 168\,a^3b^4x^{\frac{5}{2}} + 210\,a^4b^3x^2 - 280}{b^8}\right.$$

input
```
integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

output
```
1/4*x^4*log((b*sqrt(x) + a)^p*c) - 1/3360*b*p*(840*a^8*log(b*sqrt(x) + a)/
b^9 + (105*b^7*x^4 - 120*a*b^6*x^(7/2) + 140*a^2*b^5*x^3 - 168*a^3*b^4*x^(
5/2) + 210*a^4*b^3*x^2 - 280*a^5*b^2*x^(3/2) + 420*a^6*b*x - 840*a^7*sqrt(
x))/b^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(121) = 242$.

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.22

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{840\, bx^4 \log(c) + \left(\frac{840\,(b\sqrt{x}+a)^8 \log(b\sqrt{x}+a)}{b^7} - \frac{6720\,(b\sqrt{x}+a)^7 a \log(b\sqrt{x}+a)}{b^7} + \frac{23520\,(b\sqrt{x}+a)^6 a^2 \log(b\sqrt{x}+a)}{b^7} - \frac{47040\,(b\sqrt{x}+$$

input
```
integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

output
```
1/3360*(840*b*x^4*log(c) + (840*(b*sqrt(x) + a)^8*log(b*sqrt(x) + a)/b^7 -
 6720*(b*sqrt(x) + a)^7*a*log(b*sqrt(x) + a)/b^7 + 23520*(b*sqrt(x) + a)^6
*a^2*log(b*sqrt(x) + a)/b^7 - 47040*(b*sqrt(x) + a)^5*a^3*log(b*sqrt(x) +
a)/b^7 + 58800*(b*sqrt(x) + a)^4*a^4*log(b*sqrt(x) + a)/b^7 - 47040*(b*sqr
t(x) + a)^3*a^5*log(b*sqrt(x) + a)/b^7 + 23520*(b*sqrt(x) + a)^2*a^6*log(b
*sqrt(x) + a)/b^7 - 6720*(b*sqrt(x) + a)*a^7*log(b*sqrt(x) + a)/b^7 - 105*
(b*sqrt(x) + a)^8/b^7 + 960*(b*sqrt(x) + a)^7*a/b^7 - 3920*(b*sqrt(x) + a)
^6*a^2/b^7 + 9408*(b*sqrt(x) + a)^5*a^3/b^7 - 14700*(b*sqrt(x) + a)^4*a^4/
b^7 + 15680*(b*sqrt(x) + a)^3*a^5/b^7 - 11760*(b*sqrt(x) + a)^2*a^6/b^7 +
6720*(b*sqrt(x) + a)*a^7/b^7)*p)/b
```

Mupad [B] (verification not implemented)

Time = 15.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) dx = \frac{x^4 \ln \left(c \left(a+b\sqrt{x}\right)^p\right)}{4} - \frac{p\,x^4}{32} - \frac{a^8\,p\,\ln \left(a+b\sqrt{x}\right)}{4\,b^8} - \frac{a^2\,p\,x^3}{24\,b^2} - \frac{a^4\,p\,x^2}{16\,b^4} + \frac{a^3\,p\,x^{5/2}}{20\,b^3} + \frac{a^5\,p\,x^{3/2}}{12\,b^5} + \frac{a^7\,p\,\sqrt{x}}{4\,b^7} + \frac{a\,p\,x^{7/2}}{28\,b} - \frac{a^6\,p\,x}{8\,b^6}$$

input
```
int(x^3*log(c*(a + b*x^(1/2))^p),x)
```

output
```
(x^4*log(c*(a + b*x^(1/2))^p))/4 - (p*x^4)/32 - (a^8*p*log(a + b*x^(1/2)))
/(4*b^8) - (a^2*p*x^3)/(24*b^2) - (a^4*p*x^2)/(16*b^4) + (a^3*p*x^(5/2))/(
20*b^3) + (a^5*p*x^(3/2))/(12*b^5) + (a^7*p*x^(1/2))/(4*b^7) + (a*p*x^(7/2
))/(28*b) - (a^6*p*x)/(8*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) dx$$

$$= \frac{840\sqrt{x}\,a^7bp + 280\sqrt{x}\,a^5b^3px + 168\sqrt{x}\,a^3b^5p\,x^2 + 120\sqrt{x}\,a\,b^7p\,x^3 - 840\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)a^8 + 840\log(}{3360b^8}$$

input
```
int(x^3*log(c*(a+b*x^(1/2))^p),x)
```

output
```
(840*sqrt(x)*a**7*b*p + 280*sqrt(x)*a**5*b**3*p*x + 168*sqrt(x)*a**3*b**5*
p*x**2 + 120*sqrt(x)*a*b**7*p*x**3 - 840*log((sqrt(x)*b + a)**p*c)*a**8 +
840*log((sqrt(x)*b + a)**p*c)*b**8*x**4 - 420*a**6*b**2*p*x - 210*a**4*b**
4*p*x**2 - 140*a**2*b**6*p*x**3 - 105*b**8*p*x**4)/(3360*b**8)
```

3.47 $\int x^2 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int x^2 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{a^5p\sqrt{x}}{3b^5}-\frac{a^4px}{6b^4}+\frac{a^3px^{3/2}}{9b^3}-\frac{a^2px^2}{12b^2}+\frac{apx^{5/2}}{15b}-\frac{px^3}{18}-\frac{a^6p\log\left(a+b\sqrt{x}\right)}{3b^6}+\frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

output
```
1/3*a^5*p*x^(1/2)/b^5-1/6*a^4*p*x/b^4+1/9*a^3*p*x^(3/2)/b^3-1/12*a^2*p*x^2
/b^2+1/15*a*p*x^(5/2)/b-1/18*p*x^3-1/3*a^6*p*ln(a+b*x^(1/2))/b^6+1/3*x^3*l
n(c*(a+b*x^(1/2))^p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91

$$\int x^2 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{bp\sqrt{x}\left(60a^5-30a^4b\sqrt{x}+20a^3b^2x-15a^2b^3x^{3/2}+12ab^4x^2-10b^5x^{5/2}\right)-60a^6p\log\left(a+b\sqrt{x}\right)+60b^6x^3}{180b^6}$$

input
```
Integrate[x^2*Log[c*(a + b*Sqrt[x])^p],x]
```

output
```
(b*p*Sqrt[x]*(60*a^5 - 30*a^4*b*Sqrt[x] + 20*a^3*b^2*x - 15*a^2*b^3*x^(3/2
) + 12*a*b^4*x^2 - 10*b^5*x^(5/2)) - 60*a^6*p*Log[a + b*Sqrt[x]] + 60*b^6*
x^3*Log[c*(a + b*Sqrt[x])^p])/(180*b^6)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int x^{5/2} \log \left(c\left(a+b\sqrt{x}\right)^p\right) d\sqrt{x}$$

$$\Big\downarrow \text{ 2842}$$

$$2\left(\frac{1}{6}x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{6}bp\int \frac{x^3}{a+b\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 49}$$

$$2\left(\frac{1}{6}x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{6}bp\int \left(\frac{a^6}{b^6\left(a+b\sqrt{x}\right)}-\frac{a^5}{b^6}+\frac{\sqrt{x}a^4}{b^5}-\frac{xa^3}{b^4}+\frac{x^{3/2}a^2}{b^3}-\frac{x^2a}{b^2}+\frac{x^{5/2}}{b}\right)d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{1}{6}x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{6}bp\left(\frac{a^6 \log \left(a+b\sqrt{x}\right)}{b^7}-\frac{a^5\sqrt{x}}{b^6}+\frac{a^4x}{2b^5}-\frac{a^3x^{3/2}}{3b^4}+\frac{a^2x^2}{4b^3}-\frac{ax^{5/2}}{5b^2}+\frac{x^3}{6b}\right)\right)$$

input
```
Int[x^2*Log[c*(a + b*Sqrt[x])^p],x]
```

output
```
2*(-1/6*(b*p*(-((a^5*Sqrt[x])/b^6) + (a^4*x)/(2*b^5) - (a^3*x^(3/2))/(3*b^
4) + (a^2*x^2)/(4*b^3) - (a*x^(5/2))/(5*b^2) + x^3/(6*b) + (a^6*Log[a + b*
Sqrt[x]])/b^7)) + (x^3*Log[c*(a + b*Sqrt[x])^p])/6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

method	result	size
parts	$\frac{x^{3} \ln\left(c \left(a+b\sqrt{x}\right)^{p}\right)}{3}-\frac{p b \left(-\frac{2 \left(-\frac{b^{5} x^{3}}{6}+\frac{a\, x^{\frac{5}{2}} b^{4}}{5}-\frac{x^{2} a^{2} b^{3}}{4}+\frac{a^{3} x^{\frac{3}{2}} b^{2}}{3}-\frac{a^{4} x b}{2}+a^{5} \sqrt{x}\right)}{b^{6}}+\frac{2 a^{6} \ln\left(a+b\sqrt{x}\right)}{b^{7}}\right)}{6}$	99

input
```
int(x^2*ln(c*(a+b*x^(1/2))^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(c*(a+b*x^(1/2))^p)-1/6*p*b*(-2/b^6*(-1/6*b^5*x^3+1/5*a*x^(5/2)*
b^4-1/4*x^2*a^2*b^3+1/3*a^3*x^(3/2)*b^2-1/2*a^4*x*b+a^5*x^(1/2))+2*a^6/b^7
*ln(a+b*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{10\,b^6px^3-60\,b^6x^3\log\left(c\right)+15\,a^2b^4px^2+30\,a^4b^2px-60\left(b^6px^3-a^6p\right)\log\left(b\sqrt{x}+a\right)-4\left(3\,ab^5px^2+\right.}{180\,b^6}$$

input
```
integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

output
```
-1/180*(10*b^6*p*x^3 - 60*b^6*x^3*log(c) + 15*a^2*b^4*p*x^2 + 30*a^4*b^2*p
*x - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) - 4*(3*a*b^5*p*x^2 + 5*a^3*
b^3*p*x + 15*a^5*b*p)*sqrt(x))/b^6
```

Sympy [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{bp\left(\frac{2a^6\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^6}-\frac{2a^5\sqrt{x}}{b^6}+\frac{a^4x}{b^5}-\frac{2a^3x^{\frac{3}{2}}}{3b^4}+\frac{a^2x^2}{2b^3}-\frac{2ax^{\frac{5}{2}}}{5b^2}+\frac{x^3}{3b}\right)}{6} + \frac{x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3}$$

input
```
integrate(x**2*ln(c*(a+b*x**(1/2))**p),x)
```

output
```
-b*p*(2*a**6*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True)
)/b**6 - 2*a**5*sqrt(x)/b**6 + a**4*x/b**5 - 2*a**3*x**(3/2)/(3*b**4) + a*
*2*x**2/(2*b**3) - 2*a*x**(5/2)/(5*b**2) + x**3/(3*b))/6 + x**3*log(c*(a +
 b*sqrt(x))**p)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx=\frac{1}{3}\,x^3\log\left(\left(b\sqrt{x}+a\right)^p c\right)$$
$$-\frac{1}{180}\,bp\left(\frac{60\,a^6\log\left(b\sqrt{x}+a\right)}{b^7}+\frac{10\,b^5x^3-12\,ab^4x^{\frac{5}{2}}+15\,a^2b^3x^2-20\,a^3b^2x^{\frac{3}{2}}+30\,a^4bx-60\,a^5\sqrt{x}}{b^6}\right)$$

input
```
integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

output
```
1/3*x^3*log((b*sqrt(x) + a)^p*c) - 1/180*b*p*(60*a^6*log(b*sqrt(x) + a)/b^
7 + (10*b^5*x^3 - 12*a*b^4*x^(5/2) + 15*a^2*b^3*x^2 - 20*a^3*b^2*x^(3/2) +
 30*a^4*b*x - 60*a^5*sqrt(x))/b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(97) = 194.

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.07

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$$
$$=\frac{60\,bx^3\log\left(c\right)+\left(\frac{60\,\left(b\sqrt{x}+a\right)^6\log\left(b\sqrt{x}+a\right)}{b^5}-\frac{360\,\left(b\sqrt{x}+a\right)^5a\log\left(b\sqrt{x}+a\right)}{b^5}+\frac{900\,\left(b\sqrt{x}+a\right)^4a^2\log\left(b\sqrt{x}+a\right)}{b^5}-\frac{1200\,\left(b\sqrt{x}+a\right)^3a^3\,\mathrm{l}}{b^5}\right.}{}$$

input
```
integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

output
```
1/180*(60*b*x^3*log(c) + (60*(b*sqrt(x) + a)^6*log(b*sqrt(x) + a)/b^5 - 36
0*(b*sqrt(x) + a)^5*a*log(b*sqrt(x) + a)/b^5 + 900*(b*sqrt(x) + a)^4*a^2*l
og(b*sqrt(x) + a)/b^5 - 1200*(b*sqrt(x) + a)^3*a^3*log(b*sqrt(x) + a)/b^5
+ 900*(b*sqrt(x) + a)^2*a^4*log(b*sqrt(x) + a)/b^5 - 360*(b*sqrt(x) + a)*a
^5*log(b*sqrt(x) + a)/b^5 - 10*(b*sqrt(x) + a)^6/b^5 + 72*(b*sqrt(x) + a)^
5*a/b^5 - 225*(b*sqrt(x) + a)^4*a^2/b^5 + 400*(b*sqrt(x) + a)^3*a^3/b^5 -
450*(b*sqrt(x) + a)^2*a^4/b^5 + 360*(b*sqrt(x) + a)*a^5/b^5)*p)/b
```

Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{x^3\,\ln\left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{3}-\frac{p\,x^3}{18}-\frac{a^6\,p\,\ln\left(a+b\,\sqrt{x}\right)}{3\,b^6}-\frac{a^2\,p\,x^2}{12\,b^2}+\frac{a^3\,p\,x^{3/2}}{9\,b^3}+\frac{a^5\,p\,\sqrt{x}}{3\,b^5}+\frac{a\,p\,x^{5/2}}{15\,b}-\frac{a^4\,p\,x}{6\,b^4}$$

input
```
int(x^2*log(c*(a + b*x^(1/2))^p),x)
```

output
```
(x^3*log(c*(a + b*x^(1/2))^p))/3 - (p*x^3)/18 - (a^6*p*log(a + b*x^(1/2)))
/(3*b^6) - (a^2*p*x^2)/(12*b^2) + (a^3*p*x^(3/2))/(9*b^3) + (a^5*p*x^(1/2)
)/(3*b^5) + (a*p*x^(5/2))/(15*b) - (a^4*p*x)/(6*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{60\sqrt{x}\,a^5bp+20\sqrt{x}\,a^3b^3px+12\sqrt{x}\,a\,b^5p\,x^2-60\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)a^6+60\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)b^6x^3-30}{180b^6}$$

input
```
int(x^2*log(c*(a+b*x^(1/2))^p),x)
```

output
```
(60*sqrt(x)*a**5*b*p + 20*sqrt(x)*a**3*b**3*p*x + 12*sqrt(x)*a*b**5*p*x**2
 - 60*log((sqrt(x)*b + a)**p*c)*a**6 + 60*log((sqrt(x)*b + a)**p*c)*b**6*x
**3 - 30*a**4*b**2*p*x - 15*a**2*b**4*p*x**2 - 10*b**6*p*x**3)/(180*b**6)
```

3.48 $\int x \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int x \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{a^3p\sqrt{x}}{2b^3} - \frac{a^2px}{4b^2} + \frac{apx^{3/2}}{6b} - \frac{px^2}{8} - \frac{a^4p\log\left(a+b\sqrt{x}\right)}{2b^4} + \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

output
```
1/2*a^3*p*x^(1/2)/b^3-1/4*a^2*p*x/b^2+1/6*a*p*x^(3/2)/b-1/8*p*x^2-1/2*a^4*
p*ln(a+b*x^(1/2))/b^4+1/2*x^2*ln(c*(a+b*x^(1/2))^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int x \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{bp\sqrt{x}\left(12a^3-6a^2b\sqrt{x}+4ab^2x-3b^3x^{3/2}\right)-12a^4p\log\left(a+b\sqrt{x}\right)+12b^4x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{24b^4}$$

input
```
Integrate[x*Log[c*(a + b*Sqrt[x])^p],x]
```

output
```
(b*p*Sqrt[x]*(12*a^3 - 6*a^2*b*Sqrt[x] + 4*a*b^2*x - 3*b^3*x^(3/2)) - 12*a
^4*p*Log[a + b*Sqrt[x]] + 12*b^4*x^2*Log[c*(a + b*Sqrt[x])^p])/(24*b^4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(c\left(a+b\sqrt{x}\right)^p\right) dx$$

$$\Big\downarrow \text{2904}$$

$$2\int x^{3/2} \log \left(c\left(a+b\sqrt{x}\right)^p\right) d\sqrt{x}$$

$$\Big\downarrow \text{2842}$$

$$2\left(\frac{1}{4}x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{4}bp\int \frac{x^2}{a+b\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{49}$$

$$2\left(\frac{1}{4}x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{4}bp\int \left(\frac{a^4}{b^4\left(a+b\sqrt{x}\right)}-\frac{a^3}{b^4}+\frac{\sqrt{x}a^2}{b^3}-\frac{xa}{b^2}+\frac{x^{3/2}}{b}\right)d\sqrt{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{4}x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{4}bp\left(\frac{a^4 \log \left(a+b\sqrt{x}\right)}{b^5}-\frac{a^3\sqrt{x}}{b^4}+\frac{a^2x}{2b^3}-\frac{ax^{3/2}}{3b^2}+\frac{x^2}{4b}\right)\right)$$

input
```
Int[x*Log[c*(a + b*Sqrt[x])^p],x]
```

output
```
2*(-1/4*(b*p*(-((a^3*Sqrt[x])/b^4) + (a^2*x)/(2*b^3) - (a*x^(3/2))/(3*b^2)
 + x^2/(4*b) + (a^4*Log[a + b*Sqrt[x]])/b^5)) + (x^2*Log[c*(a + b*Sqrt[x])
^p])/4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

method	result	size
parts	$\frac{x^2 \ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{2}-\frac{pb\left(-\frac{2\left(-\frac{b^3x^2}{4}+\frac{a\,x^{\frac{3}{2}}b^2}{3}-\frac{a^2bx}{2}+a^3\sqrt{x}\right)}{b^4}+\frac{2a^4\ln\left(a+b\sqrt{x}\right)}{b^5}\right)}{4}$	77

input
```
int(x*ln(c*(a+b*x^(1/2))^p),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(a+b*x^(1/2))^p)-1/4*p*b*(-2/b^4*(-1/4*b^3*x^2+1/3*a*x^(3/2)*
b^2-1/2*a^2*b*x+a^3*x^(1/2))+2*a^4/b^5*ln(a+b*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{3\,b^4px^2-12\,b^4x^2\log\left(c\right)+6\,a^2b^2px-12\left(b^4px^2-a^4p\right)\log\left(b\sqrt{x}+a\right)-4\left(ab^3px+3\,a^3bp\right)\sqrt{x}}{24\,b^4}$$

input
```
integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

output
```
-1/24*(3*b^4*p*x^2 - 12*b^4*x^2*log(c) + 6*a^2*b^2*p*x - 12*(b^4*p*x^2 - a
^4*p)*log(b*sqrt(x) + a) - 4*(a*b^3*p*x + 3*a^3*b*p)*sqrt(x))/b^4
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{bp\left(\frac{2a^4\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^4}-\frac{2a^3\sqrt{x}}{b^4}+\frac{a^2x}{b^3}-\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{x^2}{2b}\right)}{4}+\frac{x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2}$$

input
```
integrate(x*ln(c*(a+b*x**(1/2))**p),x)
```

output
```
-b*p*(2*a**4*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True)
)/b**4 - 2*a**3*sqrt(x)/b**4 + a**2*x/b**3 - 2*a*x**(3/2)/(3*b**2) + x**2/
(2*b))/4 + x**2*log(c*(a + b*sqrt(x))**p)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$$
$$= -\frac{1}{24}bp\left(\frac{12\,a^4\log\left(b\sqrt{x}+a\right)}{b^5}+\frac{3\,b^3x^2-4\,ab^2x^{\frac{3}{2}}+6\,a^2bx-12\,a^3\sqrt{x}}{b^4}\right)$$
$$+\frac{1}{2}\,x^2\log\left(\left(b\sqrt{x}+a\right)^pc\right)$$

input
```
integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

output
```
-1/24*b*p*(12*a^4*log(b*sqrt(x) + a)/b^5 + (3*b^3*x^2 - 4*a*b^2*x^(3/2) +
6*a^2*b*x - 12*a^3*sqrt(x))/b^4) + 1/2*x^2*log((b*sqrt(x) + a)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(73) = 146.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.84

$$\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$$
$$= \frac{12\,bx^2\log\left(c\right)+\left(\frac{12\,\left(b\sqrt{x}+a\right)^4\log\left(b\sqrt{x}+a\right)}{b^3}-\frac{48\,\left(b\sqrt{x}+a\right)^3a\log\left(b\sqrt{x}+a\right)}{b^3}+\frac{72\,\left(b\sqrt{x}+a\right)^2a^2\log\left(b\sqrt{x}+a\right)}{b^3}-\frac{48\,\left(b\sqrt{x}+a\right)a^3\log\left(b\sqrt{\right.}}{b^3}}{24\,b}$$

input
```
integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

output
```
1/24*(12*b*x^2*log(c) + (12*(b*sqrt(x) + a)^4*log(b*sqrt(x) + a)/b^3 - 48*
(b*sqrt(x) + a)^3*a*log(b*sqrt(x) + a)/b^3 + 72*(b*sqrt(x) + a)^2*a^2*log(
b*sqrt(x) + a)/b^3 - 48*(b*sqrt(x) + a)*a^3*log(b*sqrt(x) + a)/b^3 - 3*(b*
sqrt(x) + a)^4/b^3 + 16*(b*sqrt(x) + a)^3*a/b^3 - 36*(b*sqrt(x) + a)^2*a^2
/b^3 + 48*(b*sqrt(x) + a)*a^3/b^3)*p)/b
```

Mupad [B] (verification not implemented)

Time = 15.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int x \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{x^2 \ln \left(c\left(a+b\sqrt{x}\right)^p\right)}{2} - \frac{p\,x^2}{8} - \frac{a^4\,p\,\ln \left(a+b\sqrt{x}\right)}{2\,b^4} + \frac{a^3\,p\,\sqrt{x}}{2\,b^3} - \frac{a^2\,p\,x}{4\,b^2} + \frac{a\,p\,x^{3/2}}{6\,b}$$

input
```
int(x*log(c*(a + b*x^(1/2))^p),x)
```

output
```
(x^2*log(c*(a + b*x^(1/2))^p))/2 - (p*x^2)/8 - (a^4*p*log(a + b*x^(1/2)))/
(2*b^4) + (a^3*p*x^(1/2))/(2*b^3) - (a^2*p*x)/(4*b^2) + (a*p*x^(3/2))/(6*b
)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int x \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{12\sqrt{x}\,a^3bp + 4\sqrt{x}\,a\,b^3px - 12\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)a^4 + 12\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)b^4x^2 - 6a^2b^2px - 3b^4p\,x^2}{24b^4}$$

input
```
int(x*log(c*(a+b*x^(1/2))^p),x)
```

output
```
(12*sqrt(x)*a**3*b*p + 4*sqrt(x)*a*b**3*p*x - 12*log((sqrt(x)*b + a)**p*c)
*a**4 + 12*log((sqrt(x)*b + a)**p*c)*b**4*x**2 - 6*a**2*b**2*p*x - 3*b**4*
p*x**2)/(24*b**4)
```

3.49 $\int \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{ap\sqrt{x}}{b} - \frac{px}{2} - \frac{a^2 p \log \left(a+b\sqrt{x}\right)}{b^2} + x \log \left(c\left(a+b\sqrt{x}\right)^p\right)$$

output
```
a*p*x^(1/2)/b-1/2*p*x-a^2*p*ln(a+b*x^(1/2))/b^2+x*ln(c*(a+b*x^(1/2))^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{ap\sqrt{x}}{b} - \frac{px}{2} - \frac{a^2 p \log \left(a+b\sqrt{x}\right)}{b^2} + x \log \left(c\left(a+b\sqrt{x}\right)^p\right)$$

input
```
Integrate[Log[c*(a + b*Sqrt[x])^p],x]
```

output
```
(a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a +
b*Sqrt[x])^p]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2898, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right) dx$$

$$\Big\downarrow \text{2898}$$

$$x\log \left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{2}bp\int \frac{\sqrt{x}}{a+b\sqrt{x}}dx$$

$$\Big\downarrow \text{798}$$

$$x\log \left(c\left(a+b\sqrt{x}\right)^p\right)-bp\int \frac{x}{a+b\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$x\log \left(c\left(a+b\sqrt{x}\right)^p\right)-bp\int \left(\frac{a^2}{b^2\left(a+b\sqrt{x}\right)}-\frac{a}{b^2}+\frac{\sqrt{x}}{b}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$x\log \left(c\left(a+b\sqrt{x}\right)^p\right)-bp\left(\frac{a^2\log \left(a+b\sqrt{x}\right)}{b^3}-\frac{a\sqrt{x}}{b^2}+\frac{x}{2b}\right)$$

input
```
Int[Log[c*(a + b*Sqrt[x])^p],x]
```

output
```
-(b*p*(-((a*Sqrt[x])/b^2) + x/(2*b) + (a^2*Log[a + b*Sqrt[x]])/b^3)) + x*L
og[c*(a + b*Sqrt[x])^p]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
default	$x \ln\left(c \left(a+b \sqrt{x}\right)^{p}\right)-\frac{p b \left(-\frac{2 \left(-\frac{b x}{2}+a \sqrt{x}\right)}{b^{2}}+\frac{2 a^{2} \ln\left(a+b \sqrt{x}\right)}{b^{3}}\right)}{2}$	52
parts	$x \ln\left(c \left(a+b \sqrt{x}\right)^{p}\right)-\frac{p b \left(-\frac{2 \left(-\frac{b x}{2}+a \sqrt{x}\right)}{b^{2}}+\frac{2 a^{2} \ln\left(a+b \sqrt{x}\right)}{b^{3}}\right)}{2}$	52

input
```
int(ln(c*(a+b*x^(1/2))^p),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(a+b*x^(1/2))^p)-1/2*p*b*(-2/b^2*(-1/2*b*x+a*x^(1/2))+2*a^2/b^3*ln(
a+b*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{b^2px-2\,b^2x\log \left(c\right)-2\,abp\sqrt{x}-2\,\left(b^2px-a^2p\right)\log \left(b\sqrt{x}+a\right)}{2\,b^2}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

output
```
-1/2*(b^2*p*x - 2*b^2*x*log(c) - 2*a*b*p*sqrt(x) - 2*(b^2*p*x - a^2*p)*log
(b*sqrt(x) + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{bp\left(\frac{2a^2\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^2}-\frac{2a\sqrt{x}}{b^2}+\frac{x}{b}\right)}{2} + x\log \left(c\left(a+b\sqrt{x}\right)^p\right)$$

input
```
integrate(ln(c*(a+b*x**(1/2))**p),x)
```

output
```
-b*p*(2*a**2*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True)
)/b**2 - 2*a*sqrt(x)/b**2 + x/b)/2 + x*log(c*(a + b*sqrt(x))**p)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{1}{2}\,bp\left(\frac{2\,a^2\log\left(b\sqrt{x}+a\right)}{b^3}+\frac{bx-2\,a\sqrt{x}}{b^2}\right) + x\log\left(\left(b\sqrt{x}+a\right)^pc\right)$$

input
```
integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

output
```
-1/2*b*p*(2*a^2*log(b*sqrt(x) + a)/b^3 + (b*x - 2*a*sqrt(x))/b^2) + x*log(
(b*sqrt(x) + a)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{\frac{\left(2\left(b\sqrt{x}+a\right)^2\log\left(b\sqrt{x}+a\right)-4\left(b\sqrt{x}+a\right)a\log\left(b\sqrt{x}+a\right)-\left(b\sqrt{x}+a\right)^2+4\left(b\sqrt{x}+a\right)a\right)p}{b}+\frac{2\left(\left(b\sqrt{x}+a\right)^2-2\left(b\sqrt{x}+a\right)a\right)\log(c)}{b}}{2\,b}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

output
```
1/2*((2*(b*sqrt(x) + a)^2*log(b*sqrt(x) + a) - 4*(b*sqrt(x) + a)*a*log(b*s
qrt(x) + a) - (b*sqrt(x) + a)^2 + 4*(b*sqrt(x) + a)*a)*p/b + 2*((b*sqrt(x)
 + a)^2 - 2*(b*sqrt(x) + a)*a)*log(c)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = x\,\ln\left(c\,\left(a+b\,\sqrt{x}\right)^p\right)-\frac{p\,\left(b^2\,x+2\,a^2\,\ln\left(a+b\,\sqrt{x}\right)-2\,a\,b\,\sqrt{x}\right)}{2\,b^2}$$

input
```
int(log(c*(a + b*x^(1/2))^p),x)
```

output
```
x*log(c*(a + b*x^(1/2))^p) - (p*(b^2*x + 2*a^2*log(a + b*x^(1/2)) - 2*a*b*
x^(1/2)))/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{2\sqrt{x}\,abp-2\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)a^2+2\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)b^2x-b^2px}{2b^2}$$

input
```
int(log(c*(a+b*x^(1/2))^p),x)
```

output
```
(2*sqrt(x)*a*b*p - 2*log((sqrt(x)*b + a)**p*c)*a**2 + 2*log((sqrt(x)*b + a
)**p*c)*b**2*x - b**2*p*x)/(2*b**2)
```

3.50 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx=2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right)+2p\,\mathrm{PolyLog}\left(2,1+\frac{b\sqrt{x}}{a}\right)$$

output
```
2*ln(c*(a+b*x^(1/2))^p)*ln(-b*x^(1/2)/a)+2*p*polylog(2,1+b*x^(1/2)/a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx=2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right)+2p\,\mathrm{PolyLog}\left(2,\frac{a+b\sqrt{x}}{a}\right)$$

input
```
Integrate[Log[c*(a + b*Sqrt[x])^p]/x,x]
```

output
```
2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, (a + b*S
qrt[x])/a]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x} \, dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2841}$$

$$2\left(\log \left(-\frac{b\sqrt{x}}{a}\right)\log \left(c\left(a+b\sqrt{x}\right)^p\right)-bp\int \frac{\log \left(-\frac{b\sqrt{x}}{a}\right)}{a+b\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2752}$$

$$2\left(\log \left(-\frac{b\sqrt{x}}{a}\right)\log \left(c\left(a+b\sqrt{x}\right)^p\right)+p\,\text{PolyLog}\left(2,\frac{\sqrt{x}b}{a}+1\right)\right)$$

input
```
Int[Log[c*(a + b*Sqrt[x])^p]/x,x]
```

output
```
2*(Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + p*PolyLog[2, 1 + (b*Sq
rt[x])/a])
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

method	result	size
parts	$\ln\left(c\left(a+b\sqrt{x}\right)^{p}\right)\ln\left(x\right)-\frac{pb\left(\frac{4\,\mathrm{dilog}\left(\frac{a+b\sqrt{x}}{a}\right)}{b}+\frac{2\ln\left(x\right)\ln\left(\frac{a+b\sqrt{x}}{a}\right)}{b}\right)}{2}$	58

input
```
int(ln(c*(a+b*x^(1/2))^p)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(c*(a+b*x^(1/2))^p)*ln(x)-1/2*p*b*(4*dilog((a+b*x^(1/2))/a)/b+2*ln(x)*ln
((a+b*x^(1/2))/a)/b)
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = \int \frac{\log\left(\left(b\sqrt{x}+a\right)^p c\right)}{x}\,dx$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="fricas")
```

output
```
integral(log((b*sqrt(x) + a)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = \int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx$$

input
```
integrate(ln(c*(a+b*x**(1/2))**p)/x,x)
```

output
```
Integral(log(c*(a + b*sqrt(x))**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = bp\left(\frac{\log\left(b\sqrt{x}+a\right)\log\left(x\right)}{b} - \frac{\log\left(x\right)\log\left(\frac{b\sqrt{x}}{a}+1\right)+2\,\mathrm{Li}_2\left(-\frac{b\sqrt{x}}{a}\right)}{b}\right) - p\log\left(b\sqrt{x}+a\right)\log\left(x\right) + \log\left(\left(b\sqrt{x}+a\right)^p c\right)\log\left(x\right)$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="maxima")
```

output
```
b*p*(log(b*sqrt(x) + a)*log(x)/b - (log(x)*log(b*sqrt(x)/a + 1) + 2*dilog(
-b*sqrt(x)/a))/b) - p*log(b*sqrt(x) + a)*log(x) + log((b*sqrt(x) + a)^p*c)
*log(x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = \int \frac{\log\left(\left(b\sqrt{x}+a\right)^p c\right)}{x}\,dx$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="giac")
```

output
```
integrate(log((b*sqrt(x) + a)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = \int \frac{\ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx$$

input
```
int(log(c*(a + b*x^(1/2))^p)/x,x)
```

output
```
int(log(c*(a + b*x^(1/2))^p)/x, x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = \frac{\left(\int \frac{\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)}{-b^2x^2+a^2x}dx\right)a^2p - \left(\int \frac{\sqrt{x}\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)}{-b^2x^2+a^2x}dx\right)abp + \log\left(\left(\sqrt{x}\,b+a\right)^p c\right)^2}{p}$$

input
```
int(log(c*(a+b*x^(1/2))^p)/x,x)
```

output
```
(int(log((sqrt(x)*b + a)**p*c)/(a**2*x - b**2*x**2),x)*a**2*p - int((sqrt(
x)*log((sqrt(x)*b + a)**p*c))/(a**2*x - b**2*x**2),x)*a*b*p + log((sqrt(x)
*b + a)**p*c)**2)/p
```

3.51 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx=-\frac{bp}{a\sqrt{x}}+\frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{b^2p\log(x)}{2a^2}$$

output
```
-b*p/a/x^(1/2)+b^2*p*ln(a+b*x^(1/2))/a^2-ln(c*(a+b*x^(1/2))^p)/x-1/2*b^2*p
*ln(x)/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx=-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{bp\left(\frac{2a}{\sqrt{x}}-2b\log\left(a+b\sqrt{x}\right)+b\log(x)\right)}{2a^2}$$

input
```
Integrate[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
```

output
```
-(Log[c*(a + b*Sqrt[x])^p]/x) - (b*p*((2*a)/Sqrt[x] - 2*b*Log[a + b*Sqrt[x
]] + b*Log[x]))/(2*a^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2} \, dx$$

$$\Big\downarrow \ 2904$$

$$2\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \ 2842$$

$$2\left(\frac{1}{2}bp\int \frac{1}{\left(a+b\sqrt{x}\right)x}d\sqrt{x}-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{2x}\right)$$

$$\Big\downarrow \ 54$$

$$2\left(\frac{1}{2}bp\int \left(\frac{b^2}{a^2\left(a+b\sqrt{x}\right)}-\frac{b}{a^2\sqrt{x}}+\frac{1}{ax}\right)d\sqrt{x}-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{2x}\right)$$

$$\Big\downarrow \ 2009$$

$$2\left(\frac{1}{2}bp\left(\frac{b\log \left(a+b\sqrt{x}\right)}{a^2}-\frac{b\log \left(\sqrt{x}\right)}{a^2}-\frac{1}{a\sqrt{x}}\right)-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{2x}\right)$$

input
```
Int[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
```

output
```
2*(-1/2*Log[c*(a + b*Sqrt[x])^p]/x + (b*p*(-(1/(a*Sqrt[x])) + (b*Log[a + b
*Sqrt[x]])/a^2 - (b*Log[Sqrt[x]])/a^2))/2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result	size
parts	$-\frac{\ln\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x}+\frac{pb\left(\frac{2b\ln\left(a+b\sqrt{x}\right)}{a^{2}}-\frac{2}{a\sqrt{x}}-\frac{b\ln\left(x\right)}{a^{2}}\right)}{2}$	54

input
```
int(ln(c*(a+b*x^(1/2))^p)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(a+b*x^(1/2))^p)/x+1/2*p*b*(2/a^2*b*ln(a+b*x^(1/2))-2/a/x^(1/2)-1/a^
2*b*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = -\frac{b^2px\log\left(\sqrt{x}\right)+abp\sqrt{x}+a^2\log(c)-(b^2px-a^2p)\log\left(b\sqrt{x}+a\right)}{a^2x}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="fricas")
```

output
```
-(b^2*p*x*log(sqrt(x)) + a*b*p*sqrt(x) + a^2*log(c) - (b^2*p*x - a^2*p)*lo
g(b*sqrt(x) + a))/(a^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(56) = 112.

Time = 8.42 (sec) , antiderivative size = 352, normalized size of antiderivative = 5.59

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = \begin{cases} -\frac{\log(0^pc)}{x} \\ -\frac{p}{2x}-\frac{\log\left(c\left(b\sqrt{x}\right)^p\right)}{x} \\ -\frac{\log(0^pc)}{x} \\ -\frac{2a^3\sqrt{x}\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2a^2bpx}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2a^2bx\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{ab^2px^{\frac{3}{2}}\log(x)}{2a^3x^{\frac{3}{2}}+2a^2bx^2}-\frac{2ab^2px^{\frac{3}{2}}}{2a^3x^{\frac{3}{2}}+2a^2bx^2}+\frac{2ab^2x^{\frac{3}{2}}\log\left(c(a+b\right.}{2a^3x^{\frac{3}{2}}+2a^2bx} \end{cases}$$

input
```
integrate(ln(c*(a+b*x**(1/2))**p)/x**2,x)
```

output
```
Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-p/(2*x) - log(c*(b*sqrt
(x))**p)/x, Eq(a, 0)), (-log(0**p*c)/x, Eq(a, -b*sqrt(x))), (-2*a**3*sqrt(
x)*log(c*(a + b*sqrt(x))**p)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - 2*a**2*b*
p*x/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - 2*a**2*b*x*log(c*(a + b*sqrt(x))**
p)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - a*b**2*p*x**(3/2)*log(x)/(2*a**3*x*
*(3/2) + 2*a**2*b*x**2) - 2*a*b**2*p*x**(3/2)/(2*a**3*x**(3/2) + 2*a**2*b*
x**2) + 2*a*b**2*x**(3/2)*log(c*(a + b*sqrt(x))**p)/(2*a**3*x**(3/2) + 2*a
**2*b*x**2) - b**3*p*x**2*log(x)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) + 2*b**
3*x**2*log(c*(a + b*sqrt(x))**p)/(2*a**3*x**(3/2) + 2*a**2*b*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = \frac{1}{2}\,bp\left(\frac{2\,b\log \left(b\sqrt{x}+a\right)}{a^2}-\frac{b\log \left(x\right)}{a^2}-\frac{2}{a\sqrt{x}}\right) - \frac{\log \left(\left(b\sqrt{x}+a\right)^p c\right)}{x}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*p*(2*b*log(b*sqrt(x) + a)/a^2 - b*log(x)/a^2 - 2/(a*sqrt(x))) - log(
(b*sqrt(x) + a)^p*c)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(55) = 110.

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = -\frac{\frac{b^3p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^2-2\left(b\sqrt{x}+a\right)a+a^2}-\frac{b^3p\log\left(b\sqrt{x}+a\right)}{a^2}+\frac{b^3p\log\left(b\sqrt{x}\right)}{a^2}+\frac{\left(b\sqrt{x}+a\right)b^3p-ab^3p+ab^3\log\left(c\right)}{\left(b\sqrt{x}+a\right)^2a-2\left(b\sqrt{x}+a\right)a^2+a^3}}{b}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="giac")
```

output
```
-(b^3*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^2 - 2*(b*sqrt(x) + a)*a + a^2)
 - b^3*p*log(b*sqrt(x) + a)/a^2 + b^3*p*log(b*sqrt(x))/a^2 + ((b*sqrt(x) +
 a)*b^3*p - a*b^3*p + a*b^3*log(c))/((b*sqrt(x) + a)^2*a - 2*(b*sqrt(x) +
a)*a^2 + a^3))/b
```

Mupad [B] (verification not implemented)

Time = 15.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx=\frac{2\,b^2\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^2}-\frac{\ln \left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{x}-\frac{b\,p}{a\,\sqrt{x}}$$

input
```
int(log(c*(a + b*x^(1/2))^p)/x^2,x)
```

output
```
(2*b^2*p*atanh((2*b*x^(1/2))/a + 1))/a^2 - log(c*(a + b*x^(1/2))^p)/x - (b
*p)/(a*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx=\frac{-\sqrt{x}\,abp-\log\left(\sqrt{x}\right)b^2px-\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)a^2+\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)b^2x}{a^2x}$$

input
```
int(log(c*(a+b*x^(1/2))^p)/x^2,x)
```

output
```
( - sqrt(x)*a*b*p - log(sqrt(x))*b**2*p*x - log((sqrt(x)*b + a)**p*c)*a**2
 + log((sqrt(x)*b + a)**p*c)*b**2*x)/(a**2*x)
```

3.52 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx=-\frac{bp}{6ax^{3/2}}+\frac{b^2p}{4a^2x}-\frac{b^3p}{2a^3\sqrt{x}}+\frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{b^4p\log(x)}{4a^4}$$

output
```
-1/6*b*p/a/x^(3/2)+1/4*b^2*p/a^2/x-1/2*b^3*p/a^3/x^(1/2)+1/2*b^4*p*ln(a+b*
x^(1/2))/a^4-1/2*ln(c*(a+b*x^(1/2))^p)/x^2-1/4*b^4*p*ln(x)/a^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx=-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}+\frac{1}{4}bp\left(-\frac{2}{3ax^{3/2}}+\frac{b}{a^2x}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b^3\log\left(a+b\sqrt{x}\right)}{a^4}-\frac{b^3\log(x)}{a^4}\right)$$

input
```
Integrate[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```

output
```
-1/2*Log[c*(a + b*Sqrt[x])^p]/x^2 + (b*p*(-2/(3*a*x^(3/2)) + b/(a^2*x) - (
2*b^2)/(a^3*Sqrt[x]) + (2*b^3*Log[a + b*Sqrt[x]])/a^4 - (b^3*Log[x])/a^4))
/4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = $\{2904, 2842, 54, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$2\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{2842}$$

$$2\left(\frac{1}{4}bp\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}d\sqrt{x}-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{4x^2}\right)$$

$$\Big\downarrow \text{54}$$

$$2\left(\frac{1}{4}bp\int \left(\frac{b^4}{a^4\left(a+b\sqrt{x}\right)}-\frac{b^3}{a^4\sqrt{x}}+\frac{b^2}{a^3x}-\frac{b}{a^2x^{3/2}}+\frac{1}{ax^2}\right)d\sqrt{x}-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{4x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{4}bp\left(\frac{b^3\log \left(a+b\sqrt{x}\right)}{a^4}-\frac{b^3\log \left(\sqrt{x}\right)}{a^4}-\frac{b^2}{a^3\sqrt{x}}+\frac{b}{2a^2x}-\frac{1}{3ax^{3/2}}\right)-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{4x^2}\right)$$

input
```
Int[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```

output
```
2*(-1/4*Log[c*(a + b*Sqrt[x])^p]/x^2 + (b*p*(-1/3*1/(a*x^(3/2)) + b/(2*a^2
*x) - b^2/(a^3*Sqrt[x]) + (b^3*Log[a + b*Sqrt[x]])/a^4 - (b^3*Log[Sqrt[x]]
)/a^4))/4)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

method	result	size
parts	$-\frac{\ln\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{2x^{2}}+\frac{pb\left(\frac{2b^{3}\ln\left(a+b\sqrt{x}\right)}{a^{4}}-\frac{2}{3a\,x^{\frac{3}{2}}}-\frac{2b^{2}}{a^{3}\sqrt{x}}+\frac{b}{a^{2}x}-\frac{b^{3}\ln\left(x\right)}{a^{4}}\right)}{4}$	77

input
```
int(ln(c*(a+b*x^(1/2))^p)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(a+b*x^(1/2))^p)/x^2+1/4*p*b*(2*b^3/a^4*ln(a+b*x^(1/2))-2/3/a/x^
(3/2)-2*b^2/a^3/x^(1/2)+b/a^2/x-b^3/a^4*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = -\frac{6\,b^4px^2\log\left(\sqrt{x}\right)-3\,a^2b^2px+6\,a^4\log\left(c\right)-6\left(b^4px^2-a^4p\right)\log\left(b\sqrt{x}+a\right)+2\left(3\,ab^3px+a^3bp\right)\sqrt{x}}{12\,a^4x^2}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="fricas")
```

output
```
-1/12*(6*b^4*p*x^2*log(sqrt(x)) - 3*a^2*b^2*p*x + 6*a^4*log(c) - 6*(b^4*p*
x^2 - a^4*p)*log(b*sqrt(x) + a) + 2*(3*a*b^3*p*x + a^3*b*p)*sqrt(x))/(a^4*
x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(90) = 180.

Time = 73.06 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.35

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = \begin{cases} -\frac{\log\left(0^pc\right)}{2x^2} \\ -\frac{p}{8x^2}-\frac{\log\left(c\left(b\sqrt{x}\right)^p\right)}{2x^2} \\ -\frac{\log\left(0^pc\right)}{2x^2} \\ -\frac{6a^5\sqrt{x}\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{2a^4bpx}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{6a^4bx\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3}+\frac{a^3b^2px^{\frac{3}{2}}}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{3a^2b^3px^2}{12a^5x^{\frac{5}{2}}+12a^4bx^3}-\frac{3ab^4px^{\frac{5}{2}}\log}{12a^5x^{\frac{5}{2}}+12}\end{cases}$$

input
```
integrate(ln(c*(a+b*x**(1/2))**p)/x**3,x)
```

output
```
Piecewise((-log(0**p*c)/(2*x**2), Eq(a, 0) & Eq(b, 0)), (-p/(8*x**2) - log
(c*(b*sqrt(x))**p)/(2*x**2), Eq(a, 0)), (-log(0**p*c)/(2*x**2), Eq(a, -b*s
qrt(x))), (-6*a**5*sqrt(x)*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 1
2*a**4*b*x**3) - 2*a**4*b*p*x/(12*a**5*x**(5/2) + 12*a**4*b*x**3) - 6*a**4
*b*x*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12*a**4*b*x**3) + a**3*
b**2*p*x**(3/2)/(12*a**5*x**(5/2) + 12*a**4*b*x**3) - 3*a**2*b**3*p*x**2/(
12*a**5*x**(5/2) + 12*a**4*b*x**3) - 3*a*b**4*p*x**(5/2)*log(x)/(12*a**5*x
**(5/2) + 12*a**4*b*x**3) - 6*a*b**4*p*x**(5/2)/(12*a**5*x**(5/2) + 12*a**
4*b*x**3) + 6*a*b**4*x**(5/2)*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2)
+ 12*a**4*b*x**3) - 3*b**5*p*x**3*log(x)/(12*a**5*x**(5/2) + 12*a**4*b*x**
3) + 6*b**5*x**3*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12*a**4*b*x
**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = \frac{1}{12}\,bp\left(\frac{6\,b^3\log\left(b\sqrt{x}+a\right)}{a^4}-\frac{3\,b^3\log\left(x\right)}{a^4}-\frac{6\,b^2x-3\,ab\sqrt{x}+2\,a^2}{a^3x^{\frac{3}{2}}}\right)-\frac{\log\left(\left(b\sqrt{x}+a\right)^pc\right)}{2\,x^2}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="maxima")
```

output
```
1/12*b*p*(6*b^3*log(b*sqrt(x) + a)/a^4 - 3*b^3*log(x)/a^4 - (6*b^2*x - 3*a
*b*sqrt(x) + 2*a^2)/(a^3*x^(3/2))) - 1/2*log((b*sqrt(x) + a)^p*c)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(80) = 160$.

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.32

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = -\frac{\frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^4-4\left(b\sqrt{x}+a\right)^3a+6\left(b\sqrt{x}+a\right)^2a^2-4\left(b\sqrt{x}+a\right)a^3+a^4} - \frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{a^4} + \frac{6\,b^5p\log\left(b\sqrt{x}\right)}{a^4} + \frac{6\left(b\sqrt{x}+a\right)^3b^5p-21\left(b\sqrt{x}+a\right)^2\cdots}{\left(b\sqrt{x}+a\right)^4a^3-4\left(b\sqrt{x}+\cdots\right.}}{12\,b}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="giac")
```

output
```
-1/12*(6*b^5*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^4 - 4*(b*sqrt(x) + a)^3
*a + 6*(b*sqrt(x) + a)^2*a^2 - 4*(b*sqrt(x) + a)*a^3 + a^4) - 6*b^5*p*log(
b*sqrt(x) + a)/a^4 + 6*b^5*p*log(b*sqrt(x))/a^4 + (6*(b*sqrt(x) + a)^3*b^5
*p - 21*(b*sqrt(x) + a)^2*a*b^5*p + 26*(b*sqrt(x) + a)*a^2*b^5*p - 11*a^3*
b^5*p + 6*a^3*b^5*log(c))/((b*sqrt(x) + a)^4*a^3 - 4*(b*sqrt(x) + a)^3*a^4
 + 6*(b*sqrt(x) + a)^2*a^5 - 4*(b*sqrt(x) + a)*a^6 + a^7))/b
```

Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = \frac{b^4\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^4} - \frac{\ln\left(c\,\left(a+b\,\sqrt{x}\right)^p\right)}{2\,x^2} - \frac{\frac{b\,p}{3\,a}-\frac{b^2\,p\,\sqrt{x}}{2\,a^2}+\frac{b^3\,p\,x}{a^3}}{2\,x^{3/2}}$$

input
```
int(log(c*(a + b*x^(1/2))^p)/x^3,x)
```

output
```
(b^4*p*atanh((2*b*x^(1/2))/a + 1))/a^4 - log(c*(a + b*x^(1/2))^p)/(2*x^2)
- ((b*p)/(3*a) - (b^2*p*x^(1/2))/(2*a^2) + (b^3*p*x)/a^3)/(2*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx$$
$$= \frac{-2\sqrt{x}\,a^3bp-6\sqrt{x}\,a\,b^3px-6\log\left(\sqrt{x}\right)b^4p\,x^2-6\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)a^4+6\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)b^4x^2+3a}{12a^4x^2}$$

input
```
int(log(c*(a+b*x^(1/2))^p)/x^3,x)
```

output
```
( - 2*sqrt(x)*a**3*b*p - 6*sqrt(x)*a*b**3*p*x - 6*log(sqrt(x))*b**4*p*x**2
 - 6*log((sqrt(x)*b + a)**p*c)*a**4 + 6*log((sqrt(x)*b + a)**p*c)*b**4*x**
2 + 3*a**2*b**2*p*x)/(12*a**4*x**2)
```

3.53 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = -\frac{bp}{15ax^{5/2}}+\frac{b^2p}{12a^2x^2}-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^4p}{6a^4x}-\frac{b^5p}{3a^5\sqrt{x}} + \frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}-\frac{b^6p\log(x)}{6a^6}$$

output
```
-1/15*b*p/a/x^(5/2)+1/12*b^2*p/a^2/x^2-1/9*b^3*p/a^3/x^(3/2)+1/6*b^4*p/a^4
/x-1/3*b^5*p/a^5/x^(1/2)+1/3*b^6*p*ln(a+b*x^(1/2))/a^6-1/3*ln(c*(a+b*x^(1/
2))^p)/x^3-1/6*b^6*p*ln(x)/a^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = \frac{abp\sqrt{x}\left(-12a^4+15a^3b\sqrt{x}-20a^2b^2x+30ab^3x^{3/2}-60b^4x^2\right)+60b^6px^3\log\left(a+b\sqrt{x}\right)-60a^6\log\left(c\left(a+\right.\right.}{180a^6x^3}$$

input
```
Integrate[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

output
```
(a*b*p*Sqrt[x]*(-12*a^4 + 15*a^3*b*Sqrt[x] - 20*a^2*b^2*x + 30*a*b^3*x^(3/
2) - 60*b^4*x^2) + 60*b^6*p*x^3*Log[a + b*Sqrt[x]] - 60*a^6*Log[c*(a + b*S
qrt[x])^p] - 30*b^6*p*x^3*Log[x])/(180*a^6*x^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}dx$$

$$\Big\downarrow \text{2904}$$

$$2\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \text{2842}$$

$$2\left(\frac{1}{6}bp\int \frac{1}{\left(a+b\sqrt{x}\right)x^3}d\sqrt{x}-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{6x^3}\right)$$

$$\Big\downarrow \text{54}$$

$$2\left(\frac{1}{6}bp\int \left(\frac{b^6}{a^6\left(a+b\sqrt{x}\right)}-\frac{b^5}{a^6\sqrt{x}}+\frac{b^4}{a^5x}-\frac{b^3}{a^4x^{3/2}}+\frac{b^2}{a^3x^2}-\frac{b}{a^2x^{5/2}}+\frac{1}{ax^3}\right)d\sqrt{x}-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{6x^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{6}bp\left(\frac{b^5\log \left(a+b\sqrt{x}\right)}{a^6}-\frac{b^5\log \left(\sqrt{x}\right)}{a^6}-\frac{b^4}{a^5\sqrt{x}}+\frac{b^3}{2a^4x}-\frac{b^2}{3a^3x^{3/2}}+\frac{b}{4a^2x^2}-\frac{1}{5ax^{5/2}}\right)-\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{6x^3}\right)$$

input
```
Int[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

output
```
2*(-1/6*Log[c*(a + b*Sqrt[x])^p]/x^3 + (b*p*(-1/5*1/(a*x^(5/2)) + b/(4*a^2
*x^2) - b^2/(3*a^3*x^(3/2)) + b^3/(2*a^4*x) - b^4/(a^5*Sqrt[x]) + (b^5*Log
[a + b*Sqrt[x]])/a^6 - (b^5*Log[Sqrt[x]])/a^6))/6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

method	result	size
parts	$-\frac{\ln\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{3x^{3}}+\frac{pb\left(\frac{2b^{5}\ln\left(a+b\sqrt{x}\right)}{a^{6}}-\frac{2}{5a\,x^{\frac{5}{2}}}-\frac{2b^{4}}{a^{5}\sqrt{x}}-\frac{2b^{2}}{3a^{3}x^{\frac{3}{2}}}-\frac{b^{5}\ln\left(x\right)}{a^{6}}+\frac{b^{3}}{a^{4}x}+\frac{b}{2a^{2}x^{2}}\right)}{6}$	99

input
```
int(ln(c*(a+b*x^(1/2))^p)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(c*(a+b*x^(1/2))^p)/x^3+1/6*p*b*(2/a^6*b^5*ln(a+b*x^(1/2))-2/5/a/x^
(5/2)-2/a^5*b^4/x^(1/2)-2/3*b^2/a^3/x^(3/2)-1/a^6*b^5*ln(x)+b^3/a^4/x+1/2*
b/a^2/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = -\frac{60\,b^6px^3\log\left(\sqrt{x}\right) - 30\,a^2b^4px^2 - 15\,a^4b^2px + 60\,a^6\log\left(c\right) - 60\,\left(b^6px^3 - a^6p\right)\log\left(b\sqrt{x}+a\right) + 4\,(15\,a}{180\,a^6x^3}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="fricas")
```

output
```
-1/180*(60*b^6*p*x^3*log(sqrt(x)) - 30*a^2*b^4*p*x^2 - 15*a^4*b^2*p*x + 60
*a^6*log(c) - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) + 4*(15*a*b^5*p*x^
2 + 5*a^3*b^3*p*x + 3*a^5*b*p)*sqrt(x))/(a^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b*x**(1/2))**p)/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx$$
$$=\frac{1}{180}\,bp\left(\frac{60\,b^5\log\left(b\sqrt{x}+a\right)}{a^6}-\frac{30\,b^5\log\left(x\right)}{a^6}-\frac{60\,b^4x^2-30\,ab^3x^{\frac{3}{2}}+20\,a^2b^2x-15\,a^3b\sqrt{x}+12\,a^4}{a^5x^{\frac{5}{2}}}\right)$$
$$-\frac{\log\left(\left(b\sqrt{x}+a\right)^pc\right)}{3\,x^3}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="maxima")
```

output
```
1/180*b*p*(60*b^5*log(b*sqrt(x) + a)/a^6 - 30*b^5*log(x)/a^6 - (60*b^4*x^2
 - 30*a*b^3*x^(3/2) + 20*a^2*b^2*x - 15*a^3*b*sqrt(x) + 12*a^4)/(a^5*x^(5/
2))) - 1/3*log((b*sqrt(x) + a)^p*c)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(104) = 208.

Time = 0.12 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.49

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx =$$
$$-\frac{\frac{60\,b^7p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^6-6\,\left(b\sqrt{x}+a\right)^5a+15\,\left(b\sqrt{x}+a\right)^4a^2-20\,\left(b\sqrt{x}+a\right)^3a^3+15\,\left(b\sqrt{x}+a\right)^2a^4-6\,\left(b\sqrt{x}+a\right)a^5+a^6}-\frac{60\,b^7p\log\left(b\sqrt{x}+a\right)}{a^6}+\frac{60\,b^7p\log(b}{a^6}}{18}$$

input
```
integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="giac")
```

output
```
-1/180*(60*b^7*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^6 - 6*(b*sqrt(x) + a)
^5*a + 15*(b*sqrt(x) + a)^4*a^2 - 20*(b*sqrt(x) + a)^3*a^3 + 15*(b*sqrt(x)
 + a)^2*a^4 - 6*(b*sqrt(x) + a)*a^5 + a^6) - 60*b^7*p*log(b*sqrt(x) + a)/a
^6 + 60*b^7*p*log(b*sqrt(x))/a^6 + (60*(b*sqrt(x) + a)^5*b^7*p - 330*(b*sq
rt(x) + a)^4*a*b^7*p + 740*(b*sqrt(x) + a)^3*a^2*b^7*p - 855*(b*sqrt(x) +
a)^2*a^3*b^7*p + 522*(b*sqrt(x) + a)*a^4*b^7*p - 137*a^5*b^7*p + 60*a^5*b^
7*log(c))/((b*sqrt(x) + a)^6*a^5 - 6*(b*sqrt(x) + a)^5*a^6 + 15*(b*sqrt(x)
 + a)^4*a^7 - 20*(b*sqrt(x) + a)^3*a^8 + 15*(b*sqrt(x) + a)^2*a^9 - 6*(b*s
qrt(x) + a)*a^10 + a^11))/b
```

Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = \frac{2\,b^6\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{3\,a^6} - \frac{\frac{b\,p}{5\,a}-\frac{b^2\,p\,\sqrt{x}}{4\,a^2}+\frac{b^5\,p\,x^2}{a^5}-\frac{b^4\,p\,x^{3/2}}{2\,a^4}+\frac{b^3\,p\,x}{3\,a^3}}{3\,x^{5/2}} - \frac{\ln\left(c\,\left(a+b\,\sqrt{x}\right)^p\right)}{3\,x^3}$$

input
```
int(log(c*(a + b*x^(1/2))^p)/x^4,x)
```

output
```
(2*b^6*p*atanh((2*b*x^(1/2))/a + 1))/(3*a^6) - ((b*p)/(5*a) - (b^2*p*x^(1/
2))/(4*a^2) + (b^5*p*x^2)/a^5 - (b^4*p*x^(3/2))/(2*a^4) + (b^3*p*x)/(3*a^3
))/(3*x^(5/2)) - log(c*(a + b*x^(1/2))^p)/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = \frac{-12\sqrt{x}\,a^5bp-20\sqrt{x}\,a^3b^3px-60\sqrt{x}\,a\,b^5p\,x^2-60\log\left(\sqrt{x}\right)b^6p\,x^3-60\log\left(\left(\sqrt{x}\,b+a\right)^p c\right)a^6+60\log((\cdots}{180a^6x^3}$$

input
```
int(log(c*(a+b*x^(1/2))^p)/x^4,x)
```

output
```
( - 12*sqrt(x)*a**5*b*p - 20*sqrt(x)*a**3*b**3*p*x - 60*sqrt(x)*a*b**5*p*x
**2 - 60*log(sqrt(x))*b**6*p*x**3 - 60*log((sqrt(x)*b + a)**p*c)*a**6 + 60
*log((sqrt(x)*b + a)**p*c)*b**6*x**3 + 15*a**4*b**2*p*x + 30*a**2*b**4*p*x
**2)/(180*a**6*x**3)
```

3.54 $\int \frac{\log(a+b\sqrt{x})}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{\log \left(a+b\sqrt{x}\right)}{\sqrt{x}} \, dx = -2\sqrt{x} + \frac{2\left(a+b\sqrt{x}\right) \log \left(a+b\sqrt{x}\right)}{b}$$

output
```
-2*x^(1/2)+2*(a+b*x^(1/2))*ln(a+b*x^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{\log \left(a+b\sqrt{x}\right)}{\sqrt{x}} \, dx = 2\left(-\sqrt{x} + \frac{\left(a+b\sqrt{x}\right) \log \left(a+b\sqrt{x}\right)}{b}\right)$$

input
```
Integrate[Log[a + b*Sqrt[x]]/Sqrt[x],x]
```

output
```
2*(-Sqrt[x] + ((a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2904, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{2904}$$

$$2\int \log\left(a+b\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2836}$$

$$\frac{2\int \log\left(a+b\sqrt{x}\right)d\left(a+b\sqrt{x}\right)}{b}$$

$$\Big\downarrow \text{2732}$$

$$\frac{2\left(\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)-a-b\sqrt{x}\right)}{b}$$

input
```
Int[Log[a + b*Sqrt[x]]/Sqrt[x],x]
```

output
```
(2*(-a - b*Sqrt[x] + (a + b*Sqrt[x])*Log[a + b*Sqrt[x]]))/b
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{2(a+b\sqrt{x})\ln(a+b\sqrt{x})-2b\sqrt{x}-2a}{b}$	32
default	$\frac{2(a+b\sqrt{x})\ln(a+b\sqrt{x})-2b\sqrt{x}-2a}{b}$	32

input
```
int(ln(a+b*x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*((a+b*x^(1/2))*ln(a+b*x^(1/2))-b*x^(1/2)-a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}\right)}{b}$$

input
```
integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.88

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } \left(a=0 \vee a=-b\sqrt{x}\right) \wedge \left(a = \right. \\ 2\sqrt{x}\log\left(a\right) & \text{for } b=0 \\ \frac{2a^2\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}} + \frac{2a^2}{ab+b^2\sqrt{x}} + \frac{4ab\sqrt{x}\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}} + \frac{2b^2x\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}} - \frac{2b^2x}{ab+b^2\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(a+b*x**(1/2))/x**(1/2),x)
```

output
```
Piecewise((zoo*sqrt(x), (Eq(a, 0) | Eq(a, -b*sqrt(x))) & (Eq(b, 0) | Eq(a,
 -b*sqrt(x)))), (2*sqrt(x)*log(a), Eq(b, 0)), (2*a**2*log(a + b*sqrt(x))/(
a*b + b**2*sqrt(x)) + 2*a**2/(a*b + b**2*sqrt(x)) + 4*a*b*sqrt(x)*log(a +
b*sqrt(x))/(a*b + b**2*sqrt(x)) + 2*b**2*x*log(a + b*sqrt(x))/(a*b + b**2*
sqrt(x)) - 2*b**2*x/(a*b + b**2*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

input
```
integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{\log \left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(\left(b\sqrt{x}+a\right)\log \left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

input
```
integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

Mupad [B] (verification not implemented)

Time = 15.52 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{\log \left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\ln \left(a+b\,\sqrt{x}\right)-2\sqrt{x}+\frac{2\,a\,\ln \left(a+b\,\sqrt{x}\right)}{b}$$

input
```
int(log(a + b*x^(1/2))/x^(1/2),x)
```

output
```
2*x^(1/2)*log(a + b*x^(1/2)) - 2*x^(1/2) + (2*a*log(a + b*x^(1/2)))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{\log \left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\log \left(\sqrt{x}\,b+a\right)b-2\sqrt{x}\,b+2\log \left(\sqrt{x}\,b+a\right)a}{b}$$

input
```
int(log(a+b*x^(1/2))/x^(1/2),x)
```

output
```
(2*(sqrt(x)*log(sqrt(x)*b + a)*b - sqrt(x)*b + log(sqrt(x)*b + a)*a))/b
```

3.55 $\int (fx)^m \log\left(c(d+ex^3)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int (fx)^m \log\left(c(d+ex^3)^p\right)\,dx = -\frac{3ep(fx)^{4+m}\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{3},\frac{7+m}{3},-\frac{ex^3}{d}\right)}{df^4(1+m)(4+m)} + \frac{(fx)^{1+m}\log\left(c(d+ex^3)^p\right)}{f(1+m)}$$

output
```
-3*e*p*(f*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],-e*x^3/d)/d/f^4/(1
+m)/(4+m)+(f*x)^(1+m)*ln(c*(e*x^3+d)^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (fx)^m \log\left(c(d+ex^3)^p\right)\,dx = \frac{x(fx)^m\left(-3epx^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{3},\frac{7+m}{3},-\frac{ex^3}{d}\right)+d(4+m)\log\left(c(d+ex^3)^p\right)\right)}{d(1+m)(4+m)}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

output
```
(x*(f*x)^m*(-3*e*p*x^3*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3
)/d)] + d*(4 + m)*Log[c*(d + e*x^3)^p]))/(d*(1 + m)*(4 + m))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2905, 8, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log\left(c\left(d+ex^3\right)^p\right)\,dx$$

$$\Big\downarrow \; 2905$$

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^3\right)^p\right)}{f(m+1)}-\frac{3ep\int \frac{x^2(fx)^{m+1}}{ex^3+d}dx}{f(m+1)}$$

$$\Big\downarrow \; 8$$

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^3\right)^p\right)}{f(m+1)}-\frac{3ep\int \frac{(fx)^{m+3}}{ex^3+d}dx}{f^3(m+1)}$$

$$\Big\downarrow \; 888$$

$$\frac{(fx)^{m+1}\log\left(c\left(d+ex^3\right)^p\right)}{f(m+1)}-\frac{3ep(fx)^{m+4}\,\text{Hypergeometric2F1}\left(1,\frac{m+4}{3},\frac{m+7}{3},-\frac{ex^3}{d}\right)}{df^4(m+1)(m+4)}$$

input
```
Int[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

output
```
(-3*e*p*(f*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)
/d)])/(d*f^4*(1 + m)*(4 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^3)^p])/(f*(1
 + m))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^{m} \ln \left(c\left(e\,x^{3}+d\right)^{p}\right) dx$$

input
```
int((f*x)^m*ln(c*(e*x^3+d)^p),x)
```

output
```
int((f*x)^m*ln(c*(e*x^3+d)^p),x)
```

Fricas [F]

$$\int (fx)^m \log \left(c\left(d+ex^3\right)^p\right) \, dx = \int (fx)^m \log \left(\left(ex^3+d\right)^p c\right) \, dx$$

input
```
integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log((e*x^3 + d)^p*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c\left(d+ex^3\right)^p\right) dx = \text{Timed out}$$

input
```
integrate((f*x)**m*ln(c*(e*x**3+d)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m \log \left(c\left(d+ex^3\right)^p\right) dx = \int (fx)^m \log \left(\left(ex^3+d\right)^p c\right) dx$$

input
```
integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
f^m*x*x^m*log((e*x^3 + d)^p)/(m + 1) + integrate(((e*f^m*(m + 1)*log(c) -
3*e*f^m*p)*x^3 + d*f^m*(m + 1)*log(c))*x^m/(e*(m + 1)*x^3 + d*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log \left(c\left(d+ex^3\right)^p\right) dx = \int (fx)^m \log \left(\left(ex^3+d\right)^p c\right) dx$$

input
```
integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log((e*x^3 + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c\left(d+ex^3\right)^p\right) \, dx = \int \ln \left(c \left(e\,x^3+d\right)^p\right) \,(f\,x)^m \, dx$$

input
```
int(log(c*(d + e*x^3)^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e*x^3)^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log \left(c\left(d+ex^3\right)^p\right) \, dx$$
$$= \frac{f^m \left(x^m \log\left(\left(e\,x^3+d\right)^p c\right) mx + x^m \log\left(\left(e\,x^3+d\right)^p c\right) x - 3x^m px + 3\left(\int \frac{x^m}{em\,x^3+e\,x^3+dm+d}dx\right) d\,m^2 p + 6\left(\int \frac{}{em}\right.\right.}{m^2+2m+1}$$

input
```
int((f*x)^m*log(c*(e*x^3+d)^p),x)
```

output
```
(f**m*(x**m*log((d + e*x**3)**p*c)*m*x + x**m*log((d + e*x**3)**p*c)*x - 3
*x**m*p*x + 3*int(x**m/(d*m + d + e*m*x**3 + e*x**3),x)*d*m**2*p + 6*int(x
**m/(d*m + d + e*m*x**3 + e*x**3),x)*d*m*p + 3*int(x**m/(d*m + d + e*m*x**
3 + e*x**3),x)*d*p))/(m**2 + 2*m + 1)
```

3.56 $\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx = -\frac{2ep(fx)^{3+m}\operatorname{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)} + \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}$$

output
```
-2*e*p*(f*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],-e*x^2/d)/d/f^3/(1
+m)/(3+m)+(f*x)^(1+m)*ln(c*(e*x^2+d)^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{x(fx)^m\left(-2epx^2\operatorname{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)+d(3+m)\log\left(c\left(d+ex^2\right)^p\right)\right)}{d(1+m)(3+m)}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

output
```
(x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2
)/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2905, 8, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep\int \frac{x(fx)^{m+1}}{ex^2+d}dx}{f(m+1)}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep\int \frac{(fx)^{m+2}}{ex^2+d}dx}{f^2(m+1)}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3} \text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

input
```
Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

output
```
(-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1
 + m))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^m \ln \left(c\left(e\,x^2+d\right)^p\right) dx$$

input
```
int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

output
```
int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

Fricas [F]

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right)\, dx = \int (fx)^m \log \left(\left(ex^2+d\right)^p c\right)\, dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Sympy [A] (verification not implemented)

Time = 34.92 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.65

$$\int (fx)^m \log\left(c(d+ex^2)^p\right) dx = \text{Too large to display}$$

input
```
integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

output
```
-2*e*p*Piecewise((0**m*sqrt(-d/e**3)*log(-e*sqrt(-d/e**3) + x)/2 - 0**m*sq
rt(-d/e**3)*log(e*sqrt(-d/e**3) + x)/2 + 0**m*x/e, Eq(f, 0) | (Eq(f, 0) &
Ne(m, -1))), (f**(m + 1)*m*x**(m + 3)*lerchphi(e*x**2*exp_polar(I*pi)/d, 1
, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2
+ 5/2)) + 3*f**(m + 1)*x**(m + 3)*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/
2 + 3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/
2)), (m > -oo) & (m < oo) & Ne(m, -1)), (-Piecewise((-polylog(2, e*x**2*ex
p_polar(I*pi)/d)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polyl
og(2, e*x**2*exp_polar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylo
g(2, e*x**2*exp_polar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), (
(0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - p
olylog(2, e*x**2*exp_polar(I*pi)/d)/2, True))/(2*e*f) + log(f*x)*log(d + e
*x**2)/(2*e*f), True)) + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**
(m + 1)/(m + 1), Ne(m, -1)), (log(f*x), True))/f, True))*log(c*(d + e*x**2
)**p)
```

Maxima [F]

$$\int (fx)^m \log\left(c(d+ex^2)^p\right) dx = \int (fx)^m \log\left((ex^2+d)^p c\right) dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
f^m*x*x^m*log((e*x^2 + d)^p)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (
e*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m/(e*(m + 1)*x^2 + d*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)\,dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int \ln\left(c\left(e\,x^2+d\right)^p\right)(f\,x)^m\,dx$$

input
```
int(log(c*(d + e*x^2)^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e*x^2)^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{f^m\left(x^m \log\left(\left(e\,x^2+d\right)^p c\right) m x + x^m \log\left(\left(e\,x^2+d\right)^p c\right) x - 2x^m p x + 2\left(\int \frac{x^m}{e m\,x^2+e\,x^2+d m+d}dx\right) d\,m^2 p + 4\left(\int \frac{}{e} \right.}{m^2+2m+1}$$

input
```
int((f*x)^m*log(c*(e*x^2+d)^p),x)
```

output
```
(f**m*(x**m*log((d + e*x**2)**p*c)*m*x + x**m*log((d + e*x**2)**p*c)*x - 2
*x**m*p*x + 2*int(x**m/(d*m + d + e*m*x**2 + e*x**2),x)*d*m**2*p + 4*int(x
**m/(d*m + d + e*m*x**2 + e*x**2),x)*d*m*p + 2*int(x**m/(d*m + d + e*m*x**
2 + e*x**2),x)*d*p))/(m**2 + 2*m + 1)
```

3.57 $\int (fx)^m \log (c(d+ex)^p) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int (fx)^m \log (c(d+ex)^p) \, dx = -\frac{ep(fx)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,-\frac{ex}{d}\right)}{df^2(1+m)(2+m)} + \frac{(fx)^{1+m} \log (c(d+ex)^p)}{f(1+m)}$$

output
```
-e*p*(f*x)^(2+m)*hypergeom([1, 2+m],[3+m],-e*x/d)/d/f^2/(1+m)/(2+m)+(f*x)^
(1+m)*ln(c*(e*x+d)^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int (fx)^m \log (c(d+ex)^p) \, dx = \frac{x(fx)^m \left(-epx \text{Hypergeometric2F1}\left(1,2+m,3+m,-\frac{ex}{d}\right) + d(2+m) \log (c(d+ex)^p)\right)}{d(1+m)(2+m)}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e*x)^p],x]
```

output
```
(x*(f*x)^m*(-(e*p*x*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)]) + d*(2
 + m)*Log[c*(d + e*x)^p]))/(d*(1 + m)*(2 + m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2842, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log \left(c(d+ex)^p\right) dx$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{(fx)^{m+1}\log \left(c(d+ex)^p\right)}{f(m+1)}-\frac{ep\int \frac{(fx)^{m+1}}{d+ex}dx}{f(m+1)}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{(fx)^{m+1}\log \left(c(d+ex)^p\right)}{f(m+1)}-\frac{ep(fx)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,-\frac{ex}{d}\right)}{df^2(m+1)(m+2)}$$

input
```
Int[(f*x)^m*Log[c*(d + e*x)^p],x]
```

output
```
-((e*p*(f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)])/(d*f^
2*(1 + m)*(2 + m))) + ((f*x)^(1 + m)*Log[c*(d + e*x)^p])/(f*(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [F]

$$\int (fx)^m \ln \left(c(ex+d)^p\right) dx$$

input
```
int((f*x)^m*ln(c*(e*x+d)^p),x)
```

output
```
int((f*x)^m*ln(c*(e*x+d)^p),x)
```

Fricas [F]

$$\int (fx)^m \log \left(c(d+ex)^p\right) \, dx = \int (fx)^m \log \left((ex+d)^p c\right) \, dx$$

input
```
integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log((e*x + d)^p*c), x)
```

Sympy [F]

$$\int (fx)^m \log \left(c(d+ex)^p\right) \, dx = \int (fx)^m \log \left(c(d+ex)^p\right) dx$$

input
```
integrate((f*x)**m*ln(c*(e*x+d)**p),x)
```

output
```
Integral((f*x)**m*log(c*(d + e*x)**p), x)
```

Maxima [F]

$$\int (fx)^m \log \left(c(d+ex)^p\right) \, dx = \int (fx)^m \log \left((ex+d)^p c\right) \, dx$$

input
```
integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="maxima")
```

output
```
f^m*x*x^m*log((e*x + d)^p)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (e*
f^m*(m + 1)*log(c) - e*f^m*p)*x)*x^m/(e*(m + 1)*x + d*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log \left(c(d+ex)^p\right) \, dx = \int (fx)^m \log \left((ex+d)^p c\right) \, dx$$

input
```
integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log((e*x + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log (c(d+ex)^p) \, dx = \int \ln (c\,(d+e\,x)^p) \,(f\,x)^m \,dx$$

input
```
int(log(c*(d + e*x)^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e*x)^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log (c(d+ex)^p) \, dx = \frac{f^m \left(x^m \log((ex+d)^p\,c)\,e\,m^2x + x^m \log((ex+d)^p\,c)\,emx + x^m dmp + x^m dp - x^m empx - \left(\int \frac{x^m}{em\,x^2+dmx+e\,}\right.\right.}{em\,(m^2+2m+1)}$$

input
```
int((f*x)^m*log(c*(e*x+d)^p),x)
```

output
```
(f**m*(x**m*log((d + e*x)**p*c)*e*m**2*x + x**m*log((d + e*x)**p*c)*e*m*x
+ x**m*d*m*p + x**m*d*p - x**m*e*m*p*x - int(x**m/(d*m*x + d*x + e*m*x**2
+ e*x**2),x)*d**2*m**3*p - 2*int(x**m/(d*m*x + d*x + e*m*x**2 + e*x**2),x)
*d**2*m**2*p - int(x**m/(d*m*x + d*x + e*m*x**2 + e*x**2),x)*d**2*m*p))/(e
*m*(m**2 + 2*m + 1))
```

3.58 $\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right) dx = \frac{ep(fx)^m \text{Hypergeometric2F1}\left(1,-m,1-m,-\frac{e}{dx}\right)}{dm(1+m)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}$$

output
```
e*p*(f*x)^m*hypergeom([1, -m],[1-m],-e/d/x)/d/m/(1+m)+(f*x)^(1+m)*ln(c*(d+
e/x)^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right) dx = \frac{(fx)^m\left(ep\,\text{Hypergeometric2F1}\left(1,-m,1-m,-\frac{e}{dx}\right)+dmx\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)}{dm(1+m)}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e/x)^p],x]
```

output
```
((f*x)^m*(e*p*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))] + d*m*x*Log[c*(d
 + e/x)^p]))/(d*m*(1 + m))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2905, 8, 862, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right) dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{ep\int \frac{(fx)^{m+1}}{\left(d+\frac{e}{x}\right)x^2}dx}{f(m+1)}+\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{efp\int \frac{(fx)^{m-1}}{d+\frac{e}{x}}dx}{m+1}+\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{ 862}$$

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)}-\frac{ep\left(\frac{1}{x}\right)^m(fx)^m\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{d+\frac{e}{x}}d\frac{1}{x}}{m+1}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)}+\frac{ep(fx)^m\text{Hypergeometric2F1}\left(1,-m,1-m,-\frac{e}{dx}\right)}{dm(m+1)}$$

input
```
Int[(f*x)^m*Log[c*(d + e/x)^p],x]
```

output
```
(e*p*(f*x)^m*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))])/(d*m*(1 + m)) +
((f*x)^(1 + m)*Log[c*(d + e/x)^p])/(f*(1 + m))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x}\right)^p\right) dx$$

input
```
int((f*x)^m*ln(c*(d+e/x)^p),x)
```

output
```
int((f*x)^m*ln(c*(d+e/x)^p),x)
```

Fricas [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right) dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right) dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log(c*((d*x + e)/x)^p), x)
```

Sympy [A] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.45

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right) dx$$

$$= ep\left(\begin{cases} \frac{0^m \log(dx+e)}{d} \\ \frac{d^{m-1} f^{m+1} m x^m \Phi\left(\frac{e e^{i\pi}}{dx}, 1, m e^{i\pi}\right)\Gamma(-m)}{d^m f m \Gamma(1-m) + d^m f \Gamma(1-m)} \\ \frac{\begin{cases} -\frac{1}{dx} \\ \frac{\begin{cases} \operatorname{Li}_2\left(\frac{e e^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(d)\log(x) + \operatorname{Li}_2\left(\frac{e e^{i\pi}}{dx}\right) & \text{for } |x| < 1 \\ -\log(d)\log\left(\frac{1}{x}\right) + \operatorname{Li}_2\left(\frac{e e^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x\right)\log(d) + \operatorname{Li}_2\left(\frac{e e^{i\pi}}{dx}\right) & \text{otherwise} \end{cases}}{e} \end{cases}}{f} \end{cases}\right) + \left(\begin{cases} 0^m x & \text{for } f = 0 \\ \frac{\begin{cases} \frac{(fx)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log(fx) & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)$$

input
```
integrate((f*x)**m*ln(c*(d+e/x)**p),x)
```

output
```
e*p*Piecewise((0**m*log(d*x + e)/d, Eq(f, 0) | (Eq(f, 0) & Ne(m, -1))), (d
**(m - 1)*f**(m + 1)*m*x***m*lerchphi(e*exp_polar(I*pi)/(d*x), 1, m*exp_pol
ar(I*pi))*gamma(-m)/(d**m*f*m*gamma(1 - m) + d**m*f*gamma(1 - m)), (m > -o
o) & (m < oo) & Ne(m, -1)), (Piecewise((-1/(d*x), Eq(e, 0)), (Piecewise((p
olylog(2, e*exp_polar(I*pi)/(d*x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d
)*log(x) + polylog(2, e*exp_polar(I*pi)/(d*x)), Abs(x) < 1), (-log(d)*log(
1/x) + polylog(2, e*exp_polar(I*pi)/(d*x)), 1/Abs(x) < 1), (-meijerg(((),
(1, 1)), ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*
log(d) + polylog(2, e*exp_polar(I*pi)/(d*x)), True))/e, True))/f - Piecewi
se((1/(d*x), Eq(e, 0)), (log(d + e/x)/e, True))*log(f*x)/f, True)) + Piece
wise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m + 1), Ne(m, -1)), (
log(f*x), True))/f, True))*log(c*(d + e/x)**p)
```

Maxima [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx=\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="maxima")
```

output
```
(f^m*x*x^m*log((d*x + e)^p) - f^m*x*x^m*log(x^p))/(m + 1) + integrate((d*f
^m*(m + 1)*x*log(c) + e*f^m*(m + 1)*log(c) + e*f^m*p)*x^m/(d*(m + 1)*x + e
*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx=\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log(c*(d + e/x)^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx=\int \ln\left(c\left(d+\frac{e}{x}\right)^p\right)(f\,x)^m\,dx$$

input
```
int(log(c*(d + e/x)^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e/x)^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx$$
$$=\frac{f^m\left(x^m\log\left(\frac{(dx+e)^p c}{x^p}\right)dmx+x^m\log\left(\frac{(dx+e)^p c}{x^p}\right)em+x^m ep-\left(\int\frac{x^m\log\left(\frac{(dx+e)^p c}{x^p}\right)}{x}dx\right)e\,m^2\right)}{dm\,(m+1)}$$

input
```
int((f*x)^m*log(c*(d+e/x)^p),x)
```

output
```
(f**m*(x**m*log(((d*x + e)**p*c)/x**p)*d*m*x + x**m*log(((d*x + e)**p*c)/x
**p)*e*m + x**m*e*p - int((x**m*log(((d*x + e)**p*c)/x**p))/x,x)*e*m**2))/
(d*m*(m + 1))
```

3.59 $\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right) dx = -\frac{2efp(fx)^{-1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{2},\frac{3-m}{2},-\frac{e}{dx^2}\right)}{d(1-m^2)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}$$

output
```
-2*e*f*p*(f*x)^(-1+m)*hypergeom([1, 1/2-1/2*m],[3/2-1/2*m],-e/d/x^2)/d/(-m
^2+1)+(f*x)^(1+m)*ln(c*(d+e/x^2)^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right) dx = \frac{(fx)^m\left(2ep\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\frac{e}{dx^2}\right)+d(-1+m)x^2\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\right)}{d(-1+m)(1+m)x}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e/x^2)^p],x]
```

output
```
((f*x)^m*(2*e*p*Hypergeometric2F1[1, 1/2 - m/2, 3/2 - m/2, -(e/(d*x^2))] +
 d*(-1 + m)*x^2*Log[c*(d + e/x^2)^p]))/(d*(-1 + m)*(1 + m)*x)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2905, 8, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{2ep\int \frac{(fx)^{m+1}}{\left(d+\frac{e}{x^2}\right)x^3}dx}{f(m+1)}+\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{8}$$

$$\frac{2ef^2p\int \frac{(fx)^{m-2}}{d+\frac{e}{x^2}}dx}{m+1}+\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{862}$$

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)}-\frac{2efp\left(\frac{1}{x}\right)^{m-1}(fx)^{m-1}\int \frac{\left(\frac{1}{x}\right)^{-m}}{d+\frac{e}{x^2}}d\frac{1}{x}}{m+1}$$

$$\Big\downarrow \text{278}$$

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)}-\frac{2efp(fx)^{m-1}\text{Hypergeometric2F1}\left(1,\frac{1-m}{2},\frac{3-m}{2},-\frac{e}{dx^2}\right)}{d(1-m)(m+1)}$$

input
```
Int[(f*x)^m*Log[c*(d + e/x^2)^p],x]
```

output
```
(-2*e*f*p*(f*x)^(-1 + m)*Hypergeometric2F1[1, (1 - m)/2, (3 - m)/2, -(e/(d
*x^2))])/(d*(1 - m)*(1 + m)) + ((f*x)^(1 + m)*Log[c*(d + e/x^2)^p])/(f*(1
+ m))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)dx$$

input
```
int((f*x)^m*ln(c*(d+e/x^2)^p),x)
```

output
```
int((f*x)^m*ln(c*(d+e/x^2)^p),x)
```

Fricas [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right) dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right) dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log(c*((d*x^2 + e)/x^2)^p), x)
```

Sympy [A] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.46

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right) dx = \text{Too large to display}$$

input
```
integrate((f*x)**m*ln(c*(d+e/x**2)**p),x)
```

output
```
2*e*p*Piecewise((-0**m*sqrt(-1/(d*e))*log(-e*sqrt(-1/(d*e)) + x)/2 + 0**m*
sqrt(-1/(d*e))*log(e*sqrt(-1/(d*e)) + x)/2, Eq(f, 0) | (Eq(f, 0) & Ne(m, -
1))), (f**(m + 1)*m*x**(m - 1)*lerchphi(e*exp_polar(I*pi)/(d*x**2), 1, 1/2
 - m/2)*gamma(1/2 - m/2)/(4*d*f*m*gamma(3/2 - m/2) + 4*d*f*gamma(3/2 - m/2
)) - f**(m + 1)*x**(m - 1)*lerchphi(e*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m
/2)*gamma(1/2 - m/2)/(4*d*f*m*gamma(3/2 - m/2) + 4*d*f*gamma(3/2 - m/2)),
(m > -oo) & (m < oo) & Ne(m, -1)), (Piecewise((polylog(2, e*exp_polar(I*pi
)/(d*x**2))/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) + polylog(2,
 e*exp_polar(I*pi)/(d*x**2))/2, Abs(x) < 1), (-log(d)*log(1/x) + polylog(2
, e*exp_polar(I*pi)/(d*x**2))/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((
0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + po
lylog(2, e*exp_polar(I*pi)/(d*x**2))/2, True))/(2*e*f) - log(f*x)*log(d +
e/x**2)/(2*e*f), True)) + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)*
*(m + 1)/(m + 1), Ne(m, -1)), (log(f*x), True))/f, True))*log(c*(d + e/x**
2)**p)
```

Maxima [F]

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^2}\right)^p\right) dx = \int (fx)^m \log \left(c\left(d+\frac{e}{x^2}\right)^p\right) dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="maxima")
```

output
```
(f^m*x*x^m*log((d*x^2 + e)^p) - 2*f^m*x*x^m*log(x^p))/(m + 1) + integrate(
(d*f^m*(m + 1)*x^2*log(c) + e*f^m*(m + 1)*log(c) + 2*e*f^m*p)*x^m/(d*(m +
1)*x^2 + e*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^2}\right)^p\right) dx = \int (fx)^m \log \left(c\left(d+\frac{e}{x^2}\right)^p\right) dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log(c*(d + e/x^2)^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^2}\right)^p\right) dx = \int \ln \left(c\left(d+\frac{e}{x^2}\right)^p\right) (f\,x)^m\, dx$$

input
```
int(log(c*(d + e/x^2)^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e/x^2)^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^2}\right)^p\right) dx = f^m\left(\int x^m \log\left(\frac{(d\,x^2+e)^p\,c}{x^{2p}}\right) dx\right)$$

input
```
int((f*x)^m*log(c*(d+e/x^2)^p),x)
```

output
```
f**m*int(x**m*log(((d*x**2 + e)**p*c)/x**(2*p)),x)
```

3.60 $\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) dx = -\frac{3ef^2p(fx)^{-2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2-m}{3},\frac{5-m}{3},-\frac{e}{dx^3}\right)}{d(2+m-m^2)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}$$

output
```
-3*e*f^2*p*(f*x)^(-2+m)*hypergeom([1, 2/3-1/3*m],[5/3-1/3*m],-e/d/x^3)/d/(
-m^2+m+2)+(f*x)^(1+m)*ln(c*(d+e/x^3)^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) dx = \frac{(fx)^m\left(3ep\,\text{Hypergeometric2F1}\left(1,\frac{2}{3}-\frac{m}{3},\frac{5}{3}-\frac{m}{3},-\frac{e}{dx^3}\right)+d(-2+m)x^3\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\right)}{d(-2+m)(1+m)x^2}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e/x^3)^p],x]
```

output
```
((f*x)^m*(3*e*p*Hypergeometric2F1[1, 2/3 - m/3, 5/3 - m/3, -(e/(d*x^3))] +
 d*(-2 + m)*x^3*Log[c*(d + e/x^3)^p]))/(d*(-2 + m)*(1 + m)*x^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2905, 8, 862, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) \, dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{3ep\int \frac{(fx)^{m+1}}{\left(d+\frac{e}{x^3}\right)x^4}dx}{f(m+1)}+\frac{(fx)^{m+1}\log \left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{8}$$

$$\frac{3ef^3p\int \frac{(fx)^{m-3}}{d+\frac{e}{x^3}}dx}{m+1}+\frac{(fx)^{m+1}\log \left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{862}$$

$$\frac{(fx)^{m+1}\log \left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)}-\frac{3ef^2p\left(\frac{1}{x}\right)^{m-2}(fx)^{m-2}\int \frac{\left(\frac{1}{x}\right)^{1-m}}{d+\frac{e}{x^3}}d\frac{1}{x}}{m+1}$$

$$\Big\downarrow \text{888}$$

$$\frac{(fx)^{m+1}\log \left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)}-\frac{3ef^2p(fx)^{m-2}\text{Hypergeometric2F1}\left(1,\frac{2-m}{3},\frac{5-m}{3},-\frac{e}{dx^3}\right)}{d(2-m)(m+1)}$$

input
```
Int[(f*x)^m*Log[c*(d + e/x^3)^p],x]
```

output
```
(-3*e*f^2*p*(f*x)^(-2 + m)*Hypergeometric2F1[1, (2 - m)/3, (5 - m)/3, -(e/
(d*x^3))])/(d*(2 - m)*(1 + m)) + ((f*x)^(1 + m)*Log[c*(d + e/x^3)^p])/(f*(
1 + m))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x^3}\right)^p\right) dx$$

input
```
int((f*x)^m*ln(c*(d+e/x^3)^p),x)
```

output
```
int((f*x)^m*ln(c*(d+e/x^3)^p),x)
```

Fricas [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log(c*((d*x^3 + e)/x^3)^p), x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*ln(c*(d+e/x**3)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="maxima")
```

output
```
(f^m*x*x^m*log((d*x^3 + e)^p) - 3*f^m*x*x^m*log(x^p))/(m + 1) + integrate(
(d*f^m*(m + 1)*x^3*log(c) + e*f^m*(m + 1)*log(c) + 3*e*f^m*p)*x^m/(d*(m +
1)*x^3 + e*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) \, dx = \int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) \, dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log(c*(d + e/x^3)^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) \, dx = \int \ln \left(c\left(d+\frac{e}{x^3}\right)^p\right) (f\,x)^m \, dx$$

input
```
int(log(c*(d + e/x^3)^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e/x^3)^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) \, dx = f^m \left(\int x^m \log\left(\frac{(d\,x^3+e)^p\,c}{x^{3p}}\right) dx\right)$$

input
```
int((f*x)^m*log(c*(d+e/x^3)^p),x)
```

output
```
f**m*int(x**m*log(((d*x**3 + e)**p*c)/x**(3*p)),x)
```

3.61 $\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right)\,dx = -\frac{epx^{3/2}(fx)^m\,\text{Hypergeometric2F1}\left(1,3+2m,2(2+m),-\frac{e\sqrt{x}}{d}\right)}{d\left(3+5m+2m^2\right)} + \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)}$$

output
```
-e*p*x^(3/2)*(f*x)^m*hypergeom([1, 3+2*m],[4+2*m],-e*x^(1/2)/d)/d/(2*m^2+5
*m+3)+(f*x)^(1+m)*ln(c*(d+e*x^(1/2))^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right)\,dx = \frac{x(fx)^m\left(-ep\sqrt{x}\,\text{Hypergeometric2F1}\left(1,3+2m,4+2m,-\frac{e\sqrt{x}}{d}\right)+d(3+2m)\log\left(c\left(d+e\sqrt{x}\right)^p\right)\right)}{d(1+m)(3+2m)}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
```

output
```
(x*(f*x)^m*(-(e*p*Sqrt[x]*Hypergeometric2F1[1, 3 + 2*m, 4 + 2*m, -((e*Sqrt
[x])/d)]) + d*(3 + 2*m)*Log[c*(d + e*Sqrt[x])^p]))/(d*(1 + m)*(3 + 2*m))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2905, 30, 864, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)}-\frac{ep\int \frac{(fx)^{m+1}}{\left(d+e\sqrt{x}\right)\sqrt{x}}dx}{2f(m+1)}$$

$$\Big\downarrow \text{ 30}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)}-\frac{epx^{-m}(fx)^m\int \frac{x^{m+\frac{1}{2}}}{d+e\sqrt{x}}dx}{2(m+1)}$$

$$\Big\downarrow \text{ 864}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)}-\frac{epx^{-m}(fx)^m\int \frac{x^{m+1}}{d+e\sqrt{x}}d\sqrt{x}}{m+1}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)}-\frac{epx^{\frac{1}{2}(2m+3)-m}(fx)^m \text{Hypergeometric2F1}\left(1,2m+3,2(m+2),-\frac{e\sqrt{x}}{d}\right)}{d(m+1)(2m+3)}$$

input
```
Int[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
```

output
```
-((e*p*x^(-m + (3 + 2*m)/2)*(f*x)^m*Hypergeometric2F1[1, 3 + 2*m, 2*(2 + m
), -((e*Sqrt[x])/d)])/(d*(1 + m)*(3 + 2*m))) + ((f*x)^(1 + m)*Log[c*(d + e
*Sqrt[x])^p])/(f*(1 + m))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^m \ln \left(c\left(d+e\sqrt{x}\right)^p\right) dx$$

input
```
int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
```

output
```
int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
```

Fricas [F]

$$\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right) dx = \int (fx)^m \log\left(\left(e\sqrt{x}+d\right)^p c\right) dx$$

input
```
integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log((e*sqrt(x) + d)^p*c), x)
```

Sympy [F]

$$\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right) dx = \int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right) dx$$

input
```
integrate((f*x)**m*ln(c*(d+e*x**(1/2))**p),x)
```

output
```
Integral((f*x)**m*log(c*(d + e*sqrt(x))**p), x)
```

Maxima [F]

$$\int (fx)^m \log\left(c\left(d+e\sqrt{x}\right)^p\right) dx = \int (fx)^m \log\left(\left(e\sqrt{x}+d\right)^p c\right) dx$$

input
```
integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="maxima")
```

output
```
e^2*f^m*p*integrate(1/2*x*x^m/(d*e*(m + 1)*sqrt(x) + d^2*(m + 1)), x) + (d
*f^m*(2*m + 3)*x*x^m*log((e*sqrt(x) + d)^p) + d*f^m*(2*m + 3)*x*x^m*log(c)
 - e*f^m*p*x^(3/2)*x^m)/((2*m^2 + 5*m + 3)*d)
```

Giac [F]

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx=\int (fx)^m \log \left(\left(e\sqrt{x}+d\right)^p c\right) \, dx$$

input
```
integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log((e*sqrt(x) + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx=\int \ln \left(c\,{\left(d+e\,\sqrt{x}\right)}^p\right)\,{\left(f\,x\right)}^m \,d x$$

input
```
int(log(c*(d + e*x^(1/2))^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e*x^(1/2))^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx$$
$$=\frac{f^m\left(2x^{m+\frac{1}{2}}de\,m^2p+2x^{m+\frac{1}{2}}demp+4x^m\log\left(\left(\sqrt{x}\,e+d\right)^p c\right)e^2m^3x+6x^m\log\left(\left(\sqrt{x}\,e+d\right)^p c\right)e^2m^2x+2x\right.}{}$$

input
```
int((f*x)^m*log(c*(d+e*x^(1/2))^p),x)
```

output
```
(f**m*(2*x**((2*m + 1)/2)*d*e*m**2*p + 2*x**((2*m + 1)/2)*d*e*m*p + 4*x**m
*log((sqrt(x)*e + d)**p*c)*e**2*m**3*x + 6*x**m*log((sqrt(x)*e + d)**p*c)*
e**2*m**2*x + 2*x**m*log((sqrt(x)*e + d)**p*c)*e**2*m*x - 2*x**m*d**2*m**2
*p - 3*x**m*d**2*m*p - x**m*d**2*p - 2*x**m*e**2*m**2*p*x - x**m*e**2*m*p*
x - 2*int(x**((2*m + 1)/2)/(d**2*m*x + d**2*x - e**2*m*x**2 - e**2*x**2),x
)*d**3*e*m**4*p - 5*int(x**((2*m + 1)/2)/(d**2*m*x + d**2*x - e**2*m*x**2
- e**2*x**2),x)*d**3*e*m**3*p - 4*int(x**((2*m + 1)/2)/(d**2*m*x + d**2*x
- e**2*m*x**2 - e**2*x**2),x)*d**3*e*m**2*p - int(x**((2*m + 1)/2)/(d**2*m
*x + d**2*x - e**2*m*x**2 - e**2*x**2),x)*d**3*e*m*p + 2*int(x**m/(d**2*m*
x + d**2*x - e**2*m*x**2 - e**2*x**2),x)*d**4*m**4*p + 5*int(x**m/(d**2*m*
x + d**2*x - e**2*m*x**2 - e**2*x**2),x)*d**4*m**3*p + 4*int(x**m/(d**2*m*
x + d**2*x - e**2*m*x**2 - e**2*x**2),x)*d**4*m**2*p + int(x**m/(d**2*m*x
+ d**2*x - e**2*m*x**2 - e**2*x**2),x)*d**4*m*p))/(2*e**2*m*(2*m**3 + 5*m*
*2 + 4*m + 1))
```

3.62 $\int (fx)^m \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 80

$$\int (fx)^m \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \frac{ep\sqrt{x}(fx)^m \text{Hypergeometric2F1}\left(1,-1-2m,-2m,-\frac{e}{d\sqrt{x}}\right)}{d(1+3m+2m^2)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)}$$

output
```
e*p*x^(1/2)*(f*x)^m*hypergeom([1, -1-2*m],[-2*m],-e/d/x^(1/2))/d/(2*m^2+3*
m+1)+(f*x)^(1+m)*ln(c*(d+e/x^(1/2))^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int (fx)^m \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \frac{\sqrt{x}(fx)^m \left(ep\,\text{Hypergeometric2F1}\left(1,-1-2m,-2m,-\frac{e}{d\sqrt{x}}\right)+d(1+2m)\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\right)}{d(1+m)(1+2m)}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
```

output
```
(Sqrt[x]*(f*x)^m*(e*p*Hypergeometric2F1[1, -1 - 2*m, -2*m, -(e/(d*Sqrt[x])
)] + d*(1 + 2*m)*Sqrt[x]*Log[c*(d + e/Sqrt[x])^p]))/(d*(1 + m)*(1 + 2*m))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2905, 30, 795, 864, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{ep\int \frac{(fx)^{m+1}}{\left(d+\frac{e}{\sqrt{x}}\right)x^{3/2}}dx}{2f(m+1)}+\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{ 30}$$

$$\frac{epx^{-m}(fx)^m\int \frac{x^{m-\frac{1}{2}}}{d+\frac{e}{\sqrt{x}}}dx}{2(m+1)}+\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{ 795}$$

$$\frac{epx^{-m}(fx)^m\int \frac{x^m}{\sqrt{x}d+e}dx}{2(m+1)}+\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{ 864}$$

$$\frac{epx^{-m}(fx)^m\int \frac{x^{\frac{1}{2}(2m+1)}}{\sqrt{x}d+e}d\sqrt{x}}{m+1}+\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)}+\frac{px(fx)^m \text{Hypergeometric2F1}\left(1,2(m+1),2m+3,-\frac{d\sqrt{x}}{e}\right)}{2(m+1)^2}$$

input
```
Int[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
```

output
```
(p*x*(f*x)^m*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((d*Sqrt[x])/e)])/(
2*(1 + m)^2) + ((f*x)^(1 + m)*Log[c*(d + e/Sqrt[x])^p])/(f*(1 + m))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)dx$$

input
```
int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
```

output
```
int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
```

Fricas [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)dx = \int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log(c*((d*x + e*sqrt(x))/x)^p), x)
```

Sympy [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)dx = \int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)dx$$

input
```
integrate((f*x)**m*ln(c*(d+e/x**(1/2))**p),x)
```

output
```
Integral((f*x)**m*log(c*(d + e/sqrt(x))**p), x)
```

Maxima [F]

$$\int (fx)^m \log \left(c \left(d + \frac{e}{\sqrt{x}} \right)^p \right) dx = \int (fx)^m \log \left(c \left(d + \frac{e}{\sqrt{x}} \right)^p \right) dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="maxima")
```

output
```
d^2*f^m*p*integrate(1/2*x*x^m/(d*e*(m + 1)*sqrt(x) + e^2*(m + 1)), x) + 1/
2*(2*(2*m^2 + 5*m + 3)*e*f^m*x*x^m*log((d*sqrt(x) + e)^p) - 2*(2*m^2 + 5*m
 + 3)*e*f^m*x*x^m*log(x^(1/2*p)) - 2*(m*p + p)*d*f^m*x^(3/2)*x^m + (2*(2*m
^2 + 5*m + 3)*e*f^m*log(c) + (2*m*p + 3*p)*e*f^m)*x*x^m)/((2*m^3 + 7*m^2 +
 8*m + 3)*e)
```

Giac [F]

$$\int (fx)^m \log \left(c \left(d + \frac{e}{\sqrt{x}} \right)^p \right) dx = \int (fx)^m \log \left(c \left(d + \frac{e}{\sqrt{x}} \right)^p \right) dx$$

input
```
integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log(c*(d + e/sqrt(x))^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c \left(d + \frac{e}{\sqrt{x}} \right)^p \right) dx = \int \ln \left(c \left(d + \frac{e}{\sqrt{x}} \right)^p \right) (f\,x)^m\,dx$$

input
```
int(log(c*(d + e/x^(1/2))^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e/x^(1/2))^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\,dx$$

$$= \frac{f^m\left(2x^{m+\frac{1}{2}}demp + 4x^m\log\left(\frac{(\sqrt{x}\,d+e)^p c}{x^{\frac{p}{2}}}\right)d^2m^2x + 2x^m\log\left(\frac{(\sqrt{x}\,d+e)^p c}{x^{\frac{p}{2}}}\right)d^2mx - 4x^m\log\left(\frac{(\sqrt{x}\,d+e)^p c}{x^{\frac{p}{2}}}\right)e^2m^2 \cdots\right.}{2d^2 \cdots}$$

input
```
int((f*x)^m*log(c*(d+e/x^(1/2))^p),x)
```

output
```
(f**m*(2*x**((2*m + 1)/2)*d*e*m*p + 4*x**m*log(((sqrt(x)*d + e)**p*c)/x**(
p/2))*d**2*m**2*x + 2*x**m*log(((sqrt(x)*d + e)**p*c)/x**(p/2))*d**2*m*x -
 4*x**m*log(((sqrt(x)*d + e)**p*c)/x**(p/2))*e**2*m**2 - 2*x**m*log(((sqrt
(x)*d + e)**p*c)/x**(p/2))*e**2*m - 2*x**m*e**2*m*p - x**m*e**2*p + 4*int(
(x**m*log(((sqrt(x)*d + e)**p*c)/x**(p/2)))/x,x)*e**2*m**3 + 2*int((x**m*l
og(((sqrt(x)*d + e)**p*c)/x**(p/2)))/x,x)*e**2*m**2))/(2*d**2*m*(2*m**2 +
3*m + 1))
```

3.63 $\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx = -\frac{enpx^{1+n}(fx)^m \text{Hypergeometric2F1}\left(1,\frac{1+m+n}{n},\frac{1+m+2n}{n},-\frac{ex^n}{d}\right)}{d(1+m)(1+m+n)} + \frac{(fx)^{1+m}\log\left(c(d+ex^n)^p\right)}{f(1+m)}$$

output
```
-e*n*p*x^(1+n)*(f*x)^m*hypergeom([1, (1+m+n)/n],[(1+m+2*n)/n],-e*x^n/d)/d/
(1+m)/(1+m+n)+(f*x)^(1+m)*ln(c*(d+e*x^n)^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx = \frac{x(fx)^m\left(-enpx^n \text{Hypergeometric2F1}\left(1,\frac{1+m+n}{n},\frac{1+m+2n}{n},-\frac{ex^n}{d}\right)+d(1+m+n)\log\left(c(d+ex^n)^p\right)\right)}{d(1+m)(1+m+n)}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e*x^n)^p],x]
```

output
```
(x*(f*x)^m*(-(e*n*p*x^n*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n)/
n, -((e*x^n)/d)]) + d*(1 + m + n)*Log[c*(d + e*x^n)^p]))/(d*(1 + m)*(1 + m
 + n))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log \left(c(d+ex^n)^p\right) \, dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{(fx)^{m+1}\log \left(c(d+ex^n)^p\right)}{f(m+1)}-\frac{enp\int \frac{x^{n-1}(fx)^{m+1}}{ex^n+d}dx}{f(m+1)}$$

$$\Big\downarrow \text{30}$$

$$\frac{(fx)^{m+1}\log \left(c(d+ex^n)^p\right)}{f(m+1)}-\frac{enpx^{-m}(fx)^m\int \frac{x^{m+n}}{ex^n+d}dx}{m+1}$$

$$\Big\downarrow \text{888}$$

$$\frac{(fx)^{m+1}\log \left(c(d+ex^n)^p\right)}{f(m+1)}-\frac{enpx^{n+1}(fx)^m \text{Hypergeometric2F1}\left(1,\frac{m+n+1}{n},\frac{m+2n+1}{n},-\frac{ex^n}{d}\right)}{d(m+1)(m+n+1)}$$

input
```
Int[(f*x)^m*Log[c*(d + e*x^n)^p],x]
```

output
```
-((e*n*p*x^(1 + n)*(f*x)^m*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*
n)/n, -((e*x^n)/d)])/(d*(1 + m)*(1 + m + n))) + ((f*x)^(1 + m)*Log[c*(d +
e*x^n)^p])/(f*(1 + m))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^m \ln \left(c(d + e\, x^n)^p\right) dx$$

input
```
int((f*x)^m*ln(c*(d+e*x^n)^p),x)
```

output
```
int((f*x)^m*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int (fx)^m \log \left(c(d+ex^n)^p\right) \, dx = \int (fx)^m \log \left((ex^n+d)^p c\right) \, dx$$

input
```
integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log((e*x^n + d)^p*c), x)
```

Sympy [F]

$$\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx = \int (fx)^m \log\left(c(d+ex^n)^p\right)dx$$

input
```
integrate((f*x)**m*ln(c*(d+e*x**n)**p),x)
```

output
```
Integral((f*x)**m*log(c*(d + e*x**n)**p), x)
```

Maxima [F]

$$\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx = \int (fx)^m \log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
d*f^m*n*p*integrate(x^m/(e*(m + 1)*x^n + d*(m + 1)), x) + (f^m*(m + 1)*x*x
^m*log((e*x^n + d)^p) - (f^m*n*p - f^m*(m + 1)*log(c))*x*x^m)/(m^2 + 2*m +
 1)
```

Giac [F]

$$\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx = \int (fx)^m \log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c(d+ex^n)^p\right) \, dx = \int \ln \left(c\left(d+e\,x^n\right)^p\right)\,\left(f\,x\right)^m \,dx$$

input
```
int(log(c*(d + e*x^n)^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e*x^n)^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log \left(c(d+ex^n)^p\right) \, dx$$
$$= \frac{f^m \left(x^m \log((x^n e+d)^p\, c)\, mx + x^m \log((x^n e+d)^p\, c)\, x - x^m npx + \left(\int \frac{x^m}{x^n em+x^n e+dm+d}dx\right) d\, m^2 np + 2\left(\int \frac{}{x^n}\right.\right.}{m^2+2m+1}$$

input
```
int((f*x)^m*log(c*(d+e*x^n)^p),x)
```

output
```
(f**m*(x**m*log((x**n*e + d)**p*c)*m*x + x**m*log((x**n*e + d)**p*c)*x - x
**m*n*p*x + int(x**m/(x**n*e*m + x**n*e + d*m + d),x)*d*m**2*n*p + 2*int(x
**m/(x**n*e*m + x**n*e + d*m + d),x)*d*m*n*p + int(x**m/(x**n*e*m + x**n*e
 + d*m + d),x)*d*n*p))/(m**2 + 2*m + 1)
```

3.64 $\int (fx)^{-1+3n}\log\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int (fx)^{-1+3n}\log\left(c(d+ex^n)^p\right)\,dx=-\frac{p(fx)^{3n}}{9fn}-\frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn}+\frac{dpx^{-n}(fx)^{3n}}{6efn}+\frac{d^3px^{-3n}(fx)^{3n}\log(d+ex^n)}{3e^3fn}+\frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn}$$

output
```
-1/9*p*(f*x)^(3*n)/f/n-1/3*d^2*p*(f*x)^(3*n)/e^2/f/n/(x^(2*n))+1/6*d*p*(f*
x)^(3*n)/e/f/n/(x^n)+1/3*d^3*p*(f*x)^(3*n)*ln(d+e*x^n)/e^3/f/n/(x^(3*n))+1
/3*(f*x)^(3*n)*ln(c*(d+e*x^n)^p)/f/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

$$\int (fx)^{-1+3n}\log\left(c(d+ex^n)^p\right)\,dx=\frac{x^{-3n}(fx)^{3n}\left(-epx^n(6d^2-3dex^n+2e^2x^{2n})+6d^3p\log(d+ex^n)+6e^3x^{3n}\log\left(c(d+ex^n)^p\right)\right)}{18e^3fn}$$

input
```
Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
```

output
```
((f*x)^(3*n)*(-(e*p*x^n*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))) + 6*d^3*p*Log
[d + e*x^n] + 6*e^3*x^(3*n)*Log[c*(d + e*x^n)^p]))/(18*e^3*f*n*x^(3*n))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2905, 30, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{3n-1}\log\left(c(d+ex^n)^p\right)dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn}-\frac{ep\int \frac{x^{n-1}(fx)^{3n}}{ex^n+d}dx}{3f}$$

$$\Big\downarrow \text{30}$$

$$\frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn}-\frac{epx^{-3n}(fx)^{3n}\int \frac{x^{4n-1}}{ex^n+d}dx}{3f}$$

$$\Big\downarrow \text{798}$$

$$\frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn}-\frac{epx^{-3n}(fx)^{3n}\int \frac{x^{3n}}{ex^n+d}dx^n}{3fn}$$

$$\Big\downarrow \text{49}$$

$$\frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn}-\frac{epx^{-3n}(fx)^{3n}\int \left(-\frac{dx^n}{e^2}+\frac{x^{2n}}{e}-\frac{d^3}{e^3(ex^n+d)}+\frac{d^2}{e^3}\right)dx^n}{3fn}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn}-\frac{epx^{-3n}(fx)^{3n}\left(-\frac{d^3\log(d+ex^n)}{e^4}+\frac{d^2x^n}{e^3}-\frac{dx^{2n}}{2e^2}+\frac{x^{3n}}{3e}\right)}{3fn}$$

input
```
Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
```

output
```
-1/3*(e*p*(f*x)^(3*n)*((d^2*x^n)/e^3 - (d*x^(2*n))/(2*e^2) + x^(3*n)/(3*e)
 - (d^3*Log[d + e*x^n])/e^4))/(f*n*x^(3*n)) + ((f*x)^(3*n)*Log[c*(d + e*x^
n)^p])/(3*f*n)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^{-1+3n} \ln \left(c(d+e\,x^n)^p\right) dx$$

input
```
int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p),x)
```

output
```
int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int (fx)^{-1+3n} \log \left(c(d+ex^n)^p\right) \, dx = \frac{3\,de^2f^{3\,n-1}px^{2\,n} - 6\,d^2ef^{3\,n-1}px^n - 2\,(e^3p - 3\,e^3\log{(c)})f^{3\,n-1}x^{3\,n} + 6\,(e^3f^{3\,n-1}px^{3\,n} + d^3f^{3\,n-1}p)\log{(ex}}{18\,e^3n}$$

input
```
integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
1/18*(3*d*e^2*f^(3*n - 1)*p*x^(2*n) - 6*d^2*e*f^(3*n - 1)*p*x^n - 2*(e^3*p
 - 3*e^3*log(c))*f^(3*n - 1)*x^(3*n) + 6*(e^3*f^(3*n - 1)*p*x^(3*n) + d^3*
f^(3*n - 1)*p)*log(e*x^n + d))/(e^3*n)
```

Sympy [F]

$$\int (fx)^{-1+3n} \log \left(c(d+ex^n)^p\right) \, dx = \int (fx)^{3n-1} \log \left(c(d+ex^n)^p\right) dx$$

input
```
integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p),x)
```

output
```
Integral((f*x)**(3*n - 1)*log(c*(d + e*x**n)**p), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int (fx)^{-1+3n} \log (c(d+ex^n)^p) \, dx$$
$$= \frac{ep\left(\frac{6\, d^3 f^{3\,n} \log\left(\frac{ex^n+d}{e}\right)}{e^4 n} - \frac{2\, e^2 f^{3\,n} x^{3\,n} - 3\, def^{3\,n} x^{2\,n} + 6\, d^2 f^{3\,n} x^n}{e^3 n}\right)}{18\, f} + \frac{(fx)^{3\,n} \log\left((ex^n+d)^p c\right)}{3\, fn}$$

input
```
integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
1/18*e*p*(6*d^3*f^(3*n)*log((e*x^n + d)/e)/(e^4*n) - (2*e^2*f^(3*n)*x^(3*n
) - 3*d*e*f^(3*n)*x^(2*n) + 6*d^2*f^(3*n)*x^n)/(e^3*n))/f + 1/3*(f*x)^(3*n
)*log((e*x^n + d)^p*c)/(f*n)
```

Giac [F]

$$\int (fx)^{-1+3n} \log (c(d+ex^n)^p) \, dx = \int (fx)^{3\,n-1} \log\left((ex^n+d)^p c\right) \, dx$$

input
```
integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^(3*n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+3n}\log(c(d+ex^n)^p)\,dx = \int \ln\left(c\left(d+e\,x^n\right)^p\right)\left(f\,x\right)^{3\,n-1}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)*(f*x)^(3*n - 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)*(f*x)^(3*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int (fx)^{-1+3n}\log(c(d+ex^n)^p)\,dx = \frac{f^{3n}(6x^{3n}\log((x^ne+d)^p\,c)\,e^3-2x^{3n}e^3p+3x^{2n}d\,e^2p-6x^nd^2ep+6\log((x^ne+d)^p\,c)\,d^3)}{18e^3fn}$$

input
```
int((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x)
```

output
```
(f**(3*n)*(6*x**(3*n)*log((x**n*e + d)**p*c)*e**3 - 2*x**(3*n)*e**3*p + 3*
x**(2*n)*d*e**2*p - 6*x**n*d**2*e*p + 6*log((x**n*e + d)**p*c)*d**3))/(18*
e**3*f*n)
```

3.65 $\int (fx)^{-1+2n}\log\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int (fx)^{-1+2n}\log\left(c(d+ex^n)^p\right)\,dx = -\frac{p(fx)^{2n}}{4fn}+\frac{dpx^{-n}(fx)^{2n}}{2efn} - \frac{d^2px^{-2n}(fx)^{2n}\log(d+ex^n)}{2e^2fn} + \frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}$$

output
```
-1/4*p*(f*x)^(2*n)/f/n+1/2*d*p*(f*x)^(2*n)/e/f/n/(x^n)-1/2*d^2*p*(f*x)^(2*
n)*ln(d+e*x^n)/e^2/f/n/(x^(2*n))+1/2*(f*x)^(2*n)*ln(c*(d+e*x^n)^p)/f/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.66

$$\int (fx)^{-1+2n}\log\left(c(d+ex^n)^p\right)\,dx = -\frac{x^{-2n}(fx)^{2n}\left(2d^2p\log(d+ex^n)+ex^n(-2dp+epx^n-2ex^n\log\left(c(d+ex^n)^p\right))\right)}{4e^2fn}$$

input
```
Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
```

output
```
-1/4*((f*x)^(2*n)*(2*d^2*p*Log[d + e*x^n] + e*x^n*(-2*d*p + e*p*x^n - 2*e*
x^n*Log[c*(d + e*x^n)^p])))/(e^2*f*n*x^(2*n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2905, 30, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{2n-1}\log\left(c(d+ex^n)^p\right)dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{ep\int \frac{x^{n-1}(fx)^{2n}}{ex^n+d}dx}{2f}$$

$$\Big\downarrow \text{30}$$

$$\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{epx^{-2n}(fx)^{2n}\int \frac{x^{3n-1}}{ex^n+d}dx}{2f}$$

$$\Big\downarrow \text{798}$$

$$\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{epx^{-2n}(fx)^{2n}\int \frac{x^{2n}}{ex^n+d}dx^n}{2fn}$$

$$\Big\downarrow \text{49}$$

$$\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{epx^{-2n}(fx)^{2n}\int \left(\frac{x^n}{e}+\frac{d^2}{e^2(ex^n+d)}-\frac{d}{e^2}\right)dx^n}{2fn}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}-\frac{epx^{-2n}(fx)^{2n}\left(\frac{d^2\log(d+ex^n)}{e^3}-\frac{dx^n}{e^2}+\frac{x^{2n}}{2e}\right)}{2fn}$$

input
```
Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
```

output
```
-1/2*(e*p*(f*x)^(2*n)*(-((d*x^n)/e^2) + x^(2*n)/(2*e) + (d^2*Log[d + e*x^n
])/e^3))/(f*n*x^(2*n)) + ((f*x)^(2*n)*Log[c*(d + e*x^n)^p])/(2*f*n)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^{2n-1} \ln \left(c(d+e\,x^n)^p\right) dx$$

input
```
int((f*x)^(2*n-1)*ln(c*(d+e*x^n)^p),x)
```

output
```
int((f*x)^(2*n-1)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int (fx)^{-1+2n}\log \left(c(d+ex^n)^p\right)\,dx = \frac{2\,def^{2\,n-1}px^n - (e^2p - 2\,e^2\log(c))f^{2\,n-1}x^{2\,n} + 2\,(e^2f^{2\,n-1}px^{2\,n} - d^2f^{2\,n-1}p)\log(ex^n + d)}{4\,e^2n}$$

input
```
integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
1/4*(2*d*e*f^(2*n - 1)*p*x^n - (e^2*p - 2*e^2*log(c))*f^(2*n - 1)*x^(2*n)
+ 2*(e^2*f^(2*n - 1)*p*x^(2*n) - d^2*f^(2*n - 1)*p)*log(e*x^n + d))/(e^2*n
)
```

Sympy [F]

$$\int (fx)^{-1+2n}\log \left(c(d+ex^n)^p\right)\,dx = \int (fx)^{2n-1}\log \left(c(d+ex^n)^p\right)dx$$

input
```
integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p),x)
```

output
```
Integral((f*x)**(2*n - 1)*log(c*(d + e*x**n)**p), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.85

$$\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) dx = -\frac{ep\left(\frac{2\,d^2 f^{2\,n} \log\left(\frac{ex^n+d}{e}\right)}{e^3 n} + \frac{ef^{2\,n}x^{2\,n} - 2\,df^{2\,n}x^n}{e^2 n}\right)}{4\,f} + \frac{(fx)^{2\,n} \log\left((ex^n+d)^p c\right)}{2\,fn}$$

input
```
integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
-1/4*e*p*(2*d^2*f^(2*n)*log((e*x^n + d)/e)/(e^3*n) + (e*f^(2*n)*x^(2*n) -
2*d*f^(2*n)*x^n)/(e^2*n))/f + 1/2*(f*x)^(2*n)*log((e*x^n + d)^p*c)/(f*n)
```

Giac [F]

$$\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) dx = \int (fx)^{2\,n-1} \log\left((ex^n+d)^p c\right) dx$$

input
```
integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^(2*n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) dx = \int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f\,x\right)}^{2\,n-1} \,dx$$

input
```
int(log(c*(d + e*x^n)^p)*(f*x)^(2*n - 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)*(f*x)^(2*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.67

$$\int (fx)^{-1+2n}\log (c(d+ex^n)^p)\ dx$$
$$=\frac{f^{2n}(2x^{2n}\log((x^ne+d)^p\,c)\,e^2-x^{2n}e^2p+2x^ndep-2\log((x^ne+d)^p\,c)\,d^2)}{4e^2fn}$$

input
```
int((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x)
```

output
```
(f**(2*n)*(2*x**(2*n)*log((x**n*e + d)**p*c)*e**2 - x**(2*n)*e**2*p + 2*x*
*n*d*e*p - 2*log((x**n*e + d)**p*c)*d**2))/(4*e**2*f*n)
```

3.66 $\int (fx)^{-1+n} \log (c(d+ex^n)^p) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int (fx)^{-1+n} \log (c(d+ex^n)^p) \, dx = -\frac{p(fx)^n}{fn} + \frac{dpx^{-n}(fx)^n \log (d+ex^n)}{efn} + \frac{(fx)^n \log (c(d+ex^n)^p)}{fn}$$

output
```
-p*(f*x)^n/f/n+d*p*(f*x)^n*ln(d+e*x^n)/e/f/n/(x^n)+(f*x)^n*ln(c*(d+e*x^n)^
p)/f/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int (fx)^{-1+n} \log (c(d+ex^n)^p) \, dx = \frac{x^{1-n}(fx)^{-1+n}\left(-px^n + \frac{(d+ex^n)\log(c(d+ex^n)^p)}{e}\right)}{n}$$

input
```
Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
```

output
```
(x^(1 - n)*(f*x)^(-1 + n)*(-(p*x^n) + ((d + e*x^n)*Log[c*(d + e*x^n)^p])/e
))/n
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2905, 30, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{n-1}\log \left(c\left(d+ex^n\right)^p\right) \, dx$$

$$\Big\downarrow \ 2905$$

$$\frac{(fx)^n \log \left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{ep\int \frac{x^{n-1}(fx)^n}{ex^n+d}dx}{f}$$

$$\Big\downarrow \ 30$$

$$\frac{(fx)^n \log \left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{epx^{-n}(fx)^n\int \frac{x^{2n-1}}{ex^n+d}dx}{f}$$

$$\Big\downarrow \ 798$$

$$\frac{(fx)^n \log \left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{epx^{-n}(fx)^n\int \frac{x^n}{ex^n+d}dx^n}{fn}$$

$$\Big\downarrow \ 49$$

$$\frac{(fx)^n \log \left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{epx^{-n}(fx)^n\int \left(\frac{1}{e}-\frac{d}{e(ex^n+d)}\right)dx^n}{fn}$$

$$\Big\downarrow \ 2009$$

$$\frac{(fx)^n \log \left(c\left(d+ex^n\right)^p\right)}{fn}-\frac{epx^{-n}(fx)^n\left(\frac{x^n}{e}-\frac{d\log(d+ex^n)}{e^2}\right)}{fn}$$

input
```
Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
```

output
```
-((e*p*(f*x)^n*(x^n/e - (d*Log[d + e*x^n])/e^2))/(f*n*x^n)) + ((f*x)^n*Log
[c*(d + e*x^n)^p])/(f*n)
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^{-1+n} \ln \left(c(d+e\,x^n)^p\right) dx$$

input
```
int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p),x)
```

output
```
int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx = -\frac{(ep-e\log(c))f^{n-1}x^n-(ef^{n-1}px^n+df^{n-1}p)\log(ex^n+d)}{en}$$

input
```
integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
-((e*p - e*log(c))*f^(n - 1)*x^n - (e*f^(n - 1)*p*x^n + d*f^(n - 1)*p)*log
(e*x^n + d))/(e*n)
```

Sympy [F]

$$\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{n-1}\log\left(c(d+ex^n)^p\right)dx$$

input
```
integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p),x)
```

output
```
Integral((f*x)**(n - 1)*log(c*(d + e*x**n)**p), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx = -\frac{ep\left(\frac{f^nx^n}{en}-\frac{df^n\log\left(\frac{ex^n+d}{e}\right)}{e^2n}\right)}{f}+\frac{(fx)^n\log\left((ex^n+d)^pc\right)}{fn}$$

input
```
integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
-e*p*(f^n*x^n/(e*n) - d*f^n*log((e*x^n + d)/e)/(e^2*n))/f + (f*x)^n*log((e
*x^n + d)^p*c)/(f*n)
```

Giac [F]

$$\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{n-1}\log\left((ex^n+d)^pc\right)\,dx$$

input
```
integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^(n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx=\int \ln\left(c\,(d+e\,x^n)^p\right)\,(f\,x)^{n-1}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)*(f*x)^(n - 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)*(f*x)^(n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx=\frac{f^n\left(x^n\log\left((x^ne+d)^p\,c\right)e-x^nep+\log\left((x^ne+d)^p\,c\right)d\right)}{efn}$$

input
```
int((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x)
```

output
```
(f**n*(x**n*log((x**n*e + d)**p*c)*e - x**n*e*p + log((x**n*e + d)**p*c)*d
))/(e*f*n)
```

3.67 $\int \frac{\log(c(d+ex^n)^p)}{fx} dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{fx} dx = \frac{\log \left(-\frac{ex^n}{d}\right) \log \left(c(d+ex^n)^p\right)}{fn} + \frac{p \operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn}$$

output
```
ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n+p*polylog(2,1+e*x^n/d)/f/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{fx} dx = \frac{\log \left(-\frac{ex^n}{d}\right) \log \left(c(d+ex^n)^p\right) + p \operatorname{PolyLog}\left(2,\frac{d+ex^n}{d}\right)}{fn}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(f*x),x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/(f*
n)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {27, 2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{fx} \, dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\log(c(ex^n+d)^p)}{x}dx}{f}$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{\int x^{-n}\log (c(ex^n+d)^p)\,dx^n}{fn}$$

$$\Big\downarrow \text{ 2841}$$

$$\frac{\log \left(-\frac{ex^n}{d}\right)\log (c(d+ex^n)^p)-ep\int \frac{\log\left(-\frac{ex^n}{d}\right)}{ex^n+d}dx^n}{fn}$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{\log \left(-\frac{ex^n}{d}\right)\log (c(d+ex^n)^p)+p\,\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}$$

input
```
Int[Log[c*(d + e*x^n)^p]/(f*x),x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d])/(f*
n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.64

method	result
risch	$\frac{\ln(x)\ln((d+e\,x^n)^p)}{f}+\frac{\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csgn}(ic)}{2}-\frac{i\pi\mathrm{csgn}(ic(d+e\,x^n)^p)}{2}\right.}{f}$

input
```
int(ln(c*(d+e*x^n)^p)/f/x,x,method=_RETURNVERBOSE)
```

output
```
1/f*ln(x)*ln((d+e*x^n)^p)+1/f*(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*
x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2
*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+l
n(c))*ln(x)-1/f*p/n*dilog((d+e*x^n)/d)-1/f*p*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{\log(c(d+ex^n)^p)}{fx}\,dx = \frac{np\log(ex^n+d)\log(x) - np\log(x)\log\left(\frac{ex^n+d}{d}\right) + n\log(c)\log(x) - p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{fn}$$

input
```
integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="fricas")
```

output
```
(n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(
x) - p*dilog(-(e*x^n + d)/d + 1))/(f*n)
```

Sympy [F]

$$\int \frac{\log(c(d+ex^n)^p)}{fx}\,dx = \frac{\int \frac{\log(c(d+ex^n)^p)}{x}\,dx}{f}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/f/x,x)
```

output
```
Integral(log(c*(d + e*x**n)**p)/x, x)/f
```

Maxima [F]

$$\int \frac{\log(c(d+ex^n)^p)}{fx}\,dx = \int \frac{\log((ex^n+d)^p c)}{fx}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="maxima")
```

output
```
1/2*(2*d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - n*p*log(x)^2 + 2*log((
e*x^n + d)^p)*log(x) + 2*log(c)*log(x))/f
```

Giac [F]

$$\int \frac{\log (c(d+ex^n)^p)}{fx} \, dx = \int \frac{\log ((ex^n+d)^p c)}{fx} \, dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/(f*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (c(d+ex^n)^p)}{fx} \, dx = \int \frac{\ln (c (d+e x^n)^p)}{f x} \, dx$$

input
```
int(log(c*(d + e*x^n)^p)/(f*x),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(f*x), x)
```

Reduce [F]

$$\int \frac{\log (c(d+ex^n)^p)}{fx} \, dx = \frac{2\left(\int \frac{\log((x^n e+d)^p c)}{x^n ex+dx} dx\right) dnp + \log((x^n e+d)^p c)^2}{2fnp}$$

input
```
int(log(c*(d+e*x^n)^p)/f/x,x)
```

output
```
(2*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d*n*p + log((x**n*e + d)
**p*c)**2)/(2*f*n*p)
```

3.68 $\int (fx)^{-1-n} \log \left(c(d+ex^n)^p\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int (fx)^{-1-n} \log \left(c(d+ex^n)^p\right) \, dx = \frac{epx^n(fx)^{-n}\log(x)}{df} - \frac{epx^n(fx)^{-n}\log(d+ex^n)}{dfn} - \frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}$$

output
```
e*p*x^n*ln(x)/d/f/((f*x)^n)-e*p*x^n*ln(d+e*x^n)/d/f/n/((f*x)^n)-ln(c*(d+e*
x^n)^p)/f/n/((f*x)^n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int (fx)^{-1-n} \log \left(c(d+ex^n)^p\right) \, dx = -\frac{(fx)^{-n}\left(-enpx^n\log(x)+epx^n\log(d+ex^n)+d\log\left(c(d+ex^n)^p\right)\right)}{dfn}$$

input
```
Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
```

output
```
-((-(e*n*p*x^n*Log[x]) + e*p*x^n*Log[d + e*x^n] + d*Log[c*(d + e*x^n)^p])/
(d*f*n*(f*x)^n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2905, 30, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{-n-1}\log\left(c(d+ex^n)^p\right)dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{ep\int \frac{x^{n-1}(fx)^{-n}}{ex^n+d}dx}{f}-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}$$

$$\Big\downarrow \text{30}$$

$$\frac{epx^n(fx)^{-n}\int \frac{1}{x(ex^n+d)}dx}{f}-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}$$

$$\Big\downarrow \text{798}$$

$$\frac{epx^n(fx)^{-n}\int \frac{x^{-n}}{ex^n+d}dx^n}{fn}-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}$$

$$\Big\downarrow \text{47}$$

$$\frac{epx^n(fx)^{-n}\left(\frac{\int x^{-n}dx^n}{d}-\frac{e\int \frac{1}{ex^n+d}dx^n}{d}\right)}{fn}-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}$$

$$\Big\downarrow \text{14}$$

$$\frac{epx^n(fx)^{-n}\left(\frac{\log(x^n)}{d}-\frac{e\int \frac{1}{ex^n+d}dx^n}{d}\right)}{fn}-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}$$

$$\Big\downarrow \text{16}$$

$$\frac{epx^n(fx)^{-n}\left(\frac{\log(x^n)}{d}-\frac{\log(d+ex^n)}{d}\right)}{fn}-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}$$

input
```
Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
```

output
```
(e*p*x^n*(Log[x^n]/d - Log[d + e*x^n]/d))/(f*n*(f*x)^n) - Log[c*(d + e*x^n
)^p]/(f*n*(f*x)^n)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^{-1-n} \ln \left(c(d+e\,x^n)^p\right) dx$$

input
```
int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p),x)
```

output
```
int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int (fx)^{-1-n} \log \left(c(d+ex^n)^p\right)\, dx = \frac{ef^{-n-1}npx^n \log (x) - df^{-n-1} \log (c) - (ef^{-n-1}px^n + df^{-n-1}p) \log (ex^n + d)}{dnx^n}$$

input
```
integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
(e*f^(-n - 1)*n*p*x^n*log(x) - d*f^(-n - 1)*log(c) - (e*f^(-n - 1)*p*x^n +
 d*f^(-n - 1)*p)*log(e*x^n + d))/(d*n*x^n)
```

Sympy [F(-2)]

Exception generated.

$$\int (fx)^{-1-n} \log \left(c(d+ex^n)^p\right)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int (fx)^{-1-n}\log\left(c(d+ex^n)^p\right)\,dx = \frac{ep\left(\frac{\log(x)}{df^n} - \frac{\log\left(\frac{ex^n+d}{e}\right)}{df^n n}\right)}{f} - \frac{\log\left((ex^n+d)^p c\right)}{(fx)^n fn}$$

input
```
integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
e*p*(log(x)/(d*f^n) - log((e*x^n + d)/e)/(d*f^n*n))/f - log((e*x^n + d)^p*
c)/((f*x)^n*f*n)
```

Giac [F]

$$\int (fx)^{-1-n}\log\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{-n-1}\log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^(-n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1-n}\log\left(c(d+ex^n)^p\right)\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{(f\,x)^{n+1}}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(f*x)^(n + 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(f*x)^(n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int (fx)^{-1-n} \log \left(c(d+ex^n)^p\right) \, dx = \frac{-x^n \log(x^n e+d) \, ep + x^n \log(x) \, enp - \log\left((x^n e+d)^p \, c\right) d}{x^n f^n dfn}$$

input
```
int((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x)
```

output
```
( - x**n*log(x**n*e + d)*e*p + x**n*log(x)*e*n*p - log((x**n*e + d)**p*c)*
d)/(x**n*f**n*d*f*n)
```

3.69 $\int (fx)^{-1-2n} \log (c(d+ex^n)^p) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int (fx)^{-1-2n} \log (c(d+ex^n)^p) \, dx = -\frac{epx^n(fx)^{-2n}}{2dfn} - \frac{e^2px^{2n}(fx)^{-2n}\log(x)}{2d^2f} + \frac{e^2px^{2n}(fx)^{-2n}\log(d+ex^n)}{2d^2fn} - \frac{(fx)^{-2n}\log(c(d+ex^n)^p)}{2fn}$$

output
```
-1/2*e*p*x^n/d/f/n/((f*x)^(2*n))-1/2*e^2*p*x^(2*n)*ln(x)/d^2/f/((f*x)^(2*n
))+1/2*e^2*p*x^(2*n)*ln(d+e*x^n)/d^2/f/n/((f*x)^(2*n))-1/2*ln(c*(d+e*x^n)^
p)/f/n/((f*x)^(2*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.63

$$\int (fx)^{-1-2n} \log (c(d+ex^n)^p) \, dx = -\frac{(fx)^{-2n}(e^2npx^{2n}\log(x) - e^2px^{2n}\log(d+ex^n) + d(epx^n + d\log(c(d+ex^n)^p)))}{2d^2fn}$$

input
```
Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

output
```
-1/2*(e^2*n*p*x^(2*n)*Log[x] - e^2*p*x^(2*n)*Log[d + e*x^n] + d*(e*p*x^n +
 d*Log[c*(d + e*x^n)^p]))/(d^2*f*n*(f*x)^(2*n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2905, 30, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{-2n-1}\log\left(c(d+ex^n)^p\right)dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{ep\int \frac{x^{n-1}(fx)^{-2n}}{ex^n+d}dx}{2f}-\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}$$

$$\Big\downarrow \text{30}$$

$$\frac{epx^{2n}(fx)^{-2n}\int \frac{x^{-n-1}}{ex^n+d}dx}{2f}-\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}$$

$$\Big\downarrow \text{798}$$

$$\frac{epx^{2n}(fx)^{-2n}\int \frac{x^{-2n}}{ex^n+d}dx^n}{2fn}-\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}$$

$$\Big\downarrow \text{54}$$

$$\frac{epx^{2n}(fx)^{-2n}\int \left(\frac{x^{-2n}}{d}-\frac{ex^{-n}}{d^2}+\frac{e^2}{d^2(ex^n+d)}\right)dx^n}{2fn}-\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}$$

$$\Big\downarrow \text{2009}$$

$$\frac{epx^{2n}(fx)^{-2n}\left(-\frac{e\log(x^n)}{d^2}+\frac{e\log(d+ex^n)}{d^2}-\frac{x^{-n}}{d}\right)}{2fn}-\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}$$

input
```
Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

output
```
(e*p*x^(2*n)*(-(1/(d*x^n)) - (e*Log[x^n])/d^2 + (e*Log[d + e*x^n])/d^2))/(
2*f*n*(f*x)^(2*n)) - Log[c*(d + e*x^n)^p]/(2*f*n*(f*x)^(2*n))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^{-1-2n} \ln\left(c(d+e\,x^n)^p\right) dx$$

input
```
int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p),x)
```

output
```
int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int (fx)^{-1-2n} \log\left(c(d+ex^n)^p\right)\, dx = -\frac{e^2f^{-2n-1}npx^{2n}\log(x) + def^{-2n-1}px^n + d^2f^{-2n-1}\log(c) - (e^2f^{-2n-1}px^{2n} - d^2f^{-2n-1}p)\log(ex^n + }{2\,d^2nx^{2n}}$$

input
```
integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
-1/2*(e^2*f^(-2*n - 1)*n*p*x^(2*n)*log(x) + d*e*f^(-2*n - 1)*p*x^n + d^2*f
^(-2*n - 1)*log(c) - (e^2*f^(-2*n - 1)*p*x^(2*n) - d^2*f^(-2*n - 1)*p)*log
(e*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F(-2)]

Exception generated.

$$\int (fx)^{-1-2n} \log\left(c(d+ex^n)^p\right)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx = -\frac{ep\left(\frac{e\log(x)}{d^2f^{2n}} - \frac{e\log\left(\frac{ex^n+d}{e}\right)}{d^2f^{2n}n} + \frac{1}{df^{2n}nx^n}\right)}{2f} - \frac{\log\left((ex^n+d)^pc\right)}{2(fx)^{2n}fn}$$

input
```
integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
-1/2*e*p*(e*log(x)/(d^2*f^(2*n)) - e*log((e*x^n + d)/e)/(d^2*f^(2*n)*n) +
1/(d*f^(2*n)*n*x^n))/f - 1/2*log((e*x^n + d)^p*c)/((f*x)^(2*n)*f*n)
```

Giac [F]

$$\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{-2n-1}\log\left((ex^n+d)^pc\right)\,dx$$

input
```
integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^(-2*n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{(f\,x)^{2\,n+1}}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(f*x)^(2*n + 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(f*x)^(2*n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int (fx)^{-1-2n} \log \left(c(d+ex^n)^p\right) dx = \frac{x^{2n}\log(x^n e+d)\, e^2 p - x^{2n}\log(x)\, e^2 np - x^n dep - \log((x^n e+d)^p\, c)\, d^2}{2x^{2n} f^{2n} d^2 fn}$$

input
```
int((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x)
```

output
```
(x**(2*n)*log(x***n*e + d)*e**2*p - x**(2*n)*log(x)*e**2*n*p - x**n*d*e*p -
 log((x***n*e + d)**p*c)*d**2)/(2*x**(2*n)*f**(2*n)*d**2*f*n)
```

3.70 $\int x^2 \log \left(c(d+ex^n)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int x^2 \log \left(c(d+ex^n)^p\right) \, dx = -\frac{enpx^{3+n} \text{Hypergeometric2F1}\left(1,\frac{3+n}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(3+n)} + \frac{1}{3}x^3 \log \left(c(d+ex^n)^p\right)$$

output
```
-1/3*e*n*p*x^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)+1/3*x^
3*ln(c*(d+e*x^n)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int x^2 \log \left(c(d+ex^n)^p\right) \, dx = \frac{1}{3}x^3\left(-\frac{enpx^n \text{Hypergeometric2F1}\left(1,\frac{3+n}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{d(3+n)} + \log \left(c(d+ex^n)^p\right)\right)$$

input
```
Integrate[x^2*Log[c*(d + e*x^n)^p],x]
```

output
```
(x^3*(-((e*n*p*x^n*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])
/(d*(3 + n))) + Log[c*(d + e*x^n)^p]))/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log \left(c(d+ex^n)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{3}x^3 \log \left(c(d+ex^n)^p\right)-\frac{1}{3}enp\int \frac{x^{n+2}}{ex^n+d}dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{1}{3}x^3 \log \left(c(d+ex^n)^p\right)-\frac{enpx^{n+3}\,\text{Hypergeometric2F1}\left(1,\frac{n+3}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(n+3)}$$

input
```
Int[x^2*Log[c*(d + e*x^n)^p],x]
```

output
```
-1/3*(e*n*p*x^(3 + n)*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d
)])/(d*(3 + n)) + (x^3*Log[c*(d + e*x^n)^p])/3
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x^2 \ln \left(c(d+e\,x^n)^p\right) dx$$

input
```
int(x^2*ln(c*(d+e*x^n)^p),x)
```

output
```
int(x^2*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int x^2 \log \left(c(d+ex^n)^p\right)\,dx = \int x^2 \log \left((ex^n+d)^p c\right)\,dx$$

input
```
integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
integral(x^2*log((e*x^n + d)^p*c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int x^2 \log \left(c(d+ex^n)^p\right) \, dx = -\frac{d^{-2-\frac{3}{n}}d^{1+\frac{3}{n}}epx^{n+3}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{3\Gamma\left(2+\frac{3}{n}\right)} - \frac{d^{-2-\frac{3}{n}}d^{1+\frac{3}{n}}epx^{n+3}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} + \frac{x^3\log\left(c(d+ex^n)^p\right)}{3}$$

input
```
integrate(x**2*ln(c*(d+e*x**n)**p),x)
```

output
```
-d**(-2 - 3/n)*d**(1 + 3/n)*e*p*x**(n + 3)*lerchphi(e*x**n*exp_polar(I*pi)
/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(3*gamma(2 + 3/n)) - d**(-2 - 3/n)*d**(1 +
3/n)*e*p*x**(n + 3)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1
 + 3/n)/(n*gamma(2 + 3/n)) + x**3*log(c*(d + e*x**n)**p)/3
```

Maxima [F]

$$\int x^2 \log \left(c(d+ex^n)^p\right) \, dx = \int x^2 \log \left((ex^n+d)^pc\right) \, dx$$

input
```
integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
-1/9*(n*p - 3*log(c))*x^3 + d*n*p*integrate(1/3*x^2/(e*x^n + d), x) + 1/3*
x^3*log((e*x^n + d)^p)
```

Giac [F]

$$\int x^2 \log\left(c(d+ex^n)^p\right)\,dx = \int x^2 \log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate(x^2*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(c(d+ex^n)^p\right)\,dx = \int x^2\,\ln\left(c\,(d+e\,x^n)^p\right)\,dx$$

input
```
int(x^2*log(c*(d + e*x^n)^p),x)
```

output
```
int(x^2*log(c*(d + e*x^n)^p), x)
```

Reduce [F]

$$\int x^2 \log\left(c(d+ex^n)^p\right)\,dx = \frac{\left(\int \frac{x^2}{x^n e+d}dx\right) dnp}{3} + \frac{\log((x^n e+d)^p\,c)\,x^3}{3} - \frac{np\,x^3}{9}$$

input
```
int(x^2*log(c*(d+e*x^n)^p),x)
```

output
```
(3*int(x**2/(x**n*e + d),x)*d*n*p + 3*log((x**n*e + d)**p*c)*x**3 - n*p*x*
*3)/9
```

3.71 $\int x \log(c(d+ex^n)^p) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int x \log(c(d+ex^n)^p) \, dx = -\frac{enpx^{2+n} \text{Hypergeometric2F1}\left(1, \frac{2+n}{n}, 2\left(1+\frac{1}{n}\right), -\frac{ex^n}{d}\right)}{2d(2+n)} + \frac{1}{2}x^2 \log(c(d+ex^n)^p)$$

output
```
-1/2*e*n*p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)+1/2*x^
2*ln(c*(d+e*x^n)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int x \log(c(d+ex^n)^p) \, dx = \frac{1}{2}x^2 \left(-\frac{enpx^n \text{Hypergeometric2F1}\left(1, \frac{2+n}{n}, 2+\frac{2}{n}, -\frac{ex^n}{d}\right)}{d(2+n)} + \log(c(d+ex^n)^p)\right)$$

input
```
Integrate[x*Log[c*(d + e*x^n)^p],x]
```

output
```
(x^2*(-((e*n*p*x^n*Hypergeometric2F1[1, (2 + n)/n, 2 + 2/n, -((e*x^n)/d)])
/(d*(2 + n))) + Log[c*(d + e*x^n)^p]))/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log \left(c(d+ex^n)^p\right) \, dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{2}x^2 \log \left(c(d+ex^n)^p\right) - \frac{1}{2}enp \int \frac{x^{n+1}}{ex^n+d}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2 \log \left(c(d+ex^n)^p\right) - \frac{enpx^{n+2} \text{Hypergeometric2F1}\left(1, \frac{n+2}{n}, 2\left(1+\frac{1}{n}\right), -\frac{ex^n}{d}\right)}{2d(n+2)}$$

input
```
Int[x*Log[c*(d + e*x^n)^p],x]
```

output
```
-1/2*(e*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(1 + n^(-1)), -((e
*x^n)/d)])/(d*(2 + n)) + (x^2*Log[c*(d + e*x^n)^p])/2
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x \ln \left(c(d + e\, x^n)^p\right) dx$$

input
```
int(x*ln(c*(d+e*x^n)^p),x)
```

output
```
int(x*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int x \log \left(c(d + ex^n)^p\right)\, dx = \int x \log \left((ex^n + d)^p c\right)\, dx$$

input
```
integrate(x*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
integral(x*log((e*x^n + d)^p*c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int x\log\left(c(d+ex^n)^p\right)\,dx = -\frac{d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}epx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2\Gamma\left(2+\frac{2}{n}\right)} - \frac{d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}epx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} + \frac{x^2\log\left(c(d+ex^n)^p\right)}{2}$$

input
```
integrate(x*ln(c*(d+e*x**n)**p),x)
```

output
```
-d**(-2 - 2/n)*d**(1 + 2/n)*e*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi)
/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*gamma(2 + 2/n)) - d**(-2 - 2/n)*d**(1 +
2/n)*e*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1
 + 2/n)/(n*gamma(2 + 2/n)) + x**2*log(c*(d + e*x**n)**p)/2
```

Maxima [F]

$$\int x\log\left(c(d+ex^n)^p\right)\,dx = \int x\log\left((ex^n+d)^pc\right)\,dx$$

input
```
integrate(x*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
d*n*p*integrate(1/2*x/(e*x^n + d), x) - 1/4*(n*p - 2*log(c))*x^2 + 1/2*x^2
*log((e*x^n + d)^p)
```

Giac [F]

$$\int x \log \left(c(d+ex^n)^p\right) \, dx = \int x \log \left((ex^n+d)^p c\right) \, dx$$

input
```
integrate(x*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate(x*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \log \left(c(d+ex^n)^p\right) \, dx = \int x \, \ln \left(c \left(d+e \, x^n\right)^p\right) \, dx$$

input
```
int(x*log(c*(d + e*x^n)^p),x)
```

output
```
int(x*log(c*(d + e*x^n)^p), x)
```

Reduce [F]

$$\int x \log \left(c(d+ex^n)^p\right) \, dx = \frac{\left(\int \frac{x}{x^n e+d} dx\right) dnp}{2} + \frac{\log\left((x^n e+d)^p \, c\right) x^2}{2} - \frac{np \, x^2}{4}$$

input
```
int(x*log(c*(d+e*x^n)^p),x)
```

output
```
(2*int(x/(x**n*e + d),x)*d*n*p + 2*log((x**n*e + d)**p*c)*x**2 - n*p*x**2)
/4
```

3.72 $\int \log (c(d+ex^n)^p) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \log (c(d+ex^n)^p) \, dx = -\frac{enpx^{1+n} \text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} + x \log (c(d+ex^n)^p)$$

output
```
-e*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)+x*ln(c*(d+e*
x^n)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \log (c(d+ex^n)^p) \, dx = x\left(-\frac{enpx^n \text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} + \log (c(d+ex^n)^p)\right)$$

input
```
Integrate[Log[c*(d + e*x^n)^p],x]
```

output
```
x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)]
)/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2898, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c(d+ex^n)^p\right)\,dx$$

$$\Big\downarrow \text{2898}$$

$$x\log \left(c(d+ex^n)^p\right)-enp\int \frac{x^n}{ex^n+d}dx$$

$$\Big\downarrow \text{888}$$

$$x\log \left(c(d+ex^n)^p\right)-\frac{enpx^{n+1}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)}$$

input
```
Int[Log[c*(d + e*x^n)^p],x]
```

output
```
-((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/
d)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [F]

$$\int \ln \left(c(d+e\,x^{n})^{p}\right)dx$$

input
```
int(ln(c*(d+e*x^n)^p),x)
```

output
```
int(ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int \log \left(c(d+ex^{n})^{p}\right)\,dx=\int \log \left((ex^{n}+d)^{p}c\right)\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \log \left(c(d+ex^{n})^{p}\right)\,dx$$
$$=x\log \left(c(d+ex^{n})^{p}\right)+\frac{d^{-\frac{1}{n}}d^{1+\frac{1}{n}}ee^{\frac{1}{n}}e^{-1-\frac{1}{n}}px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate(ln(c*(d+e*x**n)**p),x)
```

output
```
x*log(c*(d + e*x**n)**p) + d**(1 + 1/n)*e*e**(1/n)*e**(-1 - 1/n)*p*x*lerch
phi(d*exp_polar(I*pi)/(e*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(d*d**(1/
n)*n*gamma(1 + 1/n))
```

Maxima [F]

$$\int \log\left(c(d+ex^n)^p\right)\,dx = \int \log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
d*n*p*integrate(1/(e*x^n + d), x) - (n*p - log(c))*x + x*log((e*x^n + d)^p
)
```

Giac [F]

$$\int \log\left(c(d+ex^n)^p\right)\,dx = \int \log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(c(d+ex^n)^p\right)\,dx=\int \ln \left(c\,{\left(d+e\,x^n\right)}^p\right)\,dx$$

input
```
int(log(c*(d + e*x^n)^p),x)
```

output
```
int(log(c*(d + e*x^n)^p), x)
```

Reduce [F]

$$\int \log \left(c(d+ex^n)^p\right)\,dx=\left(\int \frac{1}{x^n e+d}dx\right)dnp+\log \left((x^n e+d)^p c\right)x-npx$$

input
```
int(log(c*(d+e*x^n)^p),x)
```

output
```
int(1/(x**n*e + d),x)*d*n*p + log((x**n*e + d)**p*c)*x - n*p*x
```

3.73 $\int \frac{\log(c(d+ex^n)^p)}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log(c(d+ex^n)^p)}{x} \, dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+p*polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\log(c(d+ex^n)^p)}{x} \, dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p) + p\,\mathrm{PolyLog}\left(2,\frac{d+ex^n}{d}\right)}{n}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/x,x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{x} \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{\int x^{-n} \log (c(ex^n+d)^p) \, dx^n}{n}$$

$$\Big\downarrow \text{2841}$$

$$\frac{\log \left(-\frac{ex^n}{d}\right) \log (c(d+ex^n)^p) - ep \int \frac{\log \left(-\frac{ex^n}{d}\right)}{ex^n+d} dx^n}{n}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\log \left(-\frac{ex^n}{d}\right) \log (c(d+ex^n)^p) + p \operatorname{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]/x,x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.98 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.86

method	result
risch	$\ln(x)\ln((d+e\,x^n)^p)+\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csgn}(ic)}{2}-\right.$

input
```
int(ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^
2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*cs
gn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*ln
(x)-p/n*dilog((d+e*x^n)/d)-p*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \frac{np\log(ex^n+d)\log(x) - np\log(x)\log\left(\frac{ex^n+d}{d}\right) + n\log(c)\log(x) - p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

input
```
integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
(n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(
x) - p*dilog(-(e*x^n + d)/d + 1))/n
```

Sympy [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log(c(d+ex^n)^p)}{x}\,dx$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral(log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log((ex^n+d)^p c)}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*p*log(x)^2 + log((e*x^n
 + d)^p)*log(x) + log(c)*log(x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/x,x)
```

output
```
int(log(c*(d + e*x^n)^p)/x, x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{2\left(\int \frac{\log((x^n e+d)^p c)}{x^n ex+dx}dx\right)dnp + \log\left((x^n e+d)^p\,c\right)^2}{2np}$$

input
```
int(log(c*(d+e*x^n)^p)/x,x)
```

output
```
(2*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d*n*p + log((x**n*e + d)
**p*c)**2)/(2*n*p)
```

3.74 $\int \frac{\log(c(d+ex^n)^p)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{\log(c(d+ex^n)^p)}{x^2}\,dx = -\frac{enpx^{-1+n}\,\text{Hypergeometric2F1}\left(1,-\frac{1-n}{n},2-\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1-n)} - \frac{\log(c(d+ex^n)^p)}{x}$$

output
```
-e*n*p*x^(-1+n)*hypergeom([1, -(1-n)/n],[2-1/n],-e*x^n/d)/d/(1-n)-ln(c*(d+
e*x^n)^p)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{\log(c(d+ex^n)^p)}{x^2}\,dx = \frac{\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,\frac{-1+n}{n},2-\frac{1}{n},-\frac{ex^n}{d}\right)}{d(-1+n)} - \log(c(d+ex^n)^p)}{x}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/x^2,x]
```

output
```
((e*n*p*x^n*Hypergeometric2F1[1, (-1 + n)/n, 2 - n^(-1), -((e*x^n)/d)])/(d
*(-1 + n)) - Log[c*(d + e*x^n)^p])/x
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(d+ex^n)^p)}{x^2}dx$$

$$\Big\downarrow \text{ 2905}$$

$$enp\int \frac{x^{n-2}}{ex^n+d}dx - \frac{\log(c(d+ex^n)^p)}{x}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\log(c(d+ex^n)^p)}{x} - \frac{enpx^{n-1}\,\text{Hypergeometric2F1}\left(1,-\frac{1-n}{n},2-\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1-n)}$$

input
```
Int[Log[c*(d + e*x^n)^p]/x^2,x]
```

output
```
-((e*n*p*x^(-1 + n)*Hypergeometric2F1[1, -((1 - n)/n), 2 - n^(-1), -((e*x^
n)/d)])/(d*(1 - n))) - Log[c*(d + e*x^n)^p]/x
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\ln \left(c(d + e\, x^n)^p\right)}{x^2} dx$$

input
```
int(ln(c*(d+e*x^n)^p)/x^2,x)
```

output
```
int(ln(c*(d+e*x^n)^p)/x^2,x)
```

Fricas [F]

$$\int \frac{\log \left(c(d + ex^n)^p\right)}{x^2}\, dx = \int \frac{\log \left((ex^n + d)^p c\right)}{x^2}\, dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{\log \left(c(d + ex^n)^p\right)}{x^2}\, dx = -\frac{\log \left(c(d + ex^n)^p\right)}{x} + \frac{d^{\frac{1}{n}} d^{1-\frac{1}{n}} e e^{-\frac{1}{n}} e^{-1+\frac{1}{n}} p \Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{1}{n}\right) \Gamma\left(-\frac{1}{n}\right)}{dnx\Gamma\left(1 - \frac{1}{n}\right)}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x**2,x)
```

output
```
-log(c*(d + e*x**n)**p)/x + d**(1/n)*d**(1 - 1/n)*e*e**(-1 + 1/n)*p*lerchp
hi(d*exp_polar(I*pi)/(e*x**n), 1, 1/n)*gamma(-1/n)/(d*e**(1/n)*n*x*gamma(1
 - 1/n))
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^2}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="maxima")
```

output
```
-d*n*p*integrate(1/(e*x^2*x^n + d*x^2), x) - (n*p + log((e*x^n + d)^p) + l
og(c))/x
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^2}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x^2}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/x^2,x)
```

output
```
int(log(c*(d + e*x^n)^p)/x^2, x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx = \frac{-\left(\int \frac{1}{x^n e\,x^2+d\,x^2}dx\right)dnpx - \log\left((x^n e+d)^p\,c\right) - np}{x}$$

input
```
int(log(c*(d+e*x^n)^p)/x^2,x)
```

output
```
( - (int(1/(x**n*e*x**2 + d*x**2),x)*d*n*p*x + log((x**n*e + d)**p*c) + n*
p))/x
```

3.75 $\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = -\frac{enpx^{-2+n}\,\text{Hypergeometric2F1}\left(1,-\frac{2-n}{n},2\left(1-\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(2-n)} - \frac{\log(c(d+ex^n)^p)}{2x^2}$$

output
```
-1/2*e*n*p*x^(-2+n)*hypergeom([1, -(2-n)/n],[2-2/n],-e*x^n/d)/d/(2-n)-1/2*
ln(c*(d+e*x^n)^p)/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = \frac{\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,\frac{-2+n}{n},2-\frac{2}{n},-\frac{ex^n}{d}\right)}{d(-2+n)} - \log(c(d+ex^n)^p)}{2x^2}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/x^3,x]
```

output
```
((e*n*p*x^n*Hypergeometric2F1[1, (-2 + n)/n, 2 - 2/n, -((e*x^n)/d)])/(d*(-
2 + n)) - Log[c*(d + e*x^n)^p])/(2*x^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{1}{2}enp\int \frac{x^{n-3}}{ex^n+d}dx - \frac{\log(c(d+ex^n)^p)}{2x^2}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\log(c(d+ex^n)^p)}{2x^2} - \frac{enpx^{n-2}\,\text{Hypergeometric2F1}\left(1,-\frac{2-n}{n},2\left(1-\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(2-n)}$$

input
```
Int[Log[c*(d + e*x^n)^p]/x^3,x]
```

output
```
-1/2*(e*n*p*x^(-2 + n)*Hypergeometric2F1[1, -((2 - n)/n), 2*(1 - n^(-1)),
-((e*x^n)/d)])/(d*(2 - n)) - Log[c*(d + e*x^n)^p]/(2*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\ln (c(d+e\,x^n)^p)}{x^3}dx$$

input
```
int(ln(c*(d+e*x^n)^p)/x^3,x)
```

output
```
int(ln(c*(d+e*x^n)^p)/x^3,x)
```

Fricas [F]

$$\int \frac{\log (c(d+ex^n)^p)}{x^3}\,dx = \int \frac{\log ((ex^n+d)^pc)}{x^3}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.68 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int \frac{\log (c(d+ex^n)^p)}{x^3}\,dx = -\frac{\log (c(d+ex^n)^p)}{2x^2} + \frac{d^{\frac{2}{n}}d^{1-\frac{2}{n}}ee^{-\frac{2}{n}}e^{-1+\frac{2}{n}}p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{2}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{dnx^2\Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x**3,x)
```

output
```
-log(c*(d + e*x**n)**p)/(2*x**2) + d**(2/n)*d**(1 - 2/n)*e*e**(-1 + 2/n)*p
*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, 2/n)*gamma(-2/n)/(d*e**(2/n)*n*x*
*2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = \int \frac{\log((ex^n+d)^p c)}{x^3}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="maxima")
```

output
```
-d*n*p*integrate(1/2/(e*x^3*x^n + d*x^3), x) - 1/4*(n*p + 2*log((e*x^n + d
)^p) + 2*log(c))/x^2
```

Giac [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = \int \frac{\log((ex^n+d)^p c)}{x^3}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = \int \frac{\ln(c\,(d+e\,x^n)^p)}{x^3}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/x^3,x)
```

output
```
int(log(c*(d + e*x^n)^p)/x^3, x)
```

Reduce [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = \frac{-2\left(\int \frac{1}{x^n e\,x^3+d\,x^3}dx\right)dnp\,x^2 - 2\log((x^n e+d)^p\,c) - np}{4x^2}$$

input
```
int(log(c*(d+e*x^n)^p)/x^3,x)
```

output
```
( - 2*int(1/(x**n*e*x**3 + d*x**3),x)*d*n*p*x**2 - 2*log((x**n*e + d)**p*c
) - n*p)/(4*x**2)
```

3.76 $\int \frac{\log(c(d+ex^n)^p)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 70

$$\int \frac{\log(c(d+ex^n)^p)}{x^4}\,dx = -\frac{enpx^{-3+n}\,\text{Hypergeometric2F1}\left(1,-\frac{3-n}{n},2-\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(3-n)} - \frac{\log(c(d+ex^n)^p)}{3x^3}$$

output
```
-1/3*e*n*p*x^(-3+n)*hypergeom([1, -(3-n)/n],[2-3/n],-e*x^n/d)/d/(3-n)-1/3*
ln(c*(d+e*x^n)^p)/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{\log(c(d+ex^n)^p)}{x^4}\,dx = \frac{\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,\frac{-3+n}{n},2-\frac{3}{n},-\frac{ex^n}{d}\right)}{d(-3+n)} - \log(c(d+ex^n)^p)}{3x^3}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/x^4,x]
```

output
```
((e*n*p*x^n*Hypergeometric2F1[1, (-3 + n)/n, 2 - 3/n, -((e*x^n)/d)])/(d*(-
3 + n)) - Log[c*(d + e*x^n)^p])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2905, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{x^4} dx$$

$$\Big\downarrow \ 2905$$

$$\frac{1}{3}enp\int \frac{x^{n-4}}{ex^n+d}dx-\frac{\log (c(d+ex^n)^p)}{3x^3}$$

$$\Big\downarrow \ 888$$

$$-\frac{\log (c(d+ex^n)^p)}{3x^3}-\frac{enpx^{n-3}\,\text{Hypergeometric2F1}\left(1,-\frac{3-n}{n},2-\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(3-n)}$$

input
```
Int[Log[c*(d + e*x^n)^p]/x^4,x]
```

output
```
-1/3*(e*n*p*x^(-3 + n)*Hypergeometric2F1[1, -((3 - n)/n), 2 - 3/n, -((e*x^
n)/d)])/(d*(3 - n)) - Log[c*(d + e*x^n)^p]/(3*x^3)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{\ln \left(c(d+e\,x^n)^p\right)}{x^4}dx$$

input
```
int(ln(c*(d+e*x^n)^p)/x^4,x)
```

output
```
int(ln(c*(d+e*x^n)^p)/x^4,x)
```

Fricas [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x^4}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{x^4}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x^4}\,dx = -\frac{\log \left(c(d+ex^n)^p\right)}{3x^3} + \frac{d^{\frac{3}{n}}d^{1-\frac{3}{n}}ee^{-\frac{3}{n}}e^{-1+\frac{3}{n}}p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{3}{n}\right)\Gamma\left(-\frac{3}{n}\right)}{dnx^3\Gamma\left(1-\frac{3}{n}\right)}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x**4,x)
```

output
```
-log(c*(d + e*x**n)**p)/(3*x**3) + d**(3/n)*d**(1 - 3/n)*e*e**(-1 + 3/n)*p
*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, 3/n)*gamma(-3/n)/(d*e**(3/n)*n*x**
*3*gamma(1 - 3/n))
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^4}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="maxima")
```

output
```
-d*n*p*integrate(1/3/(e*x^4*x^n + d*x^4), x) - 1/9*(n*p + 3*log((e*x^n + d
)^p) + 3*log(c))/x^3
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^4}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x^4}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/x^4,x)
```

output
```
int(log(c*(d + e*x^n)^p)/x^4, x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = \frac{-3\left(\int \frac{1}{x^n e\,x^4+d\,x^4}dx\right) dnp\,x^3 - 3\log\left((x^n e+d)^p\,c\right) - np}{9x^3}$$

input
```
int(log(c*(d+e*x^n)^p)/x^4,x)
```

output
```
( - 3*int(1/(x**n*e*x**4 + d*x**4),x)*d*n*p*x**3 - 3*log((x**n*e + d)**p*c
) - n*p)/(9*x**3)
```

3.77 $\int x^5 \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 215

$$\begin{aligned}\int x^5 \log^2\left(c(a+bx^2)^p\right)\,dx = {} & \frac{a^2p^2x^2}{b^2} - \frac{ap^2(a+bx^2)^2}{4b^3} + \frac{p^2(a+bx^2)^3}{27b^3} \\ & - \frac{a^3p^2\log^2(a+bx^2)}{6b^3} - \frac{a^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} \\ & + \frac{ap(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{2b^3} \\ & - \frac{p(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} \\ & + \frac{a^3p\log(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3b^3} \\ & + \frac{1}{6}x^6\log^2\left(c(a+bx^2)^p\right)\end{aligned}$$

output
```
a^2*p^2*x^2/b^2-1/4*a*p^2*(b*x^2+a)^2/b^3+1/27*p^2*(b*x^2+a)^3/b^3-1/6*a^3
*p^2*ln(b*x^2+a)^2/b^3-a^2*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3+1/2*a*p*(b*x^
2+a)^2*ln(c*(b*x^2+a)^p)/b^3-1/9*p*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)/b^3+1/3*a
^3*p*ln(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3+1/6*x^6*ln(c*(b*x^2+a)^p)^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.59

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{bp^2x^2(66a^2-15abx^2+4b^2x^4)-30a^3p^2\log\left(a+bx^2\right)-6p(6a^3+6a^2bx^2-3ab^2x^4+2b^3x^6)\log\left(c(a+bx\right.}{108b^3}$$

input
```
Integrate[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
(b*p^2*x^2*(66*a^2 - 15*a*b*x^2 + 4*b^2*x^4) - 30*a^3*p^2*Log[a + b*x^2] -
 6*p*(6*a^3 + 6*a^2*b*x^2 - 3*a*b^2*x^4 + 2*b^3*x^6)*Log[c*(a + b*x^2)^p]
+ 18*(a^3 + b^3*x^6)*Log[c*(a + b*x^2)^p]^2)/(108*b^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2904, 2845, 2858, 25, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int x^4 \log^2\left(c\left(bx^2+a\right)^p\right)dx^2$$

$$\Big\downarrow \text{2845}$$

$$\frac{1}{2}\left(\frac{1}{3}x^6\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{2}{3}bp\int \frac{x^6\log\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx^2\right)$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{2}\left(\frac{1}{3}x^6\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{2}{3}p\int x^4\log\left(c\left(bx^2+a\right)^p\right)d\left(bx^2+a\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{2}{3}p\int -x^4\log\left(c(bx^2+a)^p\right)d(bx^2+a)+\frac{1}{3}x^6\log^2\left(c(a+bx^2)^p\right)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2p\int -b^3x^4\log\left(c(bx^2+a)^p\right)d(bx^2+a)}{3b^3}+\frac{1}{3}x^6\log^2\left(c(a+bx^2)^p\right)\right)$$

$$\Big\downarrow \text{2772}$$

$$\frac{1}{2}\left(\frac{2p\left(-p\int\left(-\frac{x^4}{3}-3a^2+\frac{3}{2}a(bx^2+a)+\frac{a^3\log(bx^2+a)}{x^2}\right)d(bx^2+a)+a^3\log\left(a+bx^2\right)\log\left(c(a+bx^2)^p\right)-3a^2(a\right.}{3b^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2p\left(a^3\log\left(a+bx^2\right)\log\left(c(a+bx^2)^p\right)-3a^2\left(a+bx^2\right)\log\left(c(a+bx^2)^p\right)-p\left(\frac{1}{2}a^3\log^2\left(a+bx^2\right)-3a^2\left(a+bx\right.\right.\right.}{3b^3}\right.$$

input
```
Int[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
((x^6*Log[c*(a + b*x^2)^p]^2)/3 + (2*p*(-(p*((3*a*x^4)/4 - x^6/9 - 3*a^2*(
a + b*x^2) + (a^3*Log[a + b*x^2]^2)/2)) + (3*a*x^4*Log[c*(a + b*x^2)^p])/2
 - (x^6*Log[c*(a + b*x^2)^p])/3 - 3*a^2*(a + b*x^2)*Log[c*(a + b*x^2)^p] +
 a^3*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]))/(3*b^3))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{-18x^{6}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}b^{3}+12x^{6}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)b^{3}p-4x^{6}b^{3}p^{2}-18x^{4}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)a\,b^{2}p+15x^{4}a\,b^{2}p^{2}+36x^{2}\ln\left(c\left(b\,x^{2}+a\right.\right.}{108b^{3}}$
risch	Expression too large to display

input
```
int(x^5*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/108*(-18*x^6*ln(c*(b*x^2+a)^p)^2*b^3+12*x^6*ln(c*(b*x^2+a)^p)*b^3*p-4*x
^6*b^3*p^2-18*x^4*ln(c*(b*x^2+a)^p)*a*b^2*p+15*x^4*a*b^2*p^2+36*x^2*ln(c*(
b*x^2+a)^p)*a^2*b*p-66*x^2*a^2*b*p^2+102*ln(b*x^2+a)*a^3*p^2-18*ln(c*(b*x^
2+a)^p)^2*a^3-36*ln(c*(b*x^2+a)^p)*a^3*p+66*a^3*p^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{4\,b^3p^2x^6 + 18\,b^3x^6\log\left(c\right)^2 - 15\,ab^2p^2x^4 + 66\,a^2bp^2x^2 + 18\left(b^3p^2x^6 + a^3p^2\right)\log\left(bx^2+a\right)^2 - 6\left(2\,b^3p^2x^6 - \right.}{}$$

input
```
integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
1/108*(4*b^3*p^2*x^6 + 18*b^3*x^6*log(c)^2 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p
^2*x^2 + 18*(b^3*p^2*x^6 + a^3*p^2)*log(b*x^2 + a)^2 - 6*(2*b^3*p^2*x^6 -
3*a*b^2*p^2*x^4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2 - 6*(b^3*p*x^6 + a^3*p)*log
(c))*log(b*x^2 + a) - 6*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p*x^2)*log(
c))/b^3
```

Sympy [A] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \begin{cases} -\frac{11a^3p\log\left(c\left(a+bx^2\right)^p\right)}{18b^3} + \frac{a^3\log\left(c\left(a+bx^2\right)^p\right)^2}{6b^3} + \frac{11a^2p^2x^2}{18b^2} - \frac{a^2px^2\log\left(c\left(a+bx^2\right)^p\right)}{3b^2} - \frac{5ap^2x^4}{36b} + \frac{apx^4\log\left(c\left(a+bx^2\right)^p\right)}{6b} + \frac{p^2x^6}{27} \\ \frac{x^6\log\left(a^pc\right)^2}{6} \end{cases}$$

input
```
integrate(x**5*ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Piecewise((-11*a**3*p*log(c*(a + b*x**2)**p)/(18*b**3) + a**3*log(c*(a + b
*x**2)**p)**2/(6*b**3) + 11*a**2*p**2*x**2/(18*b**2) - a**2*p*x**2*log(c*(
a + b*x**2)**p)/(3*b**2) - 5*a*p**2*x**4/(36*b) + a*p*x**4*log(c*(a + b*x*
*2)**p)/(6*b) + p**2*x**6/27 - p*x**6*log(c*(a + b*x**2)**p)/9 + x**6*log(
c*(a + b*x**2)**p)**2/6, Ne(b, 0)), (x**6*log(a**p*c)**2/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

$$\begin{aligned}\int x^5\log^2\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{6}\,x^6\log\left(\left(bx^2+a\right)^pc\right)^2\\&\quad+\frac{1}{18}\,bp\left(\frac{6\,a^3\log\left(bx^2+a\right)}{b^4}-\frac{2\,b^2x^6-3\,abx^4+6\,a^2x^2}{b^3}\right)\log\left(\left(bx^2+a\right)^pc\right)\\&\quad+\frac{\left(4\,b^3x^6-15\,ab^2x^4+66\,a^2bx^2-18\,a^3\log\left(bx^2+a\right)^2-66\,a^3\log\left(bx^2+a\right)\right)p^2}{108\,b^3}\end{aligned}$$

input
```
integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
1/6*x^6*log((b*x^2 + a)^p*c)^2 + 1/18*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b
^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c) + 1/108*(4*b^3*x
^6 - 15*a*b^2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*
x^2 + a))*p^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.72

$$\int x^5 \log^2 \left(c(a+bx^2)^p\right) dx$$
$$= \frac{(bx^2+a)^3 p^2 \log (bx^2+a)^2}{6\,b^3} - \frac{(bx^2+a)^2 a p^2 \log (bx^2+a)^2}{2\,b^3} - \frac{(bx^2+a)^3 p^2 \log (bx^2+a)}{9\,b^3}$$
$$+ \frac{(bx^2+a)^2 a p^2 \log (bx^2+a)}{2\,b^3} + \frac{(bx^2+a)^3 p \log (bx^2+a) \log (c)}{3\,b^3}$$
$$- \frac{(bx^2+a)^2 a p \log (bx^2+a) \log (c)}{b^3} + \frac{(bx^2+a)^3 p^2}{27\,b^3} - \frac{(bx^2+a)^2 a p^2}{4\,b^3}$$
$$- \frac{(bx^2+a)^3 p \log (c)}{9\,b^3} + \frac{(bx^2+a)^2 a p \log (c)}{2\,b^3} + \frac{(bx^2+a)^3 \log (c)^2}{6\,b^3} - \frac{(bx^2+a)^2 a \log (c)^2}{2\,b^3}$$
$$+ \frac{\left(2\,bx^2 + (bx^2+a) \log (bx^2+a)^2 - 2\,(bx^2+a) \log (bx^2+a) + 2\,a\right) a^2 p^2 - 2\,(bx^2 - (bx^2+a) \log (bx^2 -}{2\,b^3}$$

input
```
integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
1/6*(b*x^2 + a)^3*p^2*log(b*x^2 + a)^2/b^3 - 1/2*(b*x^2 + a)^2*a*p^2*log(b
*x^2 + a)^2/b^3 - 1/9*(b*x^2 + a)^3*p^2*log(b*x^2 + a)/b^3 + 1/2*(b*x^2 +
a)^2*a*p^2*log(b*x^2 + a)/b^3 + 1/3*(b*x^2 + a)^3*p*log(b*x^2 + a)*log(c)/
b^3 - (b*x^2 + a)^2*a*p*log(b*x^2 + a)*log(c)/b^3 + 1/27*(b*x^2 + a)^3*p^2
/b^3 - 1/4*(b*x^2 + a)^2*a*p^2/b^3 - 1/9*(b*x^2 + a)^3*p*log(c)/b^3 + 1/2*
(b*x^2 + a)^2*a*p*log(c)/b^3 + 1/6*(b*x^2 + a)^3*log(c)^2/b^3 - 1/2*(b*x^2
 + a)^2*a*log(c)^2/b^3 + 1/2*((2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*
(b*x^2 + a)*log(b*x^2 + a) + 2*a)*a^2*p^2 - 2*(b*x^2 - (b*x^2 + a)*log(b*x
^2 + a) + a)*a^2*p*log(c) + (b*x^2 + a)*a^2*log(c)^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.59

$$\int x^5 \log^2 \left(c(a+bx^2)^p\right) dx = \frac{p^2\,x^6}{27} + \ln\left(c\,\left(b\,x^2+a\right)^p\right)^2 \left(\frac{x^6}{6} + \frac{a^3}{6\,b^3}\right)$$
$$- \ln\left(c\,\left(b\,x^2+a\right)^p\right) \left(\frac{p\,x^6}{9} + \frac{a^2\,p\,x^2}{3\,b^2} - \frac{a\,p\,x^4}{6\,b}\right)$$
$$- \frac{5\,a\,p^2\,x^4}{36\,b} - \frac{11\,a^3\,p^2\,\ln\left(b\,x^2+a\right)}{18\,b^3} + \frac{11\,a^2\,p^2\,x^2}{18\,b^2}$$

input
```
int(x^5*log(c*(a + b*x^2)^p)^2,x)
```

output
```
(p^2*x^6)/27 + log(c*(a + b*x^2)^p)^2*(x^6/6 + a^3/(6*b^3)) - log(c*(a + b
*x^2)^p)*((p*x^6)/9 + (a^2*p*x^2)/(3*b^2) - (a*p*x^4)/(6*b)) - (5*a*p^2*x^
4)/(36*b) - (11*a^3*p^2*log(a + b*x^2))/(18*b^3) + (11*a^2*p^2*x^2)/(18*b^
2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.77

$$\int x^5 \log^2\left(c(a+bx^2)^p\right)\,dx$$
$$= \frac{18\log\left((b\,x^2+a)^p\,c\right)^2 a^3 + 18\log\left((b\,x^2+a)^p\,c\right)^2 b^3x^6 - 66\log\left((b\,x^2+a)^p\,c\right)a^3p - 36\log\left((b\,x^2+a)^p\,c\right)a^2b}{108}$$

input
```
int(x^5*log(c*(b*x^2+a)^p)^2,x)
```

output
```
(18*log((a + b*x**2)**p*c)**2*a**3 + 18*log((a + b*x**2)**p*c)**2*b**3*x**
6 - 66*log((a + b*x**2)**p*c)*a**3*p - 36*log((a + b*x**2)**p*c)*a**2*b*p*
x**2 + 18*log((a + b*x**2)**p*c)*a*b**2*p*x**4 - 12*log((a + b*x**2)**p*c)
*b**3*p*x**6 + 66*a**2*b*p**2*x**2 - 15*a*b**2*p**2*x**4 + 4*b**3*p**2*x**
6)/(108*b**3)
```

3.78 $\int x^3 \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 145

$$\int x^3\log^2\left(c\left(a+bx^2\right)^p\right)\,dx=-\frac{ap^2x^2}{b}+\frac{p^2(a+bx^2)^2}{8b^2}+\frac{ap(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2}-\frac{p(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^2}-\frac{a(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2}+\frac{(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^2}$$

output
```
-a*p^2*x^2/b+1/8*p^2*(b*x^2+a)^2/b^2+a*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^2-1
/4*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)/b^2-1/2*a*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2
/b^2+1/4*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right) dx$$
$$= \frac{bp^2x^2(-6a+bx^2)+2a^2p^2\log\left(a+bx^2\right)+2p(2a^2+2abx^2-b^2x^4)\log\left(c\left(a+bx^2\right)^p\right)-2(a^2-b^2x^4)\log^2\left(\cdots\right.}{8b^2}$$

input
```
Integrate[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
(b*p^2*x^2*(-6*a + b*x^2) + 2*a^2*p^2*Log[a + b*x^2] + 2*p*(2*a^2 + 2*a*b*
x^2 - b^2*x^4)*Log[c*(a + b*x^2)^p] - 2*(a^2 - b^2*x^4)*Log[c*(a + b*x^2)^
p]^2)/(8*b^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right) dx$$
$$\Big\downarrow \text{ 2904}$$
$$\frac{1}{2}\int x^2 \log^2\left(c\left(bx^2+a\right)^p\right) dx^2$$
$$\Big\downarrow \text{ 2848}$$
$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)\log^2\left(c\left(bx^2+a\right)^p\right)}{b}-\frac{a\log^2\left(c\left(bx^2+a\right)^p\right)}{b}\right) dx^2$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{2b^2}-\frac{a(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{b^2}-\frac{p(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{2b^2}+\frac{2ap(a+b\right.$$

input
```
Int[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
((-2*a*p^2*x^2)/b + (p^2*(a + b*x^2)^2)/(4*b^2) + (2*a*p*(a + b*x^2)*Log[c
*(a + b*x^2)^p])/b^2 - (p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(2*b^2) - (a
*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/b^2 + ((a + b*x^2)^2*Log[c*(a + b*x^2
)^p]^2)/(2*b^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{2x^4 \ln\left(c\left(b\,x^2+a\right)^p\right)^2 b^2-2x^4 \ln\left(c\left(b\,x^2+a\right)^p\right) b^2 p+b^2 p^2 x^4+4x^2 \ln\left(c\left(b\,x^2+a\right)^p\right) abp-6ab\,p^2 x^2+10 \ln\left(b\,x^2+a\right) a^2 p^2-2\ln\left(c\left(b\,x^2\right.\right.}{8b^2}$
risch	Expression too large to display

input
```
int(x^3*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*x^4*ln(c*(b*x^2+a)^p)^2*b^2-2*x^4*ln(c*(b*x^2+a)^p)*b^2*p+b^2*p^2*x
^4+4*x^2*ln(c*(b*x^2+a)^p)*a*b*p-6*a*b*p^2*x^2+10*ln(b*x^2+a)*a^2*p^2-2*ln
(c*(b*x^2+a)^p)^2*a^2-4*ln(c*(b*x^2+a)^p)*a^2*p+6*a^2*p^2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{b^2p^2x^4 + 2\,b^2x^4\log\left(c\right)^2 - 6\,abp^2x^2 + 2\left(b^2p^2x^4 - a^2p^2\right)\log\left(bx^2+a\right)^2 - 2\left(b^2p^2x^4 - 2\,abp^2x^2 - 3\,a^2p^2 - 2\right.}{8\,b^2}$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
1/8*(b^2*p^2*x^4 + 2*b^2*x^4*log(c)^2 - 6*a*b*p^2*x^2 + 2*(b^2*p^2*x^4 - a
^2*p^2)*log(b*x^2 + a)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2 - 2*
(b^2*p*x^4 - a^2*p)*log(c))*log(b*x^2 + a) - 2*(b^2*p*x^4 - 2*a*b*p*x^2)*l
og(c))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \begin{cases} \frac{3a^2p\log\left(c(a+bx^2)^p\right)}{4b^2} - \frac{a^2\log\left(c(a+bx^2)^p\right)^2}{4b^2} - \frac{3ap^2x^2}{4b} + \frac{apx^2\log\left(c(a+bx^2)^p\right)}{2b} + \frac{p^2x^4}{8} - \frac{px^4\log\left(c(a+bx^2)^p\right)}{4} + \frac{x^4\log\left(c(a+bx\right.}{4} \\ \frac{x^4\log\left(a^pc\right)^2}{4} \end{cases}$$

input
```
integrate(x**3*ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Piecewise((3*a**2*p*log(c*(a + b*x**2)**p)/(4*b**2) - a**2*log(c*(a + b*x*
*2)**p)**2/(4*b**2) - 3*a*p**2*x**2/(4*b) + a*p*x**2*log(c*(a + b*x**2)**p
)/(2*b) + p**2*x**4/8 - p*x**4*log(c*(a + b*x**2)**p)/4 + x**4*log(c*(a +
b*x**2)**p)**2/4, Ne(b, 0)), (x**4*log(a**p*c)**2/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{1}{4}x^4\log\left(\left(bx^2+a\right)^pc\right)^2 - \frac{1}{4}bp\left(\frac{2a^2\log\left(bx^2+a\right)}{b^3} + \frac{bx^4-2ax^2}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right)$$
$$+ \frac{\left(b^2x^4 - 6abx^2 + 2a^2\log\left(bx^2+a\right)^2 + 6a^2\log\left(bx^2+a\right)\right)p^2}{8b^2}$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
1/4*x^4*log((b*x^2 + a)^p*c)^2 - 1/4*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^
4 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c) + 1/8*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*
log(b*x^2 + a)^2 + 6*a^2*log(b*x^2 + a))*p^2/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.49

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{2\left(bx^2+a\right)^2 p^2 \log\left(bx^2+a\right)^2 - 2\left(bx^2+a\right)^2 p^2 \log\left(bx^2+a\right) + 4\left(bx^2+a\right)^2 p \log\left(bx^2+a\right)\log\left(c\right) + \left(bx^2+\right.}{8\,b^2}$$
$$- \frac{\left(2\,bx^2 + \left(bx^2+a\right)\log\left(bx^2+a\right)^2 - 2\left(bx^2+a\right)\log\left(bx^2+a\right) + 2\,a\right)ap^2 - 2\left(bx^2 - \left(bx^2+a\right)\log\left(bx^2+\right.\right.}{2\,b^2}$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
1/8*(2*(b*x^2 + a)^2*p^2*log(b*x^2 + a)^2 - 2*(b*x^2 + a)^2*p^2*log(b*x^2
+ a) + 4*(b*x^2 + a)^2*p*log(b*x^2 + a)*log(c) + (b*x^2 + a)^2*p^2 - 2*(b*
x^2 + a)^2*p*log(c) + 2*(b*x^2 + a)^2*log(c)^2)/b^2 - 1/2*((2*b*x^2 + (b*x
^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*a*p^2 - 2*(
b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a*p*log(c) + (b*x^2 + a)*a*log(c)^
2)/b^2
```

Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{p^2\,x^4}{8} - \ln\left(c\left(b\,x^2+a\right)^p\right)\left(\frac{p\,x^4}{4} - \frac{a\,p\,x^2}{2\,b}\right) + \ln\left(c\left(b\,x^2+a\right)^p\right)^2\left(\frac{x^4}{4} - \frac{a^2}{4\,b^2}\right) - \frac{3\,a\,p^2\,x^2}{4\,b} + \frac{3\,a^2\,p^2\,\ln\left(b\,x^2+a\right)}{4\,b^2}$$

input
```
int(x^3*log(c*(a + b*x^2)^p)^2,x)
```

output
```
(p^2*x^4)/8 - log(c*(a + b*x^2)^p)*((p*x^4)/4 - (a*p*x^2)/(2*b)) + log(c*(
a + b*x^2)^p)^2*(x^4/4 - a^2/(4*b^2)) - (3*a*p^2*x^2)/(4*b) + (3*a^2*p^2*l
og(a + b*x^2))/(4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.87

$$\int x^3 \log^2\left(c(a+bx^2)^p\right)\,dx$$
$$= \frac{-2\log\left((b\,x^2+a)^p\,c\right)^2 a^2 + 2\log\left((b\,x^2+a)^p\,c\right)^2 b^2x^4 + 6\log\left((b\,x^2+a)^p\,c\right)a^2p + 4\log\left((b\,x^2+a)^p\,c\right)abp\,x^2}{8b^2}$$

input
```
int(x^3*log(c*(b*x^2+a)^p)^2,x)
```

output
```
( - 2*log((a + b*x**2)**p*c)**2*a**2 + 2*log((a + b*x**2)**p*c)**2*b**2*x*
*4 + 6*log((a + b*x**2)**p*c)*a**2*p + 4*log((a + b*x**2)**p*c)*a*b*p*x**2
 - 2*log((a + b*x**2)**p*c)*b**2*p*x**4 - 6*a*b*p**2*x**2 + b**2*p**2*x**4
)/(8*b**2)
```

3.79 $\int x \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int x \log^2\left(c(a+bx^2)^p\right)\,dx = p^2x^2 - \frac{p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b} + \frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b}$$

output
```
p^2*x^2-p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/
b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int x \log^2\left(c(a+bx^2)^p\right)\,dx = \frac{1}{2}\left(\frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{b} - 2p\left(-px^2 + \frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}\right)\right)$$

input
```
Integrate[x*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
(((a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/b - 2*p*(-(p*x^2) + ((a + b*x^2)*Log
[c*(a + b*x^2)^p])/b))/2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2836, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log^2\left(c\left(a+bx^2\right)^p\right) dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \log^2\left(c\left(bx^2+a\right)^p\right) dx^2$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log^2\left(c\left(bx^2+a\right)^p\right) d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{2733}$$

$$\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right) - 2p\int \log\left(c\left(bx^2+a\right)^p\right) d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{2732}$$

$$\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right) - 2p\left(\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right) - p\left(a+bx^2\right)\right)}{2b}$$

input
```
Int[x*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
((a + b*x^2)*Log[c*(a + b*x^2)^p]^2 - 2*p*(-(p*(a + b*x^2)) + (a + b*x^2)*
Log[c*(a + b*x^2)^p]))/(2*b)
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.72

method	result	size
parallelrisch	$\frac{x^{2} \ln\left(c \left(b \,x^{2}+a \right)^{p}\right)^{2} a b p -2 x^{2} \ln\left(c \left(b \,x^{2}+a \right)^{p}\right) a b \,p^{2}+2 a b \,p^{3} x^{2}+\ln\left(c \left(b \,x^{2}+a \right)^{p}\right)^{2} a^{2} p -2 \ln\left(c \left(b \,x^{2}+a \right)^{p}\right) a^{2} p^{2}}{2 a b p}$	105
risch	Expression too large to display	1034

input
```
int(x*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2*ln(c*(b*x^2+a)^p)^2*a*b*p-2*x^2*ln(c*(b*x^2+a)^p)*a*b*p^2+2*a*b*p
^3*x^2+ln(c*(b*x^2+a)^p)^2*a^2*p-2*ln(c*(b*x^2+a)^p)*a^2*p^2)/a/b/p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{2\,bp^2x^2 - 2\,bpx^2\log(c) + bx^2\log(c)^2 + (bp^2x^2+ap^2)\log(bx^2+a)^2 - 2\,(bp^2x^2+ap^2-(bpx^2+ap)\log(}{2\,b}$$

input
```
integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*p^2*x^2 - 2*b*p*x^2*log(c) + b*x^2*log(c)^2 + (b*p^2*x^2 + a*p^2)
*log(b*x^2 + a)^2 - 2*(b*p^2*x^2 + a*p^2 - (b*p*x^2 + a*p)*log(c))*log(b*x
^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.48

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \begin{cases} -\frac{ap\log\left(c(a+bx^2)^p\right)}{b} + \frac{a\log\left(c(a+bx^2)^p\right)^2}{2b} + p^2x^2 - px^2\log\left(c\left(a+bx^2\right)^p\right) + \frac{x^2\log\left(c(a+bx^2)^p\right)^2}{2} & \text{for } b\neq 0 \\ \frac{x^2\log(a^pc)^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Piecewise((-a*p*log(c*(a + b*x**2)**p)/b + a*log(c*(a + b*x**2)**p)**2/(2*
b) + p**2*x**2 - p*x**2*log(c*(a + b*x**2)**p) + x**2*log(c*(a + b*x**2)**
p)**2/2, Ne(b, 0)), (x**2*log(a**p*c)**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx = -bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right) + \frac{1}{2}x^2\log\left(\left(bx^2+a\right)^pc\right)^2 + \frac{\left(2\,bx^2-a\log\left(bx^2+a\right)^2-2\,a\log\left(bx^2+a\right)\right)p^2}{2\,b}$$

input
```
integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
-b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c) + 1/2*x^2*log((b*
x^2 + a)^p*c)^2 + 1/2*(2*b*x^2 - a*log(b*x^2 + a)^2 - 2*a*log(b*x^2 + a))*
p^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{\left(2\,bx^2+\left(bx^2+a\right)\log\left(bx^2+a\right)^2-2\left(bx^2+a\right)\log\left(bx^2+a\right)+2\,a\right)p^2-2\left(bx^2-\left(bx^2+a\right)\log\left(bx^2+a\right)\right.}{2\,b}$$

input
```
integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
1/2*((2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a
) + 2*a)*p^2 - 2*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c) + (b*x^
2 + a)*log(c)^2)/b
```

Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx = p^2\,x^2+\ln\left(c\,\left(b\,x^2+a\right)^p\right)^2\,\left(\frac{a}{2\,b}+\frac{x^2}{2}\right) - p\,x^2\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)-\frac{a\,p^2\,\ln\left(b\,x^2+a\right)}{b}$$

input
```
int(x*log(c*(a + b*x^2)^p)^2,x)
```

output
```
p^2*x^2 + log(c*(a + b*x^2)^p)^2*(a/(2*b) + x^2/2) - p*x^2*log(c*(a + b*x^
2)^p) - (a*p^2*log(a + b*x^2))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{\log\left(\left(b\,x^2+a\right)^p c\right)^2 a+\log\left(\left(b\,x^2+a\right)^p c\right)^2 b\,x^2-2\log\left(\left(b\,x^2+a\right)^p c\right)ap-2\log\left(\left(b\,x^2+a\right)^p c\right)bp\,x^2+2b\,p^2}{2b}$$

input
```
int(x*log(c*(b*x^2+a)^p)^2,x)
```

output
```
(log((a + b*x**2)**p*c)**2*a + log((a + b*x**2)**p*c)**2*b*x**2 - 2*log((a
 + b*x**2)**p*c)*a*p - 2*log((a + b*x**2)**p*c)*b*p*x**2 + 2*b*p**2*x**2)/
(2*b)
```

3.80 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}\,dx = \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right) + p\log\left(c\left(a+bx^2\right)^p\right)\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right) - p^2\,\text{PolyLog}\left(3,1+\frac{bx^2}{a}\right)$$

output
```
1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^2+p*ln(c*(b*x^2+a)^p)*polylog(2,1+b*x^2
/a)-p^2*polylog(3,1+b*x^2/a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(72) = 144.

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.26

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \log(x)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2 + 2p\Big(-p\log\left(a+bx^2\right) + \log\left(c(a+bx^2)^p\right)\Big)\left(\log(x)\left(\log\left(a+bx^2\right)-\log\left(1+\frac{bx^2}{a}\right)\right) - \frac{1}{2}\operatorname{PolyLog}\left(2,-\frac{bx^2}{a}\right)\right) + \frac{1}{2}p^2\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(a+bx^2\right) + 2\log\left(a+bx^2\right)\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right) - 2\operatorname{PolyLog}\left(3,1+\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^2/x,x]
```

output
```
Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 2*p*(-(p*Log[a + b
*x^2]) + Log[c*(a + b*x^2)^p])*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a
]) - PolyLog[2, -((b*x^2)/a)]/2) + (p^2*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^
2 + 2*Log[a + b*x^2]*PolyLog[2, 1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/
a]))/2
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx$$

$$\Big\downarrow \; 2904$$

$$\frac{1}{2}\int \frac{\log^2\left(c(bx^2+a)^p\right)}{x^2}dx^2$$

$$\Big\downarrow \textbf{2843}$$

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-2bp\int\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx^2\right)$$

$$\Big\downarrow \textbf{2881}$$

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-2p\int\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(bx^2+a\right)^p\right)}{x^2}d\left(bx^2+a\right)\right)$$

$$\Big\downarrow \textbf{2821}$$

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-2p\left(p\int\frac{\text{PolyLog}\left(2,\frac{bx^2+a}{a}\right)}{x^2}d\left(bx^2+a\right)-\text{PolyLog}\left(2,\frac{bx^2+a}{a}\right)\log\left(c\left(a\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{7143}$$

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-2p\left(p\,\text{PolyLog}\left(3,\frac{bx^2+a}{a}\right)-\text{PolyLog}\left(2,\frac{bx^2+a}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)\right)\right)$$

input
```
Int[Log[c*(a + b*x^2)^p]^2/x,x]
```

output
```
(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2 - 2*p*(-(Log[c*(a + b*x^2)^p]*Po
lyLog[2, (a + b*x^2)/a]) + p*PolyLog[3, (a + b*x^2)/a]))/2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^2}{x}dx$$

input
```
int(ln(c*(b*x^2+a)^p)^2/x,x)
```

output
```
int(ln(c*(b*x^2+a)^p)^2/x,x)
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^2/x, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**2/x,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**2/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \frac{1}{2}\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)p^2 + \left(\log\left(bx^2+a\right)\log\left(-\frac{bx^2+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\right)p\log\left(c\right)+\log\left(c\right)^2\log\left(x\right)$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="maxima")
```

output
```
1/2*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*log
(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2 + (log(b*x^2 + a)*log(-(b*x
^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c) + log(c)^2*log(x)
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^2/x,x)
```

output
```
int(log(c*(a + b*x^2)^p)^2/x, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \frac{6\left(\int \frac{\log\left((b\,x^2+a)^p c\right)^2}{b\,x^3+ax}dx\right)ap+\log\left((b\,x^2+a)^p\,c\right)^3}{6p}$$

input
```
int(log(c*(b*x^2+a)^p)^2/x,x)
```

output
```
(6*int(log((a + b*x**2)**p*c)**2/(a*x + b*x**3),x)*a*p + log((a + b*x**2)*
*p*c)**3)/(6*p)
```

3.81 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3} dx = \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} - \frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{bp^2 \operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{a}$$

output

```
b*p*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/a/x
^2+b*p^2*polylog(2,1+b*x^2/a)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3} dx = \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} - \frac{b\log^2\left(c(a+bx^2)^p\right)}{2a} - \frac{\log^2\left(c(a+bx^2)^p\right)}{2x^2} + \frac{bp^2 \operatorname{PolyLog}\left(2,\frac{a+bx^2}{a}\right)}{a}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

output
```
(b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - (b*Log[c*(a + b*x^2)^p]^2
)/(2*a) - Log[c*(a + b*x^2)^p]^2/(2*x^2) + (b*p^2*PolyLog[2, (a + b*x^2)/a
])/a
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2904, 2844, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{x^4}dx^2$$

$$\Big\downarrow \text{2844}$$

$$\frac{1}{2}\left(\frac{2bp\int \frac{\log\left(c\left(bx^2+a\right)^p\right)}{x^2}dx^2}{a}-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{ax^2}\right)$$

$$\Big\downarrow \text{2841}$$

$$\frac{1}{2}\left(\frac{2bp\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)-bp\int \frac{\log\left(-\frac{bx^2}{a}\right)}{bx^2+a}dx^2\right)}{a}-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{ax^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}\left(\frac{2bp\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)+p\,\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\right)}{a}-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{ax^2}\right)$$

input
```
Int[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

output
```
(-(((a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(a*x^2)) + (2*b*p*(Log[-((b*x^2)/a
)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 + (b*x^2)/a]))/a)/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 481, normalized size of antiderivative = 6.01

method	result
risch	$-\frac{\ln((b x^2+a)^p)^2}{2x^2}-\frac{pb\ln((b x^2+a)^p)\ln(b x^2+a)}{a}+\frac{2pb\ln((b x^2+a)^p)\ln(x)}{a}-\frac{2p^2b\ln(x)\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{a}-\frac{2p^2b\ln(x)\ln(}{a}$

input
```
int(ln(c*(b*x^2+a)^p)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln((b*x^2+a)^p)^2/x^2-p*b*ln((b*x^2+a)^p)/a*ln(b*x^2+a)+2*p*b*ln((b*x
^2+a)^p)/a*ln(x)-2*p^2*b/a*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2*p^
2*b/a*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2*p^2*b/a*dilog((-b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))-2*p^2*b/a*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1
/2*p^2*b/a*ln(b*x^2+a)^2+(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b
*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))*(-1/2/x^2*ln(
(b*x^2+a)^p)+p*b*(-1/2/a*ln(b*x^2+a)+ln(x)/a))-1/8*(I*Pi*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*
csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I
*c)+2*ln(c))^2/x^2
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^3}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^2/x^3, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^3}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**2/x**3,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**2/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{1}{2}b^2p^2\left(\frac{\log\left(bx^2+a\right)^2}{ab} - \frac{2\left(2\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{ab}\right) - bp\left(\frac{\log\left(bx^2+a\right)}{a} - \frac{\log\left(x^2\right)}{a}\right)\log\left(\left(bx^2+a\right)^pc\right) - \frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{2\,x^2}$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="maxima")
```

output
```
1/2*b^2*p^2*(log(b*x^2 + a)^2/(a*b) - 2*(2*log(b*x^2/a + 1)*log(x) + dilog
(-b*x^2/a))/(a*b)) - b*p*(log(b*x^2 + a)/a - log(x^2)/a)*log((b*x^2 + a)^p
*c) - 1/2*log((b*x^2 + a)^p*c)^2/x^2
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^3}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^3}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^2/x^3,x)
```

output
```
int(log(c*(a + b*x^2)^p)^2/x^3, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{-4\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^5+a\,x^3}dx\right) a^2 p\,x^2 - \log\left((b\,x^2+a)^p\,c\right)^2 a - 2\log\left((b\,x^2+a)^p\,c\right) ap - 2\log\left((b\,x^2+a)^p\,c\right) bp\,x^2}{2a\,x^2}$$

input
```
int(log(c*(b*x^2+a)^p)^2/x^3,x)
```

output
```
( - 4*int(log((a + b*x**2)**p*c)/(a*x**3 + b*x**5),x)*a**2*p*x**2 - log((a
 + b*x**2)**p*c)**2*a - 2*log((a + b*x**2)**p*c)*a*p - 2*log((a + b*x**2)*
*p*c)*b*p*x**2 + 4*log(x)*b*p**2*x**2)/(2*a*x**2)
```

3.82 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 129

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx = \frac{b^2p^2\log(x)}{a^2} - \frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} - \frac{b^2p\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^2} + \frac{b^2p^2\operatorname{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^2}$$

output
```
b^2*p^2*ln(x)/a^2-1/2*b*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^2/x^2-1/4*ln(c*(b*
x^2+a)^p)^2/x^4-1/2*b^2*p*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^2+1/2*b^2*
p^2*polylog(2,a/(b*x^2+a))/a^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.09

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$$

$$= \frac{-\log^2\left(c(a+bx^2)^p\right) + \frac{bx^2 (4bp^2x^2\log(x) - 2bp^2x^2\log(a+bx^2) - 2ap\log(c(a+bx^2)^p) + bx^2\log^2(c(a+bx^2)^p) - 2bpx^2 (\log (-\frac{bx^2}{a}) \log(c}{a^2}}{4x^4}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

output
```
(-Log[c*(a + b*x^2)^p]^2 + (b*x^2*(4*b*p^2*x^2*Log[x] - 2*b*p^2*x^2*Log[a
+ b*x^2] - 2*a*p*Log[c*(a + b*x^2)^p] + b*x^2*Log[c*(a + b*x^2)^p]^2 - 2*b
*p*x^2*(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 + (b*x^2)/
a])))/a^2)/(4*x^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2904, 2845, 2858, 27, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{x^6}dx^2$$

$$\Big\downarrow \text{2845}$$

$$\frac{1}{2}\left(bp\int \frac{\log\left(c\left(bx^2+a\right)^p\right)}{x^4\left(bx^2+a\right)}dx^2-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{2}\left(p\int \frac{\log\left(c\left(bx^2+a\right)^p\right)}{x^6}d\left(bx^2+a\right)-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(b^2p\int \frac{\log\left(c\left(bx^2+a\right)^p\right)}{b^2x^6}d\left(bx^2+a\right)-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{2789}$$

$$\frac{1}{2}\left(b^2p\left(\frac{\int \frac{\log(c(bx^2+a)^p)}{b^2x^4}d(bx^2+a)}{a}+\frac{\int -\frac{\log(c(bx^2+a)^p)}{bx^4}d(bx^2+a)}{a}\right)-\frac{\log^2\left(c(a+bx^2)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \; 2751$$

$$\frac{1}{2}\left(b^2p\left(\frac{-\frac{p\int -\frac{1}{bx^2}d(bx^2+a)}{a}-\frac{(a+bx^2)\log(c(a+bx^2)^p)}{abx^2}}{a}+\frac{\int -\frac{\log(c(bx^2+a)^p)}{bx^4}d(bx^2+a)}{a}\right)-\frac{\log^2\left(c(a+bx^2)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(b^2p\left(\frac{\int -\frac{\log(c(bx^2+a)^p)}{bx^4}d(bx^2+a)}{a}+\frac{\frac{p\log(-bx^2)}{a}-\frac{(a+bx^2)\log(c(a+bx^2)^p)}{abx^2}}{a}\right)-\frac{\log^2\left(c(a+bx^2)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \; 2779$$

$$\frac{1}{2}\left(b^2p\left(\frac{\frac{p\int \frac{\log\left(1-\frac{a}{x^2}\right)}{x^2}d(bx^2+a)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log(c(a+bx^2)^p)}{a}}{a}+\frac{\frac{p\log(-bx^2)}{a}-\frac{(a+bx^2)\log(c(a+bx^2)^p)}{abx^2}}{a}\right)-\frac{\log^2\left(c(a+bx^2)\right.}{2x^4}\right.$$

$$\Big\downarrow \; 2838$$

$$\frac{1}{2}\left(b^2p\left(\frac{\frac{p\,\text{PolyLog}\left(2,\frac{a}{x^2}\right)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log(c(a+bx^2)^p)}{a}}{a}+\frac{\frac{p\log(-bx^2)}{a}-\frac{(a+bx^2)\log(c(a+bx^2)^p)}{abx^2}}{a}\right)-\frac{\log^2\left(c(a+bx^2)^p\right)}{2x^4}\right)$$

input
```
Int[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

output
```
(-1/2*Log[c*(a + b*x^2)^p]^2/x^4 + b^2*p*(((p*Log[-(b*x^2)])/a - ((a + b*x
^2)*Log[c*(a + b*x^2)^p])/(a*b*x^2))/a + (-((Log[1 - a/x^2]*Log[c*(a + b*x
^2)^p])/a) + (p*PolyLog[2, a/x^2])/a)/a))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 554, normalized size of antiderivative = 4.29

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2}}{4x^{4}}+\frac{p\,b^{2}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln\left(b\,x^{2}+a\right)}{2a^{2}}-\frac{pb\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{2a\,x^{2}}-\frac{p\,b^{2}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln(x)}{a^{2}}-\frac{p^{2}b^{2}\ln\left(b\,x^{2}+a\right)^{2}}{4a^{2}}-$

input
```
int(ln(c*(b*x^2+a)^p)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln((b*x^2+a)^p)^2/x^4+1/2*p*b^2*ln((b*x^2+a)^p)/a^2*ln(b*x^2+a)-1/2*p
*b*ln((b*x^2+a)^p)/a/x^2-p*b^2*ln((b*x^2+a)^p)/a^2*ln(x)-1/4*p^2*b^2/a^2*l
n(b*x^2+a)^2-1/2*p^2*b^2/a^2*ln(b*x^2+a)+b^2*p^2*ln(x)/a^2+p^2*b^2/a^2*ln(
x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+p^2*b^2/a^2*ln(x)*ln((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))+p^2*b^2/a^2*dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+p^
2*b^2/a^2*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+(I*Pi*csgn(I*(b*x^2+a)^p)
*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c
)+2*ln(c))*(-1/4/x^4*ln((b*x^2+a)^p)+1/2*p*b*(1/2*b/a^2*ln(b*x^2+a)-1/2/a/
x^2-1/a^2*b*ln(x)))-1/16*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2
-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b
*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^4
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^5}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^2/x^5, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^5}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**2/x**5,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**2/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{1}{4}b^2p^2\left(\frac{\log\left(bx^2+a\right)^2}{a^2} - \frac{2\left(2\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{a^2} + \frac{2\log\left(bx^2+a\right)}{a^2} - \frac{4\log\left(x\right)}{a^2}\right) + \frac{1}{2}bp\left(\frac{b\log\left(bx^2+a\right)}{a^2} - \frac{b\log\left(x^2\right)}{a^2} - \frac{1}{ax^2}\right)\log\left(\left(bx^2+a\right)^pc\right) - \frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{4\,x^4}$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="maxima")
```

output
```
-1/4*b^2*p^2*(log(b*x^2 + a)^2/a^2 - 2*(2*log(b*x^2/a + 1)*log(x) + dilog(
-b*x^2/a))/a^2 + 2*log(b*x^2 + a)/a^2 - 4*log(x)/a^2) + 1/2*b*p*(b*log(b*x
^2 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2))*log((b*x^2 + a)^p*c) - 1/4*log((
b*x^2 + a)^p*c)^2/x^4
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{x^5}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^5}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^2/x^5,x)
```

output
```
int(log(c*(a + b*x^2)^p)^2/x^5, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \frac{-4\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^3+ax}dx\right) a\,b^2 p\,x^4 - \log\left((b\,x^2+a)^p\,c\right)^2 a^2 - 2\log\left((b\,x^2+a)^p\,c\right) abp\,x^2 - 2\log\left((b\,x^2+a)^p\,c\right.}{4a^2x^4}$$

input
```
int(log(c*(b*x^2+a)^p)^2/x^5,x)
```

output
```
( - 4*int(log((a + b*x**2)**p*c)/(a*x + b*x**3),x)*a*b**2*p*x**4 - log((a
+ b*x**2)**p*c)**2*a**2 - 2*log((a + b*x**2)**p*c)*a*b*p*x**2 - 2*log((a +
 b*x**2)**p*c)*b**2*p*x**4 + 4*log(x)*b**2*p**2*x**4)/(4*a**2*x**4)
```

3.83 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 193

$$\begin{aligned}\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = &-\frac{b^2p^2}{6a^2x^2}-\frac{b^3p^2\log(x)}{a^3}+\frac{b^3p^2\log\left(a+bx^2\right)}{6a^3}\\ &-\frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4}+\frac{b^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3a^3x^2}\\ &-\frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6}+\frac{b^3p\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{3a^3}\\ &-\frac{b^3p^2\,\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{3a^3}\end{aligned}$$

output
```
-1/6*b^2*p^2/a^2/x^2-b^3*p^2*ln(x)/a^3+1/6*b^3*p^2*ln(b*x^2+a)/a^3-1/6*b*p
*ln(c*(b*x^2+a)^p)/a/x^4+1/3*b^2*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^3/x^2-1/6
*ln(c*(b*x^2+a)^p)^2/x^6+1/3*b^3*p*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^3
-1/3*b^3*p^2*polylog(2,a/(b*x^2+a))/a^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx =$$
$$-\frac{ab^2p^2x^4+6b^3p^2x^6\log(x)-3b^3p^2x^6\log\left(a+bx^2\right)+a^2bpx^2\log\left(c(a+bx^2)^p\right)-2ab^2px^4\log\left(c(a+bx^2\right)}{}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

output
```
-1/6*(a*b^2*p^2*x^4 + 6*b^3*p^2*x^6*Log[x] - 3*b^3*p^2*x^6*Log[a + b*x^2]
+ a^2*b*p*x^2*Log[c*(a + b*x^2)^p] - 2*a*b^2*p*x^4*Log[c*(a + b*x^2)^p] -
2*b^3*p*x^6*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + a^3*Log[c*(a + b*x^2)
^p]^2 + b^3*x^6*Log[c*(a + b*x^2)^p]^2 - 2*b^3*p^2*x^6*PolyLog[2, 1 + (b*x
^2)/a])/(a^3*x^6)
```

Rubi [A] (warning: unable to verify)

Time = 1.41 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {2904, 2845, 2858, 25, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{\log^2\left(c(bx^2+a)^p\right)}{x^8}dx^2$$

$$\Big\downarrow \text{2845}$$

$$\frac{1}{2}\left(\frac{2}{3}bp\int \frac{\log\left(c(bx^2+a)^p\right)}{x^6\left(bx^2+a\right)}dx^2-\frac{\log^2\left(c(a+bx^2)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{2}\left(\frac{2}{3}p\int \frac{\log \left(c\left(bx^2+a\right)^p\right)}{x^8}d\left(bx^2+a\right)-\frac{\log ^2\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{2}{3}p\int -\frac{\log \left(c\left(bx^2+a\right)^p\right)}{x^8}d\left(bx^2+a\right)-\frac{\log ^2\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\int -\frac{\log \left(c\left(bx^2+a\right)^p\right)}{b^3x^8}d\left(bx^2+a\right)-\frac{\log ^2\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{2789}$$

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\left(\frac{\int -\frac{\log \left(c\left(bx^2+a\right)^p\right)}{b^3x^6}d\left(bx^2+a\right)}{a}+\frac{\int \frac{\log \left(c\left(bx^2+a\right)^p\right)}{b^2x^6}d\left(bx^2+a\right)}{a}\right)-\frac{\log ^2\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{2756}$$

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\left(\frac{\int \frac{\log \left(c\left(bx^2+a\right)^p\right)}{b^2x^6}d\left(bx^2+a\right)}{a}+\frac{\frac{\log \left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-\frac{1}{2}p\int \frac{1}{b^2x^6}d\left(bx^2+a\right)}{a}\right)-\frac{\log ^2\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\left(\frac{\frac{\log \left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-\frac{1}{2}p\int \left(\frac{1}{b^2x^4a}-\frac{1}{bx^2a^2}+\frac{1}{x^2a^2}\right)d\left(bx^2+a\right)}{a}+\frac{\int \frac{\log \left(c\left(bx^2+a\right)^p\right)}{b^2x^6}d\left(bx^2+a\right)}{a}\right)-\frac{\log ^2\left(c\left(a-\right.\right.}{3x}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\left(\frac{\int \frac{\log \left(c\left(bx^2+a\right)^p\right)}{b^2x^6}d\left(bx^2+a\right)}{a}+\frac{\frac{\log \left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-\frac{1}{2}p\left(-\frac{\log \left(-bx^2\right)}{a^2}+\frac{\log \left(a+bx^2\right)}{a^2}-\frac{1}{abx^2}\right)}{a}\right)-\frac{\log ^2\left(c\left(a+b\right.\right.}{3x^6}\right.$$

$$\Big\downarrow \text{2789}$$

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\left(\frac{\frac{\int \frac{\log \left(c\left(bx^2+a\right)^p\right)}{b^2x^4}d\left(bx^2+a\right)}{a}+\frac{\int -\frac{\log \left(c\left(bx^2+a\right)^p\right)}{bx^4}d\left(bx^2+a\right)}{a}}{a}+\frac{\frac{\log \left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-\frac{1}{2}p\left(-\frac{\log \left(-bx^2\right)}{a^2}+\frac{\log \left(a+bx^2\right)}{a^2}-\frac{}{ab}\right.}{a}\right.\right.$$

$\downarrow$ 2751

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\left(\frac{\frac{-\frac{p\int -\frac{1}{bx^2}d(bx^2+a)}{a}-\frac{(a+bx^2)\log(c(a+bx^2)^p)}{abx^2}}{a}+\frac{\int -\frac{\log(c(bx^2+a)^p)}{bx^4}d(bx^2+a)}{a}}{a}+\frac{\frac{\log(c(a+bx^2)^p)}{2b^2x^4}-\frac{1}{2}p\left(-\frac{\log(-bx^2)}{a^2}+\right.}{a}\right.\right.$$

$\downarrow$ 16

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\left(\frac{\frac{\int -\frac{\log(c(bx^2+a)^p)}{bx^4}d(bx^2+a)}{a}+\frac{\frac{p\log(-bx^2)}{a}-\frac{(a+bx^2)\log(c(a+bx^2)^p)}{abx^2}}{a}}{a}+\frac{\frac{\log(c(a+bx^2)^p)}{2b^2x^4}-\frac{1}{2}p\left(-\frac{\log(-bx^2)}{a^2}+\frac{\log(a+}{a^2}\right.}{a}\right.\right.$$

$\downarrow$ 2779

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\left(\frac{\frac{\frac{p\int\frac{\log\left(1-\frac{a}{x^2}\right)}{x^2}d(bx^2+a)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log(c(a+bx^2)^p)}{a}}{a}+\frac{\frac{p\log(-bx^2)}{a}-\frac{(a+bx^2)\log(c(a+bx^2)^p)}{abx^2}}{a}}{a}+\frac{\frac{\log(c(a+bx^2)^p)}{2b^2x^4}-\frac{1}{2}p\left(-\right.}{}\right.\right.$$

$\downarrow$ 2838

$$\frac{1}{2}\left(-\frac{2}{3}b^3p\left(\frac{\frac{\log(c(a+bx^2)^p)}{2b^2x^4}-\frac{1}{2}p\left(-\frac{\log(-bx^2)}{a^2}+\frac{\log(a+bx^2)}{a^2}-\frac{1}{abx^2}\right)}{a}+\frac{\frac{\frac{p\,\text{PolyLog}\left(2,\frac{a}{x^2}\right)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log(c(a+bx^2)^p)}{a}}{a}+\frac{p\log(-}{a}}{a}\right.\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

output
```
(-1/3*Log[c*(a + b*x^2)^p]^2/x^6 - (2*b^3*p*((-1/2*(p*(-(1/(a*b*x^2)) - Lo
g[-(b*x^2)]/a^2 + Log[a + b*x^2]/a^2)) + Log[c*(a + b*x^2)^p]/(2*b^2*x^4))
/a + (((p*Log[-(b*x^2)])/a - ((a + b*x^2)*Log[c*(a + b*x^2)^p])/(a*b*x^2))
/a + (-((Log[1 - a/x^2]*Log[c*(a + b*x^2)^p])/a) + (p*PolyLog[2, a/x^2])/a
)/a)/a))/3)/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.81 (sec) , antiderivative size = 607, normalized size of antiderivative = 3.15

method	result
risch	$-\frac{\ln\left((b\,x^2+a)^p\right)^2}{6x^6}-\frac{p\,b^3\ln\left((b\,x^2+a)^p\right)\ln(b\,x^2+a)}{3a^3}-\frac{pb\ln\left((b\,x^2+a)^p\right)}{6a\,x^4}+\frac{2p\,b^3\ln\left((b\,x^2+a)^p\right)\ln(x)}{3a^3}+\frac{p\,b^2\ln\left((b\,x^2+a)^p\right)}{3a^2x^2}$

input
```
int(ln(c*(b*x^2+a)^p)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln((b*x^2+a)^p)^2/x^6-1/3*p*b^3*ln((b*x^2+a)^p)/a^3*ln(b*x^2+a)-1/6*p
*b*ln((b*x^2+a)^p)/a/x^4+2/3*p*b^3*ln((b*x^2+a)^p)/a^3*ln(x)+1/3*p*b^2*ln(
(b*x^2+a)^p)/a^2/x^2+1/2*b^3*p^2*ln(b*x^2+a)/a^3-1/6*b^2*p^2/a^2/x^2-b^3*p
^2*ln(x)/a^3-2/3*p^2*b^3/a^3*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/
3*p^2*b^3/a^3*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/3*p^2*b^3/a^3*di
log((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/3*p^2*b^3/a^3*dilog((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))+1/6*p^2*b^3/a^3*ln(b*x^2+a)^2+(I*Pi*csgn(I*(b*x^2+a)^p)
*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c
)+2*ln(c))*(-1/6/x^6*ln((b*x^2+a)^p)+1/3*p*b*(-1/2*b^2/a^3*ln(b*x^2+a)-1/4
/a/x^4+b^2/a^3*ln(x)+1/2*b/a^2/x^2))-1/24*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)
-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(
c))^2/x^6
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx=\int \frac{\log\left((bx^2+a)^pc\right)^2}{x^7}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^2/x^7, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^7}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**2/x**7,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**2/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.90

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{1}{6}b^2p^2\left(\frac{2\left(2\log\left(\frac{bx^2}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)b}{a^3} - \frac{3b\log(bx^2+a)}{a^3} - \frac{bx^2\log(bx^2+a)^2-6bx^2\log$$
$$- \frac{1}{6}bp\left(\frac{2b^2\log(bx^2+a)}{a^3} - \frac{2b^2\log(x^2)}{a^3} - \frac{2bx^2-a}{a^2x^4}\right)\log\left((bx^2+a)^pc\right) - \frac{\log\left((bx^2+a)^pc\right)^2}{6x^6}$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="maxima")
```

output
```
-1/6*b^2*p^2*(2*(2*log(b*x^2/a + 1)*log(x) + dilog(-b*x^2/a))*b/a^3 - 3*b*
log(b*x^2 + a)/a^3 - (b*x^2*log(b*x^2 + a)^2 - 6*b*x^2*log(x) - a)/(a^3*x^
2)) - 1/6*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 -
a)/(a^2*x^4))*log((b*x^2 + a)^p*c) - 1/6*log((b*x^2 + a)^p*c)^2/x^6
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^7}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^2/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^7}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^2/x^7,x)
```

output
```
int(log(c*(a + b*x^2)^p)^2/x^7, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = \frac{4\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^3+ax}dx\right) a\,b^3 p\,x^6 - \log\left((b\,x^2+a)^p\,c\right)^2 a^3 - \log\left((b\,x^2+a)^p\,c\right) a^2 b p\,x^2 + 2\log\left((b\,x^2+a)^p\,c\right) a}{6a^3x^6}$$

input
```
int(log(c*(b*x^2+a)^p)^2/x^7,x)
```

output
```
(4*int(log((a + b*x**2)**p*c)/(a*x + b*x**3),x)*a*b**3*p*x**6 - log((a + b
*x**2)**p*c)**2*a**3 - log((a + b*x**2)**p*c)*a**2*b*p*x**2 + 2*log((a + b
*x**2)**p*c)*a*b**2*p*x**4 + 3*log((a + b*x**2)**p*c)*b**3*p*x**6 - 6*log(
x)*b**3*p**2*x**6 - a*b**2*p**2*x**4)/(6*a**3*x**6)
```

3.84 $\int x^4 \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 336

$$\int x^4\log^2\left(c(a+bx^2)^p\right)\,dx = \frac{184a^2p^2x}{75b^2}-\frac{64ap^2x^3}{225b}+\frac{8p^2x^5}{125}-\frac{184a^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}}+\frac{4ia^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}}+\frac{8a^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5b^{5/2}}-\frac{4a^2px\log\left(c(a+bx^2)^p\right)}{5b^2}+\frac{4apx^3\log\left(c(a+bx^2)^p\right)}{15b}-\frac{4}{25}px^5\log\left(c(a+bx^2)^p\right)+\frac{4a^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5b^{5/2}}+\frac{1}{5}x^5\log^2\left(c(a+bx^2)^p\right)+\frac{4ia^{5/2}p^2\,\mathrm{Pol}}{}$$

output
```
184/75*a^2*p^2*x/b^2-64/225*a*p^2*x^3/b+8/125*p^2*x^5-184/75*a^(5/2)*p^2*a
rctan(b^(1/2)*x/a^(1/2))/b^(5/2)+4/5*I*a^(5/2)*p^2*arctan(b^(1/2)*x/a^(1/2
))^2/b^(5/2)+8/5*a^(5/2)*p^2*arctan(b^(1/2)*x/a^(1/2))*ln(2*a^(1/2)/(a^(1/
2)+I*b^(1/2)*x))/b^(5/2)-4/5*a^2*p*x*ln(c*(b*x^2+a)^p)/b^2+4/15*a*p*x^3*ln
(c*(b*x^2+a)^p)/b-4/25*p*x^5*ln(c*(b*x^2+a)^p)+4/5*a^(5/2)*p*arctan(b^(1/2
)*x/a^(1/2))*ln(c*(b*x^2+a)^p)/b^(5/2)+1/5*x^5*ln(c*(b*x^2+a)^p)^2+4/5*I*a
^(5/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.74

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{900ia^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2+60a^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-46p+30p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)+15\log\left(c\left(a+bx^2\right)^p\right)\right)+}{}$$

input
```
Integrate[x^4*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
((900*I)*a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 60*a^(5/2)*p*ArcTan[(
Sqrt[b]*x)/Sqrt[a]]*(-46*p + 30*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)]
 + 15*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(345*a^2 - 40*a*b*x^2 + 9*b
^2*x^4) - 60*p*(15*a^2 - 5*a*b*x^2 + 3*b^2*x^4)*Log[c*(a + b*x^2)^p] + 225
*b^2*x^4*Log[c*(a + b*x^2)^p]^2) + (900*I)*a^(5/2)*p^2*PolyLog[2, (I*Sqrt[
a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(1125*b^(5/2))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{ 2907}$$

$$\frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{4}{5}bp\int \frac{x^6\log\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx$$

$$\Big\downarrow \text{ 2926}$$

$$\frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right)-$$
$$\frac{4}{5}bp\int\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)x^4}{b}-\frac{a\log\left(c\left(bx^2+a\right)^p\right)x^2}{b^2}-\frac{a^3\log\left(c\left(bx^2+a\right)^p\right)}{b^3\left(bx^2+a\right)}+\frac{a^2\log\left(c\left(bx^2+a\right)^p\right)}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right)-$$
$$\frac{4}{5}bp\left(-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^{7/2}}-\frac{ia^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{b^{7/2}}+\frac{46a^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15b^{7/2}}-\frac{2a^{5/2}p\arctan\left(\right.}{}\right.$$

input
```
Int[x^4*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
(x^5*Log[c*(a + b*x^2)^p]^2)/5 - (4*b*p*((-46*a^2*p*x)/(15*b^3) + (16*a*p*
x^3)/(45*b^2) - (2*p*x^5)/(25*b) + (46*a^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(15*b^(7/2)) - (I*a^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/b^(7/2) - (
2*a^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[
b]*x)])/b^(7/2) + (a^2*x*Log[c*(a + b*x^2)^p])/b^3 - (a*x^3*Log[c*(a + b*x
^2)^p])/(3*b^2) + (x^5*Log[c*(a + b*x^2)^p])/(5*b) - (a^(5/2)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/b^(7/2) - (I*a^(5/2)*p*PolyLog[2, 1
- (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/b^(7/2)))/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
 + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.82

method	result
risch	$\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2} x^{5}}{5}-\frac{4p\,x^{5} \ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{25}+\frac{4pa\,x^{3} \ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{15b}-\frac{4p\,a^{2} x \ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{5b^{2}}-\frac{4p^{2} a^{3} \arctan\left(\frac{bx}{\sqrt{ab}}\right) \ln\left(b\,x^{2}+a\right.}{5b^{2}\sqrt{ab}}$

input
```
int(x^4*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*ln((b*x^2+a)^p)^2*x^5-4/25*p*x^5*ln((b*x^2+a)^p)+4/15*p/b*a*x^3*ln((b*
x^2+a)^p)-4/5*p/b^2*a^2*x*ln((b*x^2+a)^p)-4/5*p^2/b^2*a^3/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))*ln(b*x^2+a)+4/5*p/b^2*a^3/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))*ln((b*x^2+a)^p)+8/125*p^2*x^5-64/225*a*p^2*x^3/b+184/75*a^2*p^2*x/
b^2-184/75*p^2/b^2*a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-4/5*p^2*b*Sum(-
1/2*(ln(x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alpha/b*ln(x-_alpha)^2+1/2*_alpha/
a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/a*dilog(1/2*(x+_alpha)
/_alpha)))*a^3/b^4/_alpha,_alpha=RootOf(_Z^2*b+a))+(I*Pi*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*
csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I
*c)+2*ln(c))*(1/5*x^5*ln((b*x^2+a)^p)-2/5*p*b*(1/b^3*(1/5*b^2*x^5-1/3*a*b*
x^3+a^2*x)-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))+1/20*(I*Pi*csgn(I
*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b
*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^
p)^2*csgn(I*c)+2*ln(c))^2*x^5
```

Fricas [F]

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right) dx = \int x^4 \log\left(\left(bx^2+a\right)^p c\right)^2 dx$$

input
```
integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
integral(x^4*log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F]

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right) dx = \int x^4 \log\left(c\left(a+bx^2\right)^p\right)^2 dx$$

input
```
integrate(x**4*ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(x**4*log(c*(a + b*x**2)**p)**2, x)
```

Maxima [F]

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right) dx = \int x^4 \log\left(\left(bx^2+a\right)^p c\right)^2 dx$$

input
```
integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
1/5*p^2*x^5*log(b*x^2 + a)^2 + integrate(1/5*(5*b*x^6*log(c)^2 + 5*a*x^4*l
og(c)^2 - 2*((2*p^2 - 5*p*log(c))*b*x^6 - 5*a*p*x^4*log(c))*log(b*x^2 + a)
)/(b*x^2 + a), x)
```

Giac [F]

$$\int x^4 \log^2\left(c(a+bx^2)^p\right)\,dx = \int x^4 \log\left((bx^2+a)^p c\right)^2\,dx$$

input
```
integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
integrate(x^4*log((b*x^2 + a)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \log^2\left(c(a+bx^2)^p\right)\,dx = \int x^4 \ln\left(c\left(b\,x^2+a\right)^p\right)^2\,dx$$

input
```
int(x^4*log(c*(a + b*x^2)^p)^2,x)
```

output
```
int(x^4*log(c*(a + b*x^2)^p)^2, x)
```

Reduce [F]

$$\int x^4 \log^2\left(c(a+bx^2)^p\right)\,dx$$
$$= \frac{-2760\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2p^2 + 900\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^2+a}dx\right)a^3bp + 225\log\left((b\,x^2+a)^p\,c\right)^2 b^3x^5 - 900\log\left((\right.}{}$$

input
```
int(x^4*log(c*(b*x^2+a)^p)^2,x)
```

output
```
( - 2760*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*p**2 + 900*int
(log((a + b*x**2)**p*c)/(a + b*x**2),x)*a**3*b*p + 225*log((a + b*x**2)**p
*c)**2*b**3*x**5 - 900*log((a + b*x**2)**p*c)*a**2*b*p*x + 300*log((a + b*
x**2)**p*c)*a*b**2*p*x**3 - 180*log((a + b*x**2)**p*c)*b**3*p*x**5 + 2760*
a**2*b*p**2*x - 320*a*b**2*p**2*x**3 + 72*b**3*p**2*x**5)/(1125*b**3)
```

3.85 $\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 294

$$\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = -\frac{32ap^2x}{9b}+\frac{8p^2x^3}{27}+\frac{32a^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}}-\frac{4ia^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}-\frac{8a^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}}+\frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b}-\frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{4a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}}+\frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{4ia^{3/2}p^2\,\mathrm{Poly}$$

output
```
-32/9*a*p^2*x/b+8/27*p^2*x^3+32/9*a^(3/2)*p^2*arctan(b^(1/2)*x/a^(1/2))/b^
(3/2)-4/3*I*a^(3/2)*p^2*arctan(b^(1/2)*x/a^(1/2))^2/b^(3/2)-8/3*a^(3/2)*p^
2*arctan(b^(1/2)*x/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/b^(3/2)+4/
3*a*p*x*ln(c*(b*x^2+a)^p)/b-4/9*p*x^3*ln(c*(b*x^2+a)^p)-4/3*a^(3/2)*p*arct
an(b^(1/2)*x/a^(1/2))*ln(c*(b*x^2+a)^p)/b^(3/2)+1/3*x^3*ln(c*(b*x^2+a)^p)^
2-4/3*I*a^(3/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.76

$$\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{-36ia^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 - 12a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-8p+6p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)+3\log\left(c\left(a+bx^2\right)^p\right)\right)+\sqrt{}}{}$$

input
```
Integrate[x^2*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
((-36*I)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 12*a^(3/2)*p*ArcTan[(
Sqrt[b]*x)/Sqrt[a]]*(-8*p + 6*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] +
 3*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(-12*a + b*x^2) + 12*p*(3*a -
b*x^2)*Log[c*(a + b*x^2)^p] + 9*b*x^2*Log[c*(a + b*x^2)^p]^2) - (36*I)*a^(
3/2)*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(
27*b^(3/2))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{2907}$$

$$\frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{4}{3}bp\int \frac{x^4\log\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx$$

$$\Big\downarrow \text{2926}$$

$$\frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-$$
$$\frac{4}{3}bp\int\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)a^2}{b^2\left(bx^2+a\right)}-\frac{\log\left(c\left(bx^2+a\right)^p\right)a}{b^2}+\frac{x^2\log\left(c\left(bx^2+a\right)^p\right)}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-$$
$$\frac{4}{3}bp\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^{5/2}}+\frac{ia^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{b^{5/2}}-\frac{8a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right.}{b^{5/}}\right.$$

input
```
Int[x^2*Log[c*(a + b*x^2)^p]^2,x]
```

output
```
(x^3*Log[c*(a + b*x^2)^p]^2)/3 - (4*b*p*((8*a*p*x)/(3*b^2) - (2*p*x^3)/(9*
b) - (8*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(5/2)) + (I*a^(3/2)*p*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/b^(5/2) + (2*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/S
qrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/b^(5/2) - (a*x*Log[c*(a
+ b*x^2)^p])/b^2 + (x^3*Log[c*(a + b*x^2)^p])/(3*b) + (a^(3/2)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/b^(5/2) + (I*a^(3/2)*p*PolyLog[2, 1
 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/b^(5/2)))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
 + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.92

method	result
risch	$\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2} x^{3}}{3}-\frac{4p\,x^{3}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{9}+\frac{4pax\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{3b}+\frac{4p^{2}a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^{2}+a\right)}{3b\sqrt{ab}}-\frac{4p\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln}{3b\sqrt{ab}}$

input
```
int(x^2*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*ln((b*x^2+a)^p)^2*x^3-4/9*p*x^3*ln((b*x^2+a)^p)+4/3*p/b*a*x*ln((b*x^2+
a)^p)+4/3*p^2/b*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln(b*x^2+a)-4/3*p/
b*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln((b*x^2+a)^p)+8/27*p^2*x^3-32/
9*a*p^2*x/b+32/9*p^2/b*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-4/3*p^2*b*S
um(1/2*(ln(x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alpha/b*ln(x-_alpha)^2+1/2*_alp
ha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/a*dilog(1/2*(x+_alp
ha)/_alpha)))*a^2/b^3/_alpha,_alpha=RootOf(_Z^2*b+a))+(I*Pi*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csg
n(I*c)+2*ln(c))*(1/3*x^3*ln((b*x^2+a)^p)-2/3*p*b*(1/b^2*(1/3*b*x^3-a*x)+a^
2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))+1/12*(I*Pi*csgn(I*(b*x^2+a)^p)
*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c
)+2*ln(c))^2*x^3
```

Fricas [F]

$$\int x^2 \log^2 \left(c\left(a+bx^2\right)^p\right) \, dx = \int x^2 \log \left(\left(bx^2+a\right)^p c\right)^2 \, dx$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
integral(x^2*log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F]

$$\int x^2 \log^2 \left(c\left(a+bx^2\right)^p\right) \, dx = \int x^2 \log \left(c\left(a+bx^2\right)^p\right)^2 dx$$

input
```
integrate(x**2*ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(x**2*log(c*(a + b*x**2)**p)**2, x)
```

Maxima [F]

$$\int x^2 \log^2 \left(c\left(a+bx^2\right)^p\right) \, dx = \int x^2 \log \left(\left(bx^2+a\right)^p c\right)^2 \, dx$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
1/3*p^2*x^3*log(b*x^2 + a)^2 + integrate(1/3*(3*b*x^4*log(c)^2 + 3*a*x^2*l
og(c)^2 - 2*((2*p^2 - 3*p*log(c))*b*x^4 - 3*a*p*x^2*log(c))*log(b*x^2 + a)
)/(b*x^2 + a), x)
```

Giac [F]

$$\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx=\int x^2 \log\left(\left(bx^2+a\right)^p c\right)^2\,dx$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
integrate(x^2*log((b*x^2 + a)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx=\int x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2\,dx$$

input
```
int(x^2*log(c*(a + b*x^2)^p)^2,x)
```

output
```
int(x^2*log(c*(a + b*x^2)^p)^2, x)
```

Reduce [F]

$$\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\frac{96\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,p^2-36\left(\int\frac{\log\left(\left(b\,x^2+a\right)^p c\right)}{b\,x^2+a}dx\right)a^2bp+9\log\left(\left(b\,x^2+a\right)^p c\right)^2 b^2x^3+36\log\left(\left(b\,x^2+a\right)^p\right.}{27b^2}$$

input
```
int(x^2*log(c*(b*x^2+a)^p)^2,x)
```

output
```
(96*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*p**2 - 36*int(log((a +
 b*x**2)**p*c)/(a + b*x**2),x)*a**2*b*p + 9*log((a + b*x**2)**p*c)**2*b**2
*x**3 + 36*log((a + b*x**2)**p*c)*a*b*p*x - 12*log((a + b*x**2)**p*c)*b**2
*p*x**3 - 96*a*b*p**2*x + 8*b**2*p**2*x**3)/(27*b**2)
```

3.86 $\int \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 237

$$\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = 8p^2x - \frac{8\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} - 4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} + x\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{4i\sqrt{a}p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}$$

output

```
8*p^2*x-8*a^(1/2)*p^2*arctan(b^(1/2)*x/a^(1/2))/b^(1/2)+4*I*a^(1/2)*p^2*ar
ctan(b^(1/2)*x/a^(1/2))^2/b^(1/2)+8*a^(1/2)*p^2*arctan(b^(1/2)*x/a^(1/2))*
ln(2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/b^(1/2)-4*p*x*ln(c*(b*x^2+a)^p)+4*a^(1
/2)*p*arctan(b^(1/2)*x/a^(1/2))*ln(c*(b*x^2+a)^p)/b^(1/2)+x*ln(c*(b*x^2+a)
^p)^2+4*I*a^(1/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.81

$$\int \log^2\left(c(a+bx^2)^p\right)\,dx$$
$$= \frac{4i\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2+4\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-2p+2p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)+\log\left(c(a+bx^2)^p\right)\right)+\sqrt{b}x(8p^2}{\sqrt{b}}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^2,x]
```

output
```
((4*I)*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*p*ArcTan[(Sqr
t[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + Lo
g[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2 - 4*p*Log[c*(a + b*x^2)^p] + Log[c*
(a + b*x^2)^p]^2) + (4*I)*Sqrt[a]*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/(
(-I)*Sqrt[a] + Sqrt[b]*x)])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2900, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(c(a+bx^2)^p\right)\,dx$$
$$\Big\downarrow \text{2900}$$
$$x\log^2\left(c(a+bx^2)^p\right)-4bp\int \frac{x^2\log\left(c(bx^2+a)^p\right)}{bx^2+a}dx$$
$$\Big\downarrow \text{2926}$$
$$x\log^2\left(c(a+bx^2)^p\right)-4bp\int \left(\frac{\log\left(c(bx^2+a)^p\right)}{b}-\frac{a\log\left(c(bx^2+a)^p\right)}{b\,(bx^2+a)}\right)dx$$

$\downarrow$ 2009

$$x\log^2\left(c\left(a+bx^2\right)^p\right)-4bp\left(-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^{3/2}}-\frac{i\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{b^{3/2}}+\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right.$$

input

```
Int[Log[c*(a + b*x^2)^p]^2,x]
```

output

```
x*Log[c*(a + b*x^2)^p]^2 - 4*b*p*((-2*p*x)/b + (2*Sqrt[a]*p*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(3/2) - (I*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/b^(3/
2) - (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I
*Sqrt[b]*x)])/b^(3/2) + (x*Log[c*(a + b*x^2)^p])/b - (Sqrt[a]*ArcTan[(Sqrt
[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/b^(3/2) - (I*Sqrt[a]*p*PolyLog[2, 1
- (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/b^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2900

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Simp[b*e*n*p*q   Int[x^n*
((a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

rule 2926

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \ln \left(c \left(b\,x^{2}+a \right)^{p}\right)^{2}dx$$

input
```
int(ln(c*(b*x^2+a)^p)^2,x)
```

output
```
int(ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [F]

$$\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \int \log\left(\left(bx^2+a\right)^p c\right)^2\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F]

$$\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \int \log\left(c\left(a+bx^2\right)^p\right)^2 dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**2, x)
```

Maxima [F]

$$\int \log^2\left(c(a+bx^2)^p\right)\,dx=\int \log\left((bx^2+a)^p c\right)^2\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
p^2*x*log(b*x^2 + a)^2 + integrate((b*x^2*log(c)^2 + a*log(c)^2 - 2*((2*p^
2 - p*log(c))*b*x^2 - a*p*log(c))*log(b*x^2 + a))/(b*x^2 + a), x)
```

Giac [F]

$$\int \log^2\left(c(a+bx^2)^p\right)\,dx=\int \log\left((bx^2+a)^p c\right)^2\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(c(a+bx^2)^p\right)\,dx=\int \ln\left(c\left(b\,x^2+a\right)^p\right)^2\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^2,x)
```

output
```
int(log(c*(a + b*x^2)^p)^2, x)
```

Reduce [F]

$$\int \log^2\left(c(a+bx^2)^p\right)\,dx$$
$$= \frac{-8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)p^2+4\left(\int \frac{\log\left((b\,x^2+a)^p\,c\right)}{b\,x^2+a}dx\right)abp+\log\left((b\,x^2+a)^p\,c\right)^2bx-4\log\left((b\,x^2+a)^p\,c\right)bpx+}{b}$$

input
```
int(log(c*(b*x^2+a)^p)^2,x)
```

output
```
( - 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*p**2 + 4*int(log((a +
b*x**2)**p*c)/(a + b*x**2),x)*a*b*p + log((a + b*x**2)**p*c)**2*b*x - 4*lo
g((a + b*x**2)**p*c)*b*p*x + 8*b*p**2*x)/b
```

3.87 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 190

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx = \frac{4i\sqrt{b}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}} + \frac{4\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} + \frac{4i\sqrt{b}p^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}}$$

output
```
4*I*b^(1/2)*p^2*arctan(b^(1/2)*x/a^(1/2))^2/a^(1/2)+8*b^(1/2)*p^2*arctan(b
^(1/2)*x/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/a^(1/2)+4*b^(1/2)*p*
arctan(b^(1/2)*x/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(1/2)-ln(c*(b*x^2+a)^p)^2/x+
4*I*b^(1/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx$$
$$= \frac{4i\sqrt{b}p^2x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2-\sqrt{a}\log^2\left(c(a+bx^2)^p\right)+4\sqrt{b}px\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(2p\log\left(\frac{2i}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\right)+\log\left(c(a+b\right.\right.}{\sqrt{a}x}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

output
```
((4*I)*Sqrt[b]*p^2*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - Sqrt[a]*Log[c*(a + b*
x^2)^p]^2 + 4*Sqrt[b]*p*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(2*p*Log[(2*I)/(I -
(Sqrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^p]) + (4*I)*Sqrt[b]*p^2*x*PolyLo
g[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*x)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2907, 2920, 27, 5455, 27, 5379, 27, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx$$
$$\Big\downarrow \ 2907$$
$$4bp\int \frac{\log\left(c(bx^2+a)^p\right)}{bx^2+a}dx-\frac{\log^2\left(c(a+bx^2)^p\right)}{x}$$
$$\Big\downarrow \ 2920$$
$$4bp\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}\sqrt{b}}-2bp\int \frac{x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,(bx^2+a)}dx\right)-\frac{\log^2\left(c(a+bx^2)^p\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$4bp\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}p\int\frac{x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{bx^2+a}dx}{\sqrt{a}}\right)-\frac{\log^2\left(c(a+bx^2)^p\right)}{x}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\log^2\left(c(a+bx^2)^p\right)}{x}+4bp\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}p\left(-\frac{\int\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i\sqrt{a}-\sqrt{b}x}dx}{\sqrt{a}\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\log^2\left(c(a+bx^2)^p\right)}{x}+4bp\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}p\left(-\frac{\int\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i\sqrt{a}-\sqrt{b}x}dx}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}\right)$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\log^2\left(c(a+bx^2)^p\right)}{x}+4bp\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}p\left(-\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}-\frac{\int\frac{a\log\left(\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{bx^2+a}dx}{\sqrt{a}}}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}+4bp\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}p\left(-\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}-\sqrt{a}\int\frac{\log\left(\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{bx^2+a}dx}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}\right)$$

$\downarrow$ 2849

$$-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}+4bp\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}p\left(-\frac{\frac{i\sqrt{a}\int\frac{\log\left(\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}}d\frac{1}{i\sqrt{b}x+\sqrt{a}}}{\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}\right)$$

$\downarrow$ 2752

$$-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}+4bp\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}p\left(-\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{2\sqrt{b}}}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}\right)$$

input
```
Int[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

output
```
-(Log[c*(a + b*x^2)^p]^2/x) + 4*b*p*((ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a
 + b*x^2)^p])/(Sqrt[a]*Sqrt[b]) - (2*Sqrt[b]*p*(((-1/2*I)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]]^2)/b - ((ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a]
+ I*Sqrt[b]*x)])/Sqrt[b] + ((I/2)*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*
Sqrt[b]*x)])/Sqrt[b])/Sqrt[b]))/Sqrt[a])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.79 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.35

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2}}{x}-\frac{4p^{2}b\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^{2}+a\right)}{\sqrt{ab}}+\frac{4pb\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{\sqrt{ab}}+p^{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{2}+a\right)}\frac{2\ln(}{}\right.$

input
```
int(ln(c*(b*x^2+a)^p)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*ln((b*x^2+a)^p)^2-4*p^2*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln(b*x^
2+a)+4*p*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln((b*x^2+a)^p)+p^2*sum(1/_
alpha*(2*ln(x-_alpha)*ln(b*x^2+a)-b*(1/_alpha/b*ln(x-_alpha)^2+2*_alpha/a*
ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/a*dilog(1/2*(x+_alpha)/_al
pha))),_alpha=RootOf(_Z^2*b+a))+(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn
(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))*(-1/x*
ln((b*x^2+a)^p)+2*p*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/4*(I*Pi*csgn(
I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)
^p)^2*csgn(I*c)+2*ln(c))^2/x
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^2}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^2}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**2/x**2,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^2}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="maxima")
```

output
```
-p^2*log(b*x^2 + a)^2/x + integrate((b*x^2*log(c)^2 + a*log(c)^2 + 2*((2*p
^2 + p*log(c))*b*x^2 + a*p*log(c))*log(b*x^2 + a))/(b*x^4 + a*x^2), x)
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^2}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^2}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^2/x^2,x)
```

output
```
int(log(c*(a + b*x^2)^p)^2/x^2, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)p^2x - 4\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^4+a\,x^2}dx\right)a^2px - \log\left((b\,x^2+a)^p c\right)^2 a - 4\log\left((b\,x^2+a)^p c\right)ap}{ax}$$

input
```
int(log(c*(b*x^2+a)^p)^2/x^2,x)
```

output
```
(8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*p**2*x - 4*int(log((a + b
*x**2)**p*c)/(a*x**2 + b*x**4),x)*a**2*p*x - log((a + b*x**2)**p*c)**2*a -
 4*log((a + b*x**2)**p*c)*a*p)/(a*x)
```

3.88 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^4} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 254

$$\begin{aligned}\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx &= \frac{8b^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} \\ &\quad - \frac{8b^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}} - \frac{4bp\log\left(c(a+bx^2)^p\right)}{3ax} \\ &\quad - \frac{4b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3a^{3/2}} \\ &\quad - \frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3} - \frac{4ib^{3/2}p^2\,\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}}\end{aligned}$$

output
```
8/3*b^(3/2)*p^2*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)-4/3*I*b^(3/2)*p^2*arctan
(b^(1/2)*x/a^(1/2))^2/a^(3/2)-8/3*b^(3/2)*p^2*arctan(b^(1/2)*x/a^(1/2))*ln
(2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/a^(3/2)-4/3*b*p*ln(c*(b*x^2+a)^p)/a/x-4/
3*b^(3/2)*p*arctan(b^(1/2)*x/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(3/2)-1/3*ln(c*(
b*x^2+a)^p)^2/x^3-4/3*I*b^(3/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*b^(1/
2)*x))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.81

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx$$
$$= \frac{-4ib^{3/2}p^2x^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2-4b^{3/2}px^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-2p+2p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)+\log\left(c(a+bx^2)^p\right)\right)-\sqrt{}}{3a^{3/2}x^3}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

output
```
((-4*I)*b^(3/2)*p^2*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 4*b^(3/2)*p*x^3*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]
*x)] + Log[c*(a + b*x^2)^p]) - Sqrt[a]*Log[c*(a + b*x^2)^p]*(4*b*p*x^2 + a
*Log[c*(a + b*x^2)^p]) - (4*I)*b^(3/2)*p^2*x^3*PolyLog[2, (I*Sqrt[a] + Sqr
t[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(3*a^(3/2)*x^3)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx$$
$$\Big\downarrow \ 2907$$
$$\frac{4}{3}bp\int \frac{\log\left(c(bx^2+a)^p\right)}{x^2\left(bx^2+a\right)}dx-\frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3}$$
$$\Big\downarrow \ 2926$$
$$\frac{4}{3}bp\int \left(\frac{\log\left(c(bx^2+a)^p\right)}{ax^2}-\frac{b\log\left(c(bx^2+a)^p\right)}{a\left(bx^2+a\right)}\right)dx-\frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3}+$$
$$\frac{4}{3}bp\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{3/2}}-\frac{i\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}}+\frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

output
```
-1/3*Log[c*(a + b*x^2)^p]^2/x^3 + (4*b*p*((2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(3/2) - (I*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) -
(2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt
[b]*x)])/a^(3/2) - Log[c*(a + b*x^2)^p]/(a*x) - (Sqrt[b]*ArcTan[(Sqrt[b]*x
)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/a^(3/2) - (I*Sqrt[b]*p*PolyLog[2, 1 - (2*
Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3/2)))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.06

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2}}{3x^{3}}-\frac{4pb\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{3ax}+\frac{4p^{2}b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^{2}+a\right)}{3a\sqrt{ab}}-\frac{4p\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{3a\sqrt{ab}}+\frac{8p^{2}b^{2}\operatorname{arc}}{3a}$

input
```
int(ln(c*(b*x^2+a)^p)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln((b*x^2+a)^p)^2/x^3-4/3*p*b*ln((b*x^2+a)^p)/a/x+4/3*p^2*b^2/a/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2))*ln(b*x^2+a)-4/3*p*b^2/a/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2))*ln((b*x^2+a)^p)+8/3*p^2*b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2))+4/3*p^2*b*Sum(-1/2*(ln(x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alpha/b*ln
(x-_alpha)^2+1/2*_alpha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alph
a/a*dilog(1/2*(x+_alpha)/_alpha)))/a/_alpha,_alpha=RootOf(_Z^2*b+a))+(I*Pi
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*
x^2+a)^p)^2*csgn(I*c)+2*ln(c))*(-1/3/x^3*ln((b*x^2+a)^p)+2/3*p*b*(-1/a/x-1
/a*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))-1/12*(I*Pi*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csg
n(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)
+2*ln(c))^2/x^3
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx=\int \frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{x^4}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^4}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**2/x**4,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^4}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*p^2*log(b*x^2 + a)^2/x^3 + integrate(1/3*(3*b*x^2*log(c)^2 + 3*a*log(
c)^2 + 2*((2*p^2 + 3*p*log(c))*b*x^2 + 3*a*p*log(c))*log(b*x^2 + a))/(b*x^
6 + a*x^4), x)
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^4}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^4}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^2/x^4,x)
```

output
```
int(log(c*(a + b*x^2)^p)^2/x^4, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \frac{-8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,p^2x^3 - 12\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^6+a\,x^4}dx\right)a^3p\,x^3 - 3\log\left((b\,x^2+a)^p\,c\right)^2a^2 - 4\log\left((b\,x^2+a)\right.}{9a^2x^3}$$

input
```
int(log(c*(b*x^2+a)^p)^2/x^4,x)
```

output
```
( - 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*p**2*x**3 - 12*int(l
og((a + b*x**2)**p*c)/(a*x**4 + b*x**6),x)*a**3*p*x**3 - 3*log((a + b*x**2
)**p*c)**2*a**2 - 4*log((a + b*x**2)**p*c)*a**2*p - 8*a*b*p**2*x**2)/(9*a*
*2*x**3)
```

3.89 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 296

$$\begin{aligned}\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = &-\frac{8b^2p^2}{15a^2x}-\frac{32b^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}}+\frac{4ib^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}}\\&+\frac{8b^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}}\\&-\frac{4bp\log\left(c(a+bx^2)^p\right)}{15ax^3}+\frac{4b^2p\log\left(c(a+bx^2)^p\right)}{5a^2x}\\&+\frac{4b^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5a^{5/2}}\\&-\frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5}+\frac{4ib^{5/2}p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}}\end{aligned}$$

output
```
-8/15*b^2*p^2/a^2/x-32/15*b^(5/2)*p^2*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)+4/
5*I*b^(5/2)*p^2*arctan(b^(1/2)*x/a^(1/2))^2/a^(5/2)+8/5*b^(5/2)*p^2*arctan
(b^(1/2)*x/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/a^(5/2)-4/15*b*p*l
n(c*(b*x^2+a)^p)/a/x^3+4/5*b^2*p*ln(c*(b*x^2+a)^p)/a^2/x+4/5*b^(5/2)*p*arc
tan(b^(1/2)*x/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(5/2)-1/5*ln(c*(b*x^2+a)^p)^2/x
^5+4/5*I*b^(5/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/a^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.99

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = -\frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5} + \frac{4}{5}bp\left(-\frac{2b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{2bp\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{bx^2}{a}\right)}{3a^2x} - \frac{\log\left(c(a+bx^2)^p\right)}{3ax^3} + \frac{b\log\left(c(a+bx^2)^p\right)}{a^2x} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{5/2}} + \frac{p\left(ib^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 + 2b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2i\sqrt{a}}{i\sqrt{a}-\sqrt{b}x}\right) + ib^{3/2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}+\sqrt{b}x}{i\sqrt{a}-\sqrt{b}x}\right)\right)}{a^{5/2}}\right)$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^2/x^6,x]
```

output
```
-1/5*Log[c*(a + b*x^2)^p]^2/x^5 + (4*b*p*((-2*b^(3/2)*p*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/a^(5/2) - (2*b*p*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)])
/(3*a^2*x) - Log[c*(a + b*x^2)^p]/(3*a*x^3) + (b*Log[c*(a + b*x^2)^p])/(a^
2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/a^(5/2)
+ (p*(I*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 2*b^(3/2)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]]*Log[((2*I)*Sqrt[a])/(I*Sqrt[a] - Sqrt[b]*x)] + I*b^(3/2)*PolyL
og[2, -((I*Sqrt[a] + Sqrt[b]*x)/(I*Sqrt[a] - Sqrt[b]*x))]))/a^(5/2)))/5
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^6}\,dx$$

$$\Big\downarrow \text{2907}$$

$$\frac{4}{5}bp\int \frac{\log\left(c\left(bx^2+a\right)^p\right)}{x^4\left(bx^2+a\right)}dx-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

$$\Big\downarrow \text{2926}$$

$$\frac{4}{5}bp\int \left(\frac{\log\left(c\left(bx^2+a\right)^p\right)b^2}{a^2\left(bx^2+a\right)}-\frac{\log\left(c\left(bx^2+a\right)^p\right)b}{a^2x^2}+\frac{\log\left(c\left(bx^2+a\right)^p\right)}{ax^4}\right)dx-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}+\frac{4}{5}bp\left(\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{5/2}}+\frac{ib^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{5/2}}-\frac{8b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^2/x^6,x]
```

output
```
-1/5*Log[c*(a + b*x^2)^p]^2/x^5 + (4*b*p*((-2*b*p)/(3*a^2*x) - (8*b^(3/2)*
p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(5/2)) + (I*b^(3/2)*p*ArcTan[(Sqrt[b]*
x)/Sqrt[a]]^2)/a^(5/2) + (2*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*S
qrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(5/2) - Log[c*(a + b*x^2)^p]/(3*a*x^3)
 + (b*Log[c*(a + b*x^2)^p])/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/a^(5/2) + (I*b^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[a])/(
Sqrt[a] + I*Sqrt[b]*x)])/a^(5/2)))/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.48 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)^2}{5x^5}-\frac{4p^2b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^2+a\right)}{5a^2\sqrt{ab}}+\frac{4p\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(\left(b\,x^2+a\right)^p\right)}{5a^2\sqrt{ab}}-\frac{4pb\ln\left(\left(b\,x^2+a\right)^p\right)}{15a\,x^3}+\frac{4p\,b^2\ln\left(\left(\right.\right.}{5a}$

input
```
int(ln(c*(b*x^2+a)^p)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*ln((b*x^2+a)^p)^2/x^5-4/5*p^2*b^3/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2))*ln(b*x^2+a)+4/5*p*b^3/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln((b*x
^2+a)^p)-4/15*p*b*ln((b*x^2+a)^p)/a/x^3+4/5*p*b^2*ln((b*x^2+a)^p)/a^2/x-32
/15*p^2*b^3/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+4/5*p^2*b*Sum(1/2*(ln(
x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alpha/b*ln(x-_alpha)^2+1/2*_alpha/a*ln(x-_
alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/a*dilog(1/2*(x+_alpha)/_alpha)
))*b/a^2/_alpha,_alpha=RootOf(_Z^2*b+a))-8/15*b^2*p^2/a^2/x+(I*Pi*csgn(I*(
b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x
^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)
^2*csgn(I*c)+2*ln(c))*(-1/5/x^5*ln((b*x^2+a)^p)+2/5*p*b*(b^2/a^2/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))-1/3/a/x^3+b/a^2/x))-1/20*(I*Pi*csgn(I*(b*x^2+a)
^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)
*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(
I*c)+2*ln(c))^2/x^5
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\log\left((bx^2+a)^pc\right)^2}{x^6}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^2/x^6, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^6}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**2/x**6,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**2/x**6, x)
```

Maxima [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^6}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="maxima")
```

output
```
-1/5*p^2*log(b*x^2 + a)^2/x^5 + integrate(1/5*(5*b*x^2*log(c)^2 + 5*a*log(
c)^2 + 2*((2*p^2 + 5*p*log(c))*b*x^2 + 5*a*p*log(c))*log(b*x^2 + a))/(b*x^
8 + a*x^6), x)
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^6}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^6}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^2/x^6,x)
```

output
```
int(log(c*(a + b*x^2)^p)^2/x^6, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx$$
$$= \frac{24\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2p^2x^5 - 60\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^8+a\,x^6}dx\right)a^4p\,x^5 - 15\log\left((b\,x^2+a)^p\,c\right)^2a^3 - 12\log\left((b\,x^2+\right.}{75a^3x^5}$$

input
```
int(log(c*(b*x^2+a)^p)^2/x^6,x)
```

output
```
(24*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*p**2*x**5 - 60*int(
log((a + b*x**2)**p*c)/(a*x**6 + b*x**8),x)*a**4*p*x**5 - 15*log((a + b*x*
*2)**p*c)**2*a**3 - 12*log((a + b*x**2)**p*c)*a**3*p - 8*a**2*b*p**2*x**2
+ 24*a*b**2*p**2*x**4)/(75*a**3*x**5)
```

3.90 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 338

$$
\begin{aligned}
\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx =& -\frac{8b^2p^2}{105a^2x^3}+\frac{64b^3p^2}{105a^3x}+\frac{184b^{7/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{105a^{7/2}} \\
&-\frac{4ib^{7/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}} \\
&-\frac{8b^{7/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}}-\frac{4bp\log\left(c(a+bx^2)^p\right)}{35ax^5} \\
&+\frac{4b^2p\log\left(c(a+bx^2)^p\right)}{21a^2x^3}-\frac{4b^3p\log\left(c(a+bx^2)^p\right)}{7a^3x} \\
&-\frac{4b^{7/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{7a^{7/2}} \\
&-\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7}-\frac{4ib^{7/2}p^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}}
\end{aligned}
$$

output
```
-8/105*b^2*p^2/a^2/x^3+64/105*b^3*p^2/a^3/x+184/105*b^(7/2)*p^2*arctan(b^(
1/2)*x/a^(1/2))/a^(7/2)-4/7*I*b^(7/2)*p^2*arctan(b^(1/2)*x/a^(1/2))^2/a^(7
/2)-8/7*b^(7/2)*p^2*arctan(b^(1/2)*x/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*b^(1
/2)*x))/a^(7/2)-4/35*b*p*ln(c*(b*x^2+a)^p)/a/x^5+4/21*b^2*p*ln(c*(b*x^2+a)
^p)/a^2/x^3-4/7*b^3*p*ln(c*(b*x^2+a)^p)/a^3/x-4/7*b^(7/2)*p*arctan(b^(1/2)
*x/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(7/2)-1/7*ln(c*(b*x^2+a)^p)^2/x^7-4/7*I*b^
(7/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/a^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.24 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.04

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = -\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7} + \frac{4}{7}bp\left(\frac{2b^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}} - \frac{2bp\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{bx^2}{a}\right)}{15a^2x^3} + \frac{2b^2p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{bx^2}{a}\right)}{3a^3x} - \frac{\log\left(c(a+bx^2)^p\right)}{5ax^5} + \frac{b\log\left(c(a+bx^2)^p\right)}{3a^2x^3} - \frac{b^2\log\left(c(a+bx^2)^p\right)}{a^3x} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{7/2}} - \frac{p\left(ib^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 + 2b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2i\sqrt{a}}{i\sqrt{a}-\sqrt{b}x}\right) + ib^{5/2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}+\sqrt{b}x}{i\sqrt{a}-\sqrt{b}x}\right)\right)}{a^{7/2}}\right)$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^2/x^8,x]
```

output
```
-1/7*Log[c*(a + b*x^2)^p]^2/x^7 + (4*b*p*((2*b^(5/2)*p*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(7/2) - (2*b*p*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/a)])
/(15*a^2*x^3) + (2*b^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)])/(3
*a^3*x) - Log[c*(a + b*x^2)^p]/(5*a*x^5) + (b*Log[c*(a + b*x^2)^p])/(3*a^2
*x^3) - (b^2*Log[c*(a + b*x^2)^p])/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]]*Log[c*(a + b*x^2)^p])/a^(7/2) - (p*(I*b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]]^2 + 2*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[((2*I)*Sqrt[a])/(I*S
qrt[a] - Sqrt[b]*x)] + I*b^(5/2)*PolyLog[2, -((I*Sqrt[a] + Sqrt[b]*x)/(I*S
qrt[a] - Sqrt[b]*x))]))/a^(7/2)))/7
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^8}\,dx$$

$$\Big\downarrow \ 2907$$

$$\frac{4}{7}bp\int \frac{\log\left(c\left(bx^2+a\right)^p\right)}{x^6\left(bx^2+a\right)}dx-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7}$$

$$\Big\downarrow \ 2926$$

$$\frac{4}{7}bp\int \left(-\frac{\log\left(c\left(bx^2+a\right)^p\right)b^3}{a^3\left(bx^2+a\right)}+\frac{\log\left(c\left(bx^2+a\right)^p\right)b^2}{a^3x^2}-\frac{\log\left(c\left(bx^2+a\right)^p\right)b}{a^2x^4}+\frac{\log\left(c\left(bx^2+a\right)^p\right)}{ax^6}\right)dx-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7}+$$

$$\frac{4}{7}bp\left(-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^{7/2}}-\frac{ib^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{7/2}}+\frac{46b^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{7/2}}-\frac{2b^{5/2}p\arctan\left(\right.}{}\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^2/x^8,x]
```

output
```
-1/7*Log[c*(a + b*x^2)^p]^2/x^7 + (4*b*p*((-2*b*p)/(15*a^2*x^3) + (16*b^2*
p)/(15*a^3*x) + (46*b^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(15*a^(7/2)) -
(I*b^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(7/2) - (2*b^(5/2)*p*ArcTan[
(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(7/2) - L
og[c*(a + b*x^2)^p]/(5*a*x^5) + (b*Log[c*(a + b*x^2)^p])/(3*a^2*x^3) - (b^
2*Log[c*(a + b*x^2)^p])/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log
[c*(a + b*x^2)^p])/a^(7/2) - (I*b^(5/2)*p*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt
[a] + I*Sqrt[b]*x)])/a^(7/2)))/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.42 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2}}{7x^{7}}+\frac{4p^{2}b^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^{2}+a\right)}{7a^{3}\sqrt{ab}}-\frac{4p\,b^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{7a^{3}\sqrt{ab}}-\frac{4pb\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{35a\,x^{5}}-\frac{4p\,b^{3}\ln((}{7a}$

input
```
int(ln(c*(b*x^2+a)^p)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*ln((b*x^2+a)^p)^2/x^7+4/7*p^2*b^4/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2))*ln(b*x^2+a)-4/7*p*b^4/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln((b*x
^2+a)^p)-4/35*p*b*ln((b*x^2+a)^p)/a/x^5-4/7*p*b^3*ln((b*x^2+a)^p)/a^3/x+4/
21*p*b^2*ln((b*x^2+a)^p)/a^2/x^3+184/105*p^2*b^4/a^3/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2))+4/7*p^2*b*Sum(-1/2*(ln(x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alph
a/b*ln(x-_alpha)^2+1/2*_alpha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2
*_alpha/a*dilog(1/2*(x+_alpha)/_alpha)))/a^3*b^2/_alpha,_alpha=RootOf(_Z^2
*b+a))-8/105*b^2*p^2/a^2/x^3+64/105*b^3*p^2/a^3/x+(I*Pi*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*c
sgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*
c)+2*ln(c))*(-1/7/x^7*ln((b*x^2+a)^p)+2/7*p*b*(-b^3/a^3/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2))-1/5/a/x^5-b^2/a^3/x+1/3*b/a^2/x^3))-1/28*(I*Pi*csgn(I*(b
*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^
2*csgn(I*c)+2*ln(c))^2/x^7
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^8}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^2/x^8, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^8}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**2/x**8,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**2/x**8, x)
```

Maxima [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^8}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="maxima")
```

output
```
-1/7*p^2*log(b*x^2 + a)^2/x^7 + integrate(1/7*(7*b*x^2*log(c)^2 + 7*a*log(
c)^2 + 2*((2*p^2 + 7*p*log(c))*b*x^2 + 7*a*p*log(c))*log(b*x^2 + a))/(b*x^
10 + a*x^8), x)
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = \int \frac{\log\left((bx^2+a)^pc\right)^2}{x^8}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^2/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^8}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^2/x^8,x)
```

output
```
int(log(c*(a + b*x^2)^p)^2/x^8, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = \frac{-120\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^3p^2x^7 - 420\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^{10}+a\,x^8}dx\right)a^5p\,x^7 - 105\log\left((b\,x^2+a)^p\,c\right)^2a^4 - 60\log\left((b\right.}{735a^4x^7}$$

input
```
int(log(c*(b*x^2+a)^p)^2/x^8,x)
```

output
```
( - 120*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*p**2*x**7 - 420
*int(log((a + b*x**2)**p*c)/(a*x**8 + b*x**10),x)*a**5*p*x**7 - 105*log((a
 + b*x**2)**p*c)**2*a**4 - 60*log((a + b*x**2)**p*c)*a**4*p - 24*a**3*b*p*
*2*x**2 + 40*a**2*b**2*p**2*x**4 - 120*a*b**3*p**2*x**6)/(735*a**4*x**7)
```

3.91 $\int x^5 \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 334

$$\begin{aligned}\int x^5 \log^3\left(c(a+bx^2)^p\right)\,dx = & -\frac{3a^2p^3x^2}{b^2}+\frac{3ap^3(a+bx^2)^2}{8b^3}-\frac{p^3(a+bx^2)^3}{27b^3} \\ & +\frac{3a^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} \\ & -\frac{3ap^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^3} \\ & +\frac{p^2(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} \\ & -\frac{3a^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^3} \\ & +\frac{3ap(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^3} \\ & -\frac{p(a+bx^2)^3\log^2\left(c(a+bx^2)^p\right)}{6b^3} \\ & +\frac{a^2(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^3} \\ & -\frac{a(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{2b^3} \\ & +\frac{(a+bx^2)^3\log^3\left(c(a+bx^2)^p\right)}{6b^3}\end{aligned}$$

output
```
-3*a^2*p^3*x^2/b^2+3/8*a*p^3*(b*x^2+a)^2/b^3-1/27*p^3*(b*x^2+a)^3/b^3+3*a^
2*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3-3/4*a*p^2*(b*x^2+a)^2*ln(c*(b*x^2+a)
^p)/b^3+1/9*p^2*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)/b^3-3/2*a^2*p*(b*x^2+a)*ln(c
*(b*x^2+a)^p)^2/b^3+3/4*a*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^3-1/6*p*(b*x
^2+a)^3*ln(c*(b*x^2+a)^p)^2/b^3+1/2*a^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^3/b^3-
1/2*a*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^3/b^3+1/6*(b*x^2+a)^3*ln(c*(b*x^2+a)^p
)^3/b^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.53

$$\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{bp^3x^2(-510a^2+57abx^2-8b^2x^4)+114a^3p^3\log(a+bx^2)+6p^2(66a^3+66a^2bx^2-15ab^2x^4+4b^3x^6)\log}{2}$$

input
```
Integrate[x^5*Log[c*(a + b*x^2)^p]^3,x]
```

output
```
(b*p^3*x^2*(-510*a^2 + 57*a*b*x^2 - 8*b^2*x^4) + 114*a^3*p^3*Log[a + b*x^2
] + 6*p^2*(66*a^3 + 66*a^2*b*x^2 - 15*a*b^2*x^4 + 4*b^3*x^6)*Log[c*(a + b*
x^2)^p] - 18*p*(11*a^3 + 6*a^2*b*x^2 - 3*a*b^2*x^4 + 2*b^3*x^6)*Log[c*(a +
 b*x^2)^p]^2 + 36*(a^3 + b^3*x^6)*Log[c*(a + b*x^2)^p]^3)/(216*b^3)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int x^4\log^3\left(c\left(bx^2+a\right)^p\right)dx^2$$

$$\Big\downarrow \text{2848}$$

$$\frac{1}{2}\int\left(\frac{\left(bx^2+a\right)^2\log^3\left(c\left(bx^2+a\right)^p\right)}{b^2}-\frac{2a\left(bx^2+a\right)\log^3\left(c\left(bx^2+a\right)^p\right)}{b^2}+\frac{a^2\log^3\left(c\left(bx^2+a\right)^p\right)}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{6a^2p^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^3}+\frac{a^2\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{b^3}-\frac{3a^2p\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{b^3}-\frac{6a}{}\right.$$

input
```
Int[x^5*Log[c*(a + b*x^2)^p]^3,x]
```

output
```
((-6*a^2*p^3*x^2)/b^2 + (3*a*p^3*(a + b*x^2)^2)/(4*b^3) - (2*p^3*(a + b*x^
2)^3)/(27*b^3) + (6*a^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b^3 - (3*a*p
^2*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(2*b^3) + (2*p^2*(a + b*x^2)^3*Log[
c*(a + b*x^2)^p])/(9*b^3) - (3*a^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/b
^3 + (3*a*p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^2)/(2*b^3) - (p*(a + b*x^2)
^3*Log[c*(a + b*x^2)^p]^2)/(3*b^3) + (a^2*(a + b*x^2)*Log[c*(a + b*x^2)^p]
^3)/b^3 - (a*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/b^3 + ((a + b*x^2)^3*Lo
g[c*(a + b*x^2)^p]^3)/(3*b^3))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{36x^6\ln(c(b\,x^2+a)^p)^3b^3-36x^6\ln(c(b\,x^2+a)^p)^2b^3p+24x^6\ln(c(b\,x^2+a)^p)b^3p^2-8b^3p^3x^6+54x^4\ln(c(b\,x^2+a)^p)^2a\,b^2p-90x^4\ln}{}$
risch	Expression too large to display

input
```
int(x^5*ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

output
```
1/216*(36*x^6*ln(c*(b*x^2+a)^p)^3*b^3-36*x^6*ln(c*(b*x^2+a)^p)^2*b^3*p+24*
x^6*ln(c*(b*x^2+a)^p)*b^3*p^2-8*b^3*p^3*x^6+54*x^4*ln(c*(b*x^2+a)^p)^2*a*b
^2*p-90*x^4*ln(c*(b*x^2+a)^p)*a*b^2*p^2+57*a*b^2*p^3*x^4-108*x^2*ln(c*(b*x
^2+a)^p)^2*a^2*b*p+396*x^2*ln(c*(b*x^2+a)^p)*a^2*b*p^2-510*a^2*b*p^3*x^2+9
06*ln(b*x^2+a)*a^3*p^3+36*ln(c*(b*x^2+a)^p)^3*a^3-198*ln(c*(b*x^2+a)^p)^2*
a^3*p-396*ln(c*(b*x^2+a)^p)*a^3*p^2+510*a^3*p^3)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.07

$$\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx =$$
$$-\frac{8\,b^3p^3x^6-36\,b^3x^6\log(c)^3-57\,ab^2p^3x^4+510\,a^2bp^3x^2-36\left(b^3p^3x^6+a^3p^3\right)\log\left(bx^2+a\right)^3+18\left(2\,b^3p^3\right.}{}$$

input
```
integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
-1/216*(8*b^3*p^3*x^6 - 36*b^3*x^6*log(c)^3 - 57*a*b^2*p^3*x^4 + 510*a^2*b
*p^3*x^2 - 36*(b^3*p^3*x^6 + a^3*p^3)*log(b*x^2 + a)^3 + 18*(2*b^3*p^3*x^6
 - 3*a*b^2*p^3*x^4 + 6*a^2*b*p^3*x^2 + 11*a^3*p^3 - 6*(b^3*p^2*x^6 + a^3*p
^2)*log(c))*log(b*x^2 + a)^2 + 18*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p
*x^2)*log(c)^2 - 6*(4*b^3*p^3*x^6 - 15*a*b^2*p^3*x^4 + 66*a^2*b*p^3*x^2 +
85*a^3*p^3 + 18*(b^3*p*x^6 + a^3*p)*log(c)^2 - 6*(2*b^3*p^2*x^6 - 3*a*b^2*
p^2*x^4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2)*log(c))*log(b*x^2 + a) - 6*(4*b^3*
p^2*x^6 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p^2*x^2)*log(c))/b^3
```

Sympy [A] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.87

$$\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \begin{cases} \frac{85a^3p^2\log\left(c\left(a+bx^2\right)^p\right)}{36b^3} - \frac{11a^3p\log\left(c\left(a+bx^2\right)^p\right)^2}{12b^3} + \frac{a^3\log\left(c\left(a+bx^2\right)^p\right)^3}{6b^3} - \frac{85a^2p^3x^2}{36b^2} + \frac{11a^2p^2x^2\log\left(c\left(a+bx^2\right)^p\right)}{6b^2} - \frac{a^2px^2\log\left(c\right.}{2} \\ \frac{x^6\log\left(a^pc\right)^3}{6} \end{cases}$$

input
```
integrate(x**5*ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Piecewise((85*a**3*p**2*log(c*(a + b*x**2)**p)/(36*b**3) - 11*a**3*p*log(c
*(a + b*x**2)**p)**2/(12*b**3) + a**3*log(c*(a + b*x**2)**p)**3/(6*b**3) -
 85*a**2*p**3*x**2/(36*b**2) + 11*a**2*p**2*x**2*log(c*(a + b*x**2)**p)/(6
*b**2) - a**2*p*x**2*log(c*(a + b*x**2)**p)**2/(2*b**2) + 19*a*p**3*x**4/(
72*b) - 5*a*p**2*x**4*log(c*(a + b*x**2)**p)/(12*b) + a*p*x**4*log(c*(a +
b*x**2)**p)**2/(4*b) - p**3*x**6/27 + p**2*x**6*log(c*(a + b*x**2)**p)/9 -
 p*x**6*log(c*(a + b*x**2)**p)**2/6 + x**6*log(c*(a + b*x**2)**p)**3/6, Ne
(b, 0)), (x**6*log(a**p*c)**3/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.72

$$\int x^5 \log^3 \left(c\left(a+bx^2\right)^p\right) dx = \frac{1}{6}x^6 \log\left(\left(bx^2+a\right)^p c\right)^3 + \frac{1}{12}bp\left(\frac{6a^3\log\left(bx^2+a\right)}{b^4} - \frac{2b^2x^6-3abx^4+6a^2x^2}{b^3}\right)\log\left(\left(bx^2+a\right)^p c\right)^2 - \frac{1}{216}bp\left(\frac{\left(8b^3x^6-57ab^2x^4-36a^3\log\left(bx^2+a\right)^3+510a^2bx^2-198a^3\log\left(bx^2+a\right)^2-510a^3\log\left(bx^2\right.\right.}{b^4}\right.$$

input
```
integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
1/6*x^6*log((b*x^2 + a)^p*c)^3 + 1/12*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b
^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c)^2 - 1/216*b*p*((
8*b^3*x^6 - 57*a*b^2*x^4 - 36*a^3*log(b*x^2 + a)^3 + 510*a^2*b*x^2 - 198*a
^3*log(b*x^2 + a)^2 - 510*a^3*log(b*x^2 + a))*p^2/b^4 - 6*(4*b^3*x^6 - 15*
a*b^2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*x^2 + a)
)*p*log((b*x^2 + a)^p*c)/b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(314) = 628.

Time = 0.13 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.98

$$\int x^5 \log^3 \left(c\left(a+bx^2\right)^p\right) dx = \text{Too large to display}$$

input
```
integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
1/6*(b*x^2 + a)^3*p^3*log(b*x^2 + a)^3/b^3 - 1/2*(b*x^2 + a)^2*a*p^3*log(b
*x^2 + a)^3/b^3 - 1/6*(b*x^2 + a)^3*p^3*log(b*x^2 + a)^2/b^3 + 3/4*(b*x^2
+ a)^2*a*p^3*log(b*x^2 + a)^2/b^3 + 1/2*(b*x^2 + a)^3*p^2*log(b*x^2 + a)^2
*log(c)/b^3 - 3/2*(b*x^2 + a)^2*a*p^2*log(b*x^2 + a)^2*log(c)/b^3 + 1/9*(b
*x^2 + a)^3*p^3*log(b*x^2 + a)/b^3 - 3/4*(b*x^2 + a)^2*a*p^3*log(b*x^2 + a
)/b^3 - 1/3*(b*x^2 + a)^3*p^2*log(b*x^2 + a)*log(c)/b^3 + 3/2*(b*x^2 + a)^
2*a*p^2*log(b*x^2 + a)*log(c)/b^3 + 1/2*(b*x^2 + a)^3*p*log(b*x^2 + a)*log
(c)^2/b^3 - 3/2*(b*x^2 + a)^2*a*p*log(b*x^2 + a)*log(c)^2/b^3 - 1/27*(b*x^
2 + a)^3*p^3/b^3 + 3/8*(b*x^2 + a)^2*a*p^3/b^3 + 1/9*(b*x^2 + a)^3*p^2*log
(c)/b^3 - 3/4*(b*x^2 + a)^2*a*p^2*log(c)/b^3 - 1/6*(b*x^2 + a)^3*p*log(c)^
2/b^3 + 3/4*(b*x^2 + a)^2*a*p*log(c)^2/b^3 + 1/6*(b*x^2 + a)^3*log(c)^3/b^
3 - 1/2*(b*x^2 + a)^2*a*log(c)^3/b^3 + 1/2*(((b*x^2 + a)*log(b*x^2 + a)^3
- 6*b*x^2 - 3*(b*x^2 + a)*log(b*x^2 + a)^2 + 6*(b*x^2 + a)*log(b*x^2 + a)
- 6*a)*a^2*p^3 + 3*(2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)
*log(b*x^2 + a) + 2*a)*a^2*p^2*log(c) - 3*(b*x^2 - (b*x^2 + a)*log(b*x^2 +
 a) + a)*a^2*p*log(c)^2 + (b*x^2 + a)*a^2*log(c)^3)/b^3
```

Mupad [B] (verification not implemented)

Time = 14.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.56

$$\int x^5 \log^3\left(c(a+bx^2)^p\right) dx = \ln\left(c\left(b\,x^2+a\right)^p\right)^3\left(\frac{x^6}{6}+\frac{a^3}{6\,b^3}\right) - \ln\left(c\left(b\,x^2+a\right)^p\right)^2\left(\frac{p\,x^6}{6}+\frac{11\,a^3\,p}{12\,b^3}+\frac{a^2\,p\,x^2}{2\,b^2}-\frac{a\,p\,x^4}{4\,b}\right) - \frac{p^3\,x^6}{27}+\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)\left(\frac{b\,p^2\,x^6}{3}-\frac{5\,a\,p^2\,x^4}{4}+\frac{11\,a^2\,p^2\,x^2}{2\,b}\right)}{3\,b} + \frac{19\,a\,p^3\,x^4}{72\,b}+\frac{85\,a^3\,p^3\,\ln\left(b\,x^2+a\right)}{36\,b^3}-\frac{85\,a^2\,p^3\,x^2}{36\,b^2}$$

input
```
int(x^5*log(c*(a + b*x^2)^p)^3,x)
```

output
```
log(c*(a + b*x^2)^p)^3*(x^6/6 + a^3/(6*b^3)) - log(c*(a + b*x^2)^p)^2*((p*
x^6)/6 + (11*a^3*p)/(12*b^3) + (a^2*p*x^2)/(2*b^2) - (a*p*x^4)/(4*b)) - (p
^3*x^6)/27 + (log(c*(a + b*x^2)^p)*((b*p^2*x^6)/3 - (5*a*p^2*x^4)/4 + (11*
a^2*p^2*x^2)/(2*b)))/(3*b) + (19*a*p^3*x^4)/(72*b) + (85*a^3*p^3*log(a + b
*x^2))/(36*b^3) - (85*a^2*p^3*x^2)/(36*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.79

$$\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{36\log\left(\left(b\,x^2+a\right)^p c\right)^3 a^3+36\log\left(\left(b\,x^2+a\right)^p c\right)^3 b^3x^6-198\log\left(\left(b\,x^2+a\right)^p c\right)^2 a^3p-108\log\left(\left(b\,x^2+a\right)^p c\right)^2}{}$$

input
```
int(x^5*log(c*(b*x^2+a)^p)^3,x)
```

output
```
(36*log((a + b*x**2)**p*c)**3*a**3 + 36*log((a + b*x**2)**p*c)**3*b**3*x**
6 - 198*log((a + b*x**2)**p*c)**2*a**3*p - 108*log((a + b*x**2)**p*c)**2*a
**2*b*p*x**2 + 54*log((a + b*x**2)**p*c)**2*a*b**2*p*x**4 - 36*log((a + b*
x**2)**p*c)**2*b**3*p*x**6 + 510*log((a + b*x**2)**p*c)*a**3*p**2 + 396*lo
g((a + b*x**2)**p*c)*a**2*b*p**2*x**2 - 90*log((a + b*x**2)**p*c)*a*b**2*p
**2*x**4 + 24*log((a + b*x**2)**p*c)*b**3*p**2*x**6 - 510*a**2*b*p**3*x**2
 + 57*a*b**2*p**3*x**4 - 8*b**3*p**3*x**6)/(216*b**3)
```

3.92 $\int x^3 \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 211

$$\int x^3 \log^3\left(c(a+bx^2)^p\right)\,dx = \frac{3ap^3x^2}{b} - \frac{3p^3(a+bx^2)^2}{16b^2} - \frac{3ap^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2} + \frac{3p^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{8b^2} + \frac{3ap(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2} - \frac{3p(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{8b^2} - \frac{a(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^2} + \frac{(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{4b^2}$$

output
```
3*a*p^3*x^2/b-3/16*p^3*(b*x^2+a)^2/b^2-3*a*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)
/b^2+3/8*p^2*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)/b^2+3/2*a*p*(b*x^2+a)*ln(c*(b*x
^2+a)^p)^2/b^2-3/8*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^2-1/2*a*(b*x^2+a)*l
n(c*(b*x^2+a)^p)^3/b^2+1/4*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^3/b^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.69

$$\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx =$$
$$-\frac{3bp^3x^2(-14a+bx^2)+6a^2p^3\log\left(a+bx^2\right)+6p^2(6a^2+6abx^2-b^2x^4)\log\left(c\left(a+bx^2\right)^p\right)-6p(3a^2+2a}{16b^2}$$

input
```
Integrate[x^3*Log[c*(a + b*x^2)^p]^3,x]
```

output
```
-1/16*(3*b*p^3*x^2*(-14*a + b*x^2) + 6*a^2*p^3*Log[a + b*x^2] + 6*p^2*(6*a
^2 + 6*a*b*x^2 - b^2*x^4)*Log[c*(a + b*x^2)^p] - 6*p*(3*a^2 + 2*a*b*x^2 -
b^2*x^4)*Log[c*(a + b*x^2)^p]^2 + 4*(a^2 - b^2*x^4)*Log[c*(a + b*x^2)^p]^3
)/b^2
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int x^2 \log^3\left(c\left(bx^2+a\right)^p\right)dx^2$$

$$\Big\downarrow \text{2848}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)\log^3\left(c\left(bx^2+a\right)^p\right)}{b}-\frac{a\log^3\left(c\left(bx^2+a\right)^p\right)}{b}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3p^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^2}-\frac{6ap^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2}+\frac{(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{2b^2}-\frac{a(a+\right.$$

input
```
Int[x^3*Log[c*(a + b*x^2)^p]^3,x]
```

output
```
((6*a*p^3*x^2)/b - (3*p^3*(a + b*x^2)^2)/(8*b^2) - (6*a*p^2*(a + b*x^2)*Lo
g[c*(a + b*x^2)^p])/b^2 + (3*p^2*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^
2) + (3*a*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/b^2 - (3*p*(a + b*x^2)^2*L
og[c*(a + b*x^2)^p]^2)/(4*b^2) - (a*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/b^
2 + ((a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(2*b^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 15.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{-4x^4\ln(c(b\,x^2+a)^p)^3b^2+6x^4\ln(c(b\,x^2+a)^p)^2b^2p-6x^4\ln(c(b\,x^2+a)^p)b^2p^2+3x^4b^2p^3-12x^2\ln(c(b\,x^2+a)^p)^2abp+36x^2\ln}{}$
risch	Expression too large to display

input
```
int(x^3*ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-4*x^4*ln(c*(b*x^2+a)^p)^3*b^2+6*x^4*ln(c*(b*x^2+a)^p)^2*b^2*p-6*x^
4*ln(c*(b*x^2+a)^p)*b^2*p^2+3*x^4*b^2*p^3-12*x^2*ln(c*(b*x^2+a)^p)^2*a*b*p
+36*x^2*ln(c*(b*x^2+a)^p)*a*b*p^2-42*a*b*p^3*x^2+78*ln(b*x^2+a)*a^2*p^3+4*
ln(c*(b*x^2+a)^p)^3*a^2-18*ln(c*(b*x^2+a)^p)^2*a^2*p-36*ln(c*(b*x^2+a)^p)*
a^2*p^2+42*a^2*p^3)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.30

$$\int x^3 \log^3\left(c(a+bx^2)^p\right)\,dx = -\frac{3\,b^2p^3x^4-4\,b^2x^4\log(c)^3-42\,abp^3x^2-4\,(b^2p^3x^4-a^2p^3)\log(bx^2+a)^3+6\,(b^2p^3x^4-2\,abp^3x^2-3\,a^2p}{}$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
-1/16*(3*b^2*p^3*x^4 - 4*b^2*x^4*log(c)^3 - 42*a*b*p^3*x^2 - 4*(b^2*p^3*x^
4 - a^2*p^3)*log(b*x^2 + a)^3 + 6*(b^2*p^3*x^4 - 2*a*b*p^3*x^2 - 3*a^2*p^3
 - 2*(b^2*p^2*x^4 - a^2*p^2)*log(c))*log(b*x^2 + a)^2 + 6*(b^2*p*x^4 - 2*a
*b*p*x^2)*log(c)^2 - 6*(b^2*p^3*x^4 - 6*a*b*p^3*x^2 - 7*a^2*p^3 + 2*(b^2*p
*x^4 - a^2*p)*log(c)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2)*log(c
))*log(b*x^2 + a) - 6*(b^2*p^2*x^4 - 6*a*b*p^2*x^2)*log(c))/b^2
```

Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.06

$$\int x^3 \log^3\left(c(a+bx^2)^p\right)\,dx$$
$$= \begin{cases} -\frac{21a^2p^2\log\left(c(a+bx^2)^p\right)}{8b^2} + \frac{9a^2p\log\left(c(a+bx^2)^p\right)^2}{8b^2} - \frac{a^2\log\left(c(a+bx^2)^p\right)^3}{4b^2} + \frac{21ap^3x^2}{8b} - \frac{9ap^2x^2\log\left(c(a+bx^2)^p\right)}{4b} + \frac{3apx^2\log\left(c(a\right.}{4b} \\ \frac{x^4\log\left(a^pc\right)^3}{4} \end{cases}$$

input
```
integrate(x**3*ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Piecewise((-21*a**2*p**2*log(c*(a + b*x**2)**p)/(8*b**2) + 9*a**2*p*log(c*
(a + b*x**2)**p)**2/(8*b**2) - a**2*log(c*(a + b*x**2)**p)**3/(4*b**2) + 2
1*a*p**3*x**2/(8*b) - 9*a*p**2*x**2*log(c*(a + b*x**2)**p)/(4*b) + 3*a*p*x
**2*log(c*(a + b*x**2)**p)**2/(4*b) - 3*p**3*x**4/16 + 3*p**2*x**4*log(c*(
a + b*x**2)**p)/8 - 3*p*x**4*log(c*(a + b*x**2)**p)**2/8 + x**4*log(c*(a +
 b*x**2)**p)**3/4, Ne(b, 0)), (x**4*log(a**p*c)**3/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.96

$$\int x^3 \log^3\left(c(a+bx^2)^p\right)\,dx$$
$$= \frac{1}{4}x^4\log\left((bx^2+a)^pc\right)^3 - \frac{3}{8}bp\left(\frac{2a^2\log(bx^2+a)}{b^3} + \frac{bx^4-2ax^2}{b^2}\right)\log\left((bx^2+a)^pc\right)^2$$
$$-\frac{1}{16}bp\left(\frac{\left(3b^2x^4+4a^2\log(bx^2+a)^3-42abx^2+18a^2\log(bx^2+a)^2+42a^2\log(bx^2+a)\right)p^2}{b^3} - \frac{6\left(b^2\right.}{}\right.$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
1/4*x^4*log((b*x^2 + a)^p*c)^3 - 3/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^
4 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c)^2 - 1/16*b*p*((3*b^2*x^4 + 4*a^2*lo
g(b*x^2 + a)^3 - 42*a*b*x^2 + 18*a^2*log(b*x^2 + a)^2 + 42*a^2*log(b*x^2 +
 a))*p^2/b^3 - 6*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*log(b*x^2 + a)^2 + 6*a^2*log
(b*x^2 + a))*p*log((b*x^2 + a)^p*c)/b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.82

$$\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{4\left(bx^2+a\right)^2p^3\log\left(bx^2+a\right)^3-6\left(bx^2+a\right)^2p^3\log\left(bx^2+a\right)^2+12\left(bx^2+a\right)^2p^2\log\left(bx^2+a\right)^2\log\left(c\right)+6\left(\right.}{}$$
$$-\frac{\left(\left(bx^2+a\right)\log\left(bx^2+a\right)^3-6\,bx^2-3\left(bx^2+a\right)\log\left(bx^2+a\right)^2+6\left(bx^2+a\right)\log\left(bx^2+a\right)-6\,a\right)ap^3+}{}$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
1/16*(4*(b*x^2 + a)^2*p^3*log(b*x^2 + a)^3 - 6*(b*x^2 + a)^2*p^3*log(b*x^2
 + a)^2 + 12*(b*x^2 + a)^2*p^2*log(b*x^2 + a)^2*log(c) + 6*(b*x^2 + a)^2*p
^3*log(b*x^2 + a) - 12*(b*x^2 + a)^2*p^2*log(b*x^2 + a)*log(c) + 12*(b*x^2
 + a)^2*p*log(b*x^2 + a)*log(c)^2 - 3*(b*x^2 + a)^2*p^3 + 6*(b*x^2 + a)^2*
p^2*log(c) - 6*(b*x^2 + a)^2*p*log(c)^2 + 4*(b*x^2 + a)^2*log(c)^3)/b^2 -
1/2*(((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 - 3*(b*x^2 + a)*log(b*x^2 + a
)^2 + 6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*a*p^3 + 3*(2*b*x^2 + (b*x^2 + a)
*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*a*p^2*log(c) - 3*(
b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a*p*log(c)^2 + (b*x^2 + a)*a*log(c
)^3)/b^2
```

Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.68

$$\int x^3 \log^3\left(c(a+bx^2)^p\right)\,dx = \ln\left(c\left(b\,x^2+a\right)^p\right)^2\left(\frac{9\,a^2\,p}{8\,b^2}-\frac{3\,p\,x^4}{8}+\frac{3\,a\,p\,x^2}{4\,b}\right) - \frac{3\,p^3\,x^4}{16}+\ln\left(c\left(b\,x^2+a\right)^p\right)\left(\frac{3\,p^2\,x^4}{8}-\frac{9\,a\,p^2\,x^2}{4\,b}\right) + \ln\left(c\left(b\,x^2+a\right)^p\right)^3\left(\frac{x^4}{4}-\frac{a^2}{4\,b^2}\right) + \frac{21\,a\,p^3\,x^2}{8\,b}-\frac{21\,a^2\,p^3\,\ln\left(b\,x^2+a\right)}{8\,b^2}$$

input
```
int(x^3*log(c*(a + b*x^2)^p)^3,x)
```

output
```
log(c*(a + b*x^2)^p)^2*((9*a^2*p)/(8*b^2) - (3*p*x^4)/8 + (3*a*p*x^2)/(4*b
)) - (3*p^3*x^4)/16 + log(c*(a + b*x^2)^p)*((3*p^2*x^4)/8 - (9*a*p^2*x^2)/
(4*b)) + log(c*(a + b*x^2)^p)^3*(x^4/4 - a^2/(4*b^2)) + (21*a*p^3*x^2)/(8*
b) - (21*a^2*p^3*log(a + b*x^2))/(8*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.94

$$\int x^3 \log^3\left(c(a+bx^2)^p\right)\,dx = \frac{-4\log\left(\left(b\,x^2+a\right)^p c\right)^3 a^2+4\log\left(\left(b\,x^2+a\right)^p c\right)^3 b^2x^4+18\log\left(\left(b\,x^2+a\right)^p c\right)^2 a^2p+12\log\left(\left(b\,x^2+a\right)^p c\right)^2 ab}{}$$

input
```
int(x^3*log(c*(b*x^2+a)^p)^3,x)
```

output
```
( - 4*log((a + b*x**2)**p*c)**3*a**2 + 4*log((a + b*x**2)**p*c)**3*b**2*x*
*4 + 18*log((a + b*x**2)**p*c)**2*a**2*p + 12*log((a + b*x**2)**p*c)**2*a*
b*p*x**2 - 6*log((a + b*x**2)**p*c)**2*b**2*p*x**4 - 42*log((a + b*x**2)**
p*c)*a**2*p**2 - 36*log((a + b*x**2)**p*c)*a*b*p**2*x**2 + 6*log((a + b*x*
*2)**p*c)*b**2*p**2*x**4 + 42*a*b*p**3*x**2 - 3*b**2*p**3*x**4)/(16*b**2)
```

3.93 $\int x \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int x \log^3\left(c(a+bx^2)^p\right)\,dx = -3p^3x^2+\frac{3p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}-\frac{3p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b}+\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b}$$

output
```
-3*p^3*x^2+3*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b-3/2*p*(b*x^2+a)*ln(c*(b*x^2
+a)^p)^2/b+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^3/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int x \log^3\left(c(a+bx^2)^p\right)\,dx = \frac{-6bp^3x^2+6p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)-3p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)+(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b}$$

input
```
Integrate[x*Log[c*(a + b*x^2)^p]^3,x]
```

output
```
(-6*b*p^3*x^2 + 6*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p] - 3*p*(a + b*x^2)*L
og[c*(a + b*x^2)^p]^2 + (a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2904, 2836, 2733, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log^3\left(c\left(a+bx^2\right)^p\right) dx$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{2}\int \log^3\left(c\left(bx^2+a\right)^p\right) dx^2$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \log^3\left(c\left(bx^2+a\right)^p\right) d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{ 2733}$$

$$\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)-3p\int \log^2\left(c\left(bx^2+a\right)^p\right) d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{ 2733}$$

$$\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)-3p\left(\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)-2p\int \log\left(c\left(bx^2+a\right)^p\right) d\left(bx^2+a\right)\right)}{2b}$$

$$\Big\downarrow \text{ 2732}$$

$$\frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)-3p\left(\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)-2p\left(\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)-p\left(a+bx^2\right)\right)\right)}{2b}$$

input
```
Int[x*Log[c*(a + b*x^2)^p]^3,x]
```

output
```
((a + b*x^2)*Log[c*(a + b*x^2)^p]^3 - 3*p*((a + b*x^2)*Log[c*(a + b*x^2)^p
]^2 - 2*p*(-(p*(a + b*x^2)) + (a + b*x^2)*Log[c*(a + b*x^2)^p])))/(2*b)
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.62

method	result
parallelrisch	$\frac{x^2\ln\left(c\left(b x^2+a\right)^p\right)^3 abp-3x^2\ln\left(c\left(b x^2+a\right)^p\right)^2 ab p^2+6x^2\ln\left(c\left(b x^2+a\right)^p\right) ab p^3-6x^2 ab p^4+\ln\left(c\left(b x^2+a\right)^p\right)^3 a^2 p-3\ln\left(c\left(b x^2+a\right.\right.}{2abp}$
risch	Expression too large to display

input
```
int(x*ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2*ln(c*(b*x^2+a)^p)^3*a*b*p-3*x^2*ln(c*(b*x^2+a)^p)^2*a*b*p^2+6*x^2
*ln(c*(b*x^2+a)^p)*a*b*p^3-6*x^2*a*b*p^4+ln(c*(b*x^2+a)^p)^3*a^2*p-3*ln(c*
(b*x^2+a)^p)^2*a^2*p^2+6*ln(c*(b*x^2+a)^p)*a^2*p^3)/a/b/p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.89

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx =$$

$$-\frac{6\,bp^3x^2 - 6\,bp^2x^2\log\left(c\right) + 3\,bpx^2\log\left(c\right)^2 - bx^2\log\left(c\right)^3 - \left(bp^3x^2 + ap^3\right)\log\left(bx^2 + a\right)^3 + 3\left(bp^3x^2 + ap\right.}{}$$

input
```
integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
-1/2*(6*b*p^3*x^2 - 6*b*p^2*x^2*log(c) + 3*b*p*x^2*log(c)^2 - b*x^2*log(c)
^3 - (b*p^3*x^2 + a*p^3)*log(b*x^2 + a)^3 + 3*(b*p^3*x^2 + a*p^3 - (b*p^2*
x^2 + a*p^2)*log(c))*log(b*x^2 + a)^2 - 3*(2*b*p^3*x^2 + 2*a*p^3 + (b*p*x^
2 + a*p)*log(c)^2 - 2*(b*p^2*x^2 + a*p^2)*log(c))*log(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$= \begin{cases} \frac{3ap^2\log\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{3ap\log\left(c\left(a+bx^2\right)^p\right)^2}{2b} + \frac{a\log\left(c\left(a+bx^2\right)^p\right)^3}{2b} - 3p^3x^2 + 3p^2x^2\log\left(c\left(a+bx^2\right)^p\right) - \frac{3px^2\log\left(c\left(a+b\right.\right.}{2} \\ \frac{x^2\log\left(a^pc\right)^3}{2} \end{cases}$$

input
```
integrate(x*ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Piecewise((3*a*p**2*log(c*(a + b*x**2)**p)/b - 3*a*p*log(c*(a + b*x**2)**p
)**2/(2*b) + a*log(c*(a + b*x**2)**p)**3/(2*b) - 3*p**3*x**2 + 3*p**2*x**2
*log(c*(a + b*x**2)**p) - 3*p*x**2*log(c*(a + b*x**2)**p)**2/2 + x**2*log(
c*(a + b*x**2)**p)**3/2, Ne(b, 0)), (x**2*log(a**p*c)**3/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.76

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=-\frac{3}{2}\,bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right)^2+\frac{1}{2}\,x^2\log\left(\left(bx^2+a\right)^pc\right)^3$$
$$+\frac{1}{2}\,bp\left(\frac{\left(a\log\left(bx^2+a\right)^3-6\,bx^2+3\,a\log\left(bx^2+a\right)^2+6\,a\log\left(bx^2+a\right)\right)p^2}{b^2}+\frac{3\left(2\,bx^2-a\log\left(bx^2+a\right)\right.}{}\right.$$

input
```
integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
-3/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c)^2 + 1/2*x^2*l
og((b*x^2 + a)^p*c)^3 + 1/2*b*p*((a*log(b*x^2 + a)^3 - 6*b*x^2 + 3*a*log(b
*x^2 + a)^2 + 6*a*log(b*x^2 + a))*p^2/b^2 + 3*(2*b*x^2 - a*log(b*x^2 + a)^
2 - 2*a*log(b*x^2 + a))*p*log((b*x^2 + a)^p*c)/b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.82

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\frac{\left(\left(bx^2+a\right)\log\left(bx^2+a\right)^3-6\,bx^2-3\left(bx^2+a\right)\log\left(bx^2+a\right)^2+6\left(bx^2+a\right)\log\left(bx^2+a\right)-6\,a\right)p^3+3\left(2\right.}{}$$

input
```
integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
1/2*(((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 - 3*(b*x^2 + a)*log(b*x^2 + a
)^2 + 6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*p^3 + 3*(2*b*x^2 + (b*x^2 + a)*l
og(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*p^2*log(c) - 3*(b*x^
2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c)^2 + (b*x^2 + a)*log(c)^3)/b
```

Mupad [B] (verification not implemented)

Time = 15.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \ln\left(c\left(b\,x^2+a\right)^p\right)^3\left(\frac{a}{2\,b}+\frac{x^2}{2}\right) - \ln\left(c\left(b\,x^2+a\right)^p\right)^2\left(\frac{3\,p\,x^2}{2}+\frac{3\,a\,p}{2\,b}\right) - 3\,p^3\,x^2 + 3\,p^2\,x^2\,\ln\left(c\left(b\,x^2+a\right)^p\right) + \frac{3\,a\,p^3\,\ln\left(b\,x^2+a\right)}{b}$$

input
```
int(x*log(c*(a + b*x^2)^p)^3,x)
```

output
```
log(c*(a + b*x^2)^p)^3*(a/(2*b) + x^2/2) - log(c*(a + b*x^2)^p)^2*((3*p*x^
2)/2 + (3*a*p)/(2*b)) - 3*p^3*x^2 + 3*p^2*x^2*log(c*(a + b*x^2)^p) + (3*a*
p^3*log(a + b*x^2))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.38

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{\log\left(\left(b\,x^2+a\right)^p c\right)^3 a + \log\left(\left(b\,x^2+a\right)^p c\right)^3 b\,x^2 - 3\log\left(\left(b\,x^2+a\right)^p c\right)^2 ap - 3\log\left(\left(b\,x^2+a\right)^p c\right)^2 bp\,x^2 + 6\log}{2b}$$

input
```
int(x*log(c*(b*x^2+a)^p)^3,x)
```

output
```
(log((a + b*x**2)**p*c)**3*a + log((a + b*x**2)**p*c)**3*b*x**2 - 3*log((a
 + b*x**2)**p*c)**2*a*p - 3*log((a + b*x**2)**p*c)**2*b*p*x**2 + 6*log((a
+ b*x**2)**p*c)*a*p**2 + 6*log((a + b*x**2)**p*c)*b*p**2*x**2 - 6*b*p**3*x
**2)/(2*b)
```

3.94 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 106

$$\begin{aligned}\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx &= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)\\ &\quad+\frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)\\ &\quad-3p^2\log\left(c\left(a+bx^2\right)^p\right)\operatorname{PolyLog}\left(3,1+\frac{bx^2}{a}\right)\\ &\quad+3p^3\operatorname{PolyLog}\left(4,1+\frac{bx^2}{a}\right)\end{aligned}$$

output
```
1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^3+3/2*p*ln(c*(b*x^2+a)^p)^2*polylog(2,1
+b*x^2/a)-3*p^2*ln(c*(b*x^2+a)^p)*polylog(3,1+b*x^2/a)+3*p^3*polylog(4,1+b
*x^2/a)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 279 vs. $2(106) = 212$.

Time = 0.23 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.63

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \log(x)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^3 + 3p\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2\left(\log(x)\left(\log\left(a+bx^2\right)-\log\left(1+\frac{bx^2}{a}\right)\right)-\frac{1}{2}\text{PolyLog}\left(2,-\frac{bx^2}{a}\right)\right) - \frac{3}{2}p^2\left(p\log\left(a+bx^2\right)-\log\left(c(a+bx^2)^p\right)\right)\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(a+bx^2\right)+2\log\left(a+bx^2\right)\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right)-2\,\text{PolyLog}\left(3,1+\frac{bx^2}{a}\right)\right) + \frac{1}{2}p^3\left(\log\left(-\frac{bx^2}{a}\right)\log^3\left(a+bx^2\right)+3\log^2\left(a+bx^2\right)\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right)-6\log\left(a+bx^2\right)\text{PolyLog}\left(3,1+\frac{bx^2}{a}\right)+6\,\text{PolyLog}\left(4,1+\frac{bx^2}{a}\right)\right)$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^3/x,x]
```

output
```
Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3 + 3*p*(-(p*Log[a + b
*x^2]) + Log[c*(a + b*x^2)^p])^2*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)
/a]) - PolyLog[2, -((b*x^2)/a)]/2) - (3*p^2*(p*Log[a + b*x^2] - Log[c*(a +
 b*x^2)^p])*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*Log[a + b*x^2]*PolyLog
[2, 1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/a]))/2 + (p^3*(Log[-((b*x^2)
/a)]*Log[a + b*x^2]^3 + 3*Log[a + b*x^2]^2*PolyLog[2, 1 + (b*x^2)/a] - 6*L
og[a + b*x^2]*PolyLog[3, 1 + (b*x^2)/a] + 6*PolyLog[4, 1 + (b*x^2)/a]))/2
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2904, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{\log^3\left(c\left(bx^2+a\right)^p\right)}{x^2}dx^2$$

$$\Big\downarrow \text{2843}$$

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)-3bp\int \frac{\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx^2\right)$$

$$\Big\downarrow \text{2881}$$

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)-3p\int \frac{\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(bx^2+a\right)^p\right)}{x^2}d\left(bx^2+a\right)\right)$$

$$\Big\downarrow \text{2821}$$

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)-3p\left(2p\int \frac{\log\left(c\left(bx^2+a\right)^p\right)\text{PolyLog}\left(2,\frac{bx^2+a}{a}\right)}{x^2}d\left(bx^2+a\right)-\text{PolyLog}\left(2,\right.\right.\right.$$

$$\Big\downarrow \text{2830}$$

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)-3p\left(2p\left(\text{PolyLog}\left(3,\frac{bx^2+a}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)-p\int \frac{\text{PolyLog}\left(3,\frac{bx^2+a}{a}\right)}{x^2}\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)-3p\left(2p\left(\text{PolyLog}\left(3,\frac{bx^2+a}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)-p\,\text{PolyLog}\left(4,\frac{bx^2+a}{a}\right)\right.\right.\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^3/x,x]
```

output
```
(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^3 - 3*p*(-(Log[c*(a + b*x^2)^p]^2*
PolyLog[2, (a + b*x^2)/a]) + 2*p*(Log[c*(a + b*x^2)^p]*PolyLog[3, (a + b*x
^2)/a] - p*PolyLog[4, (a + b*x^2)/a])))/2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln \left(c \left(b \,x^{2}+a \right)^{p}\right)^{3}}{x}d x$$

input
```
int(ln(c*(b*x^2+a)^p)^3/x,x)
```

output
```
int(ln(c*(b*x^2+a)^p)^3/x,x)
```

Fricas [F]

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}\,dx=\int \frac{\log\left(\left(bx^2+a\right)^pc\right)^3}{x}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^3/x, x)
```

Sympy [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^3}{x}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**3/x,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**3/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(101) = 202$.

Time = 0.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.05

$$\begin{aligned}
&\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx \\
&= \frac{1}{2}\left(\log\left(bx^2+a\right)^3\log\left(-\frac{bx^2+a}{a}+1\right)+3\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)^2-6\,\log\left(bx^2+a\right)\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right. \\
&\quad +\frac{3}{2}\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)p^2\log\left(c\right) \\
&\quad +\frac{3}{2}\left(\log\left(bx^2+a\right)\log\left(-\frac{bx^2+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\right)p\log\left(c\right)^2+\log\left(c\right)^3\log\left(x\right)
\end{aligned}$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="maxima")
```

output
```
1/2*(log(b*x^2 + a)^3*log(-(b*x^2 + a)/a + 1) + 3*dilog((b*x^2 + a)/a)*log
(b*x^2 + a)^2 - 6*log(b*x^2 + a)*polylog(3, (b*x^2 + a)/a) + 6*polylog(4,
(b*x^2 + a)/a))*p^3 + 3/2*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*di
log((b*x^2 + a)/a)*log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2*log(c
) + 3/2*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*
log(c)^2 + log(c)^3*log(x)
```

Giac [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^3}{x}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^3/x,x)
```

output
```
int(log(c*(a + b*x^2)^p)^3/x, x)
```

Reduce [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \frac{8\left(\int \frac{\log\left((b\,x^2+a)^p c\right)^3}{b\,x^3+ax}dx\right)ap+\log\left((b\,x^2+a)^p\,c\right)^4}{8p}$$

input
```
int(log(c*(b*x^2+a)^p)^3/x,x)
```

output
```
(8*int(log((a + b*x**2)**p*c)**3/(a*x + b*x**3),x)*a*p + log((a + b*x**2)*
*p*c)**4)/(8*p)
```

3.95 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx = \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a} - \frac{\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2ax^2} + \frac{3bp^2\log\left(c\left(a+bx^2\right)^p\right)\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{a} - \frac{3bp^3\,\text{PolyLog}\left(3,1+\frac{bx^2}{a}\right)}{a}$$

output

```
3/2*b*p*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^2/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)
^3/a/x^2+3*b*p^2*ln(c*(b*x^2+a)^p)*polylog(2,1+b*x^2/a)/a-3*b*p^3*polylog(
3,1+b*x^2/a)/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 302 vs. $2(119) = 238$.

Time = 0.42 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.54

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx =$$
$$-\frac{-6bp^3x^2\log(x)\log^2(a+bx^2)+3bp^3x^2\log\left(-\frac{bx^2}{a}\right)\log^2(a+bx^2)+bp^3x^2\log^3(a+bx^2)+12bp^2x^2\log}{}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^3/x^3,x]
```

output
```
-1/2*(-6*b*p^3*x^2*Log[x]*Log[a + b*x^2]^2 + 3*b*p^3*x^2*Log[-((b*x^2)/a)]
*Log[a + b*x^2]^2 + b*p^3*x^2*Log[a + b*x^2]^3 + 12*b*p^2*x^2*Log[x]*Log[a
 + b*x^2]*Log[c*(a + b*x^2)^p] - 6*b*p^2*x^2*Log[-((b*x^2)/a)]*Log[a + b*x
^2]*Log[c*(a + b*x^2)^p] - 3*b*p^2*x^2*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^
p] - 6*b*p*x^2*Log[x]*Log[c*(a + b*x^2)^p]^2 + 3*b*p*x^2*Log[a + b*x^2]*Lo
g[c*(a + b*x^2)^p]^2 + a*Log[c*(a + b*x^2)^p]^3 - 6*b*p^2*x^2*Log[c*(a + b
*x^2)^p]*PolyLog[2, 1 + (b*x^2)/a] + 6*b*p^3*x^2*PolyLog[3, 1 + (b*x^2)/a]
)/(a*x^2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2904, 2844, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}dx$$
$$\Big\downarrow \text{2904}$$
$$\frac{1}{2}\int \frac{\log^3\left(c(bx^2+a)^p\right)}{x^4}dx^2$$

$$\Big\downarrow \text{2844}$$

$$\frac{1}{2}\left(\frac{3bp\int \frac{\log^2(c(bx^2+a)^p)}{x^2}dx^2}{a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{ax^2}\right)$$

$$\Big\downarrow \text{2843}$$

$$\frac{1}{2}\left(\frac{3bp\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)-2bp\int \frac{\log\left(-\frac{bx^2}{a}\right)\log(c(bx^2+a)^p)}{bx^2+a}dx^2\right)}{a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{ax^2}\right)$$

$$\Big\downarrow \text{2881}$$

$$\frac{1}{2}\left(\frac{3bp\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)-2p\int \frac{\log\left(-\frac{bx^2}{a}\right)\log(c(bx^2+a)^p)}{x^2}d(bx^2+a)\right)}{a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{ax^2}\right)$$

$$\Big\downarrow \text{2821}$$

$$\frac{1}{2}\left(\frac{3bp\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)-2p\left(p\int \frac{\text{PolyLog}\left(2,\frac{bx^2+a}{a}\right)}{x^2}d(bx^2+a)-\text{PolyLog}\left(2,\frac{bx^2+a}{a}\right)\log\left(c(a+bx^2)^p\right)\right)\right)}{a}\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(\frac{3bp\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)-2p\left(p\,\text{PolyLog}\left(3,\frac{bx^2+a}{a}\right)-\text{PolyLog}\left(2,\frac{bx^2+a}{a}\right)\log\left(c(a+bx^2)^p\right)\right)\right)}{a}-\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^3/x^3,x]
```

output
```
(-(((a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(a*x^2)) + (3*b*p*(Log[-((b*x^2)/a
)]*Log[c*(a + b*x^2)^p]^2 - 2*p*(-(Log[c*(a + b*x^2)^p]*PolyLog[2, (a + b*
x^2)/a]) + p*PolyLog[3, (a + b*x^2)/a])))/a)/2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln \left(c(b\,x^2+a)^p\right)^3}{x^3}dx$$

input
```
int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

output
```
int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

Fricas [F]

$$\int \frac{\log^3 \left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)^3}{x^3}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^3/x^3, x)
```

Sympy [F]

$$\int \frac{\log^3 \left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log \left(c(a+bx^2)^p\right)^3}{x^3}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**3/x**3,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**3/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.70

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx$$
$$= \frac{1}{2}\left(\frac{3\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)p^2}{a}+\frac{6\left(\log\left(bx^2+a\right.\right.}{}\right.$$
$$-\frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{2\,x^2}$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="maxima")
```

output
```
1/2*(3*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*
log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2/a + 6*(log(b*x^2 + a)*lo
g(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c)/a + 6*log(c)^2*log(
x)/a - (p^2*log(b*x^2 + a)^3 + 3*p*log(b*x^2 + a)^2*log(c) + 3*log(b*x^2 +
 a)*log(c)^2)/a)*b*p - 1/2*log((b*x^2 + a)^p*c)^3/x^2
```

Giac [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left(\left(bx^2+a\right)^p c\right)^3}{x^3}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^3}{x^3}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^3/x^3,x)
```

output
```
int(log(c*(a + b*x^2)^p)^3/x^3, x)
```

Reduce [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{-6\left(\int \frac{\log\left((b\,x^2+a)^p c\right)^2}{b\,x^5+a\,x^3}dx\right)a^2p\,x^2-12\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^5+a\,x^3}dx\right)a^2p^2x^2-\log\left((b\,x^2+a)^p\,c\right)^3a-3\log\left((b\,x^2+a)^p\right.}{2a\,x^2}$$

input
```
int(log(c*(b*x^2+a)^p)^3/x^3,x)
```

output
```
( - 6*int(log((a + b*x**2)**p*c)**2/(a*x**3 + b*x**5),x)*a**2*p*x**2 - 12*
int(log((a + b*x**2)**p*c)/(a*x**3 + b*x**5),x)*a**2*p**2*x**2 - log((a +
b*x**2)**p*c)**3*a - 3*log((a + b*x**2)**p*c)**2*a*p - 6*log((a + b*x**2)*
*p*c)*a*p**2 - 6*log((a + b*x**2)**p*c)*b*p**2*x**2 + 12*log(x)*b*p**3*x**
2)/(2*a*x**2)
```

3.96 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 219

$$\begin{aligned}\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx &= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} \\ &\quad -\frac{3bp(a+bx^2)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2}-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} \\ &\quad -\frac{3b^2p\log^2\left(c\left(a+bx^2\right)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{4a^2} \\ &\quad +\frac{3b^2p^2\log\left(c\left(a+bx^2\right)^p\right)\operatorname{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^2} \\ &\quad +\frac{3b^2p^3\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{2a^2}+\frac{3b^2p^3\operatorname{PolyLog}\left(3,\frac{a}{a+bx^2}\right)}{2a^2}\end{aligned}$$

output

```
3/2*b^2*p^2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a^2-3/4*b*p*(b*x^2+a)*ln(c*(b*x
^2+a)^p)^2/a^2/x^2-1/4*ln(c*(b*x^2+a)^p)^3/x^4-3/4*b^2*p*ln(c*(b*x^2+a)^p)
^2*ln(1-a/(b*x^2+a))/a^2+3/2*b^2*p^2*ln(c*(b*x^2+a)^p)*polylog(2,a/(b*x^2+
a))/a^2+3/2*b^2*p^3*polylog(2,1+b*x^2/a)/a^2+3/2*b^2*p^3*polylog(3,a/(b*x^
2+a))/a^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 477 vs. 2(219) = 438.

Time = 0.47 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.18

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{3bp\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{4ax^2}$$
$$-\frac{3b^2p\log(x)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{2a^2}$$
$$+\frac{3b^2p\log\left(a+bx^2\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{4a^2}$$
$$-\frac{3p\log\left(a+bx^2\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{4x^4}$$
$$-\frac{\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^3}{4x^4}$$
$$+3p^2\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)\left(-\frac{\log^2\left(a+bx^2\right)}{4x^4}\right.$$
$$+\frac{b\left(4bx^2\log(x)+\log\left(a+bx^2\right)\left(-2(a+bx^2)-2bx^2\log\left(-\frac{bx^2}{a}\right)+bx^2\log\left(a+bx^2\right)\right)-2bx^2\,\text{PolyLog}\left(\right.\right.}{4a^2x^2}$$
$$+\frac{b^2p^3\left(\frac{(a+bx^2)\left(a\left(3-2\log\left(a+bx^2\right)\right)+(a+bx^2)\left(-3+\log\left(a+bx^2\right)\right)\right)\log^2\left(a+bx^2\right)}{b^2x^4}-3\left(-2+\log\left(a+bx^2\right)\right)\log\left(a+bx^2\right)\log\right.}{4a^2}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

output
```
(-3*b*p*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/(4*a*x^2) - (3*b^2
*p*Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/(2*a^2) + (3*b^2
*p*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/(4*a^2)
- (3*p*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/(4*x
^4) - (-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3/(4*x^4) + 3*p^2*(-(p*
Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])*(-1/4*Log[a + b*x^2]^2/x^4 + (b*(4
*b*x^2*Log[x] + Log[a + b*x^2]*(-2*(a + b*x^2) - 2*b*x^2*Log[-((b*x^2)/a)]
 + b*x^2*Log[a + b*x^2]) - 2*b*x^2*PolyLog[2, 1 + (b*x^2)/a]))/(4*a^2*x^2)
) + (b^2*p^3*(((a + b*x^2)*(a*(3 - 2*Log[a + b*x^2]) + (a + b*x^2)*(-3 + L
og[a + b*x^2]))*Log[a + b*x^2]^2)/(b^2*x^4) - 3*(-2 + Log[a + b*x^2])*Log[
a + b*x^2]*Log[1 - (a + b*x^2)/a] - 6*(-1 + Log[a + b*x^2])*PolyLog[2, (a
+ b*x^2)/a] + 6*PolyLog[3, (a + b*x^2)/a]))/(4*a^2)
```

Rubi [A] (warning: unable to verify)

Time = 1.65 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {2904, 2845, 2858, 27, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{\log^3\left(c\left(bx^2+a\right)^p\right)}{x^6}dx^2$$

$$\Big\downarrow \text{2845}$$

$$\frac{1}{2}\left(\frac{3}{2}bp\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{x^4\left(bx^2+a\right)}dx^2-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{2}\left(\frac{3}{2}p\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{x^6}d\left(bx^2+a\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{3}{2}b^2p\int\frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{b^2x^6}d\left(bx^2+a\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{2789}$$

$$\frac{1}{2}\left(\frac{3}{2}b^2p\left(\frac{\int\frac{\log^2(c(bx^2+a)^p)}{b^2x^4}d\left(bx^2+a\right)}{a}+\frac{\int-\frac{\log^2(c(bx^2+a)^p)}{bx^4}d\left(bx^2+a\right)}{a}\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{2755}$$

$$\frac{1}{2}\left(\frac{3}{2}b^2p\left(\frac{-\frac{2p\int-\frac{\log\left(c\left(bx^2+a\right)^p\right)}{bx^2}d(bx^2+a)}{a}-\frac{(a+bx^2)\log^2(c(a+bx^2)^p)}{abx^2}}{a}+\frac{\int-\frac{\log^2(c(bx^2+a)^p)}{bx^4}d\left(bx^2+a\right)}{a}\right)-\frac{\log^3\left(c\left(a+b\right.\right.}{2x^4}\right.$$

$$\Big\downarrow \text{2754}$$

$$\frac{1}{2}\left(\frac{3}{2}b^2p\left(\frac{-\frac{2p\left(p\int\frac{\log\left(1-\frac{bx^2+a}{a}\right)}{x^2}d(bx^2+a)-\log\left(1-\frac{a+bx^2}{a}\right)\log(c(a+bx^2)^p)\right)}{a}-\frac{(a+bx^2)\log^2(c(a+bx^2)^p)}{abx^2}}{a}+\frac{\int-\frac{\log^2(c(bx^2+a)^p)}{bx^4}}{a}\right.\right.$$

$$\Big\downarrow \text{2779}$$

$$\frac{1}{2}\left(\frac{3}{2}b^2p\left(\frac{\frac{2p\int\frac{\log\left(1-\frac{a}{x^2}\right)\log\left(c\left(bx^2+a\right)^p\right)}{x^2}d(bx^2+a)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log^2(c(a+bx^2)^p)}{a}}{a}+\frac{-\frac{2p\left(p\int\frac{\log\left(1-\frac{bx^2+a}{a}\right)}{x^2}d(bx^2+a)-\log\left(1-\frac{a+b}{a}\right.\right.}{a}}{}\right.\right.$$

$$\Big\downarrow \text{2821}$$

$$\frac{1}{2}\left(\frac{3}{2}b^2p\left(\frac{\frac{2p\left(\text{PolyLog}\left(2,\frac{a}{x^2}\right)\log(c(a+bx^2)^p)-p\int\frac{\text{PolyLog}\left(2,\frac{a}{x^2}\right)}{x^2}d(bx^2+a)\right)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log^2(c(a+bx^2)^p)}{a}}{a}+\frac{-\frac{2p\left(p\int\frac{\log\left(1-\frac{bx^2}{a}\right.}{x^2}\right.}{}}{}\right.\right.$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(\frac{3}{2}b^2p\left(\frac{\frac{2p\left(\text{PolyLog}\left(2,\frac{a}{x^2}\right)\log\left(c\left(a+bx^2\right)^p\right)-p\int\frac{\text{PolyLog}\left(2,\frac{a}{x^2}\right)}{x^2}d\left(bx^2+a\right)\right)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{a}}{a}+\frac{-\frac{2p\left(\log\left(1-\frac{a+bx^2}{a}\right)\right.}{}}{}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(\frac{3}{2}b^2p\left(\frac{-\frac{2p\left(\log\left(1-\frac{a+bx^2}{a}\right)\left(-\log\left(c\left(a+bx^2\right)^p\right)\right)-p\,\text{PolyLog}\left(2,\frac{bx^2+a}{a}\right)\right)}{a}-\frac{\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{abx^2}}{a}+\frac{2p\left(\text{PolyLog}\left(2,\frac{a}{x^2}\right)\log\left(c\left(a\right.\right.\right.}{}\right.\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

output
```
(-1/2*Log[c*(a + b*x^2)^p]^3/x^4 + (3*b^2*p*((-(((a + b*x^2)*Log[c*(a + b*
x^2)^p]^2)/(a*b*x^2)) - (2*p*(-(Log[c*(a + b*x^2)^p]*Log[1 - (a + b*x^2)/a
]) - p*PolyLog[2, (a + b*x^2)/a]))/a)/a + (-((Log[1 - a/x^2]*Log[c*(a + b*
x^2)^p]^2)/a) + (2*p*(Log[c*(a + b*x^2)^p]*PolyLog[2, a/x^2] + p*PolyLog[3
, a/x^2]))/a)/a))/2)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln \left(c(b\,x^2+a)^p\right)^3}{x^5}dx$$

input
```
int(ln(c*(b*x^2+a)^p)^3/x^5,x)
```

output
```
int(ln(c*(b*x^2+a)^p)^3/x^5,x)
```

Fricas [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^5}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^3/x^5, x)
```

Sympy [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^3}{x^5}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**3/x**5,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**3/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.23

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{1}{4}\left(\frac{3\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)bp^2}{a^2} - \frac{6\left(p^2-p\log\right.}{} \right. - \frac{\log\left(\left(bx^2+a\right)^pc\right)^3}{4\,x^4}$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="maxima")
```

output
```
-1/4*(3*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)
*log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*b*p^2/a^2 - 6*(p^2 - p*log(
c))*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*b/a^2
- 6*(2*p*log(c) - log(c)^2)*b*log(x)/a^2 - (b*p^2*x^2*log(b*x^2 + a)^3 - 3
*((p^2 - p*log(c))*b*x^2 + a*p^2)*log(b*x^2 + a)^2 - 3*a*log(c)^2 - 3*((2*
p*log(c) - log(c)^2)*b*x^2 + 2*a*p*log(c))*log(b*x^2 + a))/(a^2*x^2))*b*p
- 1/4*log((b*x^2 + a)^p*c)^3/x^4
```

Giac [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^5}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^3}{x^5}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^3/x^5,x)
```

output
```
int(log(c*(a + b*x^2)^p)^3/x^5, x)
```

Reduce [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx$$
$$= \frac{-6\left(\int \frac{\log\left((b\,x^2+a)^p c\right)^2}{b\,x^3+ax}dx\right) b^2 p\,x^4 + 12\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^3+ax}dx\right) b^2 p^2 x^4 - \log\left((b\,x^2+a)^p\,c\right)^3 a - 3\log\left((b\,x^2+a)^p\right.}{4a\,x^4}$$

input
```
int(log(c*(b*x^2+a)^p)^3/x^5,x)
```

output
```
( - 6*int(log((a + b*x**2)**p*c)**2/(a*x + b*x**3),x)*b**2*p*x**4 + 12*int
(log((a + b*x**2)**p*c)/(a*x + b*x**3),x)*b**2*p**2*x**4 - log((a + b*x**2
)**p*c)**3*a - 3*log((a + b*x**2)**p*c)**2*b*p*x**2)/(4*a*x**4)
```

3.97 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 352

$$\begin{aligned}\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx &= \frac{b^3p^3\log(x)}{a^3}-\frac{b^2p^2(a+bx^2)\log\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2}\\ &\quad-\frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a^3}-\frac{bp\log^2\left(c\left(a+bx^2\right)^p\right)}{4ax^4}\\ &\quad+\frac{b^2p(a+bx^2)\log^2\left(c\left(a+bx^2\right)^p\right)}{2a^3x^2}-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{6x^6}\\ &\quad-\frac{b^3p^2\log\left(c\left(a+bx^2\right)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^3}\\ &\quad+\frac{b^3p\log^2\left(c\left(a+bx^2\right)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^3}\\ &\quad+\frac{b^3p^3\operatorname{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^3}\\ &\quad-\frac{b^3p^2\log\left(c\left(a+bx^2\right)^p\right)\operatorname{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{a^3}\\ &\quad-\frac{b^3p^3\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{a^3}-\frac{b^3p^3\operatorname{PolyLog}\left(3,\frac{a}{a+bx^2}\right)}{a^3}\end{aligned}$$

output
```
b^3*p^3*ln(x)/a^3-1/2*b^2*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^3/x^2-b^3*p^2*
ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a^3-1/4*b*p*ln(c*(b*x^2+a)^p)^2/a/x^4+1/2*b
^2*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/a^3/x^2-1/6*ln(c*(b*x^2+a)^p)^3/x^6-1/2
*b^3*p^2*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^3+1/2*b^3*p*ln(c*(b*x^2+a)^
p)^2*ln(1-a/(b*x^2+a))/a^3+1/2*b^3*p^3*polylog(2,a/(b*x^2+a))/a^3-b^3*p^2*
ln(c*(b*x^2+a)^p)*polylog(2,a/(b*x^2+a))/a^3-b^3*p^3*polylog(2,1+b*x^2/a)/
a^3-b^3*p^3*polylog(3,a/(b*x^2+a))/a^3
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.62

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{-6b^3p^3x^6\log\left(-\frac{bx^2}{a}\right)+6b^3p^3x^6\log\left(a+bx^2\right)-36b^3p^3x^6\log(x)\log\left(a+bx^2\right)+18b^3p^3x^6\log\left(-\frac{bx^2}{a}\right)\mathrm{lo}}{}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

output
```
-1/12*(-6*b^3*p^3*x^6*Log[-((b*x^2)/a)] + 6*b^3*p^3*x^6*Log[a + b*x^2] - 3
6*b^3*p^3*x^6*Log[x]*Log[a + b*x^2] + 18*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log
[a + b*x^2] + 9*b^3*p^3*x^6*Log[a + b*x^2]^2 - 12*b^3*p^3*x^6*Log[x]*Log[a
 + b*x^2]^2 + 6*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*b^3*p^3
*x^6*Log[a + b*x^2]^3 + 6*a*b^2*p^2*x^4*Log[c*(a + b*x^2)^p] + 36*b^3*p^2*
x^6*Log[x]*Log[c*(a + b*x^2)^p] - 18*b^3*p^2*x^6*Log[a + b*x^2]*Log[c*(a +
 b*x^2)^p] + 24*b^3*p^2*x^6*Log[x]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 1
2*b^3*p^2*x^6*Log[-((b*x^2)/a)]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 6*b^
3*p^2*x^6*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] + 3*a^2*b*p*x^2*Log[c*(a +
 b*x^2)^p]^2 - 6*a*b^2*p*x^4*Log[c*(a + b*x^2)^p]^2 - 12*b^3*p*x^6*Log[x]*
Log[c*(a + b*x^2)^p]^2 + 6*b^3*p*x^6*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]^2
 + 2*a^3*Log[c*(a + b*x^2)^p]^3 + 6*b^3*p^2*x^6*(3*p - 2*Log[c*(a + b*x^2)
^p])*PolyLog[2, 1 + (b*x^2)/a] + 12*b^3*p^3*x^6*PolyLog[3, 1 + (b*x^2)/a])
/(a^3*x^6)
```

Rubi [A] (warning: unable to verify)

Time = 2.81 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.91, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {2904, 2845, 2858, 25, 27, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^7} \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{\log^3\left(c\left(bx^2+a\right)^p\right)}{x^8}dx^2$$

$$\Big\downarrow \text{2845}$$

$$\frac{1}{2}\left(bp\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{x^6\left(bx^2+a\right)}dx^2-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{2858}$$

$$\frac{1}{2}\left(p\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{x^8}d\left(bx^2+a\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-p\int -\frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{x^8}d\left(bx^2+a\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(b^3(-p)\int -\frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{b^3x^8}d\left(bx^2+a\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{2789}$$

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\int -\frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{b^3x^6}d\left(bx^2+a\right)}{a}+\frac{\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{b^2x^6}d\left(bx^2+a\right)}{a}\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{2756}$$

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{b^2x^6}d\left(bx^2+a\right)}{a}+\frac{\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-p\int \frac{\log\left(c\left(bx^2+a\right)^p\right)}{b^2x^6}d\left(bx^2+a\right)}{a}\right)-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^6}\right.$$

$\downarrow$ 2789

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-p\left(\frac{\int \frac{\log\left(c\left(bx^2+a\right)^p\right)}{b^2x^4}d\left(bx^2+a\right)}{a}+\frac{\int -\frac{\log\left(c\left(bx^2+a\right)^p\right)}{bx^4}d\left(bx^2+a\right)}{a}\right)}{a}+\frac{\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{b^2x^4}d\left(bx^2+a\right)}{a}+\right.\right.$$

$\downarrow$ 2751

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-p\left(\frac{-\frac{p\int -\frac{1}{bx^2}d\left(bx^2+a\right)}{a}-\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{abx^2}}{a}+\frac{\int -\frac{\log\left(c\left(bx^2+a\right)^p\right)}{bx^4}d\left(bx^2+a\right)}{a}\right)}{a}+\frac{\int \frac{\log^2\left(c\left(bx^2\right.\right.}{b^2x^4}}{}\right.\right.$$

$\downarrow$ 16

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-p\left(\frac{\int -\frac{\log\left(c\left(bx^2+a\right)^p\right)}{bx^4}d\left(bx^2+a\right)}{a}+\frac{\frac{p\log\left(-bx^2\right)}{a}-\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{abx^2}}{a}\right)}{a}+\frac{\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{b^2x^4}d\left(\right.}{a}\right.\right.$$

$\downarrow$ 2755

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-p\left(\frac{\int -\frac{\log\left(c\left(bx^2+a\right)^p\right)}{bx^4}d\left(bx^2+a\right)}{a}+\frac{\frac{p\log\left(-bx^2\right)}{a}-\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{abx^2}}{a}\right)}{a}+\frac{-\frac{2p\int -\frac{\log\left(c\left(bx^2+a\right)\right.}{bx^2}}{a}}{}\right.\right.$$

$\downarrow$ 2754

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-p\left(\frac{\int -\frac{\log\left(c\left(bx^2+a\right)^p\right)}{bx^4}d\left(bx^2+a\right)}{a}+\frac{\frac{p\log\left(-bx^2\right)}{a}-\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{abx^2}}{a}\right)}{a}+\frac{-\frac{2p\left(p\int\frac{\log\left(1-\frac{bx^2+\cdots}{a}\right)}{x^2}\cdots\right.}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 2779

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-p\left(\frac{\frac{p\int\frac{\log\left(1-\frac{a}{x^2}\right)}{x^2}d\left(bx^2+a\right)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}}{a}+\frac{\frac{p\log\left(-bx^2\right)}{a}-\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{abx^2}}{a}\right)}{a}\cdots\right.\right.$$

$\downarrow$ 2821

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-p\left(\frac{\frac{p\int\frac{\log\left(1-\frac{a}{x^2}\right)}{x^2}d\left(bx^2+a\right)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}}{a}+\frac{\frac{p\log\left(-bx^2\right)}{a}-\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{abx^2}}{a}\right)}{a}\cdots\right.\right.$$

$\downarrow$ 2838

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\frac{\frac{2p\left(\text{PolyLog}\left(2,\frac{a}{x^2}\right)\log\left(c\left(a+bx^2\right)^p\right)-p\int\frac{\text{PolyLog}\left(2,\frac{a}{x^2}\right)}{x^2}d\left(bx^2+a\right)\right)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{a}}{a}+\frac{-\frac{2p\left(\log\left(1-\frac{a+bx^2}{a}\right)\left(-\log\left(\cdots\right)\right)\right)}{\cdots}}{\cdots}}{a}\right.\right.$$

$\downarrow$ 7143

$$\frac{1}{2}\left(b^3(-p)\left(\frac{\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2x^4}-p\left(\frac{\frac{p\,\text{PolyLog}\left(2,\frac{a}{x^2}\right)}{a}-\frac{\log\left(1-\frac{a}{x^2}\right)\log\left(c\left(a+bx^2\right)^p\right)}{a}}{a}+\frac{\frac{p\log\left(-bx^2\right)}{a}-\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{abx^2}}{a}\right)}{a}+\frac{-\frac{2p(\cdots}{\cdots}}{\cdots}\cdots\right.\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

output
```
(-1/3*Log[c*(a + b*x^2)^p]^3/x^6 - b^3*p*((Log[c*(a + b*x^2)^p]^2/(2*b^2*x
^4) - p*(((p*Log[-(b*x^2)])/a - ((a + b*x^2)*Log[c*(a + b*x^2)^p])/(a*b*x^
2))/a + (-((Log[1 - a/x^2]*Log[c*(a + b*x^2)^p])/a) + (p*PolyLog[2, a/x^2]
)/a)/a))/a + ((-(((a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(a*b*x^2)) - (2*p*(-
(Log[c*(a + b*x^2)^p]*Log[1 - (a + b*x^2)/a]) - p*PolyLog[2, (a + b*x^2)/a
]))/a)/a + (-((Log[1 - a/x^2]*Log[c*(a + b*x^2)^p]^2)/a) + (2*p*(Log[c*(a
+ b*x^2)^p]*PolyLog[2, a/x^2] + p*PolyLog[3, a/x^2]))/a)/a)/a))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^3}{x^7}dx$$

input
```
int(ln(c*(b*x^2+a)^p)^3/x^7,x)
```

output
```
int(ln(c*(b*x^2+a)^p)^3/x^7,x)
```

Fricas [F]

$$\int \frac{\log^3(c(a+bx^2)^p)}{x^7}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^7}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^3/x^7, x)
```

Sympy [F]

$$\int \frac{\log^3(c(a+bx^2)^p)}{x^7}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^3}{x^7}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**3/x**7,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**3/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.96

$$\int \frac{\log^3(c(a+bx^2)^p)}{x^7}\,dx = \frac{1}{12}\left(\frac{6\left(\log(bx^2+a)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log(bx^2+a)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)b^2p^2}{a^3} - \frac{6\,(3\,p^2-2\right. \\ - \frac{\log\left((bx^2+a)^p c\right)^3}{6\,x^6}$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="maxima")
```

output
```
1/12*(6*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)
*log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*b^2*p^2/a^3 - 6*(3*p^2 - 2*
p*log(c))*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*
b^2/a^3 + 12*(p^2 - 3*p*log(c) + log(c)^2)*b^2*log(x)/a^3 - (2*b^2*p^2*x^4
*log(b*x^2 + a)^3 + 6*(p*log(c) - log(c)^2)*a*b*x^2 + 3*a^2*log(c)^2 - 3*(
(3*p^2 - 2*p*log(c))*b^2*x^4 + 2*a*b*p^2*x^2 - a^2*p^2)*log(b*x^2 + a)^2 +
 6*((p^2 - 3*p*log(c) + log(c)^2)*b^2*x^4 + (p^2 - 2*p*log(c))*a*b*x^2 + a
^2*p*log(c))*log(b*x^2 + a))/(a^3*x^4))*b*p - 1/6*log((b*x^2 + a)^p*c)^3/x
^6
```

Giac [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^7}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^3/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}{x^7}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^3/x^7,x)
```

output
```
int(log(c*(a + b*x^2)^p)^3/x^7, x)
```

Reduce [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx$$
$$= \frac{12\left(\int \frac{\log\left((b\,x^2+a)^p c\right)^2}{b\,x^3+ax}dx\right) a\,b^3 p\,x^6 - 36\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^3+ax}dx\right) a\,b^3 p^2 x^6 - 2\log\left((b\,x^2+a)^p\,c\right)^3 a^3 - 3\log\left((b\,x^2\right.}{}$$

input
```
int(log(c*(b*x^2+a)^p)^3/x^7,x)
```

output
```
(12*int(log((a + b*x**2)**p*c)**2/(a*x + b*x**3),x)*a*b**3*p*x**6 - 36*int
(log((a + b*x**2)**p*c)/(a*x + b*x**3),x)*a*b**3*p**2*x**6 - 2*log((a + b*
x**2)**p*c)**3*a**3 - 3*log((a + b*x**2)**p*c)**2*a**2*b*p*x**2 + 6*log((a
 + b*x**2)**p*c)**2*a*b**2*p*x**4 - 6*log((a + b*x**2)**p*c)*a*b**2*p**2*x
**4 - 6*log((a + b*x**2)**p*c)*b**3*p**2*x**6 + 12*log(x)*b**3*p**3*x**6)/
(12*a**3*x**6)
```

3.98 $\int x^2 \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\begin{aligned}&\int x^2\log^3\left(c(a+bx^2)^p\right)\,dx\\&=\frac{208ap^3x}{9b}-\frac{16p^3x^3}{27}-\frac{208a^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}}+\frac{32ia^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}\\&+\frac{64a^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}}-\frac{32ap^2x\log\left(c(a+bx^2)^p\right)}{3b}\\&+\frac{8}{9}p^2x^3\log\left(c(a+bx^2)^p\right)+\frac{32a^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3b^{3/2}}+\frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b}-\frac{2}{3}px^3\log\end{aligned}$$

output
```
208/9*a*p^3*x/b-16/27*p^3*x^3-208/9*a^(3/2)*p^3*arctan(b^(1/2)*x/a^(1/2))/
b^(3/2)+32/3*I*a^(3/2)*p^3*arctan(b^(1/2)*x/a^(1/2))^2/b^(3/2)+64/3*a^(3/2
)*p^3*arctan(b^(1/2)*x/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/b^(3/2
)-32/3*a*p^2*x*ln(c*(b*x^2+a)^p)/b+8/9*p^2*x^3*ln(c*(b*x^2+a)^p)+32/3*a^(3
/2)*p^2*arctan(b^(1/2)*x/a^(1/2))*ln(c*(b*x^2+a)^p)/b^(3/2)+2*a*p*x*ln(c*(
b*x^2+a)^p)^2/b-2/3*p*x^3*ln(c*(b*x^2+a)^p)^2+1/3*x^3*ln(c*(b*x^2+a)^p)^3+
32/3*I*a^(3/2)*p^3*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/b^(3/2)-2*
a^2*p*Defer(Int)(ln(c*(b*x^2+a)^p)^2/(b*x^2+a),x)/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 909 vs. 2(380) = 760.

Time = 5.43 (sec) , antiderivative size = 909, normalized size of antiderivative = 50.50

$$\int x^2 \log^3\left(c(a+bx^2)^p\right)\,dx = \text{Too large to display}$$

input
```
Integrate[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

output
```
(2*a*p*x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/b - (2*a^(3/2)*p*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2
)/b^(3/2) + p*x^3*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^
p])^2 + (x^3*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-2*p - p*Log[
a + b*x^2] + Log[c*(a + b*x^2)^p]))/3 + 3*p^2*(-(p*Log[a + b*x^2]) + Log[c
*(a + b*x^2)^p])*((x^3*Log[a + b*x^2]^2)/3 - (4*((9*I)*a^(3/2)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]]^2 + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-8 + 6*Log[(2*
Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + 3*Log[a + b*x^2]) + Sqrt[b]*x*(24*a -
2*b*x^2 + (-9*a + 3*b*x^2)*Log[a + b*x^2]) + (9*I)*a^(3/2)*PolyLog[2, (I*S
qrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/(27*b^(3/2))) + (p^3*(41
6*Sqrt[-a]*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcSin[Sqrt[a
]/Sqrt[a + b*x^2]] + (2*Sqrt[-a]*b*x^2*(624*a - 16*b*x^2 + (-288*a + 24*b*
x^2)*Log[a + b*x^2] + 18*(3*a - b*x^2)*Log[a + b*x^2]^2 + 9*b*x^2*Log[a +
b*x^2]^3))/3 + 36*Sqrt[-a]*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*(8*Sqrt[a]*Hy
pergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + Lo
g[a + b*x^2]*(4*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(
a + b*x^2)] + Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]]*Log[a + b*x^
2])) - 48*a^2*(4*Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(Log[a + b*x^2]
 - Log[1 + (b*x^2)/a]) - Sqrt[-a]*Sqrt[-((b*x^2)/a)]*(Log[1 + (b*x^2)/a]^2
 - 4*Log[1 + (b*x^2)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2*Log[(1 + Sq...
```

Rubi [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right) \, dx$$

$$\Big\downarrow \ 2907$$

$$\frac{1}{3}x^3 \log^3\left(c\left(a+bx^2\right)^p\right) - 2bp\int \frac{x^4 \log^2\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx$$

$$\Big\downarrow \ 2926$$

$$\frac{1}{3}x^3 \log^3\left(c\left(a+bx^2\right)^p\right) - 2bp\int \left(\frac{x^2 \log^2\left(c\left(bx^2+a\right)^p\right)}{b}+\frac{a^2 \log^2\left(c\left(bx^2+a\right)^p\right)}{b^2\left(bx^2+a\right)}-\frac{a \log^2\left(c\left(bx^2+a\right)^p\right)}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}x^3 \log^3\left(c\left(a+bx^2\right)^p\right) - 2bp\left(\frac{a^2\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx}{b^2}-\frac{16a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{5/2}}-\frac{16ia^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{5/2}}+\frac{104a^{3/2}p^2 a}{9b}\right.$$

input
```
Int[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3 dx$$

input
```
int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

output
```
int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 \log^3 \left(c\left(a+bx^2\right)^p\right) \, dx = \int x^2 \log \left(\left(bx^2+a\right)^p c\right)^3 \, dx$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
integral(x^2*log((b*x^2 + a)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^2 \log^3 \left(c\left(a+bx^2\right)^p\right) \, dx = \int x^2 \log \left(c\left(a+bx^2\right)^p\right)^3 \, dx$$

input
```
integrate(x**2*ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Integral(x**2*log(c*(a + b*x**2)**p)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 123, normalized size of antiderivative = 6.83

$$\int x^2 \log^3 \left(c\left(a+bx^2\right)^p\right) \, dx = \int x^2 \log \left(\left(bx^2+a\right)^p c\right)^3 \, dx$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
1/3*p^3*x^3*log(b*x^2 + a)^3 + integrate((b*x^4*log(c)^3 + a*x^2*log(c)^3
- ((2*p^3 - 3*p^2*log(c))*b*x^4 - 3*a*p^2*x^2*log(c))*log(b*x^2 + a)^2 + 3
*(b*p*x^4*log(c)^2 + a*p*x^2*log(c)^2)*log(b*x^2 + a))/(b*x^2 + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right) dx = \int x^2 \log\left(\left(bx^2+a\right)^p c\right)^3 dx$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
integrate(x^2*log((b*x^2 + a)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right) dx = \int x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3 dx$$

input
```
int(x^2*log(c*(a + b*x^2)^p)^3,x)
```

output
```
int(x^2*log(c*(a + b*x^2)^p)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 12.28

$$\int x^2 \log^3 \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$= \frac{-624\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a\,p^3 - 54\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)^2}{b\,x^2+a}dx\right) a^2bp + 288\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)}{b\,x^2+a}dx\right) a^2b\,p^2 + 9\log\left(\left(b\right.\right.}{}$$

input
```
int(x^2*log(c*(b*x^2+a)^p)^3,x)
```

output
```
( - 624*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*p**3 - 54*int(log(
(a + b*x**2)**p*c)**2/(a + b*x**2),x)*a**2*b*p + 288*int(log((a + b*x**2)*
*p*c)/(a + b*x**2),x)*a**2*b*p**2 + 9*log((a + b*x**2)**p*c)**3*b**2*x**3
+ 54*log((a + b*x**2)**p*c)**2*a*b*p*x - 18*log((a + b*x**2)**p*c)**2*b**2
*p*x**3 - 288*log((a + b*x**2)**p*c)*a*b*p**2*x + 24*log((a + b*x**2)**p*c
)*b**2*p**2*x**3 + 624*a*b*p**3*x - 16*b**2*p**3*x**3)/(27*b**2)
```

3.99 $\int \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\begin{aligned}\int \log^3\left(c(a+bx^2)^p\right)\,dx &= -48p^3x+\frac{48\sqrt{a}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{24i\sqrt{a}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}}\\
&\quad-\frac{48\sqrt{a}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}\\
&\quad+24p^2x\log\left(c(a+bx^2)^p\right)\\
&\quad-\frac{24\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}}\\
&\quad-6px\log^2\left(c(a+bx^2)^p\right)+x\log^3\left(c(a+bx^2)^p\right)\\
&\quad-\frac{24i\sqrt{a}p^3\,\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}\\
&\quad+6ap\,\text{Int}\left(\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2},x\right)\end{aligned}$$

output
```
-48*p^3*x+48*a^(1/2)*p^3*arctan(b^(1/2)*x/a^(1/2))/b^(1/2)-24*I*a^(1/2)*p^
3*arctan(b^(1/2)*x/a^(1/2))^2/b^(1/2)-48*a^(1/2)*p^3*arctan(b^(1/2)*x/a^(1
/2))*ln(2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/b^(1/2)+24*p^2*x*ln(c*(b*x^2+a)^p
)-24*a^(1/2)*p^2*arctan(b^(1/2)*x/a^(1/2))*ln(c*(b*x^2+a)^p)/b^(1/2)-6*p*x
*ln(c*(b*x^2+a)^p)^2+x*ln(c*(b*x^2+a)^p)^3-24*I*a^(1/2)*p^3*polylog(2,1-2*
a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/b^(1/2)+6*a*p*Defer(Int)(ln(c*(b*x^2+a)^p)^
2/(b*x^2+a),x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 789 vs. 2(290) = 580.

Time = 4.65 (sec) , antiderivative size = 789, normalized size of antiderivative = 56.36

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^3,x]
```

output
```
(6*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a +
 b*x^2)^p])^2)/Sqrt[b] + 3*p*x*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c
*(a + b*x^2)^p])^2 + x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-6*
p - p*Log[a + b*x^2] + Log[c*(a + b*x^2)^p]) - (3*p^2*(p*Log[a + b*x^2] -
Log[c*(a + b*x^2)^p])*((4*I)*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqr
t[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt
[b]*x)] + Log[a + b*x^2]) + Sqrt[b]*x*(8 - 4*Log[a + b*x^2] + Log[a + b*x^
2]^2) + (4*I)*Sqrt[a]*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + S
qrt[b]*x)]))/Sqrt[b] + (p^3*(-48*Sqrt[-a^2]*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt
[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]] + Sqrt[-a]*b*x^2*(-48 + 24*Log
[a + b*x^2] - 6*Log[a + b*x^2]^2 + Log[a + b*x^2]^3) - 6*Sqrt[-a^2]*Sqrt[(
b*x^2)/(a + b*x^2)]*(8*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/
2, 3/2, 3/2}, a/(a + b*x^2)] + Log[a + b*x^2]*(4*Sqrt[a]*HypergeometricPFQ
[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sqrt[a + b*x^2]*ArcSin[Sqrt
[a]/Sqrt[a + b*x^2]]*Log[a + b*x^2])) + 24*a*Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^
2]/Sqrt[-a]]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a]) + 6*(-a)^(3/2)*Sqrt[-((
b*x^2)/a)]*(Log[1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2)/a]*Log[(1 + Sqrt[-((b
*x^2)/a)])/2] + 2*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2 - 4*PolyLog[2, 1/2 - S
qrt[-((b*x^2)/a)]/2])))/(Sqrt[-a]*b*x)
```

Rubi [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2900, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{ 2900}$$

$$x\log^3\left(c\left(a+bx^2\right)^p\right)-6bp\int \frac{x^2\log^2\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx$$

$$\Big\downarrow \text{ 2926}$$

$$x\log^3\left(c\left(a+bx^2\right)^p\right)-6bp\int\left(\frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{b}-\frac{a\log^2\left(c\left(bx^2+a\right)^p\right)}{b\left(bx^2+a\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\log^3\left(c\left(a+bx^2\right)^p\right)-$$

$$6bp\left(-\frac{a\int\frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx}{b}+\frac{4\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^{3/2}}+\frac{4i\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{b^{3/2}}-\frac{8\sqrt{a}p^2\arctan\left(\right.}{b^{3/2}}\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2900
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Simp[b*e*n*p*q   Int[x^n*
((a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \ln \left(c\left(b\,x^{2}+a\right)^{p}\right)^{3}dx$$

input
```
int(ln(c*(b*x^2+a)^p)^3,x)
```

output
```
int(ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx=\int \log\left(\left(bx^2+a\right)^pc\right)^3\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx=\int \log\left(c\left(a+bx^2\right)^p\right)^3dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 111, normalized size of antiderivative = 7.93

$$\int \log^3\left(c(a+bx^2)^p\right)\,dx = \int \log\left(\left(bx^2+a\right)^p c\right)^3\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
p^3*x*log(b*x^2 + a)^3 + integrate((b*x^2*log(c)^3 + a*log(c)^3 - 3*((2*p^
3 - p^2*log(c))*b*x^2 - a*p^2*log(c))*log(b*x^2 + a)^2 + 3*(b*p*x^2*log(c)
^2 + a*p*log(c)^2)*log(b*x^2 + a))/(b*x^2 + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \log^3\left(c(a+bx^2)^p\right)\,dx = \int \log\left(\left(bx^2+a\right)^p c\right)^3\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \log^3\left(c(a+bx^2)^p\right)\,dx = \int \ln\left(c\left(b\,x^2+a\right)^p\right)^3\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^3,x)
```

output
```
int(log(c*(a + b*x^2)^p)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 10.71

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{48\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)p^3+6\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)^2}{b\,x^2+a}dx\right)abp-24\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)}{b\,x^2+a}dx\right)ab\,p^2+\log\left(\left(b\,x^2+a\right)^p c\right.}{b}$$

input
```
int(log(c*(b*x^2+a)^p)^3,x)
```

output
```
(48*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*p**3 + 6*int(log((a + b*
x**2)**p*c)**2/(a + b*x**2),x)*a*b*p - 24*int(log((a + b*x**2)**p*c)/(a +
b*x**2),x)*a*b*p**2 + log((a + b*x**2)**p*c)**3*b*x - 6*log((a + b*x**2)**
p*c)**2*b*p*x + 24*log((a + b*x**2)**p*c)*b*p**2*x - 48*b*p**3*x)/b
```

3.100 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = -\frac{\log^3\left(c(a+bx^2)^p\right)}{x} + 6bp\text{Int}\left(\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2},x\right)$$

output
```
-ln(c*(b*x^2+a)^p)^3/x+6*b*p*Defer(Int)(ln(c*(b*x^2+a)^p)^2/(b*x^2+a),x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 505, normalized size of antiderivative = 28.06

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx$$
$$= \frac{p^3\left(-96\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{a+bx^2}\right)-48\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{a+bx^2}\right)\log(a+\dots}{2\sqrt{a}x}$$
$$+\frac{6\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-p\log(a+bx^2)+\log\left(c(a+bx^2)^p\right)\right)^2}{\sqrt{a}}$$
$$-\frac{3p\log(a+bx^2)\left(-p\log(a+bx^2)+\log\left(c(a+bx^2)^p\right)\right)^2}{x}$$
$$-\frac{\left(-p\log(a+bx^2)+\log\left(c(a+bx^2)^p\right)\right)^3}{x}$$
$$+3p^2\left(-p\log(a+bx^2)+\log\left(c(a+bx^2)^p\right)\right)\left(-\frac{\log^2(a+bx^2)}{x}\right.$$
$$\left.+\frac{4\sqrt{b}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+2\log\left(\frac{2i}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\right)+\log(a+bx^2)\right)+i\,\text{PolyLog}\left(2,\frac{i\sqrt{a}+\sqrt{b}x}{-i\sqrt{a}+\sqrt{b}x}\right)\right)}{\sqrt{a}}\right)$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

output
```
(p^3*(-96*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2
, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] - 48*Sqrt[a]*Sqrt[1 - a/(a + b*x^2
)]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)]*Log[a + b
*x^2] - 2*Log[a + b*x^2]^2*(6*Sqrt[a + b*x^2]*Sqrt[1 - a/(a + b*x^2)]*ArcS
in[Sqrt[a]/Sqrt[a + b*x^2]] + Sqrt[a]*Log[a + b*x^2])))/(2*Sqrt[a]*x) + (6
*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b
*x^2)^p])^2)/Sqrt[a] - (3*p*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a
 + b*x^2)^p])^2)/x - (-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3/x + 3*
p^2*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])*(-(Log[a + b*x^2]^2/x) +
(4*Sqrt[b]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2
*Log[(2*I)/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[a + b*x^2]) + I*PolyLog[2, (I*
Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/Sqrt[a])
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2907, 2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$$

$$\Big\downarrow \text{2907}$$

$$6bp\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}$$

$$\Big\downarrow \text{2923}$$

$$6bp\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{bx^2+a}dx-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}$$

input
```
Int[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c(b\,x^{2}+a)^{p}\right)^{3}}{x^{2}}dx$$

input
```
int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

output
```
int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx=\int \frac{\log\left((bx^2+a)^pc\right)^3}{x^2}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^3}{x^2}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**3/x**2,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**3/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^2}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x^2}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^3/x^2,x)
```

output
```
int(log(c*(a + b*x^2)^p)^3/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 8.72

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx$$
$$= \frac{48\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)p^3x - 6\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)^2}{b\,x^4+a\,x^2}dx\right)a^2px - 24\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)}{b\,x^4+a\,x^2}dx\right)a^2p^2x - \log\left(\left(b\,x^2+a\right.\right.}{ax}$$

input

```
int(log(c*(b*x^2+a)^p)^3/x^2,x)
```

output

```
(48*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*p**3*x - 6*int(log((a +
b*x**2)**p*c)**2/(a*x**2 + b*x**4),x)*a**2*p*x - 24*int(log((a + b*x**2)**
p*c)/(a*x**2 + b*x**4),x)*a**2*p**2*x - log((a + b*x**2)**p*c)**3*a - 6*lo
g((a + b*x**2)**p*c)**2*a*p - 24*log((a + b*x**2)**p*c)*a*p**2)/(a*x)
```

3.101 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\begin{aligned}\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx =& \frac{8ib^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}}\\
&+\frac{16b^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}}\\
&+\frac{8b^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}}\\
&-\frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax}-\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3}\\
&+\frac{8ib^{3/2}p^3\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}}\\
&-\frac{2b^2p\operatorname{Int}\left(\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2},x\right)}{a}\end{aligned}$$

output
```
8*I*b^(3/2)*p^3*arctan(b^(1/2)*x/a^(1/2))^2/a^(3/2)+16*b^(3/2)*p^3*arctan(
b^(1/2)*x/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/a^(3/2)+8*b^(3/2)*p
^2*arctan(b^(1/2)*x/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(3/2)-2*b*p*ln(c*(b*x^2+a
)^p)^2/a/x-1/3*ln(c*(b*x^2+a)^p)^3/x^3+8*I*b^(3/2)*p^3*polylog(2,1-2*a^(1/
2)/(a^(1/2)+I*b^(1/2)*x))/a^(3/2)-2*b^2*p*Defer(Int)(ln(c*(b*x^2+a)^p)^2/(
b*x^2+a),x)/a
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 851 vs. 2(254) = 508.

Time = 3.71 (sec) , antiderivative size = 851, normalized size of antiderivative = 47.28

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

output
```
(a^2*(p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])^3 - 6*a*b*p*x^2*(-(p*Log[a
+ b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 6*Sqrt[a]*b^(3/2)*p*x^3*ArcTan[(Sqrt
[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 3*a^2*p*L
og[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 3*Sqrt[a]*p
^2*(p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])*(a^(3/2)*Log[a + b*x^2]^2 + 4
*b*x^2*(I*Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + Sqrt[a]*Log[a + b*x^2]
 + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a]
+ I*Sqrt[b]*x)] + Log[a + b*x^2]) + I*Sqrt[b]*x*PolyLog[2, (I*Sqrt[a] + Sq
rt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])) + p^3*(48*a*b*x^2*Sqrt[(b*x^2)/(a +
 b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b
*x^2)] + 24*Sqrt[-a]*(b*x^2)^(3/2)*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[a + b
*x^2] + 24*a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2,
1/2}, {3/2, 3/2}, a/(a + b*x^2)]*Log[a + b*x^2] - 6*a*b*x^2*Log[a + b*x^2]
^2 + 6*Sqrt[a]*((b*x^2)/(a + b*x^2))^(3/2)*(a + b*x^2)^(3/2)*ArcSin[Sqrt[a
]/Sqrt[a + b*x^2]]*Log[a + b*x^2]^2 - a^2*Log[a + b*x^2]^3 - 24*Sqrt[-a]*(
b*x^2)^(3/2)*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[1 + (b*x^2)/a] - 6*a^2*(-((
b*x^2)/a))^(3/2)*Log[1 + (b*x^2)/a]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*Log[1
+ (b*x^2)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] - 12*a^2*(-((b*x^2)/a))^(3/2)
*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*PolyLog[2
, 1/2 - Sqrt[-((b*x^2)/a)]/2]))/(3*a^2*x^3)
```

Rubi [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$$

$$\Big\downarrow \text{2907}$$

$$2bp\int \frac{\log^2\left(c\left(bx^2+a\right)^p\right)}{x^2\left(bx^2+a\right)}dx-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{3x^3}$$

$$\Big\downarrow \text{2926}$$

$$2bp\int \left(\frac{\log^2\left(c(bx^2+a)^p\right)}{ax^2}-\frac{b\log^2\left(c(bx^2+a)^p\right)}{a\left(bx^2+a\right)}\right)dx-\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3}+$$

$$2bp\left(-\frac{b\int \frac{\log^2\left(c(bx^2+a)^p\right)}{bx^2+a}dx}{a}+\frac{4\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}}+\frac{4i\sqrt{b}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}}+\frac{8\sqrt{b}p^2\arctan\left(\right.}{}\right.$$

input
```
Int[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c(b\,x^{2}+a)^{p}\right)^{3}}{x^{4}}dx$$

input
```
int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

output
```
int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^{3}\left(c(a+bx^{2})^{p}\right)}{x^{4}}\,dx = \int \frac{\log\left((bx^{2}+a)^{p}c\right)^{3}}{x^{4}}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^3/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\log^{3}\left(c(a+bx^{2})^{p}\right)}{x^{4}}\,dx = \int \frac{\log\left(c(a+bx^{2})^{p}\right)^{3}}{x^{4}}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)**3/x**4,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**3/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 6.50

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^4}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="maxima")
```

output
```
-1/3*p^3*log(b*x^2 + a)^3/x^3 + integrate((b*x^2*log(c)^3 + a*log(c)^3 + (
(2*p^3 + 3*p^2*log(c))*b*x^2 + 3*a*p^2*log(c))*log(b*x^2 + a)^2 + 3*(b*p*x
^2*log(c)^2 + a*p*log(c)^2)*log(b*x^2 + a))/(b*x^6 + a*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^4}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^3/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 25.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^3}{x^4}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)^3/x^4,x)
```

output
```
int(log(c*(a + b*x^2)^p)^3/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 10.06

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \frac{-16\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,p^3x^3-18\left(\int \frac{\log\left((b\,x^2+a)^p c\right)^2}{b\,x^6+a\,x^4}dx\right)a^3p\,x^3-24\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{b\,x^6+a\,x^4}dx\right)a^3p^2x^3-3\log(}{9a^2x^3}$$

input
```
int(log(c*(b*x^2+a)^p)^3/x^4,x)
```

output
```
( - 16*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*p**3*x**3 - 18*int(
log((a + b*x**2)**p*c)**2/(a*x**4 + b*x**6),x)*a**3*p*x**3 - 24*int(log((a
 + b*x**2)**p*c)/(a*x**4 + b*x**6),x)*a**3*p**2*x**3 - 3*log((a + b*x**2)*
*p*c)**3*a**2 - 6*log((a + b*x**2)**p*c)**2*a**2*p - 8*log((a + b*x**2)**p
*c)*a**2*p**2 - 16*a*b*p**3*x**2)/(9*a**2*x**3)
```

3.102 $\int \frac{x^3}{\log(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2b^2p} + \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(a+bx^2)^p)}{p}\right)}{2b^2p}$$

output
```
-1/2*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p/((c*(b*x^2+a)^p)^(1/p))+1/2
*(b*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p/((c*(b*x^2+a)^p)^(2/p))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-2/p}\left(a\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)-(a+bx^2)\,\text{ExpIntegralE}\right.}{2b^2p}$$

input
```
Integrate[x^3/Log[c*(a + b*x^2)^p],x]
```

output
```
-1/2*((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x
^2)^p]/p] - (a + b*x^2)*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]))/(b^2*p
*(c*(a + b*x^2)^p)^(2/p))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2904, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\log \left(c \left(a+b x^2\right)^p\right)} \, dx$$
$$\Big\downarrow \; 2904$$
$$\frac{1}{2} \int \frac{x^2}{\log \left(c \left(b x^2+a\right)^p\right)}dx^2$$
$$\Big\downarrow \; 2846$$
$$\frac{1}{2} \int \left(\frac{b x^2+a}{b \log \left(c \left(b x^2+a\right)^p\right)}-\frac{a}{b \log \left(c \left(b x^2+a\right)^p\right)}\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^2 \left(c\left(a+bx^2\right)^p\right)^{-2/p} \text{ExpIntegralEi}\left(\frac{2\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2 p}-\frac{a\left(a+bx^2\right) \left(c\left(a+bx^2\right)^p\right)^{-1/p} \text{ExpIntegralE}}{b^2 p}\right.$$

input
```
Int[x^3/Log[c*(a + b*x^2)^p],x]
```

output
```
(-((a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^2*p*(c*(a + b*
x^2)^p)^p^(-1))) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p
])/(b^2*p*(c*(a + b*x^2)^p)^(2/p)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 547, normalized size of antiderivative = 5.11

method	result
risch	$-\frac{(b x^2+a)^2 c^{-\frac{2}{p}} \left((b x^2+a)^p\right)^{-\frac{2}{p}} \mathrm{e}^{\frac{i\pi \operatorname{csgn}\left(ic\left(b x^2+a\right)^p\right)\left(-\operatorname{csgn}\left(ic\left(b x^2+a\right)^p\right)+\operatorname{csgn}(ic)\right)\left(-\operatorname{csgn}\left(ic\left(b x^2+a\right)^p\right)+\operatorname{csgn}\left(i\left(b x^2+a\right)^p\right)\right)}{p}} \exp$

input
```
int(x^3/ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2/p*(b*x^2+a)^2*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*
x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(
I*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*
Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+
2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)+1/2/b^2*a/p*(b*x^2+a)*c^(-1/p)*((b*x
^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)
+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2
+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi
*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+
a))/p)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{ac^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left((bx^2+a)c^{\left(\frac{1}{p}\right)}\right)-\operatorname{log\_integral}\left((b^2x^4+2\,abx^2+a^2)c^{\frac{2}{p}}\right)}{2\,b^2c^{\frac{2}{p}}p}$$

input
```
integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
-1/2*(a*c^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) - log_integral((b^2*x^4
+ 2*a*b*x^2 + a^2)*c^(2/p)))/(b^2*c^(2/p)*p)
```

Sympy [F]

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

input
```
integrate(x**3/ln(c*(b*x**2+a)**p),x)
```

output
```
Integral(x**3/log(c*(a + b*x**2)**p), x)
```

Maxima [F]

$$\int \frac{x^3}{\log(c(a+bx^2)^p)}\,dx = \int \frac{x^3}{\log((bx^2+a)^p c)}\,dx$$

input
```
integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
integrate(x^3/log((b*x^2 + a)^p*c), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\log(c(a+bx^2)^p)}\,dx = -\frac{a\mathrm{Ei}\left(\frac{\log(c)}{p}+\log(bx^2+a)\right)}{2\,b^2c^{\left(\frac{1}{p}\right)}p} + \frac{\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log(bx^2+a)\right)}{2\,b^2c^{\frac{2}{p}}p}$$

input
```
integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
-1/2*a*Ei(log(c)/p + log(b*x^2 + a))/(b^2*c^(1/p)*p) + 1/2*Ei(2*log(c)/p +
 2*log(b*x^2 + a))/(b^2*c^(2/p)*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log(c(a+bx^2)^p)}\,dx = \int \frac{x^3}{\ln(c(b\,x^2+a)^p)}\,dx$$

input
```
int(x^3/log(c*(a + b*x^2)^p),x)
```

output
```
int(x^3/log(c*(a + b*x^2)^p), x)
```

Reduce [F]

$$\int \frac{x^3}{\log \left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{x^3}{\log \left(\left(b\,x^2+a\right)^p c\right)}dx$$

input
```
int(x^3/log(c*(b*x^2+a)^p),x)
```

output
```
int(x**3/log((a + b*x**2)**p*c),x)
```

3.103 $\int \frac{x}{\log(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{x}{\log(c(a+bx^2)^p)}\,dx = \frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\,\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2bp}$$

output
```
1/2*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p/((c*(b*x^2+a)^p)^(1/p))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log(c(a+bx^2)^p)}\,dx = \frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\,\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2bp}$$

input
```
Integrate[x/Log[c*(a + b*x^2)^p],x]
```

output
```
((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^
p)^p^(-1))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2836, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{1}{\log\left(c\left(bx^2+a\right)^p\right)}dx^2$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\log\left(c\left(bx^2+a\right)^p\right)}d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\int \frac{\left(c\left(bx^2+a\right)^p\right)^{\frac{1}{p}}}{x^2}d\log\left(c\left(bx^2+a\right)^p\right)}{2bp}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp}$$

input
```
Int[x/Log[c*(a + b*x^2)^p],x]
```

output
```
((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^
p)^p^(-1))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.88 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.33

method	result
risch	$-\frac{(b x^2+a)\left((b x^2+a)^p\right)^{-\frac{1}{p}} c^{-\frac{1}{p}} \mathrm{e}^{\frac{i\pi \operatorname{csgn}\left(ic\left(b x^2+a\right)^p\right)\left(-\operatorname{csgn}\left(ic\left(b x^2+a\right)^p\right)+\operatorname{csgn}(ic)\right)\left(-\operatorname{csgn}\left(ic\left(b x^2+a\right)^p\right)+\operatorname{csgn}\left(i\left(b x^2+a\right)^p\right)\right)}{2p}} \operatorname{expI}}{}$

input
```
int(x/ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/p*(b*x^2+a)*((b*x^2+a)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*
x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(
I*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*
c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-
I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c
)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{\text{log\_integral}\left((bx^2+a)c^{\left(\frac{1}{p}\right)}\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

input
```
integrate(x/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
1/2*log_integral((b*x^2 + a)*c^(1/p))/(b*c^(1/p)*p)
```

Sympy [F]

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

input
```
integrate(x/ln(c*(b*x**2+a)**p),x)
```

output
```
Integral(x/log(c*(a + b*x**2)**p), x)
```

Maxima [F]

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

input
```
integrate(x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
integrate(x/log((b*x^2 + a)^p*c), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

input
```
integrate(x/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
1/2*Ei(log(c)/p + log(b*x^2 + a))/(b*c^(1/p)*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

input
```
int(x/log(c*(a + b*x^2)^p),x)
```

output
```
int(x/log(c*(a + b*x^2)^p), x)
```

Reduce [F]

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$
$$=\frac{2\left(\int \frac{x^3}{\log((b\,x^2+a)^p c)a+\log((b\,x^2+a)^p c)b\,x^2}dx\right)b^2p+\log\left(\log\left((b\,x^2+a)^p\,c\right)\right)a}{2bp}$$

input
```
int(x/log(c*(b*x^2+a)^p),x)
```

output
```
(2*int(x**3/(log((a + b*x**2)**p*c)*a + log((a + b*x**2)**p*c)*b*x**2),x)*
b**2*p + log(log((a + b*x**2)**p*c))*a)/(2*b*p)
```

3.104 $\int \frac{1}{x \log(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \text{Int}\left(\frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/x/ln(c*(b*x^2+a)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

input
```
Integrate[1/(x*Log[c*(a + b*x^2)^p]),x]
```

output
```
Integrate[1/(x*Log[c*(a + b*x^2)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \log \left(c \left(a+b x^2\right)^p\right)} \, dx$$
$$\Big\downarrow \text{ 2910}$$
$$\int \frac{1}{x \log \left(c \left(a+b x^2\right)^p\right)}dx$$

input
```
Int[1/(x*Log[c*(a + b*x^2)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}dx$$

input
```
int(1/x/ln(c*(b*x^2+a)^p),x)
```

output
```
int(1/x/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

input
```
integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
integral(1/(x*log((b*x^2 + a)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

input
```
integrate(1/x/ln(c*(b*x**2+a)**p),x)
```

output
```
Integral(1/(x*log(c*(a + b*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

input
```
integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

input
```
integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\,\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

input
```
int(1/(x*log(c*(a + b*x^2)^p)),x)
```

output
```
int(1/(x*log(c*(a + b*x^2)^p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.33

$$\int \frac{1}{x \log \left(c \left(a+bx^2\right)^p\right)} \, dx = \frac{2\left(\int \frac{1}{\log((b\,x^2+a)^p c)ax+\log((b\,x^2+a)^p c)b\,x^3}dx\right) ap+\log\left(\log\left((b\,x^2+a)^p\,c\right)\right)}{2p}$$

input
```
int(1/x/log(c*(b*x^2+a)^p),x)
```

output
```
(2*int(1/(log((a + b*x**2)**p*c)*a*x + log((a + b*x**2)**p*c)*b*x**3),x)*a
*p + log(log((a + b*x**2)**p*c)))/(2*p)
```

3.105 $\int \frac{1}{x^3 \log(c(a+bx^2)^p)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)} dx=\text{Int}\left(\frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/x^3/ln(c*(b*x^2+a)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)} dx=\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)} dx$$

input
```
Integrate[1/(x^3*Log[c*(a + b*x^2)^p]),x]
```

output
```
Integrate[1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)} \, dx$$
$$\Big\downarrow \text{ 2910}$$
$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)}dx$$

input
```
Int[1/(x^3*Log[c*(a + b*x^2)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \ln \left(c \left(b\,x^2+a\right)^p\right)}dx$$

input
```
int(1/x^3/ln(c*(b*x^2+a)^p),x)
```

output
```
int(1/x^3/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log (c (a + bx^2)^p)} \, dx = \int \frac{1}{x^3 \log ((bx^2 + a)^p c)} \, dx$$

input
```
integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
integral(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 \log (c (a + bx^2)^p)} \, dx = \int \frac{1}{x^3 \log (c (a + bx^2)^p)} \, dx$$

input
```
integrate(1/x**3/ln(c*(b*x**2+a)**p),x)
```

output
```
Integral(1/(x**3*log(c*(a + b*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left((bx^2+a)^p c\right)} \, dx$$

input
```
integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left((bx^2+a)^p c\right)} \, dx$$

input
```
integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \, \ln \left(c \left(b \, x^2+a\right)^p\right)} \, dx$$

input
```
int(1/(x^3*log(c*(a + b*x^2)^p)),x)
```

output
```
int(1/(x^3*log(c*(a + b*x^2)^p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{\log \left(\left(b\,x^2+a\right)^p c\right) x^3}dx$$

input
```
int(1/x^3/log(c*(b*x^2+a)^p),x)
```

output
```
int(1/(log((a + b*x**2)**p*c)*x**3),x)
```

3.106 $\int \frac{x^2}{\log(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\operatorname{Int}\left(\frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

output
```
Defer(Int)(x^2/ln(c*(b*x^2+a)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

input
```
Integrate[x^2/Log[c*(a + b*x^2)^p],x]
```

output
```
Integrate[x^2/Log[c*(a + b*x^2)^p], x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

$$\Big\downarrow \textit{2910}$$

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[x^2/Log[c*(a + b*x^2)^p],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)}dx$$

input
```
int(x^2/ln(c*(b*x^2+a)^p),x)
```

output
```
int(x^2/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log(c(a+bx^2)^p)}\,dx = \int \frac{x^2}{\log((bx^2+a)^p c)}\,dx$$

input
```
integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
integral(x^2/log((b*x^2 + a)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\log(c(a+bx^2)^p)}\,dx = \int \frac{x^2}{\log(c(a+bx^2)^p)}\,dx$$

input
```
integrate(x**2/ln(c*(b*x**2+a)**p),x)
```

output
```
Integral(x**2/log(c*(a + b*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

input
```
integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
integrate(x^2/log((b*x^2 + a)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

input
```
integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
integrate(x^2/log((b*x^2 + a)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 24.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

input
```
int(x^2/log(c*(a + b*x^2)^p),x)
```

output
```
int(x^2/log(c*(a + b*x^2)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(b\,x^2+a\right)^p c\right)}dx$$

input
```
int(x^2/log(c*(b*x^2+a)^p),x)
```

output
```
int(x**2/log((a + b*x**2)**p*c),x)
```

3.107 $\int \frac{1}{\log(c(a+bx^2)^p)} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log(c(a+bx^2)^p)} dx = \operatorname{Int}\left(\frac{1}{\log(c(a+bx^2)^p)}, x\right)$$

output
```
Defer(Int)(1/ln(c*(b*x^2+a)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log(c(a+bx^2)^p)} dx = \int \frac{1}{\log(c(a+bx^2)^p)} dx$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^(-1),x]
```

output
```
Integrate[Log[c*(a + b*x^2)^p]^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2902}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log \left(c \left(a+b x^2\right)^p\right)} \, dx$$

$$\Big\downarrow \text{ 2902}$$

$$\int \frac{1}{\log \left(c \left(a+b x^2\right)^p\right)}dx$$

input
```
Int[Log[c*(a + b*x^2)^p]^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2902
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Unintegrable[(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d,
 e, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}dx$$

input
```
int(1/ln(c*(b*x^2+a)^p),x)
```

output
```
int(1/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

input
```
integrate(1/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
integral(1/log((b*x^2 + a)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

input
```
integrate(1/ln(c*(b*x**2+a)**p),x)
```

output
```
Integral(1/log(c*(a + b*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

input
```
integrate(1/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
integrate(1/log((b*x^2 + a)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

input
```
integrate(1/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
integrate(1/log((b*x^2 + a)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 25.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

input
```
int(1/log(c*(a + b*x^2)^p),x)
```

output
```
int(1/log(c*(a + b*x^2)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log (c (a+bx^2)^p)}\,dx=\int \frac{1}{\log \left(\left(b\,x^2+a\right)^p c\right)}dx$$

input
```
int(1/log(c*(b*x^2+a)^p),x)
```

output
```
int(1/log((a + b*x**2)**p*c),x)
```

3.108 $\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x^2/ln(c*(b*x^2+a)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

input
```
Integrate[1/(x^2*Log[c*(a + b*x^2)^p]),x]
```

output
```
Integrate[1/(x^2*Log[c*(a + b*x^2)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx$$
$$\Big\downarrow \text{ 2910}$$
$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)}dx$$

input
```
Int[1/(x^2*Log[c*(a + b*x^2)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}dx$$

input
```
int(1/x^2/ln(c*(b*x^2+a)^p),x)
```

output
```
int(1/x^2/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log(c(a+bx^2)^p)}\,dx = \int \frac{1}{x^2 \log((bx^2+a)^p c)}\,dx$$

input
```
integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
integral(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 \log(c(a+bx^2)^p)}\,dx = \int \frac{1}{x^2 \log(c(a+bx^2)^p)}\,dx$$

input
```
integrate(1/x**2/ln(c*(b*x**2+a)**p),x)
```

output
```
Integral(1/(x**2*log(c*(a + b*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log \left(\left(b x^2+a\right)^p c\right)} \, dx$$

input
```
integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log \left(\left(b x^2+a\right)^p c\right)} \, dx$$

input
```
integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \ln \left(c \left(b x^2+a\right)^p\right)} \, dx$$

input
```
int(1/(x^2*log(c*(a + b*x^2)^p)),x)
```

output
```
int(1/(x^2*log(c*(a + b*x^2)^p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{\log \left(\left(b\, x^2+a\right)^p c\right) x^2} dx$$

input
```
int(1/x^2/log(c*(b*x^2+a)^p),x)
```

output
```
int(1/(log((a + b*x**2)**p*c)*x**2),x)
```

3.109 $\int \frac{x^3}{\log^2(c(a+bx^2)^p)} dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)} dx = -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2b^2p^2} + \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(a+bx^2)^p)}{p}\right)}{b^2p^2} - \frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

output

```
-1/2*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p^2/((c*(b*x^2+a)^p)^(1/p))+(
b*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p^2/((c*(b*x^2+a)^p)^(2/p))-1/2*x
^2*(b*x^2+a)/b/p/ln(c*(b*x^2+a)^p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx =$$
$$-\frac{\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-2/p}\left(bpx^2\left(c(a+bx^2)^p\right)^{2/p}+a\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)\text{lo}\right.}{2b^2p^2\log\left(c\left(a+bx^2\right)^p\right)}$$

input
```
Integrate[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

output
```
-1/2*((a + b*x^2)*(b*p*x^2*(c*(a + b*x^2)^p)^(2/p) + a*(c*(a + b*x^2)^p)^p
^(-1)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^2)^p] - 2*(a +
b*x^2)*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]*Log[c*(a + b*x^2)^p]))/(b
^2*p^2*(c*(a + b*x^2)^p)^(2/p)*Log[c*(a + b*x^2)^p])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.40, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2904, 2847, 2836, 2737, 2609, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}dx$$
$$\Big\downarrow \; 2904$$
$$\frac{1}{2}\int \frac{x^2}{\log^2\left(c\left(bx^2+a\right)^p\right)}dx^2$$
$$\Big\downarrow \; 2847$$
$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log(c(bx^2+a)^p)}dx^2}{bp}+\frac{2\int \frac{x^2}{\log(c(bx^2+a)^p)}dx^2}{p}-\frac{x^2\left(a+bx^2\right)}{bp\log\left(c\left(a+bx^2\right)^p\right)}\right)$$
$$\Big\downarrow \; 2836$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log(c(bx^2+a)^p)}d(bx^2+a)}{b^2p}+\frac{2\int \frac{x^2}{\log(c(bx^2+a)^p)}dx^2}{p}-\frac{x^2(a+bx^2)}{bp\log(c(a+bx^2)^p)}\right)$$

$\big\downarrow$ 2737

$$\frac{1}{2}\left(\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\int \frac{(c(bx^2+a)^p)^{\frac{1}{p}}}{x^2}d\log\left(c(bx^2+a)^p\right)}{b^2p^2}+\frac{2\int \frac{x^2}{\log(c(bx^2+a)^p)}dx^2}{p}-\frac{x^2(a+bx^2)}{bp\log(c(a+bx^2)^p}\right.$$

$\big\downarrow$ 2609

$$\frac{1}{2}\left(\frac{2\int \frac{x^2}{\log(c(bx^2+a)^p)}dx^2}{p}+\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(bx^2+a)^p)}{p}\right)}{b^2p^2}-\frac{x^2(a+bx^2)}{bp\log(c(a+bx^2)^p)}\right)$$

$\big\downarrow$ 2846

$$\frac{1}{2}\left(\frac{2\int \left(\frac{bx^2+a}{b\log(c(bx^2+a)^p)}-\frac{a}{b\log(c(bx^2+a)^p)}\right)dx^2}{p}+\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(bx^2+a)^p)}{p}\right)}{b^2p^2}-\right.$$

$\big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(bx^2+a)^p)}{p}\right)}{b^2p^2}+\frac{2\left(\frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(\ldots}{}\right.}{b^2p}\right.}{}\right.$$

input
```
Int[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

output
```
((a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^2*p^2*(c*(a + b*
x^2)^p)^p^(-1)) + (2*(-((a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/
p])/(b^2*p*(c*(a + b*x^2)^p)^p^(-1))) + ((a + b*x^2)^2*ExpIntegralEi[(2*Lo
g[c*(a + b*x^2)^p])/p])/(b^2*p*(c*(a + b*x^2)^p)^(2/p))))/p - (x^2*(a + b*
x^2))/(b*p*Log[c*(a + b*x^2)^p]))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.06 (sec) , antiderivative size = 1487, normalized size of antiderivative = 10.78

method	result	size
risch	Expression too large to display	1487

input
```
int(x^3/ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/p/b*x^2*(b*x^2+a)/(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*P
i*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2
+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p))
-1/p^2*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn
(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/
p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I
*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x
^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p
)-2*p*ln(b*x^2+a))/p)*x^4-2/p^2/b*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*c
sgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+
a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*c
sgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*
c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*a*x^2-1/p^2/b^2*c^(-2/p)*
((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p
)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*
x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^
2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*
csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a
))/p)*a^2+1/2/p^2/b*a*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{(ap\log\left(bx^2+a\right)+a\log\left(c\right))c^{\left(\frac{1}{p}\right)}\,\mathrm{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)+\left(b^2px^4+abpx^2\right)c^{\frac{2}{p}}-2\left(p\log\left(bx^2+\right.\right.}{2\left(b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)\right)c^{\frac{2}{p}}}$$

input
```
integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
-1/2*((a*p*log(b*x^2 + a) + a*log(c))*c^(1/p)*log_integral((b*x^2 + a)*c^(
1/p)) + (b^2*p*x^4 + a*b*p*x^2)*c^(2/p) - 2*(p*log(b*x^2 + a) + log(c))*lo
g_integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/p)))/((b^2*p^3*log(b*x^2 + a)
+ b^2*p^2*log(c))*c^(2/p))
```

Sympy [F]

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

input
```
integrate(x**3/ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(x**3/log(c*(a + b*x**2)**p)**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

input

```
integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output

```
-1/2*(b*x^4 + a*x^2)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate((2*b*
x^3 + a*x)/(b*p^2*log(b*x^2 + a) + b*p*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(136) = 272$.

Time = 0.12 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.27

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

$$=\frac{1}{2}a\left(\frac{(bx^2+a)p}{b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)}-\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{(b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right))c^{\left(\frac{1}{p}\right)}}-\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx\right.\right.}{(b^2p^3\log\left(bx^2+a\right)+}\right.$$

$$-\frac{\frac{\left(bx^2+a\right)^2p}{bp^3\log(bx^2+a)+bp^2\log(c)}-\frac{2\,p\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log(bx^2+a)\right)\log(bx^2+a)}{(bp^3\log(bx^2+a)+bp^2\log(c))c^{\frac{2}{p}}}-\frac{2\,\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log(bx^2+a)\right)\log(c)}{(bp^3\log(bx^2+a)+bp^2\log(c))c^{\frac{2}{p}}}}{2\,b}$$

input

```
integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output

```
1/2*a*((b*x^2 + a)*p/(b^2*p^3*log(b*x^2 + a) + b^2*p^2*log(c)) - p*Ei(log(
c)/p + log(b*x^2 + a))*log(b*x^2 + a)/((b^2*p^3*log(b*x^2 + a) + b^2*p^2*l
og(c))*c^(1/p)) - Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b^2*p^3*log(b*x^2
 + a) + b^2*p^2*log(c))*c^(1/p))) - 1/2*((b*x^2 + a)^2*p/(b*p^3*log(b*x^2
+ a) + b*p^2*log(c)) - 2*p*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a
)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(2/p)) - 2*Ei(2*log(c)/p + 2*lo
g(b*x^2 + a))*log(c)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(2/p)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^2 \left(c \left(a+bx^2\right)^p\right)}\, dx = \int \frac{x^3}{\ln \left(c \left(b\, x^2+a\right)^p\right)^2}\, dx$$

input
```
int(x^3/log(c*(a + b*x^2)^p)^2,x)
```

output
```
int(x^3/log(c*(a + b*x^2)^p)^2, x)
```

Reduce [F]

$$\int \frac{x^3}{\log^2 \left(c \left(a+bx^2\right)^p\right)}\, dx = \int \frac{x^3}{\log \left(\left(b\, x^2+a\right)^p c\right)^2} dx$$

input
```
int(x^3/log(c*(b*x^2+a)^p)^2,x)
```

output
```
int(x**3/log((a + b*x**2)**p*c)**2,x)
```

3.110 $\int \frac{x}{\log^2(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{x}{\log^2(c(a+bx^2)^p)}\,dx = \frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2bp^2} - \frac{a+bx^2}{2bp\log(c(a+bx^2)^p)}$$

output
```
1/2*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p^2/((c*(b*x^2+a)^p)^(1/p))-1/2*(b
*x^2+a)/b/p/ln(c*(b*x^2+a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{x}{\log^2(c(a+bx^2)^p)}\,dx = -\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\left(p\left(c(a+bx^2)^p\right)^{\frac{1}{p}} - \operatorname{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)\log\left(c(a+bx^2)^p\right)\right)}{2bp^2\log(c(a+bx^2)^p)}$$

input
```
Integrate[x/Log[c*(a + b*x^2)^p]^2,x]
```

output
```
-1/2*((a + b*x^2)*(p*(c*(a + b*x^2)^p)^p^(-1) - ExpIntegralEi[Log[c*(a + b
*x^2)^p]/p]*Log[c*(a + b*x^2)^p]))/(b*p^2*(c*(a + b*x^2)^p)^p^(-1)*Log[c*(
a + b*x^2)^p])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2904, 2836, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{1}{\log^2\left(c\left(bx^2+a\right)^p\right)}dx^2$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\log^2\left(c\left(bx^2+a\right)^p\right)}d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{\log\left(c\left(bx^2+a\right)^p\right)}d\left(bx^2+a\right)}{p}-\frac{a+bx^2}{p\log\left(c\left(a+bx^2\right)^p\right)}}{2b}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\int \frac{\left(c\left(bx^2+a\right)^p\right)^{\frac{1}{p}}}{x^2}d\log\left(c\left(bx^2+a\right)^p\right)}{p^2}-\frac{a+bx^2}{p\log\left(c\left(a+bx^2\right)^p\right)}}{2b}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{p^2}-\frac{a+bx^2}{p\log\left(c\left(a+bx^2\right)^p\right)}}{2b}$$

input
```
Int[x/Log[c*(a + b*x^2)^p]^2,x]
```

output
```
(((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(p^2*(c*(a + b*x^2)^p
)^p^(-1)) - (a + b*x^2)/(p*Log[c*(a + b*x^2)^p]))/(2*b)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.82 (sec) , antiderivative size = 421, normalized size of antiderivative = 5.07

method	result
risch	$-\frac{b x^2+a}{\left(i\pi\ \mathrm{csgn}(i(b x^2+a)^p)\mathrm{csgn}(ic(b x^2+a)^p)^2-i\pi\ \mathrm{csgn}(i(b x^2+a)^p)\ \mathrm{csgn}(ic(b x^2+a)^p)\ \mathrm{csgn}(ic)-i\pi\mathrm{csgn}(ic(b x^2+a)^p)^3+i\pi\mathrm{csgn}(ic(b x\right.}$

input
```
int(x/ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)
^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn
(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p))/p/b*(b*x^2+a)-1/2
/p^2/b*(b*x^2+a)*((b*x^2+a)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^
2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*
(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*
Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+
2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{(bpx^2+ap)c^{\left(\frac{1}{p}\right)}-\left(p\log\left(bx^2+a\right)+\log\left(c\right)\right)\mathrm{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

input
```
integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*p*x^2 + a*p)*c^(1/p) - (p*log(b*x^2 + a) + log(c))*log_integral((
b*x^2 + a)*c^(1/p)))/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(1/p))
```

Sympy [F]

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

input
```
integrate(x/ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(x/log(c*(a + b*x**2)**p)**2, x)
```

Maxima [F]

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x}{\log\left(\left(bx^2+a\right)^pc\right)^2}\,dx$$

input
```
integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*x^2 + a)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate(x/(p^2*lo
g(b*x^2 + a) + p*log(c)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=-\frac{(bx^2+a)p}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)}+\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}+\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(c\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

input
```
integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
-1/2*(b*x^2 + a)*p/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)) + 1/2*p*Ei(log(c)
/p + log(b*x^2 + a))*log(b*x^2 + a)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))
*c^(1/p)) + 1/2*Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b*p^3*log(b*x^2 + a
) + b*p^2*log(c))*c^(1/p))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

input
```
int(x/log(c*(a + b*x^2)^p)^2,x)
```

output
```
int(x/log(c*(a + b*x^2)^p)^2, x)
```

Reduce [F]

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{2\left(\int \frac{x^3}{\log\left(\left(b\,x^2+a\right)^p c\right)^2 a+\log\left(\left(b\,x^2+a\right)^p c\right)^2 b\,x^2}dx\right)\log\left(\left(b\,x^2+a\right)^p c\right) b^2 p-a}{2\log\left(\left(b\,x^2+a\right)^p c\right) bp}$$

input
```
int(x/log(c*(b*x^2+a)^p)^2,x)
```

output
```
(2*int(x**3/(log((a + b*x**2)**p*c)**2*a + log((a + b*x**2)**p*c)**2*b*x**
2),x)*log((a + b*x**2)**p*c)*b**2*p - a)/(2*log((a + b*x**2)**p*c)*b*p)
```

3.111 $\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

input
```
Integrate[1/(x*Log[c*(a + b*x^2)^p]^2),x]
```

output
```
Integrate[1/(x*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$
$$\Big\downarrow \text{ 2910}$$
$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[1/(x*Log[c*(a + b*x^2)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \ln \left(c \left(b\,x^{2}+a\right)^{p}\right)^{2}}dx$$

input
```
int(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

output
```
int(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/(x*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

input
```
integrate(1/x/ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(1/(x*log(c*(a + b*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.83

$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
-a*integrate(1/(b*p^2*x^3*log(b*x^2 + a) + b*p*x^3*log(c)), x) - 1/2*(b*x^
2 + a)/(b*p^2*x^2*log(b*x^2 + a) + b*p*x^2*log(c))
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/(x*log((b*x^2 + a)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x \ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

input
```
int(1/(x*log(c*(a + b*x^2)^p)^2),x)
```

output
```
int(1/(x*log(c*(a + b*x^2)^p)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.33

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$
$$=\frac{2\left(\int \frac{1}{\log\left(\left(b\,x^2+a\right)^p c\right)^2 a x+\log\left(\left(b\,x^2+a\right)^p c\right)^2 b\,x^3}dx\right)\log\left(\left(b\,x^2+a\right)^p c\right) a p-1}{2\log\left(\left(b\,x^2+a\right)^p c\right) p}$$

input
```
int(1/x/log(c*(b*x^2+a)^p)^2,x)
```

output
```
(2*int(1/(log((a + b*x**2)**p*c)**2*a*x + log((a + b*x**2)**p*c)**2*b*x**3
),x)*log((a + b*x**2)**p*c)*a*p - 1)/(2*log((a + b*x**2)**p*c)*p)
```

3.112 $\int \frac{1}{x^3 \log^2(c(a+bx^2)^p)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

input
```
Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]
```

output
```
Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

$$\Big\downarrow \text{2910}$$

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}dx$$

input
```
int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

output
```
int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2 \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(\left(bx^2+a\right)^p c\right)^2} \, dx$$

input
```
integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 \log^2 \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(c \left(a+bx^2\right)^p\right)^2} \, dx$$

input
```
integrate(1/x**3/ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(1/(x**3*log(c*(a + b*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.33

$$\int \frac{1}{x^3 \log^2 \left(c \left(a+b x^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(\left(b x^2+a\right)^p c\right)^2} \, dx$$

input
```
integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*x^2 + a)/(b*p^2*x^4*log(b*x^2 + a) + b*p*x^4*log(c)) - integrate((
b*x^2 + 2*a)/(b*p^2*x^5*log(b*x^2 + a) + b*p*x^5*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2 \left(c \left(a+b x^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(\left(b x^2+a\right)^p c\right)^2} \, dx$$

input
```
integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2 \left(c \left(a+b x^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \ln \left(c \left(b\,x^2+a\right)^p\right)^2} \, dx$$

input
```
int(1/(x^3*log(c*(a + b*x^2)^p)^2),x)
```

output
```
int(1/(x^3*log(c*(a + b*x^2)^p)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2 \left(c \left(a+bx^2\right)^p\right)}\, dx = \int \frac{1}{\log \left(\left(b\,x^2+a\right)^p c\right)^2 x^3} dx$$

input
```
int(1/x^3/log(c*(b*x^2+a)^p)^2,x)
```

output
```
int(1/(log((a + b*x**2)**p*c)**2*x**3),x)
```

3.113 $\int \frac{x^2}{\log^2(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \text{Int}\left(\frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

output
```
Defer(Int)(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

input
```
Integrate[x^2/Log[c*(a + b*x^2)^p]^2,x]
```

output
```
Integrate[x^2/Log[c*(a + b*x^2)^p]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$
$$\Big\downarrow \; 2910$$
$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[x^2/Log[c*(a + b*x^2)^p]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}dx$$

input
```
int(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

output
```
int(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(bx^2+a\right)^pc\right)^2}\,dx$$

input
```
integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
integral(x^2/log((b*x^2 + a)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

input
```
integrate(x**2/ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(x**2/log(c*(a + b*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.67

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(bx^2+a\right)^pc\right)^2}\,dx$$

input
```
integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*x^3 + a*x)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate(1/2*(3*
b*x^2 + a)/(b*p^2*log(b*x^2 + a) + b*p*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(bx^2+a\right)^pc\right)^2}\,dx$$

input
```
integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
integrate(x^2/log((b*x^2 + a)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

input
```
int(x^2/log(c*(a + b*x^2)^p)^2,x)
```

output
```
int(x^2/log(c*(a + b*x^2)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(b\,x^2+a\right)^p c\right)^2}dx$$

input
```
int(x^2/log(c*(b*x^2+a)^p)^2,x)
```

output
```
int(x**2/log((a + b*x**2)**p*c)**2,x)
```

3.114 $\int \frac{1}{\log^2(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log^2(c(a+bx^2)^p)}\,dx = \text{Int}\left(\frac{1}{\log^2(c(a+bx^2)^p)},x\right)$$

output
```
Defer(Int)(1/ln(c*(b*x^2+a)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2(c(a+bx^2)^p)}\,dx = \int \frac{1}{\log^2(c(a+bx^2)^p)}\,dx$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^(-2),x]
```

output
```
Integrate[Log[c*(a + b*x^2)^p]^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2902}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

$$\Big\downarrow \; 2902$$

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[Log[c*(a + b*x^2)^p]^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2902
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Unintegrable[(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d,
 e, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}dx$$

input
```
int(1/ln(c*(b*x^2+a)^p)^2,x)
```

output
```
int(1/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(bx^2+a\right)^pc\right)^2}\,dx$$

input
```
integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^(-2), x)
```

Sympy [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

input
```
integrate(1/ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 5.21

$$\int \frac{1}{\log^2(c(a+bx^2)^p)}\,dx = \int \frac{1}{\log\left((bx^2+a)^p c\right)^2}\,dx$$

input
```
integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*x^2 + a)/(b*p^2*x*log(b*x^2 + a) + b*p*x*log(c)) + integrate(1/2*(
b*x^2 - a)/(b*p^2*x^2*log(b*x^2 + a) + b*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2(c(a+bx^2)^p)}\,dx = \int \frac{1}{\log\left((bx^2+a)^p c\right)^2}\,dx$$

input
```
integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 25.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2(c(a+bx^2)^p)}\,dx = \int \frac{1}{\ln\left(c\,(b\,x^2+a)^p\right)^2}\,dx$$

input
```
int(1/log(c*(a + b*x^2)^p)^2,x)
```

output
```
int(1/log(c*(a + b*x^2)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(b\,x^2+a\right)^p c\right)^2}dx$$

input
```
int(1/log(c*(b*x^2+a)^p)^2,x)
```

output
```
int(1/log((a + b*x**2)**p*c)**2,x)
```

3.115 $\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

input
```
Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
```

output
```
Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$
$$\Big\downarrow \text{2910}$$
$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}dx$$

input
```
int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

output
```
int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2 \left(c \left(a+b x^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log \left(\left(b x^2+a\right)^p c\right)^2} \, dx$$

input
```
integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 \log^2 \left(c \left(a+b x^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)^2} \, dx$$

input
```
integrate(1/x**2/ln(c*(b*x**2+a)**p)**2,x)
```

output
```
Integral(1/(x**2*log(c*(a + b*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.39

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*x^2 + a)/(b*p^2*x^3*log(b*x^2 + a) + b*p*x^3*log(c)) - integrate(1
/2*(b*x^2 + 3*a)/(b*p^2*x^4*log(b*x^2 + a) + b*p*x^4*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

input
```
int(1/(x^2*log(c*(a + b*x^2)^p)^2),x)
```

output
```
int(1/(x^2*log(c*(a + b*x^2)^p)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2 \left(c \left(a+b x^2\right)^p\right)}\, dx = \int \frac{1}{\log \left(\left(b\, x^2+a\right)^p c\right)^2 x^2} dx$$

input
```
int(1/x^2/log(c*(b*x^2+a)^p)^2,x)
```

output
```
int(1/(log((a + b*x**2)**p*c)**2*x**2),x)
```

3.116 $\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal result

Integrand size = 18, antiderivative size = 204

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx = -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{4b^2p^3} + \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(a+bx^2)^p)}{p}\right)}{b^2p^3} - \frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

output
```
-1/4*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p^3/((c*(b*x^2+a)^p)^(1/p))+(
b*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p^3/((c*(b*x^2+a)^p)^(2/p))-1/4*x
^2*(b*x^2+a)/b/p/ln(c*(b*x^2+a)^p)^2-1/4*a*(b*x^2+a)/b^2/p^2/ln(c*(b*x^2+a
)^p)-1/2*x^2*(b*x^2+a)/b/p^2/ln(c*(b*x^2+a)^p)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx =$$
$$-\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-2/p}\left(a\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\text{ExpIntegralEi}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)\log^2\left(c(a+bx^2)^p\right)-4(a\right.}{4b^2p^3}$$

input
```
Integrate[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

output
```
-1/4*((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x
^2)^p]/p]*Log[c*(a + b*x^2)^p]^2 - 4*(a + b*x^2)*ExpIntegralEi[(2*Log[c*(a
 + b*x^2)^p])/p]*Log[c*(a + b*x^2)^p]^2 + p*(c*(a + b*x^2)^p)^(2/p)*(b*p*x
^2 + (a + 2*b*x^2)*Log[c*(a + b*x^2)^p])))/(b^2*p^3*(c*(a + b*x^2)^p)^(2/p
)*Log[c*(a + b*x^2)^p]^2)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.54, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2904, 2847, 2836, 2734, 2737, 2609, 2847, 2836, 2737, 2609, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$
$$\Big\downarrow \text{2904}$$
$$\frac{1}{2}\int \frac{x^2}{\log^3\left(c\left(bx^2+a\right)^p\right)}dx^2$$
$$\Big\downarrow \text{2847}$$
$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log^2\left(c\left(bx^2+a\right)^p\right)}dx^2}{2bp}+\frac{\int \frac{x^2}{\log^2\left(c\left(bx^2+a\right)^p\right)}dx^2}{p}-\frac{x^2\left(a+bx^2\right)}{2bp\log^2\left(c\left(a+bx^2\right)^p\right)}\right)$$

$$\Big\downarrow \text{2836}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log^2(c(bx^2+a)^p)}d(bx^2+a)}{2b^2p}+\frac{\int \frac{x^2}{\log^2(c(bx^2+a)^p)}dx^2}{p}-\frac{x^2(a+bx^2)}{2bp\log^2(c(a+bx^2)^p)}\right)$$

$$\Big\downarrow \text{2734}$$

$$\frac{1}{2}\left(\frac{a\left(\frac{\int \frac{1}{\log\left(c\left(bx^2+a\right)^p\right)}d(bx^2+a)}{p}-\frac{a+bx^2}{p\log(c(a+bx^2)^p)}\right)}{2b^2p}+\frac{\int \frac{x^2}{\log^2(c(bx^2+a)^p)}dx^2}{p}-\frac{x^2(a+bx^2)}{2bp\log^2(c(a+bx^2)^p)}\right)$$

$$\Big\downarrow \text{2737}$$

$$\frac{1}{2}\left(\frac{a\left(\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\int \frac{\left(c\left(bx^2+a\right)^p\right)^{\frac{1}{p}}}{x^2}d\log\left(c(bx^2+a)^p\right)}{p^2}-\frac{a+bx^2}{p\log(c(a+bx^2)^p)}\right)}{2b^2p}+\frac{\int \frac{x^2}{\log^2(c(bx^2+a)^p)}dx^2}{p}-\frac{x^2(a+}{2bp\log^2(c(}\right.$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^2}{\log^2(c(bx^2+a)^p)}dx^2}{p}+\frac{a\left(\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{p^2}-\frac{a+bx^2}{p\log(c(a+bx^2)^p)}\right)}{2b^2p}-\frac{x^2(a+}{2bp\log^2(c(a}\right.$$

$$\Big\downarrow \text{2847}$$

$$\frac{1}{2}\left(\frac{\frac{a\int \frac{1}{\log\left(c\left(bx^2+a\right)^p\right)}dx^2}{bp}+\frac{2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)^p\right)}dx^2}{p}-\frac{x^2(a+bx^2)}{bp\log(c(a+bx^2)^p)}}{p}+\frac{a\left(\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(bx^2+a\right)\right.}{p}\right.}{p^2}\right.}{2b^2p}\right.$$

$$\Big\downarrow \text{2836}$$

$$\frac{1}{2}\left(\frac{\frac{a\int \frac{1}{\log\left(c\left(bx^2+a\right)^p\right)}d\left(bx^2+a\right)}{b^2p}+\frac{2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)^p\right)}dx^2}{p}-\frac{x^2\left(a+bx^2\right)}{bp\log\left(c\left(a+bx^2\right)^p\right)}}{p}+\frac{a\left(\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(b\right.\right.}{}\right.}{p^2}\right.}{2b^2p}\right.$$

$$\Big\downarrow \text{2737}$$

$$\frac{1}{2}\left(\frac{\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\int \frac{\left(c\left(bx^2+a\right)^p\right)^{\frac{1}{p}}}{x^2}d\log\left(c\left(bx^2+a\right)^p\right)}{b^2p^2}+\frac{2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)^p\right)}dx^2}{p}-\frac{x^2\left(a+bx^2\right)}{bp\log\left(c\left(a+bx^2\right)^p\right)}}{p}+\frac{a\left(\frac{\left(a+bx^2\right)\left(c\left(a+bx\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)^p\right)}dx^2}{p}+\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^2}-\frac{x^2\left(a+bx^2\right)}{bp\log\left(c\left(a+bx^2\right)^p\right)}}{p}+\frac{a\left(\frac{\left(a+bx^2\right)\left(c\left(a+bx\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2846}$$

$$\frac{1}{2}\left(\frac{\frac{2\int \left(\frac{bx^2+a}{b\log\left(c\left(bx^2+a\right)^p\right)}-\frac{a}{b\log\left(c\left(bx^2+a\right)^p\right)}\right)dx^2}{p}+\frac{a\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{b^2p^2}-\frac{x^2\left(a+bx^2\right)}{bp\log\left(c\left(a+bx^2\right)^p\right)}}{p}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a\left(\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{p^2}-\frac{a+bx^2}{p\log(c(a+bx^2)^p)}\right)}{2b^2p}+\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\right.}{b^2p^2}\right.$$

input
```
Int[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

output
```
((a*(((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(p^2*(c*(a + b*x^
2)^p)^p^(-1)) - (a + b*x^2)/(p*Log[c*(a + b*x^2)^p])))/(2*b^2*p) + ((a*(a
+ b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^2*p^2*(c*(a + b*x^2)^p)
^p^(-1)) + (2*(-((a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^
2*p*(c*(a + b*x^2)^p)^p^(-1))) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a
+ b*x^2)^p])/p])/(b^2*p*(c*(a + b*x^2)^p)^(2/p))))/p - (x^2*(a + b*x^2))/(
b*p*Log[c*(a + b*x^2)^p]))/p - (x^2*(a + b*x^2))/(2*b*p*Log[c*(a + b*x^2)^
p]^2))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.22 (sec) , antiderivative size = 1969, normalized size of antiderivative = 9.65

method	result	size
risch	Expression too large to display	1969

input
```
int(x^3/ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b^2*p*x^4+2*a*b*p*x^2+I*Pi*a^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2
+a)^p)^2+I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*I*Pi*b^2*x^4*csgn(I*
(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3*I*Pi*a*b*x^2*csgn(I*c*(b*x^
2+a)^p)^3+2*I*Pi*b^2*x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+2*I*P
i*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*I*Pi*b^2*x^4*csgn(I*c*(b*x^2
+a)^p)^3-I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^3+4*ln(c)*b^2*x^4-3*I*Pi*a*b*x^2*c
sgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+3*I*Pi*a*b*x^2*csgn(I*(
b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)*csgn(I*c)+3*I*Pi*a*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+4*
b^2*x^4*ln((b*x^2+a)^p)+6*ln(c)*a*b*x^2+6*a*b*x^2*ln((b*x^2+a)^p)+2*ln(c)*
a^2+2*a^2*ln((b*x^2+a)^p))/(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)
^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*
(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+
a)^p))^2/p^2/b^2-1/p^3*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*
x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(
I*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*
Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+
2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*x^4-2/b/p^3*c^(-2/p)*((b*x^2+a)^p)^(
-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.32

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx =$$

$$-\frac{\left(ap^2\log\left(bx^2+a\right)^2+2\,ap\log\left(bx^2+a\right)\log\left(c\right)+a\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)+\left(b^2p\right.}{}$$

input
```
integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
-1/4*((a*p^2*log(b*x^2 + a)^2 + 2*a*p*log(b*x^2 + a)*log(c) + a*log(c)^2)*
c^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) + (b^2*p^2*x^4 + a*b*p^2*x^2 + (
2*b^2*p^2*x^4 + 3*a*b*p^2*x^2 + a^2*p^2)*log(b*x^2 + a) + (2*b^2*p*x^4 + 3
*a*b*p*x^2 + a^2*p)*log(c))*c^(2/p) - 4*(p^2*log(b*x^2 + a)^2 + 2*p*log(b*
x^2 + a)*log(c) + log(c)^2)*log_integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/
p)))/((b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^
3*log(c)^2)*c^(2/p))
```

Sympy [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

input
```
integrate(x**3/ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Integral(x**3/log(c*(a + b*x**2)**p)**3, x)
```

Maxima [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(p + 2*log(c))*x^4 + a*b*(p + 3*log(c))*x^2 + a^2*log(c) + (2*b^
2*p*x^4 + 3*a*b*p*x^2 + a^2*p)*log(b*x^2 + a))/(b^2*p^4*log(b*x^2 + a)^2 +
 2*b^2*p^3*log(b*x^2 + a)*log(c) + b^2*p^2*log(c)^2) + integrate(1/2*(4*b*
x^3 + 3*a*x)/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(198) = 396$.

Time = 0.14 (sec) , antiderivative size = 874, normalized size of antiderivative = 4.28

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \text{Too large to display}$$

input

```
integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output

```
1/4*((b*x^2 + a)*p^2*log(b*x^2 + a)/(b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*
log(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2) - p^2*Ei(log(c)/p + log(b*x^2 +
a))*log(b*x^2 + a)^2/((b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)
*log(c) + b^2*p^3*log(c)^2)*c^(1/p)) + (b*x^2 + a)*p^2/(b^2*p^5*log(b*x^2
+ a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2) + (b*x^2 + a)
*p*log(c)/(b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^
2*p^3*log(c)^2) - 2*p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/
((b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log
(c)^2)*c^(1/p)) - Ei(log(c)/p + log(b*x^2 + a))*log(c)^2/((b^2*p^5*log(b*x
^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2)*c^(1/p)))*
a - 1/4*(2*(b*x^2 + a)^2*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*
p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) + (b*x^2 + a)^2*p^2/(b*p^5*log
(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 4*p^2*Ei
(2*log(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a)^2
+ 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)) + 2*(b*x^2 + a)
^2*p*log(c)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^
3*log(c)^2) - 8*p*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)*log(c)/
((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)
*c^(2/p)) - 4*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2
 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^3}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

input
```
int(x^3/log(c*(a + b*x^2)^p)^3,x)
```

output
```
int(x^3/log(c*(a + b*x^2)^p)^3, x)
```

Reduce [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(\left(b\,x^2+a\right)^p c\right)^3}dx$$

input
```
int(x^3/log(c*(b*x^2+a)^p)^3,x)
```

output
```
int(x**3/log((a + b*x**2)**p*c)**3,x)
```

3.117 $\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p} \text{ExpIntegralEi}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{4bp^3} - \frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

output
```
1/4*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p^3/((c*(b*x^2+a)^p)^(1/p))-1/4*(b
*x^2+a)/b/p/ln(c*(b*x^2+a)^p)^2-1/4*(b*x^2+a)/b/p^2/ln(c*(b*x^2+a)^p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = -\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\left(-\text{ExpIntegralEi}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)\log^2\left(c\left(a+bx^2\right)^p\right) + p\left(c(a+bx^2)^p\right)^{\frac{1}{p}}(p}{4bp^3\log^2\left(c\left(a+bx^2\right)^p\right)}$$

input
```
Integrate[x/Log[c*(a + b*x^2)^p]^3,x]
```

output
```
-1/4*((a + b*x^2)*(-(ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^
2)^p]^2) + p*(c*(a + b*x^2)^p)^p^(-1)*(p + Log[c*(a + b*x^2)^p])))/(b*p^3*
(c*(a + b*x^2)^p)^p^(-1)*Log[c*(a + b*x^2)^p]^2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2904, 2836, 2734, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{1}{\log^3\left(c\left(bx^2+a\right)^p\right)}dx^2$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\log^3\left(c\left(bx^2+a\right)^p\right)}d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{\log^2\left(c\left(bx^2+a\right)^p\right)}d\left(bx^2+a\right)}{2p}-\frac{a+bx^2}{2p\log^2\left(c\left(a+bx^2\right)^p\right)}}{2b}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\frac{\int \frac{1}{\log\left(c\left(bx^2+a\right)^p\right)}d\left(bx^2+a\right)}{p}-\frac{a+bx^2}{p\log\left(c\left(a+bx^2\right)^p\right)}}{2p}-\frac{a+bx^2}{2p\log^2\left(c\left(a+bx^2\right)^p\right)}}{2b}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\int \frac{\left(c\left(bx^2+a\right)^p\right)^{\frac{1}{p}}}{x^2}d\log\left(c\left(bx^2+a\right)^p\right)}{p^2}-\frac{a+bx^2}{p\log\left(c\left(a+bx^2\right)^p\right)}}{2p}-\frac{a+bx^2}{2p\log^2\left(c\left(a+bx^2\right)^p\right)}}{2b}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{\frac{(a+bx^2)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{p^2}-\frac{a+bx^2}{p\log\left(c\left(a+bx^2\right)^p\right)}}{2p}-\frac{a+bx^2}{2p\log^2(c(a+bx^2)^p)}}{2b}$$

input
```
Int[x/Log[c*(a + b*x^2)^p]^3,x]
```

output
```
((((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(p^2*(c*(a + b*x^2)^
p)^p^(-1)) - (a + b*x^2)/(p*Log[c*(a + b*x^2)^p]))/(2*p) - (a + b*x^2)/(2*
p*Log[c*(a + b*x^2)^p]^2))/(2*b)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.07 (sec) , antiderivative size = 716, normalized size of antiderivative = 6.28

method	result
risch	$-\frac{i\pi b\,x^2 \operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2-i\pi b\,x^2 \operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\operatorname{csgn}(ic)-i\pi b\,x^2\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^3+}{2p^2\left(i\pi\ \operatorname{csgn}\left(i(b\,x^2+\right.\right.}$

input
```
int(x/ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x
^2+a)^p)^3+I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+I*Pi*a*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^
2+a)^p)*csgn(I*c)-I*Pi*a*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)
^p)^2*csgn(I*c)+2*ln(c)*b*x^2+2*b*x^2*ln((b*x^2+a)^p)+2*ln(c)*a+2*a*ln((b*
x^2+a)^p)+2*x^2*p*b+2*a*p)/p^2/(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a
)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(
I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*
x^2+a)^p))^2/b-1/4/p^3/b*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I
*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b
*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*
csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx =$$

$$-\frac{\left(bp^2x^2+ap^2+\left(bp^2x^2+ap^2\right)\log\left(bx^2+a\right)+\left(bpx^2+ap\right)\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}-\left(p^2\log\left(bx^2+a\right)^2+2\,p\log\left(b\right.\right.}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\,\mathrm{lo}\right.}$$

input
```
integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
-1/4*((b*p^2*x^2 + a*p^2 + (b*p^2*x^2 + a*p^2)*log(b*x^2 + a) + (b*p*x^2 +
 a*p)*log(c))*c^(1/p) - (p^2*log(b*x^2 + a)^2 + 2*p*log(b*x^2 + a)*log(c)
+ log(c)^2)*log_integral((b*x^2 + a)*c^(1/p)))/((b*p^5*log(b*x^2 + a)^2 +
2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
```

Sympy [F]

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

input
```
integrate(x/ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Integral(x/log(c*(a + b*x**2)**p)**3, x)
```

Maxima [F]

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
-1/4*(b*(p + log(c))*x^2 + a*(p + log(c)) + (b*p*x^2 + a*p)*log(b*x^2 + a)
)/(b*p^4*log(b*x^2 + a)^2 + 2*b*p^3*log(b*x^2 + a)*log(c) + b*p^2*log(c)^2
) + integrate(1/2*x/(p^3*log(b*x^2 + a) + p^2*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(108) = 216.

Time = 0.12 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.56

$$\begin{aligned}
&\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx \\
&= -\frac{(bx^2+a)p^2\log\left(bx^2+a\right)}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)} \\
&\quad+\frac{p^2\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)^2}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}} \\
&\quad-\frac{(bx^2+a)p^2}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)} \\
&\quad-\frac{(bx^2+a)p\log\left(c\right)}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)} \\
&\quad+\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)\log\left(c\right)}{2\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}} \\
&\quad+\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(c\right)^2}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}}
\end{aligned}$$

input
```
integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
-1/4*(b*x^2 + a)*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(
b*x^2 + a)*log(c) + b*p^3*log(c)^2) + 1/4*p^2*Ei(log(c)/p + log(b*x^2 + a)
)*log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c
) + b*p^3*log(c)^2)*c^(1/p)) - 1/4*(b*x^2 + a)*p^2/(b*p^5*log(b*x^2 + a)^2
 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 1/4*(b*x^2 + a)*p*log
(c)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)
^2) + 1/2*p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*lo
g(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
+ 1/4*Ei(log(c)/p + log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*
b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

input
```
int(x/log(c*(a + b*x^2)^p)^3,x)
```

output
```
int(x/log(c*(a + b*x^2)^p)^3, x)
```

Reduce [F]

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x}{\log\left(\left(b\,x^2+a\right)^p c\right)^3}dx$$

input
```
int(x/log(c*(b*x^2+a)^p)^3,x)
```

output
```
int(x/log((a + b*x**2)**p*c)**3,x)
```

3.118 $\int \frac{1}{x \log^3(c(a+bx^2)^p)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x \log^3\left(c\left(a+bx^2\right)^p\right)} dx = \text{Int}\left(\frac{1}{x \log^3\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^3\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{x \log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

input
```
Integrate[1/(x*Log[c*(a + b*x^2)^p]^3),x]
```

output
```
Integrate[1/(x*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \log^3 \left(c \left(a+b x^2\right)^p\right)} \, dx$$

$$\Big\downarrow \; 2910$$

$$\int \frac{1}{x \log^3 \left(c \left(a+b x^2\right)^p\right)}dx$$

input
```
Int[1/(x*Log[c*(a + b*x^2)^p]^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)^{3}}dx$$

input
```
int(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

output
```
int(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
integral(1/(x*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [N/A]

Not integrable

Time = 4.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

input
```
integrate(1/x/ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Integral(1/(x*log(c*(a + b*x**2)**p)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 8.94

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

input
```
integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
-1/4*(b^2*p*x^4 + a*b*(p - log(c))*x^2 - a^2*log(c) - (a*b*p*x^2 + a^2*p)*
log(b*x^2 + a))/(b^2*p^4*x^4*log(b*x^2 + a)^2 + 2*b^2*p^3*x^4*log(b*x^2 +
a)*log(c) + b^2*p^2*x^4*log(c)^2) + integrate(1/2*(a*b*x^2 + 2*a^2)/(b^2*p
^3*x^5*log(b*x^2 + a) + b^2*p^2*x^5*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

input
```
integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
integrate(1/(x*log((b*x^2 + a)^p*c)^3), x)
```

Mupad [N/A]

Not integrable

Time = 25.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^3 \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{x\,\ln \left(c\,{\left(b\,x^2+a\right)}^p\right)^3} \,d x$$

input
```
int(1/(x*log(c*(a + b*x^2)^p)^3),x)
```

output
```
int(1/(x*log(c*(a + b*x^2)^p)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.44

$$\int \frac{1}{x \log^3 \left(c \left(a+b x^2\right)^p\right)} \, dx =\frac{4 \left(\int \frac{1}{\mathrm{log}\left(\left(b \,x^{2}+a \right)^{p} c \right)^{3} a x +\mathrm{log}\left(\left(b \,x^{2}+a \right)^{p} c \right)^{3} b \,x^{3}}d x \right) \mathrm{log}\left(\left(b \,x^{2}+a \right)^{p} c \right)^{2} a p -1}{4 \mathrm{log}\left(\left(b \,x^{2}+a \right)^{p} c \right)^{2} p}$$

input
```
int(1/x/log(c*(b*x^2+a)^p)^3,x)
```

output
```
(4*int(1/(log((a + b*x**2)**p*c)**3*a*x + log((a + b*x**2)**p*c)**3*b*x**3
),x)*log((a + b*x**2)**p*c)**2*a*p - 1)/(4*log((a + b*x**2)**p*c)**2*p)
```

3.119 $\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \text{Int}\left(\frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

input
```
Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
```

output
```
Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

$$\Big\downarrow \ 2910$$

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \ln\left(c\left(b\,x^2+a\right)^p\right)^3}dx$$

input
```
int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

output
```
int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
integral(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [N/A]

Not integrable

Time = 7.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

input
```
integrate(1/x**3/ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Integral(1/(x**3*log(c*(a + b*x**2)**p)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 184, normalized size of antiderivative = 10.22

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(p - log(c))*x^4 + a*b*(p - 3*log(c))*x^2 - 2*a^2*log(c) - (b^2*
p*x^4 + 3*a*b*p*x^2 + 2*a^2*p)*log(b*x^2 + a))/(b^2*p^4*x^6*log(b*x^2 + a)
^2 + 2*b^2*p^3*x^6*log(b*x^2 + a)*log(c) + b^2*p^2*x^6*log(c)^2) + integra
te(1/2*(b^2*x^4 + 6*a*b*x^2 + 6*a^2)/(b^2*p^3*x^7*log(b*x^2 + a) + b^2*p^2
*x^7*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
integrate(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

Mupad [N/A]

Not integrable

Time = 25.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^3 \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \ln \left(c \left(b\,x^2+a\right)^p\right)^3} \, dx$$

input
```
int(1/(x^3*log(c*(a + b*x^2)^p)^3),x)
```

output
```
int(1/(x^3*log(c*(a + b*x^2)^p)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^3 \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{\log \left(\left(b\,x^2+a\right)^p c\right)^3 x^3} dx$$

input
```
int(1/x^3/log(c*(b*x^2+a)^p)^3,x)
```

output
```
int(1/(log((a + b*x**2)**p*c)**3*x**3),x)
```

3.120 $\int \frac{x^2}{\log^3(c(a+bx^2)^p)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \operatorname{Int}\left(\frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

output
```
Defer(Int)(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

input
```
Integrate[x^2/Log[c*(a + b*x^2)^p]^3,x]
```

output
```
Integrate[x^2/Log[c*(a + b*x^2)^p]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$
$$\Big\downarrow \text{ 2910}$$
$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[x^2/Log[c*(a + b*x^2)^p]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}dx$$

input
```
int(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

output
```
int(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
integral(x^2/log((b*x^2 + a)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 4.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

input
```
integrate(x**2/ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Integral(x**2/log(c*(a + b*x**2)**p)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 10.00

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
-1/8*(b^2*(2*p + 3*log(c))*x^4 + 2*a*b*(p + 2*log(c))*x^2 + a^2*log(c) + (
3*b^2*p*x^4 + 4*a*b*p*x^2 + a^2*p)*log(b*x^2 + a))/(b^2*p^4*x*log(b*x^2 +
a)^2 + 2*b^2*p^3*x*log(b*x^2 + a)*log(c) + b^2*p^2*x*log(c)^2) + integrate
(1/8*(9*b^2*x^4 + 4*a*b*x^2 - a^2)/(b^2*p^3*x^2*log(b*x^2 + a) + b^2*p^2*x
^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
integrate(x^2/log((b*x^2 + a)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

input
```
int(x^2/log(c*(a + b*x^2)^p)^3,x)
```

output
```
int(x^2/log(c*(a + b*x^2)^p)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(b\,x^2+a\right)^p c\right)^3}dx$$

input
```
int(x^2/log(c*(b*x^2+a)^p)^3,x)
```

output
```
int(x**2/log((a + b*x**2)**p*c)**3,x)
```

3.121 $\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx = \text{Int}\left(\frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/ln(c*(b*x^2+a)^p)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

input
```
Integrate[Log[c*(a + b*x^2)^p]^(-3),x]
```

output
```
Integrate[Log[c*(a + b*x^2)^p]^(-3), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2902}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$
$$\Big\downarrow \; 2902$$
$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[Log[c*(a + b*x^2)^p]^(-3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2902
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Unintegrable[(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d,
 e, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}dx$$

input
```
int(1/ln(c*(b*x^2+a)^p)^3,x)
```

output
```
int(1/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)^(-3), x)
```

Sympy [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

input
```
integrate(1/ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Integral(log(c*(a + b*x**2)**p)**(-3), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 164, normalized size of antiderivative = 11.71

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

input
```
integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
-1/8*(b^2*(2*p + log(c))*x^4 + 2*a*b*p*x^2 - a^2*log(c) + (b^2*p*x^4 - a^2
*p)*log(b*x^2 + a))/(b^2*p^4*x^3*log(b*x^2 + a)^2 + 2*b^2*p^3*x^3*log(b*x^
2 + a)*log(c) + b^2*p^2*x^3*log(c)^2) + integrate(1/8*(b^2*x^4 + 3*a^2)/(b
^2*p^3*x^4*log(b*x^2 + a) + b^2*p^2*x^4*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

input
```
integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)^(-3), x)
```

Mupad [N/A]

Not integrable

Time = 25.71 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}\,dx$$

input
```
int(1/log(c*(a + b*x^2)^p)^3,x)
```

output
```
int(1/log(c*(a + b*x^2)^p)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(b\,x^2+a\right)^p c\right)^3}dx$$

input
```
int(1/log(c*(b*x^2+a)^p)^3,x)
```

output
```
int(1/log((a + b*x**2)**p*c)**3,x)
```

3.122 $\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

input
```
Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
```

output
```
Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

$$\Big\downarrow \text{ 2910}$$

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}dx$$

input
```
Int[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3}dx$$

input
```
int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

output
```
int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^3 (c (a + bx^2)^p)} \, dx = \int \frac{1}{x^2 \log \left((bx^2 + a)^p c\right)^3} \, dx$$

input
```
integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

output
```
integral(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [N/A]

Not integrable

Time = 5.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 \log^3 (c (a + bx^2)^p)} \, dx = \int \frac{1}{x^2 \log \left(c (a + bx^2)^p\right)^3} \, dx$$

input
```
integrate(1/x**2/ln(c*(b*x**2+a)**p)**3,x)
```

output
```
Integral(1/(x**2*log(c*(a + b*x**2)**p)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 10.39

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

output
```
-1/8*(b^2*(2*p - log(c))*x^4 + 2*a*b*(p - 2*log(c))*x^2 - 3*a^2*log(c) - (
b^2*p*x^4 + 4*a*b*p*x^2 + 3*a^2*p)*log(b*x^2 + a))/(b^2*p^4*x^5*log(b*x^2
+ a)^2 + 2*b^2*p^3*x^5*log(b*x^2 + a)*log(c) + b^2*p^2*x^5*log(c)^2) + int
egrate(1/8*(b^2*x^4 + 12*a*b*x^2 + 15*a^2)/(b^2*p^3*x^6*log(b*x^2 + a) + b
^2*p^2*x^6*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

input
```
integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

output
```
integrate(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

Mupad [N/A]

Not integrable

Time = 25.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

input
```
int(1/(x^2*log(c*(a + b*x^2)^p)^3),x)
```

output
```
int(1/(x^2*log(c*(a + b*x^2)^p)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(b\,x^2+a\right)^p c\right)^3 x^2}dx$$

input
```
int(1/x^2/log(c*(b*x^2+a)^p)^3,x)
```

output
```
int(1/(log((a + b*x**2)**p*c)**3*x**2),x)
```

3.123 $\int \frac{x^3}{\log(c(a+bx^2))} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{x^3}{\log(c(a+bx^2))} \, dx = \frac{\text{ExpIntegralEi}(2\log(c(a+bx^2)))}{2b^2c^2} - \frac{a \,\text{LogIntegral}(c(a+bx^2))}{2b^2c}$$

output
```
1/2*Ei(2*ln((b*x^2+a)*c))/b^2/c^2-1/2*a*Li((b*x^2+a)*c)/b^2/c
```

Mathematica [F]

$$\int \frac{x^3}{\log(c(a+bx^2))} \, dx = \int \frac{x^3}{\log(c(a+bx^2))} \, dx$$

input
```
Integrate[x^3/Log[c*(a + b*x^2)],x]
```

output
```
Integrate[x^3/Log[c*(a + b*x^2)], x]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2904, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\log(c(a+bx^2))}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{x^2}{\log(c(bx^2+a))}dx^2$$

$$\Big\downarrow \text{2846}$$

$$\frac{1}{2}\int \left(\frac{bx^2+a}{b\log(c(bx^2+a))}-\frac{a}{b\log(c(bx^2+a))}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\text{ExpIntegralEi}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\,\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{b^2c}\right)$$

input
```
Int[x^3/Log[c*(a + b*x^2)],x]
```

output
```
(ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (a*LogIntegral[c*(a + b*x
^2)])/(b^2*c))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{-\operatorname{expIntegral}_1\left(-2\ln\left(c\left(b\,x^2+a\right)\right)\right)+ac\,\operatorname{expIntegral}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2b^2c^2}$	43
risch	$\frac{a\,\operatorname{expIntegral}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2b^2c}-\frac{\operatorname{expIntegral}_1\left(-2\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2b^2c^2}$	47

input
```
int(x^3/ln(c*(b*x^2+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2/c^2*(-Ei(1,-2*ln(c*(b*x^2+a)))+a*c*Ei(1,-ln(c*(b*x^2+a))))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{ac\operatorname{log\_integral}\left(bcx^2+ac\right)-\operatorname{log\_integral}\left(b^2c^2x^4+2\,abc^2x^2+a^2c^2\right)}{2\,b^2c^2}$$

input
```
integrate(x^3/log((b*x^2+a)*c),x, algorithm="fricas")
```

output
```
-1/2*(a*c*log_integral(b*c*x^2 + a*c) - log_integral(b^2*c^2*x^4 + 2*a*b*c
^2*x^2 + a^2*c^2))/(b^2*c^2)
```

Sympy [F]

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx=\int \frac{x^3}{\log\left(ac+bcx^2\right)}\,dx$$

input
```
integrate(x**3/ln((b*x**2+a)*c),x)
```

output
```
Integral(x**3/log(a*c + b*c*x**2), x)
```

Maxima [F]

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx=\int \frac{x^3}{\log\left((bx^2+a)c\right)}\,dx$$

input
```
integrate(x^3/log((b*x^2+a)*c),x, algorithm="maxima")
```

output
```
integrate(x^3/log((b*x^2 + a)*c), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx=-\frac{a\mathrm{Ei}(\log\left(bcx^2+ac\right))}{2\,b^2c}+\frac{\mathrm{Ei}(2\,\log\left(bcx^2+ac\right))}{2\,b^2c^2}$$

input
```
integrate(x^3/log((b*x^2+a)*c),x, algorithm="giac")
```

output
```
-1/2*a*Ei(log(b*c*x^2 + a*c))/(b^2*c) + 1/2*Ei(2*log(b*c*x^2 + a*c))/(b^2*
c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log \left(c \left(a+b x^2\right)\right)} \, dx=\int \frac{x^3}{\ln \left(c\,\left(b\,x^2+a\right)\right)} \, dx$$

input
```
int(x^3/log(c*(a + b*x^2)),x)
```

output
```
int(x^3/log(c*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\log \left(c \left(a+b x^2\right)\right)} \, dx=\int \frac{x^3}{\log \left(bc\,x^2+ac\right)}dx$$

input
```
int(x^3/log((b*x^2+a)*c),x)
```

output
```
int(x**3/log(a*c + b*c*x**2),x)
```

3.124 $\int \frac{x}{\log(c(a+bx^2))} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \frac{x}{\log(c(a+bx^2))} \, dx = \frac{\text{LogIntegral}(c(a+bx^2))}{2bc}$$

output
```
1/2*Li((b*x^2+a)*c)/b/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log(c(a+bx^2))} \, dx = \frac{\text{LogIntegral}(c(a+bx^2))}{2bc}$$

input
```
Integrate[x/Log[c*(a + b*x^2)],x]
```

output
```
LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2904, 2836, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\log(c(a+bx^2))}dx$$
$$\Big\downarrow \ 2904$$
$$\frac{1}{2}\int \frac{1}{\log(c(bx^2+a))}dx^2$$
$$\Big\downarrow \ 2836$$
$$\frac{\int \frac{1}{\log(c(bx^2+a))}d(bx^2+a)}{2b}$$
$$\Big\downarrow \ 2735$$
$$\frac{\text{LogIntegral}\left(c(bx^2+a)\right)}{2bc}$$

input
```
Int[x/Log[c*(a + b*x^2)],x]
```

output
```
LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Defintions of rubi rules used

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$-\frac{\text{expIntegral}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2bc}$	23
default	$-\frac{\text{expIntegral}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2bc}$	23
risch	$-\frac{\text{expIntegral}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2bc}$	23

input
```
int(x/ln(c*(b*x^2+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/c*Ei(1,-ln(c*(b*x^2+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\text{log\_integral}\left(bcx^2+ac\right)}{2\,bc}$$

input
```
integrate(x/log((b*x^2+a)*c),x, algorithm="fricas")
```

output
```
1/2*log_integral(b*c*x^2 + a*c)/(b*c)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)}\,dx=\begin{cases}\frac{x^2}{2\log\left(ac\right)} & \text{for}\ b=0\\ 0 & \text{for}\ c=0\\ \frac{\operatorname{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise}\end{cases}$$

input
```
integrate(x/ln((b*x**2+a)*c),x)
```

output
```
Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*
x**2))/(2*b*c), True))
```

Maxima [F]

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)}\,dx=\int \frac{x}{\log\left((bx^2+a)c\right)}\,dx$$

input
```
integrate(x/log((b*x^2+a)*c),x, algorithm="maxima")
```

output
```
integrate(x/log((b*x^2 + a)*c), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)}\,dx=\frac{\operatorname{Ei}(\log\left(bcx^2+ac\right))}{2\,bc}$$

input
```
integrate(x/log((b*x^2+a)*c),x, algorithm="giac")
```

output
```
1/2*Ei(log(b*c*x^2 + a*c))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 26.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{\log \left(c \left(a+b x^2\right)\right)} \, dx = \frac{\operatorname{logint}(c\,(b\,x^2+a))}{2\,b\,c}$$

input
```
int(x/log(c*(a + b*x^2)),x)
```

output
```
logint(c*(a + b*x^2))/(2*b*c)
```

Reduce [F]

$$\int \frac{x}{\log \left(c \left(a+b x^2\right)\right)} \, dx = \frac{2\left(\int \frac{x^3}{\log(bc\,x^2+ac)a+\log(bc\,x^2+ac)b\,x^2}dx\right)b^2+\log(\log(bc\,x^2+ac))\,a}{2b}$$

input
```
int(x/log((b*x^2+a)*c),x)
```

output
```
(2*int(x**3/(log(a*c + b*c*x**2)*a + log(a*c + b*c*x**2)*b*x**2),x)*b**2 +
 log(log(a*c + b*c*x**2))*a)/(2*b)
```

3.125 $\int \frac{x^3}{\log^2(c(a+bx^2))} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{x^3}{\log^2(c(a+bx^2))} \, dx = \frac{\text{ExpIntegralEi}\left(2\log\left(c(a+bx^2)\right)\right)}{b^2c^2} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\,\text{LogIntegral}\left(c(a+bx^2)\right)}{2b^2c}$$

output
```
Ei(2*ln((b*x^2+a)*c))/b^2/c^2-1/2*x^2*(b*x^2+a)/b/ln((b*x^2+a)*c)-1/2*a*Li
((b*x^2+a)*c)/b^2/c
```

Mathematica [F]

$$\int \frac{x^3}{\log^2(c(a+bx^2))} \, dx = \int \frac{x^3}{\log^2(c(a+bx^2))} \, dx$$

input
```
Integrate[x^3/Log[c*(a + b*x^2)]^2,x]
```

output
```
Integrate[x^3/Log[c*(a + b*x^2)]^2, x]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2904, 2847, 2836, 2735, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{x^2}{\log^2\left(c\left(bx^2+a\right)\right)}dx^2$$

$$\Big\downarrow \text{2847}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log(c(bx^2+a))}dx^2}{b}+2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)\right)}dx^2-\frac{x^2\left(a+bx^2\right)}{b\log\left(c\left(a+bx^2\right)\right)}\right)$$

$$\Big\downarrow \text{2836}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log(c(bx^2+a))}d\left(bx^2+a\right)}{b^2}+2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)\right)}dx^2-\frac{x^2\left(a+bx^2\right)}{b\log\left(c\left(a+bx^2\right)\right)}\right)$$

$$\Big\downarrow \text{2735}$$

$$\frac{1}{2}\left(2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)\right)}dx^2+\frac{a\,\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{b^2c}-\frac{x^2\left(a+bx^2\right)}{b\log\left(c\left(a+bx^2\right)\right)}\right)$$

$$\Big\downarrow \text{2846}$$

$$\frac{1}{2}\left(2\int \left(\frac{bx^2+a}{b\log\left(c\left(bx^2+a\right)\right)}-\frac{a}{b\log\left(c\left(bx^2+a\right)\right)}\right)dx^2+\frac{a\,\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{b^2c}-\frac{x^2\left(a+bx^2\right)}{b\log\left(c\left(a+bx^2\right)\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2\left(\frac{\text{ExpIntegralEi}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\,\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{b^2c}\right)+\frac{a\,\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{b^2c}-\frac{x^2}{b\log\left(\right.}\right.$$

input
```
Int[x^3/Log[c*(a + b*x^2)]^2,x]
```

output
```
(-((x^2*(a + b*x^2))/(b*Log[c*(a + b*x^2)])) + (a*LogIntegral[c*(a + b*x^2
)])/(b^2*c) + 2*(ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (a*LogInt
egral[c*(a + b*x^2)])/(b^2*c)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x^{2}\left(b\,x^{2}+a\right)}{2 b \ln\left(c\left(b\,x^{2}+a\right)\right)}+\frac{a \,\mathrm{expIntegral}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2 b^{2} c}-\frac{\mathrm{expIntegral}_{1}\left(-2 \ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{b^{2} c^{2}}$	74
default	$\frac{-\frac{c^{2}\left(b\,x^{2}+a\right)^{2}}{\ln\left(c\left(b\,x^{2}+a\right)\right)}-2\,\mathrm{expIntegral}_{1}\left(-2 \ln\left(c\left(b\,x^{2}+a\right)\right)\right)-a c\left(-\frac{c\left(b\,x^{2}+a\right)}{\ln\left(c\left(b\,x^{2}+a\right)\right)}-\mathrm{expIntegral}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)\right)}{2 b^{2} c^{2}}$	95

input
```
int(x^3/ln(c*(b*x^2+a))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*(b*x^2+a)/b/ln(c*(b*x^2+a))+1/2/b^2/c*a*Ei(1,-ln(c*(b*x^2+a)))-1/
b^2/c^2*Ei(1,-2*ln(c*(b*x^2+a)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{b^2c^2x^4+abc^2x^2+\left(ac\,\mathrm{log\_integral}\left(bcx^2+ac\right)-2\,\mathrm{log\_integral}\left(b^2c^2x^4+2\,abc^2x^2+a^2c^2\right)\right)\log\left(bcx^2\right.}{2\,b^2c^2\log\left(bcx^2+ac\right)}$$

input
```
integrate(x^3/log((b*x^2+a)*c)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 2*log
_integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c))/(b^2
*c^2*log(b*c*x^2 + a*c))
```

Sympy [F]

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{-ax^2-bx^4}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{\int \frac{ax}{\log\left(ac+bcx^2\right)}\,dx + \int \frac{2bx^3}{\log\left(ac+bcx^2\right)}\,dx}{b}$$

input
```
integrate(x**3/ln((b*x**2+a)*c)**2,x)
```

output
```
(-a*x**2 - b*x**4)/(2*b*log(c*(a + b*x**2))) + (Integral(a*x/log(a*c + b*c
*x**2), x) + Integral(2*b*x**3/log(a*c + b*c*x**2), x))/b
```

Maxima [F]

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\log\left((bx^2+a)c\right)^2}\,dx$$

input
```
integrate(x^3/log((b*x^2+a)*c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*x^4 + a*x^2)/(b*log(b*x^2 + a) + b*log(c)) + integrate((2*b*x^3 +
a*x)/(b*log(b*x^2 + a) + b*log(c)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{a\left(\frac{bcx^2+ac}{\log(bcx^2+ac)} - \mathrm{Ei}\left(\log\left(bcx^2+ac\right)\right)\right)}{2\,b^2c} - \frac{\frac{\left(bcx^2+ac\right)^2}{\log(bcx^2+ac)} - 2\,\mathrm{Ei}\left(2\,\log\left(bcx^2+ac\right)\right)}{2\,b^2c^2}$$

input
```
integrate(x^3/log((b*x^2+a)*c)^2,x, algorithm="giac")
```

output
```
1/2*a*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) - Ei(log(b*c*x^2 + a*c)))/(b^2*c
) - 1/2*((b*c*x^2 + a*c)^2/log(b*c*x^2 + a*c) - 2*Ei(2*log(b*c*x^2 + a*c))
)/(b^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^2(c(a+bx^2))}\,dx = \int \frac{x^3}{\ln\left(c\left(b\,x^2+a\right)\right)^2}\,dx$$

input
```
int(x^3/log(c*(a + b*x^2))^2,x)
```

output
```
int(x^3/log(c*(a + b*x^2))^2, x)
```

Reduce [F]

$$\int \frac{x^3}{\log^2(c(a+bx^2))}\,dx = \int \frac{x^3}{\log\left(bc\,x^2+ac\right)^2}dx$$

input
```
int(x^3/log((b*x^2+a)*c)^2,x)
```

output
```
int(x**3/log(a*c + b*c*x**2)**2,x)
```

3.126 $\int \frac{x}{\log^2(c(a+bx^2))} dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{x}{\log^2(c(a+bx^2))} dx = -\frac{a+bx^2}{2b\log(c(a+bx^2))} + \frac{\text{LogIntegral}(c(a+bx^2))}{2bc}$$

output
```
-1/2*(b*x^2+a)/b/ln((b*x^2+a)*c)+1/2*Li((b*x^2+a)*c)/b/c
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{x}{\log^2(c(a+bx^2))} dx = \frac{-\frac{a+bx^2}{\log(c(a+bx^2))} + \frac{\text{LogIntegral}(c(a+bx^2))}{c}}{2b}$$

input
```
Integrate[x/Log[c*(a + b*x^2)]^2,x]
```

output
```
(-((a + b*x^2)/Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/c)/(2*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2904, 2836, 2734, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}dx$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{2}\int \frac{1}{\log^2\left(c\left(bx^2+a\right)\right)}dx^2$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \frac{1}{\log^2(c(bx^2+a))}d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{\int \frac{1}{\log(c(bx^2+a))}d\left(bx^2+a\right)-\frac{a+bx^2}{\log(c(a+bx^2))}}{2b}$$

$$\Big\downarrow \text{ 2735}$$

$$\frac{\frac{\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{c}-\frac{a+bx^2}{\log(c(a+bx^2))}}{2b}$$

input
```
Int[x/Log[c*(a + b*x^2)]^2,x]
```

output
```
(-((a + b*x^2)/Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/c)/(2*b)
```

Defintions of rubi rules used

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{c\left(b\,x^{2}+a\right)}{\ln\left(c\left(b\,x^{2}+a\right)\right)}-\text{expIntegral}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2bc}$	48
default	$\frac{-\frac{c\left(b\,x^{2}+a\right)}{\ln\left(c\left(b\,x^{2}+a\right)\right)}-\text{expIntegral}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2bc}$	48
risch	$-\frac{b\,x^{2}+a}{2b\ln\left(c\left(b\,x^{2}+a\right)\right)}-\frac{\text{expIntegral}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2bc}$	48

input
```
int(x/ln(c*(b*x^2+a))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b/c*(-1/ln(c*(b*x^2+a))*c*(b*x^2+a)-Ei(1,-ln(c*(b*x^2+a))))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{x}{\log^2(c(a+bx^2))}\,dx = -\frac{bcx^2+ac-\log(bcx^2+ac)\,\text{log\_integral}(bcx^2+ac)}{2\,bc\log(bcx^2+ac)}$$

input
```
integrate(x/log((b*x^2+a)*c)^2,x, algorithm="fricas")
```

output
```
-1/2*(b*c*x^2 + a*c - log(b*c*x^2 + a*c)*log_integral(b*c*x^2 + a*c))/(b*c
*log(b*c*x^2 + a*c))
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{x}{\log^2(c(a+bx^2))}\,dx = \begin{cases} \frac{x^2}{2\log(ac)} & \text{for } b=0 \\ 0 & \text{for } c=0 \\ \frac{\text{Ei}(\log(ac+bcx^2))}{2bc} & \text{otherwise} \end{cases} + \frac{-a-bx^2}{2b\log(c(a+bx^2))}$$

input
```
integrate(x/ln((b*x**2+a)*c)**2,x)
```

output
```
Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*
x**2))/(2*b*c), True)) + (-a - b*x**2)/(2*b*log(c*(a + b*x**2)))
```

Maxima [F]

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx=\int \frac{x}{\log\left((bx^2+a)c\right)^2}\,dx$$

input
```
integrate(x/log((b*x^2+a)*c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*x^2 + a)/(b*log(b*x^2 + a) + b*log(c)) + integrate(x/(log(b*x^2 +
a) + log(c)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx=-\frac{\frac{bcx^2+ac}{\log(bcx^2+ac)}-\mathrm{Ei}\left(\log\left(bcx^2+ac\right)\right)}{2\,bc}$$

input
```
integrate(x/log((b*x^2+a)*c)^2,x, algorithm="giac")
```

output
```
-1/2*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) - Ei(log(b*c*x^2 + a*c)))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 25.67 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx=\frac{\mathrm{logint}(c\,(b\,x^2+a))}{2\,b\,c}-\frac{\frac{b\,x^2}{2}+\frac{a}{2}}{b\,\ln\left(c\,\left(b\,x^2+a\right)\right)}$$

input
```
int(x/log(c*(a + b*x^2))^2,x)
```

output
```
logint(c*(a + b*x^2))/(2*b*c) - (a/2 + (b*x^2)/2)/(b*log(c*(a + b*x^2)))
```

Reduce [F]

$$\int \frac{x}{\log^2(c(a+bx^2))}\,dx = \frac{2\left(\int \frac{x^3}{\log(bc\,x^2+ac)^2 a+\log(bc\,x^2+ac)^2 b\,x^2}dx\right)\log(bc\,x^2+ac)\,b^2-a}{2\log(bc\,x^2+ac)\,b}$$

input
```
int(x/log((b*x^2+a)*c)^2,x)
```

output
```
(2*int(x**3/(log(a*c + b*c*x**2)**2*a + log(a*c + b*c*x**2)**2*b*x**2),x)*
log(a*c + b*c*x**2)*b**2 - a)/(2*log(a*c + b*c*x**2)*b)
```

3.127 $\int \frac{x^3}{\log^3(c(a+bx^2))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\text{ExpIntegralEi}\left(2\log\left(c(a+bx^2)\right)\right)}{b^2c^2} - \frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\,\text{LogIntegral}\left(c(a+bx^2)\right)}{4b^2c}$$

output
```
Ei(2*ln((b*x^2+a)*c))/b^2/c^2-1/4*x^2*(b*x^2+a)/b/ln((b*x^2+a)*c)^2-1/4*a*
(b*x^2+a)/b^2/ln((b*x^2+a)*c)-1/2*x^2*(b*x^2+a)/b/ln((b*x^2+a)*c)-1/4*a*Li
((b*x^2+a)*c)/b^2/c
```

Mathematica [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx$$

input
```
Integrate[x^3/Log[c*(a + b*x^2)]^3,x]
```

output
```
Integrate[x^3/Log[c*(a + b*x^2)]^3, x]
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2904, 2847, 2836, 2734, 2735, 2847, 2836, 2735, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{2}\int \frac{x^2}{\log^3\left(c\left(bx^2+a\right)\right)}dx^2$$

$$\Big\downarrow \text{2847}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log^2(c(bx^2+a))}dx^2}{2b}+\int \frac{x^2}{\log^2\left(c\left(bx^2+a\right)\right)}dx^2-\frac{x^2\left(a+bx^2\right)}{2b\log^2\left(c\left(a+bx^2\right)\right)}\right)$$

$$\Big\downarrow \text{2836}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log^2(c(bx^2+a))}d\left(bx^2+a\right)}{2b^2}+\int \frac{x^2}{\log^2\left(c\left(bx^2+a\right)\right)}dx^2-\frac{x^2\left(a+bx^2\right)}{2b\log^2\left(c\left(a+bx^2\right)\right)}\right)$$

$$\Big\downarrow \text{2734}$$

$$\frac{1}{2}\left(\frac{a\left(\int \frac{1}{\log(c(bx^2+a))}d\left(bx^2+a\right)-\frac{a+bx^2}{\log(c(a+bx^2))}\right)}{2b^2}+\int \frac{x^2}{\log^2\left(c\left(bx^2+a\right)\right)}dx^2-\frac{x^2\left(a+bx^2\right)}{2b\log^2\left(c\left(a+bx^2\right)\right)}\right)$$

$$\Big\downarrow \text{2735}$$

$$\frac{1}{2}\left(\int \frac{x^2}{\log^2\left(c\left(bx^2+a\right)\right)}dx^2+\frac{a\left(\frac{\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{c}-\frac{a+bx^2}{\log(c(a+bx^2))}\right)}{2b^2}-\frac{x^2\left(a+bx^2\right)}{2b\log^2\left(c\left(a+bx^2\right)\right)}\right)$$

$$\Big\downarrow \text{2847}$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log(c(bx^2+a))}dx^2}{b}+2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)\right)}dx^2+\frac{a\left(\frac{\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{c}-\frac{a+bx^2}{\log(c(a+bx^2))}\right)}{2b^2}-\frac{x^2\left(a+bx^2\right)}{2b\log^2\left(c\left(a+bx^2\right.\right.}\right.$$

$\downarrow$ 2836

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\log(c(bx^2+a))}d\left(bx^2+a\right)}{b^2}+2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)\right)}dx^2+\frac{a\left(\frac{\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{c}-\frac{a+bx^2}{\log(c(a+bx^2))}\right)}{2b^2}-\frac{x^2(a+}{2b\log^2(c(}\right.$$

$\downarrow$ 2735

$$\frac{1}{2}\left(2\int \frac{x^2}{\log\left(c\left(bx^2+a\right)\right)}dx^2+\frac{a\,\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{b^2c}+\frac{a\left(\frac{\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{c}-\frac{a+bx^2}{\log(c(a+bx^2))}\right)}{2b^2}-\frac{x^2(a+}{2b\log^2(c(}\right.$$

$\downarrow$ 2846

$$\frac{1}{2}\left(2\int \left(\frac{bx^2+a}{b\log\left(c\left(bx^2+a\right)\right)}-\frac{a}{b\log\left(c\left(bx^2+a\right)\right)}\right)dx^2+\frac{a\,\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{b^2c}+\frac{a\left(\frac{\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{c}-\right.}{2b^2}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(2\left(\frac{\text{ExpIntegralEi}\left(2\log\left(c\left(bx^2+a\right)\right)\right)}{b^2c^2}-\frac{a\,\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{b^2c}\right)+\frac{a\,\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{b^2c}+\frac{a\left(\text{Lo}\right.}{}\right.$$

input
```
Int[x^3/Log[c*(a + b*x^2)]^3,x]
```

output
```
(-1/2*(x^2*(a + b*x^2))/(b*Log[c*(a + b*x^2)]^2) - (x^2*(a + b*x^2))/(b*Lo
g[c*(a + b*x^2)]) + (a*LogIntegral[c*(a + b*x^2)])/(b^2*c) + (a*(-((a + b*
x^2)/Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/c))/(2*b^2) + 2*(Exp
IntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (a*LogIntegral[c*(a + b*x^2)]
)/(b^2*c)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{(b\,x^2+a)(2\ln(c(b\,x^2+a))b\,x^2+b\,x^2+\ln(c(b\,x^2+a))a)}{4b^2\ln(c(b\,x^2+a))^2}+\frac{a\,\mathrm{expIntegral}_1(-\ln(c(b\,x^2+a)))}{4b^2c}-\frac{\mathrm{expIntegral}_1(-2\ln(c(b\,x^2+a)}{b^2c^2}$
default	$\frac{-\frac{c^2(b\,x^2+a)^2}{2\ln(c(b\,x^2+a))^2}-\frac{c^2(b\,x^2+a)^2}{\ln(c(b\,x^2+a))}-2\,\mathrm{expIntegral}_1(-2\ln(c(b\,x^2+a)))-ac\left(-\frac{c(b\,x^2+a)}{2\ln(c(b\,x^2+a))^2}-\frac{c(b\,x^2+a)}{2\ln(c(b\,x^2+a))}-\frac{\mathrm{expIntegral}_1(-\ln}{2}\right.}{2b^2c^2}$

input
```
int(x^3/ln(c*(b*x^2+a))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x^2+a)*(2*ln(c*(b*x^2+a))*b*x^2+b*x^2+ln(c*(b*x^2+a))*a)/b^2/ln(c*
(b*x^2+a))^2+1/4/b^2/c*a*Ei(1,-ln(c*(b*x^2+a)))-1/b^2/c^2*Ei(1,-2*ln(c*(b*
x^2+a)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{b^2c^2x^4+abc^2x^2+\left(ac\,\mathrm{log\_integral}\left(bcx^2+ac\right)-4\,\mathrm{log\_integral}\left(b^2c^2x^4+2\,abc^2x^2+a^2c^2\right)\right)\log\left(bcx^2\right.}{4\,b^2c^2\log\left(bcx^2+ac\right)^2}$$

input
```
integrate(x^3/log((b*x^2+a)*c)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 4*log
_integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c)^2 + (
2*b^2*c^2*x^4 + 3*a*b*c^2*x^2 + a^2*c^2)*log(b*c*x^2 + a*c))/(b^2*c^2*log(
b*c*x^2 + a*c)^2)
```

Sympy [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\int \frac{3ax}{\log\left(ac+bcx^2\right)}\,dx + \int \frac{4bx^3}{\log\left(ac+bcx^2\right)}\,dx}{2b} + \frac{-abx^2 - b^2x^4 + \left(-a^2 - 3abx^2 - 2b^2x^4\right)\log\left(c(a+bx^2)\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)^2}$$

input
```
integrate(x**3/ln((b*x**2+a)*c)**3,x)
```

output
```
(Integral(3*a*x/log(a*c + b*c*x**2), x) + Integral(4*b*x**3/log(a*c + b*c*
x**2), x))/(2*b) + (-a*b*x**2 - b**2*x**4 + (-a**2 - 3*a*b*x**2 - 2*b**2*x
**4)*log(c*(a + b*x**2)))/(4*b**2*log(c*(a + b*x**2))**2)
```

Maxima [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\log\left((bx^2+a)c\right)^3}\,dx$$

input
```
integrate(x^3/log((b*x^2+a)*c)^3,x, algorithm="maxima")
```

output
```
-1/4*(b^2*x^4*(2*log(c) + 1) + a*b*x^2*(3*log(c) + 1) + a^2*log(c) + (2*b^
2*x^4 + 3*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^2*log(b*x^2 + a)^2 + 2*b^2*log
(b*x^2 + a)*log(c) + b^2*log(c)^2) + integrate(1/2*(4*b*x^3 + 3*a*x)/(b*lo
g(b*x^2 + a) + b*log(c)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\log^3(c(a+bx^2))}\,dx = \frac{a\left(\frac{bcx^2+ac}{\log(bcx^2+ac)} + \frac{bcx^2+ac}{\log(bcx^2+ac)^2} - \operatorname{Ei}(\log(bcx^2+ac))\right)}{4\,b^2c} - \frac{\frac{2\,(bcx^2+ac)^2}{\log(bcx^2+ac)} + \frac{(bcx^2+ac)^2}{\log(bcx^2+ac)^2} - 4\operatorname{Ei}(2\log(bcx^2+ac))}{4\,b^2c^2}$$

input
```
integrate(x^3/log((b*x^2+a)*c)^3,x, algorithm="giac")
```

output
```
1/4*a*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) + (b*c*x^2 + a*c)/log(b*c*x^2 +
a*c)^2 - Ei(log(b*c*x^2 + a*c)))/(b^2*c) - 1/4*(2*(b*c*x^2 + a*c)^2/log(b*
c*x^2 + a*c) + (b*c*x^2 + a*c)^2/log(b*c*x^2 + a*c)^2 - 4*Ei(2*log(b*c*x^2
 + a*c)))/(b^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^3(c(a+bx^2))}\,dx = \int \frac{x^3}{\ln\left(c\,\left(b\,x^2+a\right)\right)^3}\,dx$$

input
```
int(x^3/log(c*(a + b*x^2))^3,x)
```

output
```
int(x^3/log(c*(a + b*x^2))^3, x)
```

Reduce [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx=\int \frac{x^3}{\log\left(bc\,x^2+ac\right)^3}dx$$

input
```
int(x^3/log((b*x^2+a)*c)^3,x)
```

output
```
int(x**3/log(a*c + b*c*x**2)**3,x)
```

3.128 $\int \frac{x}{\log^3(c(a+bx^2))} dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \frac{x}{\log^3(c(a+bx^2))} dx = -\frac{a+bx^2}{4b\log^2(c(a+bx^2))} - \frac{a+bx^2}{4b\log(c(a+bx^2))} + \frac{\text{LogIntegral}(c(a+bx^2))}{4bc}$$

output
```
-1/4*(b*x^2+a)/b/ln((b*x^2+a)*c)^2-1/4*(b*x^2+a)/b/ln((b*x^2+a)*c)+1/4*Li(
(b*x^2+a)*c)/b/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{x}{\log^3(c(a+bx^2))} dx = \frac{-\frac{(a+bx^2)(1+\log(c(a+bx^2)))}{\log^2(c(a+bx^2))} + \frac{\text{LogIntegral}(c(a+bx^2))}{c}}{4b}$$

input
```
Integrate[x/Log[c*(a + b*x^2)]^3,x]
```

output
```
(-(((a + b*x^2)*(1 + Log[c*(a + b*x^2)]))/Log[c*(a + b*x^2)]^2) + LogInteg
ral[c*(a + b*x^2)]/c)/(4*b)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2904, 2836, 2734, 2734, 2735}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}dx$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{2}\int \frac{1}{\log^3\left(c\left(bx^2+a\right)\right)}dx^2$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \frac{1}{\log^3(c(bx^2+a))}d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{\frac{1}{2}\int \frac{1}{\log^2(c(bx^2+a))}d\left(bx^2+a\right)-\frac{a+bx^2}{2\log^2(c(a+bx^2))}}{2b}$$

$$\Big\downarrow \text{ 2734}$$

$$\frac{\frac{1}{2}\left(\int \frac{1}{\log(c(bx^2+a))}d\left(bx^2+a\right)-\frac{a+bx^2}{\log(c(a+bx^2))}\right)-\frac{a+bx^2}{2\log^2(c(a+bx^2))}}{2b}$$

$$\Big\downarrow \text{ 2735}$$

$$\frac{\frac{1}{2}\left(\frac{\text{LogIntegral}\left(c\left(bx^2+a\right)\right)}{c}-\frac{a+bx^2}{\log(c(a+bx^2))}\right)-\frac{a+bx^2}{2\log^2(c(a+bx^2))}}{2b}$$

input
```
Int[x/Log[c*(a + b*x^2)]^3,x]
```

output
```
(-1/2*(a + b*x^2)/Log[c*(a + b*x^2)]^2 + (-((a + b*x^2)/Log[c*(a + b*x^2)]
) + LogIntegral[c*(a + b*x^2)]/c)/2)/(2*b)
```

Defintions of rubi rules used

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2735
```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{c\left(b\,x^{2}+a\right)}{2\ln\left(c\left(b\,x^{2}+a\right)\right)^{2}}-\frac{c\left(b\,x^{2}+a\right)}{2\ln\left(c\left(b\,x^{2}+a\right)\right)}-\frac{\operatorname{expIntegral}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2}}{2bc}$	70
default	$\frac{-\frac{c\left(b\,x^{2}+a\right)}{2\ln\left(c\left(b\,x^{2}+a\right)\right)^{2}}-\frac{c\left(b\,x^{2}+a\right)}{2\ln\left(c\left(b\,x^{2}+a\right)\right)}-\frac{\operatorname{expIntegral}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2}}{2bc}$	70
risch	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)\right)b\,x^{2}+b\,x^{2}+\ln\left(c\left(b\,x^{2}+a\right)\right)a+a}{4b\ln\left(c\left(b\,x^{2}+a\right)\right)^{2}}-\frac{\operatorname{expIntegral}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{4bc}$	75

input
```
int(x/ln(c*(b*x^2+a))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/b/c*(-1/2/ln(c*(b*x^2+a))^2*c*(b*x^2+a)-1/2/ln(c*(b*x^2+a))*c*(b*x^2+a
)-1/2*Ei(1,-ln(c*(b*x^2+a))))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{x}{\log^3(c(a+bx^2))}\,dx = -\frac{bcx^2 - \log(bcx^2+ac)^2 \operatorname{log\_integral}(bcx^2+ac) + ac + (bcx^2+ac)\log(bcx^2+ac)}{4\,bc\log(bcx^2+ac)^2}$$

input
```
integrate(x/log((b*x^2+a)*c)^3,x, algorithm="fricas")
```

output
```
-1/4*(b*c*x^2 - log(b*c*x^2 + a*c)^2*log_integral(b*c*x^2 + a*c) + a*c + (
b*c*x^2 + a*c)*log(b*c*x^2 + a*c))/(b*c*log(b*c*x^2 + a*c)^2)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{x}{\log^3(c(a+bx^2))}\,dx = \frac{\begin{cases} \frac{x^2}{2\log(ac)} & \text{for}\ b=0 \\ 0 & \text{for}\ c=0 \\ \frac{\operatorname{Ei}(\log(ac+bcx^2))}{2bc} & \text{otherwise} \end{cases}}{2} + \frac{-a-bx^2+(-a-bx^2)\log(c(a+bx^2))}{4b\log(c(a+bx^2))^2}$$

input
```
integrate(x/ln((b*x**2+a)*c)**3,x)
```

output
```
Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*
x**2))/(2*b*c), True))/2 + (-a - b*x**2 + (-a - b*x**2)*log(c*(a + b*x**2)
))/(4*b*log(c*(a + b*x**2))**2)
```

Maxima [F]

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x}{\log\left((bx^2+a)c\right)^3}\,dx$$

input
```
integrate(x/log((b*x^2+a)*c)^3,x, algorithm="maxima")
```

output
```
-1/4*(b*x^2*(log(c) + 1) + a*(log(c) + 1) + (b*x^2 + a)*log(b*x^2 + a))/(b
*log(b*x^2 + a)^2 + 2*b*log(b*x^2 + a)*log(c) + b*log(c)^2) + integrate(1/
2*x/(log(b*x^2 + a) + log(c)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{\frac{bcx^2+ac}{\log(bcx^2+ac)}+\frac{bcx^2+ac}{\log(bcx^2+ac)^2}-\operatorname{Ei}\left(\log\left(bcx^2+ac\right)\right)}{4\,bc}$$

input
```
integrate(x/log((b*x^2+a)*c)^3,x, algorithm="giac")
```

output
```
-1/4*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) + (b*c*x^2 + a*c)/log(b*c*x^2 + a
*c)^2 - Ei(log(b*c*x^2 + a*c)))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 25.74 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\mathrm{logint}\left(c\left(b\,x^2+a\right)\right)}{4\,b\,c}-\frac{\frac{a\,c}{4}+\ln\left(c\left(b\,x^2+a\right)\right)\left(\frac{b\,c\,x^2}{4}+\frac{a\,c}{4}\right)+\frac{b\,c\,x^2}{4}}{b\,c\ln\left(c\,\left(b\,x^2+a\right)\right)^2}$$

input
```
int(x/log(c*(a + b*x^2))^3,x)
```

output
```
logint(c*(a + b*x^2))/(4*b*c) - ((a*c)/4 + log(c*(a + b*x^2))*((a*c)/4 + (
b*c*x^2)/4) + (b*c*x^2)/4)/(b*c*log(c*(a + b*x^2))^2)
```

Reduce [F]

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx=\int \frac{x}{\log\left(bc\,x^2+ac\right)^3}dx$$

input
```
int(x/log((b*x^2+a)*c)^3,x)
```

output
```
int(x/log(a*c + b*c*x**2)**3,x)
```

3.129 $\int x^5 \log^2\left(c(d+ex^3)^p\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int x^5 \log^2\left(c(d+ex^3)^p\right) dx = -\frac{2dp^2x^3}{3e} + \frac{p^2(d+ex^3)^2}{12e^2} + \frac{2dp(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e^2} - \frac{p(d+ex^3)^2\log\left(c(d+ex^3)^p\right)}{6e^2} - \frac{d(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e^2} + \frac{(d+ex^3)^2\log^2\left(c(d+ex^3)^p\right)}{6e^2}$$

output
```
-2/3*d*p^2*x^3/e+1/12*p^2*(e*x^3+d)^2/e^2+2/3*d*p*(e*x^3+d)*ln(c*(e*x^3+d)
^p)/e^2-1/6*p*(e*x^3+d)^2*ln(c*(e*x^3+d)^p)/e^2-1/3*d*(e*x^3+d)*ln(c*(e*x^
3+d)^p)^2/e^2+1/6*(e*x^3+d)^2*ln(c*(e*x^3+d)^p)^2/e^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.70

$$\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right) dx = \frac{ep^2x^3(-6d+ex^3)+2d^2p^2\log\left(d+ex^3\right)+2p(2d^2+2dex^3-e^2x^6)\log\left(c(d+ex^3)^p\right)-2(d^2-e^2x^6)\log^2}{12e^2}$$

input
```
Integrate[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

output
```
(e*p^2*x^3*(-6*d + e*x^3) + 2*d^2*p^2*Log[d + e*x^3] + 2*p*(2*d^2 + 2*d*e*
x^3 - e^2*x^6)*Log[c*(d + e*x^3)^p] - 2*(d^2 - e^2*x^6)*Log[c*(d + e*x^3)^
p]^2)/(12*e^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right) dx$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{3}\int x^3 \log^2\left(c\left(ex^3+d\right)^p\right) dx^3$$

$$\Big\downarrow \text{ 2848}$$

$$\frac{1}{3}\int \left(\frac{(ex^3+d)\log^2\left(c\left(ex^3+d\right)^p\right)}{e}-\frac{d\log^2\left(c\left(ex^3+d\right)^p\right)}{e}\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{(d+ex^3)^2\log^2\left(c(d+ex^3)^p\right)}{2e^2}-\frac{d(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{e^2}-\frac{p(d+ex^3)^2\log\left(c(d+ex^3)^p\right)}{2e^2}+\frac{2dp(d+e\right.$$

input
```
Int[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

output
```
((-2*d*p^2*x^3)/e + (p^2*(d + e*x^3)^2)/(4*e^2) + (2*d*p*(d + e*x^3)*Log[c
*(d + e*x^3)^p])/e^2 - (p*(d + e*x^3)^2*Log[c*(d + e*x^3)^p])/(2*e^2) - (d
*(d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/e^2 + ((d + e*x^3)^2*Log[c*(d + e*x^3
)^p]^2)/(2*e^2))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 6.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{2x^6\ln\left(c\left(e\,x^3+d\right)^p\right)^2e^2-2x^6\ln\left(c\left(e\,x^3+d\right)^p\right)e^2p+e^2p^2x^6+4x^3\ln\left(c\left(e\,x^3+d\right)^p\right)dep-6de\,p^2x^3+10d^2p^2\ln\left(e\,x^3+d\right)-2\ln\left(c\left(e\,x^3\right.\right.}{12e^2}$
risch	Expression too large to display

input
```
int(x^5*ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*x^6*ln(c*(e*x^3+d)^p)^2*e^2-2*x^6*ln(c*(e*x^3+d)^p)*e^2*p+e^2*p^2*
x^6+4*x^3*ln(c*(e*x^3+d)^p)*d*e*p-6*d*e*p^2*x^3+10*d^2*p^2*ln(e*x^3+d)-2*l
n(c*(e*x^3+d)^p)^2*d^2-4*ln(c*(e*x^3+d)^p)*d^2*p+6*d^2*p^2)/e^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int x^5\log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$
$$=\frac{e^2p^2x^6+2\,e^2x^6\log\left(c\right)^2-6\,dep^2x^3+2\left(e^2p^2x^6-d^2p^2\right)\log\left(ex^3+d\right)^2-2\left(e^2p^2x^6-2\,dep^2x^3-3\,d^2p^2-\right.}{12\,e^2}$$

input
```
integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
1/12*(e^2*p^2*x^6 + 2*e^2*x^6*log(c)^2 - 6*d*e*p^2*x^3 + 2*(e^2*p^2*x^6 -
d^2*p^2)*log(e*x^3 + d)^2 - 2*(e^2*p^2*x^6 - 2*d*e*p^2*x^3 - 3*d^2*p^2 - 2
*(e^2*p*x^6 - d^2*p)*log(c))*log(e*x^3 + d) - 2*(e^2*p*x^6 - 2*d*e*p*x^3)*
log(c))/e^2
```

Sympy [A] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int x^5 \log^2\left(c(d+ex^3)^p\right)\,dx$$
$$= \begin{cases} \frac{d^2 p \log\left(c(d+ex^3)^p\right)}{2e^2} - \frac{d^2 \log\left(c(d+ex^3)^p\right)^2}{6e^2} - \frac{dp^2x^3}{2e} + \frac{dpx^3\log\left(c(d+ex^3)^p\right)}{3e} + \frac{p^2x^6}{12} - \frac{px^6\log\left(c(d+ex^3)^p\right)}{6} + \frac{x^6\log\left(c(d+ex^3)\right.}{6} \\ \frac{x^6\log\left(cd^p\right)^2}{6} \end{cases}$$

input
```
integrate(x**5*ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Piecewise((d**2*p*log(c*(d + e*x**3)**p)/(2*e**2) - d**2*log(c*(d + e*x**3
)**p)**2/(6*e**2) - d*p**2*x**3/(2*e) + d*p*x**3*log(c*(d + e*x**3)**p)/(3
*e) + p**2*x**6/12 - p*x**6*log(c*(d + e*x**3)**p)/6 + x**6*log(c*(d + e*x
**3)**p)**2/6, Ne(e, 0)), (x**6*log(c*d**p)**2/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.80

$$\int x^5 \log^2\left(c(d+ex^3)^p\right)\,dx$$
$$= \frac{1}{6}x^6\log\left((ex^3+d)^pc\right)^2 - \frac{1}{6}ep\left(\frac{2d^2\log(ex^3+d)}{e^3} + \frac{ex^6-2dx^3}{e^2}\right)\log\left((ex^3+d)^pc\right) + \frac{\left(e^2x^6 - 6dex^3 + 2d^2\log(ex^3+d)^2 + 6d^2\log(ex^3+d)\right)p^2}{12e^2}$$

input
```
integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
1/6*x^6*log((e*x^3 + d)^p*c)^2 - 1/6*e*p*(2*d^2*log(e*x^3 + d)/e^3 + (e*x^
6 - 2*d*x^3)/e^2)*log((e*x^3 + d)^p*c) + 1/12*(e^2*x^6 - 6*d*e*x^3 + 2*d^2
*log(e*x^3 + d)^2 + 6*d^2*log(e*x^3 + d))*p^2/e^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.44

$$\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{2\left(ex^3+d\right)^2p^2\log\left(ex^3+d\right)^2-2\left(ex^3+d\right)^2p^2\log\left(ex^3+d\right)+4\left(ex^3+d\right)^2p\log\left(ex^3+d\right)\log\left(c\right)+\left(ex^3+\right.}{12\,e^2}$$
$$-\frac{\left(2\,ex^3+\left(ex^3+d\right)\log\left(ex^3+d\right)^2-2\left(ex^3+d\right)\log\left(ex^3+d\right)+2\,d\right)dp^2-2\left(ex^3-\left(ex^3+d\right)\log\left(ex^3+\right.\right.}{3\,e^2}$$

input
```
integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
1/12*(2*(e*x^3 + d)^2*p^2*log(e*x^3 + d)^2 - 2*(e*x^3 + d)^2*p^2*log(e*x^3
 + d) + 4*(e*x^3 + d)^2*p*log(e*x^3 + d)*log(c) + (e*x^3 + d)^2*p^2 - 2*(e
*x^3 + d)^2*p*log(c) + 2*(e*x^3 + d)^2*log(c)^2)/e^2 - 1/3*((2*e*x^3 + (e*
x^3 + d)*log(e*x^3 + d)^2 - 2*(e*x^3 + d)*log(e*x^3 + d) + 2*d)*d*p^2 - 2*
(e*x^3 - (e*x^3 + d)*log(e*x^3 + d) + d)*d*p*log(c) + (e*x^3 + d)*d*log(c)
^2)/e^2
```

Mupad [B] (verification not implemented)

Time = 25.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.67

$$\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{p^2\,x^6}{12}-\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)\,\left(\frac{p\,x^6}{6}-\frac{d\,p\,x^3}{3\,e}\right) + {\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)}^2\,\left(\frac{x^6}{6}-\frac{d^2}{6\,e^2}\right) - \frac{d\,p^2\,x^3}{2\,e}+\frac{d^2\,p^2\,\ln\left(e\,x^3+d\right)}{2\,e^2}$$

input
```
int(x^5*log(c*(d + e*x^3)^p)^2,x)
```

output
```
(p^2*x^6)/12 - log(c*(d + e*x^3)^p)*((p*x^6)/6 - (d*p*x^3)/(3*e)) + log(c*
(d + e*x^3)^p)^2*(x^6/6 - d^2/(6*e^2)) - (d*p^2*x^3)/(2*e) + (d^2*p^2*log(
d + e*x^3))/(2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$
$$= \frac{-2\log\left(\left(e\,x^3+d\right)^p c\right)^2 d^2 + 2\log\left(\left(e\,x^3+d\right)^p c\right)^2 e^2x^6 + 6\log\left(\left(e\,x^3+d\right)^p c\right) d^2p + 4\log\left(\left(e\,x^3+d\right)^p c\right) dep\,x}{12e^2}$$

input
```
int(x^5*log(c*(e*x^3+d)^p)^2,x)
```

output
```
( - 2*log((d + e*x**3)**p*c)**2*d**2 + 2*log((d + e*x**3)**p*c)**2*e**2*x*
*6 + 6*log((d + e*x**3)**p*c)*d**2*p + 4*log((d + e*x**3)**p*c)*d*e*p*x**3
 - 2*log((d + e*x**3)**p*c)*e**2*p*x**6 - 6*d*e*p**2*x**3 + e**2*p**2*x**6
)/(12*e**2)
```

3.130 $\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{2p^2x^3}{3} - \frac{2p(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e} + \frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e}$$

output
```
2/3*p^2*x^3-2/3*p*(e*x^3+d)*ln(c*(e*x^3+d)^p)/e+1/3*(e*x^3+d)*ln(c*(e*x^3+
d)^p)^2/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{1}{3}\left(\frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{e} - 2p\left(-px^3+\frac{(d+ex^3)\log\left(c(d+ex^3)^p\right)}{e}\right)\right)$$

input
```
Integrate[x^2*Log[c*(d + e*x^3)^p]^2,x]
```

output
```
(((d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/e - 2*p*(-(p*x^3) + ((d + e*x^3)*Log
[c*(d + e*x^3)^p])/e))/3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2904, 2836, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{3}\int \log^2\left(c\left(ex^3+d\right)^p\right)dx^3$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \log^2\left(c\left(ex^3+d\right)^p\right)d\left(ex^3+d\right)}{3e}$$

$$\Big\downarrow \text{ 2733}$$

$$\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)-2p\int \log\left(c\left(ex^3+d\right)^p\right)d\left(ex^3+d\right)}{3e}$$

$$\Big\downarrow \text{ 2732}$$

$$\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)-2p\left(\left(d+ex^3\right)\log\left(c\left(d+ex^3\right)^p\right)-p\left(d+ex^3\right)\right)}{3e}$$

input
```
Int[x^2*Log[c*(d + e*x^3)^p]^2,x]
```

output
```
((d + e*x^3)*Log[c*(d + e*x^3)^p]^2 - 2*p*(-(p*(d + e*x^3)) + (d + e*x^3)*
Log[c*(d + e*x^3)^p]))/(3*e)
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

method	result	size
parallelrisch	$\frac{x^{3}\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2} e p-2 x^{3} \ln\left(c\left(e\,x^{3}+d\right)^{p}\right) e\,p^{2}+2 x^{3} e\,p^{3}+\ln\left(c\left(e\,x^{3}+d\right)^{p}\right)^{2} d p-2 \ln\left(c\left(e\,x^{3}+d\right)^{p}\right) d\,p^{2}-2 d\,p^{3}}{3 e p}$	101
risch	Expression too large to display	1036

input
```
int(x^2*ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^3*ln(c*(e*x^3+d)^p)^2*e*p-2*x^3*ln(c*(e*x^3+d)^p)*e*p^2+2*x^3*e*p^3
+ln(c*(e*x^3+d)^p)^2*d*p-2*ln(c*(e*x^3+d)^p)*d*p^2-2*d*p^3)/e/p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.45

$$\int x^2 \log^2\left(c(d+ex^3)^p\right)\,dx$$
$$= \frac{2\,ep^2x^3 - 2\,epx^3\log(c) + ex^3\log(c)^2 + (ep^2x^3 + dp^2)\log(ex^3+d)^2 - 2\,(ep^2x^3 + dp^2 - (epx^3 + dp)\log}{3\,e}$$

input
```
integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
1/3*(2*e*p^2*x^3 - 2*e*p*x^3*log(c) + e*x^3*log(c)^2 + (e*p^2*x^3 + d*p^2)
*log(e*x^3 + d)^2 - 2*(e*p^2*x^3 + d*p^2 - (e*p*x^3 + d*p)*log(c))*log(e*x
^3 + d))/e
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int x^2 \log^2\left(c(d+ex^3)^p\right)\,dx$$
$$= \begin{cases} -\frac{2dp\log\left(c(d+ex^3)^p\right)}{3e} + \frac{d\log\left(c(d+ex^3)^p\right)^2}{3e} + \frac{2p^2x^3}{3} - \frac{2px^3\log\left(c(d+ex^3)^p\right)}{3} + \frac{x^3\log\left(c(d+ex^3)^p\right)^2}{3} & \text{for } e \neq 0 \\ \frac{x^3\log(cd^p)^2}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Piecewise((-2*d*p*log(c*(d + e*x**3)**p)/(3*e) + d*log(c*(d + e*x**3)**p)*
*2/(3*e) + 2*p**2*x**3/3 - 2*p*x**3*log(c*(d + e*x**3)**p)/3 + x**3*log(c*
(d + e*x**3)**p)**2/3, Ne(e, 0)), (x**3*log(c*d**p)**2/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.47

$$\int x^2 \log^2 \left(c\left(d+ex^3\right)^p\right) \, dx = \frac{1}{3}\,x^3 \log \left(\left(ex^3+d\right)^p c\right)^2 - \frac{2}{3}\left(\frac{x^3}{e} - \frac{d \log \left(ex^3+d\right)}{e^2}\right) ep \log \left(\left(ex^3+d\right)^p c\right) + \frac{\left(2\,ex^3 - d \log \left(ex^3+d\right)^2 - 2\,d \log \left(ex^3+d\right)\right)p^2}{3\,e}$$

input
```
integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
1/3*x^3*log((e*x^3 + d)^p*c)^2 - 2/3*(x^3/e - d*log(e*x^3 + d)/e^2)*e*p*lo
g((e*x^3 + d)^p*c) + 1/3*(2*e*x^3 - d*log(e*x^3 + d)^2 - 2*d*log(e*x^3 + d
))*p^2/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.45

$$\int x^2 \log^2 \left(c\left(d+ex^3\right)^p\right) \, dx = \frac{\left(2\,ex^3 + \left(ex^3+d\right) \log \left(ex^3+d\right)^2 - 2\left(ex^3+d\right) \log \left(ex^3+d\right) + 2\,d\right)p^2 - 2\left(ex^3 - \left(ex^3+d\right) \log \left(ex^3+d\right)\right.}{3\,e}$$

input
```
integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
1/3*((2*e*x^3 + (e*x^3 + d)*log(e*x^3 + d)^2 - 2*(e*x^3 + d)*log(e*x^3 + d
) + 2*d)*p^2 - 2*(e*x^3 - (e*x^3 + d)*log(e*x^3 + d) + d)*p*log(c) + (e*x^
3 + d)*log(c)^2)/e
```

Mupad [B] (verification not implemented)

Time = 25.56 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{2\,p^2\,x^3}{3} + \ln\left(c\,\left(e\,x^3+d\right)^p\right)^2\,\left(\frac{d}{3\,e}+\frac{x^3}{3}\right) - \frac{2\,p\,x^3\,\ln\left(c\,\left(e\,x^3+d\right)^p\right)}{3} - \frac{2\,d\,p^2\,\ln\left(e\,x^3+d\right)}{3\,e}$$

input
```
int(x^2*log(c*(d + e*x^3)^p)^2,x)
```

output
```
(2*p^2*x^3)/3 + log(c*(d + e*x^3)^p)^2*(d/(3*e) + x^3/3) - (2*p*x^3*log(c*
(d + e*x^3)^p))/3 - (2*d*p^2*log(d + e*x^3))/(3*e)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{\log\left(\left(e\,x^3+d\right)^p c\right)^2 d + \log\left(\left(e\,x^3+d\right)^p c\right)^2 e\,x^3 - 2\log\left(\left(e\,x^3+d\right)^p c\right) dp - 2\log\left(\left(e\,x^3+d\right)^p c\right) ep\,x^3 + 2e\,p}{3e}$$

input
```
int(x^2*log(c*(e*x^3+d)^p)^2,x)
```

output
```
(log((d + e*x**3)**p*c)**2*d + log((d + e*x**3)**p*c)**2*e*x**3 - 2*log((d
 + e*x**3)**p*c)*d*p - 2*log((d + e*x**3)**p*c)*e*p*x**3 + 2*e*p**2*x**3)/
(3*e)
```

3.131 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx = \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) + \frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{PolyLog}\left(2,1+\frac{ex^3}{d}\right) - \frac{2}{3}p^2\,\text{PolyLog}\left(3,1+\frac{ex^3}{d}\right)$$

output
```
1/3*ln(-e*x^3/d)*ln(c*(e*x^3+d)^p)^2+2/3*p*ln(c*(e*x^3+d)^p)*polylog(2,1+e
*x^3/d)-2/3*p^2*polylog(3,1+e*x^3/d)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 2965, normalized size of antiderivative = 38.51

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx = \text{Result too large to show}$$

input
```
Integrate[Log[c*(d + e*x^3)^p]^2/x,x]
```

output
```
Log[x]*(-(p*Log[d + e*x^3]) + Log[c*(d + e*x^3)^p])^2 + 2*p*(-(p*Log[d + e
*x^3]) + Log[c*(d + e*x^3)^p])*(Log[x]*(Log[d + e*x^3] - Log[1 + (e*x^3)/d
]) - PolyLog[2, -((e*x^3)/d)]/3) + p^2*(Log[-((e^(1/3)*x)/d^(1/3))]*Log[d^
(1/3)/e^(1/3) + x]^2 + 2*Log[-((e^(1/3)*x)/d^(1/3))]*Log[d^(1/3)/e^(1/3) +
 x]*Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x] + Log[-(((-1)^(2/3)*e^(1/3)*x
)/d^(1/3))]*Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x]^2 + 2*Log[-((e^(1/3)*
x)/d^(1/3))]*Log[d^(1/3)/e^(1/3) + x]*Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x
] + 2*Log[-(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))]*Log[-(((-1)^(1/3)*d^(1/3))/e^
(1/3)) + x]*Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x] + Log[((-1)^(1/3)*e^(1/3
)*x)/d^(1/3)]*Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x]^2 + Log[((-1)^(2/3)*((
(-1)^(2/3)*d^(1/3))/e^(1/3) + x))/(-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x)]^2
*(Log[-(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))] + Log[(I*Sqrt[3]*d^(1/3))/((-1)^(
1/3)*d^(1/3) - e^(1/3)*x)] - Log[((-1)^(2/3)*(1 + (-1)^(1/3))*e^(1/3)*x)/(
(-1)^(1/3)*d^(1/3) - e^(1/3)*x)]) + (Log[-((e^(1/3)*x)/d^(1/3))] + Log[-((
(-1 + (-1)^(2/3))*d^(1/3))/(d^(1/3) + e^(1/3)*x))] - Log[((1 + (-1)^(1/3))
*e^(1/3)*x)/(d^(1/3) + e^(1/3)*x)])*Log[(d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/(
d^(1/3) + e^(1/3)*x)]^2 + (Log[2] + Log[-((e^(1/3)*x)/d^(1/3))] + Log[((1
+ (-1)^(1/3))*d^(1/3))/(d^(1/3) + e^(1/3)*x)] - Log[((3 - I*Sqrt[3])*e^(1/
3)*x)/(d^(1/3) + e^(1/3)*x)])*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/(d^(1/3
) + e^(1/3)*x)]^2 + 2*(Log[((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] - Log[-(((-1...
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x} \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{3} \int \frac{\log^2\left(c\left(ex^3+d\right)^p\right)}{x^3} dx^3$$

$$\Big\downarrow \text{2843}$$

$$\frac{1}{3}\left(\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)-2ep\int\frac{\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(ex^3+d\right)^p\right)}{ex^3+d}dx^3\right)$$

$$\Big\downarrow \text{2881}$$

$$\frac{1}{3}\left(\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)-2p\int\frac{\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(ex^3+d\right)^p\right)}{x^3}d\left(ex^3+d\right)\right)$$

$$\Big\downarrow \text{2821}$$

$$\frac{1}{3}\left(\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)-2p\left(p\int\frac{\text{PolyLog}\left(2,\frac{ex^3+d}{d}\right)}{x^3}d\left(ex^3+d\right)-\text{PolyLog}\left(2,\frac{ex^3+d}{d}\right)\log\left(c\left(d\right.\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}\left(\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right)-2p\left(p\,\text{PolyLog}\left(3,\frac{ex^3+d}{d}\right)-\text{PolyLog}\left(2,\frac{ex^3+d}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)\right)\right)$$

input
```
Int[Log[c*(d + e*x^3)^p]^2/x,x]
```

output
```
(Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p]^2 - 2*p*(-(Log[c*(d + e*x^3)^p]*Po
lyLog[2, (d + e*x^3)/d]) + p*PolyLog[3, (d + e*x^3)/d]))/3
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln\left(c(e\,x^{3}+d)^{p}\right)^{2}}{x}dx$$

input
```
int(ln(c*(e*x^3+d)^p)^2/x,x)
```

output
```
int(ln(c*(e*x^3+d)^p)^2/x,x)
```

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="fricas")
```

output
```
integral(log((e*x^3 + d)^p*c)^2/x, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx = \int \frac{\log\left(c(d+ex^3)^p\right)^2}{x}\,dx$$

input
```
integrate(ln(c*(e*x**3+d)**p)**2/x,x)
```

output
```
Integral(log(c*(d + e*x**3)**p)**2/x, x)
```

Maxima [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="maxima")
```

output
```
integrate(log((e*x^3 + d)^p*c)^2/x, x)
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="giac")
```

output
```
integrate(log((e*x^3 + d)^p*c)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx = \int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x}\,dx$$

input
```
int(log(c*(d + e*x^3)^p)^2/x,x)
```

output
```
int(log(c*(d + e*x^3)^p)^2/x, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx = \frac{9\left(\int \frac{\log\left((e\,x^3+d)^p c\right)^2}{e\,x^4+dx}dx\right)dp+\log\left((e\,x^3+d)^p\,c\right)^3}{9p}$$

input
```
int(log(c*(e*x^3+d)^p)^2/x,x)
```

output
```
(9*int(log((d + e*x**3)**p*c)**2/(d*x + e*x**4),x)*d*p + log((d + e*x**3)*
*p*c)**3)/(9*p)
```

3.132 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^4}\,dx=\frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d}-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{3dx^3}+\frac{2ep^2\operatorname{PolyLog}\left(2,1+\frac{ex^3}{d}\right)}{3d}$$

output
```
2/3*e*p*ln(-e*x^3/d)*ln(c*(e*x^3+d)^p)/d-1/3*(e*x^3+d)*ln(c*(e*x^3+d)^p)^2
/d/x^3+2/3*e*p^2*polylog(2,1+e*x^3/d)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^4}\,dx=\frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)}{3d}-\frac{e\log^2\left(c\left(d+ex^3\right)^p\right)}{3d}-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{3x^3}+\frac{2ep^2\operatorname{PolyLog}\left(2,\frac{d+ex^3}{d}\right)}{3d}$$

input
```
Integrate[Log[c*(d + e*x^3)^p]^2/x^4,x]
```

output
```
(2*e*p*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p])/(3*d) - (e*Log[c*(d + e*x^3
)^p]^2)/(3*d) - Log[c*(d + e*x^3)^p]^2/(3*x^3) + (2*e*p^2*PolyLog[2, (d +
e*x^3)/d])/(3*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2904, 2844, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^4}dx$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{3}\int \frac{\log^2\left(c\left(ex^3+d\right)^p\right)}{x^6}dx^3$$

$$\Big\downarrow \text{ 2844}$$

$$\frac{1}{3}\left(\frac{2ep\int \frac{\log\left(c\left(ex^3+d\right)^p\right)}{x^3}dx^3}{d}-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{dx^3}\right)$$

$$\Big\downarrow \text{ 2841}$$

$$\frac{1}{3}\left(\frac{2ep\left(\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)-ep\int \frac{\log\left(-\frac{ex^3}{d}\right)}{ex^3+d}dx^3\right)}{d}-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{dx^3}\right)$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{1}{3}\left(\frac{2ep\left(\log\left(-\frac{ex^3}{d}\right)\log\left(c\left(d+ex^3\right)^p\right)+p\,\text{PolyLog}\left(2,\frac{ex^3}{d}+1\right)\right)}{d}-\frac{\left(d+ex^3\right)\log^2\left(c\left(d+ex^3\right)^p\right)}{dx^3}\right)$$

input
```
Int[Log[c*(d + e*x^3)^p]^2/x^4,x]
```

output
```
(-(((d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/(d*x^3)) + (2*e*p*(Log[-((e*x^3)/d
)]*Log[c*(d + e*x^3)^p] + p*PolyLog[2, 1 + (e*x^3)/d]))/d)/3
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.16 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.78

method	result
risch	$-\frac{\ln\left(\left(e\,x^{3}+d\right)^{p}\right)^{2}}{3x^{3}}+\frac{2ep\ln\left(\left(e\,x^{3}+d\right)^{p}\right)\ln\left(x\right)}{d}-\frac{2ep\ln\left(\left(e\,x^{3}+d\right)^{p}\right)\ln\left(e\,x^{3}+d\right)}{3d}-\frac{2e\,p^{2}\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^{3}+d\right)}\left(\ln\left(x\right)\ln\left(-\right.\right.\right.}{d}$

input
```
int(ln(c*(e*x^3+d)^p)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln((e*x^3+d)^p)^2/x^3+2*e*p*ln((e*x^3+d)^p)/d*ln(x)-2/3*e*p*ln((e*x^3
+d)^p)/d*ln(e*x^3+d)-2*e*p^2/d*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1
),_R1=RootOf(_Z^3*e+d))+1/3*e*p^2/d*ln(e*x^3+d)^2+(I*Pi*csgn(I*(e*x^3+d)^p
)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*c
sgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*
c)+2*ln(c))*(-1/3*ln((e*x^3+d)^p)/x^3+e*p*(1/d*ln(x)-1/3/d*ln(e*x^3+d)))-1
/12*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d
)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csg
n(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c))^2/x^3
```

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx=\int \frac{\log\left((ex^3+d)^p c\right)^2}{x^4}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="fricas")
```

output
```
integral(log((e*x^3 + d)^p*c)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \int \frac{\log\left(c(d+ex^3)^p\right)^2}{x^4}\,dx$$

input
```
integrate(ln(c*(e*x**3+d)**p)**2/x**4,x)
```

output
```
Integral(log(c*(d + e*x**3)**p)**2/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \frac{1}{3}\,e^2p^2\left(\frac{\log\left(ex^3+d\right)^2}{de} - \frac{2\left(3\,\log\left(\frac{ex^3}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^3}{d}\right)\right)}{de}\right) - \frac{2}{3}\,ep\left(\frac{\log\left(ex^3+d\right)}{d} - \frac{\log\left(x^3\right)}{d}\right)\log\left(\left(ex^3+d\right)^pc\right) - \frac{\log\left(\left(ex^3+d\right)^pc\right)^2}{3\,x^3}$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="maxima")
```

output
```
1/3*e^2*p^2*(log(e*x^3 + d)^2/(d*e) - 2*(3*log(e*x^3/d + 1)*log(x) + dilog
(-e*x^3/d))/(d*e)) - 2/3*e*p*(log(e*x^3 + d)/d - log(x^3)/d)*log((e*x^3 +
d)^p*c) - 1/3*log((e*x^3 + d)^p*c)^2/x^3
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \int \frac{\log\left((ex^3+d)^pc\right)^2}{x^4}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="giac")
```

output
```
integrate(log((e*x^3 + d)^p*c)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \int \frac{\ln\left(c\,(e\,x^3+d)^p\right)^2}{x^4}\,dx$$

input
```
int(log(c*(d + e*x^3)^p)^2/x^4,x)
```

output
```
int(log(c*(d + e*x^3)^p)^2/x^4, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \frac{-6\left(\int \frac{\log\left((e\,x^3+d)^p c\right)}{e\,x^7+d\,x^4}dx\right)d^2p\,x^3 - \log\left((e\,x^3+d)^p\,c\right)^2 d - 2\log\left((e\,x^3+d)^p\,c\right)dp - 2\log\left((e\,x^3+d)^p\,c\right)ep\,x^3}{3d\,x^3}$$

input
```
int(log(c*(e*x^3+d)^p)^2/x^4,x)
```

output
```
( - 6*int(log((d + e*x**3)**p*c)/(d*x**4 + e*x**7),x)*d**2*p*x**3 - log((d
 + e*x**3)**p*c)**2*d - 2*log((d + e*x**3)**p*c)*d*p - 2*log((d + e*x**3)*
*p*c)*e*p*x**3 + 6*log(x)*e*p**2*x**3)/(3*d*x**3)
```

3.133 $\int x \log^2\left(c(d+ex^3)^p\right)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 1294

$$\int x \log^2\left(c(d+ex^3)^p\right)\, dx = \text{Too large to display}$$

output
```
d^(2/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)+
1/2*x^2*ln(c*(e*x^3+d)^p)^2+d^(2/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/(3
-I*3^(1/2))/d^(1/3))/e^(2/3)-(-1)^(2/3)*d^(2/3)*p*ln(d^(1/3)+(-1)^(2/3)*e^
(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(2/3)+(-1)^(1/3)*d^(2/3)*p*ln(d^(1/3)-(-1)^(1
/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(2/3)-(-1)^(2/3)*d^(2/3)*p^2*ln(-(-1)^(
2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^
(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(2/3)+(-1)^(2/3)*d^(2/3)*p^2*ln((-1)^(1
/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)
^(2/3)*e^(1/3)*x)/e^(2/3)+(-1)^(2/3)*d^(2/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e
^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/e^(2/3)
-(-1)^(1/3)*d^(2/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d
^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(2/3)-(-1)^(1/3)*d^
(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(
1/3)-(-1)^(1/3)*e^(1/3)*x)/e^(2/3)+1/2*(-1)^(2/3)*d^(2/3)*p^2*ln(d^(1/3)+(
-1)^(2/3)*e^(1/3)*x)^2/e^(2/3)-1/2*(-1)^(1/3)*d^(2/3)*p^2*ln(d^(1/3)-(-1)^
(1/3)*e^(1/3)*x)^2/e^(2/3)-(-1)^(1/3)*d^(2/3)*p^2*polylog(2,-(-1)^(1/3)*((
-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(2/3)-(-1)^(2/3)*d^
(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3)
)/e^(2/3)+(-1)^(2/3)*d^(2/3)*p^2*polylog(2,(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/
(1+(-1)^(1/3))/d^(1/3))/e^(2/3)-(-1)^(1/3)*d^(2/3)*p^2*polylog(2,(d^(1/...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.88 (sec) , antiderivative size = 1041, normalized size of antiderivative = 0.80

$$\int x \log^2 \left(c\left(d+ex^3\right)^p\right) dx = \text{Too large to display}$$

input
```
Integrate[x*Log[c*(d + e*x^3)^p]^2,x]
```

output
```
(x^2*Log[c*(d + e*x^3)^p]^2)/2 - 3*e*p*((-3*p*x^2)/(4*e) + (3*p*x^2*Hyperg
eometric2F1[2/3, 1, 5/3, -((e*x^3)/d)])/(4*e) - (d^(2/3)*p*Log[-d^(1/3) -
e^(1/3)*x]^2)/(6*e^(5/3)) - (d^(2/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[-(((-
1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3)))])/(3*e^(5/3)) -
(d^(2/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)
*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(3*e^(5/3)) + (x^2*Log[c*(d + e*
x^3)^p])/(2*e) + (d^(2/3)*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p])/
(3*e^(5/3)) - ((-1)^(1/3)*d^(2/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log
[c*(d + e*x^3)^p])/(3*e^(5/3)) + ((-1)^(2/3)*d^(2/3)*Log[-d^(1/3) - (-1)^(
2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(5/3)) - (d^(2/3)*p*PolyLog[2,
(d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(3*e^(5/3)) - (d^(2/3)*
p*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/(3*e^(5/3)
) + ((-1)^(1/3)*d^(2/3)*p*((2*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 +
 (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e^(2/3) + Log
[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^2/e^(2/3) + (2*Log[-d^(1/3) + (-1)^(1/3)
*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)
^(2/3))*d^(1/3)))])/e^(2/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*
x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1
/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(2/3)))/(6*e) - ((-1)^(2/3)*
d^(2/3)*p*((2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3)...
```

Rubi [A] (verified)

Time = 3.36 (sec) , antiderivative size = 1307, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log^2\left(c\left(d+ex^3\right)^p\right) dx$$

$$\Big\downarrow \text{2907}$$

$$\frac{1}{2}x^2 \log^2\left(c\left(d+ex^3\right)^p\right) - 3ep \int \frac{x^4 \log\left(c\left(ex^3+d\right)^p\right)}{ex^3+d}dx$$

$$\Big\downarrow \text{2926}$$

$$\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-3ep\int\left(\frac{x\log\left(c\left(ex^3+d\right)^p\right)}{e}-\frac{dx\log\left(c\left(ex^3+d\right)^p\right)}{e\left(ex^3+d\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2\log^2\left(c\left(ex^3+d\right)^p\right)-$$

$$3ep\left(-\frac{3px^2}{4e}+\frac{\log\left(c\left(ex^3+d\right)^p\right)x^2}{2e}-\frac{d^{2/3}p\log^2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{6e^{5/3}}+\frac{\sqrt[3]{-1}d^{2/3}p\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{6e^{5/3}}-\frac{(-1)^{2/3}d\right.$$

input
```
Int[x*Log[c*(d + e*x^3)^p]^2,x]
```

output
```
(x^2*Log[c*(d + e*x^3)^p]^2)/2 - 3*e*p*((-3*p*x^2)/(4*e) - (Sqrt[3]*d^(2/3
)*p*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(2*e^(5/3)) - (d^(2
/3)*p*Log[d^(1/3) + e^(1/3)*x])/(2*e^(5/3)) - (d^(2/3)*p*Log[d^(1/3) + e^(
1/3)*x]^2)/(6*e^(5/3)) - (d^(2/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[-(((-1)^(
2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3)))])/(3*e^(5/3)) + ((-1
)^(1/3)*d^(2/3)*p*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))
*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/(3*e^(5/3)) + ((-1)^(1/3)*
d^(2/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/(6*e^(5/3)) - ((-1)^(2/3)
*d^(2/3)*p*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1
/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(3*e^(5/3)) - ((-1)^(2/3)*d^(2
/3)*p*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*
d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(3*e^(5/3)) - ((-1)^(2/3)*d
^(2/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/(6*e^(5/3)) + ((-1)^(2/3)*
d^(2/3)*p*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/
3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/
3))])/(3*e^(5/3)) - (d^(2/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[((-1)^(1/3)*(d
^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(3*e^(5/3)) +
 ((-1)^(1/3)*d^(2/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[-(((-1)^(2/
3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/(3*e^(5
/3)) + (d^(2/3)*p*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(4*e^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.35 (sec) , antiderivative size = 1957, normalized size of antiderivative = 1.51

method	result	size
risch	Expression too large to display	1957

input
```
int(x*ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln((e*x^3+d)^p)^2*x^2-3/2*p*x^2*ln((e*x^3+d)^p)+1/e*p^2*d/(d/e)^(1/3)*
ln(x+(d/e)^(1/3))*ln(e*x^3+d)-1/e*p*d/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*ln((e*
x^3+d)^p)-1/2/e*p^2*d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*ln(e*x
^3+d)+1/2/e*p*d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*ln((e*x^3+d)
^p)-1/e*p^2*d*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*
ln(e*x^3+d)+1/e*p*d*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*
x-1))*ln((e*x^3+d)^p)+9/4*p^2*x^2+3/2/e*p^2*d/(d/e)^(1/3)*ln(x+(d/e)^(1/3)
)-3/4/e*p^2*d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))-3/2/e*p^2*d*3^
(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))-3*e*p^2*Sum(-1/3
*(ln(x-_alpha)*ln(e*x^3+d)-3*e*(1/6/_alpha^2/e*ln(x-_alpha)^2+1/3*_alpha*l
n(x-_alpha)*(9*_alpha^2*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+
_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))+9*_alpha^2*ln((RootOf
(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*
_alpha^2,index=2))+6*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln
((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_
alpha+3*_alpha^2,index=1))+3*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,ind
ex=2)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^
2+3*_Z*_alpha+3*_alpha^2,index=2))+3*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alp
ha^2,index=1)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/Ro
otOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))+6*_alpha*RootOf(_Z^2+3*_Z*_a...
```

Fricas [F]

$$\int x \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \int x \log\left(\left(ex^3+d\right)^p c\right)^2\,dx$$

input
```
integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(x*log((e*x^3 + d)^p*c)^2, x)
```

Sympy [F]

$$\int x\log^2\left(c\left(d+ex^3\right)^p\right)\,dx=\int x\log\left(c\left(d+ex^3\right)^p\right)^2\,dx$$

input
```
integrate(x*ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(x*log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\log^2\left(c\left(d+ex^3\right)^p\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x\log^2\left(c\left(d+ex^3\right)^p\right)\,dx=\int x\log\left(\left(ex^3+d\right)^pc\right)^2\,dx$$

input
```
integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(x*log((e*x^3 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\log^2\left(c(d+ex^3)^p\right)\,dx=\int x\ln\left(c\left(e\,x^3+d\right)^p\right)^2\,dx$$

input
```
int(x*log(c*(d + e*x^3)^p)^2,x)
```

output
```
int(x*log(c*(d + e*x^3)^p)^2, x)
```

Reduce [F]

$$\int x\log^2\left(c(d+ex^3)^p\right)\,dx = \frac{6\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)d\,p^2+12e^{\frac{2}{3}}d^{\frac{4}{3}}\left(\int\frac{\log\left(\left(e\,x^3+d\right)^p c\right)x}{e\,x^3+d}dx\right)p+2e^{\frac{2}{3}}d^{\frac{1}{3}}\log\left(\left(e\,x^3+d\right)^p c\right)^2x^2-6e^{\frac{2}{3}}d^{\frac{1}{3}}\log\left(\left(e\,x\right.\right.}{4e^{\frac{2}{3}}d^{\frac{1}{3}}}$$

input
```
int(x*log(c*(e*x^3+d)^p)^2,x)
```

output
```
(6*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*d*p**2 + 12*
e**(2/3)*d**(1/3)*int((log((d + e*x**3)**p*c)*x)/(d + e*x**3),x)*d*p + 2*e
**(2/3)*d**(1/3)*log((d + e*x**3)**p*c)**2*x**2 - 6*e**(2/3)*d**(1/3)*log(
(d + e*x**3)**p*c)*p*x**2 + 9*e**(2/3)*d**(1/3)*p**2*x**2 + 9*log(d**(1/3)
 + e**(1/3)*x)*d*p**2 - 3*log((d + e*x**3)**p*c)*d*p)/(4*e**(2/3)*d**(1/3)
)
```

3.134 $\int \log^2\left(c(d+ex^3)^p\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 1304

$$\int \log^2\left(c(d+ex^3)^p\right)\,dx = \text{Too large to display}$$

output
```
(-1)^(1/3)*d^(1/3)*p^2*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)^2/e^(1/3)-(-1)^(2
/3)*d^(1/3)*p^2*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)^2/e^(1/3)-d^(1/3)*p^2*ln
(-d^(1/3)-e^(1/3)*x)^2/e^(1/3)+x*ln(c*(e*x^3+d)^p)^2-2*d^(1/3)*p^2*polylog
(2,2*(d^(1/3)+e^(1/3)*x)/(3-I*3^(1/2))/d^(1/3))/e^(1/3)-2*(-1)^(1/3)*d^(1/
3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1
/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)+2*(-1)^(1/3)*d^(
1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/
3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)/e^(1/3)+2*(-1)^(1/3)*d^(1/3)*p^2*ln(
-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(
2/3)*e^(1/3)*x)/e^(1/3)-2*(-1)^(2/3)*d^(1/3)*p^2*ln(-d^(1/3)+(-1)^(1/3)*e^
(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1
/3))/e^(1/3)-2*(-1)^(2/3)*d^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1
+(-1)^(1/3))/d^(1/3))*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)/e^(1/3)-2*(-1)^(1/
3)*d^(1/3)*p*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(1/3)+2
*(-1)^(2/3)*d^(1/3)*p*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/
e^(1/3)+18*p^2*x-2*(-1)^(2/3)*d^(1/3)*p^2*polylog(2,-(-1)^(1/3)*((-1)^(2/3
)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*
p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(1
/3)+2*(-1)^(1/3)*d^(1/3)*p^2*polylog(2,(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(
-1)^(1/3))/d^(1/3))/e^(1/3)-2*(-1)^(2/3)*d^(1/3)*p^2*polylog(2,(d^(1/3)...
```

Mathematica [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 1101, normalized size of antiderivative = 0.84

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(d + e*x^3)^p]^2,x]
```

output
```
(18*e^(1/3)*p^2*x + 6*Sqrt[3]*d^(1/3)*p^2*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3
))/Sqrt[3]] - d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2 - 2*d^(1/3)*p^2*Log[
-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*d^(1/3) - e^(1/3)*x)/((1 + (-1)^(1/3
))*d^(1/3))] - 6*d^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x] - 2*(-1)^(2/3)*d^(1/
3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*
Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x] - (-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3)
 + (-1)^(1/3)*e^(1/3)*x]^2 + 2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(2/3)*(d^(
1/3) + e^(1/3)*x))/((-1 + (-1)^(2/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*
e^(1/3)*x] + 2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3
)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3
)*x] + (-1)^(1/3)*d^(1/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2 - 2*(
-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[((-1)^(2/3)
*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((-1 + (-1)^(2/3))*d^(1/3))] - 2*d^(1/3
)*p^2*Log[-d^(1/3) - e^(1/3)*x]*Log[(I + Sqrt[3] - ((2*I)*e^(1/3)*x)/d^(1/
3))/(3*I + Sqrt[3])] + 3*d^(1/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(
2/3)*x^2] - 6*e^(1/3)*p*x*Log[c*(d + e*x^3)^p] + 2*d^(1/3)*p*Log[-d^(1/3)
- e^(1/3)*x]*Log[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*d^(1/3)*p*Log[-d^(1/3) +
(-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 2*(-1)^(1/3)*d^(1/3)*p*Log[-d
^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] + e^(1/3)*x*Log[c*(d +
 e*x^3)^p]^2 - 2*d^(1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1...
```

Rubi [A] (verified)

Time = 3.29 (sec) , antiderivative size = 1316, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2900, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$

$$\Big\downarrow \text{2900}$$

$$x\log^2\left(c\left(d+ex^3\right)^p\right)-6ep\int \frac{x^3\log\left(c\left(ex^3+d\right)^p\right)}{ex^3+d}dx$$

$$\Big\downarrow \text{2926}$$

$$x\log^2\left(c\left(d+ex^3\right)^p\right)-6ep\int\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{e}-\frac{d\log\left(c\left(ex^3+d\right)^p\right)}{e\left(ex^3+d\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x\log^2\left(c\left(ex^3+d\right)^p\right)-$$

$$6ep\left(\frac{\sqrt[3]{d}p\log^2\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)}{6e^{4/3}}+\frac{\sqrt[3]{d}p\log\left(-\frac{\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)}{3e^{4/3}}+\frac{\sqrt[3]{d}p\log\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{\ldots}\right.}{\left(1+\sqrt[3]{-1}\right)\ldots}\right.}{3\ldots}\right.$$

input
```
Int[Log[c*(d + e*x^3)^p]^2,x]
```

output
```
x*Log[c*(d + e*x^3)^p]^2 - 6*e*p*((-3*p*x)/e - (Sqrt[3]*d^(1/3)*p*ArcTan[(
d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/e^(4/3) + (d^(1/3)*p*Log[-d^(1/
3) - e^(1/3)*x]^2)/(6*e^(4/3)) + (d^(1/3)*p*Log[d^(1/3) + e^(1/3)*x])/e^(4
/3) + (d^(1/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(
1/3)*x)/((1 - (-1)^(2/3))*d^(1/3)))])/(3*e^(4/3)) + ((-1)^(2/3)*d^(1/3)*p*
Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^
(1/3) + (-1)^(1/3)*e^(1/3)*x])/(3*e^(4/3)) + ((-1)^(2/3)*d^(1/3)*p*Log[-d^
(1/3) + (-1)^(1/3)*e^(1/3)*x]^2)/(6*e^(4/3)) - ((-1)^(1/3)*d^(1/3)*p*Log[-
(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1
/3) - (-1)^(2/3)*e^(1/3)*x])/(3*e^(4/3)) - ((-1)^(1/3)*d^(1/3)*p*Log[((-1)
^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-
d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/(3*e^(4/3)) - ((-1)^(1/3)*d^(1/3)*p*Log[-
d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/(6*e^(4/3)) + ((-1)^(1/3)*d^(1/3)*p*Log
[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]
*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(3*e^(4
/3)) + (d^(1/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1
)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(3*e^(4/3)) + ((-1)^(2/3)
*d^(1/3)*p*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3)
 + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/(3*e^(4/3)) - (d^(
1/3)*p*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(2*e^(4/3)) + (x...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2900
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Simp[b*e*n*p*q   Int[x^n*
((a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \ln \left(c \left(e \,x^{3}+d \right)^{p}\right)^{2}d x$$

input
```
int(ln(c*(e*x^3+d)^p)^2,x)
```

output
```
int(ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [F]

$$\int \log ^2\left (c\left (d+e x^3\right)^p\right) \, dx=\int { \log \left ({\left (e x^{3} + d\right)}^{p} c\right)^{2} \,d x }$$

input
```
integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^3 + d)^p*c)^2, x)
```

Sympy [F]

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \int \log\left(c\left(d+ex^3\right)^p\right)^2\,dx$$

input
```
integrate(ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \int \log\left(\left(ex^3+d\right)^p c\right)^2\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^3 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \int \ln\left(c\left(e\,x^3+d\right)^p\right)^2\,dx$$

input
```
int(log(c*(d + e*x^3)^p)^2,x)
```

output
```
int(log(c*(d + e*x^3)^p)^2, x)
```

Reduce [F]

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{6d^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)p^2 - 9d^{\frac{1}{3}}\log\left(d^{\frac{1}{3}}+e^{\frac{1}{3}}x\right)p^2 + 3d^{\frac{1}{3}}\log\left(\left(e\,x^3+d\right)^p c\right)p + 6e^{\frac{1}{3}}\left(\int \frac{\log\left(\left(e\,x^3+d\right)^p c\right)}{e\,x^3+d}dx\right)d}{e^{\frac{1}{3}}}$$

input
```
int(log(c*(e*x^3+d)^p)^2,x)
```

output
```
(6*d**(1/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*p**
2 - 9*d**(1/3)*log(d**(1/3) + e**(1/3)*x)*p**2 + 3*d**(1/3)*log((d + e*x**
3)**p*c)*p + 6*e**(1/3)*int(log((d + e*x**3)**p*c)/(d + e*x**3),x)*d*p + e
**(1/3)*log((d + e*x**3)**p*c)**2*x - 6*e**(1/3)*log((d + e*x**3)**p*c)*p*
x + 18*e**(1/3)*p**2*x)/e**(1/3)
```

3.135 $\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 1137

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2} dx = \text{Too large to display}$$

output
```
e^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)^2/d^(1/3)+2*e^(1/3)*p^2*ln(d^(1/3)+e^(1/
3)*x)*ln(-((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2
*(-1)^(1/3)*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d
^(1/3))*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/d^(1/3)-(-1)^(1/3)*e^(1/3)*p^2*ln
(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)^2/d^(1/3)+2*(-1)^(2/3)*e^(1/3)*p^2*ln(-(-1)
^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e
^(1/3)*x)/d^(1/3)+2*(-1)^(2/3)*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/
3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(
1/3)+(-1)^(2/3)*e^(1/3)*p^2*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)^2/d^(1/3)+2*e
^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p^2*ln(-(-1)^(2/3
)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/
3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*(-1)^(1/3)*e^(1/3)*p^2*ln(d^(1/3)-
(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1
)^(2/3))/d^(1/3))/d^(1/3)-2*e^(1/3)*p*ln(d^(1/3)+e^(1/3)*x)*ln(c*(e*x^3+d)
^p)/d^(1/3)+2*(-1)^(1/3)*e^(1/3)*p*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(c*(
e*x^3+d)^p)/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x
)*ln(c*(e*x^3+d)^p)/d^(1/3)-ln(c*(e*x^3+d)^p)^2/x+2*e^(1/3)*p^2*polylog(2,
(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p
^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^...
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 972, normalized size of antiderivative = 0.85

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(d + e*x^3)^p]^2/x^2,x]
```

output
```
-(Log[c*(d + e*x^3)^p]^2/x) + 6*e*p*((p*Log[-d^(1/3) - e^(1/3)*x]^2)/(6*d^
(1/3)*e^(2/3)) + (p*Log[-d^(1/3) - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) +
e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3)))])/(3*d^(1/3)*e^(2/3)) + (p*Log[-d^(
1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 +
(-1)^(1/3))*d^(1/3))])/(3*d^(1/3)*e^(2/3)) - (Log[-d^(1/3) - e^(1/3)*x]*Lo
g[c*(d + e*x^3)^p])/(3*d^(1/3)*e^(2/3)) + ((-1)^(1/3)*Log[-d^(1/3) + (-1)^
(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*d^(1/3)*e^(2/3)) - ((-1)^(2/3)*L
og[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*d^(1/3)*e^(2/
3)) + (p*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(3*
d^(1/3)*e^(2/3)) + (p*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d
^(1/3))])/(3*d^(1/3)*e^(2/3)) - ((-1)^(1/3)*p*((2*Log[((-1)^(1/3)*(d^(1/3)
 + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/
3)*x])/e^(2/3) + Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^2/e^(2/3) + (2*Log[-
d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^
(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/e^(2/3) + (2*PolyLog[2, (d^(1/3) -
 (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (2*PolyLog[2
, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(2/3)))/
(6*d^(1/3)) + ((-1)^(2/3)*p*((2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/(
(1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(2/3)
+ (2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3)...
```

Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 1153, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^2}dx$$

$$\Big\downarrow \text{ 2907}$$

$$6ep\int \frac{x\log\left(c\left(ex^3+d\right)^p\right)}{ex^3+d}dx-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}$$

$$\Big\downarrow \text{ 2926}$$

$$6ep\int\left(-\frac{\log\left(c\left(ex^3+d\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}-\frac{(-1)^{2/3}\log\left(c\left(ex^3+d\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(ex^3+d\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}\right)dx-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$6ep\left(\frac{p\log^2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{6\sqrt[3]{d}e^{2/3}}+\frac{p\log\left(-\frac{\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{3\sqrt[3]{d}e^{2/3}}+\frac{p\log\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log}{3\sqrt[3]{d}e^{2/3}}\right.$$

$$\frac{\log^2\left(c\left(ex^3+d\right)^p\right)}{x}$$

input
```
Int[Log[c*(d + e*x^3)^p]^2/x^2,x]
```

output
```
-(Log[c*(d + e*x^3)^p]^2/x) + 6*e*p*((p*Log[d^(1/3) + e^(1/3)*x]^2)/(6*d^(
1/3)*e^(2/3)) + (p*Log[d^(1/3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^
(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3)))])/(3*d^(1/3)*e^(2/3)) - ((-1)^(1/3)*p
*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^
(1/3) - (-1)^(1/3)*e^(1/3)*x])/(3*d^(1/3)*e^(2/3)) - ((-1)^(1/3)*p*Log[d^(
1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/(6*d^(1/3)*e^(2/3)) + ((-1)^(2/3)*p*Log[-(
((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3
) + (-1)^(2/3)*e^(1/3)*x])/(3*d^(1/3)*e^(2/3)) + ((-1)^(2/3)*p*Log[((-1)^(
1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(
1/3) + (-1)^(2/3)*e^(1/3)*x])/(3*d^(1/3)*e^(2/3)) + ((-1)^(2/3)*p*Log[d^(1
/3) + (-1)^(2/3)*e^(1/3)*x]^2)/(6*d^(1/3)*e^(2/3)) - ((-1)^(2/3)*p*Log[((-
1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log
[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(3*d^(1/3)*
e^(2/3)) + (p*Log[d^(1/3) + e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/
3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(3*d^(1/3)*e^(2/3)) - ((-1)^(1
/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1
)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/(3*d^(1/3)*e^(2/3)) - (L
og[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*d^(1/3)*e^(2/3)) + ((-1)^
(1/3)*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*d^(1/3)
*e^(2/3)) - ((-1)^(2/3)*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 1787, normalized size of antiderivative = 1.57

method	result	size
risch	Expression too large to display	1787

input
```
int(ln(c*(e*x^3+d)^p)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*ln((e*x^3+d)^p)^2+2*p^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*ln(e*x^3+d)-2*p
/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*ln((e*x^3+d)^p)-p^2/(d/e)^(1/3)*ln(x^2-(d/e
)^(1/3)*x+(d/e)^(2/3))*ln(e*x^3+d)+p/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e
)^(2/3))*ln((e*x^3+d)^p)-2*p^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(
d/e)^(1/3)*x-1))*ln(e*x^3+d)+2*p*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2
/(d/e)^(1/3)*x-1))*ln((e*x^3+d)^p)+p^2*sum(1/_alpha*(2*ln(x-_alpha)*ln(e*x
^3+d)-e*(1/_alpha^2/e*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*(9*_alpha^2*ln(
(RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_a
lpha+3*_alpha^2,index=1))+9*_alpha^2*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^
2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))+6*_alpha
*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z^2+3*_Z*_alpha+3
*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))+
3*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z^2+3*_Z*
_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,in
dex=2))+3*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z
^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_al
pha^2,index=1))+6*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((R
ootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alp
ha+3*_alpha^2,index=2))+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*Root
Of(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_...
```

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^2}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^3 + d)^p*c)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \int \frac{\log\left(c(d+ex^3)^p\right)^2}{x^2}\,dx$$

input
```
integrate(ln(c*(e*x**3+d)**p)**2/x**2,x)
```

output
```
Integral(log(c*(d + e*x**3)**p)**2/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^2}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^3 + d)^p*c)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \int \frac{\ln\left(c\,(e\,x^3+d)^p\right)^2}{x^2}\,dx$$

input
```
int(log(c*(d + e*x^3)^p)^2/x^2,x)
```

output
```
int(log(c*(d + e*x^3)^p)^2/x^2, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \frac{-6\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)e\,p^2x - 6e^{\frac{2}{3}}d^{\frac{4}{3}}\left(\int \frac{\log\left((e\,x^3+d)^p c\right)}{e\,x^5+d\,x^2}dx\right)px - e^{\frac{2}{3}}d^{\frac{1}{3}}\log\left((e\,x^3+d)^p\,c\right)^2 - 6e^{\frac{2}{3}}d^{\frac{1}{3}}\log\left((e\,x^3\right.}{e^{\frac{2}{3}}d^{\frac{1}{3}}x}$$

input
```
int(log(c*(e*x^3+d)^p)^2/x^2,x)
```

output
```
( - 6*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*e*p**2*x
- 6*e**(2/3)*d**(1/3)*int(log((d + e*x**3)**p*c)/(d*x**2 + e*x**5),x)*d*p*
x - e**(2/3)*d**(1/3)*log((d + e*x**3)**p*c)**2 - 6*e**(2/3)*d**(1/3)*log(
(d + e*x**3)**p*c)*p - 9*log(d**(1/3) + e**(1/3)*x)*e*p**2*x + 3*log((d +
e*x**3)**p*c)*e*p*x)/(e**(2/3)*d**(1/3)*x)
```

3.136 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1170

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3}\,dx = \text{Too large to display}$$

output
```
-1/2*e^(2/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)^2/d^(2/3)-e^(2/3)*p^2*ln(-d^(1/3)-
e^(1/3)*x)*ln(-((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2
/3)-(-1)^(2/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3)
)/d^(1/3))*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)/d^(2/3)-1/2*(-1)^(2/3)*e^(2/3
)*p^2*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)^2/d^(2/3)+(-1)^(1/3)*e^(2/3)*p^2*l
n(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-d^(1/3)-(-1)
^(2/3)*e^(1/3)*x)/d^(2/3)+(-1)^(1/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-
1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)
*x)/d^(2/3)+1/2*(-1)^(1/3)*e^(2/3)*p^2*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)^2
/d^(2/3)-e^(2/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2
/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1)^(1/3)*e^(2/3)*p^2*ln(-
(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2
/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)-(-1)^(2/3)*e^(2/3)*p^2*ln(-
d^(1/3)+(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x
)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)+e^(2/3)*p*ln(-d^(1/3)-e^(1/3)*x)*ln(c*(e
*x^3+d)^p)/d^(2/3)+(-1)^(2/3)*e^(2/3)*p*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)*
ln(c*(e*x^3+d)^p)/d^(2/3)-(-1)^(1/3)*e^(2/3)*p*ln(-d^(1/3)-(-1)^(2/3)*e^(1
/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)-1/2*ln(c*(e*x^3+d)^p)^2/x^2-e^(2/3)*p^2*p
olylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1)^(1/3)*e^
(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1...
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 766, normalized size of antiderivative = 0.65

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(d + e*x^3)^p]^2/x^3,x]
```

output
```
-1/2*Log[c*(d + e*x^3)^p]^2/x^2 - (e^(2/3)*p*(p*Log[-d^(1/3) - e^(1/3)*x]^
2 + 2*p*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*d^(1/3) - e^(1/3)*x)/((1
 + (-1)^(1/3))*d^(1/3))] + 2*p*Log[-d^(1/3) - e^(1/3)*x]*Log[(I + Sqrt[3]
- ((2*I)*e^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])] - 2*Log[-d^(1/3) - e^(1/3)*x
]*Log[c*(d + e*x^3)^p] - 2*(-1)^(2/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]
*Log[c*(d + e*x^3)^p] + 2*(-1)^(1/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*
Log[c*(d + e*x^3)^p] + 2*p*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/
3))*d^(1/3))] + (-1)^(2/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[
((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1
/3) + (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1
/3)*x))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)
*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1
/3)*e^(1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(1/3)*p*(Log[-d^(1/3)
- (-1)^(2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (
-1)^(2/3))*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))
/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*P
olyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2
*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])
+ 2*p*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(3*I + Sqrt[3])]))/(2*d
^(2/3))
```

Rubi [A] (verified)

Time = 2.81 (sec) , antiderivative size = 1183, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2907, 2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3}dx$$

$$\Big\downarrow \text{2907}$$

$$3ep\int \frac{\log\left(c\left(ex^3+d\right)^p\right)}{ex^3+d}dx-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{2x^2}$$

$$\Big\downarrow \text{2921}$$

$$3ep\int \left(-\frac{\log \left(c\left(ex^3+d\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)}-\frac{\log \left(c\left(ex^3+d\right)^p\right)}{3d^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)}-\frac{\log \left(c\left(ex^3+d\right)^p\right)}{3d^{2/3}\left(-(-1)^{2/3}\sqrt[3]{e}x-\sqrt[3]{d}\right)}\right)dx-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$3ep\left(-\frac{p\log^2\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)}{6d^{2/3}\sqrt[3]{e}}-\frac{p\log\left(-\frac{\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)}{3d^{2/3}\sqrt[3]{e}}-\frac{p\log\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right.}{3d^{2/3}\sqrt[3]{e}}\right.$$

$$\frac{\log^2\left(c\left(ex^3+d\right)^p\right)}{2x^2}$$

input
```
Int[Log[c*(d + e*x^3)^p]^2/x^3,x]
```

output
```
-1/2*Log[c*(d + e*x^3)^p]^2/x^2 + 3*e*p*(-1/6*(p*Log[-d^(1/3) - e^(1/3)*x]
^2)/(d^(2/3)*e^(1/3)) - (p*Log[-d^(1/3) - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(
1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3)))])/(3*d^(2/3)*e^(1/3)) - ((-1
)^(2/3)*p*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3)
)]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) - ((-1)^(2/3)
*p*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^2)/(6*d^(2/3)*e^(1/3)) + ((-1)^(1/
3)*p*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]
*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) + ((-1)^(1/3)*p
*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/
3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) + ((-1)^(1/
3)*p*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/(6*d^(2/3)*e^(1/3)) - ((-1)^(
1/3)*p*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))
*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))
])/(3*d^(2/3)*e^(1/3)) - (p*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(
1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(3*d^(2/3)*e^(1
/3)) - ((-1)^(2/3)*p*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[-(((-1)^(2/3
)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/(3*d^(2/
3)*e^(1/3)) + (Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*d^(2/3)*
e^(1/3)) + ((-1)^(2/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x
^3)^p])/(3*d^(2/3)*e^(1/3)) - ((-1)^(1/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.16 (sec) , antiderivative size = 1787, normalized size of antiderivative = 1.53

method	result	size
risch	Expression too large to display	1787

input
```
int(ln(c*(e*x^3+d)^p)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2*ln((e*x^3+d)^p)^2-p^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*ln(e*x^3+d)+p
/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*ln((e*x^3+d)^p)+1/2*p^2/(d/e)^(2/3)*ln(x^2-
(d/e)^(1/3)*x+(d/e)^(2/3))*ln(e*x^3+d)-1/2*p/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3
)*x+(d/e)^(2/3))*ln((e*x^3+d)^p)-p^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(d/e)^(1/3)*x-1))*ln(e*x^3+d)+p/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(d/e)^(1/3)*x-1))*ln((e*x^3+d)^p)+1/2*p^2*sum(1/_alpha^2*(2*ln(x-_alp
ha)*ln(e*x^3+d)-e*(1/_alpha^2/e*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*(9*_a
lpha^2*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z
^2+3*_Z*_alpha+3*_alpha^2,index=1))+9*_alpha^2*ln((RootOf(_Z^2+3*_Z*_alpha
+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)
)+6*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z^2+3*_
Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,
index=1))+3*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(
_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_
alpha^2,index=2))+3*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln(
(RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_a
lpha+3*_alpha^2,index=1))+6*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,inde
x=1)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2
+3*_Z*_alpha+3*_alpha^2,index=2))+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,ind
ex=2)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z^2+3*_Z*...
```

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^3}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="fricas")
```

output
```
integral(log((e*x^3 + d)^p*c)^2/x^3, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \int \frac{\log\left(c(d+ex^3)^p\right)^2}{x^3}\,dx$$

input
```
integrate(ln(c*(e*x**3+d)**p)**2/x**3,x)
```

output
```
Integral(log(c*(d + e*x**3)**p)**2/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^3}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="giac")
```

output
```
integrate(log((e*x^3 + d)^p*c)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx=\int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x^3}\,dx$$

input
```
int(log(c*(d + e*x^3)^p)^2/x^3,x)
```

output
```
int(log(c*(d + e*x^3)^p)^2/x^3, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx$$
$$=\frac{-6\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)e\,p^2x^2-12e^{\frac{1}{3}}d^{\frac{5}{3}}\left(\int \frac{\log\left(\left(e\,x^3+d\right)^p c\right)}{e\,x^6+d\,x^3}dx\right)p\,x^2-2e^{\frac{1}{3}}d^{\frac{2}{3}}\log\left(\left(e\,x^3+d\right)^p c\right)^2-6e^{\frac{1}{3}}d^{\frac{2}{3}}\log(}{4e^{\frac{1}{3}}d^{\frac{2}{3}}x^2}$$

input
```
int(log(c*(e*x^3+d)^p)^2/x^3,x)
```

output
```
( - 6*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*e*p**2*x*
*2 - 12*e**(1/3)*d**(2/3)*int(log((d + e*x**3)**p*c)/(d*x**3 + e*x**6),x)*
d*p*x**2 - 2*e**(1/3)*d**(2/3)*log((d + e*x**3)**p*c)**2 - 6*e**(1/3)*d**(
2/3)*log((d + e*x**3)**p*c)*p + 9*log(d**(1/3) + e**(1/3)*x)*e*p**2*x**2 -
 3*log((d + e*x**3)**p*c)*e*p*x**2)/(4*e**(1/3)*d**(2/3)*x**2)
```

3.137 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1328

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^5}\,dx = \text{Too large to display}$$

output
```
-3/2*3^(1/2)*e^(4/3)*p^2*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))
/d^(4/3)-3/2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)/d^(4/3)-1/4*e^(4/3)*p^2*ln(
d^(1/3)+e^(1/3)*x)^2/d^(4/3)-1/2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln(-((-
1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(1/3)
*e^(4/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(
d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/d^(4/3)+1/4*(-1)^(1/3)*e^(4/3)*p^2*ln(d^(1/3
)-(-1)^(1/3)*e^(1/3)*x)^2/d^(4/3)-1/2*(-1)^(2/3)*e^(4/3)*p^2*ln(-(-1)^(2/3
)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3
)*x)/d^(4/3)-1/2*(-1)^(2/3)*e^(4/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*
e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(4/3
)-1/4*(-1)^(2/3)*e^(4/3)*p^2*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)^2/d^(4/3)-1/
2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3)*e^(1
/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(2/3)*e^(4/3)*p^2*ln(-(-1)
^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*
e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(1/3)*e^(4/3)*p^2*ln(d
^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)
/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)+3/4*e^(4/3)*p^2*ln(d^(2/3)-d^(1/3)*e^(1/3
)*x+e^(2/3)*x^2)/d^(4/3)-3/2*e*p*ln(c*(e*x^3+d)^p)/d/x+1/2*e^(4/3)*p*ln(d^
(1/3)+e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(4/3)-1/2*(-1)^(1/3)*e^(4/3)*p*ln(d^(
1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(4/3)+1/2*(-1)^(2/3)*e^(...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.76 (sec) , antiderivative size = 912, normalized size of antiderivative = 0.69

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(d + e*x^3)^p]^2/x^5,x]
```

output
```
(-Log[c*(d + e*x^3)^p]^2 + (e*p*x^3*(9*e*p*x^3*Hypergeometric2F1[2/3, 1, 5
/3, -((e*x^3)/d)] - d^(2/3)*e^(1/3)*p*x*Log[-d^(1/3) - e^(1/3)*x]^2 - 2*d^
(2/3)*e^(1/3)*p*x*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*d^(1/3) - e^(1
/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] - 2*d^(2/3)*e^(1/3)*p*x*Log[-d^(1/3) -
e^(1/3)*x]*Log[(I + Sqrt[3] - ((2*I)*e^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])]
- 6*d*Log[c*(d + e*x^3)^p] + 2*d^(2/3)*e^(1/3)*x*Log[-d^(1/3) - e^(1/3)*x]
*Log[c*(d + e*x^3)^p] - 2*(-1)^(1/3)*d^(2/3)*e^(1/3)*x*Log[-d^(1/3) + (-1)
^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*d^(2/3)*e^(1/3)*x*Lo
g[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 2*d^(2/3)*e^(1/3
)*p*x*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + (-1)^
(1/3)*d^(2/3)*e^(1/3)*p*x*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((-
1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3)
 + (-1)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)
*x))/((-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^
(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)
*e^(1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(2/3)*d^(2/3)*e^(1/3)*p*x
*(Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/
3)*x))/((-1 + (-1)^(2/3))*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1
/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^
(1/3)*x]) + 2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1...
```

Rubi [A] (verified)

Time = 3.32 (sec) , antiderivative size = 1292, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^5}\,dx$$

$$\Big\downarrow \text{2907}$$

$$\frac{3}{2}ep\int \frac{\log\left(c\left(ex^3+d\right)^p\right)}{x^2\left(ex^3+d\right)}dx-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4}$$

$$\Big\downarrow \text{2926}$$

$$\frac{3}{2}ep\int \left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{dx^2}-\frac{ex\log\left(c\left(ex^3+d\right)^p\right)}{d\left(ex^3+d\right)}\right)dx-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4}$$

$\downarrow$ 2009

$$\frac{3}{2}ep\left(-\frac{\sqrt[3]{e}p\log^2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{6d^{4/3}}-\frac{\sqrt[3]{e}p\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{d^{4/3}}-\frac{\sqrt[3]{e}p\log\left(-\frac{\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{3d^{4/3}}-\ldots\right.$$

$$\frac{\log^2\left(c\left(ex^3+d\right)^p\right)}{4x^4}$$

input
```
Int[Log[c*(d + e*x^3)^p]^2/x^5,x]
```

output
```
-1/4*Log[c*(d + e*x^3)^p]^2/x^4 + (3*e*p*(-((Sqrt[3]*e^(1/3)*p*ArcTan[(d^(
1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/d^(4/3)) - (e^(1/3)*p*Log[d^(1/3)
+ e^(1/3)*x])/d^(4/3) - (e^(1/3)*p*Log[d^(1/3) + e^(1/3)*x]^2)/(6*d^(4/3))
 - (e^(1/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)
*x)/((1 - (-1)^(2/3))*d^(1/3)))])/(3*d^(4/3)) + ((-1)^(1/3)*e^(1/3)*p*Log[
((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3)
 - (-1)^(1/3)*e^(1/3)*x])/(3*d^(4/3)) + ((-1)^(1/3)*e^(1/3)*p*Log[d^(1/3)
- (-1)^(1/3)*e^(1/3)*x]^2)/(6*d^(4/3)) - ((-1)^(2/3)*e^(1/3)*p*Log[-(((-1)
^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (
-1)^(2/3)*e^(1/3)*x])/(3*d^(4/3)) - ((-1)^(2/3)*e^(1/3)*p*Log[((-1)^(1/3)*
(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3)
+ (-1)^(2/3)*e^(1/3)*x])/(3*d^(4/3)) - ((-1)^(2/3)*e^(1/3)*p*Log[d^(1/3) +
 (-1)^(2/3)*e^(1/3)*x]^2)/(6*d^(4/3)) + ((-1)^(2/3)*e^(1/3)*p*Log[((-1)^(1
/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(
1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(3*d^(4/3)) - (e
^(1/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^
(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(3*d^(4/3)) + ((-1)^(1/3)*e^(1/3)*p
*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/
3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/(3*d^(4/3)) + (e^(1/3)*p*Log[
d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(2*d^(4/3)) - Log[c*(d + e*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 1954, normalized size of antiderivative = 1.47

method	result	size
risch	Expression too large to display	1954

input
```
int(ln(c*(e*x^3+d)^p)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln((e*x^3+d)^p)^2/x^4-3/2*e*p*ln((e*x^3+d)^p)/d/x-1/2*e*p^2/d/(d/e)^(
1/3)*ln(x+(d/e)^(1/3))*ln(e*x^3+d)+1/2*e*p/d/(d/e)^(1/3)*ln(x+(d/e)^(1/3))
*ln((e*x^3+d)^p)+1/4*e*p^2/d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))
*ln(e*x^3+d)-1/4*e*p/d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*ln((e
*x^3+d)^p)+1/2*e*p^2/d*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/
3)*x-1))*ln(e*x^3+d)-1/2*e*p/d*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(
d/e)^(1/3)*x-1))*ln((e*x^3+d)^p)-3/2*e*p^2/d/(d/e)^(1/3)*ln(x+(d/e)^(1/3))
+3/4*e*p^2/d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+3/2*e*p^2/d*3^(
1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))+3/2*e*p^2*Sum(-1/
3*(ln(x-_alpha)*ln(e*x^3+d)-3*e*(1/6/_alpha^2/e*ln(x-_alpha)^2+1/3*_alpha*
ln(x-_alpha)*(9*_alpha^2*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x
+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))+9*_alpha^2*ln((RootO
f(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3
*_alpha^2,index=2))+6*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*l
n((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*
_alpha+3*_alpha^2,index=1))+3*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,in
dex=2)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z
^2+3*_Z*_alpha+3*_alpha^2,index=2))+3*_alpha*RootOf(_Z^2+3*_Z*_alpha+3*_al
pha^2,index=1)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/R
ootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))+6*_alpha*RootOf(_Z^2+3*_Z*_...
```

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \int \frac{\log\left((ex^3+d)^pc\right)^2}{x^5}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="fricas")
```

output
```
integral(log((e*x^3 + d)^p*c)^2/x^5, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \int \frac{\log\left(c(d+ex^3)^p\right)^2}{x^5}\,dx$$

input
```
integrate(ln(c*(e*x**3+d)**p)**2/x**5,x)
```

output
```
Integral(log(c*(d + e*x**3)**p)**2/x**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^5}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="giac")
```

output
```
integrate(log((e*x^3 + d)^p*c)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \int \frac{\ln\left(c\,(e\,x^3+d)^p\right)^2}{x^5}\,dx$$

input
```
int(log(c*(d + e*x^3)^p)^2/x^5,x)
```

output
```
int(log(c*(d + e*x^3)^p)^2/x^5, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \frac{6\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)e^2p^2x^4 - 24e^{\frac{2}{3}}d^{\frac{7}{3}}\left(\int \frac{\log\left((e\,x^3+d)^p c\right)}{e\,x^8+d\,x^5}dx\right)p\,x^4 - 4e^{\frac{2}{3}}d^{\frac{4}{3}}\log\left((e\,x^3+d)^p\,c\right)^2 - 6e^{\frac{2}{3}}d^{\frac{4}{3}}\log\left((e\right.}{16e^{\frac{2}{3}}d^{\frac{4}{3}}x^4}$$

input
```
int(log(c*(e*x^3+d)^p)^2/x^5,x)
```

output
```
(6*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*e**2*p**2*x*
*4 - 24*e**(2/3)*d**(1/3)*int(log((d + e*x**3)**p*c)/(d*x**5 + e*x**8),x)*
d**2*p*x**4 - 4*e**(2/3)*d**(1/3)*log((d + e*x**3)**p*c)**2*d - 6*e**(2/3)
*d**(1/3)*log((d + e*x**3)**p*c)*d*p - 18*e**(2/3)*d**(1/3)*e*p**2*x**3 +
9*log(d**(1/3) + e**(1/3)*x)*e**2*p**2*x**4 - 3*log((d + e*x**3)**p*c)*e**
2*p*x**4)/(16*e**(2/3)*d**(1/3)*d*x**4)
```

3.138 $\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 164

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p} - \frac{2d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p} + \frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\text{ExpIntegralEi}\left(\frac{3\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p}$$

output
```
1/3*d^2*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(1/p))-2/
3*d*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(2/p))+1/
3*(e*x^3+d)^3*Ei(3*ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(3/p))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-3/p}\left(d^2\left(c(d+ex^3)^p\right)^{2/p}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)-(d+ex^3)\left(2d\left(c(d+ex\right.\right.\right.}{3e^3p}$$

input

```
Integrate[x^8/Log[c*(d + e*x^3)^p],x]
```

output

```
((d + e*x^3)*(d^2*(c*(d + e*x^3)^p)^(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^
p]/p] - (d + e*x^3)*(2*d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(
d + e*x^3)^p])/p] - (d + e*x^3)*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p])
))/(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2904, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}dx$$

$$\Big\downarrow \; 2904$$

$$\frac{1}{3}\int \frac{x^6}{\log\left(c\left(ex^3+d\right)^p\right)}dx^3$$

$$\Big\downarrow \; 2846$$

$$\frac{1}{3}\int \left(\frac{d^2}{e^2\log\left(c\left(ex^3+d\right)^p\right)}-\frac{2\left(ex^3+d\right)d}{e^2\log\left(c\left(ex^3+d\right)^p\right)}+\frac{\left(ex^3+d\right)^2}{e^2\log\left(c\left(ex^3+d\right)^p\right)}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(ex^3+d)^p)}{p}\right)}{e^3p}+\frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\text{ExpIntegralE}}{e^3p}\right.$$

input
```
Int[x^8/Log[c*(d + e*x^3)^p],x]
```

output
```
((d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(e^3*p*(c*(d + e*
x^3)^p)^p^(-1)) - (2*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p]
)/p])/(e^3*p*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(3*Lo
g[c*(d + e*x^3)^p])/p])/(e^3*p*(c*(d + e*x^3)^p)^(3/p)))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.29 (sec) , antiderivative size = 823, normalized size of antiderivative = 5.02

method	result	size
risch	Expression too large to display	823

input
```
int(x^8/ln(c*(e*x^3+d)^p),x,method=_RETURNVERBOSE)
```

output
```
-1/3/e^3/p*(e*x^3+d)^3*c^(-3/p)*((e*x^3+d)^p)^(-3/p)*exp(3/2*I*Pi*csgn(I*c
*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+c
sgn(I*(e*x^3+d)^p))/p)*Ei(1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*c
sgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn
(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+
2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)-1/3/e^3*d^2/p*(e*x^3+d)*c^(-
1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e
*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1
,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*c
sgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)
^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p
*ln(e*x^3+d))/p)+2/3/e^3*d/p*(e*x^3+d)^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp
(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*
(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x
^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+
d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*
csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{\log \left(c\left(d+ex^3\right)^p\right)}\,dx$$
$$= \frac{c^{\frac{2}{p}} d^2 \,\mathrm{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right) - 2\,c^{\left(\frac{1}{p}\right)} d \,\mathrm{log\_integral}\left(\left(e^2x^6+2\,dex^3+d^2\right)c^{\frac{2}{p}}\right) + \mathrm{log\_integral}\left(\left(e\right.\right.}{3\,c^{\frac{3}{p}} e^3 p}$$

input
```
integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
1/3*(c^(2/p)*d^2*log_integral((e*x^3 + d)*c^(1/p)) - 2*c^(1/p)*d*log_integ
ral((e^2*x^6 + 2*d*e*x^3 + d^2)*c^(2/p)) + log_integral((e^3*x^9 + 3*d*e^2
*x^6 + 3*d^2*e*x^3 + d^3)*c^(3/p)))/(c^(3/p)*e^3*p)
```

Sympy [F]

$$\int \frac{x^8}{\log(c(d+ex^3)^p)}\,dx = \int \frac{x^8}{\log(c(d+ex^3)^p)}\,dx$$

input
```
integrate(x**8/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(x**8/log(c*(d + e*x**3)**p), x)
```

Maxima [F]

$$\int \frac{x^8}{\log(c(d+ex^3)^p)}\,dx = \int \frac{x^8}{\log((ex^3+d)^p c)}\,dx$$

input
```
integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(x^8/log((e*x^3 + d)^p*c), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{d^2\operatorname{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)}{3\,c^{\left(\frac{1}{p}\right)}e^3p} - \frac{2\,d\operatorname{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(ex^3+d\right)\right)}{3\,c^{\frac{2}{p}}e^3p} + \frac{\operatorname{Ei}\left(\frac{3\,\log(c)}{p}+3\,\log\left(ex^3+d\right)\right)}{3\,c^{\frac{3}{p}}e^3p}$$

input
```
integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
1/3*d^2*Ei(log(c)/p + log(e*x^3 + d))/(c^(1/p)*e^3*p) - 2/3*d*Ei(2*log(c)/
p + 2*log(e*x^3 + d))/(c^(2/p)*e^3*p) + 1/3*Ei(3*log(c)/p + 3*log(e*x^3 +
d))/(c^(3/p)*e^3*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^8}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

input
```
int(x^8/log(c*(d + e*x^3)^p),x)
```

output
```
int(x^8/log(c*(d + e*x^3)^p), x)
```

Reduce [F]

$$\int \frac{x^8}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{x^8}{\log \left(\left(e\,x^3+d\right)^p c\right)}dx$$

input
```
int(x^8/log(c*(e*x^3+d)^p),x)
```

output
```
int(x**8/log((d + e*x**3)**p*c),x)
```

3.139 $\int \frac{x^5}{\log(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^2p} + \frac{(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)}{3e^2p}$$

output
```
-1/3*d*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^2/p/((c*(e*x^3+d)^p)^(1/p))+1/3
*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^2/p/((c*(e*x^3+d)^p)^(2/p))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-2/p}\left(d\left(c(d+ex^3)^p\right)^{\frac{1}{p}}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)-(d+ex^3)\,\text{ExpIntegralE}\right.}{3e^2p}$$

input
```
Integrate[x^5/Log[c*(d + e*x^3)^p],x]
```

output
```
-1/3*((d + e*x^3)*(d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[Log[c*(d + e*x
^3)^p]/p] - (d + e*x^3)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]))/(e^2*p
*(c*(d + e*x^3)^p)^(2/p))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2904, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{3}\int \frac{x^3}{\log\left(c\left(ex^3+d\right)^p\right)}dx^3$$

$$\Big\downarrow \text{2846}$$

$$\frac{1}{3}\int \left(\frac{ex^3+d}{e\log\left(c\left(ex^3+d\right)^p\right)}-\frac{d}{e\log\left(c\left(ex^3+d\right)^p\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{e^2p}-\frac{d\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{ExpIntegralE}}{e^2p}\right.$$

input
```
Int[x^5/Log[c*(d + e*x^3)^p],x]
```

output
```
(-((d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(e^2*p*(c*(d + e*
x^3)^p)^p^(-1))) + ((d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p
])/(e^2*p*(c*(d + e*x^3)^p)^(2/p)))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.12 (sec) , antiderivative size = 547, normalized size of antiderivative = 5.11

method	result
risch	$-\frac{(e\,x^3+d)^2 c^{-\frac{2}{p}} \left((e\,x^3+d)^p\right)^{-\frac{2}{p}} \mathrm{e}^{\frac{i\pi\ \mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}(ic)\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}\left(i\left(e\,x^3+d\right)^p\right)\right)}{p}} \mathrm{exp}}{}$

input
```
int(x^5/ln(c*(e*x^3+d)^p),x,method=_RETURNVERBOSE)
```

output
```
-1/3/e^2/p*(e*x^3+d)^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*
x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(
I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*
(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*
Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+
2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)+1/3/e^2*d/p*(e*x^3+d)*c^(-1/p)*((e*x
^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)
+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3
+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x
^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi
*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+
d))/p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{c^{\left(\frac{1}{p}\right)} d \operatorname{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right) - \operatorname{log\_integral}\left(\left(e^2x^6+2\,dex^3+d^2\right)c^{\frac{2}{p}}\right)}{3\,c^{\frac{2}{p}}e^2p}$$

input
```
integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
-1/3*(c^(1/p)*d*log_integral((e*x^3 + d)*c^(1/p)) - log_integral((e^2*x^6
+ 2*d*e*x^3 + d^2)*c^(2/p)))/(c^(2/p)*e^2*p)
```

Sympy [F]

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
integrate(x**5/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(x**5/log(c*(d + e*x**3)**p), x)
```

Maxima [F]

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^5}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

input
```
integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(x^5/log((e*x^3 + d)^p*c), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{d\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)}{3\,c^{\left(\frac{1}{p}\right)}e^2p}+\frac{\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(ex^3+d\right)\right)}{3\,c^{\frac{2}{p}}e^2p}$$

input
```
integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
-1/3*d*Ei(log(c)/p + log(e*x^3 + d))/(c^(1/p)*e^2*p) + 1/3*Ei(2*log(c)/p +
 2*log(e*x^3 + d))/(c^(2/p)*e^2*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^5}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

input
```
int(x^5/log(c*(d + e*x^3)^p),x)
```

output
```
int(x^5/log(c*(d + e*x^3)^p), x)
```

Reduce [F]

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^5}{\log\left(\left(e\,x^3+d\right)^p c\right)}dx$$

input
```
int(x^5/log(c*(e*x^3+d)^p),x)
```

output
```
int(x**5/log((d + e*x**3)**p*c),x)
```

3.140 $\int \frac{x^2}{\log(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3ep}$$

output
```
1/3*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e/p/((c*(e*x^3+d)^p)^(1/p))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3ep}$$

input
```
Integrate[x^2/Log[c*(d + e*x^3)^p],x]
```

output
```
((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^
p)^p^(-1))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2904, 2836, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}dx$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{1}{3}\int \frac{1}{\log\left(c\left(ex^3+d\right)^p\right)}dx^3$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \frac{1}{\log(c(ex^3+d)^p)}d\left(ex^3+d\right)}{3e}$$

$$\Big\downarrow \text{ 2737}$$

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\int \frac{\left(c(ex^3+d)^p\right)^{\frac{1}{p}}}{x^3}d\log\left(c\left(ex^3+d\right)^p\right)}{3ep}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(ex^3+d)^p)}{p}\right)}{3ep}$$

input
```
Int[x^2/Log[c*(d + e*x^3)^p],x]
```

output
```
((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^
p)^p^(-1))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.91 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.33

method	result
risch	$-\frac{(e\,x^3+d)\left((e\,x^3+d)^p\right)^{-\frac{1}{p}} c^{-\frac{1}{p}} \mathrm{e}^{\frac{i\pi\,\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}(ic)\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}\left(i\left(e\,x^3+d\right)^p\right)\right)}{2p}} \mathrm{expI}}{}$

input
```
int(x^2/ln(c*(e*x^3+d)^p),x,method=_RETURNVERBOSE)
```

output
```
-1/3/e/p*(e*x^3+d)*((e*x^3+d)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*
x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(
I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*
c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-
I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c
)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{\log(c(d+ex^3)^p)}\,dx = \frac{\text{log\_integral}\left((ex^3+d)c^{\left(\frac{1}{p}\right)}\right)}{3\,c^{\left(\frac{1}{p}\right)}ep}$$

input
```
integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
1/3*log_integral((e*x^3 + d)*c^(1/p))/(c^(1/p)*e*p)
```

Sympy [F]

$$\int \frac{x^2}{\log(c(d+ex^3)^p)}\,dx = \int \frac{x^2}{\log(c(d+ex^3)^p)}\,dx$$

input
```
integrate(x**2/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(x**2/log(c*(d + e*x**3)**p), x)
```

Maxima [F]

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(ex^3+d\right)^pc\right)}\,dx$$

input
```
integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(x^2/log((e*x^3 + d)^p*c), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\frac{\operatorname{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)}{3\,c^{\left(\frac{1}{p}\right)}ep}$$

input
```
integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
1/3*Ei(log(c)/p + log(e*x^3 + d))/(c^(1/p)*e*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^2}{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)}\,dx$$

input
```
int(x^2/log(c*(d + e*x^3)^p),x)
```

output
```
int(x^2/log(c*(d + e*x^3)^p), x)
```

Reduce [F]

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{3\left(\int \frac{x^5}{\log((e\,x^3+d)^p c)d+\log((e\,x^3+d)^p c)e\,x^3}dx\right)e^2p+\log\left(\log\left(\left(e\,x^3+d\right)^p c\right)\right)d}{3ep}$$

input
```
int(x^2/log(c*(e*x^3+d)^p),x)
```

output
```
(3*int(x**5/(log((d + e*x**3)**p*c)*d + log((d + e*x**3)**p*c)*e*x**3),x)*
e**2*p + log(log((d + e*x**3)**p*c))*d)/(3*e*p)
```

3.141 $\int \frac{1}{x \log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x \log\left(c\left(d+ex^3\right)^p\right)} dx = \text{Int}\left(\frac{1}{x \log\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x/ln(c*(e*x^3+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x \log\left(c\left(d+ex^3\right)^p\right)} dx$$

input
```
Integrate[1/(x*Log[c*(d + e*x^3)^p]),x]
```

output
```
Integrate[1/(x*Log[c*(d + e*x^3)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

$$\Big\downarrow \text{ 2910}$$

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)}dx$$

input
```
Int[1/(x*Log[c*(d + e*x^3)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \ln \left(c \left(e \,x^{3}+d \right)^{p}\right)}d x$$

input
```
int(1/x/ln(c*(e*x^3+d)^p),x)
```

output
```
int(1/x/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log \left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log \left(\left(ex^3+d\right)^pc\right)}\,dx$$

input
```
integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
integral(1/(x*log((e*x^3 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 8.89 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\log \left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log \left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
integrate(1/x/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(1/(x*log(c*(d + e*x**3)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log \left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log \left(\left(ex^3+d\right)^pc\right)}\,dx$$

input
```
integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(1/(x*log((e*x^3 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log \left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log \left(\left(ex^3+d\right)^pc\right)}\,dx$$

input
```
integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
integrate(1/(x*log((e*x^3 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log \left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\,\ln \left(c\,{\left(e\,x^3+d\right)}^p\right)}\,dx$$

input
```
int(1/(x*log(c*(d + e*x^3)^p)),x)
```

output
```
int(1/(x*log(c*(d + e*x^3)^p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.33

$$\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{3\left(\int \frac{1}{\log\left(\left(e\,x^3+d\right)^p c\right)dx+\log\left(\left(e\,x^3+d\right)^p c\right)e\,x^4}dx\right)dp+\log\left(\log\left(\left(e\,x^3+d\right)^p c\right)\right)}{3p}$$

input
```
int(1/x/log(c*(e*x^3+d)^p),x)
```

output
```
(3*int(1/(log((d + e*x**3)**p*c)*d*x + log((d + e*x**3)**p*c)*e*x**4),x)*d
*p + log(log((d + e*x**3)**p*c)))/(3*p)
```

3.142 $\int \frac{1}{x^4 \log(c(d+ex^3)^p)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\text{Int}\left(\frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/x^4/ln(c*(e*x^3+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

input
```
Integrate[1/(x^4*Log[c*(d + e*x^3)^p]),x]
```

output
```
Integrate[1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$
$$\Big\downarrow \; 2910$$
$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)}dx$$

input
```
Int[1/(x^4*Log[c*(d + e*x^3)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \ln \left(c \left(e \,x^3+d\right)^p\right)}dx$$

input
```
int(1/x^4/ln(c*(e*x^3+d)^p),x)
```

output
```
int(1/x^4/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log \left(c \left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log \left(\left(ex^3+d\right)^p c\right)}\,dx$$

input
```
integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
integral(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 32.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 \log \left(c \left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log \left(c \left(d+ex^3\right)^p\right)}\,dx$$

input
```
integrate(1/x**4/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(1/(x**4*log(c*(d + e*x**3)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x^4 \log \left((e x^3+d)^p c\right)} \, dx$$

input
```
integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x^4 \log \left((e x^3+d)^p c\right)} \, dx$$

input
```
integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
integrate(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x^4 \, \ln \left(c \, {\left(e \, x^3+d\right)}^p\right)} \, dx$$

input
```
int(1/(x^4*log(c*(d + e*x^3)^p)),x)
```

output
```
int(1/(x^4*log(c*(d + e*x^3)^p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{\log \left(\left(e\,x^3+d\right)^p c\right) x^4}dx$$

input
```
int(1/x^4/log(c*(e*x^3+d)^p),x)
```

output
```
int(1/(log((d + e*x**3)**p*c)*x**4),x)
```

3.143 $\int \frac{x^3}{\log(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \text{Int}\left(\frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

output
```
Defer(Int)(x^3/ln(c*(e*x^3+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
Integrate[x^3/Log[c*(d + e*x^3)^p],x]
```

output
```
Integrate[x^3/Log[c*(d + e*x^3)^p], x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$
$$\Big\downarrow \text{2910}$$
$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}dx$$

input
```
Int[x^3/Log[c*(d + e*x^3)^p],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)}dx$$

input
```
int(x^3/ln(c*(e*x^3+d)^p),x)
```

output
```
int(x^3/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{x^3}{\log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

input
```
integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
integral(x^3/log((e*x^3 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 10.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{x^3}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

input
```
integrate(x**3/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(x**3/log(c*(d + e*x**3)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{x^3}{\log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

input
```
integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(x^3/log((e*x^3 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{x^3}{\log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

input
```
integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
integrate(x^3/log((e*x^3 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 25.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{x^3}{\ln \left(c \left(e\,x^3+d\right)^p\right)} \, dx$$

input
```
int(x^3/log(c*(d + e*x^3)^p),x)
```

output
```
int(x^3/log(c*(d + e*x^3)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log \left(c \left(d+e x^3\right)^p\right)}\, dx=\int \frac{x^3}{\log \left(\left(e\, x^3+d\right)^p c\right)}dx$$

input
```
int(x^3/log(c*(e*x^3+d)^p),x)
```

output
```
int(x**3/log((d + e*x**3)**p*c),x)
```

3.144 $\int \frac{x}{\log(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \text{Int}\left(\frac{x}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

output
```
Defer(Int)(x/ln(c*(e*x^3+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
Integrate[x/Log[c*(d + e*x^3)^p],x]
```

output
```
Integrate[x/Log[c*(d + e*x^3)^p], x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx$$
$$\Big\downarrow \text{2910}$$
$$\int \frac{x}{\log \left(c \left(d+e x^3\right)^p\right)}dx$$

input
```
Int[x/Log[c*(d + e*x^3)^p],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\ln \left(c \left(e \,x^{3}+d \right)^{p}\right)}dx$$

input
```
int(x/ln(c*(e*x^3+d)^p),x)
```

output
```
int(x/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

input
```
integrate(x/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
integral(x/log((e*x^3 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
integrate(x/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(x/log(c*(d + e*x**3)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log \left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{x}{\log \left((ex^3+d)^p c\right)} dx$$

input
```
integrate(x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(x/log((e*x^3 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log \left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{x}{\log \left((ex^3+d)^p c\right)} dx$$

input
```
integrate(x/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
integrate(x/log((e*x^3 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 25.64 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log \left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{x}{\ln \left(c\left(e\,x^3+d\right)^p\right)} dx$$

input
```
int(x/log(c*(d + e*x^3)^p),x)
```

output
```
int(x/log(c*(d + e*x^3)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(\left(e\,x^3+d\right)^p c\right)}dx$$

input
```
int(x/log(c*(e*x^3+d)^p),x)
```

output
```
int(x/log((d + e*x**3)**p*c),x)
```

3.145 $\int \frac{1}{\log(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\text{Int}\left(\frac{1}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/ln(c*(e*x^3+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
Integrate[Log[c*(d + e*x^3)^p]^(-1),x]
```

output
```
Integrate[Log[c*(d + e*x^3)^p]^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2902}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx$$
$$\Big\downarrow \text{2902}$$
$$\int \frac{1}{\log \left(c \left(d+e x^3\right)^p\right)}dx$$

input
```
Int[Log[c*(d + e*x^3)^p]^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2902
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Unintegrable[(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d,
 e, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln \left(c \left(e \,x^{3}+d \right)^{p}\right)}dx$$

input
```
int(1/ln(c*(e*x^3+d)^p),x)
```

output
```
int(1/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{\log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

input
```
integrate(1/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
integral(1/log((e*x^3 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 4.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

input
```
integrate(1/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(1/log(c*(d + e*x**3)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(ex^3+d\right)^pc\right)}\,dx$$

input
```
integrate(1/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(1/log((e*x^3 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(ex^3+d\right)^pc\right)}\,dx$$

input
```
integrate(1/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
integrate(1/log((e*x^3 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 25.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

input
```
int(1/log(c*(d + e*x^3)^p),x)
```

output
```
int(1/log(c*(d + e*x^3)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log (c (d+ex^3)^p)} \, dx = \int \frac{1}{\log ((e\,x^3+d)^p\,c)}dx$$

input
```
int(1/log(c*(e*x^3+d)^p),x)
```

output
```
int(1/log((d + e*x**3)**p*c),x)
```

3.146 $\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\text{Int}\left(\frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/x^2/ln(c*(e*x^3+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

input
```
Integrate[1/(x^2*Log[c*(d + e*x^3)^p]),x]
```

output
```
Integrate[1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$
$$\Big\downarrow \text{2910}$$
$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)}dx$$

input
```
Int[1/(x^2*Log[c*(d + e*x^3)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \ln \left(c \left(e \,x^{3}+d \right)^{p}\right)}d x$$

input
```
int(1/x^2/ln(c*(e*x^3+d)^p),x)
```

output
```
int(1/x^2/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

input
```
integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
integral(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 17.85 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

input
```
integrate(1/x**2/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(1/(x**2*log(c*(d + e*x**3)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \log \left(\left(ex^3+d\right)^p c\right)} \, dx$$

input
```
integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \log \left(\left(ex^3+d\right)^p c\right)} \, dx$$

input
```
integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
integrate(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \, \ln \left(c \left(e\, x^3+d\right)^p\right)} \, dx$$

input
```
int(1/(x^2*log(c*(d + e*x^3)^p)),x)
```

output
```
int(1/(x^2*log(c*(d + e*x^3)^p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{\log \left(\left(e\,x^3+d\right)^p c\right) x^2}dx$$

input
```
int(1/x^2/log(c*(e*x^3+d)^p),x)
```

output
```
int(1/(log((d + e*x**3)**p*c)*x**2),x)
```

3.147 $\int \frac{1}{x^3 \log(c(d+ex^3)^p)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x^3/ln(c*(e*x^3+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

input
```
Integrate[1/(x^3*Log[c*(d + e*x^3)^p]),x]
```

output
```
Integrate[1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$
$$\Big\downarrow \text{ 2910}$$
$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)}dx$$

input
```
Int[1/(x^3*Log[c*(d + e*x^3)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \ln \left(c \left(e\,x^3+d\right)^p\right)}dx$$

input
```
int(1/x^3/ln(c*(e*x^3+d)^p),x)
```

output
```
int(1/x^3/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

input
```
integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

output
```
integral(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 23.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
integrate(1/x**3/ln(c*(e*x**3+d)**p),x)
```

output
```
Integral(1/(x**3*log(c*(d + e*x**3)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x^3\log\left(\left(ex^3+d\right)^pc\right)}\,dx$$

input
```
integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

output
```
integrate(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x^3\log\left(\left(ex^3+d\right)^pc\right)}\,dx$$

input
```
integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
```

output
```
integrate(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x^3\,\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)}\,dx$$

input
```
int(1/(x^3*log(c*(d + e*x^3)^p)),x)
```

output
```
int(1/(x^3*log(c*(d + e*x^3)^p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{\log \left(\left(e\, x^3+d\right)^p c\right) x^3} dx$$

input
```
int(1/x^3/log(c*(e*x^3+d)^p),x)
```

output
```
int(1/(log((d + e*x**3)**p*c)*x**3),x)
```

3.148 $\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 195

$$\begin{aligned}\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx &= \frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^3p^2}\\ &\quad-\frac{4d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\operatorname{ExpIntegralEi}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)}{3e^3p^2}\\ &\quad+\frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\operatorname{ExpIntegralEi}\left(\frac{3\log(c(d+ex^3)^p)}{p}\right)}{e^3p^2}\\ &\quad-\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}\end{aligned}$$

output
```
1/3*d^2*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(1/p))-
4/3*d*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(2/p)
)+(e*x^3+d)^3*Ei(3*ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(3/p))-1/
3*x^6*(e*x^3+d)/e/p/ln(c*(e*x^3+d)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.49

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$
$$= \frac{\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-3/p}\left(-e^2px^6\left(c(d+ex^3)^p\right)^{3/p}+d^2\left(c(d+ex^3)^p\right)^{2/p}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)\right.}{}$$

input
```
Integrate[x^8/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
((d + e*x^3)*(-(e^2*p*x^6*(c*(d + e*x^3)^p)^(3/p)) + d^2*(c*(d + e*x^3)^p)
^(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 4*d*(d
 + e*x^3)*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/
p]*Log[c*(d + e*x^3)^p] + 3*d^2*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*
Log[c*(d + e*x^3)^p] + 6*d*e*x^3*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]
*Log[c*(d + e*x^3)^p] + 3*e^2*x^6*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p
]*Log[c*(d + e*x^3)^p]))/(3*e^3*p^2*(c*(d + e*x^3)^p)^(3/p)*Log[c*(d + e*x
^3)^p])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.58, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2904, 2847, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$
$$\Big\downarrow \text{2904}$$
$$\frac{1}{3}\int \frac{x^6}{\log^2\left(c\left(ex^3+d\right)^p\right)}dx^3$$
$$\Big\downarrow \text{2847}$$

$$\frac{1}{3}\left(\frac{2d\int \frac{x^3}{\log(c(ex^3+d)^p)}dx^3}{ep}+\frac{3\int \frac{x^6}{\log(c(ex^3+d)^p)}dx^3}{p}-\frac{x^6(d+ex^3)}{ep\log\left(c\left(d+ex^3\right)^p\right)}\right)$$

$\Big\downarrow$ 2846

$$\frac{1}{3}\left(\frac{3\int \left(\frac{d^2}{e^2\log(c(ex^3+d)^p)}-\frac{2(ex^3+d)d}{e^2\log(c(ex^3+d)^p)}+\frac{(ex^3+d)^2}{e^2\log(c(ex^3+d)^p)}\right)dx^3}{p}+\frac{2d\int \left(\frac{ex^3+d}{e\log(c(ex^3+d)^p)}-\frac{d}{e\log(c(ex^3+d)^p)}\right)dx^3}{ep}-\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{3}\left(\frac{3\left(\frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{e^3p}+\frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\text{ExpIntegralEi}\left(\frac{3\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{e^3p}-\right.}{p}\right.$$

input
```
Int[x^8/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
((2*d*(-((d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(e^2*p*(c*(
d + e*x^3)^p)^p^(-1))) + ((d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)
^p])/p])/(e^2*p*(c*(d + e*x^3)^p)^(2/p))))/(e*p) + (3*((d^2*(d + e*x^3)*Ex
pIntegralEi[Log[c*(d + e*x^3)^p]/p])/(e^3*p*(c*(d + e*x^3)^p)^p^(-1)) - (2
*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p])/(e^3*p*(c*(d +
 e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/
p])/(e^3*p*(c*(d + e*x^3)^p)^(3/p))))/p - (x^6*(d + e*x^3))/(e*p*Log[c*(d
+ e*x^3)^p]))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.42 (sec) , antiderivative size = 2564, normalized size of antiderivative = 13.15

method	result	size
risch	Expression too large to display	2564

input
```
int(x^8/ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/p/e*x^6*(e*x^3+d)/(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I
*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x
^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p
))-1/3/p^2/e^2*d^2*((e*x^3+d)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*
x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(
I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*
c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-
I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c
)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*x^3-1/3/p^2/e^3*d^3*((e*x^3+d)^p)^
(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)
+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3
+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x
^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi
*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+
d))/p)-1/p^2*((e*x^3+d)^p)^(-3/p)*c^(-3/p)*exp(3/2*I*Pi*csgn(I*c*(e*x^3+d)
^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x
^3+d)^p))/p)*Ei(1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e
*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi
*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*
ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*x^9-3/p^2/e*((e*x^3+d)^p)^(-3/p)*c^...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.08

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx =$$

$$-\frac{4\left(dp\log\left(ex^3+d\right)+d\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left(\left(e^2x^6+2\,dex^3+d^2\right)c^{\frac{2}{p}}\right)-\left(d^2p\log\left(ex^3+d\right)+d^2\operatorname{lo}\right.}{}$$

input
```
integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
-1/3*(4*(d*p*log(e*x^3 + d) + d*log(c))*c^(1/p)*log_integral((e^2*x^6 + 2*
d*e*x^3 + d^2)*c^(2/p)) - (d^2*p*log(e*x^3 + d) + d^2*log(c))*c^(2/p)*log_
integral((e*x^3 + d)*c^(1/p)) + (e^3*p*x^9 + d*e^2*p*x^6)*c^(3/p) - 3*(p*l
og(e*x^3 + d) + log(c))*log_integral((e^3*x^9 + 3*d*e^2*x^6 + 3*d^2*e*x^3
+ d^3)*c^(3/p)))/((e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c))*c^(3/p))
```

Sympy [F]

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(x**8/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(x**8/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F]

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^8}{\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

input
```
integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^9 + d*x^6)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((3*
e*x^8 + 2*d*x^5)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(193) = 386$.

Time = 0.15 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.50

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx =$$

$$-\frac{1}{3}d^2\left(\frac{(ex^3+d)p}{e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)}-\frac{p\text{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)\log\left(ex^3+d\right)}{\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}-\frac{\text{Ei}\left(\frac{\log(c)}{p}+\log\right.}{\left(e^3p^3\log\left(ex^3+\right.\right.}\right.$$

$$-\frac{\left(ex^3+d\right)^3p}{3\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)}+\frac{2\left(ex^3+d\right)^2dp}{3\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)}$$

$$-\frac{4\,dp\text{Ei}\left(\frac{2\log(c)}{p}+2\,\log\left(ex^3+d\right)\right)\log\left(ex^3+d\right)}{3\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\frac{2}{p}}}$$

$$+\frac{p\text{Ei}\left(\frac{3\log(c)}{p}+3\,\log\left(ex^3+d\right)\right)\log\left(ex^3+d\right)}{\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\frac{3}{p}}}$$

$$-\frac{4\,d\text{Ei}\left(\frac{2\log(c)}{p}+2\,\log\left(ex^3+d\right)\right)\log\left(c\right)}{3\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\frac{2}{p}}}+\frac{\text{Ei}\left(\frac{3\log(c)}{p}+3\,\log\left(ex^3+d\right)\right)\log\left(c\right)}{\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\frac{3}{p}}}$$

input
```
integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
-1/3*d^2*((e*x^3 + d)*p/(e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c)) - p*Ei(l
og(c)/p + log(e*x^3 + d))*log(e*x^3 + d)/((e^3*p^3*log(e*x^3 + d) + e^3*p^
2*log(c))*c^(1/p)) - Ei(log(c)/p + log(e*x^3 + d))*log(c)/((e^3*p^3*log(e*
x^3 + d) + e^3*p^2*log(c))*c^(1/p))) - 1/3*(e*x^3 + d)^3*p/(e^3*p^3*log(e*
x^3 + d) + e^3*p^2*log(c)) + 2/3*(e*x^3 + d)^2*d*p/(e^3*p^3*log(e*x^3 + d)
 + e^3*p^2*log(c)) - 4/3*d*p*Ei(2*log(c)/p + 2*log(e*x^3 + d))*log(e*x^3 +
 d)/((e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c))*c^(2/p)) + p*Ei(3*log(c)/p
+ 3*log(e*x^3 + d))*log(e*x^3 + d)/((e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(
c))*c^(3/p)) - 4/3*d*Ei(2*log(c)/p + 2*log(e*x^3 + d))*log(c)/((e^3*p^3*lo
g(e*x^3 + d) + e^3*p^2*log(c))*c^(2/p)) + Ei(3*log(c)/p + 3*log(e*x^3 + d)
)*log(c)/((e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c))*c^(3/p))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^8}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

input
```
int(x^8/log(c*(d + e*x^3)^p)^2,x)
```

output
```
int(x^8/log(c*(d + e*x^3)^p)^2, x)
```

Reduce [F]

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^8}{\log\left(\left(e\,x^3+d\right)^p c\right)^2}dx$$

input
```
int(x^8/log(c*(e*x^3+d)^p)^2,x)
```

output
```
int(x**8/log((d + e*x**3)**p*c)**2,x)
```

3.149 $\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 141

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\,\text{ExpIntegralEi}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^2p^2} + \frac{2(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\,\text{ExpIntegralEi}\left(\frac{2\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^2p^2} - \frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

output
```
-1/3*d*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^2/p^2/((c*(e*x^3+d)^p)^(1/p))+2
/3*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^2/p^2/((c*(e*x^3+d)^p)^(2/p))-1
/3*x^3*(e*x^3+d)/e/p/ln(c*(e*x^3+d)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx =$$
$$-\frac{\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-2/p}\left(epx^3\left(c(d+ex^3)^p\right)^{2/p}+d\left(c(d+ex^3)^p\right)^{\frac{1}{p}}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)\text{lo}\cdots\right.}{3e^2p^2\log\left(c\left(d+ex^3\right)^{p}\right.\cdots}$$

input
```
Integrate[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
-1/3*((d + e*x^3)*(e*p*x^3*(c*(d + e*x^3)^p)^(2/p) + d*(c*(d + e*x^3)^p)^p
^(-1)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 2*(d +
e*x^3)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]*Log[c*(d + e*x^3)^p]))/(e
^2*p^2*(c*(d + e*x^3)^p)^(2/p)*Log[c*(d + e*x^3)^p])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.37, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2904, 2847, 2836, 2737, 2609, 2846, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}dx$$

$$\Big\downarrow \; 2904$$

$$\frac{1}{3}\int \frac{x^3}{\log^2\left(c\left(ex^3+d\right)^p\right)}dx^3$$

$$\Big\downarrow \; 2847$$

$$\frac{1}{3}\left(\frac{d\int \frac{1}{\log(c(ex^3+d)^p)}dx^3}{ep}+\frac{2\int \frac{x^3}{\log(c(ex^3+d)^p)}dx^3}{p}-\frac{x^3\left(d+ex^3\right)}{ep\log\left(c\left(d+ex^3\right)^p\right)}\right)$$

$$\Big\downarrow \; 2836$$

$$\frac{1}{3}\left(\frac{d\int \frac{1}{\log(c(ex^3+d)^p)}d(ex^3+d)}{e^2p}+\frac{2\int \frac{x^3}{\log(c(ex^3+d)^p)}dx^3}{p}-\frac{x^3(d+ex^3)}{ep\log\left(c\left(d+ex^3\right)^p\right)}\right)$$

$\big\downarrow$ 2737

$$\frac{1}{3}\left(\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\int \frac{\left(c(ex^3+d)^p\right)^{\frac{1}{p}}}{x^3}d\log\left(c(ex^3+d)^p\right)}{e^2p^2}+\frac{2\int \frac{x^3}{\log(c(ex^3+d)^p)}dx^3}{p}-\frac{x^3(d+ex^3)}{ep\log\left(c\left(d+ex^3\right)^p\right.}\right.$$

$\big\downarrow$ 2609

$$\frac{1}{3}\left(\frac{2\int \frac{x^3}{\log(c(ex^3+d)^p)}dx^3}{p}+\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(ex^3+d)^p)}{p}\right)}{e^2p^2}-\frac{x^3(d+ex^3)}{ep\log\left(c\left(d+ex^3\right)^p\right)}\right)$$

$\big\downarrow$ 2846

$$\frac{1}{3}\left(\frac{2\int \left(\frac{ex^3+d}{e\log(c(ex^3+d)^p)}-\frac{d}{e\log(c(ex^3+d)^p)}\right)dx^3}{p}+\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(ex^3+d)^p)}{p}\right)}{e^2p^2}-\right.$$

$\big\downarrow$ 2009

$$\frac{1}{3}\left(\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(ex^3+d)^p)}{p}\right)}{e^2p^2}+\frac{2\left(\frac{(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(\right.}{}\right.}{e^2p}\right.$$

input
```
Int[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
((d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(e^2*p^2*(c*(d + e*
x^3)^p)^p^(-1)) + (2*(-((d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/
p])/(e^2*p*(c*(d + e*x^3)^p)^p^(-1))) + ((d + e*x^3)^2*ExpIntegralEi[(2*Lo
g[c*(d + e*x^3)^p])/p])/(e^2*p*(c*(d + e*x^3)^p)^(2/p))))/p - (x^3*(d + e*
x^3))/(e*p*Log[c*(d + e*x^3)^p]))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2846
```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

rule 2847
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Simp[(q + 1)/(b*n*(p + 1))   Int[(
f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Simp[q*((e*f - d*g)
/(b*e*n*(p + 1)))   Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1
), x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && Lt
Q[p, -1] && GtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.23 (sec) , antiderivative size = 1487, normalized size of antiderivative = 10.55

method	result	size
risch	Expression too large to display	1487

input
```
int(x^5/ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/p/e*x^3*(e*x^3+d)/(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I
*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x
^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p
))-2/3/p^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-
csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^
p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)
^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*
(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+
d)^p)-2*p*ln(e*x^3+d))/p)*x^6-4/3/p^2/e*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(
I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(
e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^
3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d
)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*c
sgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*d*x^3-2/3/p^2/e^2*c
^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*
x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,
-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn
(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)
^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln
(e*x^3+d))/p)*d^2+1/3/p^2/e*d*c^(-1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*P...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{(dp\log\left(ex^3+d\right)+d\log\left(c\right))c^{\left(\frac{1}{p}\right)}\,\mathrm{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right)+\left(e^2px^6+depx^3\right)c^{\frac{2}{p}}-2\left(p\log\left(ex^3+\right.\right.}{3\left(e^2p^3\log\left(ex^3+d\right)+e^2p^2\log\left(c\right)\right)c^{\frac{2}{p}}}$$

input
```
integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
-1/3*((d*p*log(e*x^3 + d) + d*log(c))*c^(1/p)*log_integral((e*x^3 + d)*c^(
1/p)) + (e^2*p*x^6 + d*e*p*x^3)*c^(2/p) - 2*(p*log(e*x^3 + d) + log(c))*lo
g_integral((e^2*x^6 + 2*d*e*x^3 + d^2)*c^(2/p)))/((e^2*p^3*log(e*x^3 + d)
+ e^2*p^2*log(c))*c^(2/p))
```

Sympy [F]

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(x**5/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(x**5/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F]

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^5}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^6 + d*x^3)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((2*
e*x^5 + d*x^2)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(137) = 274$.

Time = 0.13 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.22

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$
$$= \frac{1}{3}d\left(\frac{(ex^3+d)p}{e^2p^3\log(ex^3+d)+e^2p^2\log(c)} - \frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log(ex^3+d)\right)\log(ex^3+d)}{(e^2p^3\log(ex^3+d)+e^2p^2\log(c))c^{\left(\frac{1}{p}\right)}} - \frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log(e\right.}{(e^2p^3\log(ex^3+d)+}\right.$$
$$-\frac{\frac{(ex^3+d)^2p}{ep^3\log(ex^3+d)+ep^2\log(c)} - \frac{2\,p\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log(ex^3+d)\right)\log(ex^3+d)}{(ep^3\log(ex^3+d)+ep^2\log(c))c^{\frac{2}{p}}} - \frac{2\,\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\log(ex^3+d)\right)\log(c)}{(ep^3\log(ex^3+d)+ep^2\log(c))c^{\frac{2}{p}}}}{3\,e}$$

input
```
integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
1/3*d*((e*x^3 + d)*p/(e^2*p^3*log(e*x^3 + d) + e^2*p^2*log(c)) - p*Ei(log(
c)/p + log(e*x^3 + d))*log(e*x^3 + d)/((e^2*p^3*log(e*x^3 + d) + e^2*p^2*l
og(c))*c^(1/p)) - Ei(log(c)/p + log(e*x^3 + d))*log(c)/((e^2*p^3*log(e*x^3
 + d) + e^2*p^2*log(c))*c^(1/p))) - 1/3*((e*x^3 + d)^2*p/(e*p^3*log(e*x^3
+ d) + e*p^2*log(c)) - 2*p*Ei(2*log(c)/p + 2*log(e*x^3 + d))*log(e*x^3 + d
)/((e*p^3*log(e*x^3 + d) + e*p^2*log(c))*c^(2/p)) - 2*Ei(2*log(c)/p + 2*lo
g(e*x^3 + d))*log(c)/((e*p^3*log(e*x^3 + d) + e*p^2*log(c))*c^(2/p)))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^5}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

input
```
int(x^5/log(c*(d + e*x^3)^p)^2,x)
```

output
```
int(x^5/log(c*(d + e*x^3)^p)^2, x)
```

Reduce [F]

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^5}{\log\left(\left(e\,x^3+d\right)^p c\right)^2}dx$$

input
```
int(x^5/log(c*(e*x^3+d)^p)^2,x)
```

output
```
int(x**5/log((d + e*x**3)**p*c)**2,x)
```

3.150 $\int \frac{x^2}{\log^2(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3ep^2} - \frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

output
```
1/3*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e/p^2/((c*(e*x^3+d)^p)^(1/p))-1/3*(e
*x^3+d)/e/p/ln(c*(e*x^3+d)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-1/p}\left(p\left(c(d+ex^3)^p\right)^{\frac{1}{p}} - \text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)\log\left(c(d+ex^3)^p\right)\right)}{3ep^2\log\left(c\left(d+ex^3\right)^p\right)}$$

input
```
Integrate[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
-1/3*((d + e*x^3)*(p*(c*(d + e*x^3)^p)^p^(-1) - ExpIntegralEi[Log[c*(d + e
*x^3)^p]/p]*Log[c*(d + e*x^3)^p]))/(e*p^2*(c*(d + e*x^3)^p)^p^(-1)*Log[c*(
d + e*x^3)^p])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2904, 2836, 2734, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{1}{3}\int \frac{1}{\log^2\left(c\left(ex^3+d\right)^p\right)}dx^3$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \frac{1}{\log^2(c(ex^3+d)^p)}d\left(ex^3+d\right)}{3e}$$

$$\Big\downarrow \text{2734}$$

$$\frac{\frac{\int \frac{1}{\log\left(c\left(ex^3+d\right)^p\right)}d(ex^3+d)}{p}-\frac{d+ex^3}{p\log(c(d+ex^3)^p)}}{3e}$$

$$\Big\downarrow \text{2737}$$

$$\frac{\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\int \frac{\left(c\left(ex^3+d\right)^p\right)^{\frac{1}{p}}}{x^3}d\log\left(c\left(ex^3+d\right)^p\right)}{p^2}-\frac{d+ex^3}{p\log(c(d+ex^3)^p)}}{3e}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{p^2}-\frac{d+ex^3}{p\log(c(d+ex^3)^p)}}{3e}$$

input
```
Int[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
(((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(p^2*(c*(d + e*x^3)^p
)^p^(-1)) - (d + e*x^3)/(p*Log[c*(d + e*x^3)^p]))/(3*e)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2734
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[(a + b
*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Int
egerQ[2*p]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.16 (sec) , antiderivative size = 421, normalized size of antiderivative = 5.07

method	result
risch	$-\frac{2(e\,x^3+d)}{3\left(i\pi\ \operatorname{csgn}(i(e\,x^3+d)^p)\operatorname{csgn}(ic(e\,x^3+d)^p)^2-i\pi\ \operatorname{csgn}(i(e\,x^3+d)^p)\ \operatorname{csgn}(ic(e\,x^3+d)^p)\ \operatorname{csgn}(ic)-i\pi\operatorname{csgn}(ic(e\,x^3+d)^p)^3+i\pi\operatorname{csgn}(ic(e\right.}$

input
```
int(x^2/ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+
d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*cs
gn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p))/p/e*(e*x^3+d)-1
/3/p^2/e*(e*x^3+d)*((e*x^3+d)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*
x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(
I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*
c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-
I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c
)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{(epx^3+dp)c^{\left(\frac{1}{p}\right)} - (p\log\left(ex^3+d\right)+\log\left(c\right))\,\mathrm{log\_integral}\left((ex^3+d)c^{\left(\frac{1}{p}\right)}\right)}{3\left(ep^3\log\left(ex^3+d\right)+ep^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

input
```
integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
-1/3*((e*p*x^3 + d*p)*c^(1/p) - (p*log(e*x^3 + d) + log(c))*log_integral((
e*x^3 + d)*c^(1/p)))/((e*p^3*log(e*x^3 + d) + e*p^2*log(c))*c^(1/p))
```

Sympy [F]

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(x**2/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(x**2/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(x^2/(p*
log((e*x^3 + d)^p) + p*log(c)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=-\frac{\left(ex^3+d\right)p}{3\left(ep^3\log\left(ex^3+d\right)+ep^2\log\left(c\right)\right)}+\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)\log\left(ex^3+d\right)}{3\left(ep^3\log\left(ex^3+d\right)+ep^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}+\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)\log\left(c\right)}{3\left(ep^3\log\left(ex^3+d\right)+ep^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

input
```
integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
-1/3*(e*x^3 + d)*p/(e*p^3*log(e*x^3 + d) + e*p^2*log(c)) + 1/3*p*Ei(log(c)
/p + log(e*x^3 + d))*log(e*x^3 + d)/((e*p^3*log(e*x^3 + d) + e*p^2*log(c))
*c^(1/p)) + 1/3*Ei(log(c)/p + log(e*x^3 + d))*log(c)/((e*p^3*log(e*x^3 + d
) + e*p^2*log(c))*c^(1/p))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^2}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

input
```
int(x^2/log(c*(d + e*x^3)^p)^2,x)
```

output
```
int(x^2/log(c*(d + e*x^3)^p)^2, x)
```

Reduce [F]

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$
$$= \frac{3\left(\int \frac{x^5}{\log\left(\left(e\,x^3+d\right)^p c\right)^2 d+\log\left(\left(e\,x^3+d\right)^p c\right)^2 e\,x^3}dx\right)\log\left(\left(e\,x^3+d\right)^p c\right)e^2p-d}{3\log\left(\left(e\,x^3+d\right)^p c\right)ep}$$

input
```
int(x^2/log(c*(e*x^3+d)^p)^2,x)
```

output
```
(3*int(x**5/(log((d + e*x**3)**p*c)**2*d + log((d + e*x**3)**p*c)**2*e*x**
3),x)*log((d + e*x**3)**p*c)*e**2*p - d)/(3*log((d + e*x**3)**p*c)*e*p)
```

3.151 $\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

input
```
Integrate[1/(x*Log[c*(d + e*x^3)^p]^2),x]
```

output
```
Integrate[1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}dx$$
$$\Big\downarrow \text{2910}$$
$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}dx$$

input
```
Int[1/(x*Log[c*(d + e*x^3)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

input
```
int(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

output
```
int(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

input
```
integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/(x*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 16.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(1/x/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(1/(x*log(c*(d + e*x**3)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.83

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x \log\left(\left(ex^3+d\right)^p c\right)^2} \, dx$$

input
```
integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-d*integrate(1/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)), x) - 1/3*(e*
x^3 + d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c))
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x \log\left(\left(ex^3+d\right)^p c\right)^2} \, dx$$

input
```
integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/(x*log((e*x^3 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x \ln\left(c\left(e\,x^3+d\right)^p\right)^2} \, dx$$

input
```
int(1/(x*log(c*(d + e*x^3)^p)^2),x)
```

output
```
int(1/(x*log(c*(d + e*x^3)^p)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.33

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{3\left(\int \frac{1}{\log\left(\left(e\,x^3+d\right)^p c\right)^2 dx+\log\left(\left(e\,x^3+d\right)^p c\right)^2 e\,x^4}dx\right)\log\left(\left(e\,x^3+d\right)^p c\right)dp-1}{3\log\left(\left(e\,x^3+d\right)^p c\right)p}$$

input
```
int(1/x/log(c*(e*x^3+d)^p)^2,x)
```

output
```
(3*int(1/(log((d + e*x**3)**p*c)**2*d*x + log((d + e*x**3)**p*c)**2*e*x**4
),x)*log((d + e*x**3)**p*c)*d*p - 1)/(3*log((d + e*x**3)**p*c)*p)
```

3.152 $\int \frac{1}{x^4 \log^2(c(d+ex^3)^p)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx = \text{Int}\left(\frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

input
```
Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]
```

output
```
Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$
$$\Big\downarrow \; 2910$$
$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}dx$$

input
```
Int[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

input
```
int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

output
```
int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^4*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 38.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(1/x**4/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(1/(x**4*log(c*(d + e*x**3)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.33

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^3 + d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)) - integrate
((e*x^3 + 2*d)/(e*p*x^7*log((e*x^3 + d)^p) + e*p*x^7*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^4*log((e*x^3 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

input
```
int(1/(x^4*log(c*(d + e*x^3)^p)^2),x)
```

output
```
int(1/(x^4*log(c*(d + e*x^3)^p)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log^2 \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{\log \left(\left(e\,x^3+d\right)^p c\right)^2 x^4} dx$$

input
```
int(1/x^4/log(c*(e*x^3+d)^p)^2,x)
```

output
```
int(1/(log((d + e*x**3)**p*c)**2*x**4),x)
```

3.153 $\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \text{Int}\left(\frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

output
```
Defer(Int)(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
Integrate[x^3/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
Integrate[x^3/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$
$$\Big\downarrow \; 2910$$
$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}dx$$

input
```
Int[x^3/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

input
```
int(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

output
```
int(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

input
```
integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(x^3/log((e*x^3 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 14.73 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(x**3/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(x**3/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^4 + d*x)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(1/3*(
4*e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(x^3/log((e*x^3 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

input
```
int(x^3/log(c*(d + e*x^3)^p)^2,x)
```

output
```
int(x^3/log(c*(d + e*x^3)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(\left(e\,x^3+d\right)^p c\right)^2}dx$$

input
```
int(x^3/log(c*(e*x^3+d)^p)^2,x)
```

output
```
int(x**3/log((d + e*x**3)**p*c)**2,x)
```

3.154 $\int \frac{x}{\log^2(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \text{Int}\left(\frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

output
```
Defer(Int)(x/ln(c*(e*x^3+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
Integrate[x/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
Integrate[x/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

$$\Big\downarrow \text{2910}$$

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}dx$$

input
```
Int[x/Log[c*(d + e*x^3)^p]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

input
```
int(x/ln(c*(e*x^3+d)^p)^2,x)
```

output
```
int(x/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(x/log((e*x^3 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 8.73 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(x/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(x/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 4.62

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^3 + d)/(e*p*x*log((e*x^3 + d)^p) + e*p*x*log(c)) + integrate(1/3
*(2*e*x^3 - d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(x/log((e*x^3 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

input
```
int(x/log(c*(d + e*x^3)^p)^2,x)
```

output
```
int(x/log(c*(d + e*x^3)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(\left(e\,x^3+d\right)^p c\right)^2}dx$$

input
```
int(x/log(c*(e*x^3+d)^p)^2,x)
```

output
```
int(x/log((d + e*x**3)**p*c)**2,x)
```

3.155 $\int \frac{1}{\log^2(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\text{Int}\left(\frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/ln(c*(e*x^3+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

input
```
Integrate[Log[c*(d + e*x^3)^p]^(-2),x]
```

output
```
Integrate[Log[c*(d + e*x^3)^p]^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2902}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$
$$\Big\downarrow \text{2902}$$
$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}dx$$

input
```
Int[Log[c*(d + e*x^3)^p]^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2902
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Unintegrable[(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d,
 e, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

input
```
int(1/ln(c*(e*x^3+d)^p)^2,x)
```

output
```
int(1/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^3 + d)^p*c)^(-2), x)
```

Sympy [N/A]

Not integrable

Time = 8.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(1/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(log(c*(d + e*x**3)**p)**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 5.50

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^3 + d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)) + integrate
(1/3*(e*x^3 - 2*d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^3 + d)^p*c)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 25.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

input
```
int(1/log(c*(d + e*x^3)^p)^2,x)
```

output
```
int(1/log(c*(d + e*x^3)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(e\,x^3+d\right)^p c\right)^2}dx$$

input
```
int(1/log(c*(e*x^3+d)^p)^2,x)
```

output
```
int(1/log((d + e*x**3)**p*c)**2,x)
```

3.156 $\int \frac{1}{x^2 \log^2(c(d+ex^3)^p)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log^2(c(d+ex^3)^p)} dx = \text{Int}\left(\frac{1}{x^2 \log^2(c(d+ex^3)^p)}, x\right)$$

output
```
Defer(Int)(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2(c(d+ex^3)^p)} dx = \int \frac{1}{x^2 \log^2(c(d+ex^3)^p)} dx$$

input
```
Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
```

output
```
Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \log^2\left(c\left(d+e x^3\right)^p\right)} \, dx$$

$$\Big\downarrow \; 2910$$

$$\int \frac{1}{x^2 \log^2\left(c\left(d+e x^3\right)^p\right)}dx$$

input
```
Int[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

input
```
int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

output
```
int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^2*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 21.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(1/x**2/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(1/(x**2*log(c*(d + e*x**3)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.39

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^3 + d)/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)) - integrate
(1/3*(e*x^3 + 4*d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^2*log((e*x^3 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

input
```
int(1/(x^2*log(c*(d + e*x^3)^p)^2),x)
```

output
```
int(1/(x^2*log(c*(d + e*x^3)^p)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2 (c (d + e x^3)^p)} \, dx = \int \frac{1}{\log \left((e\, x^3 + d)^p\, c \right)^2 x^2} dx$$

input
```
int(1/x^2/log(c*(e*x^3+d)^p)^2,x)
```

output
```
int(1/(log((d + e*x**3)**p*c)**2*x**2),x)
```

3.157 $\int \frac{1}{x^3 \log^2(c(d+ex^3)^p)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} dx = \text{Int}\left(\frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

output
```
Defer(Int)(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

input
```
Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
```

output
```
Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2910\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$
$$\big\downarrow \text{ 2910}$$
$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}dx$$

input
```
Int[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

input
```
int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

output
```
int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x^3\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

input
```
integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/(x^3*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 29.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x^3\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

input
```
integrate(1/x**3/ln(c*(e*x**3+d)**p)**2,x)
```

output
```
Integral(1/(x**3*log(c*(d + e*x**3)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.44

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/3*(e*x^3 + d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)) - integrate
(1/3*(2*e*x^3 + 5*d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

input
```
integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/(x^3*log((e*x^3 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^3 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

input
```
int(1/(x^3*log(c*(d + e*x^3)^p)^2),x)
```

output
```
int(1/(x^3*log(c*(d + e*x^3)^p)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(e\,x^3+d\right)^p c\right)^2 x^3}dx$$

input
```
int(1/x^3/log(c*(e*x^3+d)^p)^2,x)
```

output
```
int(1/(log((d + e*x**3)**p*c)**2*x**3),x)
```

3.158 $\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{(fx)^{1+m}\log^3\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{6ep\text{Int}\left(\frac{(fx)^{2+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{f^2(1+m)}$$

output
```
(f*x)^(1+m)*ln(c*(e*x^2+d)^p)^3/f/(1+m)-6*e*p*Defer(Int)((f*x)^(2+m)*ln(c*
(e*x^2+d)^p)^2/(e*x^2+d),x)/f^2/(1+m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 994 vs. 2(77) = 154.

Time = 3.10 (sec) , antiderivative size = 994, normalized size of antiderivative = 49.70

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \text{Too large to display}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

output
```
((f*x)^m*((1 + m)*p^3*x^2*Log[d + e*x^2]^3 + (6*p^3*(-((e*x^2)/d))^((1 - m
)/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2,
 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1/2 -
m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^((1 +
 m)/2))*Log[d + e*x^2]^2))/e + (6*d*(1 + m)*p^3*((e*x^2)/(d + e*x^2))^(1/2
 - m/2)*(8*HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2, 1/2 - m/2},
 {3/2 - m/2, 3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] + (-1 + m)*Log[d + e*x^
2]*(-4*HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/
2 - m/2}, d/(d + e*x^2)] + (-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2
, 3/2 - m/2, d/(d + e*x^2)]*Log[d + e*x^2])))/(e*(-1 + m)^3) - (3*p^2*(-((
e*x^2)/d))^((1 - m)/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1,
1/2 - m/2}, {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*Hypergeometri
cPFQ[{1, 1, 1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-
((e*x^2)/d))^((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*x^2]) + Log[c*(
d + e*x^2)^p]))/e - (3*m*p^2*(-((e*x^2)/d))^((1 - m)/2)*(-((1 + m)*(d + e*
x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2}, 1 + (e*x^2)/d]) +
(1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2}, {2, 2}, 1 + (e*x^
2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/2))*Log[d + e*x^2]^
2)*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/e + (3*p*x^2*(-2*e*x^2*Hy
pergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[...
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2907, 2929}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right) dx$$

$$\Big\downarrow \text{2907}$$

$$\frac{(fx)^{m+1}\log^3\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{6ep\int \frac{(fx)^{m+2}\log^2\left(c\left(ex^2+d\right)^p\right)}{ex^2+d}dx}{f^2(m+1)}$$

$$\Big\downarrow \text{2929}$$

$$\frac{(fx)^{m+1}\log^3\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{6ep\int\frac{(fx)^{m+2}\log^2\left(c\left(ex^2+d\right)^p\right)}{ex^2+d}dx}{f^2(m+1)}$$

input
```
Int[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx)^m \ln\left(c\left(e\,x^2+d\right)^p\right)^3 dx$$

input
```
int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)
```

output
```
int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^3\,dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

output
```
integral((f*x)^m*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 109.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)^3 dx$$

input
```
integrate((f*x)**m*ln(c*(e*x**2+d)**p)**3,x)
```

output
```
Integral((f*x)**m*log(c*(d + e*x**2)**p)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 8.55

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^3\,dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

output
```
f^m*x*x^m*log((e*x^2 + d)^p)^3/(m + 1) + integrate((3*(d*f^m*(m + 1)*log(c
) + (e*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m*log((e*x^2 + d)^p)^2 + 3*(
e*f^m*(m + 1)*x^2*log(c)^2 + d*f^m*(m + 1)*log(c)^2)*x^m*log((e*x^2 + d)^p
) + (e*f^m*(m + 1)*x^2*log(c)^3 + d*f^m*(m + 1)*log(c)^3)*x^m)/(e*(m + 1)*
x^2 + d*(m + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^3 \left(c(d+ex^2)^p\right) \, dx = \int (fx)^m \log \left(\left(ex^2+d\right)^p c\right)^3 \, dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

output
```
integrate((f*x)^m*log((e*x^2 + d)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^3 \left(c(d+ex^2)^p\right) \, dx = \int \ln \left(c\left(e\,x^2+d\right)^p\right)^3 (f\,x)^m \, dx$$

input
```
int(log(c*(d + e*x^2)^p)^3*(f*x)^m,x)
```

output
```
int(log(c*(d + e*x^2)^p)^3*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 1504, normalized size of antiderivative = 75.20

$$\int (fx)^m \log^3 \left(c(d+ex^2)^p\right) \, dx = \text{Too large to display}$$

input
```
int((f*x)^m*log(c*(e*x^2+d)^p)^3,x)
```

output
```
(f**m*(x**m*log((d + e*x**2)**p*c)**3*m**3*x + 3*x**m*log((d + e*x**2)**p*
c)**3*m**2*x + 3*x**m*log((d + e*x**2)**p*c)**3*m*x + x**m*log((d + e*x**2
)**p*c)**3*x - 6*x**m*log((d + e*x**2)**p*c)**2*m**2*p*x - 12*x**m*log((d
+ e*x**2)**p*c)**2*m*p*x - 6*x**m*log((d + e*x**2)**p*c)**2*p*x + 24*x**m*
log((d + e*x**2)**p*c)*m*p**2*x + 24*x**m*log((d + e*x**2)**p*c)*p**2*x -
48*x**m*p**3*x + 48*int(x**m/(d*m**3 + 3*d*m**2 + 3*d*m + d + e*m**3*x**2
+ 3*e*m**2*x**2 + 3*e*m*x**2 + e*x**2),x)*d*m**4*p**3 + 192*int(x**m/(d*m*
*3 + 3*d*m**2 + 3*d*m + d + e*m**3*x**2 + 3*e*m**2*x**2 + 3*e*m*x**2 + e*x
**2),x)*d*m**3*p**3 + 288*int(x**m/(d*m**3 + 3*d*m**2 + 3*d*m + d + e*m**3
*x**2 + 3*e*m**2*x**2 + 3*e*m*x**2 + e*x**2),x)*d*m**2*p**3 + 192*int(x**m
/(d*m**3 + 3*d*m**2 + 3*d*m + d + e*m**3*x**2 + 3*e*m**2*x**2 + 3*e*m*x**2
 + e*x**2),x)*d*m*p**3 + 48*int(x**m/(d*m**3 + 3*d*m**2 + 3*d*m + d + e*m*
*3*x**2 + 3*e*m**2*x**2 + 3*e*m*x**2 + e*x**2),x)*d*p**3 + 6*int((x**m*log
((d + e*x**2)**p*c)**2)/(d*m**3 + 3*d*m**2 + 3*d*m + d + e*m**3*x**2 + 3*e
*m**2*x**2 + 3*e*m*x**2 + e*x**2),x)*d*m**6*p + 36*int((x**m*log((d + e*x*
*2)**p*c)**2)/(d*m**3 + 3*d*m**2 + 3*d*m + d + e*m**3*x**2 + 3*e*m**2*x**2
 + 3*e*m*x**2 + e*x**2),x)*d*m**5*p + 90*int((x**m*log((d + e*x**2)**p*c)*
*2)/(d*m**3 + 3*d*m**2 + 3*d*m + d + e*m**3*x**2 + 3*e*m**2*x**2 + 3*e*m*x
**2 + e*x**2),x)*d*m**4*p + 120*int((x**m*log((d + e*x**2)**p*c)**2)/(d*m*
*3 + 3*d*m**2 + 3*d*m + d + e*m**3*x**2 + 3*e*m**2*x**2 + 3*e*m*x**2 + ...
```

3.159 $\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{(fx)^{1+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{4ep\text{Int}\left(\frac{(fx)^{2+m}\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{f^2(1+m)}$$

output
```
(f*x)^(1+m)*ln(c*(e*x^2+d)^p)^2/f/(1+m)-4*e*p*Defer(Int)((f*x)^(2+m)*ln(c*
(e*x^2+d)^p)/(e*x^2+d),x)/f^2/(1+m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 466 vs. 2(75) = 150.

Time = 0.90 (sec) , antiderivative size = 466, normalized size of antiderivative = 23.30

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$= \frac{(fx)^m\left(4p^2x\left(\frac{2ex^2\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)}{d(3+m)}-\log\left(d+ex^2\right)\right)+(1+m)p^2x\log^2\left(d+ex^2\right)+\frac{4d(1+\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
((f*x)^m*(4*p^2*x*((2*e*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((
e*x^2)/d)])/(d*(3 + m)) - Log[d + e*x^2]) + (1 + m)*p^2*x*Log[d + e*x^2]^2
 + (4*d*(1 + m)*p^2*((e*x^2)/(d + e*x^2))^(1/2 - m/2)*(-2*HypergeometricPF
Q[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)
] + (-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m/2, d/(d + e*x
^2)]*Log[d + e*x^2]))/(e*(-1 + m)^2*x) + (2*p*(2*e*x^3*Hypergeometric2F1[1
, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] - d*(3 + m)*x*Log[d + e*x^2])*(p*Log
[d + e*x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) - (2*m*p*(-2*e*x^3*Hyperg
eometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*x*Log[d + e
*x^2])*(p*Log[d + e*x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) + x*(-(p*Log
[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + m*x*(-(p*Log[d + e*x^2]) + Log[c*
(d + e*x^2)^p])^2))/(1 + m)^2
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2907, 2929}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{2907}$$

$$\frac{(fx)^{m+1}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{4ep\int \frac{(fx)^{m+2}\log\left(c\left(ex^2+d\right)^p\right)}{ex^2+d}dx}{f^2(m+1)}$$

$$\Big\downarrow \text{2929}$$

$$\frac{(fx)^{m+1}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{4ep\int \frac{(fx)^{m+2}\log\left(c\left(ex^2+d\right)^p\right)}{ex^2+d}dx}{f^2(m+1)}$$

input
```
Int[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx)^m \ln \left(c\left(e\,x^2+d\right)^p\right)^2 dx$$

input
```
int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) \, dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^2 \, dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((f*x)^m*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 58.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) \, dx = \int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)^2 dx$$

input
```
integrate((f*x)**m*ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f*x)**m*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 6.30

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right) \, dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^2 \, dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
f^m*x*x^m*log((e*x^2 + d)^p)^2/(m + 1) + integrate((2*(d*f^m*(m + 1)*log(c
) + (e*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m*log((e*x^2 + d)^p) + (e*f^
m*(m + 1)*x^2*log(c)^2 + d*f^m*(m + 1)*log(c)^2)*x^m)/(e*(m + 1)*x^2 + d*(
m + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^2\,dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((f*x)^m*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\,(f\,x)^m\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^2*(f*x)^m,x)
```

output
```
int(log(c*(d + e*x^2)^p)^2*(f*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 602, normalized size of antiderivative = 30.10

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= \frac{f^m\left(x^m\log\left(\left(e\,x^2+d\right)^p c\right)^2 m^2x + 2x^m\log\left(\left(e\,x^2+d\right)^p c\right)^2 mx + x^m\log\left(\left(e\,x^2+d\right)^p c\right)^2 x - 4x^m\log\left(\left(e\,x^2+\right.\right.\right.}{}$$

input
```
int((f*x)^m*log(c*(e*x^2+d)^p)^2,x)
```

output
```
(f**m*(x**m*log((d + e*x**2)**p*c)**2*m**2*x + 2*x**m*log((d + e*x**2)**p*
c)**2*m*x + x**m*log((d + e*x**2)**p*c)**2*x - 4*x**m*log((d + e*x**2)**p*
c)*m*p*x - 4*x**m*log((d + e*x**2)**p*c)*p*x + 8*x**m*p**2*x - 8*int(x**m/
(d*m**2 + 2*d*m + d + e*m**2*x**2 + 2*e*m*x**2 + e*x**2),x)*d*m**3*p**2 -
24*int(x**m/(d*m**2 + 2*d*m + d + e*m**2*x**2 + 2*e*m*x**2 + e*x**2),x)*d*
m**2*p**2 - 24*int(x**m/(d*m**2 + 2*d*m + d + e*m**2*x**2 + 2*e*m*x**2 + e
*x**2),x)*d*m*p**2 - 8*int(x**m/(d*m**2 + 2*d*m + d + e*m**2*x**2 + 2*e*m*
x**2 + e*x**2),x)*d*p**2 + 4*int((x**m*log((d + e*x**2)**p*c))/(d*m**2 + 2
*d*m + d + e*m**2*x**2 + 2*e*m*x**2 + e*x**2),x)*d*m**4*p + 16*int((x**m*l
og((d + e*x**2)**p*c))/(d*m**2 + 2*d*m + d + e*m**2*x**2 + 2*e*m*x**2 + e*
x**2),x)*d*m**3*p + 24*int((x**m*log((d + e*x**2)**p*c))/(d*m**2 + 2*d*m +
 d + e*m**2*x**2 + 2*e*m*x**2 + e*x**2),x)*d*m**2*p + 16*int((x**m*log((d
+ e*x**2)**p*c))/(d*m**2 + 2*d*m + d + e*m**2*x**2 + 2*e*m*x**2 + e*x**2),
x)*d*m*p + 4*int((x**m*log((d + e*x**2)**p*c))/(d*m**2 + 2*d*m + d + e*m**
2*x**2 + 2*e*m*x**2 + e*x**2),x)*d*p))/(m**3 + 3*m**2 + 3*m + 1)
```

3.160 $\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx = -\frac{2ep(fx)^{3+m} \text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)} + \frac{(fx)^{1+m} \log \left(c\left(d+ex^2\right)^p\right)}{f(1+m)}$$

output
```
-2*e*p*(f*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],-e*x^2/d)/d/f^3/(1
+m)/(3+m)+(f*x)^(1+m)*ln(c*(e*x^2+d)^p)/f/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx = \frac{x(fx)^m \left(-2epx^2 \text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)+d(3+m)\log \left(c\left(d+ex^2\right)^p\right)\right)}{d(1+m)(3+m)}$$

input
```
Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

output
```
(x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2
)/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2905, 8, 278\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep\int \frac{x(fx)^{m+1}}{ex^2+d}dx}{f(m+1)}$$

$$\Big\downarrow \text{ 8}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep\int \frac{(fx)^{m+2}}{ex^2+d}dx}{f^2(m+1)}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(fx)^{m+1} \log \left(c\left(d+ex^2\right)^p\right)}{f(m+1)}-\frac{2ep(fx)^{m+3} \text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

input
```
Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

output
```
(-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1
 + m))
```

Defintions of rubi rules used

rule 8
```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_))^(p_), x_Symbol] :> Simp[1/a^m   Int[u*(a*
x)^(m + p), x], x] /; FreeQ[{a, m, p}, x] && IntegerQ[m]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^m \ln \left(c\left(e\,x^2+d\right)^p\right) dx$$

input
```
int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

output
```
int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

Fricas [F]

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right)\,dx=\int (fx)^m \log \left(\left(ex^2+d\right)^p c\right)\,dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Sympy [A] (verification not implemented)

Time = 34.61 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.65

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx = \text{Too large to display}$$

input
```
integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

output
```
-2*e*p*Piecewise((0**m*sqrt(-d/e**3)*log(-e*sqrt(-d/e**3) + x)/2 - 0**m*sq
rt(-d/e**3)*log(e*sqrt(-d/e**3) + x)/2 + 0**m*x/e, Eq(f, 0) | (Eq(f, 0) &
Ne(m, -1))), (f**(m + 1)*m*x**(m + 3)*lerchphi(e*x**2*exp_polar(I*pi)/d, 1
, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2
+ 5/2)) + 3*f**(m + 1)*x**(m + 3)*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/
2 + 3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/
2)), (m > -oo) & (m < oo) & Ne(m, -1)), (-Piecewise((-polylog(2, e*x**2*ex
p_polar(I*pi)/d)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polyl
og(2, e*x**2*exp_polar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylo
g(2, e*x**2*exp_polar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), (
(0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - p
olylog(2, e*x**2*exp_polar(I*pi)/d)/2, True))/(2*e*f) + log(f*x)*log(d + e
*x**2)/(2*e*f), True)) + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**
(m + 1)/(m + 1), Ne(m, -1)), (log(f*x), True))/f, True))*log(c*(d + e*x**2
)**p)
```

Maxima [F]

$$\int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)\,dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
f^m*x*x^m*log((e*x^2 + d)^p)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (
e*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m/(e*(m + 1)*x^2 + d*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log \left(c(d+ex^2)^p\right) \, dx = \int (fx)^m \log \left((ex^2+d)^p c\right) \, dx$$

input
```
integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c(d+ex^2)^p\right) \, dx = \int \ln \left(c\,(e\,x^2+d)^p\right) \,(f\,x)^m \, dx$$

input
```
int(log(c*(d + e*x^2)^p)*(f*x)^m,x)
```

output
```
int(log(c*(d + e*x^2)^p)*(f*x)^m, x)
```

Reduce [F]

$$\int (fx)^m \log \left(c(d+ex^2)^p\right) \, dx = \frac{f^m \left(x^m \log \left((e\,x^2+d)^p\,c\right) m x + x^m \log \left((e\,x^2+d)^p\,c\right) x - 2x^m p x + 2\left(\int \frac{x^m}{em\,x^2+e\,x^2+dm+d} dx\right) d\,m^2 p + 4\left(\int \frac{}{em}\right.\right.}{m^2+2m+1}$$

input
```
int((f*x)^m*log(c*(e*x^2+d)^p),x)
```

output
```
(f**m*(x**m*log((d + e*x**2)**p*c)*m*x + x**m*log((d + e*x**2)**p*c)*x - 2
*x**m*p*x + 2*int(x**m/(d*m + d + e*m*x**2 + e*x**2),x)*d*m**2*p + 4*int(x
**m/(d*m + d + e*m*x**2 + e*x**2),x)*d*m*p + 2*int(x**m/(d*m + d + e*m*x**
2 + e*x**2),x)*d*p))/(m**2 + 2*m + 1)
```

3.161 $\int \frac{(fx)^m}{\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \text{Int}\left(\frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

output
```
Defer(Int)((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

input
```
Integrate[(f*x)^m/Log[c*(d + e*x^2)^p],x]
```

output
```
Integrate[(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m}{\log(c(d+ex^2)^p)}\,dx$$

$$\Big\downarrow \text{2910}$$

$$\int \frac{(fx)^m}{\log(c(d+ex^2)^p)}dx$$

input
```
Int[(f*x)^m/Log[c*(d + e*x^2)^p],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{\ln(c(e\,x^2+d)^p)}dx$$

input
```
int((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

output
```
int((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

input
```
integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral((f*x)^m/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 12.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

input
```
integrate((f*x)**m/ln(c*(e*x**2+d)**p),x)
```

output
```
Integral((f*x)**m/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(fx)^m}{\log((ex^2+d)^p c)}\,dx$$

input
```
integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
integrate((f*x)^m/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(fx)^m}{\log((ex^2+d)^p c)}\,dx$$

input
```
integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate((f*x)^m/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 25.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(f\,x)^m}{\ln\left(c\,(e\,x^2+d)^p\right)}\,dx$$

input
```
int((f*x)^m/log(c*(d + e*x^2)^p),x)
```

output
```
int((f*x)^m/log(c*(d + e*x^2)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{(fx)^m}{\log(c(d+ex^2)^p)}\,dx = f^m\left(\int \frac{x^m}{\log((e\,x^2+d)^p\,c)}dx\right)$$

input
```
int((f*x)^m/log(c*(e*x^2+d)^p),x)
```

output
```
f**m*int(x**m/log((d + e*x**2)**p*c),x)
```

3.162 $\int \frac{(fx)^m}{\log^2(c(d+ex^2)^p)} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \text{Int}\left(\frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}, x\right)$$

output
```
Defer(Int)((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

input
```
Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

$$\Big\downarrow \ 2910$$

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}dx$$

input
```
Int[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}dx$$

input
```
int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

input
```
integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((f*x)^m/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 30.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

input
```
integrate((f*x)**m/ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f*x)**m/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.85

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

input
```
integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(e*f^m*x^2 + d*f^m)*x^m/(e*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + i
ntegrate(1/2*(e*f^m*(m + 1)*x^2 + d*f^m*(m - 1))*x^m/(e*p*x^2*log((e*x^2 +
 d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

input
```
integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((f*x)^m/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(f\,x)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}\,dx$$

input
```
int((f*x)^m/log(c*(d + e*x^2)^p)^2,x)
```

output
```
int((f*x)^m/log(c*(d + e*x^2)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=f^m\left(\int \frac{x^m}{\log\left(\left(e\,x^2+d\right)^p c\right)^2}dx\right)$$

input
```
int((f*x)^m/log(c*(e*x^2+d)^p)^2,x)
```

output
```
f**m*int(x**m/log((d + e*x**2)**p*c)**2,x)
```

3.163 $\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 372

$$\begin{aligned}\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx &= \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n}-\frac{dp^2x^{1-3n}(fx)^{-1+3n}(d+ex^n)^2}{2e^3n}\\
&\quad+\frac{2p^2x^{1-3n}(fx)^{-1+3n}(d+ex^n)^3}{27e^3n}-\frac{d^3p^2x^{1-3n}(fx)^{-1+3n}\log^2(d+ex^n)}{3e^3n}\\
&\quad-\frac{2d^2px^{1-3n}(fx)^{-1+3n}(d+ex^n)\log\left(c(d+ex^n)^p\right)}{e^3n}\\
&\quad+\frac{dpx^{1-3n}(fx)^{-1+3n}(d+ex^n)^2\log\left(c(d+ex^n)^p\right)}{e^3n}\\
&\quad-\frac{2px^{1-3n}(fx)^{-1+3n}(d+ex^n)^3\log\left(c(d+ex^n)^p\right)}{9e^3n}\\
&\quad+\frac{2d^3px^{1-3n}(fx)^{-1+3n}\log(d+ex^n)\log\left(c(d+ex^n)^p\right)}{3e^3n}\\
&\quad+\frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n}\end{aligned}$$

output
```
2*d^2*p^2*x^(1-2*n)*(f*x)^(-1+3*n)/e^2/n-1/2*d*p^2*x^(1-3*n)*(f*x)^(-1+3*n
)*(d+e*x^n)^2/e^3/n+2/27*p^2*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)^3/e^3/n-1/
3*d^3*p^2*x^(1-3*n)*(f*x)^(-1+3*n)*ln(d+e*x^n)^2/e^3/n-2*d^2*p*x^(1-3*n)*(
f*x)^(-1+3*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e^3/n+d*p*x^(1-3*n)*(f*x)^(-1+3*
n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)/e^3/n-2/9*p*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e
*x^n)^3*ln(c*(d+e*x^n)^p)/e^3/n+2/3*d^3*p*x^(1-3*n)*(f*x)^(-1+3*n)*ln(d+e*
x^n)*ln(c*(d+e*x^n)^p)/e^3/n+1/3*x*(f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2/n
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.46

$$\int (fx)^{-1+3n} \log^2\left(c(d+ex^n)^p\right)\,dx$$
$$= \frac{x^{-3n}(fx)^{3n}\left(-18d^3p^2\log^2\left(d+ex^n\right)+6d^3p\log\left(d+ex^n\right)\left(-11p+6\log\left(c(d+ex^n)^p\right)\right)+ex^n\left(p^2(66d^2-\right.\right.}{54e^3fn}$$

input
```
Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
((f*x)^(3*n)*(-18*d^3*p^2*Log[d + e*x^n]^2 + 6*d^3*p*Log[d + e*x^n]*(-11*p
 + 6*Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(66*d^2 - 15*d*e*x^n + 4*e^2*x^(2*
n)) - 6*p*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))*Log[c*(d + e*x^n)^p] + 18*e^
2*x^(2*n)*Log[c*(d + e*x^n)^p]^2)))/(54*e^3*f*n*x^(3*n))
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.53, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2906, 2904, 2845, 2858, 25, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{3n-1} \log^2\left(c(d+ex^n)^p\right)\,dx$$

$$\Big\downarrow \text{ 2906}$$

$$x^{1-3n}(fx)^{3n-1}\int x^{3n-1}\log^2\left(c(ex^n+d)^p\right)dx$$

$\Big\downarrow$ 2904

$$\frac{x^{1-3n}(fx)^{3n-1}\int x^{2n}\log^2\left(c(ex^n+d)^p\right)dx^n}{n}$$

$\Big\downarrow$ 2845

$$\frac{x^{1-3n}(fx)^{3n-1}\left(\frac{1}{3}x^{3n}\log^2\left(c(d+ex^n)^p\right)-\frac{2}{3}ep\int\frac{x^{3n}\log(c(ex^n+d)^p)}{ex^n+d}dx^n\right)}{n}$$

$\Big\downarrow$ 2858

$$\frac{x^{1-3n}(fx)^{3n-1}\left(\frac{1}{3}x^{3n}\log^2\left(c(d+ex^n)^p\right)-\frac{2}{3}p\int x^{2n}\log\left(c(ex^n+d)^p\right)d(ex^n+d)\right)}{n}$$

$\Big\downarrow$ 25

$$\frac{x^{1-3n}(fx)^{3n-1}\left(\frac{2}{3}p\int -x^{2n}\log\left(c(ex^n+d)^p\right)d(ex^n+d)+\frac{1}{3}x^{3n}\log^2\left(c(d+ex^n)^p\right)\right)}{n}$$

$\Big\downarrow$ 27

$$\frac{x^{1-3n}(fx)^{3n-1}\left(\frac{2p\int -e^3x^{2n}\log(c(ex^n+d)^p)d(ex^n+d)}{3e^3}+\frac{1}{3}x^{3n}\log^2\left(c(d+ex^n)^p\right)\right)}{n}$$

$\Big\downarrow$ 2772

$$\frac{x^{1-3n}(fx)^{3n-1}\left(\frac{2p\left(-p\int\left(d^3\log(ex^n+d)x^{-n}-\frac{x^{2n}}{3}-3d^2+\frac{3}{2}d(ex^n+d)\right)d(ex^n+d)+d^3\log(d+ex^n)\log(c(d+ex^n)^p)-3d^2(d+ex^n)\log(c(d+e\right.}{3e^3}\right.}{n}$$

$\Big\downarrow$ 2009

$$\frac{x^{1-3n}(fx)^{3n-1}\left(\frac{2p\left(d^3\log(d+ex^n)\log(c(d+ex^n)^p)-3d^2(d+ex^n)\log(c(d+ex^n)^p)+\frac{3}{2}dx^{2n}\log(c(d+ex^n)^p)-\frac{1}{3}x^{3n}\log(c(d+ex^n)^p)-p\left(\frac{1}{2}d^3\right.\right.}{3e^3}\right.}{n}$$

input
```
Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
(x^(1 - 3*n)*(f*x)^(-1 + 3*n)*((x^(3*n)*Log[c*(d + e*x^n)^p]^2)/3 + (2*p*(
-(p*((3*d*x^(2*n))/4 - x^(3*n)/9 - 3*d^2*(d + e*x^n) + (d^3*Log[d + e*x^n]
^2)/2)) + (3*d*x^(2*n)*Log[c*(d + e*x^n)^p])/2 - (x^(3*n)*Log[c*(d + e*x^n
)^p])/3 - 3*d^2*(d + e*x^n)*Log[c*(d + e*x^n)^p] + d^3*Log[d + e*x^n]*Log[
c*(d + e*x^n)^p]))/(3*e^3)))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2906
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Simp[(f*x)^m/x^m   Int[x^m*(a + b*Log[c*(d + e*x^n)
^p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simp
lify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Maple [F]

$$\int (fx)^{-1+3n} \ln \left(c(d+e\,x^{n})^{p}\right)^{2} dx$$

input
```
int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2,x)
```

output
```
int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.72

$$\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx = \frac{2\left(2\,e^3p^2-6\,e^3p\log\left(c\right)+9\,e^3\log\left(c\right)^2\right)f^{3\,n-1}x^{3\,n}-3\left(5\,de^2p^2-6\,de^2p\log\left(c\right)\right)f^{3\,n-1}x^{2\,n}+6\left(11\,d^2ep^2-6\right.}{}$$

input
```
integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

output
```
1/54*(2*(2*e^3*p^2 - 6*e^3*p*log(c) + 9*e^3*log(c)^2)*f^(3*n - 1)*x^(3*n)
- 3*(5*d*e^2*p^2 - 6*d*e^2*p*log(c))*f^(3*n - 1)*x^(2*n) + 6*(11*d^2*e*p^2
 - 6*d^2*e*p*log(c))*f^(3*n - 1)*x^n + 18*(e^3*f^(3*n - 1)*p^2*x^(3*n) + d
^3*f^(3*n - 1)*p^2)*log(e*x^n + d)^2 + 6*(3*d*e^2*f^(3*n - 1)*p^2*x^(2*n)
- 6*d^2*e*f^(3*n - 1)*p^2*x^n - 2*(e^3*p^2 - 3*e^3*p*log(c))*f^(3*n - 1)*x
^(3*n) - (11*d^3*p^2 - 6*d^3*p*log(c))*f^(3*n - 1))*log(e*x^n + d))/(e^3*n
)
```

Sympy [F]

$$\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{3n-1}\log\left(c(d+ex^n)^p\right)^2\,dx$$

input
```
integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p)**2,x)
```

output
```
Integral((f*x)**(3*n - 1)*log(c*(d + e*x**n)**p)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.64

$$\begin{aligned}&\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx\\&=\frac{ep\left(\frac{6\,d^3f^{3\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^4n}-\frac{2\,e^2f^{3\,n}x^{3\,n}-3\,def^{3\,n}x^{2\,n}+6\,d^2f^{3\,n}x^n}{e^3n}\right)\log\left((ex^n+d)^pc\right)}{9\,f}\\&\quad+\frac{(fx)^{3\,n}\log\left((ex^n+d)^pc\right)^2}{3\,fn}\\&\quad-\frac{\left(18\,d^3f^{3\,n}\log\left(ex^n+d\right)^2-4\,e^3f^{3\,n}x^{3\,n}+15\,de^2f^{3\,n}x^{2\,n}-66\,d^2ef^{3\,n}x^n-6\,\left(6\,f^{3\,n}\log\left(e\right)-11\,f^{3\,n}\right)d^3\,\mathrm{lo}\right.}{54\,e^3fn}\end{aligned}$$

input
```
integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

output
```
1/9*e*p*(6*d^3*f^(3*n)*log((e*x^n + d)/e)/(e^4*n) - (2*e^2*f^(3*n)*x^(3*n)
 - 3*d*e*f^(3*n)*x^(2*n) + 6*d^2*f^(3*n)*x^n)/(e^3*n))*log((e*x^n + d)^p*c
)/f + 1/3*(f*x)^(3*n)*log((e*x^n + d)^p*c)^2/(f*n) - 1/54*(18*d^3*f^(3*n)*
log(e*x^n + d)^2 - 4*e^3*f^(3*n)*x^(3*n) + 15*d*e^2*f^(3*n)*x^(2*n) - 66*d
^2*e*f^(3*n)*x^n - 6*(6*f^(3*n)*log(e) - 11*f^(3*n))*d^3*log(e*x^n + d))*p
^2/(e^3*f*n)
```

Giac [F]

$$\int (fx)^{-1+3n}\log^2(c(d+ex^n)^p)\,dx=\int (fx)^{3n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

input
```
integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

output
```
integrate((f*x)^(3*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+3n}\log^2(c(d+ex^n)^p)\,dx=\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2\,{\left(f\,x\right)}^{3\,n-1}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^2*(f*x)^(3*n - 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)^2*(f*x)^(3*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.50

$$\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{f^{3n}\left(18x^{3n}\log((x^ne+d)^p\,c)^2e^3-12x^{3n}\log((x^ne+d)^p\,c)\,e^3p+4x^{3n}e^3p^2+18x^{2n}\log((x^ne+d)^p\,c)\,d\,e^2p-\right.}{5}$$

input
```
int((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x)
```

output
```
(f**(3*n)*(18*x**(3*n)*log((x**n*e + d)**p*c)**2*e**3 - 12*x**(3*n)*log((x
**n*e + d)**p*c)*e**3*p + 4*x**(3*n)*e**3*p**2 + 18*x**(2*n)*log((x**n*e +
 d)**p*c)*d*e**2*p - 15*x**(2*n)*d*e**2*p**2 - 36*x**n*log((x**n*e + d)**p
*c)*d**2*e*p + 66*x**n*d**2*e*p**2 + 18*log((x**n*e + d)**p*c)**2*d**3 - 6
6*log((x**n*e + d)**p*c)*d**3*p))/(54*e**3*f*n)
```

3.164 $\int (fx)^{-1+2n} \log^2\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 255

$$\int (fx)^{-1+2n} \log^2\left(c(d+ex^n)^p\right)\,dx = -\frac{2dp^2x^{1-n}(fx)^{-1+2n}}{en}+\frac{p^2x^{1-2n}(fx)^{-1+2n}(d+ex^n)^2}{4e^2n}+\frac{2dpx^{1-2n}(fx)^{-1+2n}(d+ex^n)\log\left(c(d+ex^n)^p\right)}{e^2n}-\frac{px^{1-2n}(fx)^{-1+2n}(d+ex^n)^2\log\left(c(d+ex^n)^p\right)}{2e^2n}-\frac{dx^{1-2n}(fx)^{-1+2n}(d+ex^n)\log^2\left(c(d+ex^n)^p\right)}{e^2n}+\frac{x^{1-2n}(fx)^{-1+2n}(d+ex^n)^2\log^2\left(c(d+ex^n)^p\right)}{2e^2n}$$

output
```
-2*d*p^2*x^(1-n)*(f*x)^(-1+2*n)/e/n+1/4*p^2*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*
x^n)^2/e^2/n+2*d*p*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e^
2/n-1/2*p*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)/e^2/n-d*x
^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/e^2/n+1/2*x^(1-2*n)*
(f*x)^(-1+2*n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)^2/e^2/n
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.55

$$\int (fx)^{-1+2n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{x^{-2n}(fx)^{2n}\left(2d^2p^2\log^2\left(d+ex^n\right)+2d^2p\log\left(d+ex^n\right)\left(3p-2\log\left(c(d+ex^n)^p\right)\right)+ex^n\left(p^2(-6d+ex^n)+\right.\right.}{4e^2fn}$$

input
```
Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
((f*x)^(2*n)*(2*d^2*p^2*Log[d + e*x^n]^2 + 2*d^2*p*Log[d + e*x^n]*(3*p - 2
*Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(-6*d + e*x^n) + 2*p*(2*d - e*x^n)*Log
[c*(d + e*x^n)^p] + 2*e*x^n*Log[c*(d + e*x^n)^p]^2)))/(4*e^2*f*n*x^(2*n))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.64, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2906, 2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{2n-1}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$\Big\downarrow \text{ 2906}$$
$$x^{1-2n}(fx)^{2n-1}\int x^{2n-1}\log^2\left(c(ex^n+d)^p\right)dx$$
$$\Big\downarrow \text{ 2904}$$
$$\frac{x^{1-2n}(fx)^{2n-1}\int x^n\log^2\left(c(ex^n+d)^p\right)dx^n}{n}$$
$$\Big\downarrow \text{ 2848}$$
$$\frac{x^{1-2n}(fx)^{2n-1}\int \left(\frac{(ex^n+d)\log^2(c(ex^n+d)^p)}{e}-\frac{d\log^2(c(ex^n+d)^p)}{e}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{1-2n}(fx)^{2n-1}\left(\frac{(d+ex^n)^2\log^2(c(d+ex^n)^p)}{2e^2}-\frac{d(d+ex^n)\log^2(c(d+ex^n)^p)}{e^2}-\frac{p(d+ex^n)^2\log(c(d+ex^n)^p)}{2e^2}+\frac{2dp(d+ex^n)\log(c(d+ex^n)^p}{e^2}\right.}{n}$$

input
```
Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
(x^(1 - 2*n)*(f*x)^(-1 + 2*n)*((-2*d*p^2*x^n)/e + (p^2*(d + e*x^n)^2)/(4*e
^2) + (2*d*p*(d + e*x^n)*Log[c*(d + e*x^n)^p])/e^2 - (p*(d + e*x^n)^2*Log[
c*(d + e*x^n)^p])/(2*e^2) - (d*(d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/e^2 + (
(d + e*x^n)^2*Log[c*(d + e*x^n)^p]^2)/(2*e^2)))/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2906
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Simp[(f*x)^m/x^m   Int[x^m*(a + b*Log[c*(d + e*x^n)
^p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simp
lify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Maple [F]

$$\int (fx)^{2n-1} \ln \left(c(d+e\,x^n)^p\right)^2 dx$$

input
```
int((f*x)^(2*n-1)*ln(c*(d+e*x^n)^p)^2,x)
```

output
```
int((f*x)^(2*n-1)*ln(c*(d+e*x^n)^p)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.80

$$\int (fx)^{-1+2n} \log^2 \left(c(d+ex^n)^p\right) dx = \frac{\left(e^2p^2 - 2\,e^2p\log\left(c\right) + 2\,e^2\log\left(c\right)^2\right)f^{2\,n-1}x^{2\,n} - 2\left(3\,dep^2 - 2\,dep\log\left(c\right)\right)f^{2\,n-1}x^n + 2\left(e^2f^{2\,n-1}p^2x^{2\,n} - d^2\right.}{}$$

input
```
integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

output
```
1/4*((e^2*p^2 - 2*e^2*p*log(c) + 2*e^2*log(c)^2)*f^(2*n - 1)*x^(2*n) - 2*(
3*d*e*p^2 - 2*d*e*p*log(c))*f^(2*n - 1)*x^n + 2*(e^2*f^(2*n - 1)*p^2*x^(2*
n) - d^2*f^(2*n - 1)*p^2)*log(e*x^n + d)^2 + 2*(2*d*e*f^(2*n - 1)*p^2*x^n
- (e^2*p^2 - 2*e^2*p*log(c))*f^(2*n - 1)*x^(2*n) + (3*d^2*p^2 - 2*d^2*p*lo
g(c))*f^(2*n - 1))*log(e*x^n + d))/(e^2*n)
```

Sympy [F]

$$\int (fx)^{-1+2n} \log^2 \left(c(d+ex^n)^p\right) dx = \int (fx)^{2n-1} \log \left(c(d+ex^n)^p\right)^2 dx$$

input
```
integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p)**2,x)
```

output
```
Integral((f*x)**(2*n - 1)*log(c*(d + e*x**n)**p)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.78

$$\int (fx)^{-1+2n} \log^2(c(d+ex^n)^p) \, dx$$
$$= -\frac{ep\left(\frac{2\,d^2f^{2\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^3n} + \frac{ef^{2\,n}x^{2\,n}-2\,df^{2\,n}x^n}{e^2n}\right)\log\left((ex^n+d)^pc\right)}{2\,f}$$
$$+ \frac{(fx)^{2\,n}\log\left((ex^n+d)^pc\right)^2}{2\,fn}$$
$$+ \frac{\left(2\,d^2f^{2\,n}\log\left(ex^n+d\right)^2 + e^2f^{2\,n}x^{2\,n} - 6\,def^{2\,n}x^n - 2\left(2\,f^{2\,n}\log\left(e\right) - 3\,f^{2\,n}\right)d^2\log\left(ex^n+d\right)\right)p^2}{4\,e^2fn}$$

input
```
integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*e*p*(2*d^2*f^(2*n)*log((e*x^n + d)/e)/(e^3*n) + (e*f^(2*n)*x^(2*n) -
2*d*f^(2*n)*x^n)/(e^2*n))*log((e*x^n + d)^p*c)/f + 1/2*(f*x)^(2*n)*log((e*
x^n + d)^p*c)^2/(f*n) + 1/4*(2*d^2*f^(2*n)*log(e*x^n + d)^2 + e^2*f^(2*n)*
x^(2*n) - 6*d*e*f^(2*n)*x^n - 2*(2*f^(2*n)*log(e) - 3*f^(2*n))*d^2*log(e*x
^n + d))*p^2/(e^2*f*n)
```

Giac [F]

$$\int (fx)^{-1+2n} \log^2(c(d+ex^n)^p) \, dx = \int (fx)^{2\,n-1}\log\left((ex^n+d)^pc\right)^2 \, dx$$

input
```
integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

output
```
integrate((f*x)^(2*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+2n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int \ln\left(c\,(d+e\,x^n)^p\right)^2\,(f\,x)^{2\,n-1}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^2*(f*x)^(2*n - 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)^2*(f*x)^(2*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.56

$$\int (fx)^{-1+2n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{f^{2n}\Big(2x^{2n}\log((x^ne+d)^p\,c)^2e^2-2x^{2n}\log((x^ne+d)^p\,c)\,e^2p+x^{2n}e^2p^2+4x^n\log((x^ne+d)^p\,c)\,dep-6x^nde}{4e^2fn}$$

input
```
int((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x)
```

output
```
(f**(2*n)*(2*x**(2*n)*log((x**n*e + d)**p*c)**2*e**2 - 2*x**(2*n)*log((x**
n*e + d)**p*c)*e**2*p + x**(2*n)*e**2*p**2 + 4*x**n*log((x**n*e + d)**p*c)
*d*e*p - 6*x**n*d*e*p**2 - 2*log((x**n*e + d)**p*c)**2*d**2 + 6*log((x**n*
e + d)**p*c)*d**2*p))/(4*e**2*f*n)
```

3.165 $\int (fx)^{-1+n} \log^2\left(c(d+ex^n)^p\right) dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int (fx)^{-1+n} \log^2\left(c(d+ex^n)^p\right) dx = \frac{2p^2x(fx)^{-1+n}}{n} - \frac{2px^{1-n}(fx)^{-1+n}(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en} + \frac{x^{1-n}(fx)^{-1+n}(d+ex^n)\log^2\left(c(d+ex^n)^p\right)}{en}$$

output
```
2*p^2*x*(f*x)^(-1+n)/n-2*p*x^(1-n)*(f*x)^(-1+n)*(d+e*x^n)*ln(c*(d+e*x^n)^p
)/e/n+x^(1-n)*(f*x)^(-1+n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/e/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int (fx)^{-1+n} \log^2\left(c(d+ex^n)^p\right) dx = \frac{x^{-n}(fx)^n\left(2ep^2x^n - 2p(d+ex^n)\log\left(c(d+ex^n)^p\right) + (d+ex^n)\log^2\left(c(d+ex^n)^p\right)\right)}{efn}$$

input
```
Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
((f*x)^n*(2*e*p^2*x^n - 2*p*(d + e*x^n)*Log[c*(d + e*x^n)^p] + (d + e*x^n)
*Log[c*(d + e*x^n)^p]^2))/(e*f*n*x^n)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2906, 2904, 2836, 2733, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{n-1}\log^2\left(c(d+ex^n)^p\right)\,dx$$

$$\Big\downarrow \text{2906}$$

$$x^{1-n}(fx)^{n-1}\int x^{n-1}\log^2\left(c(ex^n+d)^p\right)dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{x^{1-n}(fx)^{n-1}\int \log^2\left(c(ex^n+d)^p\right)dx^n}{n}$$

$$\Big\downarrow \text{2836}$$

$$\frac{x^{1-n}(fx)^{n-1}\int \log^2\left(c(ex^n+d)^p\right)d(ex^n+d)}{en}$$

$$\Big\downarrow \text{2733}$$

$$\frac{x^{1-n}(fx)^{n-1}\left((d+ex^n)\log^2\left(c(d+ex^n)^p\right)-2p\int \log\left(c(ex^n+d)^p\right)d(ex^n+d)\right)}{en}$$

$$\Big\downarrow \text{2732}$$

$$\frac{x^{1-n}(fx)^{n-1}\left((d+ex^n)\log^2\left(c(d+ex^n)^p\right)-2p((d+ex^n)\log\left(c(d+ex^n)^p\right)-p(d+ex^n))\right)}{en}$$

input
```
Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
(x^(1 - n)*(f*x)^(-1 + n)*((d + e*x^n)*Log[c*(d + e*x^n)^p]^2 - 2*p*(-(p*(
d + e*x^n)) + (d + e*x^n)*Log[c*(d + e*x^n)^p])))/(e*n)
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2906
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Simp[(f*x)^m/x^m   Int[x^m*(a + b*Log[c*(d + e*x^n)
^p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simp
lify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Maple [F]

$$\int (fx)^{-1+n} \ln\left(c(d+e\,x^n)^p\right)^2 dx$$

input
```
int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p)^2,x)
```

output
```
int((f*x)^(-1+n)*ln(c*(d+e*x^n)^p)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int (fx)^{-1+n} \log^2\left(c(d+ex^n)^p\right) dx = \frac{\left(2\,ep^2 - 2\,ep\log(c) + e\log(c)^2\right)f^{n-1}x^n + \left(ef^{n-1}p^2x^n + df^{n-1}p^2\right)\log(ex^n+d)^2 - 2\left((ep^2 - ep\log(c))f\right.}{en}$$

input
```
integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

output
```
((2*e*p^2 - 2*e*p*log(c) + e*log(c)^2)*f^(n - 1)*x^n + (e*f^(n - 1)*p^2*x^
n + d*f^(n - 1)*p^2)*log(e*x^n + d)^2 - 2*((e*p^2 - e*p*log(c))*f^(n - 1)*
x^n + (d*p^2 - d*p*log(c))*f^(n - 1))*log(e*x^n + d))/(e*n)
```

Sympy [F]

$$\int (fx)^{-1+n} \log^2\left(c(d+ex^n)^p\right) dx = \int (fx)^{n-1} \log\left(c(d+ex^n)^p\right)^2 dx$$

input
```
integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p)**2,x)
```

output
```
Integral((f*x)**(n - 1)*log(c*(d + e*x**n)**p)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.45

$$\int (fx)^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$= -\frac{2\,ep\left(\frac{f^nx^n}{en}-\frac{df^n\log\left(\frac{ex^n+d}{e}\right)}{e^2n}\right)\log\left((ex^n+d)^pc\right)}{f}+\frac{(fx)^n\log\left((ex^n+d)^pc\right)^2}{fn}$$
$$-\frac{\left(df^n\log\left(ex^n+d\right)^2-2\,ef^nx^n-2\left(f^n\log\left(e\right)-f^n\right)d\log\left(ex^n+d\right)\right)p^2}{efn}$$

input
```
integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

output
```
-2*e*p*(f^n*x^n/(e*n) - d*f^n*log((e*x^n + d)/e)/(e^2*n))*log((e*x^n + d)^
p*c)/f + (f*x)^n*log((e*x^n + d)^p*c)^2/(f*n) - (d*f^n*log(e*x^n + d)^2 -
2*e*f^n*x^n - 2*(f^n*log(e) - f^n)*d*log(e*x^n + d))*p^2/(e*f*n)
```

Giac [F]

$$\int (fx)^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

input
```
integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

output
```
integrate((f*x)^(n - 1)*log((e*x^n + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int \ln\left(c\left(d+e\,x^n\right)^p\right)^2\left(f\,x\right)^{n-1}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^2*(f*x)^(n - 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)^2*(f*x)^(n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int (fx)^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{f^n\Big(x^n\log((x^ne+d)^p\,c)^2e-2x^n\log((x^ne+d)^p\,c)\,ep+2x^ne\,p^2+\log((x^ne+d)^p\,c)^2d-2\log((x^ne+d)^p\,c}{efn}$$

input
```
int((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x)
```

output
```
(f**n*(x**n*log((x**n*e + d)**p*c)**2*e - 2*x**n*log((x**n*e + d)**p*c)*e*
p + 2*x**n*e*p**2 + log((x**n*e + d)**p*c)**2*d - 2*log((x**n*e + d)**p*c)
*d*p))/(e*f*n)
```

3.166 $\int \frac{\log^2(c(d+ex^n)^p)}{fx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn} + \frac{2p\log\left(c(d+ex^n)^p\right)\operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn} - \frac{2p^2\operatorname{PolyLog}\left(3,1+\frac{ex^n}{d}\right)}{fn}$$

output
```
ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^2/f/n+2*p*ln(c*(d+e*x^n)^p)*polylog(2,1+e*x
^n/d)/f/n-2*p^2*polylog(3,1+e*x^n/d)/f/n
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.91

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx$$

$$= \frac{\log(x)\left(-p\log\left(d+ex^n\right)+\log\left(c(d+ex^n)^p\right)\right)^2 + 2p(-p\log\left(d+ex^n\right)+\log\left(c(d+ex^n)^p\right))\left(\log(x)\left(\log\left(\right.\right.\right.}{}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

output
```
(Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d +
e*x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/
d]) - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]
^2 + 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)
/d]))/n)/f
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {27, 2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\log^2(c(ex^n+d)^p)}{x}dx}{f}$$

$$\Big\downarrow \text{2904}$$

$$\frac{\int x^{-n}\log^2\left(c(ex^n+d)^p\right)dx^n}{fn}$$

$$\Big\downarrow \text{2843}$$

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)-2ep\int \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(ex^n+d)^p)}{ex^n+d}dx^n}{fn}$$

$$\Big\downarrow \text{2881}$$

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)-2p\int x^{-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c(ex^n+d)^p\right)d(ex^n+d)}{fn}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)-2p\left(p\int x^{-n}\,\text{PolyLog}\left(2,\frac{ex^n+d}{d}\right)d(ex^n+d)-\text{PolyLog}\left(2,\frac{ex^n+d}{d}\right)\log\left(c(d+ex^n)^p\right.\right.}{fn}$$

$$\Big\downarrow \; 7143$$

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)-2p\left(p\,\text{PolyLog}\left(3,\frac{ex^n+d}{d}\right)-\text{PolyLog}\left(2,\frac{ex^n+d}{d}\right)\log\left(c(d+ex^n)^p\right)\right)}{fn}$$

input
```
Int[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2 - 2*p*(-(Log[c*(d + e*x^n)^p]*Po
lyLog[2, (d + e*x^n)/d]) + p*PolyLog[3, (d + e*x^n)/d]))/(f*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.92 (sec) , antiderivative size = 614, normalized size of antiderivative = 6.98

method	result
risch	$\frac{\ln(e\,x^{n})\ln(d+e\,x^{n})^{2}p^{2}}{nf}-\frac{2\ln\left(-\frac{e\,x^{n}}{d}\right)\ln(d+e\,x^{n})^{2}p^{2}}{nf}+\frac{\ln(d+e\,x^{n})^{2}\ln\left(1-\frac{d+e\,x^{n}}{d}\right)p^{2}}{nf}-\frac{2\ln(e\,x^{n})\ln((d+e\,x^{n})^{p})\ln(d+e\,x^{n}}{nf}$

input
```
int(ln(c*(d+e*x^n)^p)^2/f/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/f*ln(e*x^n)*ln(d+e*x^n)^2*p^2-2/n/f*ln(-e*x^n/d)*ln(d+e*x^n)^2*p^2+1/n
/f*ln(d+e*x^n)^2*ln(1-(d+e*x^n)/d)*p^2-2/n/f*ln(e*x^n)*ln((d+e*x^n)^p)*ln(
d+e*x^n)*p-2/n/f*dilog(-e*x^n/d)*ln(d+e*x^n)*p^2+2/n/f*ln(-e*x^n/d)*ln((d+
e*x^n)^p)*ln(d+e*x^n)*p+2/n/f*ln(d+e*x^n)*polylog(2,(d+e*x^n)/d)*p^2+1/n/f
*ln(e*x^n)*ln((d+e*x^n)^p)^2+2/n/f*dilog(-e*x^n/d)*ln((d+e*x^n)^p)*p-2/n/f
*polylog(3,(d+e*x^n)/d)*p^2+1/f*(I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^
n)^p)^2-I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-I*Pi*csgn
(I*c*(d+e*x^n)^p)^3+I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+2*ln(c))/n*(ln(
x^n)*ln((d+e*x^n)^p)-e*p*(dilog((d+e*x^n)/d)/e+ln(x^n)*ln((d+e*x^n)/d)/e))
+1/4/f*(I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I*Pi*csgn(I*(d+e*
x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-I*Pi*csgn(I*c*(d+e*x^n)^p)^3+I*Pi*
csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+2*ln(c))^2*ln(x)
```

Fricas [F]

$$\int \frac{\log^2(c(d+ex^n)^p)}{fx}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)^2}{fx}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)^2/(f*x), x)
```

Sympy [F]

$$\int \frac{\log^2(c(d+ex^n)^p)}{fx}\,dx=\frac{\int \frac{\log\left(c(d+ex^n)^p\right)^2}{x}\,dx}{f}$$

input
```
integrate(ln(c*(d+e*x**n)**p)**2/f/x,x)
```

output
```
Integral(log(c*(d + e*x**n)**p)**2/x, x)/f
```

Maxima [F]

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^2}{fx}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="maxima")
```

output
```
(log((e*x^n + d)^p)^2*log(x) - integrate(-(e*x^n*log(c)^2 + d*log(c)^2 - 2
*((e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p))/(e*x*x^n +
 d*x), x))/f
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^2}{fx}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)^2/(f*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^2}{f\,x}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^2/(f*x),x)
```

output
```
int(log(c*(d + e*x^n)^p)^2/(f*x), x)
```

Reduce [F]

$$\int \frac{\log^2(c(d+ex^n)^p)}{fx}\,dx = \frac{3\left(\int \frac{\log((x^n e+d)^p c)^2}{x^n e x+dx}dx\right)dnp + \log((x^n e+d)^p\,c)^3}{3fnp}$$

input
```
int(log(c*(d+e*x^n)^p)^2/f/x,x)
```

output
```
(3*int(log((x**n*e + d)**p*c)**2/(x**n*e*x + d*x),x)*d*n*p + log((x**n*e +
 d)**p*c)**3)/(3*f*n*p)
```

3.167 $\int (fx)^{-1-n} \log^2\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int (fx)^{-1-n} \log^2\left(c(d+ex^n)^p\right)\,dx = \frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{dn} - \frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c(d+ex^n)^p\right)}{dn} + \frac{2ep^2x^{1+n}(fx)^{-1-n}\operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{dn}$$

output
```
2*e*p*x^(1+n)*(f*x)^(-1-n)*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/d/n-x*(f*x)^(-1-
n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/d/n+2*e*p^2*x^(1+n)*(f*x)^(-1-n)*polylog(
2,1+e*x^n/d)/d/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int (fx)^{-1-n} \log^2\left(c(d+ex^n)^p\right)\,dx = -\frac{x^{1+n}(fx)^{-1-n}\left(x^{-n}\log^2\left(c(d+ex^n)^p\right) - 2ep\left(-\frac{p\log\left(-\frac{dx^{-n}}{e}\right)\log\left(-e-dx^{-n}\right)}{d} + \frac{p\log^2\left(-e-dx^{-n}\right)}{2d} - \frac{\log\left(-e-dx^{-n}\right.}{}\right.\right.}{n}$$

input
```
Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
-((x^(1 + n)*(f*x)^(-1 - n)*(Log[c*(d + e*x^n)^p]^2/x^n - 2*e*p*(-((p*Log[
-(d/(e*x^n))]*Log[-e - d/x^n])/d) + (p*Log[-e - d/x^n]^2)/(2*d) - (Log[-e
- d/x^n]*Log[c*(d + e*x^n)^p])/d - (p*PolyLog[2, (e + d/x^n)/e])/d)))/n)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2906, 2904, 2844, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{-n-1}\log^2\left(c(d+ex^n)^p\right)\,dx$$

$$\Big\downarrow \text{2906}$$

$$x^{n+1}(fx)^{-n-1}\int x^{-n-1}\log^2\left(c(ex^n+d)^p\right)dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{x^{n+1}(fx)^{-n-1}\int x^{-2n}\log^2\left(c(ex^n+d)^p\right)dx^n}{n}$$

$$\Big\downarrow \text{2844}$$

$$\frac{x^{n+1}(fx)^{-n-1}\left(\frac{2ep\int x^{-n}\log(c(ex^n+d)^p)dx^n}{d}-\frac{x^{-n}(d+ex^n)\log^2(c(d+ex^n)^p)}{d}\right)}{n}$$

$$\Big\downarrow \text{2841}$$

$$\frac{x^{n+1}(fx)^{-n-1}\left(\frac{2ep\left(\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)-ep\int\frac{\log\left(-\frac{ex^n}{d}\right)}{ex^n+d}dx^n\right)}{d}-\frac{x^{-n}(d+ex^n)\log^2(c(d+ex^n)^p)}{d}\right)}{n}$$

$$\Big\downarrow \text{2752}$$

$$\frac{x^{n+1}(fx)^{-n-1}\left(\frac{2ep\left(\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)+p\operatorname{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\right)}{d}-\frac{x^{-n}(d+ex^n)\log^2(c(d+ex^n)^p)}{d}\right)}{n}$$

input
```
Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
(x^(1 + n)*(f*x)^(-1 - n)*(-(((d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(d*x^n))
 + (2*e*p*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^
n)/d]))/d))/n
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2844
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Simp[b*e*n*(p/(e*f - d*g))   Int[(a + b*Log[c*(d +
 e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &
& NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2906
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Simp[(f*x)^m/x^m   Int[x^m*(a + b*Log[c*(d + e*x^n)
^p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simp
lify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Maple [F]

$$\int (fx)^{-1-n} \ln\left(c(d+e\,x^n)^p\right)^2 dx$$

input
```
int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p)^2,x)
```

output
```
int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.59

$$\int (fx)^{-1-n} \log^2\left(c(d+ex^n)^p\right)\,dx = -\frac{2\,ef^{-n-1}np^2x^n\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)-2\,ef^{-n-1}npx^n\log\left(c\right)\log\left(x\right)+2\,ef^{-n-1}p^2x^n\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+d}{}$$

input
```
integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

output
```
-(2*e*f^(-n - 1)*n*p^2*x^n*log(x)*log((e*x^n + d)/d) - 2*e*f^(-n - 1)*n*p*
x^n*log(c)*log(x) + 2*e*f^(-n - 1)*p^2*x^n*dilog(-(e*x^n + d)/d + 1) + d*f
^(-n - 1)*log(c)^2 + (e*f^(-n - 1)*p^2*x^n + d*f^(-n - 1)*p^2)*log(e*x^n +
 d)^2 + 2*(d*f^(-n - 1)*p*log(c) - (e*n*p^2*log(x) - e*p*log(c))*f^(-n - 1
)*x^n)*log(e*x^n + d))/(d*n*x^n)
```

Sympy [F]

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{-n-1}\log\left(c(d+ex^n)^p\right)^2\,dx$$

input
```
integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p)**2,x)
```

output
```
Integral((f*x)**(-n - 1)*log(c*(d + e*x**n)**p)**2, x)
```

Maxima [F]

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{-n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

input
```
integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

output
```
-(e*n^2*p^2*x^n*log(x)^2 - e*p^2*x^n*log(e*x^n + d)^2 + d*log((e*x^n + d)^
p)^2 + d*log(c)^2 - 2*(e*n*p*x^n*log(x) - e*p*x^n*log(e*x^n + d) - d*log(c
))*log((e*x^n + d)^p))*f^(-n - 1)/(d*n*x^n) + integrate(2*(e*n*p^2*log(x)
+ e*p*log(c))/(e*f^(n + 1)*x*x^n + d*f^(n + 1)*x), x)
```

Giac [F]

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{-n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

input
```
integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

output
```
integrate((f*x)^(-n - 1)*log((e*x^n + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^2}{(f\,x)^{n+1}}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^2/(f*x)^(n + 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)^2/(f*x)^(n + 1), x)
```

Reduce [F]

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$= \frac{-2x^n\left(\int \frac{\log((x^ne+d)^pc)}{x^{2n}ex+x^ndx}dx\right)d^2np - 2x^n\log(x^ne+d)\,e\,p^2 + 2x^n\log(x)\,en\,p^2 - \log\left((x^ne+d)^p\,c\right)^2d - 2\log((x}{x^nf^ndfn}$$

input
```
int((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x)
```

output
```
( - 2*x**n*int(log((x**n*e + d)**p*c)/(x**(2*n)*e*x + x**n*d*x),x)*d**2*n*
p - 2*x**n*log(x**n*e + d)*e*p**2 + 2*x**n*log(x)*e*n*p**2 - log((x**n*e +
 d)**p*c)**2*d - 2*log((x**n*e + d)**p*c)*d*p)/(x**n*f**n*d*f*n)
```

3.168 $\int (fx)^{-1-2n} \log^2(c(d+ex^n)^p)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 200

$$\begin{aligned}\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx &= \frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2}-\frac{epx^{1+n}(fx)^{-1-2n}(d+ex^n)\log\left(c(d+ex^n)^p\right)}{d^2n}\\ &\quad-\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n}\\ &\quad-\frac{e^2px^{1+2n}(fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\log\left(1-\frac{d}{d+ex^n}\right)}{d^2n}\\ &\quad+\frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\operatorname{PolyLog}\left(2,\frac{d}{d+ex^n}\right)}{d^2n}\end{aligned}$$

output
```
e^2*p^2*x^(1+2*n)*(f*x)^(-1-2*n)*ln(x)/d^2-e*p*x^(1+n)*(f*x)^(-1-2*n)*(d+e
*x^n)*ln(c*(d+e*x^n)^p)/d^2/n-1/2*x*(f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2/n-e
^2*p*x^(1+2*n)*(f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)*ln(1-d/(d+e*x^n))/d^2/n+e^
2*p^2*x^(1+2*n)*(f*x)^(-1-2*n)*polylog(2,d/(d+e*x^n))/d^2/n
```

Mathematica [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.44

$$\int (fx)^{-1-2n} \log^2 (c(d+ex^n)^p) \, dx$$
$$= \frac{(fx)^{-2n} \left(e^2n^2p^2x^{2n} \log^2(x) + e^2p^2x^{2n} \log^2 (e+dx^{-n}) - 2e^2p^2x^{2n} \log (e-ex^{-n}) - 2e^2p^2x^{2n} \log (e+dx^-\right.}{}$$

input
```
Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
(e^2*n^2*p^2*x^(2*n)*Log[x]^2 + e^2*p^2*x^(2*n)*Log[e + d/x^n]^2 - 2*e^2*p
^2*x^(2*n)*Log[e - e/x^n] - 2*e^2*p^2*x^(2*n)*Log[e + d/x^n]*Log[e - e/x^n
] - 2*d*e*p*x^n*Log[c*(d + e*x^n)^p] + 2*e^2*p*x^(2*n)*Log[e - e/x^n]*Log[
c*(d + e*x^n)^p] - d^2*Log[c*(d + e*x^n)^p]^2 + 2*e^2*n*p*x^(2*n)*Log[x]*(
p + p*Log[e + d/x^n] - p*Log[e - e/x^n] - Log[c*(d + e*x^n)^p] + p*Log[1 +
 (e*x^n)/d]) + 2*e^2*p^2*x^(2*n)*PolyLog[2, -((e*x^n)/d)])/(2*d^2*f*n*(f*x
)^(2*n))
```

Rubi [A] (warning: unable to verify)

Time = 1.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.73, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2906, 2904, 2845, 2858, 27, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^{-2n-1} \log^2 (c(d+ex^n)^p) \, dx$$
$$\Big\downarrow \; 2906$$
$$x^{2n+1}(fx)^{-2n-1} \int x^{-2n-1} \log^2 (c(ex^n+d)^p) \, dx$$
$$\Big\downarrow \; 2904$$
$$\frac{x^{2n+1}(fx)^{-2n-1} \int x^{-3n} \log^2 (c(ex^n+d)^p) \, dx^n}{n}$$

$$\Big\downarrow \text{2845}$$

$$\frac{x^{2n+1}(fx)^{-2n-1}\left(ep\int \frac{x^{-2n}\log(c(ex^n+d)^p)}{ex^n+d}dx^n-\frac{1}{2}x^{-2n}\log^2\left(c(d+ex^n)^p\right)\right)}{n}$$

$$\Big\downarrow \text{2858}$$

$$\frac{x^{2n+1}(fx)^{-2n-1}\left(p\int x^{-3n}\log\left(c(ex^n+d)^p\right)d(ex^n+d)-\frac{1}{2}x^{-2n}\log^2\left(c(d+ex^n)^p\right)\right)}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{2n+1}(fx)^{-2n-1}\left(e^2p\int \frac{x^{-3n}\log(c(ex^n+d)^p)}{e^2}d(ex^n+d)-\frac{1}{2}x^{-2n}\log^2\left(c(d+ex^n)^p\right)\right)}{n}$$

$$\Big\downarrow \text{2789}$$

$$\frac{x^{2n+1}(fx)^{-2n-1}\left(e^2p\left(\frac{\int \frac{x^{-2n}\log(c(ex^n+d)^p)}{e^2}d(ex^n+d)}{d}+\frac{\int -\frac{x^{-2n}\log(c(ex^n+d)^p)}{e}d(ex^n+d)}{d}\right)-\frac{1}{2}x^{-2n}\log^2\left(c(d+ex^n)^p\right)\right)}{n}$$

$$\Big\downarrow \text{2751}$$

$$\frac{x^{2n+1}(fx)^{-2n-1}\left(e^2p\left(\frac{-\frac{p\int -\frac{x^{-n}}{e}d(ex^n+d)}{d}-\frac{x^{-n}(d+ex^n)\log(c(d+ex^n)^p)}{de}}{d}+\frac{\int -\frac{x^{-2n}\log(c(ex^n+d)^p)}{e}d(ex^n+d)}{d}\right)-\frac{1}{2}x^{-2n}\log^2\left(c\right.\right.}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{x^{2n+1}(fx)^{-2n-1}\left(e^2p\left(\frac{\int -\frac{x^{-2n}\log(c(ex^n+d)^p)}{e}d(ex^n+d)}{d}+\frac{\frac{p\log(-ex^n)}{d}-\frac{x^{-n}(d+ex^n)\log(c(d+ex^n)^p)}{de}}{d}\right)-\frac{1}{2}x^{-2n}\log^2\left(c(d+ex\right.\right.}{n}$$

$$\Big\downarrow \text{2779}$$

$$\frac{x^{2n+1}(fx)^{-2n-1}\left(e^2p\left(\frac{\frac{p\int x^{-n}\log\left(1-dx^{-n}\right)d(ex^n+d)}{d}-\frac{\log\left(1-dx^{-n}\right)\log(c(d+ex^n)^p)}{d}}{d}+\frac{\frac{p\log(-ex^n)}{d}-\frac{x^{-n}(d+ex^n)\log(c(d+ex^n)^p)}{de}}{d}\right)-\frac{1}{2}\right.}{n}$$

$$\Big\downarrow \text{2838}$$

$$\frac{x^{2n+1}(fx)^{-2n-1}\left(e^2p\left(\frac{\frac{p\,\text{PolyLog}\left(2,dx^{-n}\right)}{d}-\frac{\log\left(1-dx^{-n}\right)\log(c(d+ex^n)^p)}{d}}{d}+\frac{\frac{p\log(-ex^n)}{d}-\frac{x^{-n}(d+ex^n)\log(c(d+ex^n)^p)}{de}}{d}\right)-\frac{1}{2}x^{-2n}\log\right.}{n}$$

input
```
Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

output
```
(x^(1 + 2*n)*(f*x)^(-1 - 2*n)*(-1/2*Log[c*(d + e*x^n)^p]^2/x^(2*n) + e^2*p
*(((p*Log[-(e*x^n)])/d - ((d + e*x^n)*Log[c*(d + e*x^n)^p])/(d*e*x^n))/d +
 (-((Log[1 - d/x^n]*Log[c*(d + e*x^n)^p])/d) + (p*PolyLog[2, d/x^n])/d)/d)
))/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2906
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Simp[(f*x)^m/x^m   Int[x^m*(a + b*Log[c*(d + e*x^n)
^p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simp
lify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Maple [F]

$$\int (fx)^{-1-2n} \ln \left(c(d+e\,x^n)^p\right)^2 dx$$

input
```
int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

output
```
int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.40

$$\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{2\,e^2f^{-2\,n-1}np^2x^{2\,n}\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)+2\,e^2f^{-2\,n-1}p^2x^{2\,n}\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)-2\,def^{-2\,n-1}px^n\log\left(c\right)-d^2f$$

input
```
integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

output
```
1/2*(2*e^2*f^(-2*n - 1)*n*p^2*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*e^2*f^
(-2*n - 1)*p^2*x^(2*n)*dilog(-(e*x^n + d)/d + 1) - 2*d*e*f^(-2*n - 1)*p*x^
n*log(c) - d^2*f^(-2*n - 1)*log(c)^2 + 2*(e^2*n*p^2 - e^2*n*p*log(c))*f^(-
2*n - 1)*x^(2*n)*log(x) + (e^2*f^(-2*n - 1)*p^2*x^(2*n) - d^2*f^(-2*n - 1)
*p^2)*log(e*x^n + d)^2 - 2*(d*e*f^(-2*n - 1)*p^2*x^n + d^2*f^(-2*n - 1)*p*
log(c) + (e^2*n*p^2*log(x) + e^2*p^2 - e^2*p*log(c))*f^(-2*n - 1)*x^(2*n))
*log(e*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F]

$$\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{-2n-1}\log\left(c(d+ex^n)^p\right)^2\,dx$$

input
```
integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p)**2,x)
```

output
```
Integral((f*x)**(-2*n - 1)*log(c*(d + e*x**n)**p)**2, x)
```

Maxima [F]

$$\int (fx)^{-1-2n} \log^2 (c(d+ex^n)^p) \, dx = \int (fx)^{-2n-1} \log \left((ex^n+d)^p c \right)^2 \, dx$$

input
```
integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

output
```
1/2*(e^2*n^2*p^2*x^(2*n)*log(x)^2 - e^2*p^2*x^(2*n)*log(e*x^n + d)^2 - 2*d
*e*p*x^n*log(c) - d^2*log((e*x^n + d)^p)^2 - d^2*log(c)^2 - 2*(e^2*n*p*x^(
2*n)*log(x) - e^2*p*x^(2*n)*log(e*x^n + d) + d*e*p*x^n + d^2*log(c))*log((
e*x^n + d)^p))*f^(-2*n - 1)/(d^2*n*x^(2*n)) - integrate((e^2*n*p^2*log(x)
- e^2*p^2 + e^2*p*log(c))/(d*e*f^(2*n + 1)*x*x^n + d^2*f^(2*n + 1)*x), x)
```

Giac [F]

$$\int (fx)^{-1-2n} \log^2 (c(d+ex^n)^p) \, dx = \int (fx)^{-2n-1} \log \left((ex^n+d)^p c \right)^2 \, dx$$

input
```
integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

output
```
integrate((f*x)^(-2*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1-2n} \log^2 (c(d+ex^n)^p) \, dx = \int \frac{\ln \left(c \left(d + e \, x^n \right)^p \right)^2}{(f \, x)^{2n+1}} \, dx$$

input
```
int(log(c*(d + e*x^n)^p)^2/(f*x)^(2*n + 1),x)
```

output
```
int(log(c*(d + e*x^n)^p)^2/(f*x)^(2*n + 1), x)
```

Reduce [F]

$$\int (fx)^{-1-2n} \log^2\left(c(d+ex^n)^p\right) dx$$
$$= \frac{-2x^{2n}\left(\int \frac{\log((x^n e+d)^p c)}{x^n ex+dx}dx\right) d\, e^2np + x^{2n}\log(x^n e+d)\, e^2p^2 - 3x^{2n}\log((x^n e+d)^p\, c)\, e^2p + 2x^{2n}\log(x)\, e^2n\, p^2}{2x^{2n}f^{2n}d^2fn}$$

input
```
int((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x)
```

output
```
( - 2*x**(2*n)*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d*e**2*n*p +
 x**(2*n)*log(x**n*e + d)*e**2*p**2 - 3*x**(2*n)*log((x**n*e + d)**p*c)*e*
*2*p + 2*x**(2*n)*log(x)*e**2*n*p**2 - 2*x**n*log((x**n*e + d)**p*c)*d*e*p
 - log((x**n*e + d)**p*c)**2*d**2)/(2*x**(2*n)*f**(2*n)*d**2*f*n)
```

3.169 $\int \frac{\log(1+ex^n)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\log(1+ex^n)}{x}\,dx = -\frac{\text{PolyLog}(2,-ex^n)}{n}$$

output
```
-polylog(2,-e*x^n)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+ex^n)}{x}\,dx = -\frac{\text{PolyLog}(2,-ex^n)}{n}$$

input
```
Integrate[Log[1 + e*x^n]/x,x]
```

output
```
-(PolyLog[2, -(e*x^n)]/n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex^n+1)}{x}\,dx$$
$$\Big\downarrow \text{ 2838}$$
$$-\frac{\text{PolyLog}(2,-ex^n)}{n}$$

input
```
Int[Log[1 + e*x^n]/x,x]
```

output
```
-(PolyLog[2, -(e*x^n)]/n)
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\operatorname{dilog}(1+e\,x^{n})}{n}$	14
default	$-\frac{\operatorname{dilog}(1+e\,x^{n})}{n}$	14
meijerg	$-\frac{\operatorname{polylog}(2,-e\,x^{n})}{n}$	14
risch	$-\frac{\operatorname{dilog}(1+e\,x^{n})}{n}$	14

input
```
int(ln(1+e*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/n*dilog(1+e*x^n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\log(1+ex^n)}{x}\,dx = -\frac{\mathrm{Li}_2(-ex^n)}{n}$$

input
```
integrate(log(1+e*x^n)/x,x, algorithm="fricas")
```

output
```
-dilog(-e*x^n)/n
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log(1+ex^n)}{x}\,dx = -\frac{\mathrm{Li}_2\left(ex^n e^{i\pi}\right)}{n}$$

input
```
integrate(ln(1+e*x**n)/x,x)
```

output
```
-polylog(2, e*x**n*exp_polar(I*pi))/n
```

Maxima [F]

$$\int \frac{\log\left(1+ex^n\right)}{x}\,dx=\int \frac{\log\left(ex^n+1\right)}{x}\,dx$$

input
```
integrate(log(1+e*x^n)/x,x, algorithm="maxima")
```

output
```
-1/2*n*log(x)^2 + n*integrate(log(x)/(e*x*x^n + x), x) + log(e*x^n + 1)*lo
g(x)
```

Giac [F]

$$\int \frac{\log\left(1+ex^n\right)}{x}\,dx=\int \frac{\log\left(ex^n+1\right)}{x}\,dx$$

input
```
integrate(log(1+e*x^n)/x,x, algorithm="giac")
```

output
```
integrate(log(e*x^n + 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1+ex^n\right)}{x}\,dx=\int \frac{\ln\left(e\,x^n+1\right)}{x}\,dx$$

input
```
int(log(e*x^n + 1)/x,x)
```

output
```
int(log(e*x^n + 1)/x, x)
```

Reduce [F]

$$\int \frac{\log (1 + e x^n)}{x} \, dx = \frac{2\left(\int \frac{\log(x^n e+1)}{x^n e x+x} dx\right) n + \log(x^n e + 1)^2}{2n}$$

input
```
int(log(1+e*x^n)/x,x)
```

output
```
(2*int(log(x**n*e + 1)/(x**n*e*x + x),x)*n + log(x**n*e + 1)**2)/(2*n)
```

3.170 $\int \frac{\log(2+ex^n)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{\log(2+ex^n)}{x} dx = \log(2)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{2}\right)}{n}$$

output
```
ln(2)*ln(x)-polylog(2,-1/2*e*x^n)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log(2+ex^n)}{x} dx = \log(2)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{2}\right)}{n}$$

input
```
Integrate[Log[2 + e*x^n]/x,x]
```

output
```
Log[2]*Log[x] - PolyLog[2, -1/2*(e*x^n)]/n
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2839, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex^n+2)}{x}dx$$
$$\Big\downarrow \text{2904}$$
$$\frac{\int x^{-n}\log(ex^n+2)\,dx^n}{n}$$
$$\Big\downarrow \text{2839}$$
$$\frac{\int x^{-n}\log\left(\frac{ex^n}{2}+1\right)dx^n+\log(2)\log(x^n)}{n}$$
$$\Big\downarrow \text{2838}$$
$$\frac{\log(2)\log(x^n)-\text{PolyLog}\left(2,-\frac{ex^n}{2}\right)}{n}$$

input
```
Int[Log[2 + e*x^n]/x,x]
```

output
```
(Log[2]*Log[x^n] - PolyLog[2, -1/2*(e*x^n)])/n
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2839
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Simp[b   Int[Log[1 + e*(x/d)]/x, x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 1.55 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

method	result	size
risch	$\ln(x)\ln(2+e\,x^n)-\frac{\operatorname{dilog}\left(\frac{e\,x^n}{2}+1\right)}{n}-\ln(x)\ln\left(\frac{e\,x^n}{2}+1\right)$	40
derivativedivides	$\frac{\left(\ln(2+e\,x^n)-\ln\left(\frac{e\,x^n}{2}+1\right)\right)\ln\left(-\frac{e\,x^n}{2}\right)-\operatorname{dilog}\left(\frac{e\,x^n}{2}+1\right)}{n}$	45
default	$\frac{\left(\ln(2+e\,x^n)-\ln\left(\frac{e\,x^n}{2}+1\right)\right)\ln\left(-\frac{e\,x^n}{2}\right)-\operatorname{dilog}\left(\frac{e\,x^n}{2}+1\right)}{n}$	45

input
```
int(ln(2+e*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln(2+e*x^n)-1/n*dilog(1/2*e*x^n+1)-ln(x)*ln(1/2*e*x^n+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{\log(2+ex^n)}{x}\,dx = \frac{n\log(ex^n+2)\log(x)-n\log\left(\frac{1}{2}\,ex^n+1\right)\log(x)-\mathrm{Li}_2\left(-\frac{1}{2}\,ex^n\right)}{n}$$

input
```
integrate(log(2+e*x^n)/x,x, algorithm="fricas")
```

output
```
(n*log(e*x^n + 2)*log(x) - n*log(1/2*e*x^n + 1)*log(x) - dilog(-1/2*e*x^n)
)/n
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.76

$$\int \frac{\log(2+ex^n)}{x}\,dx = \begin{cases} -\frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{2}\right)}{n} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(2)\log(x)-\frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{2}\right)}{n} & \text{for } |x|<1 \\ -\log(2)\log\left(\frac{1}{x}\right)-\frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{2}\right)}{n} & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(2)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(2)-\frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{2}\right)}{n} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(2+e*x**n)/x,x)
```

output
```
Piecewise((-polylog(2, e*x**n*exp_polar(I*pi)/2)/n, (Abs(x) < 1) & (1/Abs(
x) < 1)), (log(2)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, Abs(x)
< 1), (-log(2)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, 1/Abs(x)
 < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(2) + meijerg(((1, 1),
()), ((), (0, 0)), x)*log(2) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, Tru
e))
```

Maxima [F]

$$\int \frac{\log(2+ex^n)}{x}\,dx = \int \frac{\log(ex^n+2)}{x}\,dx$$

input
```
integrate(log(2+e*x^n)/x,x, algorithm="maxima")
```

output
```
-1/2*n*log(x)^2 + 2*n*integrate(log(x)/(e*x*x^n + 2*x), x) + log(e*x^n + 2
)*log(x)
```

Giac [F]

$$\int \frac{\log(2+ex^n)}{x}\,dx=\int \frac{\log(ex^n+2)}{x}\,dx$$

input
```
integrate(log(2+e*x^n)/x,x, algorithm="giac")
```

output
```
integrate(log(e*x^n + 2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(2+ex^n)}{x}\,dx=\int \frac{\ln(e\,x^n+2)}{x}\,dx$$

input
```
int(log(e*x^n + 2)/x,x)
```

output
```
int(log(e*x^n + 2)/x, x)
```

Reduce [F]

$$\int \frac{\log(2+ex^n)}{x}\,dx=\frac{4\left(\int \frac{\log(x^n e+2)}{x^n ex+2x}dx\right)n+\log(x^n e+2)^2}{2n}$$

input
```
int(log(2+e*x^n)/x,x)
```

output
```
(4*int(log(x**n*e + 2)/(x**n*e*x + 2*x),x)*n + log(x**n*e + 2)**2)/(2*n)
```

3.171 $\int \frac{\log(2(3+ex^n))}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{\log\left(2(3+ex^n)\right)}{x} \, dx = \log(6)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

output
```
ln(6)*ln(x)-polylog(2,-1/3*e*x^n)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(2(3+ex^n)\right)}{x} \, dx = \log(6)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

input
```
Integrate[Log[2*(3 + e*x^n)]/x,x]
```

output
```
Log[6]*Log[x] - PolyLog[2, -1/3*(e*x^n)]/n
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2904, 2839, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (2(ex^n+3))}{x} \, dx$$
$$\Big\downarrow \text{2904}$$
$$\frac{\int x^{-n} \log (2(ex^n+3)) \, dx^n}{n}$$
$$\Big\downarrow \text{2839}$$
$$\frac{\int x^{-n} \log \left(\frac{ex^n}{3}+1\right) \, dx^n + \log(6) \log (x^n)}{n}$$
$$\Big\downarrow \text{2838}$$
$$\frac{\log(6) \log (x^n) - \text{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

input
```
Int[Log[2*(3 + e*x^n)]/x,x]
```

output
```
(Log[6]*Log[x^n] - PolyLog[2, -1/3*(e*x^n)])/n
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2839
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Simp[b   Int[Log[1 + e*(x/d)]/x, x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 1.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

method	result	size
risch	$\ln\left(x\right)\ln\left(6+2e\,x^{n}\right)-\frac{\operatorname{dilog}\left(\frac{e\,x^{n}}{3}+1\right)}{n}-\ln\left(x\right)\ln\left(\frac{e\,x^{n}}{3}+1\right)$	41
derivativedivides	$\frac{\left(\ln\left(6+2e\,x^{n}\right)-\ln\left(\frac{e\,x^{n}}{3}+1\right)\right)\ln\left(-\frac{e\,x^{n}}{3}\right)-\operatorname{dilog}\left(\frac{e\,x^{n}}{3}+1\right)}{n}$	46
default	$\frac{\left(\ln\left(6+2e\,x^{n}\right)-\ln\left(\frac{e\,x^{n}}{3}+1\right)\right)\ln\left(-\frac{e\,x^{n}}{3}\right)-\operatorname{dilog}\left(\frac{e\,x^{n}}{3}+1\right)}{n}$	46

input
```
int(ln(6+2*e*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln(6+2*e*x^n)-1/n*dilog(1/3*e*x^n+1)-ln(x)*ln(1/3*e*x^n+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(18) = 36.

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{\log\left(2(3+ex^{n})\right)}{x}\,dx = \frac{n\log\left(2\,ex^{n}+6\right)\log\left(x\right)-n\log\left(\frac{1}{3}\,ex^{n}+1\right)\log\left(x\right)-\mathrm{Li}_{2}\left(-\frac{1}{3}\,ex^{n}\right)}{n}$$

input
```
integrate(log(6+2*e*x^n)/x,x, algorithm="fricas")
```

output
```
(n*log(2*e*x^n + 6)*log(x) - n*log(1/3*e*x^n + 1)*log(x) - dilog(-1/3*e*x^
n))/n
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.76

$$\int \frac{\log(2(3+ex^n))}{x}\,dx = \begin{cases} -\frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{3}\right)}{n} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(6)\log(x) - \frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{3}\right)}{n} & \text{for } |x|<1 \\ -\log(6)\log\left(\frac{1}{x}\right) - \frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{3}\right)}{n} & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(6) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(6) - \frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{3}\right)}{n} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(6+2*e*x**n)/x,x)
```

output
```
Piecewise((-polylog(2, e*x**n*exp_polar(I*pi)/3)/n, (Abs(x) < 1) & (1/Abs(
x) < 1)), (log(6)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, Abs(x)
< 1), (-log(6)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, 1/Abs(x)
 < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(6) + meijerg(((1, 1),
()), ((), (0, 0)), x)*log(6) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, Tru
e))
```

Maxima [F]

$$\int \frac{\log (2(3+ex^n))}{x}\,dx=\int \frac{\log (2\,ex^n+6)}{x}\,dx$$

input
```
integrate(log(6+2*e*x^n)/x,x, algorithm="maxima")
```

output
```
-1/2*n*log(x)^2 + 3*n*integrate(log(x)/(e*x*x^n + 3*x), x) + log(2)*log(x)
 + log(e*x^n + 3)*log(x)
```

Giac [F]

$$\int \frac{\log (2(3+ex^n))}{x}\,dx=\int \frac{\log (2\,ex^n+6)}{x}\,dx$$

input
```
integrate(log(6+2*e*x^n)/x,x, algorithm="giac")
```

output
```
integrate(log(2*e*x^n + 6)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (2(3+ex^n))}{x}\,dx=\int \frac{\ln (2\,e\,x^n+6)}{x}\,dx$$

input
```
int(log(2*e*x^n + 6)/x,x)
```

output
```
int(log(2*e*x^n + 6)/x, x)
```

Reduce [F]

$$\int \frac{\log(2(3+ex^n))}{x}\,dx = \frac{6\left(\int \frac{\log(2x^n e+6)}{x^n e x+3x}dx\right)n+\log(2x^n e+6)^2}{2n}$$

input
```
int(log(6+2*e*x^n)/x,x)
```

output
```
(6*int(log(2*x**n*e + 6)/(x**n*e*x + 3*x),x)*n + log(2*x**n*e + 6)**2)/(2*
n)
```

3.172 $\int \frac{\log(c(d+ex^n))}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{\log(c(d+ex^n))}{x}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n))}{n} + \frac{\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
ln(-e*x^n/d)*ln(c*(d+e*x^n))/n+polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{\log(c(d+ex^n))}{x}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)) + \text{PolyLog}\left(2,\frac{d+ex^n}{d}\right)}{n}$$

input
```
Integrate[Log[c*(d + e*x^n)]/x,x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)] + PolyLog[2, (d + e*x^n)/d])/n
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n))}{x} \, dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{\int x^{-n} \log (c(ex^n+d)) \, dx^n}{n}$$

$$\Big\downarrow \text{2841}$$

$$\frac{\log \left(-\frac{ex^n}{d}\right) \log (c(d+ex^n)) - e \int \frac{\log\left(-\frac{ex^n}{d}\right)}{ex^n+d} dx^n}{n}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\log \left(-\frac{ex^n}{d}\right) \log (c(d+ex^n)) + \text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}$$

input
```
Int[Log[c*(d + e*x^n)]/x,x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)] + PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(-\frac{e\,x^{n}}{d}\right)+\ln\left(c e\,x^{n}+c d\right)\ln\left(-\frac{e\,x^{n}}{d}\right)}{n}$
default	$\frac{\operatorname{dilog}\left(-\frac{e\,x^{n}}{d}\right)+\ln\left(c e\,x^{n}+c d\right)\ln\left(-\frac{e\,x^{n}}{d}\right)}{n}$
risch	$\ln\left(x\right)\ln\left(d+e\,x^{n}\right)+\left(\frac{i\pi\ \operatorname{csgn}\left(i\left(d+e\,x^{n}\right)\right)\operatorname{csgn}\left(i c\left(d+e\,x^{n}\right)\right)^{2}}{2}-\frac{i\pi\ \operatorname{csgn}\left(i\left(d+e\,x^{n}\right)\right)\operatorname{csgn}\left(i c\left(d+e\,x^{n}\right)\right)\operatorname{csgn}(i}{2}\right.$

input
```
int(ln(c*(d+e*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(dilog(-e*x^n/d)+ln(c*e*x^n+c*d)*ln(-e*x^n/d))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{\log\left(c(d+ex^n)\right)}{x}\,dx = \frac{n\log\left(cex^n+cd\right)\log\left(x\right)-n\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)-\operatorname{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

input
```
integrate(log(c*(d+e*x^n))/x,x, algorithm="fricas")
```

output
```
(n*log(c*e*x^n + c*d)*log(x) - n*log(x)*log((e*x^n + d)/d) - dilog(-(e*x^n
 + d)/d + 1))/n
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex^n)\right)}{x}\,dx = \int \frac{\log\left(cd+cex^n\right)}{x}\,dx$$

input
```
integrate(ln(c*(d+e*x**n))/x,x)
```

output
```
Integral(log(c*d + c*e*x**n)/x, x)
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)c\right)}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n))/x,x, algorithm="maxima")
```

output
```
d*n*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*log(x)^2 + log(e*x^n + d)
*log(x) + log(c)*log(x)
```

Giac [F]

$$\int \frac{\log (c(d+ex^n))}{x} \, dx = \int \frac{\log ((ex^n+d)c)}{x} \, dx$$

input
```
integrate(log(c*(d+e*x^n))/x,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (c(d+ex^n))}{x} \, dx = \int \frac{\ln (c (d+e x^n))}{x} \, dx$$

input
```
int(log(c*(d + e*x^n))/x,x)
```

output
```
int(log(c*(d + e*x^n))/x, x)
```

Reduce [F]

$$\int \frac{\log (c(d+ex^n))}{x} \, dx = \frac{2\left(\int \frac{\log(x^n ce+cd)}{x^n ex+dx} dx\right) dn + \log(x^n ce+cd)^2}{2n}$$

input
```
int(log(c*(d+e*x^n))/x,x)
```

output
```
(2*int(log(x**n*c*e + c*d)/(x**n*e*x + d*x),x)*d*n + log(x**n*c*e + c*d)**
2)/(2*n)
```

3.173 $\int \frac{\log(c(d+ex^n)^p)}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log(c(d+ex^n)^p)}{x} \, dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{p\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+p*polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\log(c(d+ex^n)^p)}{x} \, dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p) + p\,\text{PolyLog}\left(2,\frac{d+ex^n}{d}\right)}{n}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/x,x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{x} dx$$
$$\Big\downarrow \text{ 2904}$$
$$\frac{\int x^{-n} \log (c(ex^n+d)^p) dx^n}{n}$$
$$\Big\downarrow \text{ 2841}$$
$$\frac{\log \left(-\frac{ex^n}{d}\right) \log (c(d+ex^n)^p) - ep \int \frac{\log\left(-\frac{ex^n}{d}\right)}{ex^n+d} dx^n}{n}$$
$$\Big\downarrow \text{ 2752}$$
$$\frac{\log \left(-\frac{ex^n}{d}\right) \log (c(d+ex^n)^p) + p \operatorname{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]/x,x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.86

method	result
risch	$\ln\left(x\right)\ln\left(\left(d+e\,x^{n}\right)^{p}\right)+\left(\frac{i\pi\ \mathrm{csgn}\left(i\left(d+e\,x^{n}\right)^{p}\right)\mathrm{csgn}\left(ic\left(d+e\,x^{n}\right)^{p}\right)^{2}}{2}-\frac{i\pi\ \mathrm{csgn}\left(i\left(d+e\,x^{n}\right)^{p}\right)\mathrm{csgn}\left(ic\left(d+e\,x^{n}\right)^{p}\right)\mathrm{csgn}\left(ic\right)}{2}-$

input
```
int(ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^
2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*cs
gn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*ln
(x)-p/n*dilog((d+e*x^n)/d)-p*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{np\log\left(ex^n+d\right)\log\left(x\right)-np\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)+n\log\left(c\right)\log\left(x\right)-p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

input
```
integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
(n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(
x) - p*dilog(-(e*x^n + d)/d + 1))/n
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral(log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*p*log(x)^2 + log((e*x^n
 + d)^p)*log(x) + log(c)*log(x)
```

Giac [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log((ex^n+d)^p c)}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln(c\,(d+e\,x^n)^p)}{x}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/x,x)
```

output
```
int(log(c*(d + e*x^n)^p)/x, x)
```

Reduce [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \frac{2\left(\int \frac{\log((x^n e+d)^p c)}{x^n ex+dx}dx\right)dnp+\log((x^n e+d)^p\,c)^2}{2np}$$

input
```
int(log(c*(d+e*x^n)^p)/x,x)
```

output
```
(2*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d*n*p + log((x**n*e + d)
**p*c)**2)/(2*n*p)
```

3.174 $\int \frac{\log^2(c(d+ex^n)^p)}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 79

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{x}\,dx=\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{n}+\frac{2p\log\left(c(d+ex^n)^p\right)\operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}-\frac{2p^2\operatorname{PolyLog}\left(3,1+\frac{ex^n}{d}\right)}{n}$$

output
```
ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^2/n+2*p*ln(c*(d+e*x^n)^p)*polylog(2,1+e*x^n
/d)/n-2*p^2*polylog(3,1+e*x^n/d)/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 164 vs. 2(79) = 158.

Time = 0.06 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.08

$$\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx$$
$$= \log(x)\left(-p\log(d+ex^n)+\log(c(d+ex^n)^p)\right)^2 + 2p(-p\log(d+ex^n) + \log(c(d+ex^n)^p))\left(\log(x)\left(\log(d+ex^n)-\log\left(1+\frac{ex^n}{d}\right)\right)-\frac{\text{PolyLog}\left(2,-\frac{ex^n}{d}\right)}{n}\right) + \frac{p^2\left(\log\left(-\frac{ex^n}{d}\right)\log^2(d+ex^n)+2\log(d+ex^n)\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)-2\,\text{PolyLog}\left(3,1+\frac{ex^n}{d}\right)\right)}{n}$$

input

```
Integrate[Log[c*(d + e*x^n)^p]^2/x,x]
```

output

```
Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d + e
*x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/d
]) - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]^
2 + 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)/
d]))/n
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx$$
$$\Big\downarrow \text{2904}$$
$$\frac{\int x^{-n}\log^2(c(ex^n+d)^p)\,dx^n}{n}$$
$$\Big\downarrow \text{2843}$$
$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2(c(d+ex^n)^p)-2ep\int \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(ex^n+d)^p)}{ex^n+d}dx^n}{n}$$

$\Big\downarrow$ 2881

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)-2p\int x^{-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c(ex^n+d)^p\right)d(ex^n+d)}{n}$$

$\Big\downarrow$ 2821

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)-2p\left(p\int x^{-n}\operatorname{PolyLog}\left(2,\frac{ex^n+d}{d}\right)d(ex^n+d)-\operatorname{PolyLog}\left(2,\frac{ex^n+d}{d}\right)\log\left(c(d+ex^n)^p\right)\right)}{n}$$

$\Big\downarrow$ 7143

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)-2p\left(p\operatorname{PolyLog}\left(3,\frac{ex^n+d}{d}\right)-\operatorname{PolyLog}\left(2,\frac{ex^n+d}{d}\right)\log\left(c(d+ex^n)^p\right)\right)}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]^2/x,x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2 - 2*p*(-(Log[c*(d + e*x^n)^p]*Po
lyLog[2, (d + e*x^n)/d]) + p*PolyLog[3, (d + e*x^n)/d]))/n
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.16 (sec) , antiderivative size = 578, normalized size of antiderivative = 7.32

method	result
risch	$\frac{\ln\left(1-\frac{d+e\,x^{n}}{d}\right)\ln\left(d+e\,x^{n}\right)^{2}p^{2}}{n}-\frac{2\ln\left(-\frac{e\,x^{n}}{d}\right)\ln\left(d+e\,x^{n}\right)^{2}p^{2}}{n}+\frac{\ln\left(e\,x^{n}\right)\ln\left(d+e\,x^{n}\right)^{2}p^{2}}{n}+\frac{2\,\mathrm{polylog}\left(2,\frac{d+e\,x^{n}}{d}\right)\ln\left(d+e\,x^{n}\right)p^{2}}{n}$

input
```
int(ln(c*(d+e*x^n)^p)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*ln(1-(d+e*x^n)/d)*ln(d+e*x^n)^2*p^2-2/n*ln(-e*x^n/d)*ln(d+e*x^n)^2*p^2
+1/n*ln(e*x^n)*ln(d+e*x^n)^2*p^2+2/n*polylog(2,(d+e*x^n)/d)*ln(d+e*x^n)*p^
2-2/n*dilog(-e*x^n/d)*ln(d+e*x^n)*p^2+2/n*ln(-e*x^n/d)*ln((d+e*x^n)^p)*ln(
d+e*x^n)*p-2/n*ln(e*x^n)*ln((d+e*x^n)^p)*ln(d+e*x^n)*p-2/n*polylog(3,(d+e*
x^n)/d)*p^2+2/n*dilog(-e*x^n/d)*ln((d+e*x^n)^p)*p+1/n*ln(e*x^n)*ln((d+e*x^
n)^p)^2+(I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I*Pi*csgn(I*(d+e
*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-I*Pi*csgn(I*c*(d+e*x^n)^p)^3+I*Pi
*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+2*ln(c))/n*(ln(x^n)*ln((d+e*x^n)^p)-e*p
*(dilog((d+e*x^n)/d)/e+ln(x^n)*ln((d+e*x^n)/d)/e))+1/4*(I*Pi*csgn(I*(d+e*x
^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)
^p)*csgn(I*c)-I*Pi*csgn(I*c*(d+e*x^n)^p)^3+I*Pi*csgn(I*c*(d+e*x^n)^p)^2*cs
gn(I*c)+2*ln(c))^2*ln(x)
```

Fricas [F]

$$\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)^2}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)^2/x, x)
```

Sympy [F]

$$\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)^2}{x}\,dx$$

input
```
integrate(ln(c*(d+e*x**n)**p)**2/x,x)
```

output
```
Integral(log(c*(d + e*x**n)**p)**2/x, x)
```

Maxima [F]

$$\int \frac{\log^2 (c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^2}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="maxima")
```

output
```
log((e*x^n + d)^p)^2*log(x) - integrate(-(e*x^n*log(c)^2 + d*log(c)^2 - 2*
((e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p))/(e*x*x^n +
d*x), x)
```

Giac [F]

$$\int \frac{\log^2 (c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^2}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2 (c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^2}{x}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^2/x,x)
```

output
```
int(log(c*(d + e*x^n)^p)^2/x, x)
```

Reduce [F]

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{3\left(\int \frac{\log((x^n e+d)^p c)^2}{x^n ex+dx}dx\right)dnp + \log((x^n e+d)^p\,c)^3}{3np}$$

input
```
int(log(c*(d+e*x^n)^p)^2/x,x)
```

output
```
(3*int(log((x**n*e + d)**p*c)**2/(x**n*e*x + d*x),x)*d*n*p + log((x**n*e +
 d)**p*c)**3)/(3*n*p)
```

3.175 $\int \frac{\log^3(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int \frac{\log^3(c(d+ex^n)^p)}{x} dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log^3(c(d+ex^n)^p)}{n} + \frac{3p\log^2(c(d+ex^n)^p)\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n} - \frac{6p^2\log(c(d+ex^n)^p)\,\text{PolyLog}\left(3,1+\frac{ex^n}{d}\right)}{n} + \frac{6p^3\,\text{PolyLog}\left(4,1+\frac{ex^n}{d}\right)}{n}$$

output
```
ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^3/n+3*p*ln(c*(d+e*x^n)^p)^2*polylog(2,1+e*x
^n/d)/n-6*p^2*ln(c*(d+e*x^n)^p)*polylog(3,1+e*x^n/d)/n+6*p^3*polylog(4,1+e
*x^n/d)/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 270 vs. $2(113) = 226$.

Time = 0.18 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.39

$$\int \frac{\log^3\left(c(d+ex^n)^p\right)}{x}\,dx$$
$$= \frac{-np^3\log(x)\log^3(d+ex^n)+p^3\log\left(-\frac{ex^n}{d}\right)\log^3(d+ex^n)+3np^2\log(x)\log^2(d+ex^n)\log\left(c(d+ex^n)^p\right)}{}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]^3/x,x]
```

output
```
(-(n*p^3*Log[x]*Log[d + e*x^n]^3) + p^3*Log[-((e*x^n)/d)]*Log[d + e*x^n]^3
 + 3*n*p^2*Log[x]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*p^2*Log[-((e*x
^n)/d)]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*n*p*Log[x]*Log[d + e*x^n
]*Log[c*(d + e*x^n)^p]^2 + 3*p*Log[-((e*x^n)/d)]*Log[d + e*x^n]*Log[c*(d +
 e*x^n)^p]^2 + n*Log[x]*Log[c*(d + e*x^n)^p]^3 + 3*p*Log[c*(d + e*x^n)^p]^
2*PolyLog[2, 1 + (e*x^n)/d] - 6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (e
*x^n)/d] + 6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2904, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c(d+ex^n)^p\right)}{x}\,dx$$
$$\Big\downarrow \text{ 2904}$$
$$\frac{\int x^{-n}\log^3\left(c(ex^n+d)^p\right)dx^n}{n}$$
$$\Big\downarrow \text{ 2843}$$

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)-3ep\int\frac{\log\left(-\frac{ex^n}{d}\right)\log^2(c(ex^n+d)^p)}{ex^n+d}dx^n}{n}$$

$$\Big\downarrow \; 2881$$

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)-3p\int x^{-n}\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(ex^n+d)^p\right)d(ex^n+d)}{n}$$

$$\Big\downarrow \; 2821$$

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)-3p\left(2p\int x^{-n}\log\left(c(ex^n+d)^p\right)\text{PolyLog}\left(2,\frac{ex^n+d}{d}\right)d(ex^n+d)-\text{PolyLog}\left(2,\frac{ex^n+}{d}\right.\right.}{n}$$

$$\Big\downarrow \; 2830$$

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)-3p\left(2p\left(\text{PolyLog}\left(3,\frac{ex^n+d}{d}\right)\log\left(c(d+ex^n)^p\right)-p\int x^{-n}\text{PolyLog}\left(3,\frac{ex^n+d}{d}\right)d(ex^n\right.\right.}{n}$$

$$\Big\downarrow \; 7143$$

$$\frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)-3p\left(2p\left(\text{PolyLog}\left(3,\frac{ex^n+d}{d}\right)\log\left(c(d+ex^n)^p\right)-p\,\text{PolyLog}\left(4,\frac{ex^n+d}{d}\right)\right)-\text{PolyLog}\right.}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]^3/x,x]
```

output
```
(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^3 - 3*p*(-(Log[c*(d + e*x^n)^p]^2*
PolyLog[2, (d + e*x^n)/d]) + 2*p*(Log[c*(d + e*x^n)^p]*PolyLog[3, (d + e*x
^n)/d] - p*PolyLog[4, (d + e*x^n)/d])))/n
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.04 (sec) , antiderivative size = 1409, normalized size of antiderivative = 12.47

method	result	size
risch	Expression too large to display	1409

input
```
int(ln(c*(d+e*x^n)^p)^3/x,x,method=_RETURNVERBOSE)
```

output
```
3/n*ln(d+e*x^n)^3*ln(-e*x^n/d)*p^3-2/n*ln(d+e*x^n)^3*ln(1-(d+e*x^n)/d)*p^3
-1/n*ln(d+e*x^n)^3*ln(e*x^n)*p^3-6/n*ln((d+e*x^n)^p)*ln(d+e*x^n)^2*ln(-e*x
^n/d)*p^2+3/n*ln((d+e*x^n)^p)*ln(d+e*x^n)^2*ln(1-(d+e*x^n)/d)*p^2+3/n*ln((
d+e*x^n)^p)*ln(d+e*x^n)^2*ln(e*x^n)*p^2+3/n*ln(d+e*x^n)^2*dilog(-e*x^n/d)*
p^3-3/n*ln(d+e*x^n)^2*polylog(2,(d+e*x^n)/d)*p^3+3/n*ln((d+e*x^n)^p)^2*ln(
d+e*x^n)*ln(-e*x^n/d)*p-3/n*ln((d+e*x^n)^p)^2*ln(d+e*x^n)*ln(e*x^n)*p-6/n*
ln((d+e*x^n)^p)*ln(d+e*x^n)*dilog(-e*x^n/d)*p^2+6/n*ln((d+e*x^n)^p)*ln(d+e
*x^n)*polylog(2,(d+e*x^n)/d)*p^2+1/n*ln((d+e*x^n)^p)^3*ln(e*x^n)+3/n*ln((d
+e*x^n)^p)^2*dilog(-e*x^n/d)*p-6/n*ln((d+e*x^n)^p)*polylog(3,(d+e*x^n)/d)*
p^2+6/n*polylog(4,(d+e*x^n)/d)*p^3+1/8*(I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*
(d+e*x^n)^p)^2-I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-I*
Pi*csgn(I*c*(d+e*x^n)^p)^3+I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+2*ln(c))
^3*ln(x)+(3/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-3/2*I*Pi*cs
gn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-3/2*I*Pi*csgn(I*c*(d+e*x
^n)^p)^3+3/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+3*ln(c))/n*((ln((d+e*x
^n)^p)-p*ln(d+e*x^n))^2*ln(e*x^n)+p^2*(ln(d+e*x^n)^2*ln(1-(d+e*x^n)/d)+2*l
n(d+e*x^n)*polylog(2,(d+e*x^n)/d)-2*polylog(3,(d+e*x^n)/d))+2*p*(ln((d+e*x
^n)^p)-p*ln(d+e*x^n))*(dilog(-e*x^n/d)+ln(d+e*x^n)*ln(-e*x^n/d)))+(-3/4*Pi
^2*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*x^n)^p)^4+3/2*Pi^2*csgn(I*(d+e*x^n)
^p)^2*csgn(I*c*(d+e*x^n)^p)^3*csgn(I*c)-3/4*Pi^2*csgn(I*(d+e*x^n)^p)^2*...
```

Fricas [F]

$$\int \frac{\log^3\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^3}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)^3/x, x)
```

Sympy [F]

$$\int \frac{\log^3\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)^3}{x}\,dx$$

input
```
integrate(ln(c*(d+e*x**n)**p)**3/x,x)
```

output
```
Integral(log(c*(d + e*x**n)**p)**3/x, x)
```

Maxima [F]

$$\int \frac{\log^3\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^3}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="maxima")
```

output
```
log((e*x^n + d)^p)^3*log(x) - integrate(-(e*x^n*log(c)^3 + d*log(c)^3 - 3*
((e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p)^2 + 3*(e*x^n
*log(c)^2 + d*log(c)^2)*log((e*x^n + d)^p))/(e*x*x^n + d*x), x)
```

Giac [F]

$$\int \frac{\log^3\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^3}{x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^3}{x}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^3/x,x)
```

output
```
int(log(c*(d + e*x^n)^p)^3/x, x)
```

Reduce [F]

$$\int \frac{\log^3(c(d+ex^n)^p)}{x}\,dx = \frac{4\left(\int \frac{\log((x^n e+d)^p c)^3}{x^n e x+dx}dx\right)dnp+\log((x^n e+d)^p\,c)^4}{4np}$$

input
```
int(log(c*(d+e*x^n)^p)^3/x,x)
```

output
```
(4*int(log((x**n*e + d)**p*c)**3/(x**n*e*x + d*x),x)*d*n*p + log((x**n*e +
 d)**p*c)**4)/(4*n*p)
```

3.176 $\int (d+ex)^3 \log \left(c(a+bx)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\int (d+ex)^3 \log \left(c(a+bx)^p\right) \, dx = -\frac{(bd-ae)^3 px}{4b^3} - \frac{(bd-ae)^2 p(d+ex)^2}{8b^2 e} - \frac{(bd-ae)p(d+ex)^3}{12be} - \frac{p(d+ex)^4}{16e} - \frac{(bd-ae)^4 p \log(a+bx)}{4b^4 e} + \frac{(d+ex)^4 \log \left(c(a+bx)^p\right)}{4e}$$

output
```
-1/4*(-a*e+b*d)^3*p*x/b^3-1/8*(-a*e+b*d)^2*p*(e*x+d)^2/b^2/e-1/12*(-a*e+b*
d)*p*(e*x+d)^3/b/e-1/16*p*(e*x+d)^4/e-1/4*(-a*e+b*d)^4*p*ln(b*x+a)/b^4/e+1
/4*(e*x+d)^4*ln(c*(b*x+a)^p)/e
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.32

$$\int (d+ex)^3 \log \left(c(a+bx)^p\right) \, dx = -\frac{bpx(-12a^3e^3 + 6a^2be^2(8d+ex) - 4ab^2e(18d^2+6dex+e^2x^2) + b^3(48d^3+36d^2ex+16de^2x^2+3e^3x^3$$

input
```
Integrate[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

output
```
-1/48*(b*p*x*(-12*a^3*e^3 + 6*a^2*b*e^2*(8*d + e*x) - 4*a*b^2*e*(18*d^2 +
6*d*e*x + e^2*x^2) + b^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3))
 + 12*a^2*e*(6*b^2*d^2 - 4*a*b*d*e + a^2*e^2)*p*Log[a + b*x] - 12*b^3*(4*a
*d^3 + b*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*Log[c*(a + b*x)^p]
)/b^4
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \log \left(c(a+bx)^p\right) \, dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(d+ex)^4 \log \left(c(a+bx)^p\right)}{4e}-\frac{bp \int \frac{(d+ex)^4}{a+bx}dx}{4e}$$

$$\Big\downarrow \text{49}$$

$$\frac{(d+ex)^4 \log \left(c(a+bx)^p\right)}{4e}-\frac{bp \int \left(\frac{(bd-ae)^4}{b^4(a+bx)}+\frac{e(bd-ae)^3}{b^4}+\frac{e(d+ex)(bd-ae)^2}{b^3}+\frac{e(d+ex)^2(bd-ae)}{b^2}+\frac{e(d+ex)^3}{b}\right)dx}{4e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^4 \log \left(c(a+bx)^p\right)}{4e}-\frac{bp \left(\frac{(bd-ae)^4 \log(a+bx)}{b^5}+\frac{ex(bd-ae)^3}{b^4}+\frac{(d+ex)^2(bd-ae)^2}{2b^3}+\frac{(d+ex)^3(bd-ae)}{3b^2}+\frac{(d+ex)^4}{4b}\right)}{4e}$$

input
```
Int[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

output
```
-1/4*(b*p*((e*(b*d - a*e)^3*x)/b^4 + ((b*d - a*e)^2*(d + e*x)^2)/(2*b^3) +
 ((b*d - a*e)*(d + e*x)^3)/(3*b^2) + (d + e*x)^4/(4*b) + ((b*d - a*e)^4*Lo
g[a + b*x])/b^5))/e + ((d + e*x)^4*Log[c*(a + b*x)^p])/(4*e)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(128) = 256$.

Time = 1.72 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.03

method	result
parts	$\frac{\ln(c(bx+a)^p)e^3x^4}{4} + \ln\left(c(bx+a)^p\right) e^2 d\, x^3 + \frac{3\ln(c(bx+a)^p)e\,d^2x^2}{2} + d^3x\ln\left(c(bx+a)^p\right) + \frac{\ln(c(bx+a)^p}{4e}$
parallelrisch	$-\frac{-12x^4\ln(c(bx+a)^p)a\,b^4e^3+3x^4a\,b^4e^3p-48x^3\ln(c(bx+a)^p)a\,b^4d\,e^2-4x^3a^2b^3e^3p+16x^3a\,b^4d\,e^2p-72x^2\ln(c(bx+a)^p)a\,b^4c}{}$
risch	$-d^3px + \frac{d^3pa\ln(bx+a)}{b} - \frac{ie^3\pi\,x^4\,\mathrm{csgn}(ic(bx+a)^p)^3}{8} - \frac{i\pi\,d^3x\,\mathrm{csgn}(ic(bx+a)^p)^3}{2} - \frac{d\,e^2p\,x^3}{3} - \frac{3d^2ep\,x^2}{4} + \frac{e^3a^3p}{4b^3}$

input
```
int((e*x+d)^3*ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(c*(b*x+a)^p)*e^3*x^4+ln(c*(b*x+a)^p)*e^2*d*x^3+3/2*ln(c*(b*x+a)^p)*
e*d^2*x^2+d^3*x*ln(c*(b*x+a)^p)+1/4*ln(c*(b*x+a)^p)/e*d^4-1/4/e*p*b*(-e/b^
4*(-1/4*b^3*e^3*x^4+1/3*((a*e-2*b*d)*b^2*e^2-2*b^3*e^2*d)*x^3+1/2*(2*(a*e-
2*b*d)*d*e*b^2-b*e*(a^2*e^2-2*a*b*d*e+2*b^2*d^2))*x^2+x*(a*e-2*b*d)*(a^2*e
^2-2*a*b*d*e+2*b^2*d^2))+(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*
d^3*e+b^4*d^4)/b^5*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(128) = 256$.

Time = 0.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.92

$$\int (d+ex)^3\log\left(c(a+bx)^p\right)\,dx = -\frac{3\,b^4e^3px^4+4\,(4\,b^4de^2-ab^3e^3)px^3+6\,(6\,b^4d^2e-4\,ab^3de^2+a^2b^2e^3)px^2+12\,(4\,b^4d^3-6\,ab^3d^2e+4\,a^2b$$

input
```
integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="fricas")
```

output
```
-1/48*(3*b^4*e^3*p*x^4 + 4*(4*b^4*d*e^2 - a*b^3*e^3)*p*x^3 + 6*(6*b^4*d^2*
e - 4*a*b^3*d*e^2 + a^2*b^2*e^3)*p*x^2 + 12*(4*b^4*d^3 - 6*a*b^3*d^2*e + 4
*a^2*b^2*d*e^2 - a^3*b*e^3)*p*x - 12*(b^4*e^3*p*x^4 + 4*b^4*d*e^2*p*x^3 +
6*b^4*d^2*e*p*x^2 + 4*b^4*d^3*p*x + (4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3
*b*d*e^2 - a^4*e^3)*p)*log(b*x + a) - 12*(b^4*e^3*x^4 + 4*b^4*d*e^2*x^3 +
6*b^4*d^2*e*x^2 + 4*b^4*d^3*x)*log(c))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(117) = 234$.

Time = 0.97 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.39

$$\int (d+ex)^3\log\left(c(a+bx)^p\right)\,dx = \begin{cases} -\frac{a^4e^3\log\left(c(a+bx)^p\right)}{4b^4}+\frac{a^3de^2\log\left(c(a+bx)^p\right)}{b^3}+\frac{a^3e^3px}{4b^3}-\frac{3a^2d^2e\log\left(c(a+bx)^p\right)}{2b^2}-\frac{a^2de^2px}{b^2}-\frac{a^2e^3px^2}{8b^2}+\frac{ad^3\log\left(c(a+bx)^p\right)}{b}+ \\ \left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)\log\left(a^pc\right) \end{cases}$$

input
```
integrate((e*x+d)**3*ln(c*(b*x+a)**p),x)
```

output
```
Piecewise((-a**4*e**3*log(c*(a + b*x)**p)/(4*b**4) + a**3*d*e**2*log(c*(a
+ b*x)**p)/b**3 + a**3*e**3*p*x/(4*b**3) - 3*a**2*d**2*e*log(c*(a + b*x)**
p)/(2*b**2) - a**2*d*e**2*p*x/b**2 - a**2*e**3*p*x**2/(8*b**2) + a*d**3*lo
g(c*(a + b*x)**p)/b + 3*a*d**2*e*p*x/(2*b) + a*d*e**2*p*x**2/(2*b) + a*e**
3*p*x**3/(12*b) - d**3*p*x + d**3*x*log(c*(a + b*x)**p) - 3*d**2*e*p*x**2/
4 + 3*d**2*e*x**2*log(c*(a + b*x)**p)/2 - d*e**2*p*x**3/3 + d*e**2*x**3*lo
g(c*(a + b*x)**p) - e**3*p*x**4/16 + e**3*x**4*log(c*(a + b*x)**p)/4, Ne(b
, 0)), ((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)*log(a**p*c)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.53

$$\int (d+ex)^3\log\left(c(a+bx)^p\right)\,dx =$$
$$-\frac{1}{48}\,bp\Bigg(\frac{3\,b^3e^3x^4+4\,(4\,b^3de^2-ab^2e^3)x^3+6\,(6\,b^3d^2e-4\,ab^2de^2+a^2be^3)x^2+12\,(4\,b^3d^3-6\,ab^2d^2e+4}{b^4}$$
$$+\frac{1}{4}\left(e^3x^4+4\,de^2x^3+6\,d^2ex^2+4\,d^3x\right)\log\left((bx+a)^pc\right)$$

input
```
integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="maxima")
```

output
```
-1/48*b*p*((3*b^3*e^3*x^4 + 4*(4*b^3*d*e^2 - a*b^2*e^3)*x^3 + 6*(6*b^3*d^2
*e - 4*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + 12*(4*b^3*d^3 - 6*a*b^2*d^2*e + 4*a^
2*b*d*e^2 - a^3*e^3)*x)/b^4 - 12*(4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*
d*e^2 - a^4*e^3)*log(b*x + a)/b^5) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*
x^2 + 4*d^3*x)*log((b*x + a)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(128) = 256$.

Time = 0.13 (sec) , antiderivative size = 573, normalized size of antiderivative = 4.09

$$\int (d+ex)^3 \log \left(c(a+bx)^p\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="giac")
```

output
```
(b*x + a)*d^3*p*log(b*x + a)/b + 3/2*(b*x + a)^2*d^2*e*p*log(b*x + a)/b^2
- 3*(b*x + a)*a*d^2*e*p*log(b*x + a)/b^2 + (b*x + a)^3*d*e^2*p*log(b*x + a
)/b^3 - 3*(b*x + a)^2*a*d*e^2*p*log(b*x + a)/b^3 + 3*(b*x + a)*a^2*d*e^2*p
*log(b*x + a)/b^3 + 1/4*(b*x + a)^4*e^3*p*log(b*x + a)/b^4 - (b*x + a)^3*a
*e^3*p*log(b*x + a)/b^4 + 3/2*(b*x + a)^2*a^2*e^3*p*log(b*x + a)/b^4 - (b*
x + a)*a^3*e^3*p*log(b*x + a)/b^4 - (b*x + a)*d^3*p/b - 3/4*(b*x + a)^2*d^
2*e*p/b^2 + 3*(b*x + a)*a*d^2*e*p/b^2 - 1/3*(b*x + a)^3*d*e^2*p/b^3 + 3/2*
(b*x + a)^2*a*d*e^2*p/b^3 - 3*(b*x + a)*a^2*d*e^2*p/b^3 - 1/16*(b*x + a)^4
*e^3*p/b^4 + 1/3*(b*x + a)^3*a*e^3*p/b^4 - 3/4*(b*x + a)^2*a^2*e^3*p/b^4 +
 (b*x + a)*a^3*e^3*p/b^4 + (b*x + a)*d^3*log(c)/b + 3/2*(b*x + a)^2*d^2*e*
log(c)/b^2 - 3*(b*x + a)*a*d^2*e*log(c)/b^2 + (b*x + a)^3*d*e^2*log(c)/b^3
 - 3*(b*x + a)^2*a*d*e^2*log(c)/b^3 + 3*(b*x + a)*a^2*d*e^2*log(c)/b^3 + 1
/4*(b*x + a)^4*e^3*log(c)/b^4 - (b*x + a)^3*a*e^3*log(c)/b^4 + 3/2*(b*x +
a)^2*a^2*e^3*log(c)/b^4 - (b*x + a)*a^3*e^3*log(c)/b^4
```

Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.49

$$\int (d+ex)^3 \log\left(c(a+bx)^p\right)\,dx$$
$$= \ln\left(c\left(a+b\,x\right)^p\right)\left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)$$
$$+x^2\left(\frac{a\left(d\,e^2\,p-\frac{a\,e^3\,p}{4\,b}\right)}{2\,b}-\frac{3\,d^2\,e\,p}{4}\right)$$
$$-x\left(d^3\,p+\frac{a\left(\frac{a\left(d\,e^2\,p-\frac{a\,e^3\,p}{4\,b}\right)}{b}-\frac{3\,d^2\,e\,p}{2}\right)}{b}\right)-x^3\left(\frac{d\,e^2\,p}{3}-\frac{a\,e^3\,p}{12\,b}\right)$$
$$-\frac{e^3\,p\,x^4}{16}-\frac{\ln\left(a+b\,x\right)\left(p\,a^4\,e^3-4\,p\,a^3\,b\,d\,e^2+6\,p\,a^2\,b^2\,d^2\,e-4\,p\,a\,b^3\,d^3\right)}{4\,b^4}$$

input
```
int(log(c*(a + b*x)^p)*(d + e*x)^3,x)
```

output
```
log(c*(a + b*x)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) + x
^2*((a*(d*e^2*p - (a*e^3*p)/(4*b)))/(2*b) - (3*d^2*e*p)/4) - x*(d^3*p + (a
*((a*(d*e^2*p - (a*e^3*p)/(4*b)))/b - (3*d^2*e*p)/2))/b) - x^3*((d*e^2*p)/
3 - (a*e^3*p)/(12*b)) - (e^3*p*x^4)/16 - (log(a + b*x)*(a^4*e^3*p - 4*a*b^
3*d^3*p - 4*a^3*b*d*e^2*p + 6*a^2*b^2*d^2*e*p))/(4*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.11

$$\int (d+ex)^3 \log\left(c(a+bx)^p\right)\,dx$$
$$= \frac{-12\log((bx+a)^p\,c)\,a^4e^3+48\log((bx+a)^p\,c)\,a^3bd\,e^2-72\log((bx+a)^p\,c)\,a^2b^2d^2e+48\log((bx+a)^p\,c)}{}$$

input
```
int((e*x+d)^3*log(c*(b*x+a)^p),x)
```

output
```
( - 12*log((a + b*x)**p*c)*a**4*e**3 + 48*log((a + b*x)**p*c)*a**3*b*d*e**
2 - 72*log((a + b*x)**p*c)*a**2*b**2*d**2*e + 48*log((a + b*x)**p*c)*a*b**
3*d**3 + 48*log((a + b*x)**p*c)*b**4*d**3*x + 72*log((a + b*x)**p*c)*b**4*
d**2*e*x**2 + 48*log((a + b*x)**p*c)*b**4*d*e**2*x**3 + 12*log((a + b*x)**
p*c)*b**4*e**3*x**4 + 12*a**3*b*e**3*p*x - 48*a**2*b**2*d*e**2*p*x - 6*a**
2*b**2*e**3*p*x**2 + 72*a*b**3*d**2*e*p*x + 24*a*b**3*d*e**2*p*x**2 + 4*a*
b**3*e**3*p*x**3 - 48*b**4*d**3*p*x - 36*b**4*d**2*e*p*x**2 - 16*b**4*d*e*
*2*p*x**3 - 3*b**4*e**3*p*x**4)/(48*b**4)
```

3.177 $\int (d+ex)^2 \log(c(a+bx)^p)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 112

$$\int (d+ex)^2 \log(c(a+bx)^p)\,dx = -\frac{(bd-ae)^2px}{3b^2} - \frac{(bd-ae)p(d+ex)^2}{6be} - \frac{p(d+ex)^3}{9e} - \frac{(bd-ae)^3p\log(a+bx)}{3b^3e} + \frac{(d+ex)^3\log(c(a+bx)^p)}{3e}$$

output
```
-1/3*(-a*e+b*d)^2*p*x/b^2-1/6*(-a*e+b*d)*p*(e*x+d)^2/b/e-1/9*p*(e*x+d)^3/e
-1/3*(-a*e+b*d)^3*p*ln(b*x+a)/b^3/e+1/3*(e*x+d)^3*ln(c*(b*x+a)^p)/e
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int (d+ex)^2 \log(c(a+bx)^p)\,dx = \frac{6a^2e(-3bd+ae)p\log(a+bx)+b(-px(6a^2e^2-3abe(6d+ex)+b^2(18d^2+9dex+2e^2x^2))+6b(3ad^2+}{18b^3}$$

input
```
Integrate[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

output
```
(6*a^2*e*(-3*b*d + a*e)*p*Log[a + b*x] + b*(-(p*x*(6*a^2*e^2 - 3*a*b*e*(6*
d + e*x) + b^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*b*(3*a*d^2 + b*x*(3*d^
2 + 3*d*e*x + e^2*x^2))*Log[c*(a + b*x)^p]))/(18*b^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{(d+ex)^3 \log \left(c(a+bx)^p\right)}{3e}-\frac{bp\int \frac{(d+ex)^3}{a+bx}dx}{3e}$$

$$\Big\downarrow \text{49}$$

$$\frac{(d+ex)^3 \log \left(c(a+bx)^p\right)}{3e}-\frac{bp\int \left(\frac{(bd-ae)^3}{b^3(a+bx)}+\frac{e(bd-ae)^2}{b^3}+\frac{e(d+ex)(bd-ae)}{b^2}+\frac{e(d+ex)^2}{b}\right)dx}{3e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^3 \log \left(c(a+bx)^p\right)}{3e}-\frac{bp\left(\frac{(bd-ae)^3\log(a+bx)}{b^4}+\frac{ex(bd-ae)^2}{b^3}+\frac{(d+ex)^2(bd-ae)}{2b^2}+\frac{(d+ex)^3}{3b}\right)}{3e}$$

input
```
Int[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

output
```
-1/3*(b*p*((e*(b*d - a*e)^2*x)/b^3 + ((b*d - a*e)*(d + e*x)^2)/(2*b^2) + (
d + e*x)^3/(3*b) + ((b*d - a*e)^3*Log[a + b*x])/b^4))/e + ((d + e*x)^3*Log
[c*(a + b*x)^p])/(3*e)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.64

method	result
parts	$\frac{\ln(c(bx+a)^p)e^2x^3}{3}+\ln\left(c(bx+a)^p\right)ed\,x^2+d^2x\ln\left(c(bx+a)^p\right)+\frac{\ln(c(bx+a)^p)d^3}{3e}-\frac{pb\left(\frac{e\left(\frac{1}{3}x^3b^2e^2-\frac{1}{2}ab\right.}{}\right.}{}$
parallelrisch	$\frac{6x^3\ln(c(bx+a)^p)b^3e^2-2x^3b^3e^2p+18x^2\ln(c(bx+a)^p)b^3de+3x^2a\,b^2e^2p-9x^2b^3dep+6\ln(bx+a)a^3e^2p-18\ln(bx+a)a^2bdep+3}{}$
risch	$-\frac{e^2a^2px}{3b^2}+\frac{e^2\ln(bx+a)a^3p}{3b^3}+\frac{e^2ap\,x^2}{6b}-\frac{dep\,x^2}{2}+\frac{d^2pa\ln(bx+a)}{b}+e\ln\left(c\right)d\,x^2-\frac{\ln(bx+a)d^3p}{3e}+\frac{eadpx}{b}-$

input
```
int((e*x+d)^2*ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(c*(b*x+a)^p)*e^2*x^3+ln(c*(b*x+a)^p)*e*d*x^2+d^2*x*ln(c*(b*x+a)^p)+
1/3*ln(c*(b*x+a)^p)/e*d^3-1/3/e*p*b*(e/b^3*(1/3*x^3*b^2*e^2-1/2*a*b*e^2*x^
2+3/2*d*e*b^2*x^2+x*a^2*e^2-3*a*b*d*e*x+3*b^2*x*d^2)+(-a^3*e^3+3*a^2*b*d*e
^2-3*a*b^2*d^2*e+b^3*d^3)/b^4*ln(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.54

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx =$$
$$-\frac{2\,b^3e^2px^3+3\,(3\,b^3de-ab^2e^2)px^2+6\,(3\,b^3d^2-3\,ab^2de+a^2be^2)px-6\,(b^3e^2px^3+3\,b^3depx^2+3\,b^3d^2p\ldots}{18\,b^3}$$

input
```
integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="fricas")
```

output
```
-1/18*(2*b^3*e^2*p*x^3 + 3*(3*b^3*d*e - a*b^2*e^2)*p*x^2 + 6*(3*b^3*d^2 -
3*a*b^2*d*e + a^2*b*e^2)*p*x - 6*(b^3*e^2*p*x^3 + 3*b^3*d*e*p*x^2 + 3*b^3*
d^2*p*x + (3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*p)*log(b*x + a) - 6*(b^3*e
^2*x^3 + 3*b^3*d*e*x^2 + 3*b^3*d^2*x)*log(c))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(94) = 188.

Time = 0.56 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.80

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx$$
$$= \begin{cases} \frac{a^3e^2\log(c(a+bx)^p)}{3b^3} - \frac{a^2de\log(c(a+bx)^p)}{b^2} - \frac{a^2e^2px}{3b^2} + \frac{ad^2\log(c(a+bx)^p)}{b} + \frac{adepx}{b} + \frac{ae^2px^2}{6b} - d^2px + d^2x\log(c(a+bx\ldots \\ \left(d^2x + dex^2 + \frac{e^2x^3}{3}\right)\log(a^pc) \end{cases}$$

input
```
integrate((e*x+d)**2*ln(c*(b*x+a)**p),x)
```

output
```
Piecewise((a**3*e**2*log(c*(a + b*x)**p)/(3*b**3) - a**2*d*e*log(c*(a + b*
x)**p)/b**2 - a**2*e**2*p*x/(3*b**2) + a*d**2*log(c*(a + b*x)**p)/b + a*d*
e*p*x/b + a*e**2*p*x**2/(6*b) - d**2*p*x + d**2*x*log(c*(a + b*x)**p) - d*
e*p*x**2/2 + d*e*x**2*log(c*(a + b*x)**p) - e**2*p*x**3/9 + e**2*x**3*log(
c*(a + b*x)**p)/3, Ne(b, 0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)*log(a**p*
c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx = -\frac{1}{18} bp \left(\frac{2 b^2 e^2 x^3 + 3 (3 b^2 de - abe^2) x^2 + 6 (3 b^2 d^2 - 3 abde + a^2 e^2) x}{b^3} - \frac{6 (3 ab^2 d^2 - 3 a^2 bde + a^3 e^2) \log}{b^4} \right. + \frac{1}{3} \left(e^2 x^3 + 3 dex^2 + 3 d^2 x\right) \log \left((bx+a)^p c\right)$$

input
```
integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="maxima")
```

output
```
-1/18*b*p*((2*b^2*e^2*x^3 + 3*(3*b^2*d*e - a*b*e^2)*x^2 + 6*(3*b^2*d^2 - 3
*a*b*d*e + a^2*e^2)*x)/b^3 - 6*(3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*log(b
*x + a)/b^4) + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x + a)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(102) = 204.

Time = 0.12 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.82

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx = \frac{(bx+a)d^2p\log(bx+a)}{b} + \frac{(bx+a)^2dep\log(bx+a)}{b^2} - \frac{2(bx+a)adep\log(bx+a)}{b^2} + \frac{(bx+a)^3e^2p\log(bx+a)}{3b^3} - \frac{(bx+a)^2ae^2p\log(bx+a)}{b^3} + \frac{(bx+a)a^2e^2p\log(bx+a)}{b^3} - \frac{(bx+a)d^2p}{b} - \frac{(bx+a)^2dep}{2b^2} + \frac{2(bx+a)adep}{b^2} - \frac{(bx+a)^3e^2p}{9b^3} + \frac{(bx+a)^2ae^2p}{2b^3} - \frac{(bx+a)a^2e^2p}{b^3} + \frac{(bx+a)d^2\log(c)}{b} + \frac{(bx+a)^2de\log(c)}{b^2} - \frac{2(bx+a)ade\log(c)}{b^2} + \frac{(bx+a)^3e^2\log(c)}{3b^3} - \frac{(bx+a)^2ae^2\log(c)}{b^3} + \frac{(bx+a)a^2e^2\log(c)}{b^3}$$

input
```
integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="giac")
```

output
```
(b*x + a)*d^2*p*log(b*x + a)/b + (b*x + a)^2*d*e*p*log(b*x + a)/b^2 - 2*(b
*x + a)*a*d*e*p*log(b*x + a)/b^2 + 1/3*(b*x + a)^3*e^2*p*log(b*x + a)/b^3
- (b*x + a)^2*a*e^2*p*log(b*x + a)/b^3 + (b*x + a)*a^2*e^2*p*log(b*x + a)/
b^3 - (b*x + a)*d^2*p/b - 1/2*(b*x + a)^2*d*e*p/b^2 + 2*(b*x + a)*a*d*e*p/
b^2 - 1/9*(b*x + a)^3*e^2*p/b^3 + 1/2*(b*x + a)^2*a*e^2*p/b^3 - (b*x + a)*
a^2*e^2*p/b^3 + (b*x + a)*d^2*log(c)/b + (b*x + a)^2*d*e*log(c)/b^2 - 2*(b
*x + a)*a*d*e*log(c)/b^2 + 1/3*(b*x + a)^3*e^2*log(c)/b^3 - (b*x + a)^2*a*
e^2*log(c)/b^3 + (b*x + a)*a^2*e^2*log(c)/b^3
```

Mupad [B] (verification not implemented)

Time = 25.73 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.17

$$\int (d+ex)^2 \log(c(a+bx)^p)\, dx = \ln(c\,(a+b\,x)^p)\,\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right) - x^2\left(\frac{d\,e\,p}{2}-\frac{a\,e^2\,p}{6\,b}\right) - x\left(d^2\,p-\frac{a\left(d\,e\,p-\frac{a\,e^2\,p}{3\,b}\right)}{b}\right) - \frac{e^2\,p\,x^3}{9} + \frac{\ln(a+b\,x)\,\left(p\,a^3\,e^2-3\,p\,a^2\,b\,d\,e+3\,p\,a\,b^2\,d^2\right)}{3\,b^3}$$

input
```
int(log(c*(a + b*x)^p)*(d + e*x)^2,x)
```

output
```
log(c*(a + b*x)^p)*(d^2*x + (e^2*x^3)/3 + d*e*x^2) - x^2*((d*e*p)/2 - (a*e
^2*p)/(6*b)) - x*(d^2*p - (a*(d*e*p - (a*e^2*p)/(3*b)))/b) - (e^2*p*x^3)/9
 + (log(a + b*x)*(a^3*e^2*p + 3*a*b^2*d^2*p - 3*a^2*b*d*e*p))/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.68

$$\int (d+ex)^2 \log(c(a+bx)^p)\, dx = \frac{6\log((bx+a)^p c)\,a^3e^2 - 18\log((bx+a)^p c)\,a^2bde + 18\log((bx+a)^p c)\,a\,b^2d^2 + 18\log((bx+a)^p c)\,b^3d^2x}{}$$

input
```
int((e*x+d)^2*log(c*(b*x+a)^p),x)
```

output
```
(6*log((a + b*x)**p*c)*a**3*e**2 - 18*log((a + b*x)**p*c)*a**2*b*d*e + 18*
log((a + b*x)**p*c)*a*b**2*d**2 + 18*log((a + b*x)**p*c)*b**3*d**2*x + 18*
log((a + b*x)**p*c)*b**3*d*e*x**2 + 6*log((a + b*x)**p*c)*b**3*e**2*x**3 -
 6*a**2*b*e**2*p*x + 18*a*b**2*d*e*p*x + 3*a*b**2*e**2*p*x**2 - 18*b**3*d*
*2*p*x - 9*b**3*d*e*p*x**2 - 2*b**3*e**2*p*x**3)/(18*b**3)
```

3.178 $\int (d+ex)\log \left (c(a+bx)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int (d+ex)\log \left (c(a+bx)^p\right) \, dx=-\frac {(bd-ae)px}{2b}-\frac {p(d+ex)^2}{4e}-\frac {(bd-ae)^2p\log (a+bx)}{2b^2e}+\frac {(d+ex)^2\log \left (c(a+bx)^p\right)}{2e}$$

output
```
-1/2*(-a*e+b*d)*p*x/b-1/4*p*(e*x+d)^2/e-1/2*(-a*e+b*d)^2*p*ln(b*x+a)/b^2/e
+1/2*(e*x+d)^2*ln(c*(b*x+a)^p)/e
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int (d+ex)\log \left (c(a+bx)^p\right) \, dx=-dpx-\frac {1}{2}ep\left (-\frac {ax}{b}+\frac {x^2}{2}+\frac {a^2\log (a+bx)}{b^2}\right)+\frac {1}{2}ex^2\log \left (c(a+bx)^p\right)+\frac {d(a+bx)\log \left (c(a+bx)^p\right)}{b}$$

input
```
Integrate[(d + e*x)*Log[c*(a + b*x)^p],x]
```

output
```
-(d*p*x) - (e*p*(-((a*x)/b) + x^2/2 + (a^2*Log[a + b*x])/b^2))/2 + (e*x^2*
Log[c*(a + b*x)^p])/2 + (d*(a + b*x)*Log[c*(a + b*x)^p])/b
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\log \left(c(a+bx)^p\right) \, dx$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{(d+ex)^2\log \left(c(a+bx)^p\right)}{2e}-\frac{bp\int \frac{(d+ex)^2}{a+bx}dx}{2e}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{(d+ex)^2\log \left(c(a+bx)^p\right)}{2e}-\frac{bp\int \left(\frac{(bd-ae)^2}{b^2(a+bx)}+\frac{e(bd-ae)}{b^2}+\frac{e(d+ex)}{b}\right)dx}{2e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(d+ex)^2\log \left(c(a+bx)^p\right)}{2e}-\frac{bp\left(\frac{(bd-ae)^2\log(a+bx)}{b^3}+\frac{ex(bd-ae)}{b^2}+\frac{(d+ex)^2}{2b}\right)}{2e}$$

input
```
Int[(d + e*x)*Log[c*(a + b*x)^p],x]
```

output
```
-1/2*(b*p*((e*(b*d - a*e)*x)/b^2 + (d + e*x)^2/(2*b) + ((b*d - a*e)^2*Log[
a + b*x])/b^3))/e + ((d + e*x)^2*Log[c*(a + b*x)^p])/(2*e)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result
parts	$\frac{\ln(c(bx+a)^p)e\,x^2}{2}+dx\ln\left(c(bx+a)^p\right)-\frac{pb\left(-\frac{-\frac{1}{2}be\,x^2+aex-2bdx}{b^2}+\frac{a(ea-2bd)\ln(bx+a)}{b^3}\right)}{2}$
norman	$dx\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)-\frac{ep\,x^2}{4}+\frac{x^2e\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)}{2}+\frac{p(ea-2bd)x}{2b}-\frac{p\left(a^2e-2dab\right)\ln(bx+a)}{2b^2}$
default	$dx\ln\left(c(bx+a)^p\right)-xdp+\frac{dpa\ln(bx+a)}{b}+\frac{x^2e\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)}{2}-\frac{ep\,x^2}{4}-\frac{p\,a^2e\ln(bx+a)}{2b^2}+\frac{eapx}{2b}$
parallelrisch	$-\frac{-2x^2\ln(c(bx+a)^p)b^2e+b^2ep\,x^2+2\ln(bx+a)a^2ep-8\ln(bx+a)abdp-4x\ln(c(bx+a)^p)b^2d-2abepx+4b^2dpx+4\ln(c(bx+a)^p}{4b^2}$
risch	$\left(\frac{1}{2}e\,x^2+dx\right)\ln\left((bx+a)^p\right)-\frac{i\pi e\,x^2\,\mathrm{csgn}(ic(bx+a)^p)^3}{4}+\frac{i\pi dx\,\mathrm{csgn}(ic(bx+a)^p)^2\,\mathrm{csgn}(ic)}{2}+\frac{i\pi e\,x^2\,\mathrm{csgn}(ic(bx}{4}$

input
```
int((e*x+d)*ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*(b*x+a)^p)*e*x^2+d*x*ln(c*(b*x+a)^p)-1/2*p*b*(-1/b^2*(-1/2*b*e*x^
2+a*e*x-2*b*d*x)+a*(a*e-2*b*d)/b^3*ln(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08

$$\int (d+ex)\log\left(c(a+bx)^p\right)\,dx = -\frac{b^2epx^2+2\,(2\,b^2d-abe)px-2\,(b^2epx^2+2\,b^2dpx+(2\,abd-a^2e)p)\log\left(bx+a\right)-2\,(b^2ex^2+2\,b^2dx)\,\mathrm{lo}}{4\,b^2}$$

input
```
integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="fricas")
```

output
```
-1/4*(b^2*e*p*x^2 + 2*(2*b^2*d - a*b*e)*p*x - 2*(b^2*e*p*x^2 + 2*b^2*d*p*x
 + (2*a*b*d - a^2*e)*p)*log(b*x + a) - 2*(b^2*e*x^2 + 2*b^2*d*x)*log(c))/b
^2
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int (d+ex)\log\left(c(a+bx)^p\right)\,dx = \begin{cases} -\frac{a^2e\log\left(c(a+bx)^p\right)}{2b^2}+\frac{ad\log\left(c(a+bx)^p\right)}{b}+\frac{aepx}{2b}-dpx+dx\log\left(c(a+bx)^p\right)-\frac{epx^2}{4}+\frac{ex^2\log\left(c(a+bx)^p\right)}{2} & \text{for } b\neq 0 \\ \left(dx+\frac{ex^2}{2}\right)\log\left(a^pc\right) & \text{otherwis} \end{cases}$$

input
```
integrate((e*x+d)*ln(c*(b*x+a)**p),x)
```

output
```
Piecewise((-a**2*e*log(c*(a + b*x)**p)/(2*b**2) + a*d*log(c*(a + b*x)**p)/
b + a*e*p*x/(2*b) - d*p*x + d*x*log(c*(a + b*x)**p) - e*p*x**2/4 + e*x**2*
log(c*(a + b*x)**p)/2, Ne(b, 0)), ((d*x + e*x**2/2)*log(a**p*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int (d+ex)\log \left(c(a+bx)^p\right)\,dx = -\frac{1}{4}\,bp\left(\frac{bex^2+2\,(2\,bd-ae)x}{b^2}-\frac{2\,(2\,abd-a^2e)\log (bx+a)}{b^3}\right) + \frac{1}{2}\left(ex^2+2\,dx\right)\log \left((bx+a)^pc\right)$$

input
```
integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="maxima")
```

output
```
-1/4*b*p*((b*e*x^2 + 2*(2*b*d - a*e)*x)/b^2 - 2*(2*a*b*d - a^2*e)*log(b*x
+ a)/b^3) + 1/2*(e*x^2 + 2*d*x)*log((b*x + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int (d+ex)\log \left(c(a+bx)^p\right)\,dx = \frac{(bx+a)dp\log (bx+a)}{b}+\frac{(bx+a)^2ep\log (bx+a)}{2\,b^2} - \frac{(bx+a)aep\log (bx+a)}{b^2}-\frac{(bx+a)dp}{b} - \frac{(bx+a)^2ep}{4\,b^2}+\frac{(bx+a)aep}{b^2}+\frac{(bx+a)d\log (c)}{b} + \frac{(bx+a)^2e\log (c)}{2\,b^2}-\frac{(bx+a)ae\log (c)}{b^2}$$

input
```
integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="giac")
```

output
```
(b*x + a)*d*p*log(b*x + a)/b + 1/2*(b*x + a)^2*e*p*log(b*x + a)/b^2 - (b*x
 + a)*a*e*p*log(b*x + a)/b^2 - (b*x + a)*d*p/b - 1/4*(b*x + a)^2*e*p/b^2 +
 (b*x + a)*a*e*p/b^2 + (b*x + a)*d*log(c)/b + 1/2*(b*x + a)^2*e*log(c)/b^2
 - (b*x + a)*a*e*log(c)/b^2
```

Mupad [B] (verification not implemented)

Time = 25.71 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int (d+ex)\log \left(c(a+bx)^p\right)\,dx = \ln \left(c\,{\left(a+b\,x\right)}^p\right)\,\left(\frac{e\,x^2}{2}+d\,x\right)-x\,\left(d\,p-\frac{a\,e\,p}{2\,b}\right)-\frac{e\,p\,x^2}{4}-\frac{\ln \left(a+b\,x\right)\,\left(a^2\,e\,p-2\,a\,b\,d\,p\right)}{2\,b^2}$$

input
```
int(log(c*(a + b*x)^p)*(d + e*x),x)
```

output
```
log(c*(a + b*x)^p)*(d*x + (e*x^2)/2) - x*(d*p - (a*e*p)/(2*b)) - (e*p*x^2)
/4 - (log(a + b*x)*(a^2*e*p - 2*a*b*d*p))/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.17

$$\int (d+ex)\log \left(c(a+bx)^p\right)\,dx = \frac{-2\log((bx+a)^p\,c)\,a^2e+4\log((bx+a)^p\,c)\,abd+4\log((bx+a)^p\,c)\,b^2dx+2\log((bx+a)^p\,c)\,b^2e\,x^2+2a}{4b^2}$$

input
```
int((e*x+d)*log(c*(b*x+a)^p),x)
```

output
```
( - 2*log((a + b*x)**p*c)*a**2*e + 4*log((a + b*x)**p*c)*a*b*d + 4*log((a
+ b*x)**p*c)*b**2*d*x + 2*log((a + b*x)**p*c)*b**2*e*x**2 + 2*a*b*e*p*x -
4*b**2*d*p*x - b**2*e*p*x**2)/(4*b**2)
```

3.179 $\int \log (c(a+bx)^p)\ dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \log (c(a+bx)^p)\ dx = -px + \frac{(a+bx)\log (c(a+bx)^p)}{b}$$

output
```
-p*x+(b*x+a)*ln(c*(b*x+a)^p)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \log (c(a+bx)^p)\ dx = -px + \frac{(a+bx)\log (c(a+bx)^p)}{b}$$

input
```
Integrate[Log[c*(a + b*x)^p],x]
```

output
```
-(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c(a+bx)^p\right) \, dx$$

$$\Big\downarrow \text{2836}$$

$$\frac{\int \log \left(c(a+bx)^p\right) d(a+bx)}{b}$$

$$\Big\downarrow \text{2732}$$

$$\frac{(a+bx)\log \left(c(a+bx)^p\right)-p(a+bx)}{b}$$

input
```
Int[Log[c*(a + b*x)^p],x]
```

output
```
(-(p*(a + b*x)) + (a + b*x)*Log[c*(a + b*x)^p])/b
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

method	result
norman	$x \ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)+\frac{pa\ln(bx+a)}{b}-px$
default	$x\ln\left(c(bx+a)^{p}\right)-pb\left(\frac{x}{b}-\frac{a\ln(bx+a)}{b^{2}}\right)$
parts	$x\ln\left(c(bx+a)^{p}\right)-pb\left(\frac{x}{b}-\frac{a\ln(bx+a)}{b^{2}}\right)$
parallelrisch	$\frac{x\ln(c(bx+a)^{p})abp-xab\,p^{2}+\ln(c(bx+a)^{p})a^{2}p}{abp}$
risch	$x\ln\left((bx+a)^{p}\right)+\frac{i\pi x\ \mathrm{csgn}(i(bx+a)^{p})\,\mathrm{csgn}(ic(bx+a)^{p})^{2}}{2}-\frac{i\pi x\ \mathrm{csgn}(i(bx+a)^{p})\,\mathrm{csgn}(ic(bx+a)^{p})\,\mathrm{csgn}(ic)}{2}-\frac{i\pi x\ \mathrm{cs}}{}$

input
```
int(ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*exp(p*ln(b*x+a)))+p*a/b*ln(b*x+a)-p*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \log\left(c(a+bx)^{p}\right)\,dx=-\frac{bpx-bx\log\left(c\right)-(bpx+ap)\log\left(bx+a\right)}{b}$$

input
```
integrate(log(c*(b*x+a)^p),x, algorithm="fricas")
```

output
```
-(b*p*x - b*x*log(c) - (b*p*x + a*p)*log(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \log \left(c(a+bx)^p\right)\,dx=\begin{cases}\frac{a\log\left(c(a+bx)^p\right)}{b}-px+x\log\left(c(a+bx)^p\right) & \text{for } b\neq 0\\ x\log\left(a^pc\right) & \text{otherwise}\end{cases}$$

input
```
integrate(ln(c*(b*x+a)**p),x)
```

output
```
Piecewise((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*x)**p), Ne(b, 0)
), (x*log(a**p*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \log \left(c(a+bx)^p\right)\,dx=-bp\left(\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}\right)+x\log\left((bx+a)^pc\right)$$

input
```
integrate(log(c*(b*x+a)^p),x, algorithm="maxima")
```

output
```
-b*p*(x/b - a*log(b*x + a)/b^2) + x*log((b*x + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \log \left(c(a+bx)^p\right)\,dx=\frac{(bx+a)p\log\left(bx+a\right)}{b}-\frac{(bx+a)p}{b}+\frac{(bx+a)\log\left(c\right)}{b}$$

input
```
integrate(log(c*(b*x+a)^p),x, algorithm="giac")
```

output
```
(b*x + a)*p*log(b*x + a)/b - (b*x + a)*p/b + (b*x + a)*log(c)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \log \left(c(a+bx)^p\right) \, dx = x \, \ln \left(c \left(a+b\,x\right)^p\right) - p\,x + \frac{a\,p\,\ln\left(a+b\,x\right)}{b}$$

input
```
int(log(c*(a + b*x)^p),x)
```

output
```
x*log(c*(a + b*x)^p) - p*x + (a*p*log(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \log \left(c(a+bx)^p\right) \, dx = \frac{\log\left((bx+a)^p\,c\right)a + \log\left((bx+a)^p\,c\right)bx - bpx}{b}$$

input
```
int(log(c*(b*x+a)^p),x)
```

output
```
(log((a + b*x)**p*c)*a + log((a + b*x)**p*c)*b*x - b*p*x)/b
```

3.180 $\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

output
```
ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e+p*polylog(2,-e*(b*x+a)/(-a*e+b*
d))/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\,\text{PolyLog}\left(2,\frac{e(a+bx)}{-bd+ae}\right)}{e}$$

input
```
Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(
a + b*x))/(-(b*d) + a*e)])/e
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+bx)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{2841}$$

$$\frac{\log \left(c(a+bx)^p\right) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{bp \int \frac{\log \left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}dx}{e}$$

$$\Big\downarrow \text{2840}$$

$$\frac{\log \left(c(a+bx)^p\right) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{p \int \frac{\log \left(\frac{e(a+bx)}{bd-ae}+1\right)}{a+bx}d(a+bx)}{e}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\log \left(c(a+bx)^p\right) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p \operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

input
```
Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e
*(a + b*x))/(b*d - a*e))])/e
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

method	result
parts	$\frac{\ln\left(c\left(bx+a\right)^{p}\right)\ln\left(ex+d\right)}{e}-\frac{pb\left(\frac{\operatorname{dilog}\left(\frac{\left(ex+d\right)b+ea-bd}{ea-bd}\right)}{b}+\frac{\ln\left(ex+d\right)\ln\left(\frac{\left(ex+d\right)b+ea-bd}{ea-bd}\right)}{b}\right)}{e}$
risch	$\frac{\ln\left(\left(bx+a\right)^{p}\right)\ln\left(ex+d\right)}{e}-\frac{p\operatorname{dilog}\left(\frac{\left(ex+d\right)b+ea-bd}{ea-bd}\right)}{e}-\frac{p\ln\left(ex+d\right)\ln\left(\frac{\left(ex+d\right)b+ea-bd}{ea-bd}\right)}{e}+\frac{\left(\frac{i\pi\ \operatorname{csgn}\left(i\left(bx+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(bx+a\right)^{p}\right)^{2}}{2}-\right.}{}$

input
```
int(ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(b*x+a)^p)/e*ln(e*x+d)-1/e*p*b*(dilog(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/
b+ln(e*x+d)*ln(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b)
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx$$

input
```
integrate(ln(c*(b*x+a)**p)/(e*x+d),x)
```

output
```
Integral(log(c*(a + b*x)**p)/(d + e*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(57) = 114$.

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.03

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{bp\left(\frac{\log(bx+a)\log(ex+d)}{b} - \frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{b}\right)}{e} - \frac{p\log\left(bx+a\right)\log\left(ex+d\right)}{e} + \frac{\log\left((bx+a)^p c\right)\log\left(ex+d\right)}{e}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
b*p*(log(b*x + a)*log(e*x + d)/b - (log(e*x + d)*log(-(b*e*x + b*d)/(b*d -
 a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/b)/e - p*log(b*x + a)*log(e
*x + d)/e + log((b*x + a)^p*c)*log(e*x + d)/e
```

Giac [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\int \frac{\log\left((bx+a)^pc\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\int \frac{\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x}\,dx$$

input
```
int(log(c*(a + b*x)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b*x)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx$$
$$=\frac{2\left(\int \frac{\log\left((bx+a)^pc\right)}{be\,x^2+aex+bdx+ad}dx\right)aep-2\left(\int \frac{\log\left((bx+a)^pc\right)}{be\,x^2+aex+bdx+ad}dx\right)bdp+\log\left((bx+a)^p\,c\right)^2}{2ep}$$

input
```
int(log(c*(b*x+a)^p)/(e*x+d),x)
```

output
```
(2*int(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*a*e*p - 2*i
nt(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*b*d*p + log((a
+ b*x)**p*c)**2)/(2*e*p)
```

3.181 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 68

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2} dx = \frac{bp\log(a+bx)}{e(bd-ae)} - \frac{\log\left(c(a+bx)^p\right)}{e(d+ex)} - \frac{bp\log(d+ex)}{e(bd-ae)}$$

output
```
b*p*ln(b*x+a)/e/(-a*e+b*d)-ln(c*(b*x+a)^p)/e/(e*x+d)-b*p*ln(e*x+d)/e/(-a*e
+b*d)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2} dx = \frac{\frac{bp\log(a+bx)}{bd-ae} - \frac{\log(c(a+bx)^p)}{d+ex} + \frac{bp\log(d+ex)}{-bd+ae}}{e}$$

input
```
Integrate[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
```

output
```
((b*p*Log[a + b*x])/(b*d - a*e) - Log[c*(a + b*x)^p]/(d + e*x) + (b*p*Log[
d + e*x])/(-(b*d) + a*e))/e
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2842, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(a+bx)^p)}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{bp \int \frac{1}{(a+bx)(d+ex)}dx}{e} - \frac{\log (c(a+bx)^p)}{e(d+ex)}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{bp \left(\frac{b \int \frac{1}{a+bx}dx}{bd-ae} - \frac{e \int \frac{1}{d+ex}dx}{bd-ae}\right)}{e} - \frac{\log (c(a+bx)^p)}{e(d+ex)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{bp \left(\frac{\log(a+bx)}{bd-ae} - \frac{\log(d+ex)}{bd-ae}\right)}{e} - \frac{\log (c(a+bx)^p)}{e(d+ex)}$$

input
```
Int[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
```

output
```
-(Log[c*(a + b*x)^p]/(e*(d + e*x))) + (b*p*(Log[a + b*x]/(b*d - a*e) - Log
[d + e*x]/(b*d - a*e)))/e
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

method	result
parts	$-\frac{\ln(c(bx+a)^{p})}{e(ex+d)}+\frac{pb\left(-\frac{\ln(bx+a)}{ea-bd}+\frac{\ln(ex+d)}{ea-bd}\right)}{e}$
parallelrisch	$-\frac{\ln(bx+a)x\,b^{2}ep-\ln(ex+d)x\,b^{2}ep+\ln(bx+a)b^{2}dp-\ln(ex+d)b^{2}dp+\ln(c(bx+a)^{p})abe-\ln(c(bx+a)^{p})b^{2}d}{(ea-bd)(ex+d)be}$
risch	$-\frac{\ln((bx+a)^{p})}{e(ex+d)}-\frac{i\pi ae\ \mathrm{csgn}(i(bx+a)^{p})\,\mathrm{csgn}(ic(bx+a)^{p})^{2}-i\pi ae\ \mathrm{csgn}(i(bx+a)^{p})\,\mathrm{csgn}(ic(bx+a)^{p})\,\mathrm{csgn}(ic)-i\pi ae\ \mathrm{csgn}(ic(bx}{}$

input
```
int(ln(c*(b*x+a)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(b*x+a)^p)/e/(e*x+d)+p*b/e*(-1/(a*e-b*d)*ln(b*x+a)+1/(a*e-b*d)*ln(e*
x+d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = \frac{(bepx+aep)\log(bx+a)-(bepx+bdp)\log(ex+d)-(bd-ae)\log(c)}{bd^2e-ade^2+(bde^2-ae^3)x}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
((b*e*p*x + a*e*p)*log(b*x + a) - (b*e*p*x + b*d*p)*log(e*x + d) - (b*d -
a*e)*log(c))/(b*d^2*e - a*d*e^2 + (b*d*e^2 - a*e^3)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(53) = 106$.

Time = 1.84 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.47

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = \begin{cases} \frac{\frac{a\log\left(c(a+bx)^p\right)}{b}-px+x\log\left(c(a+bx)^p\right)}{d^2} & \text{for } e=0 \\ -\frac{p}{de+e^2x}-\frac{\log\left(c\left(\frac{bd}{e}+bx\right)^p\right)}{de+e^2x} & \text{for } a=\frac{bd}{e} \\ -\frac{ae\log\left(c(a+bx)^p\right)}{ade^2+ae^3x-bd^2e-bde^2x}+\frac{bdp\log\left(\frac{d}{e}+x\right)}{ade^2+ae^3x-bd^2e-bde^2x}+\frac{bepx\log\left(\frac{d}{e}+x\right)}{ade^2+ae^3x-bd^2e-bde^2x}-\frac{bex\log\left(c(a+bx)^p\right)}{ade^2+ae^3x-bd^2e-bde^2x} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x+a)**p)/(e*x+d)**2,x)
```

output
```
Piecewise(((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*x)**p))/d**2, E
q(e, 0)), (-p/(d*e + e**2*x) - log(c*(b*d/e + b*x)**p)/(d*e + e**2*x), Eq(
a, b*d/e)), (-a*e*log(c*(a + b*x)**p)/(a*d*e**2 + a*e**3*x - b*d**2*e - b*
d*e**2*x) + b*d*p*log(d/e + x)/(a*d*e**2 + a*e**3*x - b*d**2*e - b*d*e**2*
x) + b*e*p*x*log(d/e + x)/(a*d*e**2 + a*e**3*x - b*d**2*e - b*d*e**2*x) -
b*e*x*log(c*(a + b*x)**p)/(a*d*e**2 + a*e**3*x - b*d**2*e - b*d*e**2*x), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{\log \left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = \frac{bp\left(\frac{\log(bx+a)}{bd-ae} - \frac{\log(ex+d)}{bd-ae}\right)}{e} - \frac{\log\left((bx+a)^p c\right)}{(ex+d)e}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
b*p*(log(b*x + a)/(b*d - a*e) - log(e*x + d)/(b*d - a*e))/e - log((b*x + a
)^p*c)/((e*x + d)*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{\log \left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = \frac{bp\log(bx+a)}{bde-ae^2} - \frac{bp\log(ex+d)}{bde-ae^2} - \frac{p\log(bx+a)}{e^2x+de} - \frac{\log(c)}{e^2x+de}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

output
```
b*p*log(b*x + a)/(b*d*e - a*e^2) - b*p*log(e*x + d)/(b*d*e - a*e^2) - p*lo
g(b*x + a)/(e^2*x + d*e) - log(c)/(e^2*x + d*e)
```

Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{\log \left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = -\frac{\ln\left(c\,(a+b\,x)^p\right)}{e\,\left(d+e\,x\right)} + \frac{b\,p\,\mathrm{atan}\left(\frac{a\,e\,1\mathrm{i}+b\,d\,1\mathrm{i}+b\,e\,x\,2\mathrm{i}}{a\,e-b\,d}\right)\,2\mathrm{i}}{a\,e^2-b\,d\,e}$$

input
```
int(log(c*(a + b*x)^p)/(d + e*x)^2,x)
```

output
```
(b*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*2i)/(a*e^2 - b*d*e) -
log(c*(a + b*x)^p)/(e*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.69

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx$$
$$= \frac{-\log(bx+a)\,adep-\log(bx+a)\,a\,e^2px+\log(ex+d)\,b\,d^2p+\log(ex+d)\,bdepx+\log((bx+a)^p\,c)\,a\,e^2x}{de\left(a\,e^2x-bdex+ade-b\,d^2\right)}$$

input
```
int(log(c*(b*x+a)^p)/(e*x+d)^2,x)
```

output
```
( - log(a + b*x)*a*d*e*p - log(a + b*x)*a*e**2*p*x + log(d + e*x)*b*d**2*p
 + log(d + e*x)*b*d*e*p*x + log((a + b*x)**p*c)*a*e**2*x - log((a + b*x)**
p*c)*b*d*e*x)/(d*e*(a*d*e + a*e**2*x - b*d**2 - b*d*e*x))
```

3.182 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^3} dx = \frac{bp}{2e(bd-ae)(d+ex)} + \frac{b^2p\log(a+bx)}{2e(bd-ae)^2} - \frac{\log(c(a+bx)^p)}{2e(d+ex)^2} - \frac{b^2p\log(d+ex)}{2e(bd-ae)^2}$$

output
```
1/2*b*p/e/(-a*e+b*d)/(e*x+d)+1/2*b^2*p*ln(b*x+a)/e/(-a*e+b*d)^2-1/2*ln(c*(
b*x+a)^p)/e/(e*x+d)^2-1/2*b^2*p*ln(e*x+d)/e/(-a*e+b*d)^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^3} dx = \frac{-\log(c(a+bx)^p) + \frac{bp(d+ex)(bd-ae+b(d+ex)\log(a+bx)-b(d+ex)\log(d+ex))}{(bd-ae)^2}}{2e(d+ex)^2}$$

input
```
Integrate[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
```

output
```
(-Log[c*(a + b*x)^p] + (b*p*(d + e*x)*(b*d - a*e + b*(d + e*x)*Log[a + b*x
] - b*(d + e*x)*Log[d + e*x]))/(b*d - a*e)^2)/(2*e*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{bp\int \frac{1}{(a+bx)(d+ex)^2}dx}{2e}-\frac{\log(c(a+bx)^p)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{bp\int \left(\frac{b^2}{(bd-ae)^2(a+bx)}-\frac{eb}{(bd-ae)^2(d+ex)}-\frac{e}{(bd-ae)(d+ex)^2}\right)dx}{2e}-\frac{\log(c(a+bx)^p)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bp\left(\frac{1}{(d+ex)(bd-ae)}+\frac{b\log(a+bx)}{(bd-ae)^2}-\frac{b\log(d+ex)}{(bd-ae)^2}\right)}{2e}-\frac{\log(c(a+bx)^p)}{2e(d+ex)^2}$$

input
```
Int[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
```

output
```
-1/2*Log[c*(a + b*x)^p]/(e*(d + e*x)^2) + (b*p*(1/((b*d - a*e)*(d + e*x))
+ (b*Log[a + b*x])/(b*d - a*e)^2 - (b*Log[d + e*x])/(b*d - a*e)^2))/(2*e)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{\ln\left(c \left(b x+a\right)^{p}\right)}{2 e \left(e x+d\right)^{2}}+\frac{p b \left(\frac{b \ln\left(b x+a\right)}{\left(e a-b d\right)^{2}}-\frac{1}{\left(e a-b d\right) \left(e x+d\right)}-\frac{b \ln\left(e x+d\right)}{\left(e a-b d\right)^{2}}\right)}{2 e}$
parallelrisch	$-\frac{-x\, b^{3} d\, e^{2} p-2 \ln\left(c \left(b x+a\right)^{p}\right) a\, b^{2} d\, e^{2}-\ln\left(b x+a\right) x^{2} b^{3} e^{3} p+\ln\left(e x+d\right) x^{2} b^{3} e^{3} p-b^{3} d^{2} e p+\ln\left(c \left(b x+a\right)^{p}\right) a^{2} b\, e^{3}+\ln\left(c \left(b x+a\right)^{p}\right) b^{3} d \cdots}{2 \left(a^{2} e^{2}-2 d e a b+d^{2} b^{2}\right) \left(e x+\cdots\right.}$
risch	$-\frac{\ln\left(\left(b x+a\right)^{p}\right)}{2 e \left(e x+d\right)^{2}}-\frac{2 a b\, e^{2} p x-2 b^{2} d e p x+2 \ln\left(e x+d\right) b^{2} d^{2} p-2 \ln\left(-b x-a\right) b^{2} d^{2} p-2 i \pi a b d e \,\mathrm{csgn}\left(i \left(b x+a\right)^{p}\right) \mathrm{csgn}\left(i c \left(b x+a\right)^{p}\right)^{2}-2 \cdots}{\cdots}$

input
```
int(ln(c*(b*x+a)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(b*x+a)^p)/e/(e*x+d)^2+1/2*p*b/e*(b/(a*e-b*d)^2*ln(b*x+a)-1/(a*e
-b*d)/(e*x+d)-b/(a*e-b*d)^2*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(97) = 194$.

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.25

$$\int \frac{\log \left(c(a+bx)^p\right)}{(d+ex)^3}\,dx = \frac{(b^2de-abe^2)px+(b^2d^2-abde)p+(b^2e^2px^2+2\,b^2depx+(2\,abde-a^2e^2)p)\log\left(bx+a\right)-(b^2e^2px^2+2}{2\,(b^2d^4e-2\,abd^3e^2+a^2d^2e^3+(b^2d^2e^3-2\,abde^4+a^2e^5)x^2+2\,(b^2d^3}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*((b^2*d*e - a*b*e^2)*p*x + (b^2*d^2 - a*b*d*e)*p + (b^2*e^2*p*x^2 + 2*
b^2*d*e*p*x + (2*a*b*d*e - a^2*e^2)*p)*log(b*x + a) - (b^2*e^2*p*x^2 + 2*b
^2*d*e*p*x + b^2*d^2*p)*log(e*x + d) - (b^2*d^2 - 2*a*b*d*e + a^2*e^2)*log
(c))/(b^2*d^4*e - 2*a*b*d^3*e^2 + a^2*d^2*e^3 + (b^2*d^2*e^3 - 2*a*b*d*e^4
 + a^2*e^5)*x^2 + 2*(b^2*d^3*e^2 - 2*a*b*d^2*e^3 + a^2*d*e^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(85) = 170$.

Time = 6.21 (sec) , antiderivative size = 1518, normalized size of antiderivative = 14.46

$$\int \frac{\log \left(c(a+bx)^p\right)}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate(ln(c*(b*x+a)**p)/(e*x+d)**3,x)
```

output
```
Piecewise(((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*x)**p))/d**3, E
q(e, 0)), (-p/(4*d**2*e + 8*d*e**2*x + 4*e**3*x**2) - 2*log(c*(b*d/e + b*x
)**p)/(4*d**2*e + 8*d*e**2*x + 4*e**3*x**2), Eq(a, b*d/e)), (-a**2*e**2*lo
g(c*(a + b*x)**p)/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 -
 4*a*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e +
 4*b**2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2) - a*b*d*e*p/(2*a**2*d**2*e**3
 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b*d**2*e**3*
x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b**2*d**2*e
**3*x**2) + 2*a*b*d*e*log(c*(a + b*x)**p)/(2*a**2*d**2*e**3 + 4*a**2*d*e**
4*x + 2*a**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*b*d*e**
4*x**2 + 2*b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2) - a*b
*e**2*p*x/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a*b*d
**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b**2*
d**3*e**2*x + 2*b**2*d**2*e**3*x**2) - b**2*d**2*p*log(d/e + x)/(2*a**2*d*
*2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b*d**
2*e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b**2
*d**2*e**3*x**2) + b**2*d**2*p/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**
2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2*
b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2) - 2*b**2*d*e*p*x
*log(d/e + x)/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - ...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

$$\int \frac{\log (c(a+bx)^p)}{(d+ex)^3}\,dx = \frac{bp\left(\frac{b\log(bx+a)}{b^2d^2-2\,abde+a^2e^2} - \frac{b\log(ex+d)}{b^2d^2-2\,abde+a^2e^2} + \frac{1}{bd^2-ade+(bde-ae^2)x}\right)}{2\,e} - \frac{\log\left((bx+a)^pc\right)}{2\,(ex+d)^2e}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*b*p*(b*log(b*x + a)/(b^2*d^2 - 2*a*b*d*e + a^2*e^2) - b*log(e*x + d)/(
b^2*d^2 - 2*a*b*d*e + a^2*e^2) + 1/(b*d^2 - a*d*e + (b*d*e - a*e^2)*x))/e
- 1/2*log((b*x + a)^p*c)/((e*x + d)^2*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.76

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}\,dx = \frac{b^2p\log(bx+a)}{2\,(b^2d^2e-2\,abde^2+a^2e^3)} - \frac{b^2p\log(ex+d)}{2\,(b^2d^2e-2\,abde^2+a^2e^3)} - \frac{p\log(bx+a)}{2\,(e^3x^2+2\,de^2x+d^2e)} + \frac{bepx+bdp-bd\log(c)+ae\log(c)}{2\,(bde^3x^2-ae^4x^2+2\,bd^2e^2x-2\,ade^3x+bd^3e-ad^2e^2)}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

output
```
1/2*b^2*p*log(b*x + a)/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) - 1/2*b^2*p*log
(e*x + d)/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) - 1/2*p*log(b*x + a)/(e^3*x^
2 + 2*d*e^2*x + d^2*e) + 1/2*(b*e*p*x + b*d*p - b*d*log(c) + a*e*log(c))/(
b*d*e^3*x^2 - a*e^4*x^2 + 2*b*d^2*e^2*x - 2*a*d*e^3*x + b*d^3*e - a*d^2*e^
2)
```

Mupad [B] (verification not implemented)

Time = 26.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}\,dx = -\frac{\ln\left(c\,(a+b\,x)^p\right)}{2\,e\,(d+e\,x)^2} - \frac{b\,p}{2\,e\,(a\,e-b\,d)\,(d+e\,x)} - \frac{b^2\,p\,\mathrm{atan}\left(\frac{a\,e\,1\mathrm{i}+b\,d\,1\mathrm{i}+b\,e\,x\,2\mathrm{i}}{a\,e-b\,d}\right)\,1\mathrm{i}}{e\,(a\,e-b\,d)^2}$$

input
```
int(log(c*(a + b*x)^p)/(d + e*x)^3,x)
```

output
```
- log(c*(a + b*x)^p)/(2*e*(d + e*x)^2) - (b*p)/(2*e*(a*e - b*d)*(d + e*x))
 - (b^2*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*1i)/(e*(a*e - b*d
)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 429, normalized size of antiderivative = 4.09

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^3}\,dx$$
$$= \frac{-2\log(bx+a)\,a^2d^2e^2p-4\log(bx+a)\,a^2d\,e^3px-2\log(bx+a)\,a^2e^4p\,x^2+4\log(bx+a)\,ab\,d^3ep+8\log(}{}$$

input
```
int(log(c*(b*x+a)^p)/(e*x+d)^3,x)
```

output
```
( - 2*log(a + b*x)*a**2*d**2*e**2*p - 4*log(a + b*x)*a**2*d*e**3*p*x - 2*l
og(a + b*x)*a**2*e**4*p*x**2 + 4*log(a + b*x)*a*b*d**3*e*p + 8*log(a + b*x
)*a*b*d**2*e**2*p*x + 4*log(a + b*x)*a*b*d*e**3*p*x**2 - 2*log(d + e*x)*b*
*2*d**4*p - 4*log(d + e*x)*b**2*d**3*e*p*x - 2*log(d + e*x)*b**2*d**2*e**2
*p*x**2 + 4*log((a + b*x)**p*c)*a**2*d*e**3*x + 2*log((a + b*x)**p*c)*a**2
*e**4*x**2 - 8*log((a + b*x)**p*c)*a*b*d**2*e**2*x - 4*log((a + b*x)**p*c)
*a*b*d*e**3*x**2 + 4*log((a + b*x)**p*c)*b**2*d**3*e*x + 2*log((a + b*x)**
p*c)*b**2*d**2*e**2*x**2 - a*b*d**3*e*p + a*b*d*e**3*p*x**2 + b**2*d**4*p
- b**2*d**2*e**2*p*x**2)/(4*d**2*e*(a**2*d**2*e**2 + 2*a**2*d*e**3*x + a**
2*e**4*x**2 - 2*a*b*d**3*e - 4*a*b*d**2*e**2*x - 2*a*b*d*e**3*x**2 + b**2*
d**4 + 2*b**2*d**3*e*x + b**2*d**2*e**2*x**2))
```

3.183 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^4} dx = \frac{bp}{6e(bd-ae)(d+ex)^2} + \frac{b^2p}{3e(bd-ae)^2(d+ex)} + \frac{b^3p\log(a+bx)}{3e(bd-ae)^3} - \frac{\log(c(a+bx)^p)}{3e(d+ex)^3} - \frac{b^3p\log(d+ex)}{3e(bd-ae)^3}$$

output
```
1/6*b*p/e/(-a*e+b*d)/(e*x+d)^2+1/3*b^2*p/e/(-a*e+b*d)^2/(e*x+d)+1/3*b^3*p*
ln(b*x+a)/e/(-a*e+b*d)^3-1/3*ln(c*(b*x+a)^p)/e/(e*x+d)^3-1/3*b^3*p*ln(e*x+
d)/e/(-a*e+b*d)^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^4} dx = \frac{-2\log(c(a+bx)^p) + \frac{bp(d+ex)((bd-ae)(3bd-ae+2bex)+2b^2(d+ex)^2\log(a+bx)-2b^2(d+ex)^2\log(d+ex))}{(bd-ae)^3}}{6e(d+ex)^3}$$

input
```
Integrate[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

output
```
(-2*Log[c*(a + b*x)^p] + (b*p*(d + e*x)*((b*d - a*e)*(3*b*d - a*e + 2*b*e*
x) + 2*b^2*(d + e*x)^2*Log[a + b*x] - 2*b^2*(d + e*x)^2*Log[d + e*x]))/(b*
d - a*e)^3)/(6*e*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^4}dx$$

$$\Big\downarrow \text{2842}$$

$$\frac{bp\int \frac{1}{(a+bx)(d+ex)^3}dx}{3e}-\frac{\log(c(a+bx)^p)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{bp\int \left(\frac{b^3}{(bd-ae)^3(a+bx)}-\frac{eb^2}{(bd-ae)^3(d+ex)}-\frac{eb}{(bd-ae)^2(d+ex)^2}-\frac{e}{(bd-ae)(d+ex)^3}\right)dx}{3e}-\frac{\log(c(a+bx)^p)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bp\left(\frac{b^2\log(a+bx)}{(bd-ae)^3}-\frac{b^2\log(d+ex)}{(bd-ae)^3}+\frac{b}{(d+ex)(bd-ae)^2}+\frac{1}{2(d+ex)^2(bd-ae)}\right)}{3e}-\frac{\log(c(a+bx)^p)}{3e(d+ex)^3}$$

input
```
Int[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

output
```
-1/3*Log[c*(a + b*x)^p]/(e*(d + e*x)^3) + (b*p*(1/(2*(b*d - a*e)*(d + e*x)
^2) + b/((b*d - a*e)^2*(d + e*x)) + (b^2*Log[a + b*x])/(b*d - a*e)^3 - (b^
2*Log[d + e*x])/(b*d - a*e)^3))/(3*e)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

method	result
parts	$-\frac{\ln\left(c \left(b x +a \right)^{p}\right)}{3 e \left(e x +d \right)^{3}}+\frac{p b \left(-\frac{b^{2} \ln\left(b x +a \right)}{\left(e a -b d \right)^{3}}-\frac{1}{2 \left(e a -b d \right) \left(e x +d \right)^{2}}+\frac{b^{2} \ln\left(e x +d \right)}{\left(e a -b d \right)^{3}}+\frac{b}{\left(e a -b d \right)^{2} \left(e x +d \right)}\right)}{3 e}$
parallelrisch	$-\frac{3 b^{4} d^{3} e^{2} p +2 \ln\left(c \left(b x +a \right)^{p}\right) a^{3} b \,e^{5}-2 \ln\left(c \left(b x +a \right)^{p}\right) b^{4} d^{3} e^{2}-2 x^{2} a \,b^{3} e^{5} p +2 x^{2} b^{4} d \,e^{4} p +x \,a^{2} b^{2} e^{5} p +5 x \,b^{4} d^{2} e^{3} p -6 \ln\left(c \left(b x +a \right)^{p}\right)}{}$
risch	$-\frac{\ln\left(\left(b x +a \right)^{p}\right)}{3 e \left(e x +d \right)^{3}}+\frac{6 a \,b^{2} d \,e^{2} p x -a^{2} b \,e^{3} p x -2 \ln\left(b x +a \right) b^{3} d^{3} p +2 a \,b^{2} e^{3} p \,x^{2}-2 b^{3} d \,e^{2} p \,x^{2}+2 \ln\left(-e x -d \right) b^{3} d^{3} p -a^{2} b d p \,e^{2}+4 a \,b^{2}}{}$

input
```
int(ln(c*(b*x+a)^p)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(c*(b*x+a)^p)/e/(e*x+d)^3+1/3*p*b/e*(-b^2/(a*e-b*d)^3*ln(b*x+a)-1/2
/(a*e-b*d)/(e*x+d)^2+b^2/(a*e-b*d)^3*ln(e*x+d)+b/(a*e-b*d)^2/(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(123) = 246.

Time = 0.09 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.33

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^4}\,dx = \frac{2\,(b^3de^2-ab^2e^3)px^2+(5\,b^3d^2e-6\,ab^2de^2+a^2be^3)px+(3\,b^3d^3-4\,ab^2d^2e+a^2bde^2)p+2\,(b^3e^3px^3+3\,b}{6\,(b^3d^6e-3\,ab^2d^5e^2+3\,a^2bd^4e^3-a^3d^3e^4+(b^3d^3e^4-3}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/6*(2*(b^3*d*e^2 - a*b^2*e^3)*p*x^2 + (5*b^3*d^2*e - 6*a*b^2*d*e^2 + a^2*
b*e^3)*p*x + (3*b^3*d^3 - 4*a*b^2*d^2*e + a^2*b*d*e^2)*p + 2*(b^3*e^3*p*x^
3 + 3*b^3*d*e^2*p*x^2 + 3*b^3*d^2*e*p*x + (3*a*b^2*d^2*e - 3*a^2*b*d*e^2 +
 a^3*e^3)*p)*log(b*x + a) - 2*(b^3*e^3*p*x^3 + 3*b^3*d*e^2*p*x^2 + 3*b^3*d
^2*e*p*x + b^3*d^3*p)*log(e*x + d) - 2*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*
d*e^2 - a^3*e^3)*log(c))/(b^3*d^6*e - 3*a*b^2*d^5*e^2 + 3*a^2*b*d^4*e^3 -
a^3*d^3*e^4 + (b^3*d^3*e^4 - 3*a*b^2*d^2*e^5 + 3*a^2*b*d*e^6 - a^3*e^7)*x^
3 + 3*(b^3*d^4*e^3 - 3*a*b^2*d^3*e^4 + 3*a^2*b*d^2*e^5 - a^3*d*e^6)*x^2 +
3*(b^3*d^5*e^2 - 3*a*b^2*d^4*e^3 + 3*a^2*b*d^3*e^4 - a^3*d^2*e^5)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4571 vs. 2(109) = 218.

Time = 18.40 (sec) , antiderivative size = 4571, normalized size of antiderivative = 34.37

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate(ln(c*(b*x+a)**p)/(e*x+d)**4,x)
```

output
```
Piecewise(((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*x)**p))/d**4, E
q(e, 0)), (-p/(9*d**3*e + 27*d**2*e**2*x + 27*d*e**3*x**2 + 9*e**4*x**3) -
 3*log(c*(b*d/e + b*x)**p)/(9*d**3*e + 27*d**2*e**2*x + 27*d*e**3*x**2 + 9
*e**4*x**3), Eq(a, b*d/e)), (-2*a**3*e**3*log(c*(a + b*x)**p)/(6*a**3*d**3
*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*
a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a
**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**
2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5
*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - a**2*b*d*e**2*
p/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e
**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e*
*5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**
3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e
- 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) +
6*a**2*b*d*e**2*log(c*(a + b*x)**p)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*
x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2
*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b
**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b*
*2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**
3*x**2 - 6*b**3*d**3*e**4*x**3) - a**2*b*e**3*p*x/(6*a**3*d**3*e**4 + 1...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.74

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^4}\,dx = \frac{\left(\frac{2\,b^2\log(bx+a)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3} - \frac{2\,b^2\log(ex+d)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3} + \frac{2\,bex+3\,bd-ae}{b^2d^4-2\,abd^3e+a^2d^2e^2+(b^2d^2e^2-2\,abde^3+a^2e^4)x^2+2\,(b^2d^3e-2\,abd}\right.}{6\,e} - \frac{\log\left((bx+a)^pc\right)}{3\,(ex+d)^3e}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/6*(2*b^2*log(b*x + a)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3
) - 2*b^2*log(e*x + d)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)
 + (2*b*e*x + 3*b*d - a*e)/(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2
*e^2 - 2*a*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e
^3)*x))*b*p/e - 1/3*log((b*x + a)^p*c)/((e*x + d)^3*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(123) = 246$.

Time = 0.12 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.74

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^4}\,dx = \frac{b^3p\log\left(bx+a\right)}{3\left(b^3d^3e-3\,ab^2d^2e^2+3\,a^2bde^3-a^3e^4\right)} - \frac{b^3p\log\left(ex+d\right)}{3\left(b^3d^3e-3\,ab^2d^2e^2+3\,a^2bde^3-a^3e^4\right)} - \frac{p\log\left(bx+a\right)}{3\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)} + \frac{2\,b^2e^2px^2+5\,b^2depx-abe^2px+3\,b^2d^2p-abdep-2\,b^2d^2\log\left(c\right)+4\,abde\log\left(c\right)}{6\left(b^2d^2e^4x^3-2\,abde^5x^3+a^2e^6x^3+3\,b^2d^3e^3x^2-6\,abd^2e^4x^2+3\,a^2de^5x^2+3\,b^2d^4e^2x-6\,abd^3e^3x+3\,a\right.}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/3*b^3*p*log(b*x + a)/(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*
e^4) - 1/3*b^3*p*log(e*x + d)/(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3
 - a^3*e^4) - 1/3*p*log(b*x + a)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^
3*e) + 1/6*(2*b^2*e^2*p*x^2 + 5*b^2*d*e*p*x - a*b*e^2*p*x + 3*b^2*d^2*p -
a*b*d*e*p - 2*b^2*d^2*log(c) + 4*a*b*d*e*log(c) - 2*a^2*e^2*log(c))/(b^2*d
^2*e^4*x^3 - 2*a*b*d*e^5*x^3 + a^2*e^6*x^3 + 3*b^2*d^3*e^3*x^2 - 6*a*b*d^2
*e^4*x^2 + 3*a^2*d*e^5*x^2 + 3*b^2*d^4*e^2*x - 6*a*b*d^3*e^3*x + 3*a^2*d^2
*e^4*x + b^2*d^5*e - 2*a*b*d^4*e^2 + a^2*d^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 26.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.09

$$\int \frac{\log (c(a+bx)^p)}{(d+ex)^4} \, dx = \frac{b^2\,p\,x}{3\,(a\,e-b\,d)^2\,(d+e\,x)^2} - \frac{\ln\left(c\,(a+b\,x)^p\right)}{3\,e\,(d+e\,x)^3} - \frac{a\,b\,p}{6\,(a\,e-b\,d)^2\,(d+e\,x)^2} + \frac{b^2\,d\,p}{2\,e\,(a\,e-b\,d)^2\,(d+e\,x)^2} + \frac{b^3\,p\,\mathrm{atan}\left(\frac{a\,e\,1\mathrm{i}+b\,d\,1\mathrm{i}+b\,e\,x\,2\mathrm{i}}{a\,e-b\,d}\right)\,2\mathrm{i}}{3\,e\,(a\,e-b\,d)^3}$$

input
```
int(log(c*(a + b*x)^p)/(d + e*x)^4,x)
```

output
```
(b^2*p*x)/(3*(a*e - b*d)^2*(d + e*x)^2) - log(c*(a + b*x)^p)/(3*e*(d + e*x
)^3) + (b^3*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*2i)/(3*e*(a*e
 - b*d)^3) - (a*b*p)/(6*(a*e - b*d)^2*(d + e*x)^2) + (b^2*d*p)/(2*e*(a*e -
 b*d)^2*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 898, normalized size of antiderivative = 6.75

$$\int \frac{\log (c(a+bx)^p)}{(d+ex)^4} \, dx = \text{Too large to display}$$

input
```
int(log(c*(b*x+a)^p)/(e*x+d)^4,x)
```

output

```
( - 6*log(a + b*x)*a**3*d**3*e**3*p - 18*log(a + b*x)*a**3*d**2*e**4*p*x -
 18*log(a + b*x)*a**3*d*e**5*p*x**2 - 6*log(a + b*x)*a**3*e**6*p*x**3 + 18
*log(a + b*x)*a**2*b*d**4*e**2*p + 54*log(a + b*x)*a**2*b*d**3*e**3*p*x +
54*log(a + b*x)*a**2*b*d**2*e**4*p*x**2 + 18*log(a + b*x)*a**2*b*d*e**5*p*
x**3 - 18*log(a + b*x)*a*b**2*d**5*e*p - 54*log(a + b*x)*a*b**2*d**4*e**2*
p*x - 54*log(a + b*x)*a*b**2*d**3*e**3*p*x**2 - 18*log(a + b*x)*a*b**2*d**
2*e**4*p*x**3 + 6*log(d + e*x)*b**3*d**6*p + 18*log(d + e*x)*b**3*d**5*e*p
*x + 18*log(d + e*x)*b**3*d**4*e**2*p*x**2 + 6*log(d + e*x)*b**3*d**3*e**3
*p*x**3 + 18*log((a + b*x)**p*c)*a**3*d**2*e**4*x + 18*log((a + b*x)**p*c)
*a**3*d*e**5*x**2 + 6*log((a + b*x)**p*c)*a**3*e**6*x**3 - 54*log((a + b*x
)**p*c)*a**2*b*d**3*e**3*x - 54*log((a + b*x)**p*c)*a**2*b*d**2*e**4*x**2
- 18*log((a + b*x)**p*c)*a**2*b*d*e**5*x**3 + 54*log((a + b*x)**p*c)*a*b**
2*d**4*e**2*x + 54*log((a + b*x)**p*c)*a*b**2*d**3*e**3*x**2 + 18*log((a +
 b*x)**p*c)*a*b**2*d**2*e**4*x**3 - 18*log((a + b*x)**p*c)*b**3*d**5*e*x -
 18*log((a + b*x)**p*c)*b**3*d**4*e**2*x**2 - 6*log((a + b*x)**p*c)*b**3*d
**3*e**3*x**3 - 3*a**2*b*d**4*e**2*p - 3*a**2*b*d**3*e**3*p*x + 10*a*b**2*
d**5*e*p + 12*a*b**2*d**4*e**2*p*x - 2*a*b**2*d**2*e**4*p*x**3 - 7*b**3*d*
*6*p - 9*b**3*d**5*e*p*x + 2*b**3*d**3*e**3*p*x**3)/(18*d**3*e*(a**3*d**3*
e**3 + 3*a**3*d**2*e**4*x + 3*a**3*d*e**5*x**2 + a**3*e**6*x**3 - 3*a**2*b
*d**4*e**2 - 9*a**2*b*d**3*e**3*x - 9*a**2*b*d**2*e**4*x**2 - 3*a**2*b*...
```

3.184 $\int (d+ex)^3 \log\left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 178

$$\begin{aligned}\int (d+ex)^3 \log\left(c(a+bx^2)^p\right) \, dx = & -\frac{2d(bd^2-ae^2)\,px}{b} - \frac{e(6bd^2-ae^2)\,px^2}{4b} - \frac{2}{3}de^2px^3 \\ & - \frac{1}{8}e^3px^4 + \frac{2\sqrt{a}d(bd^2-ae^2)\,p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} \\ & - \frac{(b^2d^4-6abd^2e^2+a^2e^4)\,p\log\left(a+bx^2\right)}{4b^2e} \\ & + \frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}\end{aligned}$$

output
```
-2*d*(-a*e^2+b*d^2)*p*x/b-1/4*e*(-a*e^2+6*b*d^2)*p*x^2/b-2/3*d*e^2*p*x^3-1
/8*e^3*p*x^4+2*a^(1/2)*d*(-a*e^2+b*d^2)*p*arctan(b^(1/2)*x/a^(1/2))/b^(3/2
)-1/4*(a^2*e^4-6*a*b*d^2*e^2+b^2*d^4)*p*ln(b*x^2+a)/b^2/e+1/4*(e*x+d)^4*ln
(c*(b*x^2+a)^p)/e
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.40

$$\int (d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$= \frac{-6\left(b^2d^4+4\sqrt{-a}b^{3/2}d^3e-6abd^2e^2+4(-a)^{3/2}\sqrt{b}de^3+a^2e^4\right)p\log\left(\sqrt{-a}-\sqrt{b}x\right)-6\left(b^2d^4-4\sqrt{-a}b^3\right.}{}$$

input
```
Integrate[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

output
```
(-6*(b^2*d^4 + 4*Sqrt[-a]*b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*(-a)^(3/2)*Sqr
t[b]*d*e^3 + a^2*e^4)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 6*(b^2*d^4 - 4*Sqrt[-a
]*b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*Sqrt[-a]*a*Sqrt[b]*d*e^3 + a^2*e^4)*p*
Log[Sqrt[-a] + Sqrt[b]*x] + b*(6*a*e^3*p*x*(8*d + e*x) - b*e*p*x*(48*d^3 +
 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3) + 6*b*(d + e*x)^4*Log[c*(a + b*x^2
)^p]))/(24*b^2*e)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2913, 525, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$\Big\downarrow \; 2913$$
$$\frac{(d+ex)^4 \log \left(c\left(a+bx^2\right)^p\right)}{4e}-\frac{bp\int \frac{x(d+ex)^4}{bx^2+a}dx}{2e}$$
$$\Big\downarrow \; 525$$

$$\frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}-\frac{bp\left(\frac{\int\frac{x\left(bd^4+4bexd^3+4be^3x^3d+e^2\left(6bd^2-ae^2\right)x^2\right)}{bx^2+a}dx}{b}+\frac{e^4x^4}{4b}\right)}{2e}$$

$$\Big\downarrow \text{2333}$$

$$\frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}-\frac{bp\left(\frac{\int\left(4dx^2e^3+\frac{\left(6bd^2-ae^2\right)xe^2}{b}+4d\left(d^2-\frac{ae^2}{b}\right)e-\frac{4ade\left(bd^2-ae^2\right)-\left(b^2d^4-6abe^2d^2+a^2e^4\right)x}{b\left(bx^2+a\right)}\right)dx}{b}+\frac{e^4x^4}{4b}\right)}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}-\frac{bp\left(\frac{\frac{\left(a^2e^4-6abd^2e^2+b^2d^4\right)\log\left(a+bx^2\right)}{2b^2}-\frac{4\sqrt{a}de\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(bd^2-ae^2\right)}{b^{3/2}}+\frac{e^2x^2\left(6bd^2-ae^2\right)}{2b}+4dex\left(d^2-\frac{ae^2}{b}\right)+\frac{4}{3}de^3x^3}{b}+\frac{e^4x^4}{4b}\right)}{2e}$$

input
```
Int[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

output
```
-1/2*(b*p*((e^4*x^4)/(4*b) + (4*d*e*(d^2 - (a*e^2)/b)*x + (e^2*(6*b*d^2 -
a*e^2)*x^2)/(2*b) + (4*d*e^3*x^3)/3 - (4*Sqrt[a]*d*e*(b*d^2 - a*e^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + ((b^2*d^4 - 6*a*b*d^2*e^2 + a^2*e^4)*Log
[a + b*x^2])/(2*b^2))/b))/e + ((d + e*x)^4*Log[c*(a + b*x^2)^p])/(4*e)
```

Defintions of rubi rules used

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.37

method	result
parts	$\frac{\ln(c(b x^2+a)^p) e^3 x^4}{4} + \ln\left(c(b x^2+a)^p\right) e^2 d x^3 + \frac{3\ln(c(b x^2+a)^p) e d^2 x^2}{2} + d^3 x \ln\left(c(b x^2+a)^p\right) + \frac{\ln(c(b x^2+}{4e}$
risch	Expression too large to display

input
```
int((e*x+d)^3*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(c*(b*x^2+a)^p)*e^3*x^4+ln(c*(b*x^2+a)^p)*e^2*d*x^3+3/2*ln(c*(b*x^2+
a)^p)*e*d^2*x^2+d^3*x*ln(c*(b*x^2+a)^p)+1/4*ln(c*(b*x^2+a)^p)/e*d^4-1/2*p*
b/e*(-e/b^2*(-1/4*x^4*b*e^3-4/3*x^3*b*d*e^2+1/2*a*e^3*x^2-3*e*d^2*b*x^2+4*
x*e^2*d*a-4*b*x*d^3)+1/b^2*(1/2*(a^2*e^4-6*a*b*d^2*e^2+b^2*d^4)/b*ln(b*x^2
+a)+(4*a^2*d*e^3-4*a*b*d^3*e)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.80

$$\int (d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right)\,dx$$

$$= \left[-\frac{3\,b^2e^3px^4 + 16\,b^2de^2px^3 + 6\,(6\,b^2d^2e - abe^3)px^2 + 24\,(b^2d^3 - abde^2)p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2+a}\right) + 48}{} \right.$$

$$-\frac{3\,b^2e^3px^4 + 16\,b^2de^2px^3 + 6\,(6\,b^2d^2e - abe^3)px^2 - 48\,(b^2d^3 - abde^2)p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 48\,(b^2d^3 -}{}$$

input

```
integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output

```
[-1/24*(3*b^2*e^3*p*x^4 + 16*b^2*d*e^2*p*x^3 + 6*(6*b^2*d^2*e - a*b*e^3)*p
*x^2 + 24*(b^2*d^3 - a*b*d*e^2)*p*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b)
 - a)/(b*x^2 + a)) + 48*(b^2*d^3 - a*b*d*e^2)*p*x - 6*(b^2*e^3*p*x^4 + 4*b
^2*d*e^2*p*x^3 + 6*b^2*d^2*e*p*x^2 + 4*b^2*d^3*p*x + (6*a*b*d^2*e - a^2*e^
3)*p)*log(b*x^2 + a) - 6*(b^2*e^3*x^4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2
+ 4*b^2*d^3*x)*log(c))/b^2, -1/24*(3*b^2*e^3*p*x^4 + 16*b^2*d*e^2*p*x^3 +
6*(6*b^2*d^2*e - a*b*e^3)*p*x^2 - 48*(b^2*d^3 - a*b*d*e^2)*p*sqrt(a/b)*arc
tan(b*x*sqrt(a/b)/a) + 48*(b^2*d^3 - a*b*d*e^2)*p*x - 6*(b^2*e^3*p*x^4 + 4
*b^2*d*e^2*p*x^3 + 6*b^2*d^2*e*p*x^2 + 4*b^2*d^3*p*x + (6*a*b*d^2*e - a^2*
e^3)*p)*log(b*x^2 + a) - 6*(b^2*e^3*x^4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^
2 + 4*b^2*d^3*x)*log(c))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(170) = 340$.

Time = 17.51 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.96

$$\int (d+ex)^3 \log \left(c(a+bx^2)^p\right) \, dx$$
$$= \begin{cases} \left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right)\log\left(0^pc\right) \\ \left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right)\log\left(a^pc\right) \\ -2d^3px + d^3x\log\left(c(bx^2)^p\right) - \frac{3d^2epx^2}{2} + \frac{3d^2ex^2\log\left(c(bx^2)^p\right)}{2} - \frac{2de^2px^3}{3} + de^2x^3\log\left(c(bx^2)^p\right) - \frac{e^3px^4}{8} + \frac{e^3x^4\log \cdots}{} \\ -\frac{2a^2de^2p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{a^2de^2\log\left(c(a+bx^2)^p\right)}{b^2\sqrt{-\frac{a}{b}}} - \frac{a^2e^3\log\left(c(a+bx^2)^p\right)}{4b^2} + \frac{2ad^3p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{ad^3\log\left(c(a+bx^2)^p\right)}{b\sqrt{-\frac{a}{b}}} + \frac{3a\cdots}{} \end{cases}$$

input
```
integrate((e*x+d)**3*ln(c*(b*x**2+a)**p),x)
```

output
```
Piecewise(((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)*log(0**p
*c), Eq(a, 0) & Eq(b, 0)), ((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3
*x**4/4)*log(a**p*c), Eq(b, 0)), (-2*d**3*p*x + d**3*x*log(c*(b*x**2)**p)
- 3*d**2*e*p*x**2/2 + 3*d**2*e*x**2*log(c*(b*x**2)**p)/2 - 2*d*e**2*p*x**3
/3 + d*e**2*x**3*log(c*(b*x**2)**p) - e**3*p*x**4/8 + e**3*x**4*log(c*(b*x
**2)**p)/4, Eq(a, 0)), (-2*a**2*d*e**2*p*log(x - sqrt(-a/b))/(b**2*sqrt(-a
/b)) + a**2*d*e**2*log(c*(a + b*x**2)**p)/(b**2*sqrt(-a/b)) - a**2*e**3*lo
g(c*(a + b*x**2)**p)/(4*b**2) + 2*a*d**3*p*log(x - sqrt(-a/b))/(b*sqrt(-a/
b)) - a*d**3*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) + 3*a*d**2*e*log(c*(a +
 b*x**2)**p)/(2*b) + 2*a*d*e**2*p*x/b + a*e**3*p*x**2/(4*b) - 2*d**3*p*x +
 d**3*x*log(c*(a + b*x**2)**p) - 3*d**2*e*p*x**2/2 + 3*d**2*e*x**2*log(c*(
a + b*x**2)**p)/2 - 2*d*e**2*p*x**3/3 + d*e**2*x**3*log(c*(a + b*x**2)**p)
 - e**3*p*x**4/8 + e**3*x**4*log(c*(a + b*x**2)**p)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.99

$$\int (d+ex)^3 \log \left(c(a+bx^2)^p\right) \, dx$$
$$= \frac{1}{24}bp\left(\frac{48\,(abd^3 - a^2de^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} - \frac{3\,be^3x^4 + 16\,bde^2x^3 + 6\,(6\,bd^2e - ae^3)x^2 + 48\,(bd^3 - ade^2)x}{b^2} + \cdots\right.$$
$$+ \frac{1}{4}\left(e^3x^4 + 4\,de^2x^3 + 6\,d^2ex^2 + 4\,d^3x\right)\log\left(\left(bx^2+a\right)^pc\right)$$

input
```
integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
1/24*b*p*(48*(a*b*d^3 - a^2*d*e^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) -
 (3*b*e^3*x^4 + 16*b*d*e^2*x^3 + 6*(6*b*d^2*e - a*e^3)*x^2 + 48*(b*d^3 - a
*d*e^2)*x)/b^2 + 6*(6*a*b*d^2*e - a^2*e^3)*log(b*x^2 + a)/b^3) + 1/4*(e^3*
x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*log((b*x^2 + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.22

$$\int (d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx = -\frac{1}{8} \left(e^3p - 2\,e^3 \log\left(c\right)\right)x^4 - \frac{1}{3}\left(2\,de^2p - 3\,de^2 \log\left(c\right)\right)x^3 - \frac{(6\,bd^2ep - ae^3p - 6\,bd^2e\log\left(c\right))x^2}{4\,b} + \frac{1}{4}\left(e^3px^4 + 4\,de^2px^3 + 6\,d^2epx^2 + 4\,d^3px\right)\log\left(bx^2 + a\right) - \frac{(2\,bd^3p - 2\,ade^2p - bd^3\log\left(c\right))x}{b} + \frac{2\left(abd^3p - a^2de^2p\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{\left(6\,abd^2ep - a^2e^3p\right)\log\left(bx^2 + a\right)}{4\,b^2}$$

input
```
integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
-1/8*(e^3*p - 2*e^3*log(c))*x^4 - 1/3*(2*d*e^2*p - 3*d*e^2*log(c))*x^3 - 1
/4*(6*b*d^2*e*p - a*e^3*p - 6*b*d^2*e*log(c))*x^2/b + 1/4*(e^3*p*x^4 + 4*d
*e^2*p*x^3 + 6*d^2*e*p*x^2 + 4*d^3*p*x)*log(b*x^2 + a) - (2*b*d^3*p - 2*a*
d*e^2*p - b*d^3*log(c))*x/b + 2*(a*b*d^3*p - a^2*d*e^2*p)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b) + 1/4*(6*a*b*d^2*e*p - a^2*e^3*p)*log(b*x^2 + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 26.02 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.25

$$\int (d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{e^3\,x^4\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{4} - 2\,d^3\,p\,x - \frac{e^3\,p\,x^4}{8} + d^3\,x\,\ln\left(c\,\left(b\,x^2+a\right)^p\right) + \frac{3\,d^2\,e\,x^2\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{2} + d\,e^2\,x^3\,\ln\left(c\,\left(b\,x^2+a\right)^p\right) - \frac{3\,d^2\,e\,p\,x^2}{2} - \frac{2\,d\,e^2\,p\,x^3}{3} + \frac{a\,e^3\,p\,x^2}{4\,b} + \frac{2\,\sqrt{a}\,d^3\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{a^2\,e^3\,p\,\ln\left(b\,x^2+a\right)}{4\,b^2} - \frac{2\,a^{3/2}\,d\,e^2\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}} + \frac{2\,a\,d\,e^2\,p\,x}{b} + \frac{3\,a\,d^2\,e\,p\,\ln\left(b\,x^2+a\right)}{2\,b}$$

input
```
int(log(c*(a + b*x^2)^p)*(d + e*x)^3,x)
```

output
```
(e^3*x^4*log(c*(a + b*x^2)^p))/4 - 2*d^3*p*x - (e^3*p*x^4)/8 + d^3*x*log(c
*(a + b*x^2)^p) + (3*d^2*e*x^2*log(c*(a + b*x^2)^p))/2 + d*e^2*x^3*log(c*(
a + b*x^2)^p) - (3*d^2*e*p*x^2)/2 - (2*d*e^2*p*x^3)/3 + (a*e^3*p*x^2)/(4*b
) + (2*a^(1/2)*d^3*p*atan((b^(1/2)*x)/a^(1/2)))/b^(1/2) - (a^2*e^3*p*log(a
 + b*x^2))/(4*b^2) - (2*a^(3/2)*d*e^2*p*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
 + (2*a*d*e^2*p*x)/b + (3*a*d^2*e*p*log(a + b*x^2))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.43

$$\int (d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$= \frac{-48\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) ad\,e^2p + 48\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b\,d^3p - 6\log\left(\left(b\,x^2+a\right)^p c\right) a^2e^3 + 36\log\left(\left(b\,x^2+\right.\right.}{}$$

input
```
int((e*x+d)^3*log(c*(b*x^2+a)^p),x)
```

output
```
( - 48*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*d*e**2*p + 48*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*d**3*p - 6*log((a + b*x**2)**p*
c)*a**2*e**3 + 36*log((a + b*x**2)**p*c)*a*b*d**2*e + 24*log((a + b*x**2)*
*p*c)*b**2*d**3*x + 36*log((a + b*x**2)**p*c)*b**2*d**2*e*x**2 + 24*log((a
 + b*x**2)**p*c)*b**2*d*e**2*x**3 + 6*log((a + b*x**2)**p*c)*b**2*e**3*x**
4 + 48*a*b*d*e**2*p*x + 6*a*b*e**3*p*x**2 - 48*b**2*d**3*p*x - 36*b**2*d**
2*e*p*x**2 - 16*b**2*d*e**2*p*x**3 - 3*b**2*e**3*p*x**4)/(24*b**2)
```

3.185 $\int (d+ex)^2 \log \left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = -\frac{2(3bd^2-ae^2)\,px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{2\sqrt{a}(3bd^2-ae^2)\,p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} - \frac{d(bd^2-3ae^2)\,p \log\left(a+bx^2\right)}{3be} + \frac{(d+ex)^3 \log\left(c(a+bx^2)^p\right)}{3e}$$

output
```
-2/3*(-a*e^2+3*b*d^2)*p*x/b-d*e*p*x^2-2/9*e^2*p*x^3+2/3*a^(1/2)*(-a*e^2+3*
b*d^2)*p*arctan(b^(1/2)*x/a^(1/2))/b^(3/2)-1/3*d*(-3*a*e^2+b*d^2)*p*ln(b*x
^2+a)/b/e+1/3*(e*x+d)^3*ln(c*(b*x^2+a)^p)/e
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.50

$$\int (d+ex)^2 \log\left(c\left(a+bx^2\right)^p\right)\, dx$$
$$= \frac{3\left(-b^{3/2}d^3-3\sqrt{-a}bd^2e+3a\sqrt{b}de^2+\sqrt{-a}ae^3\right)p\log\left(\sqrt{-a}-\sqrt{b}x\right)-3\left(b^{3/2}d^3-3\sqrt{-a}bd^2e-3a\sqrt{b}d\right.}{}$$

input
```
Integrate[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

output
```
(3*(-(b^(3/2)*d^3) - 3*Sqrt[-a]*b*d^2*e + 3*a*Sqrt[b]*d*e^2 + Sqrt[-a]*a*e
^3)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 3*(b^(3/2)*d^3 - 3*Sqrt[-a]*b*d^2*e - 3*
a*Sqrt[b]*d*e^2 + Sqrt[-a]*a*e^3)*p*Log[Sqrt[-a] + Sqrt[b]*x] + Sqrt[b]*(6
*a*e^3*p*x - b*e*p*x*(18*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*b*(d + e*x)^3*Log[
c*(a + b*x^2)^p]))/(9*b^(3/2)*e)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2913, 525, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \log\left(c\left(a+bx^2\right)^p\right)\, dx$$

$$\Big\downarrow \; 2913$$

$$\frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{2bp\int \frac{x(d+ex)^3}{bx^2+a}dx}{3e}$$

$$\Big\downarrow \; 525$$

$$\frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{2bp\left(\frac{\int \frac{x\left(bd^3+3be^2x^2d+e\left(3bd^2-ae^2\right)x\right)}{bx^2+a}dx}{b}+\frac{e^3x^3}{3b}\right)}{3e}$$

$$\Big\downarrow \ 2333$$

$$\frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{2bp\left(\frac{\int\left(3dxe^2+\left(3d^2-\frac{ae^2}{b}\right)e-\frac{ae\left(3bd^2-ae^2\right)-bd\left(bd^2-3ae^2\right)x}{b\left(bx^2+a\right)}\right)dx}{b}+\frac{e^3x^3}{3b}\right)}{3e}$$

$$\Big\downarrow \ 2009$$

$$\frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{2bp\left(\frac{-\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3bd^2-ae^2\right)}{b^{3/2}}+\frac{d\left(bd^2-3ae^2\right)\log\left(a+bx^2\right)}{2b}+ex\left(3d^2-\frac{ae^2}{b}\right)+\frac{3}{2}de^2x^2}{b}+\frac{e^3x^3}{3b}\right)}{3e}$$

input
```
Int[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

output
```
(-2*b*p*((e^3*x^3)/(3*b) + (e*(3*d^2 - (a*e^2)/b)*x + (3*d*e^2*x^2)/2 - (S
qrt[a]*e*(3*b*d^2 - a*e^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + (d*(b*d^
2 - 3*a*e^2)*Log[a + b*x^2])/(2*b))/b))/(3*e) + ((d + e*x)^3*Log[c*(a + b*
x^2)^p])/(3*e)
```

Defintions of rubi rules used

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.33

method	result
parts	$\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)e^{2}x^{3}}{3}+\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)ed\,x^{2}+d^{2}x\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)+\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)d^{3}}{3e}-\frac{2pb\left(-\frac{e\left(-\frac{1}{3}x^{3}b\right.}{}\right.}{}$
risch	$-dep\,x^{2}-\frac{ix\pi\,d^{2}\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{3}}{2}-\frac{ie^{2}\pi\,x^{3}\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{3}}{6}+\frac{2xap\,e^{2}}{3b}+\frac{ie^{2}\pi\,x^{3}\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}\operatorname{csgn}\left(ic\right)}{6}-$

input
```
int((e*x+d)^2*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(c*(b*x^2+a)^p)*e^2*x^3+ln(c*(b*x^2+a)^p)*e*d*x^2+d^2*x*ln(c*(b*x^2+
a)^p)+1/3*ln(c*(b*x^2+a)^p)/e*d^3-2/3*p*b/e*(-e/b^2*(-1/3*x^3*b*e^2-3/2*b*
d*e*x^2+x*a*e^2-3*b*x*d^2)+1/b^2*(1/2*(-3*a*b*d*e^2+b^2*d^3)/b*ln(b*x^2+a)
+(a^2*e^3-3*a*b*d^2*e)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.27

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$= \left[-\frac{2\,be^2px^3 + 9\,bdepx^2 + 3\,(3\,bd^2 - ae^2)p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 6\,(3\,bd^2 - ae^2)px - 3\,(be^2px^3 + 3}{9\,b}\right.$$
$$-\frac{2\,be^2px^3 + 9\,bdepx^2 - 6\,(3\,bd^2 - ae^2)p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 6\,(3\,bd^2 - ae^2)px - 3\,(be^2px^3 + 3\,bdepx^2}{9\,b}$$

input
```
integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
[-1/9*(2*b*e^2*p*x^3 + 9*b*d*e*p*x^2 + 3*(3*b*d^2 - a*e^2)*p*sqrt(-a/b)*lo
g((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*(3*b*d^2 - a*e^2)*p*x -
3*(b*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x + 3*a*d*e*p)*log(b*x^2 + a) -
 3*(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(c))/b, -1/9*(2*b*e^2*p*x^3 +
9*b*d*e*p*x^2 - 6*(3*b*d^2 - a*e^2)*p*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
6*(3*b*d^2 - a*e^2)*p*x - 3*(b*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x + 3
*a*d*e*p)*log(b*x^2 + a) - 3*(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(c))
/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(131) = 262$.

Time = 8.72 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.65

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \begin{cases} \left(d^2x + dex^2 + \frac{e^2x^3}{3}\right)\log\left(0^pc\right) \\ \left(d^2x + dex^2 + \frac{e^2x^3}{3}\right)\log\left(a^pc\right) \\ -2d^2px + d^2x\log\left(c\left(bx^2\right)^p\right) - depx^2 + dex^2\log\left(c\left(bx^2\right)^p\right) - \frac{2e^2px^3}{9} + \frac{e^2x^3\log\left(c\left(bx^2\right)^p\right)}{3} \\ -\frac{2a^2e^2p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}} + \frac{a^2e^2\log\left(c\left(a+bx^2\right)^p\right)}{3b^2\sqrt{-\frac{a}{b}}} + \frac{2ad^2p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{ad^2\log\left(c\left(a+bx^2\right)^p\right)}{b\sqrt{-\frac{a}{b}}} + \frac{ade\log\left(c\left(a+bx^2\right)^p\right)}{b} + \frac{2ae^2p}{3b} \end{cases}$$

input
```
integrate((e*x+d)**2*ln(c*(b*x**2+a)**p),x)
```

output
```
Piecewise(((d**2*x + d*e*x**2 + e**2*x**3/3)*log(0**p*c), Eq(a, 0) & Eq(b,
 0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)*log(a**p*c), Eq(b, 0)), (-2*d**2*
p*x + d**2*x*log(c*(b*x**2)**p) - d*e*p*x**2 + d*e*x**2*log(c*(b*x**2)**p)
 - 2*e**2*p*x**3/9 + e**2*x**3*log(c*(b*x**2)**p)/3, Eq(a, 0)), (-2*a**2*e
**2*p*log(x - sqrt(-a/b))/(3*b**2*sqrt(-a/b)) + a**2*e**2*log(c*(a + b*x**
2)**p)/(3*b**2*sqrt(-a/b)) + 2*a*d**2*p*log(x - sqrt(-a/b))/(b*sqrt(-a/b))
 - a*d**2*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) + a*d*e*log(c*(a + b*x**2)
**p)/b + 2*a*e**2*p*x/(3*b) - 2*d**2*p*x + d**2*x*log(c*(a + b*x**2)**p) -
 d*e*p*x**2 + d*e*x**2*log(c*(a + b*x**2)**p) - 2*e**2*p*x**3/9 + e**2*x**
3*log(c*(a + b*x**2)**p)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{1}{9}\left(\frac{9\,ade\log\left(bx^2+a\right)}{b^2} + \frac{6\left(3\,abd^2 - a^2e^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} - \frac{2\,be^2x^3 + 9\,bdex^2 + 6\left(3\,bd^2 - ae^2\right)x}{b^2}\right)bp$$
$$+ \frac{1}{3}\left(e^2x^3 + 3\,dex^2 + 3\,d^2x\right)\log\left(\left(bx^2+a\right)^pc\right)$$

input
```
integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
1/9*(9*a*d*e*log(b*x^2 + a)/b^2 + 6*(3*a*b*d^2 - a^2*e^2)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^2) - (2*b*e^2*x^3 + 9*b*d*e*x^2 + 6*(3*b*d^2 - a*e^2)*x
)/b^2)*b*p + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x^2 + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.08

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = -\frac{1}{9} \left(2 e^2 p - 3 e^2 \log(c)\right) x^3 + \frac{adep \log \left(bx^2+a\right)}{b} - (dep - de \log(c)) x^2 + \frac{1}{3} \left(e^2 p x^3 + 3 depx^2 + 3 d^2 px\right) \log \left(bx^2+a\right) - \frac{\left(6 bd^2 p - 2 ae^2 p - 3 bd^2 \log(c)\right) x}{3 b} + \frac{2 \left(3 abd^2 p - a^2 e^2 p\right) \arctan \left(\frac{bx}{\sqrt{ab}}\right)}{3 \sqrt{ab} b}$$

input
```
integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
-1/9*(2*e^2*p - 3*e^2*log(c))*x^3 + a*d*e*p*log(b*x^2 + a)/b - (d*e*p - d*
e*log(c))*x^2 + 1/3*(e^2*p*x^3 + 3*d*e*p*x^2 + 3*d^2*p*x)*log(b*x^2 + a) -
 1/3*(6*b*d^2*p - 2*a*e^2*p - 3*b*d^2*log(c))*x/b + 2/3*(3*a*b*d^2*p - a^2
*e^2*p)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [B] (verification not implemented)

Time = 28.78 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.87

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{e^2\,x^3\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{3} - 2\,d^2\,p\,x - \frac{2\,e^2\,p\,x^3}{9} + d^2\,x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right) + d\,e\,x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right) - d\,e\,p\,x^2 + \frac{2\,a\,e^2\,p\,x}{3\,b} - \frac{2\,\sqrt{a}\,d^2\,p\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,d^2\,p\,x}{a^2\,e^2\,p-3\,a\,b\,d^2\,p}-\frac{a^{3/2}\,\sqrt{b}\,e^2\,p\,x}{a^2\,e^2\,p-3\,a\,b\,d^2\,p}\right)}{\sqrt{b}} + \frac{2\,a^{3/2}\,e^2\,p\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,d^2\,p\,x}{a^2\,e^2\,p-3\,a\,b\,d^2\,p}-\frac{a^{3/2}\,\sqrt{b}\,e^2\,p\,x}{a^2\,e^2\,p-3\,a\,b\,d^2\,p}\right)}{3\,b^{3/2}} + \frac{a\,d\,e\,p\,\ln\left(b\,x^2+a\right)}{b}$$

input
```
int(log(c*(a + b*x^2)^p)*(d + e*x)^2,x)
```

output
```
(e^2*x^3*log(c*(a + b*x^2)^p))/3 - 2*d^2*p*x - (2*e^2*p*x^3)/9 + d^2*x*log
(c*(a + b*x^2)^p) + d*e*x^2*log(c*(a + b*x^2)^p) - d*e*p*x^2 + (2*a*e^2*p*
x)/(3*b) - (2*a^(1/2)*d^2*p*atan((3*a^(1/2)*b^(3/2)*d^2*p*x)/(a^2*e^2*p -
3*a*b*d^2*p) - (a^(3/2)*b^(1/2)*e^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p)))/b^(1/
2) + (2*a^(3/2)*e^2*p*atan((3*a^(1/2)*b^(3/2)*d^2*p*x)/(a^2*e^2*p - 3*a*b*
d^2*p) - (a^(3/2)*b^(1/2)*e^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p)))/(3*b^(3/2))
 + (a*d*e*p*log(a + b*x^2))/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.26

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{-6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a\,e^2p + 18\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b\,d^2p + 9\log\left(\left(b\,x^2+a\right)^p c\right) abde + 9\log\left(\left(b\,x^2+a\right)\right.}{}$$

input
```
int((e*x+d)^2*log(c*(b*x^2+a)^p),x)
```

output
```
( - 6*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*e**2*p + 18*sqrt(b)*
sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*d**2*p + 9*log((a + b*x**2)**p*c)*
a*b*d*e + 9*log((a + b*x**2)**p*c)*b**2*d**2*x + 9*log((a + b*x**2)**p*c)*
b**2*d*e*x**2 + 3*log((a + b*x**2)**p*c)*b**2*e**2*x**3 + 6*a*b*e**2*p*x -
 18*b**2*d**2*p*x - 9*b**2*d*e*p*x**2 - 2*b**2*e**2*p*x**3)/(9*b**2)
```

3.186 $\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 99

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx = -2dpx-\frac{1}{2}epx^2+\frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{\left(bd^2-ae^2\right)p\log\left(a+bx^2\right)}{2be}+\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}$$

output
```
-2*d*p*x-1/2*e*p*x^2+2*a^(1/2)*d*p*arctan(b^(1/2)*x/a^(1/2))/b^(1/2)-1/2*(
-a*e^2+b*d^2)*p*ln(b*x^2+a)/b/e+1/2*(e*x+d)^2*ln(c*(b*x^2+a)^p)/e
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx = -2dpx+\frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+dx\log\left(c\left(a+bx^2\right)^p\right)+\frac{1}{2}e\left(-px^2+\frac{\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b}\right)$$

input
```
Integrate[(d + e*x)*Log[c*(a + b*x^2)^p],x]
```

output
```
-2*d*p*x + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + d*x*Log[c
*(a + b*x^2)^p] + (e*(-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b))/2
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2913, 525, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{2913}$$

$$\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{bp\int \frac{x(d+ex)^2}{bx^2+a}dx}{e}$$

$$\Big\downarrow \text{525}$$

$$\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{bp\left(\frac{\int \frac{x\left(bd^2+2bexd-ae^2\right)}{bx^2+a}dx}{b}+\frac{e^2x^2}{2b}\right)}{e}$$

$$\Big\downarrow \text{523}$$

$$\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{bp\left(\frac{\int \left(2de-\frac{2ade-\left(bd^2-ae^2\right)x}{bx^2+a}\right)dx}{b}+\frac{e^2x^2}{2b}\right)}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{bp\left(\frac{-\frac{2\sqrt{a}de\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2b}+2dex}{b}+\frac{e^2x^2}{2b}\right)}{e}$$

input
```
Int[(d + e*x)*Log[c*(a + b*x^2)^p],x]
```

output
```
-((b*p*((e^2*x^2)/(2*b) + (2*d*e*x - (2*Sqrt[a]*d*e*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/Sqrt[b] + ((b*d^2 - a*e^2)*Log[a + b*x^2])/(2*b))/b))/e) + ((d + e*
x)^2*Log[c*(a + b*x^2)^p])/(2*e)
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

method	result
default	$dx\ln\left(c(b\,x^2+a)^p\right)-2xdp+\frac{2dpa\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{e\left(\ln\left(c(b\,x^2+a)^p\right)(b\,x^2+a)-p(b\,x^2+a)\right)}{2b}$
parts	$\frac{\ln\left(c(b\,x^2+a)^p\right)e\,x^2}{2}+dx\ln\left(c(b\,x^2+a)^p\right)-pb\left(\frac{\frac{1}{2}e\,x^2+2dx}{b}-\frac{a\left(\frac{e\ln\left(b\,x^2+a\right)}{2b}+\frac{2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b}\right)$
risch	$\left(\frac{1}{2}e\,x^2+dx\right)\ln\left((b\,x^2+a)^p\right)+\frac{ix\pi d\ \mathrm{csgn}\left(i(b\,x^2+a)^p\right)\mathrm{csgn}\left(ic(b\,x^2+a)^p\right)^2}{2}-\frac{i\pi e\,x^2\mathrm{csgn}\left(ic(b\,x^2+a)^p\right)^3}{4}+\frac{i\,\mathrm{csgn}(ic}{}$

input
```
int((e*x+d)*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
d*x*ln(c*(b*x^2+a)^p)-2*x*d*p+2*d*p*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+
1/2*e/b*(ln(c*(b*x^2+a)^p)*(b*x^2+a)-p*(b*x^2+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.00

$$\int(d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$=\left[-\frac{bepx^2-2\,bdp\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+4\,bdpx-\left(bepx^2+2\,bdpx+aep\right)\log\left(bx^2+a\right)-\left(bex^2+\right.}{2\,b}\right.$$

$$-\frac{bepx^2-4\,bdp\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+4\,bdpx-\left(bepx^2+2\,bdpx+aep\right)\log\left(bx^2+a\right)-\left(bex^2+2\,bdx\right)\mathrm{lo}}{2\,b}$$

input
```
integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
[-1/2*(b*e*p*x^2 - 2*b*d*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)) + 4*b*d*p*x - (b*e*p*x^2 + 2*b*d*p*x + a*e*p)*log(b*x^2 + a) -
 (b*e*x^2 + 2*b*d*x)*log(c))/b, -1/2*(b*e*p*x^2 - 4*b*d*p*sqrt(a/b)*arctan
(b*x*sqrt(a/b)/a) + 4*b*d*p*x - (b*e*p*x^2 + 2*b*d*p*x + a*e*p)*log(b*x^2
+ a) - (b*e*x^2 + 2*b*d*x)*log(c))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(92) = 184$.

Time = 4.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.01

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \begin{cases} \left(dx+\frac{ex^2}{2}\right)\log\left(0^pc\right) \\ \left(dx+\frac{ex^2}{2}\right)\log\left(a^pc\right) \\ -2dpx+dx\log\left(c\left(bx^2\right)^p\right)-\frac{epx^2}{2}+\frac{ex^2\log\left(c\left(bx^2\right)^p\right)}{2} \\ \frac{2adp\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}}-\frac{ad\log\left(c\left(a+bx^2\right)^p\right)}{b\sqrt{-\frac{a}{b}}}+\frac{ae\log\left(c\left(a+bx^2\right)^p\right)}{2b}-2dpx+dx\log\left(c\left(a+bx^2\right)^p\right)-\frac{epx^2}{2}+\frac{ex^2\log\left(c\left(a+b\right.\right.}{2} \end{cases}$$

input
```
integrate((e*x+d)*ln(c*(b*x**2+a)**p),x)
```

output
```
Piecewise(((d*x + e*x**2/2)*log(0**p*c), Eq(a, 0) & Eq(b, 0)), ((d*x + e*x
**2/2)*log(a**p*c), Eq(b, 0)), (-2*d*p*x + d*x*log(c*(b*x**2)**p) - e*p*x*
*2/2 + e*x**2*log(c*(b*x**2)**p)/2, Eq(a, 0)), (2*a*d*p*log(x - sqrt(-a/b)
)/(b*sqrt(-a/b)) - a*d*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) + a*e*log(c*(
a + b*x**2)**p)/(2*b) - 2*d*p*x + d*x*log(c*(a + b*x**2)**p) - e*p*x**2/2
+ e*x**2*log(c*(a + b*x**2)**p)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{2}\left(\frac{4\,ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{ae\log\left(bx^2+a\right)}{b^2}-\frac{ex^2+4\,dx}{b}\right)bp + \frac{1}{2}\left(ex^2+2\,dx\right)\log\left(\left(bx^2+a\right)^p c\right)$$

input
```
integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
1/2*(4*a*d*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + a*e*log(b*x^2 + a)/b^2 -
(e*x^2 + 4*d*x)/b)*b*p + 1/2*(e*x^2 + 2*d*x)*log((b*x^2 + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{2\,adp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{1}{2}\left(ep-e\log\left(c\right)\right)x^2 + \frac{aep\log\left(bx^2+a\right)}{2\,b} - \left(2\,dp-d\log\left(c\right)\right)x + \frac{1}{2}\left(epx^2+2\,dpx\right)\log\left(bx^2+a\right)$$

input
```
integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
2*a*d*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - 1/2*(e*p - e*log(c))*x^2 + 1/2*a
*e*p*log(b*x^2 + a)/b - (2*d*p - d*log(c))*x + 1/2*(e*p*x^2 + 2*d*p*x)*log
(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 26.66 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx = d\,x\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)-\frac{e\,p\,x^2}{2}-2\,d\,p\,x+\frac{e\,x^2\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{2}+\frac{a\,e\,p\,\ln\left(b\,x^2+a\right)}{2\,b}+\frac{2\,\sqrt{a}\,d\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

input

```
int(log(c*(a + b*x^2)^p)*(d + e*x),x)
```

output

```
d*x*log(c*(a + b*x^2)^p) - (e*p*x^2)/2 - 2*d*p*x + (e*x^2*log(c*(a + b*x^2
)^p))/2 + (a*e*p*log(a + b*x^2))/(2*b) + (2*a^(1/2)*d*p*atan((b^(1/2)*x)/a
^(1/2)))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{4\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)dp+\log\left(\left(b\,x^2+a\right)^p c\right)ae+2\log\left(\left(b\,x^2+a\right)^p c\right)bdx+\log\left(\left(b\,x^2+a\right)^p c\right)be\,x^2-4bc}{2b}$$

input

```
int((e*x+d)*log(c*(b*x^2+a)^p),x)
```

output

```
(4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*d*p + log((a + b*x**2)**p
*c)*a*e + 2*log((a + b*x**2)**p*c)*b*d*x + log((a + b*x**2)**p*c)*b*e*x**2
 - 4*b*d*p*x - b*e*p*x**2)/(2*b)
```

3.187 $\int \log \left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \log \left(c(a+bx^2)^p\right) \, dx = -2px + \frac{2\sqrt{a}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + x \log \left(c(a+bx^2)^p\right)$$

output
```
-2*p*x+2*a^(1/2)*p*arctan(b^(1/2)*x/a^(1/2))/b^(1/2)+x*ln(c*(b*x^2+a)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log \left(c(a+bx^2)^p\right) \, dx = -2px + \frac{2\sqrt{a}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + x \log \left(c(a+bx^2)^p\right)$$

input
```
Integrate[Log[c*(a + b*x^2)^p],x]
```

output
```
-2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + 
b*x^2)^p]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2898, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c(a+bx^2)^p\right) \, dx$$

$$\Big\downarrow \text{2898}$$

$$x \log \left(c(a+bx^2)^p\right) - 2bp \int \frac{x^2}{bx^2+a}dx$$

$$\Big\downarrow \text{262}$$

$$x \log \left(c(a+bx^2)^p\right) - 2bp \left(\frac{x}{b} - \frac{a \int \frac{1}{bx^2+a}dx}{b}\right)$$

$$\Big\downarrow \text{218}$$

$$x \log \left(c(a+bx^2)^p\right) - 2bp \left(\frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)$$

input
```
Int[Log[c*(a + b*x^2)^p],x]
```

output
```
-2*b*p*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)) + x*Log[c*(a
+ b*x^2)^p]
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result
default	$x \ln\left(c\left(b\,x^2+a\right)^p\right)-2pb\left(\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}\right)$
parts	$x \ln\left(c\left(b\,x^2+a\right)^p\right)-2pb\left(\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}\right)$
risch	$x \ln\left(\left(b\,x^2+a\right)^p\right)+\frac{i\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2\operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)\pi x}{2}-\frac{i\pi x\ \operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\operatorname{csgn}(ic)}{2}-$

input
```
int(ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(b*x^2+a)^p)-2*p*b*(x/b-1/b*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int \log \left(c\left(a+bx^2\right)^p\right) \, dx = \left[px \log \left(bx^2 + a\right) + p\sqrt{-\frac{a}{b}} \log \left(\frac{bx^2 + 2\, bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 2\, px + x \log \left(c\right), px \log \left(bx^2 + a\right) + 2\, p\sqrt{\frac{a}{b}} \arctan \left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 2\, px + x \log \left(c\right)\right]$$

input
```
integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
[p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x
^2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) - 2*p*x + x*log(c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 2.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \log \left(c\left(a+bx^2\right)^p\right) \, dx = \begin{cases} x \log \left(0^p c\right) & \text{for}\: a = 0 \wedge b = 0 \\ x \log \left(a^p c\right) & \text{for}\: b = 0 \\ -2px + x \log \left(c\left(bx^2\right)^p\right) & \text{for}\: a = 0 \\ \frac{2ap \log \left(x - \sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{a \log \left(c\left(a+bx^2\right)^p\right)}{b\sqrt{-\frac{a}{b}}} - 2px + x \log \left(c\left(a+bx^2\right)^p\right) & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(b*x**2+a)**p),x)
```

output
```
Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (x*log(a**p*c), Eq(b, 0)),
 (-2*p*x + x*log(c*(b*x**2)**p), Eq(a, 0)), (2*a*p*log(x - sqrt(-a/b))/(b*
sqrt(-a/b)) - a*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) - 2*p*x + x*log(c*(a
 + b*x**2)**p), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(a+bx^2\right)^p\right) \, dx = 2\, bp \left(\frac{a \arctan \left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{x}{b}\right) + x \log \left(\left(bx^2+a\right)^p c\right)$$

input
```
integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
2*b*p*(a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - x/b) + x*log((b*x^2 + a)^p*
c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \log \left(c\left(a+bx^2\right)^p\right) \, dx = px \log \left(bx^2+a\right) + \frac{2\, ap \arctan \left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - (2\, p - \log(c))x$$

input
```
integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c)
)*x
```

Mupad [B] (verification not implemented)

Time = 25.95 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \log \left(c\left(a+bx^2\right)^p\right) \, dx = x \ln \left(c \left(b x^2+a\right)^p\right)-2 p x+\frac{2 \sqrt{a} p \operatorname{atan}\left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
int(log(c*(a + b*x^2)^p),x)
```

output
```
x*log(c*(a + b*x^2)^p) - 2*p*x + (2*a^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/b
^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{2\sqrt{b}\, \sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\, \sqrt{a}}\right) p+\log \left(\left(b x^2+a\right)^p c\right) bx-2bpx}{b}$$

input
```
int(log(c*(b*x^2+a)^p),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*p + log((a + b*x**2)**p*c
)*b*x - 2*b*p*x)/b
```

3.188 $\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 201

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}$$

output

```
-p*ln(e*((-a)^(1/2)-b^(1/2)*x)/(b^(1/2)*d+(-a)^(1/2)*e))*ln(e*x+d)/e-p*ln(
-e*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*d-(-a)^(1/2)*e))*ln(e*x+d)/e+ln(e*x+d)*
ln(c*(b*x^2+a)^p)/e-p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d-(-a)^(1/2)*e))/
e-p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d+(-a)^(1/2)*e))/e
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

output
```
-((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x]
)/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[
d + e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[
b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e
*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2912, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

$$\Big\downarrow \; 2912$$

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{2bp\int\frac{x\log(d+ex)}{bx^2+a}dx}{e}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{2bp\int\left(\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{2bp\left(\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{2b}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{2b}+\frac{\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{2b}+\frac{\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{2b}\right)}{e}$$

input
```
Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

output
```
(Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (2*b*p*((Log[(e*(Sqrt[-a] - Sqrt[b
]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/(2*b) + (Log[-((e*(Sqrt[-a]
+ Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/(2*b) + PolyLog[2,
(Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]/(2*b) + PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)]/(2*b)))/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98

method	result
parts	$\frac{\ln(ex+d)\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{e}-\frac{2pb\left(\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\ln\left(\frac{e\sqrt{-ab}+(ex+d)b-bd}{e\sqrt{-ab}-bd}\right)\right)}{2b}+\frac{\operatorname{dilog}\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\operatorname{dilo}}{2b}\right.}{e}$
risch	$\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln(ex+d)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}+(ex+d)b-bd}{e\sqrt{-ab}-bd}\right)}{e}-\frac{p\operatorname{dilog}\left(\frac{e\sqrt{-ab}-(ex+}{e\sqrt{-ab}+}\right.}{e}$

input
```
int(ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-2*p*b/e*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(
e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(
-a*b)^(1/2)-b*d)))/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(
1/2)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b)
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx=\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

output
```
Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)}{d+e\,x}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b*x^2)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{4\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{be\,x^3+bd\,x^2+aex+ad}dx\right)aep-4\left(\int \frac{\log\left((b\,x^2+a)^p c\right)x}{be\,x^3+bd\,x^2+aex+ad}dx\right)bdp+\log\left((b\,x^2+a)^p\,c\right)^2}{4ep}$$

input
```
int(log(c*(b*x^2+a)^p)/(e*x+d),x)
```

output
```
(4*int(log((a + b*x**2)**p*c)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)*a*e*p
 - 4*int((log((a + b*x**2)**p*c)*x)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)
*b*d*p + log((a + b*x**2)**p*c)**2)/(4*e*p)
```

3.189 $\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 119

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \frac{2\sqrt{a}\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{bd^2+ae^2} - \frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)} + \frac{bdp\log\left(a+bx^2\right)}{e\left(bd^2+ae^2\right)} - \frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}$$

output
```
2*a^(1/2)*b^(1/2)*p*arctan(b^(1/2)*x/a^(1/2))/(a*e^2+b*d^2)-2*b*d*p*ln(e*x
+d)/e/(a*e^2+b*d^2)+b*d*p*ln(b*x^2+a)/e/(a*e^2+b*d^2)-ln(c*(b*x^2+a)^p)/e/
(e*x+d)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \frac{2\sqrt{a}\sqrt{b}ep(d+ex)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - 2bdp(d+ex)\log(d+ex) + bd^2p\log\left(a+bx^2\right) + bdepx\log\left(a+bx^2\right) -}{e\left(bd^2+ae^2\right)(d+ex)}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

output
```
(2*Sqrt[a]*Sqrt[b]*e*p*(d + e*x)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 2*b*d*p*(d
+ e*x)*Log[d + e*x] + b*d^2*p*Log[a + b*x^2] + b*d*e*p*x*Log[a + b*x^2] -
b*d^2*Log[c*(a + b*x^2)^p] - a*e^2*Log[c*(a + b*x^2)^p])/(e*(b*d^2 + a*e^2
)*(d + e*x))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2913, 587, 16, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 2913}$$

$$\frac{2bp\int \frac{x}{(d+ex)(bx^2+a)}dx}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}$$

$$\Big\downarrow \text{ 587}$$

$$\frac{2bp\left(\frac{\int \frac{ae+bdx}{bx^2+a}dx}{ae^2+bd^2}-\frac{de\int \frac{1}{d+ex}dx}{ae^2+bd^2}\right)}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{2bp\left(\frac{\int \frac{ae+bdx}{bx^2+a}dx}{ae^2+bd^2}-\frac{d\log(d+ex)}{ae^2+bd^2}\right)}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}$$

$$\Big\downarrow \text{ 452}$$

$$\frac{2bp\left(\frac{bd\int \frac{x}{bx^2+a}dx+ae\int \frac{1}{bx^2+a}dx}{ae^2+bd^2}-\frac{d\log(d+ex)}{ae^2+bd^2}\right)}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2bp\left(\frac{bd\int\frac{x}{bx^2+a}dx+\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}{ae^2+bd^2}-\frac{d\log(d+ex)}{ae^2+bd^2}\right)}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{2bp\left(\frac{\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{1}{2}d\log\left(a+bx^2\right)}{ae^2+bd^2}-\frac{d\log(d+ex)}{ae^2+bd^2}\right)}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}$$

input
```
Int[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

output
```
(2*b*p*(-((d*Log[d + e*x])/(b*d^2 + a*e^2)) + ((Sqrt[a]*e*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/Sqrt[b] + (d*Log[a + b*x^2])/2)/(b*d^2 + a*e^2)))/e - Log[c*(
a + b*x^2)^p]/(e*(d + e*x))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

method	result
parts	$-\frac{\ln\left(c\left(b x^2+a\right)^p\right)}{e(ex+d)}+\frac{2pb\left(-\frac{d\ln(ex+d)}{a e^2+b d^2}+\frac{\frac{d\ln\left(b x^2+a\right)}{2}+\frac{ea\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a e^2+b d^2}\right)}{e}$
risch	$-\frac{\ln\left(\left(b x^2+a\right)^p\right)}{e(ex+d)}+\frac{i\pi a\,\mathrm{csgn}\left(i\left(b x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b x^2+a\right)^p\right)\mathrm{csgn}(ic)e^2-i\pi a\,\mathrm{csgn}\left(ic\left(b x^2+a\right)^p\right)^2\mathrm{csgn}(ic)e^2+i\pi a\,\mathrm{csgn}\left(ic\left(b x^2+a\right.\right.}{}$

input
```
int(ln(c*(b*x^2+a)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(b*x^2+a)^p)/e/(e*x+d)+2*p*b/e*(-d/(a*e^2+b*d^2)*ln(e*x+d)+1/(a*e^2+
b*d^2)*(1/2*d*ln(b*x^2+a)+e*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.19

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx$$
$$= \left[\frac{(e^2px+dep)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+(bdepx-ae^2p)\log(bx^2+a)-2(bdepx+bd^2p)\log(ex+d)}{bd^3e+ade^3+(bd^2e^2+ae^4)x}\right.$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[((e^2*p*x + d*e*p)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)) + (b*d*e*p*x - a*e^2*p)*log(b*x^2 + a) - 2*(b*d*e*p*x + b*d^2*p)*log(e*
x + d) - (b*d^2 + a*e^2)*log(c))/(b*d^3*e + a*d*e^3 + (b*d^2*e^2 + a*e^4)*
x), (2*(e^2*p*x + d*e*p)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (b*d*e*p*x - a*
e^2*p)*log(b*x^2 + a) - 2*(b*d*e*p*x + b*d^2*p)*log(e*x + d) - (b*d^2 + a*
e^2)*log(c))/(b*d^3*e + a*d*e^3 + (b*d^2*e^2 + a*e^4)*x)]
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**2,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \frac{\left(\frac{2\,ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bd^2+ae^2)\sqrt{ab}} + \frac{d\log(bx^2+a)}{bd^2+ae^2} - \frac{2\,d\log(ex+d)}{bd^2+ae^2}\right)bp}{e} - \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)e}$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
(2*a*e*arctan(b*x/sqrt(a*b))/((b*d^2 + a*e^2)*sqrt(a*b)) + d*log(b*x^2 + a
)/(b*d^2 + a*e^2) - 2*d*log(e*x + d)/(b*d^2 + a*e^2))*b*p/e - log((b*x^2 +
 a)^p*c)/((e*x + d)*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \frac{bdp\log\left(bx^2+a\right)}{bd^2e+ae^3} - \frac{2\,bdp\log\left(ex+d\right)}{bd^2e+ae^3} + \frac{2\,abp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bd^2+ae^2)\sqrt{ab}} - \frac{p\log\left(bx^2+a\right)}{e^2x+de} - \frac{\log\left(c\right)}{e^2x+de}$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

output
```
b*d*p*log(b*x^2 + a)/(b*d^2*e + a*e^3) - 2*b*d*p*log(e*x + d)/(b*d^2*e + a
*e^3) + 2*a*b*p*arctan(b*x/sqrt(a*b))/((b*d^2 + a*e^2)*sqrt(a*b)) - p*log(
b*x^2 + a)/(e^2*x + d*e) - log(c)/(e^2*x + d*e)
```

Mupad [B] (verification not implemented)

Time = 26.91 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.83

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx$$
$$= \frac{\ln\left(\frac{4\,b^3\,p^2\,x}{e}-\frac{p\,\left(b\,d+e\,\sqrt{-a\,b}\right)\,\left(2\,a\,b^2\,e\,p+2\,b^3\,d\,p\,x-\frac{2\,b^2\,e\,p\,\left(b\,d+e\,\sqrt{-a\,b}\right)\,\left(-b\,x\,d^2+4\,a\,d\,e+3\,a\,x\,e^2\right)}{b\,d^2\,e+a\,e^3}\right)}{b\,d^2\,e+a\,e^3}\right)\,\left(b\,d\,p+e\,p\,\sqrt{-a\,b}\right)}{b\,d^2\,e+a\,e^3}$$
$$-\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{e\,\left(d+e\,x\right)}$$
$$+\frac{\ln\left(\frac{4\,b^3\,p^2\,x}{e}-\frac{p\,\left(b\,d-e\,\sqrt{-a\,b}\right)\,\left(2\,a\,b^2\,e\,p+2\,b^3\,d\,p\,x-\frac{2\,b^2\,e\,p\,\left(b\,d-e\,\sqrt{-a\,b}\right)\,\left(-b\,x\,d^2+4\,a\,d\,e+3\,a\,x\,e^2\right)}{b\,d^2\,e+a\,e^3}\right)}{b\,d^2\,e+a\,e^3}\right)\,\left(b\,d\,p-e\,p\,\sqrt{-a\,b}\right)}{b\,d^2\,e+a\,e^3}$$
$$-\frac{2\,b\,d\,p\,\ln\left(d+e\,x\right)}{b\,d^2\,e+a\,e^3}$$

input
```
int(log(c*(a + b*x^2)^p)/(d + e*x)^2,x)
```

output
```
(log((4*b^3*p^2*x)/e - (p*(b*d + e*(-a*b)^(1/2))*(2*a*b^2*e*p + 2*b^3*d*p*
x - (2*b^2*e*p*(b*d + e*(-a*b)^(1/2))*(4*a*d*e + 3*a*e^2*x - b*d^2*x))/(a*
e^3 + b*d^2*e)))/(a*e^3 + b*d^2*e))*(b*d*p + e*p*(-a*b)^(1/2)))/(a*e^3 + b
*d^2*e) - log(c*(a + b*x^2)^p)/(e*(d + e*x)) + (log((4*b^3*p^2*x)/e - (p*(
b*d - e*(-a*b)^(1/2))*(2*a*b^2*e*p + 2*b^3*d*p*x - (2*b^2*e*p*(b*d - e*(-a
*b)^(1/2))*(4*a*d*e + 3*a*e^2*x - b*d^2*x))/(a*e^3 + b*d^2*e)))/(a*e^3 + b
*d^2*e))*(b*d*p - e*p*(-a*b)^(1/2)))/(a*e^3 + b*d^2*e) - (2*b*d*p*log(d +
e*x))/(a*e^3 + b*d^2*e)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.50

$$\int \frac{\log (c(a+bx^2)^p)}{(d+ex)^2}\,dx$$
$$= \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)d^2ep + 2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)d\,e^2px - \log(b\,x^2+a)\,ad\,e^2p - \log(b\,x^2+a)\,a\,e^3px - \cdots}{de\,(a\,e^3x + b\,d^2ex + \cdots}$$

input
```
int(log(c*(b*x^2+a)^p)/(e*x+d)^2,x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*d**2*e*p + 2*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*d*e**2*p*x - log(a + b*x**2)*a*d*e**2*p
- log(a + b*x**2)*a*e**3*p*x - 2*log(d + e*x)*b*d**3*p - 2*log(d + e*x)*b*
d**2*e*p*x + log((a + b*x**2)**p*c)*a*e**3*x + log((a + b*x**2)**p*c)*b*d*
*2*e*x)/(d*e*(a*d*e**2 + a*e**3*x + b*d**3 + b*d**2*e*x))
```

3.190 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 174

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)}+\frac{2\sqrt{a}b^{3/2}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\left(bd^2+ae^2\right)^2} - \frac{b(bd^2-ae^2)\,p\log(d+ex)}{e\left(bd^2+ae^2\right)^2} + \frac{b(bd^2-ae^2)\,p\log\left(a+bx^2\right)}{2e\left(bd^2+ae^2\right)^2}-\frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2}$$

output
```
b*d*p/e/(a*e^2+b*d^2)/(e*x+d)+2*a^(1/2)*b^(3/2)*d*p*arctan(b^(1/2)*x/a^(1/
2))/(a*e^2+b*d^2)^2-b*(-a*e^2+b*d^2)*p*ln(e*x+d)/e/(a*e^2+b*d^2)^2+1/2*b*(
-a*e^2+b*d^2)*p*ln(b*x^2+a)/e/(a*e^2+b*d^2)^2-1/2*ln(c*(b*x^2+a)^p)/e/(e*x
+d)^2
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.25

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx$$
$$= \frac{\frac{bp(d+ex)\left(\left(\sqrt{-a}bd^2+2a\sqrt{b}de+(-a)^{3/2}e^2\right)(d+ex)\log\left(\sqrt{-a}-\sqrt{b}x\right)+\left(\sqrt{-a}bd^2-2a\sqrt{b}de+(-a)^{3/2}e^2\right)(d+ex)\log\left(\sqrt{-a}+\sqrt{b}x\right)+2\sqrt{-a}(bd^3+}{\sqrt{-a}(bd^2+ae^2)^2}}{2e(d+ex)^2}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

output
```
((b*p*(d + e*x)*((Sqrt[-a]*b*d^2 + 2*a*Sqrt[b]*d*e + (-a)^(3/2)*e^2)*(d +
e*x)*Log[Sqrt[-a] - Sqrt[b]*x] + (Sqrt[-a]*b*d^2 - 2*a*Sqrt[b]*d*e + (-a)^
(3/2)*e^2)*(d + e*x)*Log[Sqrt[-a] + Sqrt[b]*x] + 2*Sqrt[-a]*(b*d^3 + a*d*e
^2 - (b*d^2 - a*e^2)*(d + e*x)*Log[d + e*x])))/(Sqrt[-a]*(b*d^2 + a*e^2)^2
) - Log[c*(a + b*x^2)^p])/(2*e*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2913, 594, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{ 2913}$$

$$\frac{bp\int \frac{x}{(d+ex)^2(bx^2+a)}dx}{e}-\frac{\log \left(c(a+bx^2)^p\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 594}$$

$$\frac{bp\left(\frac{d}{(d+ex)(ae^2+bd^2)}-\frac{\int -\frac{ae+bdx}{(d+ex)\left(bx^2+a\right)}dx}{ae^2+bd^2}\right)}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{bp\left(\frac{\int \frac{ae+bdx}{(d+ex)\left(bx^2+a\right)}dx}{ae^2+bd^2}+\frac{d}{(d+ex)(ae^2+bd^2)}\right)}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \; 657$$

$$\frac{bp\left(\frac{\int \left(\frac{e\left(ae^2-bd^2\right)}{\left(bd^2+ae^2\right)(d+ex)}+\frac{b\left(2ade+\left(bd^2-ae^2\right)x\right)}{\left(bd^2+ae^2\right)\left(bx^2+a\right)}\right)dx}{ae^2+bd^2}+\frac{d}{(d+ex)(ae^2+bd^2)}\right)}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{bp\left(\frac{\frac{2\sqrt{a}\sqrt{b}de\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{ae^2+bd^2}+\frac{\left(bd^2-ae^2\right)\log\left(a+bx^2\right)}{2\left(ae^2+bd^2\right)}-\frac{\left(bd^2-ae^2\right)\log(d+ex)}{ae^2+bd^2}}{ae^2+bd^2}+\frac{d}{(d+ex)(ae^2+bd^2)}\right)}{e}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}$$

input
```
Int[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

output
```
(b*p*(d/((b*d^2 + a*e^2)*(d + e*x)) + ((2*Sqrt[a]*Sqrt[b]*d*e*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2) - ((b*d^2 - a*e^2)*Log[d + e*x])/(b*d^2 +
 a*e^2) + ((b*d^2 - a*e^2)*Log[a + b*x^2])/(2*(b*d^2 + a*e^2)))/(b*d^2 + a
*e^2)))/e - Log[c*(a + b*x^2)^p]/(2*e*(d + e*x)^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

method	result	size
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{2e\left(ex+d\right)^{2}}+\frac{pb\left(\frac{\left(a\,e^{2}-b\,d^{2}\right)\ln\left(ex+d\right)}{\left(a\,e^{2}+b\,d^{2}\right)^{2}}+\frac{d}{\left(a\,e^{2}+b\,d^{2}\right)\left(ex+d\right)}+\frac{b\left(\frac{\left(-a\,e^{2}+b\,d^{2}\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{2dea\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{\left(a\,e^{2}+b\,d^{2}\right)^{2}}\right)}{e}$	147
risch	Expression too large to display	3183

input
```
int(ln(c*(b*x^2+a)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(b*x^2+a)^p)/e/(e*x+d)^2+p*b/e*((a*e^2-b*d^2)/(a*e^2+b*d^2)^2*ln
(e*x+d)+d/(a*e^2+b*d^2)/(e*x+d)+b/(a*e^2+b*d^2)^2*(1/2*(-a*e^2+b*d^2)/b*ln
(b*x^2+a)+2*d*e*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(162) = 324$.

Time = 0.14 (sec) , antiderivative size = 744, normalized size of antiderivative = 4.28

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx$$
$$= \left[\frac{2\,(b^2d^3e + abde^3)px + 2\,(bde^3px^2 + 2\,bd^2e^2px + bd^3ep)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right) + 2\,(b^2d^4 + abd^2e^2)p}{2}\right.$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2*d^3*e + a*b*d*e^3)*p*x + 2*(b*d*e^3*p*x^2 + 2*b*d^2*e^2*p*x +
 b*d^3*e*p)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(
b^2*d^4 + a*b*d^2*e^2)*p + ((b^2*d^2*e^2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e -
 a*b*d*e^3)*p*x - (3*a*b*d^2*e^2 + a^2*e^4)*p)*log(b*x^2 + a) - 2*((b^2*d^
2*e^2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x + (b^2*d^4 - a*b*d^
2*e^2)*p)*log(e*x + d) - (b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*log(c))/(b^2*
d^6*e + 2*a*b*d^4*e^3 + a^2*d^2*e^5 + (b^2*d^4*e^3 + 2*a*b*d^2*e^5 + a^2*e
^7)*x^2 + 2*(b^2*d^5*e^2 + 2*a*b*d^3*e^4 + a^2*d*e^6)*x), 1/2*(2*(b^2*d^3*
e + a*b*d*e^3)*p*x + 4*(b*d*e^3*p*x^2 + 2*b*d^2*e^2*p*x + b*d^3*e*p)*sqrt(
a*b)*arctan(sqrt(a*b)*x/a) + 2*(b^2*d^4 + a*b*d^2*e^2)*p + ((b^2*d^2*e^2 -
 a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x - (3*a*b*d^2*e^2 + a^2*e^4
)*p)*log(b*x^2 + a) - 2*((b^2*d^2*e^2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*
b*d*e^3)*p*x + (b^2*d^4 - a*b*d^2*e^2)*p)*log(e*x + d) - (b^2*d^4 + 2*a*b*
d^2*e^2 + a^2*e^4)*log(c))/(b^2*d^6*e + 2*a*b*d^4*e^3 + a^2*d^2*e^5 + (b^2
*d^4*e^3 + 2*a*b*d^2*e^5 + a^2*e^7)*x^2 + 2*(b^2*d^5*e^2 + 2*a*b*d^3*e^4 +
 a^2*d*e^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{\left(\frac{4\,abde\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2d^4+2\,abd^2e^2+a^2e^4)\sqrt{ab}} + \frac{(bd^2-ae^2)\log(bx^2+a)}{b^2d^4+2\,abd^2e^2+a^2e^4} - \frac{2\,(bd^2-ae^2)\log(ex+d)}{b^2d^4+2\,abd^2e^2+a^2e^4} + \frac{2\,d}{bd^3+ade^2+(bd^2e+ae^3)x}\right)bp}{2\,e} - \frac{\log\left((bx^2+a)^pc\right)}{2\,(ex+d)^2e}$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(4*a*b*d*e*arctan(b*x/sqrt(a*b))/((b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*
sqrt(a*b)) + (b*d^2 - a*e^2)*log(b*x^2 + a)/(b^2*d^4 + 2*a*b*d^2*e^2 + a^2
*e^4) - 2*(b*d^2 - a*e^2)*log(e*x + d)/(b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)
 + 2*d/(b*d^3 + a*d*e^2 + (b*d^2*e + a*e^3)*x))*b*p/e - 1/2*log((b*x^2 + a
)^p*c)/((e*x + d)^2*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.60

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{2\,ab^2dp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2d^4+2\,abd^2e^2+a^2e^4)\sqrt{ab}} + \frac{(b^2d^2p-abe^2p)\log\left(bx^2+a\right)}{2\left(b^2d^4e+2\,abd^2e^3+a^2e^5\right)} - \frac{p\log\left(bx^2+a\right)}{2\left(e^3x^2+2\,de^2x+d^2e\right)} - \frac{(b^2d^2p-abe^2p)\log\left(ex+d\right)}{b^2d^4e+2\,abd^2e^3+a^2e^5} + \frac{2\,bdepx+2\,bd^2p-bd^2\log\left(c\right)-ae^2\log\left(c\right)}{2\left(bd^2e^3x^2+ae^5x^2+2\,bd^3e^2x+2\,ade^4x+bd^4e+ad^2e^3\right)}$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

output
```
2*a*b^2*d*p*arctan(b*x/sqrt(a*b))/((b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*sqr
t(a*b)) + 1/2*(b^2*d^2*p - a*b*e^2*p)*log(b*x^2 + a)/(b^2*d^4*e + 2*a*b*d^
2*e^3 + a^2*e^5) - 1/2*p*log(b*x^2 + a)/(e^3*x^2 + 2*d*e^2*x + d^2*e) - (b
^2*d^2*p - a*b*e^2*p)*log(e*x + d)/(b^2*d^4*e + 2*a*b*d^2*e^3 + a^2*e^5) +
 1/2*(2*b*d*e*p*x + 2*b*d^2*p - b*d^2*log(c) - a*e^2*log(c))/(b*d^2*e^3*x^
2 + a*e^5*x^2 + 2*b*d^3*e^2*x + 2*a*d*e^4*x + b*d^4*e + a*d^2*e^3)
```

Mupad [B] (verification not implemented)

Time = 26.63 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.56

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{\ln\left(b^2\,x+\sqrt{-a\,b^3}\right)\,\left(b^2\,d^2\,p-a\,b\,e^2\,p+2\,d\,e\,p\,\sqrt{-a\,b^3}\right)}{2\,\left(a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e\right)} - \frac{\ln\left(d+e\,x\right)\,\left(b^2\,d^2\,p-a\,b\,e^2\,p\right)}{a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e} - \frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{2\,e\,\left(d^2+2\,d\,e\,x+e^2\,x^2\right)} - \frac{\ln\left(b^2\,x-\sqrt{-a\,b^3}\right)\,\left(a\,b\,e^2\,p-b^2\,d^2\,p+2\,d\,e\,p\,\sqrt{-a\,b^3}\right)}{2\,\left(a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e\right)} + \frac{b\,d\,p}{\left(x\,e^2+d\,e\right)\,\left(b\,d^2+a\,e^2\right)}$$

input
```
int(log(c*(a + b*x^2)^p)/(d + e*x)^3,x)
```

output
```
(log(b^2*x + (-a*b^3)^(1/2))*(b^2*d^2*p - a*b*e^2*p + 2*d*e*p*(-a*b^3)^(1/
2)))/(2*(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3)) - (log(d + e*x)*(b^2*d^2*p
- a*b*e^2*p))/(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3) - log(c*(a + b*x^2)^p)
/(2*e*(d^2 + e^2*x^2 + 2*d*e*x)) - (log(b^2*x - (-a*b^3)^(1/2))*(a*b*e^2*p
 - b^2*d^2*p + 2*d*e*p*(-a*b^3)^(1/2)))/(2*(a^2*e^5 + b^2*d^4*e + 2*a*b*d^
2*e^3)) + (b*d*p)/((d*e + e^2*x)*(a*e^2 + b*d^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.45

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx$$
$$= \frac{8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,d^4e^2px + 4\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,d^3e^3p\,x^2 - \log(b\,x^2+a)\,a^2d^2e^4p - \log(b\,x^2+a)\,a}{}$$

input
```
int(log(c*(b*x^2+a)^p)/(e*x+d)^3,x)
```

output
```
(4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*d**5*e*p + 8*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*d**4*e**2*p*x + 4*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*b*d**3*e**3*p*x**2 - log(a + b*x**2)*a**2*d**2
*e**4*p - 2*log(a + b*x**2)*a**2*d*e**5*p*x - log(a + b*x**2)*a**2*e**6*p*
x**2 - 3*log(a + b*x**2)*a*b*d**4*e**2*p - 6*log(a + b*x**2)*a*b*d**3*e**3
*p*x - 3*log(a + b*x**2)*a*b*d**2*e**4*p*x**2 + 2*log(d + e*x)*a*b*d**4*e*
*2*p + 4*log(d + e*x)*a*b*d**3*e**3*p*x + 2*log(d + e*x)*a*b*d**2*e**4*p*x
**2 - 2*log(d + e*x)*b**2*d**6*p - 4*log(d + e*x)*b**2*d**5*e*p*x - 2*log(
d + e*x)*b**2*d**4*e**2*p*x**2 + 2*log((a + b*x**2)**p*c)*a**2*d*e**5*x +
log((a + b*x**2)**p*c)*a**2*e**6*x**2 + 4*log((a + b*x**2)**p*c)*a*b*d**3*
e**3*x + 2*log((a + b*x**2)**p*c)*a*b*d**2*e**4*x**2 + 2*log((a + b*x**2)*
*p*c)*b**2*d**5*e*x + log((a + b*x**2)**p*c)*b**2*d**4*e**2*x**2 + a*b*d**
4*e**2*p - a*b*d**2*e**4*p*x**2 + b**2*d**6*p - b**2*d**4*e**2*p*x**2)/(2*
d**2*e*(a**2*d**2*e**4 + 2*a**2*d*e**5*x + a**2*e**6*x**2 + 2*a*b*d**4*e**
2 + 4*a*b*d**3*e**3*x + 2*a*b*d**2*e**4*x**2 + b**2*d**6 + 2*b**2*d**5*e*x
 + b**2*d**4*e**2*x**2))
```

3.191 $\int (d+ex)^3 \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 320

$$\begin{aligned}
&\int (d+ex)^3 \log \left(c(a+bx^3)^p\right) \, dx \\
&= -\frac{3(4bd^3-ae^3)\,px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 \\
&\quad - \frac{\sqrt{3}\sqrt[3]{a}\left(4bd^3+6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}} \\
&\quad + \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}} \\
&\quad - \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}} \\
&\quad - \frac{d(bd^3-4ae^3)\,p\log(a+bx^3)}{4be} + \frac{(d+ex)^4\log\left(c(a+bx^3)^p\right)}{4e}
\end{aligned}$$

output
```
-3/4*(-a*e^3+4*b*d^3)*p*x/b-9/4*d^2*e*p*x^2-d*e^2*p*x^3-3/16*e^3*p*x^4-1/4
*3^(1/2)*a^(1/3)*(4*b*d^3+6*a^(1/3)*b^(2/3)*d^2*e-a*e^3)*p*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))/b^(4/3)+1/4*a^(1/3)*(4*b*d^3-6*a^(1/3)*
b^(2/3)*d^2*e-a*e^3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)-1/8*a^(1/3)*(4*b*d^3-
6*a^(1/3)*b^(2/3)*d^2*e-a*e^3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/b^(4/3)-1/4*d*(-4*a*e^3+b*d^3)*p*ln(b*x^3+a)/b/e+1/4*(e*x+d)^4*ln(c*(b*x^
3+a)^p)/e
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.89

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right)\, dx$$

$$= \frac{\frac{3e(-4bd^3+ae^3)px}{b} - 9d^2e^2px^2 - 4de^3px^3 - \frac{3}{4}e^4px^4 + \frac{\sqrt{3}\sqrt[3]{a}e(-4bd^3+ae^3)p\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} + 9d^2e^2px^2 \text{ Hyperge}}{}$$

input
```
Integrate[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```

output
```
((3*e*(-4*b*d^3 + a*e^3)*p*x)/b - 9*d^2*e^2*p*x^2 - 4*d*e^3*p*x^3 - (3*e^4
*p*x^4)/4 + (Sqrt[3]*a^(1/3)*e*(-4*b*d^3 + a*e^3)*p*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]])/b^(4/3) + 9*d^2*e^2*p*x^2*Hypergeometric2F1[2/3, 1,
 5/3, -((b*x^3)/a)] + (a^(1/3)*e*(4*b*d^3 - a*e^3)*p*Log[a^(1/3) + b^(1/3)
*x])/b^(4/3) + (a^(1/3)*e*(-4*b*d^3 + a*e^3)*p*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(2*b^(4/3)) - (d*(b*d^3 - 4*a*e^3)*p*Log[a + b*x^3])/
b + (d + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2913, 2375, 27, 2375, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right)\, dx$$

$$\Big\downarrow \; 2913$$

$$\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}-\frac{3bp\int\frac{x^2(d+ex)^4}{bx^3+a}dx}{4e}$$

$\downarrow$ 2375

$$\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}-\frac{3bp\left(\frac{\int\frac{4x^2\left(bd^4+6be^2x^2d^2+4be^3x^3d+e\left(4bd^3-ae^3\right)x\right)}{bx^3+a}dx}{4b}+\frac{e^4x^4}{4b}\right)}{4e}$$

$\downarrow$ 27

$$\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}-\frac{3bp\left(\frac{\int\frac{x^2\left(bd^4+6be^2x^2d^2+4be^3x^3d+e\left(4bd^3-ae^3\right)x\right)}{bx^3+a}dx}{b}+\frac{e^4x^4}{4b}\right)}{4e}$$

$\downarrow$ 2375

$$\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}-\frac{3bp\left(\frac{\frac{\int\frac{3x^2\left(6b^2d^2e^2x^2+be\left(4bd^3-ae^3\right)x+bd\left(bd^3-4ae^3\right)\right)}{bx^3+a}dx}{3b}+\frac{4}{3}de^3x^3}{b}+\frac{e^4x^4}{4b}\right)}{4e}$$

$\downarrow$ 27

$$\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}-\frac{3bp\left(\frac{\frac{\int\frac{x^2\left(6b^2d^2e^2x^2+be\left(4bd^3-ae^3\right)x+bd\left(bd^3-4ae^3\right)\right)}{bx^3+a}dx}{b}+\frac{4}{3}de^3x^3}{b}+\frac{e^4x^4}{4b}\right)}{4e}$$

$\downarrow$ 2426

$$\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}-\frac{3bp\left(\frac{\frac{\int\left(-ae^4+6bd^2xe^2+4bd^3e-\frac{6abd^2xe^2+a\left(4bd^3-ae^3\right)e-bd\left(bd^3-4ae^3\right)x^2}{bx^3+a}\right)dx}{b}+\frac{4}{3}de^3x^3}{b}+\frac{e^4x^4}{4b}\right)}{4e}$$

$\downarrow$ 2009

$$\frac{(d+ex)^4\log\left(c(a+bx^3)^p\right)}{4e} - \frac{3bp\left(\frac{\frac{\frac{\sqrt[3]{a}e\left(-6\sqrt[3]{a}b^{2/3}d^2e-ae^3+4bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}}+\frac{\sqrt[3]{a}e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(6\sqrt[3]{a}b^{2/3}d^2e-ae^3+4bd^3\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\sqrt[3]{a}e\left(-6\sqrt[3]{a}\cdots\right.}{}}{b}}{b}\right.}{4e}$$

input
```
Int[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```

output
```
(-3*b*p*((e^4*x^4)/(4*b) + ((4*d*e^3*x^3)/3 + (e*(4*b*d^3 - a*e^3)*x + 3*b
*d^2*e^2*x^2 + (a^(1/3)*e*(4*b*d^3 + 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*ArcT
an[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3)) - (a^(1/3
)*e*(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(1/3)) + (a^(1/3)*e*(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(1/3)) + (d*(b*d^3 - 4*a*e
^3)*Log[a + b*x^3])/3)/b)/b))/(4*e) + ((d + e*x)^4*Log[c*(a + b*x^3)^p])/(
4*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.24

method	result
parts	$\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)e^{3}x^{4}}{4}+\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)e^{2}d\,x^{3}+\frac{3\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)e\,d^{2}x^{2}}{2}+d^{3}x\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)+\frac{\ln\left(c\left(b\,x^{3}+\right.\right.}{4e}$
risch	$\frac{p\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3}b+a\right)}\frac{\left(bd\left(4a\,e^{3}-b\,d^{3}\right)\_R^{2}+6ab\,d^{2}e^{2}\_R-a^{2}e^{4}+4ab\,d^{3}e\right)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{4b^{2}e}-3d^{3}px-d\,e^{2}p\,x^{3}-\frac{9d^{2}ep}{4}$

input
```
int((e*x+d)^3*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(c*(b*x^3+a)^p)*e^3*x^4+ln(c*(b*x^3+a)^p)*e^2*d*x^3+3/2*ln(c*(b*x^3+
a)^p)*e*d^2*x^2+d^3*x*ln(c*(b*x^3+a)^p)+1/4*ln(c*(b*x^3+a)^p)/e*d^4-3/4*p*
b/e*(-e/b^2*(-1/4*x^4*b*e^3-4/3*x^3*b*d*e^2-3*e*d^2*b*x^2+x*a*e^3-4*b*x*d^
3)+((a^2*e^4-4*a*b*d^3*e)*(1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6/b/(
1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))-6*a*b*d^2*e^2*(-1/3/b/(1/b
*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(
1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1
/3)*x-1)))+1/3*(-4*a*b*d*e^3+b^2*d^4)/b*ln(b*x^3+a))/b^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.62 (sec) , antiderivative size = 8840, normalized size of antiderivative = 27.62

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 22.09 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.83

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right) \, dx = -\frac{3a^2e^3p\,\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)}{4b} + 3ad^3p\,\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right) + \frac{9ad^2ep\,\mathrm{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)}{2} + ade^2p\left(\begin{cases}\frac{x^3}{a} & \text{for}\ b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right) + \frac{3ae^3px}{4b} - 3d^3px + d^3x\log\left(c\left(a+bx^3\right)^p\right) - \frac{9d^2epx^2}{4} + \frac{3d^2ex^2\log\left(c\left(a+bx^3\right)^p\right)}{2} - de^2px^3 + de^2x^3\log\left(c\left(a+bx^3\right)^p\right) - \frac{3e^3px^4}{16} + \frac{e^3x^4\log\left(c\left(a+bx^3\right)^p\right)}{4}$$

input
```
integrate((e*x+d)**3*ln(c*(b*x**3+a)**p),x)
```

output
```
-3*a**2*e**3*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x))
)/(4*b) + 3*a*d**3*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a
 + x))) + 9*a*d**2*e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t
**2*a*b + x)))/2 + a*d*e**2*p*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**
3)/b, True)) + 3*a*e**3*p*x/(4*b) - 3*d**3*p*x + d**3*x*log(c*(a + b*x**3)
**p) - 9*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c*(a + b*x**3)**p)/2 - d*e**2
*p*x**3 + d*e**2*x**3*log(c*(a + b*x**3)**p) - 3*e**3*p*x**4/16 + e**3*x**
4*log(c*(a + b*x**3)**p)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.04

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right)\, dx$$
$$= \frac{1}{16}\, bp\left(\frac{4\sqrt{3}\left(6\, abd^2e\left(\frac{a}{b}\right)^{\frac{2}{3}} + 4\, abd^3\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2e^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2} - \frac{3\, be^3x^4 + 16\, bde^2x^3 + 3\cdots}{\cdots}\right.$$
$$+ \frac{1}{4}\left(e^3x^4 + 4\, de^2x^3 + 6\, d^2ex^2 + 4\, d^3x\right)\log\left(\left(bx^3+a\right)^p c\right)$$

input
```
integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
1/16*b*p*(4*sqrt(3)*(6*a*b*d^2*e*(a/b)^(2/3) + 4*a*b*d^3*(a/b)^(1/3) - a^2
*e^3*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b
^2) - (3*b*e^3*x^4 + 16*b*d*e^2*x^3 + 36*b*d^2*e*x^2 + 12*(4*b*d^3 - a*e^3
)*x)/b^2 + 2*(8*a*b*d*e^2*(a/b)^(2/3) + 6*a*b*d^2*e*(a/b)^(1/3) - 4*a*b*d^
3 + a^2*e^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 4*
(4*a*b*d*e^2*(a/b)^(2/3) - 6*a*b*d^2*e*(a/b)^(1/3) + 4*a*b*d^3 - a^2*e^3)*
log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d
^2*e*x^2 + 4*d^3*x)*log((b*x^3 + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.19

$$\int (d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)\,dx$$
$$= -\frac{1}{16}\left(3e^3p-4e^3\log(c)\right)x^4 + \frac{ade^2p\log\left(|bx^3+a|\right)}{b} - \left(de^2p-de^2\log(c)\right)x^3 - \frac{3}{4}\left(3d^2ep-2d^2e\log(c)\right)x^2 + \frac{1}{4}\left(e^3px^4+4de^2px^3+6d^2epx^2+4d^3px\right)\log\left(bx^3+a\right) - \frac{\left(12bd^3p-3ae^3p-4bd^3\log(c)\right)x}{4b} + \frac{\sqrt{3}\left(4\left(-ab^2\right)^{\frac{1}{3}}bd^3p-\left(-ab^2\right)^{\frac{1}{3}}ae^3p-6\left(-ab^2\right)^{\frac{2}{3}}d^2ep\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{4b^2} + \frac{\left(4\left(-ab^2\right)^{\frac{1}{3}}bd^3p-\left(-ab^2\right)^{\frac{1}{3}}ae^3p+6\left(-ab^2\right)^{\frac{2}{3}}d^2ep\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{8b^2} - \frac{\left(6ab^3d^2ep\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4ab^3d^3p-a^2b^2e^3p\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{4ab^3}$$

input
```
integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
-1/16*(3*e^3*p - 4*e^3*log(c))*x^4 + a*d*e^2*p*log(abs(b*x^3 + a))/b - (d*
e^2*p - d*e^2*log(c))*x^3 - 3/4*(3*d^2*e*p - 2*d^2*e*log(c))*x^2 + 1/4*(e^
3*p*x^4 + 4*d*e^2*p*x^3 + 6*d^2*e*p*x^2 + 4*d^3*p*x)*log(b*x^3 + a) - 1/4*
(12*b*d^3*p - 3*a*e^3*p - 4*b*d^3*log(c))*x/b + 1/4*sqrt(3)*(4*(-a*b^2)^(1
/3)*b*d^3*p - (-a*b^2)^(1/3)*a*e^3*p - 6*(-a*b^2)^(2/3)*d^2*e*p)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 1/8*(4*(-a*b^2)^(1/3)*b
*d^3*p - (-a*b^2)^(1/3)*a*e^3*p + 6*(-a*b^2)^(2/3)*d^2*e*p)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))/b^2 - 1/4*(6*a*b^3*d^2*e*p*(-a/b)^(1/3) + 4*a*b
^3*d^3*p - a^2*b^2*e^3*p)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 25.89 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.68

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \ln \left(c \left(b\,x^3+a\right)^p\right) \left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right) - x\left(3\,d^3\,p-\frac{3\,a\,e^3\,p}{4\,b}\right)+\left(\sum_{k=1}^{3}\ln\left(x\left(\frac{9\,a^3\,d\,e^5\,p^2}{4}+\frac{45\,b\,a^2\,d^4\,e^2\,p^2}{4}\right) + \mathrm{root}\left(64\,b^4\,c^3-192\,a\,b^3\,c^2\,d\,e^2\,p+288\,a\,b^3\,c\,d^5\,e\,p^2+120\,a^2\,b^2\,c\,d^2\,e^4\,p^2-4\,a^3\,b\,d^3\,e^6\,p^3-24\,a^2\,b^2\,d^6\,e^3\,p^3- \right. \right. \right. + \frac{45\,a^3\,d^2\,e^4\,p^2}{8}+\frac{27\,a^2\,b\,d^5\,e\,p^2}{2}\right)\mathrm{root}\left(64\,b^4\,c^3-192\,a\,b^3\,c^2\,d\,e^2\,p+288\,a\,b^3\,c\,d^5\,e\,p^2 + 120\,a^2\,b^2\,c\,d^2\,e^4\,p^2-4\,a^3\,b\,d^3\,e^6\,p^3-24\,a^2\,b^2\,d^6\,e^3\,p^3-64\,a\,b^3\,d^9\,p^3+a^4\,e^9\,p^3,c,k\right)\Bigg) - \frac{3\,e^3\,p\,x^4}{16}-\frac{9\,d^2\,e\,p\,x^2}{4}-d\,e^2\,p\,x^3$$

input
```
int(log(c*(a + b*x^3)^p)*(d + e*x)^3,x)
```

output
```
log(c*(a + b*x^3)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) -
 x*(3*d^3*p - (3*a*e^3*p)/(4*b)) + symsum(log(x*((9*a^3*d*e^5*p^2)/4 + (45
*a^2*b*d^4*e^2*p^2)/4) + root(64*b^4*c^3 - 192*a*b^3*c^2*d*e^2*p + 288*a*b
^3*c*d^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3*b*d^3*e^6*p^3 - 24*a^2*
b^2*d^6*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c, k)*(x*(9*a*b^2*d^3*p
- (9*a^2*b*e^3*p)/4) + 9*root(64*b^4*c^3 - 192*a*b^3*c^2*d*e^2*p + 288*a*b
^3*c*d^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3*b*d^3*e^6*p^3 - 24*a^2*
b^2*d^6*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c, k)*a*b^2 - 18*a^2*b*d
*e^2*p) + (45*a^3*d^2*e^4*p^2)/8 + (27*a^2*b*d^5*e*p^2)/2)*root(64*b^4*c^3
 - 192*a*b^3*c^2*d*e^2*p + 288*a*b^3*c*d^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p
^2 - 4*a^3*b*d^3*e^6*p^3 - 24*a^2*b^2*d^6*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4
*e^9*p^3, c, k), k, 1, 3) - (3*e^3*p*x^4)/16 - (9*d^2*e*p*x^2)/4 - d*e^2*p
*x^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.36

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right)\, dx$$

$$= \frac{4b^{\frac{1}{3}}a^{\frac{5}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) e^3p - 16b^{\frac{4}{3}}a^{\frac{2}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) d^3p - 24\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) ab\, d^2ep - 6b^{\frac{1}{3}}a^{\frac{5}{3}}\mathrm{lo}}{}$$

input
```
int((e*x+d)^3*log(c*(b*x^3+a)^p),x)
```

output
```
(4*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*a*e**3*p - 16*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
x)/(a**(1/3)*sqrt(3)))*b*d**3*p - 24*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x
)/(a**(1/3)*sqrt(3)))*a*b*d**2*e*p - 6*b**(1/3)*a**(2/3)*log(a**(1/3) + b*
*(1/3)*x)*a*e**3*p + 24*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*b*d**
3*p + 2*b**(1/3)*a**(2/3)*log((a + b*x**3)**p*c)*a*e**3 - 8*b**(1/3)*a**(2
/3)*log((a + b*x**3)**p*c)*b*d**3 + 16*b**(2/3)*a**(1/3)*log((a + b*x**3)*
*p*c)*a*d*e**2 + 16*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*b*d**3*x + 24
*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*b*d**2*e*x**2 + 16*b**(2/3)*a**(
1/3)*log((a + b*x**3)**p*c)*b*d*e**2*x**3 + 4*b**(2/3)*a**(1/3)*log((a + b
*x**3)**p*c)*b*e**3*x**4 + 12*b**(2/3)*a**(1/3)*a*e**3*p*x - 48*b**(2/3)*a
**(1/3)*b*d**3*p*x - 36*b**(2/3)*a**(1/3)*b*d**2*e*p*x**2 - 16*b**(2/3)*a*
*(1/3)*b*d*e**2*p*x**3 - 3*b**(2/3)*a**(1/3)*b*e**3*p*x**4 - 36*log(a**(1/
3) + b**(1/3)*x)*a*b*d**2*e*p + 12*log((a + b*x**3)**p*c)*a*b*d**2*e)/(16*
b**(2/3)*a**(1/3)*b)
```

3.192 $\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 250

$$\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx = -3d^2px-\frac{3}{2}depx^2-\frac{1}{3}e^2px^3 -\frac{\sqrt{3}\sqrt[3]{a}d\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} +\frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} -\frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b^{2/3}} -\frac{\left(bd^3-ae^3\right)p\log\left(a+bx^3\right)}{3be} +\frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e}$$

output
```
-3*d^2*p*x-3/2*d*e*p*x^2-1/3*e^2*p*x^3-3^(1/2)*a^(1/3)*d*(b^(1/3)*d+a^(1/3
)*e)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))/b^(2/3)+a^(1/3)*d
*(b^(1/3)*d-a^(1/3)*e)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)-1/2*a^(1/3)*d*(b^(1
/3)*d-a^(1/3)*e)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)-1/3*(
-a*e^3+b*d^3)*p*ln(b*x^3+a)/b/e+1/3*(e*x+d)^3*ln(c*(b*x^3+a)^p)/e
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.92

$$\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$= \frac{-\frac{p\left(18bd^2ex+9bde^2x^2+2be^3x^3+6\sqrt{3}\sqrt[3]{a}b^{2/3}d^2e\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-9bde^2x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)-6\sqrt[3]{a}b^{2/3}d^2e\log\left(\sqrt[3]{\right.}\right.}{2b}}{3e}$$

input
```
Integrate[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```

output
```
(-1/2*(p*(18*b*d^2*e*x + 9*b*d*e^2*x^2 + 2*b*e^3*x^3 + 6*Sqrt[3]*a^(1/3)*b
^(2/3)*d^2*e*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 9*b*d*e^2*x^2*H
ypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] - 6*a^(1/3)*b^(2/3)*d^2*e*Log[
a^(1/3) + b^(1/3)*x] + 3*a^(1/3)*b^(2/3)*d^2*e*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2] + 2*(b*d^3 - a*e^3)*Log[a + b*x^3]))/b + (d + e*x)^3*L
og[c*(a + b*x^3)^p])/(3*e)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2913, 2375, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$\Big\downarrow \; 2913$$
$$\frac{(d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right)}{3e}-\frac{bp\int \frac{x^2(d+ex)^3}{bx^3+a}dx}{e}$$

$$\Big\downarrow \text{2375}$$

$$\frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e}-\frac{bp\left(\frac{\int\frac{3x^2\left(bd^3+3bexd^2+3be^2x^2d-ae^3\right)}{bx^3+a}dx}{3b}+\frac{e^3x^3}{3b}\right)}{e}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e}-\frac{bp\left(\frac{\int\frac{x^2\left(bd^3+3bexd^2+3be^2x^2d-ae^3\right)}{bx^3+a}dx}{b}+\frac{e^3x^3}{3b}\right)}{e}$$

$$\Big\downarrow \text{2426}$$

$$\frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e}-\frac{bp\left(\frac{\int\left(3ed^2+3e^2xd-\frac{3aed^2+3ae^2xd-\left(bd^3-ae^3\right)x^2}{bx^3+a}\right)dx}{b}+\frac{e^3x^3}{3b}\right)}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e}-$$

$$\frac{bp\left(\frac{\frac{\sqrt[3]{a}de\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b^{2/3}}+\frac{\sqrt{3}\sqrt[3]{a}de\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{b^{2/3}}-\frac{\sqrt[3]{a}de\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\right.}{b^{2/3}}}{b}\right.}{e}$$

input
```
Int[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```

output
```
-((b*p*((e^3*x^3)/(3*b) + (3*d^2*e*x + (3*d*e^2*x^2)/2 + (Sqrt[3]*a^(1/3)*
d*e*(b^(1/3)*d + a^(1/3)*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/b^(2/3) - (a^(1/3)*d*e*(b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*
x])/b^(2/3) + (a^(1/3)*d*e*(b^(1/3)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(2*b^(2/3)) + ((b*d^3 - a*e^3)*Log[a + b*x^3])/(3
*b))/b))/e) + ((d + e*x)^3*Log[c*(a + b*x^3)^p])/(3*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.34

method	result
parts	$\frac{\ln(c(b\,x^3+a)^p)e^2x^3}{3}+\ln\left(c(b\,x^3+a)^p\right)ed\,x^2+d^2x\ln\left(c(b\,x^3+a)^p\right)+\frac{\ln(c(b\,x^3+a)^p)d^3}{3e}-\frac{pb\left(\frac{e\left(\frac{1}{3}e^2x^3+\frac{3}{2}e\right.}{b}\right.}{}$
risch	$\frac{(ex+d)^3\ln((b\,x^3+a)^p)}{3e}-\frac{ie^2\pi\,x^3\mathrm{csgn}(ic(b\,x^3+a)^p)^3}{6}-\frac{ix\pi\,d^2\mathrm{csgn}(ic(b\,x^3+a)^p)^3}{2}+\frac{ie\pi d\,x^2\,\mathrm{csgn}(i(b\,x^3+a)^p)\mathrm{csgn}(ic(b\,x^3+}{2}$

input
```
int((e*x+d)^2*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(c*(b*x^3+a)^p)*e^2*x^3+ln(c*(b*x^3+a)^p)*e*d*x^2+d^2*x*ln(c*(b*x^3+
a)^p)+1/3*ln(c*(b*x^3+a)^p)/e*d^3-p*b/e*(e/b*(1/3*e^2*x^3+3/2*e*x^2*d+3*d^
2*x)+(-3*e*d^2*a*(1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2
/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))-3*e^2*d*a*(-1/3/b/(1/b*a)^(1/3)*ln(
x+(1/b*a)^(1/3))+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))
+1/3*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))+1/
3*(-a*e^3+b*d^3)/b*ln(b*x^3+a))/b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.29 (sec) , antiderivative size = 5799, normalized size of antiderivative = 23.20

$$\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 15.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.69

$$\begin{aligned}\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx &= 3ad^2p \operatorname{RootSum}\left(27t^3a^2b-1, \left(t \mapsto t\log\left(3ta+x\right)\right)\right) \\ &+ 3adep \operatorname{RootSum}\left(27t^3ab^2+1, \left(t \mapsto t\log\left(9t^2ab+x\right)\right)\right) \\ &+ \frac{ae^2p\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0 \\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3} - 3d^2px + d^2x\log\left(c\left(a+bx^3\right)^p\right) \\ &- \frac{3depx^2}{2} + dex^2\log\left(c\left(a+bx^3\right)^p\right) - \frac{e^2px^3}{3} + \frac{e^2x^3\log\left(c\left(a+bx^3\right)^p\right)}{3}\end{aligned}$$

input
```
integrate((e*x+d)**2*ln(c*(b*x**3+a)**p),x)
```

output
```
3*a*d**2*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x))) +
3*a*d*e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x))
) + a*e**2*p*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b, True))/3 -
3*d**2*p*x + d**2*x*log(c*(a + b*x**3)**p) - 3*d*e*p*x**2/2 + d*e*x**2*log
(c*(a + b*x**3)**p) - e**2*p*x**3/3 + e**2*x**3*log(c*(a + b*x**3)**p)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx =$$

$$-\frac{1}{6}bp\left(\frac{2e^2x^3+9dex^2+18d^2x}{b} - \frac{6\sqrt{3}\left(abde\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2} - \frac{\left(2ae^2\left(\frac{a}{b}\right)\right.}{}\right.$$

$$+\frac{1}{3}\left(e^2x^3+3dex^2+3d^2x\right)\log\left(\left(bx^3+a\right)^pc\right)$$

input
```
integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
-1/6*b*p*((2*e^2*x^3 + 9*d*e*x^2 + 18*d^2*x)/b - 6*sqrt(3)*(a*b*d*e*(a/b)^
(2/3) + a*b*d^2*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(a*b^2) - (2*a*e^2*(a/b)^(2/3) + 3*a*d*e*(a/b)^(1/3) - 3*a*d^2)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 2*(a*e^2*(a/b)^(2/
3) - 3*a*d*e*(a/b)^(1/3) + 3*a*d^2)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
) + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x^3 + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.10

$$\int (d+ex)^2\log \left(c\left(a+bx^3\right)^p\right)\,dx$$
$$= -\frac{1}{3}\left(e^2p-e^2\log\left(c\right)\right)x^3+\frac{ae^2p\log\left(|bx^3+a|\right)}{3\,b}-\frac{1}{2}\left(3\,dep-2\,de\log\left(c\right)\right)x^2 - \left(3\,d^2p-d^2\log\left(c\right)\right)x+\frac{1}{3}\left(e^2px^3+3\,depx^2+3\,d^2px\right)\log\left(bx^3+a\right) + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd^2p-\left(-ab^2\right)^{\frac{2}{3}}dep\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2} - \frac{\left(abdep\left(-\frac{a}{b}\right)^{\frac{1}{3}}+abd^2p\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bd^2p+\left(-ab^2\right)^{\frac{2}{3}}dep\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^2}$$

input
```
integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
-1/3*(e^2*p - e^2*log(c))*x^3 + 1/3*a*e^2*p*log(abs(b*x^3 + a))/b - 1/2*(3
*d*e*p - 2*d*e*log(c))*x^2 - (3*d^2*p - d^2*log(c))*x + 1/3*(e^2*p*x^3 + 3
*d*e*p*x^2 + 3*d^2*p*x)*log(b*x^3 + a) + sqrt(3)*((-a*b^2)^(1/3)*b*d^2*p -
 (-a*b^2)^(2/3)*d*e*p)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/b^2 - (a*b*d*e*p*(-a/b)^(1/3) + a*b*d^2*p)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a*b) + 1/2*((-a*b^2)^(1/3)*b*d^2*p + (-a*b^2)^(2/3)*d*e*p)*lo
g(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

Mupad [B] (verification not implemented)

Time = 25.62 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.43

$$\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$= \left(\sum_{k=1}^{3} \ln \left(\mathrm{root}\left(27\,b^3\,c^3-27\,a\,b^2\,c^2\,e^2\,p+81\,a\,b^2\,c\,d^3\,e\,p^2+9\,a^2\,b\,c\,e^4\,p^2-27\,a\,b^2\,d^6\,p^3-a^3\,e^6\,p^3,c,k\right)\,\left(\mathrm{root}\left(2\right.\right.\right.\right.$$
$$+\,a^3\,e^4\,p^2+9\,a^2\,b\,d^3\,e\,p^2+6\,a^2\,b\,d^2\,e^2\,p^2\,x\Big)\ \mathrm{root}\big(27\,b^3\,c^3-27\,a\,b^2\,c^2\,e^2\,p$$
$$\left.+\,81\,a\,b^2\,c\,d^3\,e\,p^2+9\,a^2\,b\,c\,e^4\,p^2-27\,a\,b^2\,d^6\,p^3-a^3\,e^6\,p^3,c,k\big)\right)$$
$$+\ln\left(c\left(b\,x^3+a\right)^p\right)\,\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right)-3\,d^2\,p\,x-\frac{e^2\,p\,x^3}{3}-\frac{3\,d\,e\,p\,x^2}{2}$$

input
```
int(log(c*(a + b*x^3)^p)*(d + e*x)^2,x)
```

output
```
symsum(log(root(27*b^3*c^3 - 27*a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9
*a^2*b*c*e^4*p^2 - 27*a*b^2*d^6*p^3 - a^3*e^6*p^3, c, k)*(9*root(27*b^3*c^
3 - 27*a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*a^2*b*c*e^4*p^2 - 27*a*b
^2*d^6*p^3 - a^3*e^6*p^3, c, k)*a*b^2 - 6*a^2*b*e^2*p + 9*a*b^2*d^2*p*x) +
 a^3*e^4*p^2 + 9*a^2*b*d^3*e*p^2 + 6*a^2*b*d^2*e^2*p^2*x)*root(27*b^3*c^3
- 27*a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*a^2*b*c*e^4*p^2 - 27*a*b^2
*d^6*p^3 - a^3*e^6*p^3, c, k), k, 1, 3) + log(c*(a + b*x^3)^p)*(d^2*x + (e
^2*x^3)/3 + d*e*x^2) - 3*d^2*p*x - (e^2*p*x^3)/3 - (3*d*e*p*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.16

$$\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$= \frac{-6b^{\frac{4}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)d^2p-6\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)abdep+9b^{\frac{4}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)d^2p-3b^{\frac{4}{3}}a^{\frac{2}{3}}\log\left((b\right.}{}$$

input
```
int((e*x+d)^2*log(c*(b*x^3+a)^p),x)
```

output

```
( - 6*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*b*d**2*p - 6*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqr
t(3)))*a*b*d*e*p + 9*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*b*d**2*p
 - 3*b**(1/3)*a**(2/3)*log((a + b*x**3)**p*c)*b*d**2 + 2*b**(2/3)*a**(1/3)
*log((a + b*x**3)**p*c)*a*e**2 + 6*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c
)*b*d**2*x + 6*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*b*d*e*x**2 + 2*b**
(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*b*e**2*x**3 - 18*b**(2/3)*a**(1/3)*b
*d**2*p*x - 9*b**(2/3)*a**(1/3)*b*d*e*p*x**2 - 2*b**(2/3)*a**(1/3)*b*e**2*
p*x**3 - 9*log(a**(1/3) + b**(1/3)*x)*a*b*d*e*p + 3*log((a + b*x**3)**p*c)
*a*b*d*e)/(6*b**(2/3)*a**(1/3)*b)
```

3.193 $\int (d+ex)\log\left(c(a+bx^3)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 229

$$\int (d+ex)\log\left(c(a+bx^3)^p\right)\,dx = -3dpx-\frac{3}{4}epx^2 - \frac{\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}} - \frac{d^2p\log\left(a+bx^3\right)}{2e} + \frac{(d+ex)^2\log\left(c(a+bx^3)^p\right)}{2e}$$

output
```
-3*d*p*x-3/4*e*p*x^2-1/2*3^(1/2)*a^(1/3)*(2*b^(1/3)*d+a^(1/3)*e)*p*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))/b^(2/3)+1/2*a^(1/3)*(2*b^(1/3)*
d-a^(1/3)*e)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)-1/4*a^(1/3)*(2*b^(1/3)*d-a^(1
/3)*e)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)-1/2*d^2*p*ln(b*
x^3+a)/e+1/2*(e*x+d)^2*ln(c*(b*x^3+a)^p)/e
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.89

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx=-3dpx-\frac{3}{4}epx^2+\frac{\sqrt{3}\sqrt[3]{a}dp\arctan\left(\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}\right)}{\sqrt[3]{b}}+\frac{3}{4}epx^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)+\frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+dx\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{2}ex^2\log\left(c\left(a+bx^3\right)^p\right)$$

input
```
Integrate[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

output
```
-3*d*p*x - (3*e*p*x^2)/4 + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(-(a^(1/3)*b^(1/3))
 + 2*b^(2/3)*x)/(Sqrt[3]*a^(1/3)*b^(1/3))])/b^(1/3) + (3*e*p*x^2*Hypergeom
etric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 + (a^(1/3)*d*p*Log[a^(1/3) + b^(1/3
)*x])/b^(1/3) - (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(2*b^(1/3)) + d*x*Log[c*(a + b*x^3)^p] + (e*x^2*Log[c*(a + b*x^3)^p])/2
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {2913, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx$$

$$\Big\downarrow \; 2913$$

$$\frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}-\frac{3bp\int \frac{x^2(d+ex)^2}{bx^3+a}dx}{2e}$$

$$\Big\downarrow \; 2426$$

$$\frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}-\frac{3bp\int \left(\frac{xe^2}{b}+\frac{2de}{b}-\frac{axe^2+2ade-bd^2x^2}{b(bx^3+a)}\right)dx}{2e}$$

$$\Big\downarrow \; 2009$$

$$\frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}-$$

$$\frac{3bp\left(\frac{\sqrt[3]{a}e\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}+\frac{\sqrt[3]{a}e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)}{\sqrt{3}b^{5/3}}-\frac{\sqrt[3]{a}e\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log}{3b^{5/3}}\right.}{2e}$$

input
```
Int[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

output
```
(-3*b*p*((2*d*e*x)/b + (e^2*x^2)/(2*b) + (a^(1/3)*e*(2*b^(1/3)*d + a^(1/3)
*e)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) -
 (a^(1/3)*e*(2*b^(1/3)*d - a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)
) + (a^(1/3)*e*(2*b^(1/3)*d - a^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*b^(5/3)) + (d^2*Log[a + b*x^3])/(3*b)))/(2*e) + ((d + e*
x)^2*Log[c*(a + b*x^3)^p])/(2*e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.08

method	result
parts	$\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right) e\,x^{2}}{2}+d x \ln\left(c\left(b\,x^{3}+a\right)^{p}\right)-\frac{3 p b \left(\frac{\frac{1}{2} e\,x^{2}+2 d x}{b}-\frac{\left(2 d \left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$
risch	$\left(\frac{1}{2} e\,x^{2}+d x\right) \ln\left(\left(b\,x^{3}+a\right)^{p}\right)+\frac{i c \operatorname{sgn}\left(i c \left(b\,x^{3}+a\right)^{p}\right)^{2} \operatorname{csgn}\left(i \left(b\,x^{3}+a\right)^{p}\right) x^{2} e \pi}{4}-\frac{i \pi e\,x^{2} \operatorname{csgn}\left(i \left(b\,x^{3}+a\right)^{p}\right) \operatorname{csgn}\left(i c \left(b\,x^{3}+\cdots\right.\right.}{4}$

input
```
int((e*x+d)*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*(b*x^3+a)^p)*e*x^2+d*x*ln(c*(b*x^3+a)^p)-3/2*p*b*(1/b*(1/2*e*x^2+
2*d*x)-(2*d*(1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*l
n(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))+e*(-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/
3))+1/6/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*3^(1/2)/
b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))))/b*a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 2284, normalized size of antiderivative = 9.97

$$\int (d+ex)\log \left(c(a+bx^3)^p\right)\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
-3/4*e*p*x^2 - 3*d*p*x + 1/4*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((
8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) -
 (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/
b^2)^(1/3)*(I*sqrt(3) + 1))*log(4*a*d*e^2*p^2 + 2*(4*(1/2)^(2/3)*a*d*e*p^2
*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3
*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^
3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p + 1/4*(4*(1/2)^(2
/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*
p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^
2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b*e + (8*b
*d^3 + a*e^3)*p^2*x) - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*
b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (
1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^
2)^(1/3)*(I*sqrt(3) + 1) - sqrt(3)*sqrt(-(32*a*d*e*p^2 + (4*(1/2)^(2/3)*a*
d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 -
a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8
*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b)/b))*log(-2*a*
d*e^2*p^2 - (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*
a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*
b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*...
```

Sympy [A] (verification not implemented)

Time = 10.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.49

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx = 3adp\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right) + \frac{3aep\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)}{2} - 3dpx + dx\log\left(c\left(a+bx^3\right)^p\right) - \frac{3epx^2}{4} + \frac{ex^2\log\left(c\left(a+bx^3\right)^p\right)}{2}$$

input
```
integrate((e*x+d)*ln(c*(b*x**3+a)**p),x)
```

output
```
3*a*d*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x))) + 3*a
*e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))/2 -
 3*d*p*x + d*x*log(c*(a + b*x**3)**p) - 3*e*p*x**2/4 + e*x**2*log(c*(a + b
*x**3)**p)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.82

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{4}bp\left(\frac{3\left(ex^2+4\,dx\right)}{b} - \frac{2\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ad\right)\log\left(x^2-\right.}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right. + \frac{1}{2}\left(ex^2+2\,dx\right)\log\left(\left(bx^3+a\right)^pc\right)$$

input
```
integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
-1/4*b*p*(3*(e*x^2 + 4*d*x)/b - 2*sqrt(3)*(a*e*(a/b)^(1/3) + 2*a*d)*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - (a*e*(a/
b)^(1/3) - 2*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3))
 + 2*(a*e*(a/b)^(1/3) - 2*a*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + 1
/2*(e*x^2 + 2*d*x)*log((b*x^3 + a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.93

$$\int (d+ex)\log \left(c\left(a+bx^3\right)^p\right)\,dx$$
$$= -\frac{1}{4}\left(3\,ep-2\,e\log\left(c\right)\right)x^2-\left(3\,dp-d\log\left(c\right)\right)x+\frac{1}{2}\left(epx^2+2\,dpx\right)\log\left(bx^3+a\right)$$
$$-\frac{\left(aep\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,adp\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{2\,a}$$
$$+\frac{\left(2\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}bdp-\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}ep\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{2\,b^2}$$
$$+\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bdp+\left(-ab^2\right)^{\frac{2}{3}}ep\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{4\,b^2}$$

input
```
integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
-1/4*(3*e*p - 2*e*log(c))*x^2 - (3*d*p - d*log(c))*x + 1/2*(e*p*x^2 + 2*d*
p*x)*log(b*x^3 + a) - 1/2*(a*e*p*(-a/b)^(1/3) + 2*a*d*p)*(-a/b)^(1/3)*log(
abs(x - (-a/b)^(1/3)))/a + 1/2*(2*sqrt(3)*(-a*b^2)^(1/3)*b*d*p - sqrt(3)*(
-a*b^2)^(2/3)*e*p)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b
^2 + 1/4*(2*(-a*b^2)^(1/3)*b*d*p + (-a*b^2)^(2/3)*e*p)*log(x^2 + x*(-a/b)^
(1/3) + (-a/b)^(2/3))/b^2
```

Mupad [B] (verification not implemented)

Time = 25.53 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.92

$$\int (d+ex)\log \left(c\left(a+bx^3\right)^p\right)\,dx$$
$$=\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(8\,b^2\,c^3+12\,a\,b\,c\,d\,e\,p^2-8\,a\,b\,d^3\,p^3+a^2\,e^3\,p^3,c,k\right)\,\left(\mathrm{root}\left(8\,b^2\,c^3+12\,a\,b\,c\,d\,e\,p^2-8\,a\,b\,d^3\,p^3+a\right.\right.\right.\right.$$
$$\left.\left.+\frac{9\,a^2\,b\,d\,e\,p^2}{2}+\frac{9\,a^2\,b\,e^2\,p^2\,x}{4}\right)\,\mathrm{root}\left(8\,b^2\,c^3+12\,a\,b\,c\,d\,e\,p^2-8\,a\,b\,d^3\,p^3+a^2\,e^3\,p^3,c,k\right)\right)$$
$$+\ln\left(c\,\left(b\,x^3+a\right)^p\right)\,\left(\frac{e\,x^2}{2}+d\,x\right)-\frac{3\,e\,p\,x^2}{4}-3\,d\,p\,x$$

input
```
int(log(c*(a + b*x^3)^p)*(d + e*x),x)
```

output
```
symsum(log(root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3
, c, k)*(9*root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3
, c, k)*a*b^2 + 9*a*b^2*d*p*x) + (9*a^2*b*d*e*p^2)/2 + (9*a^2*b*e^2*p^2*x)
/4)*root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3, c, k)
, k, 1, 3) + log(c*(a + b*x^3)^p)*(d*x + (e*x^2)/2) - (3*e*p*x^2)/4 - 3*d*
p*x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90

$$\int (d+ex)\log \left(c\left(a+bx^3\right)^p\right)\,dx$$
$$=\frac{-4b^{\frac{1}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)dp-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)aep+6b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)dp-2b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(\left(b\,x^3+\right.\right.}{}$$

input
```
int((e*x+d)*log(c*(b*x^3+a)^p),x)
```

output
```
( - 4*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*d*p - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))
)*a*e*p + 6*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*d*p - 2*b**(1/3)*
a**(2/3)*log((a + b*x**3)**p*c)*d + 4*b**(2/3)*a**(1/3)*log((a + b*x**3)**
p*c)*d*x + 2*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*e*x**2 - 12*b**(2/3)
*a**(1/3)*d*p*x - 3*b**(2/3)*a**(1/3)*e*p*x**2 - 3*log(a**(1/3) + b**(1/3)
*x)*a*e*p + log((a + b*x**3)**p*c)*a*e)/(4*b**(2/3)*a**(1/3))
```

3.194 $\int \log\left(c(a+bx^3)^p\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \log\left(c\left(a+bx^3\right)^p\right) dx = -3px - \frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x\log\left(c\left(a+bx^3\right)^p\right)$$

output
```
-3*p*x-3^(1/2)*a^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))
/b^(1/3)+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-1/2*a^(1/3)*p*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)+x*ln(c*(b*x^3+a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \log\left(c\left(a+bx^3\right)^p\right) dx = -3px - \frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x\log\left(c\left(a+bx^3\right)^p\right)$$

input
```
Integrate[Log[c*(a + b*x^3)^p],x]
```

output
```
-3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b
^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)
^p]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {2898, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c\left(a+bx^3\right)^p\right) \, dx$$

$$\Big\downarrow \text{2898}$$

$$x \log \left(c\left(a+bx^3\right)^p\right)-3bp \int \frac{x^3}{bx^3+a}dx$$

$$\Big\downarrow \text{843}$$

$$x \log \left(c\left(a+bx^3\right)^p\right)-3bp \left(\frac{x}{b}-\frac{a \int \frac{1}{bx^3+a}dx}{b}\right)$$

$$\Big\downarrow \text{750}$$

$$x \log \left(c\left(a+bx^3\right)^p\right)-3bp \left(\frac{x}{b}-\frac{a \left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)$$

$$\Big\downarrow \text{16}$$

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\downarrow$ 1142

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\downarrow$ 25

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\downarrow$ 27

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\big\downarrow$ 1082

$$x\log\left(c\left(a+bx^3\right)^p\right)-3bp\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\big\downarrow$ 217

$$3bp\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$\downarrow$ 1103

$$x\log\left(c\left(a+bx^3\right)^p\right)-$$

$$3bp\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

input
```
Int[Log[c*(a + b*x^3)^p],x]
```

output
```
-3*b*p*(x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[
3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b) + x*Log[c*(a
 + b*x^3)^p]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

method	result
default	$x\ln\left(c(b\,x^3+a)^p\right)-3pb\left(\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}\right)$
parts	$x\ln\left(c(b\,x^3+a)^p\right)-3pb\left(\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}\right)$
risch	$x\ln\left((b\,x^3+a)^p\right)+\frac{i\mathrm{csgn}\left(ic(b\,x^3+a)^p\right)^2\mathrm{csgn}\left(i(b\,x^3+a)^p\right)\pi x}{2}-\frac{i\pi x\,\mathrm{csgn}\left(i(b\,x^3+a)^p\right)\mathrm{csgn}\left(ic(b\,x^3+a)^p\right)\mathrm{csgn}(ic)}{2}-\frac{i\pi}{}$

input
```
int(ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(b*x^3+a)^p)-3*p*b*(x/b-(1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/
6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))/b*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = px\log\left(bx^3+a\right)+\sqrt{3}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right) - \frac{1}{2}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-3\,px+x\log\left(c\right)$$

input
```
integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)
^(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)
^(2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

Sympy [A] (verification not implemented)

Time = 24.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = \begin{cases} x\log\left(0^p c\right) \\ -3px+x\log\left(c\left(bx^3\right)^p\right) \\ x\log\left(a^p c\right) \\ -3px+x\log\left(c\left(a+bx^3\right)^p\right)-\frac{3bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\sqrt{3}bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a}+\frac{b\left(-\frac{a}{b}\right)\cdots}{} \end{cases}$$

input
```
integrate(ln(c*(b*x**3+a)**p),x)
```

output
```
Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (-3*p*x + x*log(c*(b*x**3)
**p), Eq(a, 0)), (x*log(a**p*c), Eq(b, 0)), (-3*p*x + x*log(c*(a + b*x**3)
**p) - 3*b*p*(-a/b)**(4/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3
))/(2*a) - sqrt(3)*b*p*(-a/b)**(4/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) +
sqrt(3)/3)/a + b*(-a/b)**(4/3)*log(c*(a + b*x**3)**p)/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{2}\,bp\left(\frac{6\,x}{b}-\frac{2\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) + x\log\left(\left(bx^3+a\right)^pc\right)$$

input
```
integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
-1/2*b*p*(6*x/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(b^2*(a/b)^(2/3)) + a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(2/3)) - 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + x*log((b*x^3
+ a)^p*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \log \left(c\left(a+bx^3\right)^p\right)\, dx = -\frac{1}{2}\, abp\left(\frac{2\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab} - \frac{2\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\right.}{ab^2}\right. + px\log\left(bx^3+a\right) - (3\,p-\log\left(c\right))x$$

input
```
integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
-1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2)
 - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*
log(b*x^3 + a) - (3*p - log(c))*x
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int \log \left(c\left(a+bx^3\right)^p\right)\, dx = x\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right) - 3\,p\,x - \frac{{(-a)}^{1/3}\,p\,\ln\left({(-a)}^{4/3}+a\,b^{1/3}\,x\right)}{b^{1/3}} + \frac{{(-a)}^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-{(-a)}^{4/3}-\sqrt{3}\,{(-a)}^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{1/3}} - \frac{{(-a)}^{1/3}\,p\,\ln\left(2\,a\,b^{1/3}\,x-{(-a)}^{4/3}+\sqrt{3}\,{(-a)}^{4/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{1/3}}$$

input
```
int(log(c*(a + b*x^3)^p),x)
```

output
```
x*log(c*(a + b*x^3)^p) - 3*p*x - ((-a)^(1/3)*p*log((-a)^(4/3) + a*b^(1/3)*
x))/b^(1/3) + ((-a)^(1/3)*p*log(2*a*b^(1/3)*x - 3^(1/2)*(-a)^(4/3)*1i - (-
a)^(4/3))*((3^(1/2)*1i)/2 + 1/2))/b^(1/3) - ((-a)^(1/3)*p*log(3^(1/2)*(-a)
^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2))/b^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.68

$$\int \log \left(c\left(a+bx^3\right)^p\right)\,dx$$
$$=\frac{-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)p+3a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)p-a^{\frac{1}{3}}\log\left(\left(b\,x^{3}+a\right)^{p}c\right)+2b^{\frac{1}{3}}\log\left(\left(b\,x^{3}+a\right)^{p}c\right)x-6b^{\frac{1}{3}}}{2b^{\frac{1}{3}}}$$

input
```
int(log(c*(b*x^3+a)^p),x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
p + 3*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*p - a**(1/3)*log((a + b*x**3)**p
*c) + 2*b**(1/3)*log((a + b*x**3)**p*c)*x - 6*b**(1/3)*p*x)/(2*b**(1/3))
```

3.195 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 308

$$\begin{aligned}\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx &= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} \\ &\quad -\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} \\ &\quad -\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} \\ &\quad +\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} \\ &\quad -\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} \\ &\quad -\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}\end{aligned}$$

output
```
-p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((
-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e
-p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^
(1/3)*e))*ln(e*x+d)/e+ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p*polylog(2,b^(1/3)*(e
*x+d)/(b^(1/3)*d-a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)
^(1/3)*a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(
1/3)*e))/e
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.02

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

output
```
-((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x
])/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x
])/e + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d +
 e*x))/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^
(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b
^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2912, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$$

$$\Big\downarrow \text{2912}$$

$$\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{3bp\int \frac{x^2\log(d+ex)}{bx^3+a}dx}{e}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}-$$
$$\frac{3bp\int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}+\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{b}x+(-1)^{2/3}\sqrt[3]{a}\right)}\right)dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{3bp\left(\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{3b} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{3b} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{3b} + \frac{\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\right.}{\sqrt[3]{b}d-}\right.}{3b}\right.}{e}$$

input
```
Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

output
```
(Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (3*b*p*((Log[-((e*(a^(1/3) + b^(1/
3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/(3*b) + (Log[-((e*((-1)^(2/
3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x]
)/(3*b) + (Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d
+ (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/(3*b) + PolyLog[2, (b^(1/3)*(d + e*
x))/(b^(1/3)*d - a^(1/3)*e)]/(3*b) + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/
3)*d + (-1)^(1/3)*a^(1/3)*e)]/(3*b) + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1
/3)*d - (-1)^(2/3)*a^(1/3)*e)]/(3*b)))/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.88 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.33

method	result
parts	$\frac{\ln(ex+d)\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b-3bd\_Z^{2}+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_}{\_}\right.\right.\right.}{e}$
risch	$\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)\ln(ex+d)}{e}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b-3bd\_Z^{2}+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_}{\_R}\right.\right.\right.}{e}$

input
```
int(ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog
((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx=\int \frac{\log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx=\int \frac{\ln \left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x} \,dx$$

input
```
int(log(c*(a + b*x^3)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b*x^3)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{6\left(\int \frac{\log\left(\left(b\,x^3+a\right)^p c\right)}{be\,x^4+bd\,x^3+aex+ad}dx\right) aep-6\left(\int \frac{\log\left(\left(b\,x^3+a\right)^p c\right) x^2}{be\,x^4+bd\,x^3+aex+ad}dx\right) bdp+\log\left(\left(b\,x^3+a\right)^p c\right)^2}{6ep}$$

input
```
int(log(c*(b*x^3+a)^p)/(e*x+d),x)
```

output
```
(6*int(log((a + b*x**3)**p*c)/(a*d + a*e*x + b*d*x**3 + b*e*x**4),x)*a*e*p
 - 6*int((log((a + b*x**3)**p*c)*x**2)/(a*d + a*e*x + b*d*x**3 + b*e*x**4)
,x)*b*d*p + log((a + b*x**3)**p*c)**2)/(6*e*p)
```

3.196 $\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 292

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = -\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}de+a^{2/3}e^2} + \frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)}$$

output
```
-3^(1/2)*a^(1/3)*b^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3
))/(b^(2/3)*d^2+a^(1/3)*b^(1/3)*d*e+a^(2/3)*e^2)+a^(1/3)*b^(1/3)*(b^(1/3)*
d+a^(1/3)*e)*p*ln(a^(1/3)+b^(1/3)*x)/(-a*e^3+b*d^3)-3*b*d^2*p*ln(e*x+d)/e/
(-a*e^3+b*d^3)-a^(1/3)*b^(1/3)*(b^(1/3)*d+a^(1/3)*e)*p*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(-2*a*e^3+2*b*d^3)+b*d^2*p*ln(b*x^3+a)/e/(-a*e^3+b*
d^3)-ln(c*(b*x^3+a)^p)/e/(e*x+d)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = -\frac{2\sqrt{3}\sqrt[3]{a}b^{2/3}dep(d+ex)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+3be^2px^2(d+ex)\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)-2}{}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

output
```
-1/2*(2*Sqrt[3]*a^(1/3)*b^(2/3)*d*e*p*(d + e*x)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + 3*b*e^2*p*x^2*(d + e*x)*Hypergeometric2F1[2/3, 1, 5/3,
 -((b*x^3)/a)] - 2*a^(1/3)*b^(2/3)*d*e*p*(d + e*x)*Log[a^(1/3) + b^(1/3)*x
] + 6*b*d^2*p*(d + e*x)*Log[d + e*x] + a^(1/3)*b^(2/3)*d*e*p*(d + e*x)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b*d^2*p*(d + e*x)*Log[a +
b*x^3] + 2*(b*d^3 - a*e^3)*Log[c*(a + b*x^3)^p])/(e*(b*d^3 - a*e^3)*(d + e
*x))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2913, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 2913}$$

$$\frac{3bp\int \frac{x^2}{(d+ex)(bx^3+a)}dx}{e}-\frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)}$$

$\Big\downarrow$ 7276

$$\frac{3bp\int \left(\frac{-axe^2+ade+bd^2x^2}{(bd^3-ae^3)(bx^3+a)}-\frac{d^2e}{(bd^3-ae^3)(d+ex)}\right)dx}{e}-\frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)}$$

$\Big\downarrow$ 2009

$$\frac{3bp\left(-\frac{\sqrt[3]{a}e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(a^{2/3}e^2+\sqrt[3]{a}\sqrt[3]{b}de+b^{2/3}d^2\right)}-\frac{\sqrt[3]{a}e\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bd^3-ae^3)}+\frac{\sqrt[3]{a}e\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\right.}{3b^{2/3}(bd^3-ae^3)}\right.}{e}$$

$$\frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)}$$

input
```
Int[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

output
```
(3*b*p*(-((a^(1/3)*e*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(S
qrt[3]*b^(2/3)*(b^(2/3)*d^2 + a^(1/3)*b^(1/3)*d*e + a^(2/3)*e^2))) + (a^(1
/3)*e*(b^(1/3)*d + a^(1/3)*e)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*d^3
- a*e^3)) - (d^2*Log[d + e*x])/(b*d^3 - a*e^3) - (a^(1/3)*e*(b^(1/3)*d + a
^(1/3)*e)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*d^
3 - a*e^3)) + (d^2*Log[a + b*x^3])/(3*(b*d^3 - a*e^3))))/e - Log[c*(a + b*
x^3)^p]/(e*(d + e*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.95

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e(ex+d)}+\frac{3pb\left(\frac{d^{2}\ln(ex+d)}{a\,e^{3}-b\,d^{3}}+\frac{-dea\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+a\,e^{2}\left(\cdots\right.}{a\,e^{3}-\cdots}\right.}{e}$
risch	Expression too large to display

input
```
int(ln(c*(b*x^3+a)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(b*x^3+a)^p)/e/(e*x+d)+3*p*b/e*(d^2/(a*e^3-b*d^3)*ln(e*x+d)+(-d*e*a*
(1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a
)^(1/3)*x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1)))+a*e^2*(-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6/
b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a
)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))-1/3*d^2*ln(b*x^3+a))/(a
*e^3-b*d^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 7010, normalized size of antiderivative = 24.01

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = -\frac{\left(\frac{6\,d^2\log(ex+d)}{bd^3-ae^3} + \frac{2\,\sqrt{3}\left(ae^2\left(\frac{a}{b}\right)^{\frac{2}{3}} - ade\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(b^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(2\,bd^2\left(\frac{a}{b}\right)^{\frac{2}{3}} - ae^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - ade\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{2\,e} - \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)e}$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/2*(6*d^2*log(e*x + d)/(b*d^3 - a*e^3) + 2*sqrt(3)*(a*e^2*(a/b)^(2/3) -
a*d*e*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b
^2*d^3*(a/b)^(2/3) - a*b*e^3*(a/b)^(2/3))*(a/b)^(1/3)) - (2*b*d^2*(a/b)^(2
/3) - a*e^2*(a/b)^(1/3) - a*d*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b
^2*d^3*(a/b)^(2/3) - a*b*e^3*(a/b)^(2/3)) - 2*(b*d^2*(a/b)^(2/3) + a*e^2*(
a/b)^(1/3) + a*d*e)*log(x + (a/b)^(1/3))/(b^2*d^3*(a/b)^(2/3) - a*b*e^3*(a
/b)^(2/3)))*b*p/e - log((b*x^3 + a)^p*c)/((e*x + d)*e)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.24

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx$$

$$= -\frac{3\,bd^2p\log\left(ex+d\right)}{bd^3e-ae^4} + \frac{bd^2p\log\left(\left|bx^3+a\right|\right)}{bd^3e-ae^4} + \frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\,bp\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2d^2-\left(-ab^2\right)^{\frac{1}{3}}\,bde+\left(-ab^2\right)^{\frac{2}{3}}\,e^2}$$
$$+ \frac{\left(ab^3d^3e^3p\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2b^2e^6p\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^3d^4e^2p + a^2b^2de^5p\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^3d^6e^2-2\,a^2b^2d^3e^5+a^3be^8}$$
$$- \frac{p\log\left(bx^3+a\right)}{e^2x+de}$$
$$+ \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}\,bdp-\left(-ab^2\right)^{\frac{2}{3}}\,ep\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^2d^3-abe^3\right)} - \frac{\log\left(c\right)}{e^2x+de}$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-3*b*d^2*p*log(e*x + d)/(b*d^3*e - a*e^4) + b*d^2*p*log(abs(b*x^3 + a))/(b
*d^3*e - a*e^4) + sqrt(3)*(-a*b^2)^(1/3)*b*p*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/(b^2*d^2 - (-a*b^2)^(1/3)*b*d*e + (-a*b^2)^(2/3)*
e^2) + (a*b^3*d^3*e^3*p*(-a/b)^(1/3) - a^2*b^2*e^6*p*(-a/b)^(1/3) - a*b^3*
d^4*e^2*p + a^2*b^2*d*e^5*p)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^
3*d^6*e^2 - 2*a^2*b^2*d^3*e^5 + a^3*b*e^8) - p*log(b*x^3 + a)/(e^2*x + d*e
) + 1/2*((-a*b^2)^(1/3)*b*d*p - (-a*b^2)^(2/3)*e*p)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(b^2*d^3 - a*b*e^3) - log(c)/(e^2*x + d*e)
```

Mupad [B] (verification not implemented)

Time = 25.83 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.52

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(-\frac{27\,a\,b^4\,d\,p^3 + 27\,a\,b^4\,e\,p^3\,x + \mathrm{root}(b\,d^3\,e^3\,z^3 - a\,e^6\,z^3 - 3\,b\,d^2\,e^2\,p\,z^2 + 3\,b\,d\,e\,p^2\,z - b\,p^3, z, k)^3}{} \right.\right.$$
$$\left.\left. - a\,e^6\,z^3 - 3\,b\,d^2\,e^2\,p\,z^2 + 3\,b\,d\,e\,p^2\,z - b\,p^3, z, k)\right)\right)$$
$$- \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{x\,e^2 + d\,e} + \frac{3\,b\,d^2\,p\,\ln(d+e\,x)}{a\,e^4 - b\,d^3\,e}$$

input
```
int(log(c*(a + b*x^3)^p)/(d + e*x)^2,x)
```

output
```
symsum(log(-(27*a*b^4*d*p^3 + 27*a*b^4*e*p^3*x + 9*root(b*d^3*e^3*z^3 - a*
e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a*b^4*d^4*e^3
 + 45*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z -
 b*p^3, z, k)^3*a^2*b^3*d*e^6 - 9*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2
*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^2*a^2*b^3*e^5*p + 36*root(b*d^3*
e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a
^2*b^3*e^7*x + 9*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*
d*e*p^2*z - b*p^3, z, k)^2*a*b^4*d^3*e^2*p + 18*root(b*d^3*e^3*z^3 - a*e^6
*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a*b^4*d^3*e^4*x
- 45*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z -
b*p^3, z, k)*a*b^4*d^2*e*p^2 - 72*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2
*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)*a*b^4*d*e^2*p^2*x + 27*root(b*d^
3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^2
*a*b^4*d^2*e^3*p*x)/e^2)*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^
2 + 3*b*d*e*p^2*z - b*p^3, z, k), k, 1, 3) - log(c*(a + b*x^3)^p)/(d*e + e
^2*x) + (3*b*d^2*p*log(d + e*x))/(a*e^4 - b*d^3*e)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6006, normalized size of antiderivative = 20.57

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^2} \, dx = \text{Too large to display}$$

input
```
int(log(c*(b*x^3+a)^p)/(e*x+d)^2,x)
```

output

```
( - 206*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))
)*a**6*b*d**4*e**18*p - 206*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x
)/(a**(1/3)*sqrt(3)))*a**6*b*d**3*e**19*p*x + 4216*a**(2/3)*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**5*b**2*d**7*e**15*p + 4216
*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**5*
b**2*d**6*e**16*p*x + 4420*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)
/(a**(1/3)*sqrt(3)))*a**4*b**3*d**10*e**12*p + 4420*a**(2/3)*sqrt(3)*atan(
(a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**4*b**3*d**9*e**13*p*x - 4
420*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*
*3*b**4*d**13*e**9*p - 4420*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x
)/(a**(1/3)*sqrt(3)))*a**3*b**4*d**12*e**10*p*x - 4216*a**(2/3)*sqrt(3)*at
an((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**5*d**16*e**6*p -
4216*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a
**2*b**5*d**15*e**7*p*x + 206*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)
*x)/(a**(1/3)*sqrt(3)))*a*b**6*d**19*e**3*p + 206*a**(2/3)*sqrt(3)*atan((a
**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**6*d**18*e**4*p*x - 2*b**(
1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a
**7*d**2*e**20*p - 2*b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)
*x)/(a**(1/3)*sqrt(3)))*a**7*d*e**21*p*x + 850*b**(1/3)*a**(1/3)*sqrt(3)*a
tan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**6*b*d**5*e**17*p +...
```

3.197 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 391

$$\begin{aligned}
\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} \\
&\quad - \frac{\sqrt{3}\sqrt[3]{a}b^{2/3}\left(2bd^3-3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\left(bd^3-ae^3\right)^2} \\
&\quad + \frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2} \\
&\quad - \frac{3bd(bd^3+2ae^3)\,p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2} \\
&\quad - \frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2} \\
&\quad + \frac{bd(bd^3+2ae^3)\,p\log\left(a+bx^3\right)}{2e\left(bd^3-ae^3\right)^2} - \frac{\log\left(c(a+bx^3)^p\right)}{2e(d+ex)^2}
\end{aligned}$$

output
```
3/2*b*d^2*p/e/(-a*e^3+b*d^3)/(e*x+d)-1/2*3^(1/2)*a^(1/3)*b^(2/3)*(2*b*d^3-
3*a^(1/3)*b^(2/3)*d^2*e+a*e^3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/
a^(1/3))/(-a*e^3+b*d^3)^2+1/2*a^(1/3)*b^(2/3)*(2*b*d^3+3*a^(1/3)*b^(2/3)*d
^2*e+a*e^3)*p*ln(a^(1/3)+b^(1/3)*x)/(-a*e^3+b*d^3)^2-3/2*b*d*(2*a*e^3+b*d^
3)*p*ln(e*x+d)/e/(-a*e^3+b*d^3)^2-1/4*a^(1/3)*b^(2/3)*(2*b*d^3+3*a^(1/3)*b
^(2/3)*d^2*e+a*e^3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-a*e^3+b*
d^3)^2+1/2*b*d*(2*a*e^3+b*d^3)*p*ln(b*x^3+a)/e/(-a*e^3+b*d^3)^2-1/2*ln(c*(
b*x^3+a)^p)/e/(e*x+d)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.63 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.83

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx$$

$$= \frac{\frac{b^{2/3}p(d+ex)\left(6\sqrt[3]{b}d^2\left(bd^3-ae^3\right)-2\sqrt{3}\sqrt[3]{a}e\left(2bd^3+ae^3\right)(d+ex)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-9b^{4/3}d^2e^2x^2(d+ex)\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right.}{}}{}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```

output
```
((b^(2/3)*p*(d + e*x)*(6*b^(1/3)*d^2*(b*d^3 - a*e^3) - 2*Sqrt[3]*a^(1/3)*e
*(2*b*d^3 + a*e^3)*(d + e*x)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] -
 9*b^(4/3)*d^2*e^2*x^2*(d + e*x)*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/
a)] + 2*a^(1/3)*e*(2*b*d^3 + a*e^3)*(d + e*x)*Log[a^(1/3) + b^(1/3)*x] - 6
*b^(1/3)*d*(b*d^3 + 2*a*e^3)*(d + e*x)*Log[d + e*x] - a^(1/3)*e*(2*b*d^3 +
 a*e^3)*(d + e*x)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(1/
3)*d*(b*d^3 + 2*a*e^3)*(d + e*x)*Log[a + b*x^3]))/(b*d^3 - a*e^3)^2 - 2*Lo
g[c*(a + b*x^3)^p])/(4*e*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2913, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3} \, dx$$

$$\Big\downarrow \ 2913$$

$$\frac{3bp\int \frac{x^2}{(d+ex)^2(bx^3+a)}dx}{2e}-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \ 7276$$

$$\frac{3bp\int \left(-\frac{ed^2}{(bd^3-ae^3)(d+ex)^2}-\frac{e\left(bd^3+2ae^3\right)d}{(bd^3-ae^3)^2(d+ex)}+\frac{-3abd^2xe^2+a\left(2bd^3+ae^3\right)e+bd\left(bd^3+2ae^3\right)x^2}{(bd^3-ae^3)^2(bx^3+a)}\right)dx}{2e}-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{3bp\left(-\frac{\sqrt[3]{a}e\left(3\sqrt[3]{a}b^{2/3}d^2e+ae^3+2bd^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bd^3-ae^3)^2}-\frac{\sqrt[3]{a}e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-3\sqrt[3]{a}b^{2/3}d^2e+ae^3+2bd^3\right)}{\sqrt{3}\sqrt[3]{b}(bd^3-ae^3)^2}+ \cdots\right)}{2e}-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}$$

input
```
Int[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```

output
```
(3*b*p*(d^2/((b*d^3 - a*e^3)*(d + e*x)) - (a^(1/3)*e*(2*b*d^3 - 3*a^(1/3)*
b^(2/3)*d^2*e + a*e^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*b^(1/3)*(b*d^3 - a*e^3)^2) + (a^(1/3)*e*(2*b*d^3 + 3*a^(1/3)*b^(2
/3)*d^2*e + a*e^3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(1/3)*(b*d^3 - a*e^3)^2)
 - (d*(b*d^3 + 2*a*e^3)*Log[d + e*x])/(b*d^3 - a*e^3)^2 - (a^(1/3)*e*(2*b*
d^3 + 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*b^(1/3)*(b*d^3 - a*e^3)^2) + (d*(b*d^3 + 2*a*e^3)*Log[a +
b*x^3])/(3*(b*d^3 - a*e^3)^2)))/(2*e) - Log[c*(a + b*x^3)^p]/(2*e*(d + e*x
)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.89

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{2e\left(ex+d\right)^{2}}+\frac{3pb\left(-\frac{d^{2}}{\left(a\,e^{3}-b\,d^{3}\right)\left(ex+d\right)}-\frac{d\left(2a\,e^{3}+b\,d^{3}\right)\ln\left(ex+d\right)}{\left(a\,e^{3}-b\,d^{3}\right)^{2}}+\frac{\left(a^{2}e^{4}+2ab\,d^{3}e\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(ln(c*(b*x^3+a)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(b*x^3+a)^p)/e/(e*x+d)^2+3/2*p*b/e*(-d^2/(a*e^3-b*d^3)/(e*x+d)-d
*(2*a*e^3+b*d^3)/(a*e^3-b*d^3)^2*ln(e*x+d)+((a^2*e^4+2*a*b*d^3*e)*(1/3/b/(
1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*
x+(1/b*a)^(2/3))+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x-1)))-3*a*b*d^2*e^2*(-1/3/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/6/
b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*3^(1/2)/b/(1/b*a
)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)))+1/3*(2*a*b*d*e^3+b^2*d^
4)/b*ln(b*x^3+a))/(a*e^3-b*d^3)^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.60 (sec) , antiderivative size = 13236, normalized size of antiderivative = 33.85

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.32

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx = -\frac{\left(\frac{2\sqrt{3}\left(3\,abd^2e^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abd^3e\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2e^4\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(b^3d^6\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ab^2d^3e^3\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2be^6\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{6\,d^2}{bd^4-ade^3+(bd^3e-ae^4)x}+\frac{6\left(bd^4+2\,ade^3\right)\log(ex+d}{b^2d^6-2\,abd^3e^3+a^2e^6}\right.}{} - \frac{\log\left((bx^3+a)^pc\right)}{2\,(ex+d)^2e}$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(3)*(3*a*b*d^2*e^2*(a/b)^(2/3) - 2*a*b*d^3*e*(a/b)^(1/3) - a^2
*e^4*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^
3*d^6*(a/b)^(2/3) - 2*a*b^2*d^3*e^3*(a/b)^(2/3) + a^2*b*e^6*(a/b)^(2/3))*(
a/b)^(1/3)) - 6*d^2/(b*d^4 - a*d*e^3 + (b*d^3*e - a*e^4)*x) + 6*(b*d^4 + 2
*a*d*e^3)*log(e*x + d)/(b^2*d^6 - 2*a*b*d^3*e^3 + a^2*e^6) - (2*b^2*d^4*(a
/b)^(2/3) + 4*a*b*d*e^3*(a/b)^(2/3) - 3*a*b*d^2*e^2*(a/b)^(1/3) - 2*a*b*d^
3*e - a^2*e^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*d^6*(a/b)^(2/3)
 - 2*a*b^2*d^3*e^3*(a/b)^(2/3) + a^2*b*e^6*(a/b)^(2/3)) - 2*(b^2*d^4*(a/b)
^(2/3) + 2*a*b*d*e^3*(a/b)^(2/3) + 3*a*b*d^2*e^2*(a/b)^(1/3) + 2*a*b*d^3*e
 + a^2*e^4)*log(x + (a/b)^(1/3))/(b^3*d^6*(a/b)^(2/3) - 2*a*b^2*d^3*e^3*(a
/b)^(2/3) + a^2*b*e^6*(a/b)^(2/3)))*b*p/e - 1/2*log((b*x^3 + a)^p*c)/((e*x
 + d)^2*e)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.69

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^3} \, dx = \text{Too large to display}$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

output
```
1/2*(3*a*b^5*d^8*e^3*p*(-a/b)^(1/3) - 6*a^2*b^4*d^5*e^6*p*(-a/b)^(1/3) + 3
*a^3*b^3*d^2*e^9*p*(-a/b)^(1/3) - 2*a*b^5*d^9*e^2*p + 3*a^2*b^4*d^6*e^5*p
- a^4*b^2*e^11*p)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5*d^12*e^2
- 4*a^2*b^4*d^9*e^5 + 6*a^3*b^3*d^6*e^8 - 4*a^4*b^2*d^3*e^11 + a^5*b*e^14)
 + 3/2*(2*(-a*b^2)^(1/3)*b*d*p - (-a*b^2)^(2/3)*e*p)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*d^4 + 2*sqrt(3)*a*b*d*e^3 -
2*sqrt(3)*(-a*b^2)^(1/3)*b*d^3*e - sqrt(3)*(-a*b^2)^(1/3)*a*e^4 + 3*sqrt(3
)*(-a*b^2)^(2/3)*d^2*e^2) - 1/2*p*log(b*x^3 + a)/(e^3*x^2 + 2*d*e^2*x + d^
2*e) - 3/2*(b^2*d^4*p + 2*a*b*d*e^3*p)*log(e*x + d)/(b^2*d^6*e - 2*a*b*d^3
*e^4 + a^2*e^7) + 1/4*(2*(-a*b^2)^(1/3)*b*d^3*p + (-a*b^2)^(1/3)*a*e^3*p -
 3*(-a*b^2)^(2/3)*d^2*e*p)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^2*d
^6 - 2*a*b*d^3*e^3 + a^2*e^6) + 1/2*(b^2*d^4*p + 2*a*b*d*e^3*p)*log(abs(b*
x^3 + a))/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7) + 1/2*(3*b*d^2*e*p*x + 3*b
*d^3*p - b*d^3*log(c) + a*e^3*log(c))/(b*d^3*e^3*x^2 - a*e^6*x^2 + 2*b*d^4
*e^2*x - 2*a*d*e^5*x + b*d^5*e - a*d^2*e^4)
```

Mupad [B] (verification not implemented)

Time = 26.27 (sec) , antiderivative size = 2227, normalized size of antiderivative = 5.70

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^3} \, dx = \text{Too large to display}$$

input
```
int(log(c*(a + b*x^3)^p)/(d + e*x)^3,x)
```

output
```
symsum(log(-(27*a*b^6*d^4*p^3 + 216*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^
3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*
d^2*e*p^2*z + b^2*p^3, z, k)^3*a^2*b^5*d^7*e^6 - 648*root(16*a*b*d^3*e^6*z
^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e
^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a^3*b^4*d^4*e^9 + 72*root(
16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^
2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a*b^6*d^10
*e^3 + 360*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 2
4*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z,
 k)^3*a^4*b^3*d*e^12 + 18*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*
a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*
z + b^2*p^3, z, k)*a^3*b^4*e^7*p^2 + 288*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d
^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6
*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a^4*b^3*e^13*x + 27*a^2*b^5*d*e^3*p^3
- 27*a^2*b^5*e^4*p^3*x + 36*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 -
8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^
2*z + b^2*p^3, z, k)^2*a*b^6*d^8*e^2*p + 144*root(16*a*b*d^3*e^6*z^3 - 8*b
^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2
 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a*b^6*d^9*e^4*x - 90*root(16*a*b*d
^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 1...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16189, normalized size of antiderivative = 41.40

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^3}\, dx = \text{Too large to display}$$

input
```
int(log(c*(b*x^3+a)^p)/(e*x+d)^3,x)
```

output

```
(30*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*
*10*b*d**5*e**30*p + 60*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a
**(1/3)*sqrt(3)))*a**10*b*d**4*e**31*p*x + 30*a**(2/3)*sqrt(3)*atan((a**(1
/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**10*b*d**3*e**32*p*x**2 + 756*a*
*(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**9*b**
2*d**8*e**27*p + 1512*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**
(1/3)*sqrt(3)))*a**9*b**2*d**7*e**28*p*x + 756*a**(2/3)*sqrt(3)*atan((a**(
1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**9*b**2*d**6*e**29*p*x**2 - 242
46*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**
8*b**3*d**11*e**24*p - 48492*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
x)/(a**(1/3)*sqrt(3)))*a**8*b**3*d**10*e**25*p*x - 24246*a**(2/3)*sqrt(3)*
atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**8*b**3*d**9*e**26*p*
x**2 - 760518*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sq
rt(3)))*a**7*b**4*d**14*e**21*p - 1521036*a**(2/3)*sqrt(3)*atan((a**(1/3)
- 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**7*b**4*d**13*e**22*p*x - 760518*a**
(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**7*b**4
*d**12*e**23*p*x**2 - 3657690*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)
*x)/(a**(1/3)*sqrt(3)))*a**6*b**5*d**17*e**18*p - 7315380*a**(2/3)*sqrt(3)
*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**6*b**5*d**16*e**19*
p*x - 3657690*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)...
```

3.198 $\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{be(6a^2d^2-4abde+b^2e^2)\,px}{4a^3}+\frac{be^2(4ad-be)px^2}{8a^2}+\frac{be^3px^3}{12a}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}+\frac{d^4p\log(x)}{4e}-\frac{(ad-be)^4p\log(b+ax)}{4a^4e}$$

output
```
1/4*b*e*(6*a^2*d^2-4*a*b*d*e+b^2*e^2)*p*x/a^3+1/8*b*e^2*(4*a*d-b*e)*p*x^2/
a^2+1/12*b*e^3*p*x^3/a+1/4*(e*x+d)^4*ln(c*(a+b/x)^p)/e+1/4*d^4*p*ln(x)/e-1
/4*(a*d-b*e)^4*p*ln(a*x+b)/a^4/e
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.82

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{\frac{be^2px(6b^2e^2-3abe(8d+ex)+2a^2(18d^2+6dex+e^2x^2))}{6a^3}+(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+d^4p\log(x)-\frac{(ad-be)^4p\log(b+ax)}{a^4}}{4e}$$

input
```
Integrate[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

output
```
((b*e^2*p*x*(6*b^2*e^2 - 3*a*b*e*(8*d + e*x) + 2*a^2*(18*d^2 + 6*d*e*x + e
^2*x^2)))/(6*a^3) + (d + e*x)^4*Log[c*(a + b/x)^p] + d^4*p*Log[x] - ((a*d
- b*e)^4*p*Log[b + a*x])/a^4)/(4*e)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2913, 1016, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

$$\Big\downarrow \text{2913}$$

$$\frac{bp\int \frac{(d+ex)^4}{\left(a+\frac{b}{x}\right)x^2}dx}{4e}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}$$

$$\Big\downarrow \text{1016}$$

$$\frac{bp\int \frac{(d+ex)^4}{x(b+ax)}dx}{4e}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}$$

$$\Big\downarrow \text{93}$$

$$\frac{bp\int \left(\frac{d^4}{bx}+\frac{e^4x^2}{a}+\frac{e^2\left(6a^2d^2-4abed+b^2e^2\right)}{a^3}+\frac{e^3(4ad-be)x}{a^2}-\frac{(ad-be)^4}{a^3b(b+ax)}\right)dx}{4e}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bp\left(-\frac{(ad-be)^4\log(ax+b)}{a^4b}+\frac{e^3x^2(4ad-be)}{2a^2}+\frac{e^2x\left(6a^2d^2-4abde+b^2e^2\right)}{a^3}+\frac{e^4x^3}{3a}+\frac{d^4\log(x)}{b}\right)}{4e}+\frac{(d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e}$$

input
```
Int[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

output
```
((d + e*x)^4*Log[c*(a + b/x)^p])/(4*e) + (b*p*((e^2*(6*a^2*d^2 - 4*a*b*d*e
 + b^2*e^2)*x)/a^3 + (e^3*(4*a*d - b*e)*x^2)/(2*a^2) + (e^4*x^3)/(3*a) + (
d^4*Log[x])/b - ((a*d - b*e)^4*Log[b + a*x])/(a^4*b)))/(4*e)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.76

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)e^{3}x^{4}}{4}+\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)e^{2}d\,x^{3}+\frac{3\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)e\,d^{2}x^{2}}{2}+\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)d^{3}x+\frac{\ln\left(c\left(a+\frac{b}{x}\right.\right.}{4e}$
parallelrisch	$-\frac{-6x^{4}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{4}e^{3}-24x^{3}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{4}d\,e^{2}-2x^{3}a^{3}b\,e^{3}p-36x^{2}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{4}d^{2}e-12x^{2}a^{3}bd\,e^{2}p+3x^{2}a^{2}b^{2}e}{}$

input
```
int((e*x+d)^3*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(c*(a+b/x)^p)*e^3*x^4+ln(c*(a+b/x)^p)*e^2*d*x^3+3/2*ln(c*(a+b/x)^p)*
e*d^2*x^2+ln(c*(a+b/x)^p)*d^3*x+1/4*ln(c*(a+b/x)^p)/e*d^4+1/4*p*b/e*(e^2/a
^3*(1/3*a^2*e^2*x^3+2*a^2*e*x^2*d-1/2*a*b*e^2*x^2+6*x*a^2*d^2-4*a*b*d*e*x+
x*b^2*e^2)+(-a^4*d^4+4*a^3*b*d^3*e-6*a^2*b^2*d^2*e^2+4*a*b^3*d*e^3-b^4*e^4
)/a^4/b*ln(a*x+b)+d^4/b*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.72

$$\int (d+ex)^{3}\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)dx$$
$$=\frac{2\,a^{3}be^{3}px^{3}+3\,(4\,a^{3}bde^{2}-a^{2}b^{2}e^{3})px^{2}+6\,(6\,a^{3}bd^{2}e-4\,a^{2}b^{2}de^{2}+ab^{3}e^{3})px+6\,(4\,a^{3}bd^{3}-6\,a^{2}b^{2}d^{2}e+4\,a}{}$$

input
```
integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

output
```
1/24*(2*a^3*b*e^3*p*x^3 + 3*(4*a^3*b*d*e^2 - a^2*b^2*e^3)*p*x^2 + 6*(6*a^3
*b*d^2*e - 4*a^2*b^2*d*e^2 + a*b^3*e^3)*p*x + 6*(4*a^3*b*d^3 - 6*a^2*b^2*d
^2*e + 4*a*b^3*d*e^2 - b^4*e^3)*p*log(a*x + b) + 6*(a^4*e^3*x^4 + 4*a^4*d*
e^2*x^3 + 6*a^4*d^2*e*x^2 + 4*a^4*d^3*x)*log(c) + 6*(a^4*e^3*p*x^4 + 4*a^4
*d*e^2*p*x^3 + 6*a^4*d^2*e*p*x^2 + 4*a^4*d^3*p*x)*log((a*x + b)/x))/a^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(128) = 256.

Time = 1.44 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.55

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$= \begin{cases} d^3x\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{3d^2ex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2} + de^2x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{e^3x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4} + \frac{bd^3p\log\left(x+\frac{b}{a}\right)}{a} + 3 \\ d^3px + d^3x\log\left(c\left(\frac{b}{x}\right)^p\right) + \frac{3d^2epx^2}{4} + \frac{3d^2ex^2\log\left(c\left(\frac{b}{x}\right)^p\right)}{2} + \frac{de^2px^3}{3} + de^2x^3\log\left(c\left(\frac{b}{x}\right)^p\right) + \frac{e^3px^4}{16} + \frac{e^3x^4\log\left(c\left(\frac{b}{x}\right)^2\right.}{4} \end{cases}$$

input
```
integrate((e*x+d)**3*ln(c*(a+b/x)**p),x)
```

output
```
Piecewise((d**3*x*log(c*(a + b/x)**p) + 3*d**2*e*x**2*log(c*(a + b/x)**p)/
2 + d*e**2*x**3*log(c*(a + b/x)**p) + e**3*x**4*log(c*(a + b/x)**p)/4 + b*
d**3*p*log(x + b/a)/a + 3*b*d**2*e*p*x/(2*a) + b*d*e**2*p*x**2/(2*a) + b*e
**3*p*x**3/(12*a) - 3*b**2*d**2*e*p*log(x + b/a)/(2*a**2) - b**2*d*e**2*p*
x/a**2 - b**2*e**3*p*x**2/(8*a**2) + b**3*d*e**2*p*log(x + b/a)/a**3 + b**
3*e**3*p*x/(4*a**3) - b**4*e**3*p*log(x + b/a)/(4*a**4), Ne(a, 0)), (d**3*
p*x + d**3*x*log(c*(b/x)**p) + 3*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c*(b/
x)**p)/2 + d*e**2*p*x**3/3 + d*e**2*x**3*log(c*(b/x)**p) + e**3*p*x**4/16
+ e**3*x**4*log(c*(b/x)**p)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$= \frac{1}{24}bp\left(\frac{2a^2e^3x^3 + 3(4a^2de^2 - abe^3)x^2 + 6(6a^2d^2e - 4abde^2 + b^2e^3)x}{a^3} + \frac{6(4a^3d^3 - 6a^2bd^2e + 4ab^2de^2}{a^4}\right.$$
$$+ \frac{1}{4}\left(e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

input
```
integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="maxima")
```

output
```
1/24*b*p*((2*a^2*e^3*x^3 + 3*(4*a^2*d*e^2 - a*b*e^3)*x^2 + 6*(6*a^2*d^2*e
- 4*a*b*d*e^2 + b^2*e^3)*x)/a^3 + 6*(4*a^3*d^3 - 6*a^2*b*d^2*e + 4*a*b^2*d
*e^2 - b^3*e^3)*log(a*x + b)/a^4) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x
^2 + 4*d^3*x)*log((a + b/x)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(127) = 254$.

Time = 0.14 (sec) , antiderivative size = 847, normalized size of antiderivative = 6.09

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="giac")
```

output
```
-1/24*(6*(4*a^3*b^2*d^3*p - 6*a^2*b^3*d^2*e*p + 4*a*b^4*d*e^2*p - b^5*e^3*
p - 12*(a*x + b)*a^2*b^2*d^3*p/x + 12*(a*x + b)*a*b^3*d^2*e*p/x - 4*(a*x +
 b)*b^4*d*e^2*p/x + 12*(a*x + b)^2*a*b^2*d^3*p/x^2 - 6*(a*x + b)^2*b^3*d^2
*e*p/x^2 - 4*(a*x + b)^3*b^2*d^3*p/x^3)*log((a*x + b)/x)/(a^4 - 4*(a*x + b
)*a^3/x + 6*(a*x + b)^2*a^2/x^2 - 4*(a*x + b)^3*a/x^3 + (a*x + b)^4/x^4) +
 (36*a^5*b^3*d^2*e*p - 36*a^4*b^4*d*e^2*p + 11*a^3*b^5*e^3*p + 24*a^6*b^2*
d^3*log(c) - 36*a^5*b^3*d^2*e*log(c) + 24*a^4*b^4*d*e^2*log(c) - 6*a^3*b^5
*e^3*log(c) - 108*(a*x + b)*a^4*b^3*d^2*e*p/x + 96*(a*x + b)*a^3*b^4*d*e^2
*p/x - 26*(a*x + b)*a^2*b^5*e^3*p/x - 72*(a*x + b)*a^5*b^2*d^3*log(c)/x +
72*(a*x + b)*a^4*b^3*d^2*e*log(c)/x - 24*(a*x + b)*a^3*b^4*d*e^2*log(c)/x
+ 108*(a*x + b)^2*a^3*b^3*d^2*e*p/x^2 - 84*(a*x + b)^2*a^2*b^4*d*e^2*p/x^2
 + 21*(a*x + b)^2*a*b^5*e^3*p/x^2 + 72*(a*x + b)^2*a^4*b^2*d^3*log(c)/x^2
- 36*(a*x + b)^2*a^3*b^3*d^2*e*log(c)/x^2 - 36*(a*x + b)^3*a^2*b^3*d^2*e*p
/x^3 + 24*(a*x + b)^3*a*b^4*d*e^2*p/x^3 - 6*(a*x + b)^3*b^5*e^3*p/x^3 - 24
*(a*x + b)^3*a^3*b^2*d^3*log(c)/x^3)/(a^7 - 4*(a*x + b)*a^6/x + 6*(a*x + b
)^2*a^5/x^2 - 4*(a*x + b)^3*a^4/x^3 + (a*x + b)^4*a^3/x^4) + 6*(4*a^3*b^2*
d^3*p - 6*a^2*b^3*d^2*e*p + 4*a*b^4*d*e^2*p - b^5*e^3*p)*log(-a + (a*x + b
)/x)/a^4 - 6*(4*a^3*b^2*d^3*p - 6*a^2*b^3*d^2*e*p + 4*a*b^4*d*e^2*p - b^5*
e^3*p)*log((a*x + b)/x)/a^4)/b
```

Mupad [B] (verification not implemented)

Time = 25.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.32

$$\int (d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$$
$$= x\left(\frac{b\left(\frac{b^2 e^3 p}{4 a^2}-\frac{b d e^2 p}{a}\right)}{a}+\frac{3 b d^2 e p}{2 a}\right)$$
$$+\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(d^3 x+\frac{3 d^2 e x^2}{2}+d e^2 x^3+\frac{e^3 x^4}{4}\right)-x^2\left(\frac{b^2 e^3 p}{8 a^2}-\frac{b d e^2 p}{2 a}\right)$$
$$-\frac{\ln(b+a x)\left(-4 p a^3 b d^3+6 p a^2 b^2 d^2 e-4 p a b^3 d e^2+p b^4 e^3\right)}{4 a^4}+\frac{b e^3 p x^3}{12 a}$$

input
```
int(log(c*(a + b/x)^p)*(d + e*x)^3,x)
```

output
```
x*((b*((b^2*e^3*p)/(4*a^2) - (b*d*e^2*p)/a))/a + (3*b*d^2*e*p)/(2*a)) + lo
g(c*(a + b/x)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) - x^2
*((b^2*e^3*p)/(8*a^2) - (b*d*e^2*p)/(2*a)) - (log(b + a*x)*(b^4*e^3*p - 4*
a^3*b*d^3*p - 4*a*b^3*d*e^2*p + 6*a^2*b^2*d^2*e*p))/(4*a^4) + (b*e^3*p*x^3
)/(12*a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.44

$$\int (d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$$
$$= \frac{24 \log\left(\frac{(ax+b)^p c}{x^p}\right) a^4 d^3 x + 36 \log\left(\frac{(ax+b)^p c}{x^p}\right) a^4 d^2 e x^2 + 24 \log\left(\frac{(ax+b)^p c}{x^p}\right) a^4 d e^2 x^3 + 6 \log\left(\frac{(ax+b)^p c}{x^p}\right) a^4 e^3 x^4 +}{}$$

input
```
int((e*x+d)^3*log(c*(a+b/x)^p),x)
```

output

```
(24*log(((a*x + b)**p*c)/x**p)*a**4*d**3*x + 36*log(((a*x + b)**p*c)/x**p)
*a**4*d**2*e*x**2 + 24*log(((a*x + b)**p*c)/x**p)*a**4*d*e**2*x**3 + 6*log
(((a*x + b)**p*c)/x**p)*a**4*e**3*x**4 + 24*log(((a*x + b)**p*c)/x**p)*a**
3*b*d**3 - 36*log(((a*x + b)**p*c)/x**p)*a**2*b**2*d**2*e + 24*log(((a*x +
 b)**p*c)/x**p)*a*b**3*d*e**2 - 6*log(((a*x + b)**p*c)/x**p)*b**4*e**3 + 2
4*log(x)*a**3*b*d**3*p - 36*log(x)*a**2*b**2*d**2*e*p + 24*log(x)*a*b**3*d
*e**2*p - 6*log(x)*b**4*e**3*p + 36*a**3*b*d**2*e*p*x + 12*a**3*b*d*e**2*p
*x**2 + 2*a**3*b*e**3*p*x**3 - 24*a**2*b**2*d*e**2*p*x - 3*a**2*b**2*e**3*
p*x**2 + 6*a*b**3*e**3*p*x)/(24*a**4)
```

3.199 $\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{be(3ad-be)px}{3a^2}+\frac{be^2px^2}{6a}+\frac{(d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{d^3p\log(x)}{3e}-\frac{(ad-be)^3p\log(b+ax)}{3a^3e}$$

output
```
1/3*b*e*(3*a*d-b*e)*p*x/a^2+1/6*b*e^2*p*x^2/a+1/3*(e*x+d)^3*ln(c*(a+b/x)^p
)/e+1/3*d^3*p*ln(x)/e-1/3*(a*d-b*e)^3*p*ln(a*x+b)/a^3/e
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.84

$$\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{2a^3(d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)+p(abe^2x(6ad-2be+aex)+2a^3d^3\log(x)-2(ad-be)^3\log(b+ax))}{6a^3e}$$

input
```
Integrate[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```

output
```
(2*a^3*(d + e*x)^3*Log[c*(a + b/x)^p] + p*(a*b*e^2*x*(6*a*d - 2*b*e + a*e*
x) + 2*a^3*d^3*Log[x] - 2*(a*d - b*e)^3*Log[b + a*x]))/(6*a^3*e)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2913, 1016, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

$$\Big\downarrow \text{2913}$$

$$\frac{bp\int \frac{(d+ex)^3}{\left(a+\frac{b}{x}\right)x^2}dx}{3e}+\frac{(d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}$$

$$\Big\downarrow \text{1016}$$

$$\frac{bp\int \frac{(d+ex)^3}{x(b+ax)}dx}{3e}+\frac{(d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}$$

$$\Big\downarrow \text{93}$$

$$\frac{bp\int \left(\frac{d^3}{bx}+\frac{e^2(3ad-be)}{a^2}+\frac{e^3x}{a}-\frac{(ad-be)^3}{a^2b(b+ax)}\right)dx}{3e}+\frac{(d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bp\left(-\frac{(ad-be)^3 \log(ax+b)}{a^3b}+\frac{e^2x(3ad-be)}{a^2}+\frac{e^3x^2}{2a}+\frac{d^3 \log(x)}{b}\right)}{3e}+\frac{(d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}$$

input
```
Int[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```

output
```
((d + e*x)^3*Log[c*(a + b/x)^p])/(3*e) + (b*p*((e^2*(3*a*d - b*e)*x)/a^2 +
 (e^3*x^2)/(2*a) + (d^3*Log[x])/b - ((a*d - b*e)^3*Log[b + a*x])/(a^3*b)))
/(3*e)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.65

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)e^{2}x^{3}}{3}+\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)ed\,x^{2}+\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)d^{2}x+\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)d^{3}}{3e}+\frac{pb\left(\frac{e^{2}\left(\frac{1}{2}ae\,x^{2}+3\right.}{a^{2}}\right.}{}$
parallelrisch	$-\frac{-2x^{3}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{3}e^{2}-6x^{2}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{3}de-x^{2}a^{2}b\,e^{2}p+6\ln(x)a^{2}b\,d^{2}p-12\ln(ax+b)a^{2}b\,d^{2}p+6\ln(ax+b)a\,b^{2}dep-}{6a^{3}}$

input
```
int((e*x+d)^2*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(c*(a+b/x)^p)*e^2*x^3+ln(c*(a+b/x)^p)*e*d*x^2+ln(c*(a+b/x)^p)*d^2*x+
1/3*ln(c*(a+b/x)^p)/e*d^3+1/3*p*b/e*(e^2/a^2*(1/2*a*e*x^2+3*x*d*a-b*e*x)+(
-a^3*d^3+3*a^2*b*d^2*e-3*a*b^2*d*e^2+b^3*e^3)/a^3/b*ln(a*x+b)+d^3/b*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

$$\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$$
$$= \frac{a^2be^2px^2 + 2\,(3\,a^2bde - ab^2e^2)px + 2\,(3\,a^2bd^2 - 3\,ab^2de + b^3e^2)p\log\left(ax+b\right) + 2\,(a^3e^2x^3 + 3\,a^3dex^2 + 3}{6\,a^3}$$

input
```
integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

output
```
1/6*(a^2*b*e^2*p*x^2 + 2*(3*a^2*b*d*e - a*b^2*e^2)*p*x + 2*(3*a^2*b*d^2 -
3*a*b^2*d*e + b^3*e^2)*p*log(a*x + b) + 2*(a^3*e^2*x^3 + 3*a^3*d*e*x^2 + 3
*a^3*d^2*x)*log(c) + 2*(a^3*e^2*p*x^3 + 3*a^3*d*e*p*x^2 + 3*a^3*d^2*p*x)*l
og((a*x + b)/x))/a^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(88) = 176.

Time = 0.87 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.12

$$\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$$
$$= \begin{cases} d^2x\log\left(c\left(a+\frac{b}{x}\right)^p\right) + dex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{e^2x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3} + \frac{bd^2p\log\left(x+\frac{b}{a}\right)}{a} + \frac{bdepx}{a} + \frac{be^2px^2}{6a} - \frac{b^2dep\log}{} \\ d^2px + d^2x\log\left(c\left(\frac{b}{x}\right)^p\right) + \frac{depx^2}{2} + dex^2\log\left(c\left(\frac{b}{x}\right)^p\right) + \frac{e^2px^3}{9} + \frac{e^2x^3\log\left(c\left(\frac{b}{x}\right)^p\right)}{3} \end{cases}$$

input
```
integrate((e*x+d)**2*ln(c*(a+b/x)**p),x)
```

output
```
Piecewise((d**2*x*log(c*(a + b/x)**p) + d*e*x**2*log(c*(a + b/x)**p) + e**
2*x**3*log(c*(a + b/x)**p)/3 + b*d**2*p*log(x + b/a)/a + b*d*e*p*x/a + b*e
**2*p*x**2/(6*a) - b**2*d*e*p*log(x + b/a)/a**2 - b**2*e**2*p*x/(3*a**2) +
 b**3*e**2*p*log(x + b/a)/(3*a**3), Ne(a, 0)), (d**2*p*x + d**2*x*log(c*(b
/x)**p) + d*e*p*x**2/2 + d*e*x**2*log(c*(b/x)**p) + e**2*p*x**3/9 + e**2*x
**3*log(c*(b/x)**p)/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{1}{6}\,bp\left(\frac{ae^2x^2+2\,(3\,ade-be^2)x}{a^2}+\frac{2\,(3\,a^2d^2-3\,abde+b^2e^2)\log\left(ax+b\right)}{a^3}\right) + \frac{1}{3}\left(e^2x^3+3\,dex^2+3\,d^2x\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

input
```
integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

output
```
1/6*b*p*((a*e^2*x^2 + 2*(3*a*d*e - b*e^2)*x)/a^2 + 2*(3*a^2*d^2 - 3*a*b*d*
e + b^2*e^2)*log(a*x + b)/a^3) + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((
a + b/x)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(92) = 184$.

Time = 0.13 (sec) , antiderivative size = 490, normalized size of antiderivative = 4.80

$$\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = -\frac{\frac{2\left(3\,a^2b^2d^2p-3\,ab^3dep+b^4e^2p-\frac{6\,(ax+b)ab^2d^2p}{x}+\frac{3\,(ax+b)b^3dep}{x}+\frac{3\,(ax+b)^2b^2d^2p}{x^2}\right)\log\left(\frac{ax+b}{x}\right)}{a^3-\frac{3\,(ax+b)a^2}{x}+\frac{3\,(ax+b)^2a}{x^2}-\frac{(ax+b)^3}{x^3}}+\frac{6\,a^3b^3dep-3\,a^2b^4e^2p+6\,a^4b^2d^2\log(c)-6\,\cdots}{\cdots}}{\cdots}$$

input
```
integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

output
```
-1/6*(2*(3*a^2*b^2*d^2*p - 3*a*b^3*d*e*p + b^4*e^2*p - 6*(a*x + b)*a*b^2*d
^2*p/x + 3*(a*x + b)*b^3*d*e*p/x + 3*(a*x + b)^2*b^2*d^2*p/x^2)*log((a*x +
 b)/x)/(a^3 - 3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3) +
 (6*a^3*b^3*d*e*p - 3*a^2*b^4*e^2*p + 6*a^4*b^2*d^2*log(c) - 6*a^3*b^3*d*e
*log(c) + 2*a^2*b^4*e^2*log(c) - 12*(a*x + b)*a^2*b^3*d*e*p/x + 5*(a*x + b
)*a*b^4*e^2*p/x - 12*(a*x + b)*a^3*b^2*d^2*log(c)/x + 6*(a*x + b)*a^2*b^3*
d*e*log(c)/x + 6*(a*x + b)^2*a*b^3*d*e*p/x^2 - 2*(a*x + b)^2*b^4*e^2*p/x^2
 + 6*(a*x + b)^2*a^2*b^2*d^2*log(c)/x^2)/(a^5 - 3*(a*x + b)*a^4/x + 3*(a*x
 + b)^2*a^3/x^2 - (a*x + b)^3*a^2/x^3) + 2*(3*a^2*b^2*d^2*p - 3*a*b^3*d*e*
p + b^4*e^2*p)*log(-a + (a*x + b)/x)/a^3 - 2*(3*a^2*b^2*d^2*p - 3*a*b^3*d*
e*p + b^4*e^2*p)*log((a*x + b)/x)/a^3)/b
```

Mupad [B] (verification not implemented)

Time = 25.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int (d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right) - x\left(\frac{b^2\,e^2\,p}{3\,a^2}-\frac{b\,d\,e\,p}{a}\right) + \frac{\ln(b+a\,x)\,\left(3\,p\,a^2\,b\,d^2-3\,p\,a\,b^2\,d\,e+p\,b^3\,e^2\right)}{3\,a^3} + \frac{b\,e^2\,p\,x^2}{6\,a}$$

input
```
int(log(c*(a + b/x)^p)*(d + e*x)^2,x)
```

output
```
log(c*(a + b/x)^p)*(d^2*x + (e^2*x^3)/3 + d*e*x^2) - x*((b^2*e^2*p)/(3*a^2
) - (b*d*e*p)/a) + (log(b + a*x)*(b^3*e^2*p + 3*a^2*b*d^2*p - 3*a*b^2*d*e*
p))/(3*a^3) + (b*e^2*p*x^2)/(6*a)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.14

$$\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

$$= \frac{6\log\left(\frac{(ax+b)^p c}{x^p}\right) a^3 d^2 x + 6\log\left(\frac{(ax+b)^p c}{x^p}\right) a^3 de\, x^2 + 2\log\left(\frac{(ax+b)^p c}{x^p}\right) a^3 e^2 x^3 + 6\log\left(\frac{(ax+b)^p c}{x^p}\right) a^2 b\, d^2 - 6\log\left(\right.}{}$$

input
```
int((e*x+d)^2*log(c*(a+b/x)^p),x)
```

output
```
(6*log(((a*x + b)**p*c)/x**p)*a**3*d**2*x + 6*log(((a*x + b)**p*c)/x**p)*a
**3*d*e*x**2 + 2*log(((a*x + b)**p*c)/x**p)*a**3*e**2*x**3 + 6*log(((a*x +
 b)**p*c)/x**p)*a**2*b*d**2 - 6*log(((a*x + b)**p*c)/x**p)*a*b**2*d*e + 2*
log(((a*x + b)**p*c)/x**p)*b**3*e**2 + 6*log(x)*a**2*b*d**2*p - 6*log(x)*a
*b**2*d*e*p + 2*log(x)*b**3*e**2*p + 6*a**2*b*d*e*p*x + a**2*b*e**2*p*x**2
 - 2*a*b**2*e**2*p*x)/(6*a**3)
```

3.200 $\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx=\frac{bepx}{2a}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{d^2p\log(x)}{2e}-\frac{(ad-be)^2p\log(b+ax)}{2a^2e}$$

output
```
1/2*b*e*p*x/a+1/2*(e*x+d)^2*ln(c*(a+b/x)^p)/e+1/2*d^2*p*ln(x)/e-1/2*(a*d-b
*e)^2*p*ln(a*x+b)/a^2/e
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx=\frac{bdp\log\left(a+\frac{b}{x}\right)}{a}+dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2}ex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bdp\log(x)}{a}+\frac{1}{2}bep\left(\frac{x}{a}-\frac{b\log(b+ax)}{a^2}\right)$$

input
```
Integrate[(d + e*x)*Log[c*(a + b/x)^p],x]
```

output
```
(b*d*p*Log[a + b/x])/a + d*x*Log[c*(a + b/x)^p] + (e*x^2*Log[c*(a + b/x)^p
])/2 + (b*d*p*Log[x])/a + (b*e*p*(x/a - (b*Log[b + a*x])/a^2))/2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2913, 1016, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$

$$\Big\downarrow \text{2913}$$

$$\frac{bp\int \frac{(d+ex)^2}{\left(a+\frac{b}{x}\right)x^2}dx}{2e}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}$$

$$\Big\downarrow \text{1016}$$

$$\frac{bp\int \frac{(d+ex)^2}{x(b+ax)}dx}{2e}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}$$

$$\Big\downarrow \text{93}$$

$$\frac{bp\int \left(\frac{d^2}{bx}+\frac{e^2}{a}-\frac{(ad-be)^2}{ab(b+ax)}\right)dx}{2e}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bp\left(-\frac{(ad-be)^2\log(ax+b)}{a^2b}+\frac{e^2x}{a}+\frac{d^2\log(x)}{b}\right)}{2e}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}$$

input
```
Int[(d + e*x)*Log[c*(a + b/x)^p],x]
```

output
```
((d + e*x)^2*Log[c*(a + b/x)^p])/(2*e) + (b*p*((e^2*x)/a + (d^2*Log[x])/b
- ((a*d - b*e)^2*Log[b + a*x])/(a^2*b)))/(2*e)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)e\,x^{2}}{2}+\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)dx+\frac{pb\left(\frac{ex}{a}+\frac{(2da-be)\ln(ax+b)}{a^{2}}\right)}{2}$
parallelrisch	$-\frac{-x^{2}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{2}e+2\ln(x)abdp-4\ln(ax+b)abdp+\ln(ax+b)b^{2}ep-2x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{2}d-abepx+2\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)ab}{2a^{2}}$

input
```
int((e*x+d)*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*(a+b/x)^p)*e*x^2+ln(c*(a+b/x)^p)*d*x+1/2*p*b*(e*x/a+(2*a*d-b*e)/a
^2*ln(a*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$=\frac{abepx+(2\,abd-b^2e)p\log(ax+b)+(a^2ex^2+2\,a^2dx)\log(c)+(a^2epx^2+2\,a^2dpx)\log\left(\frac{ax+b}{x}\right)}{2\,a^2}$$

input
```
integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="fricas")
```

output
```
1/2*(a*b*e*p*x + (2*a*b*d - b^2*e)*p*log(a*x + b) + (a^2*e*x^2 + 2*a^2*d*x
)*log(c) + (a^2*e*p*x^2 + 2*a^2*d*p*x)*log((a*x + b)/x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$=\begin{cases} dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{ex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2}+\frac{bdp\log\left(x+\frac{b}{a}\right)}{a}+\frac{bepx}{2a}-\frac{b^2ep\log\left(x+\frac{b}{a}\right)}{2a^2} & \text{for } a\neq 0\\ dpx+dx\log\left(c\left(\frac{b}{x}\right)^p\right)+\frac{epx^2}{4}+\frac{ex^2\log\left(c\left(\frac{b}{x}\right)^p\right)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)*ln(c*(a+b/x)**p),x)
```

output
```
Piecewise((d*x*log(c*(a + b/x)**p) + e*x**2*log(c*(a + b/x)**p)/2 + b*d*p*
log(x + b/a)/a + b*e*p*x/(2*a) - b**2*e*p*log(x + b/a)/(2*a**2), Ne(a, 0))
, (d*p*x + d*x*log(c*(b/x)**p) + e*p*x**2/4 + e*x**2*log(c*(b/x)**p)/2, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{1}{2}\,bp\left(\frac{ex}{a}+\frac{(2\,ad-be)\log\left(ax+b\right)}{a^2}\right) + \frac{1}{2}\left(ex^2+2\,dx\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

input
```
integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="maxima")
```

output
```
1/2*b*p*(e*x/a + (2*a*d - b*e)*log(a*x + b)/a^2) + 1/2*(e*x^2 + 2*d*x)*log
((a + b/x)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(70) = 140$.

Time = 0.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.99

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = -\frac{\frac{\left(2\,ab^2dp-b^3ep-\frac{2\,(ax+b)b^2dp}{x}\right)\log\left(\frac{ax+b}{x}\right)}{a^2-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}}+\frac{ab^3ep+2\,a^2b^2d\log(c)-ab^3e\log(c)-\frac{(ax+b)b^3ep}{x}-\frac{2\,(ax+b)ab^2d\log(c)}{x}}{a^3-\frac{2\,(ax+b)a^2}{x}+\frac{(ax+b)^2a}{x^2}}+\frac{(2\,ab^2dp-b^3ep)\log(}{a^2}}{2\,b}$$

input
```
integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="giac")
```

output
```
-1/2*((2*a*b^2*d*p - b^3*e*p - 2*(a*x + b)*b^2*d*p/x)*log((a*x + b)/x)/(a^
2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + (a*b^3*e*p + 2*a^2*b^2*d*log(c) -
 a*b^3*e*log(c) - (a*x + b)*b^3*e*p/x - 2*(a*x + b)*a*b^2*d*log(c)/x)/(a^3
 - 2*(a*x + b)*a^2/x + (a*x + b)^2*a/x^2) + (2*a*b^2*d*p - b^3*e*p)*log(-a
 + (a*x + b)/x)/a^2 - (2*a*b^2*d*p - b^3*e*p)*log((a*x + b)/x)/a^2)/b
```

Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int (d+ex)\log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx=\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\,\left(\frac{e\,x^2}{2}+d\,x\right)-\frac{\ln\left(b+a\,x\right)\,\left(b^2\,e\,p-2\,a\,b\,d\,p\right)}{2\,a^2}+\frac{b\,e\,p\,x}{2\,a}$$

input
```
int(log(c*(a + b/x)^p)*(d + e*x),x)
```

output
```
log(c*(a + b/x)^p)*(d*x + (e*x^2)/2) - (log(b + a*x)*(b^2*e*p - 2*a*b*d*p)
)/(2*a^2) + (b*e*p*x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.47

$$\int (d+ex)\log \left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$=\frac{2\log\left(\frac{(ax+b)^p c}{x^p}\right)a^2dx+\log\left(\frac{(ax+b)^p c}{x^p}\right)a^2e\,x^2+2\log\left(\frac{(ax+b)^p c}{x^p}\right)abd-\log\left(\frac{(ax+b)^p c}{x^p}\right)b^2e+2\log(x)\,abdp-\mathrm{l}}{2a^2}$$

input
```
int((e*x+d)*log(c*(a+b/x)^p),x)
```

output
```
(2*log(((a*x + b)**p*c)/x**p)*a**2*d*x + log(((a*x + b)**p*c)/x**p)*a**2*e
*x**2 + 2*log(((a*x + b)**p*c)/x**p)*a*b*d - log(((a*x + b)**p*c)/x**p)*b*
*2*e + 2*log(x)*a*b*d*p - log(x)*b**2*e*p + a*b*e*p*x)/(2*a**2)
```

3.201 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}$$

output
```
ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(a*x+b)/(a*d-
b*e))*ln(e*x+d)/e-p*polylog(2,a*(e*x+d)/(a*d-b*e))/e+p*polylog(2,1+e*x/d)/
e
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} dx = \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log \left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log \left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} + \frac{p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}$$

input
```
Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
 (p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d +
 e*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2912, 2005, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} dx$$

$$\Big\downarrow \text{2912}$$

$$\frac{bp\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}dx}{e} + \frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}$$

$$\Big\downarrow \text{2005}$$

$$\frac{bp\int \frac{\log(d+ex)}{x(b+ax)}dx}{e} + \frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}$$

$$\Big\downarrow \text{2863}$$

$$\frac{bp\int\left(\frac{\log(d+ex)}{bx}-\frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{bp\left(-\frac{\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{b}-\frac{\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{b}+\frac{\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{b}+\frac{\log\left(-\frac{ex}{d}\right)\log(d+ex)}{b}\right)}{e}$$

input
```
Int[Log[c*(a + b/x)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b/x)^p]*Log[d + e*x])/e + (b*p*((Log[-((e*x)/d)]*Log[d + e*x])
/b - (Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/b - PolyLog[2, (a*(d
 + e*x))/(a*d - b*e)]/b + PolyLog[2, 1 + (e*x)/d]/b))/e
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln(ex+d)}{e}+pb\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{-da+a(ex+d)+be}{-da+be}\right)}{a}+\frac{\ln(ex+d)\ln\left(\frac{-da+a(ex+d)+be}{-da+be}\right)}{a}\right)a}{be}+\frac{\operatorname{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln}{be}\right.$

input
```
int(ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*b*(-(dilog((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))/
a+ln(e*x+d)*ln((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))/a)*a/b/e+(dilog(-e*x/d)+ln
(e*x+d)*ln(-e*x/d))/b/e)
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx = \int \frac{\log \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx$$

input
```
integrate(ln(c*(a+b/x)**p)/(e*x+d),x)
```

output
```
Integral(log(c*(a + b/x)**p)/(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.41

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx = \frac{bp\left(\frac{\log(ex+d)\log\left(a+\frac{b}{x}\right)}{b} - \frac{\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+1\right)+\mathrm{Li}_2\left(\frac{aex+ad}{ad-be}\right)}{b} + \frac{\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)}{b}\right)}{e} - \frac{p\log(ex+d)\log\left(a+\frac{b}{x}\right)}{e} + \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)\log(ex+d)}{e}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
b*p*(log(e*x + d)*log(a + b/x)/b - (log(e*x + d)*log(-(a*e*x + a*d)/(a*d -
 b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))/b + (log(e*x + d)*log(-(e*x
 + d)/d + 1) + dilog((e*x + d)/d))/b)/e - p*log(e*x + d)*log(a + b/x)/e +
log((a + b/x)^p*c)*log(e*x + d)/e
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

input
```
int(log(c*(a + b/x)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b/x)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)}{ex+d}dx$$

input
```
int(log(c*(a+b/x)^p)/(e*x+d),x)
```

output
```
int(log(((a*x + b)**p*c)/x**p)/(d + e*x),x)
```

3.202 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx=-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}-\frac{p\log(x)}{de}+\frac{ap\log(b+ax)}{e(ad-be)}-\frac{bp\log(d+ex)}{d(ad-be)}$$

output
```
-ln(c*(a+b/x)^p)/e/(e*x+d)-p*ln(x)/d/e+a*p*ln(a*x+b)/e/(a*d-b*e)-b*p*ln(e*
x+d)/d/(a*d-b*e)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx=-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}-\frac{p\log(x)}{de}+\frac{ap\log(b+ax)}{e(ad-be)}-\frac{bp\log(d+ex)}{d(ad-be)}$$

input
```
Integrate[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
```

output
```
-(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x]
)/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2913, 1016, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}dx$$

$$\Big\downarrow \ 2913$$

$$-\frac{bp\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)}dx}{e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}$$

$$\Big\downarrow \ 1016$$

$$-\frac{bp\int \frac{1}{x(b+ax)(d+ex)}dx}{e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}$$

$$\Big\downarrow \ 93$$

$$-\frac{bp\int \left(\frac{a^2}{b(be-ad)(b+ax)}+\frac{1}{bdx}+\frac{e^2}{d(ad-be)(d+ex)}\right)dx}{e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}-\frac{bp\left(-\frac{a\log(ax+b)}{b(ad-be)}+\frac{e\log(d+ex)}{d(ad-be)}+\frac{\log(x)}{bd}\right)}{e}$$

input
```
Int[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
```

output
```
-(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (b*p*(Log[x]/(b*d) - (a*Log[b + a*x]
)/(b*(a*d - b*e)) + (e*Log[d + e*x])/(d*(a*d - b*e))))/e
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{e(ex+d)}-\frac{pb\left(-\frac{a\ln(ax+b)}{b(da-be)}+\frac{e\ln(ex+d)}{d(da-be)}+\frac{\ln(x)}{bd}\right)}{e}$	86
parallelrisch	$-\frac{-\ln(x)x\,b^{2}e\,p^{2}+\ln(ex+d)x\,b^{2}e\,p^{2}-\ln(x)b^{2}d\,p^{2}+\ln(ex+d)b^{2}d\,p^{2}-x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)abdp-\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{2}dp}{(ex+d)pbd(da-be)}$	124

input
```
int(ln(c*(a+b/x)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(a+b/x)^p)/e/(e*x+d)-p*b/e*(-a/b/(a*d-b*e)*ln(a*x+b)+e/d/(a*d-b*e)*l
n(e*x+d)+1/b/d*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.83

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx = -\frac{(ad^2-bde)p\log\left(\frac{ax+b}{x}\right)-(adepx+ad^2p)\log(ax+b)+(be^2px+bdep)\log(ex+d)+(ad^2-bde)\log}{ad^3e-bd^2e^2+(ad^2e^2-bde^3)x}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-((a*d^2 - b*d*e)*p*log((a*x + b)/x) - (a*d*e*p*x + a*d^2*p)*log(a*x + b)
+ (b*e^2*p*x + b*d*e*p)*log(e*x + d) + (a*d^2 - b*d*e)*log(c) + ((a*d*e -
b*e^2)*p*x + (a*d^2 - b*d*e)*p)*log(x))/(a*d^3*e - b*d^2*e^2 + (a*d^2*e^2
- b*d*e^3)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(61) = 122.

Time = 2.21 (sec) , antiderivative size = 425, normalized size of antiderivative = 5.25

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx = \begin{cases} \frac{dp\log\left(\frac{d}{e}+x\right)}{d^2e+de^2x}+\frac{epx\log\left(\frac{d}{e}+x\right)}{d^2e+de^2x}+\frac{ex\log\left(c\left(\frac{b}{x}\right)^p\right)}{d^2e+de^2x} \\ \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\log(ax+b)}{a}}{d^2} \\ -\frac{dp}{d^2e+de^2x}+\frac{ex\log\left(c\left(\frac{b}{x}+\frac{be}{d}\right)^p\right)}{d^2e+de^2x} \\ \frac{-\frac{a\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}+\frac{p}{x}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}}{e^2} \\ \frac{adx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{ad^3+ad^2ex-bd^2e-bde^2x}+\frac{bdp\log\left(x+\frac{b}{a}\right)}{ad^3+ad^2ex-bd^2e-bde^2x}-\frac{bdp\log\left(\frac{d}{e}+x\right)}{ad^3+ad^2ex-bd^2e-bde^2x}+\frac{bepx\log\left(x+\frac{b}{a}\right)}{ad^3+ad^2ex-bd^2e-bde^2x}-\frac{bepx\log\left(\frac{d}{e}+x\right)}{ad^3+ad^2ex-bd^2e-} \end{cases}$$

input
```
integrate(ln(c*(a+b/x)**p)/(e*x+d)**2,x)
```

output
```
Piecewise((d*p*log(d/e + x)/(d**2*e + d*e**2*x) + e*p*x*log(d/e + x)/(d**2
*e + d*e**2*x) + e*x*log(c*(b/x)**p)/(d**2*e + d*e**2*x), Eq(a, 0)), ((x*l
og(c*(a + b/x)**p) + b*p*log(a*x + b)/a)/d**2, Eq(e, 0)), (-d*p/(d**2*e +
d*e**2*x) + e*x*log(c*(b/x + b*e/d)**p)/(d**2*e + d*e**2*x), Eq(a, b*e/d))
, ((-a*log(c*(a + b/x)**p)/b + p/x - log(c*(a + b/x)**p)/x)/e**2, Eq(d, 0)
), (a*d*x*log(c*(a + b/x)**p)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x
) + b*d*p*log(x + b/a)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*d
*p*log(d/e + x)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) + b*e*p*x*lo
g(x + b/a)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*e*p*x*log(d/e
 + x)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*e*x*log(c*(a + b/x
)**p)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx=\frac{bp\left(\frac{a\log(ax+b)}{abd-b^2e}-\frac{e\log(ex+d)}{ad^2-bde}-\frac{\log(x)}{bd}\right)}{e}-\frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)e}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
b*p*(a*log(a*x + b)/(a*b*d - b^2*e) - e*log(e*x + d)/(a*d^2 - b*d*e) - log
(x)/(b*d))/e - log((a + b/x)^p*c)/((e*x + d)*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.79

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx$$
$$=-\frac{\frac{b^2p\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{ad^2-bde}+\frac{b^2p\log\left(\frac{ax+b}{x}\right)}{ad^2-bde-\frac{(ax+b)d^2}{x}}-\frac{b^2p\log\left(\frac{ax+b}{x}\right)}{ad^2-bde}+\frac{b^2\log(c)}{ad^2-bde-\frac{(ax+b)d^2}{x}}}{b}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-(b^2*p*log(-a*d + b*e + (a*x + b)*d/x)/(a*d^2 - b*d*e) + b^2*p*log((a*x +
 b)/x)/(a*d^2 - b*d*e - (a*x + b)*d^2/x) - b^2*p*log((a*x + b)/x)/(a*d^2 -
 b*d*e) + b^2*log(c)/(a*d^2 - b*d*e - (a*x + b)*d^2/x))/b
```

Mupad [B] (verification not implemented)

Time = 25.80 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx=-\frac{\ln \left(c\,{\left(\frac{b+a\,x}{x}\right)}^p\right)}{x\,e^2+d\,e}-\frac{p\,\ln \left(x\right)}{d\,e}-\frac{a\,p\,\ln \left(b+a\,x\right)}{b\,e^2-a\,d\,e}-\frac{b\,p\,\ln \left(d+e\,x\right)}{a\,d^2-b\,d\,e}$$

input
```
int(log(c*(a + b/x)^p)/(d + e*x)^2,x)
```

output
```
- log(c*((b + a*x)/x)^p)/(d*e + e^2*x) - (p*log(x))/(d*e) - (a*p*log(b + a
*x))/(b*e^2 - a*d*e) - (b*p*log(d + e*x))/(a*d^2 - b*d*e)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx$$
$$=\frac{\log(ax+b)\,bdp+\log(ax+b)\,bepx-\log(ex+d)\,bdp-\log(ex+d)\,bepx+\log\left(\frac{(ax+b)^p c}{x^p}\right)adx-\log\left(\frac{(ax+}{x}\right.}{d\left(adex-b\,e^2x+a\,d^2-bde\right)}$$

input
```
int(log(c*(a+b/x)^p)/(e*x+d)^2,x)
```

output
```
(log(a*x + b)*b*d*p + log(a*x + b)*b*e*p*x - log(d + e*x)*b*d*p - log(d +
e*x)*b*e*p*x + log(((a*x + b)**p*c)/x**p)*a*d*x - log(((a*x + b)**p*c)/x**
p)*b*e*x)/(d*(a*d**2 + a*d*e*x - b*d*e - b*e**2*x))
```

3.203 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \frac{bp}{2d(ad-be)(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{p\log(x)}{2d^2e} + \frac{a^2p\log(b+ax)}{2e(ad-be)^2} - \frac{b(2ad-be)p\log(d+ex)}{2d^2(ad-be)^2}$$

output
```
1/2*b*p/d/(a*d-b*e)/(e*x+d)-1/2*ln(c*(a+b/x)^p)/e/(e*x+d)^2-1/2*p*ln(x)/d^
2/e+1/2*a^2*p*ln(a*x+b)/e/(a*d-b*e)^2-1/2*b*(2*a*d-b*e)*p*ln(e*x+d)/d^2/(a
*d-b*e)^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \frac{\frac{bep}{d(ad-be)(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2} - \frac{p\log(x)}{d^2} + \frac{a^2p\log(b+ax)}{(ad-be)^2} + \frac{be(-2ad+be)p\log(d+ex)}{d^2(ad-be)^2}}{2e}$$

input
```
Integrate[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
```

output
```
((b*e*p)/(d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^2 - (p*L
og[x])/d^2 + (a^2*p*Log[b + a*x])/(a*d - b*e)^2 + (b*e*(-2*a*d + b*e)*p*Lo
g[d + e*x])/(d^2*(a*d - b*e)^2))/(2*e)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2913, 1016, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 2913}$$

$$-\frac{bp\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^2}dx}{2e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 1016}$$

$$-\frac{bp\int \frac{1}{x(b+ax)(d+ex)^2}dx}{2e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 93}$$

$$-\frac{bp\int \left(-\frac{a^3}{b(be-ad)^2(b+ax)}+\frac{1}{bd^2x}+\frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex)}+\frac{e^2}{d(ad-be)(d+ex)^2}\right)dx}{2e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{bp\left(-\frac{a^2\log(ax+b)}{b(ad-be)^2}+\frac{e(2ad-be)\log(d+ex)}{d^2(ad-be)^2}-\frac{e}{d(d+ex)(ad-be)}+\frac{\log(x)}{bd^2}\right)}{2e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}$$

input
```
Int[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
```

output
```
-1/2*Log[c*(a + b/x)^p]/(e*(d + e*x)^2) - (b*p*(-(e/(d*(a*d - b*e)*(d + e*
x))) + Log[x]/(b*d^2) - (a^2*Log[b + a*x])/(b*(a*d - b*e)^2) + (e*(2*a*d -
 b*e)*Log[d + e*x])/(d^2*(a*d - b*e)^2)))/(2*e)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{2e\left(ex+d\right)^{2}}-\frac{pb\left(-\frac{a^{2}\ln\left(ax+b\right)}{b\left(da-be\right)^{2}}-\frac{e}{d\left(da-be\right)\left(ex+d\right)}+\frac{e\left(2da-be\right)\ln\left(ex+d\right)}{d^{2}\left(da-be\right)^{2}}+\frac{\ln\left(x\right)}{b\,d^{2}}\right)}{2e}$
parallelrisch	$-\frac{-a^{2}b\,d^{3}e^{2}p^{2}+a\,b^{2}d^{2}e^{3}p^{2}+\ln\left(x\right)x^{2}a\,b^{2}e^{5}p^{2}-\ln\left(ex+d\right)x^{2}a\,b^{2}e^{5}p^{2}-2\ln\left(x\right)a^{2}b\,d^{3}e^{2}p^{2}+\ln\left(x\right)a\,b^{2}d^{2}e^{3}p^{2}+2\ln\left(ex+d\right)a^{2}b\,d^{3}}{}$

input
```
int(ln(c*(a+b/x)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(a+b/x)^p)/e/(e*x+d)^2-1/2*p*b/e*(-1/b*a^2/(a*d-b*e)^2*ln(a*x+b)
-e/d/(a*d-b*e)/(e*x+d)+e*(2*a*d-b*e)/d^2/(a*d-b*e)^2*ln(e*x+d)+1/b/d^2*ln(
x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(117) = 234$.

Time = 0.56 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.37

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \frac{(abd^2e^2 - b^2de^3)px - (a^2d^4 - 2\,abd^3e + b^2d^2e^2)p\log\left(\frac{ax+b}{x}\right) + (abd^3e - b^2d^2e^2)p + (a^2d^2e^2px^2 + 2\,a^2d^3e}{}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*((a*b*d^2*e^2 - b^2*d*e^3)*p*x - (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)
*p*log((a*x + b)/x) + (a*b*d^3*e - b^2*d^2*e^2)*p + (a^2*d^2*e^2*p*x^2 + 2
*a^2*d^3*e*p*x + a^2*d^4*p)*log(a*x + b) - ((2*a*b*d*e^3 - b^2*e^4)*p*x^2
+ 2*(2*a*b*d^2*e^2 - b^2*d*e^3)*p*x + (2*a*b*d^3*e - b^2*d^2*e^2)*p)*log(e
*x + d) - (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*log(c) - ((a^2*d^2*e^2 - 2
*a*b*d*e^3 + b^2*e^4)*p*x^2 + 2*(a^2*d^3*e - 2*a*b*d^2*e^2 + b^2*d*e^3)*p*
x + (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*p)*log(x))/(a^2*d^6*e - 2*a*b*d^
5*e^2 + b^2*d^4*e^3 + (a^2*d^4*e^3 - 2*a*b*d^3*e^4 + b^2*d^2*e^5)*x^2 + 2*
(a^2*d^5*e^2 - 2*a*b*d^4*e^3 + b^2*d^3*e^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3485 vs. $2(105) = 210$.

Time = 9.18 (sec) , antiderivative size = 3485, normalized size of antiderivative = 27.44

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate(ln(c*(a+b/x)**p)/(e*x+d)**3,x)
```

output
```
Piecewise((d**2*p*log(d/e + x)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**
2) - d**2*p/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) + 2*d*e*p*x*log(
d/e + x)/(2*d**4*e + 4*d**3*e**2*x + 2*d**2*e**3*x**2) - d*e*p*x/(2*d**4*e
 + 4*d**3*e**2*x + 2*d**2*e**3*x**2) + 2*d*e*x*log(c*(b/x)**p)/(2*d**4*e +
 4*d**3*e**2*x + 2*d**2*e**3*x**2) + e**2*p*x**2*log(d/e + x)/(2*d**4*e +
4*d**3*e**2*x + 2*d**2*e**3*x**2) + e**2*x**2*log(c*(b/x)**p)/(2*d**4*e +
4*d**3*e**2*x + 2*d**2*e**3*x**2), Eq(a, 0)), ((x*log(c*(a + b/x)**p) + b*
p*log(a*x + b)/a)/d**3, Eq(e, 0)), (-3*d**2*p/(4*d**4*e + 8*d**3*e**2*x +
4*d**2*e**3*x**2) - 2*d*e*p*x/(4*d**4*e + 8*d**3*e**2*x + 4*d**2*e**3*x**2
) + 4*d*e*x*log(c*(b/x + b*e/d)**p)/(4*d**4*e + 8*d**3*e**2*x + 4*d**2*e**
3*x**2) + 2*e**2*x**2*log(c*(b/x + b*e/d)**p)/(4*d**4*e + 8*d**3*e**2*x +
4*d**2*e**3*x**2), Eq(a, b*e/d)), ((a**2*log(c*(a + b/x)**p)/(2*b**2) - a*
p/(2*b*x) + p/(4*x**2) - log(c*(a + b/x)**p)/(2*x**2))/e**3, Eq(d, 0)), (2
*a**2*d**3*x*log(c*(a + b/x)**p)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d
**4*e**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 +
2*b**2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + a**2*d**2
*e*x**2*log(c*(a + b/x)**p)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e
**2*x**2 - 4*a*b*d**5*e - 8*a*b*d**4*e**2*x - 4*a*b*d**3*e**3*x**2 + 2*b**
2*d**4*e**2 + 4*b**2*d**3*e**3*x + 2*b**2*d**2*e**4*x**2) + 2*a*b*d**3*p*l
og(x + b/a)/(2*a**2*d**6 + 4*a**2*d**5*e*x + 2*a**2*d**4*e**2*x**2 - 4*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \frac{\left(\frac{a^2\log(ax+b)}{a^2bd^2-2\,ab^2de+b^3e^2} - \frac{(2\,ade-be^2)\log(ex+d)}{a^2d^4-2\,abd^3e+b^2d^2e^2} + \frac{e}{ad^3-bd^2e+(ad^2e-bde^2)x} - \frac{\log(x)}{bd^2}\right)bp}{2\,e} - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{2\,(ex+d)^2e}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(a^2*log(a*x + b)/(a^2*b*d^2 - 2*a*b^2*d*e + b^3*e^2) - (2*a*d*e - b*e
^2)*log(e*x + d)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2) + e/(a*d^3 - b*d^2*
e + (a*d^2*e - b*d*e^2)*x) - log(x)/(b*d^2))*b*p/e - 1/2*log((a + b/x)^p*c
)/((e*x + d)^2*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(117) = 234.

Time = 0.14 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.70

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\frac{(2\,ab^2dp-b^3ep)\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{a^2d^4-2\,abd^3e+b^2d^2e^2} + \frac{\left(2\,ab^2dp-b^3ep-\frac{2\,(ax+b)b^2dp}{x}\right)\log\left(\frac{ax+b}{x}\right)}{a^2d^4-2\,abd^3e+b^2d^2e^2-\frac{2\,(ax+b)ad^4}{x}+\frac{2\,(ax+b)bd^3e}{x}+\frac{(ax+b)^2d^4}{x^2}} - \frac{(2\,ab^2dp-b^3ep)\log\left(\frac{ax+b}{x}\right)}{a^2d^4-2\,abd^3e+b^2d^2e^2}}{2\,b}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/2*((2*a*b^2*d*p - b^3*e*p)*log(-a*d + b*e + (a*x + b)*d/x)/(a^2*d^4 - 2
*a*b*d^3*e + b^2*d^2*e^2) + (2*a*b^2*d*p - b^3*e*p - 2*(a*x + b)*b^2*d*p/x
)*log((a*x + b)/x)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 - 2*(a*x + b)*a*d^
4/x + 2*(a*x + b)*b*d^3*e/x + (a*x + b)^2*d^4/x^2) - (2*a*b^2*d*p - b^3*e*
p)*log((a*x + b)/x)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2) - (a*b^3*d*e*p -
 b^4*e^2*p - 2*a^2*b^2*d^2*log(c) + 3*a*b^3*d*e*log(c) - b^4*e^2*log(c) -
(a*x + b)*b^3*d*e*p/x + 2*(a*x + b)*a*b^2*d^2*log(c)/x - 2*(a*x + b)*b^3*d
*e*log(c)/x)/(a^3*d^5 - 3*a^2*b*d^4*e + 3*a*b^2*d^3*e^2 - b^3*d^2*e^3 - 2*
(a*x + b)*a^2*d^5/x + 4*(a*x + b)*a*b*d^4*e/x - 2*(a*x + b)*b^2*d^3*e^2/x
+ (a*x + b)^2*a*d^5/x^2 - (a*x + b)^2*b*d^4*e/x^2))/b
```

Mupad [B] (verification not implemented)

Time = 26.54 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.71

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \frac{a^2\,p\,\ln\left(b+a\,x\right)}{2\,a^2\,d^2\,e-4\,a\,b\,d\,e^2+2\,b^2\,e^3}-\frac{\ln\left(c\,{\left(\frac{b+a\,x}{x}\right)}^p\right)}{2\,\left(d^2\,e+2\,d\,e^2\,x+e^3\,x^2\right)} -\frac{p\,\ln\left(x\right)}{2\,d^2\,e}-\frac{b\,e\,p}{2\,b\,d^2\,e^2-2\,a\,d^3\,e+2\,b\,d\,e^3\,x-2\,a\,d^2\,e^2\,x} +\frac{b^2\,e\,p\,\ln\left(d+e\,x\right)}{2\,a^2\,d^4-4\,a\,b\,d^3\,e+2\,b^2\,d^2\,e^2}-\frac{2\,a\,b\,d\,p\,\ln\left(d+e\,x\right)}{2\,a^2\,d^4-4\,a\,b\,d^3\,e+2\,b^2\,d^2\,e^2}$$

input
```
int(log(c*(a + b/x)^p)/(d + e*x)^3,x)
```

output
```
(a^2*p*log(b + a*x))/(2*b^2*e^3 + 2*a^2*d^2*e - 4*a*b*d*e^2) - log(c*((b +
 a*x)/x)^p)/(2*(d^2*e + e^3*x^2 + 2*d*e^2*x)) - (p*log(x))/(2*d^2*e) - (b*
e*p)/(2*b*d^2*e^2 - 2*a*d^3*e + 2*b*d*e^3*x - 2*a*d^2*e^2*x) + (b^2*e*p*lo
g(d + e*x))/(2*a^2*d^4 + 2*b^2*d^2*e^2 - 4*a*b*d^3*e) - (2*a*b*d*p*log(d +
 e*x))/(2*a^2*d^4 + 2*b^2*d^2*e^2 - 4*a*b*d^3*e)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 489, normalized size of antiderivative = 3.85

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx$$
$$= \frac{4\log(ax+b)\,ab\,d^3p+8\log(ax+b)\,ab\,d^2epx+4\log(ax+b)\,abd\,e^2p\,x^2-2\log(ax+b)\,b^2d^2ep-4\log(ax$$

input
```
int(log(c*(a+b/x)^p)/(e*x+d)^3,x)
```

output
```
(4*log(a*x + b)*a*b*d**3*p + 8*log(a*x + b)*a*b*d**2*e*p*x + 4*log(a*x + b
)*a*b*d*e**2*p*x**2 - 2*log(a*x + b)*b**2*d**2*e*p - 4*log(a*x + b)*b**2*d
*e**2*p*x - 2*log(a*x + b)*b**2*e**3*p*x**2 - 4*log(d + e*x)*a*b*d**3*p -
8*log(d + e*x)*a*b*d**2*e*p*x - 4*log(d + e*x)*a*b*d*e**2*p*x**2 + 2*log(d
 + e*x)*b**2*d**2*e*p + 4*log(d + e*x)*b**2*d*e**2*p*x + 2*log(d + e*x)*b*
*2*e**3*p*x**2 + 4*log(((a*x + b)**p*c)/x**p)*a**2*d**3*x + 2*log(((a*x +
b)**p*c)/x**p)*a**2*d**2*e*x**2 - 8*log(((a*x + b)**p*c)/x**p)*a*b*d**2*e*
x - 4*log(((a*x + b)**p*c)/x**p)*a*b*d*e**2*x**2 + 4*log(((a*x + b)**p*c)/
x**p)*b**2*d*e**2*x + 2*log(((a*x + b)**p*c)/x**p)*b**2*e**3*x**2 + a*b*d*
*3*p - a*b*d*e**2*p*x**2 - b**2*d**2*e*p + b**2*e**3*p*x**2)/(4*d**2*(a**2
*d**4 + 2*a**2*d**3*e*x + a**2*d**2*e**2*x**2 - 2*a*b*d**3*e - 4*a*b*d**2*
e**2*x - 2*a*b*d*e**3*x**2 + b**2*d**2*e**2 + 2*b**2*d*e**3*x + b**2*e**4*
x**2))
```

3.204 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 175

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \frac{bp}{6d(ad-be)(d+ex)^2} + \frac{b(2ad-be)p}{3d^2(ad-be)^2(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} - \frac{p\log(x)}{3d^3e} + \frac{a^3p\log(b+ax)}{3e(ad-be)^3} - \frac{b(3a^2d^2-3abde+b^2e^2)\,p\log(d+ex)}{3d^3(ad-be)^3}$$

output
```
1/6*b*p/d/(a*d-b*e)/(e*x+d)^2+1/3*b*(2*a*d-b*e)*p/d^2/(a*d-b*e)^2/(e*x+d)-
1/3*ln(c*(a+b/x)^p)/e/(e*x+d)^3-1/3*p*ln(x)/d^3/e+1/3*a^3*p*ln(a*x+b)/e/(a
*d-b*e)^3-1/3*b*(3*a^2*d^2-3*a*b*d*e+b^2*e^2)*p*ln(e*x+d)/d^3/(a*d-b*e)^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx$$
$$= \frac{\frac{bep}{2d(ad-be)(d+ex)^2}+\frac{be(2ad-be)p}{d^2(ad-be)^2(d+ex)}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}-\frac{p\log(x)}{d^3}+\frac{a^3p\log(b+ax)}{(ad-be)^3}-\frac{be(3a^2d^2-3abde+b^2e^2)p\log(d+ex)}{d^3(ad-be)^3}}{3e}$$

input
```
Integrate[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```

output
```
((b*e*p)/(2*d*(a*d - b*e)*(d + e*x)^2) + (b*e*(2*a*d - b*e)*p)/(d^2*(a*d -
 b*e)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^3 - (p*Log[x])/d^3 + (a^
3*p*Log[b + a*x])/(a*d - b*e)^3 - (b*e*(3*a^2*d^2 - 3*a*b*d*e + b^2*e^2)*p
*Log[d + e*x])/(d^3*(a*d - b*e)^3))/(3*e)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2913, 1016, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx$$
$$\Big\downarrow \; 2913$$
$$-\frac{bp\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^3}dx}{3e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}$$
$$\Big\downarrow \; 1016$$
$$-\frac{bp\int \frac{1}{x(b+ax)(d+ex)^3}dx}{3e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{ 93}$$

$$-\frac{bp\int \left(\frac{a^4}{b(be-ad)^3(b+ax)}+\frac{1}{bd^3x}+\frac{e^2(3a^2d^2-3abed+b^2e^2)}{d^3(ad-be)^3(d+ex)}+\frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex)^2}+\frac{e^2}{d(ad-be)(d+ex)^3}\right)dx}{3e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{bp\left(-\frac{a^3\log(ax+b)}{b(ad-be)^3}+\frac{e(3a^2d^2-3abde+b^2e^2)\log(d+ex)}{d^3(ad-be)^3}-\frac{e(2ad-be)}{d^2(d+ex)(ad-be)^2}-\frac{e}{2d(d+ex)^2(ad-be)}+\frac{\log(x)}{bd^3}\right)}{3e}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}$$

input
```
Int[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```

output
```
-1/3*Log[c*(a + b/x)^p]/(e*(d + e*x)^3) - (b*p*(-1/2*e/(d*(a*d - b*e)*(d +
 e*x)^2) - (e*(2*a*d - b*e))/(d^2*(a*d - b*e)^2*(d + e*x)) + Log[x]/(b*d^3
) - (a^3*Log[b + a*x])/(b*(a*d - b*e)^3) + (e*(3*a^2*d^2 - 3*a*b*d*e + b^2
*e^2)*Log[d + e*x])/(d^3*(a*d - b*e)^3)))/(3*e)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{3e(ex+d)^{3}}-\frac{pb\left(-\frac{a^{3}\ln(ax+b)}{b(da-be)^{3}}-\frac{e}{2d(da-be)(ex+d)^{2}}-\frac{e(2da-be)}{d^{2}(da-be)^{2}(ex+d)}+\frac{e\left(3a^{2}d^{2}-3deab+b^{2}e^{2}\right)\ln(ex+d)}{d^{3}(da-be)^{3}}+\frac{\ln(x)}{b\,d^{3}}\right)}{3e}$
parallelrisch	$-\frac{3x^{3}b^{4}e^{5}p^{2}-6\ln(x)x^{2}b^{4}d\,e^{4}p^{2}+6\ln(ex+d)x^{2}b^{4}d\,e^{4}p^{2}-6\ln(x)x\,b^{4}d^{2}e^{3}p^{2}+6\ln(ex+d)x\,b^{4}d^{2}e^{3}p^{2}+6\ln(x)a\,b^{3}d^{4}e\,p^{2}-6\ln(}{}$

input
```
int(ln(c*(a+b/x)^p)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(c*(a+b/x)^p)/e/(e*x+d)^3-1/3*p*b/e*(-1/b*a^3/(a*d-b*e)^3*ln(a*x+b)
-1/2*e/d/(a*d-b*e)/(e*x+d)^2-e*(2*a*d-b*e)/d^2/(a*d-b*e)^2/(e*x+d)+e*(3*a^
2*d^2-3*a*b*d*e+b^2*e^2)/d^3/(a*d-b*e)^3*ln(e*x+d)+1/b/d^3*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(163) = 326.

Time = 3.47 (sec) , antiderivative size = 818, normalized size of antiderivative = 4.67

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{(d+ex)^{4}}\,dx = \text{Too large to display}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/6*(2*(2*a^2*b*d^3*e^3 - 3*a*b^2*d^2*e^4 + b^3*d*e^5)*p*x^2 + (9*a^2*b*d^
4*e^2 - 14*a*b^2*d^3*e^3 + 5*b^3*d^2*e^4)*p*x - 2*(a^3*d^6 - 3*a^2*b*d^5*e
 + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*p*log((a*x + b)/x) + (5*a^2*b*d^5*e - 8*
a*b^2*d^4*e^2 + 3*b^3*d^3*e^3)*p + 2*(a^3*d^3*e^3*p*x^3 + 3*a^3*d^4*e^2*p*
x^2 + 3*a^3*d^5*e*p*x + a^3*d^6*p)*log(a*x + b) - 2*((3*a^2*b*d^2*e^4 - 3*
a*b^2*d*e^5 + b^3*e^6)*p*x^3 + 3*(3*a^2*b*d^3*e^3 - 3*a*b^2*d^2*e^4 + b^3*
d*e^5)*p*x^2 + 3*(3*a^2*b*d^4*e^2 - 3*a*b^2*d^3*e^3 + b^3*d^2*e^4)*p*x + (
3*a^2*b*d^5*e - 3*a*b^2*d^4*e^2 + b^3*d^3*e^3)*p)*log(e*x + d) - 2*(a^3*d^
6 - 3*a^2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*log(c) - 2*((a^3*d^3*e^
3 - 3*a^2*b*d^2*e^4 + 3*a*b^2*d*e^5 - b^3*e^6)*p*x^3 + 3*(a^3*d^4*e^2 - 3*
a^2*b*d^3*e^3 + 3*a*b^2*d^2*e^4 - b^3*d*e^5)*p*x^2 + 3*(a^3*d^5*e - 3*a^2*
b*d^4*e^2 + 3*a*b^2*d^3*e^3 - b^3*d^2*e^4)*p*x + (a^3*d^6 - 3*a^2*b*d^5*e
 + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*p)*log(x))/(a^3*d^9*e - 3*a^2*b*d^8*e^2 +
 3*a*b^2*d^7*e^3 - b^3*d^6*e^4 + (a^3*d^6*e^4 - 3*a^2*b*d^5*e^5 + 3*a*b^2*
d^4*e^6 - b^3*d^3*e^7)*x^3 + 3*(a^3*d^7*e^3 - 3*a^2*b*d^6*e^4 + 3*a*b^2*d^
5*e^5 - b^3*d^4*e^6)*x^2 + 3*(a^3*d^8*e^2 - 3*a^2*b*d^7*e^3 + 3*a*b^2*d^6*
e^4 - b^3*d^5*e^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b/x)**p)/(e*x+d)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.71

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx$$
$$= \frac{\left(\frac{2a^3\log(ax+b)}{a^3bd^3-3a^2b^2d^2e+3ab^3de^2-b^4e^3} - \frac{2(3a^2d^2e-3abde^2+b^2e^3)\log(ex+d)}{a^3d^6-3a^2bd^5e+3ab^2d^4e^2-b^3d^3e^3} + \frac{5ad^2e-3bde^2+2(2ade^2-be^3)x}{a^2d^6-2abd^5e+b^2d^4e^2+(a^2d^4e^2-2abd^3e^3+b^2d^2e^4)x^2+2(a}\right.}{6e}$$
$$- \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{3(ex+d)^3e}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
1/6*(2*a^3*log(a*x + b)/(a^3*b*d^3 - 3*a^2*b^2*d^2*e + 3*a*b^3*d*e^2 - b^4
*e^3) - 2*(3*a^2*d^2*e - 3*a*b*d*e^2 + b^2*e^3)*log(e*x + d)/(a^3*d^6 - 3*
a^2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3) + (5*a*d^2*e - 3*b*d*e^2 + 2*
(2*a*d*e^2 - b*e^3)*x)/(a^2*d^6 - 2*a*b*d^5*e + b^2*d^4*e^2 + (a^2*d^4*e^2
 - 2*a*b*d^3*e^3 + b^2*d^2*e^4)*x^2 + 2*(a^2*d^5*e - 2*a*b*d^4*e^2 + b^2*d
^3*e^3)*x) - 2*log(x)/(b*d^3))*b*p/e - 1/3*log((a + b/x)^p*c)/((e*x + d)^3
*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(163) = 326$.

Time = 0.14 (sec) , antiderivative size = 1097, normalized size of antiderivative = 6.27

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="giac")
```

output

```
-1/6*(2*(3*a^2*b^2*d^2*p - 3*a*b^3*d*e*p + b^4*e^2*p)*log(-a*d + b*e + (a*
x + b)*d/x)/(a^3*d^6 - 3*a^2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3) + 2*
(3*a^2*b^2*d^2*p - 3*a*b^3*d*e*p + b^4*e^2*p - 6*(a*x + b)*a*b^2*d^2*p/x +
 3*(a*x + b)*b^3*d*e*p/x + 3*(a*x + b)^2*b^2*d^2*p/x^2)*log((a*x + b)/x)/(
a^3*d^6 - 3*a^2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3 - 3*(a*x + b)*a^2*
d^6/x + 6*(a*x + b)*a*b*d^5*e/x - 3*(a*x + b)*b^2*d^4*e^2/x + 3*(a*x + b)^
2*a*d^6/x^2 - 3*(a*x + b)^2*b*d^5*e/x^2 - (a*x + b)^3*d^6/x^3) - 2*(3*a^2*
b^2*d^2*p - 3*a*b^3*d*e*p + b^4*e^2*p)*log((a*x + b)/x)/(a^3*d^6 - 3*a^2*b
*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3) - (6*a^3*b^3*d^3*e*p - 15*a^2*b^4*
d^2*e^2*p + 12*a*b^5*d*e^3*p - 3*b^6*e^4*p - 6*a^4*b^2*d^4*log(c) + 18*a^3
*b^3*d^3*e*log(c) - 20*a^2*b^4*d^2*e^2*log(c) + 10*a*b^5*d*e^3*log(c) - 2*
b^6*e^4*log(c) - 12*(a*x + b)*a^2*b^3*d^3*e*p/x + 19*(a*x + b)*a*b^4*d^2*e
^2*p/x - 7*(a*x + b)*b^5*d*e^3*p/x + 12*(a*x + b)*a^3*b^2*d^4*log(c)/x - 3
0*(a*x + b)*a^2*b^3*d^3*e*log(c)/x + 24*(a*x + b)*a*b^4*d^2*e^2*log(c)/x -
 6*(a*x + b)*b^5*d*e^3*log(c)/x + 6*(a*x + b)^2*a*b^3*d^3*e*p/x^2 - 4*(a*x
 + b)^2*b^4*d^2*e^2*p/x^2 - 6*(a*x + b)^2*a^2*b^2*d^4*log(c)/x^2 + 12*(a*x
 + b)^2*a*b^3*d^3*e*log(c)/x^2 - 6*(a*x + b)^2*b^4*d^2*e^2*log(c)/x^2)/(a^
5*d^8 - 5*a^4*b*d^7*e + 10*a^3*b^2*d^6*e^2 - 10*a^2*b^3*d^5*e^3 + 5*a*b^4*
d^4*e^4 - b^5*d^3*e^5 - 3*(a*x + b)*a^4*d^8/x + 12*(a*x + b)*a^3*b*d^7*e/x
 - 18*(a*x + b)*a^2*b^2*d^6*e^2/x + 12*(a*x + b)*a*b^3*d^5*e^3/x - 3*(a...
```

Mupad [B] (verification not implemented)

Time = 27.22 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.78

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \frac{p\,\ln(d+e\,x)}{3\,d^3\,e} - \frac{3\,b^2\,e^2\,p}{2\,(3\,a^2\,d^5\,e+6\,a^2\,d^4\,e^2\,x+3\,a^2\,d^3\,e^3\,x^2-6\,a\,b\,d^4\,e^2-12\,a\,b\,d^3\,e^3\,x-6\,a\,b\,d^2\,e^4\,x^2+3\,b^2\,d^3\,e^3+6\,b^2\,d^2} - \frac{p\,\ln(x)}{3\,d^3\,e} - \frac{a^3\,p\,\ln(b+a\,x)}{-3\,a^3\,d^3\,e+9\,a^2\,b\,d^2\,e^2-9\,a\,b^2\,d\,e^3+3\,b^3\,e^4} - \frac{\ln\left(c\left(\frac{b+a\,x}{x}\right)^p\right)}{3\,(d^3\,e+3\,d^2\,e^2\,x+3\,d\,e^3\,x^2+e^4\,x^3)} - \frac{b^2\,e^3\,p\,x}{3\,a^2\,d^6\,e+6\,a^2\,d^5\,e^2\,x+3\,a^2\,d^4\,e^3\,x^2-6\,a\,b\,d^5\,e^2-12\,a\,b\,d^4\,e^3\,x-6\,a\,b\,d^3\,e^4\,x^2+3\,b^2\,d^4\,e^3+6\,b^2\,d^3\,e^4} - \frac{a^3\,d^3\,p\,\ln(d+e\,x)}{3\,a^3\,d^6\,e-9\,a^2\,b\,d^5\,e^2+9\,a\,b^2\,d^4\,e^3-3\,b^3\,d^3\,e^4} + \frac{5\,a\,b\,d\,e\,p}{2\,(3\,a^2\,d^5\,e+6\,a^2\,d^4\,e^2\,x+3\,a^2\,d^3\,e^3\,x^2-6\,a\,b\,d^4\,e^2-12\,a\,b\,d^3\,e^3\,x-6\,a\,b\,d^2\,e^4\,x^2+3\,b^2\,d^3\,e^3+6\,b^2\,d^2} + \frac{2\,a\,b\,d\,e^2\,p\,x}{3\,a^2\,d^6\,e+6\,a^2\,d^5\,e^2\,x+3\,a^2\,d^4\,e^3\,x^2-6\,a\,b\,d^5\,e^2-12\,a\,b\,d^4\,e^3\,x-6\,a\,b\,d^3\,e^4\,x^2+3\,b^2\,d^4\,e^3+6\,b^2\,d^3\,e^4}$$

input
```
int(log(c*(a + b/x)^p)/(d + e*x)^4,x)
```

output
```
(p*log(d + e*x))/(3*d^3*e) - (3*b^2*e^2*p)/(2*(3*a^2*d^5*e + 3*b^2*d^3*e^3
 + 6*a^2*d^4*e^2*x + 6*b^2*d^2*e^4*x + 3*b^2*d*e^5*x^2 + 3*a^2*d^3*e^3*x^2
 - 6*a*b*d^4*e^2 - 12*a*b*d^3*e^3*x - 6*a*b*d^2*e^4*x^2)) - (p*log(x))/(3*
d^3*e) - (a^3*p*log(b + a*x))/(3*b^3*e^4 - 3*a^3*d^3*e + 9*a^2*b*d^2*e^2 -
 9*a*b^2*d*e^3) - log(c*((b + a*x)/x)^p)/(3*(d^3*e + e^4*x^3 + 3*d^2*e^2*x
 + 3*d*e^3*x^2)) - (b^2*e^3*p*x)/(3*a^2*d^6*e + 3*b^2*d^4*e^3 + 6*a^2*d^5*
e^2*x + 6*b^2*d^3*e^4*x + 3*a^2*d^4*e^3*x^2 + 3*b^2*d^2*e^5*x^2 - 6*a*b*d^
5*e^2 - 12*a*b*d^4*e^3*x - 6*a*b*d^3*e^4*x^2) - (a^3*d^3*p*log(d + e*x))/(
3*a^3*d^6*e - 3*b^3*d^3*e^4 + 9*a*b^2*d^4*e^3 - 9*a^2*b*d^5*e^2) + (5*a*b*
d*e*p)/(2*(3*a^2*d^5*e + 3*b^2*d^3*e^3 + 6*a^2*d^4*e^2*x + 6*b^2*d^2*e^4*x
 + 3*b^2*d*e^5*x^2 + 3*a^2*d^3*e^3*x^2 - 6*a*b*d^4*e^2 - 12*a*b*d^3*e^3*x
- 6*a*b*d^2*e^4*x^2)) + (2*a*b*d*e^2*p*x)/(3*a^2*d^6*e + 3*b^2*d^4*e^3 + 6
*a^2*d^5*e^2*x + 6*b^2*d^3*e^4*x + 3*a^2*d^4*e^3*x^2 + 3*b^2*d^2*e^5*x^2 -
 6*a*b*d^5*e^2 - 12*a*b*d^4*e^3*x - 6*a*b*d^3*e^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 984, normalized size of antiderivative = 5.62

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
int(log(c*(a+b/x)^p)/(e*x+d)^4,x)
```

output
```
(6*log(a*x + b)*a**3*d**6*p + 18*log(a*x + b)*a**3*d**5*e*p*x + 18*log(a*x
 + b)*a**3*d**4*e**2*p*x**2 + 6*log(a*x + b)*a**3*d**3*e**3*p*x**3 - 18*lo
g(d + e*x)*a**2*b*d**5*e*p - 54*log(d + e*x)*a**2*b*d**4*e**2*p*x - 54*log
(d + e*x)*a**2*b*d**3*e**3*p*x**2 - 18*log(d + e*x)*a**2*b*d**2*e**4*p*x**
3 + 18*log(d + e*x)*a*b**2*d**4*e**2*p + 54*log(d + e*x)*a*b**2*d**3*e**3*
p*x + 54*log(d + e*x)*a*b**2*d**2*e**4*p*x**2 + 18*log(d + e*x)*a*b**2*d*e
**5*p*x**3 - 6*log(d + e*x)*b**3*d**3*e**3*p - 18*log(d + e*x)*b**3*d**2*e
**4*p*x - 18*log(d + e*x)*b**3*d*e**5*p*x**2 - 6*log(d + e*x)*b**3*e**6*p*
x**3 - 6*log(((a*x + b)**p*c)/x**p)*a**3*d**6 + 18*log(((a*x + b)**p*c)/x*
*p)*a**2*b*d**5*e - 18*log(((a*x + b)**p*c)/x**p)*a*b**2*d**4*e**2 + 6*log
(((a*x + b)**p*c)/x**p)*b**3*d**3*e**3 - 6*log(x)*a**3*d**6*p - 18*log(x)*
a**3*d**5*e*p*x - 18*log(x)*a**3*d**4*e**2*p*x**2 - 6*log(x)*a**3*d**3*e**
3*p*x**3 + 18*log(x)*a**2*b*d**5*e*p + 54*log(x)*a**2*b*d**4*e**2*p*x + 54
*log(x)*a**2*b*d**3*e**3*p*x**2 + 18*log(x)*a**2*b*d**2*e**4*p*x**3 - 18*l
og(x)*a*b**2*d**4*e**2*p - 54*log(x)*a*b**2*d**3*e**3*p*x - 54*log(x)*a*b*
*2*d**2*e**4*p*x**2 - 18*log(x)*a*b**2*d*e**5*p*x**3 + 6*log(x)*b**3*d**3*
e**3*p + 18*log(x)*b**3*d**2*e**4*p*x + 18*log(x)*b**3*d*e**5*p*x**2 + 6*l
og(x)*b**3*e**6*p*x**3 + 11*a**2*b*d**5*e*p + 15*a**2*b*d**4*e**2*p*x - 4*
a**2*b*d**2*e**4*p*x**3 - 18*a*b**2*d**4*e**2*p - 24*a*b**2*d**3*e**3*p*x
+ 6*a*b**2*d*e**5*p*x**3 + 7*b**3*d**3*e**3*p + 9*b**3*d**2*e**4*p*x - ...
```

3.205 $\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d} - \frac{\text{PolyLog}\left(2,\frac{a(c+dx)}{ac-bd}\right)}{d} + \frac{\text{PolyLog}\left(2,1+\frac{dx}{c}\right)}{d}$$

output
```
ln(a+b/x)*ln(d*x+c)/d+ln(-d*x/c)*ln(d*x+c)/d-ln(-d*(a*x+b)/(a*c-b*d))*ln(d
*x+c)/d-polylog(2,a*(d*x+c)/(a*c-b*d))/d+polylog(2,1+d*x/c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)+\log(x)\log(c+dx)-\log\left(\frac{b}{a}+x\right)\log(c+dx)+\log\left(\frac{b}{a}+x\right)\log\left(\frac{a(c+dx)}{ac-bd}\right)-\log}{d}$$

input
```
Integrate[Log[a + b/x]/(c + d*x),x]
```

output
```
(Log[a + b/x]*Log[c + d*x] + Log[x]*Log[c + d*x] - Log[b/a + x]*Log[c + d*
x] + Log[b/a + x]*Log[(a*(c + d*x))/(a*c - b*d)] - Log[x]*Log[1 + (d*x)/c]
 - PolyLog[2, -((d*x)/c)] + PolyLog[2, (d*(b + a*x))/(-(a*c) + b*d)])/d
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2912, 2005, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(a+\frac{b}{x}\right)}{c+dx} \, dx$$

$$\Big\downarrow \text{2912}$$

$$\frac{b \int \frac{\log (c+dx)}{\left(a+\frac{b}{x}\right) x^2}dx}{d}+\frac{\log \left(a+\frac{b}{x}\right) \log (c+dx)}{d}$$

$$\Big\downarrow \text{2005}$$

$$\frac{b \int \frac{\log (c+dx)}{x (b+a x)}dx}{d}+\frac{\log \left(a+\frac{b}{x}\right) \log (c+dx)}{d}$$

$$\Big\downarrow \text{2863}$$

$$\frac{b \int \left(\frac{\log (c+dx)}{b x}-\frac{a \log (c+dx)}{b (b+a x)}\right)dx}{d}+\frac{\log \left(a+\frac{b}{x}\right) \log (c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b \left(-\frac{\text{PolyLog}\left(2,\frac{a (c+dx)}{a c-b d}\right)}{b}-\frac{\log (c+dx) \log \left(-\frac{d (a x+b)}{a c-b d}\right)}{b}+\frac{\text{PolyLog}\left(2,\frac{d x}{c}+1\right)}{b}+\frac{\log \left(-\frac{d x}{c}\right) \log (c+dx)}{b}\right)}{d}+\frac{\log \left(a+\frac{b}{x}\right) \log (c+dx)}{d}$$

input
```
Int[Log[a + b/x]/(c + d*x),x]
```

output
```
(Log[a + b/x]*Log[c + d*x])/d + (b*((Log[-((d*x)/c)]*Log[c + d*x])/b - (Lo
g[-((d*(b + a*x))/(a*c - b*d))]*Log[c + d*x])/b - PolyLog[2, (a*(c + d*x))
/(a*c - b*d)]/b + PolyLog[2, 1 + (d*x)/c]/b))/d
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Maple [A] (verified)

Time = 6.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\right)}{d}-\frac{\operatorname{dilog}\left(-\frac{b}{ax}\right)}{d}+\frac{\operatorname{dilog}\left(\frac{-ac+bd+c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{d}+\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{-ac+bd+c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{d}$
derivativedivides	$-b\left(\frac{\operatorname{dilog}\left(-\frac{b}{ax}\right)+\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\right)}{db}-\frac{\left(\frac{\operatorname{dilog}\left(\frac{-ac+bd+c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{c}+\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{-ac+bd+c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{c}\right)c}{db}\right)$
default	$-b\left(\frac{\operatorname{dilog}\left(-\frac{b}{ax}\right)+\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\right)}{db}-\frac{\left(\frac{\operatorname{dilog}\left(\frac{-ac+bd+c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{c}+\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{-ac+bd+c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{c}\right)c}{db}\right)$
parts	$\frac{\ln\left(a+\frac{b}{x}\right)\ln(dx+c)}{d}+b\left(\frac{\operatorname{dilog}\left(-\frac{dx}{c}\right)+\ln(dx+c)\ln\left(-\frac{dx}{c}\right)}{bd}-\frac{\left(\frac{\operatorname{dilog}\left(\frac{-ac+a(dx+c)+bd}{-ac+bd}\right)}{a}+\frac{\ln(dx+c)\ln\left(\frac{-ac+a(dx\right.}{-ac+\right.}}{a}\right.}{bd}\right.$

input
```
int(ln(a+b/x)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d*ln(a+b/x)*ln(-b/a/x)-1/d*dilog(-b/a/x)+1/d*dilog((-a*c+b*d+c*(a+b/x))
/(-a*c+b*d))+1/d*ln(a+b/x)*ln((-a*c+b*d+c*(a+b/x))/(-a*c+b*d))
```

Fricas [F]

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx=\int \frac{\log\left(a+\frac{b}{x}\right)}{dx+c}\,dx$$

input
```
integrate(log(a+b/x)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(log((a*x + b)/x)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\log\left(a + \frac{b}{x}\right)}{c + dx}\, dx = \int \frac{\log\left(a + \frac{b}{x}\right)}{c + dx}\, dx$$

input
```
integrate(ln(a+b/x)/(d*x+c),x)
```

output
```
Integral(log(a + b/x)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

$$\int \frac{\log\left(a + \frac{b}{x}\right)}{c + dx}\, dx = -\frac{\log\left(\frac{dx}{c} + 1\right)\log(x) + \text{Li}_2\left(-\frac{dx}{c}\right)}{d} + \frac{\log(ax + b)\log\left(\frac{adx+bd}{ac-bd} + 1\right) + \text{Li}_2\left(-\frac{adx+bd}{ac-bd}\right)}{d}$$

input
```
integrate(log(a+b/x)/(d*x+c),x, algorithm="maxima")
```

output
```
-(log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(a*x + b)*log((a*d*x + b*
d)/(a*c - b*d) + 1) + dilog(-(a*d*x + b*d)/(a*c - b*d)))/d
```

Giac [F]

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(a+\frac{b}{x}\right)}{dx+c}\,dx$$

input
```
integrate(log(a+b/x)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log(a + b/x)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = \int \frac{\ln\left(a+\frac{b}{x}\right)}{c+d\,x}\,dx$$

input
```
int(log(a + b/x)/(c + d*x),x)
```

output
```
int(log(a + b/x)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{ax+b}{x}\right)}{dx+c}dx$$

input
```
int(log(a+b/x)/(d*x+c),x)
```

output
```
int(log((a*x + b)/x)/(c + d*x),x)
```

3.206 $\int (d+ex)^m \log \left(c \left(a+bx^3\right)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 301

$$\begin{aligned}\int (d+ex)^m \log \left(c \left(a+bx^3\right)^p\right) \, dx &= \frac{\sqrt[3]{b}p(d+ex)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)(1+m)(2+m)} \\ &+ \frac{\sqrt[3]{b}p(d+ex)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e\right)(1+m)(2+m)} \\ &+ \frac{\sqrt[3]{b}p(d+ex)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e\right)(1+m)(2+m)} \\ &+ \frac{(d+ex)^{1+m} \log \left(c \left(a+bx^3\right)^p\right)}{e(1+m)}\end{aligned}$$

output

```
b^(1/3)*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*
d-a^(1/3)*e))/e/(b^(1/3)*d-a^(1/3)*e)/(1+m)/(2+m)+b^(1/3)*p*(e*x+d)^(2+m)*
hypergeom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))
/e/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e)/(1+m)/(2+m)+b^(1/3)*p*(e*x+d)^(2+m)*hy
pergeom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e
/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e)/(1+m)/(2+m)+(e*x+d)^(1+m)*ln(c*(b*x^3+a)
^p)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.79

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right) \, dx$$

$$= \frac{(d+ex)^{1+m}\left(-\frac{\sqrt[3]{b}p(d+ex)\left(-\frac{\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}-\frac{\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right.}{2+m}\right.}{e(1+m)}$$

input

```
Integrate[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
```

output

```
((d + e*x)^(1 + m)*(-((b^(1/3)*p*(d + e*x)*(-(Hypergeometric2F1[1, 2 + m,
3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)]/(b^(1/3)*d - a^(1/3)*e
)) - Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (
-1)^(1/3)*a^(1/3)*e)]/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e) - Hypergeometric2
F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)
]/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)))/(2 + m)) + Log[c*(a + b*x^3)^p]))/(
e*(1 + m))
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2913, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right) dx$$

$$\Big\downarrow \text{2913}$$

$$\frac{(d+ex)^{m+1}\log \left(c\left(a+bx^3\right)^p\right)}{e(m+1)}-\frac{3bp\int \frac{x^2(d+ex)^{m+1}}{bx^3+a}dx}{e(m+1)}$$

$$\Big\downarrow \text{7276}$$

$$\frac{(d+ex)^{m+1}\log \left(c\left(a+bx^3\right)^p\right)}{e(m+1)}-\frac{3bp\int \left(\frac{(d+ex)^{m+1}}{3b^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}+\frac{(d+ex)^{m+1}}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{(d+ex)^{m+1}}{3b^{2/3}\left(\sqrt[3]{b}x+(-1)^{2/3}\sqrt[3]{a}\right)}\right)dx}{e(m+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+1}\log \left(c\left(a+bx^3\right)^p\right)}{e(m+1)}-\frac{3bp\left(-\frac{(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{3b^{2/3}(m+2)\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)}-\frac{(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{3b^{2/3}(m+2)\left(\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d\right)}\right.}{e(m+1)}$$

input
```
Int[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
```

output
```
(-3*b*p*(-1/3*((d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/
3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/(b^(2/3)*(b^(1/3)*d - a^(1/3)*e)*(
2 + m)) - ((d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(
d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/(3*b^(2/3)*(b^(1/3)*d + (-1
)^(1/3)*a^(1/3)*e)*(2 + m)) - ((d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 +
m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/(3*b^(2
/3)*(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)*(2 + m))))/(e*(1 + m)) + ((d + e*x)
^(1 + m)*Log[c*(a + b*x^3)^p])/(e*(1 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int (ex+d)^m \ln\left(c\left(b\,x^3+a\right)^p\right) dx$$

input
```
int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
```

output
```
int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log\left(c\left(a+bx^3\right)^p\right)\,dx = \int (ex+d)^m \log\left(\left(bx^3+a\right)^p c\right)\,dx$$

input
```
integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m*log((b*x^3 + a)^p*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \log\left(c\left(a+bx^3\right)^p\right)\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*ln(c*(b*x**3+a)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m \log\left(c\left(a+bx^3\right)^p\right)\,dx = \int (ex+d)^m \log\left(\left(bx^3+a\right)^p c\right)\,dx$$

input
```
integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

output
```
(e*x + d)*(e*x + d)^m*log((b*x^3 + a)^p)/(e*(m + 1)) + integrate(-(3*b*d*p
*x^2 - (e*(m + 1)*log(c) - 3*e*p)*b*x^3 - a*e*(m + 1)*log(c))*(e*x + d)^m/
(b*e*(m + 1)*x^3 + a*e*(m + 1)), x)
```

Giac [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right) \, dx = \int (ex+d)^m \log \left(\left(bx^3+a\right)^p c\right) \, dx$$

input
```
integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m*log((b*x^3 + a)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right) \, dx = \int \ln \left(c\,\left(b\,x^3+a\right)^p\right)\,\left(d+e\,x\right)^m \,dx$$

input
```
int(log(c*(a + b*x^3)^p)*(d + e*x)^m,x)
```

output
```
int(log(c*(a + b*x^3)^p)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$= \frac{(ex+d)^m \log\left(\left(b\,x^3+a\right)^p c\right) d\,m^2 + (ex+d)^m \log\left(\left(b\,x^3+a\right)^p c\right) dm + (ex+d)^m \log\left(\left(b\,x^3+a\right)^p c\right) e\,m^2 x}{}$$

input
```
int((e*x+d)^m*log(c*(b*x^3+a)^p),x)
```

output
```
((d + e*x)**m*log((a + b*x**3)**p*c)*d*m**2 + (d + e*x)**m*log((a + b*x**3
)**p*c)*d*m + (d + e*x)**m*log((a + b*x**3)**p*c)*e*m**2*x + (d + e*x)**m*
log((a + b*x**3)**p*c)*e*m*x - 6*(d + e*x)**m*d*m*p - 3*(d + e*x)**m*d*p -
 3*(d + e*x)**m*e*m*p*x + 6*int((d + e*x)**m/(a*d*m + a*d + a*e*m*x + a*e*
x + b*d*m*x**3 + b*d*x**3 + b*e*m*x**4 + b*e*x**4),x)*a*d*e*m**3*p + 12*in
t((d + e*x)**m/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x**3 + b*d*x**3 + b*
e*m*x**4 + b*e*x**4),x)*a*d*e*m**2*p + 6*int((d + e*x)**m/(a*d*m + a*d + a
*e*m*x + a*e*x + b*d*m*x**3 + b*d*x**3 + b*e*m*x**4 + b*e*x**4),x)*a*d*e*m
*p - 3*int(((d + e*x)**m*x**2)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x**3
 + b*d*x**3 + b*e*m*x**4 + b*e*x**4),x)*b*d**2*m**3*p - 6*int(((d + e*x)**
m*x**2)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x**3 + b*d*x**3 + b*e*m*x**
4 + b*e*x**4),x)*b*d**2*m**2*p - 3*int(((d + e*x)**m*x**2)/(a*d*m + a*d +
a*e*m*x + a*e*x + b*d*m*x**3 + b*d*x**3 + b*e*m*x**4 + b*e*x**4),x)*b*d**2
*m*p + 3*int(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x**3
+ b*d*x**3 + b*e*m*x**4 + b*e*x**4),x)*a*e**2*m**3*p + 6*int(((d + e*x)**m
*x)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x**3 + b*d*x**3 + b*e*m*x**4 +
b*e*x**4),x)*a*e**2*m**2*p + 3*int(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x
 + a*e*x + b*d*m*x**3 + b*d*x**3 + b*e*m*x**4 + b*e*x**4),x)*a*e**2*m*p)/(
e*m*(m**2 + 2*m + 1))
```

3.207 $\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 205

$$\begin{aligned}\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{\sqrt{b}p(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e\left(\sqrt{b}d-\sqrt{-a}e\right)(1+m)(2+m)} \\ &\quad+\frac{\sqrt{b}p(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e\left(\sqrt{b}d+\sqrt{-a}e\right)(1+m)(2+m)} \\ &\quad+\frac{(d+ex)^{1+m}\log\left(c\left(a+bx^2\right)^p\right)}{e(1+m)}\end{aligned}$$

output
```
b^(1/2)*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],b^(1/2)*(e*x+d)/(b^(1/2)*
d-(-a)^(1/2)*e))/e/(b^(1/2)*d-(-a)^(1/2)*e)/(1+m)/(2+m)+b^(1/2)*p*(e*x+d)^
(2+m)*hypergeom([1, 2+m],[3+m],b^(1/2)*(e*x+d)/(b^(1/2)*d+(-a)^(1/2)*e))/e
/(b^(1/2)*d+(-a)^(1/2)*e)/(1+m)/(2+m)+(e*x+d)^(1+m)*ln(c*(b*x^2+a)^p)/e/(1
+m)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.86

$$\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{\sqrt{b}p(d+ex)\left(\left(\sqrt{b}d+\sqrt{-a}e\right)\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\left(\sqrt{b}d-\sqrt{-a}e\right)\text{Hypergeometric2F1}\left(1,2+m,3\right.\right.}{(bd^2+ae^2)(2+m)}}{e(1+m)}$$

input
```
Integrate[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

output
```
((d + e*x)^(1 + m)*((Sqrt[b]*p*(d + e*x)*((Sqrt[b]*d + Sqrt[-a]*e)*Hyperge
ometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]
+ (Sqrt[b]*d - Sqrt[-a]*e)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d
+ e*x))/(Sqrt[b]*d + Sqrt[-a]*e)]))/((b*d^2 + a*e^2)*(2 + m)) + Log[c*(a +
 b*x^2)^p]))/(e*(1 + m))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2913, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$\Big\downarrow \text{2913}$$
$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^2\right)^p\right)}{e(m+1)}-\frac{2bp\int \frac{x(d+ex)^{m+1}}{bx^2+a}dx}{e(m+1)}$$
$$\Big\downarrow \text{615}$$

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^2\right)^p\right)}{e(m+1)}-\frac{2bp\int\left(\frac{(d+ex)^{m+1}}{2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{(d+ex)^{m+1}}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx}{e(m+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+bx^2\right)^p\right)}{e(m+1)}-\frac{2bp\left(-\frac{(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{2\sqrt{b}(m+2)\left(\sqrt{b}d-\sqrt{-a}e\right)}-\frac{(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{2\sqrt{b}(m+2)\left(\sqrt{-a}e+\sqrt{b}d\right)}\right)}{e(m+1)}$$

input
```
Int[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

output
```
(-2*b*p*(-1/2*((d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[
b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/(Sqrt[b]*(Sqrt[b]*d - Sqrt[-a]*e)
*(2 + m)) - ((d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]
*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/(2*Sqrt[b]*(Sqrt[b]*d + Sqrt[-a]*e)
*(2 + m))))/(e*(1 + m)) + ((d + e*x)^(1 + m)*Log[c*(a + b*x^2)^p])/(e*(1 +
 m))
```

Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [F]

$$\int (ex+d)^m \ln\left(c\left(b\,x^2+a\right)^p\right) dx$$

input
```
int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

output
```
int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)\,dx = \int (ex+d)^m \log\left(\left(bx^2+a\right)^p c\right)\,dx$$

input
```
integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m*log((b*x^2 + a)^p*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \log\left(c\left(a+bx^2\right)^p\right)\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*ln(c*(b*x**2+a)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^2\right)^p\right) \, dx = \int (ex+d)^m \log \left(\left(bx^2+a\right)^p c\right) \, dx$$

input
```
integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

output
```
(e*p*x + d*p)*(e*x + d)^m*log(b*x^2 + a)/(e*(m + 1)) + integrate(-(2*b*d*p
*x - (e*(m + 1)*log(c) - 2*e*p)*b*x^2 - a*e*(m + 1)*log(c))*(e*x + d)^m/(b
*e*(m + 1)*x^2 + a*e*(m + 1)), x)
```

Giac [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^2\right)^p\right) \, dx = \int (ex+d)^m \log \left(\left(bx^2+a\right)^p c\right) \, dx$$

input
```
integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m*log((b*x^2 + a)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c\left(a+bx^2\right)^p\right) \, dx = \int \ln \left(c\left(b\,x^2+a\right)^p\right) \left(d+e\,x\right)^m \, dx$$

input
```
int(log(c*(a + b*x^2)^p)*(d + e*x)^m,x)
```

output
```
int(log(c*(a + b*x^2)^p)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$= \frac{(ex+d)^m \log\left(\left(b\,x^2+a\right)^p c\right) d\,m^2 + (ex+d)^m \log\left(\left(b\,x^2+a\right)^p c\right) dm + (ex+d)^m \log\left(\left(b\,x^2+a\right)^p c\right) e\,m^2 x}{}$$

input
```
int((e*x+d)^m*log(c*(b*x^2+a)^p),x)
```

output
```
((d + e*x)**m*log((a + b*x**2)**p*c)*d*m**2 + (d + e*x)**m*log((a + b*x**2
)**p*c)*d*m + (d + e*x)**m*log((a + b*x**2)**p*c)*e*m**2*x + (d + e*x)**m*
log((a + b*x**2)**p*c)*e*m*x - 4*(d + e*x)**m*d*m*p - 2*(d + e*x)**m*d*p -
 2*(d + e*x)**m*e*m*p*x + 4*int((d + e*x)**m/(a*d*m + a*d + a*e*m*x + a*e*
x + b*d*m*x**2 + b*d*x**2 + b*e*m*x**3 + b*e*x**3),x)*a*d*e*m**3*p + 8*int
((d + e*x)**m/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x**2 + b*d*x**2 + b*e
*m*x**3 + b*e*x**3),x)*a*d*e*m**2*p + 4*int((d + e*x)**m/(a*d*m + a*d + a*
e*m*x + a*e*x + b*d*m*x**2 + b*d*x**2 + b*e*m*x**3 + b*e*x**3),x)*a*d*e*m*
p + 2*int(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x**2 + b
*d*x**2 + b*e*m*x**3 + b*e*x**3),x)*a*e**2*m**3*p + 4*int(((d + e*x)**m*x)
/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x**2 + b*d*x**2 + b*e*m*x**3 + b*e
*x**3),x)*a*e**2*m**2*p + 2*int(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x +
a*e*x + b*d*m*x**2 + b*d*x**2 + b*e*m*x**3 + b*e*x**3),x)*a*e**2*m*p - 2*i
nt(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x**2 + b*d*x**2
 + b*e*m*x**3 + b*e*x**3),x)*b*d**2*m**3*p - 4*int(((d + e*x)**m*x)/(a*d*m
 + a*d + a*e*m*x + a*e*x + b*d*m*x**2 + b*d*x**2 + b*e*m*x**3 + b*e*x**3),
x)*b*d**2*m**2*p - 2*int(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x + a*e*x +
 b*d*m*x**2 + b*d*x**2 + b*e*m*x**3 + b*e*x**3),x)*b*d**2*m*p)/(e*m*(m**2
+ 2*m + 1))
```

3.208 $\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \frac{bp(d+ex)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{b(d+ex)}{bd-ae}\right)}{e(bd-ae)(1+m)(2+m)} + \frac{(d+ex)^{1+m} \log \left(c(a+bx)^p\right)}{e(1+m)}$$

output
```
b*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],b*(e*x+d)/(-a*e+b*d))/e/(-a*e+b
*d)/(1+m)/(2+m)+(e*x+d)^(1+m)*ln(c*(b*x+a)^p)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \frac{(d+ex)^{1+m} \left(\frac{bp(d+ex) \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)(2+m)} + \log \left(c(a+bx)^p\right)\right)}{e(1+m)}$$

input
```
Integrate[(d + e*x)^m*Log[c*(a + b*x)^p],x]
```

output
```
((d + e*x)^(1 + m)*((b*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(
d + e*x))/(b*d - a*e)])/((b*d - a*e)*(2 + m)) + Log[c*(a + b*x)^p]))/(e*(1
 + m))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2842, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \log\left(c(a+bx)^p\right) dx$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{(d+ex)^{m+1}\log\left(c(a+bx)^p\right)}{e(m+1)} - \frac{bp\int \frac{(d+ex)^{m+1}}{a+bx}dx}{e(m+1)}$$

$$\Big\downarrow \text{ 78}$$

$$\frac{(d+ex)^{m+1}\log\left(c(a+bx)^p\right)}{e(m+1)} + \frac{bp(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)(m+2)(bd-ae)}$$

input
```
Int[(d + e*x)^m*Log[c*(a + b*x)^p],x]
```

output
```
(b*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d + e*x))/(b
*d - a*e)])/(e*(b*d - a*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a
+ b*x)^p])/(e*(1 + m))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

Maple [F]

$$\int (ex+d)^m \ln \left(c(bx+a)^p\right) dx$$

input
```
int((e*x+d)^m*ln(c*(b*x+a)^p),x)
```

output
```
int((e*x+d)^m*ln(c*(b*x+a)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log \left(c(a+bx)^p\right)\, dx = \int (ex+d)^m \log \left((bx+a)^p c\right)\, dx$$

input
```
integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m*log((b*x + a)^p*c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*ln(c*(b*x+a)**p),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \int (ex+d)^m \log \left((bx+a)^p c\right) \, dx$$

input
```
integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="maxima")
```

output
```
(e*x + d)*(e*x + d)^m*log((b*x + a)^p)/(e*(m + 1)) + integrate((a*e*(m + 1
)*log(c) - b*d*p + (e*(m + 1)*log(c) - e*p)*b*x)*(e*x + d)^m/(b*e*(m + 1)*
x + a*e*(m + 1)), x)
```

Giac [F]

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \int (ex+d)^m \log \left((bx+a)^p c\right) \, dx$$

input
```
integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m*log((b*x + a)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \int \ln \left(c\,{\left(a+b\,x\right)}^p\right)\,{\left(d+e\,x\right)}^m \,dx$$

input
```
int(log(c*(a + b*x)^p)*(d + e*x)^m,x)
```

output
```
int(log(c*(a + b*x)^p)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx$$
$$= \frac{(ex+d)^m \log((bx+a)^p\, c)\, ade\, m^2 + (ex+d)^m \log((bx+a)^p\, c)\, adem + (ex+d)^m \log((bx+a)^p\, c)\, a\, e^2 m}{}$$

input
```
int((e*x+d)^m*log(c*(b*x+a)^p),x)
```

output
```
((d + e*x)**m*log((a + b*x)**p*c)*a*d*e*m**2 + (d + e*x)**m*log((a + b*x)*
*p*c)*a*d*e*m + (d + e*x)**m*log((a + b*x)**p*c)*a*e**2*m**2*x + (d + e*x)
**m*log((a + b*x)**p*c)*a*e**2*m*x + (d + e*x)**m*a*d*e*p - (d + e*x)**m*a
*e**2*m*p*x - (d + e*x)**m*b*d**2*m*p - (d + e*x)**m*b*d**2*p + int(((d +
e*x)**m*x)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x + b*d*x + b*e*m*x**2 +
 b*e*x**2),x)*a**2*e**3*m**3*p + 2*int(((d + e*x)**m*x)/(a*d*m + a*d + a*e
*m*x + a*e*x + b*d*m*x + b*d*x + b*e*m*x**2 + b*e*x**2),x)*a**2*e**3*m**2*
p + int(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x + b*d*x
+ b*e*m*x**2 + b*e*x**2),x)*a**2*e**3*m*p - 2*int(((d + e*x)**m*x)/(a*d*m
+ a*d + a*e*m*x + a*e*x + b*d*m*x + b*d*x + b*e*m*x**2 + b*e*x**2),x)*a*b*
d*e**2*m**3*p - 4*int(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x + a*e*x + b*
d*m*x + b*d*x + b*e*m*x**2 + b*e*x**2),x)*a*b*d*e**2*m**2*p - 2*int(((d +
e*x)**m*x)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x + b*d*x + b*e*m*x**2 +
 b*e*x**2),x)*a*b*d*e**2*m*p + int(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x
 + a*e*x + b*d*m*x + b*d*x + b*e*m*x**2 + b*e*x**2),x)*b**2*d**2*e*m**3*p
+ 2*int(((d + e*x)**m*x)/(a*d*m + a*d + a*e*m*x + a*e*x + b*d*m*x + b*d*x
+ b*e*m*x**2 + b*e*x**2),x)*b**2*d**2*e*m**2*p + int(((d + e*x)**m*x)/(a*d
*m + a*d + a*e*m*x + a*e*x + b*d*m*x + b*d*x + b*e*m*x**2 + b*e*x**2),x)*b
**2*d**2*e*m*p)/(a*e**2*m*(m**2 + 2*m + 1))
```

3.209 $\int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\begin{aligned}\int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx &= \frac{ap(d+ex)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{a(d+ex)}{ad-be}\right)}{e(ad-be)(1+m)(2+m)} \\ &\quad - \frac{p(d+ex)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)} \\ &\quad + \frac{(d+ex)^{1+m} \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}\end{aligned}$$

output
```
a*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],a*(e*x+d)/(a*d-b*e))/e/(a*d-b*e
)/(1+m)/(2+m)-p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],1+e*x/d)/d/e/(m^2+3
*m+2)+(e*x+d)^(1+m)*ln(c*(a+b/x)^p)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.91

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$= \frac{(d+ex)^{1+m}\left(-adp(d+ex)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{a(d+ex)}{ad-be}\right)+(ad-be)\left(p(d+ex)\,\text{Hyp}\right.\right.}{de(-ad+be)(1+m)(2+m)}$$

input
```
Integrate[(d + e*x)^m*Log[c*(a + b/x)^p],x]
```

output
```
((d + e*x)^(1 + m)*(-(a*d*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (
a*(d + e*x))/(a*d - b*e)]) + (a*d - b*e)*(p*(d + e*x)*Hypergeometric2F1[1,
 2 + m, 3 + m, 1 + (e*x)/d] - d*(2 + m)*Log[c*(a + b/x)^p])))/(d*e*(-(a*d)
 + b*e)*(1 + m)*(2 + m))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2913, 1016, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$\Big\downarrow \text{2913}$$
$$\frac{bp\int \frac{(d+ex)^{m+1}}{\left(a+\frac{b}{x}\right)x^2}dx}{e(m+1)}+\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}$$
$$\Big\downarrow \text{1016}$$
$$\frac{bp\int \frac{(d+ex)^{m+1}}{x(b+ax)}dx}{e(m+1)}+\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}$$

$$\Big\downarrow \textbf{97}$$

$$\frac{bp\left(\frac{\int \frac{(d+ex)^{m+1}}{x}dx}{b}-\frac{a\int \frac{(d+ex)^{m+1}}{b+ax}dx}{b}\right)}{e(m+1)}+\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}$$

$$\Big\downarrow \textbf{75}$$

$$\frac{bp\left(-\frac{a\int \frac{(d+ex)^{m+1}}{b+ax}dx}{b}-\frac{(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{ex}{d}+1\right)}{bd(m+2)}\right)}{e(m+1)}+\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}$$

$$\Big\downarrow \textbf{78}$$

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)}+\frac{bp\left(\frac{a(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{a(d+ex)}{ad-be}\right)}{b(m+2)(ad-be)}-\frac{(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{ex}{d}+1\right)}{bd(m+2)}\right)}{e(m+1)}$$

input
```
Int[(d + e*x)^m*Log[c*(a + b/x)^p],x]
```

output
```
(b*p*((a*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (a*(d + e*x)
)/(a*d - b*e)])/(b*(a*d - b*e)*(2 + m)) - ((d + e*x)^(2 + m)*Hypergeometri
c2F1[1, 2 + m, 3 + m, 1 + (e*x)/d])/(b*d*(2 + m))))/(e*(1 + m)) + ((d + e*
x)^(1 + m)*Log[c*(a + b/x)^p])/(e*(1 + m))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] && !IntegerQ[p]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [F]

$$\int (ex+d)^m \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

input
```
int((e*x+d)^m*ln(c*(a+b/x)^p),x)
```

output
```
int((e*x+d)^m*ln(c*(a+b/x)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \int (ex+d)^m \log \left(\left(a+\frac{b}{x}\right)^p c\right) dx$$

input
```
integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m*log(c*((a*x + b)/x)^p), x)
```

Sympy [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

input
```
integrate((e*x+d)**m*ln(c*(a+b/x)**p),x)
```

output
```
Integral((d + e*x)**m*log(c*(a + b/x)**p), x)
```

Maxima [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \int (ex+d)^m \log \left(\left(a+\frac{b}{x}\right)^p c\right) dx$$

input
```
integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="maxima")
```

output
```
(e*x + d)*(e*x + d)^m*log((a*x + b)^p)/(e*(m + 1)) - integrate(-(b*e*(m +
1)*log(c) - a*d*p + (e*(m + 1)*log(c) - e*p)*a*x - (a*e*(m + 1)*x + b*e*(m
 + 1))*log(x^p))*(e*x + d)^m/(a*e*(m + 1)*x + b*e*(m + 1)), x)
```

Giac [F]

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \int (ex+d)^m \log\left(\left(a+\frac{b}{x}\right)^p c\right)\,dx$$

input
```
integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m*log((a + b/x)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \int \ln\left(c\left(a+\frac{b}{x}\right)^p\right)(d+e\,x)^m\,dx$$

input
```
int(log(c*(a + b/x)^p)*(d + e*x)^m,x)
```

output
```
int(log(c*(a + b/x)^p)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$= \frac{(ex+d)^m \log\left(\frac{(ax+b)^p c}{x^p}\right) dm + (ex+d)^m \log\left(\frac{(ax+b)^p c}{x^p}\right) emx + 2(ex+d)^m dp + \left(\int \frac{(ex\cdots}{aem\,x^3+adm\,x^2+ae\,x^3+bem\cdots}\right.}{}$$

input
```
int((e*x+d)^m*log(c*(a+b/x)^p),x)
```

output
```
((d + e*x)**m*log(((a*x + b)**p*c)/x**p)*d*m + (d + e*x)**m*log(((a*x + b)
**p*c)/x**p)*e*m*x + 2*(d + e*x)**m*d*p + int((d + e*x)**m/(a*d*m*x**2 + a
*d*x**2 + a*e*m*x**3 + a*e*x**3 + b*d*m*x + b*d*x + b*e*m*x**2 + b*e*x**2)
,x)*b*d**2*m**2*p + int((d + e*x)**m/(a*d*m*x**2 + a*d*x**2 + a*e*m*x**3 +
 a*e*x**3 + b*d*m*x + b*d*x + b*e*m*x**2 + b*e*x**2),x)*b*d**2*m*p - 2*int
(((d + e*x)**m*x)/(a*d*m*x + a*d*x + a*e*m*x**2 + a*e*x**2 + b*d*m + b*d +
 b*e*m*x + b*e*x),x)*a*d*e*m**2*p - 2*int(((d + e*x)**m*x)/(a*d*m*x + a*d*
x + a*e*m*x**2 + a*e*x**2 + b*d*m + b*d + b*e*m*x + b*e*x),x)*a*d*e*m*p +
int(((d + e*x)**m*x)/(a*d*m*x + a*d*x + a*e*m*x**2 + a*e*x**2 + b*d*m + b*
d + b*e*m*x + b*e*x),x)*b*e**2*m**2*p + int(((d + e*x)**m*x)/(a*d*m*x + a*
d*x + a*e*m*x**2 + a*e*x**2 + b*d*m + b*d + b*e*m*x + b*e*x),x)*b*e**2*m*p
)/(e*m*(m + 1))
```

3.210 $\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 257

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{\sqrt{-a}p(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e\left(\sqrt{-a}d-\sqrt{b}e\right)(1+m)(2+m)} + \frac{\sqrt{-a}p(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e\left(\sqrt{-a}d+\sqrt{b}e\right)(1+m)(2+m)} - \frac{2p(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)} + \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}$$

output
```
(-a)^(1/2)*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],(-a)^(1/2)*(e*x+d)/((-
a)^(1/2)*d-b^(1/2)*e))/e/((-a)^(1/2)*d-b^(1/2)*e)/(1+m)/(2+m)+(-a)^(1/2)*p
*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],(-a)^(1/2)*(e*x+d)/((-a)^(1/2)*d+b
^(1/2)*e))/e/((-a)^(1/2)*d+b^(1/2)*e)/(1+m)/(2+m)-2*p*(e*x+d)^(2+m)*hyperg
eom([1, 2+m],[3+m],1+e*x/d)/d/e/(m^2+3*m+2)+(e*x+d)^(1+m)*ln(c*(a+b/x^2)^p
)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.82

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{p(d+ex)\left(d\left(ad-\sqrt{-a}\sqrt{b}e\right)\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)+d\left(ad+\sqrt{-a}\sqrt{b}e\right)\text{Hypergeometric2F1}\left(1,2+m\right.}{d(ad^2+be^2)(2+m)}\right.}{e(1+m)}$$

input
```
Integrate[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

output
```
((d + e*x)^(1 + m)*((p*(d + e*x)*(d*(a*d - Sqrt[-a]*Sqrt[b]*e)*Hypergeomet
ric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] + d
*(a*d + Sqrt[-a]*Sqrt[b]*e)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*(
d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 2*(a*d^2 + b*e^2)*Hypergeometric2F1[
1, 2 + m, 3 + m, 1 + (e*x)/d]))/(d*(a*d^2 + b*e^2)*(2 + m)) + Log[c*(a + b
/x^2)^p]))/(e*(1 + m))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2913, 1894, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

$$\Big\downarrow \ 2913$$

$$\frac{2bp\int \frac{(d+ex)^{m+1}}{\left(a+\frac{b}{x^2}\right)x^3}dx}{e(m+1)}+\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)}$$

$$\Big\downarrow \ 1894$$

$$\frac{2bp\int \frac{(d+ex)^{m+1}}{x(ax^2+b)}dx}{e(m+1)}+\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)}$$

$\big\downarrow$ 615

$$\frac{2bp\int\left(\frac{(d+ex)^{m+1}}{bx}-\frac{ax(d+ex)^{m+1}}{b(ax^2+b)}\right)dx}{e(m+1)}+\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)}$$

$\big\downarrow$ 2009

$$\frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)}+$$
$$\frac{2bp\left(\frac{\sqrt{-a}(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{2b(m+2)\left(\sqrt{-a}d-\sqrt{b}e\right)}+\frac{\sqrt{-a}(d+ex)^{m+2}\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{2b(m+2)\left(\sqrt{-a}d+\sqrt{b}e\right)}-\right.}{e(m+1)}$$

input
```
Int[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

output
```
(2*b*p*((Sqrt[-a]*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sq
rt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/(2*b*(Sqrt[-a]*d - Sqrt[b]*e)
*(2 + m)) + (Sqrt[-a]*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m,
 (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/(2*b*(Sqrt[-a]*d + Sqrt[b
]*e)*(2 + m)) - ((d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, 1 +
(e*x)/d])/(b*d*(2 + m))))/(e*(1 + m)) + ((d + e*x)^(1 + m)*Log[c*(a + b/x^
2)^p])/(e*(1 + m))
```

Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [F]

$$\int (ex+d)^m \ln \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$

input
```
int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

output
```
int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \int (ex+d)^m \log \left(\left(a+\frac{b}{x^2}\right)^p c\right) dx$$

input
```
integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m*log(c*((a*x^2 + b)/x^2)^p), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)**m*ln(c*(a+b/x**2)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\int (ex+d)^m \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)\,dx$$

input
```
integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

output
```
(e*p*x + d*p)*(e*x + d)^m*log(a*x^2 + b)/(e*(m + 1)) - integrate((2*a*d*p*
x - (e*(m + 1)*log(c) - 2*e*p)*a*x^2 - b*e*(m + 1)*log(c) + 2*(a*e*(m + 1)
*x^2 + b*e*(m + 1))*log(x^p))*(e*x + d)^m/(a*e*(m + 1)*x^2 + b*e*(m + 1)),
 x)
```

Giac [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\int (ex+d)^m \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)\,dx$$

input
```
integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m*log((a + b/x^2)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \int \ln \left(c\left(a+\frac{b}{x^2}\right)^p\right) (d+e\,x)^m\, dx$$

input
```
int(log(c*(a + b/x^2)^p)*(d + e*x)^m,x)
```

output
```
int(log(c*(a + b/x^2)^p)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$
$$= \frac{(ex+d)^m \log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right) d + (ex+d)^m \log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right) ex + 2\left(\int \frac{(ex+d)^m}{am\,x^3+a\,x^3+bmx+bx}dx\right) bdmp + 2\left(\int \frac{}{am\,x^3}\right.}{e\,(m+1)}$$

input
```
int((e*x+d)^m*log(c*(a+b/x^2)^p),x)
```

output
```
((d + e*x)**m*log(((a*x**2 + b)**p*c)/x**(2*p))*d + (d + e*x)**m*log(((a*x
**2 + b)**p*c)/x**(2*p))*e*x + 2*int((d + e*x)**m/(a*m*x**3 + a*x**3 + b*m
*x + b*x),x)*b*d*m*p + 2*int((d + e*x)**m/(a*m*x**3 + a*x**3 + b*m*x + b*x
),x)*b*d*p + 2*int((d + e*x)**m/(a*m*x**2 + a*x**2 + b*m + b),x)*b*e*m*p +
 2*int((d + e*x)**m/(a*m*x**2 + a*x**2 + b*m + b),x)*b*e*p)/(e*(m + 1))
```

3.211 $\int (f+gx)^m \log\left(c(d+ex^n)^p\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right) dx = \text{Int}\left((f+gx)^m \log\left(c(d+ex^n)^p\right), x\right)$$

output
```
Defer(Int)((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right) dx = \int (f+gx)^m \log\left(c(d+ex^n)^p\right) dx$$

input
```
Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]
```

output
```
Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2914}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right)\, dx$$

$$\Big\downarrow \ 2914$$

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right)dx$$

input
```
Int[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2914
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)
+ (g_.)*(x_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x)^r*(a + b*Log[c*(d
+ e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x]
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (gx+f)^m \ln\left(c(d+e\,x^n)^p\right)dx$$

input
```
int((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

output
```
int((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right)\,dx = \int (gx+f)^m \log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
integral((g*x + f)^m*log((e*x^n + d)^p*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right)\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**m*ln(c*(d+e*x**n)**p),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.00

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right)\,dx = \int (gx+f)^m \log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
(g*x + f)*(g*x + f)^m*log((e*x^n + d)^p)/(g*(m + 1)) + integrate((d*g*(m +
 1)*x*log(c) - (e*f*n*p + (e*g*n*p - e*g*(m + 1)*log(c))*x)*x^n)*(g*x + f)
^m/(e*g*(m + 1)*x*x^n + d*g*(m + 1)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)^m \log (c(d+ex^n)^p) \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{-1,[
0,0,6,3,6,0,2,2,0,1,0]%%%}+%%%{1,[0,0,6,2,6,1,2,2,0,0,1]%%%}+%%%{1,[0,0,6,
2,6,0,2,2,0,
```

Mupad [N/A]

Not integrable

Time = 25.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (f+gx)^m \log (c(d+ex^n)^p) \, dx = \int \ln \left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+g\,x\right)}^m \,d x$$

input
```
int(log(c*(d + e*x^n)^p)*(f + g*x)^m,x)
```

output
```
int(log(c*(d + e*x^n)^p)*(f + g*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 800, normalized size of antiderivative = 40.00

$$\int (f+gx)^m \log\left(c(d+ex^n)^p\right)\,dx$$

$$= \frac{(gx+f)^m \log((x^n e+d)^p c)\,f\,m^2 + (gx+f)^m \log((x^n e+d)^p c)\,fm + (gx+f)^m \log((x^n e+d)^p c)\,g\,m^2 x}{}$$

input
```
int((g*x+f)^m*log(c*(d+e*x^n)^p),x)
```

output
```
((f + g*x)**m*log((x**n*e + d)**p*c)*f*m**2 + (f + g*x)**m*log((x**n*e + d
)**p*c)*f*m + (f + g*x)**m*log((x**n*e + d)**p*c)*g*m**2*x + (f + g*x)**m*
log((x**n*e + d)**p*c)*g*m*x - 2*(f + g*x)**m*f*m*n*p - (f + g*x)**m*f*n*p
 - (f + g*x)**m*g*m*n*p*x + 2*int((f + g*x)**m/(x**n*e*f*m + x**n*e*f + x*
*n*e*g*m*x + x**n*e*g*x + d*f*m + d*f + d*g*m*x + d*g*x),x)*d*f*g*m**3*n*p
 + 4*int((f + g*x)**m/(x**n*e*f*m + x**n*e*f + x**n*e*g*m*x + x**n*e*g*x +
 d*f*m + d*f + d*g*m*x + d*g*x),x)*d*f*g*m**2*n*p + 2*int((f + g*x)**m/(x*
*n*e*f*m + x**n*e*f + x**n*e*g*m*x + x**n*e*g*x + d*f*m + d*f + d*g*m*x +
d*g*x),x)*d*f*g*m*n*p + int(((f + g*x)**m*x)/(x**n*e*f*m + x**n*e*f + x**n
*e*g*m*x + x**n*e*g*x + d*f*m + d*f + d*g*m*x + d*g*x),x)*d*g**2*m**3*n*p
+ 2*int(((f + g*x)**m*x)/(x**n*e*f*m + x**n*e*f + x**n*e*g*m*x + x**n*e*g*
x + d*f*m + d*f + d*g*m*x + d*g*x),x)*d*g**2*m**2*n*p + int(((f + g*x)**m*
x)/(x**n*e*f*m + x**n*e*f + x**n*e*g*m*x + x**n*e*g*x + d*f*m + d*f + d*g*
m*x + d*g*x),x)*d*g**2*m*n*p - int((x**n*(f + g*x)**m)/(x**n*e*f*m*x + x**
n*e*f*x + x**n*e*g*m*x**2 + x**n*e*g*x**2 + d*f*m*x + d*f*x + d*g*m*x**2 +
 d*g*x**2),x)*e*f**2*m**3*n*p - 2*int((x**n*(f + g*x)**m)/(x**n*e*f*m*x +
x**n*e*f*x + x**n*e*g*m*x**2 + x**n*e*g*x**2 + d*f*m*x + d*f*x + d*g*m*x**
2 + d*g*x**2),x)*e*f**2*m**2*n*p - int((x**n*(f + g*x)**m)/(x**n*e*f*m*x +
 x**n*e*f*x + x**n*e*g*m*x**2 + x**n*e*g*x**2 + d*f*m*x + d*f*x + d*g*m*x*
*2 + d*g*x**2),x)*e*f**2*m*n*p)/(g*m*(m**2 + 2*m + 1))
```

3.212 $\int (f+gx)^3 \log\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 234

$$\begin{aligned}
\int (f+gx)^3 \log\left(c(d+ex^n)^p\right)\,dx
&= -\frac{ef^3npx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} \\
&\quad -\frac{3ef^2gnpx^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(2+n)} \\
&\quad -\frac{efg^2npx^{3+n}\,\text{Hypergeometric2F1}\left(1,\frac{3+n}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{d(3+n)} \\
&\quad -\frac{eg^3npx^{4+n}\,\text{Hypergeometric2F1}\left(1,\frac{4+n}{n},2\left(1+\frac{2}{n}\right),-\frac{ex^n}{d}\right)}{4d(4+n)} \\
&\quad -\frac{f^4p\log(d+ex^n)}{4g}+\frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}
\end{aligned}$$

output
```
-e*f^3*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-3/2*e*f^
2*g*n*p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-e*f*g^2*n
*p*x^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)-1/4*e*g^3*n*p*
x^(4+n)*hypergeom([1, (4+n)/n],[2+4/n],-e*x^n/d)/d/(4+n)-1/4*f^4*p*ln(d+e*
x^n)/g+1/4*(g*x+f)^4*ln(c*(d+e*x^n)^p)/g
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96

$$\int (f+gx)^3 \log \left(c(d+ex^n)^p\right) dx$$
$$= \frac{-enp\left(\frac{4f^3gx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)}+\frac{6f^2g^2x^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(2+n)}+\frac{4fg^3x^{3+n}\,\text{Hy}}{}\right.}{}$$

input
```
Integrate[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```

output
```
(-(e*n*p*((4*f^3*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1),
-((e*x^n)/d)])/(d*(1 + n)) + (6*f^2*g^2*x^(2 + n)*Hypergeometric2F1[1, (2
+ n)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (4*f*g^3*x^(3 + n)*Hy
pergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (g^4*x
^(4 + n)*Hypergeometric2F1[1, (4 + n)/n, 2 + 4/n, -((e*x^n)/d)])/(d*(4 + n
)) + (f^4*Log[d + e*x^n])/(e*n))) + (f + g*x)^4*Log[c*(d + e*x^n)^p])/(4*g
)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2913, 2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \log \left(c(d+ex^n)^p\right) dx$$
$$\Big\downarrow \text{ 2913}$$
$$\frac{(f+gx)^4 \log \left(c(d+ex^n)^p\right)}{4g}-\frac{enp\int \frac{x^{n-1}(f+gx)^4}{ex^n+d}dx}{4g}$$
$$\Big\downarrow \text{ 2383}$$

$$\frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}-\frac{enp\int\left(\frac{f^4x^{n-1}}{ex^n+d}+\frac{4f^3gx^n}{ex^n+d}+\frac{6f^2g^2x^{n+1}}{ex^n+d}+\frac{4fg^3x^{n+2}}{ex^n+d}+\frac{g^4x^{n+3}}{ex^n+d}\right)dx}{4g}$$

$\Big\downarrow$ 2009

$$\frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}-\frac{enp\left(\frac{f^4\log(d+ex^n)}{en}+\frac{4f^3gx^{n+1}\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)}+\frac{6f^2g^2x^{n+2}\text{Hypergeometric2F1}\left(1,\frac{n+2}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(n+2)}+\right.}{4g}$$

input
```
Int[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```

output
```
-1/4*(e*n*p*((4*f^3*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1
), -((e*x^n)/d)])/(d*(1 + n)) + (6*f^2*g^2*x^(2 + n)*Hypergeometric2F1[1,
(2 + n)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (4*f*g^3*x^(3 + n)
*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (g^
4*x^(4 + n)*Hypergeometric2F1[1, (4 + n)/n, 2*(1 + 2/n), -((e*x^n)/d)])/(d
*(4 + n)) + (f^4*Log[d + e*x^n])/(e*n)))/g + ((f + g*x)^4*Log[c*(d + e*x^n
)^p])/(4*g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [F]

$$\int (gx+f)^3 \ln\left(c(d+e\,x^n)^p\right)dx$$

input

```
int((g*x+f)^3*ln(c*(d+e*x^n)^p),x)
```

output

```
int((g*x+f)^3*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int (f+gx)^3 \log\left(c(d+ex^n)^p\right)\,dx = \int (gx+f)^3 \log\left((ex^n+d)^p c\right)\,dx$$

input

```
integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output

```
integral((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*log((e*x^n + d)^p*c), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.49 (sec) , antiderivative size = 515, normalized size of antiderivative = 2.20

$$\int (f+gx)^3\log\left(c(d+ex^n)^p\right)\,dx$$
$$= -\frac{d^{-2-\frac{4}{n}}d^{1+\frac{4}{n}}eg^3px^{n+4}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{4}{n}\right)\Gamma\left(1+\frac{4}{n}\right)}{4\Gamma\left(2+\frac{4}{n}\right)}$$
$$-\frac{d^{-2-\frac{4}{n}}d^{1+\frac{4}{n}}eg^3px^{n+4}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{4}{n}\right)\Gamma\left(1+\frac{4}{n}\right)}{n\Gamma\left(2+\frac{4}{n}\right)}$$
$$-\frac{d^{-2-\frac{3}{n}}d^{1+\frac{3}{n}}efg^2px^{n+3}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{\Gamma\left(2+\frac{3}{n}\right)}$$
$$-\frac{3d^{-2-\frac{3}{n}}d^{1+\frac{3}{n}}efg^2px^{n+3}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{n\Gamma\left(2+\frac{3}{n}\right)}$$
$$-\frac{3d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}ef^2gpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2\Gamma\left(2+\frac{2}{n}\right)}$$
$$-\frac{3d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}ef^2gpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}$$
$$+f^3x\log\left(c(d+ex^n)^p\right)+\frac{3f^2gx^2\log\left(c(d+ex^n)^p\right)}{2}+fg^2x^3\log\left(c(d+ex^n)^p\right)$$
$$+\frac{g^3x^4\log\left(c(d+ex^n)^p\right)}{4}+\frac{d^{-\frac{1}{n}}d^{1+\frac{1}{n}}ee^{\frac{1}{n}}e^{-1-\frac{1}{n}}f^3px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((g*x+f)**3*ln(c*(d+e*x**n)**p),x)
```

output
```
-d**(-2 - 4/n)*d**(1 + 4/n)*e*g**3*p*x**(n + 4)*lerchphi(e*x**n*exp_polar(
I*pi)/d, 1, 1 + 4/n)*gamma(1 + 4/n)/(4*gamma(2 + 4/n)) - d**(-2 - 4/n)*d**
(1 + 4/n)*e*g**3*p*x**(n + 4)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 4/
n)*gamma(1 + 4/n)/(n*gamma(2 + 4/n)) - d**(-2 - 3/n)*d**(1 + 3/n)*e*f*g**2
*p*x**(n + 3)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n
)/gamma(2 + 3/n) - 3*d**(-2 - 3/n)*d**(1 + 3/n)*e*f*g**2*p*x**(n + 3)*lerc
hphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(n*gamma(2 + 3/n
)) - 3*d**(-2 - 2/n)*d**(1 + 2/n)*e*f**2*g*p*x**(n + 2)*lerchphi(e*x**n*ex
p_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*gamma(2 + 2/n)) - 3*d**(-2
- 2/n)*d**(1 + 2/n)*e*f**2*g*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi)/
d, 1, 1 + 2/n)*gamma(1 + 2/n)/(n*gamma(2 + 2/n)) + f**3*x*log(c*(d + e*x**
n)**p) + 3*f**2*g*x**2*log(c*(d + e*x**n)**p)/2 + f*g**2*x**3*log(c*(d + e
*x**n)**p) + g**3*x**4*log(c*(d + e*x**n)**p)/4 + d**(1 + 1/n)*e*e**(1/n)*
e**(-1 - 1/n)*f**3*p*x*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, exp_polar(I
*pi)/n)*gamma(1/n)/(d*d**(1/n)*n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (f+gx)^3\log\left(c(d+ex^n)^p\right)\,dx=\int (gx+f)^3\log\left((ex^n+d)^pc\right)\,dx$$

input
```
integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
-1/16*(g^3*n*p - 4*g^3*log(c))*x^4 - 1/3*(f*g^2*n*p - 3*f*g^2*log(c))*x^3
- 3/4*(f^2*g*n*p - 2*f^2*g*log(c))*x^2 - (f^3*n*p - f^3*log(c))*x + 1/4*(g
^3*x^4 + 4*f*g^2*x^3 + 6*f^2*g*x^2 + 4*f^3*x)*log((e*x^n + d)^p) + integra
te(1/4*(d*g^3*n*p*x^3 + 4*d*f*g^2*n*p*x^2 + 6*d*f^2*g*n*p*x + 4*d*f^3*n*p)
/(e*x^n + d), x)
```

Giac [F]

$$\int (f+gx)^3 \log \left(c(d+ex^n)^p\right) \, dx = \int (gx+f)^3 \log \left((ex^n+d)^p c\right) \, dx$$

input
```
integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate((g*x + f)^3*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \log \left(c(d+ex^n)^p\right) \, dx = \int \ln \left(c\,(d+e\,x^n)^p\right)\,(f+g\,x)^3 \, dx$$

input
```
int(log(c*(d + e*x^n)^p)*(f + g*x)^3,x)
```

output
```
int(log(c*(d + e*x^n)^p)*(f + g*x)^3, x)
```

Reduce [F]

$$\begin{aligned}\int (f+gx)^3 \log \left(c(d+ex^n)^p\right) \, dx = {} & \frac{\left(\int \frac{x^3}{x^n e+d}dx\right) d\, g^3 np}{4} + \left(\int \frac{x^2}{x^n e+d}dx\right) df\, g^2 np \\ & + \frac{3\left(\int \frac{x}{x^n e+d}dx\right) d\, f^2 gnp}{2} + \left(\int \frac{1}{x^n e+d}dx\right) d\, f^3 np \\ & + \log((x^n e+d)^p\, c)\, f^3 x + \frac{3\log((x^n e+d)^p\, c)\, f^2 g\, x^2}{2} \\ & + \log((x^n e+d)^p\, c)\, f\, g^2 x^3 + \frac{\log((x^n e+d)^p\, c)\, g^3 x^4}{4} \\ & - f^3 npx - \frac{3f^2 gnp\, x^2}{4} - \frac{f\, g^2 np\, x^3}{3} - \frac{g^3 np\, x^4}{16}\end{aligned}$$

input
```
int((g*x+f)^3*log(c*(d+e*x^n)^p),x)
```

output
```
(12*int(x**3/(x**n*e + d),x)*d*g**3*n*p + 48*int(x**2/(x**n*e + d),x)*d*f*
g**2*n*p + 72*int(x/(x**n*e + d),x)*d*f**2*g*n*p + 48*int(1/(x**n*e + d),x
)*d*f**3*n*p + 48*log((x**n*e + d)**p*c)*f**3*x + 72*log((x**n*e + d)**p*c
)*f**2*g*x**2 + 48*log((x**n*e + d)**p*c)*f*g**2*x**3 + 12*log((x**n*e + d
)**p*c)*g**3*x**4 - 48*f**3*n*p*x - 36*f**2*g*n*p*x**2 - 16*f*g**2*n*p*x**
3 - 3*g**3*n*p*x**4)/48
```

3.213 $\int (f+gx)^2 \log(c(d+ex^n)^p) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 181

$$\int (f+gx)^2 \log(c(d+ex^n)^p) \, dx = -\frac{ef^2npx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{efgnpx^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(2+n)} - \frac{eg^2npx^{3+n}\,\text{Hypergeometric2F1}\left(1,\frac{3+n}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(3+n)} - \frac{f^3p\log(d+ex^n)}{3g} + \frac{(f+gx)^3\log(c(d+ex^n)^p)}{3g}$$

output
```
-e*f^2*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-e*f*g*n*
p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-1/3*e*g^2*n*p*x
^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)-1/3*f^3*p*ln(d+e*x
^n)/g+1/3*(g*x+f)^3*ln(c*(d+e*x^n)^p)/g
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int (f+gx)^2 \log (c(d+ex^n)^p) \, dx$$
$$= \frac{-enp\left(\frac{3f^2gx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)}+\frac{3fg^2x^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(2+n)}+\frac{g^3x^{3+n}\,\text{Hyperg}}{}\right.}{3g}$$

input
```
Integrate[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```

output
```
(-(e*n*p*((3*f^2*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1),
-((e*x^n)/d)])/(d*(1 + n)) + (3*f*g^2*x^(2 + n)*Hypergeometric2F1[1, (2 +
n)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (g^3*x^(3 + n)*Hypergeo
metric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (f^3*Log[d +
 e*x^n])/(e*n))) + (f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2913, 2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \log (c(d+ex^n)^p) \, dx$$
$$\Big\downarrow \; 2913$$
$$\frac{(f+gx)^3 \log (c(d+ex^n)^p)}{3g}-\frac{enp\int \frac{x^{n-1}(f+gx)^3}{ex^n+d}dx}{3g}$$
$$\Big\downarrow \; 2383$$
$$\frac{(f+gx)^3 \log (c(d+ex^n)^p)}{3g}-\frac{enp\int \left(\frac{f^3x^{n-1}}{ex^n+d}+\frac{3f^2gx^n}{ex^n+d}+\frac{3fg^2x^{n+1}}{ex^n+d}+\frac{g^3x^{n+2}}{ex^n+d}\right)dx}{3g}$$

$$\downarrow \text{2009}$$

$$\frac{(f+gx)^3\log\left(c(d+ex^n)^p\right)}{3g}-\frac{enp\left(\frac{f^3\log(d+ex^n)}{en}+\frac{3f^2gx^{n+1}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)}+\frac{3fg^2x^{n+2}\,\text{Hypergeometric2F1}\left(1,\frac{n+2}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(n+2)}+\right.}{3g}$$

input
```
Int[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```

output
```
-1/3*(e*n*p*((3*f^2*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1
), -((e*x^n)/d)])/(d*(1 + n)) + (3*f*g^2*x^(2 + n)*Hypergeometric2F1[1, (2
 + n)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (g^3*x^(3 + n)*Hyper
geometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (f^3*Log[
d + e*x^n])/(e*n)))/g + ((f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [F]

$$\int (gx+f)^2 \ln \left(c(d+e\,x^n)^p\right) dx$$

input
```
int((g*x+f)^2*ln(c*(d+e*x^n)^p),x)
```

output
```
int((g*x+f)^2*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int (f+gx)^2 \log \left(c(d+ex^n)^p\right)\,dx = \int (gx+f)^2 \log \left((ex^n+d)^p c\right)\,dx$$

input
```
integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
integral((g^2*x^2 + 2*f*g*x + f^2)*log((e*x^n + d)^p*c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.41 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.99

$$\int (f+gx)^2 \log \left(c(d+ex^n)^p\right)\,dx = -\frac{d^{-2-\frac{3}{n}}d^{1+\frac{3}{n}}eg^2px^{n+3}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{3\Gamma\left(2+\frac{3}{n}\right)} - \frac{d^{-2-\frac{3}{n}}d^{1+\frac{3}{n}}eg^2px^{n+3}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} - \frac{d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}efgpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{\Gamma\left(2+\frac{2}{n}\right)} - \frac{2d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}efgpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} + f^2x\log\left(c(d+ex^n)^p\right) + fgx^2\log\left(c(d+ex^n)^p\right) + \frac{g^2x^3\log\left(c(d+ex^n)^p\right)}{3} + \frac{d^{-\frac{1}{n}}d^{1+\frac{1}{n}}ee^{\frac{1}{n}}e^{-1-\frac{1}{n}}f^2px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((g*x+f)**2*ln(c*(d+e*x**n)**p),x)
```

output
```
-d**(-2 - 3/n)*d**(1 + 3/n)*e*g**2*p*x**(n + 3)*lerchphi(e*x**n*exp_polar(
I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(3*gamma(2 + 3/n)) - d**(-2 - 3/n)*d**
(1 + 3/n)*e*g**2*p*x**(n + 3)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/
n)*gamma(1 + 3/n)/(n*gamma(2 + 3/n)) - d**(-2 - 2/n)*d**(1 + 2/n)*e*f*g*p*
x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/g
amma(2 + 2/n) - 2*d**(-2 - 2/n)*d**(1 + 2/n)*e*f*g*p*x**(n + 2)*lerchphi(e
*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(n*gamma(2 + 2/n)) + f
**2*x*log(c*(d + e*x**n)**p) + f*g*x**2*log(c*(d + e*x**n)**p) + g**2*x**3
*log(c*(d + e*x**n)**p)/3 + d**(1 + 1/n)*e*e**(1/n)*e**(-1 - 1/n)*f**2*p*x
*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(d*
d**(1/n)*n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (f+gx)^2 \log \left(c(d+ex^n)^p\right) \, dx = \int (gx+f)^2 \log \left((ex^n+d)^p c\right) \, dx$$

input
```
integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
-1/9*(g^2*n*p - 3*g^2*log(c))*x^3 - 1/2*(f*g*n*p - 2*f*g*log(c))*x^2 - (f^
2*n*p - f^2*log(c))*x + 1/3*(g^2*x^3 + 3*f*g*x^2 + 3*f^2*x)*log((e*x^n + d
)^p) + integrate(1/3*(d*g^2*n*p*x^2 + 3*d*f*g*n*p*x + 3*d*f^2*n*p)/(e*x^n
+ d), x)
```

Giac [F]

$$\int (f+gx)^2 \log \left(c(d+ex^n)^p\right) \, dx = \int (gx+f)^2 \log \left((ex^n+d)^p c\right) \, dx$$

input
```
integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \log \left(c(d+ex^n)^p\right) \, dx = \int \ln \left(c\,(d+e\,x^n)^p\right)\,(f+g\,x)^2 \, dx$$

input
```
int(log(c*(d + e*x^n)^p)*(f + g*x)^2,x)
```

output
```
int(log(c*(d + e*x^n)^p)*(f + g*x)^2, x)
```

Reduce [F]

$$\int (f+gx)^2\log\left(c(d+ex^n)^p\right)\,dx = \frac{\left(\int \frac{x^2}{x^n e+d}dx\right) d\,g^2 np}{3} + \left(\int \frac{x}{x^n e+d}dx\right) df\,gnp + \left(\int \frac{1}{x^n e+d}dx\right) d\,f^2 np + \log((x^n e+d)^p\,c)\,f^2 x + \log((x^n e+d)^p\,c)\,fg\,x^2 + \frac{\log((x^n e+d)^p\,c)\,g^2 x^3}{3} - f^2 npx - \frac{fgnp\,x^2}{2} - \frac{g^2 np\,x^3}{9}$$

input
```
int((g*x+f)^2*log(c*(d+e*x^n)^p),x)
```

output
```
(6*int(x**2/(x**n*e + d),x)*d*g**2*n*p + 18*int(x/(x**n*e + d),x)*d*f*g*n*
p + 18*int(1/(x**n*e + d),x)*d*f**2*n*p + 18*log((x**n*e + d)**p*c)*f**2*x
 + 18*log((x**n*e + d)**p*c)*f*g*x**2 + 6*log((x**n*e + d)**p*c)*g**2*x**3
 - 18*f**2*n*p*x - 9*f*g*n*p*x**2 - 2*g**2*n*p*x**3)/18
```

3.214 $\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\begin{aligned}\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx &= -\frac{efnpx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)}\\ &\quad-\frac{egnpx^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(2+n)}\\ &\quad-\frac{f^2p\log(d+ex^n)}{2g}+\frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g}\end{aligned}$$

output
```
-e*f*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-1/2*e*g*n*
p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-1/2*f^2*p*ln(d+
e*x^n)/g+1/2*(g*x+f)^2*ln(c*(d+e*x^n)^p)/g
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int (f+gx)\log \left(c(d+ex^n)^p\right)\,dx$$
$$= -\frac{efnpx^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{n},1+\frac{1+n}{n},-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{egnpx^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},1+\frac{2+n}{n},-\frac{ex^n}{d}\right)}{2d(2+n)} + fx\log \left(c(d+ex^n)^p\right)+\frac{1}{2}gx^2\log \left(c(d+ex^n)^p\right)$$

input
```
Integrate[(f + g*x)*Log[c*(d + e*x^n)^p],x]
```

output
```
-((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/n, 1 + (1 + n)/n, -((e*x
^n)/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n,
 1 + (2 + n)/n, -((e*x^n)/d)])/(2*d*(2 + n)) + f*x*Log[c*(d + e*x^n)^p] +
(g*x^2*Log[c*(d + e*x^n)^p])/2
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2913, 2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\log \left(c(d+ex^n)^p\right)\,dx$$
$$\Big\downarrow \text{2913}$$
$$\frac{(f+gx)^2\log \left(c(d+ex^n)^p\right)}{2g}-\frac{enp\int \frac{x^{n-1}(f+gx)^2}{ex^n+d}dx}{2g}$$
$$\Big\downarrow \text{2383}$$

$$\frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g}-\frac{enp\int\left(\frac{f^2x^{n-1}}{ex^n+d}+\frac{2fgx^n}{ex^n+d}+\frac{g^2x^{n+1}}{ex^n+d}\right)dx}{2g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g}-\frac{enp\left(\frac{f^2\log(d+ex^n)}{en}+\frac{2fgx^{n+1}\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)}+\frac{g^2x^{n+2}\text{Hypergeometric2F1}\left(1,\frac{n+2}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(n+2)}\right)}{2g}$$

input
```
Int[(f + g*x)*Log[c*(d + e*x^n)^p],x]
```

output
```
-1/2*(e*n*p*((2*f*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1),
 -((e*x^n)/d)])/(d*(1 + n)) + (g^2*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/
n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (f^2*Log[d + e*x^n])/(e*n)
))/g + ((f + g*x)^2*Log[c*(d + e*x^n)^p])/(2*g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

rule 2913
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Simp[b*e*n*(p/(g*(r + 1)))   Int[x^(n - 1)*((f + g
*x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x
] && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Maple [F]

$$\int (gx+f)\ln\left(c(d+e\,x^n)^p\right)dx$$

input
```
int((g*x+f)*ln(c*(d+e*x^n)^p),x)
```

output
```
int((g*x+f)*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx=\int (gx+f)\log\left((ex^n+d)^pc\right)\,dx$$

input
```
integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
integral((g*x + f)*log((e*x^n + d)^p*c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.74 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.62

$$\begin{aligned}\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx=&-\frac{d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}egpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2\Gamma\left(2+\frac{2}{n}\right)}\\&-\frac{d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}egpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}\\&+fx\log\left(c(d+ex^n)^p\right)+\frac{gx^2\log\left(c(d+ex^n)^p\right)}{2}\\&+\frac{d^{-\frac{1}{n}}d^{1+\frac{1}{n}}ee^{\frac{1}{n}}e^{-1-\frac{1}{n}}fpx\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}\end{aligned}$$

input
```
integrate((g*x+f)*ln(c*(d+e*x**n)**p),x)
```

output
```
-d**(-2 - 2/n)*d**(1 + 2/n)*e*g*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*p
i)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*gamma(2 + 2/n)) - d**(-2 - 2/n)*d**(1
+ 2/n)*e*g*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gam
ma(1 + 2/n)/(n*gamma(2 + 2/n)) + f*x*log(c*(d + e*x**n)**p) + g*x**2*log(c
*(d + e*x**n)**p)/2 + d**(1 + 1/n)*e*e**(1/n)*e**(-1 - 1/n)*f*p*x*lerchphi
(d*exp_polar(I*pi)/(e*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(d*d**(1/n)*
n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx=\int (gx+f)\log\left((ex^n+d)^pc\right)\,dx$$

input
```
integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
-1/4*(g*n*p - 2*g*log(c))*x^2 - (f*n*p - f*log(c))*x + 1/2*(g*x^2 + 2*f*x)
*log((e*x^n + d)^p) + integrate(1/2*(d*g*n*p*x + 2*d*f*n*p)/(e*x^n + d), x
)
```

Giac [F]

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx=\int (gx+f)\log\left((ex^n+d)^pc\right)\,dx$$

input
```
integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate((g*x + f)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx=\int \ln\left(c\,\left(d+e\,x^n\right)^p\right)\,\left(f+g\,x\right)\,dx$$

input
```
int(log(c*(d + e*x^n)^p)*(f + g*x),x)
```

output
```
int(log(c*(d + e*x^n)^p)*(f + g*x), x)
```

Reduce [F]

$$\begin{aligned}\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx&=\frac{\left(\int \frac{x}{x^n e+d}dx\right)dgnp}{2}+\left(\int \frac{1}{x^n e+d}dx\right)dfnp\\&\quad+\log\left(\left(x^n e+d\right)^p c\right)fx\\&\quad+\frac{\log\left(\left(x^n e+d\right)^p c\right)g\,x^2}{2}-fnpx-\frac{gnp\,x^2}{4}\end{aligned}$$

input
```
int((g*x+f)*log(c*(d+e*x^n)^p),x)
```

output
```
(2*int(x/(x**n*e + d),x)*d*g*n*p + 4*int(1/(x**n*e + d),x)*d*f*n*p + 4*log
((x**n*e + d)**p*c)*f*x + 2*log((x**n*e + d)**p*c)*g*x**2 - 4*f*n*p*x - g*
n*p*x**2)/4
```

3.215 $\int \log \left(c(d+ex^n)^p\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \log \left(c(d+ex^n)^p\right) dx = -\frac{enpx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} + x\log\left(c(d+ex^n)^p\right)$$

output
```
-e*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)+x*ln(c*(d+e*
x^n)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \log \left(c(d+ex^n)^p\right) dx = x\left(-\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} + \log\left(c(d+ex^n)^p\right)\right)$$

input
```
Integrate[Log[c*(d + e*x^n)^p],x]
```

output
```
x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)]
)/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2898, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(c(d+ex^n)^p\right)\,dx$$

$$\Big\downarrow \text{2898}$$

$$x\log\left(c(d+ex^n)^p\right)-enp\int \frac{x^n}{ex^n+d}dx$$

$$\Big\downarrow \text{888}$$

$$x\log\left(c(d+ex^n)^p\right)-\frac{enpx^{n+1}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)}$$

input
```
Int[Log[c*(d + e*x^n)^p],x]
```

output
```
-((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/
d)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

Maple [F]

$$\int \ln \left(c(d+e\,x^{n})^{p}\right) dx$$

input
```
int(ln(c*(d+e*x^n)^p),x)
```

output
```
int(ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int \log \left(c(d+ex^{n})^{p}\right)\,dx=\int \log \left((ex^{n}+d)^{p}c\right)\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \log \left(c(d+ex^{n})^{p}\right)\,dx$$
$$=x\log \left(c(d+ex^{n})^{p}\right)+\frac{d^{-\frac{1}{n}}d^{1+\frac{1}{n}}ee^{\frac{1}{n}}e^{-1-\frac{1}{n}}px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate(ln(c*(d+e*x**n)**p),x)
```

output
```
x*log(c*(d + e*x**n)**p) + d**(1 + 1/n)*e*e**(1/n)*e**(-1 - 1/n)*p*x*lerch
phi(d*exp_polar(I*pi)/(e*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(d*d**(1/
n)*n*gamma(1 + 1/n))
```

Maxima [F]

$$\int \log\left(c(d+ex^n)^p\right)\,dx = \int \log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

output
```
d*n*p*integrate(1/(e*x^n + d), x) - (n*p - log(c))*x + x*log((e*x^n + d)^p
)
```

Giac [F]

$$\int \log\left(c(d+ex^n)^p\right)\,dx = \int \log\left((ex^n+d)^p c\right)\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log\left(c(d+ex^n)^p\right)\,dx = \int \ln\left(c\left(d+e\,x^n\right)^p\right)\,dx$$

input
```
int(log(c*(d + e*x^n)^p),x)
```

output
```
int(log(c*(d + e*x^n)^p), x)
```

Reduce [F]

$$\int \log\left(c(d+ex^n)^p\right)\,dx = \left(\int \frac{1}{x^n e+d}dx\right) dnp + \log\left(\left(x^n e+d\right)^p c\right) x - npx$$

input
```
int(log(c*(d+e*x^n)^p),x)
```

output
```
int(1/(x**n*e + d),x)*d*n*p + log((x**n*e + d)**p*c)*x - n*p*x
```

3.216 $\int \frac{\log(c(d+ex^n)^p)}{f+gx} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log(c(d+ex^n)^p)}{f+gx} dx = \text{Int}\left(\frac{\log(c(d+ex^n)^p)}{f+gx}, x\right)$$

output
```
Defer(Int)(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

Mathematica [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{f+gx} dx = \int \frac{\log(c(d+ex^n)^p)}{f+gx} dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(f + g*x),x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]/(f + g*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2914}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{f+gx} \, dx$$

$$\Big\downarrow \ 2914$$

$$\int \frac{\log (c(d+ex^n)^p)}{f+gx}dx$$

input
```
Int[Log[c*(d + e*x^n)^p]/(f + g*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2914
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)
+ (g_.)*(x_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x)^r*(a + b*Log[c*(d
+ e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x]
```

Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c(d+e\,x^n)^p\right)}{gx+f}dx$$

input
```
int(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

output
```
int(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{f+gx}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{gx+f}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/(g*x + f), x)
```

Sympy [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{f+gx}\,dx = \int \frac{\log \left(c(d+ex^n)^p\right)}{f+gx}\,dx$$

input
```
integrate(ln(c*(d+e*x**n)**p)/(g*x+f),x)
```

output
```
Integral(log(c*(d + e*x**n)**p)/(f + g*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{f+gx}\,dx = \int \frac{\log((ex^n+d)^pc)}{gx+f}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^n + d)^p*c)/(g*x + f), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{f+gx}\,dx = \int \frac{\log((ex^n+d)^pc)}{gx+f}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/(g*x + f), x)
```

Mupad [N/A]

Not integrable

Time = 25.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{f+gx}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{f+g\,x}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(f + g*x),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(f + g*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 6.15

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{f+gx}\,dx = \frac{2\left(\int \frac{\log((x^n e+d)^p c)}{x^n ef+x^n egx+df+dgx}dx\right) dgnp-2\left(\int \frac{x^n \log((x^n e+d)^p c)}{x^n efx+x^n eg\,x^2+dfx+dg\,x^2}dx\right) efnp+\log((x^n e+d)^p\,c)^2}{2gnp}$$

input
```
int(log(c*(d+e*x^n)^p)/(g*x+f),x)
```

output
```
(2*int(log((x**n*e + d)**p*c)/(x**n*e*f + x**n*e*g*x + d*f + d*g*x),x)*d*g
*n*p - 2*int((x**n*log((x**n*e + d)**p*c))/(x**n*e*f*x + x**n*e*g*x**2 + d
*f*x + d*g*x**2),x)*e*f*n*p + log((x**n*e + d)**p*c)**2)/(2*g*n*p)
```

3.217 $\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx = \text{Int}\left(\frac{\log(c(d+ex^n)^p)}{(f+gx)^2},x\right)$$

output
```
Defer(Int)(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx = \int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2914}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{(f+gx)^2} \, dx$$
$$\Big\downarrow \text{ 2914}$$
$$\int \frac{\log \left(c(d+ex^n)^p\right)}{(f+gx)^2}dx$$

input
```
Int[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2914
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)
+ (g_.)*(x_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x)^r*(a + b*Log[c*(d
+ e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x]
```

Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c(d+e\,x^{n})^{p}\right)}{\left(gx+f\right)^{2}}dx$$

input
```
int(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

output
```
int(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{(gx+f)^2}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [N/A]

Not integrable

Time = 45.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx$$

input
```
integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**2,x)
```

output
```
Integral(log(c*(d + e*x**n)**p)/(f + g*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.25

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{(gx+f)^2}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
-d*n*p*integrate(1/(d*g^2*x^2 + d*f*g*x + (e*g^2*x^2 + e*f*g*x)*x^n), x) -
 n*p*log(g*x + f)/(f*g) - (f*log((e*x^n + d)^p) + f*log(c) - (g*n*p*x + f*
n*p)*log(x))/(f*g^2*x + f^2*g)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{(gx+f)^2}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/(g*x + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{(f+g\,x)^2}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(f + g*x)^2,x)
```

output
```
int(log(c*(d + e*x^n)^p)/(f + g*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 8.45

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx$$
$$= \frac{-\left(\int \frac{1}{x^n efx+x^n eg\,x^2+dfx+dg\,x^2}dx\right) d\,f^2np - \left(\int \frac{1}{x^n efx+x^n eg\,x^2+dfx+dg\,x^2}dx\right) dfgnpx - \log(x^n e+d)\,fp - \log(x}{fg\,(gx}$$

input
```
int(log(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

output
```
( - int(1/(x**n*e*f*x + x**n*e*g*x**2 + d*f*x + d*g*x**2),x)*d*f**2*n*p -
int(1/(x**n*e*f*x + x**n*e*g*x**2 + d*f*x + d*g*x**2),x)*d*f*g*n*p*x - log
(x**n*e + d)*f*p - log(x**n*e + d)*g*p*x - log(f + g*x)*f*n*p - log(f + g*
x)*g*n*p*x + log((x**n*e + d)**p*c)*g*x + log(x)*f*n*p + log(x)*g*n*p*x)/(
f*g*(f + g*x))
```

3.218 $\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log (c(d+ex^n)^p)}{(f+gx)^3}\,dx = \text{Int}\left(\frac{\log (c(d+ex^n)^p)}{(f+gx)^3},x\right)$$

output
```
Defer(Int)(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log (c(d+ex^n)^p)}{(f+gx)^3}\,dx = \int \frac{\log (c(d+ex^n)^p)}{(f+gx)^3}\,dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2914}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{(f+gx)^3} dx$$
$$\Big\downarrow \text{2914}$$
$$\int \frac{\log (c(d+ex^n)^p)}{(f+gx)^3} dx$$

input
```
Int[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2914
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)
+ (g_.)*(x_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x)^r*(a + b*Log[c*(d
+ e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x]
```

Maple [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c \left(d+e\,x^{n}\right)^{p}\right)}{\left(g x+f\right)^{3}}d x$$

input
```
int(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

output
```
int(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{(gx+f)^3}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 8.70

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx = \int \frac{\log((ex^n+d)^p c)}{(gx+f)^3}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
-d*n*p*integrate(1/2/(d*g^3*x^3 + 2*d*f*g^2*x^2 + d*f^2*g*x + (e*g^3*x^3 +
 2*e*f*g^2*x^2 + e*f^2*g*x)*x^n), x) + 1/2*(f*g*n*p*x + f^2*n*p - f^2*log(
(e*x^n + d)^p) - f^2*log(c) + (g^2*n*p*x^2 + 2*f*g*n*p*x + f^2*n*p)*log(x)
)/(f^2*g^3*x^2 + 2*f^3*g^2*x + f^4*g) - 1/2*n*p*log(g*x + f)/(f^2*g)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx = \int \frac{\log((ex^n+d)^p c)}{(gx+f)^3}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/(g*x + f)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{(f+g\,x)^3}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(f + g*x)^3,x)
```

output
```
int(log(c*(d + e*x^n)^p)/(f + g*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 401, normalized size of antiderivative = 20.05

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3}\,dx$$
$$= \frac{-\left(\int \frac{1}{x^n e\,f^2x+2x^n efg\,x^2+x^n e\,g^2x^3+d\,f^2x+2dfg\,x^2+d\,g^2x^3}dx\right) d\,f^4np - 2\left(\int \frac{1}{x^n e\,f^2x+2x^n efg\,x^2+x^n e\,g^2x^3+d\,f^2x+2dfg\,x^2+d\,g^2}\right.}{}$$

input
```
int(log(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

output
```
( - int(1/(x**n*e*f**2*x + 2*x**n*e*f*g*x**2 + x**n*e*g**2*x**3 + d*f**2*x
 + 2*d*f*g*x**2 + d*g**2*x**3),x)*d*f**4*n*p - 2*int(1/(x**n*e*f**2*x + 2*
x**n*e*f*g*x**2 + x**n*e*g**2*x**3 + d*f**2*x + 2*d*f*g*x**2 + d*g**2*x**3
),x)*d*f**3*g*n*p*x - int(1/(x**n*e*f**2*x + 2*x**n*e*f*g*x**2 + x**n*e*g*
*2*x**3 + d*f**2*x + 2*d*f*g*x**2 + d*g**2*x**3),x)*d*f**2*g**2*n*p*x**2 -
 log(x**n*e + d)*f**2*p - 2*log(x**n*e + d)*f*g*p*x - log(x**n*e + d)*g**2
*p*x**2 - log(f + g*x)*f**2*n*p - 2*log(f + g*x)*f*g*n*p*x - log(f + g*x)*
g**2*n*p*x**2 + 2*log((x**n*e + d)**p*c)*f*g*x + log((x**n*e + d)**p*c)*g*
*2*x**2 + log(x)*f**2*n*p + 2*log(x)*f*g*n*p*x + log(x)*g**2*n*p*x**2 - f*
g*n*p*x - g**2*n*p*x**2)/(2*f**2*g*(f**2 + 2*f*g*x + g**2*x**2))
```

3.219 $\int \frac{x^3 \log(c(a+bx)^p)}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 250

$$\int \frac{x^3 \log(c(a+bx)^p)}{d+ex}\,dx = -\frac{d^2px}{e^3} - \frac{adpx}{2be^2} - \frac{a^2px}{3b^2e} + \frac{dpx^2}{4e^2} + \frac{apx^2}{6be} - \frac{px^3}{9e} + \frac{a^2dp\log(a+bx)}{2b^2e^2} + \frac{a^3p\log(a+bx)}{3b^3e} - \frac{dx^2\log(c(a+bx)^p)}{2e^2} + \frac{x^3\log(c(a+bx)^p)}{3e} + \frac{d^2(a+bx)\log(c(a+bx)^p)}{be^3} - \frac{d^3\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^4} - \frac{d^3p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^4}$$

output
```
-d^2*p*x/e^3-1/2*a*d*p*x/b/e^2-1/3*a^2*p*x/b^2/e+1/4*d*p*x^2/e^2+1/6*a*p*x
^2/b/e-1/9*p*x^3/e+1/2*a^2*d*p*ln(b*x+a)/b^2/e^2+1/3*a^3*p*ln(b*x+a)/b^3/e
-1/2*d*x^2*ln(c*(b*x+a)^p)/e^2+1/3*x^3*ln(c*(b*x+a)^p)/e+d^2*(b*x+a)*ln(c*
(b*x+a)^p)/b/e^3-d^3*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e^4-d^3*p*po
lylog(2,-e*(b*x+a)/(-a*e+b*d))/e^4
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.73

$$\int \frac{x^3 \log (c(a+bx)^p)}{d+ex}\,dx = \frac{6a^2e^2(3bd+2ae)p\log(a+bx)+b\Big(-epx(12a^2e^2-6abe(-3d+ex)+b^2(36d^2-9dex+4e^2x^2))+6b\log}{36b}$$

input
```
Integrate[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

output
```
(6*a^2*e^2*(3*b*d + 2*a*e)*p*Log[a + b*x] + b*(-(e*p*x*(12*a^2*e^2 - 6*a*b
*e*(-3*d + e*x) + b^2*(36*d^2 - 9*d*e*x + 4*e^2*x^2))) + 6*b*Log[c*(a + b*
x)^p]*(6*a*d^2*e + b*e*x*(6*d^2 - 3*d*e*x + 2*e^2*x^2) - 6*b*d^3*Log[(b*(d
 + e*x))/(b*d - a*e)])) - 36*b^3*d^3*p*PolyLog[2, (e*(a + b*x))/(-(b*d) +
a*e)])/(36*b^3*e^4)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log (c(a+bx)^p)}{d+ex}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(-\frac{d^3 \log (c(a+bx)^p)}{e^3(d+ex)}+\frac{d^2 \log (c(a+bx)^p)}{e^3}-\frac{dx \log (c(a+bx)^p)}{e^2}+\frac{x^2 \log (c(a+bx)^p)}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3p\log(a+bx)}{3b^3e}+\frac{a^2dp\log(a+bx)}{2b^2e^2}-\frac{a^2px}{3b^2e}-\frac{d^3\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^4}+\frac{d^2(a+bx)\log\left(c(a+bx)^p\right)}{be^3}-\frac{dx^2\log\left(c(a+bx)^p\right)}{2e^2}+\frac{x^3\log\left(c(a+bx)^p\right)}{3e}-\frac{d^3p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^4}-\frac{adpx}{2be^2}+\frac{apx^2}{6be}-\frac{d^2px}{e^3}+\frac{dpx^2}{4e^2}-\frac{px^3}{9e}$$

input
```
Int[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

output
```
-((d^2*p*x)/e^3) - (a*d*p*x)/(2*b*e^2) - (a^2*p*x)/(3*b^2*e) + (d*p*x^2)/(
4*e^2) + (a*p*x^2)/(6*b*e) - (p*x^3)/(9*e) + (a^2*d*p*Log[a + b*x])/(2*b^2
*e^2) + (a^3*p*Log[a + b*x])/(3*b^3*e) - (d*x^2*Log[c*(a + b*x)^p])/(2*e^2
) + (x^3*Log[c*(a + b*x)^p])/(3*e) + (d^2*(a + b*x)*Log[c*(a + b*x)^p])/(b
*e^3) - (d^3*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^4 - (d^3
*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.19

method	result
parts	$\frac{x^3 \ln(c(bx+a)^p)}{3e} - \frac{d\,x^2 \ln(c(bx+a)^p)}{2e^2} + \frac{\ln(c(bx+a)^p) d^2 x}{e^3} - \frac{\ln(c(bx+a)^p) d^3 \ln(ex+d)}{e^4} - \frac{pb\left(-\frac{-\frac{2(ex+d)^3 b^2}{3} - (ex+d)^2 abe - 7}{}\right.}{}$
risch	$\frac{\ln((bx+a)^p)x^3}{3e} - \frac{\ln((bx+a)^p)d\,x^2}{2e^2} + \frac{\ln((bx+a)^p)x\,d^2}{e^3} - \frac{\ln((bx+a)^p)d^3 \ln(ex+d)}{e^4} - \frac{p\,x^3}{9e} + \frac{dp\,x^2}{4e^2} - \frac{d^2 px}{e^3} - \frac{49p\,d^3}{36e^4} +$

input
```
int(x^3*ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(c*(b*x+a)^p)/e-1/2*d*x^2*ln(c*(b*x+a)^p)/e^2+ln(c*(b*x+a)^p)/e^
3*d^2*x-ln(c*(b*x+a)^p)*d^3/e^4*ln(e*x+d)-p*b/e*(-1/6/e^3*(-1/b^3*(2/3*(e*
x+d)^3*b^2-(e*x+d)^2*a*b*e-7/2*(e*x+d)^2*b^2*d+2*(e*x+d)*a^2*e^2+5*(e*x+d)
*a*b*d*e+11*(e*x+d)*b^2*d^2)+e*a*(2*a^2*e^2+3*a*b*d*e+6*b^2*d^2)/b^4*ln((e
*x+d)*b+e*a-b*d))-1/e^3*d^3*(dilog(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b+ln(e*x
+d)*ln(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b))
```

Fricas [F]

$$\int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\, dx = \int \frac{x^3 \log \left((bx+a)^p c\right)}{ex+d}\, dx$$

input
```
integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^3 \log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left(c(a+bx)^p\right)}{d+ex}\,dx$$

input
```
integrate(x**3*ln(c*(b*x+a)**p)/(e*x+d),x)
```

output
```
Integral(x**3*log(c*(a + b*x)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^3 \log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left((bx+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left((bx+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^3\,\ln\left(c\,(a+b\,x)^p\right)}{d+e\,x}\,dx$$

input
```
int((x^3*log(c*(a + b*x)^p))/(d + e*x),x)
```

output
```
int((x^3*log(c*(a + b*x)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{-36\left(\int \frac{\log\left((bx+a)^p c\right)}{be\,x^2+aex+bdx+ad}dx\right) a\,b^3 d^3 ep + 36\left(\int \frac{\log\left((bx+a)^p c\right)}{be\,x^2+aex+bdx+ad}dx\right) b^4 d^4 p - 18 \log\left((bx+a)^p\,c\right)^2 b^3 d^3 + 12 \log}{}$$

input
```
int(x^3*log(c*(b*x+a)^p)/(e*x+d),x)
```

output
```
( - 36*int(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*a*b**3*
d**3*e*p + 36*int(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*
b**4*d**4*p - 18*log((a + b*x)**p*c)**2*b**3*d**3 + 12*log((a + b*x)**p*c)
*a**3*e**3*p + 18*log((a + b*x)**p*c)*a**2*b*d*e**2*p + 36*log((a + b*x)**
p*c)*a*b**2*d**2*e*p + 36*log((a + b*x)**p*c)*b**3*d**2*e*p*x - 18*log((a
+ b*x)**p*c)*b**3*d*e**2*p*x**2 + 12*log((a + b*x)**p*c)*b**3*e**3*p*x**3
- 12*a**2*b*e**3*p**2*x - 18*a*b**2*d*e**2*p**2*x + 6*a*b**2*e**3*p**2*x**
2 - 36*b**3*d**2*e*p**2*x + 9*b**3*d*e**2*p**2*x**2 - 4*b**3*e**3*p**2*x**
3)/(36*b**3*e**4*p)
```

3.220 $\int \frac{x^2 \log(c(a+bx)^p)}{d+ex} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex} \, dx = \frac{dpx}{e^2}+\frac{apx}{2be}-\frac{px^2}{4e}-\frac{a^2p\log(a+bx)}{2b^2e} +\frac{x^2 \log \left(c(a+bx)^p\right)}{2e}-\frac{d(a+bx)\log \left(c(a+bx)^p\right)}{be^2} +\frac{d^2 \log \left(c(a+bx)^p\right)\log \left(\frac{b(d+ex)}{bd-ae}\right)}{e^3} +\frac{d^2p \operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^3}$$

output
```
d*p*x/e^2+1/2*a*p*x/b/e-1/4*p*x^2/e-1/2*a^2*p*ln(b*x+a)/b^2/e+1/2*x^2*ln(c
*(b*x+a)^p)/e-d*(b*x+a)*ln(c*(b*x+a)^p)/b/e^2+d^2*ln(c*(b*x+a)^p)*ln(b*(e*
x+d)/(-a*e+b*d))/e^3+d^2*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/e^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex} \, dx$$
$$= \frac{bepx(4bd+2ae-bex)-2a^2e^2p\log(a+bx)+b\log\left(c(a+bx)^p\right)\left(-4ade+2bex(-2d+ex)+4bd^2\log\left(\right.\right.}{4b^2e^3}$$

input
```
Integrate[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]
```

output
```
(b*e*p*x*(4*b*d + 2*a*e - b*e*x) - 2*a^2*e^2*p*Log[a + b*x] + b*Log[c*(a +
 b*x)^p]*(-4*a*d*e + 2*b*e*x*(-2*d + e*x) + 4*b*d^2*Log[(b*(d + e*x))/(b*d
 - a*e)]) + 4*b^2*d^2*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(4*b^2*e
^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex} \, dx$$
$$\Big\downarrow \; 2863$$
$$\int \left(\frac{d^2 \log \left(c(a+bx)^p\right)}{e^2(d+ex)}-\frac{d \log \left(c(a+bx)^p\right)}{e^2}+\frac{x \log \left(c(a+bx)^p\right)}{e}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{a^2p\log(a+bx)}{2b^2e}+\frac{d^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3}-\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{be^2}+\frac{x^2\log\left(c(a+bx)^p\right)}{2e}+\frac{d^2p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^3}+\frac{apx}{2be}+\frac{dpx}{e^2}-\frac{px^2}{4e}$$

input
```
Int[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]
```

output
```
(d*p*x)/e^2 + (a*p*x)/(2*b*e) - (p*x^2)/(4*e) - (a^2*p*Log[a + b*x])/(2*b^
2*e) + (x^2*Log[c*(a + b*x)^p])/(2*e) - (d*(a + b*x)*Log[c*(a + b*x)^p])/(
b*e^2) + (d^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^3 + (d^
2*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.34

method	result
parts	$\frac{x^2\ln(c(bx+a)^p)}{2e} - \frac{dx\ln(c(bx+a)^p)}{e^2} + \frac{\ln(c(bx+a)^p)d^2\ln(ex+d)}{e^3} - \frac{pb\left(\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{b}+\frac{\ln(ex+d)\ln\left(\frac{(ex+d)b+ea-b\cdots}{ea-bd}\right)}{b}\cdots\right)}{e^2}\cdots\right)}{\cdots}$
risch	$\frac{\ln((bx+a)^p)x^2}{2e} - \frac{\ln((bx+a)^p)dx}{e^2} + \frac{\ln((bx+a)^p)d^2\ln(ex+d)}{e^3} - \frac{p\,d^2\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{e^3} - \frac{p\,d^2\ln(ex+d)\ln\left(\frac{(ex+d)b+ea-\cdots}{ea-bd}\cdots\right)}{e^3}\cdots$

input
```
int(x^2*ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(b*x+a)^p)/e-d*x*ln(c*(b*x+a)^p)/e^2+ln(c*(b*x+a)^p)*d^2/e^3*
ln(e*x+d)-p*b/e*(d^2/e^2*(dilog(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b+ln(e*x+d)
*ln(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b)+1/2/e^2*(-1/b^2*((e*x+d)*a*e+3*d*(e*
x+d)*b-1/2*(e*x+d)^2*b)+e*a*(a*e+2*b*d)/b^3*ln((e*x+d)*b+e*a-b*d)))
```

Fricas [F]

$$\int \frac{x^2 \log (c(a+bx)^p)}{d+ex}\,dx = \int \frac{x^2 \log ((bx+a)^p c)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^2 \log (c(a+bx)^p)}{d+ex}\,dx = \int \frac{x^2 \log (c(a+bx)^p)}{d+ex}\,dx$$

input
```
integrate(x**2*ln(c*(b*x+a)**p)/(e*x+d),x)
```

output
```
Integral(x**2*log(c*(a + b*x)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^2 \log (c(a+bx)^p)}{d+ex}\,dx = \int \frac{x^2 \log ((bx+a)^p c)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^2 \ln \left(c\,(a+b\,x)^p\right)}{d+e\,x}\,dx$$

input
```
int((x^2*log(c*(a + b*x)^p))/(d + e*x),x)
```

output
```
int((x^2*log(c*(a + b*x)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{4\left(\int \frac{\log((bx+a)^p c)}{be\,x^2+aex+bdx+ad}dx\right) a\,b^2 d^2 ep - 4\left(\int \frac{\log((bx+a)^p c)}{be\,x^2+aex+bdx+ad}dx\right) b^3 d^3 p + 2\log((bx+a)^p\,c)^2\,b^2 d^2 - 2\log((bx+}{}$$

input
```
int(x^2*log(c*(b*x+a)^p)/(e*x+d),x)
```

output
```
(4*int(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*a*b**2*d**2
*e*p - 4*int(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*b**3*
d**3*p + 2*log((a + b*x)**p*c)**2*b**2*d**2 - 2*log((a + b*x)**p*c)*a**2*e
**2*p - 4*log((a + b*x)**p*c)*a*b*d*e*p - 4*log((a + b*x)**p*c)*b**2*d*e*p
*x + 2*log((a + b*x)**p*c)*b**2*e**2*p*x**2 + 2*a*b*e**2*p**2*x + 4*b**2*d
*e*p**2*x - b**2*e**2*p**2*x**2)/(4*b**2*e**3*p)
```

3.221 $\int \frac{x\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{x\log\left(c(a+bx)^p\right)}{d+ex}\,dx = -\frac{px}{e}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be} - \frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} - \frac{dp\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^2}$$

output
```
-p*x/e+(b*x+a)*ln(c*(b*x+a)^p)/b/e-d*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*
d))/e^2-d*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/e^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int \frac{x\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{-bepx+\log\left(c(a+bx)^p\right)\left(ae+bex-bd\log\left(\frac{b(d+ex)}{bd-ae}\right)\right)-bdp\,\text{PolyLog}\left(2,\frac{e(a+bx)}{-bd+ae}\right)}{be^2}$$

input
```
Integrate[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
```

output
```
(-(b*e*p*x) + Log[c*(a + b*x)^p]*(a*e + b*e*x - b*d*Log[(b*(d + e*x))/(b*d
 - a*e)]) - b*d*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(b*e^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\log\left(c(a+bx)^p\right)}{d+ex}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{\log\left(c(a+bx)^p\right)}{e}-\frac{d\log\left(c(a+bx)^p\right)}{e(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be}-\frac{dp\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^2}-\frac{px}{e}$$

input
```
Int[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
```

output
```
-((p*x)/e) + ((a + b*x)*Log[c*(a + b*x)^p])/(b*e) - (d*Log[c*(a + b*x)^p]*
Log[(b*(d + e*x))/(b*d - a*e)])/e^2 - (d*p*PolyLog[2, -((e*(a + b*x))/(b*d
 - a*e))])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.71

method	result
parts	$\frac{x\ln\left(c(bx+a)^p\right)}{e}-\frac{\ln\left(c(bx+a)^p\right)d\ln(ex+d)}{e^2}-\frac{pb\left(\frac{ex+d}{eb}-\frac{a\ln((ex+d)b+ea-bd)}{b^2}-\frac{d\left(\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{b}+\frac{\ln(ex+d)\ln\left(\frac{(ex+d)b+\cdots}{ea-\cdots}\right)}{b}\right.}{e}\right.}{e}$
risch	$\frac{\ln\left((bx+a)^p\right)x}{e}-\frac{\ln\left((bx+a)^p\right)d\ln(ex+d)}{e^2}-\frac{px}{e}-\frac{pd}{e^2}+\frac{pa\ln((ex+d)b+ea-bd)}{be}+\frac{pd\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{e^2}+\frac{pd\ln(ex+d\cdots}{}$

input
```
int(x*ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(b*x+a)^p)/e-ln(c*(b*x+a)^p)*d/e^2*ln(e*x+d)-p*b/e*(1/e*(e*x+d)/b-a
/b^2*ln((e*x+d)*b+e*a-b*d)-1/e*d*(dilog(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b+l
n(e*x+d)*ln(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b))
```

Fricas [F]

$$\int \frac{x \log (c(a+bx)^p)}{d+ex}\,dx = \int \frac{x \log ((bx+a)^p c)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x*log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x \log (c(a+bx)^p)}{d+ex}\,dx = \int \frac{x \log (c(a+bx)^p)}{d+ex}\,dx$$

input
```
integrate(x*ln(c*(b*x+a)**p)/(e*x+d),x)
```

output
```
Integral(x*log(c*(a + b*x)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x \log (c(a+bx)^p)}{d+ex}\,dx = \int \frac{x \log ((bx+a)^p c)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x*log((b*x + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx=\int \frac{x \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x*log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx=\int \frac{x\,\ln \left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x}\,dx$$

input
```
int((x*log(c*(a + b*x)^p))/(d + e*x),x)
```

output
```
int((x*log(c*(a + b*x)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{-2\left(\int \frac{\log((bx+a)^p c)}{be\,x^2+aex+bdx+ad}dx\right) abdep+2\left(\int \frac{\log((bx+a)^p c)}{be\,x^2+aex+bdx+ad}dx\right) b^2d^2p-\log(\left(bx+a\right)^p c)^2\, bd+2\log(\left(bx+a\right)^p}{2b\,e^2p}$$

input
```
int(x*log(c*(b*x+a)^p)/(e*x+d),x)
```

output
```
( - 2*int(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*a*b*d*e*
p + 2*int(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*b**2*d**
2*p - log((a + b*x)**p*c)**2*b*d + 2*log((a + b*x)**p*c)*a*e*p + 2*log((a
+ b*x)**p*c)*b*e*p*x - 2*b*e*p**2*x)/(2*b*e**2*p)
```

3.222 $\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx=\frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

output
```
ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e+p*polylog(2,-e*(b*x+a)/(-a*e+b*
d))/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx=\frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\,\text{PolyLog}\left(2,\frac{e(a+bx)}{-bd+ae}\right)}{e}$$

input
```
Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(
a + b*x))/(-(b*d) + a*e)])/e
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+bx)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{ 2841}$$

$$\frac{\log \left(c(a+bx)^p\right) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{bp \int \frac{\log \left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}dx}{e}$$

$$\Big\downarrow \text{ 2840}$$

$$\frac{\log \left(c(a+bx)^p\right) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e}-\frac{p \int \frac{\log \left(\frac{e(a+bx)}{bd-ae}+1\right)}{a+bx}d(a+bx)}{e}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\log \left(c(a+bx)^p\right) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p \,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

input
```
Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e
*(a + b*x))/(b*d - a*e))])/e
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

method	result
parts	$\frac{\ln(c(bx+a)^p)\ln(ex+d)}{e}-\frac{pb\left(\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{b}+\frac{\ln(ex+d)\ln\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{b}\right)}{e}$
risch	$\frac{\ln((bx+a)^p)\ln(ex+d)}{e}-\frac{p\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{e}+\frac{\left(\frac{i\pi\ \operatorname{csgn}(i(bx+a)^p)\operatorname{csgn}(ic(bx+a)^p)^2}{2}-\right.}{}$

input
```
int(ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(b*x+a)^p)*ln(e*x+d)/e-p*b/e*(dilog(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b+
ln(e*x+d)*ln(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b)
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\int \frac{\log\left((bx+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx$$

input
```
integrate(ln(c*(b*x+a)**p)/(e*x+d),x)
```

output
```
Integral(log(c*(a + b*x)**p)/(d + e*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(57) = 114.

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.03

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\frac{bp\left(\frac{\log(bx+a)\log(ex+d)}{b}-\frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{b}\right)}{e}-\frac{p\log\left(bx+a\right)\log\left(ex+d\right)}{e}+\frac{\log\left((bx+a)^p c\right)\log\left(ex+d\right)}{e}$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
b*p*(log(b*x + a)*log(e*x + d)/b - (log(e*x + d)*log(-(b*e*x + b*d)/(b*d -
 a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/b)/e - p*log(b*x + a)*log(e
*x + d)/e + log((b*x + a)^p*c)*log(e*x + d)/e
```

Giac [F]

$$\int \frac{\log (c(a+bx)^p)}{d+ex} \, dx = \int \frac{\log ((bx+a)^p c)}{ex+d} \, dx$$

input
```
integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (c(a+bx)^p)}{d+ex} \, dx = \int \frac{\ln \left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x} \, dx$$

input
```
int(log(c*(a + b*x)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b*x)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log (c(a+bx)^p)}{d+ex} \, dx = \frac{2\left(\int \frac{\log((bx+a)^p c)}{be\,x^2+aex+bdx+ad}dx\right) aep - 2\left(\int \frac{\log((bx+a)^p c)}{be\,x^2+aex+bdx+ad}dx\right) bdp + \log((bx+a)^p\, c)^2}{2ep}$$

input
```
int(log(c*(b*x+a)^p)/(e*x+d),x)
```

output
```
(2*int(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*a*e*p - 2*i
nt(log((a + b*x)**p*c)/(a*d + a*e*x + b*d*x + b*e*x**2),x)*b*d*p + log((a
+ b*x)**p*c)**2)/(2*e*p)
```

3.223 $\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx = \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d}$$

output
```
ln(-b*x/a)*ln(c*(b*x+a)^p)/d-ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/d-p*
polylog(2,-e*(b*x+a)/(-a*e+b*d))/d+p*polylog(2,1+b*x/a)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx = \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{a+bx}{a}\right)}{d} - \frac{p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}$$

input
```
Integrate[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

output
```
(Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e
*x))/(b*d - a*e)])/d + (p*PolyLog[2, (a + b*x)/a])/d - (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/d
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(a+bx)^p)}{x(d+ex)} \, dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{\log (c(a+bx)^p)}{dx}-\frac{e\log (c(a+bx)^p)}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log (c(a+bx)^p)\log \left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{\log \left(-\frac{bx}{a}\right)\log (c(a+bx)^p)}{d}-\frac{p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d}$$

input
```
Int[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

output
```
(Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e
*x))/(b*d - a*e)])/d - (p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d + (p
*PolyLog[2, 1 + (b*x)/a])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

method	result
parts	$-\frac{\ln(c(bx+a)^p)\ln(ex+d)}{d}+\frac{\ln(c(bx+a)^p)\ln(x)}{d}-pb\left(\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{db}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{db}-\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{db}-\frac{\ln(ex$
risch	$-\frac{\ln((bx+a)^p)\ln(ex+d)}{d}+\frac{\ln((bx+a)^p)\ln(x)}{d}-\frac{p\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{d}-\frac{p\ln(x)\ln\left(\frac{bx+a}{a}\right)}{d}+\frac{p\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{d}+\frac{p\ln(ex-$

input
```
int(ln(c*(b*x+a)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(b*x+a)^p)/d*ln(e*x+d)+ln(c*(b*x+a)^p)/d*ln(x)-p*b*(1/d*dilog((b*x+a
)/a)/b+1/d*ln(x)*ln((b*x+a)/a)/b-1/d*dilog(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/
b-1/d*ln(e*x+d)*ln(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b)
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{x(d+ex)}\,dx=\int \frac{\log\left((bx+a)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^p*c)/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(c(a+bx)^p\right)}{x\,(d+ex)}\,dx$$

input
```
integrate(ln(c*(b*x+a)**p)/x/(e*x+d),x)
```

output
```
Integral(log(c*(a + b*x)**p)/(x*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.27

$$\int \frac{\log\left(c(a+bx)^p\right)}{x(d+ex)}\,dx = -bp\left(\frac{\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{bd} - \frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{bd}\right) - \left(\frac{\log(ex+d)}{d}-\frac{\log(x)}{d}\right)\log\left((bx+a)^p c\right)$$

input
```
integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

output
```
-b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))/(b*d) - (log(e*x + d)*log(-(
b*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/(b*d)) -
 (log(e*x + d)/d - log(x)/d)*log((b*x + a)^p*c)
```

Giac [F]

$$\int \frac{\log (c(a+bx)^p)}{x(d+ex)} \, dx = \int \frac{\log \left((bx+a)^p c\right)}{(ex+d)x} \, dx$$

input
```
integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (c(a+bx)^p)}{x(d+ex)} \, dx = \int \frac{\ln \left(c \, (a+b\,x)^p\right)}{x\, (d+e\,x)} \, dx$$

input
```
int(log(c*(a + b*x)^p)/(x*(d + e*x)),x)
```

output
```
int(log(c*(a + b*x)^p)/(x*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log (c(a+bx)^p)}{x(d+ex)} \, dx = \int \frac{\log \left((bx+a)^p\, c\right)}{e\,x^2+dx} dx$$

input
```
int(log(c*(b*x+a)^p)/x/(e*x+d),x)
```

output
```
int(log((a + b*x)**p*c)/(d*x + e*x**2),x)
```

3.224 $\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \frac{bp\log(x)}{ad} - \frac{bp\log(a+bx)}{ad} - \frac{\log(c(a+bx)^p)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} + \frac{ep\operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2} - \frac{ep\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d^2}$$

output
```
b*p*ln(x)/a/d-b*p*ln(b*x+a)/a/d-ln(c*(b*x+a)^p)/d/x-e*ln(-b*x/a)*ln(c*(b*x
+a)^p)/d^2+e*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/d^2+e*p*polylog(2,-e
*(b*x+a)/(-a*e+b*d))/d^2-e*p*polylog(2,1+b*x/a)/d^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \frac{bp\log(x)}{ad} - \frac{bp\log(a+bx)}{ad} - \frac{\log(c(a+bx)^p)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{a+bx}{a}\right)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2}$$

input
```
Integrate[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

output
```
(b*p*Log[x])/(a*d) - (b*p*Log[a + b*x])/(a*d) - Log[c*(a + b*x)^p]/(d*x) -
 (e*Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d^2 + (e*Log[c*(a + b*x)^p]*Log[(b
*(d + e*x))/(b*d - a*e)])/d^2 - (e*p*PolyLog[2, (a + b*x)/a])/d^2 + (e*p*P
olyLog[2, -((e*(a + b*x))/(b*d - a*e))]])/d^2
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(\frac{e^2\log(c(a+bx)^p)}{d^2(d+ex)} - \frac{e\log(c(a+bx)^p)}{d^2x} + \frac{\log(c(a+bx)^p)}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2}+\frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}-\frac{\log\left(c(a+bx)^p\right)}{dx}+\frac{ep\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^2}+\frac{bp\log(x)}{ad}-\frac{bp\log(a+bx)}{ad}$$

input
```
Int[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

output
```
(b*p*Log[x])/(a*d) - (b*p*Log[a + b*x])/(a*d) - Log[c*(a + b*x)^p]/(d*x) -
 (e*Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d^2 + (e*Log[c*(a + b*x)^p]*Log[(b
*(d + e*x))/(b*d - a*e)])/d^2 + (e*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e
))])/d^2 - (e*p*PolyLog[2, 1 + (b*x)/a])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.38

method	result
parts	$\frac{\ln(c(bx+a)^p)e\ln(ex+d)}{d^2}-\frac{\ln(c(bx+a)^p)}{dx}-\frac{\ln(c(bx+a)^p)e\ln(x)}{d^2}-pb\left(\frac{e\left(\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{b}+\frac{\ln(ex+d)\ln\left(\frac{(ex+d)b+ea-}{ea-bd}\right.}{b}\right.}{d^2}\right.$
risch	$\frac{\ln((bx+a)^p)e\ln(ex+d)}{d^2}-\frac{\ln((bx+a)^p)}{dx}-\frac{\ln((bx+a)^p)e\ln(x)}{d^2}-\frac{pe\operatorname{dilog}\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right)}{d^2}-\frac{pe\ln(ex+d)\ln\left(\frac{(ex+d)b+ea-bd}{ea-bd}\right.}{d^2}$

input
```
int(ln(c*(b*x+a)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(b*x+a)^p)*e/d^2*ln(e*x+d)-ln(c*(b*x+a)^p)/d/x-ln(c*(b*x+a)^p)*e/d^2*
ln(x)-p*b*(e/d^2*(dilog(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b+ln(e*x+d)*ln(((e*
x+d)*b+e*a-b*d)/(a*e-b*d))/b)-1/d/a*ln(x)+1/d/a*ln(b*x+a)-e/d^2*dilog((b*x
+a)/a)/b-e/d^2*ln(x)*ln((b*x+a)/a)/b)
```

Fricas [F]

$$\int \frac{\log \left(c(a+bx)^p\right)}{x^2(d+ex)}\,dx=\int \frac{\log \left((bx+a)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^p*c)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx)^p\right)}{x^2(d+ex)}\,dx=\text{Timed out}$$

input
```
integrate(ln(c*(b*x+a)**p)/x**2/(e*x+d),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.07

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx$$
$$= bp\left(\frac{\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\text{Li}_2\left(-\frac{bx}{a}\right)\right)e}{bd^2}-\frac{\left(\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\text{Li}_2\left(\frac{bex+bd}{bd-ae}\right)\right)e}{bd^2}-\frac{\log(bx+a}{ad}\right.$$
$$+\left(\frac{e\log(ex+d)}{d^2}-\frac{e\log(x)}{d^2}-\frac{1}{dx}\right)\log\left((bx+a)^pc\right)$$

input
```
integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))*e/(b*d^2) - (log(e*x + d)*log
(-(b*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))*e/(b*
d^2) - log(b*x + a)/(a*d) + log(x)/(a*d)) + (e*log(e*x + d)/d^2 - e*log(x)
/d^2 - 1/(d*x))*log((b*x + a)^p*c)
```

Giac [F]

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \int \frac{\log\left((bx+a)^pc\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

input
```
int(log(c*(a + b*x)^p)/(x^2*(d + e*x)),x)
```

output
```
int(log(c*(a + b*x)^p)/(x^2*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \int \frac{\log\left((bx+a)^p\,c\right)}{e\,x^3+d\,x^2}dx$$

input
```
int(log(c*(b*x+a)^p)/x^2/(e*x+d),x)
```

output
```
int(log((a + b*x)**p*c)/(d*x**2 + e*x**3),x)
```

3.225 $\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 227

$$\int \frac{\log (c(a+bx)^p)}{x^3(d+ex)}\,dx = -\frac{bp}{2adx}-\frac{b^2p\log(x)}{2a^2d}-\frac{bep\log(x)}{ad^2}+\frac{b^2p\log(a+bx)}{2a^2d}+\frac{bep\log(a+bx)}{ad^2}-\frac{\log (c(a+bx)^p)}{2dx^2}+\frac{e\log (c(a+bx)^p)}{d^2x}+\frac{e^2\log \left(-\frac{bx}{a}\right)\log (c(a+bx)^p)}{d^3}-\frac{e^2\log (c(a+bx)^p)\log \left(\frac{b(d+ex)}{bd-ae}\right)}{d^3}-\frac{e^2p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d^3}$$

output

```
-1/2*b*p/a/d/x-1/2*b^2*p*ln(x)/a^2/d-b*e*p*ln(x)/a/d^2+1/2*b^2*p*ln(b*x+a)
/a^2/d+b*e*p*ln(b*x+a)/a/d^2-1/2*ln(c*(b*x+a)^p)/d/x^2+e*ln(c*(b*x+a)^p)/d
^2/x+e^2*ln(-b*x/a)*ln(c*(b*x+a)^p)/d^3-e^2*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(
-a*e+b*d))/d^3-e^2*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/d^3+e^2*p*polylog(2,
1+b*x/a)/d^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \frac{\log \left(c(a+bx)^p\right)}{x^3(d+ex)}\,dx = -\frac{\frac{2bdep\log(x)}{a}-\frac{2bdep\log(a+bx)}{a}+\frac{bd^2p(a+bx\log(x)-bx\log(a+bx))}{a^2x}+\frac{d^2\log(c(a+bx)^p)}{x^2}-\frac{2de\log(c(a+bx)^p)}{x}-2e^2\log\left(-\frac{bx}{a}\right)}{}$$

input
```
Integrate[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

output
```
-1/2*((2*b*d*e*p*Log[x])/a - (2*b*d*e*p*Log[a + b*x])/a + (b*d^2*p*(a + b*
x*Log[x] - b*x*Log[a + b*x]))/(a^2*x) + (d^2*Log[c*(a + b*x)^p])/x^2 - (2*
d*e*Log[c*(a + b*x)^p])/x - 2*e^2*Log[-((b*x)/a)]*Log[c*(a + b*x)^p] + 2*e
^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)] + 2*e^2*p*PolyLog[2,
(e*(a + b*x))/(-(b*d) + a*e)] - 2*e^2*p*PolyLog[2, 1 + (b*x)/a])/d^3
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+bx)^p\right)}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{2863}$$

$$\int \left(-\frac{e^3\log \left(c(a+bx)^p\right)}{d^3(d+ex)}+\frac{e^2\log \left(c(a+bx)^p\right)}{d^3x}-\frac{e\log \left(c(a+bx)^p\right)}{d^2x^2}+\frac{\log \left(c(a+bx)^p\right)}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2p\log(x)}{2a^2d}+\frac{b^2p\log(a+bx)}{2a^2d}+\frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^3}-\frac{e^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^3}+\frac{e\log\left(c(a+bx)^p\right)}{d^2x}-\frac{\log\left(c(a+bx)^p\right)}{2dx^2}-\frac{e^2p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^3}+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^3}-\frac{bep\log(x)}{ad^2}+\frac{bep\log(a+bx)}{ad^2}-\frac{bp}{2adx}$$

input
```
Int[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

output
```
-1/2*(b*p)/(a*d*x) - (b^2*p*Log[x])/(2*a^2*d) - (b*e*p*Log[x])/(a*d^2) + (
b^2*p*Log[a + b*x])/(2*a^2*d) + (b*e*p*Log[a + b*x])/(a*d^2) - Log[c*(a +
b*x)^p]/(2*d*x^2) + (e*Log[c*(a + b*x)^p])/(d^2*x) + (e^2*Log[-((b*x)/a)]*
Log[c*(a + b*x)^p])/d^3 - (e^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d -
 a*e)])/d^3 - (e^2*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d^3 + (e^2*
p*PolyLog[2, 1 + (b*x)/a])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.15

method	result
parts	$-\frac{\ln(c(bx+a)^p)e^2\ln(ex+d)}{d^3}-\frac{\ln(c(bx+a)^p)}{2d\,x^2}+\frac{\ln(c(bx+a)^p)e^2\ln(x)}{d^3}+\frac{e\ln(c(bx+a)^p)}{d^2x}-\frac{pb\left(\frac{2e^2\,\mathrm{dilog}\left(\frac{bx+a}{a}\right)}{d^3b}+\frac{2e^2\ln(x)\ln(\ldots}{d^3b}\right.}{}$
risch	$-\frac{\ln((bx+a)^p)e^2\ln(ex+d)}{d^3}-\frac{\ln((bx+a)^p)}{2d\,x^2}+\frac{\ln((bx+a)^p)e^2\ln(x)}{d^3}+\frac{\ln((bx+a)^p)e}{d^2x}-\frac{bep\ln(x)}{a\,d^2}-\frac{b^2p\ln(x)}{2a^2d}-\frac{bp}{2adx}+\ldots$

input
```
int(ln(c*(b*x+a)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(b*x+a)^p)*e^2/d^3*ln(e*x+d)-1/2*ln(c*(b*x+a)^p)/d/x^2+ln(c*(b*x+a)^
p)*e^2/d^3*ln(x)+e*ln(c*(b*x+a)^p)/d^2/x-1/2*p*b*(2*e^2/d^3*dilog((b*x+a)/
a)/b+2*e^2/d^3*ln(x)*ln((b*x+a)/a)/b-2*e^2/d^3*dilog(((e*x+d)*b+e*a-b*d)/(
a*e-b*d))/b-2*e^2/d^3*ln(e*x+d)*ln(((e*x+d)*b+e*a-b*d)/(a*e-b*d))/b-1/d^2*
(-d/a/x+1/a^2*(-2*a*e-b*d)*ln(x)+(2*a*e+b*d)/a^2*ln(b*x+a)))
```

Fricas [F]

$$\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx = \int \frac{\log((bx+a)^p c)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)^p*c)/(e*x^4 + d*x^3), x)
```

Sympy [F]

$$\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx = \int \frac{\log(c(a+bx)^p)}{x^3\,(d+ex)}\,dx$$

input
```
integrate(ln(c*(b*x+a)**p)/x**3/(e*x+d),x)
```

output
```
Integral(log(c*(a + b*x)**p)/(x**3*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95

$$\int \frac{\log \left(c(a+bx)^p\right)}{x^3(d+ex)}\,dx$$
$$= \frac{1}{2}\left(2\,e\left(\frac{\log \left(bx+a\right)}{ad^2}-\frac{\log \left(x\right)}{ad^2}\right)-\frac{2\left(\log \left(\frac{bx}{a}+1\right)\log \left(x\right)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)e^2}{bd^3}+\frac{2\left(\log \left(ex+d\right)\log \left(-\frac{bex+bd}{bd-ae}\right.\right.}{bd^3}\right.$$
$$-\frac{1}{2}\left(\frac{2\,e^2\log \left(ex+d\right)}{d^3}-\frac{2\,e^2\log \left(x\right)}{d^3}-\frac{2\,ex-d}{d^2x^2}\right)\log \left(\left(bx+a\right)^pc\right)$$

input
```
integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*(2*e*(log(b*x + a)/(a*d^2) - log(x)/(a*d^2)) - 2*(log(b*x/a + 1)*log(x
) + dilog(-b*x/a))*e^2/(b*d^3) + 2*(log(e*x + d)*log(-(b*e*x + b*d)/(b*d -
 a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))*e^2/(b*d^3) + b*log(b*x + a
)/(a^2*d) - b*log(x)/(a^2*d) - 1/(a*d*x))*b*p - 1/2*(2*e^2*log(e*x + d)/d^
3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))*log((b*x + a)^p*c)
```

Giac [F]

$$\int \frac{\log \left(c(a+bx)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log \left(\left(bx+a\right)^pc\right)}{\left(ex+d\right)x^3}\,dx$$

input
```
integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^p\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

input
```
int(log(c*(a + b*x)^p)/(x^3*(d + e*x)),x)
```

output
```
int(log(c*(a + b*x)^p)/(x^3*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx = \int \frac{\log\left((bx+a)^p\,c\right)}{e\,x^4+d\,x^3}dx$$

input
```
int(log(c*(b*x+a)^p)/x^3/(e*x+d),x)
```

output
```
int(log((a + b*x)**p*c)/(d*x**3 + e*x**4),x)
```

3.226 $\int \frac{x^3 \log\left(c(a+bx^2)^p\right)}{d+ex} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 394

$$\int \frac{x^3 \log\left(c(a+bx^2)^p\right)}{d+ex} \, dx = -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}d^2p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} - \frac{2a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e} + \frac{d^3p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^4} + \frac{d^2x\log\left(c(a+bx^2)^p\right)}{e^3} + \frac{x^3\log\left(c(a+bx^2)^p\right)}{3e} - \frac{d(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be^2} - \frac{d^3\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^4} + \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^4} + \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^4}$$

output
```
-2*d^2*p*x/e^3+2/3*a*p*x/b/e+1/2*d*p*x^2/e^2-2/9*p*x^3/e+2*a^(1/2)*d^2*p*a
rctan(b^(1/2)*x/a^(1/2))/b^(1/2)/e^3-2/3*a^(3/2)*p*arctan(b^(1/2)*x/a^(1/2
))/b^(3/2)/e+d^3*p*ln(e*((-a)^(1/2)-b^(1/2)*x)/(b^(1/2)*d+(-a)^(1/2)*e))*l
n(e*x+d)/e^4+d^3*p*ln(-e*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*d-(-a)^(1/2)*e))*
ln(e*x+d)/e^4+d^2*x*ln(c*(b*x^2+a)^p)/e^3+1/3*x^3*ln(c*(b*x^2+a)^p)/e-1/2*
d*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b/e^2-d^3*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^4+d^
3*p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d-(-a)^(1/2)*e))/e^4+d^3*p*polylog(
2,b^(1/2)*(e*x+d)/(b^(1/2)*d+(-a)^(1/2)*e))/e^4
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.94

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex} \, dx$$
$$= \frac{-36d^2epx + \frac{12ae^3px}{b} - 4e^3px^3 + \frac{36\sqrt{a}d^2ep\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{12a^{3/2}e^3p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} + 18d^3p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(e}{}$$

input
```
Integrate[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

output
```
(-36*d^2*e*p*x + (12*a*e^3*p*x)/b - 4*e^3*p*x^3 + (36*Sqrt[a]*d^2*e*p*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] - (12*a^(3/2)*e^3*p*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/b^(3/2) + 18*d^3*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqr
t[-a]*e)]*Log[d + e*x] + 18*d^3*p*Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b
]*d) + Sqrt[-a]*e)]*Log[d + e*x] + 18*d^2*e*x*Log[c*(a + b*x^2)^p] + 6*e^3
*x^3*Log[c*(a + b*x^2)^p] - 18*d^3*Log[d + e*x]*Log[c*(a + b*x^2)^p] + 9*d
*e^2*(p*x^2 - ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b) + 18*d^3*p*PolyLog[2,
(Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + 18*d^3*p*PolyLog[2, (Sqrt[
b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/(18*e^4)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log \left(c \left(a+b x^2\right)^p\right)}{d+e x} \, dx$$

$$\Big\downarrow \text{ 2916}$$

$$\int \left(-\frac{d^3 \log \left(c \left(a+b x^2\right)^p\right)}{e^3 (d+e x)}+\frac{d^2 \log \left(c \left(a+b x^2\right)^p\right)}{e^3}-\frac{d x \log \left(c \left(a+b x^2\right)^p\right)}{e^2}+\frac{x^2 \log \left(c \left(a+b x^2\right)^p\right)}{e}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2 a^{3/2} p \arctan \left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{3 b^{3/2} e}+\frac{2 \sqrt{a} d^2 p \arctan \left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b} e^3}-\frac{d^3 \log (d+e x) \log \left(c \left(a+b x^2\right)^p\right)}{e^4}+\frac{d^2 x \log \left(c \left(a+b x^2\right)^p\right)}{e^3}-\frac{d \left(a+b x^2\right) \log \left(c \left(a+b x^2\right)^p\right)}{2 b e^2}+\frac{x^3 \log \left(c \left(a+b x^2\right)^p\right)}{3 e}+\frac{d^3 p \operatorname{PolyLog}\left(2,\frac{\sqrt{b} (d+e x)}{\sqrt{b} d-\sqrt{-a} e}\right)}{e^4}+\frac{d^3 p \operatorname{PolyLog}\left(2,\frac{\sqrt{b} (d+e x)}{\sqrt{b} d+\sqrt{-a} e}\right)}{e^4}+\frac{d^3 p \log (d+e x) \log \left(\frac{e \left(\sqrt{-a}-\sqrt{b} x\right)}{\sqrt{-a} e+\sqrt{b} d}\right)}{e^4}+\frac{d^3 p \log (d+e x) \log \left(-\frac{e \left(\sqrt{-a}+\sqrt{b} x\right)}{\sqrt{b} d-\sqrt{-a} e}\right)}{e^4}+\frac{2 a p x}{3 b e}-\frac{2 d^2 p x}{e^3}+\frac{d p x^2}{2 e^2}-\frac{2 p x^3}{9 e}$$

input
```
Int[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

output
```
(-2*d^2*p*x)/e^3 + (2*a*p*x)/(3*b*e) + (d*p*x^2)/(2*e^2) - (2*p*x^3)/(9*e)
 + (2*Sqrt[a]*d^2*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e^3) - (2*a^(3/2
)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)*e) + (d^3*p*Log[(e*(Sqrt[-a] -
 Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e
*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/e^4 + (d
^2*x*Log[c*(a + b*x^2)^p])/e^3 + (x^3*Log[c*(a + b*x^2)^p])/(3*e) - (d*(a
+ b*x^2)*Log[c*(a + b*x^2)^p])/(2*b*e^2) - (d^3*Log[d + e*x]*Log[c*(a + b*
x^2)^p])/e^4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]
*e)])/e^4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)
])/e^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.05

method	result
parts	$\frac{x^3\ln\left(c\left(bx^2+a\right)^p\right)}{3e} - \frac{\ln\left(c\left(bx^2+a\right)^p\right)x^2 d}{2e^2} + \frac{d^2 x\ln\left(c\left(bx^2+a\right)^p\right)}{e^3} - \frac{d^3\ln(ex+d)\ln\left(c\left(bx^2+a\right)^p\right)}{e^4} - \frac{2pb\left(d^3\left(-\frac{\ln(ex+d)\left(\ln\left(\frac{e}{}\right.\right.}{}\right.\right.}{}$
risch	$\frac{\ln\left(\left(bx^2+a\right)^p\right)x^3}{3e} - \frac{\ln\left(\left(bx^2+a\right)^p\right)d x^2}{2e^2} + \frac{\ln\left(\left(bx^2+a\right)^p\right)x d^2}{e^3} - \frac{\ln\left(\left(bx^2+a\right)^p\right)d^3\ln(ex+d)}{e^4} - \frac{2p x^3}{9e} + \frac{dp x^2}{2e^2} - \frac{2d^2 p x}{e^3} -$

input
```
int(x^3*ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(c*(b*x^2+a)^p)/e-1/2*ln(c*(b*x^2+a)^p)/e^2*x^2*d+d^2*x*ln(c*(b*
x^2+a)^p)/e^3-d^3*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^4-2*p*b/e^2*(d^3/e^2*(-1/2
*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*
(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b-1/2*(dilog((e*(-a*b)^
(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b
-b*d)/(e*(-a*b)^(1/2)-b*d)))/b)+1/6/e^2*(-1/b^2*(2*(e*x+d)*a*e^2-11*(e*x+d
)*b*d^2+7/2*d*(e*x+d)^2*b-2/3*(e*x+d)^3*b)+a*e^2/b^2*(3/2*d*ln((e*x+d)^2*b
-2*d*(e*x+d)*b+a*e^2+b*d^2)+(2*a*e^2-6*b*d^2)/e/(a*b)^(1/2)*arctan(1/2*(2*
(e*x+d)*b-2*b*d)/e/(a*b)^(1/2)))))
```

Fricas [F]

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

input
```
integrate(x**3*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

output
```
Integral(x**3*log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^3\ \ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x}\,dx$$

input
```
int((x^3*log(c*(a + b*x^2)^p))/(d + e*x),x)
```

output
```
int((x^3*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$
$$= \frac{-24\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a\,e^3p^2 + 72\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b\,d^2e\,p^2 - 36\left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{be\,x^3+bd\,x^2+aex+ad}dx\right) a\,b^2d^3ep + \cdots}{}$$

input
```
int(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x)
```

output
```
( - 24*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*e**3*p**2 + 72*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*d**2*e*p**2 - 36*int(log((a +
b*x**2)**p*c)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)*a*b**2*d**3*e*p + 36*
int((log((a + b*x**2)**p*c)*x)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)*b**3
*d**4*p - 9*log((a + b*x**2)**p*c)**2*b**2*d**3 - 18*log((a + b*x**2)**p*c
)*a*b*d*e**2*p + 36*log((a + b*x**2)**p*c)*b**2*d**2*e*p*x - 18*log((a + b
*x**2)**p*c)*b**2*d*e**2*p*x**2 + 12*log((a + b*x**2)**p*c)*b**2*e**3*p*x*
*3 + 24*a*b*e**3*p**2*x - 72*b**2*d**2*e*p**2*x + 18*b**2*d*e**2*p**2*x**2
 - 8*b**2*e**3*p**2*x**3)/(36*b**2*e**4*p)
```

3.227 $\int \frac{x^2 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 313

$$\int \frac{x^2 \log\left(c\left(a+bx^2\right)^p\right)}{d+ex} \, dx = \frac{2dpx}{e^2} - \frac{px^2}{2e} - \frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2} - \frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^3} - \frac{dx\log\left(c\left(a+bx^2\right)^p\right)}{e^2} + \frac{(a+bx^2)\log\left(c\left(a+bx^2\right)^p\right)}{2be} + \frac{d^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^3} - \frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3} - \frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^3}$$

output
```
2*d*p*x/e^2-1/2*p*x^2/e-2*a^(1/2)*d*p*arctan(b^(1/2)*x/a^(1/2))/b^(1/2)/e^
2-d^2*p*ln(e*((-a)^(1/2)-b^(1/2)*x)/(b^(1/2)*d+(-a)^(1/2)*e))*ln(e*x+d)/e^
3-d^2*p*ln(-e*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*d-(-a)^(1/2)*e))*ln(e*x+d)/e
^3-d*x*ln(c*(b*x^2+a)^p)/e^2+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b/e+d^2*ln(e*
x+d)*ln(c*(b*x^2+a)^p)/e^3-d^2*p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d-(-a)
^(1/2)*e))/e^3-d^2*p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d+(-a)^(1/2)*e))/e
^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.99

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{2dpx}{e^2} - \frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2} - \frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^3} - \frac{dx\log\left(c(a+bx^2)^p\right)}{e^2} + \frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3} - \frac{px^2 - \frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}}{2e} - \frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3} - \frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^3}$$

input
```
Integrate[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

output
```
(2*d*p*x)/e^2 - (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e^2)
- (d^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d +
e*x])/e^3 - (d^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*
e))]*Log[d + e*x])/e^3 - (d*x*Log[c*(a + b*x^2)^p])/e^2 + (d^2*Log[d + e*x
]*Log[c*(a + b*x^2)^p])/e^3 - (p*x^2 - ((a + b*x^2)*Log[c*(a + b*x^2)^p])/
b)/(2*e) - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]
)/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e
^3
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(c \left(a+b x^2\right)^p\right)}{d+e x} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{d^2 \log \left(c \left(a+b x^2\right)^p\right)}{e^2 (d+e x)}-\frac{d \log \left(c \left(a+b x^2\right)^p\right)}{e^2}+\frac{x \log \left(c \left(a+b x^2\right)^p\right)}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2 \sqrt{a} d p \arctan \left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{b} e^2}+\frac{d^2 \log (d+e x) \log \left(c \left(a+b x^2\right)^p\right)}{e^3}-\frac{d x \log \left(c \left(a+b x^2\right)^p\right)}{e^2}+\frac{\left(a+b x^2\right) \log \left(c \left(a+b x^2\right)^p\right)}{2 b e}-\frac{d^2 p \operatorname{PolyLog}\left(2,\frac{\sqrt{b} (d+e x)}{\sqrt{b} d-\sqrt{-a} e}\right)}{e^3}-\frac{d^2 p \operatorname{PolyLog}\left(2,\frac{\sqrt{b} (d+e x)}{\sqrt{b} d+\sqrt{-a} e}\right)}{e^3}-\frac{d^2 p \log (d+e x) \log \left(\frac{e \left(\sqrt{-a}-\sqrt{b} x\right)}{\sqrt{-a} e+\sqrt{b} d}\right)}{e^3}-\frac{d^2 p \log (d+e x) \log \left(-\frac{e \left(\sqrt{-a}+\sqrt{b} x\right)}{\sqrt{b} d-\sqrt{-a} e}\right)}{e^3}+\frac{2 d p x}{e^2}-\frac{p x^2}{2 e}$$

input
```
Int[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

output
```
(2*d*p*x)/e^2 - (p*x^2)/(2*e) - (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[b]*e^2) - (d^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[
-a]*e)]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[
b]*d - Sqrt[-a]*e))]*Log[d + e*x])/e^3 - (d*x*Log[c*(a + b*x^2)^p])/e^2 +
((a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*b*e) + (d^2*Log[d + e*x]*Log[c*(a +
b*x^2)^p])/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-
a]*e)])/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*
e)])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.11

method	result
parts	$\frac{x^{2} \ln\left(c \left(b \,x^{2}+a \right)^{p}\right)}{2 e}-\frac{d x \ln\left(c \left(b \,x^{2}+a \right)^{p}\right)}{e^{2}}+\frac{d^{2} \ln\left(e x +d \right) \ln\left(c \left(b \,x^{2}+a \right)^{p}\right)}{e^{3}}-\frac{2 p b \left(\frac{\left(e x +d \right)^{2}}{4 e b}-\frac{3 d \left(e x +d \right)}{2 b e}-\frac{e a \ln\left(\left(e x +d \right)^{2} b -2 d \left(e x +d \right) b\right.}{4 b^{2}}\right.}{}$
risch	$\frac{\ln\left(\left(b \,x^{2}+a \right)^{p}\right) x^{2}}{2 e}-\frac{\ln\left(\left(b \,x^{2}+a \right)^{p}\right) d x}{e^{2}}+\frac{\ln\left(\left(b \,x^{2}+a \right)^{p}\right) d^{2} \ln\left(e x +d \right)}{e^{3}}-\frac{p \,d^{2} \ln\left(e x +d \right) \ln\left(\frac{e \sqrt{-a b}-\left(e x +d \right) b +b d}{e \sqrt{-a b}+b d}\right)}{e^{3}}-\frac{p \,d^{2} \ln\left(e x +d\right.}{}$

input
```
int(x^2*ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(b*x^2+a)^p)/e-d*x*ln(c*(b*x^2+a)^p)/e^2+d^2*ln(e*x+d)*ln(c*(
b*x^2+a)^p)/e^3-2*p*b/e^2*(1/4/e/b*(e*x+d)^2-3/2/b/e*d*(e*x+d)-1/4*e*a/b^2
*ln((e*x+d)^2*b-2*d*(e*x+d)*b+a*e^2+b*d^2)+a/b*d/(a*b)^(1/2)*arctan(1/2*(2
*(e*x+d)*b-2*b*d)/e/(a*b)^(1/2))-d^2/e*(-1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)
-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e
*(-a*b)^(1/2)-b*d)))/b-1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)
^(1/2)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b
))
```

Fricas [F]

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

input
```
integrate(x**2*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

output
```
Integral(x**2*log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^2 \log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^2 \ln\left(c\,(b\,x^2+a)^p\right)}{d+e\,x}\,dx$$

input
```
int((x^2*log(c*(a + b*x^2)^p))/(d + e*x),x)
```

output
```
int((x^2*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$
$$= \frac{-8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)de\,p^2 + 4\left(\int \frac{\log((b\,x^2+a)^p c)}{be\,x^3+bd\,x^2+aex+ad}dx\right)ab\,d^2ep - 4\left(\int \frac{\log((b\,x^2+a)^p c)x}{be\,x^3+bd\,x^2+aex+ad}dx\right)b^2d^3p + \log(}{}$$

input
```
int(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x)
```

output
```
( - 8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*d*e*p**2 + 4*int(log((
a + b*x**2)**p*c)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)*a*b*d**2*e*p - 4*
int((log((a + b*x**2)**p*c)*x)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)*b**2
*d**3*p + log((a + b*x**2)**p*c)**2*b*d**2 + 2*log((a + b*x**2)**p*c)*a*e*
*2*p - 4*log((a + b*x**2)**p*c)*b*d*e*p*x + 2*log((a + b*x**2)**p*c)*b*e**
2*p*x**2 + 8*b*d*e*p**2*x - 2*b*e**2*p**2*x**2)/(4*b*e**3*p)
```

3.228 $\int \frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 256

$$\begin{aligned}\int \frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx &= -\frac{2px}{e}+\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e}\\ &\quad+\frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^2}\\ &\quad+\frac{dp\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^2}\\ &\quad+\frac{x\log\left(c(a+bx^2)^p\right)}{e}-\frac{d\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^2}\\ &\quad+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^2}\end{aligned}$$

output
```
-2*p*x/e+2*a^(1/2)*p*arctan(b^(1/2)*x/a^(1/2))/b^(1/2)/e+d*p*ln(e*((-a)^(1
/2)-b^(1/2)*x)/(b^(1/2)*d+(-a)^(1/2)*e))*ln(e*x+d)/e^2+d*p*ln(-e*((-a)^(1/
2)+b^(1/2)*x)/(b^(1/2)*d-(-a)^(1/2)*e))*ln(e*x+d)/e^2+x*ln(c*(b*x^2+a)^p)/
e-d*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^2+d*p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)
*d-(-a)^(1/2)*e))/e^2+d*p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d+(-a)^(1/2)*
e))/e^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.93

$$\int \frac{x \log \left(c(a+bx^2)^p\right)}{d+ex} \, dx = \frac{-2epx + \frac{2\sqrt{a}ep \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + dp \log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + dp\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + ex\,\mathrm{l}}{}$$

input
```
Integrate[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

output
```
(-2*e*p*x + (2*Sqrt[a]*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + d*p*Log[
(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x] + d*p*Lo
g[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)]*Log[d + e*x] + e
*x*Log[c*(a + b*x^2)^p] - d*Log[d + e*x]*Log[c*(a + b*x^2)^p] + d*p*PolyLo
g[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + d*p*PolyLog[2, (Sqrt[
b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^2
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log \left(c(a+bx^2)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{\log \left(c(a+bx^2)^p\right)}{e} - \frac{d \log \left(c(a+bx^2)^p\right)}{e(d+ex)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e}-\frac{d\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{x\log\left(c(a+bx^2)^p\right)}{e}+\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}+\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}-\frac{2px}{e}$$

input
```
Int[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

output
```
(-2*p*x)/e + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e) + (d*p*
Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^2
 + (d*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e^2 + (x*Log[c*(a + b*x^2)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^
2)^p])/e^2 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]
)/e^2 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.03

method	result
parts	$\frac{x\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{e}-\frac{d\ln(ex+d)\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{e^{2}}-\frac{2pb\left(\frac{ex+d}{b}-\frac{ae\arctan\left(\frac{2(ex+d)b-2bd}{2e\sqrt{ab}}\right)}{b\sqrt{ab}}+d\left(-\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right.\right.}{2b}\right.\right.}{}$
risch	$\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)x}{e}-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)d\ln(ex+d)}{e^{2}}-\frac{2px}{e}-\frac{2pd}{e^{2}}+\frac{2pa\arctan\left(\frac{2(ex+d)b-2bd}{2e\sqrt{ab}}\right)}{e\sqrt{ab}}+\frac{pd\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}-(ex+d)b}{e\sqrt{-ab}+bd}\right.}{e^{2}}$

input
```
int(x*ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(b*x^2+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^2-2*p*b/e^2*(1/b*(e*
x+d)-a*e/b/(a*b)^(1/2)*arctan(1/2*(2*(e*x+d)*b-2*b*d)/e/(a*b)^(1/2))+d*(-1
/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((
e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b-1/2*(dilog((e*(-a*b
)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)
*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b))
```

Fricas [F]

$$\int \frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx=\int \frac{x\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

input
```
integrate(x*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

output
```
Integral(x*log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x}\,dx$$

input
```
int((x*log(c*(a + b*x^2)^p))/(d + e*x),x)
```

output
```
int((x*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) e\,p^2 - 4\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)}{be\,x^3+bd\,x^2+aex+ad}dx\right) abdep + 4\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)x}{be\,x^3+bd\,x^2+aex+ad}dx\right) b^2d^2p - \log\left((b\,x^2\right.}{4b\,e^2p}$$

input
```
int(x*log(c*(b*x^2+a)^p)/(e*x+d),x)
```

output
```
(8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*e*p**2 - 4*int(log((a + b
*x**2)**p*c)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)*a*b*d*e*p + 4*int((log
((a + b*x**2)**p*c)*x)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)*b**2*d**2*p
- log((a + b*x**2)**p*c)**2*b*d + 4*log((a + b*x**2)**p*c)*b*e*p*x - 8*b*e
*p**2*x)/(4*b*e**2*p)
```

3.229 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 201

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}$$

output
```
-p*ln(e*((-a)^(1/2)-b^(1/2)*x)/(b^(1/2)*d+(-a)^(1/2)*e))*ln(e*x+d)/e-p*ln(
-e*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*d-(-a)^(1/2)*e))*ln(e*x+d)/e+ln(e*x+d)*
ln(c*(b*x^2+a)^p)/e-p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d-(-a)^(1/2)*e))/
e-p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d+(-a)^(1/2)*e))/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(c \left(a+b x^2\right)^p\right)}{d+e x} \, dx=-\frac{p \log \left(\frac{e \left(\sqrt{-a}-\sqrt{b} x\right)}{\sqrt{b} d+\sqrt{-a} e}\right) \log (d+e x)}{e}-\frac{p \log \left(-\frac{e \left(\sqrt{-a}+\sqrt{b} x\right)}{\sqrt{b} d-\sqrt{-a} e}\right) \log (d+e x)}{e}+\frac{\log (d+e x) \log \left(c \left(a+b x^2\right)^p\right)}{e}-\frac{p \operatorname{PolyLog}\left(2,\frac{\sqrt{b} (d+e x)}{\sqrt{b} d-\sqrt{-a} e}\right)}{e}-\frac{p \operatorname{PolyLog}\left(2,\frac{\sqrt{b} (d+e x)}{\sqrt{b} d+\sqrt{-a} e}\right)}{e}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

output
```
-((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x]
)/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[
d + e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[
b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e
*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2912, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c \left(a+b x^2\right)^p\right)}{d+e x} \, dx$$

$$\Big\downarrow \text{ 2912}$$

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{2bp\int\frac{x\log(d+ex)}{bx^2+a}dx}{e}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{2bp\int\left(\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e}-\frac{2bp\left(\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{2b}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{2b}+\frac{\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{2b}+\frac{\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{2b}\right)}{e}$$

input
```
Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

output
```
(Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (2*b*p*((Log[(e*(Sqrt[-a] - Sqrt[b
]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/(2*b) + (Log[-((e*(Sqrt[-a]
+ Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/(2*b) + PolyLog[2,
(Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]/(2*b) + PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)]/(2*b)))/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98

method	result
parts	$\frac{\ln(ex+d)\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{e}-\frac{2pb\left(\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\ln\left(\frac{e\sqrt{-ab}+(ex+d)b-bd}{e\sqrt{-ab}-bd}\right)\right)}{2b}+\frac{\operatorname{dilog}\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\operatorname{dilog}}{2b}\right.}{e}$
risch	$\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln(ex+d)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}+(ex+d)b-bd}{e\sqrt{-ab}-bd}\right)}{e}-\frac{p\operatorname{dilog}\left(\frac{e\sqrt{-ab}-(ex+}{e\sqrt{-ab}+}\right.}{e}$

input
```
int(ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-2*p*b/e*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(
e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(
-a*b)^(1/2)-b*d)))/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(
1/2)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b)
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx=\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

output
```
Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx=\int \frac{\ln \left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x} \,dx$$

input
```
int(log(c*(a + b*x^2)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b*x^2)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{4\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)}{be\,x^3+bd\,x^2+aex+ad}dx\right) aep-4\left(\int \frac{\log\left(\left(b\,x^2+a\right)^p c\right)x}{be\,x^3+bd\,x^2+aex+ad}dx\right) bdp+\log\left(\left(b\,x^2+a\right)^p c\right)^2}{4ep}$$

input
```
int(log(c*(b*x^2+a)^p)/(e*x+d),x)
```

output
```
(4*int(log((a + b*x**2)**p*c)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)*a*e*p
 - 4*int((log((a + b*x**2)**p*c)*x)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),x)
*b*d*p + log((a + b*x**2)**p*c)**2)/(4*e*p)
```

3.230 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x(d+ex)}\,dx = \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d} - \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{2d}$$

output
```
p*ln(e*((-a)^(1/2)-b^(1/2)*x)/(b^(1/2)*d+(-a)^(1/2)*e))*ln(e*x+d)/d+p*ln(-
e*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*d-(-a)^(1/2)*e))*ln(e*x+d)/d+1/2*ln(-b*x
^2/a)*ln(c*(b*x^2+a)^p)/d-ln(e*x+d)*ln(c*(b*x^2+a)^p)/d+p*polylog(2,b^(1/2
)*(e*x+d)/(b^(1/2)*d-(-a)^(1/2)*e))/d+p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)
*d+(-a)^(1/2)*e))/d+1/2*p*polylog(2,1+b*x^2/a)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.93

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx$$
$$= \frac{2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)+2p\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)+\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)-}{}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```

output
```
(2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x]
 + 2*p*Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)]*Log[d +
 e*x] + Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] - 2*Log[d + e*x]*Log[c*(a +
 b*x^2)^p] + 2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]
+ 2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + p*PolyLog
[2, 1 + (b*x^2)/a])/(2*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx$$
$$\Big\downarrow \text{2916}$$
$$\int \left(\frac{\log\left(c(a+bx^2)^p\right)}{dx}-\frac{e\log\left(c(a+bx^2)^p\right)}{d(d+ex)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d}+\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d}+\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d}$$

input
```
Int[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```

output
```
(p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/
d + (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d +
 e*x])/d + (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d) - (Log[d + e*x]*
Log[c*(a + b*x^2)^p])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - S
qrt[-a]*e)])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e
)])/d + (p*PolyLog[2, 1 + (b*x^2)/a])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

method	result
parts	$-\frac{\ln(ex+d)\ln\left(c\left(b\,x^2+a\right)^p\right)}{d}+\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)\ln(x)}{d}-2pb\left(\frac{\frac{\ln(x)\left(\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2b}+\frac{\operatorname{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilo}}{2b}}{d}\right.$
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)\ln(ex+d)}{d}+\frac{\ln\left(\left(b\,x^2+a\right)^p\right)\ln(x)}{d}-\frac{p\ln(x)\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}-\frac{p\ln(x)\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}-\frac{p\operatorname{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}$

input
```
int(ln(c*(b*x^2+a)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-ln(e*x+d)*ln(c*(b*x^2+a)^p)/d+ln(c*(b*x^2+a)^p)/d*ln(x)-2*p*b*(1/d*(1/2*l
n(x)*(ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln((b*x+(-a*b)^(1/2))/(-a*b)^(1
/2)))/b+1/2*(dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2)))/b)-1/d*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d
)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-
b*d)))/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+d
ilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx=\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)}{x\left(d+ex\right)}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x/(e*x+d),x)
```

output
```
Integral(log(c*(a + b*x**2)**p)/(x*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)/(x*(d + e*x)),x)
```

output
```
int(log(c*(a + b*x^2)^p)/(x*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left((b\,x^2+a)^p\,c\right)}{e\,x^2+dx}dx$$

input
```
int(log(c*(b*x^2+a)^p)/x/(e*x+d),x)
```

output
```
int(log((a + b*x**2)**p*c)/(d*x + e*x**2),x)
```

3.231 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 306

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{ep\log\left(\frac{e(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(\sqrt{-a}+\sqrt{b}x)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{\log\left(c(a+bx^2)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2} - \frac{ep\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^2} - \frac{ep\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d^2} - \frac{ep\,\mathrm{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{2d^2}$$

output
```
2*b^(1/2)*p*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/d-e*p*ln(e*((-a)^(1/2)-b^(1/
2)*x)/(b^(1/2)*d+(-a)^(1/2)*e))*ln(e*x+d)/d^2-e*p*ln(-e*((-a)^(1/2)+b^(1/2
)*x)/(b^(1/2)*d-(-a)^(1/2)*e))*ln(e*x+d)/d^2-ln(c*(b*x^2+a)^p)/d/x-1/2*e*l
n(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d^2+e*ln(e*x+d)*ln(c*(b*x^2+a)^p)/d^2-e*p*po
lylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d-(-a)^(1/2)*e))/d^2-e*p*polylog(2,b^(1/2
)*(e*x+d)/(b^(1/2)*d+(-a)^(1/2)*e))/d^2-1/2*e*p*polylog(2,1+b*x^2/a)/d^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.92

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx =$$

$$-\frac{-\frac{4\sqrt{b}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + 2ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + 2ep\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + \frac{2d\log(c(}{}}{}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```

output
```
-1/2*((-4*Sqrt[b]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + 2*e*p*Log[(e*
(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x] + 2*e*p*Log
[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)]*Log[d + e*x] + (2
*d*Log[c*(a + b*x^2)^p])/x + e*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] - 2*
e*Log[d + e*x]*Log[c*(a + b*x^2)^p] + 2*e*p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d - Sqrt[-a]*e)] + 2*e*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]
*d + Sqrt[-a]*e)] + e*p*PolyLog[2, 1 + (b*x^2)/a])/d^2
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{ 2916}$$

$$\int \left(\frac{e^2\log\left(c(a+bx^2)^p\right)}{d^2(d+ex)} - \frac{e\log\left(c(a+bx^2)^p\right)}{d^2x} + \frac{\log\left(c(a+bx^2)^p\right)}{dx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2}+\frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2}-\frac{\log\left(c(a+bx^2)^p\right)}{dx}-\frac{ep\operatorname{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^2}$$

input
```
Int[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```

output
```
(2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d) - (e*p*Log[(e*(Sqrt[
-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d^2 - (e*p*Log[-
((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/d^2 -
 Log[c*(a + b*x^2)^p]/(d*x) - (e*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(
2*d^2) + (e*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^2 - (e*p*PolyLog[2, (Sqrt
[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(
d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^2)/a])
/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.16

method	result
parts	$\frac{e\ln(ex+d)\ln\left(c\left(bx^2+a\right)^p\right)}{d^2}-\frac{\ln\left(c\left(bx^2+a\right)^p\right)}{dx}-\frac{\ln\left(c\left(bx^2+a\right)^p\right)e\ln(x)}{d^2}-2pb\left(e\left(\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\ln\left(e\sqrt{\ldots}\right.\right.}{2b}\right.\right.$
risch	$\frac{\ln\left(\left(bx^2+a\right)^p\right)e\ln(ex+d)}{d^2}-\frac{\ln\left(\left(bx^2+a\right)^p\right)}{dx}-\frac{\ln\left(\left(bx^2+a\right)^p\right)e\ln(x)}{d^2}-\frac{pe\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)}{d^2}-\frac{pe\ln(ex+d)\ln\ldots}{}$

input
```
int(ln(c*(b*x^2+a)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
e*ln(e*x+d)*ln(c*(b*x^2+a)^p)/d^2-ln(c*(b*x^2+a)^p)/d/x-ln(c*(b*x^2+a)^p)*
e/d^2*ln(x)-2*p*b*(e/d^2*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)
/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b
*d)))/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+di
log((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b)-1/d/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))-e/d^2*(1/2*ln(x)*(ln((-b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))+ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b+1/2*(dilog((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))+dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2(d+ex)}\,dx=\int \frac{\log\left(\left(bx^2+a\right)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x**2/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)}{x^2\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)/(x^2*(d + e*x)),x)
```

output
```
int(log(c*(a + b*x^2)^p)/(x^2*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)px - \left(\int \frac{\log\left((b\,x^2+a)^p c\right)}{e\,x^2+dx}dx\right)aex - \log\left((b\,x^2+a)^p\,c\right)a}{adx}$$

input
```
int(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*p*x - int(log((a + b*x**2
)**p*c)/(d*x + e*x**2),x)*a*e*x - log((a + b*x**2)**p*c)*a)/(a*d*x)
```

3.232 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 371

$$\begin{aligned}\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3(d+ex)}\,dx = &-\frac{2\sqrt{b}ep\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2}+\frac{bp\log(x)}{ad} \\ &+\frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^3} \\ &+\frac{e^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^3} \\ &-\frac{bp\log\left(a+bx^2\right)}{2ad}-\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2dx^2} \\ &+\frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x}+\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^3} \\ &-\frac{e^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^3} \\ &+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3} \\ &+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{2d^3}\end{aligned}$$

output
```
-2*b^(1/2)*e*p*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/d^2+b*p*ln(x)/a/d+e^2*p*l
n(e*((-a)^(1/2)-b^(1/2)*x)/(b^(1/2)*d+(-a)^(1/2)*e))*ln(e*x+d)/d^3+e^2*p*l
n(-e*((-a)^(1/2)+b^(1/2)*x)/(b^(1/2)*d-(-a)^(1/2)*e))*ln(e*x+d)/d^3-1/2*b*
p*ln(b*x^2+a)/a/d-1/2*ln(c*(b*x^2+a)^p)/d/x^2+e*ln(c*(b*x^2+a)^p)/d^2/x+1/
2*e^2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d^3-e^2*ln(e*x+d)*ln(c*(b*x^2+a)^p)/d
^3+e^2*p*polylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d-(-a)^(1/2)*e))/d^3+e^2*p*pol
ylog(2,b^(1/2)*(e*x+d)/(b^(1/2)*d+(-a)^(1/2)*e))/d^3+1/2*e^2*p*polylog(2,1
+b*x^2/a)/d^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.94

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^3(d+ex)} \, dx$$
$$= \frac{-\frac{4\sqrt{b}dep\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{2bd^2p\log(x)}{a} + 2e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + 2e^2p\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)\log(d+e}{}$$

input
```
Integrate[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

output
```
((-4*Sqrt[b]*d*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (2*b*d^2*p*Log[x
])/a + 2*e^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Lo
g[d + e*x] + 2*e^2*p*Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-
a]*e)]*Log[d + e*x] - (b*d^2*p*Log[a + b*x^2])/a - (d^2*Log[c*(a + b*x^2)^
p])/x^2 + (2*d*e*Log[c*(a + b*x^2)^p])/x - 2*e^2*Log[d + e*x]*Log[c*(a + b
*x^2)^p] + 2*e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)
] + 2*e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + e^2
*(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 + (b*x^2)/a]))/(
2*d^3)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^2\right)^p\right)}{x^3(d+ex)} \, dx$$

$$\Big\downarrow \text{ 2916}$$

$$\int \left(-\frac{e^3 \log \left(c\left(a+bx^2\right)^p\right)}{d^3(d+ex)}+\frac{e^2 \log \left(c\left(a+bx^2\right)^p\right)}{d^3 x}-\frac{e \log \left(c\left(a+bx^2\right)^p\right)}{d^2 x^2}+\frac{\log \left(c\left(a+bx^2\right)^p\right)}{d x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\sqrt{b}ep \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2}+\frac{e^2 \log\left(-\frac{bx^2}{a}\right)\log \left(c\left(a+bx^2\right)^p\right)}{2d^3}-\frac{e^2\log(d+ex)\log \left(c\left(a+bx^2\right)^p\right)}{d^3}+\frac{e \log \left(c\left(a+bx^2\right)^p\right)}{d^2 x}-\frac{\log \left(c\left(a+bx^2\right)^p\right)}{2dx^2}+\frac{e^2 p \operatorname{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^3}+\frac{e^2 p \operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}+\frac{e^2 p \operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d^3}+\frac{e^2 p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d^3}+\frac{e^2 p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}-\frac{bp\log\left(a+bx^2\right)}{2ad}+\frac{bp\log(x)}{ad}$$

input
```
Int[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

output
```
(-2*Sqrt[b]*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d^2) + (b*p*Log[x])/
(a*d) + (e^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Lo
g[d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqr
t[-a]*e))]*Log[d + e*x])/d^3 - (b*p*Log[a + b*x^2])/(2*a*d) - Log[c*(a + b
*x^2)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^2)^p])/(d^2*x) + (e^2*Log[-((b*x^2)
/a)]*Log[c*(a + b*x^2)^p])/(2*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*x^2)^p
])/d^3 + (e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/
d^3 + (e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^3
 + (e^2*p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.11

method	result
parts	$-\frac{e^{2} \ln \left(e x +d \right) \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}{d^{3}}-\frac{\ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}{2 d \,x^{2}}+\frac{\ln \left(c \left(b \,x^{2}+a \right)^{p}\right) e^{2} \ln \left(x \right)}{d^{3}}+\frac{e \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}{d^{2} x}-p b \left(\frac{\ln \left(b \,x^{2}+a \right)}{2 d a}+\frac{2 e a}{}\right.$
risch	$-\frac{\ln \left(\left(b \,x^{2}+a \right)^{p}\right) e^{2} \ln \left(e x +d \right)}{d^{3}}-\frac{\ln \left(\left(b \,x^{2}+a \right)^{p}\right)}{2 d \,x^{2}}+\frac{\ln \left(\left(b \,x^{2}+a \right)^{p}\right) e^{2} \ln \left(x \right)}{d^{3}}+\frac{\ln \left(\left(b \,x^{2}+a \right)^{p}\right) e}{d^{2} x}-\frac{b p \ln \left(b \,x^{2}+a \right)}{2 a d}-\frac{2 p b e \arctan}{d^{2} \sqrt{a}}$

input
```
int(ln(c*(b*x^2+a)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-e^2*ln(e*x+d)*ln(c*(b*x^2+a)^p)/d^3-1/2*ln(c*(b*x^2+a)^p)/d/x^2+ln(c*(b*x
^2+a)^p)*e^2/d^3*ln(x)+e*ln(c*(b*x^2+a)^p)/d^2/x-p*b*(1/2/d/a*ln(b*x^2+a)+
2/d^2*e/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/d/a*ln(x)+2*e^2/d^3*(1/2*ln(
x)*(ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
)))/b+1/2*(dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2)))/b)-2*e^2/d^3*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b
+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1
/2)-b*d)))/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d
))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b))
```

Fricas [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^p*c)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**2+a)**p)/x**3/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)}{x^3\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b*x^2)^p)/(x^3*(d + e*x)),x)
```

output
```
int(log(c*(a + b*x^2)^p)/(x^3*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log(c(a+bx^2)^p)}{x^3(d+ex)}\,dx$$
$$= \frac{-4\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ep\,x^2 + 2\left(\int \frac{\log((b\,x^2+a)^p c)}{be\,x^4+bd\,x^3+ae\,x^2+adx}dx\right)a^2e^2x^2 + 2\left(\int \frac{\log((b\,x^2+a)^p c)x}{be\,x^3+bd\,x^2+aex+ad}dx\right)ab\,e^2x^2 - \cdots}{2a\,d^2x^2}$$

input
```
int(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x)
```

output
```
( - 4*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*e*p*x**2 + 2*int(log((
a + b*x**2)**p*c)/(a*d*x + a*e*x**2 + b*d*x**3 + b*e*x**4),x)*a**2*e**2*x*
*2 + 2*int((log((a + b*x**2)**p*c)*x)/(a*d + a*e*x + b*d*x**2 + b*e*x**3),
x)*a*b*e**2*x**2 - log((a + b*x**2)**p*c)*a*d + 2*log((a + b*x**2)**p*c)*a
*e*x - log((a + b*x**2)**p*c)*b*d*x**2 + 2*log(x)*b*d*p*x**2)/(2*a*d**2*x*
*2)
```

3.233 $\int \frac{x^3 \log \left(c \left(a+b x^3\right)^p\right)}{d+e x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 692

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{3d^2px}{e^3}+\frac{3dpx^2}{4e^2}-\frac{px^3}{3e}-\frac{\sqrt{3}\sqrt[3]{a}d^2p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^3}$$
$$+\frac{\sqrt{3}a^{2/3}dp\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e^2}$$
$$+\frac{\sqrt[3]{a}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^3}+\frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2}$$
$$+\frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$
$$+\frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$
$$+\frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$
$$-\frac{\sqrt[3]{a}d^2p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^3}$$
$$-\frac{a^{2/3}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e^2}$$
$$+\frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3}-\frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2}$$
$$+\frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3be}$$
$$-\frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4}$$
$$+\frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4}$$
$$+\frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^4}$$
$$+\frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4}$$

output
```
-3*d^2*p*x/e^3+3/4*d*p*x^2/e^2-1/3*p*x^3/e-3^(1/2)*a^(1/3)*d^2*p*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))/b^(1/3)/e^3+1/2*3^(1/2)*a^(2/3)*d
*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))/b^(2/3)/e^2+a^(1/3)*d
^2*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/e^3+1/2*a^(2/3)*d*p*ln(a^(1/3)+b^(1/3)*
x)/b^(2/3)/e^2+d^3*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e
*x+d)/e^4+d^3*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)
*a^(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/
3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/e^4-1/2*a^(1/3)*d^2*p*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)/e^3-1/4*a^(2/3)*d*p*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/e^2+d^2*x*ln(c*(b*x^3+a)^p)/e^
3-1/2*d*x^2*ln(c*(b*x^3+a)^p)/e^2+1/3*(b*x^3+a)*ln(c*(b*x^3+a)^p)/b/e-d^3*
ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^4+d^3*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d
-a^(1/3)*e))/e^4+d^3*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(
1/3)*e))/e^4+d^3*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)
*e))/e^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.47 (sec) , antiderivative size = 581, normalized size of antiderivative = 0.84

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx$$

$$= \frac{-36bd^2epx + 9bde^2px^2 - 4be^3px^3 - 12\sqrt{3}\sqrt[3]{a}b^{2/3}d^2ep\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 9bde^2px^2\,\text{Hypergeometric2F}}{}$$

input
```
Integrate[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

output
```
(-36*b*d^2*e*p*x + 9*b*d*e^2*p*x^2 - 4*b*e^3*p*x^3 - 12*Sqrt[3]*a^(1/3)*b^
(2/3)*d^2*e*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 9*b*d*e^2*p*x^
2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + 12*a^(1/3)*b^(2/3)*d^2*e*
p*Log[a^(1/3) + b^(1/3)*x] + 12*b*d^3*p*Log[(e*((-1)^(1/3)*a^(1/3) - b^(1/
3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x] + 12*b*d^3*p*Log[(
e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*Log[d + e*x] + 12*b*d
^3*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + (-1)^(2/3)*a
^(1/3)*e)]*Log[d + e*x] - 6*a^(1/3)*b^(2/3)*d^2*e*p*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2] + 4*a*e^3*Log[c*(a + b*x^3)^p] + 12*b*d^2*e*x*Log
[c*(a + b*x^3)^p] - 6*b*d*e^2*x^2*Log[c*(a + b*x^3)^p] + 4*b*e^3*x^3*Log[c
*(a + b*x^3)^p] - 12*b*d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 12*b*d^3*p*
PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)] + 12*b*d^3*p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + 12*b*d^3*p
*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/(12*b
*e^4)
```

Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log \left(c\left(a+bx^3\right)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(-\frac{d^3 \log \left(c\left(a+bx^3\right)^p\right)}{e^3 (d+ex)}+\frac{d^2 \log \left(c\left(a+bx^3\right)^p\right)}{e^3}-\frac{dx \log \left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+bx^3\right)^p\right)}{e}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{3}a^{2/3}dp\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e^2}-\frac{\sqrt[3]{a}d^2p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^3}-\frac{a^{2/3}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e^2}+\frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2}-\frac{\sqrt{3}\sqrt[3]{a}d^2p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^3}-\frac{d^3\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^4}+\frac{d^2x\log\left(c\left(a+bx^3\right)^p\right)}{e^3}-\frac{dx^2\log\left(c\left(a+bx^3\right)^p\right)}{2e^2}+\frac{\left(a+bx^3\right)\log\left(c\left(a+bx^3\right)^p\right)}{3be}+\frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4}+\frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^4}+\frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^4}+\frac{\sqrt[3]{a}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^3}-\frac{3d^2px}{e^3}+\frac{3dpx^2}{4e^2}-\frac{px^3}{3e}$$

input
```
Int[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

output
```
(-3*d^2*p*x)/e^3 + (3*d*p*x^2)/(4*e^2) - (p*x^3)/(3*e) - (Sqrt[3]*a^(1/3)*
d^2*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^3) + (
Sqrt[3]*a^(2/3)*d*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*
b^(2/3)*e^2) + (a^(1/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^3) + (a
^(2/3)*d*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)*e^2) + (d^3*p*Log[-((e*(a^
(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*L
og[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*
e))]*Log[d + e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^4 - (a^(1/3)
*d^2*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e^3) - (
a^(2/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)*e^2
) + (d^2*x*Log[c*(a + b*x^3)^p])/e^3 - (d*x^2*Log[c*(a + b*x^3)^p])/(2*e^2
) + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/(3*b*e) - (d^3*Log[d + e*x]*Log[c*(
a + b*x^3)^p])/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^
(1/3)*e)])/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(
1/3)*a^(1/3)*e)])/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d -
 (-1)^(2/3)*a^(1/3)*e)])/e^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.30 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.44

method	result
parts	$\frac{\ln(c(b\,x^3+a)^p)x^3}{3e} - \frac{d\,x^2\ln(c(b\,x^3+a)^p)}{2e^2} + \frac{d^2x\ln(c(b\,x^3+a)^p)}{e^3} - \frac{d^3\ln(ex+d)\ln(c(b\,x^3+a)^p)}{e^4} - \frac{3pb\left(-\frac{-\frac{2(ex+d)^3}{3}-\frac{7d(ex}{2}}{}\right.}{}$
risch	$\frac{\ln((b\,x^3+a)^p)x^3}{3e} - \frac{\ln((b\,x^3+a)^p)d\,x^2}{2e^2} + \frac{\ln((b\,x^3+a)^p)x\,d^2}{e^3} - \frac{\ln((b\,x^3+a)^p)d^3\ln(ex+d)}{e^4} - \frac{p\,x^3}{3e} + \frac{3dp\,x^2}{4e^2} - \frac{3d^2px}{e^3} -$

input
```
int(x^3*ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(c*(b*x^3+a)^p)/e*x^3-1/2*d*x^2*ln(c*(b*x^3+a)^p)/e^2+d^2*x*ln(c*(b*
x^3+a)^p)/e^3-d^3*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^4-3*p*b/e^3*(-1/6/e*(-1/b*
(2/3*(e*x+d)^3-7/2*d*(e*x+d)^2+11*d^2*(e*x+d))+1/3/b^2*sum((2*_R^2-7*_R*d+
11*d^2)/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*
d^2+a*e^3-b*d^3))*a*e^3)-1/3*d^3/e/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+di
log((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3)
))
```

Fricas [F]

$$\int \frac{x^3\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^3\log\left((bx^3+a)^pc\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex} \, dx = \text{Timed out}$$

input
```
integrate(x**3*ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex} \, dx = \int \frac{x^3 \log \left((bx^3+a)^p c\right)}{ex+d} \, dx$$

input
```
integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex} \, dx = \int \frac{x^3 \log \left((bx^3+a)^p c\right)}{ex+d} \, dx$$

input
```
integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^3\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x}\,dx$$

input
```
int((x^3*log(c*(a + b*x^3)^p))/(d + e*x),x)
```

output
```
int((x^3*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{-12b^{\frac{4}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)d^2e\,p^2+6\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)abd\,e^2p^2+18b^{\frac{4}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)d^2e\,p^2-6b^{\frac{4}{3}}}{}$$

input
```
int(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x)
```

output
```
( - 12*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*b*d**2*e*p**2 + 6*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/
3)*sqrt(3)))*a*b*d*e**2*p**2 + 18*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3
)*x)*b*d**2*e*p**2 - 6*b**(1/3)*a**(2/3)*log((a + b*x**3)**p*c)*b*d**2*e*p
 - 12*b**(2/3)*a**(1/3)*int(log((a + b*x**3)**p*c)/(a*d + a*e*x + b*d*x**3
 + b*e*x**4),x)*a*b*d**3*e*p + 12*b**(2/3)*a**(1/3)*int((log((a + b*x**3)*
*p*c)*x**2)/(a*d + a*e*x + b*d*x**3 + b*e*x**4),x)*b**2*d**4*p - 2*b**(2/3
)*a**(1/3)*log((a + b*x**3)**p*c)**2*b*d**3 + 4*b**(2/3)*a**(1/3)*log((a +
 b*x**3)**p*c)*a*e**3*p + 12*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*b*d*
*2*e*p*x - 6*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*b*d*e**2*p*x**2 + 4*
b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*b*e**3*p*x**3 - 36*b**(2/3)*a**(1
/3)*b*d**2*e*p**2*x + 9*b**(2/3)*a**(1/3)*b*d*e**2*p**2*x**2 - 4*b**(2/3)*
a**(1/3)*b*e**3*p**2*x**3 + 9*log(a**(1/3) + b**(1/3)*x)*a*b*d*e**2*p**2 -
 3*log((a + b*x**3)**p*c)*a*b*d*e**2*p)/(12*b**(2/3)*a**(1/3)*b*e**4*p)
```

3.234 $\int \frac{x^2 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 643

$$\int \frac{x^2 \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{3dpx}{e^2} - \frac{3px^2}{4e} + \frac{\sqrt{3}\sqrt[3]{a}dp\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^2} - \frac{\sqrt{3}a^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e} - \frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^2} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e} - \frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^3} + \frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^2} + \frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e} - \frac{dx\log\left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2\log\left(c(a+bx^3)^p\right)}{2e} + \frac{d^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3}$$

output
```
3*d*p*x/e^2-3/4*p*x^2/e+3^(1/2)*a^(1/3)*d*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)*3^(1/2)/a^(1/3))/b^(1/3)/e^2-1/2*3^(1/2)*a^(2/3)*p*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)*3^(1/2)/a^(1/3))/b^(2/3)/e-a^(1/3)*d*p*ln(a^(1/3)+b^(1/3)*x)/
b^(1/3)/e^2-1/2*a^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/e-d^2*p*ln(-e*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln(-e*((-1)^(2/
3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^3-d^2*
p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(
1/3)*e))*ln(e*x+d)/e^3+1/2*a^(1/3)*d*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/b^(1/3)/e^2+1/4*a^(2/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/b^(2/3)/e-d*x*ln(c*(b*x^3+a)^p)/e^2+1/2*x^2*ln(c*(b*x^3+a)^p)/e+d^2*ln(e*
x+d)*ln(c*(b*x^3+a)^p)/e^3-d^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1
/3)*e))/e^3-d^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*
e))/e^3-d^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/
e^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.45 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.78

$$\int \frac{x^2 \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx =$$

$$-\frac{-12depx+3e^2px^2-\frac{4\sqrt{3}\sqrt[3]{a}dep\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-3e^2px^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)+\frac{4\sqrt[3]{a}d\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

output
```
-1/4*(-12*d*e*p*x + 3*e^2*p*x^2 - (4*Sqrt[3]*a^(1/3)*d*e*p*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 3*e^2*p*x^2*Hypergeometric2F1[2/3,
 1, 5/3, -((b*x^3)/a)] + (4*a^(1/3)*d*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3
) + 4*d^2*p*Log[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/
3)*a^(1/3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(
1/3)*d) + a^(1/3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*((-1)^(2/3)*a^(1/3) +
b^(1/3)*x))/(-(b^(1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (2*a^(1/
3)*d*e*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 4*d*e*x
*Log[c*(a + b*x^3)^p] - 2*e^2*x^2*Log[c*(a + b*x^3)^p] - 4*d^2*Log[d + e*x
]*Log[c*(a + b*x^3)^p] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d
 - a^(1/3)*e)] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^
(1/3)*a^(1/3)*e)] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-
1)^(2/3)*a^(1/3)*e)])/e^3
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(c\left(a+bx^3\right)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{d^2 \log \left(c\left(a+bx^3\right)^p\right)}{e^2(d+ex)}-\frac{d \log \left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x \log \left(c\left(a+bx^3\right)^p\right)}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{3}a^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e}+\frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^2}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e}+\frac{\sqrt{3}\sqrt[3]{a}dp\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^2}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}-\frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^3}-\frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^2}+\frac{3dpx}{e^2}-\frac{3px^2}{4e}$$

input
```
Int[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

output
```
(3*d*p*x)/e^2 - (3*p*x^2)/(4*e) + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(a^(1/3) - 2
*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^2) - (Sqrt[3]*a^(2/3)*p*ArcTan[
(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)*e) - (a^(1/3)*d*p*L
og[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^2) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*
x])/(2*b^(2/3)*e) - (d^2*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^
(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/
3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log
[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a
^(1/3)*e)]*Log[d + e*x])/e^3 + (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(2*b^(1/3)*e^2) + (a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2])/(4*b^(2/3)*e) - (d*x*Log[c*(a + b*x^3)^p])/e^2 + (x^2
*Log[c*(a + b*x^3)^p])/(2*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^3
 - (d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^3 - (
d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/
e^3 - (d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3
)*e)])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.39

method	result
parts	$\frac{x^2 \ln\left(c\left(b\,x^3+a\right)^p\right)}{2e} - \frac{dx \ln\left(c\left(b\,x^3+a\right)^p\right)}{e^2} + \frac{d^2 \ln(ex+d) \ln\left(c\left(b\,x^3+a\right)^p\right)}{e^3} - \frac{3pb\left(d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3 b-3\_Z^2 bd+3b\,d^2\_Z+a\,e\right.}\right.\right.}{}$
risch	$\frac{\ln\left(\left(b\,x^3+a\right)^p\right)x^2}{2e} - \frac{\ln\left(\left(b\,x^3+a\right)^p\right)dx}{e^2} + \frac{\ln\left(\left(b\,x^3+a\right)^p\right)d^2 \ln(ex+d)}{e^3} - \frac{p\,d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3 b-3\_Z^2 bd+3b\,d^2\_Z+a\,e^3-b\,d^3\right)}\right.}{}$

input
```
int(x^2*ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(b*x^3+a)^p)/e-d*x*ln(c*(b*x^3+a)^p)/e^2+d^2*ln(e*x+d)*ln(c*(
b*x^3+a)^p)/e^3-3*p*b/e^3*(1/3*d^2/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+di
log((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3)
)+1/4/b*(e*x+d)^2-3/2/b*(e*x+d)*d+1/6/b^2*sum((-_R+3*d)/(_R^2-2*_R*d+d^2)*
ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))*a*e^3)
```

Fricas [F]

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x**2*ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^2\ \ln\left(c\,(b\,x^3+a)^p\right)}{d+e\,x}\,dx$$

input
```
int((x^2*log(c*(a + b*x^3)^p))/(d + e*x),x)
```

output
```
int((x^2*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{12b^{\frac{1}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)de\,p^2-6\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a\,e^2p^2-18b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)de\,p^2+6b^{\frac{1}{3}}a^{\frac{2}{3}}\log}{}$$

input
```
int(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x)
```

output
```
(12*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqr
t(3)))*d*e*p**2 - 6*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(
3)))*a*e**2*p**2 - 18*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*d*e*p**
2 + 6*b**(1/3)*a**(2/3)*log((a + b*x**3)**p*c)*d*e*p + 12*b**(2/3)*a**(1/3
)*int(log((a + b*x**3)**p*c)/(a*d + a*e*x + b*d*x**3 + b*e*x**4),x)*a*d**2
*e*p - 12*b**(2/3)*a**(1/3)*int((log((a + b*x**3)**p*c)*x**2)/(a*d + a*e*x
 + b*d*x**3 + b*e*x**4),x)*b*d**3*p + 2*b**(2/3)*a**(1/3)*log((a + b*x**3)
**p*c)**2*d**2 - 12*b**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*d*e*p*x + 6*b
**(2/3)*a**(1/3)*log((a + b*x**3)**p*c)*e**2*p*x**2 + 36*b**(2/3)*a**(1/3)
*d*e*p**2*x - 9*b**(2/3)*a**(1/3)*e**2*p**2*x**2 - 9*log(a**(1/3) + b**(1/
3)*x)*a*e**2*p**2 + 3*log((a + b*x**3)**p*c)*a*e**2*p)/(12*b**(2/3)*a**(1/
3)*e**3*p)
```

3.235 $\int \frac{x \log \left(c \left(a+b x^3\right)^p\right)}{d+e x} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 457

$$\int \frac{x\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{3px}{e} - \frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2} - \frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e} + \frac{x\log\left(c(a+bx^3)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^2} + \frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2} + \frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^2} + \frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2}$$

output
```
-3*p*x/e-3^(1/2)*a^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3
))/b^(1/3)/e+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/e+d*p*ln(-e*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln(-e*((-1)^(2/3)*a^(1
/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln((-1)
^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*
ln(e*x+d)/e^2-1/2*a^(1/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1
/3)/e+x*ln(c*(b*x^3+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^2+d*p*polylog(
2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^2+d*p*polylog(2,b^(1/3)*(e*x+d)
/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e^2+d*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1
/3)*d-(-1)^(2/3)*a^(1/3)*e))/e^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.94

$$\int \frac{x \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{-6epx - \frac{2\sqrt{3}\sqrt[3]{a}ep\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2\sqrt[3]{a}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + 2dp\log\left(\frac{e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex) -}{}$$

input
```
Integrate[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

output
```
(-6*e*p*x - (2*Sqrt[3]*a^(1/3)*e*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]])/b^(1/3) + (2*a^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + 2*d*p*L
og[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)
]*Log[d + e*x] + 2*d*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/
3)*e)]*Log[d + e*x] + 2*d*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^
(1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (a^(1/3)*e*p*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*e*x*Log[c*(a + b*x^3)^p] -
 2*d*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*
x))/(b^(1/3)*d - a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/
3)*d + (-1)^(1/3)*a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1
/3)*d - (-1)^(2/3)*a^(1/3)*e)])/(2*e^2)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{d\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e}-\frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e}-\frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^2}+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e}-\frac{3px}{e}$$

input
```
Int[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

output
```
(-3*p*x)/e - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(b^(1/3)*e) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e) +
(d*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x
])/e^2 + (d*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)
^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[((-1)^(1/3)*e*(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e
^2 - (a^(1/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)
*e) + (x*Log[c*(a + b*x^3)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e
^2 + (d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^2 + (
d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^
2 + (d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)
])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.80 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.45

method	result
parts	$\frac{x\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e}-\frac{d\ln\left(ex+d\right)\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e^{2}}-\frac{3pb\left(\frac{(ex+d)e}{b}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b-3\_Z^{2}bd+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\frac{\ln\left(ex-\_\right.}{\_R^{2}-2\_}\right.}{3b^{2}}\right.}{}$
risch	$\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)x}{e}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)d\ln\left(ex+d\right)}{e^{2}}-\frac{3px}{e}-\frac{3pd}{e^{2}}+\frac{pe\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b-3\_Z^{2}bd+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\frac{\ln\left(ex-\_\right.}{\_R^{2}-2\_}\right.}{b}$

input
```
int(x*ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(b*x^3+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^2-3*p*b/e^3*(1/b*(e*
x+d)*e-1/3/b^2*sum(1/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^
2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))*a*e^4-1/3*d*e/b*sum(ln(e*x+d)*ln((-e*x+_R1-
d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*
e^3-b*d^3)))
```

Fricas [F]

$$\int \frac{x \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x*ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x \log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x \log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x\,\ln\left(c\,(b\,x^3+a)^p\right)}{d+e\,x}\,dx$$

input
```
int((x*log(c*(a + b*x^3)^p))/(d + e*x),x)
```

output
```
int((x*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx$$
$$= \frac{-6a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)e\,p^2 + 9a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)e\,p^2 - 3a^{\frac{1}{3}}\log\left(\left(b\,x^3+a\right)^p c\right)ep - 6b^{\frac{1}{3}}\left(\int \frac{\log\left(\left(b\,x^3+a\right)^p}{be\,x^4+bd\,x^3+aex}\right.}{}$$

input
```
int(x*log(c*(b*x^3+a)^p)/(e*x+d),x)
```

output
```
( - 6*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
e*p**2 + 9*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*e*p**2 - 3*a**(1/3)*log((a
+ b*x**3)**p*c)*e*p - 6*b**(1/3)*int(log((a + b*x**3)**p*c)/(a*d + a*e*x +
 b*d*x**3 + b*e*x**4),x)*a*d*e*p + 6*b**(1/3)*int((log((a + b*x**3)**p*c)*
x**2)/(a*d + a*e*x + b*d*x**3 + b*e*x**4),x)*b*d**2*p - b**(1/3)*log((a +
b*x**3)**p*c)**2*d + 6*b**(1/3)*log((a + b*x**3)**p*c)*e*p*x - 18*b**(1/3)
*e*p**2*x)/(6*b**(1/3)*e**2*p)
```

3.236 $\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 308

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}$$

output
```
-p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((
-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e
-p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^
(1/3)*e))*ln(e*x+d)/e+ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p*polylog(2,b^(1/3)*(e
*x+d)/(b^(1/3)*d-a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)
^(1/3)*a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(
1/3)*e))/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.02

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

output
```
-((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x
])/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x
])/e + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d +
 e*x))/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^
(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b
^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2912, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{2912}$$

$$\frac{\log(d+ex)\log \left(c\left(a+bx^3\right)^p\right)}{e}-\frac{3bp\int \frac{x^2\log(d+ex)}{bx^3+a}dx}{e}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\log(d+ex)\log \left(c\left(a+bx^3\right)^p\right)}{e}-$$

$$\frac{3bp\int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}+\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{b}x+(-1)^{2/3}\sqrt[3]{a}\right)}\right)dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{3bp\left(\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{3b} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{3b} + \frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{3b} + \frac{\log(d+ex)\log\left(-\frac{e(\sqrt[3]{a}+}{\sqrt[3]{b}d-}\right.}{3b}\right.}{e}$$

input
```
Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

output
```
(Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (3*b*p*((Log[-((e*(a^(1/3) + b^(1/
3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/(3*b) + (Log[-((e*((-1)^(2/
3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x]
)/(3*b) + (Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d
+ (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/(3*b) + PolyLog[2, (b^(1/3)*(d + e*
x))/(b^(1/3)*d - a^(1/3)*e)]/(3*b) + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/
3)*d + (-1)^(1/3)*a^(1/3)*e)]/(3*b) + PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1
/3)*d - (-1)^(2/3)*a^(1/3)*e)]/(3*b)))/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.53 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.33

method	result
parts	$\frac{\ln(ex+d)\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b-3\_Z^{2}bd+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_R1-d}{\_R1}\right)\right)\right)}{e}$
risch	$\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)\ln(ex+d)}{e}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b-3\_Z^{2}bd+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_R1-d}{\_R1}\right)\right)\right)}{e}$

input
```
int(ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog
((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx=\int \frac{\log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{d+e\,x}\,dx$$

input
```
int(log(c*(a + b*x^3)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b*x^3)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{6\left(\int \frac{\log\left((b\,x^3+a)^p c\right)}{be\,x^4+bd\,x^3+aex+ad}dx\right)aep-6\left(\int \frac{\log\left((b\,x^3+a)^p c\right)x^2}{be\,x^4+bd\,x^3+aex+ad}dx\right)bdp+\log\left((b\,x^3+a)^p\,c\right)^2}{6ep}$$

input
```
int(log(c*(b*x^3+a)^p)/(e*x+d),x)
```

output
```
(6*int(log((a + b*x**3)**p*c)/(a*d + a*e*x + b*d*x**3 + b*e*x**4),x)*a*e*p
 - 6*int((log((a + b*x**3)**p*c)*x**2)/(a*d + a*e*x + b*d*x**3 + b*e*x**4)
,x)*b*d*p + log((a + b*x**3)**p*c)**2)/(6*e*p)
```

3.237 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 352

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{bx^3}{a}\right)}{3d}$$

output
```
p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/d+p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/d+
p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(
1/3)*e))*ln(e*x+d)/d+1/3*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d-ln(e*x+d)*ln(c*(
b*x^3+a)^p)/d+p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/d+p*polyl
og(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d+p*polylog(2,b^(1/
3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d+1/3*p*polylog(2,1+b*x^3/a)/
d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.93

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx$$
$$= \frac{3p\log\left(\frac{e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)+3p\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)\log(d+ex)+3p\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{}\right.}{-\sqrt[3]{b}d+(-1)^{2/3}}\right.}{}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```

output
```
(3*p*Log[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1
/3)*e)]*Log[d + e*x] + 3*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a
^(1/3)*e)]*Log[d + e*x] + 3*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(
b^(1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] + Log[-((b*x^3)/a)]*Log[c
*(a + b*x^3)^p] - 3*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 3*p*PolyLog[2, (b^
(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)] + 3*p*PolyLog[2, (b^(1/3)*(d + e
*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + 3*p*PolyLog[2, (b^(1/3)*(d + e*
x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)] + p*PolyLog[2, 1 + (b*x^3)/a])/(3*
d)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx$$
$$\Big\downarrow \; 2916$$

$$\int \left(\frac{\log \left(c\left(a+bx^3\right)^p\right)}{dx}-\frac{e\log \left(c\left(a+bx^3\right)^p\right)}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log(d+ex)\log \left(c\left(a+bx^3\right)^p\right)}{d}+\frac{\log \left(-\frac{bx^3}{a}\right)\log \left(c\left(a+bx^3\right)^p\right)}{3d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}+\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d}$$

input
```
Int[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```

output
```
(p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])
/d + (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)
*a^(1/3)*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d + (Log[-(
(b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d) - (Log[d + e*x]*Log[c*(a + b*x^3)^
p])/d + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d + (p
*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d + (
p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d +
(p*PolyLog[2, 1 + (b*x^3)/a])/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.38 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.48

method	result
parts	$-\frac{\ln(ex+d)\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{d}+\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)\ln(x)}{d}-3pb\left(\frac{\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b+a\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right.\right.}{3db}\right.$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)\ln(ex+d)}{d}+\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)\ln(x)}{d}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b+a\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{d}$

input
```
int(ln(c*(b*x^3+a)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-ln(e*x+d)*ln(c*(b*x^3+a)^p)/d+ln(c*(b*x^3+a)^p)/d*ln(x)-3*p*b*(1/3/d/b*su
m(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))-1/3/d/b*s
um(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*
b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3)))
```

Fricas [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \int \frac{\log \left((bx^3+a)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^3 + a)^p*c)/(e*x^2 + d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \int \frac{\log \left((bx^3+a)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \int \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{x\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b*x^3)^p)/(x*(d + e*x)),x)
```

output
```
int(log(c*(a + b*x^3)^p)/(x*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left((b\,x^3+a)^p\,c\right)}{e\,x^2+dx}dx$$

input
```
int(log(c*(b*x^3+a)^p)/x/(e*x+d),x)
```

output
```
int(log((a + b*x**3)**p*c)/(d*x + e*x**2),x)
```

3.238 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 510

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d} - \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} + \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d} - \frac{\log\left(c(a+bx^3)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,1+\frac{bx^3}{a}\right)}{3d^2}$$

output
```
-3^(1/2)*b^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))/a^(1/
3)/d-b^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/d-e*p*ln(-e*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1
/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln((-1)^(1/3)*e
*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d
)/d^2+1/2*b^(1/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/d-ln
(c*(b*x^3+a)^p)/d/x-1/3*e*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d^2+e*ln(e*x+d)*l
n(c*(b*x^3+a)^p)/d^2-e*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/
d^2-e*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d^2-e*
p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d^2-1/3*e*p*
polylog(2,1+b*x^3/a)/d^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.77

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx$$

$$= \frac{9bdpx^3\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)-2a\left(3epx\log\left(\frac{e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)+3epx\log\right.}{}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

output
```
(9*b*d*p*x^3*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] - 2*a*(3*e*p*x*L
og[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)
]*Log[d + e*x] + 3*e*p*x*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(
1/3)*e)]*Log[d + e*x] + 3*e*p*x*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(
-(b^(1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] + 3*d*Log[c*(a + b*x^3)
^p] + e*x*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p] - 3*e*x*Log[d + e*x]*Log[
c*(a + b*x^3)^p] + 3*e*p*x*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(
1/3)*e)] + 3*e*p*x*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*
a^(1/3)*e)] + 3*e*p*x*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/
3)*a^(1/3)*e)] + e*p*x*PolyLog[2, 1 + (b*x^3)/a]))/(6*a*d^2*x)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2(d+ex)}dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{e^2\log\left(c\left(a+bx^3\right)^p\right)}{d^2(d+ex)}-\frac{e\log\left(c\left(a+bx^3\right)^p\right)}{d^2x}+\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d}-\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d}-\frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^2}+\frac{e\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx}-\frac{ep\operatorname{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^2}-\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d}$$

input
```
Int[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

output
```
-((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(a
^(1/3)*d)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d) - (e*p*Log[-
((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^2 - (
e*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(
1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^2 + (b^(1
/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d) - Log[
c*(a + b*x^3)^p]/(d*x) - (e*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^2
) + (e*Log[d + e*x]*Log[c*(a + b*x^3)^p])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(
d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*
x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d
 + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, 1 + (b
*x^3)/a])/(3*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.56 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.57

method	result
parts	$\frac{e\ln(ex+d)\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{d^{2}}-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{dx}-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)e\ln(x)}{d^{2}}-3pb\left(\frac{e\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b-3\_Z^{2}bd+3b\,d^{2}\_Z+a\right.}\right.}{}\right.$
risch	$\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)e\ln(ex+d)}{d^{2}}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{dx}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)e\ln(x)}{d^{2}}-\frac{pe\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b-3\_Z^{2}bd+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\right.}{}$

input
```
int(ln(c*(b*x^3+a)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
e*ln(e*x+d)*ln(c*(b*x^3+a)^p)/d^2-ln(c*(b*x^3+a)^p)/d/x-ln(c*(b*x^3+a)^p)*
e/d^2*ln(x)-3*p*b*(1/3*e/d^2/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-
e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/3/
d/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))-1/6/d/b/(1/b*a)^(1/3)*ln(x^2-(1/b*a)
^(1/3)*x+(1/b*a)^(2/3))-1/3/d*3^(1/2)/b/(1/b*a)^(1/3)*arctan(1/3*3^(1/2)*(
2/(1/b*a)^(1/3)*x-1))-1/3*e/d^2/b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1),_R1=RootOf(_Z^3*b+a)))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx=\int \frac{\log\left(\left(bx^3+a\right)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^3 + a)^p*c)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x**2/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{x^2\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b*x^3)^p)/(x^2*(d + e*x)),x)
```

output
```
int(log(c*(a + b*x^3)^p)/(x^2*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bpx-2b^{\frac{2}{3}}a^{\frac{1}{3}}\left(\int\frac{\log\left((b\,x^3+a)^p\,c\right)}{e\,x^2+dx}dx\right)ex-2b^{\frac{2}{3}}a^{\frac{1}{3}}\log\left((b\,x^3+a)^p\,c\right)-3\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{2b^{\frac{2}{3}}a^{\frac{1}{3}}dx}$$

input
```
int(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*p*x - 2
*b**(2/3)*a**(1/3)*int(log((a + b*x**3)**p*c)/(d*x + e*x**2),x)*e*x - 2*b*
*(2/3)*a**(1/3)*log((a + b*x**3)**p*c) - 3*log(a**(1/3) + b**(1/3)*x)*b*p*
x + log((a + b*x**3)**p*c)*b*x)/(2*b**(2/3)*a**(1/3)*d*x)
```

3.239 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 674

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = -\frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d} + \frac{\sqrt{3}\sqrt[3]{b}ep\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2} + \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2} + \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3} - \frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}d} - \frac{\sqrt[3]{b}ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d^2} - \frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^3} - \frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,1+\frac{bx^3}{a}\right)}{3d^3}$$

output
```
-1/2*3^(1/2)*b^(2/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))/a
^(2/3)/d+3^(1/2)*b^(1/3)*e*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1
/3))/a^(1/3)/d^2+1/2*b^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/d+b^(1/3)*e*p
*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/d^2+e^2*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*d-a^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(
b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln((-1)^(1/3)*e*(a^(1
/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/d^3-
1/4*b^(2/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/d-1/2*b^(1
/3)*e*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/d^2-1/2*ln(c*(b*
x^3+a)^p)/d/x^2+e*ln(c*(b*x^3+a)^p)/d^2/x+1/3*e^2*ln(-b*x^3/a)*ln(c*(b*x^3
+a)^p)/d^3-e^2*ln(e*x+d)*ln(c*(b*x^3+a)^p)/d^3+e^2*p*polylog(2,b^(1/3)*(e*
x+d)/(b^(1/3)*d-a^(1/3)*e))/d^3+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d
+(-1)^(1/3)*a^(1/3)*e))/d^3+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1
)^(2/3)*a^(1/3)*e))/d^3+1/3*e^2*p*polylog(2,1+b*x^3/a)/d^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.50 (sec) , antiderivative size = 542, normalized size of antiderivative = 0.80

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx$$

$$= \frac{-\frac{6\sqrt{3}b^{2/3}d^2p\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{18bdepx^2\operatorname{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)}{a} + \frac{6b^{2/3}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + 12e^2p\log\left(\frac{e(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```

output
```
((-6*Sqrt[3]*b^(2/3)*d^2*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^
(2/3) - (18*b*d*e*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/a +
(6*b^(2/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + 12*e^2*p*Log[(e*((-1)
^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e
*x] + 12*e^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*L
og[d + e*x] + 12*e^2*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)
*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (3*b^(2/3)*d^2*p*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) - (6*d^2*Log[c*(a + b*x^3)^p])/
x^2 + (12*d*e*Log[c*(a + b*x^3)^p])/x + 4*e^2*Log[-((b*x^3)/a)]*Log[c*(a +
 b*x^3)^p] - 12*e^2*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 12*e^2*p*PolyLog[2
, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (b^(1
/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (
b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)] + 4*e^2*p*PolyLog[2
, 1 + (b*x^3)/a])/(12*d^3)
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{x^3(d+ex)} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(-\frac{e^3 \log \left(c\left(a+bx^3\right)^p\right)}{d^3(d+ex)}+\frac{e^2 \log \left(c\left(a+bx^3\right)^p\right)}{d^3x}-\frac{e \log \left(c\left(a+bx^3\right)^p\right)}{d^2x^2}+\frac{\log \left(c\left(a+bx^3\right)^p\right)}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d}-\frac{\sqrt[3]{b}ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d^2}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}d}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d}+\frac{\sqrt{3}\sqrt[3]{b}ep\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2}+\frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^3}-\frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3}+\frac{e\log\left(c(a+bx^3)^p\right)}{d^2x}-\frac{\log\left(c(a+bx^3)^p\right)}{2dx^2}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^3}+\frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2}$$

input
```
Int[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```

output
```
-1/2*(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])
/(a^(2/3)*d) + (Sqrt[3]*b^(1/3)*e*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(a^(1/3)*d^2) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/
3)*d) + (b^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d^2) + (e^2*p*Log[
-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^3 +
(e^2*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*
a^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^3 -
(b^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)*d) -
 (b^(1/3)*e*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d
^2) - Log[c*(a + b*x^3)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^3)^p])/(d^2*x) +
(e^2*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^3) - (e^2*Log[d + e*x]*L
og[c*(a + b*x^3)^p])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d +
(-1)^(1/3)*a^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/
3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2, 1 + (b*x^3)/a])/(3*
d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.40 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.62

method	result
parts	$-\frac{e^{2}\ln(ex+d)\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{d^{3}}-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{2d\,x^{2}}+\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)e^{2}\ln(x)}{d^{3}}+\frac{e\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{d^{2}x}-\frac{3pb\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3db\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln}{}\right.}{}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)e^{2}\ln(ex+d)}{d^{3}}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{2d\,x^{2}}+\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)e^{2}\ln(x)}{d^{3}}+\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)e}{d^{2}x}+\frac{p\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{2d\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{p\ln\left(x^{2}-\right.}{}$

input
```
int(ln(c*(b*x^3+a)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-e^2*ln(e*x+d)*ln(c*(b*x^3+a)^p)/d^3-1/2*ln(c*(b*x^3+a)^p)/d/x^2+ln(c*(b*x
^3+a)^p)*e^2/d^3*ln(x)+e*ln(c*(b*x^3+a)^p)/d^2/x-3/2*p*b*(-1/3/d/b/(1/b*a)
^(2/3)*ln(x+(1/b*a)^(1/3))+1/6/d/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))-1/3/d/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(
1/3)*x-1))-2/3/d^2*e/b/(1/b*a)^(1/3)*ln(x+(1/b*a)^(1/3))+1/3/d^2*e/b/(1/b*
a)^(1/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+2/3/d^2*e*3^(1/2)/b/(1/b*a)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+2/3*e^2/d^3/b*sum(ln(x)*l
n((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))-2/3*e^2/d^3/b*sum(
ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3
*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3)))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log((b*x^3 + a)^p*c)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(b*x**3+a)**p)/x**3/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log \left((bx^3+a)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\ln \left(c\,{\left(b\,x^3+a\right)}^p\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

input
```
int(log(c*(a + b*x^3)^p)/(x^3*(d + e*x)),x)
```

output
```
int(log(c*(a + b*x^3)^p)/(x^3*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx$$
$$=\frac{4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bep\,x^2-2b^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)dp\,x^2+4b^{\frac{2}{3}}a^{\frac{2}{3}}\left(\int \frac{\log\left(\left(b\,x^3+a\right)^p c\right)}{e\,x^2+dx}dx\right)e^2x^2-2b^{\frac{2}{3}}a^{\frac{2}{3}}\mathrm{l}}{}$$

input
```
int(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x)
```

output

```
(4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*e
*p*x**2 - 2*b**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*b*d*p*x**2 + 4*b**(2/3)*a**(2/3)*int(log((a + b*x**3)**p*c)/(d*x + e
*x**2),x)*e**2*x**2 - 2*b**(2/3)*a**(2/3)*log((a + b*x**3)**p*c)*d + 4*b**
(2/3)*a**(2/3)*log((a + b*x**3)**p*c)*e*x + 6*a**(1/3)*log(a**(1/3) + b**(
1/3)*x)*b*e*p*x**2 - 2*a**(1/3)*log((a + b*x**3)**p*c)*b*e*x**2 + 3*b**(1/
3)*log(a**(1/3) + b**(1/3)*x)*b*d*p*x**2 - b**(1/3)*log((a + b*x**3)**p*c)
*b*d*x**2)/(4*b**(2/3)*a**(2/3)*d**2*x**2)
```

3.240 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 297

$$\begin{aligned}\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} \, dx = & -\frac{bdpx}{2ae^2}-\frac{b^2px}{3a^2e}+\frac{bpx^2}{6ae}+\frac{d^2x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} \\ & -\frac{dx^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2}+\frac{x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} \\ & +\frac{bd^2p\log(b+ax)}{ae^3}+\frac{b^2dp\log(b+ax)}{2a^2e^2} \\ & +\frac{b^3p\log(b+ax)}{3a^3e}-\frac{d^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^4} \\ & -\frac{d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4}+\frac{d^3p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^4} \\ & +\frac{d^3p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^4}-\frac{d^3p\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^4}\end{aligned}$$

output
```
-1/2*b*d*p*x/a/e^2-1/3*b^2*p*x/a^2/e+1/6*b*p*x^2/a/e+d^2*x*ln(c*(a+b/x)^p)
/e^3-1/2*d*x^2*ln(c*(a+b/x)^p)/e^2+1/3*x^3*ln(c*(a+b/x)^p)/e+b*d^2*p*ln(a*
x+b)/a/e^3+1/2*b^2*d*p*ln(a*x+b)/a^2/e^2+1/3*b^3*p*ln(a*x+b)/a^3/e-d^3*ln(
c*(a+b/x)^p)*ln(e*x+d)/e^4-d^3*p*ln(-e*x/d)*ln(e*x+d)/e^4+d^3*p*ln(-e*(a*x
+b)/(a*d-b*e))*ln(e*x+d)/e^4+d^3*p*polylog(2,a*(e*x+d)/(a*d-b*e))/e^4-d^3*
p*polylog(2,1+e*x/d)/e^4
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.04

$$\int \frac{x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{d+ex} \, dx = \frac{bd^2 p \log \left(a+\frac{b}{x}\right)}{ae^3} + \frac{d^2 x \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{bd^2 p \log(x)}{ae^3} - \frac{bp\left(\frac{2bx}{a^2}-\frac{x^2}{a}-\frac{2b^2\log\left(a+\frac{b}{x}\right)}{a^3}-\frac{2b^2\log(x)}{a^3}\right)}{6e} - \frac{bdp\left(\frac{x}{a}-\frac{b\log(b+ax)}{a^2}\right)}{2e^2} - \frac{d^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right) \log(d+ex)}{e^4} - \frac{d^3 p \log \left(-\frac{ex}{d}\right) \log(d+ex)}{e^4} + \frac{d^3 p \log \left(-\frac{e(b+ax)}{ad-be}\right) \log(d+ex)}{e^4} - \frac{d^3 p \operatorname{PolyLog}\left(2, \frac{d+ex}{d}\right)}{e^4} + \frac{d^3 p \operatorname{PolyLog}\left(2, \frac{a(d+ex)}{ad-be}\right)}{e^4}$$

input
```
Integrate[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

output
```
(b*d^2*p*Log[a + b/x])/(a*e^3) + (d^2*x*Log[c*(a + b/x)^p])/e^3 - (d*x^2*L
og[c*(a + b/x)^p])/(2*e^2) + (x^3*Log[c*(a + b/x)^p])/(3*e) + (b*d^2*p*Log
[x])/(a*e^3) - (b*p*((2*b*x)/a^2 - x^2/a - (2*b^2*Log[a + b/x])/a^3 - (2*b
^2*Log[x])/a^3))/(6*e) - (b*d*p*(x/a - (b*Log[b + a*x])/a^2))/(2*e^2) - (d
^3*Log[c*(a + b/x)^p]*Log[d + e*x])/e^4 - (d^3*p*Log[-((e*x)/d)]*Log[d + e
*x])/e^4 + (d^3*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^4 - (d
^3*p*PolyLog[2, (d + e*x)/d])/e^4 + (d^3*p*PolyLog[2, (a*(d + e*x))/(a*d -
 b*e)])/e^4
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{ 2916}$$

$$\int \left(-\frac{d^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3 (d+ex)}+\frac{d^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^3 p \log (ax+b)}{3a^3 e}+\frac{b^2 dp \log (ax+b)}{2a^2 e^2}-\frac{b^2 px}{3a^2 e}-\frac{d^3 \log (d+ex) \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^4}+\frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2}+\frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{d^3 p \operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^4}+\frac{d^3 p \log (d+ex) \log \left(-\frac{e(ax+b)}{ad-be}\right)}{e^4}+\frac{bd^2 p \log (ax+b)}{ae^3}-\frac{bdpx}{2ae^2}+\frac{bpx^2}{6ae}-\frac{d^3 p \operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4}-\frac{d^3 p \log \left(-\frac{ex}{d}\right) \log (d+ex)}{e^4}$$

input
```
Int[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

output
```
-1/2*(b*d*p*x)/(a*e^2) - (b^2*p*x)/(3*a^2*e) + (b*p*x^2)/(6*a*e) + (d^2*x*
Log[c*(a + b/x)^p])/e^3 - (d*x^2*Log[c*(a + b/x)^p])/(2*e^2) + (x^3*Log[c*
(a + b/x)^p])/(3*e) + (b*d^2*p*Log[b + a*x])/(a*e^3) + (b^2*d*p*Log[b + a*
x])/(2*a^2*e^2) + (b^3*p*Log[b + a*x])/(3*a^3*e) - (d^3*Log[c*(a + b/x)^p]
*Log[d + e*x])/e^4 - (d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log
[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^4 + (d^3*p*PolyLog[2, (a*(d
 + e*x))/(a*d - b*e)])/e^4 - (d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.01

method	result
parts	$\frac{x^3 \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} - \frac{d x^2 \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{d^2 x \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{d^3 \ln\left(c\left(a+\frac{b}{x}\right)^p\right) \ln(ex+d)}{e^4} + pbe\left(-\frac{5ad(ex+d)-a(ex+d)}{a^2}\right.$

input
```
int(x^3*ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(c*(a+b/x)^p)/e-1/2*d*x^2*ln(c*(a+b/x)^p)/e^2+d^2*x*ln(c*(a+b/x)
^p)/e^3-d^3*ln(c*(a+b/x)^p)*ln(e*x+d)/e^4+p*b*e*(-1/6/e^4*(1/a^2*(5*a*d*(e
*x+d)-a*(e*x+d)^2+2*b*e*(e*x+d))+(-6*a^2*d^2-3*a*b*d*e-2*b^2*e^2)/a^3*ln(d
*a-a*(e*x+d)-b*e))-1/e^5*d^3/b*ln(e*x+d)*ln(-e*x/d)-1/e^5*d^3/b*dilog(-e*x
/d)+1/e^5*d^3/b*dilog((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))+1/e^5*d^3/b*ln(e*x+
d)*ln((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e)))
```

Fricas [F]

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^3*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

input
```
integrate(x**3*ln(c*(a+b/x)**p)/(e*x+d),x)
```

output
```
Integral(x**3*log(c*(a + b/x)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^3 \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^3*log((a + b/x)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^3 \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^3*log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{d+e x} \, dx=\int \frac{x^3\,\ln \left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{d+e\,x} \,d x$$

input
```
int((x^3*log(c*(a + b/x)^p))/(d + e*x),x)
```

output
```
int((x^3*log(c*(a + b/x)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{d+e x} \, dx=\int \frac{\log \left(\frac{(a x+b)^p c}{x^p}\right) x^3}{e x+d}d x$$

input
```
int(x^3*log(c*(a+b/x)^p)/(e*x+d),x)
```

output
```
int((log(((a*x + b)**p*c)/x**p)*x**3)/(d + e*x),x)
```

3.241 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{bpx}{2ae} - \frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} - \frac{b^2p\log(b+ax)}{2a^2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} + \frac{d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^3} - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3} + \frac{d^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^3}$$

output
```
1/2*b*p*x/a/e-d*x*ln(c*(a+b/x)^p)/e^2+1/2*x^2*ln(c*(a+b/x)^p)/e-b*d*p*ln(a
*x+b)/a/e^2-1/2*b^2*p*ln(a*x+b)/a^2/e+d^2*ln(c*(a+b/x)^p)*ln(e*x+d)/e^3+d^
2*p*ln(-e*x/d)*ln(e*x+d)/e^3-d^2*p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/e^3-
d^2*p*polylog(2,a*(e*x+d)/(a*d-b*e))/e^3+d^2*p*polylog(2,1+e*x/d)/e^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = -\frac{bdp\log\left(a+\frac{b}{x}\right)}{ae^2} - \frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(x)}{ae^2} + \frac{bp\left(\frac{x}{a}-\frac{b\log(b+ax)}{a^2}\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} + \frac{d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^3} + \frac{d^2p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3}$$

input
```
Integrate[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```

output
```
-((b*d*p*Log[a + b/x])/(a*e^2)) - (d*x*Log[c*(a + b/x)^p])/e^2 + (x^2*Log[
c*(a + b/x)^p])/(2*e) - (b*d*p*Log[x])/(a*e^2) + (b*p*(x/a - (b*Log[b + a*
x])/a^2))/(2*e) + (d^2*Log[c*(a + b/x)^p]*Log[d + e*x])/e^3 + (d^2*p*Log[-
((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Lo
g[d + e*x])/e^3 + (d^2*p*PolyLog[2, (d + e*x)/d])/e^3 - (d^2*p*PolyLog[2,
(a*(d + e*x))/(a*d - b*e)])/e^3
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{d^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2(d+ex)}-\frac{d \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2 p \log(ax+b)}{2a^2 e}+\frac{d^2 \log(d+ex) \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{d^2 p \operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3}-\frac{d^2 p \log(d+ex) \log\left(-\frac{e(ax+b)}{ad-be}\right)}{e^3}-\frac{bdp \log(ax+b)}{ae^2}+\frac{bpx}{2ae}+\frac{d^2 p \operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}+\frac{d^2 p \log\left(-\frac{ex}{d}\right) \log(d+ex)}{e^3}$$

input
```
Int[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```

output
```
(b*p*x)/(2*a*e) - (d*x*Log[c*(a + b/x)^p])/e^2 + (x^2*Log[c*(a + b/x)^p])/
(2*e) - (b*d*p*Log[b + a*x])/(a*e^2) - (b^2*p*Log[b + a*x])/(2*a^2*e) + (d
^2*Log[c*(a + b/x)^p]*Log[d + e*x])/e^3 + (d^2*p*Log[-((e*x)/d)]*Log[d + e
*x])/e^3 - (d^2*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^3 - (d
^2*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e^3 + (d^2*p*PolyLog[2, 1 + (e
*x)/d])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.10

method	result
parts	$\frac{x^2 \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{dx \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{d^2 \ln\left(c\left(a+\frac{b}{x}\right)^p\right) \ln(ex+d)}{e^3} + pbe\left(-\frac{d^2 \operatorname{dilog}\left(\frac{-da+a(ex+d)+be}{-da+be}\right)}{e^4 b} - \frac{d^2 \ln(ex+d)}{}\right.$

input
```
int(x^2*ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(a+b/x)^p)/e-d*x*ln(c*(a+b/x)^p)/e^2+d^2*ln(c*(a+b/x)^p)*ln(e
*x+d)/e^3+p*b*e*(-1/e^4*d^2/b*dilog((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))-1/e^4
*d^2/b*ln(e*x+d)*ln((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))+1/e^4*d^2/b*ln(e*x+d)
*ln(-e*x/d)+1/e^4*d^2/b*dilog(-e*x/d)+1/2/e^3*((e*x+d)/a+(-2*a*d-b*e)/a^2*
ln(d*a-a*(e*x+d)-b*e)))
```

Fricas [F]

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^2*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

input
```
integrate(x**2*ln(c*(a+b/x)**p)/(e*x+d),x)
```

output
```
Integral(x**2*log(c*(a + b/x)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^2 \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^2*log((a + b/x)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^2 \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^2*log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^2\,\ln \left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{d+e\,x}\,dx$$

input
```
int((x^2*log(c*(a + b/x)^p))/(d + e*x),x)
```

output
```
int((x^2*log(c*(a + b/x)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{\log \left(\frac{(ax+b)^p c}{x^p}\right) x^2}{ex+d}dx$$

input
```
int(x^2*log(c*(a+b/x)^p)/(e*x+d),x)
```

output
```
int((log(((a*x + b)**p*c)/x**p)*x**2)/(d + e*x),x)
```

3.242 $\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned}\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = {} & \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{bp\log(b+ax)}{ae} \\ & -\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2}-\frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} \\ & +\frac{dp\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^2} \\ & +\frac{dp\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^2}-\frac{dp\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^2}\end{aligned}$$

output
```
x*ln(c*(a+b/x)^p)/e+b*p*ln(a*x+b)/a/e-d*ln(c*(a+b/x)^p)*ln(e*x+d)/e^2-d*p*
ln(-e*x/d)*ln(e*x+d)/e^2+d*p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/e^2+d*p*po
lylog(2,a*(e*x+d)/(a*d-b*e))/e^2-d*p*polylog(2,1+e*x/d)/e^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09

$$\int \frac{x \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{bp \log \left(a+\frac{b}{x}\right)}{ae}+\frac{x \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{bp \log(x)}{ae}-\frac{d \log \left(c \left(a+\frac{b}{x}\right)^p\right) \log(d+ex)}{e^2}-\frac{dp \log \left(-\frac{ex}{d}\right) \log(d+ex)}{e^2}+\frac{dp \log \left(-\frac{e(b+ax)}{ad-be}\right) \log(d+ex)}{e^2}-\frac{dp \operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^2}+\frac{dp \operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^2}$$

input
```
Integrate[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

output
```
(b*p*Log[a + b/x])/(a*e) + (x*Log[c*(a + b/x)^p])/e + (b*p*Log[x])/(a*e) -
 (d*Log[c*(a + b/x)^p]*Log[d + e*x])/e^2 - (d*p*Log[-((e*x)/d)]*Log[d + e*
x])/e^2 + (d*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^2 - (d*p*
PolyLog[2, (d + e*x)/d])/e^2 + (d*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])
/e^2
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

$$\Big\downarrow \; 2916$$

$$\int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{dp\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e^2}+\frac{bp\log(ax+b)}{ae}-\frac{dp\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}$$

input
```
Int[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

output
```
(x*Log[c*(a + b/x)^p])/e + (b*p*Log[b + a*x])/(a*e) - (d*Log[c*(a + b/x)^p
]*Log[d + e*x])/e^2 - (d*p*Log[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-(
(e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (a*(d + e*
x))/(a*d - b*e)])/e^2 - (d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

method	result
parts	$\frac{x\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{e}-\frac{d\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln(ex+d)}{e^{2}}+pbe\left(\frac{\ln(da-a(ex+d)-be)}{e^{2}a}+\frac{d\,\mathrm{dilog}\left(\frac{-da+a(ex+d)+be}{-da+be}\right)}{e^{3}b}+\frac{d\ln(ex+d)\ln\left(\right.}{}\right.$

input
```
int(x*ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(a+b/x)^p)/e-d*ln(c*(a+b/x)^p)*ln(e*x+d)/e^2+p*b*e*(1/e^2*ln(d*a-a*
(e*x+d)-b*e)/a+1/e^3*d/b*dilog((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))+1/e^3*d/b*
ln(e*x+d)*ln((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))-1/e^3*d/b*ln(e*x+d)*ln(-e*x/
d)-1/e^3*d/b*dilog(-e*x/d))
```

Fricas [F]

$$\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx=\int \frac{x\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx=\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx$$

input
```
integrate(x*ln(c*(a+b/x)**p)/(e*x+d),x)
```

output
```
Integral(x*log(c*(a + b/x)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x*log((a + b/x)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x*log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x \ln \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

input
```
int((x*log(c*(a + b/x)^p))/(d + e*x),x)
```

output
```
int((x*log(c*(a + b/x)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)x}{ex+d}dx$$

input
```
int(x*log(c*(a+b/x)^p)/(e*x+d),x)
```

output
```
int((log(((a*x + b)**p*c)/x**p)*x)/(d + e*x),x)
```

3.243 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e} + \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}$$

output

```
ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(a*x+b)/(a*d-
b*e))*ln(e*x+d)/e-p*polylog(2,a*(e*x+d)/(a*d-b*e))/e+p*polylog(2,1+e*x/d)/
e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} + \frac{p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}$$

input
```
Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
 (p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d +
 e*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2912, 2005, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

$$\Big\downarrow \text{ 2912}$$

$$\frac{bp\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{bp\int \frac{\log(d+ex)}{x(b+ax)}dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}$$

$$\Big\downarrow \text{2863}$$

$$\frac{bp\int\left(\frac{\log(d+ex)}{bx}-\frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{bp\left(-\frac{\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{b}-\frac{\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{b}+\frac{\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{b}+\frac{\log\left(-\frac{ex}{d}\right)\log(d+ex)}{b}\right)}{e}$$

input
```
Int[Log[c*(a + b/x)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b/x)^p]*Log[d + e*x])/e + (b*p*((Log[-((e*x)/d)]*Log[d + e*x])
/b - (Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/b - PolyLog[2, (a*(d
 + e*x))/(a*d - b*e)]/b + PolyLog[2, 1 + (e*x)/d]/b))/e
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln(ex+d)}{e}+pb\left(\frac{\operatorname{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{be}-\frac{\left(\frac{\operatorname{dilog}\left(\frac{-da+a(ex+d)+be}{-da+be}\right)}{a}+\frac{\ln(ex+d)\ln\left(\frac{-da+a(ex+d)+be}{-da+be}\right.}{a}\right.}{be}\right.$

input
```
int(ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*b*((dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b/e-
(dilog((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))/a+ln(e*x+d)*ln((-d*a+a*(e*x+d)+b*e
)/(-a*d+b*e))/a)*a/b/e)
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

input
```
integrate(ln(c*(a+b/x)**p)/(e*x+d),x)
```

output
```
Integral(log(c*(a + b/x)**p)/(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{bp\left(\frac{\log(ex+d)\log\left(a+\frac{b}{x}\right)}{b} - \frac{\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+1\right)+\mathrm{Li}_2\left(\frac{aex+ad}{ad-be}\right)}{b} + \frac{\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)}{b}\right)}{e} - \frac{p\log(ex+d)\log\left(a+\frac{b}{x}\right)}{e} + \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)\log(ex+d)}{e}$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
b*p*(log(e*x + d)*log(a + b/x)/b - (log(e*x + d)*log(-(a*e*x + a*d)/(a*d -
 b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))/b + (log(e*x + d)*log(-(e*x
 + d)/d + 1) + dilog((e*x + d)/d))/b)/e - p*log(e*x + d)*log(a + b/x)/e +
log((a + b/x)^p*c)*log(e*x + d)/e
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

input
```
int(log(c*(a + b/x)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b/x)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)}{ex+d}dx$$

input
```
int(log(c*(a+b/x)^p)/(e*x+d),x)
```

output
```
int(log(((a*x + b)**p*c)/x**p)/(d + e*x),x)
```

3.244 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d} - \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{b}{ax}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d} - \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d}$$

output
```
-ln(c*(a+b/x)^p)*ln(-b/a/x)/d-ln(c*(a+b/x)^p)*ln(e*x+d)/d-p*ln(-e*x/d)*ln(
e*x+d)/d+p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/d-p*polylog(2,1+b/a/x)/d+p*p
olylog(2,a*(e*x+d)/(a*d-b*e))/d-p*polylog(2,1+e*x/d)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d} - \frac{p\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x}}{a}\right)}{d} - \frac{p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d}$$

input
```
Integrate[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
```

output
```
-((Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d) - (Log[c*(a + b/x)^p]*Log[d + e*
x])/d - (p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b + a*x))/(a*d -
 b*e))]*Log[d + e*x])/d - (p*PolyLog[2, (a + b/x)/a])/d - (p*PolyLog[2, (d
 + e*x)/d])/d + (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d(d+ex)}\right)dx$$

$\Big\downarrow$ 2009

$$-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{d}-\frac{p\,\mathrm{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d}-\frac{p\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}$$

input
```
Int[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
```

output
```
-((Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d) - (Log[c*(a + b/x)^p]*Log[d + e*
x])/d - (p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b + a*x))/(a*d -
 b*e))]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x)])/d + (p*PolyLog[2, (a
*(d + e*x))/(a*d - b*e)])/d - (p*PolyLog[2, 1 + (e*x)/d])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.31

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\ln(ex+d)}{d}+\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\ln(x)}{d}+pb\left(\frac{\ln(x)^2}{2db}-\frac{\mathrm{dilog}\left(\frac{ax+b}{b}\right)}{db}-\frac{\ln(x)\ln\left(\frac{ax+b}{b}\right)}{db}+\frac{\mathrm{dilog}\left(\frac{-da+a(ex+d}{-da+be}\right.}{db}\right.$

input
```
int(ln(c*(a+b/x)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(a+b/x)^p)*ln(e*x+d)/d+ln(c*(a+b/x)^p)/d*ln(x)+p*b*(1/2/d*ln(x)^2/b-
1/d/b*dilog((a*x+b)/b)-1/d/b*ln(x)*ln((a*x+b)/b)+1/d/b*dilog((-d*a+a*(e*x+
d)+b*e)/(-a*d+b*e))+1/d/b*ln(e*x+d)*ln((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))-1/
d/b*ln(e*x+d)*ln(-e*x/d)-1/d/b*dilog(-e*x/d))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x + b)/x)^p)/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x\,(d+ex)}\,dx$$

input
```
integrate(ln(c*(a+b/x)**p)/x/(e*x+d),x)
```

output
```
Integral(log(c*(a + b/x)**p)/(x*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = -\frac{1}{2}\,bp\left(\frac{2\,\log\left(ex+d\right)\log\left(x\right)-\log\left(x\right)^2}{bd}+\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)}{bd}-\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log\left(x\right)\right.}{bd}\right. - \left(\frac{\log\left(ex+d\right)}{d}-\frac{\log\left(x\right)}{d}\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

input
```
integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*b*p*((2*log(e*x + d)*log(x) - log(x)^2)/(b*d) + 2*(log(a*x/b + 1)*log
(x) + dilog(-a*x/b))/(b*d) - 2*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))/(b*
d) - 2*(log(e*x + d)*log(-(a*e*x + a*d)/(a*d - b*e) + 1) + dilog((a*e*x +
a*d)/(a*d - b*e)))/(b*d)) - (log(e*x + d)/d - log(x)/d)*log((a + b/x)^p*c)
```

Giac [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)} \, dx = \int \frac{\log \left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)x} \, dx$$

input
```
integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)} \, dx = \int \frac{\ln \left(c\,\left(a+\frac{b}{x}\right)^p\right)}{x\,\left(d+e\,x\right)} \, dx$$

input
```
int(log(c*(a + b/x)^p)/(x*(d + e*x)),x)
```

output
```
int(log(c*(a + b/x)^p)/(x*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx$$
$$=\frac{-2\left(\int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)}{ae\,x^3+ad\,x^2+be\,x^2+bdx}dx\right)abdp+2\left(\int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)}{ae\,x^3+ad\,x^2+be\,x^2+bdx}dx\right)b^2ep-\log\left(\frac{(ax+b)^p c}{x^p}\right)^2 a}{2bep}$$

input
```
int(log(c*(a+b/x)^p)/x/(e*x+d),x)
```

output
```
( - 2*int(log(((a*x + b)**p*c)/x**p)/(a*d*x**2 + a*e*x**3 + b*d*x + b*e*x*
*2),x)*a*b*d*p + 2*int(log(((a*x + b)**p*c)/x**p)/(a*d*x**2 + a*e*x**3 + b
*d*x + b*e*x**2),x)*b**2*e*p - log(((a*x + b)**p*c)/x**p)**2*a)/(2*b*e*p)
```

3.245 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 198

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} + \frac{ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,1+\frac{b}{ax}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^2}$$

output

```
p/d/x-(a+b/x)*ln(c*(a+b/x)^p)/b/d+e*ln(c*(a+b/x)^p)*ln(-b/a/x)/d^2+e*ln(c*
(a+b/x)^p)*ln(e*x+d)/d^2+e*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(-e*(a*x+b)/(a
*d-b*e))*ln(e*x+d)/d^2+e*p*polylog(2,1+b/a/x)/d^2-e*p*polylog(2,a*(e*x+d)/
(a*d-b*e))/d^2+e*p*polylog(2,1+e*x/d)/d^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} + \frac{ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x}}{a}\right)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^2}$$

input
```
Integrate[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```

output
```
p/(d*x) - ((a + b/x)*Log[c*(a + b/x)^p])/(b*d) + (e*Log[c*(a + b/x)^p]*Log
[-(b/(a*x))])/d^2 + (e*Log[c*(a + b/x)^p]*Log[d + e*x])/d^2 + (e*p*Log[-((
e*x)/d)]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d
+ e*x])/d^2 + (e*p*PolyLog[2, (a + b/x)/a])/d^2 + (e*p*PolyLog[2, (d + e*x
)/d])/d^2 - (e*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d^2
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{ 2916}$$

$$\int \left(\frac{e^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2(d+ex)} - \frac{e \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2 x} + \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e \log \left(-\frac{b}{ax}\right) \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2} + \frac{e \log(d+ex) \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2} - \frac{\left(a+\frac{b}{x}\right) \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{ep \operatorname{PolyLog}\left(2, \frac{b}{ax}+1\right)}{d^2} - \frac{ep \operatorname{PolyLog}\left(2, \frac{a(d+ex)}{ad-be}\right)}{d^2} - \frac{ep \log(d+ex) \log \left(-\frac{e(ax+b)}{ad-be}\right)}{d^2} + \frac{ep \operatorname{PolyLog}\left(2, \frac{ex}{d}+1\right)}{d^2} + \frac{ep \log \left(-\frac{ex}{d}\right) \log(d+ex)}{d^2} + \frac{p}{dx}$$

input
```
Int[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```

output
```
p/(d*x) - ((a + b/x)*Log[c*(a + b/x)^p])/(b*d) + (e*Log[c*(a + b/x)^p]*Log
[-(b/(a*x))])/d^2 + (e*Log[c*(a + b/x)^p]*Log[d + e*x])/d^2 + (e*p*Log[-((
e*x)/d)]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d
+ e*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x)])/d^2 - (e*p*PolyLog[2, (a*(d +
e*x))/(a*d - b*e)])/d^2 + (e*p*PolyLog[2, 1 + (e*x)/d])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.33

method	result
parts	$\frac{e\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln(ex+d)}{d^{2}}-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{dx}-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)e\ln(x)}{d^{2}}+pb\left(\frac{e\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{-da+a(ex+d)+be}{-da+be}\right)}{a}+\frac{\ln(ex+d)\ln\left(\frac{-d}{}\right.}{}\right.}{b}\right.}{}\right.$

input
```
int(ln(c*(a+b/x)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
e*ln(c*(a+b/x)^p)*ln(e*x+d)/d^2-ln(c*(a+b/x)^p)/d/x-ln(c*(a+b/x)^p)*e/d^2*
ln(x)+p*b*(e/d^2*(-(dilog((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))/a+ln(e*x+d)*ln(
(-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))/a)/b*a+(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d
))/b)+1/d/b/x+1/d/b^2*a*ln(x)-1/d/b^2*a*ln(a*x+b)-1/2*e/d^2*ln(x)^2/b+e/d^
2/b*dilog((a*x+b)/b)+e/d^2/b*ln(x)*ln((a*x+b)/b))
```

Fricas [F]

$$\int\frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{2}(d+ex)}\,dx=\int\frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{(ex+d)x^{2}}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x + b)/x)^p)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b/x)**p)/x**2/(e*x+d),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.16

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx$$
$$= \frac{1}{2}bp\left(\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\text{Li}_2\left(-\frac{ax}{b}\right)\right)e}{bd^2} - \frac{2\left(\log\left(\frac{ex}{d}+1\right)\log(x)+\text{Li}_2\left(-\frac{ex}{d}\right)\right)e}{bd^2} - \frac{2\left(\log(ex+d)\log\right.}{}\right.$$
$$+\left(\frac{e\log(ex+d)}{d^2} - \frac{e\log(x)}{d^2} - \frac{1}{dx}\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

input
```
integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*b*p*(2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*e/(b*d^2) - 2*(log(e*x/
d + 1)*log(x) + dilog(-e*x/d))*e/(b*d^2) - 2*(log(e*x + d)*log(-(a*e*x + a
*d)/(a*d - b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))*e/(b*d^2) - 2*a*l
og(a*x + b)/(b^2*d) + 2*a*log(x)/(b^2*d) + (2*e*log(e*x + d)*log(x) - e*lo
g(x)^2)/(b*d^2) + 2/(b*d*x)) + (e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x
))*log((a + b/x)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b/x)^p)/(x^2*(d + e*x)),x)
```

output
```
int(log(c*(a + b/x)^p)/(x^2*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{2\left(\int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)}{ae\,x^3+ad\,x^2+be\,x^2+bdx}dx\right)abdpx - 2\left(\int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)}{ae\,x^3+ad\,x^2+be\,x^2+bdx}dx\right)b^2epx + \log\left(\frac{(ax+b)^p c}{x^p}\right)^2 ax - 2\log\left(\frac{(ax+}{x}\right.}{2bdpx}$$

input
```
int(log(c*(a+b/x)^p)/x^2/(e*x+d),x)
```

output
```
(2*int(log(((a*x + b)**p*c)/x**p)/(a*d*x**2 + a*e*x**3 + b*d*x + b*e*x**2)
,x)*a*b*d*p*x - 2*int(log(((a*x + b)**p*c)/x**p)/(a*d*x**2 + a*e*x**3 + b*
d*x + b*e*x**2),x)*b**2*e*p*x + log(((a*x + b)**p*c)/x**p)**2*a*x - 2*log(
((a*x + b)**p*c)/x**p)*a*p*x - 2*log(((a*x + b)**p*c)/x**p)*b*p + 2*b*p**2
)/(2*b*d*p*x)
```

3.246 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx &= \frac{p}{4dx^2}-\frac{ap}{2bdx}-\frac{ep}{d^2x}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}\\ &\quad-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3}\\ &\quad-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3}-\frac{e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}\\ &\quad+\frac{e^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^3}-\frac{e^2p\,\mathrm{PolyLog}\left(2,1+\frac{b}{ax}\right)}{d^3}\\ &\quad+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^3}-\frac{e^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^3}\end{aligned}$$

output

```
1/4*p/d/x^2-1/2*a*p/b/d/x-e*p/d^2/x+1/2*a^2*p*ln(a+b/x)/b^2/d+e*(a+b/x)*ln
(c*(a+b/x)^p)/b/d^2-1/2*ln(c*(a+b/x)^p)/d/x^2-e^2*ln(c*(a+b/x)^p)*ln(-b/a/
x)/d^3-e^2*ln(c*(a+b/x)^p)*ln(e*x+d)/d^3-e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^
2*p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/d^3-e^2*p*polylog(2,1+b/a/x)/d^3+e^
2*p*polylog(2,a*(e*x+d)/(a*d-b*e))/d^3-e^2*p*polylog(2,1+e*x/d)/d^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.92

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx =$$

$$-\frac{-\frac{d^2p}{x^2}+\frac{2ad^2p}{bx}+\frac{4dep}{x}-\frac{2a^2d^2p\log\left(a+\frac{b}{x}\right)}{b^2}-\frac{4de\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}+\frac{2d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}+4e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\text{lo}$$

input
```
Integrate[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```

output
```
-1/4*(-((d^2*p)/x^2) + (2*a*d^2*p)/(b*x) + (4*d*e*p)/x - (2*a^2*d^2*p*Log[
a + b/x])/b^2 - (4*d*e*(a + b/x)*Log[c*(a + b/x)^p])/b + (2*d^2*Log[c*(a +
 b/x)^p])/x^2 + 4*e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))] + 4*e^2*Log[c*(a
+ b/x)^p]*Log[d + e*x] + 4*e^2*p*Log[-((e*x)/d)]*Log[d + e*x] - 4*e^2*p*Lo
g[(e*(b + a*x))/(-(a*d) + b*e)]*Log[d + e*x] + 4*e^2*p*PolyLog[2, 1 + b/(a
*x)] - 4*e^2*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)] + 4*e^2*p*PolyLog[2,
1 + (e*x)/d])/d^3
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \; 2916$$

$$\int \left(-\frac{e^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3(d+ex)}+\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3x}-\frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2x^2}+\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}-\frac{e^2\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}-\frac{e^2p\,\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{d^3}-\frac{ap}{2bdx}-\frac{e^2p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}-\frac{e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}-\frac{ep}{d^2x}+\frac{p}{4dx^2}$$

input
```
Int[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```

output
```
p/(4*d*x^2) - (a*p)/(2*b*d*x) - (e*p)/(d^2*x) + (a^2*p*Log[a + b/x])/(2*b^
2*d) + (e*(a + b/x)*Log[c*(a + b/x)^p])/(b*d^2) - Log[c*(a + b/x)^p]/(2*d*
x^2) - (e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d^3 - (e^2*Log[c*(a + b/x)
^p]*Log[d + e*x])/d^3 - (e^2*p*Log[-((e*x)/d)]*Log[d + e*x])/d^3 + (e^2*p*
Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1
+ b/(a*x)])/d^3 + (e^2*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d^3 - (e^2
*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.20

method	result
parts	$-\frac{e^{2} \ln\left(c\left(a+\frac{b}{x}\right)^{p}\right) \ln(ex+d)}{d^{3}}-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{2d\,x^{2}}+\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right) e^{2} \ln(x)}{d^{3}}+\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right) e}{d^{2} x}+\frac{pb\left(-\frac{-\frac{d}{2b\,x^{2}}-\frac{-da-2be}{b^{2} x}+\frac{(da}{}}{}\right.}{}$

input
```
int(ln(c*(a+b/x)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-e^2*ln(c*(a+b/x)^p)*ln(e*x+d)/d^3-1/2*ln(c*(a+b/x)^p)/d/x^2+ln(c*(a+b/x)^
p)*e^2/d^3*ln(x)+ln(c*(a+b/x)^p)*e/d^2/x+1/2*p*b*(-1/d^2*(-1/2*d/b/x^2-(-a
*d-2*b*e)/b^2/x+(a*d+2*b*e)/b^3*a*ln(x)-(a*d+2*b*e)/b^3*a*ln(a*x+b))+e^2/d
^3*ln(x)^2/b-2*e^2/d^3/b*dilog((a*x+b)/b)-2*e^2/d^3/b*ln(x)*ln((a*x+b)/b)+
2*e^2/d^3/b*dilog((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))+2*e^2/d^3/b*ln(e*x+d)*l
n((-d*a+a*(e*x+d)+b*e)/(-a*d+b*e))-2*e^2/d^3/b*ln(e*x+d)*ln(-e*x/d)-2*e^2/
d^3/b*dilog(-e*x/d))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{3}(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p} c\right)}{(ex+d)x^{3}}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x + b)/x)^p)/(e*x^4 + d*x^3), x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3\,(d+ex)}\,dx$$

input
```
integrate(ln(c*(a+b/x)**p)/x**3/(e*x+d),x)
```

output
```
Integral(log(c*(a + b/x)**p)/(x**3*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx = \frac{1}{4}\left(4\,e\left(\frac{a\log\left(ax+b\right)}{b^2d^2}-\frac{a\log\left(x\right)}{b^2d^2}-\frac{1}{bd^2x}\right)-\frac{4\left(\log\left(\frac{ax}{b}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)e^2}{bd^3}+\frac{4\left(\log\left(\frac{ex}{d}+1\right)\log\right.}{} \right. - \frac{1}{2}\left(\frac{2\,e^2\log\left(ex+d\right)}{d^3}-\frac{2\,e^2\log\left(x\right)}{d^3}-\frac{2\,ex-d}{d^2x^2}\right)\log\left(\left(a+\frac{b}{x}\right)^pc\right)$$

input
```
integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
1/4*(4*e*(a*log(a*x + b)/(b^2*d^2) - a*log(x)/(b^2*d^2) - 1/(b*d^2*x)) - 4
*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*e^2/(b*d^3) + 4*(log(e*x/d + 1)*l
og(x) + dilog(-e*x/d))*e^2/(b*d^3) + 4*(log(e*x + d)*log(-(a*e*x + a*d)/(a
*d - b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))*e^2/(b*d^3) + 2*a^2*log
(a*x + b)/(b^3*d) - 2*a^2*log(x)/(b^3*d) - 2*(2*e^2*log(e*x + d)*log(x) -
e^2*log(x)^2)/(b*d^3) - (2*a*x - b)/(b^2*d*x^2))*b*p - 1/2*(2*e^2*log(e*x
+ d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))*log((a + b/x)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b/x)^p)/(x^3*(d + e*x)),x)
```

output
```
int(log(c*(a + b/x)^p)/(x^3*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx$$
$$=\frac{-4\left(\int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)}{ae\,x^3+ad\,x^2+be\,x^2+bdx}dx\right)a\,b^2dep\,x^2+4\left(\int \frac{\log\left(\frac{(ax+b)^p c}{x^p}\right)}{ae\,x^3+ad\,x^2+be\,x^2+bdx}dx\right)b^3e^2p\,x^2-2\log\left(\frac{(ax+b)^p c}{x^p}\right)^2 abe\,x^2}{}$$

input
```
int(log(c*(a+b/x)^p)/x^3/(e*x+d),x)
```

output
```
( - 4*int(log(((a*x + b)**p*c)/x**p)/(a*d*x**2 + a*e*x**3 + b*d*x + b*e*x*
*2),x)*a*b**2*d*e*p*x**2 + 4*int(log(((a*x + b)**p*c)/x**p)/(a*d*x**2 + a*
e*x**3 + b*d*x + b*e*x**2),x)*b**3*e**2*p*x**2 - 2*log(((a*x + b)**p*c)/x*
*p)**2*a*b*e*x**2 + 2*log(((a*x + b)**p*c)/x**p)*a**2*d*p*x**2 + 4*log(((a
*x + b)**p*c)/x**p)*a*b*e*p*x**2 - 2*log(((a*x + b)**p*c)/x**p)*b**2*d*p +
 4*log(((a*x + b)**p*c)/x**p)*b**2*e*p*x - 2*a*b*d*p**2*x + b**2*d*p**2 -
4*b**2*e*p**2*x)/(4*b**2*d**2*p*x**2)
```

3.247 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 421

$$
\begin{aligned}
\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx =\ & \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2 p \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2} p \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} \\
& + \frac{d^2 x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} \\
& + \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} \\
& - \frac{2d^3 p \log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} \\
& + \frac{d^3 p \log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^4} \\
& + \frac{d^3 p \log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^4} \\
& - \frac{bdp \log\left(b+ax^2\right)}{2ae^2} + \frac{d^3 p \operatorname{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^4} \\
& + \frac{d^3 p \operatorname{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^4} - \frac{2d^3 p \operatorname{PolyLog}\left(2, 1+\frac{ex}{d}\right)}{e^4}
\end{aligned}
$$

output
```
2/3*b*p*x/a/e+2*b^(1/2)*d^2*p*arctan(a^(1/2)*x/b^(1/2))/a^(1/2)/e^3-2/3*b^
(3/2)*p*arctan(a^(1/2)*x/b^(1/2))/a^(3/2)/e+d^2*x*ln(c*(a+b/x^2)^p)/e^3-1/
2*d*x^2*ln(c*(a+b/x^2)^p)/e^2+1/3*x^3*ln(c*(a+b/x^2)^p)/e-d^3*ln(c*(a+b/x^
2)^p)*ln(e*x+d)/e^4-2*d^3*p*ln(-e*x/d)*ln(e*x+d)/e^4+d^3*p*ln(e*(b^(1/2)-(
-a)^(1/2)*x)/((-a)^(1/2)*d+b^(1/2)*e))*ln(e*x+d)/e^4+d^3*p*ln(-e*(b^(1/2)+
(-a)^(1/2)*x)/((-a)^(1/2)*d-b^(1/2)*e))*ln(e*x+d)/e^4-1/2*b*d*p*ln(a*x^2+b
)/a/e^2+d^3*p*polylog(2,(-a)^(1/2)*(e*x+d)/((-a)^(1/2)*d-b^(1/2)*e))/e^4+d
^3*p*polylog(2,(-a)^(1/2)*(e*x+d)/((-a)^(1/2)*d+b^(1/2)*e))/e^4-2*d^3*p*po
lylog(2,1+e*x/d)/e^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.96

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \frac{-12\sqrt{a}\sqrt{b}d^2ep\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)+4be^3px\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b}{ax^2}\right)-3bde^2p\log\left(a+\frac{b}{x^2}\right)+6a\cdots}{\cdots}$$

input
```
Integrate[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

output
```
(-12*Sqrt[a]*Sqrt[b]*d^2*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)] + 4*b*e^3*p*x*Hyp
ergeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))] - 3*b*d*e^2*p*Log[a + b/x^2] +
6*a*d^2*e*x*Log[c*(a + b/x^2)^p] - 3*a*d*e^2*x^2*Log[c*(a + b/x^2)^p] + 2*
a*e^3*x^3*Log[c*(a + b/x^2)^p] - 6*b*d*e^2*p*Log[x] - 6*a*d^3*Log[c*(a + b
/x^2)^p]*Log[d + e*x] - 12*a*d^3*p*Log[-((e*x)/d)]*Log[d + e*x] + 6*a*d^3*
p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] +
6*a*d^3*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[
d + e*x] + 6*a*d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]
*e)] + 6*a*d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)]
 - 12*a*d^3*p*PolyLog[2, 1 + (e*x)/d])/(6*a*e^4)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{d+e x} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(-\frac{d^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e^3 (d+e x)}+\frac{d^2 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e^3}-\frac{d x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2 b^{3/2} p \arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{3 a^{3/2} e}+\frac{2 \sqrt{b} d^2 p \arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{a} e^3}-\frac{d^3 \log (d+e x) \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e^4}+\frac{d^2 x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e^3}-\frac{d x^2 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{2 e^2}+\frac{x^3 \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{3 e}+\frac{d^3 p \text{PolyLog}\left(2,\frac{\sqrt{-a} (d+e x)}{\sqrt{-a} d-\sqrt{b} e}\right)}{e^4}+\frac{d^3 p \text{PolyLog}\left(2,\frac{\sqrt{-a} (d+e x)}{\sqrt{-a} d+\sqrt{b} e}\right)}{e^4}+\frac{d^3 p \log (d+e x) \log \left(\frac{e \left(\sqrt{b}-\sqrt{-a} x\right)}{\sqrt{-a} d+\sqrt{b} e}\right)}{e^4}+\frac{d^3 p \log (d+e x) \log \left(-\frac{e \left(\sqrt{-a} x+\sqrt{b}\right)}{\sqrt{-a} d-\sqrt{b} e}\right)}{e^4}-\frac{b d p \log \left(a x^2+b\right)}{2 a e^2}+\frac{2 b p x}{3 a e}-\frac{2 d^3 p \text{PolyLog}\left(2,\frac{e x}{d}+1\right)}{e^4}-\frac{2 d^3 p \log \left(-\frac{e x}{d}\right) \log (d+e x)}{e^4}$$

input
```
Int[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

output
```
(2*b*p*x)/(3*a*e) + (2*Sqrt[b]*d^2*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]
*e^3) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)*e) + (d^2*x*L
og[c*(a + b/x^2)^p])/e^3 - (d*x^2*Log[c*(a + b/x^2)^p])/(2*e^2) + (x^3*Log
[c*(a + b/x^2)^p])/(3*e) - (d^3*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^4 - (
2*d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[(e*(Sqrt[b] - Sqrt[
-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sqr
t[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/e^4 - (b*d*p*
Log[b + a*x^2])/(2*a*e^2) + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-
a]*d - Sqrt[b]*e)])/e^4 + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]
*d + Sqrt[b]*e)])/e^4 - (2*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.98

method	result
parts	$\frac{x^3 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d\,x^2 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{d^2 x \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{d^3 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) \ln(ex+d)}{e^4} + 2pb\,e^2 \left(\frac{d^3\left(-\frac{\mathrm{dilog}(-}{}\right.}{}\right.$

input
```
int(x^3*ln(c*(a+b/x^2)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(c*(a+b/x^2)^p)/e-1/2*d*x^2*ln(c*(a+b/x^2)^p)/e^2+d^2*x*ln(c*(a+
b/x^2)^p)/e^3-d^3*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^4+2*p*b*e^2*(1/e^4*d^3*(-(
dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b/e^2-(-1/2*ln(e*x+d)*(ln((e*(-a*b)^(1
/2)+d*a-a*(e*x+d))/(e*(-a*b)^(1/2)+d*a))+ln((e*(-a*b)^(1/2)-d*a+a*(e*x+d))
/(e*(-a*b)^(1/2)-d*a)))/a-1/2*(dilog((e*(-a*b)^(1/2)+d*a-a*(e*x+d))/(e*(-a
*b)^(1/2)+d*a))+dilog((e*(-a*b)^(1/2)-d*a+a*(e*x+d))/(e*(-a*b)^(1/2)-d*a))
)/a)/b*a/e^2)+1/6/e^4*(2*(e*x+d)/a+1/a*(-3/2*d*ln(a*d^2-2*a*d*(e*x+d)+a*(e
*x+d)^2+b*e^2)+(6*a*d^2-2*b*e^2)/e/(a*b)^(1/2)*arctan(1/2*(-2*d*a+2*a*(e*x
+d))/e/(a*b)^(1/2)))))
```

Fricas [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^3*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$$

input
```
integrate(x**3*ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

output
```
Integral(x**3*log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^3*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^3*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x}\,dx$$

input
```
int((x^3*log(c*(a + b/x^2)^p))/(d + e*x),x)
```

output
```
int((x^3*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)x^3}{ex+d}dx$$

input
```
int(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x)
```

output
```
int((log(((a*x**2 + b)**p*c)/x**(2*p))*x**3)/(d + e*x),x)
```

3.248 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 353

$$\begin{aligned}\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx &= -\frac{2\sqrt{b}dp\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} \\
&\quad + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} \\
&\quad + \frac{2d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} \\
&\quad - \frac{d^2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^3} \\
&\quad - \frac{d^2p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^3} \\
&\quad + \frac{bp\log\left(b+ax^2\right)}{2ae} - \frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^3} \\
&\quad - \frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^3} + \frac{2d^2p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^3}\end{aligned}$$

output
```
-2*b^(1/2)*d*p*arctan(a^(1/2)*x/b^(1/2))/a^(1/2)/e^2-d*x*ln(c*(a+b/x^2)^p)
/e^2+1/2*x^2*ln(c*(a+b/x^2)^p)/e+d^2*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^3+2*d^2
*p*ln(-e*x/d)*ln(e*x+d)/e^3-d^2*p*ln(e*(b^(1/2)-(-a)^(1/2)*x)/((-a)^(1/2)*
d+b^(1/2)*e))*ln(e*x+d)/e^3-d^2*p*ln(-e*(b^(1/2)+(-a)^(1/2)*x)/((-a)^(1/2)
*d-b^(1/2)*e))*ln(e*x+d)/e^3+1/2*b*p*ln(a*x^2+b)/a/e-d^2*p*polylog(2,(-a)^
(1/2)*(e*x+d)/((-a)^(1/2)*d-b^(1/2)*e))/e^3-d^2*p*polylog(2,(-a)^(1/2)*(e*
x+d)/((-a)^(1/2)*d+b^(1/2)*e))/e^3+2*d^2*p*polylog(2,1+e*x/d)/e^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.99

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx$$
$$= \frac{4\sqrt{a}\sqrt{b}dep \arctan \left(\frac{\sqrt{b}}{\sqrt{a}x}\right) + be^2p \log \left(a+\frac{b}{x^2}\right) - 2adex \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) + ae^2x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) + 2be^2p}{}$$

input
```
Integrate[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

output
```
(4*Sqrt[a]*Sqrt[b]*d*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)] + b*e^2*p*Log[a + b/x
^2] - 2*a*d*e*x*Log[c*(a + b/x^2)^p] + a*e^2*x^2*Log[c*(a + b/x^2)^p] + 2*
b*e^2*p*Log[x] + 2*a*d^2*Log[c*(a + b/x^2)^p]*Log[d + e*x] + 4*a*d^2*p*Log
[-((e*x)/d)]*Log[d + e*x] - 2*a*d^2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt
[-a]*d + Sqrt[b]*e)]*Log[d + e*x] - 2*a*d^2*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x
))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x] - 2*a*d^2*p*PolyLog[2, (Sqrt[
-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - 2*a*d^2*p*PolyLog[2, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 4*a*d^2*p*PolyLog[2, 1 + (e*x)/d])/
(2*a*e^3)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{d^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2 (d+ex)} - \frac{d \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{b} d p \arctan\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{a} e^2} + \frac{d^2 \log(d+ex) \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} - \frac{d^2 p \operatorname{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a} d-\sqrt{b} e}\right)}{e^3} - \frac{d^2 p \operatorname{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a} d+\sqrt{b} e}\right)}{e^3} - \frac{d^2 p \log(d+ex) \log\left(\frac{e\left(\sqrt{b}-\sqrt{-a} x\right)}{\sqrt{-a} d+\sqrt{b} e}\right)}{e^3} - \frac{d^2 p \log(d+ex) \log\left(-\frac{e\left(\sqrt{-a} x+\sqrt{b}\right)}{\sqrt{-a} d-\sqrt{b} e}\right)}{e^3} + \frac{b p \log\left(a x^2+b\right)}{2 a e} + \frac{2 d^2 p \operatorname{PolyLog}\left(2, \frac{ex}{d}+1\right)}{e^3} + \frac{2 d^2 p \log\left(-\frac{ex}{d}\right) \log(d+ex)}{e^3}$$

input
```
Int[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

output
```
(-2*Sqrt[b]*d*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e^2) - (d*x*Log[c*(a
 + b/x^2)^p])/e^2 + (x^2*Log[c*(a + b/x^2)^p])/(2*e) + (d^2*Log[c*(a + b/x
^2)^p]*Log[d + e*x])/e^3 + (2*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d
^2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x]
)/e^3 - (d^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]
*Log[d + e*x])/e^3 + (b*p*Log[b + a*x^2])/(2*a*e) - (d^2*p*PolyLog[2, (Sqr
t[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e^3 - (d^2*p*PolyLog[2, (Sqrt[
-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e^3 + (2*d^2*p*PolyLog[2, 1 + (e
*x)/d])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.03

method	result
parts	$\frac{x^2 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} - \frac{dx \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{d^2 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right) \ln(ex+d)}{e^3} + 2pb\, e^2 \left(\frac{\ln\left(a\, d^2-2ad(ex+d)+a(ex+d)^2+b\, e^2\right)}{4e^3 a}\right.$

input
```
int(x^2*ln(c*(a+b/x^2)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(a+b/x^2)^p)/e-d*x*ln(c*(a+b/x^2)^p)/e^2+d^2*ln(c*(a+b/x^2)^p
)*ln(e*x+d)/e^3+2*p*b*e^2*(1/4/e^3/a*ln(a*d^2-2*a*d*(e*x+d)+a*(e*x+d)^2+b*
e^2)-1/e^4*d/(a*b)^(1/2)*arctan(1/2*(-2*d*a+2*a*(e*x+d))/e/(a*b)^(1/2))-1/
e^3*d^2*(-(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b/e^2-(-1/2*ln(e*x+d)*(ln((
e*(-a*b)^(1/2)+d*a-a*(e*x+d))/(e*(-a*b)^(1/2)+d*a))+ln((e*(-a*b)^(1/2)-d*a
+a*(e*x+d))/(e*(-a*b)^(1/2)-d*a)))/a-1/2*(dilog((e*(-a*b)^(1/2)+d*a-a*(e*x
+d))/(e*(-a*b)^(1/2)+d*a))+dilog((e*(-a*b)^(1/2)-d*a+a*(e*x+d))/(e*(-a*b)^
(1/2)-d*a)))/a)/b*a/e^2))
```

Fricas [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^2*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$$

input
```
integrate(x**2*ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

output
```
Integral(x**2*log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^2 \log \left(c \left(a + \frac{b}{x^2}\right)^p\right)}{d + ex} \, dx = \int \frac{x^2 \log \left(\left(a + \frac{b}{x^2}\right)^p c\right)}{ex + d} \, dx$$

input
```
integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^2*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c \left(a + \frac{b}{x^2}\right)^p\right)}{d + ex} \, dx = \int \frac{x^2 \log \left(\left(a + \frac{b}{x^2}\right)^p c\right)}{ex + d} \, dx$$

input
```
integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^2*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c \left(a + \frac{b}{x^2}\right)^p\right)}{d + ex} \, dx = \int \frac{x^2 \ln \left(c \left(a + \frac{b}{x^2}\right)^p\right)}{d + e\,x} \, dx$$

input
```
int((x^2*log(c*(a + b/x^2)^p))/(d + e*x),x)
```

output
```
int((x^2*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx=\int \frac{\log \left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)x^2}{ex+d}dx$$

input
```
int(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x)
```

output
```
int((log(((a*x**2 + b)**p*c)/x**(2*p))*x**2)/(d + e*x),x)
```

3.249 $\int \frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 291

$$\begin{aligned}
\int \frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = {} & \frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} \\
& - \frac{d \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} \\
& + \frac{dp\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^2} \\
& + \frac{dp\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^2} \\
& + \frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^2} \\
& + \frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^2} - \frac{2dp\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^2}
\end{aligned}$$

output
```
2*b^(1/2)*p*arctan(a^(1/2)*x/b^(1/2))/a^(1/2)/e+x*ln(c*(a+b/x^2)^p)/e-d*ln
(c*(a+b/x^2)^p)*ln(e*x+d)/e^2-2*d*p*ln(-e*x/d)*ln(e*x+d)/e^2+d*p*ln(e*(b^(
1/2)-(-a)^(1/2)*x)/((-a)^(1/2)*d+b^(1/2)*e))*ln(e*x+d)/e^2+d*p*ln(-e*(b^(1
/2)+(-a)^(1/2)*x)/((-a)^(1/2)*d-b^(1/2)*e))*ln(e*x+d)/e^2+d*p*polylog(2,(-
a)^(1/2)*(e*x+d)/((-a)^(1/2)*d-b^(1/2)*e))/e^2+d*p*polylog(2,(-a)^(1/2)*(e
*x+d)/((-a)^(1/2)*d+b^(1/2)*e))/e^2-2*d*p*polylog(2,1+e*x/d)/e^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.93

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx$$
$$= \frac{-\frac{2\sqrt{b}ep \arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{a}} + ex \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) - d \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \log(d+ex) - 2dp \log \left(-\frac{ex}{d}\right) \log(d+ex) +}{}$$

input
```
Integrate[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

output
```
((-2*Sqrt[b]*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + e*x*Log[c*(a + b/x
^2)^p] - d*Log[c*(a + b/x^2)^p]*Log[d + e*x] - 2*d*p*Log[-((e*x)/d)]*Log[d
 + e*x] + d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log
[d + e*x] + d*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)
]*Log[d + e*x] + d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]
*e)] + d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 2*d
*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx$$

$\Big\downarrow$ 2916

$$\int \left(\frac{\log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{d \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e(d+ex)}\right)dx$$

$\Big\downarrow$ 2009

$$\frac{2\sqrt{b} p \arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{\sqrt{a} e}-\frac{d \log (d+ex) \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{e}+\frac{dp \operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a} d-\sqrt{b} e}\right)}{e^2}+\frac{dp \operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a} d+\sqrt{b} e}\right)}{e^2}+\frac{dp \log (d+ex) \log \left(\frac{e\left(\sqrt{b}-\sqrt{-a} x\right)}{\sqrt{-a} d+\sqrt{b} e}\right)}{e^2}+\frac{dp \log (d+ex) \log \left(-\frac{e\left(\sqrt{-a} x+\sqrt{b}\right)}{\sqrt{-a} d-\sqrt{b} e}\right)}{e^2}-\frac{2 dp \operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{2 dp \log \left(-\frac{ex}{d}\right) \log (d+ex)}{e^2}$$

input
```
Int[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

output
```
(2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e) + (x*Log[c*(a + b/x^
2)^p])/e - (d*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^2 - (2*d*p*Log[-((e*x)/
d)]*Log[d + e*x])/e^2 + (d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d +
Sqrt[b]*e)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqr
t[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d +
e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x)
)/(Sqrt[-a]*d + Sqrt[b]*e)])/e^2 - (2*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.02

method	result
parts	$\frac{x\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)}{e}-\frac{d\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)\ln(ex+d)}{e^{2}}+2pb\,e^{2}\left(\frac{\arctan\left(\frac{-2da+2a(ex+d)}{2e\sqrt{ab}}\right)}{e^{3}\sqrt{ab}}+\frac{d\left(-\frac{\operatorname{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{b\,e^{2}}\right.}{}\right.$

input
```
int(x*ln(c*(a+b/x^2)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(a+b/x^2)^p)/e-d*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^2+2*p*b*e^2*(1/e^3/(
a*b)^(1/2)*arctan(1/2*(-2*d*a+2*a*(e*x+d))/e/(a*b)^(1/2))+1/e^2*d*(-(dilog
(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b/e^2-(-1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)+d
*a-a*(e*x+d))/(e*(-a*b)^(1/2)+d*a))+ln((e*(-a*b)^(1/2)-d*a+a*(e*x+d))/(e*(
-a*b)^(1/2)-d*a)))/a-1/2*(dilog((e*(-a*b)^(1/2)+d*a-a*(e*x+d))/(e*(-a*b)^(
1/2)+d*a))+dilog((e*(-a*b)^(1/2)-d*a+a*(e*x+d))/(e*(-a*b)^(1/2)-d*a)))/a)/
b*a/e^2))
```

Fricas [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{x \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d} \, dx$$

input
```
integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx$$

input
```
integrate(x*ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

output
```
Integral(x*log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{x \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d} \, dx$$

input
```
integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{x \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d} \, dx$$

input
```
integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{x \ln \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x} \, dx$$

input
```
int((x*log(c*(a + b/x^2)^p))/(d + e*x),x)
```

output
```
int((x*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{\log \left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right) x}{ex+d} dx$$

input
```
int(x*log(c*(a+b/x^2)^p)/(e*x+d),x)
```

output
```
int((log(((a*x**2 + b)**p*c)/x**(2*p))*x)/(d + e*x),x)
```

3.250 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 241

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e}+\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}+\frac{2p\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}$$

output

```
ln(c*(a+b/x^2)^p)*ln(e*x+d)/e+2*p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(e*(b^(1/2)-(
-a)^(1/2)*x)/((-a)^(1/2)*d+b^(1/2)*e))*ln(e*x+d)/e-p*ln(-e*(b^(1/2)+(-a)^(
1/2)*x)/((-a)^(1/2)*d-b^(1/2)*e))*ln(e*x+d)/e-p*polylog(2,(-a)^(1/2)*(e*x+
d)/((-a)^(1/2)*d-b^(1/2)*e))/e-p*polylog(2,(-a)^(1/2)*(e*x+d)/((-a)^(1/2)*
d+b^(1/2)*e))/e+2*p*polylog(2,1+e*x/d)/e
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e} + \frac{2p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}$$

input
```
Integrate[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])
/e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e
*x])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Lo
g[d + e*x])/e + (2*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x
))/(Sqrt[-a]*d + Sqrt[b]*e)])/e
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2912, 2005, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$$

$$\Big\downarrow \; 2912$$

$$\frac{2bp\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{2bp\int \frac{\log(d+ex)}{x(ax^2+b)}dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}$$

$$\Big\downarrow \text{ 2863}$$

$$\frac{2bp\int \left(\frac{\log(d+ex)}{bx}-\frac{ax\log(d+ex)}{b(ax^2+b)}\right)dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}+$$

$$\frac{2bp\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{2b}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{2b}-\frac{\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{2b}-\frac{\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{2b}+\text{Poly}\right.}{e}$$

input
```
Int[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*b*p*((Log[-((e*x)/d)]*Log[d + e
*x])/b - (Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d +
 e*x])/(2*b) - (Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))
]*Log[d + e*x])/(2*b) - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt
[b]*e)]/(2*b) - PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)]/
(2*b) + PolyLog[2, 1 + (e*x)/d]/b))/e
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
 + n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)\ln(ex+d)}{e}+2pbe\left(\frac{\operatorname{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{b\,e^{2}}+\frac{\left(-\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}+da-a(ex+d)}{e\sqrt{-ab}+da}\right)+\ln\left(\frac{e\sqrt{-ab}-da+}{e\sqrt{-ab}}\right.\right.}{2a}\right.}{}\right.$

input
```
int(ln(c*(a+b/x^2)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(a+b/x^2)^p)*ln(e*x+d)/e+2*p*b*e*((dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d)
)/b/e^2+(-1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)+d*a-a*(e*x+d))/(e*(-a*b)^(1/2)
+d*a))+ln((e*(-a*b)^(1/2)-d*a+a*(e*x+d))/(e*(-a*b)^(1/2)-d*a)))/a-1/2*(dil
og((e*(-a*b)^(1/2)+d*a-a*(e*x+d))/(e*(-a*b)^(1/2)+d*a))+dilog((e*(-a*b)^(1
/2)-d*a+a*(e*x+d))/(e*(-a*b)^(1/2)-d*a)))/a)/b*a/e^2)
```

Fricas [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx=\int \frac{\log \left(\left(a+\frac{b}{x^2}\right)^pc\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx=\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$$

input
```
integrate(ln(c*(a+b/x**2)**p)/(e*x+d),x)
```

output
```
Integral(log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx=\int \frac{\log \left(\left(a+\frac{b}{x^2}\right)^pc\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((a + b/x^2)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x^2)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x}\,dx$$

input
```
int(log(c*(a + b/x^2)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b/x^2)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)}{ex+d}dx$$

input
```
int(log(c*(a+b/x^2)^p)/(e*x+d),x)
```

output
```
int(log(((a*x**2 + b)**p*c)/x**(2*p))/(d + e*x),x)
```

3.251 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx =& -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d}\\ &-\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}+\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d}\\ &+\frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d}\\ &-\frac{p\operatorname{PolyLog}\left(2,1+\frac{b}{ax^2}\right)}{2d}+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}\\ &+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}-\frac{2p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d}\end{aligned}$$

output

```
-1/2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d-ln(c*(a+b/x^2)^p)*ln(e*x+d)/d-2*p*ln
(-e*x/d)*ln(e*x+d)/d+p*ln(e*(b^(1/2)-(-a)^(1/2)*x)/((-a)^(1/2)*d+b^(1/2)*e
))*ln(e*x+d)/d+p*ln(-e*(b^(1/2)+(-a)^(1/2)*x)/((-a)^(1/2)*d-b^(1/2)*e))*ln
(e*x+d)/d-1/2*p*polylog(2,1+b/a/x^2)/d+p*polylog(2,(-a)^(1/2)*(e*x+d)/((-a
)^(1/2)*d-b^(1/2)*e))/d+p*polylog(2,(-a)^(1/2)*(e*x+d)/((-a)^(1/2)*d+b^(1/
2)*e))/d-2*p*polylog(2,1+e*x/d)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.01

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = -\frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\log \left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log \left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log \left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d} + \frac{p\log \left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d} - \frac{p\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x^2}}{a}\right)}{2d} - \frac{2p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}$$

input
```
Integrate[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

output
```
-1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/d - (Log[c*(a + b/x^2)^p]*Lo
g[d + e*x])/d - (2*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[(e*(Sqrt[b]
- Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/d + (p*Log[-((e*(Sq
rt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/d - (p*PolyL
og[2, (a + b/x^2)/a])/(2*d) - (2*p*PolyLog[2, (d + e*x)/d])/d + (p*PolyLog
[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d + (p*PolyLog[2, (Sqr
t[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}-\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}+\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}-\frac{p\,\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d}-\frac{2p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}$$

input
```
Int[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

output
```
-1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/d - (Log[c*(a + b/x^2)^p]*Lo
g[d + e*x])/d - (2*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[(e*(Sqrt[b]
- Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/d + (p*Log[-((e*(Sq
rt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/d - (p*PolyL
og[2, 1 + b/(a*x^2)])/(2*d) + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]
*d - Sqrt[b]*e)])/d + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqr
t[b]*e)])/d - (2*p*PolyLog[2, 1 + (e*x)/d])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.26

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(ex+d)}{d}+\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(x)}{d}+2pb\left(\frac{\frac{\ln(x)^2}{2b}-\frac{\left(\frac{\ln(x)\left(\ln\left(\frac{-ax+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{ax+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\mathrm{dilog}\left(\frac{-ax+}{\sqrt{-}}\right.}{}\right.}{b}}{d}\right.$

input
```
int(ln(c*(a+b/x^2)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(a+b/x^2)^p)*ln(e*x+d)/d+ln(c*(a+b/x^2)^p)/d*ln(x)+2*p*b*(1/d*(1/2*l
n(x)^2/b-(1/2*ln(x)*(ln((-a*x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln((a*x+(-a*b)^(
1/2))/(-a*b)^(1/2)))/a+1/2*(dilog((-a*x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog(
(a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a)/b*a)-1/d*((dilog(-e*x/d)+ln(e*x+d)*ln
(-e*x/d))/b-(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)+d*a-a*(e*x+d))/(e*(-a*b)^(1
/2)+d*a))+ln((e*(-a*b)^(1/2)-d*a+a*(e*x+d))/(e*(-a*b)^(1/2)-d*a)))/a+1/2*(
dilog((e*(-a*b)^(1/2)+d*a-a*(e*x+d))/(e*(-a*b)^(1/2)+d*a))+dilog((e*(-a*b)
^(1/2)-d*a+a*(e*x+d))/(e*(-a*b)^(1/2)-d*a)))/a)/b*a))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^2 + d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b/x**2)**p)/x/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((a + b/x^2)^p*c)/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx=\int \frac{\log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x^2)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx=\int \frac{\ln \left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

input
```
int(log(c*(a + b/x^2)^p)/(x*(d + e*x)),x)
```

output
```
int(log(c*(a + b/x^2)^p)/(x*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx=\int \frac{\log \left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)}{e\,x^2+dx}dx$$

input
```
int(log(c*(a+b/x^2)^p)/x/(e*x+d),x)
```

output
```
int(log(((a*x**2 + b)**p*c)/x**(2*p))/(d*x + e*x**2),x)
```

3.252 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 357

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx &= \frac{2p}{dx}+\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}\\ &\quad+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2}\\ &\quad+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2}+\frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}\\ &\quad-\frac{ep\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^2}\\ &\quad-\frac{ep\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^2}\\ &\quad+\frac{ep\,\text{PolyLog}\left(2,1+\frac{b}{ax^2}\right)}{2d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2}\\ &\quad-\frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2}+\frac{2ep\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^2}\end{aligned}$$

output
```
2*p/d/x+2*a^(1/2)*p*arctan(a^(1/2)*x/b^(1/2))/b^(1/2)/d-ln(c*(a+b/x^2)^p)/
d/x+1/2*e*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d^2+e*ln(c*(a+b/x^2)^p)*ln(e*x+d)
/d^2+2*e*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(e*(b^(1/2)-(-a)^(1/2)*x)/((-a)^
(1/2)*d+b^(1/2)*e))*ln(e*x+d)/d^2-e*p*ln(-e*(b^(1/2)+(-a)^(1/2)*x)/((-a)^(
1/2)*d-b^(1/2)*e))*ln(e*x+d)/d^2+1/2*e*p*polylog(2,1+b/a/x^2)/d^2-e*p*poly
log(2,(-a)^(1/2)*(e*x+d)/((-a)^(1/2)*d-b^(1/2)*e))/d^2-e*p*polylog(2,(-a)^
(1/2)*(e*x+d)/((-a)^(1/2)*d+b^(1/2)*e))/d^2+2*e*p*polylog(2,1+e*x/d)/d^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.92

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx$$
$$= \frac{\frac{4dp}{x} - \frac{4\sqrt{a}dp\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{b}} - \frac{2d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} + e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right) + 2e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex}{}$$

input
```
Integrate[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```

output
```
((4*d*p)/x - (4*Sqrt[a]*d*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b] - (2*d*Lo
g[c*(a + b/x^2)^p])/x + e*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))] + 2*e*Log
[c*(a + b/x^2)^p]*Log[d + e*x] + 4*e*p*Log[-((e*x)/d)]*Log[d + e*x] - 2*e*
p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] -
2*e*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d +
e*x] + e*p*PolyLog[2, 1 + b/(a*x^2)] - 2*e*p*PolyLog[2, (Sqrt[-a]*(d + e*x
))/(Sqrt[-a]*d - Sqrt[b]*e)] - 2*e*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt
[-a]*d + Sqrt[b]*e)] + 4*e*p*PolyLog[2, 1 + (e*x)/d])/(2*d^2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \; 2916$$

$$\int \left(\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2(d+ex)}-\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}+\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d}+\frac{e\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{ep\,\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2}-\frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2}+\frac{2ep\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}+\frac{2p}{dx}$$

input
```
Int[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```

output
```
(2*p)/(d*x) + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[b]*d) - Log[
c*(a + b/x^2)^p]/(d*x) + (e*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^2
) + (e*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^2 + (2*e*p*Log[-((e*x)/d)]*Log
[d + e*x])/d^2 - (e*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]
*e)]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d
 - Sqrt[b]*e))]*Log[d + e*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x^2)])/(2*d^2
) - (e*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d^2 -
(e*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^2 + (2*e
*p*PolyLog[2, 1 + (e*x)/d])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.17

method	result
parts	$\frac{e\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)\ln(ex+d)}{d^{2}}-\frac{\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)}{dx}-\frac{\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)e\ln(x)}{d^{2}}+2pb\left(\frac{e\left(\frac{\operatorname{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{b}-\frac{\left(\frac{\ln(ex+}{}\right.}{}\right.}{}\right.$

input
```
int(ln(c*(a+b/x^2)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
e*ln(c*(a+b/x^2)^p)*ln(e*x+d)/d^2-ln(c*(a+b/x^2)^p)/d/x-ln(c*(a+b/x^2)^p)*
e/d^2*ln(x)+2*p*b*(e/d^2*((dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b-(1/2*ln(e
*x+d)*(ln((e*(-a*b)^(1/2)+d*a-a*(e*x+d))/(e*(-a*b)^(1/2)+d*a))+ln((e*(-a*b
)^(1/2)-d*a+a*(e*x+d))/(e*(-a*b)^(1/2)-d*a)))/a+1/2*(dilog((e*(-a*b)^(1/2)
+d*a-a*(e*x+d))/(e*(-a*b)^(1/2)+d*a))+dilog((e*(-a*b)^(1/2)-d*a+a*(e*x+d))
/(e*(-a*b)^(1/2)-d*a)))/a)/b*a)+1/d/b/x+1/d/b*a/(a*b)^(1/2)*arctan(a*x/(a*
b)^(1/2))-e/d^2*(1/2*ln(x)^2/b-(1/2*ln(x)*(ln((-a*x+(-a*b)^(1/2))/(-a*b)^(
1/2))+ln((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a+1/2*(dilog((-a*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))+dilog((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a)/b*a))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b/x**2)**p)/x**2/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b/x^2)^p)/(x^2*(d + e*x)),x)
```

output
```
int(log(c*(a + b/x^2)^p)/(x^2*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\frac{(a\,x^2+b)^p c}{x^{2p}}\right)}{e\,x^3+d\,x^2}dx$$

input
```
int(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x)
```

output
```
int(log(((a*x**2 + b)**p*c)/x**(2*p))/(d*x**2 + e*x**3),x)
```

3.253 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 414

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx &= \frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}ep\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}\\
&\quad+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3}\\
&\quad-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3}-\frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}\\
&\quad+\frac{e^2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^3}\\
&\quad+\frac{e^2p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^3}\\
&\quad-\frac{e^2p\operatorname{PolyLog}\left(2,1+\frac{b}{ax^2}\right)}{2d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3}\\
&\quad+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3}-\frac{2e^2p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^3}\end{aligned}$$

output
```
1/2*p/d/x^2-2*e*p/d^2/x-2*a^(1/2)*e*p*arctan(a^(1/2)*x/b^(1/2))/b^(1/2)/d^
2-1/2*(a+b/x^2)*ln(c*(a+b/x^2)^p)/b/d+e*ln(c*(a+b/x^2)^p)/d^2/x-1/2*e^2*ln
(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d^3-e^2*ln(c*(a+b/x^2)^p)*ln(e*x+d)/d^3-2*e^2
*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p*ln(e*(b^(1/2)-(-a)^(1/2)*x)/((-a)^(1/2)*
d+b^(1/2)*e))*ln(e*x+d)/d^3+e^2*p*ln(-e*(b^(1/2)+(-a)^(1/2)*x)/((-a)^(1/2)
*d-b^(1/2)*e))*ln(e*x+d)/d^3-1/2*e^2*p*polylog(2,1+b/a/x^2)/d^3+e^2*p*poly
log(2,(-a)^(1/2)*(e*x+d)/((-a)^(1/2)*d-b^(1/2)*e))/d^3+e^2*p*polylog(2,(-a
)^(1/2)*(e*x+d)/((-a)^(1/2)*d+b^(1/2)*e))/d^3-2*e^2*p*polylog(2,1+e*x/d)/d
^3
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx = \frac{-\frac{4dep}{x}+\frac{4\sqrt{a}dep\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{b}}+\frac{2de\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}+d^2\left(\frac{p}{x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{b}\right)-e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(\right.}{}$$

input
```
Integrate[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```

output
```
((-4*d*e*p)/x + (4*Sqrt[a]*d*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b] + (2
*d*e*Log[c*(a + b/x^2)^p])/x + d^2*(p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)
^p])/b) - e^2*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))] - 2*e^2*Log[c*(a + b/
x^2)^p]*Log[d + e*x] - 4*e^2*p*Log[-((e*x)/d)]*Log[d + e*x] + 2*e^2*p*Log[
(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] + 2*e^2*
p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x]
 - e^2*p*PolyLog[2, 1 + b/(a*x^2)] + 2*e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x
))/(Sqrt[-a]*d - Sqrt[b]*e)] + 2*e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sq
rt[-a]*d + Sqrt[b]*e)] - 4*e^2*p*PolyLog[2, 1 + (e*x)/d])/(2*d^3)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \ 2916$$

$$\int \left(-\frac{e^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3(d+ex)}+\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3x}-\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x^2}+\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2\sqrt{a}ep\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2}-\frac{e^2\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}-\frac{e^2p\operatorname{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3}-\frac{2e^2p\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}-\frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}-\frac{2ep}{d^2x}+\frac{p}{2dx^2}$$

input
```
Int[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```

output
```
p/(2*d*x^2) - (2*e*p)/(d^2*x) - (2*Sqrt[a]*e*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]]
)/(Sqrt[b]*d^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b*d) + (e*Log[c*(a
 + b/x^2)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^
3) - (e^2*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^3 - (2*e^2*p*Log[-((e*x)/d)
]*Log[d + e*x])/d^3 + (e^2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d +
Sqrt[b]*e)]*Log[d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(S
qrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(a*x^
2)])/(2*d^3) + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b
]*e)])/d^3 + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*
e)])/d^3 - (2*e^2*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.18

method	result
parts	$-\frac{e^{2} \ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right) \ln\left(e x+d\right)}{d^{3}}-\frac{\ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)}{2 d\, x^{2}}+\frac{\ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right) e^{2} \ln\left(x\right)}{d^{3}}+\frac{e \ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)}{d^{2} x}+p b \left(\frac{1}{2 d b\, x^{2}}-\frac{2 e}{d^{2} b x}\right.$

input
```
int(ln(c*(a+b/x^2)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-e^2*ln(c*(a+b/x^2)^p)*ln(e*x+d)/d^3-1/2*ln(c*(a+b/x^2)^p)/d/x^2+ln(c*(a+b
/x^2)^p)*e^2/d^3*ln(x)+e*ln(c*(a+b/x^2)^p)/d^2/x+p*b*(1/2/d/b/x^2-2/d^2*e/
b/x+1/d/b^2*a*ln(x)-1/2/d/b^2*a*ln(a*x^2+b)-2/d^2/b*a*e/(a*b)^(1/2)*arctan
(a*x/(a*b)^(1/2))+2*e^2/d^3*(1/2*ln(x)^2/b-(1/2*ln(x)*(ln((-a*x+(-a*b)^(1/
2))/(-a*b)^(1/2))+ln((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a+1/2*(dilog((-a*x+
(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a)/b*a
)-2*e^2/d^3*((dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b-(1/2*ln(e*x+d)*(ln((e*
(-a*b)^(1/2)+d*a-a*(e*x+d))/(e*(-a*b)^(1/2)+d*a))+ln((e*(-a*b)^(1/2)-d*a+a
*(e*x+d))/(e*(-a*b)^(1/2)-d*a)))/a+1/2*(dilog((e*(-a*b)^(1/2)+d*a-a*(e*x+d
))/(e*(-a*b)^(1/2)+d*a))+dilog((e*(-a*b)^(1/2)-d*a+a*(e*x+d))/(e*(-a*b)^(1
/2)-d*a)))/a)/b*a))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b/x**2)**p)/x**3/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b/x^2)^p)/(x^3*(d + e*x)),x)
```

output
```
int(log(c*(a + b/x^2)^p)/(x^3*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx$$
$$= \frac{-4\left(\int \frac{\log\left(\frac{\left(a\,x^2+b\right)^p c}{x^{2p}}\right)}{ae\,x^5+ad\,x^4+be\,x^3+bd\,x^2}dx\right)b^2ep\,x^2+4\left(\int \frac{\log\left(\frac{\left(a\,x^2+b\right)^p c}{x^{2p}}\right)}{ae\,x^4+ad\,x^3+be\,x^2+bdx}dx\right)abdp\,x^2+\log\left(\frac{\left(a\,x^2+b\right)^p c}{x^{2p}}\right)^2 a\,x^2-2}{4bdp\,x^2}$$

input
```
int(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x)
```

output
```
( - 4*int(log(((a*x**2 + b)**p*c)/x**(2*p))/(a*d*x**4 + a*e*x**5 + b*d*x**
2 + b*e*x**3),x)*b**2*e*p*x**2 + 4*int(log(((a*x**2 + b)**p*c)/x**(2*p))/(
a*d*x**3 + a*e*x**4 + b*d*x + b*e*x**2),x)*a*b*d*p*x**2 + log(((a*x**2 + b
)**p*c)/x**(2*p))**2*a*x**2 - 2*log(((a*x**2 + b)**p*c)/x**(2*p))*a*p*x**2
 - 2*log(((a*x**2 + b)**p*c)/x**(2*p))*b*p + 2*b*p**2)/(4*b*d*p*x**2)
```

3.254 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 714

$$\int \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}d^2p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} + \frac{\sqrt{3}b^{2/3}dp\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^3} + \frac{b^{2/3}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e^2} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} - \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} - \frac{\sqrt[3]{b}d^2p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^3} - \frac{b^{2/3}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e^2} + \frac{bp\log\left(b+ax^3\right)}{3ae} + \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4} - \frac{3d^3p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^4}$$

output

```
-3^(1/2)*b^(1/3)*d^2*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))/a
^(1/3)/e^3+1/2*3^(1/2)*b^(2/3)*d*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2
)/b^(1/3))/a^(2/3)/e^2+d^2*x*ln(c*(a+b/x^3)^p)/e^3-1/2*d*x^2*ln(c*(a+b/x^3
)^p)/e^2+1/3*x^3*ln(c*(a+b/x^3)^p)/e+b^(1/3)*d^2*p*ln(b^(1/3)+a^(1/3)*x)/a
^(1/3)/e^3+1/2*b^(2/3)*d*p*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/e^2-d^3*ln(c*(a+b
/x^3)^p)*ln(e*x+d)/e^4-3*d^3*p*ln(-e*x/d)*ln(e*x+d)/e^4+d^3*p*ln(-e*(b^(1/
3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln(-e*((-1)^(2/3)
*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^4+d^3*p*
ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/
3)*e))*ln(e*x+d)/e^4-1/2*b^(1/3)*d^2*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3
)*x^2)/a^(1/3)/e^3-1/4*b^(2/3)*d*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^
2)/a^(2/3)/e^2+1/3*b*p*ln(a*x^3+b)/a/e+d^3*p*polylog(2,a^(1/3)*(e*x+d)/(a^
(1/3)*d-b^(1/3)*e))/e^4+d^3*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1
/3)*b^(1/3)*e))/e^4+d^3*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*
b^(1/3)*e))/e^4-3*d^3*p*polylog(2,1+e*x/d)/e^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.36 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.75

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{3bdp\,\text{Hypergeometric2F1}\left(\frac{1}{3},1,\frac{4}{3},-\frac{b}{ax^3}\right)}{2ae^2x} - \frac{3bd^2p\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{b}{ax^3}\right)}{2ae^3x^2} + \frac{bp\log\left(a+\frac{b}{x^3}\right)}{3ae} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{bp\log(x)}{ae} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} - \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} - \frac{3d^3p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^4} + \frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4}$$

input
```
Integrate[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

output
```
(3*b*d*p*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))])/(2*a*e^2*x) - (3*b*
d^2*p*Hypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))])/(2*a*e^3*x^2) + (b*p*L
og[a + b/x^3])/(3*a*e) + (d^2*x*Log[c*(a + b/x^3)^p])/e^3 - (d*x^2*Log[c*(
a + b/x^3)^p])/(2*e^2) + (x^3*Log[c*(a + b/x^3)^p])/(3*e) + (b*p*Log[x])/(
a*e) - (d^3*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^4 - (3*d^3*p*Log[-((e*x)/
d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d
 - b^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (
-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e
^4 + (d^3*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
 + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^4 - (3*d^3*p*PolyLog[2, (d + e*x
)/d])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)]
)/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1
/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/
3)*b^(1/3)*e)])/e^4
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(-\frac{d^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3 (d+ex)}+\frac{d^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3}-\frac{dx \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{3}b^{2/3}dp\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2}-\frac{\sqrt[3]{b}d^2p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}e^3}-\frac{b^{2/3}dp\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{4a^{2/3}e^2}+\frac{b^{2/3}dp\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{2a^{2/3}e^2}-\frac{\sqrt{3}\sqrt[3]{b}d^2p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3}-\frac{d^3\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^4}+\frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3}-\frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2}+\frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e}+\frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4}+\frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}e^3}+\frac{bp\log\left(ax^3+b\right)}{3ae}-\frac{3d^3p\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4}-\frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4}$$

input
```
Int[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

output
```
-((Sqrt[3]*b^(1/3)*d^2*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]
)/(a^(1/3)*e^3)) + (Sqrt[3]*b^(2/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sq
rt[3]*b^(1/3))])/(2*a^(2/3)*e^2) + (d^2*x*Log[c*(a + b/x^3)^p])/e^3 - (d*x
^2*Log[c*(a + b/x^3)^p])/(2*e^2) + (x^3*Log[c*(a + b/x^3)^p])/(3*e) + (b^(
1/3)*d^2*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^3) + (b^(2/3)*d*p*Log[b^(1
/3) + a^(1/3)*x])/(2*a^(2/3)*e^2) - (d^3*Log[c*(a + b/x^3)^p]*Log[d + e*x]
)/e^4 - (3*d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(b^(1
/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log
[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)
)]*Log[d + e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1
/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^4 - (b^(1/3)*d
^2*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^3) - (b^
(2/3)*d*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e^2)
+ (b*p*Log[b + a*x^3])/(3*a*e) + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^
(1/3)*d - b^(1/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3
)*d + (-1)^(1/3)*b^(1/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/
(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/e^4 - (3*d^3*p*PolyLog[2, 1 + (e*x)/d
])/e^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.09 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.42

method	result
parts	$\frac{x^{3} \ln\left(c \left(a+\frac{b}{x^{3}}\right)^{p}\right)}{3e}-\frac{d\,x^{2} \ln\left(c \left(a+\frac{b}{x^{3}}\right)^{p}\right)}{2e^{2}}+\frac{d^{2} x \ln\left(c \left(a+\frac{b}{x^{3}}\right)^{p}\right)}{e^{3}}-\frac{d^{3} \ln\left(c \left(a+\frac{b}{x^{3}}\right)^{p}\right) \ln\left(ex+d\right)}{e^{4}}+3pb\,e^{3}\left(-\frac{\_R\text{=RootO}}{}\right.$

input
```
int(x^3*ln(c*(a+b/x^3)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*ln(c*(a+b/x^3)^p)/e-1/2*d*x^2*ln(c*(a+b/x^3)^p)/e^2+d^2*x*ln(c*(a+
b/x^3)^p)/e^3-d^3*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^4+3*p*b*e^3*(-1/18/e^4/a*s
um((2*_R^2-7*_R*d+11*d^2)/(-_R^2+2*_R*d-d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*a
-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))-1/e^4*d^3*(-1/3*sum(ln(e*x+d)*ln((-e*
x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*
d^2-a*d^3+b*e^3))/b/e^3+(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b/e^3))
```

Fricas [F]

$$\int \frac{x^{3} \log\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right)}{d+ex}\,dx=\int \frac{x^{3} \log\left(\left(a+\frac{b}{x^{3}}\right)^{p} c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^3*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x**3*ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^3*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^3*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\int \frac{x^3\,\ln \left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{d+e\,x} \,dx$$

input
```
int((x^3*log(c*(a + b/x^3)^p))/(d + e*x),x)
```

output
```
int((x^3*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^3 \log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\int \frac{\log \left(\frac{\left(a\,x^{3}+b\right)^{p} c}{x^{3 p}}\right) x^{3}}{e x +d}d x$$

input
```
int(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x)
```

output
```
int((log(((a*x**3 + b)**p*c)/x**(3*p))*x**3)/(d + e*x),x)
```

3.255 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 666

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\sqrt{3}\sqrt[3]{b}dp\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2} - \frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e} - \frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{\sqrt[3]{b}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^2} - \frac{b^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} + \frac{\sqrt[3]{b}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^2} + \frac{b^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e} - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3} + \frac{3d^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^3}$$

output
```
3^(1/2)*b^(1/3)*d*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))/a^(1
/3)/e^2-1/2*3^(1/2)*b^(2/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(
1/3))/a^(2/3)/e-d*x*ln(c*(a+b/x^3)^p)/e^2+1/2*x^2*ln(c*(a+b/x^3)^p)/e-b^(1
/3)*d*p*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/e^2-1/2*b^(2/3)*p*ln(b^(1/3)+a^(1/3)
*x)/a^(2/3)/e+d^2*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^3+3*d^2*p*ln(-e*x/d)*ln(e*
x+d)/e^3-d^2*p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/
e^3-d^2*p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/
3)*e))*ln(e*x+d)/e^3-d^2*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/
(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/e^3+1/2*b^(1/3)*d*p*ln(b^(2/3)
-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(1/3)/e^2+1/4*b^(2/3)*p*ln(b^(2/3)-a^(1/
3)*b^(1/3)*x+a^(2/3)*x^2)/a^(2/3)/e-d^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/
3)*d-b^(1/3)*e))/e^3-d^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)
*b^(1/3)*e))/e^3-d^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(
1/3)*e))/e^3+3*d^2*p*polylog(2,1+e*x/d)/e^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.73

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = -\frac{3bp\,\text{Hypergeometric2F1}\left(\frac{1}{3},1,\frac{4}{3},-\frac{b}{ax^3}\right)}{2aex} + \frac{3bdp\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{b}{ax^3}\right)}{2ae^2x^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} + \frac{3d^2p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3}$$

input
```
Integrate[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

output
```
(-3*b*p*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))])/(2*a*e*x) + (3*b*d*p
*Hypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))])/(2*a*e^2*x^2) - (d*x*Log[c*
(a + b/x^3)^p])/e^2 + (x^2*Log[c*(a + b/x^3)^p])/(2*e) + (d^2*Log[c*(a + b
/x^3)^p]*Log[d + e*x])/e^3 + (3*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 -
(d^2*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e
*x])/e^3 - (d^2*p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a
^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^(1
/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)
]*Log[d + e*x])/e^3 + (3*d^2*p*PolyLog[2, (d + e*x)/d])/e^3 - (d^2*p*PolyL
og[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^3 - (d^2*p*PolyLog[2
, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e^3 - (d^2*p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/e^3
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{d+e x} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{d^2 \log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{e^2 (d+e x)}-\frac{d \log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x \log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e}+\frac{\sqrt[3]{b}dp\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}e^2}+\frac{b^{2/3}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{4a^{2/3}e}-\frac{b^{2/3}p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{2a^{2/3}e}+\frac{\sqrt{3}\sqrt[3]{b}dp\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2}+\frac{d^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3}-\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}-\frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3}-\frac{\sqrt[3]{b}dp\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}e^2}+\frac{3d^2p\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}+\frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3}$$

input
```
Int[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

output
```
(Sqrt[3]*b^(1/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a
^(1/3)*e^2) - (Sqrt[3]*b^(2/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b
^(1/3))])/(2*a^(2/3)*e) - (d*x*Log[c*(a + b/x^3)^p])/e^2 + (x^2*Log[c*(a +
 b/x^3)^p])/(2*e) - (b^(1/3)*d*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^2) -
 (b^(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*a^(2/3)*e) + (d^2*Log[c*(a + b/x^
3)^p]*Log[d + e*x])/e^3 + (3*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^
2*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x]
)/e^3 - (d^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1
)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^(1/3)*e*(b^(1/3)
 + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x]
)/e^3 + (b^(1/3)*d*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^
(1/3)*e^2) + (b^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4
*a^(2/3)*e) - (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e
)])/e^3 - (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^
(1/3)*e)])/e^3 - (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(
2/3)*b^(1/3)*e)])/e^3 + (3*d^2*p*PolyLog[2, 1 + (e*x)/d])/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.60 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.40

method	result
parts	$\frac{x^2 \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{dx \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{d^2 \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right) \ln(ex+d)}{e^3} + 3pb\,e^3 \left(\frac{d^2 \left(-\sum\_{\_R1=\mathrm{RootOf}\left(\_Z^3 a-3\_Z^2 ad+3\_\right.} \right.}{} \right.$

input
```
int(x^2*ln(c*(a+b/x^3)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(c*(a+b/x^3)^p)/e-d*x*ln(c*(a+b/x^3)^p)/e^2+d^2*ln(c*(a+b/x^3)^p
)*ln(e*x+d)/e^3+3*p*b*e^3*(1/e^3*d^2*(-1/3*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_
R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+
b*e^3))/b/e^3+(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b/e^3)+1/6/e^3/a*sum((-
_R+3*d)/(-_R^2+2*_R*d-d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a
*d^2-a*d^3+b*e^3)))
```

Fricas [F]

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x^2*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(x**2*ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x^2*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x^2*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\int \frac{x^2\ \ln \left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{d+e\,x} \,dx$$

input
```
int((x^2*log(c*(a + b/x^3)^p))/(d + e*x),x)
```

output
```
int((x^2*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\int \frac{\log \left(\frac{\left(a\,x^3+b\right)^p c}{x^{3p}}\right) x^2}{ex+d}dx$$

input
```
int(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x)
```

output
```
int((log(((a*x**3 + b)**p*c)/x**(3*p))*x**2)/(d + e*x),x)
```

3.256 $\int \frac{x \log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 488

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e}+\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e}-\frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2}-\frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2}-\frac{\sqrt[3]{b}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e}+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}-\frac{3dp\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^2}$$

output
```
-3^(1/2)*b^(1/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))/a^(1/
3)/e+x*ln(c*(a+b/x^3)^p)/e+b^(1/3)*p*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/e-d*ln(
c*(a+b/x^3)^p)*ln(e*x+d)/e^2-3*d*p*ln(-e*x/d)*ln(e*x+d)/e^2+d*p*ln(-e*(b^(
1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln(-e*((-1)^(2/3)
*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln
((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)
*e))*ln(e*x+d)/e^2-1/2*b^(1/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)
/a^(1/3)/e+d*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^2+d*p*po
lylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^2+d*p*polylog(
2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^2-3*d*p*polylog(2,1+
e*x/d)/e^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.83

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = -\frac{3bp\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{b}{ax^3}\right)}{2aex^2} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} - \frac{3dp\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^2} + \frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}$$

input
```
Integrate[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

output
```
(-3*b*p*Hypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))])/(2*a*e*x^2) + (x*Log
[c*(a + b/x^3)^p])/e - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*
Log[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(
a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-(((-1)^(2/3)*e*(b^(
1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d +
 e*x])/e^2 + (d*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(
1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^2 - (3*d*p*PolyLog[2, (d +
 e*x)/d])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e
)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1
/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)
*b^(1/3)*e)])/e^2
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{\log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{e}-\frac{d \log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{e(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[3]{b}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}e}-\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2}+\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}e}-\frac{3dp\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2}-\frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}$$

input
```
Int[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

output
```
-((Sqrt[3]*b^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a
^(1/3)*e)) + (x*Log[c*(a + b/x^3)^p])/e + (b^(1/3)*p*Log[b^(1/3) + a^(1/3)
*x])/(a^(1/3)*e) - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Log[
-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1
/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-((e*((-1)^(2/3)*b^(1/3)
 + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^2 + (d
*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(
1/3)*b^(1/3)*e)]*Log[d + e*x])/e^2 - (b^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1
/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e) + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/
(a^(1/3)*d - b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/
3)*d + (-1)^(1/3)*b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/e^2 - (3*d*p*PolyLog[2, 1 + (e*x)/d])/
e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.53 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.49

method	result
parts	$\frac{x\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}-\frac{d\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(ex+d)}{e^2}+3pb\,e^3\left(-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3a-3\_Z^2ad+3\_Zad^2-ad^3+be^3\right)}\frac{\ln\left(ex-\_R\right.}{-\_R^2+2\_}}{3e^2a}\right.$

input
```
int(x*ln(c*(a+b/x^3)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*(a+b/x^3)^p)/e-d*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^2+3*p*b*e^3*(-1/3/e^
2/a*sum(1/(-_R^2+2*_R*d-d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z
*a*d^2-a*d^3+b*e^3))-1/e^2*d*(-1/3*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilo
g((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))/
b/e^3+(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b/e^3))
```

Fricas [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx = \int \frac{x \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d} \, dx$$

input
```
integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(x*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx = \text{Timed out}$$

input
```
integrate(x*ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx = \int \frac{x \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d} \, dx$$

input
```
integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(x*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(x*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x\,\ln\left(c\,\left(a+\frac{b}{x^3}\right)^p\right)}{d+e\,x}\,dx$$

input
```
int((x*log(c*(a + b/x^3)^p))/(d + e*x),x)
```

output
```
int((x*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

Reduce [F]

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\frac{(a\,x^3+b)^p c}{x^{3p}}\right)x}{ex+d}dx$$

input
```
int(x*log(c*(a+b/x^3)^p)/(e*x+d),x)
```

output
```
int((log(((a*x**3 + b)**p*c)/x**(3*p))*x)/(d + e*x),x)
```

3.257 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 344

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e}+\frac{3p\log \left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log \left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p\log \left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p\log \left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}+\frac{3p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}$$

output

```
ln(c*(a+b/x^3)^p)*ln(e*x+d)/e+3*p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(b^(1/3)+
a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-1)^(2/3)*b^(1/3)+
a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e-p*ln((-1)^(1/3)*e
*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d
)/e-p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e-p*polylog(2,a^(1/
3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e-p*polylog(2,a^(1/3)*(e*x+d)
/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e+3*p*polylog(2,1+e*x/d)/e
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e}+\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e} + \frac{3p\,\mathrm{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}$$

input
```
Integrate[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])
/e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d +
e*x])/e - (p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3
)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1
/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e
*x])/e + (3*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x
))/(a^(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3
)*d + (-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/
3)*d - (-1)^(2/3)*b^(1/3)*e)])/e
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2912, 2005, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx$$

$$\Big\downarrow \text{2912}$$

$$\frac{3bp\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}$$

$$\Big\downarrow \text{2005}$$

$$\frac{3bp\int \frac{\log(d+ex)}{x(ax^3+b)}dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}$$

$$\Big\downarrow \text{2863}$$

$$\frac{3bp\int \left(\frac{\log(d+ex)}{bx}-\frac{ax^2\log(d+ex)}{b(ax^3+b)}\right)dx}{e}+\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+$$

$$3bp\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{3b}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{3b}-\frac{\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{3b}-\frac{\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}\right.}{\sqrt[3]{a}}\right.}{3b}\right.$$

input
```
Int[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

output
```
(Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*b*p*((Log[-((e*x)/d)]*Log[d + e
*x])/b - (Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d
+ e*x])/(3*b) - (Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (
-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/(3*b) - (Log[((-1)^(1/3)*e*(b^(1/3) +
 (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/
(3*b) - PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)]/(3*b) - Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]/(3*b) - P
olyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)]/(3*b) +
PolyLog[2, 1 + (e*x)/d]/b))/e
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.41

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(ex+d)}{e}+3pb\,e^2\left(-\frac{\sum_{\_R1=\text{RootOf}\left(\_Z^3a-3\_Z^2ad+3\_Za\,d^2-a\,d^3+b\,e^3\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\text{dil}\right.}{3b\,e^3}\right.$

input
```
int(ln(c*(a+b/x^3)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(a+b/x^3)^p)*ln(e*x+d)/e+3*p*b*e^2*(-1/3*sum(ln(e*x+d)*ln((-e*x+_R1-d
)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d
^3+b*e^3))/b/e^3+(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b/e^3)
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^pc\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{\log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((a + b/x^3)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{\log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x^3)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+e\,x}\,dx$$

input
```
int(log(c*(a + b/x^3)^p)/(d + e*x),x)
```

output
```
int(log(c*(a + b/x^3)^p)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\int \frac{\log\left(\frac{(a\,x^3+b)^p c}{x^{3p}}\right)}{ex+d}dx$$

input
```
int(log(c*(a+b/x^3)^p)/(e*x+d),x)
```

output
```
int(log(((a*x**3 + b)**p*c)/x**(3*p))/(d + e*x),x)
```

3.258 $\int \frac{\log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 388

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d} + \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d} - \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{b}{ax^3}\right)}{3d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d} - \frac{3p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d}$$

output
```
-1/3*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d-ln(c*(a+b/x^3)^p)*ln(e*x+d)/d-3*p*ln
(-e*x/d)*ln(e*x+d)/d+p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln
(e*x+d)/d+p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(
1/3)*e))*ln(e*x+d)/d+p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(
1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/d-1/3*p*polylog(2,1+b/a/x^3)/d+p*p
olylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/d+p*polylog(2,a^(1/3)*(e*x
+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/d+p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/
3)*d-(-1)^(2/3)*b^(1/3)*e))/d-3*p*polylog(2,1+e*x/d)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.02

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = -\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log \left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{3p\log \left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log \left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d} + \frac{p\log \left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d} + \frac{p\log \left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d} - \frac{p\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x^3}}{a}\right)}{3d} - \frac{3p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}$$

input
```
Integrate[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```

output
```
-1/3*(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/d - (Log[c*(a + b/x^3)^p]*Lo
g[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3
) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-(((-1)
^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)
*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)
*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, (
a + b/x^3)/a])/(3*d) - (3*p*PolyLog[2, (d + e*x)/d])/d + (p*PolyLog[2, (a^
(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d +
 e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d
+ e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)} \, dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx}-\frac{e \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}+\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}-\frac{p\,\mathrm{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d}-\frac{3p\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}$$

input
```
Int[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```

output
```
-1/3*(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/d - (Log[c*(a + b/x^3)^p]*Lo
g[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3
) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-((e*((
-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d
 + e*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/
3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x^3
)])/(3*d) + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d
+ (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d
 + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/
d - (3*p*PolyLog[2, 1 + (e*x)/d])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.88 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.57

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(ex+d)}{d}+\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(x)}{d}+3pb\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3a+b\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1}{\_R}\right.\right.}{3db}\right.$

input
```
int(ln(c*(a+b/x^3)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-ln(c*(a+b/x^3)^p)*ln(e*x+d)/d+ln(c*(a+b/x^3)^p)/d*ln(x)+3*p*b*(-1/3/d*sum
(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*a+b))/b+1/2/d*ln
(x)^2/b-1/d/b*ln(e*x+d)*ln(-e*x/d)-1/d/b*dilog(-e*x/d)+1/3/d*sum(ln(e*x+d)
*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d
+3*_Z*a*d^2-a*d^3+b*e^3))/b)
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^2 + d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b/x**3)**p)/x/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((a + b/x^3)^p*c)/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x^3)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx=\int \frac{\ln \left(c\,\left(a+\frac{b}{x^3}\right)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

input
```
int(log(c*(a + b/x^3)^p)/(x*(d + e*x)),x)
```

output
```
int(log(c*(a + b/x^3)^p)/(x*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx=\int \frac{\log \left(\frac{(a\,x^3+b)^p c}{x^{3p}}\right)}{e\,x^2+dx}dx$$

input
```
int(log(c*(a+b/x^3)^p)/x/(e*x+d),x)
```

output
```
int(log(((a*x**3 + b)**p*c)/x**(3*p))/(d*x + e*x**2),x)
```

3.259 $\int \frac{\log \left(c \left(a+\frac{b}{x^3}\right)^p\right)}{x^2 (d+e x)} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 557

$$
\begin{aligned}
\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx &= \frac{3p}{dx}-\frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} \\
&+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2}-\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d} \\
&+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2}+\frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} \\
&-\frac{ep\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&-\frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&-\frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&+\frac{\sqrt[3]{a}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d} \\
&+\frac{ep\operatorname{PolyLog}\left(2,1+\frac{b}{ax^3}\right)}{3d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2} \\
&-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2} \\
&-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}+\frac{3ep\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^2}
\end{aligned}
$$

output

```
3*p/d/x-3^(1/2)*a^(1/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3)
)/b^(1/3)/d-ln(c*(a+b/x^3)^p)/d/x+1/3*e*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d^2
-a^(1/3)*p*ln(b^(1/3)+a^(1/3)*x)/b^(1/3)/d+e*ln(c*(a+b/x^3)^p)*ln(e*x+d)/d
^2+3*e*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d
-b^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/
3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1
)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/d^2+1/2*a^(
1/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(1/3)/d+1/3*e*p*polylog
(2,1+b/a/x^3)/d^2-e*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/d^2
-e*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/d^2-e*p*p
olylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/d^2+3*e*p*polyl
og(2,1+e*x/d)/d^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.83

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{3bp\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-\frac{b}{ax^3}\right)}{4adx^4} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x^3}}{a}\right)}{3d^2} + \frac{3ep\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}$$

input
```
Integrate[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```

output
```
(3*b*p*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))])/(4*a*d*x^4) - Log[c*(
a + b/x^3)^p]/(d*x) + (e*Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/(3*d^2) +
 (e*Log[c*(a + b/x^3)^p]*Log[d + e*x])/d^2 + (3*e*p*Log[-((e*x)/d)]*Log[d
+ e*x])/d^2 - (e*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e)
)]*Log[d + e*x])/d^2 - (e*p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1
/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[
((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^
(1/3)*e)]*Log[d + e*x])/d^2 + (e*p*PolyLog[2, (a + b/x^3)/a])/(3*d^2) + (3
*e*p*PolyLog[2, (d + e*x)/d])/d^2 - (e*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a
^(1/3)*d - b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)
*d + (-1)^(1/3)*b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^
(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^2
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2(d+ex)}-\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}+\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[3]{a}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{b}d}-\frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d}+\frac{e\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx}+\frac{ep\operatorname{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2}-\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{b}d}+\frac{3ep\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}+\frac{3p}{dx}$$

input

```
Int[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```

output

```
(3*p)/(d*x) - (Sqrt[3]*a^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b
^(1/3))])/(b^(1/3)*d) - Log[c*(a + b/x^3)^p]/(d*x) + (e*Log[c*(a + b/x^3)^
p]*Log[-(b/(a*x^3))])/(3*d^2) - (a^(1/3)*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1
/3)*d) + (e*Log[c*(a + b/x^3)^p]*Log[d + e*x])/d^2 + (3*e*p*Log[-((e*x)/d)
]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^
(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)
*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-
1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/
3)*e)]*Log[d + e*x])/d^2 + (a^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^
(2/3)*x^2])/(2*b^(1/3)*d) + (e*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^2) - (e*p
*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^2 - (e*p*PolyL
og[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^2 - (e*p*
PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^2 +
(3*e*p*PolyLog[2, 1 + (e*x)/d])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.33 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.62

method	result
parts	$\frac{e\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(ex+d)}{d^2}-\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx}-\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)e\ln(x)}{d^2}+3pb\left(\frac{e\left(\frac{\operatorname{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{b}-\frac{\_R1=R}{}\right.}{}\right.$

input
```
int(ln(c*(a+b/x^3)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
e*ln(c*(a+b/x^3)^p)*ln(e*x+d)/d^2-ln(c*(a+b/x^3)^p)/d/x-ln(c*(a+b/x^3)^p)*
e/d^2*ln(x)+3*p*b*(e/d^2*((dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b-1/3*sum(l
n(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*
_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))/b)+1/d/b/x-1/3/d/b/(1/a*b)^(1/3)*ln(x+(1
/a*b)^(1/3))+1/6/d/b/(1/a*b)^(1/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))+1
/3/d/b*3^(1/2)/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))+1/3
*e/d^2*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*a+b))/
b-1/2*e/d^2*ln(x)^2/b)
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b/x**3)**p)/x**2/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^2}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b/x^3)^p)/(x^2*(d + e*x)),x)
```

output
```
int(log(c*(a + b/x^3)^p)/(x^2*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\frac{(a\,x^3+b)^p c}{x^{3p}}\right)}{e\,x^3+d\,x^2}dx$$

input
```
int(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x)
```

output
```
int(log(((a*x**3 + b)**p*c)/x**(3*p))/(d*x**2 + e*x**3),x)
```

3.260 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 737

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\sqrt{3}a^{2/3}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d}+\frac{\sqrt{3}\sqrt[3]{a}ep\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3}+\frac{a^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2b^{2/3}d}+\frac{\sqrt[3]{a}ep\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d^2}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3}-\frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3}-\frac{a^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4b^{2/3}d}-\frac{\sqrt[3]{a}ep\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d^2}-\frac{e^2p\,\text{PolyLog}\left(2,1+\frac{b}{ax^3}\right)}{3d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3}-\frac{3e^2p\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^3}$$

output
```
3/4*p/d/x^2-3*e*p/d^2/x-1/2*3^(1/2)*a^(2/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3
)*x)*3^(1/2)/b^(1/3))/b^(2/3)/d+3^(1/2)*a^(1/3)*e*p*arctan(1/3*(b^(1/3)-2*
a^(1/3)*x)*3^(1/2)/b^(1/3))/b^(1/3)/d^2-1/2*ln(c*(a+b/x^3)^p)/d/x^2+e*ln(c
*(a+b/x^3)^p)/d^2/x-1/3*e^2*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d^3+1/2*a^(2/3)
*p*ln(b^(1/3)+a^(1/3)*x)/b^(2/3)/d+a^(1/3)*e*p*ln(b^(1/3)+a^(1/3)*x)/b^(1/
3)/d^2-e^2*ln(c*(a+b/x^3)^p)*ln(e*x+d)/d^3-3*e^2*p*ln(-e*x/d)*ln(e*x+d)/d^
3+e^2*p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/d^3+e^2
*p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*
ln(e*x+d)/d^3+e^2*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3
)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/d^3-1/4*a^(2/3)*p*ln(b^(2/3)-a^(1/3)*
b^(1/3)*x+a^(2/3)*x^2)/b^(2/3)/d-1/2*a^(1/3)*e*p*ln(b^(2/3)-a^(1/3)*b^(1/3
)*x+a^(2/3)*x^2)/b^(1/3)/d^2-1/3*e^2*p*polylog(2,1+b/a/x^3)/d^3+e^2*p*poly
log(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/d^3+e^2*p*polylog(2,a^(1/3)*(
e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+
d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/d^3-3*e^2*p*polylog(2,1+e*x/d)/d^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.73

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = -\frac{3bep\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-\frac{b}{ax^3}\right)}{4ad^2x^4} + \frac{3bp\,\text{Hypergeometric2F1}\left(1,\frac{5}{3},\frac{8}{3},-\frac{b}{ax^3}\right)}{10adx^5} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3} - \frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} - \frac{e^2p\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x^3}}{a}\right)}{3d^3} - \frac{3e^2p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^3} + \frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3}$$

input
```
Integrate[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```

output
```
(-3*b*e*p*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))])/(4*a*d^2*x^4) + (3
*b*p*Hypergeometric2F1[1, 5/3, 8/3, -(b/(a*x^3))])/(10*a*d*x^5) - Log[c*(a
 + b/x^3)^p]/(2*d*x^2) + (e*Log[c*(a + b/x^3)^p])/(d^2*x) - (e^2*Log[c*(a
+ b/x^3)^p]*Log[-(b/(a*x^3))])/(3*d^3) - (e^2*Log[c*(a + b/x^3)^p]*Log[d +
 e*x])/d^3 - (3*e^2*p*Log[-((e*x)/d)]*Log[d + e*x])/d^3 + (e^2*p*Log[-((e*
(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*
p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^
(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(b^(1/3) +
 (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/
d^3 - (e^2*p*PolyLog[2, (a + b/x^3)/a])/(3*d^3) - (3*e^2*p*PolyLog[2, (d +
 e*x)/d])/d^3 + (e^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)
*e)])/d^3 + (e^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*
b^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)
^(2/3)*b^(1/3)*e)])/d^3
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2916, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{2916}$$

$$\int \left(-\frac{e^3\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3(d+ex)}+\frac{e^2\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3x}-\frac{e\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x^2}+\frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{3}a^{2/3}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d}-\frac{\sqrt[3]{a}ep\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{b}d^2}-\frac{a^{2/3}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{4b^{2/3}d}+\frac{a^{2/3}p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{2b^{2/3}d}+\frac{\sqrt{3}\sqrt[3]{a}ep\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2}-\frac{e^2\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}-\frac{e^2p\operatorname{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3}+\frac{\sqrt[3]{a}ep\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{b}d^2}-\frac{3e^2p\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}-\frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}-\frac{3ep}{d^2x}+\frac{3p}{4dx^2}$$

input
```
Int[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```

output
```
(3*p)/(4*d*x^2) - (3*e*p)/(d^2*x) - (Sqrt[3]*a^(2/3)*p*ArcTan[(b^(1/3) - 2
*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(2*b^(2/3)*d) + (Sqrt[3]*a^(1/3)*e*p*ArcTa
n[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(b^(1/3)*d^2) - Log[c*(a + b
/x^3)^p]/(2*d*x^2) + (e*Log[c*(a + b/x^3)^p])/(d^2*x) - (e^2*Log[c*(a + b/
x^3)^p]*Log[-(b/(a*x^3))])/(3*d^3) + (a^(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/
(2*b^(2/3)*d) + (a^(1/3)*e*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)*d^2) - (e^
2*Log[c*(a + b/x^3)^p]*Log[d + e*x])/d^3 - (3*e^2*p*Log[-((e*x)/d)]*Log[d
+ e*x])/d^3 + (e^2*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*
e))]*Log[d + e*x])/d^3 + (e^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))
/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)
^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)
*e)]*Log[d + e*x])/d^3 - (a^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2
/3)*x^2])/(4*b^(2/3)*d) - (a^(1/3)*e*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a
^(2/3)*x^2])/(2*b^(1/3)*d^2) - (e^2*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^3) +
 (e^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^3 + (e^
2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^
3 + (e^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*
e)])/d^3 - (3*e^2*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2916
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.19 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.66

method	result
parts	$-\frac{e^{2} \ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right) \ln(ex+d)}{d^{3}}-\frac{\ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right)}{2d\,x^{2}}+\frac{\ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right) e^{2} \ln(x)}{d^{3}}+\frac{e\ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right)}{d^{2}x}+\frac{3pb\left(\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3db\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln \ldots}{\ldots}\right.}{\ldots}$

input
```
int(ln(c*(a+b/x^3)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-e^2*ln(c*(a+b/x^3)^p)*ln(e*x+d)/d^3-1/2*ln(c*(a+b/x^3)^p)/d/x^2+ln(c*(a+b
/x^3)^p)*e^2/d^3*ln(x)+e*ln(c*(a+b/x^3)^p)/d^2/x+3/2*p*b*(1/3/d/b/(1/a*b)^
(2/3)*ln(x+(1/a*b)^(1/3))-1/6/d/b/(1/a*b)^(2/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/
a*b)^(2/3))+1/3/d/b/(1/a*b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1
/3)*x-1))+2/3/d^2/b*e/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1/3))-1/3/d^2/b*e/(1/a*b
)^(1/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))-2/3/d^2/b*e*3^(1/2)/(1/a*b)^
(1/3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))+1/2/d/b/x^2-2/d^2*e/b/x-2/
3*e^2/d^3*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*a+b
))/b+e^2/d^3*ln(x)^2/b-2*e^2/d^3*((dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))/b-1
/3*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_
Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))/b))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(a+b/x**3)**p)/x**3/(e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^3}\,dx$$

input
```
integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3\,(d+e\,x)}\,dx$$

input
```
int(log(c*(a + b/x^3)^p)/(x^3*(d + e*x)),x)
```

output
```
int(log(c*(a + b/x^3)^p)/(x^3*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(\frac{(a\,x^3+b)^p c}{x^{3p}}\right)}{e\,x^4+d\,x^3}dx$$

input
```
int(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x)
```

output
```
int(log(((a*x**3 + b)**p*c)/x**(3*p))/(d*x**3 + e*x**4),x)
```

3.261 $\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 749

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \frac{3p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{3ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}$$

output
```
3*p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/
g^(1/2)-p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*(d^(1/3)+e^(1/3)*
x)/(I*e^(1/3)*f^(1/2)+d^(1/3)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1
/2)-p*arctan(g^(1/2)*x/f^(1/2))*ln(-2*I*f^(1/2)*g^(1/2)*((-1)^(2/3)*d^(1/3
)+e^(1/3)*x)/(e^(1/3)*f^(1/2)+(-1)^(1/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*g^(1/
2)*x))/f^(1/2)/g^(1/2)-p*arctan(g^(1/2)*x/f^(1/2))*ln(2*(-1)^(5/6)*f^(1/2)
*g^(1/2)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(e^(1/3)*f^(1/2)+(-1)^(5/6)*d^(1/3
)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+arctan(g^(1/2)*x/f^(1/2)
)*ln(c*(e*x^3+d)^p)/f^(1/2)/g^(1/2)-3/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)
-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*f^(1/2)*g^(1/2)*(d^(1
/3)+e^(1/3)*x)/(I*e^(1/3)*f^(1/2)+d^(1/3)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/
f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1+2*I*f^(1/2)*g^(1/2)*((-1)^(2/3)*d^(1/3
)+e^(1/3)*x)/(e^(1/3)*f^(1/2)+(-1)^(1/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*g^(1/
2)*x))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*(-1)^(5/6)*f^(1/2)*g^(1/2)*(d
^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(e^(1/3)*f^(1/2)+(-1)^(5/6)*d^(1/3)*g^(1/2))/
(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.16

$$\int \frac{\log \left(c(d+ex^3)^p\right)}{f+gx^2} \, dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]
```

output
```
(-(p*Log[(Sqrt[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] + d^(1/3)*Sqrt[
g])]*Log[Sqrt[-f] - Sqrt[g]*x]) - p*Log[(Sqrt[g]*(-((-1)^(1/3)*d^(1/3)) +
e^(1/3)*x))/(e^(1/3)*Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f]
- Sqrt[g]*x] - p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/(e^(1/3)*S
qrt[-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[-
((Sqrt[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] - d^(1/3)*Sqrt[g]))]*Lo
g[Sqrt[-f] + Sqrt[g]*x] + p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))
/(-(e^(1/3)*Sqrt[-f]) + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g
]*x] + p*Log[((-1)^(1/3)*Sqrt[g]*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/(e^(1/3
)*Sqrt[-f] + (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*x] + Log[
Sqrt[-f] - Sqrt[g]*x]*Log[c*(d + e*x^3)^p] - Log[Sqrt[-f] + Sqrt[g]*x]*Log
[c*(d + e*x^3)^p] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)
*Sqrt[-f] + d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*
x))/(e^(1/3)*Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/
3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])
] + p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] - d^(1
/3)*Sqrt[g])] + p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqr
t[-f] + (-1)^(1/3)*d^(1/3)*Sqrt[g])] + p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + S
qrt[g]*x))/(e^(1/3)*Sqrt[-f] - (-1)^(2/3)*d^(1/3)*Sqrt[g])])/(2*Sqrt[-f]*S
qrt[g])
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 706, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2920, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^3\right)^p\right)}{f+gx^2} \, dx$$

$$\Big\downarrow \text{ 2920}$$

$$\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}}-3ep\int \frac{x^2\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(ex^3+d\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{3ep\int\frac{x^2\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex^3+d}dx}{\sqrt{f}\sqrt{g}}$$

$\Big\downarrow$ 7276

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{3ep\int\left(\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{3e^{2/3}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{3e^{2/3}\left(\sqrt[3]{e}x-\sqrt[3]{-1}\sqrt[3]{d}\right)}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{3e^{2/3}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$\Big\downarrow$ 2009

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{3ep\left(\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt[3]{d}\sqrt{g}+i\sqrt[3]{e}\sqrt{f}\right)}\right)}{3e}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}+\sqrt[3]{e}\sqrt{f}\right)}\right)}{3e}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\ldots}{\ldots}\right.}{}$$

input
```
Int[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]
```

output
```
(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^3)^p])/(Sqrt[f]*Sqrt[g]) - (3*
e*p*(-((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x
)])/e) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(d^(1/3) + e^
(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))]
)/(3*e) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[((-2*I)*Sqrt[f]*Sqrt[g]*((-1)^(
2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(1/6)*d^(1/3)*Sqrt[g])
*(Sqrt[f] - I*Sqrt[g]*x))])/(3*e) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*(-
1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((e^(1/3)*Sqrt[
f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(3*e) + ((I/2)
*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/e - ((I/6)*PolyLog[2
, 1 - (2*Sqrt[f]*Sqrt[g]*(d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1
/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e - ((I/6)*PolyLog[2, 1 + ((2*I)*S
qrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^
(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e - ((I/6)*PolyLog[2, 1
- (2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((e^(1/3
)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e))/(Sq
rt[f]*Sqrt[g])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.91 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\left(\ln\left(\left(e\,x^{3}+d\right)^{p}\right)-p\ln\left(e\,x^{3}+d\right)\right)\arctan\left(\frac{g x}{\sqrt{g f}}\right)}{\sqrt{g f}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(g\_Z^{2}+f\right)}\frac{\ln\left(x-\_\alpha\right)\ln\left(e\,x^{3}+d\right)-\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^{3}g+3\_\alpha\_Z\right.}\right.}{}\right.}{}$

input
```
int(ln(c*(e*x^3+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
(ln((e*x^3+d)^p)-p*ln(e*x^3+d))/(g*f)^(1/2)*arctan(g*x/(g*f)^(1/2))+1/2*p/
g*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^3+d)-sum(ln(x-_alpha)*ln((_R1-x+_alpha
)/_R1)+dilog((_R1-x+_alpha)/_R1),_R1=RootOf(_Z^3*e*g+3*_Z^2*_alpha*e*g-3*_
Z*e*f-_alpha*e*f+d*g))),_alpha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^3+d
)^p)*csgn(I*c*(e*x^3+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+
d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^3+
d)^p)^2*csgn(I*c)+ln(c))/(g*f)^(1/2)*arctan(g*x/(g*f)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**3+d)**p)/(g*x**2+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left((ex^3+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left((ex^3+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,(e\,x^3+d)^p\right)}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e*x^3)^p)/(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x^3)^p)/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((e\,x^3+d)^p\,c\right)}{g\,x^2+f}dx$$

input
```
int(log(c*(e*x^3+d)^p)/(g*x^2+f),x)
```

output
```
int(log((d + e*x**3)**p*c)/(f + g*x**2),x)
```

3.262 $\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 533

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}$$

output
```
2*p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/
g^(1/2)-p*arctan(g^(1/2)*x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1
/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/
2)/g^(1/2)-p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^
(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(
1/2)/g^(1/2)+arctan(g^(1/2)*x/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)-I
*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+1/2*I*p*po
lylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)
^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1
-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^
(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = -\frac{i\left(p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)+p\log\left(\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{\cdots}}\right.\right.}{}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

output
```
((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqr
t[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d]
+ Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g
]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[
f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*S
qrt[g]*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p
] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sq
rt[-d]*Sqrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]
*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[
g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sq
rt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*S
qrt[g])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2920, 27, 5463, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx$$

$$\Big\downarrow \ 2920$$

$$\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-2ep\int \frac{x \arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(ex^2+d\right)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{2ep\int \frac{x \arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex^2+d}dx}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 5463$$

$$\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{2ep\int \left(\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-$$
$$\frac{2ep\left(\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2e}+\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2e}-\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right.}{e}\right.}{\sqrt{f}\sqrt{g}}$$

input
```
Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

output
```
(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (2*
e*p*(-((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x
)])/e) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] -
Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x
))])/(2*e) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d]
 + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g
]*x))])/(2*e) + ((I/2)*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)]
)/e - ((I/4)*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I
*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e - ((I/4)
*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqr
t[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e))/(Sqrt[f]*Sqrt[g])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5463
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.77 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(\ln((e\,x^2+d)^p)-p\ln(e\,x^2+d))\arctan\left(\frac{gx}{\sqrt{gf}}\right)}{\sqrt{gf}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}(g\_Z^2+f)}\frac{\ln(x-\_\alpha)\ln(e\,x^2+d)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}(\_Z^2 eg+}{\mathrm{RootOf}(\_Z^2}\right.\right.}{}\right.}{}$

input
```
int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(g*f)^(1/2)*arctan(g*x/(g*f)^(1/2))+1/2*p/
g*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x
+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f
,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_al
pha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^
2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*cs
gn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))/(g
*f)^(1/2)*arctan(g*x/(g*f)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

input
```
integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

output
```
Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log \left(\left(e\,x^2+d\right)^p c\right)}{g\,x^2+f}dx$$

input
```
int(log(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

output
```
int(log((d + e*x**2)**p*c)/(f + g*x**2),x)
```

3.263 $\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 229

$$\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx = \frac{\log(c(d+ex)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log(c(d+ex)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

output
```
1/2*ln(c*(e*x+d)^p)*ln(e*((-f)^(1/2)-g^(1/2)*x)/(e*(-f)^(1/2)+d*g^(1/2)))/
(-f)^(1/2)/g^(1/2)-1/2*ln(c*(e*x+d)^p)*ln(e*((-f)^(1/2)+g^(1/2)*x)/(e*(-f)
^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,-g^(1/2)*(e*x+d)/(e*
(-f)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(2,g^(1/2)*(e*x+d)/
(e*(-f)^(1/2)+d*g^(1/2)))/(-f)^(1/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{\log \left(c(d+ex)^p\right)}{f+gx^2}\,dx$$
$$= \frac{\log \left(c(d+ex)^p\right) \left(\log \left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right) - \log \left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)\right) - p\,\text{PolyLog}\left(2, -\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right) + p\,\text{PolyLog}\left(2,\right.}{2\sqrt{-f}\sqrt{g}}$$

input
```
Integrate[Log[c*(d + e*x)^p]/(f + g*x^2),x]
```

output
```
(Log[c*(d + e*x)^p]*(Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g
])] - Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]) - p*PolyLo
g[2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + p*PolyLog[2, (Sqrt
[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(d+ex)^p\right)}{f+gx^2}\,dx$$
$$\Big\downarrow \; 2856$$
$$\int \left(\frac{\sqrt{-f}\log \left(c(d+ex)^p\right)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\sqrt{-f}\log \left(c(d+ex)^p\right)}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}-\frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}$$

input
```
Int[Log[c*(d + e*x)^p]/(f + g*x^2),x]
```

output
```
(Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g]
)])/(2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*
x))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((Sqr
t[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*Sqrt[g]) + (p*Poly
Log[2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.27 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{gf}}\right)p\ln(ex+d)}{\sqrt{gf}}+\frac{\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{gf}}\right)\ln\left((ex+d)^p\right)}{\sqrt{gf}}+\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-gf}-g(ex+d)+dg}{e\sqrt{-gf}+dg}\right)}{2\sqrt{-gf}}-\frac{p\ln(ex\text{-}}{}$

input
```
int(ln(c*(e*x+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
-1/(g*f)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(g*f)^(1/2))*p*ln(e*x+d)+1
/(g*f)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(g*f)^(1/2))*ln((e*x+d)^p)+1
/2*p*ln(e*x+d)/(-g*f)^(1/2)*ln((e*(-g*f)^(1/2)-g*(e*x+d)+d*g)/(e*(-g*f)^(1
/2)+d*g))-1/2*p*ln(e*x+d)/(-g*f)^(1/2)*ln((e*(-g*f)^(1/2)+g*(e*x+d)-d*g)/(
e*(-g*f)^(1/2)-d*g))+1/2*p/(-g*f)^(1/2)*dilog((e*(-g*f)^(1/2)-g*(e*x+d)+d*
g)/(e*(-g*f)^(1/2)+d*g))-1/2*p/(-g*f)^(1/2)*dilog((e*(-g*f)^(1/2)+g*(e*x+d
)-d*g)/(e*(-g*f)^(1/2)-d*g))+(1/2*I*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^
p)^2-1/2*I*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^p)*csgn(I*c)-1/2*I*Pi*csg
n(I*c*(e*x+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x+d)^p)^2*csgn(I*c)+ln(c))/(g*f)^(
1/2)*arctan(g*x/(g*f)^(1/2))
```

Fricas [F]

$$\int \frac{\log (c(d+ex)^p)}{f+gx^2}\,dx=\int \frac{\log ((ex+d)^pc)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{\log (c(d+ex)^p)}{f+gx^2}\,dx=\int \frac{\log (c(d+ex)^p)}{f+gx^2}\,dx$$

input
```
integrate(ln(c*(e*x+d)**p)/(g*x**2+f),x)
```

output
```
Integral(log(c*(d + e*x)**p)/(f + g*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.35

$$\int \frac{\log \left(c(d+ex)^p\right)}{f+gx^2}\,dx = \frac{ep\left(\frac{2\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log(ex+d)}{e}+\frac{\arctan\left(\frac{\left(e^2x+de\right)\sqrt{f}\sqrt{g}}{e^2f+d^2g},\frac{degx+d^2g}{e^2f+d^2g}\right)\log\left(gx^2+f\right)-\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{e^2gx^2+2\,degx+d^2g}{e^2f+d^2g}\right)-i\,\mathrm{Li}_2\left(\frac{degx+e^{\cdots}}{e^2f\cdots}\right.\cdots}{e}\cdots\right)}{2\sqrt{fg}} - \frac{p\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log(ex+d)}{\sqrt{fg}} + \frac{\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left((ex+d)^pc\right)}{\sqrt{fg}}$$

input
```
integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*e*p*(2*arctan(g*x/sqrt(f*g))*log(e*x + d)/e + (arctan2((e^2*x + d*e)*s
qrt(f)*sqrt(g)/(e^2*f + d^2*g), (d*e*g*x + d^2*g)/(e^2*f + d^2*g))*log(g*x
^2 + f) - arctan(sqrt(g)*x/sqrt(f))*log((e^2*g*x^2 + 2*d*e*g*x + d^2*g)/(e
^2*f + d^2*g)) - I*dilog((d*e*g*x + e^2*f - (I*e^2*x - I*d*e)*sqrt(f)*sqrt
(g))/(e^2*f + 2*I*d*e*sqrt(f)*sqrt(g) - d^2*g)) + I*dilog((d*e*g*x + e^2*f
 + (I*e^2*x - I*d*e)*sqrt(f)*sqrt(g))/(e^2*f - 2*I*d*e*sqrt(f)*sqrt(g) - d
^2*g)))/e)/sqrt(f*g) - p*arctan(g*x/sqrt(f*g))*log(e*x + d)/sqrt(f*g) + ar
ctan(g*x/sqrt(f*g))*log((e*x + d)^p*c)/sqrt(f*g)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left((ex+d)^pc\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,(d+e\,x)^p\right)}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e*x)^p)/(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x)^p)/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex+d)^p\,c\right)}{g\,x^2+f}dx$$

input
```
int(log(c*(e*x+d)^p)/(g*x^2+f),x)
```

output
```
int(log((d + e*x)**p*c)/(f + g*x**2),x)
```

3.264 $\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 360

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(e+dx)}{(id\sqrt{f}+e\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\,\text{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{ip\,\text{PolyLog}\left(2,\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(e+dx)}{(id\sqrt{f}+e\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}$$

output
```
arctan(g^(1/2)*x/f^(1/2))*ln(c*(d+e/x)^p)/f^(1/2)/g^(1/2)+p*arctan(g^(1/2)
*x/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)-p*arctan(g
^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*(d*x+e)/(I*d*f^(1/2)+e*g^(1/2))/(f^
(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,-I*g^(1/2)*x/f^(1/2)
)/f^(1/2)/g^(1/2)-1/2*I*p*polylog(2,I*g^(1/2)*x/f^(1/2))/f^(1/2)/g^(1/2)-1
/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+1/2*I*
p*polylog(2,1-2*f^(1/2)*g^(1/2)*(d*x+e)/(I*d*f^(1/2)+e*g^(1/2))/(f^(1/2)-I
*g^(1/2)*x))/f^(1/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.04

$$\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx$$
$$= \frac{\log \left(c\left(d+\frac{e}{x}\right)^p\right)\log \left(\sqrt{-f}-\sqrt{g}x\right)+p\log \left(\frac{\sqrt{g}x}{\sqrt{-f}}\right)\log \left(\sqrt{-f}-\sqrt{g}x\right)-p\log \left(\frac{\sqrt{g}(e+dx)}{d\sqrt{-f}+e\sqrt{g}}\right)\log \left(\sqrt{-f}-\sqrt{\right.}}{}$$

input
```
Integrate[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

output
```
(Log[c*(d + e/x)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[(Sqrt[g]*x)/Sqrt[-f]
]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(e + d*x))/(d*Sqrt[-f] + e*Sq
rt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - Log[c*(d + e/x)^p]*Log[Sqrt[-f] + Sqrt
[g]*x] - p*Log[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log
[-((Sqrt[g]*(e + d*x))/(d*Sqrt[-f] - e*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x
] - p*PolyLog[2, (d*(Sqrt[-f] - Sqrt[g]*x))/(d*Sqrt[-f] + e*Sqrt[g])] + p*
PolyLog[2, (d*(Sqrt[-f] + Sqrt[g]*x))/(d*Sqrt[-f] - e*Sqrt[g])] - p*PolyLo
g[2, 1 + (Sqrt[g]*x)/Sqrt[-f]] + p*PolyLog[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)
])/(2*Sqrt[-f]*Sqrt[g])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2920, 27, 2005, 5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}dx$$

$$\Big\downarrow \ 2920$$

$$ep\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{e}{x}\right)x^2}dx+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 27$$

$$\frac{ep\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x}\right)x^2}dx}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 2005$$

$$\frac{ep\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(e+dx)}dx}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 5411$$

$$\frac{ep\int \left(\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex}-\frac{d\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e(e+dx)}\right)dx}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}}+$$

$$\frac{ep\left(-\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(dx+e)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(e\sqrt{g}+id\sqrt{f}\right)}\right)}{e}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{e}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(e+dx)}{\left(i\sqrt{f}d+e\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2e}+\frac{i\,\text{PolyLog}}{}\right.}{\sqrt{f}\sqrt{g}}$$

input
```
Int[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

output
```
(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x)^p])/(Sqrt[f]*Sqrt[g]) + (e*p*
((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/e
- (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(e + d*x))/((I*d*Sqr
t[f] + e*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e + ((I/2)*PolyLog[2, ((-I)*S
qrt[g]*x)/Sqrt[f]])/e - ((I/2)*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/e - ((I/
2)*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/e + ((I/2)*PolyLog
[2, 1 - (2*Sqrt[f]*Sqrt[g]*(e + d*x))/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt[f]
- I*Sqrt[g]*x))])/e))/(Sqrt[f]*Sqrt[g])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Maple [F]

$$\int \frac{\ln \left(c\left(d+\frac{e}{x}\right)^{p}\right)}{g\,x^{2}+f}dx$$

input
```
int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
```

output
```
int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^{p}\right)}{f+gx^{2}}\,dx=\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^{p}\right)}{gx^{2}+f}\,dx$$

input
```
integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log(c*((d*x + e)/x)^p)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^{p}\right)}{f+gx^{2}}\,dx=\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^{p}\right)}{f+gx^{2}}\,dx$$

input
```
integrate(ln(c*(d+e/x)**p)/(g*x**2+f),x)
```

output
```
Integral(log(c*(d + e/x)**p)/(f + g*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx$$
$$= \frac{ep\left(\frac{4\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left(d+\frac{e}{x}\right)}{e} - \frac{\left(\pi-2\arctan\left(\frac{(d^2x+de)\sqrt{f}\sqrt{g}}{d^2f+e^2g},\frac{degx+e^2g}{d^2f+e^2g}\right)\right)\log(gx^2+f)-4\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)+2\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\mathrm{lo}}{}\right.}{4}$$
$$- \frac{p\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left(d+\frac{e}{x}\right)}{\sqrt{fg}} + \frac{\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{fg}}$$

input
```
integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/4*e*p*(4*arctan(g*x/sqrt(f*g))*log(d + e/x)/e - ((pi - 2*arctan2((d^2*x
+ d*e)*sqrt(f)*sqrt(g)/(d^2*f + e^2*g), (d*e*g*x + e^2*g)/(d^2*f + e^2*g))
)*log(g*x^2 + f) - 4*arctan(sqrt(g)*x/sqrt(f))*log(sqrt(g)*x/sqrt(f)) + 2*
arctan(sqrt(g)*x/sqrt(f))*log((d^2*g*x^2 + 2*d*e*g*x + e^2*g)/(d^2*f + e^2
*g)) + 2*I*dilog((I*sqrt(g)*x + sqrt(f))/sqrt(f)) - 2*I*dilog(-(I*sqrt(g)*
x - sqrt(f))/sqrt(f)) + 2*I*dilog((d*e*g*x + d^2*f - (I*d^2*x - I*d*e)*sqr
t(f)*sqrt(g))/(d^2*f + 2*I*d*e*sqrt(f)*sqrt(g) - e^2*g)) - 2*I*dilog((d*e*
g*x + d^2*f + (I*d^2*x - I*d*e)*sqrt(f)*sqrt(g))/(d^2*f - 2*I*d*e*sqrt(f)*
sqrt(g) - e^2*g)))/e)/sqrt(f*g) - p*arctan(g*x/sqrt(f*g))*log(d + e/x)/sqr
t(f*g) + arctan(g*x/sqrt(f*g))*log(c*(d + e/x)^p)/sqrt(f*g)
```

Giac [F]

$$\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left(c\left(d+\frac{e}{x}\right)^p\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log(c*(d + e/x)^p)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\ln \left(c\,{\left(d+\frac{e}{x}\right)}^p\right)}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e/x)^p)/(f + g*x^2),x)
```

output
```
int(log(c*(d + e/x)^p)/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{\log \left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log \left(\frac{(dx+e)^p c}{x^p}\right)}{g\,x^2+f}dx$$

input
```
int(log(c*(d+e/x)^p)/(g*x^2+f),x)
```

output
```
int(log(((d*x + e)**p*c)/x**p)/(f + g*x**2),x)
```

3.265 $\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 597

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}$$

output
```
arctan(g^(1/2)*x/f^(1/2))*ln(c*(d+e/x^2)^p)/f^(1/2)/g^(1/2)+2*p*arctan(g^(
1/2)*x/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)-p*arct
an(g^(1/2)*x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*(e^(1/2)-(-d)^(1/2)*x)/(I*(-d)
^(1/2)*f^(1/2)-e^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)-p*a
rctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*(e^(1/2)+(-d)^(1/2)*x)/(I*(-
d)^(1/2)*f^(1/2)+e^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+I
*p*polylog(2,-I*g^(1/2)*x/f^(1/2))/f^(1/2)/g^(1/2)-I*p*polylog(2,I*g^(1/2)
*x/f^(1/2))/f^(1/2)/g^(1/2)-I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x
))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1+2*f^(1/2)*g^(1/2)*(e^(1/2)-(-d)^(1/
2)*x)/(I*(-d)^(1/2)*f^(1/2)-e^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2
)/g^(1/2)+1/2*I*p*polylog(2,1-2*f^(1/2)*g^(1/2)*(e^(1/2)+(-d)^(1/2)*x)/(I*
(-d)^(1/2)*f^(1/2)+e^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.18

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx$$
$$= \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\log\left(\sqrt{-f}-\sqrt{g}x\right)+2p\log\left(\frac{\sqrt{g}x}{\sqrt{-f}}\right)\log\left(\sqrt{-f}-\sqrt{g}x\right)-p\log\left(\frac{\sqrt{g}(-\sqrt{e}+\sqrt{-d}x)}{\sqrt{-d}\sqrt{-f}-\sqrt{e}\sqrt{g}}\right)\log\left(\sqrt{-\right.}{}$$

input
```
Integrate[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]
```

output
```
(Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + 2*p*Log[(Sqrt[g]*x)/Sqrt
[-f]]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(-Sqrt[e] + Sqrt[-d]*x))/
(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(
Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log
[Sqrt[-f] - Sqrt[g]*x] - Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] + Sqrt[g]*x] -
2*p*Log[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log[(Sqrt[
g]*(Sqrt[e] - Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log[Sqrt
[-f] + Sqrt[g]*x] + p*Log[-((Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqr
t[-f] - Sqrt[e]*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x] - p*PolyLog[2, (Sqrt[
-d]*(Sqrt[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])] - p*Pol
yLog[2, (Sqrt[-d]*(Sqrt[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqr
t[g])] + p*PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f]
 - Sqrt[e]*Sqrt[g])] + p*PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqr
t[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])] - 2*p*PolyLog[2, 1 + (Sqrt[g]*x)/Sqrt[-
f]] + 2*p*PolyLog[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)])/(2*Sqrt[-f]*Sqrt[g])
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 557, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2920, 27, 2005, 5463, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx$$

$\Big\downarrow$ 2920

$$2ep\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{e}{x^2}\right)x^3}dx+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}$$

$\Big\downarrow$ 27

$$\frac{2ep\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x^2}\right)x^3}dx}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}$$

$\Big\downarrow$ 2005

$$\frac{2ep\int \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(dx^2+e)}dx}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}$$

$\Big\downarrow$ 5463

$$\frac{2ep\int \left(\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex}-\frac{dx\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e(dx^2+e)}\right)dx}{\sqrt{f}\sqrt{g}}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}$$

$\Big\downarrow$ 2009

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+$$

$$\frac{2ep\left(-\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f})}\right)}{2e}-\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}x+\sqrt{e})}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f})}\right)}{2e}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{}}\right.}{e}\right.}{}$$

input

```
Int[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]
```

output
```
(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x^2)^p])/(Sqrt[f]*Sqrt[g]) + (2*
e*p*((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)]
)/e - (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[e] - Sqrt
[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))]
)/(2*e) - (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[e] + S
qrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x
))])/(2*e) + ((I/2)*PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]])/e - ((I/2)*PolyL
og[2, (I*Sqrt[g]*x)/Sqrt[f]])/e - ((I/2)*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[
f] - I*Sqrt[g]*x)])/e + ((I/4)*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[e]
- Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g
]*x))])/e + ((I/4)*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x
))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e))/
(Sqrt[f]*Sqrt[g])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5463
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

Maple [F]

$$\int \frac{\ln \left(c\left(d+\frac{e}{x^2}\right)^p\right)}{g\,x^2+f}dx$$

input
```
int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

output
```
int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log \left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left(c\left(d+\frac{e}{x^2}\right)^p\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log(c*((d*x^2 + e)/x^2)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e/x**2)**p)/(g*x**2+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(log(c*(d + e/x^2)^p)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log(c*(d + e/x^2)^p)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e/x^2)^p)/(f + g*x^2),x)
```

output
```
int(log(c*(d + e/x^2)^p)/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(\frac{(d\,x^2+e)^p c}{x^{2p}}\right)}{g\,x^2+f}dx$$

input
```
int(log(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

output
```
int(log(((d*x**2 + e)**p*c)/x**(2*p))/(f + g*x**2),x)
```

3.266 $\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 541

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

output
```
-1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*((-(-f)^(1/2))^(1/2)-g^(1/4)*x^(1/2))/(e*(
-(-f)^(1/2))^(1/2)+d*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e*x^(1/2))^p
)*ln(e*((-f)^(1/4)-g^(1/4)*x^(1/2))/(e*(-f)^(1/4)+d*g^(1/4)))/(-f)^(1/2)/g
^(1/2)-1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*((-(-f)^(1/2))^(1/2)+g^(1/4)*x^(1/2)
)/(e*(-(-f)^(1/2))^(1/2)-d*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e*x^(1
/2))^p)*ln(e*((-f)^(1/4)+g^(1/4)*x^(1/2))/(e*(-f)^(1/4)-d*g^(1/4)))/(-f)^(
1/2)/g^(1/2)-1/2*p*polylog(2,-g^(1/4)*(d+e*x^(1/2))/(e*(-(-f)^(1/2))^(1/2)
-d*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(2,-g^(1/4)*(d+e*x^(1/2))/(e*
(-f)^(1/4)-d*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,g^(1/4)*(d+e*x^(
1/2))/(e*(-(-f)^(1/2))^(1/2)+d*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(
2,g^(1/4)*(d+e*x^(1/2))/(e*(-f)^(1/4)+d*g^(1/4)))/(-f)^(1/2)/g^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.78

$$\int \frac{\log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx$$

$$= \frac{\log \left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)-\log \left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-i\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+id\sqrt[4]{g}}\right)-\log\left(c\left(d+\right.\right.}{}$$

input
```
Integrate[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

output
```
(Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(
1/4) + d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - I*g^(1/
4)*Sqrt[x]))/(e*(-f)^(1/4) + I*d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[
(e*((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) - I*d*g^(1/4))] + Log[c
*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) -
 d*g^(1/4))] + p*PolyLog[2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*
g^(1/4)))] - p*PolyLog[2, (I*g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + I*d*
g^(1/4))] - p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(I*e*(-f)^(1/4) + d*g^(
1/4))] + p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))
])/(2*Sqrt[-f]*Sqrt[g])
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2922, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2} \, dx$$

$$\Big\downarrow \text{ 2922}$$

$$2\int \frac{\sqrt{x}\log \left(c\left(d+e\sqrt{x}\right)^p\right)}{gx^2+f}d\sqrt{x}$$

$$\Big\downarrow \text{ 2863}$$

$$2\int \left(-\frac{\sqrt{g}\sqrt{x}\log \left(c\left(d+e\sqrt{x}\right)^p\right)}{2\sqrt{-f}\left(\sqrt{-f}\sqrt{g}-gx\right)}-\frac{\sqrt{g}\sqrt{x}\log \left(c\left(d+e\sqrt{x}\right)^p\right)}{2\sqrt{-f}\left(gx+\sqrt{-f}\sqrt{g}\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{\log \left(c\left(d+e\sqrt{x}\right)^p\right)\log \left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt{-\sqrt{-f}}}\right)}{4\sqrt{-f}\sqrt{g}}+\frac{\log \left(c\left(d+e\sqrt{x}\right)^p\right)\log \left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt[4]{-f}}\right)}{4\sqrt{-f}\sqrt{g}}-\frac{\log \left(c\left(d+e\right.\right.}{}\right.$$

input
```
Int[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

output
```
2*(-1/4*(Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] - g^(1/4)*Sqrt[x
]))/(e*Sqrt[-Sqrt[-f]] + d*g^(1/4))])/(Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e*S
qrt[x])^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4
))])/(4*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-
f]] + g^(1/4)*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4))])/(4*Sqrt[-f]*Sqrt
[g]) + (Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e
*(-f)^(1/4) - d*g^(1/4))])/(4*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((g^(1/4)
*(d + e*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4)))])/(4*Sqrt[-f]*Sqrt[g])
+ (p*PolyLog[2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4)))])/
(4*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*Sqrt[-Sq
rt[-f]] + d*g^(1/4))])/(4*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, (g^(1/4)*(d +
e*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(4*Sqrt[-f]*Sqrt[g]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2922
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   S
ubst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x]
, x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
 r, s}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{\ln\left(c\left(d+e\sqrt{x}\right)^{p}\right)}{g\,x^{2}+f}dx$$

input
```
int(ln(c*(d+e*x^(1/2))^p)/(g*x^2+f),x)
```

output
```
int(ln(c*(d+e*x^(1/2))^p)/(g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(\left(e\sqrt{x}+d\right)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**(1/2))**p)/(g*x**2+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(\left(e\sqrt{x}+d\right)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(\left(e\sqrt{x}+d\right)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\ln\left(c\left(d+e\sqrt{x}\right)^p\right)}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e*x^(1/2))^p)/(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x^(1/2))^p)/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(\left(\sqrt{x}\,e+d\right)^p c\right)}{g\,x^2+f}dx$$

input
```
int(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x)
```

output
```
int(log((sqrt(x)*e + d)**p*c)/(f + g*x**2),x)
```

3.267 $\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 561

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx = -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

output
```
-1/2*ln(c*(d+e/x^(1/2))^p)*ln(e*(g^(1/4)-(-(-f)^(1/2))^(1/2)/x^(1/2))/(d*(
-(-f)^(1/2))^(1/2)+e*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e/x^(1/2))^p
)*ln(-e*(g^(1/4)+(-(-f)^(1/2))^(1/2)/x^(1/2))/(d*(-(-f)^(1/2))^(1/2)-e*g^(
1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e/x^(1/2))^p)*ln(e*(g^(1/4)-(-f)^(1/
4)/x^(1/2))/(d*(-f)^(1/4)+e*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e/x^(
1/2))^p)*ln(-e*(g^(1/4)+(-f)^(1/4)/x^(1/2))/(d*(-f)^(1/4)-e*g^(1/4)))/(-f)
^(1/2)/g^(1/2)-1/2*p*polylog(2,(-(-f)^(1/2))^(1/2)*(d+e/x^(1/2))/(d*(-(-f)
^(1/2))^(1/2)-e*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(2,(-f)^(1/4)*(d
+e/x^(1/2))/(d*(-f)^(1/4)-e*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,(
-(-f)^(1/2))^(1/2)*(d+e/x^(1/2))/(d*(-(-f)^(1/2))^(1/2)+e*g^(1/4)))/(-f)^(
1/2)/g^(1/2)+1/2*p*polylog(2,(-f)^(1/4)*(d+e/x^(1/2))/(d*(-f)^(1/4)+e*g^(1
/4)))/(-f)^(1/2)/g^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.63

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

output
```
(Log[c*(d + e/Sqrt[x])^p]*Log[-(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[-((g^
(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4)))]*Log[-(-f)^(1/4) - g^(1
/4)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[(-I)*(-f)^(1/4) - g^(1/4)*Sqrt
[x]] + p*Log[(I*g^(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(1/4))]*Log
[(-I)*(-f)^(1/4) - g^(1/4)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[I*(-f)^
(1/4) - g^(1/4)*Sqrt[x]] + p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(I*d*(-f)^(1/4)
 + e*g^(1/4))]*Log[I*(-f)^(1/4) - g^(1/4)*Sqrt[x]] + Log[c*(d + e/Sqrt[x])
^p]*Log[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(d
*(-f)^(1/4) + e*g^(1/4))]*Log[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[I*(-f)
^(1/4) - g^(1/4)*Sqrt[x]]*Log[((-I)*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*Log[(
-I)*(-f)^(1/4) - g^(1/4)*Sqrt[x]]*Log[(I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*
Log[(-f)^(1/4) - g^(1/4)*Sqrt[x]]*Log[(g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*Lo
g[-(-f)^(1/4) - g^(1/4)*Sqrt[x]]*Log[(f*g^(1/4)*Sqrt[x])/(-f)^(5/4)] - p*P
olyLog[2, (d*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))] +
 p*PolyLog[2, (d*((-f)^(1/4) - I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(
1/4))] + p*PolyLog[2, (d*((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) -
 I*e*g^(1/4))] - p*PolyLog[2, (d*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(d*(-f)^(
1/4) - e*g^(1/4))] - p*PolyLog[2, 1 - (I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*
PolyLog[2, 1 + (I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*PolyLog[2, 1 + (g^(1/4)
*Sqrt[x])/(-f)^(1/4)] + p*PolyLog[2, 1 + (f*g^(1/4)*Sqrt[x])/(-f)^(5/4)...
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2922, 2925, 2005, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx$$

$$\Big\downarrow \ 2922$$

$$2\int \frac{\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{gx^2+f}d\sqrt{x}$$

$$\Big\downarrow \ 2925$$

$$-2\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{\left(f+\frac{g}{x^2}\right)x^{3/2}}d\frac{1}{\sqrt{x}}$$

$\Big\downarrow$ 2005

$$-2\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{\sqrt{x}\left(fx^2+g\right)}d\frac{1}{\sqrt{x}}$$

$\Big\downarrow$ 2863

$$-2\int \left(-\frac{f\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{2\sqrt{-f}\sqrt{g}\sqrt{x}\left(\sqrt{-f}\sqrt{g}-fx\right)}-\frac{f\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{2\sqrt{-f}\sqrt{g}\sqrt{x}\left(fx+\sqrt{-f}\sqrt{g}\right)}\right)d\frac{1}{\sqrt{x}}$$

$\Big\downarrow$ 2009

$$-2\left(\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{4\sqrt{-f}\sqrt{g}}+\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}+\sqrt[4]{g}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{4\sqrt{-f}\sqrt{g}}-\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.}{}\right.$$

input
```
Int[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

output
```
-2*((Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4) - Sqrt[-Sqrt[-f]]/Sqrt[x]))/
(d*Sqrt[-Sqrt[-f]] + e*g^(1/4))])/(4*Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e/Sqr
t[x])^p]*Log[-((e*(g^(1/4) + Sqrt[-Sqrt[-f]]/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]]
- e*g^(1/4)))])/(4*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g
^(1/4) - (-f)^(1/4)/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(4*Sqrt[-f]*Sqr
t[g]) - (Log[c*(d + e/Sqrt[x])^p]*Log[-((e*(g^(1/4) + (-f)^(1/4)/Sqrt[x]))
/(d*(-f)^(1/4) - e*g^(1/4)))])/(4*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, (Sqrt[
-Sqrt[-f]]*(d + e/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*g^(1/4))])/(4*Sqrt[-f]*
Sqrt[g]) - (p*PolyLog[2, ((-f)^(1/4)*(d + e/Sqrt[x]))/(d*(-f)^(1/4) - e*g^
(1/4))])/(4*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, (Sqrt[-Sqrt[-f]]*(d + e/Sqrt
[x]))/(d*Sqrt[-Sqrt[-f]] + e*g^(1/4))])/(4*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[
2, ((-f)^(1/4)*(d + e/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(4*Sqrt[-f]*S
qrt[g]))
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2922
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   S
ubst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x]
, x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
 r, s}, x] && FractionQ[n]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [F]

$$\int \frac{\ln \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)}{g\,x^{2}+f}dx$$

input
```
int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

output
```
int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log(c*((d*x + e*sqrt(x))/x)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx=\text{Timed out}$$

input
```
integrate(ln(c*(d+e/x**(1/2))**p)/(g*x**2+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(log(c*(d + e/sqrt(x))^p)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log(c*(d + e/sqrt(x))^p)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e/x^(1/2))^p)/(f + g*x^2),x)
```

output
```
int(log(c*(d + e/x^(1/2))^p)/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{g\,x^2+f}dx$$

input
```
int(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

output
```
int(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

3.268 $\int (f + gx^2)^3 \log\left(c(d + ex^2)^p\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 338

$$\int (f+gx^2)^3 \log\left(c(d+ex^2)^p\right) dx = -2f^3px + \frac{2df^2gpx}{e} - \frac{6d^2fg^2px}{5e^2} + \frac{2d^3g^3px}{7e^3} - \frac{2}{3}f^2gpx^3 + \frac{2dfg^2px^3}{5e} - \frac{2d^2g^3px^3}{21e^2} - \frac{6}{25}fg^2px^5 + \frac{2dg^3px^5}{35e} - \frac{2}{49}g^3px^7 + \frac{2\sqrt{d}f^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{3/2}f^2gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} + \frac{6d^{5/2}fg^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}} - \frac{2d^{7/2}g^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} + f^3x\log\left(c(d+ex^2)^p\right) + f^2gx^3\log\left(c(d+ex^2)^p\right) + \frac{3}{5}fg^2x^5\log\left(c(d+ex^2)^p\right) + \frac{1}{7}g^3x^7\log\left(c(d+ex^2)^p\right)$$

output

```
-2*f^3*p*x+2*d*f^2*g*p*x/e-6/5*d^2*f*g^2*p*x/e^2+2/7*d^3*g^3*p*x/e^3-2/3*f
^2*g*p*x^3+2/5*d*f*g^2*p*x^3/e-2/21*d^2*g^3*p*x^3/e^2-6/25*f*g^2*p*x^5+2/3
5*d*g^3*p*x^5/e-2/49*g^3*p*x^7+2*d^(1/2)*f^3*p*arctan(e^(1/2)*x/d^(1/2))/e
^(1/2)-2*d^(3/2)*f^2*g*p*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)+6/5*d^(5/2)*f*g
^2*p*arctan(e^(1/2)*x/d^(1/2))/e^(5/2)-2/7*d^(7/2)*g^3*p*arctan(e^(1/2)*x/
d^(1/2))/e^(7/2)+f^3*x*ln(c*(e*x^2+d)^p)+f^2*g*x^3*ln(c*(e*x^2+d)^p)+3/5*f
*g^2*x^5*ln(c*(e*x^2+d)^p)+1/7*g^3*x^7*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.64

$$\int \left(f+gx^2\right)^3 \log \left(c\left(d+ex^2\right)^p\right)\, dx = -\frac{2px(-525d^3g^3+35d^2eg^2(63f+5gx^2)-105de^2g(35f^2+7fgx^2+g^2x^4)+e^3(3675f^3+1225f^2gx^2+}{3675e^3} - \frac{2\sqrt{d}(-35e^3f^3+35de^2f^2g-21d^2efg^2+5d^3g^3)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{35e^{7/2}} + \frac{1}{35}x\left(35f^3+35f^2gx^2+21fg^2x^4+5g^3x^6\right)\log\left(c\left(d+ex^2\right)^p\right)$$

input
```
Integrate[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```

output
```
(-2*p*x*(-525*d^3*g^3 + 35*d^2*e*g^2*(63*f + 5*g*x^2) - 105*d*e^2*g*(35*f^
2 + 7*f*g*x^2 + g^2*x^4) + e^3*(3675*f^3 + 1225*f^2*g*x^2 + 441*f*g^2*x^4
+ 75*g^3*x^6)))/(3675*e^3) - (2*Sqrt[d]*(-35*e^3*f^3 + 35*d*e^2*f^2*g - 21
*d^2*e*f*g^2 + 5*d^3*g^3)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(35*e^(7/2)) + (x
*(35*f^3 + 35*f^2*g*x^2 + 21*f*g^2*x^4 + 5*g^3*x^6)*Log[c*(d + e*x^2)^p])/
35
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+gx^2\right)^3 \log \left(c\left(d+ex^2\right)^p\right)\, dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f^3 \log \left(c\left(d+ex^2\right)^p\right)+3f^2gx^2 \log \left(c\left(d+ex^2\right)^p\right)+3fg^2x^4 \log \left(c\left(d+ex^2\right)^p\right)+g^3x^6 \log \left(c\left(d+ex^2\right)^p\right)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d^{3/2}f^2gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}+\frac{6d^{5/2}fg^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}-\frac{2d^{7/2}g^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}+\frac{2\sqrt{d}f^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+f^3x\log\left(c\left(d+ex^2\right)^p\right)+f^2gx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{5}fg^2x^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^3x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{2d^3g^3px}{7e^3}-\frac{6d^2fg^2px}{5e^2}-\frac{2d^2g^3px^3}{21e^2}+\frac{2df^2gpx}{e}+\frac{2dfg^2px^3}{5e}+\frac{2dg^3px^5}{35e}-2f^3px-\frac{2}{3}f^2gpx^3-\frac{6}{25}fg^2px^5-\frac{2}{49}g^3px^7$$

input
```
Int[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```

output
```
-2*f^3*p*x + (2*d*f^2*g*p*x)/e - (6*d^2*f*g^2*p*x)/(5*e^2) + (2*d^3*g^3*p*
x)/(7*e^3) - (2*f^2*g*p*x^3)/3 + (2*d*f*g^2*p*x^3)/(5*e) - (2*d^2*g^3*p*x^
3)/(21*e^2) - (6*f*g^2*p*x^5)/25 + (2*d*g^3*p*x^5)/(35*e) - (2*g^3*p*x^7)/
49 + (2*Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*f^
2*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (6*d^(5/2)*f*g^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(7*e^(7/2)) + f^3*x*Log[c*(d + e*x^2)^p] + f^2*g*x^3*Log[c*(d + e*x^2
)^p] + (3*f*g^2*x^5*Log[c*(d + e*x^2)^p])/5 + (g^3*x^7*Log[c*(d + e*x^2)^p
])/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g^3x^7\ln(c(e\,x^2+d)^p)}{7}+\frac{3f\,g^2x^5\ln(c(e\,x^2+d)^p)}{5}+f^2g\,x^3\ln\left(c(e\,x^2+d)^p\right)+f^3x\ln\left(c(e\,x^2+d)^p\right)-\frac{2ep\left(-\right.}{}$
risch	Expression too large to display

input
```
int((g*x^2+f)^3*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/7*g^3*x^7*ln(c*(e*x^2+d)^p)+3/5*f*g^2*x^5*ln(c*(e*x^2+d)^p)+f^2*g*x^3*ln
(c*(e*x^2+d)^p)+f^3*x*ln(c*(e*x^2+d)^p)-2/35*e*p*(-1/e^4*(-5/7*e^3*g^3*x^7
+d*e^2*g^3*x^5-21/5*e^3*f*g^2*x^5-5/3*d^2*e*g^3*x^3+7*d*e^2*f*g^2*x^3-35/3
*e^3*f^2*g*x^3+5*x*d^3*g^3-21*x*e*f*g^2*d^2+35*x*d*e^2*f^2*g-35*x*e^3*f^3)
+d*(5*d^3*g^3-21*d^2*e*f*g^2+35*d*e^2*f^2*g-35*e^3*f^3)/e^4/(d*e)^(1/2)*ar
ctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.76

$$\int \left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$= \left[-\frac{150\,e^3g^3px^7+42\,(21\,e^3fg^2-5\,de^2g^3)px^5+70\,(35\,e^3f^2g-21\,de^2fg^2+5\,d^2eg^3)px^3+105\,(35\,e^3f^3-}{}\right.$$

$$-\frac{150\,e^3g^3px^7+42\,(21\,e^3fg^2-5\,de^2g^3)px^5+70\,(35\,e^3f^2g-21\,de^2fg^2+5\,d^2eg^3)px^3-210\,(35\,e^3f^3-}{}$$

input
```
integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/3675*(150*e^3*g^3*p*x^7 + 42*(21*e^3*f*g^2 - 5*d*e^2*g^3)*p*x^5 + 70*(
35*e^3*f^2*g - 21*d*e^2*f*g^2 + 5*d^2*e*g^3)*p*x^3 + 105*(35*e^3*f^3 - 35*
d*e^2*f^2*g + 21*d^2*e*f*g^2 - 5*d^3*g^3)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*
sqrt(-d/e) - d)/(e*x^2 + d)) + 210*(35*e^3*f^3 - 35*d*e^2*f^2*g + 21*d^2*e
*f*g^2 - 5*d^3*g^3)*p*x - 105*(5*e^3*g^3*p*x^7 + 21*e^3*f*g^2*p*x^5 + 35*e
^3*f^2*g*p*x^3 + 35*e^3*f^3*p*x)*log(e*x^2 + d) - 105*(5*e^3*g^3*x^7 + 21*
e^3*f*g^2*x^5 + 35*e^3*f^2*g*x^3 + 35*e^3*f^3*x)*log(c))/e^3, -1/3675*(150
*e^3*g^3*p*x^7 + 42*(21*e^3*f*g^2 - 5*d*e^2*g^3)*p*x^5 + 70*(35*e^3*f^2*g
- 21*d*e^2*f*g^2 + 5*d^2*e*g^3)*p*x^3 - 210*(35*e^3*f^3 - 35*d*e^2*f^2*g +
 21*d^2*e*f*g^2 - 5*d^3*g^3)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 210*(35
*e^3*f^3 - 35*d*e^2*f^2*g + 21*d^2*e*f*g^2 - 5*d^3*g^3)*p*x - 105*(5*e^3*g
^3*p*x^7 + 21*e^3*f*g^2*p*x^5 + 35*e^3*f^2*g*p*x^3 + 35*e^3*f^3*p*x)*log(e
*x^2 + d) - 105*(5*e^3*g^3*x^7 + 21*e^3*f*g^2*x^5 + 35*e^3*f^2*g*x^3 + 35*
e^3*f^3*x)*log(c))/e^3]
```

Sympy [A] (verification not implemented)

Time = 125.04 (sec) , antiderivative size = 697, normalized size of antiderivative = 2.06

$$\int \left(f+gx^2\right)^3 \log \left(c\left(d+ex^2\right)^p\right) \, dx = \text{Too large to display}$$

input
```
integrate((g*x**2+f)**3*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise(((f**3*x + f**2*g*x**3 + 3*f*g**2*x**5/5 + g**3*x**7/7)*log(0**p
*c), Eq(d, 0) & Eq(e, 0)), ((f**3*x + f**2*g*x**3 + 3*f*g**2*x**5/5 + g**3
*x**7/7)*log(c*d**p), Eq(e, 0)), (-2*f**3*p*x + f**3*x*log(c*(e*x**2)**p)
- 2*f**2*g*p*x**3/3 + f**2*g*x**3*log(c*(e*x**2)**p) - 6*f*g**2*p*x**5/25
+ 3*f*g**2*x**5*log(c*(e*x**2)**p)/5 - 2*g**3*p*x**7/49 + g**3*x**7*log(c*
(e*x**2)**p)/7, Eq(d, 0)), (-2*d**4*g**3*p*log(x - sqrt(-d/e))/(7*e**4*sqr
t(-d/e)) + d**4*g**3*log(c*(d + e*x**2)**p)/(7*e**4*sqrt(-d/e)) + 6*d**3*f
*g**2*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - 3*d**3*f*g**2*log(c*(d +
 e*x**2)**p)/(5*e**3*sqrt(-d/e)) + 2*d**3*g**3*p*x/(7*e**3) - 2*d**2*f**2*
g*p*log(x - sqrt(-d/e))/(e**2*sqrt(-d/e)) + d**2*f**2*g*log(c*(d + e*x**2)
**p)/(e**2*sqrt(-d/e)) - 6*d**2*f*g**2*p*x/(5*e**2) - 2*d**2*g**3*p*x**3/(
21*e**2) + 2*d*f**3*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f**3*log(c*(d
 + e*x**2)**p)/(e*sqrt(-d/e)) + 2*d*f**2*g*p*x/e + 2*d*f*g**2*p*x**3/(5*e)
 + 2*d*g**3*p*x**5/(35*e) - 2*f**3*p*x + f**3*x*log(c*(d + e*x**2)**p) - 2
*f**2*g*p*x**3/3 + f**2*g*x**3*log(c*(d + e*x**2)**p) - 6*f*g**2*p*x**5/25
 + 3*f*g**2*x**5*log(c*(d + e*x**2)**p)/5 - 2*g**3*p*x**7/49 + g**3*x**7*l
og(c*(d + e*x**2)**p)/7, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (f + gx^2)^3 \log (c(d + ex^2)^p) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.79

$$\int \left(f+gx^2\right)^3 \log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= -\frac{1}{49}\left(2\,g^3p-7\,g^3\log\left(c\right)\right)x^7 - \frac{(42\,efg^2p-10\,dg^3p-105\,efg^2\log\left(c\right))x^5}{175\,e}$$
$$- \frac{(70\,e^2f^2gp-42\,defg^2p+10\,d^2g^3p-105\,e^2f^2g\log\left(c\right))x^3}{105\,e^2}$$
$$+ \frac{1}{35}\left(5\,g^3px^7+21\,fg^2px^5+35\,f^2gpx^3+35\,f^3px\right)\log\left(ex^2+d\right)$$
$$- \frac{(70\,e^3f^3p-70\,de^2f^2gp+42\,d^2efg^2p-10\,d^3g^3p-35\,e^3f^3\log\left(c\right))x}{35\,e^3}$$
$$+ \frac{2\,(35\,de^3f^3p-35\,d^2e^2f^2gp+21\,d^3efg^2p-5\,d^4g^3p)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{35\,\sqrt{de}e^3}$$

input
```
integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
-1/49*(2*g^3*p - 7*g^3*log(c))*x^7 - 1/175*(42*e*f*g^2*p - 10*d*g^3*p - 10
5*e*f*g^2*log(c))*x^5/e - 1/105*(70*e^2*f^2*g*p - 42*d*e*f*g^2*p + 10*d^2*
g^3*p - 105*e^2*f^2*g*log(c))*x^3/e^2 + 1/35*(5*g^3*p*x^7 + 21*f*g^2*p*x^5
 + 35*f^2*g*p*x^3 + 35*f^3*p*x)*log(e*x^2 + d) - 1/35*(70*e^3*f^3*p - 70*d
*e^2*f^2*g*p + 42*d^2*e*f*g^2*p - 10*d^3*g^3*p - 35*e^3*f^3*log(c))*x/e^3
+ 2/35*(35*d*e^3*f^3*p - 35*d^2*e^2*f^2*g*p + 21*d^3*e*f*g^2*p - 5*d^4*g^3
*p)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3)
```

Mupad [B] (verification not implemented)

Time = 26.43 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.88

$$\int (f+gx^2)^3 \log \left(c(d+ex^2)^p\right)\,dx = x^3\left(\frac{d\left(\frac{6fg^2p}{5}-\frac{2dg^3p}{7e}\right)}{3e}-\frac{2f^2gp}{3}\right) - x\left(2f^3p+\frac{d\left(\frac{d\left(\frac{6fg^2p}{5}-\frac{2dg^3p}{7e}\right)}{e}-2f^2gp\right)}{e}\right) - x^5\left(\frac{6fg^2p}{25}-\frac{2dg^3p}{35e}\right) + \ln\left(c\left(ex^2+d\right)^p\right)\left(f^3x+f^2gx^3+\frac{3fg^2x^5}{5}+\frac{g^3x^7}{7}\right)-\frac{2g^3px^7}{49} - \frac{2\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\sqrt{e}\,p\,x\,(5d^3g^3-21d^2efg^2+35de^2f^2g-35e^3f^3)}{5pd^4g^3-21pd^3efg^2+35pd^2e^2f^2g-35pde^3f^3}\right)(5d^3g^3-21d^2efg^2+35de^2f^2g-35e^3f^3)}{35e^{7/2}}$$

input
```
int(log(c*(d + e*x^2)^p)*(f + g*x^2)^3,x)
```

output
```
x^3*((d*((6*f*g^2*p)/5 - (2*d*g^3*p)/(7*e)))/(3*e) - (2*f^2*g*p)/3) - x*(2
*f^3*p + (d*((d*((6*f*g^2*p)/5 - (2*d*g^3*p)/(7*e)))/e - 2*f^2*g*p))/e) -
x^5*((6*f*g^2*p)/25 - (2*d*g^3*p)/(35*e)) + log(c*(d + e*x^2)^p)*(f^3*x +
(g^3*x^7)/7 + f^2*g*x^3 + (3*f*g^2*x^5)/5) - (2*g^3*p*x^7)/49 - (2*d^(1/2)
*p*atan((d^(1/2)*e^(1/2)*p*x*(5*d^3*g^3 - 35*e^3*f^3 + 35*d*e^2*f^2*g - 21
*d^2*e*f*g^2))/(5*d^4*g^3*p - 35*d*e^3*f^3*p - 21*d^3*e*f*g^2*p + 35*d^2*e
^2*f^2*g*p))*(5*d^3*g^3 - 35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^2*e*f*g^2))/(
35*e^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.97

$$\int (f+gx^2)^3 \log \left(c(d+ex^2)^p\right)\,dx = \frac{-1050\sqrt{e}\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right)d^3g^3p+4410\sqrt{e}\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right)d^2efg^2p-7350\sqrt{e}\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right)de^2f^2g}{}$$

input
```
int((g*x^2+f)^3*log(c*(e*x^2+d)^p),x)
```

output
```
( - 1050*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**3*g**3*p + 4410*
sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*e*f*g**2*p - 7350*sqrt(
e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e**2*f**2*g*p + 7350*sqrt(e)*sq
rt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**3*f**3*p + 3675*log((d + e*x**2)**p
*c)*e**4*f**3*x + 3675*log((d + e*x**2)**p*c)*e**4*f**2*g*x**3 + 2205*log(
(d + e*x**2)**p*c)*e**4*f*g**2*x**5 + 525*log((d + e*x**2)**p*c)*e**4*g**3
*x**7 + 1050*d**3*e*g**3*p*x - 4410*d**2*e**2*f*g**2*p*x - 350*d**2*e**2*g
**3*p*x**3 + 7350*d*e**3*f**2*g*p*x + 1470*d*e**3*f*g**2*p*x**3 + 210*d*e*
*3*g**3*p*x**5 - 7350*e**4*f**3*p*x - 2450*e**4*f**2*g*p*x**3 - 882*e**4*f
*g**2*p*x**5 - 150*e**4*g**3*p*x**7)/(3675*e**4)
```

3.269 $\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5 + \frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{4d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}} + f^2x\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)$$

output

```
-2*f^2*p*x+4/3*d*f*g*p*x/e-2/5*d^2*g^2*p*x/e^2-4/9*f*g*p*x^3+2/15*d*g^2*p*
x^3/e-2/25*g^2*p*x^5+2*d^(1/2)*f^2*p*arctan(e^(1/2)*x/d^(1/2))/e^(1/2)-4/3
*d^(3/2)*f*g*p*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)+2/5*d^(5/2)*g^2*p*arctan(
e^(1/2)*x/d^(1/2))/e^(5/2)+f^2*x*ln(c*(e*x^2+d)^p)+2/3*f*g*x^3*ln(c*(e*x^2
+d)^p)+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.68

$$\int (f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\frac{30\sqrt{d}(15e^2f^2-10defg+3d^2g^2)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{e}x(-2p(45d^2g^2-15deg(10f+gx^2)+e^2(225f^2+}{225e^{5/2}}$$

input
```
Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqr
t[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f
^2 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*L
og[c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$

$$\Big\downarrow \text{ 2921}$$

$$\int \left(f^2\log\left(c(d+ex^2)^p\right)+2fgx^2\log\left(c(d+ex^2)^p\right)+g^2x^4\log\left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+ f^2x\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)-\frac{2d^2g^2px}{5e^2}+ \frac{4dfgpx}{3e}+\frac{2dg^2px^3}{15e}-2f^2px-\frac{4}{9}fgpx^3-\frac{2}{25}g^2px^5$$

input
```
Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
-2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)
/9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2))
 + f^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*
x^5*Log[c*(d + e*x^2)^p])/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g^2x^5\ln(c(e\,x^2+d)^p)}{5}+\frac{2fg\,x^3\ln(c(e\,x^2+d)^p)}{3}+f^2x\ln\left(c(e\,x^2+d)^p\right)-\frac{2ep\left(\frac{\frac{3}{5}e^2g^2x^5-de\,g^2x^3+\frac{10}{3}e^2fg\,x^3+3x\,d^2g^2-1}{e^3}\right.}{}$
risch	$-\frac{i\pi\,g^2x^5\,\mathrm{csgn}(i(e\,x^2+d)^p)\,\mathrm{csgn}(ic(e\,x^2+d)^p)\,\mathrm{csgn}(ic)}{10}+\frac{i\pi fg\,x^3\mathrm{csgn}(ic(e\,x^2+d)^p)^2\,\mathrm{csgn}(ic)}{3}+\frac{i\pi fg\,x^3\,\mathrm{csgn}(i(e\,x^2+d)^p)\mathrm{csg}}{3}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2/3*f*g*x^3*ln(c*(e*x^2+d)^p)+f^2*x*ln(c*(e*
x^2+d)^p)-2/15*e*p*(1/e^3*(3/5*e^2*g^2*x^5-d*e*g^2*x^3+10/3*e^2*f*g*x^3+3*
x*d^2*g^2-10*d*e*f*g*x+15*e^2*f^2*x)-d*(3*d^2*g^2-10*d*e*f*g+15*e^2*f^2)/e
^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.83

$$\int \left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$=\left[-\frac{18\,e^2g^2px^5+10\,(10\,e^2fg-3\,deg^2)px^3-15\,(15\,e^2f^2-10\,defg+3\,d^2g^2)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)}{}\right.$$

$$-\frac{18\,e^2g^2px^5+10\,(10\,e^2fg-3\,deg^2)px^3-30\,(15\,e^2f^2-10\,defg+3\,d^2g^2)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+30\,(1}{}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/225*(18*e^2*g^2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 15*(15*e^2
*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e)
- d)/(e*x^2 + d)) + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*x - 15*(3*e
^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*log(e*x^2 + d) - 15*(3*e
^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/e^2, -1/225*(18*e^2*g^
2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 30*(15*e^2*f^2 - 10*d*e*f*g
+ 3*d^2*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 30*(15*e^2*f^2 - 10*d*e
*f*g + 3*d^2*g^2)*p*x - 15*(3*e^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^
2*p*x)*log(e*x^2 + d) - 15*(3*e^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)
*log(c))/e^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(231) = 462$.

Time = 32.85 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.16

$$\int (f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$= \begin{cases} \left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(0^pc\right) \\ \left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(cd^p\right) \\ -2f^2px+f^2x\log\left(c(ex^2)^p\right)-\frac{4fgpx^3}{9}+\frac{2fgx^3\log\left(c(ex^2)^p\right)}{3}-\frac{2g^2px^5}{25}+\frac{g^2x^5\log\left(c(ex^2)^p\right)}{5} \\ \frac{2d^3g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{d^3g^2\log\left(c(d+ex^2)^p\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{4d^2fgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{2d^2fg\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}-\frac{2d^2g^2px}{5e^2}+\frac{2df^2p\log\left(x-\sqrt{\right.}}{e\sqrt{-\frac{d}{e}}} \end{cases}$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise(((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(0**p*c), Eq(d, 0) & E
q(e, 0)), ((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(c*d**p), Eq(e, 0)), (
-2*f**2*p*x + f**2*x*log(c*(e*x**2)**p) - 4*f*g*p*x**3/9 + 2*f*g*x**3*log(
c*(e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log(c*(e*x**2)**p)/5, Eq(d
, 0)), (2*d**3*g**2*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - d**3*g**2*
log(c*(d + e*x**2)**p)/(5*e**3*sqrt(-d/e)) - 4*d**2*f*g*p*log(x - sqrt(-d/
e))/(3*e**2*sqrt(-d/e)) + 2*d**2*f*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-
d/e)) - 2*d**2*g**2*p*x/(5*e**2) + 2*d*f**2*p*log(x - sqrt(-d/e))/(e*sqrt(
-d/e)) - d*f**2*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 4*d*f*g*p*x/(3*e)
+ 2*d*g**2*p*x**3/(15*e) - 2*f**2*p*x + f**2*x*log(c*(d + e*x**2)**p) - 4*
f*g*p*x**3/9 + 2*f*g*x**3*log(c*(d + e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g*
*2*x**5*log(c*(d + e*x**2)**p)/5, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.79

$$\int \left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=-\frac{1}{25}\left(2\,g^2p-5\,g^2\log\left(c\right)\right)x^5-\frac{2\left(10\,efgp-3\,dg^2p-15\,efg\log\left(c\right)\right)x^3}{45\,e}$$
$$+\frac{1}{15}\left(3\,g^2px^5+10\,fgpx^3+15\,f^2px\right)\log\left(ex^2+d\right)$$
$$-\frac{\left(30\,e^2f^2p-20\,defgp+6\,d^2g^2p-15\,e^2f^2\log\left(c\right)\right)x}{15\,e^2}$$
$$+\frac{2\left(15\,de^2f^2p-10\,d^2efgp+3\,d^3g^2p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}e^2}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
-1/25*(2*g^2*p - 5*g^2*log(c))*x^5 - 2/45*(10*e*f*g*p - 3*d*g^2*p - 15*e*f
*g*log(c))*x^3/e + 1/15*(3*g^2*p*x^5 + 10*f*g*p*x^3 + 15*f^2*p*x)*log(e*x^
2 + d) - 1/15*(30*e^2*f^2*p - 20*d*e*f*g*p + 6*d^2*g^2*p - 15*e^2*f^2*log(
c))*x/e^2 + 2/15*(15*d*e^2*f^2*p - 10*d^2*e*f*g*p + 3*d^3*g^2*p)*arctan(e*
x/sqrt(d*e))/(sqrt(d*e)*e^2)
```

Mupad [B] (verification not implemented)

Time = 26.62 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.87

$$\int \left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\ln\left(c\left(e\,x^2+d\right)^p\right)\left(f^2\,x+\frac{2\,f\,g\,x^3}{3}+\frac{g^2\,x^5}{5}\right)$$
$$-x\left(2\,f^2\,p-\frac{d\left(\frac{4\,f\,g\,p}{3}-\frac{2\,d\,g^2\,p}{5\,e}\right)}{e}\right)-x^3\left(\frac{4\,f\,g\,p}{9}-\frac{2\,d\,g^2\,p}{15\,e}\right)-\frac{2\,g^2\,p\,x^5}{25}$$
$$+\frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{3\,p\,d^3\,g^2-10\,p\,d^2\,e\,f\,g+15\,p\,d\,e^2\,f^2}\right)\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{15\,e^{5/2}}$$

input
```
int(log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

output
```
log(c*(d + e*x^2)^p)*(f^2*x + (g^2*x^5)/5 + (2*f*g*x^3)/3) - x*(2*f^2*p -
(d*((4*f*g*p)/3 - (2*d*g^2*p)/(5*e)))/e) - x^3*((4*f*g*p)/9 - (2*d*g^2*p)/
(15*e)) - (2*g^2*p*x^5)/25 + (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(3*d^2
*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(3*d^3*g^2*p + 15*d*e^2*f^2*p - 10*d^2*e*
f*g*p))*(3*d^2*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(15*e^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.96

$$\int (f + gx^2)^2 \log (c(d + ex^2)^p) \, dx$$
$$= \frac{90\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) d^2g^2p - 300\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) defgp + 450\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) e^2f^2p + 225\log}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p),x)
```

output
```
(90*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*g**2*p - 300*sqrt(e
)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*f*g*p + 450*sqrt(e)*sqrt(d)*at
an((e*x)/(sqrt(e)*sqrt(d)))*e**2*f**2*p + 225*log((d + e*x**2)**p*c)*e**3*
f**2*x + 150*log((d + e*x**2)**p*c)*e**3*f*g*x**3 + 45*log((d + e*x**2)**p
*c)*e**3*g**2*x**5 - 90*d**2*e*g**2*p*x + 300*d*e**2*f*g*p*x + 30*d*e**2*g
**2*p*x**3 - 450*e**3*f**2*p*x - 100*e**3*f*g*p*x**3 - 18*e**3*g**2*p*x**5
)/(225*e**3)
```

3.270 $\int (f+gx^2) \log \left(c(d+ex^2)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3 + \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + fx\log\left(c(d+ex^2)^p\right)+\frac{1}{3}gx^3\log\left(c(d+ex^2)^p\right)$$

output
```
-2*f*p*x+2/3*d*g*p*x/e-2/9*g*p*x^3+2*d^(1/2)*f*p*arctan(e^(1/2)*x/d^(1/2))
/e^(1/2)-2/3*d^(3/2)*g*p*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)+f*x*ln(c*(e*x^2
+d)^p)+1/3*g*x^3*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3 + \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + fx\log\left(c(d+ex^2)^p\right)+\frac{1}{3}gx^3\log\left(c(d+ex^2)^p\right)$$

input
```
Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
-2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*
e^(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$

$$\Big\downarrow \text{ 2921}$$

$$\int \left(f\log\left(c(d+ex^2)^p\right)+gx^2\log\left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c(d+ex^2)^p\right)+\frac{1}{3}gx^3\log\left(c(d+ex^2)^p\right)+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

input
```
Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
-2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*
e^(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g\,x^{3}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{3}+fx\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)-\frac{2ep\left(-\frac{-\frac{1}{3}eg\,x^{3}+xdg-3efx}{e^{2}}+\frac{d(dg-3ef)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{e^{2}\sqrt{de}}\right)}{3}$
risch	$\left(\frac{1}{3}g\,x^{3}+fx\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)+\frac{i\pi g\,x^{3}\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}\mathrm{csgn}\left(ic\right)}{6}-\frac{i\pi g\,x^{3}\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{3}}{6}-\frac{ix\pi f\ \mathrm{csgn}\left(i(e\,x\right.}{}$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*g*x^3*ln(c*(e*x^2+d)^p)+f*x*ln(c*(e*x^2+d)^p)-2/3*e*p*(-1/e^2*(-1/3*e*
g*x^3+x*d*g-3*e*f*x)+d*(d*g-3*e*f)/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.88

$$\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\left[-\frac{2\,egpx^3+3\,(3\,ef-dg)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2-2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+6\,(3\,ef-dg)px-3\,(egpx^3+3\,efpx)\log\left(ex^2\right.}{9\,e}\right.$$
$$-\frac{2\,egpx^3-6\,(3\,ef-dg)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+6\,(3\,ef-dg)px-3\,(egpx^3+3\,efpx)\log\left(ex^2+d\right)-}{9\,e}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/9*(2*e*g*p*x^3 + 3*(3*e*f - d*g)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(
-d/e) - d)/(e*x^2 + d)) + 6*(3*e*f - d*g)*p*x - 3*(e*g*p*x^3 + 3*e*f*p*x)*
log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e, -1/9*(2*e*g*p*x^3 - 6*(3
*e*f - d*g)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 6*(3*e*f - d*g)*p*x - 3*
(e*g*p*x^3 + 3*e*f*p*x)*log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.

Time = 8.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.22

$$\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\begin{cases}\left(fx+\frac{gx^3}{3}\right)\log\left(0^pc\right) \\ \left(fx+\frac{gx^3}{3}\right)\log\left(cd^p\right) \\ -2fpx+fx\log\left(c\left(ex^2\right)^p\right)-\frac{2gpx^3}{9}+\frac{gx^3\log\left(c\left(ex^2\right)^p\right)}{3} \\ -\frac{2d^2gp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{d^2g\log\left(c\left(d+ex^2\right)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{2dfp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{df\log\left(c\left(d+ex^2\right)^p\right)}{e\sqrt{-\frac{d}{e}}}+\frac{2dgpx}{3e}-2fpx+fx\log\left(c(\right.\end{cases}$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise(((f*x + g*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f*x + g*x
**3/3)*log(c*d**p), Eq(e, 0)), (-2*f*p*x + f*x*log(c*(e*x**2)**p) - 2*g*p*
x**3/9 + g*x**3*log(c*(e*x**2)**p)/3, Eq(d, 0)), (-2*d**2*g*p*log(x - sqrt
(-d/e))/(3*e**2*sqrt(-d/e)) + d**2*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-
d/e)) + 2*d*f*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f*log(c*(d + e*x**2
)**p)/(e*sqrt(-d/e)) + 2*d*g*p*x/(3*e) - 2*f*p*x + f*x*log(c*(d + e*x**2)*
*p) - 2*g*p*x**3/9 + g*x**3*log(c*(d + e*x**2)**p)/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\begin{aligned}\int (f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)\,dx = &-\frac{1}{9}\,(2\,gp-3\,g\log\left(c\right))x^3\\&+\frac{1}{3}\,\left(gpx^3+3\,fpx\right)\log\left(ex^2+d\right)\\&-\frac{(6\,efp-2\,dgp-3\,ef\log\left(c\right))x}{3\,e}\\&+\frac{2\,\left(3\,defp-d^2gp\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{3\,\sqrt{de}e}\end{aligned}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
-1/9*(2*g*p - 3*g*log(c))*x^3 + 1/3*(g*p*x^3 + 3*f*p*x)*log(e*x^2 + d) - 1
/3*(6*e*f*p - 2*d*g*p - 3*e*f*log(c))*x/e + 2/3*(3*d*e*f*p - d^2*g*p)*arct
an(e*x/sqrt(d*e))/(sqrt(d*e)*e)
```

Mupad [B] (verification not implemented)

Time = 25.95 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int (f+gx^2)\log \left(c(d+ex^2)^p\right)\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{g\,x^3}{3}+f\,x\right) - x\left(2\,f\,p-\frac{2\,d\,g\,p}{3\,e}\right)-\frac{2\,g\,p\,x^3}{9} - \frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,(d\,g-3\,e\,f)}{d^2\,g\,p-3\,d\,e\,f\,p}\right)\,(d\,g-3\,e\,f)}{3\,e^{3/2}}$$

input
```
int(log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

output
```
log(c*(d + e*x^2)^p)*(f*x + (g*x^3)/3) - x*(2*f*p - (2*d*g*p)/(3*e)) - (2*
g*p*x^3)/9 - (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(d*g - 3*e*f))/(d^2*g*
p - 3*d*e*f*p))*(d*g - 3*e*f))/(3*e^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int (f+gx^2)\log \left(c(d+ex^2)^p\right)\,dx = \frac{-6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)dgp+18\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)efp+9\log\left(\left(e\,x^2+d\right)^p c\right)e^2fx+3\log\left(\left(e\,x^2+d\right)^p e\right.}{9e^2}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p),x)
```

output
```
( - 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*g*p + 18*sqrt(e)*sqr
t(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p + 9*log((d + e*x**2)**p*c)*e**2*f
*x + 3*log((d + e*x**2)**p*c)*e**2*g*x**3 + 6*d*e*g*p*x - 18*e**2*f*p*x -
2*e**2*g*p*x**3)/(9*e**2)
```

3.271 $\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 533

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx &= \frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} \\
&\quad - \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad + \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} \\
&\quad + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}\end{aligned}$$

output
```
2*p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/
g^(1/2)-p*arctan(g^(1/2)*x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1
/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/
2)/g^(1/2)-p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^
(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(
1/2)/g^(1/2)+arctan(g^(1/2)*x/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)-I
*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+1/2*I*p*po
lylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)
^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1
-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^
(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx =$$

$$-\frac{i\left(p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)+p\log\left(\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right.\right.}{}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

output
```
((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqr
t[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d]
+ Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g
]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[
f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*S
qrt[g]*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p
] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sq
rt[-d]*Sqrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]
*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[
g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sq
rt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*S
qrt[g])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2920, 27, 5463, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{f+gx^2} \, dx$$

$$\Big\downarrow \ 2920$$

$$\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-2ep\int \frac{x \arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(ex^2+d\right)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{2ep\int \frac{x \arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex^2+d}dx}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 5463$$

$$\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{2ep\int \left(\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-$$
$$\frac{2ep\left(\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2e}+\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2e}-\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right.}{e}\right.}{\sqrt{f}\sqrt{g}}$$

input
```
Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

output
```
(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (2*
e*p*(-((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x
)])/e) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] -
Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x
))])/(2*e) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d]
 + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g
]*x))])/(2*e) + ((I/2)*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)]
)/e - ((I/4)*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I
*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e - ((I/4)
*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqr
t[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e))/(Sqrt[f]*Sqrt[g])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5463
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.84

method	result
risch	$\frac{\left(\ln\left(\left(e\,x^2+d\right)^p\right)-p\ln\left(e\,x^2+d\right)\right)\arctan\left(\frac{gx}{\sqrt{gf}}\right)}{\sqrt{gf}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(g\_Z^2+f\right)}\frac{\ln(x-\_\alpha)\ln\left(e\,x^2+d\right)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2 eg+\cdots\right.}{\mathrm{RootOf}\left(\_Z^2\cdots\right.}\right.\right.}{\cdots}\right.}{\cdots}$

input
```
int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(g*f)^(1/2)*arctan(g*x/(g*f)^(1/2))+1/2*p/
g*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x
+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f
,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_al
pha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^
2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*cs
gn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))/(g
*f)^(1/2)*arctan(g*x/(g*f)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

input
```
integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

output
```
Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log \left((ex^2+d)^pc\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log \left((ex^2+d)^pc\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\ln \left(c\,(e\,x^2+d)^p\right)}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left((e\,x^2+d)^p\,c\right)}{g\,x^2+f}dx$$

input
```
int(log(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

output
```
int(log((d + e*x**2)**p*c)/(f + g*x**2),x)
```

3.272 $\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 751

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} - \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}} + \frac{ip\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}} + \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}$$

output
```
d^(1/2)*e^(1/2)*p*arctan(e^(1/2)*x/d^(1/2))/f/(-d*g+e*f)-1/2*e*p*ln((-f)^(
1/2)-g^(1/2)*x)/(-f)^(1/2)/g^(1/2)/(-d*g+e*f)+p*arctan(g^(1/2)*x/f^(1/2))*
ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)-1/2*p*arctan(g^(1/2)*x
/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)-
(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)-1/2*p*arctan(g^
(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^
(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)+1/2*e*p*l
n((-f)^(1/2)+g^(1/2)*x)/(-f)^(1/2)/g^(1/2)/(-d*g+e*f)-1/4*ln(c*(e*x^2+d)^p
)/f/g^(1/2)/((-f)^(1/2)-g^(1/2)*x)+1/4*ln(c*(e*x^2+d)^p)/f/g^(1/2)/((-f)^(
1/2)+g^(1/2)*x)+1/2*arctan(g^(1/2)*x/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(3/2)/g^
(1/2)-1/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)
+1/4*I*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f
^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)+1/4*I*p*
polylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-
d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.47 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.17

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$$

$$= \frac{\frac{2\sqrt{-d}\sqrt{e}\sqrt{f}p\log(\sqrt{-d}-\sqrt{e}x)}{-ef+dg} + \frac{2\sqrt{-d}\sqrt{e}\sqrt{f}p\log(\sqrt{-d}+\sqrt{e}x)}{ef-dg} + \frac{2e\sqrt{-f^2}p\log(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}(ef-dg)} + \frac{2e\sqrt{-f^2}p\log(\sqrt{-f}+\sqrt{g}x)}{\sqrt{g}(-ef+dg)} - \frac{ip\log\left(\cdots\right.}{i\cdots}}{}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

output
```
((2*Sqrt[-d]*Sqrt[e]*Sqrt[f]*p*Log[Sqrt[-d] - Sqrt[e]*x])/(-(e*f) + d*g) +
 (2*Sqrt[-d]*Sqrt[e]*Sqrt[f]*p*Log[Sqrt[-d] + Sqrt[e]*x])/(e*f - d*g) + (2
*e*Sqrt[-f^2]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(Sqrt[g]*(e*f - d*g)) + (2*e*Sq
rt[-f^2]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(Sqrt[g]*(-(e*f) + d*g)) - (I*p*Log[
(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*L
og[1 - (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] - (I*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqr
t[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/
Sqrt[f]])/Sqrt[g] + (I*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + (
I*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqr
t[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + (Sqrt[f]*Log[c*(d + e*x^2
)^p])/(-(Sqrt[-f]*Sqrt[g]) + g*x) + (Sqrt[f]*Log[c*(d + e*x^2)^p])/(Sqrt[-
f]*Sqrt[g] + g*x) + (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/S
qrt[g] - (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f
] - I*Sqrt[-d]*Sqrt[g])])/Sqrt[g] - (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*
Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/Sqrt[g] + (I*p*PolyLo
g[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[
g])])/Sqrt[g] + (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]
*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/Sqrt[g])/(4*f^(3/2))
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

$$\Big\downarrow \ 2921$$

$$\int \left(-\frac{g\log \left(c\left(d+ex^2\right)^p\right)}{2f\left(-fg-g^2x^2\right)}-\frac{g\log \left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g\log \left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2f^{3/2}\sqrt{g}}-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2f^{3/2}\sqrt{g}}+\frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}+\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}-\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{ip\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4f^{3/2}\sqrt{g}}+\frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}-\frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}}$$

input

```
Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

output

```
(Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log
[Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt
[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]
) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqr
t[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))]
)/(2*f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt
[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[
f] - I*Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])
/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqr
t[-f] - Sqrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g
]*x)) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt
[g]) - ((I/2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3
/2)*Sqrt[g]) + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt
[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])
/(f^(3/2)*Sqrt[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d]
+ Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x))])/(f^(3/2)*Sqrt[g])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [F]

$$\int \frac{\ln \left(c(e\,x^2+d)^p\right)}{\left(g\,x^2+f\right)^2}dx$$

input
```
int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \int \frac{\ln \left(c \left(e\, x^2+d\right)^p\right)}{\left(g\, x^2+f\right)^2} \, dx$$

input
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \int \frac{\log \left(\left(e\, x^2+d\right)^p c\right)}{g^2 x^4+2 f g\, x^2+f^2} dx$$

input
```
int(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
int(log((d + e*x**2)**p*c)/(f**2 + 2*f*g*x**2 + g**2*x**4),x)
```

3.273 $\int (f+gx^2)^2 \log^2\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 945

$$\int (f+gx^2)^2 \log^2\left(c(d+ex^2)^p\right)\,dx = \text{Too large to display}$$

output
```
f^2*x*ln(c*(e*x^2+d)^p)^2-16/3*d^(3/2)*f*g*p^2*arctan(e^(1/2)*x/d^(1/2))*l
n(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(3/2)-8/3*d^(3/2)*f*g*p*arctan(e^(1/2
)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(3/2)-8/3*I*d^(3/2)*f*g*p^2*polylog(2,1-2
*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(3/2)-8/3*I*d^(3/2)*f*g*p^2*arctan(e^(1/
2)*x/d^(1/2))^2/e^(3/2)+4*I*d^(1/2)*f^2*p^2*arctan(e^(1/2)*x/d^(1/2))^2/e^
(1/2)+4/5*I*d^(5/2)*g^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e
^(5/2)+4/5*I*d^(5/2)*g^2*p^2*arctan(e^(1/2)*x/d^(1/2))^2/e^(5/2)+64/9*d^(3
/2)*f*g*p^2*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)+8/5*d^(5/2)*g^2*p^2*arctan(e
^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(5/2)+4/5*d^(5/2)*
g^2*p*arctan(e^(1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(5/2)+8*d^(1/2)*f^2*p^
2*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)+4*
d^(1/2)*f^2*p*arctan(e^(1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(1/2)+8/3*d*f*
g*p*x*ln(c*(e*x^2+d)^p)/e-184/75*d^(5/2)*g^2*p^2*arctan(e^(1/2)*x/d^(1/2))
/e^(5/2)-8*d^(1/2)*f^2*p^2*arctan(e^(1/2)*x/d^(1/2))/e^(1/2)+184/75*d^2*g^
2*p^2*x/e^2-64/225*d*g^2*p^2*x^3/e-8/9*f*g*p*x^3*ln(c*(e*x^2+d)^p)-64/9*d*
f*g*p^2*x/e-4/5*d^2*g^2*p*x*ln(c*(e*x^2+d)^p)/e^2+4/15*d*g^2*p*x^3*ln(c*(e
*x^2+d)^p)/e+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)^2+8/125*g^2*p^2*x^5+8*f^2*p^2*x
+4*I*d^(1/2)*f^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)+
16/27*f*g*p^2*x^3-4*f^2*p*x*ln(c*(e*x^2+d)^p)-4/25*g^2*p*x^5*ln(c*(e*x^2+d
)^p)+2/3*f*g*x^3*ln(c*(e*x^2+d)^p)^2
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.46

$$\int (f+gx^2)^2 \log^2 \left(c(d+ex^2)^p\right) \, dx$$

$$= \frac{900i\sqrt{d}(15e^2f^2-10defg+3d^2g^2)\,p^2 \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2 + 60\sqrt{d}p \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-2(225e^2f^2-200defg+\right.}{}$$

input
```
Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
((900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p^2*ArcTan[(Sqrt[e]
*x)/Sqrt[d]]^2 + 60*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2*(225*e^2*f^2
 - 200*d*e*f*g + 69*d^2*g^2)*p + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*
p*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 15*(15*e^2*f^2 - 10*d*e*f*g +
 3*d^2*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*x*(8*p^2*(1035*d^2*g^2 - 120*d
*e*g*(25*f + g*x^2) + e^2*(3375*f^2 + 250*f*g*x^2 + 27*g^2*x^4)) - 60*p*(4
5*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2 + 50*f*g*x^2 + 9*g^2*x^
4))*Log[c*(d + e*x^2)^p] + 225*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[c
*(d + e*x^2)^p]^2) + (900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)
*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(3375
*e^(5/2))
```

Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 945, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2921}$$

$$\int \left(f^2 \log^2\left(c\left(d+ex^2\right)^p\right)+2fgx^2 \log^2\left(c\left(d+ex^2\right)^p\right)+g^2x^4 \log^2\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{8}{125}g^2p^2x^5+\frac{1}{5}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^5-\frac{4}{25}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^5-\frac{64dg^2p^2x^3}{225e}+\frac{16}{27}fgp^2x^3+\frac{2}{3}fg\log^2\left(c\left(ex^2+d\right)^p\right)x^3+\frac{4dg^2p\log\left(c\left(ex^2+d\right)^p\right)x^3}{15e}-\frac{8}{9}fgp\log\left(c\left(ex^2+d\right)^p\right)x^3+8f^2p^2x+\frac{184d^2g^2p^2x}{75e^2}-\frac{64dfgp^2x}{9e}+f^2\log^2\left(c\left(ex^2+d\right)^p\right)x-4f^2p\log\left(c\left(ex^2+d\right)^p\right)x-\frac{4d^2g^2p\log\left(c\left(ex^2+d\right)^p\right)x}{5e^2}+\frac{8dfgp\log\left(c\left(ex^2+d\right)^p\right)x}{3e}+\frac{4i\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}+\frac{4id^{5/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{5e^{5/2}}-\frac{8id^{3/2}fgp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}-\frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{184d^{5/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{75e^{5/2}}+\frac{64d^{3/2}fgp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}+\frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}+\frac{8d^{5/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{5e^{5/2}}-\frac{16d^{3/2}fgp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{3e^{3/2}}+\frac{4\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{\sqrt{e}}+\frac{4d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{5e^{5/2}}-\frac{8d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{3e^{3/2}}+\frac{4i\sqrt{d}f^2p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}+\frac{4id^{5/2}g^2p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{5e^{5/2}}-\frac{8id^{3/2}fgp^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{3e^{3/2}}$$

input
```
Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```

output

```
8*f^2*p^2*x - (64*d*f*g*p^2*x)/(9*e) + (184*d^2*g^2*p^2*x)/(75*e^2) + (16*
f*g*p^2*x^3)/27 - (64*d*g^2*p^2*x^3)/(225*e) + (8*g^2*p^2*x^5)/125 - (8*Sq
rt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (64*d^(3/2)*f*g*p^2*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*e^(3/2)) - (184*d^(5/2)*g^2*p^2*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(75*e^(5/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)
/Sqrt[d]]^2)/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[
d]]^2)/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)
/e^(5/2) + (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/
(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (16*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)
/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + (8*d^(5/
2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e
]*x)])/(5*e^(5/2)) - 4*f^2*p*x*Log[c*(d + e*x^2)^p] + (8*d*f*g*p*x*Log[c*(
d + e*x^2)^p])/(3*e) - (4*d^2*g^2*p*x*Log[c*(d + e*x^2)^p])/(5*e^2) - (8*f
*g*p*x^3*Log[c*(d + e*x^2)^p])/9 + (4*d*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(1
5*e) - (4*g^2*p*x^5*Log[c*(d + e*x^2)^p])/25 + (4*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (8*d^(3/2)*f*g*p*ArcTan[
(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(3*e^(3/2)) + (4*d^(5/2)*g^2*p*
ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(5*e^(5/2)) + f^2*x*Log[
c*(d + e*x^2)^p]^2 + (2*f*g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (g^2*x^5*Log[c
*(d + e*x^2)^p]^2)/5 + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d]...
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.42 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.14

method	result	size
risch	Expression too large to display	1077

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*ln((e*x^2+d)^p)^2*g*f*x^3+8/3/e*p^2*d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(
1/2))*f*g*ln(e*x^2+d)-8/3/e*p*d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f*g*
ln((e*x^2+d)^p)+16/27*f*g*p^2*x^3-8/9*p*f*g*x^3*ln((e*x^2+d)^p)-8*p^2*d/(d
*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^2-64/9*d*f*g*p^2*x/e-4/25*p*g^2*x^5*ln
((e*x^2+d)^p)-4*p*f^2*x*ln((e*x^2+d)^p)-4/5/e^2*p*g^2*d^2*x*ln((e*x^2+d)^p
)+4/15/e*p*g^2*d*x^3*ln((e*x^2+d)^p)+4*p*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1
/2))*f^2*ln((e*x^2+d)^p)-4*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^2*l
n(e*x^2+d)-184/75/e^2*p^2*g^2*d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-4/5/
e^2*p^2*g^2*d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(e*x^2+d)+8/3/e*p*d*
f*g*x*ln((e*x^2+d)^p)-4/15*e*p^2*Sum(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1
/4/_alpha/e*ln(x-_alpha)^2+1/2*_alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_al
pha)+1/2*_alpha/d*dilog(1/2*(x+_alpha)/_alpha)))*d*(3*d^2*g^2-10*d*e*f*g+1
5*e^2*f^2)/e^4/_alpha,_alpha=RootOf(_Z^2*e+d))+(I*Pi*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn
(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+
2*ln(c))*(1/5*ln((e*x^2+d)^p)*g^2*x^5+2/3*ln((e*x^2+d)^p)*g*f*x^3+ln((e*x^
2+d)^p)*x*f^2-2/15*e*p*(1/e^3*(3/5*e^2*g^2*x^5-d*e*g^2*x^3+10/3*e^2*f*g*x^
3+3*x*d^2*g^2-10*d*e*f*g*x+15*e^2*f^2*x)-d*(3*d^2*g^2-10*d*e*f*g+15*e^2*f^
2)/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))))+4/5/e^2*p*g^2*d^3/(d*e)^(1/2)
*arctan(x*e/(d*e)^(1/2))*ln((e*x^2+d)^p)+64/9/e*p^2*d^2/(d*e)^(1/2)*arc...
```

Fricas [F]

$$\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^2\,dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F]

$$\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)^2\,dx$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^2\,dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\left(g\,x^2+f\right)^2\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2, x)
```

Reduce [F]

$$\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\frac{-8280\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)d^2g^2p^2+24000\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)defg\,p^2-27000\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)e^2f^2$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x)
```

output

```
( - 8280*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*g**2*p**2 + 24
000*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*f*g*p**2 - 27000*sqr
t(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**2*f**2*p**2 + 2700*int(log((
d + e*x**2)**p*c)/(d + e*x**2),x)*d**3*e*g**2*p - 9000*int(log((d + e*x**2
)**p*c)/(d + e*x**2),x)*d**2*e**2*f*g*p + 13500*int(log((d + e*x**2)**p*c)
/(d + e*x**2),x)*d*e**3*f**2*p + 3375*log((d + e*x**2)**p*c)**2*e**3*f**2*
x + 2250*log((d + e*x**2)**p*c)**2*e**3*f*g*x**3 + 675*log((d + e*x**2)**p
*c)**2*e**3*g**2*x**5 - 2700*log((d + e*x**2)**p*c)*d**2*e*g**2*p*x + 9000
*log((d + e*x**2)**p*c)*d*e**2*f*g*p*x + 900*log((d + e*x**2)**p*c)*d*e**2
*g**2*p*x**3 - 13500*log((d + e*x**2)**p*c)*e**3*f**2*p*x - 3000*log((d +
e*x**2)**p*c)*e**3*f*g*p*x**3 - 540*log((d + e*x**2)**p*c)*e**3*g**2*p*x**
5 + 8280*d**2*e*g**2*p**2*x - 24000*d*e**2*f*g*p**2*x - 960*d*e**2*g**2*p*
*2*x**3 + 27000*e**3*f**2*p**2*x + 2000*e**3*f*g*p**2*x**3 + 216*e**3*g**2
*p**2*x**5)/(3375*e**3)
```

3.274 $\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 548

$$\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx = 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$
$$+\frac{32d^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}+\frac{4i\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{4id^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}$$
$$+\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}-\frac{8d^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}$$
$$-4fpx\log\left(c(d+ex^2)^p\right)+\frac{4dgpx\log\left(c(d+ex^2)^p\right)}{3e}-\frac{4}{9}gpx^3\log\left(c(d+ex^2)^p\right)+\frac{4\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(}{\sqrt{e}}$$

output
```
8*f*p^2*x-32/9*d*g*p^2*x/e+8/27*g*p^2*x^3-8*d^(1/2)*f*p^2*arctan(e^(1/2)*x
/d^(1/2))/e^(1/2)+32/9*d^(3/2)*g*p^2*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)-4/3
*I*d^(3/2)*g*p^2*arctan(e^(1/2)*x/d^(1/2))^2/e^(3/2)+4*I*d^(1/2)*f*p^2*pol
ylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)+8*d^(1/2)*f*p^2*arctan(e
^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)-8/3*d^(3/2)*
g*p^2*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(3/2
)-4*f*p*x*ln(c*(e*x^2+d)^p)+4/3*d*g*p*x*ln(c*(e*x^2+d)^p)/e-4/9*g*p*x^3*ln
(c*(e*x^2+d)^p)+4*d^(1/2)*f*p*arctan(e^(1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/
e^(1/2)-4/3*d^(3/2)*g*p*arctan(e^(1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(3/2
)+f*x*ln(c*(e*x^2+d)^p)^2+1/3*g*x^3*ln(c*(e*x^2+d)^p)^2+4*I*d^(1/2)*f*p^2*
arctan(e^(1/2)*x/d^(1/2))^2/e^(1/2)-4/3*I*d^(3/2)*g*p^2*polylog(2,1-2*d^(1
/2)/(d^(1/2)+I*e^(1/2)*x))/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.51

$$\int (f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$= \frac{-36i\sqrt{d}(-3ef+dg)p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2-12\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(2(9ef-4dg)p+6(-3ef+dg)p\log\left(\frac{}{\sqrt{d}}\right.\right.}{}$$

input
```
Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
((-36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 - 12*Sqr
t[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(9*e*f - 4*d*g)*p + 6*(-3*e*f + d*g)
*p*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + (-9*e*f + 3*d*g)*Log[c*(d +
e*x^2)^p]) + Sqrt[e]*x*(8*p^2*(27*e*f - 12*d*g + e*g*x^2) - 12*p*(9*e*f -
3*d*g + e*g*x^2)*Log[c*(d + e*x^2)^p] + 9*e*(3*f + g*x^2)*Log[c*(d + e*x^2
)^p]^2) - (36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e
]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(27*e^(3/2))
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f\log^2\left(c(d+ex^2)^p\right)+gx^2\log^2\left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{3e^{3/2}}+\frac{4\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}}-\frac{4id^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}+\frac{32d^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}-\frac{8d^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}+\frac{4i\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+fx\log^2\left(c(d+ex^2)^p\right)-4fpx\log\left(c(d+ex^2)^p\right)+\frac{4dgpx\log\left(c(d+ex^2)^p\right)}{3e}+\frac{1}{3}gx^3\log^2\left(c(d+ex^2)^p\right)-\frac{4}{9}gpx^3\log\left(c(d+ex^2)^p\right)-\frac{4id^{3/2}gp^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{3e^{3/2}}+\frac{4i\sqrt{d}fp^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}-\frac{32dgp^2x}{9e}+8fp^2x+\frac{8}{27}gp^2x^3$$

input
```
Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
8*f*p^2*x - (32*d*g*p^2*x)/(9*e) + (8*g*p^2*x^3)/27 - (8*Sqrt[d]*f*p^2*Arc
Tan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(9*e^(3/2)) + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)
/Sqrt[e] - (((4*I)/3)*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(3/2)
 + (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] +
 I*Sqrt[e]*x)])/Sqrt[e] - (8*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log
[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) - 4*f*p*x*Log[c*(d + e*
x^2)^p] + (4*d*g*p*x*Log[c*(d + e*x^2)^p])/(3*e) - (4*g*p*x^3*Log[c*(d + e
*x^2)^p])/9 + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)
^p])/Sqrt[e] - (4*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2
)^p])/(3*e^(3/2)) + f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^3*Log[c*(d + e*x^2)^
p]^2)/3 + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqr
t[e]*x)])/Sqrt[e] - (((4*I)/3)*d^(3/2)*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(S
qrt[d] + I*Sqrt[e]*x)])/e^(3/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.15 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.33

method	result	size
risch	Expression too large to display	729

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*ln((e*x^2+d)^p)^2*g*x^3+ln((e*x^2+d)^p)^2*x*f-4/9*p*g*x^3*ln((e*x^2+d)
^p)+4/3/e*p*g*d*x*ln((e*x^2+d)^p)-4*p*f*x*ln((e*x^2+d)^p)+4/3/e*p^2*g*d^2/
(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(e*x^2+d)-4/3/e*p*g*d^2/(d*e)^(1/2)*
arctan(x*e/(d*e)^(1/2))*ln((e*x^2+d)^p)-4*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*
e)^(1/2))*f*ln(e*x^2+d)+4*p*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f*ln((e*
x^2+d)^p)+8/27*g*p^2*x^3-32/9*d*g*p^2*x/e+32/9/e*p^2*g*d^2/(d*e)^(1/2)*arc
tan(x*e/(d*e)^(1/2))+8*f*p^2*x-8*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
*f-4/3*e*p^2*Sum(1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/4/_alpha/e*ln(x-_alp
ha)^2+1/2*_alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/d*dil
og(1/2*(x+_alpha)/_alpha)))*d*(d*g-3*e*f)/e^3/_alpha,_alpha=RootOf(_Z^2*e+
d))+(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d
)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csg
n(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(c))*(1/3*ln((e*x^2+d)^p)*g*x^3+ln((e*x
^2+d)^p)*x*f-2/3*e*p*(1/e^2*(1/3*e*g*x^3-x*d*g+3*e*f*x)+d*(d*g-3*e*f)/e^2/
(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))))+1/4*(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)
-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(
c))^2*(1/3*g*x^3+f*x)
```

Fricas [F]

$$\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx=\int (gx^2+f)\log\left((ex^2+d)^pc\right)^2\,dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((g*x^2 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F]

$$\int (f + gx^2) \log^2 (c(d + ex^2)^p) \, dx = \int (f + gx^2) \log (c(d + ex^2)^p)^2 \, dx$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f + g*x**2)*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f + gx^2) \log^2 (c(d + ex^2)^p) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (f + gx^2) \log^2 (c(d + ex^2)^p) \, dx = \int (gx^2 + f) \log ((ex^2 + d)^p c)^2 \, dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\left(g\,x^2+f\right)\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^2), x)
```

Reduce [F]

$$\int (f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\frac{96\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)dg\,p^2-216\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ef\,p^2-36\left(\int\frac{\log\left(\left(e\,x^2+d\right)^p c\right)}{e\,x^2+d}dx\right)d^2egp+108\left(\int\frac{\log}{}\right.}{}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x)
```

output
```
(96*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*g*p**2 - 216*sqrt(e)*s
qrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p**2 - 36*int(log((d + e*x**2)**p
*c)/(d + e*x**2),x)*d**2*e*g*p + 108*int(log((d + e*x**2)**p*c)/(d + e*x**
2),x)*d*e**2*f*p + 27*log((d + e*x**2)**p*c)**2*e**2*f*x + 9*log((d + e*x*
*2)**p*c)**2*e**2*g*x**3 + 36*log((d + e*x**2)**p*c)*d*e*g*p*x - 108*log((
d + e*x**2)**p*c)*e**2*f*p*x - 12*log((d + e*x**2)**p*c)*e**2*g*p*x**3 - 9
6*d*e*g*p**2*x + 216*e**2*f*p**2*x + 8*e**2*g*p**2*x**3)/(27*e**2)
```

3.275 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\text{Int}\left(\frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2},x\right)$$

output
```
Defer(Int)(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

Mathematica [N/A]

Not integrable

Time = 3.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

input
```
Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
```

output
```
Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$
$$\Big\downarrow \ 2923$$
$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}dx$$

input
```
Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^2}{g\,x^2+f}dx$$

input
```
int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

output
```
int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

Sympy [N/A]

Not integrable

Time = 10.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c(d+ex^2)^p\right)^2}{f+gx^2}\,dx$$

input
```
integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f),x)
```

output
```
Integral(log(c*(d + e*x**2)**p)**2/(f + g*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

Mupad [N/A]

Not integrable

Time = 25.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^2}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^2/(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)^2/(f + g*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(\left(e\,x^2+d\right)^p c\right)^2}{g\,x^2+f}dx$$

input
```
int(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

output
```
int(log((d + e*x**2)**p*c)**2/(f + g*x**2),x)
```

3.276 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2},x\right)$$

output
```
Defer(Int)(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

input
```
Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
```

output
```
Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$$
$$\Big\downarrow \ 2923$$
$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx$$

input
```
Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}{\left(g\,x^2+f\right)^2}dx$$

input
```
int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

output
```
int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{(gx^2+f)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)^2/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2 \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2 \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\, dx = \int \frac{\log \left((ex^2+d)^p c\right)^2}{(gx^2+f)^2}\, dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2 \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\, dx = \int \frac{\ln \left(c\,(e\,x^2+d)^p\right)^2}{(g\,x^2+f)^2}\, dx$$

input
```
int(log(c*(d + e*x^2)^p)^2/(f + g*x^2)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)^2/(f + g*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log\left((e\,x^2+d)^p\,c\right)^2}{g^2x^4+2fg\,x^2+f^2}dx$$

input
```
int(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

output
```
int(log((d + e*x**2)**p*c)**2/(f**2 + 2*f*g*x**2 + g**2*x**4),x)
```

3.277 $\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx$$

$$= -48fp^3x+\frac{208dgp^3x}{9e}-\frac{16}{27}gp^3x^3+\frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{208d^{3/2}gp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}-\frac{24i\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}+\frac{32id^{3/2}gp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} - \frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+\frac{64d^{3/2}gp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}} +24fp^2x\log\left(c(d+ex^2)^p\right)-\frac{32dgp^2x\log\left(c(d+ex^2)^p\right)}{3e}+\frac{8}{9}gp^2x^3\log\left(c(d+ex^2)^p\right)-\frac{24\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}\ldots}{\sqrt{\ldots}}\right.\ldots}{\sqrt{\ldots}}$$

output
```
-48*f*p^3*x+208/9*d*g*p^3*x/e-16/27*g*p^3*x^3+48*d^(1/2)*f*p^3*arctan(e^(1
/2)*x/d^(1/2))/e^(1/2)-208/9*d^(3/2)*g*p^3*arctan(e^(1/2)*x/d^(1/2))/e^(3/
2)-24*I*d^(1/2)*f*p^3*arctan(e^(1/2)*x/d^(1/2))^2/e^(1/2)-24*I*d^(1/2)*f*p
^3*polylog(2,-(d^(1/2)-I*e^(1/2)*x)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)-48*d^(1
/2)*f*p^3*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^
(1/2)+64/3*d^(3/2)*g*p^3*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I
*e^(1/2)*x))/e^(3/2)+24*f*p^2*x*ln(c*(e*x^2+d)^p)-32/3*d*g*p^2*x*ln(c*(e*x
^2+d)^p)/e+8/9*g*p^2*x^3*ln(c*(e*x^2+d)^p)-24*d^(1/2)*f*p^2*arctan(e^(1/2)
*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(1/2)+32/3*d^(3/2)*g*p^2*arctan(e^(1/2)*x/
d^(1/2))*ln(c*(e*x^2+d)^p)/e^(3/2)-6*f*p*x*ln(c*(e*x^2+d)^p)^2+2*d*g*p*x*l
n(c*(e*x^2+d)^p)^2/e-2/3*g*p*x^3*ln(c*(e*x^2+d)^p)^2+f*x*ln(c*(e*x^2+d)^p)
^3+1/3*g*x^3*ln(c*(e*x^2+d)^p)^3+32/3*I*d^(3/2)*g*p^3*polylog(2,-(d^(1/2)-
I*e^(1/2)*x)/(d^(1/2)+I*e^(1/2)*x))/e^(3/2)+32/3*I*d^(3/2)*g*p^3*arctan(e^
(1/2)*x/d^(1/2))^2/e^(3/2)-2*d*(d*g-3*e*f)*p*Defer(Int)(ln(c*(e*x^2+d)^p)^
2/(e*x^2+d),x)/e
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1772 vs. 2(683) = 1366.

Time = 12.98 (sec) , antiderivative size = 1772, normalized size of antiderivative = 80.55

$$\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx=\text{Too large to display}$$

input
```
Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

output
```
(2*d*g*p*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/e + (6*Sqrt[d]*
f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p
])^2)/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p])^2)/e^(3/2) + 3*f*p*x*Log[d + e*x^2]*(-(p*Log
[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + g*p*x^3*Log[d + e*x^2]*(-(p*Log[d
 + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + f*x*(-(p*Log[d + e*x^2]) + Log[c*(d
 + e*x^2)^p])^2*(-6*p - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]) + (g*x^3*
(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-2*p - p*Log[d + e*x^2] +
Log[c*(d + e*x^2)^p]))/3 + 3*f*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2
)^p])*(x*Log[d + e*x^2]^2 - (4*((-I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2
 + Sqrt[e]*x*(-2 + Log[d + e*x^2]) - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(
-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + Log[d + e*x^2]) - I*Sqrt
[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/Sqrt[
e]) + 3*g*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*((x^3*Log[d + e
*x^2]^2)/3 - (4*((9*I)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + 3*d^(3/2)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]]*(-8 + 6*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
] + 3*Log[d + e*x^2]) + Sqrt[e]*x*(24*d - 2*e*x^2 + (-9*d + 3*e*x^2)*Log[d
 + e*x^2]) + (9*I)*d^(3/2)*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d
] + Sqrt[e]*x)]))/(27*e^(3/2))) + (g*p^3*(416*Sqrt[-d]*d^(3/2)*Sqrt[d + e*
x^2]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]] + 36*Sqrt[...
```

Rubi [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f\log^3\left(c(d+ex^2)^p\right)+gx^2\log^3\left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d^2gp\int \frac{\log^2(c(ex^2+d)^p)}{ex^2+d}dx}{e}+6dfp\int \frac{\log^2\left(c(ex^2+d)^p\right)}{ex^2+d}dx+\frac{32d^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{3e^{3/2}}-\frac{24\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}}+\frac{32id^{3/2}gp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}-\frac{208d^{3/2}gp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}+\frac{64d^{3/2}gp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}-\frac{24i\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}+\frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+24fp^2x\log\left(c(d+ex^2)^p\right)+fx\log^3\left(c(d+ex^2)^p\right)-6fpx\log^2\left(c(d+ex^2)^p\right)-\frac{32dgp^2x\log\left(c(d+ex^2)^p\right)}{3e}+\frac{8}{9}gp^2x^3\log\left(c(d+ex^2)^p\right)+\frac{2dgpx\log^2\left(c(d+ex^2)^p\right)}{e}+\frac{1}{3}gx^3\log^3\left(c(d+ex^2)^p\right)-\frac{2}{3}gpx^3\log^2\left(c(d+ex^2)^p\right)+\frac{32id^{3/2}gp^3\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{3e^{3/2}}-\frac{24i\sqrt{d}fp^3\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}+\frac{208dgp^3x}{9e}-48fp^3x-\frac{16}{27}gp^3x^3$$

input
```
Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(g\,x^{2}+f\right) \ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{3}dx$$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
```

output
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(f+gx^2\right) \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(gx^2+f\right) \log\left(\left(ex^2+d\right)^p c\right)^3\,dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

output
```
integral((g*x^2 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 10.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(f+gx^2\right) \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f+gx^2\right) \log\left(c\left(d+ex^2\right)^p\right)^3 dx$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**3,x)
```

output
```
Integral((f + g*x**2)*log(c*(d + e*x**2)**p)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f + gx^2) \log^3 \left(c(d + ex^2)^p\right) dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (f + gx^2) \log^3 \left(c(d + ex^2)^p\right) dx = \int (gx^2 + f) \log \left((ex^2 + d)^p c\right)^3 dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (f + gx^2) \log^3 \left(c(d + ex^2)^p\right) dx = \int \ln \left(c \left(e\,x^2 + d\right)^p\right)^3 \left(g\,x^2 + f\right) dx$$

input
```
int(log(c*(d + e*x^2)^p)^3*(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)^3*(f + g*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 397, normalized size of antiderivative = 18.05

$$\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx$$

$$= \frac{-624\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)dg\,p^3+1296\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ef\,p^3-54\left(\int\frac{\log\left((e\,x^2+d)^p c\right)^2}{e\,x^2+d}dx\right)d^2egp+162\left(\right.}{}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x)
```

output
```
( - 624*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*g*p**3 + 1296*sqrt
(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p**3 - 54*int(log((d + e*x**
2)**p*c)**2/(d + e*x**2),x)*d**2*e*g*p + 162*int(log((d + e*x**2)**p*c)**2
/(d + e*x**2),x)*d*e**2*f*p + 288*int(log((d + e*x**2)**p*c)/(d + e*x**2),
x)*d**2*e*g*p**2 - 648*int(log((d + e*x**2)**p*c)/(d + e*x**2),x)*d*e**2*f
*p**2 + 27*log((d + e*x**2)**p*c)**3*e**2*f*x + 9*log((d + e*x**2)**p*c)**
3*e**2*g*x**3 + 54*log((d + e*x**2)**p*c)**2*d*e*g*p*x - 162*log((d + e*x*
*2)**p*c)**2*e**2*f*p*x - 18*log((d + e*x**2)**p*c)**2*e**2*g*p*x**3 - 288
*log((d + e*x**2)**p*c)*d*e*g*p**2*x + 648*log((d + e*x**2)**p*c)*e**2*f*p
**2*x + 24*log((d + e*x**2)**p*c)*e**2*g*p**2*x**3 + 624*d*e*g*p**3*x - 12
96*e**2*f*p**3*x - 16*e**2*g*p**3*x**3)/(27*e**2)
```

3.278 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \text{Int}\left(\frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2},x\right)$$

output
```
Defer(Int)(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

Mathematica [N/A]

Not integrable

Time = 6.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

input
```
Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
```

output
```
Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$$
$$\Big\downarrow \; 2923$$
$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx$$

input
```
Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{g\,x^2+f}dx$$

input
```
int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

output
```
int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

Sympy [N/A]

Not integrable

Time = 14.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c(d+ex^2)^p\right)^3}{f+gx^2}\,dx$$

input
```
integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f),x)
```

output
```
Integral(log(c*(d + e*x**2)**p)**3/(f + g*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

Mupad [N/A]

Not integrable

Time = 25.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^3}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^3/(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)^3/(f + g*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((e\,x^2+d)^p\,c\right)^3}{g\,x^2+f}dx$$

input
```
int(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

output
```
int(log((d + e*x**2)**p*c)**3/(f + g*x**2),x)
```

3.279 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx=\text{Int}\left(\frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2},x\right)$$

output
```
Defer(Int)(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx=\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$$

input
```
Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
```

output
```
Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx$$

$$\Big\downarrow \text{ 2923}$$

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx$$

input
```
Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{\left(g\,x^2+f\right)^2}dx$$

input
```
int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

output
```
int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{(gx^2+f)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)^3/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{(gx^2+f)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^3}{(g\,x^2+f)^2}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^3/(f + g*x^2)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)^3/(f + g*x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left(\left(e\,x^2+d\right)^p c\right)^3}{g^2x^4+2fg\,x^2+f^2}dx$$

input
```
int(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

output
```
int(log((d + e*x**2)**p*c)**3/(f**2 + 2*f*g*x**2 + g**2*x**4),x)
```

3.280 $\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \operatorname{Int}\left(\frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)},x\right)$$

output
```
Defer(Int)((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx$$

input
```
Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]
```

output
```
Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx$$

$$\Big\downarrow \; 2923$$

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}dx$$

input
```
Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(g\,x^2+f)^2}{\ln(c\,(e\,x^2+d)^p)}dx$$

input
```
int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

output
```
int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^2+f)^2}{\log((ex^2+d)^pc)}\,dx$$

input
```
integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 8.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx$$

input
```
integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
```

output
```
Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^2+f)^2}{\log((ex^2+d)^pc)}\,dx$$

input
```
integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^2+f)^2}{\log((ex^2+d)^pc)}\,dx$$

input
```
integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 25.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(g\,x^2+f)^2}{\ln(c\,(e\,x^2+d)^p)}\,dx$$

input
```
int((f + g*x^2)^2/log(c*(d + e*x^2)^p),x)
```

output
```
int((f + g*x^2)^2/log(c*(d + e*x^2)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.88

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \left(\int \frac{x^4}{\log((e\,x^2+d)^p\,c)}dx\right)g^2+2\left(\int \frac{x^2}{\log((e\,x^2+d)^p\,c)}dx\right)fg + \left(\int \frac{1}{\log((e\,x^2+d)^p\,c)}dx\right)f^2$$

input
```
int((g*x^2+f)^2/log(c*(e*x^2+d)^p),x)
```

output
```
int(x**4/log((d + e*x**2)**p*c),x)*g**2 + 2*int(x**2/log((d + e*x**2)**p*c
),x)*f*g + int(1/log((d + e*x**2)**p*c),x)*f**2
```

3.281 $\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx = \text{Int}\left(\frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

output
```
Defer(Int)((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)} dx$$

input
```
Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
```

output
```
Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$
$$\Big\downarrow \; 2923$$
$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}dx$$

input
```
Int[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{g\,x^2+f}{\ln\left(c\left(e\,x^2+d\right)^p\right)}dx$$

input
```
int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

output
```
int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

input
```
integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

input
```
integrate((g*x**2+f)/ln(c*(e*x**2+d)**p),x)
```

output
```
Integral((f + g*x**2)/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^pc\right)}\,dx$$

input
```
integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
integrate((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^pc\right)}\,dx$$

input
```
integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 25.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{g\,x^2+f}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}\,dx$$

input
```
int((f + g*x^2)/log(c*(d + e*x^2)^p),x)
```

output
```
int((f + g*x^2)/log(c*(d + e*x^2)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{f+gx^2}{\log(c(d+ex^2)^p)}\,dx = \left(\int \frac{x^2}{\log\left((e\,x^2+d)^p\,c\right)}dx\right)g + \left(\int \frac{1}{\log\left((e\,x^2+d)^p\,c\right)}dx\right)f$$

input
```
int((g*x^2+f)/log(c*(e*x^2+d)^p),x)
```

output
```
int(x**2/log((d + e*x**2)**p*c),x)*g + int(1/log((d + e*x**2)**p*c),x)*f
```

3.282 $\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)} \, dx = \text{Int}\left(\frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}, x\right)$$

output
```
Defer(Int)(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)} \, dx = \int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)} \, dx$$

input
```
Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
```

output
```
Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx^2)\log (c(d+ex^2)^p)} \, dx$$
$$\Big\downarrow \text{ 2923}$$
$$\int \frac{1}{(f+gx^2)\log (c(d+ex^2)^p)}dx$$

input
```
Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^2+f)\ln (c\,(e\,x^2+d)^p)}dx$$

input
```
int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

output
```
int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)\log((ex^2+d)^pc)}\,dx$$

input
```
integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 15.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx$$

input
```
integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p),x)
```

output
```
Integral(1/((f + g*x**2)*log(c*(d + e*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^2+f)\log\left((ex^2+d)^p c\right)}\,dx$$

input
```
integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^2+f)\log\left((ex^2+d)^p c\right)}\,dx$$

input
```
integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^2+f\right)}\,dx$$

input
```
int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)),x)
```

output
```
int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{\log((e\,x^2+d)^p\,c)\,f+\log((e\,x^2+d)^p\,c)\,g\,x^2}dx$$

input
```
int(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x)
```

output
```
int(1/(log((d + e*x**2)**p*c)*f + log((d + e*x**2)**p*c)*g*x**2),x)
```

3.283 $\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} \, dx = \text{Int}\left(\frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)}, x\right)$$

output
```
Defer(Int)(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} \, dx = \int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} \, dx$$

input
```
Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
```

output
```
Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx^2)^2 \log (c (d+ex^2)^p)} \, dx$$
$$\Big\downarrow \text{2923}$$
$$\int \frac{1}{(f+gx^2)^2 \log (c (d+ex^2)^p)}dx$$

input
```
Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^2+f)^2 \ln (c (e\,x^2+d)^p)}dx$$

input
```
int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

output
```
int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{(f+gx^2)^2\log(c(d+ex^2)^p)}\,dx=\int \frac{1}{(gx^2+f)^2\log((ex^2+d)^pc)}\,dx$$

input
```
integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 167.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx^2)^2\log(c(d+ex^2)^p)}\,dx=\int \frac{1}{(f+gx^2)^2\log(c(d+ex^2)^p)}\,dx$$

input
```
integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
```

output
```
Integral(1/((f + g*x**2)**2*log(c*(d + e*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)^2 \log((ex^2+d)^p c)}\,dx$$

input
```
integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)^2 \log((ex^2+d)^p c)}\,dx$$

input
```
integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)}\,dx = \int \frac{1}{\ln(c\,(e\,x^2+d)^p)\,(g\,x^2+f)^2}\,dx$$

input
```
int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)^2),x)
```

output
```
int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{1}{(f+gx^2)^2 \log (c (d+ex^2)^p)} \, dx = \int \frac{1}{\log \left(\left(e\,x^{2}+d\right)^{p} c\right) f^{2}+2 \log \left(\left(e\,x^{2}+d\right)^{p} c\right) f g\,x^{2}+\log \left(\left(e\,x^{2}+d\right)^{p} c\right) g^{2} x^{4}}dx$$

input
```
int(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x)
```

output
```
int(1/(log((d + e*x**2)**p*c)*f**2 + 2*log((d + e*x**2)**p*c)*f*g*x**2 + l
og((d + e*x**2)**p*c)*g**2*x**4),x)
```

3.284 $\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)} \, dx = \operatorname{Int}\left(\frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}, x\right)$$

output
```
Defer(Int)((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)} \, dx = \int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)} \, dx$$

input
```
Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)} dx$$
$$\Big\downarrow \ 2923$$
$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)} dx$$

input
```
Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(g\,x^{2}+f)^{2}}{\ln \left(c\,(e\,x^{2}+d)^{p}\right)^{2}}dx$$

input
```
int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(gx^2+f)^2}{\log\left((ex^2+d)^p c\right)^2}\,dx$$

input
```
integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 13.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

input
```
integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 148, normalized size of antiderivative = 6.17

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(gx^2+f)^2}{\log\left((ex^2+d)^p c\right)^2}\,dx$$

input
```
integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(e*g^2*x^6 + (2*e*f*g + d*g^2)*x^4 + d*f^2 + (e*f^2 + 2*d*f*g)*x^2)/(
e*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + integrate(1/2*(5*e*g^2*x^6 + 3*
(2*e*f*g + d*g^2)*x^4 - d*f^2 + (e*f^2 + 2*d*f*g)*x^2)/(e*p*x^2*log((e*x^2
 + d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(gx^2+f)^2}{\log\left((ex^2+d)^p c\right)^2}\,dx$$

input
```
integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(g\,x^2+f)^2}{\ln\left(c\,(e\,x^2+d)^p\right)^2}\,dx$$

input
```
int((f + g*x^2)^2/log(c*(d + e*x^2)^p)^2,x)
```

output
```
int((f + g*x^2)^2/log(c*(d + e*x^2)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.88

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx = \left(\int \frac{x^4}{\log\left((e\,x^2+d)^p\,c\right)^2}dx\right) g^2 + 2\left(\int \frac{x^2}{\log\left((e\,x^2+d)^p\,c\right)^2}dx\right) f g + \left(\int \frac{1}{\log\left((e\,x^2+d)^p\,c\right)^2}dx\right) f^2$$

input
```
int((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x)
```

output
```
int(x**4/log((d + e*x**2)**p*c)**2,x)*g**2 + 2*int(x**2/log((d + e*x**2)**
p*c)**2,x)*f*g + int(1/log((d + e*x**2)**p*c)**2,x)*f**2
```

3.285 $\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \text{Int}\left(\frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

output
```
Defer(Int)((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

input
```
Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

$$\Big\downarrow \text{2923}$$

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}dx$$

input
```
Int[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{g\,x^{2}+f}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}dx$$

input
```
int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

input
```
integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((g*x^2 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 7.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

input
```
integrate((g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f + g*x**2)/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.59

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

input
```
integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(e*g*x^4 + (e*f + d*g)*x^2 + d*f)/(e*p*x*log((e*x^2 + d)^p) + e*p*x*l
og(c)) + integrate(1/2*(3*e*g*x^4 + (e*f + d*g)*x^2 - d*f)/(e*p*x^2*log((e
*x^2 + d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

input
```
integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{g\,x^2+f}{\ln\left(c\,\left(e\,x^2+d\right)^p\right)^2}\,dx$$

input
```
int((f + g*x^2)/log(c*(d + e*x^2)^p)^2,x)
```

output
```
int((f + g*x^2)/log(c*(d + e*x^2)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\left(\int \frac{x^2}{\log\left(\left(e\,x^2+d\right)^p c\right)^2}dx\right)g+\left(\int \frac{1}{\log\left(\left(e\,x^2+d\right)^p c\right)^2}dx\right)f$$

input
```
int((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x)
```

output
```
int(x**2/log((d + e*x**2)**p*c)**2,x)*g + int(1/log((d + e*x**2)**p*c)**2,
x)*f
```

3.286 $\int \frac{1}{(f+gx^2)\log^2(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

input
```
Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
```

output
```
Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}dx$$
$$\Big\downarrow \text{ 2923}$$
$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}dx$$

input
```
Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^2+f)\ln\left(c\left(e\,x^2+d\right)^p\right)^2}dx$$

input
```
int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)\log\left((ex^2+d)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 22.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx^2)\log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{(f+gx^2)\log\left(c(d+ex^2)^p\right)^2}\,dx$$

input
```
integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral(1/((f + g*x**2)*log(c*(d + e*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 6.54

$$\int \frac{1}{(f+gx^2)\log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)\log\left((ex^2+d)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(e*x^2 + d)/(e*g*p*x^3*log(c) + e*f*p*x*log(c) + (e*g*p*x^3 + e*f*p*x
)*log((e*x^2 + d)^p)) - integrate(1/2*(e*g*x^4 - (e*f - 3*d*g)*x^2 + d*f)/
(e*g^2*p*x^6*log(c) + 2*e*f*g*p*x^4*log(c) + e*f^2*p*x^2*log(c) + (e*g^2*p
*x^6 + 2*e*f*g*p*x^4 + e*f^2*p*x^2)*log((e*x^2 + d)^p)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)\log\left((ex^2+d)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2\,\left(g\,x^2+f\right)}\,dx$$

input
```
int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)),x)
```

output
```
int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(e\,x^2+d\right)^p c\right)^2 f+\log\left(\left(e\,x^2+d\right)^p c\right)^2 g\,x^2}dx$$

input
```
int(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x)
```

output
```
int(1/(log((d + e*x**2)**p*c)**2*f + log((d + e*x**2)**p*c)**2*g*x**2),x)
```

3.287 $\int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)} dx = \operatorname{Int}\left(\frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)}, x\right)$$

output
```
Defer(Int)(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 6.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)} dx = \int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)} dx$$

input
```
Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]
```

output
```
Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx^2)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$
$$\Big\downarrow \ 2923$$
$$\int \frac{1}{(f+gx^2)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}dx$$

input
```
Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(g\,x^2+f\right)^2 \ln\left(c\left(e\,x^2+d\right)^p\right)^2}dx$$

input
```
int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)^2 \log\left((ex^2+d)^p c\right)^2}\,dx$$

input
```
integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 8.92

$$\int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)^2 \log\left((ex^2+d)^p c\right)^2}\,dx$$

input
```
integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(e*x^2 + d)/(e*g^2*p*x^5*log(c) + 2*e*f*g*p*x^3*log(c) + e*f^2*p*x*lo
g(c) + (e*g^2*p*x^5 + 2*e*f*g*p*x^3 + e*f^2*p*x)*log((e*x^2 + d)^p)) - int
egrate(1/2*(3*e*g*x^4 - (e*f - 5*d*g)*x^2 + d*f)/(e*g^3*p*x^8*log(c) + 3*e
*f*g^2*p*x^6*log(c) + 3*e*f^2*g*p*x^4*log(c) + e*f^3*p*x^2*log(c) + (e*g^3
*p*x^8 + 3*e*f*g^2*p*x^6 + 3*e*f^2*g*p*x^4 + e*f^3*p*x^2)*log((e*x^2 + d)^
p)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(gx^2+f)^2\log\left((ex^2+d)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2\left(g\,x^2+f\right)^2}\,dx$$

input
```
int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2),x)
```

output
```
int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{1}{(f+gx^2)^2\log^2(c(d+ex^2)^p)}\,dx$$
$$= \int \frac{1}{\log\left((e\,x^2+d)^p\,c\right)^2 f^2 + 2\log\left((e\,x^2+d)^p\,c\right)^2 f g\,x^2 + \log\left((e\,x^2+d)^p\,c\right)^2 g^2 x^4}dx$$

input

```
int(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x)
```

output

```
int(1/(log((d + e*x**2)**p*c)**2*f**2 + 2*log((d + e*x**2)**p*c)**2*f*g*x*
*2 + log((d + e*x**2)**p*c)**2*g**2*x**4),x)
```

3.288 $\int (f+gx^3)^3 \log \left(c(d+ex^2)^p\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 366

$$\begin{aligned}\int (f+gx^3)^3 \log \left(c(d+ex^2)^p\right) \, dx &= -2f^3px + \frac{6d^3fg^2px}{7e^3} + \frac{3df^2gpx^2}{4e} - \frac{d^4g^3px^2}{10e^4} \\ &\quad - \frac{2d^2fg^2px^3}{7e^2} - \frac{3}{8}f^2gpx^4 + \frac{d^3g^3px^4}{20e^3} \\ &\quad + \frac{6dfg^2px^5}{35e} - \frac{d^2g^3px^6}{30e^2} - \frac{6}{49}fg^2px^7 + \frac{dg^3px^8}{40e} \\ &\quad - \frac{1}{50}g^3px^{10} + \frac{2\sqrt{d}f^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\ &\quad - \frac{6d^{7/2}fg^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} \\ &\quad - \frac{3d^2f^2gp\log(d+ex^2)}{4e^2} + \frac{d^5g^3p\log(d+ex^2)}{10e^5} \\ &\quad + f^3x\log\left(c(d+ex^2)^p\right) + \frac{3}{4}f^2gx^4\log\left(c(d+ex^2)^p\right) \\ &\quad + \frac{3}{7}fg^2x^7\log\left(c(d+ex^2)^p\right) \\ &\quad + \frac{1}{10}g^3x^{10}\log\left(c(d+ex^2)^p\right)\end{aligned}$$

output
```
-2*f^3*p*x+6/7*d^3*f*g^2*p*x/e^3+3/4*d*f^2*g*p*x^2/e-1/10*d^4*g^3*p*x^2/e^
4-2/7*d^2*f*g^2*p*x^3/e^2-3/8*f^2*g*p*x^4+1/20*d^3*g^3*p*x^4/e^3+6/35*d*f*
g^2*p*x^5/e-1/30*d^2*g^3*p*x^6/e^2-6/49*f*g^2*p*x^7+1/40*d*g^3*p*x^8/e-1/5
0*g^3*p*x^10+2*d^(1/2)*f^3*p*arctan(e^(1/2)*x/d^(1/2))/e^(1/2)-6/7*d^(7/2)
*f*g^2*p*arctan(e^(1/2)*x/d^(1/2))/e^(7/2)-3/4*d^2*f^2*g*p*ln(e*x^2+d)/e^2
+1/10*d^5*g^3*p*ln(e*x^2+d)/e^5+f^3*x*ln(c*(e*x^2+d)^p)+3/4*f^2*g*x^4*ln(c
*(e*x^2+d)^p)+3/7*f*g^2*x^7*ln(c*(e*x^2+d)^p)+1/10*g^3*x^10*ln(c*(e*x^2+d)
^p)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.70

$$\int \left(f+gx^3\right)^3 \log \left(c\left(d+ex^2\right)^p\right)\, dx$$
$$= \frac{-epx(2940d^4g^3x+140d^2e^2g^2x^2(60f+7gx^3)-210d^3eg^2(120f+7gx^3)-105de^3gx(210f^2+48fgx^3+\cdots}{}$$

input
```
Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

output
```
(-(e*p*x*(2940*d^4*g^3*x + 140*d^2*e^2*g^2*x^2*(60*f + 7*g*x^3) - 210*d^3*
e*g^2*(120*f + 7*g*x^3) - 105*d*e^3*g*x*(210*f^2 + 48*f*g*x^3 + 7*g^2*x^6)
 + 3*e^4*(19600*f^3 + 3675*f^2*g*x^3 + 1200*f*g^2*x^6 + 196*g^3*x^9))) - 8
400*Sqrt[d]*e^(3/2)*f*(-7*e^3*f^2 + 3*d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d
]] + 1470*d^2*g*(-15*e^3*f^2 + 2*d^3*g^2)*p*Log[d + e*x^2] + 210*e^5*x*(14
0*f^3 + 105*f^2*g*x^3 + 60*f*g^2*x^6 + 14*g^3*x^9)*Log[c*(d + e*x^2)^p])/(
29400*e^5)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+gx^3\right)^3 \log \left(c\left(d+ex^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{ 2921}$$

$$\int \left(f^3 \log \left(c\left(d+ex^2\right)^p\right)+3 f^2 g x^3 \log \left(c\left(d+ex^2\right)^p\right)+3 f g^2 x^6 \log \left(c\left(d+ex^2\right)^p\right)+g^3 x^9 \log \left(c\left(d+ex^2\right)^p\right)\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{6 d^{7/2} f g^2 p \arctan \left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{7 e^{7/2}}+\frac{2 \sqrt{d} f^3 p \arctan \left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{\sqrt{e}}+f^3 x \log \left(c\left(d+ex^2\right)^p\right)+\frac{3}{4} f^2 g x^4 \log \left(c\left(d+ex^2\right)^p\right)+\frac{3}{7} f g^2 x^7 \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{10} g^3 x^{10} \log \left(c\left(d+ex^2\right)^p\right)+\frac{d^5 g^3 p \log \left(d+ex^2\right)}{10 e^5}-\frac{d^4 g^3 p x^2}{10 e^4}+\frac{6 d^3 f g^2 p x}{7 e^3}+\frac{d^3 g^3 p x^4}{20 e^3}-\frac{3 d^2 f^2 g p \log \left(d+ex^2\right)}{4 e^2}-\frac{2 d^2 f g^2 p x^3}{7 e^2}-\frac{d^2 g^3 p x^6}{30 e^2}+\frac{3 d f^2 g p x^2}{4 e}+\frac{6 d f g^2 p x^5}{35 e}+\frac{d g^3 p x^8}{40 e}-2 f^3 p x-\frac{3}{8} f^2 g p x^4-\frac{6}{49} f g^2 p x^7-\frac{1}{50} g^3 p x^{10}$$

input

```
Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

output

```
-2*f^3*p*x + (6*d^3*f*g^2*p*x)/(7*e^3) + (3*d*f^2*g*p*x^2)/(4*e) - (d^4*g^
3*p*x^2)/(10*e^4) - (2*d^2*f*g^2*p*x^3)/(7*e^2) - (3*f^2*g*p*x^4)/8 + (d^3
*g^3*p*x^4)/(20*e^3) + (6*d*f*g^2*p*x^5)/(35*e) - (d^2*g^3*p*x^6)/(30*e^2)
 - (6*f*g^2*p*x^7)/49 + (d*g^3*p*x^8)/(40*e) - (g^3*p*x^10)/50 + (2*Sqrt[d
]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (6*d^(7/2)*f*g^2*p*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (3*d^2*f^2*g*p*Log[d + e*x^2])/(4*e^2)
+ (d^5*g^3*p*Log[d + e*x^2])/(10*e^5) + f^3*x*Log[c*(d + e*x^2)^p] + (3*f^
2*g*x^4*Log[c*(d + e*x^2)^p])/4 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p])/7 + (
g^3*x^10*Log[c*(d + e*x^2)^p])/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [A] (verified)

Time = 16.82 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.84

method	result
parts	$\frac{g^3x^{10}\ln\left(c\left(e\,x^2+d\right)^p\right)}{10}+\frac{3f\,g^2x^7\ln\left(c\left(e\,x^2+d\right)^p\right)}{7}+\frac{3f^2g\,x^4\ln\left(c\left(e\,x^2+d\right)^p\right)}{4}+f^3x\ln\left(c\left(e\,x^2+d\right)^p\right)-\frac{ep\left(\frac{7}{5}e^4g^3x^{10}\ldots\right.}{}$
risch	Expression too large to display

input
```
int((g*x^3+f)^3*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/10*g^3*x^10*ln(c*(e*x^2+d)^p)+3/7*f*g^2*x^7*ln(c*(e*x^2+d)^p)+3/4*f^2*g*
x^4*ln(c*(e*x^2+d)^p)+f^3*x*ln(c*(e*x^2+d)^p)-1/70*e*p*(1/e^5*(7/5*e^4*g^3
*x^10-7/4*d*e^3*g^3*x^8+60/7*e^4*f*g^2*x^7+7/3*d^2*e^2*g^3*x^6-12*d*e^3*f*
g^2*x^5-7/2*d^3*e*g^3*x^4+105/4*e^4*f^2*g*x^4+20*d^2*e^2*f*g^2*x^3+7*d^4*g
^3*x^2-105/2*d*f^2*g*x^2*e^3-60*x*d^3*f*g^2*e+140*x*e^4*f^3)-d/e^5*(1/2*(1
4*d^4*g^3-105*d*e^3*f^2*g)/e*ln(e*x^2+d)+(-60*d^3*e*f*g^2+140*e^4*f^3)/(d*
e)^(1/2)*arctan(x*e/(d*e)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.93

$$\int \left(f+gx^3\right)^3\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$= \Bigg[-\frac{588\,e^5g^3px^{10}-735\,de^4g^3px^8+3600\,e^5fg^2px^7+980\,d^2e^3g^3px^6-5040\,de^4fg^2px^5+8400\,d^2e^3fg^2px^3}{}$$

$$-\frac{588\,e^5g^3px^{10}-735\,de^4g^3px^8+3600\,e^5fg^2px^7+980\,d^2e^3g^3px^6-5040\,de^4fg^2px^5+8400\,d^2e^3fg^2px^3}{}$$

input
```
integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/29400*(588*e^5*g^3*p*x^10 - 735*d*e^4*g^3*p*x^8 + 3600*e^5*f*g^2*p*x^7
 + 980*d^2*e^3*g^3*p*x^6 - 5040*d*e^4*f*g^2*p*x^5 + 8400*d^2*e^3*f*g^2*p*x
^3 + 735*(15*e^5*f^2*g - 2*d^3*e^2*g^3)*p*x^4 - 1470*(15*d*e^4*f^2*g - 2*d
^4*e*g^3)*p*x^2 + 4200*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*sqrt(-d/e)*log((e*x
^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^
2)*p*x - 210*(14*e^5*g^3*p*x^10 + 60*e^5*f*g^2*p*x^7 + 105*e^5*f^2*g*p*x^4
 + 140*e^5*f^3*p*x - 7*(15*d^2*e^3*f^2*g - 2*d^5*g^3)*p)*log(e*x^2 + d) -
210*(14*e^5*g^3*x^10 + 60*e^5*f*g^2*x^7 + 105*e^5*f^2*g*x^4 + 140*e^5*f^3*
x)*log(c))/e^5, -1/29400*(588*e^5*g^3*p*x^10 - 735*d*e^4*g^3*p*x^8 + 3600*
e^5*f*g^2*p*x^7 + 980*d^2*e^3*g^3*p*x^6 - 5040*d*e^4*f*g^2*p*x^5 + 8400*d^
2*e^3*f*g^2*p*x^3 + 735*(15*e^5*f^2*g - 2*d^3*e^2*g^3)*p*x^4 - 1470*(15*d*
e^4*f^2*g - 2*d^4*e*g^3)*p*x^2 - 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*sqrt
(d/e)*arctan(e*x*sqrt(d/e)/d) + 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*x - 2
10*(14*e^5*g^3*p*x^10 + 60*e^5*f*g^2*p*x^7 + 105*e^5*f^2*g*p*x^4 + 140*e^5
*f^3*p*x - 7*(15*d^2*e^3*f^2*g - 2*d^5*g^3)*p)*log(e*x^2 + d) - 210*(14*e^
5*g^3*x^10 + 60*e^5*f*g^2*x^7 + 105*e^5*f^2*g*x^4 + 140*e^5*f^3*x)*log(c))
/e^5]
```

Sympy [F(-1)]

Timed out.

$$\int \left(f+gx^3\right)^3 \log \left(c\left(d+ex^2\right)^p\right) \, dx = \text{Timed out}$$

input
```
integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^3\right)^3 \log \left(c\left(d+ex^2\right)^p\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.89

$$\int (f+gx^3)^3 \log \left(c(d+ex^2)^p\right) \, dx = \frac{dg^3px^8}{40\,e} - \frac{1}{50}\left(g^3p - 5\,g^3\log(c)\right)x^{10} - \frac{d^2g^3px^6}{30\,e^2} + \frac{6\,dfg^2px^5}{35\,e} - \frac{3}{49}\left(2\,fg^2p - 7\,fg^2\log(c)\right)x^7 - \frac{2\,d^2fg^2px^3}{7\,e^2} - \frac{(15\,e^3f^2gp - 2\,d^3g^3p - 30\,e^3f^2g\log(c))x^4}{40\,e^3} + \frac{1}{140}\left(14\,g^3px^{10} + 60\,fg^2px^7 + 105\,f^2gpx^4 + 140\,f^3px\right)\log\left(ex^2+d\right) - \frac{(14\,e^3f^3p - 6\,d^3fg^2p - 7\,e^3f^3\log(c))x}{7\,e^3} + \frac{(15\,de^3f^2gp - 2\,d^4g^3p)x^2}{20\,e^4} + \frac{2\,(7\,de^3f^3p - 3\,d^4fg^2p)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{7\,\sqrt{de}e^3} - \frac{(15\,d^2e^3f^2gp - 2\,d^5g^3p)\log\left(ex^2+d\right)}{20\,e^5}$$

input
```
integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
1/40*d*g^3*p*x^8/e - 1/50*(g^3*p - 5*g^3*log(c))*x^10 - 1/30*d^2*g^3*p*x^6
/e^2 + 6/35*d*f*g^2*p*x^5/e - 3/49*(2*f*g^2*p - 7*f*g^2*log(c))*x^7 - 2/7*
d^2*f*g^2*p*x^3/e^2 - 1/40*(15*e^3*f^2*g*p - 2*d^3*g^3*p - 30*e^3*f^2*g*lo
g(c))*x^4/e^3 + 1/140*(14*g^3*p*x^10 + 60*f*g^2*p*x^7 + 105*f^2*g*p*x^4 +
140*f^3*p*x)*log(e*x^2 + d) - 1/7*(14*e^3*f^3*p - 6*d^3*f*g^2*p - 7*e^3*f^
3*log(c))*x/e^3 + 1/20*(15*d*e^3*f^2*g*p - 2*d^4*g^3*p)*x^2/e^4 + 2/7*(7*d
*e^3*f^3*p - 3*d^4*f*g^2*p)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3) - 1/20*(
15*d^2*e^3*f^2*g*p - 2*d^5*g^3*p)*log(e*x^2 + d)/e^5
```

Mupad [B] (verification not implemented)

Time = 28.88 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.86

$$\int (f+gx^3)^3 \log \left(c(d+ex^2)^p\right)\,dx = \frac{g^3\,x^{10}\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{10}-2\,f^3\,p\,x-\frac{g^3\,p\,x^{10}}{50}+f^3\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)+\frac{3\,f^2\,g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{4}+\frac{3\,f\,g^2\,x^7\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{7}-\frac{3\,f^2\,g\,p\,x^4}{8}-\frac{6\,f\,g^2\,p\,x^7}{49}+\frac{d\,g^3\,p\,x^8}{40\,e}+\frac{2\,\sqrt{d}\,f^3\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{d^5\,g^3\,p\,\ln\left(e\,x^2+d\right)}{10\,e^5}-\frac{d^2\,g^3\,p\,x^6}{30\,e^2}+\frac{d^3\,g^3\,p\,x^4}{20\,e^3}-\frac{d^4\,g^3\,p\,x^2}{10\,e^4}-\frac{6\,d^{7/2}\,f\,g^2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7\,e^{7/2}}-\frac{3\,d^2\,f^2\,g\,p\,\ln\left(e\,x^2+d\right)}{4\,e^2}-\frac{2\,d^2\,f\,g^2\,p\,x^3}{7\,e^2}+\frac{3\,d\,f^2\,g\,p\,x^2}{4\,e}+\frac{6\,d\,f\,g^2\,p\,x^5}{35\,e}+\frac{6\,d^3\,f\,g^2\,p\,x}{7\,e^3}$$

input
```
int(log(c*(d + e*x^2)^p)*(f + g*x^3)^3,x)
```

output
```
(g^3*x^10*log(c*(d + e*x^2)^p))/10 - 2*f^3*p*x - (g^3*p*x^10)/50 + f^3*x*l
og(c*(d + e*x^2)^p) + (3*f^2*g*x^4*log(c*(d + e*x^2)^p))/4 + (3*f*g^2*x^7*
log(c*(d + e*x^2)^p))/7 - (3*f^2*g*p*x^4)/8 - (6*f*g^2*p*x^7)/49 + (d*g^3*
p*x^8)/(40*e) + (2*d^(1/2)*f^3*p*atan((e^(1/2)*x)/d^(1/2)))/e^(1/2) + (d^5
*g^3*p*log(d + e*x^2))/(10*e^5) - (d^2*g^3*p*x^6)/(30*e^2) + (d^3*g^3*p*x^
4)/(20*e^3) - (d^4*g^3*p*x^2)/(10*e^4) - (6*d^(7/2)*f*g^2*p*atan((e^(1/2)*
x)/d^(1/2)))/(7*e^(7/2)) - (3*d^2*f^2*g*p*log(d + e*x^2))/(4*e^2) - (2*d^2
*f*g^2*p*x^3)/(7*e^2) + (3*d*f^2*g*p*x^2)/(4*e) + (6*d*f*g^2*p*x^5)/(35*e)
 + (6*d^3*f*g^2*p*x)/(7*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.97

$$\int (f+gx^3)^3 \log \left(c(d+ex^2)^p\right) \, dx$$

$$= \frac{-25200\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) d^3 ef\, g^2 p + 58800\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) e^4 f^3 p + 2940 \log\left((e\,x^2+d)^p\,c\right) d^5 g^3 -}{}$$

input
```
int((g*x^3+f)^3*log(c*(e*x^2+d)^p),x)
```

output
```
( - 25200*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**3*e*f*g**2*p + 
58800*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**4*f**3*p + 2940*log
((d + e*x**2)**p*c)*d**5*g**3 - 22050*log((d + e*x**2)**p*c)*d**2*e**3*f**
2*g + 29400*log((d + e*x**2)**p*c)*e**5*f**3*x + 22050*log((d + e*x**2)**p
*c)*e**5*f**2*g*x**4 + 12600*log((d + e*x**2)**p*c)*e**5*f*g**2*x**7 + 294
0*log((d + e*x**2)**p*c)*e**5*g**3*x**10 - 2940*d**4*e*g**3*p*x**2 + 25200
*d**3*e**2*f*g**2*p*x + 1470*d**3*e**2*g**3*p*x**4 - 8400*d**2*e**3*f*g**2
*p*x**3 - 980*d**2*e**3*g**3*p*x**6 + 22050*d*e**4*f**2*g*p*x**2 + 5040*d*
e**4*f*g**2*p*x**5 + 735*d*e**4*g**3*p*x**8 - 58800*e**5*f**3*p*x - 11025*
e**5*f**2*g*p*x**4 - 3600*e**5*f*g**2*p*x**7 - 588*e**5*g**3*p*x**10)/(294
00*e**5)
```

3.289 $\int (f+gx^3)^2 \log(c(d+ex^2)^p)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 231

$$\begin{aligned}\int (f+gx^3)^2 \log\left(c(d+ex^2)^p\right)\,dx &= -2f^2px+\frac{2d^3g^2px}{7e^3}+\frac{dfgpx^2}{2e}-\frac{2d^2g^2px^3}{21e^2}-\frac{1}{4}fgpx^4\\ &\quad+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px^7+\frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\\ &\quad-\frac{2d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}-\frac{d^2fgp\log(d+ex^2)}{2e^2}\\ &\quad+f^2x\log\left(c(d+ex^2)^p\right)+\frac{1}{2}fgx^4\log\left(c(d+ex^2)^p\right)\\ &\quad+\frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right)\end{aligned}$$

output
```
-2*f^2*p*x+2/7*d^3*g^2*p*x/e^3+1/2*d*f*g*p*x^2/e-2/21*d^2*g^2*p*x^3/e^2-1/
4*f*g*p*x^4+2/35*d*g^2*p*x^5/e-2/49*g^2*p*x^7+2*d^(1/2)*f^2*p*arctan(e^(1/
2)*x/d^(1/2))/e^(1/2)-2/7*d^(7/2)*g^2*p*arctan(e^(1/2)*x/d^(1/2))/e^(7/2)-
1/2*d^2*f*g*p*ln(e*x^2+d)/e^2+f^2*x*ln(c*(e*x^2+d)^p)+1/2*f*g*x^4*ln(c*(e*
x^2+d)^p)+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.77

$$\int \left(f+gx^3\right)^2 \log \left(c\left(d+ex^2\right)^p\right)\, dx = \frac{px(840d^3g^2-280d^2eg^2x^2+42de^2gx(35f+4gx^3)-15e^3(392f^2+49fgx^3+8g^2x^6))}{2940e^3} - \frac{2\sqrt{d}(-7e^3f^2+d^3g^2)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} - \frac{d^2fgp\log\left(d+ex^2\right)}{2e^2} + \frac{1}{14}x\left(14f^2+7fgx^3+2g^2x^6\right)\log\left(c\left(d+ex^2\right)^p\right)$$

input
```
Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(p*x*(840*d^3*g^2 - 280*d^2*e*g^2*x^2 + 42*d*e^2*g*x*(35*f + 4*g*x^3) - 15
*e^3*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6)))/(2940*e^3) - (2*Sqrt[d]*(-7*e^3*
f^2 + d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log
[d + e*x^2])/(2*e^2) + (x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6)*Log[c*(d + e*x^
2)^p])/14
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+gx^3\right)^2 \log \left(c\left(d+ex^2\right)^p\right)\, dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^3\log\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}+\frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{2d^3g^2px}{7e^3}-\frac{d^2fgp\log\left(d+ex^2\right)}{2e^2}-\frac{2d^2g^2px^3}{21e^2}+\frac{dfgpx^2}{2e}+\frac{2dg^2px^5}{35e}-2f^2px-\frac{1}{4}fgpx^4-\frac{2}{49}g^2px^7$$

input
```
Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
-2*f^2*p*x + (2*d^3*g^2*p*x)/(7*e^3) + (d*f*g*p*x^2)/(2*e) - (2*d^2*g^2*p*
x^3)/(21*e^2) - (f*g*p*x^4)/4 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49
+ (2*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(7/2)*g^2*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d + e*x^2])/(2*
e^2) + f^2*x*Log[c*(d + e*x^2)^p] + (f*g*x^4*Log[c*(d + e*x^2)^p])/2 + (g^
2*x^7*Log[c*(d + e*x^2)^p])/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.84

method	result
parts	$\frac{g^2x^7\ln(c(e\,x^2+d)^p)}{7}+\frac{f g\,x^4\ln(c(e\,x^2+d)^p)}{2}+f^2x\ln\left(c(e\,x^2+d)^p\right)-\frac{ep\left(-\frac{-\frac{2}{7}e^3g^2x^7+\frac{2}{5}d\,e^2g^2x^5-\frac{7}{4}e^3f g\,x^4-\frac{2}{3}d^2e\,g}{e^4}\right.}{}$
risch	$-\frac{p\ln\left(-d^4g^2+7d\,e^3f^2+\sqrt{-d^7e\,g^4+14d^4e^4f^2g^2-49d\,e^7f^4}\,x\right)\sqrt{-d^7e\,g^4+14d^4e^4f^2g^2-49d\,e^7f^4}}{7e^4}+\frac{\ln(c)f g\,x^4}{2}+\frac{p\ln\left(-d^4g^2+\right.}{}$

input
```
int((g*x^3+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/7*g^2*x^7*ln(c*(e*x^2+d)^p)+1/2*f*g*x^4*ln(c*(e*x^2+d)^p)+f^2*x*ln(c*(e*
x^2+d)^p)-1/7*e*p*(-1/e^4*(-2/7*e^3*g^2*x^7+2/5*d*e^2*g^2*x^5-7/4*e^3*f*g*
x^4-2/3*d^2*e*g^2*x^3+7/2*d*f*g*x^2*e^2+2*d^3*x*g^2-14*x*e^3*f^2)+1/e^4*d*
(7/2*d*e*f*g*ln(e*x^2+d)+(2*d^3*g^2-14*e^3*f^2)/(d*e)^(1/2)*arctan(x*e/(d*
e)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.97

$$\int \left(f+gx^3\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$=\left[-\frac{120\,e^3g^2px^7-168\,de^2g^2px^5+735\,e^3fgpx^4+280\,d^2eg^2px^3-1470\,de^2fgpx^2+420\,(7\,e^3f^2-d^3g^2)p\sqrt{}}{}\right.$$

$$-\frac{120\,e^3g^2px^7-168\,de^2g^2px^5+735\,e^3fgpx^4+280\,d^2eg^2px^3-1470\,de^2fgpx^2-840\,(7\,e^3f^2-d^3g^2)p\sqrt{}}{}$$

input
```
integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/2940*(120*e^3*g^2*p*x^7 - 168*d*e^2*g^2*p*x^5 + 735*e^3*f*g*p*x^4 + 28
0*d^2*e*g^2*p*x^3 - 1470*d*e^2*f*g*p*x^2 + 420*(7*e^3*f^2 - d^3*g^2)*p*sqr
t(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 840*(7*e^3*f^2 -
 d^3*g^2)*p*x - 210*(2*e^3*g^2*p*x^7 + 7*e^3*f*g*p*x^4 + 14*e^3*f^2*p*x -
7*d^2*e*f*g*p)*log(e*x^2 + d) - 210*(2*e^3*g^2*x^7 + 7*e^3*f*g*x^4 + 14*e^
3*f^2*x)*log(c))/e^3, -1/2940*(120*e^3*g^2*p*x^7 - 168*d*e^2*g^2*p*x^5 + 7
35*e^3*f*g*p*x^4 + 280*d^2*e*g^2*p*x^3 - 1470*d*e^2*f*g*p*x^2 - 840*(7*e^3
*f^2 - d^3*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 840*(7*e^3*f^2 - d^3
*g^2)*p*x - 210*(2*e^3*g^2*p*x^7 + 7*e^3*f*g*p*x^4 + 14*e^3*f^2*p*x - 7*d^
2*e*f*g*p)*log(e*x^2 + d) - 210*(2*e^3*g^2*x^7 + 7*e^3*f*g*x^4 + 14*e^3*f^
2*x)*log(c))/e^3]
```

Sympy [A] (verification not implemented)

Time = 126.47 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.90

$$\int (f+gx^3)^2 \log \left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= \begin{cases} \left(f^2x+\frac{fgx^4}{2}+\frac{g^2x^7}{7}\right)\log\left(0^pc\right) \\ \left(f^2x+\frac{fgx^4}{2}+\frac{g^2x^7}{7}\right)\log\left(cd^p\right) \\ -2f^2px+f^2x\log\left(c\left(ex^2\right)^p\right)-\frac{fgpx^4}{4}+\frac{fgx^4\log\left(c\left(ex^2\right)^p\right)}{2}-\frac{2g^2px^7}{49}+\frac{g^2x^7\log\left(c\left(ex^2\right)^p\right)}{7} \\ -\frac{2d^4g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{d^4g^2\log\left(c\left(d+ex^2\right)^p\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{2d^3g^2px}{7e^3}-\frac{d^2fg\log\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{2d^2g^2px^3}{21e^2}+\frac{2df^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-df \end{cases}$$

input
```
integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise(((f**2*x + f*g*x**4/2 + g**2*x**7/7)*log(0**p*c), Eq(d, 0) & Eq(
e, 0)), ((f**2*x + f*g*x**4/2 + g**2*x**7/7)*log(c*d**p), Eq(e, 0)), (-2*f
**2*p*x + f**2*x*log(c*(e*x**2)**p) - f*g*p*x**4/4 + f*g*x**4*log(c*(e*x**
2)**p)/2 - 2*g**2*p*x**7/49 + g**2*x**7*log(c*(e*x**2)**p)/7, Eq(d, 0)), (
-2*d**4*g**2*p*log(x - sqrt(-d/e))/(7*e**4*sqrt(-d/e)) + d**4*g**2*log(c*(
d + e*x**2)**p)/(7*e**4*sqrt(-d/e)) + 2*d**3*g**2*p*x/(7*e**3) - d**2*f*g*
log(c*(d + e*x**2)**p)/(2*e**2) - 2*d**2*g**2*p*x**3/(21*e**2) + 2*d*f**2*
p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f**2*log(c*(d + e*x**2)**p)/(e*sq
rt(-d/e)) + d*f*g*p*x**2/(2*e) + 2*d*g**2*p*x**5/(35*e) - 2*f**2*p*x + f**
2*x*log(c*(d + e*x**2)**p) - f*g*p*x**4/4 + f*g*x**4*log(c*(d + e*x**2)**p
)/2 - 2*g**2*p*x**7/49 + g**2*x**7*log(c*(d + e*x**2)**p)/7, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^3)^2 \log(c(d+ex^2)^p) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int \left(f+gx^3\right)^2 \log \left(c\left(d+ex^2\right)^p\right) \, dx = \frac{2\,dg^2px^5}{35\,e} - \frac{1}{49}\left(2\,g^2p - 7\,g^2\log\left(c\right)\right)x^7 - \frac{2\,d^2g^2px^3}{21\,e^2} + \frac{dfgpx^2}{2\,e} - \frac{1}{4}\left(fgp - 2\,fg\log\left(c\right)\right)x^4 - \frac{d^2fgp\log\left(ex^2+d\right)}{2\,e^2} + \frac{1}{14}\left(2\,g^2px^7 + 7\,fgpx^4 + 14\,f^2px\right)\log\left(ex^2+d\right) - \frac{\left(14\,e^3f^2p - 2\,d^3g^2p - 7\,e^3f^2\log\left(c\right)\right)x}{7\,e^3} + \frac{2\left(7\,de^3f^2p - d^4g^2p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{7\,\sqrt{de}e^3}$$

input
```
integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
2/35*d*g^2*p*x^5/e - 1/49*(2*g^2*p - 7*g^2*log(c))*x^7 - 2/21*d^2*g^2*p*x^
3/e^2 + 1/2*d*f*g*p*x^2/e - 1/4*(f*g*p - 2*f*g*log(c))*x^4 - 1/2*d^2*f*g*p
*log(e*x^2 + d)/e^2 + 1/14*(2*g^2*p*x^7 + 7*f*g*p*x^4 + 14*f^2*p*x)*log(e*
x^2 + d) - 1/7*(14*e^3*f^2*p - 2*d^3*g^2*p - 7*e^3*f^2*log(c))*x/e^3 + 2/7
*(7*d*e^3*f^2*p - d^4*g^2*p)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3)
```

Mupad [B] (verification not implemented)

Time = 28.45 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.37

$$\int (f+gx^3)^2\log\left(c(d+ex^2)^p\right)\,dx=\frac{g^2\,x^7\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{7}-2\,f^2\,p\,x-\frac{2\,g^2\,p\,x^7}{49}+f^2\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)+\frac{f\,g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{2}-\frac{f\,g\,p\,x^4}{4}+\frac{2\,d\,g^2\,p\,x^5}{35\,e}+\frac{2\,d^3\,g^2\,p\,x}{7\,e^3}-\frac{2\,\sqrt{d}\,f^2\,p\,\mathrm{atan}\left(\frac{7\,\sqrt{d}\,e^{7/2}\,f^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}-\frac{d^{7/2}\,\sqrt{e}\,g^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}\right)}{\sqrt{e}}+\frac{2\,d^{7/2}\,g^2\,p\,\mathrm{atan}\left(\frac{7\,\sqrt{d}\,e^{7/2}\,f^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}-\frac{d^{7/2}\,\sqrt{e}\,g^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}\right)}{7\,e^{7/2}}-\frac{2\,d^2\,g^2\,p\,x^3}{21\,e^2}+\frac{d\,f\,g\,p\,x^2}{2\,e}-\frac{d^2\,f\,g\,p\,\ln\left(e\,x^2+d\right)}{2\,e^2}$$

input
```
int(log(c*(d + e*x^2)^p)*(f + g*x^3)^2,x)
```

output
```
(g^2*x^7*log(c*(d + e*x^2)^p))/7 - 2*f^2*p*x - (2*g^2*p*x^7)/49 + f^2*x*lo
g(c*(d + e*x^2)^p) + (f*g*x^4*log(c*(d + e*x^2)^p))/2 - (f*g*p*x^4)/4 + (2
*d*g^2*p*x^5)/(35*e) + (2*d^3*g^2*p*x)/(7*e^3) - (2*d^(1/2)*f^2*p*atan((7*
d^(1/2)*e^(7/2)*f^2*p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p) - (d^(7/2)*e^(1/2)*g^
2*p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p)))/e^(1/2) + (2*d^(7/2)*g^2*p*atan((7*d^
(1/2)*e^(7/2)*f^2*p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p) - (d^(7/2)*e^(1/2)*g^2*
p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p)))/(7*e^(7/2)) - (2*d^2*g^2*p*x^3)/(21*e^2
) + (d*f*g*p*x^2)/(2*e) - (d^2*f*g*p*log(d + e*x^2))/(2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.99

$$\int (f+gx^3)^2\log\left(c(d+ex^2)^p\right)\,dx$$

$$=\frac{-840\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)d^3g^2p+5880\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)e^3f^2p-1470\log\left(\left(e\,x^2+d\right)^p c\right)d^2e^2fg+294}{}$$

input
```
int((g*x^3+f)^2*log(c*(e*x^2+d)^p),x)
```

output
```
( - 840*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**3*g**2*p + 5880*s
qrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**3*f**2*p - 1470*log((d + e
*x**2)**p*c)*d**2*e**2*f*g + 2940*log((d + e*x**2)**p*c)*e**4*f**2*x + 147
0*log((d + e*x**2)**p*c)*e**4*f*g*x**4 + 420*log((d + e*x**2)**p*c)*e**4*g
**2*x**7 + 840*d**3*e*g**2*p*x - 280*d**2*e**2*g**2*p*x**3 + 1470*d*e**3*f
*g*p*x**2 + 168*d*e**3*g**2*p*x**5 - 5880*e**4*f**2*p*x - 735*e**4*f*g*p*x
**4 - 120*e**4*g**2*p*x**7)/(2940*e**4)
```

3.290 $\int (f+gx^3) \log \left(c(d+ex^2)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int (f+gx^3) \log \left(c(d+ex^2)^p\right) \, dx = -2fpx + \frac{dgpx^2}{4e} - \frac{1}{8}gpx^4 + \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{d^2gp\log(d+ex^2)}{4e^2} + fx\log\left(c(d+ex^2)^p\right) + \frac{1}{4}gx^4\log\left(c(d+ex^2)^p\right)$$

output
```
-2*f*p*x+1/4*d*g*p*x^2/e-1/8*g*p*x^4+2*d^(1/2)*f*p*arctan(e^(1/2)*x/d^(1/2
))/e^(1/2)-1/4*d^2*g*p*ln(e*x^2+d)/e^2+f*x*ln(c*(e*x^2+d)^p)+1/4*g*x^4*ln(
c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int (f+gx^3) \log \left(c(d+ex^2)^p\right) \, dx = -2fpx + \frac{dgpx^2}{4e} - \frac{1}{8}gpx^4 + \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{d^2gp\log(d+ex^2)}{4e^2} + fx\log\left(c(d+ex^2)^p\right) + \frac{1}{4}gx^4\log\left(c(d+ex^2)^p\right)$$

input
```
Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```

output
```
-2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d +
 e*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^3\log\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^2gp\log\left(d+ex^2\right)}{4e^2}+\frac{dgpx^2}{4e}-2fpx-\frac{1}{8}gpx^4$$

input
```
Int[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```

output
```
-2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d +
 e*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

method	result
parts	$\frac{g\,x^4 \ln\left(c\left(e\,x^2+d\right)^p\right)}{4}+f x \ln\left(c\left(e\,x^2+d\right)^p\right)-\frac{ep\left(-\frac{-\frac{1}{4}eg\,x^4+\frac{1}{2}dg\,x^2-4efx}{e^2}+\frac{d\left(\frac{dg\ln\left(e\,x^2+d\right)}{2e}-\frac{4ef\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}\right)}{e^2}\right)}{2}$
risch	$\left(\frac{1}{4}g\,x^4+fx\right)\ln\left(\left(e\,x^2+d\right)^p\right)+\frac{i\,\mathrm{csgn}(ic)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2 x^4 g\pi}{8}-\frac{i\pi g\,x^4\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\mathrm{csg}}{8}$

input
```
int((g*x^3+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*g*x^4*ln(c*(e*x^2+d)^p)+f*x*ln(c*(e*x^2+d)^p)-1/2*e*p*(-1/e^2*(-1/4*e*
g*x^4+1/2*d*g*x^2-4*e*f*x)+d/e^2*(1/2*d*g/e*ln(e*x^2+d)-4*e*f/(d*e)^(1/2)*
arctan(x*e/(d*e)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.27

$$\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\left[-\frac{e^2gpx^4-2\,degpx^2-8\,e^2fp\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+16\,e^2fpx-2\,(e^2gpx^4+4\,e^2fpx-d^2gp)\log\right.}{8\,e^2}$$
$$-\frac{e^2gpx^4-2\,degpx^2-16\,e^2fp\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+16\,e^2fpx-2\,(e^2gpx^4+4\,e^2fpx-d^2gp)\log\left(ex^2\right.}{8\,e^2}$$

input
```
integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/8*(e^2*g*p*x^4 - 2*d*e*g*p*x^2 - 8*e^2*f*p*sqrt(-d/e)*log((e*x^2 + 2*e
*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 16*e^2*f*p*x - 2*(e^2*g*p*x^4 + 4*e^2*f*
p*x - d^2*g*p)*log(e*x^2 + d) - 2*(e^2*g*x^4 + 4*e^2*f*x)*log(c))/e^2, -1/
8*(e^2*g*p*x^4 - 2*d*e*g*p*x^2 - 16*e^2*f*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)
/d) + 16*e^2*f*p*x - 2*(e^2*g*p*x^4 + 4*e^2*f*p*x - d^2*g*p)*log(e*x^2 + d
) - 2*(e^2*g*x^4 + 4*e^2*f*x)*log(c))/e^2]
```

Sympy [A] (verification not implemented)

Time = 16.78 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.95

$$\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\begin{cases}\left(fx+\frac{gx^4}{4}\right)\log\left(0^pc\right)\\ \left(fx+\frac{gx^4}{4}\right)\log\left(cd^p\right)\\ -2fpx+fx\log\left(c(ex^2)^p\right)-\frac{gpx^4}{8}+\frac{gx^4\log\left(c(ex^2)^p\right)}{4}\\ -\frac{d^2g\log\left(c(d+ex^2)^p\right)}{4e^2}+\frac{2dfp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{df\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}}+\frac{dgpx^2}{4e}-2fpx+fx\log\left(c(d+ex^2)^p\right)-\frac{gpx^4}{8}+\end{cases}$$

input
```
integrate((g*x**3+f)*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise(((f*x + g*x**4/4)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f*x + g*x
**4/4)*log(c*d**p), Eq(e, 0)), (-2*f*p*x + f*x*log(c*(e*x**2)**p) - g*p*x*
*4/8 + g*x**4*log(c*(e*x**2)**p)/4, Eq(d, 0)), (-d**2*g*log(c*(d + e*x**2)
**p)/(4*e**2) + 2*d*f*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f*log(c*(d
+ e*x**2)**p)/(e*sqrt(-d/e)) + d*g*p*x**2/(4*e) - 2*f*p*x + f*x*log(c*(d +
 e*x**2)**p) - g*p*x**4/8 + g*x**4*log(c*(d + e*x**2)**p)/4, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (f + gx^3) \log \left(c(d + ex^2)^p\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\begin{aligned}\int (f+gx^3) \log \left(c(d+ex^2)^p\right) \, dx = &-\frac{1}{8} \left(gp - 2\, g \log \left(c\right)\right)x^4 + \frac{dgpx^2}{4\, e} + \frac{2\, dfp \arctan \left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} \\ &- \frac{d^2 gp \log \left(ex^2 + d\right)}{4\, e^2} - \left(2\, fp - f \log \left(c\right)\right)x \\ &+ \frac{1}{4} \left(gpx^4 + 4\, fpx\right) \log \left(ex^2 + d\right)\end{aligned}$$

input
```
integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
-1/8*(g*p - 2*g*log(c))*x^4 + 1/4*d*g*p*x^2/e + 2*d*f*p*arctan(e*x/sqrt(d*
e))/sqrt(d*e) - 1/4*d^2*g*p*log(e*x^2 + d)/e^2 - (2*f*p - f*log(c))*x + 1/
4*(g*p*x^4 + 4*f*p*x)*log(e*x^2 + d)
```

Mupad [B] (verification not implemented)

Time = 26.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx = f\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)-\frac{g\,p\,x^4}{8}-2\,f\,p\,x+\frac{g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{4}+\frac{d\,g\,p\,x^2}{4\,e}+\frac{2\,\sqrt{d}\,f\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2\,g\,p\,\ln\left(e\,x^2+d\right)}{4\,e^2}$$

input
```
int(log(c*(d + e*x^2)^p)*(f + g*x^3),x)
```

output
```
f*x*log(c*(d + e*x^2)^p) - (g*p*x^4)/8 - 2*f*p*x + (g*x^4*log(c*(d + e*x^2
)^p))/4 + (d*g*p*x^2)/(4*e) + (2*d^(1/2)*f*p*atan((e^(1/2)*x)/d^(1/2)))/e^
(1/2) - (d^2*g*p*log(d + e*x^2))/(4*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx = \frac{16\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)efp-2\log\left(\left(e\,x^2+d\right)^p c\right)d^2g+8\log\left(\left(e\,x^2+d\right)^p c\right)e^2fx+2\log\left(\left(e\,x^2+d\right)^p c\right)e^2}{8e^2}$$

input
```
int((g*x^3+f)*log(c*(e*x^2+d)^p),x)
```

output
```
(16*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p - 2*log((d + e*x**
2)**p*c)*d**2*g + 8*log((d + e*x**2)**p*c)*e**2*f*x + 2*log((d + e*x**2)**
p*c)*e**2*g*x**4 + 2*d*e*g*p*x**2 - 16*e**2*f*p*x - e**2*g*p*x**4)/(8*e**2
)
```

3.291 $\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1165

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \text{Too large to display}$$

output
```
-1/3*p*ln(g^(1/3)*((-d)^(1/2)-e^(1/2)*x)/(e^(1/2)*f^(1/3)+(-d)^(1/2)*g^(1/
3)))*ln(-f^(1/3)-g^(1/3)*x)/f^(2/3)/g^(1/3)-1/3*p*ln(-g^(1/3)*((-d)^(1/2)+
e^(1/2)*x)/(e^(1/2)*f^(1/3)-(-d)^(1/2)*g^(1/3)))*ln(-f^(1/3)-g^(1/3)*x)/f^
(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*ln(-(-1)^(1/3)*g^(1/3)*((-d)^(1/2)-e^(1/2)*
x)/(e^(1/2)*f^(1/3)-(-1)^(1/3)*(-d)^(1/2)*g^(1/3)))*ln(-f^(1/3)+(-1)^(1/3)
*g^(1/3)*x)/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*ln((-1)^(1/3)*g^(1/3)*((-d)^(
1/2)+e^(1/2)*x)/(e^(1/2)*f^(1/3)+(-1)^(1/3)*(-d)^(1/2)*g^(1/3)))*ln(-f^(1/
3)+(-1)^(1/3)*g^(1/3)*x)/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*ln((-1)^(2/3)*g^
(1/3)*((-d)^(1/2)-e^(1/2)*x)/(e^(1/2)*f^(1/3)+(-1)^(2/3)*(-d)^(1/2)*g^(1/3
)))*ln(-f^(1/3)-(-1)^(2/3)*g^(1/3)*x)/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*ln(
-(-1)^(2/3)*g^(1/3)*((-d)^(1/2)+e^(1/2)*x)/(e^(1/2)*f^(1/3)-(-1)^(2/3)*(-d
)^(1/2)*g^(1/3)))*ln(-f^(1/3)-(-1)^(2/3)*g^(1/3)*x)/f^(2/3)/g^(1/3)+1/3*ln
(-f^(1/3)-g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3)+1/3*(-1)^(2/3)*ln(-
f^(1/3)+(-1)^(1/3)*g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3)-1/3*(-1)^(
1/3)*ln(-f^(1/3)-(-1)^(2/3)*g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3)-1
/3*p*polylog(2,e^(1/2)*(f^(1/3)+g^(1/3)*x)/(e^(1/2)*f^(1/3)-(-d)^(1/2)*g^(
1/3)))/f^(2/3)/g^(1/3)-1/3*p*polylog(2,e^(1/2)*(f^(1/3)+g^(1/3)*x)/(e^(1/2
)*f^(1/3)+(-d)^(1/2)*g^(1/3)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*polylog(2,
e^(1/2)*(f^(1/3)-(-1)^(1/3)*g^(1/3)*x)/(e^(1/2)*f^(1/3)-(-1)^(1/3)*(-d)^(1
/2)*g^(1/3)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*polylog(2,e^(1/2)*(f^(1/...
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 990, normalized size of antiderivative = 0.85

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

output
```
(-(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1
/3))]*Log[-f^(1/3) - g^(1/3)*x]) - p*Log[(g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/
(-(Sqrt[e]*f^(1/3)) + Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - g^(1/3)*x] - (-1)^
(2/3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(-(Sqrt[e]*f^(1/3)
) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] - (
-1)^(2/3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/
3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] +
(-1)^(1/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1
/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x] +
 (-1)^(1/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(-(Sqrt[e]*f
^(1/3)) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*
x] + Log[-f^(1/3) - g^(1/3)*x]*Log[c*(d + e*x^2)^p] + (-1)^(2/3)*Log[-f^(1
/3) + (-1)^(1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - (-1)^(1/3)*Log[-f^(1/3)
 - (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - p*PolyLog[2, (Sqrt[e]*(f^(
1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3))] - p*PolyLog[2, (S
qrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))] - (-1)
^(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^
(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))] - (-1)^(2/3)*p*PolyLog[2, (Sqrt[e]*(
f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^
(1/3))] + (-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3...
```

Rubi [A] (verified)

Time = 3.20 (sec) , antiderivative size = 1165, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{f+gx^3} \, dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(-\frac{\log \left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}-\frac{\log \left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}-\frac{\log \left(c\left(d+ex^2\right)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
\begin{aligned}
&-\frac{p\log\left(\frac{\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}-\\
&\frac{p\log\left(-\frac{\sqrt[3]{g}(\sqrt{e}x+\sqrt{-d})}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{\log\left(c(ex^2+d)^p\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}-\\
&\frac{(-1)^{2/3}p\log\left(-\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}-\\
&\frac{(-1)^{2/3}p\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{e}x+\sqrt{-d})}{\sqrt[3]{-1}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}+\\
&\frac{\sqrt[3]{-1}p\log\left(\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{(-1)^{2/3}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(-(-1)^{2/3}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}+\\
&\frac{\sqrt[3]{-1}p\log\left(-\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{e}x+\sqrt{-d})}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-(-1)^{2/3}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}+\\
&\frac{(-1)^{2/3}\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)\log\left(c(ex^2+d)^p\right)}{3f^{2/3}\sqrt[3]{g}}-\\
&\frac{\sqrt[3]{-1}\log\left(-(-1)^{2/3}\sqrt[3]{g}x-\sqrt[3]{f}\right)\log\left(c(ex^2+d)^p\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)}{3f^{2/3}\sqrt[3]{g}}-\\
&\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{(-1)^{2/3}p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)}{3f^{2/3}\sqrt[3]{g}}-\\
&\frac{(-1)^{2/3}p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt[3]{-1}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{\sqrt[3]{-1}p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)}{3f^{2/3}\sqrt[3]{g}}+\\
&\frac{\sqrt[3]{-1}p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{(-1)^{2/3}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)}{3f^{2/3}\sqrt[3]{g}}
\end{aligned}
$$

input
```
Int[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

output
```
-1/3*(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g
^(1/3))]*Log[-f^(1/3) - g^(1/3)*x])/(f^(2/3)*g^(1/3)) - (p*Log[-((g^(1/3)*
(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3)))]*Log[-f^(1/3
) - g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Log[-(((-1)^(1/3)*g^(1
/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)
))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3
)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1
)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3
)*g^(1/3)) + ((-1)^(1/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))
/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3
)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*Log[-(((-1)^(2/3)*g^(1/3
)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))
]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) + (Log[-f^(1/3
) - g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) + ((-1)^(2/3)*Log
[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)
) - ((-1)^(1/3)*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])
/(3*f^(2/3)*g^(1/3)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt
[e]*f^(1/3) - Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - (p*PolyLog[2, (Sqr
t[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))])/(3*f^(2
/3)*g^(1/3)) - ((-1)^(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.46 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.50

method	result	size
risch	Expression too large to display	577

input
```
int(ln(c*(e*x^2+d)^p)/(g*x^3+f),x,method=_RETURNVERBOSE)
```

output
```
(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(1/3/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6/g
/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))+1/3/g/(f/g)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1)))+1/3*p/g*sum(1/_alpha^2*(ln(x-_alph
a)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,
index=1)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1))+ln((
RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)/RootOf(_Z^2*e+
2*_Z*_alpha*e+_alpha^2*e+d,index=2)))-dilog((RootOf(_Z^2*e+2*_Z*_alpha*e+_
alpha^2*e+d,index=1)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,in
dex=1))-dilog((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)
/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=2))),_alpha=RootOf(_Z^3*g+
f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*csgn(I*
(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p
)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))*(1/3/g/(f/g)^(2/3)*l
n(x+(f/g)^(1/3))-1/6/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))+1/3/g
/(f/g)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1)))
```

Fricas [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log \left((ex^2+d)^p c\right)}{gx^3+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^3+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^3+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx=\int \frac{\ln\left(c\,(e\,x^2+d)^p\right)}{g\,x^3+f}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(f + g*x^3),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(f + g*x^3), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx=\int \frac{\log\left((e\,x^2+d)^p\,c\right)}{g\,x^3+f}dx$$

input
```
int(log(c*(e*x^2+d)^p)/(g*x^3+f),x)
```

output
```
int(log((d + e*x**2)**p*c)/(f + g*x**3),x)
```

3.292 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1865

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \text{Too large to display}$$

output
```
-(-1)^(1/3)*e*p*ln(e*x^2+d)/(1+(-1)^(1/3))^4/f/(e*f^(2/3)+(-1)^(2/3)*d*g^(
2/3))/g^(1/3)-2/9*p*polylog(2,e^(1/2)*(f^(1/3)+g^(1/3)*x)/(e^(1/2)*f^(1/3)
+(-d)^(1/2)*g^(1/3)))/f^(5/3)/g^(1/3)-2/9*p*polylog(2,e^(1/2)*(f^(1/3)+g^(
1/3)*x)/(e^(1/2)*f^(1/3)-(-d)^(1/2)*g^(1/3)))/f^(5/3)/g^(1/3)+2*I*3^(1/2)*
p*ln(-(-1)^(1/3)*g^(1/3)*((-d)^(1/2)-e^(1/2)*x)/(e^(1/2)*f^(1/3)-(-1)^(1/3
)*(-d)^(1/2)*g^(1/3)))*ln(-f^(1/3)+(-1)^(1/3)*g^(1/3)*x)/(1+(-1)^(1/3))^5/
f^(5/3)/g^(1/3)+2*I*3^(1/2)*p*ln((-1)^(1/3)*g^(1/3)*((-d)^(1/2)+e^(1/2)*x)
/(e^(1/2)*f^(1/3)+(-1)^(1/3)*(-d)^(1/2)*g^(1/3)))*ln(-f^(1/3)+(-1)^(1/3)*g
^(1/3)*x)/(1+(-1)^(1/3))^5/f^(5/3)/g^(1/3)+2*(-1)^(2/3)*d^(1/2)*e^(1/2)*p*
arctan(e^(1/2)*x/d^(1/2))/(1+(-1)^(1/3))^4/f^(4/3)/(e*f^(2/3)+(-1)^(2/3)*d
*g^(2/3))+2*(-1)^(1/3)*e*p*ln(f^(1/3)-(-1)^(1/3)*g^(1/3)*x)/(1+(-1)^(1/3))
^4/f/(e*f^(2/3)+(-1)^(2/3)*d*g^(2/3))/g^(1/3)-ln(c*(e*x^2+d)^p)/(1+(-1)^(1
/3))^4/f^(4/3)/g^(1/3)/((-1)^(2/3)*f^(1/3)+g^(1/3)*x)+2*I*3^(1/2)*p*polylo
g(2,e^(1/2)*(f^(1/3)-(-1)^(1/3)*g^(1/3)*x)/(e^(1/2)*f^(1/3)+(-1)^(1/3)*(-d
)^(1/2)*g^(1/3)))/(1+(-1)^(1/3))^5/f^(5/3)/g^(1/3)+2*I*3^(1/2)*p*polylog(2
,e^(1/2)*(f^(1/3)-(-1)^(1/3)*g^(1/3)*x)/(e^(1/2)*f^(1/3)-(-1)^(1/3)*(-d)^(
1/2)*g^(1/3)))/(1+(-1)^(1/3))^5/f^(5/3)/g^(1/3)-2*I*3^(1/2)*ln(-f^(1/3)+(-
1)^(1/3)*g^(1/3)*x)*ln(c*(e*x^2+d)^p)/(1+(-1)^(1/3))^5/f^(5/3)/g^(1/3)-2/9
*(-1)^(1/3)*e*p*ln(e*x^2+d)/f/(2*e*f^(2/3)+I*(I-3^(1/2))*d*g^(2/3))/g^(1/3
)+2/9*d^(1/2)*e^(1/2)*p*arctan(e^(1/2)*x/d^(1/2))/f^(4/3)/(e*f^(2/3)+d*...
```

Mathematica [A] (warning: unable to verify)

Time = 7.28 (sec) , antiderivative size = 2168, normalized size of antiderivative = 1.16

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]
```

output
```
(x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(3*f*(f + g*x^3)) + (2*Ar
cTan[(-f^(1/3) + 2*g^(1/3)*x)/(Sqrt[3]*f^(1/3))]*(-(p*Log[d + e*x^2]) + Lo
g[c*(d + e*x^2)^p]))/(3*Sqrt[3]*f^(5/3)*g^(1/3)) + (2*Log[f^(1/3) + g^(1/3
)*x]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(9*f^(5/3)*g^(1/3)) - (
(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*Log[f^(2/3) - f^(1/3)*g^(1/3)
*x + g^(2/3)*x^2])/(9*f^(5/3)*g^(1/3)) + p*(-1/3*((-1 + (-1)^(1/3))*(-(Log
[((-I)*Sqrt[d])/Sqrt[e] + x]/((-1)^(2/3)*f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*
(Log[I*Sqrt[d] - Sqrt[e]*x] - Log[-((-1)^(2/3)*f^(1/3)) - g^(1/3)*x]))/((-
1)^(2/3)*Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))))/((1 + (-1)^(1/3))^2*f^(4/3
)*g^(1/3)) - ((-1 + (-1)^(1/3))*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/((-1)^(2/3
)*f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[-((-1
)^(2/3)*f^(1/3)) - g^(1/3)*x]))/((-1)^(2/3)*Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^
(1/3))))/(3*(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[((-I)
*Sqrt[d])/Sqrt[e] + x]/(f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] -
Sqrt[e]*x] - Log[f^(1/3) + g^(1/3)*x]))/(Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/
3))))/(3*(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[(I*Sqrt[
d])/Sqrt[e] + x]/(f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e
]*x] - Log[f^(1/3) + g^(1/3)*x]))/(Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))))/
(3*(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) - (Log[((-I)*Sqrt[d])/Sqrt[e] + x]/
((-1)^(1/3)*f^(1/3) - g^(1/3)*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x...
```

Rubi [A] (warning: unable to verify)

Time = 5.07 (sec) , antiderivative size = 1867, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}dx$$

$$\Big\downarrow \text{ 2921}$$

$$\int \left(\frac{2\log \left(c\left(d+ex^2\right)^p\right)}{9f^{5/3}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}-\frac{2(-1)^{5/6}\sqrt{3}\log \left(c\left(d+ex^2\right)^p\right)}{\left(1+\sqrt[3]{-1}\right)^5f^{5/3}\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}+\frac{2(-1)^{2/3}\log \left(c\left(d+ex^2\right)^p\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{5/3}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}+\dots\right.$$

$\downarrow$ 2009

$$\frac{2\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(g^{2/3}d+ef^{2/3}\right)}+\frac{2(-1)^{2/3}\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{4/3}\left((-1)^{2/3}g^{2/3}d+ef^{2/3}\right)}+$$
$$\frac{4\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(2ef^{2/3}-\left(1+i\sqrt{3}\right)dg^{2/3}\right)}+\frac{2\sqrt[3]{-1}ep\log\left(-\sqrt[3]{g}x-(-1)^{2/3}\sqrt[3]{f}\right)}{\left(1+\sqrt[3]{-1}\right)^4f\left((-1)^{2/3}g^{2/3}d+ef^{2/3}\right)\sqrt[3]{g}}-$$
$$\frac{2ep\log\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{9f\left(g^{2/3}d+ef^{2/3}\right)\sqrt[3]{g}}-\frac{2p\log\left(\frac{\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{9f^{5/3}\sqrt[3]{g}}-$$
$$\frac{2p\log\left(-\frac{\sqrt[3]{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{9f^{5/3}\sqrt[3]{g}}+$$
$$\frac{2i\sqrt{3}p\log\left(-\frac{\sqrt[3]{-1}\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{\left(1+\sqrt[3]{-1}\right)^5f^{5/3}\sqrt[3]{g}}+$$
$$\frac{2i\sqrt{3}p\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt[3]{-1}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{\left(1+\sqrt[3]{-1}\right)^5f^{5/3}\sqrt[3]{g}}+$$
$$\frac{4\sqrt[3]{-1}ep\log\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{9f\left(i\left(i-\sqrt{3}\right)g^{2/3}d+2ef^{2/3}\right)\sqrt[3]{g}}-$$
$$\frac{2p\log\left(\frac{(-1)^{2/3}\sqrt[3]{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{(-1)^{2/3}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{5/3}\sqrt[3]{g}}-$$
$$\frac{2p\log\left(-\frac{(-1)^{2/3}\sqrt[3]{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{5/3}\sqrt[3]{g}}+\frac{ep\log\left(ex^2+d\right)}{9f\left(g^{2/3}d+ef^{2/3}\right)\sqrt[3]{g}}-$$
$$\frac{\sqrt[3]{-1}ep\log\left(ex^2+d\right)}{\left(1+\sqrt[3]{-1}\right)^4f\left((-1)^{2/3}g^{2/3}d+ef^{2/3}\right)\sqrt[3]{g}}-\frac{2\sqrt[3]{-1}ep\log\left(ex^2+d\right)}{9f\left(i\left(i-\sqrt{3}\right)g^{2/3}d+2ef^{2/3}\right)\sqrt[3]{g}}+$$
$$\frac{2\log\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)\log\left(c\left(ex^2+d\right)^p\right)}{9f^{5/3}\sqrt[3]{g}}-\frac{2i\sqrt{3}\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)\log\left(c\left(ex^2+d\right)^p\right)}{\left(1+\sqrt[3]{-1}\right)^5f^{5/3}\sqrt[3]{g}}+$$
$$\frac{2\log\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)\log\left(c\left(ex^2+d\right)^p\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{5/3}\sqrt[3]{g}}-\frac{\log\left(c\left(ex^2+d\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}-$$
$$\frac{\log\left(c\left(ex^2+d\right)^p\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{g}x+(-1)^{2/3}\sqrt[3]{f}\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(ex^2+d\right)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}-$$
$$\frac{2p\operatorname{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)}{9f^{5/3}\sqrt[3]{g}}-\frac{2p\operatorname{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)}{9f^{5/3}\sqrt[3]{g}}+$$
$$\frac{2i\sqrt{3}p\operatorname{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)}{\left(1+\sqrt[3]{-1}\right)^5f^{5/3}\sqrt[3]{g}}+\frac{2i\sqrt{3}p\operatorname{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt[3]{-1}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)}{\left(1+\sqrt[3]{-1}\right)^5f^{5/3}\sqrt[3]{g}}-$$
$$2p\operatorname{PolyLog}\left(2,\frac{\sqrt{e}\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\quad 2p\operatorname{PolyLog}\left(2,\frac{\sqrt{e}\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{(-1)^{2/3}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)$$

input
```
Int[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]
```

output
```
(2*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(e*f^(2/3) +
d*g^(2/3))) + (2*(-1)^(2/3)*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/((1 + (-1)^(1/3))^4*f^(4/3)*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))) + (4*Sqrt
[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(2*e*f^(2/3) - (1 +
I*Sqrt[3])*d*g^(2/3))) + (2*(-1)^(1/3)*e*p*Log[-((-1)^(2/3)*f^(1/3)) - g^(
1/3)*x])/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))*g^(1/3))
 - (2*e*p*Log[f^(1/3) + g^(1/3)*x])/(9*f*(e*f^(2/3) + d*g^(2/3))*g^(1/3))
- (2*p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^
(1/3))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) - (2*p*Log[-((g^(1/3
)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3)))]*Log[f^(1/
3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) + ((2*I)*Sqrt[3]*p*Log[-(((-1)^(1/3)*
g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(
1/3)))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^5*f^(5/3)*
g^(1/3)) + ((2*I)*Sqrt[3]*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x)
)/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/
3)*g^(1/3)*x])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (4*(-1)^(1/3)*e*p*Lo
g[f^(1/3) + (-1)^(2/3)*g^(1/3)*x])/(9*f*(2*e*f^(2/3) + I*(I - Sqrt[3])*d*g
^(2/3))*g^(1/3)) - (2*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(S
qrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + (-1)^(2/3)*g^
(1/3)*x])/((1 + (-1)^(1/3))^4*f^(5/3)*g^(1/3)) - (2*p*Log[-(((-1)^(2/3)...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [F]

$$\int \frac{\ln\left(c(e\,x^{2}+d)^{p}\right)}{\left(g\,x^{3}+f\right)^{2}}dx$$

input
```
int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
```

output
```
int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^{2})^{p}\right)}{\left(f+gx^{3}\right)^{2}}\,dx = \int \frac{\log\left((ex^{2}+d)^{p}c\right)}{\left(gx^{3}+f\right)^{2}}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^{2})^{p}\right)}{\left(f+gx^{3}\right)^{2}}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{\left(gx^3+f\right)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^3+f\right)^2}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(f + g*x^3)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)/(f + g*x^3)^2, x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
```

output
```
(2*g**(1/3)*f**(2/3)*sqrt(3)*atan((f**(1/3) - 2*g**(1/3)*x)/(f**(1/3)*sqrt
(3)))*e**3*f**2*p + 2*g**(1/3)*f**(2/3)*sqrt(3)*atan((f**(1/3) - 2*g**(1/3
)*x)/(f**(1/3)*sqrt(3)))*e**3*f*g*p*x**3 + 2*sqrt(3)*atan((f**(1/3) - 2*g*
*(1/3)*x)/(f**(1/3)*sqrt(3)))*d*e**2*f**2*g*p + 2*sqrt(3)*atan((f**(1/3) -
 2*g**(1/3)*x)/(f**(1/3)*sqrt(3)))*d*e**2*f*g**2*p*x**3 + 6*g**(2/3)*f**(1
/3)*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**2*f**2*p + 6*g**(2/3)
*f**(1/3)*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**2*f*g*p*x**3 +
g**(1/3)*f**(2/3)*log(f**(2/3) - g**(1/3)*f**(1/3)*x + g**(2/3)*x**2)*e**3
*f**2*p + g**(1/3)*f**(2/3)*log(f**(2/3) - g**(1/3)*f**(1/3)*x + g**(2/3)*
x**2)*e**3*f*g*p*x**3 - 2*g**(1/3)*f**(2/3)*log(f**(1/3) + g**(1/3)*x)*e**
3*f**2*p - 2*g**(1/3)*f**(2/3)*log(f**(1/3) + g**(1/3)*x)*e**3*f*g*p*x**3
+ 6*g**(2/3)*f**(1/3)*int((log((d + e*x**2)**p*c)*x**3)/(f**2 + 2*f*g*x**3
 + g**2*x**6),x)*d**3*f*g**3 + 6*g**(2/3)*f**(1/3)*int((log((d + e*x**2)**
p*c)*x**3)/(f**2 + 2*f*g*x**3 + g**2*x**6),x)*d**3*g**4*x**3 + 6*g**(2/3)*
f**(1/3)*int((log((d + e*x**2)**p*c)*x**3)/(f**2 + 2*f*g*x**3 + g**2*x**6)
,x)*e**3*f**3*g + 6*g**(2/3)*f**(1/3)*int((log((d + e*x**2)**p*c)*x**3)/(f
**2 + 2*f*g*x**3 + g**2*x**6),x)*e**3*f**2*g**2*x**3 - 2*g**(2/3)*f**(1/3)
*log(f**(2/3) - g**(1/3)*f**(1/3)*x + g**(2/3)*x**2)*d**2*e*f*g*p - 2*g**(
2/3)*f**(1/3)*log(f**(2/3) - g**(1/3)*f**(1/3)*x + g**(2/3)*x**2)*d**2*e*g
**2*p*x**3 - 2*g**(2/3)*f**(1/3)*log(f**(1/3) + g**(1/3)*x)*d**2*e*f*g*...
```

3.293 $\int (f+gx^3)^3 \log^2\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1221

$$\int (f+gx^3)^3 \log^2\left(c(d+ex^2)^p\right)\,dx = \text{Too large to display}$$

output
```
-3/4*f^2*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2-2/3*d^2*g^3*p*(e*x^2+d)^3*l
n(c*(e*x^2+d)^p)/e^5+1/4*d*g^3*p*(e*x^2+d)^4*ln(c*(e*x^2+d)^p)/e^5-3/2*d*f
^2*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2+1/5*d^5*g^3*p*ln(e*x^2+d)*ln(c*(e*x
^2+d)^p)/e^5-1408/245*d^3*f*g^2*p^2*x/e^3-3*d*f^2*g*p^2*x^2/e+568/735*d^2*
f*g^2*p^2*x^3/e^2-288/1225*d*f*g^2*p^2*x^5/e-d^4*g^3*p*(e*x^2+d)*ln(c*(e*x
^2+d)^p)/e^5+d^3*g^3*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^5+d^4*g^3*p^2*x^2/e
^4+1408/245*d^(7/2)*f*g^2*p^2*arctan(e^(1/2)*x/d^(1/2))/e^(7/2)+8*d^(1/2)*
f^3*p^2*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1
/2)+4*d^(1/2)*f^3*p*arctan(e^(1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(1/2)+4*
I*d^(1/2)*f^3*p^2*arctan(e^(1/2)*x/d^(1/2))^2/e^(1/2)+4*I*d^(1/2)*f^3*p^2*
polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)-12/7*d^(7/2)*f*g^2*p*
arctan(e^(1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(7/2)-24/7*d^(7/2)*f*g^2*p^2
*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(7/2)-12/
7*I*d^(7/2)*f*g^2*p^2*arctan(e^(1/2)*x/d^(1/2))^2/e^(7/2)-12/7*I*d^(7/2)*f
*g^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(7/2)+12/7*d^3*f*g
^2*p*x*ln(c*(e*x^2+d)^p)/e^3-4/7*d^2*f*g^2*p*x^3*ln(c*(e*x^2+d)^p)/e^2+12/
35*d*f*g^2*p*x^5*ln(c*(e*x^2+d)^p)/e+3*d*f^2*g*p*(e*x^2+d)*ln(c*(e*x^2+d)^
p)/e^2+f^3*x*ln(c*(e*x^2+d)^p)^2+3/8*f^2*g*p^2*(e*x^2+d)^2/e^2-1/2*d^3*g^3
*p^2*(e*x^2+d)^2/e^5+2/9*d^2*g^3*p^2*(e*x^2+d)^3/e^5-1/16*d*g^3*p^2*(e*x^2
+d)^4/e^5-12/49*f*g^2*p*x^7*ln(c*(e*x^2+d)^p)-1/25*g^3*p*(e*x^2+d)^5*ln...
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 780, normalized size of antiderivative = 0.64

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = f^3x\log^2\left(c\left(d+ex^2\right)^p\right)$$
$$+\frac{3}{4}f^2gx^4\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log^2\left(c\left(d+ex^2\right)^p\right)$$
$$+\frac{3f^2g\left(ep^2x^2(-6d+ex^2)+2d^2p^2\log\left(d+ex^2\right)+2p(2d^2+2dex^2-e^2x^4)\log\left(c\left(d+ex^2\right)^p\right)-2d^2\log^2\left(\right.\right.}{8e^2}$$
$$+\frac{g^3\left(ep^2x^2(8220d^4-2310d^3ex^2+940d^2e^2x^4-405de^3x^6+144e^4x^8)-4620d^5p^2\log\left(d+ex^2\right)-60p(60\right.}{18}$$
$$+\frac{4f^3p\left(i\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2+\sqrt{e}x\left(2p-\log\left(c\left(d+ex^2\right)^p\right)\right)+\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-2p+2p\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{}}\right.\right.\right.}{\sqrt{e}}$$
$$+\frac{4fg^2p\left(-11025id^{7/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2-105d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-352p+210p\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)+105\log\right.\right.}{}$$

input
```
Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
f^3*x*Log[c*(d + e*x^2)^p]^2 + (3*f^2*g*x^4*Log[c*(d + e*x^2)^p]^2)/4 + (3
*f*g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 + (g^3*x^10*Log[c*(d + e*x^2)^p]^2)/1
0 + (3*f^2*g*(e*p^2*x^2*(-6*d + e*x^2) + 2*d^2*p^2*Log[d + e*x^2] + 2*p*(2
*d^2 + 2*d*e*x^2 - e^2*x^4)*Log[c*(d + e*x^2)^p] - 2*d^2*Log[c*(d + e*x^2)
^p]^2))/(8*e^2) + (g^3*(e*p^2*x^2*(8220*d^4 - 2310*d^3*e*x^2 + 940*d^2*e^2
*x^4 - 405*d*e^3*x^6 + 144*e^4*x^8) - 4620*d^5*p^2*Log[d + e*x^2] - 60*p*(
60*d^5 + 60*d^4*e*x^2 - 30*d^3*e^2*x^4 + 20*d^2*e^3*x^6 - 15*d*e^4*x^8 + 1
2*e^5*x^10)*Log[c*(d + e*x^2)^p] + 1800*d^5*Log[c*(d + e*x^2)^p]^2))/(1800
0*e^5) + (4*f^3*p*(I*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(
2*p - Log[c*(d + e*x^2)^p]) + Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2*p +
2*p*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + Log[c*(d + e*x^2)^p]) + I*S
qrt[d]*p*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/
Sqrt[e] + (4*f*g^2*p*((-11025*I)*d^(7/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 -
 105*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-352*p + 210*p*Log[(2*Sqrt[d])/(
Sqrt[d] + I*Sqrt[e]*x)] + 105*Log[c*(d + e*x^2)^p]) + Sqrt[e]*x*(2*p*(-184
80*d^3 + 2485*d^2*e*x^2 - 756*d*e^2*x^4 + 225*e^3*x^6) + 105*(105*d^3 - 35
*d^2*e*x^2 + 21*d*e^2*x^4 - 15*e^3*x^6)*Log[c*(d + e*x^2)^p]) - (11025*I)*
d^(7/2)*p*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))
/(25725*e^(7/2))
```

Rubi [A] (verified)

Time = 3.38 (sec) , antiderivative size = 1221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right) dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f^3 \log^2\left(c\left(d+ex^2\right)^p\right)+3f^2gx^3 \log^2\left(c\left(d+ex^2\right)^p\right)+3fg^2x^6 \log^2\left(c\left(d+ex^2\right)^p\right)+g^3x^9 \log^2\left(c\left(d+ex^2\right)^p\right)\right) dx$$

$\downarrow$ 2009

$$\frac{1}{10}g^3\log^2\left(c\left(ex^2+d\right)^p\right)x^{10}+\frac{24}{343}fg^2p^2x^7+\frac{3}{7}fg^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7-\frac{12}{49}fg^2p\log\left(c\left(ex^2+d\right)^p\right)x^7-\frac{288dfg^2p^2x^5}{1225e}+\frac{12dfg^2p\log\left(c\left(ex^2+d\right)^p\right)x^5}{35e}+\frac{568d^2fg^2p^2x^3}{735e^2}-\frac{4d^2fg^2p\log\left(c\left(ex^2+d\right)^p\right)x^3}{7e^2}+\frac{d^4g^3p^2x^2}{e^4}-\frac{3df^2gp^2x^2}{e}+8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}+f^3\log^2\left(c\left(ex^2+d\right)^p\right)x-4f^3p\log\left(c\left(ex^2+d\right)^p\right)x+\frac{12d^3fg^2p\log\left(c\left(ex^2+d\right)^p\right)x}{7e^3}+\frac{g^3p^2\left(ex^2+d\right)^5}{125e^5}-\frac{dg^3p^2\left(ex^2+d\right)^4}{16e^5}+\frac{2d^2g^3p^2\left(ex^2+d\right)^3}{9e^5}-\frac{d^3g^3p^2\left(ex^2+d\right)^2}{2e^5}+\frac{3f^2gp^2\left(ex^2+d\right)^2}{8e^2}+\frac{4i\sqrt{d}f^3p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{12id^{7/2}fg^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}}-\frac{d^5g^3p^2\log^2\left(ex^2+d\right)}{10e^5}+\frac{3f^2g\left(ex^2+d\right)^2\log^2\left(c\left(ex^2+d\right)^p\right)}{4e^2}-\frac{3df^2g\left(ex^2+d\right)\log^2\left(c\left(ex^2+d\right)^p\right)}{2e^2}-\frac{8\sqrt{d}f^3p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{1408d^{7/2}fg^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{245e^{7/2}}+\frac{8\sqrt{d}f^3p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}-\frac{24d^{7/2}fg^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{7e^{7/2}}-\frac{g^3p\left(ex^2+d\right)^5\log\left(c\left(ex^2+d\right)^p\right)}{25e^5}+\frac{dg^3p\left(ex^2+d\right)^4\log\left(c\left(ex^2+d\right)^p\right)}{4e^5}-\frac{2d^2g^3p\left(ex^2+d\right)^3\log\left(c\left(ex^2+d\right)^p\right)}{3e^5}+\frac{d^3g^3p\left(ex^2+d\right)^2\log\left(c\left(ex^2+d\right)^p\right)}{e^5}-\frac{3f^2gp\left(ex^2+d\right)^2\log\left(c\left(ex^2+d\right)^p\right)}{4e^2}-\frac{d^4g^3p\left(ex^2+d\right)\log\left(c\left(ex^2+d\right)^p\right)}{e^5}+\frac{3df^2gp\left(ex^2+d\right)\log\left(c\left(ex^2+d\right)^p\right)}{e^2}+\frac{4\sqrt{d}f^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{\sqrt{e}}-\frac{12d^{7/2}fg^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{7e^{7/2}}+\frac{d^5g^3p\log\left(ex^2+d\right)\log\left(c\left(ex^2+d\right)^p\right)}{5e^5}+\frac{4i\sqrt{d}f^3p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}-\frac{12id^{7/2}fg^2p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{7e^{7/2}}$$

input
```
Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
8*f^3*p^2*x - (1408*d^3*f*g^2*p^2*x)/(245*e^3) - (3*d*f^2*g*p^2*x^2)/e + (
d^4*g^3*p^2*x^2)/e^4 + (568*d^2*f*g^2*p^2*x^3)/(735*e^2) - (288*d*f*g^2*p^
2*x^5)/(1225*e) + (24*f*g^2*p^2*x^7)/343 + (3*f^2*g*p^2*(d + e*x^2)^2)/(8*
e^2) - (d^3*g^3*p^2*(d + e*x^2)^2)/(2*e^5) + (2*d^2*g^3*p^2*(d + e*x^2)^3)
/(9*e^5) - (d*g^3*p^2*(d + e*x^2)^4)/(16*e^5) + (g^3*p^2*(d + e*x^2)^5)/(1
25*e^5) - (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (1408*
d^(7/2)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(245*e^(7/2)) + ((4*I)*Sqrt
[d]*f^3*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (((12*I)/7)*d^(7/2)*f
*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*Sqrt[d]*f^3*p^2*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] -
 (24*d^(7/2)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d
] + I*Sqrt[e]*x)])/(7*e^(7/2)) - (d^5*g^3*p^2*Log[d + e*x^2]^2)/(10*e^5) -
 4*f^3*p*x*Log[c*(d + e*x^2)^p] + (12*d^3*f*g^2*p*x*Log[c*(d + e*x^2)^p])/
(7*e^3) - (4*d^2*f*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(7*e^2) + (12*d*f*g^2*p
*x^5*Log[c*(d + e*x^2)^p])/(35*e) - (12*f*g^2*p*x^7*Log[c*(d + e*x^2)^p])/
49 + (3*d*f^2*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (d^4*g^3*p*(d +
e*x^2)*Log[c*(d + e*x^2)^p])/e^5 - (3*f^2*g*p*(d + e*x^2)^2*Log[c*(d + e*x
^2)^p])/(4*e^2) + (d^3*g^3*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/e^5 - (2*
d^2*g^3*p*(d + e*x^2)^3*Log[c*(d + e*x^2)^p])/(3*e^5) + (d*g^3*p*(d + e*x^
2)^4*Log[c*(d + e*x^2)^p])/(4*e^5) - (g^3*p*(d + e*x^2)^5*Log[c*(d + e*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 21.77 (sec) , antiderivative size = 1584, normalized size of antiderivative = 1.30

method	result	size
risch	Expression too large to display	1584

input
```
int((g*x^3+f)^3*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
137/300*d^4*g^3*p^2*x^2/e^4-9/400/e*p^2*d*g^3*x^8-1408/245*d^3*f*g^2*p^2*x
/e^3-9/4*d*f^2*g*p^2*x^2/e+568/735*d^2*f*g^2*p^2*x^3/e^2-288/1225*d*f*g^2*
p^2*x^5/e+24/343*f*g^2*p^2*x^7+47/900/e^2*p^2*d^2*g^3*x^6-77/600/e^3*p^2*x
^4*d^3*g^3+3/8*p^2*x^4*f^2*g-137/300/e^5*p^2*d^5*ln(e*x^2+d)*g^3-12/49*p*f
*g^2*x^7*ln((e*x^2+d)^p)-3/4*p*f^2*g*x^4*ln((e*x^2+d)^p)-8*p^2*d/(d*e)^(1/
2)*arctan(x*e/(d*e)^(1/2))*f^3+3/7*ln((e*x^2+d)^p)^2*g^2*f*x^7+3/4*ln((e*x
^2+d)^p)^2*f^2*g*x^4+1/125*p^2*g^3*x^10-1/25*p*g^3*x^10*ln((e*x^2+d)^p)-4*
p*x*f^3*ln((e*x^2+d)^p)-4/7/e^2*p*d^2*f*g^2*x^3*ln((e*x^2+d)^p)+(I*Pi*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*
(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d
)^p)^2*csgn(I*c)+2*ln(c))*(1/10*ln((e*x^2+d)^p)*g^3*x^10+3/7*ln((e*x^2+d)^
p)*g^2*f*x^7+3/4*ln((e*x^2+d)^p)*f^2*g*x^4+ln((e*x^2+d)^p)*x*f^3-1/70*e*p*
(1/e^5*(7/5*e^4*g^3*x^10-7/4*d*e^3*g^3*x^8+60/7*e^4*f*g^2*x^7+7/3*d^2*e^2*
g^3*x^6-12*d*e^3*f*g^2*x^5-7/2*d^3*e*g^3*x^4+105/4*e^4*f^2*g*x^4+20*d^2*e^
2*f*g^2*x^3+7*d^4*g^3*x^2-105/2*d*f^2*g*x^2*e^3-60*x*d^3*f*g^2*e+140*x*e^4
*f^3)-d/e^5*(1/2*(14*d^4*g^3-105*d*e^3*f^2*g)/e*ln(e*x^2+d)+(-60*d^3*e*f*g
^2+140*e^4*f^3)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))))+1/20/e*p*d*g^3*x^8*
ln((e*x^2+d)^p)+1/4*(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi
*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+
d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(c))^2*(1/10*g^3*x^1...
```

Fricas [F]

$$\int (f+gx^3)^3 \log^2\left(c(d+ex^2)^p\right)\,dx = \int (gx^3+f)^3 \log\left((ex^2+d)^p c\right)^2\,dx$$

input
```
integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((g^3*x^9 + 3*f*g^2*x^6 + 3*f^2*g*x^3 + f^3)*log((e*x^2 + d)^p*c)^
2, x)
```

Sympy [F]

$$\int (f+gx^3)^3 \log^2\left(c(d+ex^2)^p\right)\,dx = \int (f+gx^3)^3 \log\left(c(d+ex^2)^p\right)^2\,dx$$

input
```
integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f + g*x**3)**3*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^3)^3 \log^2\left(c(d+ex^2)^p\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^3+f\right)^3 \log\left(\left(ex^2+d\right)^p c\right)^2\,dx$$

input
```
integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((g*x^3 + f)^3*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\left(g\,x^3+f\right)^3\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^3,x)
```

output
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^3, x)
```

Reduce [F]

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\text{Too large to display}$$

input
```
int((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x)
```

output

```
(35481600*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**3*e*f*g**2*p**2
 - 49392000*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**4*f**3*p**2 -
 10584000*int(log((d + e*x**2)**p*c)/(d + e*x**2),x)*d**4*e**2*f*g**2*p +
24696000*int(log((d + e*x**2)**p*c)/(d + e*x**2),x)*d*e**5*f**3*p + 617400
*log((d + e*x**2)**p*c)**2*d**5*g**3 - 4630500*log((d + e*x**2)**p*c)**2*d
**2*e**3*f**2*g + 6174000*log((d + e*x**2)**p*c)**2*e**5*f**3*x + 4630500*
log((d + e*x**2)**p*c)**2*e**5*f**2*g*x**4 + 2646000*log((d + e*x**2)**p*c
)**2*e**5*f*g**2*x**7 + 617400*log((d + e*x**2)**p*c)**2*e**5*g**3*x**10 -
 2819460*log((d + e*x**2)**p*c)*d**5*g**3*p - 1234800*log((d + e*x**2)**p*
c)*d**4*e*g**3*p*x**2 + 10584000*log((d + e*x**2)**p*c)*d**3*e**2*f*g**2*p
*x + 617400*log((d + e*x**2)**p*c)*d**3*e**2*g**3*p*x**4 + 13891500*log((d
 + e*x**2)**p*c)*d**2*e**3*f**2*g*p - 3528000*log((d + e*x**2)**p*c)*d**2*
e**3*f*g**2*p*x**3 - 411600*log((d + e*x**2)**p*c)*d**2*e**3*g**3*p*x**6 +
 9261000*log((d + e*x**2)**p*c)*d*e**4*f**2*g*p*x**2 + 2116800*log((d + e*
x**2)**p*c)*d*e**4*f*g**2*p*x**5 + 308700*log((d + e*x**2)**p*c)*d*e**4*g*
*3*p*x**8 - 24696000*log((d + e*x**2)**p*c)*e**5*f**3*p*x - 4630500*log((d
 + e*x**2)**p*c)*e**5*f**2*g*p*x**4 - 1512000*log((d + e*x**2)**p*c)*e**5*
f*g**2*p*x**7 - 246960*log((d + e*x**2)**p*c)*e**5*g**3*p*x**10 + 2819460*
d**4*e*g**3*p**2*x**2 - 35481600*d**3*e**2*f*g**2*p**2*x - 792330*d**3*e**
2*g**3*p**2*x**4 + 4771200*d**2*e**3*f*g**2*p**2*x**3 + 322420*d**2*e**...
```

3.294 $\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 835

$$\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right) \, dx = \text{Too large to display}$$

output
```
1408/735*d^(7/2)*g^2*p^2*arctan(e^(1/2)*x/d^(1/2))/e^(7/2)+4/35*d*g^2*p*x^
5*ln(c*(e*x^2+d)^p)/e-1/2*f*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2-2*d*f*g*
p^2*x^2/e+4/7*d^3*g^2*p*x*ln(c*(e*x^2+d)^p)/e^3-4/21*d^2*g^2*p*x^3*ln(c*(e
*x^2+d)^p)/e^2+f^2*x*ln(c*(e*x^2+d)^p)^2-1408/735*d^3*g^2*p^2*x/e^3+568/22
05*d^2*g^2*p^2*x^3/e^2-96/1225*d*g^2*p^2*x^5/e+1/4*f*g*p^2*(e*x^2+d)^2/e^2
+1/2*f*g*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2-d*f*g*(e*x^2+d)*ln(c*(e*x^2+d
)^p)^2/e^2+4*I*d^(1/2)*f^2*p^2*arctan(e^(1/2)*x/d^(1/2))^2/e^(1/2)+8*d^(1/
2)*f^2*p^2*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e
^(1/2)+4*d^(1/2)*f^2*p*arctan(e^(1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(1/2)
-4/7*d^(7/2)*g^2*p*arctan(e^(1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(7/2)-8/7
*d^(7/2)*g^2*p^2*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)
*x))/e^(7/2)-4/7*I*d^(7/2)*g^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2
)*x))/e^(7/2)-4/7*I*d^(7/2)*g^2*p^2*arctan(e^(1/2)*x/d^(1/2))^2/e^(7/2)-4/
49*g^2*p*x^7*ln(c*(e*x^2+d)^p)+2*d*f*g*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2-8
*d^(1/2)*f^2*p^2*arctan(e^(1/2)*x/d^(1/2))/e^(1/2)+1/7*g^2*x^7*ln(c*(e*x^2
+d)^p)^2+8/343*g^2*p^2*x^7+8*f^2*p^2*x+4*I*d^(1/2)*f^2*p^2*polylog(2,1-2*d
^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)-4*f^2*p*x*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.57

$$\int (f+gx^3)^2 \log^2\left(c(d+ex^2)^p\right)\,dx$$
$$= \frac{-176400i\sqrt{d}(-7e^3f^2+d^3g^2)\,p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2 - 1680\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(2(735e^3f^2-176d^3g^2)\,p-21\right.}{}$$

input
```
Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
((-176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]
]^2 - 1680*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(735*e^3*f^2 - 176*d^3
*g^2)*p - 210*(7*e^3*f^2 - d^3*g^2)*p*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]
*x)] - 105*(7*e^3*f^2 - d^3*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*(p^2*x*(-
591360*d^3*g^2 + 79520*d^2*e*g^2*x^2 - 378*d*e^2*g*x*(1225*f + 64*g*x^3) +
 225*e^3*(10976*f^2 + 343*f*g*x^3 + 32*g^2*x^6)) + 154350*d^2*e*f*g*p^2*Lo
g[d + e*x^2] - 210*p*(-840*d^3*g^2*x + 70*d^2*e*g*(-21*f + 4*g*x^3) - 42*d
*e^2*g*x^2*(35*f + 4*g*x^3) + 15*e^3*x*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6))
*Log[c*(d + e*x^2)^p] + 22050*(-7*d^2*e*f*g + e^3*x*(14*f^2 + 7*f*g*x^3 +
2*g^2*x^6))*Log[c*(d + e*x^2)^p]^2) - (176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3
*g^2)*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/
(308700*e^(7/2))
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 835, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx^3)^2 \log^2\left(c(d+ex^2)^p\right)\,dx$$

$$\Big\downarrow \; 2921$$

$$\int \left(f^2 \log^2\left(c(d+ex^2)^p\right)+2fgx^3\log^2\left(c(d+ex^2)^p\right)+g^2x^6\log^2\left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8}{343}g^2p^2x^7+\frac{1}{7}g^2\log^2\left(c(ex^2+d)^p\right)x^7-\frac{4}{49}g^2p\log\left(c(ex^2+d)^p\right)x^7-\frac{96dg^2p^2x^5}{1225e}+\frac{4dg^2p\log\left(c(ex^2+d)^p\right)x^5}{35e}+\frac{568d^2g^2p^2x^3}{2205e^2}-\frac{4d^2g^2p\log\left(c(ex^2+d)^p\right)x^3}{21e^2}-\frac{2dfgp^2x^2}{e}+8f^2p^2x-\frac{1408d^3g^2p^2x}{735e^3}+f^2\log^2\left(c(ex^2+d)^p\right)x-4f^2p\log\left(c(ex^2+d)^p\right)x+\frac{4d^3g^2p\log\left(c(ex^2+d)^p\right)x}{7e^3}+\frac{fgp^2(ex^2+d)^2}{4e^2}+\frac{4i\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{4id^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}}+\frac{fg(ex^2+d)^2\log^2\left(c(ex^2+d)^p\right)}{2e^2}-\frac{dfg(ex^2+d)\log^2\left(c(ex^2+d)^p\right)}{e^2}-\frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{1408d^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{735e^{7/2}}+\frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}-\frac{8d^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{7e^{7/2}}-\frac{fgp(ex^2+d)^2\log\left(c(ex^2+d)^p\right)}{2e^2}+\frac{2dfgp(ex^2+d)\log\left(c(ex^2+d)^p\right)}{e^2}+\frac{4\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(ex^2+d)^p\right)}{\sqrt{e}}-\frac{4d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(ex^2+d)^p\right)}{7e^{7/2}}+\frac{4i\sqrt{d}f^2p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}-\frac{4id^{7/2}g^2p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{7e^{7/2}}$$

input
```
Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
8*f^2*p^2*x - (1408*d^3*g^2*p^2*x)/(735*e^3) - (2*d*f*g*p^2*x^2)/e + (568*
d^2*g^2*p^2*x^3)/(2205*e^2) - (96*d*g^2*p^2*x^5)/(1225*e) + (8*g^2*p^2*x^7
)/343 + (f*g*p^2*(d + e*x^2)^2)/(4*e^2) - (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(735*e^(7/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/S
qrt[e] - (((4*I)/7)*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2)
 + (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x)])/Sqrt[e] - (8*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(7*e^(7/2)) - 4*f^2*p*x*Log[c*(
d + e*x^2)^p] + (4*d^3*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*d^2*g^2*
p*x^3*Log[c*(d + e*x^2)^p])/(21*e^2) + (4*d*g^2*p*x^5*Log[c*(d + e*x^2)^p]
)/(35*e) - (4*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (2*d*f*g*p*(d + e*x^2)*
Log[c*(d + e*x^2)^p])/e^2 - (f*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(2*
e^2) + (4*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/
Sqrt[e] - (4*d^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p
])/(7*e^(7/2)) + f^2*x*Log[c*(d + e*x^2)^p]^2 + (g^2*x^7*Log[c*(d + e*x^2)
^p]^2)/7 - (d*f*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 + (f*g*(d + e*x^
2)^2*Log[c*(d + e*x^2)^p]^2)/(2*e^2) + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1
 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (((4*I)/7)*d^(7/2)*g^2*
p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(7/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.05 (sec) , antiderivative size = 1127, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	1127

input
```
int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/2*d*f*g*p^2*x^2/e-4/49*p*g^2*x^7*ln((e*x^2+d)^p)+1/4*p^2*g*f*x^4-1/2*p*
f*g*x^4*ln((e*x^2+d)^p)-8*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^2-4*
p*f^2*x*ln((e*x^2+d)^p)+1/2*ln((e*x^2+d)^p)^2*g*f*x^4+(I*Pi*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^
p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csg
n(I*c)+2*ln(c))*(1/7*ln((e*x^2+d)^p)*g^2*x^7+1/2*ln((e*x^2+d)^p)*g*f*x^4+1
n((e*x^2+d)^p)*x*f^2-1/7*e*p*(1/e^4*(2/7*e^3*g^2*x^7-2/5*d*e^2*g^2*x^5+7/4
*e^3*f*g*x^4+2/3*d^2*e*g^2*x^3-7/2*d*f*g*x^2*e^2-2*d^3*x*g^2+14*x*e^3*f^2)
+1/e^4*d*(7/2*d*e*f*g*ln(e*x^2+d)+(2*d^3*g^2-14*e^3*f^2)/(d*e)^(1/2)*arcta
n(x*e/(d*e)^(1/2)))))+1/4*(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^
2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(
e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(c))^2*(1/7*g^2*x
^7+1/2*f*g*x^4+f^2*x)+3/2/e^2*p^2*d^2*f*g*ln(e*x^2+d)+1408/735/e^3*p^2*g^2
*d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+4/35/e*p*d*g^2*x^5*ln((e*x^2+d)^p
)-4/21/e^2*p*d^2*g^2*x^3*ln((e*x^2+d)^p)+4/7/e^3*p*d^3*x*g^2*ln((e*x^2+d)^
p)+1/e^2*p^2*d^2*f*g*ln(e*x^2+d)^2+4*p*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2
))*f^2*ln((e*x^2+d)^p)-4*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^2*ln(
e*x^2+d)+1/e*p*d*f*g*x^2*ln((e*x^2+d)^p)-1/e^2*p*d^2*f*g*ln(e*x^2+d)*ln((e
*x^2+d)^p)-4/7/e^3*p*d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*g^2*ln((e*x^2
+d)^p)+4/7/e^3*p^2*d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*g^2*ln(e*x^2...
```

Fricas [F]

$$\int (f+gx^3)^2 \log^2\left(c(d+ex^2)^p\right)\,dx = \int (gx^3+f)^2 \log\left((ex^2+d)^p c\right)^2\,dx$$

input
```
integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F]

$$\int (f+gx^3)^2 \log^2\left(c(d+ex^2)^p\right)\,dx = \int (f+gx^3)^2 \log\left(c(d+ex^2)^p\right)^2\,dx$$

input
```
integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f + g*x**3)**2*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^3)^2 \log^2\left(c(d+ex^2)^p\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\, dx = \int \left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^2\, dx$$

input
```
integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\, dx = \int \ln\left(c\left(e\,x^2+d\right)^p\right)^2 \left(g\,x^3+f\right)^2\, dx$$

input
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2, x)
```

Reduce [F]

$$\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\, dx$$
$$= \frac{591360\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) d^3 g^2 p^2 - 2469600\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) e^3 f^2 p^2 - 176400\left(\int \frac{\log\left(\left(e\,x^2+d\right)^p c\right)}{e\,x^2+d} dx\right) d}{}$$

input
```
int((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x)
```

output

```
(591360*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**3*g**2*p**2 - 246
9600*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**3*f**2*p**2 - 176400
*int(log((d + e*x**2)**p*c)/(d + e*x**2),x)*d**4*e*g**2*p + 1234800*int(lo
g((d + e*x**2)**p*c)/(d + e*x**2),x)*d*e**4*f**2*p - 154350*log((d + e*x**
2)**p*c)**2*d**2*e**2*f*g + 308700*log((d + e*x**2)**p*c)**2*e**4*f**2*x +
 154350*log((d + e*x**2)**p*c)**2*e**4*f*g*x**4 + 44100*log((d + e*x**2)**
p*c)**2*e**4*g**2*x**7 + 176400*log((d + e*x**2)**p*c)*d**3*e*g**2*p*x + 4
63050*log((d + e*x**2)**p*c)*d**2*e**2*f*g*p - 58800*log((d + e*x**2)**p*c
)*d**2*e**2*g**2*p*x**3 + 308700*log((d + e*x**2)**p*c)*d*e**3*f*g*p*x**2
+ 35280*log((d + e*x**2)**p*c)*d*e**3*g**2*p*x**5 - 1234800*log((d + e*x**
2)**p*c)*e**4*f**2*p*x - 154350*log((d + e*x**2)**p*c)*e**4*f*g*p*x**4 - 2
5200*log((d + e*x**2)**p*c)*e**4*g**2*p*x**7 - 591360*d**3*e*g**2*p**2*x +
 79520*d**2*e**2*g**2*p**2*x**3 - 463050*d*e**3*f*g*p**2*x**2 - 24192*d*e*
*3*g**2*p**2*x**5 + 2469600*e**4*f**2*p**2*x + 77175*e**4*f*g*p**2*x**4 +
7200*e**4*g**2*p**2*x**7)/(308700*e**4)
```

3.295 $\int (f + gx^3) \log^2 \left(c(d + ex^2)^p\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 395

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx = 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2(d+ex^2)^2}{8e^2}-\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}+\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}-4fpx\log\left(c(d+ex^2)^p\right)+\frac{dgp(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2}-\frac{gp(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{4e^2}+\frac{4\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}}+fx\log^2\left(c(d+ex^2)^p\right)-\frac{dg(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2}+\frac{g(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{4e^2}+\frac{4i\sqrt{d}fp^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}$$

output

```
8*f*p^2*x-d*g*p^2*x^2/e+1/8*g*p^2*(e*x^2+d)^2/e^2-8*d^(1/2)*f*p^2*arctan(e
^(1/2)*x/d^(1/2))/e^(1/2)+4*I*d^(1/2)*f*p^2*arctan(e^(1/2)*x/d^(1/2))^2/e^
(1/2)+8*d^(1/2)*f*p^2*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^
(1/2)*x))/e^(1/2)-4*f*p*x*ln(c*(e*x^2+d)^p)+d*g*p*(e*x^2+d)*ln(c*(e*x^2+d)
^p)/e^2-1/4*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2+4*d^(1/2)*f*p*arctan(e^(
1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(1/2)+f*x*ln(c*(e*x^2+d)^p)^2-1/2*d*g*
(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2+1/4*g*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^
2+4*I*d^(1/2)*f*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.01

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx$$
$$= fx\log^2\left(c(d+ex^2)^p\right)+\frac{1}{4}gx^4\log^2\left(c(d+ex^2)^p\right) - \frac{1}{2}gp\left(\frac{3dpx^2}{2e}-\frac{px^4}{4}-\frac{d^2p\log(d+ex^2)}{2e^2}-\frac{d^2\log\left(c(d+ex^2)^p\right)}{e^2} - \frac{dx^2\log\left(c(d+ex^2)^p\right)}{e}+\frac{1}{2}x^4\log\left(c(d+ex^2)^p\right)+\frac{d^2\log^2\left(c(d+ex^2)^p\right)}{2e^2p}\right) - 4efp\left(-\frac{2px}{e}+\frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}+\frac{x\log\left(c(d+ex^2)^p\right)}{e} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{e^{3/2}} - \frac{\sqrt{d}p\left(\frac{i\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{e}+\frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2i\sqrt{d}}{i\sqrt{d}-\sqrt{e}x}\right)}{e}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{d}+\sqrt{e}x}{i\sqrt{d}-\sqrt{e}x}\right)}{e}\right)}{\sqrt{e}}\right)$$

input
```
Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^4*Log[c*(d + e*x^2)^p]^2)/4 - (g*p*((3*d
*p*x^2)/(2*e) - (p*x^4)/4 - (d^2*p*Log[d + e*x^2])/(2*e^2) - (d^2*Log[c*(d
 + e*x^2)^p])/e^2 - (d*x^2*Log[c*(d + e*x^2)^p])/e + (x^4*Log[c*(d + e*x^2
)^p])/2 + (d^2*Log[c*(d + e*x^2)^p]^2)/(2*e^2*p)))/2 - 4*e*f*p*((-2*p*x)/e
 + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (x*Log[c*(d + e*x^2
)^p])/e - (Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/e^(3/
2) - (Sqrt[d]*p*((I*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e + (2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]]*Log[((2*I)*Sqrt[d])/(I*Sqrt[d] - Sqrt[e]*x)])/e + (I*PolyLog[2
, -((I*Sqrt[d] + Sqrt[e]*x)/(I*Sqrt[d] - Sqrt[e]*x))])/e))/Sqrt[e])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+gx^3\right)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$\Big\downarrow \; 2921$$

$$\int \left(f\log^2\left(c\left(d+ex^2\right)^p\right)+gx^3\log^2\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{4\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+\frac{g\left(d+ex^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^2}-\frac{dg\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{gp\left(d+ex^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4e^2}+\frac{dgp\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e^2}+fx\log^2\left(c\left(d+ex^2\right)^p\right)-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{gp^2\left(d+ex^2\right)^2}{8e^2}+\frac{4i\sqrt{d}fp^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}-\frac{dgp^2x^2}{e}+8fp^2x$$

input
```
Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

output
```
8*f*p^2*x - (d*g*p^2*x^2)/e + (g*p^2*(d + e*x^2)^2)/(8*e^2) - (8*Sqrt[d]*f
*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(S
qrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d
]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - 4*f*p*x*Log[c*(d +
e*x^2)^p] + (d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (g*p*(d + e*x^2
)^2*Log[c*(d + e*x^2)^p])/(4*e^2) + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] + f*x*Log[c*(d + e*x^2)^p]^2 - (d*g*(d
+ e*x^2)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) + (g*(d + e*x^2)^2*Log[c*(d + e*x
^2)^p]^2)/(4*e^2) + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/Sqrt[e]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.38 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.83

method	result	size
risch	Expression too large to display	724

input
```
int((g*x^3+f)*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*ln((e*x^2+d)^p)^2*x^4*g+ln((e*x^2+d)^p)^2*x*f-1/4*p*g*x^4*ln((e*x^2+d)
^p)+1/2/e*p*d*g*x^2*ln((e*x^2+d)^p)-4*p*f*x*ln((e*x^2+d)^p)+1/2/e^2*p^2*d^
2*g*ln(e*x^2+d)^2-1/2/e^2*p*d^2*g*ln(e*x^2+d)*ln((e*x^2+d)^p)-4*p^2*d/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2))*f*ln(e*x^2+d)+4*p*d/(d*e)^(1/2)*arctan(x*e
/(d*e)^(1/2))*f*ln((e*x^2+d)^p)+1/8*p^2*g*x^4-3/4*d*g*p^2*x^2/e+3/4/e^2*p^
2*d^2*g*ln(e*x^2+d)+8*f*p^2*x-8*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*
f-e*p^2*Sum(1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/4/_alpha/e*ln(x-_alpha)^2
+1/2*_alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/d*dilog(1/
2*(x+_alpha)/_alpha)))*d*(_alpha*d*g-4*e*f)/e^3/_alpha,_alpha=RootOf(_Z^2*
e+d))+(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*c
sgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(c))*(1/4*ln((e*x^2+d)^p)*x^4*g+ln((e
*x^2+d)^p)*x*f-1/2*e*p*(1/e^2*(1/4*e*g*x^4-1/2*d*g*x^2+4*e*f*x)+d/e^2*(1/2
*d*g/e*ln(e*x^2+d)-4*e*f/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))))+1/4*(I*Pi*
csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(
I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x
^2+d)^p)^2*csgn(I*c)+2*ln(c))^2*(1/4*g*x^4+f*x)
```

Fricas [F]

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx=\int (gx^3+f)\log\left((ex^2+d)^pc\right)^2\,dx$$

input
```
integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((g*x^3 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F]

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx=\int (f+gx^3)\log\left(c(d+ex^2)^p\right)^2\,dx$$

input
```
integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f + g*x**3)*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx = \int (gx^3+f)\log\left((ex^2+d)^pc\right)^2\,dx$$

input
```
integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((g*x^3 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx=\int \ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^2\,\left(g\,x^3+f\right)\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^3),x)
```

output
```
int(log(c*(d + e*x^2)^p)^2*(f + g*x^3), x)
```

Reduce [F]

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx = \frac{-64\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ef\,p^2+32\left(\int \frac{\log\left(\left(e\,x^2+d\right)^p c\right)}{e\,x^2+d}dx\right)d\,e^2fp-2\log\left(\left(e\,x^2+d\right)^p c\right)^2d^2g+8\log\left(\left(e\,x^2+\right.\right.}{}$$

input
```
int((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x)
```

output
```
( - 64*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p**2 + 32*int(log
((d + e*x**2)**p*c)/(d + e*x**2),x)*d*e**2*f*p - 2*log((d + e*x**2)**p*c)*
*2*d**2*g + 8*log((d + e*x**2)**p*c)**2*e**2*f*x + 2*log((d + e*x**2)**p*c
)**2*e**2*g*x**4 + 6*log((d + e*x**2)**p*c)*d**2*g*p + 4*log((d + e*x**2)*
*p*c)*d*e*g*p*x**2 - 32*log((d + e*x**2)**p*c)*e**2*f*p*x - 2*log((d + e*x
**2)**p*c)*e**2*g*p*x**4 - 6*d*e*g*p**2*x**2 + 64*e**2*f*p**2*x + e**2*g*p
**2*x**4)/(8*e**2)
```

3.296 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx=\text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3},x\right)$$

output
```
Defer(Int)(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

Mathematica [N/A]

Not integrable

Time = 22.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx=\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

input
```
Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
```

output
```
Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx$$
$$\Big\downarrow \text{2923}$$
$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}dx$$

input
```
Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^2}{g\,x^3+f}dx$$

input
```
int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

output
```
int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx=\int \frac{\log\left((ex^2+d)^pc\right)^2}{gx^3+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx=\text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx=\int \frac{\log\left((ex^2+d)^pc\right)^2}{gx^3+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{gx^3+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

Mupad [N/A]

Not integrable

Time = 25.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^2}{g\,x^3+f}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^2/(f + g*x^3),x)
```

output
```
int(log(c*(d + e*x^2)^p)^2/(f + g*x^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((e\,x^2+d)^p\,c\right)^2}{g\,x^3+f}dx$$

input
```
int(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

output
```
int(log((d + e*x**2)**p*c)**2/(f + g*x**3),x)
```

3.297 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx=\text{Int}\left(\frac{\log^2\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2},x\right)$$

output
```
Defer(Int)(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 31.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx=\int \frac{\log^2\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

input
```
Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]
```

output
```
Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx$$
$$\Big\downarrow \text{2923}$$
$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}dx$$

input
```
Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 6.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^2}{(g\,x^3+f)^2}dx$$

input
```
int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

output
```
int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{(gx^3+f)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)^2/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{(gx^3+f)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^2}{(g\,x^3+f)^2}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^2/(f + g*x^3)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)^2/(f + g*x^3)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 2374, normalized size of antiderivative = 98.92

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

output
```
( - 8*g**(1/3)*f**(2/3)*sqrt(3)*atan((f**(1/3) - 2*g**(1/3)*x)/(f**(1/3)*s
qrt(3)))*e**3*f**2*p**2 - 8*g**(1/3)*f**(2/3)*sqrt(3)*atan((f**(1/3) - 2*g
**(1/3)*x)/(f**(1/3)*sqrt(3)))*e**3*f*g*p**2*x**3 - 8*sqrt(3)*atan((f**(1/
3) - 2*g**(1/3)*x)/(f**(1/3)*sqrt(3)))*d*e**2*f**2*g*p**2 - 8*sqrt(3)*atan
((f**(1/3) - 2*g**(1/3)*x)/(f**(1/3)*sqrt(3)))*d*e**2*f*g**2*p**2*x**3 - 2
4*g**(2/3)*f**(1/3)*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**2*f**
2*p**2 - 24*g**(2/3)*f**(1/3)*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d))
)*e**2*f*g*p**2*x**3 - 4*g**(1/3)*f**(2/3)*log(f**(2/3) - g**(1/3)*f**(1/3
)*x + g**(2/3)*x**2)*e**3*f**2*p**2 - 4*g**(1/3)*f**(2/3)*log(f**(2/3) - g
**(1/3)*f**(1/3)*x + g**(2/3)*x**2)*e**3*f*g*p**2*x**3 + 8*g**(1/3)*f**(2/
3)*log(f**(1/3) + g**(1/3)*x)*e**3*f**2*p**2 + 8*g**(1/3)*f**(2/3)*log(f**
(1/3) + g**(1/3)*x)*e**3*f*g*p**2*x**3 + 12*g**(2/3)*f**(1/3)*int(log((d +
 e*x**2)**p*c)/(d*f**2 + 2*d*f*g*x**3 + d*g**2*x**6 + e*f**2*x**2 + 2*e*f*
g*x**5 + e*g**2*x**8),x)*d**4*f**2*g**2*p + 12*g**(2/3)*f**(1/3)*int(log((
d + e*x**2)**p*c)/(d*f**2 + 2*d*f*g*x**3 + d*g**2*x**6 + e*f**2*x**2 + 2*e
*f*g*x**5 + e*g**2*x**8),x)*d**4*f*g**3*p*x**3 + 12*g**(2/3)*f**(1/3)*int(
log((d + e*x**2)**p*c)/(d*f**2 + 2*d*f*g*x**3 + d*g**2*x**6 + e*f**2*x**2
+ 2*e*f*g*x**5 + e*g**2*x**8),x)*d*e**3*f**4*p + 12*g**(2/3)*f**(1/3)*int(
log((d + e*x**2)**p*c)/(d*f**2 + 2*d*f*g*x**3 + d*g**2*x**6 + e*f**2*x**2
+ 2*e*f*g*x**5 + e*g**2*x**8),x)*d*e**3*f**3*g*p*x**3 + 6*g**(2/3)*f**(...
```

3.298 $\int (f+gx^3)^2 \log^3\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (f+gx^3)^2 \log^3\left(c(d+ex^2)^p\right)\,dx = \text{Too large to display}$$

output
```
-351136/25725*d^(7/2)*g^2*p^3*arctan(e^(1/2)*x/d^(1/2))/e^(7/2)+48*d^(1/2)
*f^2*p^3*arctan(e^(1/2)*x/d^(1/2))/e^(1/2)+6*d*f*g*p^3*x^2/e-1408/245*d^3*
g^2*p^2*x*ln(c*(e*x^2+d)^p)/e^3+568/735*d^2*g^2*p^2*x^3*ln(c*(e*x^2+d)^p)/
e^2+351136/25725*d^3*g^2*p^3*x/e^3-55456/77175*d^2*g^2*p^3*x^3/e^2+5232/42
875*d*g^2*p^3*x^5/e-3/8*f*g*p^3*(e*x^2+d)^2/e^2+1/2*f*g*(e*x^2+d)^2*ln(c*(
e*x^2+d)^p)^3/e^2-6/7*d^4*g^2*p*Defer(Int)(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),x
)/e^3+2816/245*d^(7/2)*g^2*p^3*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(
1/2)+I*e^(1/2)*x))/e^(7/2)+1408/245*d^(7/2)*g^2*p^2*arctan(e^(1/2)*x/d^(1/
2))*ln(c*(e*x^2+d)^p)/e^(7/2)-48*d^(1/2)*f^2*p^3*arctan(e^(1/2)*x/d^(1/2))
*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)-24*d^(1/2)*f^2*p^2*arctan(e^(
1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(1/2)-24*I*d^(1/2)*f^2*p^3*polylog(2,1
-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)-24*I*d^(1/2)*f^2*p^3*arctan(e^(1
/2)*x/d^(1/2))^2/e^(1/2)+1408/245*I*d^(7/2)*g^2*p^3*polylog(2,1-2*d^(1/2)/
(d^(1/2)+I*e^(1/2)*x))/e^(7/2)+1408/245*I*d^(7/2)*g^2*p^3*arctan(e^(1/2)*x
/d^(1/2))^2/e^(7/2)+24*f^2*p^2*x*ln(c*(e*x^2+d)^p)+24/343*g^2*p^2*x^7*ln(c
*(e*x^2+d)^p)-6*f^2*p*x*ln(c*(e*x^2+d)^p)^2-6/49*g^2*p*x^7*ln(c*(e*x^2+d)^
p)^2+6*d*f^2*p*Defer(Int)(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),x)-288/1225*d*g^2*
p^2*x^5*ln(c*(e*x^2+d)^p)/e+3/4*f*g*p^2*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2+
6/7*d^3*g^2*p*x*ln(c*(e*x^2+d)^p)^2/e^3-2/7*d^2*g^2*p*x^3*ln(c*(e*x^2+d)^p
)^2/e^2+6/35*d*g^2*p*x^5*ln(c*(e*x^2+d)^p)^2/e-3/4*f*g*p*(e*x^2+d)^2*ln...
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2385 vs. 2(1126) = 2252.

Time = 10.96 (sec) , antiderivative size = 2385, normalized size of antiderivative = 99.38

$$\int (f + gx^3)^2 \log^3 (c(d + ex^2)^p) \, dx = \text{Result too large to show}$$

input
```
Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

output

```
(f*g*p^3*(d + e*x^2)*(-8*d*(-6 + 6*Log[d + e*x^2] - 3*Log[d + e*x^2]^2 + L
og[d + e*x^2]^3) + (d + e*x^2)*(-3 + 6*Log[d + e*x^2] - 6*Log[d + e*x^2]^2
 + 4*Log[d + e*x^2]^3)))/(8*e^2) + 6*f*g*p^2*((x^4*Log[d + e*x^2]^2)/4 - e
*((3*d*x^2)/(4*e^2) - x^4/(8*e) - (3*d^2*Log[d + e*x^2])/(4*e^3) - (d*x^2*
Log[d + e*x^2])/(2*e^2) + (x^4*Log[d + e*x^2])/(4*e) + (d^2*Log[d + e*x^2]
^2)/(4*e^3)))*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]) + (3*d*f*g*p*x^
2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*e) - (2*d^2*g^2*p*x^3
*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(7*e^2) + (6*d*g^2*p*x^5*
(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(35*e) - (3*d^2*f*g*p*Log[
d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*e^2) + (3*p*
x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6)*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + L
og[c*(d + e*x^2)^p])^2)/14 + (f*g*x^4*(-(p*Log[d + e*x^2]) + Log[c*(d + e*
x^2)^p])^2*(-3*p + 2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])))/4 + (g
^2*x^7*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p + 7*(-(p*Log[d
 + e*x^2]) + Log[c*(d + e*x^2)^p])))/49 + (x*(-(p*Log[d + e*x^2]) + Log[c*
(d + e*x^2)^p])^2*(-42*e^3*f^2*p + 6*d^3*g^2*p + 7*e^3*f^2*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p])))/(7*e^3) - (6*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(
-7*d*e^3*f^2*p*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + d^4*g^2*p*
(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2))/(7*Sqrt[d]*e^(7/2)) + 3*f
^2*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^2...
```

Rubi [N/A]

Not integrable

Time = 4.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right) \, dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f^2 \log^3\left(c\left(d+ex^2\right)^p\right)+2fgx^3 \log^3\left(c\left(d+ex^2\right)^p\right)+g^2x^6 \log^3\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{48g^2p^3x^7}{2401}+\frac{1}{7}g^2\log^3\left(c(ex^2+d)^p\right)x^7-\frac{6}{49}g^2p\log^2\left(c(ex^2+d)^p\right)x^7+\frac{24}{343}g^2p^2\log\left(c(ex^2+d)^p\right)x^7+\frac{5232dg^2p^3x^5}{42875e}+\frac{6dg^2p\log^2\left(c(ex^2+d)^p\right)x^5}{35e}-\frac{288dg^2p^2\log\left(c(ex^2+d)^p\right)x^5}{1225e}-\frac{55456d^2g^2p^3x^3}{77175e^2}-\frac{2d^2g^2p\log^2\left(c(ex^2+d)^p\right)x^3}{7e^2}+\frac{568d^2g^2p^2\log\left(c(ex^2+d)^p\right)x^3}{735e^2}+\frac{6dfgp^3x^2}{e}-48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+f^2\log^3\left(c(ex^2+d)^p\right)x-6f^2p\log^2\left(c(ex^2+d)^p\right)x+\frac{6d^3g^2p\log^2\left(c(ex^2+d)^p\right)x}{7e^3}+24f^2p^2\log\left(c(ex^2+d)^p\right)x-\frac{1408d^3g^2p^2\log\left(c(ex^2+d)^p\right)x}{245e^3}+\frac{fg(ex^2+d)^2\log^3\left(c(ex^2+d)^p\right)}{2e^2}-\frac{dfg(ex^2+d)\log^3\left(c(ex^2+d)^p\right)}{e^2}-\frac{3fgp^3(ex^2+d)^2}{8e^2}-\frac{24i\sqrt{d}f^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}+\frac{1408id^{7/2}g^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{245e^{7/2}}-\frac{3fgp(ex^2+d)^2\log^2\left(c(ex^2+d)^p\right)}{4e^2}+\frac{3dfgp(ex^2+d)\log^2\left(c(ex^2+d)^p\right)}{e^2}+\frac{48\sqrt{d}f^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{351136d^{7/2}g^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{25725e^{7/2}}-\frac{48\sqrt{d}f^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}+\frac{2816d^{7/2}g^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{245e^{7/2}}+\frac{3fgp^2(ex^2+d)^2\log\left(c(ex^2+d)^p\right)}{4e^2}-\frac{6dfgp^2(ex^2+d)\log\left(c(ex^2+d)^p\right)}{e^2}-\frac{24\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(ex^2+d)^p\right)}{\sqrt{e}}+\frac{1408d^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(ex^2+d)^p\right)}{245e^{7/2}}-\frac{24i\sqrt{d}f^2p^3\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}+\frac{1408id^{7/2}g^2p^3\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{245e^{7/2}}+6df^2p\int\frac{\log^2\left(c(ex^2+d)^p\right)}{ex^2+d}dx-\frac{6d^4g^2p\int\frac{\log^2\left(c(ex^2+d)^p\right)}{ex^2+d}dx}{7e^3}$$

input
```
Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(g\,x^{3}+f\right)^{2} \ln \left(c \left(e\,x^{2}+d\right)^{p}\right)^{3}dx$$

input
```
int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
```

output
```
int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^3\,dx$$

input
```
integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

output
```
integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 33.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)^3\,dx$$

input
```
integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**3,x)
```

output
```
Integral((f + g*x**3)**2*log(c*(d + e*x**2)**p)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^3\,dx$$

input
```
integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

output
```
integrate((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 26.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(f+gx^3\right)^2\log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \ln\left(c\left(e\,x^2+d\right)^p\right)^3\left(g\,x^3+f\right)^2\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^3*(f + g*x^3)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)^3*(f + g*x^3)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 812, normalized size of antiderivative = 33.83

$$\int \left(f+gx^3\right)^2\log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\text{Too large to display}$$

input
```
int((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x)
```

output
```
( - 294954240*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**3*g**2*p**3
 + 1037232000*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**3*f**2*p**3
 - 18522000*int(log((d + e*x**2)**p*c)**2/(d + e*x**2),x)*d**4*e*g**2*p +
129654000*int(log((d + e*x**2)**p*c)**2/(d + e*x**2),x)*d*e**4*f**2*p + 12
4185600*int(log((d + e*x**2)**p*c)/(d + e*x**2),x)*d**4*e*g**2*p**2 - 5186
16000*int(log((d + e*x**2)**p*c)/(d + e*x**2),x)*d*e**4*f**2*p**2 - 108045
00*log((d + e*x**2)**p*c)**3*d**2*e**2*f*g + 21609000*log((d + e*x**2)**p*
c)**3*e**4*f**2*x + 10804500*log((d + e*x**2)**p*c)**3*e**4*f*g*x**4 + 308
7000*log((d + e*x**2)**p*c)**3*e**4*g**2*x**7 + 18522000*log((d + e*x**2)*
*p*c)**2*d**3*e*g**2*p*x + 48620250*log((d + e*x**2)**p*c)**2*d**2*e**2*f*
g*p - 6174000*log((d + e*x**2)**p*c)**2*d**2*e**2*g**2*p*x**3 + 32413500*l
og((d + e*x**2)**p*c)**2*d*e**3*f*g*p*x**2 + 3704400*log((d + e*x**2)**p*c
)**2*d*e**3*g**2*p*x**5 - 129654000*log((d + e*x**2)**p*c)**2*e**4*f**2*p*
x - 16206750*log((d + e*x**2)**p*c)**2*e**4*f*g*p*x**4 - 2646000*log((d +
e*x**2)**p*c)**2*e**4*g**2*p*x**7 - 124185600*log((d + e*x**2)**p*c)*d**3*
e*g**2*p**2*x - 113447250*log((d + e*x**2)**p*c)*d**2*e**2*f*g*p**2 + 1669
9200*log((d + e*x**2)**p*c)*d**2*e**2*g**2*p**2*x**3 - 97240500*log((d + e
*x**2)**p*c)*d*e**3*f*g*p**2*x**2 - 5080320*log((d + e*x**2)**p*c)*d*e**3*
g**2*p**2*x**5 + 518616000*log((d + e*x**2)**p*c)*e**4*f**2*p**2*x + 16206
750*log((d + e*x**2)**p*c)*e**4*f*g*p**2*x**4 + 1512000*log((d + e*x**2...
```

3.299 $\int (f+gx^3) \log^3\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (f+gx^3)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx = -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3(d+ex^2)^2}{16e^2}+\frac{48\sqrt{d}\,fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{d}\,fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{48\sqrt{d}\,fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{3dgp^2(d+ex^2)\log\left(c\left(d+ex^2\right)^p\right)}{e^2}+\frac{3gp^2(d+ex^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{8e^2}-\frac{24\sqrt{d}\,fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}-6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3dgp(d+ex^2)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{3gp(d+ex^2)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{8e^2}+fx\log^3\left(c\left(d+ex^2\right)^p\right)-\frac{dg(d+ex^2)\log^3\left(c\left(d+ex^2\right)^p\right)}{2e^2}+\frac{g(d+ex^2)^2\log^3\left(c\left(d+ex^2\right)^p\right)}{4e^2}-\frac{24i\sqrt{d}\,fp^3\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+6dfp\,\mathrm{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)$$

output
```
-48*f*p^3*x+3*d*g*p^3*x^2/e-3/16*g*p^3*(e*x^2+d)^2/e^2+48*d^(1/2)*f*p^3*ar
ctan(e^(1/2)*x/d^(1/2))/e^(1/2)-24*I*d^(1/2)*f*p^3*arctan(e^(1/2)*x/d^(1/2
))^2/e^(1/2)-48*d^(1/2)*f*p^3*arctan(e^(1/2)*x/d^(1/2))*ln(2*d^(1/2)/(d^(1
/2)+I*e^(1/2)*x))/e^(1/2)+24*f*p^2*x*ln(c*(e*x^2+d)^p)-3*d*g*p^2*(e*x^2+d)
*ln(c*(e*x^2+d)^p)/e^2+3/8*g*p^2*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2-24*d^(1
/2)*f*p^2*arctan(e^(1/2)*x/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(1/2)-6*f*p*x*ln(c
*(e*x^2+d)^p)^2+3/2*d*g*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2-3/8*g*p*(e*x^2
+d)^2*ln(c*(e*x^2+d)^p)^2/e^2+f*x*ln(c*(e*x^2+d)^p)^3-1/2*d*g*(e*x^2+d)*ln
(c*(e*x^2+d)^p)^3/e^2+1/4*g*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^3/e^2-24*I*d^(1/
2)*f*p^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x))/e^(1/2)+6*d*f*p*Defe
r(Int)(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1051 vs. 2(518) = 1036.

Time = 2.34 (sec) , antiderivative size = 1051, normalized size of antiderivative = 47.77

$$\int (f + gx^3) \log^3 \left(c(d + ex^2)^p\right) \, dx = \text{Too large to display}$$

input
```
Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

output
```
(g*x^4*Log[c*(d + e*x^2)^p]^3)/4 + (6*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[
d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] + 3*f*p*x*Log[
d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + f*x*(-(p*Log[d
 + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p - p*Log[d + e*x^2] + Log[c*(d +
 e*x^2)^p]) - (3*g*p*((-7*d*p^2*x^2)/(2*e) + (p^2*x^4)/4 + (d^2*p^2*Log[d
+ e*x^2])/(2*e^2) + (3*d^2*p*Log[c*(d + e*x^2)^p])/e^2 + (3*d*p*x^2*Log[c*
(d + e*x^2)^p])/e - (p*x^4*Log[c*(d + e*x^2)^p])/2 - (3*d^2*Log[c*(d + e*x
^2)^p]^2)/(2*e^2) - (d*x^2*Log[c*(d + e*x^2)^p]^2)/e + (x^4*Log[c*(d + e*x
^2)^p]^2)/2 + (d^2*Log[c*(d + e*x^2)^p]^3)/(3*e^2*p)))/4 + 3*f*p^2*(-(p*Lo
g[d + e*x^2]) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^2 - (4*((-I)*Sqrt[
d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(-2 + Log[d + e*x^2]) - Sqrt[
d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e
]*x)] + Log[d + e*x^2]) - I*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-
I)*Sqrt[d] + Sqrt[e]*x)]))/Sqrt[e]) + (f*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^
2]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*
Sqrt[1 - d/(d + e*x^2)]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2},
 {3/2, 3/2, 3/2}, d/(d + e*x^2)] + 4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2,
1/2}, {3/2, 3/2}, d/(d + e*x^2)]*Log[d + e*x^2] + Sqrt[d + e*x^2]*ArcSin[S
qrt[d]/Sqrt[d + e*x^2]]*Log[d + e*x^2]^2) + Sqrt[-d]*e*x^2*(-48 + 24*Log[d
 + e*x^2] - 6*Log[d + e*x^2]^2 + Log[d + e*x^2]^3) + 24*d*Sqrt[e*x^2]*A...
```

Rubi [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx^3)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f\log^3\left(c\left(d+ex^2\right)^p\right)+gx^3\log^3\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$6dfp\int \frac{\log^2\left(c\left(ex^2+d\right)^p\right)}{ex^2+d}dx - \frac{24\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}} - \frac{24i\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} + \frac{3gp^2\left(d+ex^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{8e^2} - \frac{3dgp^2\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e^2} + \frac{g\left(d+ex^2\right)^2\log^3\left(c\left(d+ex^2\right)^p\right)}{4e^2} - \frac{dg\left(d+ex^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)}{2e^2} - \frac{3gp\left(d+ex^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{8e^2} + \frac{3dgp\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2} + 24fp^2x\log\left(c\left(d+ex^2\right)^p\right) + fx\log^3\left(c\left(d+ex^2\right)^p\right) - 6fpx\log^2\left(c\left(d+ex^2\right)^p\right) - \frac{3gp^3\left(d+ex^2\right)^2}{16e^2} - \frac{24i\sqrt{d}fp^3\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}} + \frac{3dgp^3x^2}{e} - 48fp^3x$$

input
```
Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(g\,x^{3}+f \right) \ln \left(c \left(e\,x^{2}+d \right)^{p}\right)^{3}dx$$

input
```
int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

output
```
int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(gx^{3}+f\right)\log\left(\left(ex^{2}+d\right)^{p}c\right)^{3}\,dx$$

input
```
integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

output
```
integral((g*x^3 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 10.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(f + gx^{3}\right)\log\left(c\left(d + ex^{2}\right)^{p}\right)^{3}\,dx$$

input
```
integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**3,x)
```

output
```
Integral((f + g*x**3)*log(c*(d + e*x**2)**p)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^3)\log^3\left(c(d+ex^2)^p\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (f+gx^3)\log^3\left(c(d+ex^2)^p\right)\,dx = \int (gx^3+f)\log\left((ex^2+d)^pc\right)^3\,dx$$

input
```
integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

output
```
integrate((g*x^3 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (f+gx^3)\log^3\left(c(d+ex^2)^p\right)\,dx = \int \ln\left(c\,(e\,x^2+d)^p\right)^3\,(g\,x^3+f)\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^3*(f + g*x^3),x)
```

output
```
int(log(c*(d + e*x^2)^p)^3*(f + g*x^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 374, normalized size of antiderivative = 17.00

$$\int (f + gx^3) \log^3 \left(c(d + ex^2)^p\right) \, dx$$
$$= \frac{768\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) e f\, p^3 + 96\left(\int \frac{\log\left((e\,x^2+d)^p c\right)^2}{e\,x^2+d}dx\right) d\,e^2 f p - 384\left(\int \frac{\log\left((e\,x^2+d)^p c\right)}{e\,x^2+d}dx\right) d\,e^2 f\,p^2 - 4\log$$

input
```
int((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x)
```

output
```
(768*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p**3 + 96*int(log((
d + e*x**2)**p*c)**2/(d + e*x**2),x)*d*e**2*f*p - 384*int(log((d + e*x**2)
**p*c)/(d + e*x**2),x)*d*e**2*f*p**2 - 4*log((d + e*x**2)**p*c)**3*d**2*g
+ 16*log((d + e*x**2)**p*c)**3*e**2*f*x + 4*log((d + e*x**2)**p*c)**3*e**2
*g*x**4 + 18*log((d + e*x**2)**p*c)**2*d**2*g*p + 12*log((d + e*x**2)**p*c
)**2*d*e*g*p*x**2 - 96*log((d + e*x**2)**p*c)**2*e**2*f*p*x - 6*log((d + e
*x**2)**p*c)**2*e**2*g*p*x**4 - 42*log((d + e*x**2)**p*c)*d**2*g*p**2 - 36
*log((d + e*x**2)**p*c)*d*e*g*p**2*x**2 + 384*log((d + e*x**2)**p*c)*e**2*
f*p**2*x + 6*log((d + e*x**2)**p*c)*e**2*g*p**2*x**4 + 42*d*e*g*p**3*x**2
- 768*e**2*f*p**3*x - 3*e**2*g*p**3*x**4)/(16*e**2)
```

3.300 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} \, dx=\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3},x\right)$$

output
```
Defer(Int)(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

Mathematica [N/A]

Not integrable

Time = 32.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} \, dx=\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3} \, dx$$

input
```
Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
```

output
```
Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

$$\Big\downarrow \; 2923$$

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}dx$$

input
```
Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{g\,x^3+f}dx$$

input
```
int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

output
```
int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^3+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^3+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^3+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

Mupad [N/A]

Not integrable

Time = 25.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^3}{g\,x^3+f}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^3/(f + g*x^3),x)
```

output
```
int(log(c*(d + e*x^2)^p)^3/(f + g*x^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((e\,x^2+d)^p\,c\right)^3}{g\,x^3+f}dx$$

input
```
int(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

output
```
int(log((d + e*x**2)**p*c)**3/(f + g*x**3),x)
```

3.301 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx=\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2},x\right)$$

output
```
Defer(Int)(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 36.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx=\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

input
```
Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
```

output
```
Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}dx$$
$$\Big\downarrow \text{ 2923}$$
$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}dx$$

input
```
Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)^3}{\left(g\,x^3+f\right)^2}dx$$

input
```
int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

output
```
int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx=\int \frac{\log\left((ex^2+d)^pc\right)^3}{(gx^3+f)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)^3/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx=\text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{(gx^3+f)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^3}{(g\,x^3+f)^2}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)^3/(f + g*x^3)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)^3/(f + g*x^3)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 3753, normalized size of antiderivative = 156.38

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

output
```
( - 20*g**(1/3)*f**(2/3)*sqrt(3)*atan((f**(1/3) - 2*g**(1/3)*x)/(f**(1/3)*
sqrt(3)))*e**3*f**2*p**3 - 20*g**(1/3)*f**(2/3)*sqrt(3)*atan((f**(1/3) - 2
*g**(1/3)*x)/(f**(1/3)*sqrt(3)))*e**3*f*g*p**3*x**3 - 20*sqrt(3)*atan((f**
(1/3) - 2*g**(1/3)*x)/(f**(1/3)*sqrt(3)))*d*e**2*f**2*g*p**3 - 20*sqrt(3)*
atan((f**(1/3) - 2*g**(1/3)*x)/(f**(1/3)*sqrt(3)))*d*e**2*f*g**2*p**3*x**3
 - 60*g**(2/3)*f**(1/3)*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**2
*f**2*p**3 - 60*g**(2/3)*f**(1/3)*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt
(d)))*e**2*f*g*p**3*x**3 - 10*g**(1/3)*f**(2/3)*log(f**(2/3) - g**(1/3)*f*
*(1/3)*x + g**(2/3)*x**2)*e**3*f**2*p**3 - 10*g**(1/3)*f**(2/3)*log(f**(2/
3) - g**(1/3)*f**(1/3)*x + g**(2/3)*x**2)*e**3*f*g*p**3*x**3 + 20*g**(1/3)
*f**(2/3)*log(f**(1/3) + g**(1/3)*x)*e**3*f**2*p**3 + 20*g**(1/3)*f**(2/3)
*log(f**(1/3) + g**(1/3)*x)*e**3*f*g*p**3*x**3 + 6*g**(2/3)*f**(1/3)*int(l
og((d + e*x**2)**p*c)**2/(d*f**2 + 2*d*f*g*x**3 + d*g**2*x**6 + e*f**2*x**
2 + 2*e*f*g*x**5 + e*g**2*x**8),x)*d**4*f**2*g**2*p + 6*g**(2/3)*f**(1/3)*
int(log((d + e*x**2)**p*c)**2/(d*f**2 + 2*d*f*g*x**3 + d*g**2*x**6 + e*f**
2*x**2 + 2*e*f*g*x**5 + e*g**2*x**8),x)*d**4*f*g**3*p*x**3 + 6*g**(2/3)*f*
*(1/3)*int(log((d + e*x**2)**p*c)**2/(d*f**2 + 2*d*f*g*x**3 + d*g**2*x**6
 + e*f**2*x**2 + 2*e*f*g*x**5 + e*g**2*x**8),x)*d*e**3*f**4*p + 6*g**(2/3)*
f**(1/3)*int(log((d + e*x**2)**p*c)**2/(d*f**2 + 2*d*f*g*x**3 + d*g**2*x**
6 + e*f**2*x**2 + 2*e*f*g*x**5 + e*g**2*x**8),x)*d*e**3*f**3*g*p*x**3 +...
```

3.302 $\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \text{Int}\left(\frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)},x\right)$$

output
```
Defer(Int)((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx$$

input
```
Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]
```

output
```
Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx$$

$$\Big\downarrow \text{2923}$$

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}dx$$

input
```
Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(g\,x^3+f)^2}{\ln(c\,(e\,x^2+d)^p)}dx$$

input
```
int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

output
```
int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^3+f)^2}{\log((ex^2+d)^p c)}\,dx$$

input
```
integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 14.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx$$

input
```
integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
```

output
```
Integral((f + g*x**3)**2/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^3+f)^2}{\log((ex^2+d)^p c)}\,dx$$

input
```
integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^3+f)^2}{\log((ex^2+d)^p c)}\,dx$$

input
```
integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 25.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(g\,x^3+f)^2}{\ln(c\,(e\,x^2+d)^p)}\,dx$$

input
```
int((f + g*x^3)^2/log(c*(d + e*x^2)^p),x)
```

output
```
int((f + g*x^3)^2/log(c*(d + e*x^2)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.88

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \left(\int \frac{x^6}{\log((e\,x^2+d)^p\,c)}dx\right)g^2+2\left(\int \frac{x^3}{\log((e\,x^2+d)^p\,c)}dx\right)fg + \left(\int \frac{1}{\log((e\,x^2+d)^p\,c)}dx\right)f^2$$

input
```
int((g*x^3+f)^2/log(c*(e*x^2+d)^p),x)
```

output
```
int(x**6/log((d + e*x**2)**p*c),x)*g**2 + 2*int(x**3/log((d + e*x**2)**p*c
),x)*f*g + int(1/log((d + e*x**2)**p*c),x)*f**2
```

3.303 $\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx = \text{Int}\left(\frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

output
```
Defer(Int)((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

input
```
Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]
```

output
```
Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p], x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)} \, dx$$
$$\Big\downarrow \text{2923}$$
$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}dx$$

input
```
Int[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{g\,x^{3}+f}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}dx$$

input
```
int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

output
```
int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log \left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^3+f}{\log \left(\left(ex^2+d\right)^p c\right)}\,dx$$

input
```
integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 5.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{f+gx^3}{\log \left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^3}{\log \left(c\left(d+ex^2\right)^p\right)}\,dx$$

input
```
integrate((g*x**3+f)/ln(c*(e*x**2+d)**p),x)
```

output
```
Integral((f + g*x**3)/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)}\,dx = \int \frac{gx^3 + f}{\log\left(\left(ex^2 + d\right)^p c\right)}\,dx$$

input
```
integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
integrate((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)}\,dx = \int \frac{gx^3 + f}{\log\left(\left(ex^2 + d\right)^p c\right)}\,dx$$

input
```
integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 25.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)}\,dx = \int \frac{g\,x^3 + f}{\ln\left(c\,\left(e\,x^2 + d\right)^p\right)}\,dx$$

input
```
int((f + g*x^3)/log(c*(d + e*x^2)^p),x)
```

output
```
int((f + g*x^3)/log(c*(d + e*x^2)^p), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{f+gx^3}{\log (c (d+ex^2)^p)} \, dx = \left(\int \frac{x^3}{\log ((e\,x^2+d)^p\,c)}dx\right) g + \left(\int \frac{1}{\log ((e\,x^2+d)^p\,c)}dx\right) f$$

input
```
int((g*x^3+f)/log(c*(e*x^2+d)^p),x)
```

output
```
int(x**3/log((d + e*x**2)**p*c),x)*g + int(1/log((d + e*x**2)**p*c),x)*f
```

3.304 $\int \frac{1}{(f+gx^3)\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^3)\log(c(d+ex^2)^p)}\,dx = \operatorname{Int}\left(\frac{1}{(f+gx^3)\log(c(d+ex^2)^p)},x\right)$$

output
```
Defer(Int)(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 8.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(f+gx^3)\log(c(d+ex^2)^p)}\,dx$$

input
```
Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]
```

output
```
Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx^3)\log(c(d+ex^2)^p)}\,dx$$
$$\Big\downarrow \text{2923}$$
$$\int \frac{1}{(f+gx^3)\log(c(d+ex^2)^p)}dx$$

input
```
Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^3+f)\ln(c\,(e\,x^2+d)^p)}dx$$

input
```
int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

output
```
int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(gx^3+f)\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

input
```
integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 103.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

input
```
integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p),x)
```

output
```
Integral(1/((f + g*x**3)*log(c*(d + e*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^3+f)\log\left((ex^2+d)^pc\right)}\,dx$$

input
```
integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^3+f)\log\left((ex^2+d)^pc\right)}\,dx$$

input
```
integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 25.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\,\left(e\,x^2+d\right)^p\right)\,\left(g\,x^3+f\right)}\,dx$$

input
```
int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)),x)
```

output
```
int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{1}{(f+gx^3)\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{\log\left((e\,x^2+d)^p\,c\right)f+\log\left((e\,x^2+d)^p\,c\right)g\,x^3}dx$$

input
```
int(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x)
```

output
```
int(1/(log((d + e*x**2)**p*c)*f + log((d + e*x**2)**p*c)*g*x**3),x)
```

3.305 $\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} \, dx = \text{Int}\left(\frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)}, x\right)$$

output
```
Defer(Int)(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Mathematica [N/A]

Not integrable

Time = 11.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} \, dx = \int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} \, dx$$

input
```
Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
```

output
```
Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} \, dx$$
$$\big\downarrow \text{ 2923}$$
$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)}dx$$

input
```
Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^3+f)^2 \ln(c(e\,x^2+d)^p)}dx$$

input
```
int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

output
```
int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^3+f)^2 \log((ex^2+d)^p c)}\,dx$$

input
```
integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)}\,dx = \text{Timed out}$$

input
```
integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^3+f)^2 \log((ex^2+d)^p c)}\,dx$$

input
```
integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2\log(c(d+ex^2)^p)}\,dx=\int \frac{1}{(gx^3+f)^2\log((ex^2+d)^pc)}\,dx$$

input
```
integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 26.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2\log(c(d+ex^2)^p)}\,dx=\int \frac{1}{\ln\left(c\,(e\,x^2+d)^p\right)\,(g\,x^3+f)^2}\,dx$$

input
```
int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)^2),x)
```

output
```
int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{1}{(f+gx^3)^2\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{\log((e\,x^2+d)^p\,c)\,f^2+2\log((e\,x^2+d)^p\,c)\,fg\,x^3+\log((e\,x^2+d)^p\,c)\,g^2x^6}dx$$

input
```
int(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x)
```

output
```
int(1/(log((d + e*x**2)**p*c)*f**2 + 2*log((d + e*x**2)**p*c)*f*g*x**3 + l
og((d + e*x**2)**p*c)*g**2*x**6),x)
```

3.306 $\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)} \, dx = \text{Int}\left(\frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}, x\right)$$

output
```
Defer(Int)((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)} \, dx = \int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)} \, dx$$

input
```
Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}dx$$
$$\Big\downarrow \text{2923}$$
$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}dx$$

input
```
Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(g\,x^3+f)^2}{\ln\left(c\,(e\,x^2+d)^p\right)^2}dx$$

input
```
int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\,(d+ex^2)^p\right)}\,dx = \int \frac{(gx^3+f)^2}{\log\left((ex^2+d)^p c\right)^2}\,dx$$

input
```
integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 21.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\,(d+ex^2)^p\right)}\,dx = \int \frac{(f+gx^3)^2}{\log\left(c\,(d+ex^2)^p\right)^2}\,dx$$

input
```
integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f + g*x**3)**2/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 6.33

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(gx^3+f)^2}{\log\left((ex^2+d)^p c\right)^2}\,dx$$

input
```
integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(e*g^2*x^8 + d*g^2*x^6 + 2*e*f*g*x^5 + 2*d*f*g*x^3 + e*f^2*x^2 + d*f^
2)/(e*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + integrate(1/2*(7*e*g^2*x^8
+ 5*d*g^2*x^6 + 8*e*f*g*x^5 + 4*d*f*g*x^3 + e*f^2*x^2 - d*f^2)/(e*p*x^2*lo
g((e*x^2 + d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(gx^3+f)^2}{\log\left((ex^2+d)^p c\right)^2}\,dx$$

input
```
integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 26.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(g\,x^3+f)^2}{\ln\left(c\,(e\,x^2+d)^p\right)^2}\,dx$$

input
```
int((f + g*x^3)^2/log(c*(d + e*x^2)^p)^2,x)
```

output
```
int((f + g*x^3)^2/log(c*(d + e*x^2)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.88

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx = \left(\int \frac{x^6}{\log\left((e\,x^2+d)^p\,c\right)^2}dx\right) g^2 + 2\left(\int \frac{x^3}{\log\left((e\,x^2+d)^p\,c\right)^2}dx\right) fg + \left(\int \frac{1}{\log\left((e\,x^2+d)^p\,c\right)^2}dx\right) f^2$$

input
```
int((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x)
```

output
```
int(x**6/log((d + e*x**2)**p*c)**2,x)*g**2 + 2*int(x**3/log((d + e*x**2)**
p*c)**2,x)*f*g + int(1/log((d + e*x**2)**p*c)**2,x)*f**2
```

3.307 $\int \frac{f+gx^3}{\log^2(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \text{Int}\left(\frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

output
```
Defer(Int)((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

input
```
Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}dx$$

$$\Big\downarrow \text{2923}$$

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}dx$$

input
```
Int[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{g\,x^3+f}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}dx$$

input
```
int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^3+f}{\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

input
```
integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral((g*x^3 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 8.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

input
```
integrate((g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral((f + g*x**3)/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.73

$$\int \frac{f + gx^3}{\log^2 (c (d + ex^2)^p)} \, dx = \int \frac{gx^3 + f}{\log \left((ex^2 + d)^p c \right)^2} \, dx$$

input
```
integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(e*g*x^5 + d*g*x^3 + e*f*x^2 + d*f)/(e*p*x*log((e*x^2 + d)^p) + e*p*x
*log(c)) + integrate(1/2*(4*e*g*x^5 + 2*d*g*x^3 + e*f*x^2 - d*f)/(e*p*x^2*
log((e*x^2 + d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f + gx^3}{\log^2 (c (d + ex^2)^p)} \, dx = \int \frac{gx^3 + f}{\log \left((ex^2 + d)^p c \right)^2} \, dx$$

input
```
integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate((g*x^3 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 26.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f + gx^3}{\log^2 (c (d + ex^2)^p)} \, dx = \int \frac{g\,x^3 + f}{\ln \left(c\,(e\,x^2 + d)^p \right)^2} \, dx$$

input
```
int((f + g*x^3)/log(c*(d + e*x^2)^p)^2,x)
```

output
```
int((f + g*x^3)/log(c*(d + e*x^2)^p)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\left(\int \frac{x^3}{\log\left(\left(e\,x^2+d\right)^p c\right)^2}dx\right)g+\left(\int \frac{1}{\log\left(\left(e\,x^2+d\right)^p c\right)^2}dx\right)f$$

input
```
int((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x)
```

output
```
int(x**3/log((d + e*x**2)**p*c)**2,x)*g + int(1/log((d + e*x**2)**p*c)**2,
x)*f
```

3.308 $\int \frac{1}{(f+gx^3)\log^2(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\text{Int}\left(\frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

input
```
Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
```

output
```
Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}dx$$
$$\Big\downarrow \text{2923}$$
$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}dx$$

input
```
Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^3+f)\ln\left(c\left(e\,x^2+d\right)^p\right)^2}dx$$

input
```
int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(gx^3+f)\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 140.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

input
```
integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Integral(1/((f + g*x**3)*log(c*(d + e*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 6.62

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^3+f)\log\left((ex^2+d)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(e*x^2 + d)/(e*g*p*x^4*log(c) + e*f*p*x*log(c) + (e*g*p*x^4 + e*f*p*x
)*log((e*x^2 + d)^p)) - integrate(1/2*(2*e*g*x^5 + 4*d*g*x^3 - e*f*x^2 + d
*f)/(e*g^2*p*x^8*log(c) + 2*e*f*g*p*x^5*log(c) + e*f^2*p*x^2*log(c) + (e*g
^2*p*x^8 + 2*e*f*g*p*x^5 + e*f^2*p*x^2)*log((e*x^2 + d)^p)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^3+f)\log\left((ex^2+d)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 26.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{\ln\left(c\,(e\,x^2+d)^p\right)^2\,(g\,x^3+f)}\,dx$$

input
```
int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)),x)
```

output
```
int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{1}{(f+gx^3)\log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{\log\left((e\,x^2+d)^p\,c\right)^2 f+\log\left((e\,x^2+d)^p\,c\right)^2 g\,x^3}dx$$

input
```
int(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x)
```

output
```
int(1/(log((d + e*x**2)**p*c)**2*f + log((d + e*x**2)**p*c)**2*g*x**3),x)
```

3.309 $\int \frac{1}{(f+gx^3)^2 \log^2(c(d+ex^2)^p)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^3)^2 \log^2(c(d+ex^2)^p)} \, dx = \text{Int}\left(\frac{1}{(f+gx^3)^2 \log^2(c(d+ex^2)^p)}, x\right)$$

output
```
Defer(Int)(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2 \log^2(c(d+ex^2)^p)} \, dx = \int \frac{1}{(f+gx^3)^2 \log^2(c(d+ex^2)^p)} \, dx$$

input
```
Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]
```

output
```
Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx^3)^2 \log^2\left(c\left(d+ex^2\right)^p\right)} \, dx$$

$$\Big\downarrow \; 2923$$

$$\int \frac{1}{(f+gx^3)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}dx$$

input
```
Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(g\,x^{3}+f\right)^{2} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}dx$$

input
```
int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

output
```
int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{\left(f+gx^3\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{\left(gx^3+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

output
```
integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(f+gx^3\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 8.96

$$\int \frac{1}{\left(f+gx^3\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{\left(gx^3+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

output
```
-1/2*(e*x^2 + d)/(e*g^2*p*x^7*log(c) + 2*e*f*g*p*x^4*log(c) + e*f^2*p*x*lo
g(c) + (e*g^2*p*x^7 + 2*e*f*g*p*x^4 + e*f^2*p*x)*log((e*x^2 + d)^p)) - int
egrate(1/2*(5*e*g*x^5 + 7*d*g*x^3 - e*f*x^2 + d*f)/(e*g^3*p*x^11*log(c) +
3*e*f*g^2*p*x^8*log(c) + 3*e*f^2*g*p*x^5*log(c) + e*f^3*p*x^2*log(c) + (e*
g^3*p*x^11 + 3*e*f*g^2*p*x^8 + 3*e*f^2*g*p*x^5 + e*f^3*p*x^2)*log((e*x^2 +
 d)^p)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(gx^3+f)^2\log\left((ex^2+d)^pc\right)^2}\,dx$$

input
```
integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

output
```
integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 25.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^2\,{\left(g\,x^3+f\right)}^2}\,dx$$

input
```
int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2),x)
```

output
```
int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{1}{(f+gx^3)^2\log^2(c(d+ex^2)^p)}\,dx$$
$$= \int \frac{1}{\log\left((e\,x^2+d)^p\,c\right)^2 f^2+2\log\left((e\,x^2+d)^p\,c\right)^2 f g\,x^3+\log\left((e\,x^2+d)^p\,c\right)^2 g^2x^6}dx$$

input
```
int(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x)
```

output
```
int(1/(log((d + e*x**2)**p*c)**2*f**2 + 2*log((d + e*x**2)**p*c)**2*f*g*x*
*3 + log((d + e*x**2)**p*c)**2*g**2*x**6),x)
```

3.310 $\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{d^2(4ef-3dg)px^2}{24e^3}+\frac{d(4ef-3dg)px^4}{48e^2} - \frac{(4ef-3dg)px^6}{72e}-\frac{1}{32}gpx^8 + \frac{d^3(4ef-3dg)p\log(d+ex^2)}{24e^4} + \frac{1}{6}fx^6\log\left(c(d+ex^2)^p\right)+\frac{1}{8}gx^8\log\left(c(d+ex^2)^p\right)$$

output
```
-1/24*d^2*(-3*d*g+4*e*f)*p*x^2/e^3+1/48*d*(-3*d*g+4*e*f)*p*x^4/e^2-1/72*(-
3*d*g+4*e*f)*p*x^6/e-1/32*g*p*x^8+1/24*d^3*(-3*d*g+4*e*f)*p*ln(e*x^2+d)/e^
4+1/6*f*x^6*ln(c*(e*x^2+d)^p)+1/8*g*x^8*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{d^2fpx^2}{6e^2}+\frac{d^3gpx^2}{8e^3}+\frac{dfpx^4}{12e}-\frac{d^2gpx^4}{16e^2}-\frac{1}{18}fpx^6+\frac{dgpx^6}{24e}-\frac{1}{32}gpx^8+\frac{d^3fp\log(d+ex^2)}{6e^3}-\frac{d^4gp\log(d+ex^2)}{8e^4}+\frac{1}{6}fx^6\log\left(c(d+ex^2)^p\right)+\frac{1}{8}gx^8\log\left(c(d+ex^2)^p\right)$$

input

```
Integrate[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output

```
-1/6*(d^2*f*p*x^2)/e^2 + (d^3*g*p*x^2)/(8*e^3) + (d*f*p*x^4)/(12*e) - (d^2
*g*p*x^4)/(16*e^2) - (f*p*x^6)/18 + (d*g*p*x^6)/(24*e) - (g*p*x^8)/32 + (d
^3*f*p*Log[d + e*x^2])/(6*e^3) - (d^4*g*p*Log[d + e*x^2])/(8*e^4) + (f*x^6
*Log[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d + e*x^2)^p])/8
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2925, 2861, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int x^4(gx^2+f)\log\left(c(ex^2+d)^p\right)\,dx^2$$

$$\Big\downarrow \text{2861}$$

$$\frac{1}{2}\left(-ep\int \frac{x^6(3gx^2+4f)}{12(ex^2+d)}dx^2+\frac{1}{3}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^8\log\left(c\left(d+ex^2\right)^p\right)\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{1}{12}ep\int \frac{x^6(3gx^2+4f)}{ex^2+d}dx^2+\frac{1}{3}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^8\log\left(c\left(d+ex^2\right)^p\right)\right)$$

$\big\downarrow$ 86

$$\frac{1}{2}\left(-\frac{1}{12}ep\int \left(\frac{3gx^6}{e}+\frac{(4ef-3dg)x^4}{e^2}+\frac{d(3dg-4ef)x^2}{e^3}-\frac{d^2(3dg-4ef)}{e^4}+\frac{d^3(3dg-4ef)}{e^4(ex^2+d)}\right)dx^2+\frac{1}{3}fx^6\log\left(c\right.\right.$$

$\big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{3}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{12}ep\left(-\frac{d^3(4ef-3dg)\log\left(d+ex^2\right)}{e^5}+\frac{d^2x^2(4ef-3dg)}{e^4}\right.\right.$$

input
```
Int[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
(-1/12*(e*p*((d^2*(4*e*f - 3*d*g)*x^2)/e^4 - (d*(4*e*f - 3*d*g)*x^4)/(2*e^
3) + ((4*e*f - 3*d*g)*x^6)/(3*e^2) + (3*g*x^8)/(4*e) - (d^3*(4*e*f - 3*d*g
)*Log[d + e*x^2])/e^5)) + (f*x^6*Log[c*(d + e*x^2)^p])/3 + (g*x^8*Log[c*(d
 + e*x^2)^p])/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

method	result
parts	$\frac{g\,x^{8} \ln\left(c \left(e\,x^{2}+d\right)^{p}\right)}{8}+\frac{f\,x^{6} \ln\left(c \left(e\,x^{2}+d\right)^{p}\right)}{6}-\frac{ep\left(-\frac{-\frac{3}{4} g\,x^{8} e^{3}+d g\,x^{6} e^{2}-\frac{4}{3} e^{3} f\,x^{6}-\frac{3}{2} x^{4} d^{2} e g+2 x^{4} d\,e^{2} f+3 d^{3} g\,x^{2}-4 d^{2} e f\,x^{2}}{2 e^{4}}+\right.}{12}$
parallelrisch	$-\frac{-36 x^{8} \ln\left(c \left(e\,x^{2}+d\right)^{p}\right) e^{4} g+9 e^{4} g p\,x^{8}-48 x^{6} \ln\left(c \left(e\,x^{2}+d\right)^{p}\right) e^{4} f-12 d\,e^{3} g p\,x^{6}+16 e^{4} f p\,x^{6}+18 d^{2} e^{2} g p\,x^{4}-24 d\,e^{3} f p\,x^{4}-36 d}{288 e^{4}}$
risch	$\left(\frac{1}{8} g\,x^{8}+\frac{1}{6} f\,x^{6}\right) \ln\left(\left(e\,x^{2}+d\right)^{p}\right)-\frac{i \pi g\,x^{8} \operatorname{csgn}\left(i c \left(e\,x^{2}+d\right)^{p}\right)^{3}}{16}-\frac{i \pi f\,x^{6} \operatorname{csgn}\left(i \left(e\,x^{2}+d\right)^{p}\right) \operatorname{csgn}\left(i c \left(e\,x^{2}+d\right)^{p}\right)}{12}$

input
```
int(x^5*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/8*g*x^8*ln(c*(e*x^2+d)^p)+1/6*f*x^6*ln(c*(e*x^2+d)^p)-1/12*e*p*(-1/2/e^4
*(-3/4*g*x^8*e^3+d*g*x^6*e^2-4/3*e^3*f*x^6-3/2*x^4*d^2*e*g+2*x^4*d*e^2*f+3
*d^3*g*x^2-4*d^2*e*f*x^2)+1/2*d^3*(3*d*g-4*e*f)/e^5*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{9\,e^4gpx^8+4\,(4\,e^4f-3\,de^3g)px^6-6\,(4\,de^3f-3\,d^2e^2g)px^4+12\,(4\,d^2e^2f-3\,d^3eg)px^2-12\,(3\,e^4gpx^8}{288\,e^4}$$

input
```
integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
-1/288*(9*e^4*g*p*x^8 + 4*(4*e^4*f - 3*d*e^3*g)*p*x^6 - 6*(4*d*e^3*f - 3*d
^2*e^2*g)*p*x^4 + 12*(4*d^2*e^2*f - 3*d^3*e*g)*p*x^2 - 12*(3*e^4*g*p*x^8 +
 4*e^4*f*p*x^6 + (4*d^3*e*f - 3*d^4*g)*p)*log(e*x^2 + d) - 12*(3*e^4*g*x^8
 + 4*e^4*f*x^6)*log(c))/e^4
```

Sympy [F(-1)]

Timed out.

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \text{Timed out}$$

input
```
integrate(x**5*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{1}{288}\,ep\left(\frac{9\,e^3gx^8+4\,(4\,e^3f-3\,de^2g)x^6-6\,(4\,de^2f-3\,d^2eg)x^4+12\,(4\,d^2ef-3\,d^3g)x^2}{e^4}-\frac{12\,(4\,d^3ef-}{}\right.$$
$$+\frac{1}{24}\left(3\,gx^8+4\,fx^6\right)\log\left(\left(ex^2+d\right)^pc\right)$$

input
```
integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
-1/288*e*p*((9*e^3*g*x^8 + 4*(4*e^3*f - 3*d*e^2*g)*x^6 - 6*(4*d*e^2*f - 3*
d^2*e*g)*x^4 + 12*(4*d^2*e*f - 3*d^3*g)*x^2)/e^4 - 12*(4*d^3*e*f - 3*d^4*g
)*log(e*x^2 + d)/e^5) + 1/24*(3*g*x^8 + 4*f*x^6)*log((e*x^2 + d)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(128) = 256$.

Time = 0.13 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.81

$$\int x^5 (f + gx^2) \log \left(c(d + ex^2)^p\right) \, dx$$
$$= \frac{(ex^2+d)^3 fp \log(ex^2+d)}{6\,e^3} - \frac{(ex^2+d)^2 dfp \log(ex^2+d)}{2\,e^3} + \frac{(ex^2+d)^4 gp \log(ex^2+d)}{8\,e^4} - \frac{(ex^2+d)^3 dgp \log(ex^2+d)}{2\,e^4} + \frac{3\,(ex^2+d)^2 d^2 gp \log(ex^2+d)}{4\,e^4} - \frac{(ex^2+d)^3 fp}{18\,e^3} + \frac{(ex^2+d)^2 dfp}{4\,e^3} - \frac{(ex^2+d)^4 gp}{32\,e^4} + \frac{(ex^2+d)^3 dgp}{6\,e^4} - \frac{3\,(ex^2+d)^2 d^2 gp}{8\,e^4} + \frac{(ex^2+d)^3 f \log(c)}{6\,e^3} - \frac{(ex^2+d)^2 df \log(c)}{2\,e^3} + \frac{(ex^2+d)^4 g \log(c)}{8\,e^4} - \frac{(ex^2+d)^3 dg \log(c)}{2\,e^4} + \frac{3\,(ex^2+d)^2 d^2 g \log(c)}{4\,e^4} - \frac{(ex^2 - (ex^2+d)\log(ex^2+d) + d)d^2 efp - (ex^2 - (ex^2+d)\log(ex^2+d) + d)d^3 gp - (ex^2+d)d^2 ef\,\mathrm{l}}{2\,e^4}$$

input
```
integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
1/6*(e*x^2 + d)^3*f*p*log(e*x^2 + d)/e^3 - 1/2*(e*x^2 + d)^2*d*f*p*log(e*x
^2 + d)/e^3 + 1/8*(e*x^2 + d)^4*g*p*log(e*x^2 + d)/e^4 - 1/2*(e*x^2 + d)^3
*d*g*p*log(e*x^2 + d)/e^4 + 3/4*(e*x^2 + d)^2*d^2*g*p*log(e*x^2 + d)/e^4 -
 1/18*(e*x^2 + d)^3*f*p/e^3 + 1/4*(e*x^2 + d)^2*d*f*p/e^3 - 1/32*(e*x^2 +
d)^4*g*p/e^4 + 1/6*(e*x^2 + d)^3*d*g*p/e^4 - 3/8*(e*x^2 + d)^2*d^2*g*p/e^4
 + 1/6*(e*x^2 + d)^3*f*log(c)/e^3 - 1/2*(e*x^2 + d)^2*d*f*log(c)/e^3 + 1/8
*(e*x^2 + d)^4*g*log(c)/e^4 - 1/2*(e*x^2 + d)^3*d*g*log(c)/e^4 + 3/4*(e*x^
2 + d)^2*d^2*g*log(c)/e^4 - 1/2*((e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*
d^2*e*f*p - (e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^3*g*p - (e*x^2 + d)
*d^2*e*f*log(c) + (e*x^2 + d)*d^3*g*log(c))/e^4
```

Mupad [B] (verification not implemented)

Time = 26.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^8}{8}+\frac{f\,x^6}{6}\right) - x^6\left(\frac{f\,p}{18}-\frac{d\,g\,p}{24\,e}\right)-\frac{g\,p\,x^8}{32} - \frac{\ln\left(e\,x^2+d\right)\,\left(3\,d^4\,g\,p-4\,d^3\,e\,f\,p\right)}{24\,e^4} + \frac{d\,x^4\left(\frac{f\,p}{3}-\frac{d\,g\,p}{4\,e}\right)}{4\,e}-\frac{d^2\,x^2\left(\frac{f\,p}{3}-\frac{d\,g\,p}{4\,e}\right)}{2\,e^2}$$

input
```
int(x^5*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

output
```
log(c*(d + e*x^2)^p)*((f*x^6)/6 + (g*x^8)/8) - x^6*((f*p)/18 - (d*g*p)/(24
*e)) - (g*p*x^8)/32 - (log(d + e*x^2)*(3*d^4*g*p - 4*d^3*e*f*p))/(24*e^4)
+ (d*x^4*((f*p)/3 - (d*g*p)/(4*e)))/(4*e) - (d^2*x^2*((f*p)/3 - (d*g*p)/(4
*e)))/(2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.15

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$= \frac{-36\log\left((e\,x^2+d)^p\,c\right)d^4g+48\log\left((e\,x^2+d)^p\,c\right)d^3ef+48\log\left((e\,x^2+d)^p\,c\right)e^4f\,x^6+36\log\left((e\,x^2+d)^p\right.}{28}$$

input
```
int(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x)
```

output
```
( - 36*log((d + e*x**2)**p*c)*d**4*g + 48*log((d + e*x**2)**p*c)*d**3*e*f
+ 48*log((d + e*x**2)**p*c)*e**4*f*x**6 + 36*log((d + e*x**2)**p*c)*e**4*g
*x**8 + 36*d**3*e*g*p*x**2 - 48*d**2*e**2*f*p*x**2 - 18*d**2*e**2*g*p*x**4
 + 24*d*e**3*f*p*x**4 + 12*d*e**3*g*p*x**6 - 16*e**4*f*p*x**6 - 9*e**4*g*p
*x**8)/(288*e**4)
```

3.311 $\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \frac{d(3ef-2dg)px^2}{12e^2}-\frac{(3ef-2dg)px^4}{24e}-\frac{1}{18}gpx^6-\frac{d^2(3ef-2dg)p\log(d+ex^2)}{12e^3}+\frac{1}{4}fx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{6}gx^6\log\left(c(d+ex^2)^p\right)$$

output

```
1/12*d*(-2*d*g+3*e*f)*p*x^2/e^2-1/24*(-2*d*g+3*e*f)*p*x^4/e-1/18*g*p*x^6-1
/12*d^2*(-2*d*g+3*e*f)*p*ln(e*x^2+d)/e^3+1/4*f*x^4*ln(c*(e*x^2+d)^p)+1/6*g
*x^6*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \frac{dfpx^2}{4e}-\frac{d^2gpx^2}{6e^2}-\frac{1}{8}fpx^4+\frac{dgpx^4}{12e}-\frac{1}{18}gpx^6-\frac{d^2fp\log(d+ex^2)}{4e^2}+\frac{d^3gp\log(d+ex^2)}{6e^3}+\frac{1}{4}fx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{6}gx^6\log\left(c(d+ex^2)^p\right)$$

input
```
Integrate[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
(d*f*p*x^2)/(4*e) - (d^2*g*p*x^2)/(6*e^2) - (f*p*x^4)/8 + (d*g*p*x^4)/(12*
e) - (g*p*x^6)/18 - (d^2*f*p*Log[d + e*x^2])/(4*e^2) + (d^3*g*p*Log[d + e*
x^2])/(6*e^3) + (f*x^4*Log[c*(d + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^
p])/6
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2925, 2861, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int x^2(gx^2+f)\log\left(c(ex^2+d)^p\right)dx^2$$

$$\Big\downarrow \text{2861}$$

$$\frac{1}{2}\left(-ep\int \frac{x^4(2gx^2+3f)}{6(ex^2+d)}dx^2+\frac{1}{2}fx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{3}gx^6\log\left(c(d+ex^2)^p\right)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{1}{6}ep\int \frac{x^4(2gx^2+3f)}{ex^2+d}dx^2+\frac{1}{2}fx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{3}gx^6\log\left(c(d+ex^2)^p\right)\right)$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\left(-\frac{1}{6}ep\int \left(\frac{2gx^4}{e}+\frac{(3ef-2dg)x^2}{e^2}+\frac{d(2dg-3ef)}{e^3}-\frac{d^2(2dg-3ef)}{e^3(ex^2+d)}\right)dx^2+\frac{1}{2}fx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{3}gx^6 \mathrm{l}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{6}ep\left(\frac{d^2(3ef-2dg)\log\left(d+ex^2\right)}{e^4}-\frac{dx^2(3ef-2dg)}{e^3}+\right.\right.$$

input
```
Int[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
(-1/6*(e*p*(-((d*(3*e*f - 2*d*g)*x^2)/e^3) + ((3*e*f - 2*d*g)*x^4)/(2*e^2)
 + (2*g*x^6)/(3*e) + (d^2*(3*e*f - 2*d*g)*Log[d + e*x^2])/e^4)) + (f*x^4*L
og[c*(d + e*x^2)^p])/2 + (g*x^6*Log[c*(d + e*x^2)^p])/3)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

method	result
parts	$\frac{g x^6 \ln\left(c\left(e x^2+d\right)^p\right)}{6}+\frac{f x^4 \ln\left(c\left(e x^2+d\right)^p\right)}{4}-\frac{ep\left(\frac{\frac{2}{3}e^2 g x^6-x^4 dge+\frac{3}{2} f x^4 e^2+2d^2 g x^2-3def x^2}{2e^3}-\frac{d^2(2dg-3ef)\ln\left(e x^2+d\right)}{2e^4}\right)}{6}$
parallelrisch	$\frac{12x^6 \ln\left(c\left(e x^2+d\right)^p\right)e^3g-4x^6e^3gp+18x^4\ln\left(c\left(e x^2+d\right)^p\right)e^3f+6x^4de^2gp-9x^4e^3fp-12x^2d^2egp+18x^2de^2fp+12\ln\left(e x^2+d\right)}{72e^3}$
risch	$\left(\frac{1}{6}g x^6+\frac{1}{4}f x^4\right)\ln\left(\left(e x^2+d\right)^p\right)+\frac{i\pi f x^4 \,\mathrm{csgn}\left(i\left(e x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e x^2+d\right)^p\right)^2}{8}+\frac{i\pi g x^6 \,\mathrm{csgn}\left(ic\left(e x^2+d\right)^p\right)^2}{12}$

input
```
int(x^3*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/6*g*x^6*ln(c*(e*x^2+d)^p)+1/4*f*x^4*ln(c*(e*x^2+d)^p)-1/6*e*p*(1/2/e^3*(
2/3*e^2*g*x^6-x^4*d*g*e+3/2*f*x^4*e^2+2*d^2*g*x^2-3*d*e*f*x^2)-1/2*d^2*(2*
d*g-3*e*f)/e^4*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int x^3\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = -\frac{4\,e^3gpx^6+3\left(3\,e^3f-2\,de^2g\right)px^4-6\left(3\,de^2f-2\,d^2eg\right)px^2-6\left(2\,e^3gpx^6+3\,e^3fpx^4-\left(3\,d^2ef-2\,d^3g\right.\right.}{72\,e^3}$$

input
```
integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
-1/72*(4*e^3*g*p*x^6 + 3*(3*e^3*f - 2*d*e^2*g)*p*x^4 - 6*(3*d*e^2*f - 2*d^
2*e*g)*p*x^2 - 6*(2*e^3*g*p*x^6 + 3*e^3*f*p*x^4 - (3*d^2*e*f - 2*d^3*g)*p)
*log(e*x^2 + d) - 6*(2*e^3*g*x^6 + 3*e^3*f*x^4)*log(c))/e^3
```

Sympy [A] (verification not implemented)

Time = 63.91 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \begin{cases} \frac{d^3g\log\left(c(d+ex^2)^p\right)}{6e^3} - \frac{d^2f\log\left(c(d+ex^2)^p\right)}{4e^2} - \frac{d^2gpx^2}{6e^2} + \frac{dfpx^2}{4e} + \frac{dgpx^4}{12e} - \frac{fpx^4}{8} + \frac{fx^4\log\left(c(d+ex^2)^p\right)}{4} - \frac{gpx^6}{18} + \frac{gx^6\log\left(c(\right.}{6} \\ \left(\frac{fx^4}{4} + \frac{gx^6}{6}\right)\log\left(cd^p\right) \end{cases}$$

input
```
integrate(x**3*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise((d**3*g*log(c*(d + e*x**2)**p)/(6*e**3) - d**2*f*log(c*(d + e*x*
*2)**p)/(4*e**2) - d**2*g*p*x**2/(6*e**2) + d*f*p*x**2/(4*e) + d*g*p*x**4/
(12*e) - f*p*x**4/8 + f*x**4*log(c*(d + e*x**2)**p)/4 - g*p*x**6/18 + g*x*
*6*log(c*(d + e*x**2)**p)/6, Ne(e, 0)), ((f*x**4/4 + g*x**6/6)*log(c*d**p)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{1}{72}\,ep\left(\frac{4\,e^2gx^6+3\,(3\,e^2f-2\,deg)x^4-6\,(3\,def-2\,d^2g)x^2}{e^3}+\frac{6\,(3\,d^2ef-2\,d^3g)\log\left(ex^2+d\right)}{e^4}\right) + \frac{1}{12}\left(2\,gx^6+3\,fx^4\right)\log\left(\left(ex^2+d\right)^pc\right)$$

input
```
integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
-1/72*e*p*((4*e^2*g*x^6 + 3*(3*e^2*f - 2*d*e*g)*x^4 - 6*(3*d*e*f - 2*d^2*g
)*x^2)/e^3 + 6*(3*d^2*e*f - 2*d^3*g)*log(e*x^2 + d)/e^4) + 1/12*(2*g*x^6 +
 3*f*x^4)*log((e*x^2 + d)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(107) = 214$.

Time = 0.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.26

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$= \frac{(ex^2+d)^2 fp\log(ex^2+d)}{4e^2} + \frac{(ex^2+d)^3 gp\log(ex^2+d)}{6e^3} - \frac{(ex^2+d)^2 dgp\log(ex^2+d)}{2e^3} - \frac{(ex^2+d)^2 fp}{8e^2} - \frac{(ex^2+d)^3 gp}{18e^3} + \frac{(ex^2+d)^2 dgp}{4e^3} + \frac{(ex^2+d)^2 f\log(c)}{4e^2} + \frac{(ex^2+d)^3 g\log(c)}{6e^3} - \frac{(ex^2+d)^2 dg\log(c)}{2e^3} + \frac{(ex^2-(ex^2+d)\log(ex^2+d)+d)defp - (ex^2-(ex^2+d)\log(ex^2+d)+d)d^2gp - (ex^2+d)def\log}{2e^3}$$

input
```
integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
1/4*(e*x^2 + d)^2*f*p*log(e*x^2 + d)/e^2 + 1/6*(e*x^2 + d)^3*g*p*log(e*x^2
 + d)/e^3 - 1/2*(e*x^2 + d)^2*d*g*p*log(e*x^2 + d)/e^3 - 1/8*(e*x^2 + d)^2
*f*p/e^2 - 1/18*(e*x^2 + d)^3*g*p/e^3 + 1/4*(e*x^2 + d)^2*d*g*p/e^3 + 1/4*
(e*x^2 + d)^2*f*log(c)/e^2 + 1/6*(e*x^2 + d)^3*g*log(c)/e^3 - 1/2*(e*x^2 +
 d)^2*d*g*log(c)/e^3 + 1/2*((e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d*e*f
*p - (e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^2*g*p - (e*x^2 + d)*d*e*f*
log(c) + (e*x^2 + d)*d^2*g*log(c))/e^3
```

Mupad [B] (verification not implemented)

Time = 25.94 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{g\,x^6}{6}+\frac{f\,x^4}{4}\right) - x^4\left(\frac{f\,p}{8}-\frac{d\,g\,p}{12\,e}\right)-\frac{g\,p\,x^6}{18} + \frac{\ln\left(e\,x^2+d\right)\,\left(2\,d^3\,g\,p-3\,d^2\,e\,f\,p\right)}{12\,e^3} + \frac{d\,x^2\left(\frac{f\,p}{2}-\frac{d\,g\,p}{3\,e}\right)}{2\,e}$$

input
```
int(x^3*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

output
```
log(c*(d + e*x^2)^p)*((f*x^4)/4 + (g*x^6)/6) - x^4*((f*p)/8 - (d*g*p)/(12*
e)) - (g*p*x^6)/18 + (log(d + e*x^2)*(2*d^3*g*p - 3*d^2*e*f*p))/(12*e^3) +
 (d*x^2*((f*p)/2 - (d*g*p)/(3*e)))/(2*e)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.16

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \frac{12\log\left((e\,x^2+d)^p\,c\right)d^3g-18\log\left((e\,x^2+d)^p\,c\right)d^2ef+18\log\left((e\,x^2+d)^p\,c\right)e^3f\,x^4+12\log\left((e\,x^2+d)^p\,c\right.}{72e^3}$$

input
```
int(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x)
```

output
```
(12*log((d + e*x**2)**p*c)*d**3*g - 18*log((d + e*x**2)**p*c)*d**2*e*f + 1
8*log((d + e*x**2)**p*c)*e**3*f*x**4 + 12*log((d + e*x**2)**p*c)*e**3*g*x*
*6 - 12*d**2*e*g*p*x**2 + 18*d*e**2*f*p*x**2 + 6*d*e**2*g*p*x**4 - 9*e**3*
f*p*x**4 - 4*e**3*g*p*x**6)/(72*e**3)
```

3.312 $\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=-\frac{(ef-dg)px^2}{4e}-\frac{p(f+gx^2)^2}{8g}-\frac{(ef-dg)^2p\log(d+ex^2)}{4e^2g}+\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4g}$$

output
```
-1/4*(-d*g+e*f)*p*x^2/e-1/8*p*(g*x^2+f)^2/g-1/4*(-d*g+e*f)^2*p*ln(e*x^2+d)
/e^2/g+1/4*(g*x^2+f)^2*ln(c*(e*x^2+d)^p)/g
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4-\frac{d^2gp\log(d+ex^2)}{4e^2}+\frac{1}{4}gx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{2}f\left(-px^2+\frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e}\right)$$

input
```
Integrate[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
(d*g*p*x^2)/(4*e) - (g*p*x^4)/8 - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + (g*x^
4*Log[c*(d + e*x^2)^p])/4 + (f*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^
p])/e))/2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2925, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int (gx^2+f)\log\left(c(ex^2+d)^p\right)dx^2$$

$$\Big\downarrow \text{2842}$$

$$\frac{1}{2}\left(\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{2g}-\frac{ep\int \frac{(gx^2+f)^2}{ex^2+d}dx^2}{2g}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{2g}-\frac{ep\int \left(\frac{(ef-dg)^2}{e^2(ex^2+d)}+\frac{g(ef-dg)}{e^2}+\frac{g(gx^2+f)}{e}\right)dx^2}{2g}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{2g}-\frac{ep\left(\frac{(ef-dg)^2\log(d+ex^2)}{e^3}+\frac{gx^2(ef-dg)}{e^2}+\frac{(f+gx^2)^2}{2e}\right)}{2g}\right)$$

input
```
Int[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
(-1/2*(e*p*((g*(e*f - d*g)*x^2)/e^2 + (f + g*x^2)^2/(2*e) + ((e*f - d*g)^2
*Log[d + e*x^2])/e^3))/g + ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(2*g))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.35

method	result
parts	$\frac{g\,x^4 \ln\left(c\left(e\,x^2+d\right)^p\right)}{4}+\frac{\ln\left(c\left(e\,x^2+d\right)^p\right) f\,x^2}{2}+\frac{\ln\left(c\left(e\,x^2+d\right)^p\right) f^2}{4g}-\frac{pe\left(-\frac{g\left(-\frac{1}{2}eg\,x^4+dg\,x^2-2f\,x^2e\right)}{2e^2}+\frac{\left(d^2g^2-2fged+f^2e^2\right)\dots}{2e^3}\right)}{2g}$
parallelrisch	$-\frac{-2x^4\ln\left(c\left(e\,x^2+d\right)^p\right)e^2g+gp\,x^4e^2-4x^2\ln\left(c\left(e\,x^2+d\right)^p\right)e^2f-2x^2dgpe+4x^2e^2fp+2\ln\left(e\,x^2+d\right)d^2gp-8\ln\left(e\,x^2+d\right)defp+4\dots}{8e^2}$
risch	Expression too large to display

input
```
int(x*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/4*g*x^4*ln(c*(e*x^2+d)^p)+1/2*ln(c*(e*x^2+d)^p)*f*x^2+1/4*ln(c*(e*x^2+d)
^p)/g*f^2-1/2/g*p*e*(-1/2*g/e^2*(-1/2*e*g*x^4+d*g*x^2-2*f*x^2*e)+1/2*(d^2*
g^2-2*d*e*f*g+e^2*f^2)/e^3*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{e^2gpx^4+2\,(2\,e^2f-deg)px^2-2\,(e^2gpx^4+2\,e^2fpx^2+(2\,def-d^2g)p)\log\left(ex^2+d\right)-2\,(e^2gx^4+2\,e^2\dots}{8\,e^2}$$

input
```
integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
-1/8*(e^2*g*p*x^4 + 2*(2*e^2*f - d*e*g)*p*x^2 - 2*(e^2*g*p*x^4 + 2*e^2*f*p
*x^2 + (2*d*e*f - d^2*g)*p)*log(e*x^2 + d) - 2*(e^2*g*x^4 + 2*e^2*f*x^2)*l
og(c))/e^2
```

Sympy [A] (verification not implemented)

Time = 16.94 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\begin{cases} -\frac{d^2g\log\left(c(d+ex^2)^p\right)}{4e^2}+\frac{df\log\left(c(d+ex^2)^p\right)}{2e}+\frac{dgpx^2}{4e}-\frac{fpx^2}{2}+\frac{fx^2\log\left(c(d+ex^2)^p\right)}{2}-\frac{gpx^4}{8}+\frac{gx^4\log\left(c(d+ex^2)^p\right)}{4} & \text{for } e \neq 0 \\ \left(\frac{fx^2}{2}+\frac{gx^4}{4}\right)\log\left(cd^p\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise((-d**2*g*log(c*(d + e*x**2)**p)/(4*e**2) + d*f*log(c*(d + e*x**2
)**p)/(2*e) + d*g*p*x**2/(4*e) - f*p*x**2/2 + f*x**2*log(c*(d + e*x**2)**p
)/2 - g*p*x**4/8 + g*x**4*log(c*(d + e*x**2)**p)/4, Ne(e, 0)), ((f*x**2/2
+ g*x**4/4)*log(c*d**p), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=-\frac{ep\left(\frac{eg^2x^4+2\,(2\,efg-dg^2)x^2}{e^2}+\frac{2\,(e^2f^2-2\,defg+d^2g^2)\log(ex^2+d)}{e^3}\right)}{8\,g}+\frac{(gx^2+f)^2\log\left((ex^2+d)^pc\right)}{4\,g}$$

input
```
integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
-1/8*e*p*((e*g^2*x^4 + 2*(2*e*f*g - d*g^2)*x^2)/e^2 + 2*(e^2*f^2 - 2*d*e*f
*g + d^2*g^2)*log(e*x^2 + d)/e^3)/g + 1/4*(g*x^2 + f)^2*log((e*x^2 + d)^p*
c)/g
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \frac{2\,(ex^2+d)^2gp\log\left(ex^2+d\right)-(ex^2+d)^2gp+2\,(ex^2+d)^2g\log\left(c\right)}{8\,e^2} - \frac{(ex^2-(ex^2+d)\log\left(ex^2+d\right)+d)efp-(ex^2-(ex^2+d)\log\left(ex^2+d\right)+d)dgp-(ex^2+d)ef\log\left(c\right)}{2\,e^2}$$

input
```
integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
1/8*(2*(e*x^2 + d)^2*g*p*log(e*x^2 + d) - (e*x^2 + d)^2*g*p + 2*(e*x^2 + d
)^2*g*log(c))/e^2 - 1/2*((e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*e*f*p -
(e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d*g*p - (e*x^2 + d)*e*f*log(c) +
(e*x^2 + d)*d*g*log(c))/e^2
```

Mupad [B] (verification not implemented)

Time = 25.95 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^4}{4}+\frac{f\,x^2}{2}\right)-x^2\,\left(\frac{f\,p}{2}-\frac{d\,g\,p}{4\,e}\right) - \frac{g\,p\,x^4}{8}-\frac{\ln\left(e\,x^2+d\right)\,\left(d^2\,g\,p-2\,d\,e\,f\,p\right)}{4\,e^2}$$

input
```
int(x*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

output
```
log(c*(d + e*x^2)^p)*((f*x^2)/2 + (g*x^4)/4) - x^2*((f*p)/2 - (d*g*p)/(4*e
)) - (g*p*x^4)/8 - (log(d + e*x^2)*(d^2*g*p - 2*d*e*f*p))/(4*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\frac{-2\log\left((e\,x^2+d)^p\,c\right)d^2g+4\log\left((e\,x^2+d)^p\,c\right)def+4\log\left((e\,x^2+d)^p\,c\right)e^2f\,x^2+2\log\left((e\,x^2+d)^p\,c\right)e^2}{8e^2}$$

input
```
int(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x)
```

output
```
( - 2*log((d + e*x**2)**p*c)*d**2*g + 4*log((d + e*x**2)**p*c)*d*e*f + 4*l
og((d + e*x**2)**p*c)*e**2*f*x**2 + 2*log((d + e*x**2)**p*c)*e**2*g*x**4 +
 2*d*e*g*p*x**2 - 4*e**2*f*p*x**2 - e**2*g*p*x**4)/(8*e**2)
```

3.313 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = -\frac{1}{2}gpx^2 + \frac{g(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e} + \frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}fp\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)$$

output
```
-1/2*g*p*x^2+1/2*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+1/2*f*ln(-e*x^2/d)*ln(c*(
e*x^2+d)^p)+1/2*f*p*polylog(2,1+e*x^2/d)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \frac{1}{2}g\left(-px^2 + \frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e}\right) + \frac{1}{2}f\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + p\,\text{PolyLog}\left(2,\frac{d+ex^2}{d}\right)\right)$$

input
```
Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

output
```
(g*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/2 + (f*(Log[-((e*x^2
)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*x^2)/d]))/2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)\log(c(d+ex^2)^p)}{x}\,dx$$

$$\Big\downarrow \text{ 2925}$$

$$\frac{1}{2}\int \frac{(gx^2+f)\log(c(ex^2+d)^p)}{x^2}dx^2$$

$$\Big\downarrow \text{ 2863}$$

$$\frac{1}{2}\int \left(g\log(c(ex^2+d)^p)+\frac{f\log(c(ex^2+d)^p)}{x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(f\log\left(-\frac{ex^2}{d}\right)\log(c(d+ex^2)^p)+\frac{g(d+ex^2)\log(c(d+ex^2)^p)}{e}+fp\,\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-gpx^2\right)$$

input
```
Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

output
```
(-(g*p*x^2) + (g*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e + f*Log[-((e*x^2)/d)]
*Log[c*(d + e*x^2)^p] + f*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(74) = 148$.

Time = 0.81 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.91

method	result
parts	$\frac{x^2 g \ln\left(c\left(e\,x^2+d\right)^p\right)}{2}+\ln\left(c\left(e\,x^2+d\right)^p\right) f \ln\left(x\right)-ep\left(\frac{g\,x^2}{2e}-\frac{gd\ln\left(e\,x^2+d\right)}{2e^2}+2f\left(\frac{\ln(x)\left(\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)+\ln\left(\right.\right.}{2e}\right.\right.$
risch	$\frac{\ln\left(\left(e\,x^2+d\right)^p\right) g\,x^2}{2}+\ln\left(\left(e\,x^2+d\right)^p\right) f \ln\left(x\right)-\frac{gp\,x^2}{2}+\frac{pgd\ln\left(e\,x^2+d\right)}{2e}-pf\ln\left(x\right)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)-pf\,\mathrm{l}$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*g*ln(c*(e*x^2+d)^p)+ln(c*(e*x^2+d)^p)*f*ln(x)-e*p*(1/2*g*x^2/e-1/2
*g*d/e^2*ln(e*x^2+d)+2*f*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+
ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*
e)^(1/2))+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x}\,dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
```

output
```
integral((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x,x)
```

output
```
Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x}\,dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
```

output
```
integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

Giac [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx=\int \frac{(gx^2+f)\log\left((ex^2+d)^pc\right)}{x}\,dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx=\int \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,(g\,x^2+f)}{x}\,dx$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x,x)
```

output
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx$$
$$=\frac{4\left(\int\frac{\log\left((e\,x^2+d)^pc\right)}{e\,x^3+dx}dx\right)defp+\log\left((e\,x^2+d)^p\,c\right)^2ef+2\log\left((e\,x^2+d)^p\,c\right)dgp+2\log\left((e\,x^2+d)^p\,c\right)egp\,x}{4ep}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x)
```

output
```
(4*int(log((d + e*x**2)**p*c)/(d*x + e*x**3),x)*d*e*f*p + log((d + e*x**2)
**p*c)**2*e*f + 2*log((d + e*x**2)**p*c)*d*g*p + 2*log((d + e*x**2)**p*c)*
e*g*p*x**2 - 2*e*g*p**2*x**2)/(4*e*p)
```

3.314 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx=\frac{efp\log(x)}{d}-\frac{efp\log\left(d+ex^2\right)}{2d}-\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{1}{2}gp\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)$$

output
```
e*f*p*ln(x)/d-1/2*e*f*p*ln(e*x^2+d)/d-1/2*f*ln(c*(e*x^2+d)^p)/x^2+1/2*g*ln
(-e*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*g*p*polylog(2,1+e*x^2/d)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx=\frac{efp\log(x)}{d}-\frac{efp\log\left(d+ex^2\right)}{2d}-\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+p\,\text{PolyLog}\left(2,\frac{d+ex^2}{d}\right)\right)$$

input
```
Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```

output
```
(e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])
/(2*x^2) + (g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d +
e*x^2)/d]))/2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int \frac{(gx^2+f)\log\left(c(ex^2+d)^p\right)}{x^4}dx^2$$

$$\Big\downarrow \text{2863}$$

$$\frac{1}{2}\int \left(\frac{g\log\left(c(ex^2+d)^p\right)}{x^2}+\frac{f\log\left(c(ex^2+d)^p\right)}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{f\log\left(c(d+ex^2)^p\right)}{x^2}+g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{efp\log(x^2)}{d}-\frac{efp\log(d+ex^2)}{d}+gp\,\text{PolyLog}\left(\right.\right.$$

input
```
Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```

output
```
((e*f*p*Log[x^2])/d - (e*f*p*Log[d + e*x^2])/d - (f*Log[c*(d + e*x^2)^p])/
x^2 + g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + g*p*PolyLog[2, 1 + (e*x^2
)/d])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.67

method	result
parts	$-\frac{f \ln\left(c \left(e\,x^{2}+d\right)^{p}\right)}{2 x^{2}}+\ln\left(c \left(e\,x^{2}+d\right)^{p}\right) g \ln\left(x\right)-e p \left(\frac{f \ln\left(e\,x^{2}+d\right)}{2 d}-\frac{f \ln\left(x\right)}{d}+2 g \left(\frac{\ln\left(x\right) \left(\ln\left(\frac{-e x+\sqrt{-d e}}{\sqrt{-d e}}\right)+\ln\left(\right.\right.}{2 e}\right.\right.$
risch	$\ln\left(\left(e\,x^{2}+d\right)^{p}\right) g \ln\left(x\right)-\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right) f}{2 x^{2}}-\frac{e f p \ln\left(e\,x^{2}+d\right)}{2 d}+\frac{e f p \ln\left(x\right)}{d}-p g \ln\left(x\right) \ln\left(\frac{-e x+\sqrt{-d e}}{\sqrt{-d e}}\right)-p g$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f*ln(c*(e*x^2+d)^p)/x^2+ln(c*(e*x^2+d)^p)*g*ln(x)-e*p*(1/2*f/d*ln(e*x
^2+d)-f/d*ln(x)+2*g*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e
*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1
/2))+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx=\int \frac{(gx^2+f)\log\left((ex^2+d)^pc\right)}{x^3}\,dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

output
```
integral((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

Sympy [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx=\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**3,x)
```

output
```
Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x**3, x)
```

Maxima [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx=\int \frac{(gx^2+f)\log\left((ex^2+d)^pc\right)}{x^3}\,dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
```

output
```
integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

Giac [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \int \frac{(gx^2+f)\log\left((ex^2+d)^pc\right)}{x^3}\,dx$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,(g\,x^2+f)}{x^3}\,dx$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^3,x)
```

output
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx$$
$$= \frac{-4\left(\int \frac{\log\left((e\,x^2+d)^p c\right)}{e\,x^5+d\,x^3}dx\right) d^3gp\,x^2 + \log\left((e\,x^2+d)^p\,c\right)^2 deg\,x^2 - 2\log\left((e\,x^2+d)^p\,c\right) d^2gp - 2\log\left((e\,x^2+d)\right.}{4d}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x)
```

output
```
( - 4*int(log((d + e*x**2)**p*c)/(d*x**3 + e*x**5),x)*d**3*g*p*x**2 + log(
(d + e*x**2)**p*c)**2*d*e*g*x**2 - 2*log((d + e*x**2)**p*c)*d**2*g*p - 2*l
og((d + e*x**2)**p*c)*d*e*f*p - 2*log((d + e*x**2)**p*c)*d*e*g*p*x**2 - 2*
log((d + e*x**2)**p*c)*e**2*f*p*x**2 + 4*log(x)*d*e*g*p**2*x**2 + 4*log(x)
*e**2*f*p**2*x**2)/(4*d*e*p*x**2)
```

3.315 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = -\frac{efp}{4dx^2} - \frac{e(ef-2dg)p\log(x)}{2d^2} + \frac{(ef-dg)^2p\log(d+ex^2)}{4d^2f} - \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4}$$

output
```
-1/4*e*f*p/d/x^2-1/2*e*(-2*d*g+e*f)*p*ln(x)/d^2+1/4*(-d*g+e*f)^2*p*ln(e*x^
2+d)/d^2/f-1/4*(g*x^2+f)^2*ln(c*(e*x^2+d)^p)/f/x^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \frac{egp\log(x)}{d} - \frac{egp\log(d+ex^2)}{2d} + \frac{1}{4}efp\left(-\frac{1}{dx^2} - \frac{2e\log(x)}{d^2} + \frac{e\log(d+ex^2)}{d^2}\right) - \frac{f\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{g\log\left(c(d+ex^2)^p\right)}{2x^2}$$

input
```
Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

output
```
(e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^2])/(2*d) + (e*f*p*(-(1/(d*x^2)) - (
2*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 - (f*Log[c*(d + e*x^2)^p])/(4
*x^4) - (g*Log[c*(d + e*x^2)^p])/(2*x^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2925, 2861, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int \frac{(gx^2+f)\log\left(c(ex^2+d)^p\right)}{x^6}dx^2$$

$$\Big\downarrow \text{2861}$$

$$\frac{1}{2}\left(-ep\int -\frac{(gx^2+f)^2}{2fx^4(ex^2+d)}dx^2-\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{2fx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{ep\int \frac{(gx^2+f)^2}{x^4(ex^2+d)}dx^2}{2f}-\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{2fx^4}\right)$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\left(\frac{ep\int \left(\frac{f^2}{dx^4}+\frac{(2dg-ef)f}{d^2x^2}+\frac{(dg-ef)^2}{d^2(ex^2+d)}\right)dx^2}{2f}-\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{2fx^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{ep\left(-\frac{f\log(x^2)(ef-2dg)}{d^2}+\frac{(ef-dg)^2\log(d+ex^2)}{d^2e}-\frac{f^2}{dx^2}\right)}{2f}-\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{2fx^4}\right)$$

input
```
Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

output
```
((e*p*(-(f^2/(d*x^2)) - (f*(e*f - 2*d*g)*Log[x^2])/d^2 + ((e*f - d*g)^2*Lo
g[d + e*x^2])/(d^2*e)))/(2*f) - ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(2*f*
x^4))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

method	result
parts	$-\frac{g\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{2x^{2}}-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)f}{4x^{4}}-\frac{ep\left(\frac{(2dg-ef)\ln\left(e\,x^{2}+d\right)}{2d^{2}}+\frac{(-2dg+ef)\ln(x)}{d^{2}}+\frac{f}{2d\,x^{2}}\right)}{2}$
parallelrisch	$\frac{4\ln(x)x^{4}deg\,p^{2}-2\ln(x)x^{4}e^{2}f\,p^{2}-2x^{4}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)degp+x^{4}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)e^{2}fp+x^{4}e^{2}f\,p^{2}-2x^{2}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)d^{2}gp-x}{4x^{4}p\,d^{2}}$
risch	$-\frac{\left(2g\,x^{2}+f\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{4x^{4}}-\frac{2i\pi\,d^{2}g\,x^{2}\operatorname{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\operatorname{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}-2i\pi\,d^{2}g\,x^{2}\operatorname{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\operatorname{csgn}\left(ic(e\,x^{2}\right.}{}$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*g/x^2*ln(c*(e*x^2+d)^p)-1/4*ln(c*(e*x^2+d)^p)/x^4*f-1/2*e*p*(1/2*(2*d
*g-e*f)/d^2*ln(e*x^2+d)+(-2*d*g+e*f)/d^2*ln(x)+1/2/d*f/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int\frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = -\frac{2\,(e^2f-2\,deg)px^4\log\left(x\right)+defpx^2+(2\,d^2gpx^2-(e^2f-2\,deg)px^4+d^2fp)\log\left(ex^2+d\right)+(2\,d^2gx^2}{4\,d^2x^4}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")
```

output
```
-1/4*(2*(e^2*f - 2*d*e*g)*p*x^4*log(x) + d*e*f*p*x^2 + (2*d^2*g*p*x^2 - (e
^2*f - 2*d*e*g)*p*x^4 + d^2*f*p)*log(e*x^2 + d) + (2*d^2*g*x^2 + d^2*f)*lo
g(c))/(d^2*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(83) = 166.

Time = 78.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.80

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx$$
$$= \begin{cases} -\frac{f\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{g\log\left(c(d+ex^2)^p\right)}{2x^2} - \frac{efp}{4dx^2} + \frac{egp\log(x)}{d} - \frac{eg\log\left(c(d+ex^2)^p\right)}{2d} - \frac{e^2fp\log(x)}{2d^2} + \frac{e^2f\log\left(c(d+ex^2)^p\right)}{4d^2} & \text{fo} \\ -\frac{fp}{8x^4} - \frac{f\log\left(c(ex^2)^p\right)}{4x^4} - \frac{gp}{2x^2} - \frac{g\log\left(c(ex^2)^p\right)}{2x^2} & \text{o} \end{cases}$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**5,x)
```

output
```
Piecewise((-f*log(c*(d + e*x**2)**p)/(4*x**4) - g*log(c*(d + e*x**2)**p)/(
2*x**2) - e*f*p/(4*d*x**2) + e*g*p*log(x)/d - e*g*log(c*(d + e*x**2)**p)/(
2*d) - e**2*f*p*log(x)/(2*d**2) + e**2*f*log(c*(d + e*x**2)**p)/(4*d**2),
Ne(d, 0)), (-f*p/(8*x**4) - f*log(c*(e*x**2)**p)/(4*x**4) - g*p/(2*x**2) -
 g*log(c*(e*x**2)**p)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx$$
$$= \frac{1}{4}\,ep\left(\frac{(ef-2\,dg)\log\left(ex^2+d\right)}{d^2} - \frac{(ef-2\,dg)\log\left(x^2\right)}{d^2} - \frac{f}{dx^2}\right) - \frac{(2\,gx^2+f)\log\left((ex^2+d)^p c\right)}{4\,x^4}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
```

output
```
1/4*e*p*((e*f - 2*d*g)*log(e*x^2 + d)/d^2 - (e*f - 2*d*g)*log(x^2)/d^2 - f
/(d*x^2)) - 1/4*(2*g*x^2 + f)*log((e*x^2 + d)^p*c)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(85) = 170$.

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.25

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = -\frac{\frac{(e^3fp+2\,(ex^2+d)e^2gp-2\,de^2gp)\log(ex^2+d)}{(ex^2+d)^2-2\,(ex^2+d)d+d^2}+\frac{(ex^2+d)e^3fp-de^3fp+de^3f\log(c)+2\,(ex^2+d)de^2g\log(c)-2\,d^2e^2g\log(c)}{(ex^2+d)^2d-2\,(ex^2+d)d^2+d^3}-\frac{(e^3fp-2\,de}{}}{4\,e}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
```

output
```
-1/4*((e^3*f*p + 2*(e*x^2 + d)*e^2*g*p - 2*d*e^2*g*p)*log(e*x^2 + d)/((e*x
^2 + d)^2 - 2*(e*x^2 + d)*d + d^2) + ((e*x^2 + d)*e^3*f*p - d*e^3*f*p + d*
e^3*f*log(c) + 2*(e*x^2 + d)*d*e^2*g*log(c) - 2*d^2*e^2*g*log(c))/((e*x^2
+ d)^2*d - 2*(e*x^2 + d)*d^2 + d^3) - (e^3*f*p - 2*d*e^2*g*p)*log(e*x^2 +
d)/d^2 + (e^3*f*p - 2*d*e^2*g*p)*log(e*x^2)/d^2)/e
```

Mupad [B] (verification not implemented)

Time = 26.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \frac{\ln\left(e\,x^2+d\right)\,\left(e^2\,f\,p-2\,d\,e\,g\,p\right)}{4\,d^2} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^2}{2}+\frac{f}{4}\right)}{x^4} - \frac{\ln\left(x\right)\,\left(e^2\,f\,p-2\,d\,e\,g\,p\right)}{2\,d^2} - \frac{e\,f\,p}{4\,d\,x^2}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^5,x)
```

output
```
(log(d + e*x^2)*(e^2*f*p - 2*d*e*g*p))/(4*d^2) - (log(c*(d + e*x^2)^p)*(f/
4 + (g*x^2)/2))/x^4 - (log(x)*(e^2*f*p - 2*d*e*g*p))/(2*d^2) - (e*f*p)/(4*
d*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.29

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx$$
$$= \frac{-\log\left((e\,x^2+d)^p\,c\right)d^2f-2\log\left((e\,x^2+d)^p\,c\right)d^2g\,x^2-2\log\left((e\,x^2+d)^p\,c\right)deg\,x^4+\log\left((e\,x^2+d)^p\,c\right)e^2 \cdots}{4d^2x^4}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x)
```

output
```
( - log((d + e*x**2)**p*c)*d**2*f - 2*log((d + e*x**2)**p*c)*d**2*g*x**2 -
 2*log((d + e*x**2)**p*c)*d*e*g*x**4 + log((d + e*x**2)**p*c)*e**2*f*x**4
+ 4*log(x)*d*e*g*p*x**4 - 2*log(x)*e**2*f*p*x**4 - d*e*f*p*x**2)/(4*d**2*x
**4)
```

3.316 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{efp}{12dx^4}+\frac{e(2ef-3dg)p}{12d^2x^2}+\frac{e^2(2ef-3dg)p\log(x)}{6d^3} - \frac{e^2(2ef-3dg)p\log\left(d+ex^2\right)}{12d^3} - \frac{f\log\left(c(d+ex^2)^p\right)}{6x^6}-\frac{g\log\left(c(d+ex^2)^p\right)}{4x^4}$$

output
```
-1/12*e*f*p/d/x^4+1/12*e*(-3*d*g+2*e*f)*p/d^2/x^2+1/6*e^2*(-3*d*g+2*e*f)*p
*ln(x)/d^3-1/12*e^2*(-3*d*g+2*e*f)*p*ln(e*x^2+d)/d^3-1/6*f*ln(c*(e*x^2+d)^
p)/x^6-1/4*g*ln(c*(e*x^2+d)^p)/x^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \frac{1}{4}egp\left(-\frac{1}{dx^2}-\frac{2e\log(x)}{d^2}+\frac{e\log(d+ex^2)}{d^2}\right)+\frac{1}{3}efp\left(-\frac{1}{4dx^4}+\frac{e}{2d^2x^2}+\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex^2)}{2d^3}\right)-\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6}-\frac{g\log\left(c(d+ex^2)^p\right)}{4x^4}$$

input
```
Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

output
```
(e*g*p*(-(1/(d*x^2)) - (2*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 + (e*
f*p*(-1/4*1/(d*x^4) + e/(2*d^2*x^2) + (e^2*Log[x])/d^3 - (e^2*Log[d + e*x^
2])/(2*d^3)))/3 - (f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^
p])/(4*x^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2925, 2861, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int \frac{(gx^2+f)\log\left(c(ex^2+d)^p\right)}{x^8}dx^2$$

$$\Big\downarrow \text{2861}$$

$$\frac{1}{2}\left(-ep\int -\frac{3gx^2+2f}{6x^6\left(ex^2+d\right)}dx^2-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{1}{6}ep\int \frac{3gx^2+2f}{x^6\left(ex^2+d\right)}dx^2-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \ 86$$

$$\frac{1}{2}\left(\frac{1}{6}ep\int \left(\frac{(3dg-2ef)e^2}{d^3\left(ex^2+d\right)}-\frac{(3dg-2ef)e}{d^3x^2}+\frac{3dg-2ef}{d^2x^4}+\frac{2f}{dx^6}\right)dx^2-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g\log\left(c\left(d+ex^2\right)\right.}{2x^4}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2x^4}+\frac{1}{6}ep\left(\frac{e\log\left(x^2\right)(2ef-3dg)}{d^3}-\frac{e(2ef-3dg)\log\left(d+ex^2\right)}{d^3}+\right.\right.$$

input
```
Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

output
```
((e*p*(-(f/(d*x^4)) + (2*e*f - 3*d*g)/(d^2*x^2) + (e*(2*e*f - 3*d*g)*Log[x
^2])/d^3 - (e*(2*e*f - 3*d*g)*Log[d + e*x^2])/d^3))/6 - (f*Log[c*(d + e*x^
2)^p])/(3*x^6) - (g*Log[c*(d + e*x^2)^p])/(2*x^4))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

method	result
parts	$-\frac{g \ln\left(c \left(e\,x^{2}+d\right)^{p}\right)}{4 x^{4}}-\frac{f \ln\left(c \left(e\,x^{2}+d\right)^{p}\right)}{6 x^{6}}-\frac{e p \left(-\frac{e \left(3 d g -2 e f \right) \ln\left(e\,x^{2}+d \right)}{2 d^{3}}-\frac{-3 d g +2 e f}{2 d^{2} x^{2}}+\frac{e \left(3 d g -2 e f \right) \ln\left(x \right)}{d^{3}}+\frac{f}{2 d\,x^{4}}\right)}{6}$
parallelrisch	$-\frac{6 \ln\left(x \right) x^{6} d\,e^{2} g\,p^{2}-4 \ln\left(x \right) x^{6} e^{3} f\,p^{2}-3 x^{6} \ln\left(c \left(e\,x^{2}+d \right)^{p}\right) d\,e^{2} g p +2 x^{6} \ln\left(c \left(e\,x^{2}+d \right)^{p}\right) e^{3} f p -3 x^{6} d\,e^{2} g\,p^{2}+2 x^{6} e^{3} f\,p^{2}+3 x^{4} d^{2}}{12 x^{6} p\,d^{3}}$
risch	$-\frac{\left(3 g\,x^{2}+2 f \right) \ln\left(\left(e\,x^{2}+d \right)^{p}\right)}{12 x^{6}}-\frac{12 \ln\left(x \right) d\,e^{2} g p\,x^{6}-8 \ln\left(x \right) e^{3} f p\,x^{6}-6 \ln\left(-e\,x^{2}-d \right) d\,e^{2} g p\,x^{6}+4 \ln\left(-e\,x^{2}-d \right) e^{3} f p\,x^{6}-3 i \pi\,d}{}$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/4*g*ln(c*(e*x^2+d)^p)/x^4-1/6*f*ln(c*(e*x^2+d)^p)/x^6-1/6*e*p*(-1/2*e*(
3*d*g-2*e*f)/d^3*ln(e*x^2+d)-1/2*(-3*d*g+2*e*f)/d^2/x^2+e*(3*d*g-2*e*f)/d^
3*ln(x)+1/2/d*f/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \frac{2\,(2\,e^3f-3\,de^2g)px^6\log(x)-d^2efpx^2+(2\,de^2f-3\,d^2eg)px^4-((2\,e^3f-3\,de^2g)px^6+3\,d^3gpx^2+2\,d^3}{12\,d^3x^6}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

output
```
1/12*(2*(2*e^3*f - 3*d*e^2*g)*p*x^6*log(x) - d^2*e*f*p*x^2 + (2*d*e^2*f -
3*d^2*e*g)*p*x^4 - ((2*e^3*f - 3*d*e^2*g)*p*x^6 + 3*d^3*g*p*x^2 + 2*d^3*f*
p)*log(e*x^2 + d) - (3*d^3*g*x^2 + 2*d^3*f)*log(c))/(d^3*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{1}{12}\,ep\left(\frac{(2\,e^2f-3\,deg)\log(ex^2+d)}{d^3}-\frac{(2\,e^2f-3\,deg)\log(x^2)}{d^3}-\frac{(2\,ef-3\,dg)x^2-df}{d^2x^4}\right) - \frac{(3\,gx^2+2\,f)\log\left((ex^2+d)^pc\right)}{12\,x^6}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

output
```
-1/12*e*p*((2*e^2*f - 3*d*e*g)*log(e*x^2 + d)/d^3 - (2*e^2*f - 3*d*e*g)*lo
g(x^2)/d^3 - ((2*e*f - 3*d*g)*x^2 - d*f)/(d^2*x^4)) - 1/12*(3*g*x^2 + 2*f)
*log((e*x^2 + d)^p*c)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(113) = 226$.

Time = 0.12 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.53

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{\frac{(2e^4fp+3(ex^2+d)e^3gp-3de^3gp)\log(ex^2+d)}{(ex^2+d)^3-3(ex^2+d)^2d+3(ex^2+d)d^2-d^3} - \frac{2(ex^2+d)^2e^4fp-5(ex^2+d)de^4fp+3d^2e^4fp-3(ex^2+d)^2de^3gp+6(ex^2+d)d^2e^3gp-3d\cdots}{(ex^2+d)^3d^2-3(ex^2+d)^2d^3+3(ex^2\cdots}}{12e}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

output
```
-1/12*((2*e^4*f*p + 3*(e*x^2 + d)*e^3*g*p - 3*d*e^3*g*p)*log(e*x^2 + d)/((
e*x^2 + d)^3 - 3*(e*x^2 + d)^2*d + 3*(e*x^2 + d)*d^2 - d^3) - (2*(e*x^2 +
d)^2*e^4*f*p - 5*(e*x^2 + d)*d*e^4*f*p + 3*d^2*e^4*f*p - 3*(e*x^2 + d)^2*d
*e^3*g*p + 6*(e*x^2 + d)*d^2*e^3*g*p - 3*d^3*e^3*g*p - 2*d^2*e^4*f*log(c)
- 3*(e*x^2 + d)*d^2*e^3*g*log(c) + 3*d^3*e^3*g*log(c))/((e*x^2 + d)^3*d^2
- 3*(e*x^2 + d)^2*d^3 + 3*(e*x^2 + d)*d^4 - d^5) + (2*e^4*f*p - 3*d*e^3*g*
p)*log(e*x^2 + d)/d^3 - (2*e^4*f*p - 3*d*e^3*g*p)*log(e*x^2)/d^3)/e
```

Mupad [B] (verification not implemented)

Time = 26.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \frac{\ln(x)\ (2\,e^3\,f\,p-3\,d\,e^2\,g\,p)}{6\,d^3} - \frac{\ln\left(c\,(e\,x^2+d)^p\right)\ \left(\frac{g\,x^2}{4}+\frac{f}{6}\right)}{x^6} - \frac{\ln(e\,x^2+d)\ (2\,e^3\,f\,p-3\,d\,e^2\,g\,p)}{12\,d^3} - \frac{\frac{e\,f\,p}{2\,d}+\frac{e\,p\,x^2\,(3\,d\,g-2\,e\,f)}{2\,d^2}}{6\,x^4}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^7,x)
```

output
```
(log(x)*(2*e^3*f*p - 3*d*e^2*g*p))/(6*d^3) - (log(c*(d + e*x^2)^p)*(f/6 +
(g*x^2)/4))/x^6 - (log(d + e*x^2)*(2*e^3*f*p - 3*d*e^2*g*p))/(12*d^3) - ((
e*f*p)/(2*d) + (e*p*x^2*(3*d*g - 2*e*f))/(2*d^2))/(6*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.19

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \frac{-2\log\left((e\,x^2+d)^p\,c\right)d^3f-3\log\left((e\,x^2+d)^p\,c\right)d^3g\,x^2+3\log\left((e\,x^2+d)^p\,c\right)d\,e^2g\,x^6-2\log\left((e\,x^2+d)^p\,c\right.}{12d^3x^6}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x)
```

output
```
( - 2*log((d + e*x**2)**p*c)*d**3*f - 3*log((d + e*x**2)**p*c)*d**3*g*x**2
 + 3*log((d + e*x**2)**p*c)*d*e**2*g*x**6 - 2*log((d + e*x**2)**p*c)*e**3*
f*x**6 - 6*log(x)*d*e**2*g*p*x**6 + 4*log(x)*e**3*f*p*x**6 - d**2*e*f*p*x*
*2 - 3*d**2*e*g*p*x**4 + 2*d*e**2*f*p*x**4)/(12*d**3*x**6)
```

3.317 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9} dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{(f+gx^2)\log(c(d+ex^2)^p)}{x^9} dx = -\frac{efp}{24dx^6} + \frac{e(3ef-4dg)p}{48d^2x^4} - \frac{e^2(3ef-4dg)p}{24d^3x^2} - \frac{e^3(3ef-4dg)p\log(x)}{12d^4} + \frac{e^3(3ef-4dg)p\log(d+ex^2)}{24d^4} - \frac{f\log(c(d+ex^2)^p)}{8x^8} - \frac{g\log(c(d+ex^2)^p)}{6x^6}$$

output
```
-1/24*e*f*p/d/x^6+1/48*e*(-4*d*g+3*e*f)*p/d^2/x^4-1/24*e^2*(-4*d*g+3*e*f)*
p/d^3/x^2-1/12*e^3*(-4*d*g+3*e*f)*p*ln(x)/d^4+1/24*e^3*(-4*d*g+3*e*f)*p*ln
(e*x^2+d)/d^4-1/8*f*ln(c*(e*x^2+d)^p)/x^8-1/6*g*ln(c*(e*x^2+d)^p)/x^6
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.09

$$\int \frac{(f+gx^2)\log(c(d+ex^2)^p)}{x^9}\,dx = \frac{1}{3}egp\left(-\frac{1}{4dx^4}+\frac{e}{2d^2x^2}+\frac{e^2\log(x)}{d^3}-\frac{e^2\log(d+ex^2)}{2d^3}\right)+\frac{1}{8}efp\left(-\frac{1}{3dx^6}+\frac{e}{2d^2x^4}-\frac{e^2}{d^3x^2}-\frac{2e^3\log(x)}{d^4}+\frac{e^3\log(d+ex^2)}{d^4}\right)-\frac{f\log(c(d+ex^2)^p)}{8x^8}-\frac{g\log(c(d+ex^2)^p)}{6x^6}$$

input
```
Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

output
```
(e*g*p*(-1/4*1/(d*x^4) + e/(2*d^2*x^2) + (e^2*Log[x])/d^3 - (e^2*Log[d + e
*x^2])/(2*d^3)))/3 + (e*f*p*(-1/3*1/(d*x^6) + e/(2*d^2*x^4) - e^2/(d^3*x^2
) - (2*e^3*Log[x])/d^4 + (e^3*Log[d + e*x^2])/d^4))/8 - (f*Log[c*(d + e*x^
2)^p])/(8*x^8) - (g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2925, 2861, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)\log(c(d+ex^2)^p)}{x^9}\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int \frac{(gx^2+f)\log(c(ex^2+d)^p)}{x^{10}}dx^2$$

$$\Big\downarrow \text{2861}$$

$$\frac{1}{2}\left(-ep\int -\frac{4gx^2+3f}{12x^8\left(ex^2+d\right)}dx^2-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4x^8}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{1}{12}ep\int \frac{4gx^2+3f}{x^8\left(ex^2+d\right)}dx^2-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4x^8}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}\left(\frac{1}{12}ep\int \left(-\frac{(4dg-3ef)e^3}{d^4\left(ex^2+d\right)}+\frac{(4dg-3ef)e^2}{d^4x^2}-\frac{(4dg-3ef)e}{d^3x^4}+\frac{4dg-3ef}{d^2x^6}+\frac{3f}{dx^8}\right)dx^2-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4x^8}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{4x^8}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}+\frac{1}{12}ep\left(-\frac{e^2\log\left(x^2\right)(3ef-4dg)}{d^4}+\frac{e^2(3ef-4dg)\log\left(d+ex^2\right.}{d^4}\right.\right.$$

input
```
Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

output
```
((e*p*(-(f/(d*x^6)) + (3*e*f - 4*d*g)/(2*d^2*x^4) - (e*(3*e*f - 4*d*g))/(d
^3*x^2) - (e^2*(3*e*f - 4*d*g)*Log[x^2])/d^4 + (e^2*(3*e*f - 4*d*g)*Log[d
+ e*x^2])/d^4))/12 - (f*Log[c*(d + e*x^2)^p])/(4*x^8) - (g*Log[c*(d + e*x^
2)^p])/(3*x^6))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

method	result
parts	$-\frac{g\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{6x^{6}}-\frac{f\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{8x^{8}}-\frac{ep\left(\frac{e^{2}(4dg-3ef)\ln\left(e\,x^{2}+d\right)}{2d^{4}}-\frac{-4dg+3ef}{4d^{2}x^{4}}-\frac{(4dg-3ef)e}{2d^{3}x^{2}}+\frac{f}{2d\,x^{6}}-\frac{(4dg-3ef)e^{2}\ln(x}{d^{4}}\right.}{12}$
parallelrisch	$\frac{16\ln(x)x^{8}d\,e^{3}g\,p^{2}-12\ln(x)x^{8}e^{4}f\,p^{2}-8x^{8}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)d\,e^{3}gp+6x^{8}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)e^{4}fp-8x^{8}d\,e^{3}g\,p^{2}+6x^{8}e^{4}f\,p^{2}+8x^{6}d^{2}}{48x^{8}d^{4}p}$
risch	$-\frac{\left(4g\,x^{2}+3f\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{24x^{8}}-\frac{-16\ln(x)d\,e^{3}gp\,x^{8}+12\ln(x)e^{4}fp\,x^{8}+8\ln\left(e\,x^{2}+d\right)d\,e^{3}gp\,x^{8}-6\ln\left(e\,x^{2}+d\right)e^{4}fp\,x^{8}+4i\pi\,d^{4}}{}$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/6*g*ln(c*(e*x^2+d)^p)/x^6-1/8*f*ln(c*(e*x^2+d)^p)/x^8-1/12*e*p*(1/2*e^2
*(4*d*g-3*e*f)/d^4*ln(e*x^2+d)-1/4*(-4*d*g+3*e*f)/d^2/x^4-1/2*(4*d*g-3*e*f
)/d^3*e/x^2+1/2*f/d/x^6-(4*d*g-3*e*f)/d^4*e^2*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = -\frac{4\,(3\,e^4f-4\,de^3g)px^8\log\left(x\right)+2\,d^3efpx^2+2\,(3\,de^3f-4\,d^2e^2g)px^6-(3\,d^2e^2f-4\,d^3eg)px^4-2\,((3\,e^4}{48\,d^4x^8}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")
```

output
```
-1/48*(4*(3*e^4*f - 4*d*e^3*g)*p*x^8*log(x) + 2*d^3*e*f*p*x^2 + 2*(3*d*e^3
*f - 4*d^2*e^2*g)*p*x^6 - (3*d^2*e^2*f - 4*d^3*e*g)*p*x^4 - 2*((3*e^4*f -
4*d*e^3*g)*p*x^8 - 4*d^4*g*p*x^2 - 3*d^4*f*p)*log(e*x^2 + d) + 2*(4*d^4*g*
x^2 + 3*d^4*f)*log(c))/(d^4*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \text{Timed out}$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**9,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \frac{1}{48}\,ep\left(\frac{2\,(3\,e^3f-4\,de^2g)\log\left(ex^2+d\right)}{d^4}-\frac{2\,(3\,e^3f-4\,de^2g)\log\left(x^2\right)}{d^4}-\frac{2\,(3\,e^2f-4\,deg)x^4+2\,d^2f-(3\,d}{d^3x^6}\right. - \frac{(4\,gx^2+3\,f)\log\left((ex^2+d)^pc\right)}{24\,x^8}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

output
```
1/48*e*p*(2*(3*e^3*f - 4*d*e^2*g)*log(e*x^2 + d)/d^4 - 2*(3*e^3*f - 4*d*e^
2*g)*log(x^2)/d^4 - (2*(3*e^2*f - 4*d*e*g)*x^4 + 2*d^2*f - (3*d*e*f - 4*d^
2*g)*x^2)/(d^3*x^6)) - 1/24*(4*g*x^2 + 3*f)*log((e*x^2 + d)^p*c)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(134) = 268$.

Time = 0.13 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.56

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx =$$

$$-\frac{2\left(3e^5fp+4\left(ex^2+d\right)e^4gp-4de^4gp\right)\log\left(ex^2+d\right)}{\left(ex^2+d\right)^4-4\left(ex^2+d\right)^3d+6\left(ex^2+d\right)^2d^2-4\left(ex^2+d\right)d^3+d^4}+\frac{6\left(ex^2+d\right)^3e^5fp-21\left(ex^2+d\right)^2de^5fp+26\left(ex^2+d\right)d^2e^5fp-11d^3e^5fp-8\left(ex^2\right.}{\left(ex^2+d\right)}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")
```

output
```
-1/48*(2*(3*e^5*f*p + 4*(e*x^2 + d)*e^4*g*p - 4*d*e^4*g*p)*log(e*x^2 + d)/
((e*x^2 + d)^4 - 4*(e*x^2 + d)^3*d + 6*(e*x^2 + d)^2*d^2 - 4*(e*x^2 + d)*d
^3 + d^4) + (6*(e*x^2 + d)^3*e^5*f*p - 21*(e*x^2 + d)^2*d*e^5*f*p + 26*(e*
x^2 + d)*d^2*e^5*f*p - 11*d^3*e^5*f*p - 8*(e*x^2 + d)^3*d*e^4*g*p + 28*(e*
x^2 + d)^2*d^2*e^4*g*p - 32*(e*x^2 + d)*d^3*e^4*g*p + 12*d^4*e^4*g*p + 6*d
^3*e^5*f*log(c) + 8*(e*x^2 + d)*d^3*e^4*g*log(c) - 8*d^4*e^4*g*log(c))/((e
*x^2 + d)^4*d^3 - 4*(e*x^2 + d)^3*d^4 + 6*(e*x^2 + d)^2*d^5 - 4*(e*x^2 + d
)*d^6 + d^7) - 2*(3*e^5*f*p - 4*d*e^4*g*p)*log(e*x^2 + d)/d^4 + 2*(3*e^5*f
*p - 4*d*e^4*g*p)*log(e*x^2)/d^4)/e
```

Mupad [B] (verification not implemented)

Time = 26.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \frac{\ln\left(e\,x^2+d\right)\,\left(3\,e^4\,f\,p-4\,d\,e^3\,g\,p\right)}{24\,d^4} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^2}{6}+\frac{f}{8}\right)}{x^8} - \frac{\frac{e\,f\,p}{2\,d}+\frac{e\,p\,x^2\,\left(4\,d\,g-3\,e\,f\right)}{4\,d^2}-\frac{e^2\,p\,x^4\,\left(4\,d\,g-3\,e\,f\right)}{2\,d^3}}{12\,x^6} - \frac{\ln\left(x\right)\,\left(3\,e^4\,f\,p-4\,d\,e^3\,g\,p\right)}{12\,d^4}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^9,x)
```

output
```
(log(d + e*x^2)*(3*e^4*f*p - 4*d*e^3*g*p))/(24*d^4) - (log(c*(d + e*x^2)^p
)*(f/8 + (g*x^2)/6))/x^8 - ((e*f*p)/(2*d) + (e*p*x^2*(4*d*g - 3*e*f))/(4*d
^2) - (e^2*p*x^4*(4*d*g - 3*e*f))/(2*d^3))/(12*x^6) - (log(x)*(3*e^4*f*p -
 4*d*e^3*g*p))/(12*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \frac{-6\log\left((e\,x^2+d)^p\,c\right)d^4 f-8\log\left((e\,x^2+d)^p\,c\right)d^4 g\,x^2-8\log\left((e\,x^2+d)^p\,c\right)d\,e^3 g\,x^8+6\log\left((e\,x^2+d)^p\,c\right.}{}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x)
```

output
```
( - 6*log((d + e*x**2)**p*c)*d**4*f - 8*log((d + e*x**2)**p*c)*d**4*g*x**2
 - 8*log((d + e*x**2)**p*c)*d*e**3*g*x**8 + 6*log((d + e*x**2)**p*c)*e**4*
f*x**8 + 16*log(x)*d*e**3*g*p*x**8 - 12*log(x)*e**4*f*p*x**8 - 2*d**3*e*f*
p*x**2 - 4*d**3*e*g*p*x**4 + 3*d**2*e**2*f*p*x**4 + 8*d**2*e**2*g*p*x**6 -
 6*d*e**3*f*p*x**6)/(48*d**4*x**8)
```

3.318 $\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=\frac{2dfpx}{3e}-\frac{2d^2gpx}{5e^2}-\frac{2}{9}fpx^3+\frac{2dgpx^3}{15e}-\frac{2}{25}gpx^5-\frac{2d^{3/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{1}{3}fx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}gx^5\log\left(c(d+ex^2)^p\right)$$

output
```
2/3*d*f*p*x/e-2/5*d^2*g*p*x/e^2-2/9*f*p*x^3+2/15*d*g*p*x^3/e-2/25*g*p*x^5-
2/3*d^(3/2)*f*p*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)+2/5*d^(5/2)*g*p*arctan(e
^(1/2)*x/d^(1/2))/e^(5/2)+1/3*f*x^3*ln(c*(e*x^2+d)^p)+1/5*g*x^5*ln(c*(e*x^
2+d)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\frac{30d^{3/2}(-5ef+3dg)p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{e}x\left(-2p(45d^2g-15de(5f+gx^2)+e^2x^2(25f+9gx^2))+15e^2x^2\right.}{225e^{5/2}}$$

input
```
Integrate[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
(30*d^(3/2)*(-5*e*f + 3*d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[e]*x*(-2
*p*(45*d^2*g - 15*d*e*(5*f + g*x^2) + e^2*x^2*(25*f + 9*g*x^2)) + 15*e^2*x
^2*(5*f + 3*g*x^2)*Log[c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (f+gx^2) \log \left(c (d+ex^2)^p\right) \, dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(fx^2 \log \left(c (d+ex^2)^p\right)+gx^4 \log \left(c (d+ex^2)^p\right)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d^{3/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{1}{3}fx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}gx^5\log\left(c(d+ex^2)^p\right)-\frac{2d^2gpx}{5e^2}+\frac{2dfpx}{3e}+\frac{2dgpx^3}{15e}-\frac{2}{9}fpx^3-\frac{2}{25}gpx^5$$

input
```
Int[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
(2*d*f*p*x)/(3*e) - (2*d^2*g*p*x)/(5*e^2) - (2*f*p*x^3)/9 + (2*d*g*p*x^3)/
(15*e) - (2*g*p*x^5)/25 - (2*d^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e
^(3/2)) + (2*d^(5/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + (f*x^3
*Log[c*(d + e*x^2)^p])/3 + (g*x^5*Log[c*(d + e*x^2)^p])/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

method	result
parts	$\frac{g\,x^{5} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{5}+\frac{f\,x^{3} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{3}-\frac{2ep\left(\frac{\frac{3}{5} e^{2} g\,x^{5}-x^{3} d g e+\frac{5}{3} f\,x^{3} e^{2}+3 x\,d^{2} g-5 x d f e}{e^{3}}-\frac{d^{2} \left(3 d g-5 e f\right) \arctan\left(\frac{x e}{\sqrt{d e}}\right)}{e^{3} \sqrt{d e}}\right)}{15}$
risch	$\left(\frac{1}{5} g\,x^{5}+\frac{1}{3} f\,x^{3}\right) \ln\left(\left(e\,x^{2}+d\right)^{p}\right)-\frac{i \pi g\,x^{5} \operatorname{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right)^{3}}{10}-\frac{i \pi f\,x^{3} \operatorname{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right)^{3}}{6}-\frac{i \pi g\,x^{5} \operatorname{csgn}\left(i\left(e\,x^{2}+\right.\right.}{}$

input
```
int(x^2*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/5*g*x^5*ln(c*(e*x^2+d)^p)+1/3*f*x^3*ln(c*(e*x^2+d)^p)-2/15*e*p*(1/e^3*(3
/5*e^2*g*x^5-x^3*d*g*e+5/3*f*x^3*e^2+3*x*d^2*g-5*x*d*f*e)-d^2*(3*d*g-5*e*f
)/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.95

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$

$$= \left[-\frac{18\,e^2gpx^5+10\,(5\,e^2f-3\,deg)px^3+15\,(5\,def-3\,d^2g)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)-30\,(5\,def-3\,d}{225\,e^2}\right.$$

$$-\frac{18\,e^2gpx^5+10\,(5\,e^2f-3\,deg)px^3+30\,(5\,def-3\,d^2g)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)-30\,(5\,def-3\,d^2g)px-}{225\,e^2}$$

input
```
integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/225*(18*e^2*g*p*x^5 + 10*(5*e^2*f - 3*d*e*g)*p*x^3 + 15*(5*d*e*f - 3*d
^2*g)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 30*(5
*d*e*f - 3*d^2*g)*p*x - 15*(3*e^2*g*p*x^5 + 5*e^2*f*p*x^3)*log(e*x^2 + d)
- 15*(3*e^2*g*x^5 + 5*e^2*f*x^3)*log(c))/e^2, -1/225*(18*e^2*g*p*x^5 + 10*
(5*e^2*f - 3*d*e*g)*p*x^3 + 30*(5*d*e*f - 3*d^2*g)*p*sqrt(d/e)*arctan(e*x*
sqrt(d/e)/d) - 30*(5*d*e*f - 3*d^2*g)*p*x - 15*(3*e^2*g*p*x^5 + 5*e^2*f*p*
x^3)*log(e*x^2 + d) - 15*(3*e^2*g*x^5 + 5*e^2*f*x^3)*log(c))/e^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(158) = 316.

Time = 32.77 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.08

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\begin{cases}\left(\frac{fx^3}{3}+\frac{gx^5}{5}\right)\log\left(0^pc\right)\\ \left(\frac{fx^3}{3}+\frac{gx^5}{5}\right)\log\left(cd^p\right)\\ -\frac{2fpx^3}{9}+\frac{fx^3\log\left(c(ex^2)^p\right)}{3}-\frac{2gpx^5}{25}+\frac{gx^5\log\left(c(ex^2)^p\right)}{5}\\ \frac{2d^3gp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{d^3g\log\left(c(d+ex^2)^p\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{2d^2fp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{d^2f\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}-\frac{2d^2gpx}{5e^2}+\frac{2dfpx}{3e}+\frac{2dgpx^3}{15e}-\end{cases}$$

input
```
integrate(x**2*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise(((f*x**3/3 + g*x**5/5)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f*x*
*3/3 + g*x**5/5)*log(c*d**p), Eq(e, 0)), (-2*f*p*x**3/9 + f*x**3*log(c*(e*
x**2)**p)/3 - 2*g*p*x**5/25 + g*x**5*log(c*(e*x**2)**p)/5, Eq(d, 0)), (2*d
**3*g*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - d**3*g*log(c*(d + e*x**2
)**p)/(5*e**3*sqrt(-d/e)) - 2*d**2*f*p*log(x - sqrt(-d/e))/(3*e**2*sqrt(-d
/e)) + d**2*f*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)) - 2*d**2*g*p*x/(5
*e**2) + 2*d*f*p*x/(3*e) + 2*d*g*p*x**3/(15*e) - 2*f*p*x**3/9 + f*x**3*log
(c*(d + e*x**2)**p)/3 - 2*g*p*x**5/25 + g*x**5*log(c*(d + e*x**2)**p)/5, T
rue))
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int x^2\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = -\frac{1}{25}\,(2\,gp-5\,g\log\left(c\right))x^5 - \frac{(10\,efp-6\,dgp-15\,ef\log\left(c\right))x^3}{45\,e} + \frac{1}{15}\left(3\,gpx^5+5\,fpx^3\right)\log\left(ex^2+d\right) + \frac{2\,(5\,defp-3\,d^2gp)x}{15\,e^2} - \frac{2\,(5\,d^2efp-3\,d^3gp)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}e^2}$$

input
```
integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
-1/25*(2*g*p - 5*g*log(c))*x^5 - 1/45*(10*e*f*p - 6*d*g*p - 15*e*f*log(c))
*x^3/e + 1/15*(3*g*p*x^5 + 5*f*p*x^3)*log(e*x^2 + d) + 2/15*(5*d*e*f*p - 3
*d^2*g*p)*x/e^2 - 2/15*(5*d^2*e*f*p - 3*d^3*g*p)*arctan(e*x/sqrt(d*e))/(sq
rt(d*e)*e^2)
```

Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^5}{5}+\frac{f\,x^3}{3}\right) - x^3\left(\frac{2\,f\,p}{9}-\frac{2\,d\,g\,p}{15\,e}\right) - \frac{2\,g\,p\,x^5}{25}+\frac{d\,x\,\left(\frac{2\,f\,p}{3}-\frac{2\,d\,g\,p}{5\,e}\right)}{e} + \frac{2\,d^{3/2}\,p\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,p\,x\,(3\,d\,g-5\,e\,f)}{3\,d^3\,g\,p-5\,d^2\,e\,f\,p}\right)\,(3\,d\,g-5\,e\,f)}{15\,e^{5/2}}$$

input
```
int(x^2*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

output
```
log(c*(d + e*x^2)^p)*((f*x^3)/3 + (g*x^5)/5) - x^3*((2*f*p)/9 - (2*d*g*p)/
(15*e)) - (2*g*p*x^5)/25 + (d*x*((2*f*p)/3 - (2*d*g*p)/(5*e)))/e + (2*d^(3
/2)*p*atan((d^(3/2)*e^(1/2)*p*x*(3*d*g - 5*e*f))/(3*d^3*g*p - 5*d^2*e*f*p)
)*(3*d*g - 5*e*f))/(15*e^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \frac{90\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)d^2gp-150\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)defp+75\log\left(\left(e\,x^2+d\right)^p c\right)e^3f\,x^3+45\log\left(\left(e\,x^2\right.\right.}{225e^3}$$

input
```
int(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x)
```

output
```
(90*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*g*p - 150*sqrt(e)*s
qrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*f*p + 75*log((d + e*x**2)**p*c)*e
**3*f*x**3 + 45*log((d + e*x**2)**p*c)*e**3*g*x**5 - 90*d**2*e*g*p*x + 150
*d*e**2*f*p*x + 30*d*e**2*g*p*x**3 - 50*e**3*f*p*x**3 - 18*e**3*g*p*x**5)/
(225*e**3)
```

3.319 $\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=-2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+fx\log\left(c(d+ex^2)^p\right)+\frac{1}{3}gx^3\log\left(c(d+ex^2)^p\right)$$

output
```
-2*f*p*x+2/3*d*g*p*x/e-2/9*g*p*x^3+2*d^(1/2)*f*p*arctan(e^(1/2)*x/d^(1/2))
/e^(1/2)-2/3*d^(3/2)*g*p*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)+f*x*ln(c*(e*x^2
+d)^p)+1/3*g*x^3*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=-2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+fx\log\left(c(d+ex^2)^p\right)+\frac{1}{3}gx^3\log\left(c(d+ex^2)^p\right)$$

input
```
Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output
```
-2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*
e^(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f\log\left(c\left(d+ex^2\right)^p\right)+gx^2\log\left(c\left(d+ex^2\right)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx}{3e}-2fpx-\frac{2}{9}gpx^3$$

input
```
Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

output

```
-2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*
e^(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g\,x^{3} \ln \left(c \left(e\,x^{2}+d \right)^{p}\right)}{3}+f x \ln \left(c \left(e\,x^{2}+d \right)^{p}\right)-\frac{2 e p \left(-\frac{-\frac{1}{3} e g\,x^{3}+x d g -3 e f x}{e^{2}}+\frac{d \left(d g -3 e f \right) \arctan \left(\frac{x e}{\sqrt{d e}}\right)}{e^{2} \sqrt{d e}}\right)}{3}$
risch	$\left(\frac{1}{3} g\,x^{3}+f x \right) \ln \left(\left(e\,x^{2}+d \right)^{p}\right)-\frac{i \pi g\,x^{3} \operatorname{csgn}\left(i \left(e\,x^{2}+d \right)^{p}\right) \operatorname{csgn}\left(i c \left(e\,x^{2}+d \right)^{p}\right) \operatorname{csgn}\left(i c \right)}{6}+\frac{i \pi g\,x^{3} \operatorname{csgn}\left(i \left(e\,x^{2}+d \right)^{p}\right) \operatorname{csgn}}{6}$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/3*g*x^3*ln(c*(e*x^2+d)^p)+f*x*ln(c*(e*x^2+d)^p)-2/3*e*p*(-1/e^2*(-1/3*e*
g*x^3+x*d*g-3*e*f*x)+d*(d*g-3*e*f)/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.88

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\left[-\frac{2\,egpx^3+3\,(3\,ef-dg)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2-2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+6\,(3\,ef-dg)px-3\,(egpx^3+3\,efpx)\log\left(ex^2\right.}{9\,e}\right.$$
$$-\frac{2\,egpx^3-6\,(3\,ef-dg)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+6\,(3\,ef-dg)px-3\,(egpx^3+3\,efpx)\log\left(ex^2+d\right)-}{9\,e}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/9*(2*e*g*p*x^3 + 3*(3*e*f - d*g)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(
-d/e) - d)/(e*x^2 + d)) + 6*(3*e*f - d*g)*p*x - 3*(e*g*p*x^3 + 3*e*f*p*x)*
log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e, -1/9*(2*e*g*p*x^3 - 6*(3
*e*f - d*g)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 6*(3*e*f - d*g)*p*x - 3*
(e*g*p*x^3 + 3*e*f*p*x)*log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.

Time = 8.48 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.22

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\begin{cases}\left(fx+\frac{gx^3}{3}\right)\log\left(0^pc\right)\\ \left(fx+\frac{gx^3}{3}\right)\log\left(cd^p\right)\\ -2fpx+fx\log\left(c(ex^2)^p\right)-\frac{2gpx^3}{9}+\frac{gx^3\log\left(c(ex^2)^p\right)}{3}\\ -\frac{2d^2gp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{d^2g\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{2dfp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{df\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}}+\frac{2dgpx}{3e}-2fpx+fx\log\left(c(\right.\end{cases}$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise(((f*x + g*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f*x + g*x
**3/3)*log(c*d**p), Eq(e, 0)), (-2*f*p*x + f*x*log(c*(e*x**2)**p) - 2*g*p*
x**3/9 + g*x**3*log(c*(e*x**2)**p)/3, Eq(d, 0)), (-2*d**2*g*p*log(x - sqrt
(-d/e))/(3*e**2*sqrt(-d/e)) + d**2*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-
d/e)) + 2*d*f*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f*log(c*(d + e*x**2
)**p)/(e*sqrt(-d/e)) + 2*d*g*p*x/(3*e) - 2*f*p*x + f*x*log(c*(d + e*x**2)*
*p) - 2*g*p*x**3/9 + g*x**3*log(c*(d + e*x**2)**p)/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (f + gx^2) \log \left(c(d + ex^2)^p\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\begin{aligned}\int (f + gx^2) \log \left(c(d + ex^2)^p\right) \, dx = & -\frac{1}{9} (2\,gp - 3\,g\log(c))x^3 \\ & + \frac{1}{3} (gpx^3 + 3\,fpx) \log(ex^2 + d) \\ & - \frac{(6\,efp - 2\,dgp - 3\,ef\log(c))x}{3\,e} \\ & + \frac{2\,(3\,defp - d^2gp)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{3\,\sqrt{de}e}\end{aligned}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
-1/9*(2*g*p - 3*g*log(c))*x^3 + 1/3*(g*p*x^3 + 3*f*p*x)*log(e*x^2 + d) - 1
/3*(6*e*f*p - 2*d*g*p - 3*e*f*log(c))*x/e + 2/3*(3*d*e*f*p - d^2*g*p)*arct
an(e*x/sqrt(d*e))/(sqrt(d*e)*e)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int (f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^3}{3}+f\,x\right) - x\left(2\,f\,p-\frac{2\,d\,g\,p}{3\,e}\right)-\frac{2\,g\,p\,x^3}{9} - \frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,(d\,g-3\,e\,f)}{d^2\,g\,p-3\,d\,e\,f\,p}\right)\,(d\,g-3\,e\,f)}{3\,e^{3/2}}$$

input
```
int(log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

output
```
log(c*(d + e*x^2)^p)*(f*x + (g*x^3)/3) - x*(2*f*p - (2*d*g*p)/(3*e)) - (2*
g*p*x^3)/9 - (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(d*g - 3*e*f))/(d^2*g*
p - 3*d*e*f*p))*(d*g - 3*e*f))/(3*e^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int (f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{-6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)dgp+18\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)efp+9\log\left(\left(e\,x^2+d\right)^p c\right)e^2fx+3\log\left(\left(e\,x^2+d\right)^p\right.}{9e^2}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p),x)
```

output

```
( - 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*g*p + 18*sqrt(e)*sqr
t(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p + 9*log((d + e*x**2)**p*c)*e**2*f
*x + 3*log((d + e*x**2)**p*c)*e**2*g*x**3 + 6*d*e*g*p*x - 18*e**2*f*p*x -
2*e**2*g*p*x**3)/(9*e**2)
```

3.320 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = -2gpx + \frac{2(ef+dg)p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - \frac{f\log\left(c(d+ex^2)^p\right)}{x} + gx\log\left(c(d+ex^2)^p\right)$$

output
```
-2*g*p*x+2*(d*g+e*f)*p*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/e^(1/2)-f*ln(c*(e
*x^2+d)^p)/x+g*x*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = -2gpx + \frac{2(ef+dg)p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} + \left(-\frac{f}{x}+gx\right)\log\left(c(d+ex^2)^p\right)$$

input
```
Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
```

output
```
-2*g*p*x + (2*(e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])
 + (-(f/x) + g*x)*Log[c*(d + e*x^2)^p]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(\frac{f\log\left(c(d+ex^2)^p\right)}{x^2}+g\log\left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{e}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{d}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f\log\left(c(d+ex^2)^p\right)}{x}+gx\log\left(c(d+ex^2)^p\right)-2gpx$$

input
```
Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
```

output
```
-2*g*p*x + (2*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[d
]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f*Log[c*(d + e*x^2)^p])/x +
g*x*Log[c*(d + e*x^2)^p]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

method	result
parts	$gx\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)-\frac{f\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{x}-2ep\left(\frac{gx}{e}+\frac{\left(-dg-ef\right)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{e\sqrt{de}}\right)$
risch	$-\frac{\left(-g\,x^{2}+f\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{x}+\frac{i\pi g\,x^{2}\operatorname{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\operatorname{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}ed-i\pi g\,x^{2}\operatorname{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\operatorname{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)\operatorname{csgn}$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^2,x,method=_RETURNVERBOSE)
```

output
```
g*x*ln(c*(e*x^2+d)^p)-f*ln(c*(e*x^2+d)^p)/x-2*e*p*(g*x/e+(-d*g-e*f)/e/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.76

$$\int\frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx$$

$$=\left[-\frac{2\,degpx^2+\sqrt{-de}(ef+dg)px\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)-(degpx^2-defp)\log\left(ex^2+d\right)-(degx^2-def)}{dex}\right.$$

$$-\frac{2\,degpx^2-2\sqrt{de}(ef+dg)px\arctan\left(\frac{\sqrt{de}x}{d}\right)-(degpx^2-defp)\log\left(ex^2+d\right)-(degx^2-def)\log\left(c\right)}{dex}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

output
```
[-(2*d*e*g*p*x^2 + sqrt(-d*e)*(e*f + d*g)*p*x*log((e*x^2 - 2*sqrt(-d*e)*x
- d)/(e*x^2 + d)) - (d*e*g*p*x^2 - d*e*f*p)*log(e*x^2 + d) - (d*e*g*x^2 -
d*e*f)*log(c))/(d*e*x), -(2*d*e*g*p*x^2 - 2*sqrt(d*e)*(e*f + d*g)*p*x*arct
an(sqrt(d*e)*x/d) - (d*e*g*p*x^2 - d*e*f*p)*log(e*x^2 + d) - (d*e*g*x^2 -
d*e*f)*log(c))/(d*e*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(71) = 142$.

Time = 16.41 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.83

$$\int \frac{(f + gx^2)\log\left(c(d + ex^2)^p\right)}{x^2}\,dx$$
$$= \begin{cases} \left(-\frac{f}{x} + gx\right)\log\left(0^p c\right) \\ \left(-\frac{f}{x} + gx\right)\log\left(cd^p\right) \\ -\frac{2fp}{x} - \frac{f\log\left(c(ex^2)^p\right)}{x} - 2gpx + gx\log\left(c(ex^2)^p\right) \\ \frac{2dgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}} - \frac{dg\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}} + \frac{2fp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\sqrt{-\frac{d}{e}}} - \frac{f\log\left(c(d+ex^2)^p\right)}{\sqrt{-\frac{d}{e}}} - \frac{f\log\left(c(d+ex^2)^p\right)}{x} - 2gpx + gx\log \end{cases}$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**2,x)
```

output
```
Piecewise(((-f/x + g*x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f/x + g*x)*l
og(c*d**p), Eq(e, 0)), (-2*f*p/x - f*log(c*(e*x**2)**p)/x - 2*g*p*x + g*x*
log(c*(e*x**2)**p), Eq(d, 0)), (2*d*g*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e))
 - d*g*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 2*f*p*log(x - sqrt(-d/e))/s
qrt(-d/e) - f*log(c*(d + e*x**2)**p)/sqrt(-d/e) - f*log(c*(d + e*x**2)**p)
/x - 2*g*p*x + g*x*log(c*(d + e*x**2)**p), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = -(2\,gp-g\log(c))x+\left(gpx-\frac{fp}{x}\right)\log\left(ex^2+d\right) + \frac{2\,(efp+dgp)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{f\log(c)}{x}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")
```

output
```
-(2*g*p - g*log(c))*x + (g*p*x - f*p/x)*log(e*x^2 + d) + 2*(e*f*p + d*g*p)
*arctan(e*x/sqrt(d*e))/sqrt(d*e) - f*log(c)/x
```

Mupad [B] (verification not implemented)

Time = 26.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx=\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(2\,g\,x-\frac{g\,x^2+f}{x}\right)-2\,g\,p\,x+\frac{2\,p\,\mathrm{atan}\left(\frac{2\,\sqrt{e}\,p\,x\,(d\,g+e\,f)}{\sqrt{d}\,(2\,d\,g\,p+2\,e\,f\,p)}\right)\,(d\,g+e\,f)}{\sqrt{d}\,\sqrt{e}}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^2,x)
```

output
```
log(c*(d + e*x^2)^p)*(2*g*x - (f + g*x^2)/x) - 2*g*p*x + (2*p*atan((2*e^(1
/2)*p*x*(d*g + e*f))/(d^(1/2)*(2*d*g*p + 2*e*f*p)))*(d*g + e*f))/(d^(1/2)*
e^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx=\frac{2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)dgpx+2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)efpx-\log\left(\left(e\,x^2+d\right)^p c\right)def+\log\left(\left(e\,x^2+d\right)^p c\right)de}{dex}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x)
```

output
```
(2*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*g*p*x + 2*sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p*x - log((d + e*x**2)**p*c)*d*e*f +
log((d + e*x**2)**p*c)*d*e*g*x**2 - 2*d*e*g*p*x**2)/(d*e*x)
```

3.321 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = -\frac{2efp}{3dx}-\frac{2e^{3/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g\log\left(c(d+ex^2)^p\right)}{x}$$

output
```
-2/3*e*f*p/d/x-2/3*e^(3/2)*f*p*arctan(e^(1/2)*x/d^(1/2))/d^(3/2)+2*e^(1/2)
*g*p*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)-1/3*f*ln(c*(e*x^2+d)^p)/x^3-g*ln(c*
(e*x^2+d)^p)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = \frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{2efp\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} - \frac{f\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{g\log\left(c(d+ex^2)^p\right)}{x}$$

input
```
Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

output
```
(2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f*p*Hypergeomet
ric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d + e*x^2)^p])/(3*
x^3) - (g*Log[c*(d + e*x^2)^p])/x
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx$$

$$\Big\downarrow \text{ 2926}$$

$$\int \left(\frac{f\log\left(c(d+ex^2)^p\right)}{x^4} + \frac{g\log\left(c(d+ex^2)^p\right)}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2e^{3/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g\log\left(c(d+ex^2)^p\right)}{x}-\frac{2efp}{3dx}$$

input
```
Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

output
```
(-2*e*f*p)/(3*d*x) - (2*e^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2
)) + (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (f*Log[c*(d + e
*x^2)^p])/(3*x^3) - (g*Log[c*(d + e*x^2)^p])/x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

method	result
parts	$-\frac{g\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{x}-\frac{f\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{3x^{3}}-\frac{2ep\left(\frac{\left(-3dg+ef\right)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{f}{dx}\right)}{3}$
risch	$-\frac{\left(3g\,x^{2}+f\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{3x^{3}}-\frac{3i\pi\,d^{2}g\,x^{2}\operatorname{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\operatorname{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}-3i\pi\,d^{2}g\,x^{2}\operatorname{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\operatorname{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)}{}$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-g*ln(c*(e*x^2+d)^p)/x-1/3*f*ln(c*(e*x^2+d)^p)/x^3-2/3*e*p*((-3*d*g+e*f)/d
/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/d*f/x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.77

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx$$
$$= \left[-\frac{(ef-3\,dg)px^3\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2+2\,dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right)+2\,efpx^2+(3\,dgpx^2+dfp)\log\left(ex^2+d\right)+(3\,dgx^2+df)}{3\,dx^3}\right.$$
$$\left.-\frac{2\,(ef-3\,dg)px^3\sqrt{\frac{e}{d}}\arctan\left(x\sqrt{\frac{e}{d}}\right)+2\,efpx^2+(3\,dgpx^2+dfp)\log\left(ex^2+d\right)+(3\,dgx^2+df)\log\left(c\right)}{3\,dx^3}\right]$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

output
```
[-1/3*((e*f - 3*d*g)*p*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(
e*x^2 + d)) + 2*e*f*p*x^2 + (3*d*g*p*x^2 + d*f*p)*log(e*x^2 + d) + (3*d*g*
x^2 + d*f)*log(c))/(d*x^3), -1/3*(2*(e*f - 3*d*g)*p*x^3*sqrt(e/d)*arctan(x
*sqrt(e/d)) + 2*e*f*p*x^2 + (3*d*g*p*x^2 + d*f*p)*log(e*x^2 + d) + (3*d*g*
x^2 + d*f)*log(c))/(d*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. 2(104) = 208.

Time = 39.14 (sec) , antiderivative size = 901, normalized size of antiderivative = 8.34

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**4,x)
```

output
```
Piecewise(((-f/(3*x**3) - g/x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f/(3*
x**3) - g/x)*log(c*d**p), Eq(e, 0)), (-2*f*p/(9*x**3) - f*log(c*(e*x**2)**
p)/(3*x**3) - 2*g*p/x - g*log(c*(e*x**2)**p)/x, Eq(d, 0)), ((-f/(3*x**3) -
 g/x)*log(0**p*c), Eq(d, -e*x**2)), (-d**2*f*sqrt(-d/e)*log(c*(d + e*x**2)
**p)/(3*d**2*x**3*sqrt(-d/e) + 3*d*e*x**5*sqrt(-d/e)) + 6*d**2*g*p*x**3*lo
g(x - sqrt(-d/e))/(3*d**2*x**3*sqrt(-d/e) + 3*d*e*x**5*sqrt(-d/e)) - 3*d**
2*g*x**3*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e) + 3*d*e*x**5*sqrt(
-d/e)) - 3*d**2*g*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt
(-d/e) + 3*d*e*x**5*sqrt(-d/e)) - 2*d*f*p*x**3*log(x - sqrt(-d/e))/(3*d**2
*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - 2*d*f*p*x**2*sqrt(-d/e)/(3*d**
2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) + d*f*x**3*log(c*(d + e*x**2)**
p)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - d*f*x**2*sqrt(-d/e)*
log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) +
6*d*g*p*x**5*log(x - sqrt(-d/e))/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt
(-d/e)) - 3*d*g*x**5*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e)/e + 3*
d*x**5*sqrt(-d/e)) - 3*d*g*x**4*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(3*d**2*
x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - 2*e*f*p*x**5*log(x - sqrt(-d/e)
)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - 2*e*f*p*x**4*sqrt(-d/
e)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) + e*f*x**5*log(c*(d +
e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = -\frac{2\,(e^2fp-3\,degp)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{3\,\sqrt{de}d} - \frac{(3\,gpx^2+fp)\log\left(ex^2+d\right)}{3\,x^3} - \frac{2\,efpx^2+3\,dgx^2\log\left(c\right)+df\log\left(c\right)}{3\,dx^3}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

output
```
-2/3*(e^2*f*p - 3*d*e*g*p)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d) - 1/3*(3*g*
p*x^2 + f*p)*log(e*x^2 + d)/x^3 - 1/3*(2*e*f*p*x^2 + 3*d*g*x^2*log(c) + d*
f*log(c))/(d*x^3)
```

Mupad [B] (verification not implemented)

Time = 26.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = \frac{2\,\sqrt{e}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(3\,d\,g-e\,f)}{3\,d^{3/2}} - \frac{2\,e\,f\,p}{3\,d\,x} - \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(g\,x^2+\frac{f}{3}\right)}{x^3}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^4,x)
```

output
```
(2*e^(1/2)*p*atan((e^(1/2)*x)/d^(1/2))*(3*d*g - e*f))/(3*d^(3/2)) - (2*e*f
*p)/(3*d*x) - (log(c*(d + e*x^2)^p)*(f/3 + g*x^2))/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx$$
$$= \frac{6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)dgp\,x^3 - 2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)efp\,x^3 - \log\left((e\,x^2+d)^p\,c\right)d^2f - 3\log\left((e\,x^2+d)^p\,c\right)}{3d^2x^3}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x)
```

output
```
(6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*g*p*x**3 - 2*sqrt(e)*sq
rt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p*x**3 - log((d + e*x**2)**p*c)*d*
*2*f - 3*log((d + e*x**2)**p*c)*d**2*g*x**2 - 2*d*e*f*p*x**2)/(3*d**2*x**3
)
```

3.322 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = -\frac{2efp}{15dx^3}+\frac{2e^2fp}{5d^2x}-\frac{2egp}{3dx}+\frac{2e^{5/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}$$

output
```
-2/15*e*f*p/d/x^3+2/5*e^2*f*p/d^2/x-2/3*e*g*p/d/x+2/5*e^(5/2)*f*p*arctan(e
^(1/2)*x/d^(1/2))/d^(5/2)-2/3*e^(3/2)*g*p*arctan(e^(1/2)*x/d^(1/2))/d^(3/2
)-1/5*f*ln(c*(e*x^2+d)^p)/x^5-1/3*g*ln(c*(e*x^2+d)^p)/x^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = -\frac{2efp\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{ex^2}{d}\right)}{15dx^3} - \frac{2egp\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} - \frac{f\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}$$

input
```
Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

output
```
(-2*e*f*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*
e*g*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d
 + e*x^2)^p])/(5*x^5) - (g*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx$$

$$\Big\downarrow \text{ 2926}$$

$$\int \left(\frac{f\log\left(c(d+ex^2)^p\right)}{x^6}+\frac{g\log\left(c(d+ex^2)^p\right)}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2e^{5/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}+\frac{2e^2fp}{5d^2x}-\frac{2efp}{15dx^3}-\frac{2egp}{3dx}$$

input

```
Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

output

```
(-2*e*f*p)/(15*d*x^3) + (2*e^2*f*p)/(5*d^2*x) - (2*e*g*p)/(3*d*x) + (2*e^(
5/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g*p*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f*Log[c*(d + e*x^2)^p])/(5*x^5) - (g*
Log[c*(d + e*x^2)^p])/(3*x^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

method	result
parts	$-\frac{g\ln\left(c\left(e\,x^2+d\right)^p\right)}{3x^3}-\frac{f\ln\left(c\left(e\,x^2+d\right)^p\right)}{5x^5}-\frac{2ep\left(\frac{e(5dg-3ef)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{d^2\sqrt{de}}-\frac{-5dg+3ef}{d^2x}+\frac{f}{d\,x^3}\right)}{15}$
risch	$-\frac{\left(5g\,x^2+3f\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{15x^5}+\frac{-5i\pi\,d^2g\,x^2\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2+5i\pi\,d^2g\,x^2\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right.\right.}{}$

input
```
int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3*g*ln(c*(e*x^2+d)^p)/x^3-1/5*f*ln(c*(e*x^2+d)^p)/x^5-2/15*e*p*(e*(5*d*
g-3*e*f)/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-(-5*d*g+3*e*f)/d^2/x+1/d*
f/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.85

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx$$
$$= \left[-\frac{(3e^2f-5deg)px^5\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2-2dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right)+2defpx^2-2(3e^2f-5deg)px^4+(5d^2gpx^2+3d^2f\ldots}{15d^2x^5}\right.$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")
```

output
```
[-1/15*((3*e^2*f - 5*d*e*g)*p*x^5*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d)
 - d)/(e*x^2 + d)) + 2*d*e*f*p*x^2 - 2*(3*e^2*f - 5*d*e*g)*p*x^4 + (5*d^2*
g*p*x^2 + 3*d^2*f*p)*log(e*x^2 + d) + (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2
*x^5), 1/15*(2*(3*e^2*f - 5*d*e*g)*p*x^5*sqrt(e/d)*arctan(x*sqrt(e/d)) - 2
*d*e*f*p*x^2 + 2*(3*e^2*f - 5*d*e*g)*p*x^4 - (5*d^2*g*p*x^2 + 3*d^2*f*p)*l
og(e*x^2 + d) - (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(138) = 276.

Time = 156.92 (sec) , antiderivative size = 1134, normalized size of antiderivative = 8.10

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**6,x)
```

output
```
Piecewise(((-f/(5*x**5) - g/(3*x**3))*log(0**p*c), Eq(d, 0) & Eq(e, 0)), (
(-f/(5*x**5) - g/(3*x**3))*log(c*d**p), Eq(e, 0)), (-2*f*p/(25*x**5) - f*l
og(c*(e*x**2)**p)/(5*x**5) - 2*g*p/(9*x**3) - g*log(c*(e*x**2)**p)/(3*x**3
), Eq(d, 0)), ((-f/(5*x**5) - g/(3*x**3))*log(0**p*c), Eq(d, -e*x**2)), (-
3*d**3*f*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d
**2*e*x**7*sqrt(-d/e)) - 5*d**3*g*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(
15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 2*d**2*f*p*x**2*sqr
t(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*d**2*f*x
**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2
*x**7*sqrt(-d/e)) - 10*d**2*g*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqr
t(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 10*d**2*g*p*x**4*sqrt(-d/e)/(15*d**
3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 5*d**2*g*x**5*log(c*(d +
e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 5*d**2
*g*x**4*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*
d**2*x**7*sqrt(-d/e)) + 6*d*e*f*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*s
qrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 4*d*e*f*p*x**4*sqrt(-d/e)/(15*d**
3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*d*e*f*x**5*log(c*(d + e
*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 10*d*e*
g*p*x**7*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqr
t(-d/e)) - 10*d*e*g*p*x**6*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \frac{2\left(3\,e^3fp-5\,de^2gp\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}d^2} - \frac{(5\,gpx^2+3\,fp)\log\left(ex^2+d\right)}{15\,x^5} + \frac{6\,e^2fpx^4-10\,degpx^4-2\,defpx^2-5\,d^2gx^2\log\left(c\right)-3\,d^2f\log\left(c\right)}{15\,d^2x^5}$$

input
```
integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

output
```
2/15*(3*e^3*f*p - 5*d*e^2*g*p)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2) - 1/1
5*(5*g*p*x^2 + 3*f*p)*log(e*x^2 + d)/x^5 + 1/15*(6*e^2*f*p*x^4 - 10*d*e*g*
p*x^4 - 2*d*e*f*p*x^2 - 5*d^2*g*x^2*log(c) - 3*d^2*f*log(c))/(d^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 26.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = -\frac{\frac{2\,e\,f\,p}{d}+\frac{2\,e\,p\,x^2\,(5\,d\,g-3\,e\,f)}{d^2}}{15\,x^3} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^2}{3}+\frac{f}{5}\right)}{x^5} - \frac{2\,e^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,d\,g-3\,e\,f\right)}{15\,d^{5/2}}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^6,x)
```

output
```
- ((2*e*f*p)/d + (2*e*p*x^2*(5*d*g - 3*e*f))/d^2)/(15*x^3) - (log(c*(d + e
*x^2)^p)*(f/5 + (g*x^2)/3))/x^5 - (2*e^(3/2)*p*atan((e^(1/2)*x)/d^(1/2))*(
5*d*g - 3*e*f))/(15*d^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx$$
$$= \frac{-10\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)degp\,x^5+6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)e^2fp\,x^5-3\log\left((e\,x^2+d)^p\,c\right)d^3f-5\log\left((e\,x^2\right.}{15d^3x^5}$$

input
```
int((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x)
```

output
```
( - 10*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*g*p*x**5 + 6*sqrt
(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**2*f*p*x**5 - 3*log((d + e*x**
2)**p*c)*d**3*f - 5*log((d + e*x**2)**p*c)*d**3*g*x**2 - 2*d**2*e*f*p*x**2
 - 10*d**2*e*g*p*x**4 + 6*d*e**2*f*p*x**4)/(15*d**3*x**5)
```

3.323 $\int x^5(f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 251

$$\int x^5(f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx = -\frac{d^2(ef-dg)^2px^2}{2e^4} + \frac{d(ef-2dg)(ef-dg)p(d+ex^2)^2}{4e^5} - \frac{(e^2f^2-6defg+6d^2g^2)\,p(d+ex^2)^3}{18e^5} - \frac{g(ef-2dg)p(d+ex^2)^4}{16e^5} - \frac{g^2p(d+ex^2)^5}{50e^5} + \frac{d^3(10e^2f^2-15defg+6d^2g^2)\,p\log(d+ex^2)}{60e^5} + \frac{1}{6}f^2x^6\log\left(c(d+ex^2)^p\right) + \frac{1}{4}fgx^8\log\left(c(d+ex^2)^p\right) + \frac{1}{10}g^2x^{10}\log\left(c(d+ex^2)^p\right)$$

output

```
-1/2*d^2*(-d*g+e*f)^2*p*x^2/e^4+1/4*d*(-2*d*g+e*f)*(-d*g+e*f)*p*(e*x^2+d)^
2/e^5-1/18*(6*d^2*g^2-6*d*e*f*g+e^2*f^2)*p*(e*x^2+d)^3/e^5-1/16*g*(-2*d*g+
e*f)*p*(e*x^2+d)^4/e^5-1/50*g^2*p*(e*x^2+d)^5/e^5+1/60*d^3*(6*d^2*g^2-15*d
*e*f*g+10*e^2*f^2)*p*ln(e*x^2+d)/e^5+1/6*f^2*x^6*ln(c*(e*x^2+d)^p)+1/4*f*g
*x^8*ln(c*(e*x^2+d)^p)+1/10*g^2*x^10*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.82

$$\int x^5 \left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= \frac{-epx^2(360d^4g^2 - 180d^3eg(5f+gx^2) - 30de^3x^2(10f^2+10fgx^2+3g^2x^4) + 30d^2e^2(20f^2+15fgx^2+4g}{}$$

input
```
Integrate[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(-(e*p*x^2*(360*d^4*g^2 - 180*d^3*e*g*(5*f + g*x^2) - 30*d*e^3*x^2*(10*f^2
 + 10*f*g*x^2 + 3*g^2*x^4) + 30*d^2*e^2*(20*f^2 + 15*f*g*x^2 + 4*g^2*x^4)
+ e^4*x^4*(200*f^2 + 225*f*g*x^2 + 72*g^2*x^4))) + 60*d^3*(10*e^2*f^2 - 15
*d*e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2] + 60*e^5*x^6*(10*f^2 + 15*f*g*x^2 +
 6*g^2*x^4)*Log[c*(d + e*x^2)^p])/(3600*e^5)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2925, 2861, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right)\,dx$$

$$\Big\downarrow \text{ 2925}$$

$$\frac{1}{2}\int x^4 \left(gx^2+f\right)^2 \log \left(c\left(ex^2+d\right)^p\right)dx^2$$

$$\Big\downarrow \text{ 2861}$$

$$\frac{1}{2}\left(-ep\int \frac{x^6\left(6g^2x^4+15fgx^2+10f^2\right)}{30\left(ex^2+d\right)}dx^2 + \frac{1}{3}f^2x^6\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}fgx^8\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{5}g^2x^{10}\log\left(\right.\right.$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{1}{30}ep\int\frac{x^6(6g^2x^4+15fgx^2+10f^2)}{ex^2+d}dx^2+\frac{1}{3}f^2x^6\log\left(c(d+ex^2)^p\right)+\frac{1}{2}fgx^8\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^{10}\text{lo}\right.$$

$\downarrow$ 1195

$$\frac{1}{2}\left(-\frac{1}{30}ep\int\left(\frac{6g^2(ex^2+d)^4}{e^5}+\frac{15g(ef-2dg)(ex^2+d)^3}{e^5}+\frac{10(e^2f^2-6degf+6d^2g^2)(ex^2+d)^2}{e^5}+\frac{30d(ef-}{}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{3}f^2x^6\log\left(c(d+ex^2)^p\right)+\frac{1}{2}fgx^8\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^{10}\log\left(c(d+ex^2)^p\right)-\frac{1}{30}ep\left(\frac{30d^2x^2(ef-dg)^2}{e^5}\right.\right.$$

input
```
Int[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(-1/30*(e*p*((30*d^2*(e*f - d*g)^2*x^2)/e^5 - (15*d*(e*f - 2*d*g)*(e*f - d
*g)*(d + e*x^2)^2)/e^6 + (10*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)*(d + e*x^2)
^3)/(3*e^6) + (15*g*(e*f - 2*d*g)*(d + e*x^2)^4)/(4*e^6) + (6*g^2*(d + e*x
^2)^5)/(5*e^6) - (d^3*(10*e^2*f^2 - 15*d*e*f*g + 6*d^2*g^2)*Log[d + e*x^2]
)/e^6)) + (f^2*x^6*Log[c*(d + e*x^2)^p])/3 + (f*g*x^8*Log[c*(d + e*x^2)^p]
)/2 + (g^2*x^10*Log[c*(d + e*x^2)^p])/5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 6.71 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.01

method	result
parts	$\frac{g^{2} x^{10} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right)}{10}+\frac{f g \,x^{8} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right)}{4}+\frac{f^{2} x^{6} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right)}{6}-\frac{e p \left(\frac{\frac{6}{5} e^{4} g^{2} x^{10}-\frac{3}{2} x^{8} d \,e^{3} g^{2}+\frac{15}{4} e^{4} f g \,x^{8}+2 x^{6} d^{2} e^{2}}{}\right.}{}$
parallelrisch	$\frac{360 x^{10} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right) e^{5} g^{2}-72 e^{5} g^{2} p \,x^{10}+900 x^{8} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right) e^{5} f g +90 d \,e^{4} g^{2} p \,x^{8}-225 e^{5} f g p \,x^{8}+600 x^{6} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right) e}{}$
risch	$\frac{d \,g^{2} p \,x^{8}}{40 e}-\frac{d^{2} g^{2} p \,x^{6}}{30 e^{2}}+\frac{d^{3} g^{2} p \,x^{4}}{20 e^{3}}+\frac{d \,f^{2} p \,x^{4}}{12 e}-\frac{d^{4} g^{2} p \,x^{2}}{10 e^{4}}-\frac{d^{2} f^{2} p \,x^{2}}{6 e^{2}}+\frac{\ln\left(e \,x^{2}+d \right) d^{5} g^{2} p}{10 e^{5}}+\frac{\ln\left(c \right) f g \,x^{8}}{4}-\frac{i \pi \,g^{2} x^{1}}{}$

input
```
int(x^5*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/10*g^2*x^10*ln(c*(e*x^2+d)^p)+1/4*f*g*x^8*ln(c*(e*x^2+d)^p)+1/6*f^2*x^6*
ln(c*(e*x^2+d)^p)-1/30*e*p*(1/2/e^5*(6/5*e^4*g^2*x^10-3/2*x^8*d*e^3*g^2+15
/4*e^4*f*g*x^8+2*x^6*d^2*e^2*g^2-5*x^6*d*e^3*f*g+10/3*e^4*f^2*x^6-3*x^4*d^
3*e*g^2+15/2*x^4*d^2*e^2*f*g-5*x^4*d*e^3*f^2+6*d^4*g^2*x^2-15*d^3*e*f*g*x^
2+10*d^2*e^2*f^2*x^2)-1/2*d^3*(6*d^2*g^2-15*d*e*f*g+10*e^2*f^2)/e^6*ln(e*x
^2+d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.04

$$\int x^5(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = -\frac{72\,e^5g^2px^{10}+45\,(5\,e^5fg-2\,de^4g^2)px^8+20\,(10\,e^5f^2-15\,de^4fg+6\,d^2e^3g^2)px^6-30\,(10\,de^4f^2-15\,d}{}$$

input
```
integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
-1/3600*(72*e^5*g^2*p*x^10 + 45*(5*e^5*f*g - 2*d*e^4*g^2)*p*x^8 + 20*(10*e
^5*f^2 - 15*d*e^4*f*g + 6*d^2*e^3*g^2)*p*x^6 - 30*(10*d*e^4*f^2 - 15*d^2*e
^3*f*g + 6*d^3*e^2*g^2)*p*x^4 + 60*(10*d^2*e^3*f^2 - 15*d^3*e^2*f*g + 6*d^
4*e*g^2)*p*x^2 - 60*(6*e^5*g^2*p*x^10 + 15*e^5*f*g*p*x^8 + 10*e^5*f^2*p*x^
6 + (10*d^3*e^2*f^2 - 15*d^4*e*f*g + 6*d^5*g^2)*p)*log(e*x^2 + d) - 60*(6*
e^5*g^2*x^10 + 15*e^5*f*g*x^8 + 10*e^5*f^2*x^6)*log(c))/e^5
```

Sympy [F(-1)]

Timed out.

$$\int x^5(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = \text{Timed out}$$

input
```
integrate(x**5*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.89

$$\int x^5(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx =$$
$$-\frac{1}{3600}\,ep\left(\frac{72\,e^4g^2x^{10}+45\,(5\,e^4fg-2\,de^3g^2)x^8+20\,(10\,e^4f^2-15\,de^3fg+6\,d^2e^2g^2)x^6-30\,(10\,de^3f^2}{e^5}\right.$$
$$+\frac{1}{60}\left(6\,g^2x^{10}+15\,fgx^8+10\,f^2x^6\right)\log\left((ex^2+d)^pc\right)$$

input
```
integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
-1/3600*e*p*((72*e^4*g^2*x^10 + 45*(5*e^4*f*g - 2*d*e^3*g^2)*x^8 + 20*(10*
e^4*f^2 - 15*d*e^3*f*g + 6*d^2*e^2*g^2)*x^6 - 30*(10*d*e^3*f^2 - 15*d^2*e^
2*f*g + 6*d^3*e*g^2)*x^4 + 60*(10*d^2*e^2*f^2 - 15*d^3*e*f*g + 6*d^4*g^2)*
x^2)/e^5 - 60*(10*d^3*e^2*f^2 - 15*d^4*e*f*g + 6*d^5*g^2)*log(e*x^2 + d)/e
^6) + 1/60*(6*g^2*x^10 + 15*f*g*x^8 + 10*f^2*x^6)*log((e*x^2 + d)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. 2(233) = 466.

Time = 0.14 (sec) , antiderivative size = 753, normalized size of antiderivative = 3.00

$$\int x^5(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
1/6*(e*x^2 + d)^3*f^2*p*log(e*x^2 + d)/e^3 - 1/2*(e*x^2 + d)^2*d*f^2*p*log
(e*x^2 + d)/e^3 + 1/4*(e*x^2 + d)^4*f*g*p*log(e*x^2 + d)/e^4 - (e*x^2 + d)
^3*d*f*g*p*log(e*x^2 + d)/e^4 + 3/2*(e*x^2 + d)^2*d^2*f*g*p*log(e*x^2 + d)
/e^4 + 1/10*(e*x^2 + d)^5*g^2*p*log(e*x^2 + d)/e^5 - 1/2*(e*x^2 + d)^4*d*g
^2*p*log(e*x^2 + d)/e^5 + (e*x^2 + d)^3*d^2*g^2*p*log(e*x^2 + d)/e^5 - (e*
x^2 + d)^2*d^3*g^2*p*log(e*x^2 + d)/e^5 - 1/18*(e*x^2 + d)^3*f^2*p/e^3 + 1
/4*(e*x^2 + d)^2*d*f^2*p/e^3 - 1/16*(e*x^2 + d)^4*f*g*p/e^4 + 1/3*(e*x^2 +
 d)^3*d*f*g*p/e^4 - 3/4*(e*x^2 + d)^2*d^2*f*g*p/e^4 - 1/50*(e*x^2 + d)^5*g
^2*p/e^5 + 1/8*(e*x^2 + d)^4*d*g^2*p/e^5 - 1/3*(e*x^2 + d)^3*d^2*g^2*p/e^5
 + 1/2*(e*x^2 + d)^2*d^3*g^2*p/e^5 + 1/6*(e*x^2 + d)^3*f^2*log(c)/e^3 - 1/
2*(e*x^2 + d)^2*d*f^2*log(c)/e^3 + 1/4*(e*x^2 + d)^4*f*g*log(c)/e^4 - (e*x
^2 + d)^3*d*f*g*log(c)/e^4 + 3/2*(e*x^2 + d)^2*d^2*f*g*log(c)/e^4 + 1/10*(
e*x^2 + d)^5*g^2*log(c)/e^5 - 1/2*(e*x^2 + d)^4*d*g^2*log(c)/e^5 + (e*x^2
+ d)^3*d^2*g^2*log(c)/e^5 - (e*x^2 + d)^2*d^3*g^2*log(c)/e^5 - 1/2*((e*x^2
 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^2*e^2*f^2*p - 2*(e*x^2 - (e*x^2 + d)*
log(e*x^2 + d) + d)*d^3*e*f*g*p + (e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)
*d^4*g^2*p - (e*x^2 + d)*d^2*e^2*f^2*log(c) + 2*(e*x^2 + d)*d^3*e*f*g*log(
c) - (e*x^2 + d)*d^4*g^2*log(c))/e^5
```

Mupad [B] (verification not implemented)

Time = 26.05 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.89

$$\int x^5\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{f^2\,x^6}{6}+\frac{f\,g\,x^8}{4}+\frac{g^2\,x^{10}}{10}\right)-x^6\,\left(\frac{f^2\,p}{18}-\frac{d\,\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{6\,e}\right)-x^8\,\left(\frac{f\,g\,p}{16}-\frac{d\,g^2\,p}{40\,e}\right)-\frac{g^2\,p\,x^{10}}{50}+\frac{\ln\left(e\,x^2+d\right)\,\left(6\,p\,d^5\,g^2-15\,p\,d^4\,e\,f\,g+10\,p\,d^3\,e^2\,f^2\right)}{60\,e^5}+\frac{d\,x^4\,\left(\frac{f^2\,p}{3}-\frac{d\,\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{e}\right)}{4\,e}-\frac{d^2\,x^2\,\left(\frac{f^2\,p}{3}-\frac{d\,\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{e}\right)}{2\,e^2}$$

input
```
int(x^5*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

output
```
log(c*(d + e*x^2)^p)*((f^2*x^6)/6 + (g^2*x^10)/10 + (f*g*x^8)/4) - x^6*((f
^2*p)/18 - (d*((f*g*p)/2 - (d*g^2*p)/(5*e)))/(6*e)) - x^8*((f*g*p)/16 - (d
*g^2*p)/(40*e)) - (g^2*p*x^10)/50 + (log(d + e*x^2)*(6*d^5*g^2*p + 10*d^3*
e^2*f^2*p - 15*d^4*e*f*g*p))/(60*e^5) + (d*x^4*((f^2*p)/3 - (d*((f*g*p)/2
- (d*g^2*p)/(5*e)))/e))/(4*e) - (d^2*x^2*((f^2*p)/3 - (d*((f*g*p)/2 - (d*g
^2*p)/(5*e)))/e))/(2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.18

$$\int x^5 (f + gx^2)^2 \log\left(c(d + ex^2)^p\right)\,dx$$
$$= \frac{360\log\left((e\,x^2+d)^p\,c\right)d^5g^2 - 900\log\left((e\,x^2+d)^p\,c\right)d^4efg + 600\log\left((e\,x^2+d)^p\,c\right)d^3e^2f^2 + 600\log\left((e\,x^2\right.}{}$$

input
```
int(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x)
```

output
```
(360*log((d + e*x**2)**p*c)*d**5*g**2 - 900*log((d + e*x**2)**p*c)*d**4*e*
f*g + 600*log((d + e*x**2)**p*c)*d**3*e**2*f**2 + 600*log((d + e*x**2)**p*
c)*e**5*f**2*x**6 + 900*log((d + e*x**2)**p*c)*e**5*f*g*x**8 + 360*log((d
+ e*x**2)**p*c)*e**5*g**2*x**10 - 360*d**4*e*g**2*p*x**2 + 900*d**3*e**2*f
*g*p*x**2 + 180*d**3*e**2*g**2*p*x**4 - 600*d**2*e**3*f**2*p*x**2 - 450*d*
*2*e**3*f*g*p*x**4 - 120*d**2*e**3*g**2*p*x**6 + 300*d*e**4*f**2*p*x**4 +
300*d*e**4*f*g*p*x**6 + 90*d*e**4*g**2*p*x**8 - 200*e**5*f**2*p*x**6 - 225
*e**5*f*g*p*x**8 - 72*e**5*g**2*p*x**10)/(3600*e**5)
```

3.324 $\int x^3(f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 210

$$\begin{aligned}\int x^3(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx &= \frac{d(ef-dg)^2px^2}{2e^3} \\ &- \frac{(ef-3dg)(ef-dg)p(d+ex^2)^2}{8e^4} \\ &- \frac{g(2ef-3dg)p(d+ex^2)^3}{18e^4} - \frac{g^2p(d+ex^2)^4}{32e^4} \\ &- \frac{d^2(6e^2f^2-8defg+3d^2g^2)\,p\log(d+ex^2)}{24e^4} \\ &+ \frac{1}{4}f^2x^4\log\left(c(d+ex^2)^p\right) \\ &+ \frac{1}{3}fgx^6\log\left(c(d+ex^2)^p\right) \\ &+ \frac{1}{8}g^2x^8\log\left(c(d+ex^2)^p\right)\end{aligned}$$

output

```
1/2*d*(-d*g+e*f)^2*p*x^2/e^3-1/8*(-3*d*g+e*f)*(-d*g+e*f)*p*(e*x^2+d)^2/e^4
-1/18*g*(-3*d*g+2*e*f)*p*(e*x^2+d)^3/e^4-1/32*g^2*p*(e*x^2+d)^4/e^4-1/24*d
^2*(3*d^2*g^2-8*d*e*f*g+6*e^2*f^2)*p*ln(e*x^2+d)/e^4+1/4*f^2*x^4*ln(c*(e*x
^2+d)^p)+1/3*f*g*x^6*ln(c*(e*x^2+d)^p)+1/8*g^2*x^8*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.22

$$\int x^3(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = \frac{df^2px^2}{4e}-\frac{d^2fgpx^2}{3e^2}+\frac{d^3g^2px^2}{8e^3}-\frac{1}{8}f^2px^4+\frac{dfgpx^4}{6e}-\frac{d^2g^2px^4}{16e^2}-\frac{1}{9}fgpx^6+\frac{dg^2px^6}{24e}-\frac{1}{32}g^2px^8-\frac{d^2f^2p\log(d+ex^2)}{4e^2}+\frac{d^3fgp\log(d+ex^2)}{3e^3}-\frac{d^4g^2p\log(d+ex^2)}{8e^4}+\frac{1}{4}f^2x^4\log\left(c(d+ex^2)^p\right)+\frac{1}{3}fgx^6\log\left(c(d+ex^2)^p\right)+\frac{1}{8}g^2x^8\log\left(c(d+ex^2)^p\right)$$

input
```
Integrate[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(d*f^2*p*x^2)/(4*e) - (d^2*f*g*p*x^2)/(3*e^2) + (d^3*g^2*p*x^2)/(8*e^3) -
(f^2*p*x^4)/8 + (d*f*g*p*x^4)/(6*e) - (d^2*g^2*p*x^4)/(16*e^2) - (f*g*p*x^
6)/9 + (d*g^2*p*x^6)/(24*e) - (g^2*p*x^8)/32 - (d^2*f^2*p*Log[d + e*x^2])/
(4*e^2) + (d^3*f*g*p*Log[d + e*x^2])/(3*e^3) - (d^4*g^2*p*Log[d + e*x^2])/
(8*e^4) + (f^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*x^6*Log[c*(d + e*x^2)^p]
)/3 + (g^2*x^8*Log[c*(d + e*x^2)^p])/8
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2925, 2861, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int x^2\left(gx^2+f\right)^2\log\left(c\left(ex^2+d\right)^p\right)dx^2$$

$\Big\downarrow$ 2861

$$\frac{1}{2}\left(-ep\int\frac{x^4\left(3g^2x^4+8fgx^2+6f^2\right)}{12\left(ex^2+d\right)}dx^2+\frac{1}{2}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}g^2x^8\log\left(c\left(d\right.\right.\right.$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{1}{12}ep\int\frac{x^4\left(3g^2x^4+8fgx^2+6f^2\right)}{ex^2+d}dx^2+\frac{1}{2}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}g^2x^8\log\left(\right.\right.$$

$\Big\downarrow$ 1195

$$\frac{1}{2}\left(-\frac{1}{12}ep\int\left(\frac{3g^2\left(ex^2+d\right)^3}{e^4}+\frac{4g(2ef-3dg)\left(ex^2+d\right)^2}{e^4}+\frac{6(ef-3dg)(ef-dg)\left(ex^2+d\right)}{e^4}-\frac{12d(dg-ef)^2}{e^4}\right.\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{2}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{12}ep\left(\frac{d^2\left(3d^2g^2-8defg+\right.}{e}\right.\right.$$

input
```
Int[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(-1/12*(e*p*((-12*d*(e*f - d*g)^2*x^2)/e^4 + (3*(e*f - 3*d*g)*(e*f - d*g)*
(d + e*x^2)^2)/e^5 + (4*g*(2*e*f - 3*d*g)*(d + e*x^2)^3)/(3*e^5) + (3*g^2*
(d + e*x^2)^4)/(4*e^5) + (d^2*(6*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2)*Log[d +
e*x^2])/e^5)) + (f^2*x^4*Log[c*(d + e*x^2)^p])/2 + (2*f*g*x^6*Log[c*(d + e
*x^2)^p])/3 + (g^2*x^8*Log[c*(d + e*x^2)^p])/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

method	result
parts	$\frac{g^2x^8\ln(c(e\,x^2+d)^p)}{8}+\frac{fg\,x^6\ln(c(e\,x^2+d)^p)}{3}+\frac{f^2x^4\ln(c(e\,x^2+d)^p)}{4}-\frac{ep\left(-\frac{-\frac{3}{4}e^3g^2x^8+d\,e^2g^2x^6-\frac{8}{3}e^3fg\,x^6-\frac{3}{2}d^2e\,g^2x\right.}{}$
parallelrisch	$-\frac{-36x^8\ln(c(e\,x^2+d)^p)e^4g^2+9x^8e^4g^2p-96x^6\ln(c(e\,x^2+d)^p)e^4fg-12x^6d\,e^3g^2p+32x^6e^4fgp-72x^4\ln(c(e\,x^2+d)^p)e^4f^2+}{}$
risch	$\frac{\ln(c)fg\,x^6}{3}-\frac{x^6fgp}{9}-\frac{x^8g^2p}{32}-\frac{x^4f^2p}{8}+\frac{x^6d\,g^2p}{24e}-\frac{x^4d^2g^2p}{16e^2}+\frac{x^2d^3g^2p}{8e^3}+\frac{x^2d\,f^2p}{4e}-\frac{\ln(e\,x^2+d)d^4g^2p}{8e^4}-\ln($

input
```
int(x^3*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/8*g^2*x^8*ln(c*(e*x^2+d)^p)+1/3*f*g*x^6*ln(c*(e*x^2+d)^p)+1/4*f^2*x^4*ln
(c*(e*x^2+d)^p)-1/12*e*p*(-1/2/e^4*(-3/4*e^3*g^2*x^8+d*e^2*g^2*x^6-8/3*e^3
*f*g*x^6-3/2*d^2*e*g^2*x^4+4*d*f*g*x^4*e^2-3*e^3*f^2*x^4+3*d^3*x^2*g^2-8*d
^2*e*f*g*x^2+6*d*e^2*f^2*x^2)+1/2*d^2*(3*d^2*g^2-8*d*e*f*g+6*e^2*f^2)/e^5*
ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.07

$$\int x^3(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = -\frac{9\,e^4g^2px^8+4\,(8\,e^4fg-3\,de^3g^2)px^6+6\,(6\,e^4f^2-8\,de^3fg+3\,d^2e^2g^2)px^4-12\,(6\,de^3f^2-8\,d^2e^2fg+3}{}$$

input
```
integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
-1/288*(9*e^4*g^2*p*x^8 + 4*(8*e^4*f*g - 3*d*e^3*g^2)*p*x^6 + 6*(6*e^4*f^2
 - 8*d*e^3*f*g + 3*d^2*e^2*g^2)*p*x^4 - 12*(6*d*e^3*f^2 - 8*d^2*e^2*f*g +
3*d^3*e*g^2)*p*x^2 - 12*(3*e^4*g^2*p*x^8 + 8*e^4*f*g*p*x^6 + 6*e^4*f^2*p*x
^4 - (6*d^2*e^2*f^2 - 8*d^3*e*f*g + 3*d^4*g^2)*p)*log(e*x^2 + d) - 12*(3*e
^4*g^2*x^8 + 8*e^4*f*g*x^6 + 6*e^4*f^2*x^4)*log(c))/e^4
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx = \text{Timed out}$$

input
```
integrate(x**3*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int x^3 (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx = -\frac{1}{288} ep \left(\frac{9 e^3 g^2 x^8 + 4 (8 e^3 f g - 3 d e^2 g^2) x^6 + 6 (6 e^3 f^2 - 8 d e^2 f g + 3 d^2 e g^2) x^4 - 12 (6 d e^2 f^2 - 8 d^2 e f g}{e^4} \right. + \frac{1}{24} \left(3 g^2 x^8 + 8 f g x^6 + 6 f^2 x^4\right) \log \left((ex^2+d)^p c\right)$$

input
```
integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
-1/288*e*p*((9*e^3*g^2*x^8 + 4*(8*e^3*f*g - 3*d*e^2*g^2)*x^6 + 6*(6*e^3*f^
2 - 8*d*e^2*f*g + 3*d^2*e*g^2)*x^4 - 12*(6*d*e^2*f^2 - 8*d^2*e*f*g + 3*d^3
*g^2)*x^2)/e^4 + 12*(6*d^2*e^2*f^2 - 8*d^3*e*f*g + 3*d^4*g^2)*log(e*x^2 +
d)/e^5) + 1/24*(3*g^2*x^8 + 8*f*g*x^6 + 6*f^2*x^4)*log((e*x^2 + d)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(194) = 388$.

Time = 0.13 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.59

$$\int x^3 (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = \frac{(ex^2+d)^2 f^2 p \log(ex^2+d)}{4\,e^2} + \frac{(ex^2+d)^3 fgp \log(ex^2+d)}{3\,e^3} - \frac{(ex^2+d)^2 dfgp \log(ex^2+d)}{e^3} + \frac{(ex^2+d)^4 g^2 p \log(ex^2+d)}{8\,e^4} - \frac{(ex^2+d)^3 dg^2 p \log(ex^2+d)}{2\,e^4} + \frac{3\,(ex^2+d)^2 d^2 g^2 p \log(ex^2+d)}{4\,e^4} - \frac{(ex^2+d)^2 f^2 p}{8\,e^2} - \frac{(ex^2+d)^3 fgp}{9\,e^3} + \frac{(ex^2+d)^2 dfgp}{2\,e^3} - \frac{(ex^2+d)^4 g^2 p}{32\,e^4} + \frac{(ex^2+d)^3 dg^2 p}{6\,e^4} - \frac{3\,(ex^2+d)^2 d^2 g^2 p}{8\,e^4} + \frac{(ex^2+d)^2 f^2 \log(c)}{4\,e^2} + \frac{(ex^2+d)^3 fg \log(c)}{3\,e^3} - \frac{(ex^2+d)^2 dfg \log(c)}{e^3} + \frac{(ex^2+d)^4 g^2 \log(c)}{8\,e^4} - \frac{(ex^2+d)^3 dg^2 \log(c)}{2\,e^4} + \frac{3\,(ex^2+d)^2 d^2 g^2 \log(c)}{4\,e^4} + \frac{(ex^2-(ex^2+d)\log(ex^2+d)+d)de^2f^2p - 2\,(ex^2-(ex^2+d)\log(ex^2+d)+d)d^2efgp + (ex^2-(ex$$

input
```
integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
1/4*(e*x^2 + d)^2*f^2*p*log(e*x^2 + d)/e^2 + 1/3*(e*x^2 + d)^3*f*g*p*log(e
*x^2 + d)/e^3 - (e*x^2 + d)^2*d*f*g*p*log(e*x^2 + d)/e^3 + 1/8*(e*x^2 + d)
^4*g^2*p*log(e*x^2 + d)/e^4 - 1/2*(e*x^2 + d)^3*d*g^2*p*log(e*x^2 + d)/e^4
 + 3/4*(e*x^2 + d)^2*d^2*g^2*p*log(e*x^2 + d)/e^4 - 1/8*(e*x^2 + d)^2*f^2*
p/e^2 - 1/9*(e*x^2 + d)^3*f*g*p/e^3 + 1/2*(e*x^2 + d)^2*d*f*g*p/e^3 - 1/32
*(e*x^2 + d)^4*g^2*p/e^4 + 1/6*(e*x^2 + d)^3*d*g^2*p/e^4 - 3/8*(e*x^2 + d)
^2*d^2*g^2*p/e^4 + 1/4*(e*x^2 + d)^2*f^2*log(c)/e^2 + 1/3*(e*x^2 + d)^3*f*
g*log(c)/e^3 - (e*x^2 + d)^2*d*f*g*log(c)/e^3 + 1/8*(e*x^2 + d)^4*g^2*log(
c)/e^4 - 1/2*(e*x^2 + d)^3*d*g^2*log(c)/e^4 + 3/4*(e*x^2 + d)^2*d^2*g^2*lo
g(c)/e^4 + 1/2*((e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d*e^2*f^2*p - 2*(
e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^2*e*f*g*p + (e*x^2 - (e*x^2 + d)
*log(e*x^2 + d) + d)*d^3*g^2*p - (e*x^2 + d)*d*e^2*f^2*log(c) + 2*(e*x^2 +
 d)*d^2*e*f*g*log(c) - (e*x^2 + d)*d^3*g^2*log(c))/e^4
```

Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.88

$$\int x^3(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{f^2\,x^4}{4}+\frac{f\,g\,x^6}{3}+\frac{g^2\,x^8}{8}\right)$$
$$-x^4\left(\frac{f^2\,p}{8}-\frac{d\left(\frac{2\,f\,g\,p}{3}-\frac{d\,g^2\,p}{4\,e}\right)}{4\,e}\right)-x^6\left(\frac{f\,g\,p}{9}-\frac{d\,g^2\,p}{24\,e}\right)-\frac{g^2\,p\,x^8}{32}$$
$$-\frac{\ln\left(e\,x^2+d\right)\,\left(3\,p\,d^4\,g^2-8\,p\,d^3\,e\,f\,g+6\,p\,d^2\,e^2\,f^2\right)}{24\,e^4}+\frac{d\,x^2\left(\frac{f^2\,p}{2}-\frac{d\left(\frac{2\,f\,g\,p}{3}-\frac{d\,g^2\,p}{4\,e}\right)}{e}\right)}{2\,e}$$

input
```
int(x^3*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

output
```
log(c*(d + e*x^2)^p)*((f^2*x^4)/4 + (g^2*x^8)/8 + (f*g*x^6)/3) - x^4*((f^2
*p)/8 - (d*((2*f*g*p)/3 - (d*g^2*p)/(4*e)))/(4*e)) - x^6*((f*g*p)/9 - (d*g
^2*p)/(24*e)) - (g^2*p*x^8)/32 - (log(d + e*x^2)*(3*d^4*g^2*p + 6*d^2*e^2*
f^2*p - 8*d^3*e*f*g*p))/(24*e^4) + (d*x^2*((f^2*p)/2 - (d*((2*f*g*p)/3 - (
d*g^2*p)/(4*e)))/e))/(2*e)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.20

$$\int x^3(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\frac{-36\log\left(\left(e\,x^2+d\right)^p c\right)d^4g^2+96\log\left(\left(e\,x^2+d\right)^p c\right)d^3efg-72\log\left(\left(e\,x^2+d\right)^p c\right)d^2e^2f^2+72\log\left(\left(e\,x^2+\right.\right.}{}$$

input
```
int(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x)
```

output
```
( - 36*log((d + e*x**2)**p*c)*d**4*g**2 + 96*log((d + e*x**2)**p*c)*d**3*e
*f*g - 72*log((d + e*x**2)**p*c)*d**2*e**2*f**2 + 72*log((d + e*x**2)**p*c
)*e**4*f**2*x**4 + 96*log((d + e*x**2)**p*c)*e**4*f*g*x**6 + 36*log((d + e
*x**2)**p*c)*e**4*g**2*x**8 + 36*d**3*e*g**2*p*x**2 - 96*d**2*e**2*f*g*p*x
**2 - 18*d**2*e**2*g**2*p*x**4 + 72*d*e**3*f**2*p*x**2 + 48*d*e**3*f*g*p*x
**4 + 12*d*e**3*g**2*p*x**6 - 36*e**4*f**2*p*x**4 - 32*e**4*f*g*p*x**6 - 9
*e**4*g**2*p*x**8)/(288*e**4)
```

3.325 $\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = -\frac{(ef-dg)^2px^2}{6e^2} - \frac{(ef-dg)p(f+gx^2)^2}{12eg} - \frac{p(f+gx^2)^3}{18g} - \frac{(ef-dg)^3p\log(d+ex^2)}{6e^3g} + \frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6g}$$

output
```
-1/6*(-d*g+e*f)^2*p*x^2/e^2-1/12*(-d*g+e*f)*p*(g*x^2+f)^2/e/g-1/18*p*(g*x^
2+f)^3/g-1/6*(-d*g+e*f)^3*p*ln(e*x^2+d)/e^3/g+1/6*(g*x^2+f)^3*ln(c*(e*x^2+
d)^p)/g
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$$

$$= \frac{6d^2g(-3ef+dg)p\log(d+ex^2) + e\left(-px^2(6d^2g^2 - 3deg(6f+gx^2) + e^2(18f^2+9fgx^2+2g^2x^4)) + 6e(3\right.}{36e^3}$$

input
```
Integrate[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(6*d^2*g*(-3*e*f + d*g)*p*Log[d + e*x^2] + e*(-(p*x^2*(6*d^2*g^2 - 3*d*e*g
*(6*f + g*x^2) + e^2*(18*f^2 + 9*f*g*x^2 + 2*g^2*x^4))) + 6*e*(3*d*f^2 + e
*x^2*(3*f^2 + 3*f*g*x^2 + g^2*x^4))*Log[c*(d + e*x^2)^p]))/(36*e^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2925, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$\Big\downarrow \ 2925$$

$$\frac{1}{2}\int \left(gx^2+f\right)^2\log\left(c\left(ex^2+d\right)^p\right)dx^2$$

$$\Big\downarrow \ 2842$$

$$\frac{1}{2}\left(\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}-\frac{ep\int \frac{\left(gx^2+f\right)^3}{ex^2+d}dx^2}{3g}\right)$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\left(\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}-\frac{ep\int \left(\frac{(ef-dg)^3}{e^3\left(ex^2+d\right)}+\frac{g(ef-dg)^2}{e^3}+\frac{g\left(gx^2+f\right)(ef-dg)}{e^2}+\frac{g\left(gx^2+f\right)^2}{e}\right)dx^2}{3g}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)}{3g}-\frac{ep\left(\frac{(ef-dg)^3\log\left(d+ex^2\right)}{e^4}+\frac{gx^2(ef-dg)^2}{e^3}+\frac{\left(f+gx^2\right)^2(ef-dg)}{2e^2}+\frac{\left(f+gx^2\right)^3}{3e}\right)}{3g}\right)$$

input
```
Int[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(-1/3*(e*p*((g*(e*f - d*g)^2*x^2)/e^3 + ((e*f - d*g)*(f + g*x^2)^2)/(2*e^2
) + (f + g*x^2)^3/(3*e) + ((e*f - d*g)^3*Log[d + e*x^2])/e^4))/g + ((f + g
*x^2)^3*Log[c*(d + e*x^2)^p])/(3*g))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.66

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)\,g^2x^6}{6}+\frac{fg\,x^4\ln(c(e\,x^2+d)^p)}{2}+\frac{\ln(c(e\,x^2+d)^p)\,f^2x^2}{2}+\frac{\ln(c(e\,x^2+d)^p)\,f^3}{6g}-\frac{pe\left(\frac{g\left(\frac{1}{3}e^2g^2x^6-\frac{1}{2}de\,g^2x^4\right.}{}\right.}{}$
parallelrisch	$\frac{6x^6\ln(c(e\,x^2+d)^p)\,e^3g^2-2e^3g^2p\,x^6+18x^4\ln(c(e\,x^2+d)^p)\,e^3fg+3d\,e^2g^2p\,x^4-9fgp\,x^4e^3+18x^2\ln(c(e\,x^2+d)^p)\,e^3f^2-6d^2e\,g}{}$
risch	$\frac{g^2dp\,x^4}{12e}-\frac{g^2d^2p\,x^2}{6e^2}-\frac{f^2p\,x^2}{2}-\frac{g^2p\,x^6}{18}+\frac{g\ln(c)f\,x^4}{2}-\frac{\ln(e\,x^2+d)\,f^3p}{6g}-\frac{fgp\,x^4}{4}+\frac{g^2\ln(e\,x^2+d)\,d^3p}{6e^3}+\frac{\ln(e\,x^2}{}$

input
```
int(x*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(c*(e*x^2+d)^p)*g^2*x^6+1/2*f*g*x^4*ln(c*(e*x^2+d)^p)+1/2*ln(c*(e*x^
2+d)^p)*f^2*x^2+1/6*ln(c*(e*x^2+d)^p)/g*f^3-1/3/g*p*e*(1/2*g/e^3*(1/3*e^2*
g^2*x^6-1/2*d*e*g^2*x^4+3/2*e^2*f*g*x^4+x^2*d^2*g^2-3*d*e*f*g*x^2+3*e^2*f^
2*x^2)+1/2*(-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^3*f^3)/e^4*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.45

$$\int x(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = -\frac{2\,e^3g^2px^6+3\,(3\,e^3fg-de^2g^2)px^4+6\,(3\,e^3f^2-3\,de^2fg+d^2eg^2)px^2-6\,(e^3g^2px^6+3\,e^3fgpx^4+3\,e^3}{36\,e^3}$$

input
```
integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
-1/36*(2*e^3*g^2*p*x^6 + 3*(3*e^3*f*g - d*e^2*g^2)*p*x^4 + 6*(3*e^3*f^2 -
3*d*e^2*f*g + d^2*e*g^2)*p*x^2 - 6*(e^3*g^2*p*x^6 + 3*e^3*f*g*p*x^4 + 3*e^
3*f^2*p*x^2 + (3*d*e^2*f^2 - 3*d^2*e*f*g + d^3*g^2)*p)*log(e*x^2 + d) - 6*
(e^3*g^2*x^6 + 3*e^3*f*g*x^4 + 3*e^3*f^2*x^2)*log(c))/e^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(104) = 208$.

Time = 62.82 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.90

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$$
$$= \begin{cases} \frac{d^3g^2\log\left(c(d+ex^2)^p\right)}{6e^3} - \frac{d^2fg\log\left(c(d+ex^2)^p\right)}{2e^2} - \frac{d^2g^2px^2}{6e^2} + \frac{df^2\log\left(c(d+ex^2)^p\right)}{2e} + \frac{dfgpx^2}{2e} + \frac{dg^2px^4}{12e} - \frac{f^2px^2}{2} + \frac{f^2x^2\log\left(c(d+\right.}{2} \\ \left(\frac{f^2x^2}{2} + \frac{fgx^4}{2} + \frac{g^2x^6}{6}\right)\log\left(cd^p\right) \end{cases}$$

input
```
integrate(x*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise((d**3*g**2*log(c*(d + e*x**2)**p)/(6*e**3) - d**2*f*g*log(c*(d +
 e*x**2)**p)/(2*e**2) - d**2*g**2*p*x**2/(6*e**2) + d*f**2*log(c*(d + e*x*
*2)**p)/(2*e) + d*f*g*p*x**2/(2*e) + d*g**2*p*x**4/(12*e) - f**2*p*x**2/2
+ f**2*x**2*log(c*(d + e*x**2)**p)/2 - f*g*p*x**4/4 + f*g*x**4*log(c*(d +
e*x**2)**p)/2 - g**2*p*x**6/18 + g**2*x**6*log(c*(d + e*x**2)**p)/6, Ne(e,
 0)), ((f**2*x**2/2 + f*g*x**4/2 + g**2*x**6/6)*log(c*d**p), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = \frac{(gx^2+f)^3\log\left((ex^2+d)^pc\right)}{6\,g} - \frac{ep\left(\frac{2\,e^2g^3x^6+3\,(3\,e^2fg^2-deg^3)x^4+6\,(3\,e^2f^2g-3\,defg^2+d^2g^3)x^2}{e^3} + \frac{6\,(e^3f^3-3\,de^2f^2g+3\,d^2efg^2-d^3g^3)\log(ex^2+d)}{e^4}\right)}{36\,g}$$

input
```
integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
1/6*(g*x^2 + f)^3*log((e*x^2 + d)^p*c)/g - 1/36*e*p*((2*e^2*g^3*x^6 + 3*(3
*e^2*f*g^2 - d*e*g^3)*x^4 + 6*(3*e^2*f^2*g - 3*d*e*f*g^2 + d^2*g^3)*x^2)/e
^3 + 6*(e^3*f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*log(e*x^2 + d)/
e^4)/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(114) = 228$.

Time = 0.13 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.74

$$\int x(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$= \frac{(ex^2+d)^2fgp\log(ex^2+d)}{2\,e^2} + \frac{(ex^2+d)^3g^2p\log(ex^2+d)}{6\,e^3} - \frac{(ex^2+d)^2dg^2p\log(ex^2+d)}{2\,e^3} - \frac{(ex^2+d)^2fgp}{4\,e^2} - \frac{(ex^2+d)^3g^2p}{18\,e^3} + \frac{(ex^2+d)^2dg^2p}{4\,e^3} + \frac{(ex^2+d)^2fg\log(c)}{2\,e^2} + \frac{(ex^2+d)^3g^2\log(c)}{6\,e^3} - \frac{(ex^2+d)^2dg^2\log(c)}{2\,e^3} - \frac{(ex^2-(ex^2+d)\log(ex^2+d)+d)e^2f^2p - 2\,(ex^2-(ex^2+d)\log(ex^2+d)+d)defgp + (ex^2-(ex^2}{2\,e}$$

input
```
integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
1/2*(e*x^2 + d)^2*f*g*p*log(e*x^2 + d)/e^2 + 1/6*(e*x^2 + d)^3*g^2*p*log(e
*x^2 + d)/e^3 - 1/2*(e*x^2 + d)^2*d*g^2*p*log(e*x^2 + d)/e^3 - 1/4*(e*x^2
+ d)^2*f*g*p/e^2 - 1/18*(e*x^2 + d)^3*g^2*p/e^3 + 1/4*(e*x^2 + d)^2*d*g^2*
p/e^3 + 1/2*(e*x^2 + d)^2*f*g*log(c)/e^2 + 1/6*(e*x^2 + d)^3*g^2*log(c)/e^
3 - 1/2*(e*x^2 + d)^2*d*g^2*log(c)/e^3 - 1/2*((e*x^2 - (e*x^2 + d)*log(e*x
^2 + d) + d)*e^2*f^2*p - 2*(e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d*e*f*
g*p + (e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^2*g^2*p - (e*x^2 + d)*e^2
*f^2*log(c) + 2*(e*x^2 + d)*d*e*f*g*log(c) - (e*x^2 + d)*d^2*g^2*log(c))/e
^3
```

Mupad [B] (verification not implemented)

Time = 25.86 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\left(\frac{f^2\,x^2}{2}+\frac{f\,g\,x^4}{2}+\frac{g^2\,x^6}{6}\right) - x^2\left(\frac{f^2\,p}{2}-\frac{d\left(f\,g\,p-\frac{d\,g^2\,p}{3\,e}\right)}{2\,e}\right) - x^4\left(\frac{f\,g\,p}{4}-\frac{d\,g^2\,p}{12\,e}\right) + \frac{\ln\left(e\,x^2+d\right)\left(p\,d^3\,g^2-3\,p\,d^2\,e\,f\,g+3\,p\,d\,e^2\,f^2\right)}{6\,e^3} - \frac{g^2\,p\,x^6}{18}$$

input
```
int(x*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

output
```
log(c*(d + e*x^2)^p)*((f^2*x^2)/2 + (g^2*x^6)/6 + (f*g*x^4)/2) - x^2*((f^2
*p)/2 - (d*(f*g*p - (d*g^2*p)/(3*e)))/(2*e)) - x^4*((f*g*p)/4 - (d*g^2*p)/
(12*e)) + (log(d + e*x^2)*(d^3*g^2*p + 3*d*e^2*f^2*p - 3*d^2*e*f*g*p))/(6*
e^3) - (g^2*p*x^6)/18
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.68

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = \frac{6\log\left(\left(e\,x^2+d\right)^p c\right)d^3g^2 - 18\log\left(\left(e\,x^2+d\right)^p c\right)d^2efg + 18\log\left(\left(e\,x^2+d\right)^p c\right)d\,e^2f^2 + 18\log\left(\left(e\,x^2+d\right)^p\right.}{}$$

input
```
int(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x)
```

output
```
(6*log((d + e*x**2)**p*c)*d**3*g**2 - 18*log((d + e*x**2)**p*c)*d**2*e*f*g
 + 18*log((d + e*x**2)**p*c)*d*e**2*f**2 + 18*log((d + e*x**2)**p*c)*e**3*
f**2*x**2 + 18*log((d + e*x**2)**p*c)*e**3*f*g*x**4 + 6*log((d + e*x**2)**
p*c)*e**3*g**2*x**6 - 6*d**2*e*g**2*p*x**2 + 18*d*e**2*f*g*p*x**2 + 3*d*e*
*2*g**2*p*x**4 - 18*e**3*f**2*p*x**2 - 9*e**3*f*g*p*x**4 - 2*e**3*g**2*p*x
**6)/(36*e**3)
```

3.326 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x} \, dx = -fgpx^2 + \frac{dg^2px^2}{4e} - \frac{1}{8}g^2px^4 - \frac{d^2g^2p\log(d+ex^2)}{4e^2} + \frac{1}{4}g^2x^4\log\left(c(d+ex^2)^p\right) + \frac{fg(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e} + \frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}f^2p\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)$$

output
```
-f*g*p*x^2+1/4*d*g^2*p*x^2/e-1/8*g^2*p*x^4-1/4*d^2*g^2*p*ln(e*x^2+d)/e^2+1
/4*g^2*x^4*ln(c*(e*x^2+d)^p)+f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+1/2*f^2*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*f^2*p*polylog(2,1+e*x^2/d)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x} \, dx$$
$$= \frac{-egpx^2(8ef-2dg+egx^2)-2d^2g^2p\log(d+ex^2)+2e\left(g(4df+4efx^2+egx^4)+2ef^2\log\left(-\frac{ex^2}{d}\right)\right)\log}{8e^2}$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

output
```
(-(e*g*p*x^2*(8*e*f - 2*d*g + e*g*x^2)) - 2*d^2*g^2*p*Log[d + e*x^2] + 2*e
*(g*(4*d*f + 4*e*f*x^2 + e*g*x^4) + 2*e*f^2*Log[-((e*x^2)/d)])*Log[c*(d +
e*x^2)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^2)/d])/(8*e^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x} \, dx$$
$$\Big\downarrow \text{2925}$$
$$\frac{1}{2}\int \frac{(gx^2+f)^2 \log(c(ex^2+d)^p)}{x^2}dx^2$$
$$\Big\downarrow \text{2863}$$
$$\frac{1}{2}\int \left(\frac{\log(c(ex^2+d)^p)f^2}{x^2}+2g\log(c(ex^2+d)^p)f+g^2x^2\log(c(ex^2+d)^p)\right)dx^2$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(f^2 \log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{2fg(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e}+\frac{1}{2}g^2x^4\log\left(c(d+ex^2)^p\right)-\frac{d^2g^2p\log(}{2e^2}\right.$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

output
```
(-2*f*g*p*x^2 + (d*g^2*p*x^2)/(2*e) - (g^2*p*x^4)/4 - (d^2*g^2*p*Log[d + e
*x^2])/(2*e^2) + (g^2*x^4*Log[c*(d + e*x^2)^p])/2 + (2*f*g*(d + e*x^2)*Log
[c*(d + e*x^2)^p])/e + f^2*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + f^2*p*
PolyLog[2, 1 + (e*x^2)/d])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.37

method	result
parts	$\frac{g^2x^4\ln(c(e\,x^2+d)^p)}{4}+f g\,x^2\ln\left(c(e\,x^2+d)^p\right)+\ln\left(c(e\,x^2+d)^p\right)f^2\ln\left(x\right)-\frac{ep\left(g\left(\frac{\frac{1}{2}eg\,x^4-dg\,x^2+4f\,x^2e}{2e^2}+\frac{d}{}\right.\right.}{}$
risch	$\frac{\ln((e\,x^2+d)^p)g^2x^4}{4}+\ln\left((e\,x^2+d)^p\right)gf\,x^2+\ln\left((e\,x^2+d)^p\right)f^2\ln\left(x\right)-\frac{g^2p\,x^4}{8}+\frac{d\,g^2p\,x^2}{4e}-fgp\,x^2-$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*g^2*x^4*ln(c*(e*x^2+d)^p)+f*g*x^2*ln(c*(e*x^2+d)^p)+ln(c*(e*x^2+d)^p)*
f^2*ln(x)-1/2*e*p*(g*(1/2/e^2*(1/2*e*g*x^4-d*g*x^2+4*f*x^2*e)+1/2*d*(d*g-4
*e*f)/e^3*ln(e*x^2+d))+4*f^2*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/
2))+ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/
(-d*e)^(1/2))+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x}\,dx=\int \frac{(gx^2+f)^2\log\left((ex^2+d)^pc\right)}{x}\,dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
```

output
```
integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x}\,dx$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x,x)
```

output
```
Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(gx^2+f)^2 \log\left((ex^2+d)^p c\right)}{x}\,dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
```

output
```
integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x, x)
```

Giac [F]

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(gx^2+f)^2 \log\left((ex^2+d)^p c\right)}{x}\,dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{\ln \left(c\,(e\,x^2+d)^p\right)\,\left(g\,x^2+f\right)^2}{x}\,dx$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x,x)
```

output
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x}\,dx$$
$$= \frac{8\left(\int \frac{\log\left((e\,x^2+d)^p c\right)}{e\,x^3+dx}dx\right) d\,e^2 f^2 p + 2\log\left((e\,x^2+d)^p\,c\right)^2 e^2 f^2 - 2\log\left((e\,x^2+d)^p\,c\right) d^2 g^2 p + 8\log\left((e\,x^2+d)^p\right.}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x)
```

output
```
(8*int(log((d + e*x**2)**p*c)/(d*x + e*x**3),x)*d*e**2*f**2*p + 2*log((d +
 e*x**2)**p*c)**2*e**2*f**2 - 2*log((d + e*x**2)**p*c)*d**2*g**2*p + 8*log
((d + e*x**2)**p*c)*d*e*f*g*p + 8*log((d + e*x**2)**p*c)*e**2*f*g*p*x**2 +
 2*log((d + e*x**2)**p*c)*e**2*g**2*p*x**4 + 2*d*e*g**2*p**2*x**2 - 8*e**2
*f*g*p**2*x**2 - e**2*g**2*p**2*x**4)/(8*e**2*p)
```

3.327 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^3} dx = -\frac{1}{2}g^2px^2 + \frac{ef^2p\log(x)}{d} - \frac{ef^2p\log(d+ex^2)}{2d} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} + \frac{g^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e} + fg\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + fgp\operatorname{PolyLog}\left(2,1+\frac{ex^2}{d}\right)$$

output
```
-1/2*g^2*p*x^2+e*f^2*p*ln(x)/d-1/2*e*f^2*p*ln(e*x^2+d)/d-1/2*f^2*ln(c*(e*x
^2+d)^p)/x^2+1/2*g^2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+f*g*ln(-e*x^2/d)*ln(c*(
e*x^2+d)^p)+f*g*p*polylog(2,1+e*x^2/d)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^3} dx = \frac{ef^2 p \log(x)}{d} - \frac{ef^2 p \log(d+ex^2)}{2d} - \frac{f^2 \log \left(c(d+ex^2)^p\right)}{2x^2} - \frac{1}{2} g^2 \left(px^2 - \frac{(d+ex^2) \log \left(c(d+ex^2)^p\right)}{e}\right) + fg\left(\log\left(-\frac{ex^2}{d}\right) \log \left(c(d+ex^2)^p\right) + p \operatorname{PolyLog}\left(2, \frac{d+ex^2}{d}\right)\right)$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```

output
```
(e*f^2*p*Log[x])/d - (e*f^2*p*Log[d + e*x^2])/(2*d) - (f^2*Log[c*(d + e*x^
2)^p])/(2*x^2) - (g^2*(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/2 +
f*g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*x^2)/d])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^3} dx$$

$$\Big\downarrow \; 2925$$

$$\frac{1}{2} \int \frac{(gx^2+f)^2 \log \left(c(ex^2+d)^p\right)}{x^4} dx^2$$

$$\Big\downarrow \text{2863}$$

$$\frac{1}{2}\int \left(\frac{\log \left(c(ex^2+d)^p\right) f^2}{x^4}+\frac{2g \log \left(c(ex^2+d)^p\right) f}{x^2}+g^2 \log \left(c(ex^2+d)^p\right)\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{f^2 \log \left(c(d+ex^2)^p\right)}{x^2}+2fg \log \left(-\frac{ex^2}{d}\right) \log \left(c(d+ex^2)^p\right)+\frac{g^2(d+ex^2) \log \left(c(d+ex^2)^p\right)}{e}+\frac{ef^2p \log \left(x^2\right)}{d}\right.$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```

output
```
(-(g^2*p*x^2) + (e*f^2*p*Log[x^2])/d - (e*f^2*p*Log[d + e*x^2])/d - (f^2*L
og[c*(d + e*x^2)^p])/x^2 + (g^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e + 2*f*
g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + 2*f*g*p*PolyLog[2, 1 + (e*x^2)/
d])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.56

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)g^2x^2}{2}-\frac{f^2\ln(c(e\,x^2+d)^p)}{2x^2}+2\ln\left(c(e\,x^2+d)^p\right)gf\ln(x)-ep\left(\frac{g^2x^2}{2e}-\frac{(d^2g^2-f^2e^2)\ln(e\,x^2+d)}{2d\,e^2}\right.$
risch	$\frac{\ln((e\,x^2+d)^p)g^2x^2}{2}+2\ln\left((e\,x^2+d)^p\right)gf\ln(x)-\frac{\ln((e\,x^2+d)^p)f^2}{2x^2}-\frac{g^2p\,x^2}{2}+\frac{pd\ln(e\,x^2+d)g^2}{2e}-\frac{e\,f^2p\ln(e\,x^2}{2d}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*(e*x^2+d)^p)*g^2*x^2-1/2*f^2*ln(c*(e*x^2+d)^p)/x^2+2*ln(c*(e*x^2+
d)^p)*g*f*ln(x)-e*p*(1/2*g^2/e*x^2-1/2*(d^2*g^2-e^2*f^2)/d/e^2*ln(e*x^2+d)
-f^2/d*ln(x)+4*g*f*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*
x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/
2))+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^3}\,dx=\int \frac{(gx^2+f)^2\log\left((ex^2+d)^pc\right)}{x^3}\,dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

output
```
integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^3, x)
```

Sympy [F]

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^3} \, dx = \int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^3} \, dx$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**3,x)
```

output
```
Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x**3, x)
```

Maxima [F]

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^3} \, dx = \int \frac{(gx^2+f)^2 \log \left((ex^2+d)^p c\right)}{x^3} \, dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
```

output
```
integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^3, x)
```

Giac [F]

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^3} \, dx = \int \frac{(gx^2+f)^2 \log \left((ex^2+d)^p c\right)}{x^3} \, dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^3}\, dx = \int \frac{\ln \left(c \left(e\, x^2+d\right)^p\right)\, \left(g\, x^2+f\right)^2}{x^3}\, dx$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^3,x)
```

output
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^3}\, dx$$
$$= \frac{-4\left(\int \frac{\log\left(\left(e\, x^2+d\right)^p c\right)}{e\, x^5+d\, x^3} dx\right) d^3 f g p\, x^2 + \log\left(\left(e\, x^2+d\right)^p c\right)^2 d e f g\, x^2 - 2 \log\left(\left(e\, x^2+d\right)^p c\right) d^2 f g p + \log\left(\left(e\, x^2+\right.\right.}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x)
```

output
```
( - 4*int(log((d + e*x**2)**p*c)/(d*x**3 + e*x**5),x)*d**3*f*g*p*x**2 + lo
g((d + e*x**2)**p*c)**2*d*e*f*g*x**2 - 2*log((d + e*x**2)**p*c)*d**2*f*g*p
 + log((d + e*x**2)**p*c)*d**2*g**2*p*x**2 - log((d + e*x**2)**p*c)*d*e*f*
*2*p - 2*log((d + e*x**2)**p*c)*d*e*f*g*p*x**2 + log((d + e*x**2)**p*c)*d*
e*g**2*p*x**4 - log((d + e*x**2)**p*c)*e**2*f**2*p*x**2 + 4*log(x)*d*e*f*g
*p**2*x**2 + 2*log(x)*e**2*f**2*p**2*x**2 - d*e*g**2*p**2*x**4)/(2*d*e*p*x
**2)
```

3.328 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5} dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5} dx = -\frac{ef^2p}{4dx^2} - \frac{e^2f^2p\log(x)}{2d^2} + \frac{2efgp\log(x)}{d} + \frac{e^2f^2p\log(d+ex^2)}{4d^2} - \frac{efgp\log(d+ex^2)}{d} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{fg\log\left(c(d+ex^2)^p\right)}{x^2} + \frac{1}{2}g^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}g^2p\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)$$

output
```
-1/4*e*f^2*p/d/x^2-1/2*e^2*f^2*p*ln(x)/d^2+2*e*f*g*p*ln(x)/d+1/4*e^2*f^2*p
*ln(e*x^2+d)/d^2-e*f*g*p*ln(e*x^2+d)/d-1/4*f^2*ln(c*(e*x^2+d)^p)/x^4-f*g*l
n(c*(e*x^2+d)^p)/x^2+1/2*g^2*ln(-e*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*g^2*p*poly
log(2,1+e*x^2/d)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^5} dx = \frac{1}{4}\left(\frac{8efgp\log(x)}{d} - \frac{4efgp\log(d+ex^2)}{d} - \frac{ef^2p(d+2ex^2\log(x)-ex^2\log(d+ex^2))}{d^2x^2} - \frac{f^2\log(c(d+ex^2)^p)}{x^4} - \frac{4fg\log(c(d+ex^2)^p)}{x^2} + 2g^2\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + p\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)\right)\right)$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]
```

output
```
((8*e*f*g*p*Log[x])/d - (4*e*f*g*p*Log[d + e*x^2])/d - (e*f^2*p*(d + 2*e*x
^2*Log[x] - e*x^2*Log[d + e*x^2]))/(d^2*x^2) - (f^2*Log[c*(d + e*x^2)^p])/
x^4 - (4*f*g*Log[c*(d + e*x^2)^p])/x^2 + 2*g^2*(Log[-((e*x^2)/d)]*Log[c*(d
 + e*x^2)^p] + p*PolyLog[2, 1 + (e*x^2)/d]))/4
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^5} dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int \frac{(gx^2+f)^2\log\left(c(ex^2+d)^p\right)}{x^6}dx^2$$

$$\Big\downarrow \text{2863}$$

$$\frac{1}{2}\int \left(\frac{\log\left(c(ex^2+d)^p\right)f^2}{x^6}+\frac{2g\log\left(c(ex^2+d)^p\right)f}{x^4}+\frac{g^2\log\left(c(ex^2+d)^p\right)}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^4}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{x^2}+g^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)-\frac{e^2f^2p\log\left(x^2\right)}{2d^2}+\frac{e^2f^2}{}\right.$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]
```

output
```
(-1/2*(e*f^2*p)/(d*x^2) - (e^2*f^2*p*Log[x^2])/(2*d^2) + (2*e*f*g*p*Log[x^
2])/d + (e^2*f^2*p*Log[d + e*x^2])/(2*d^2) - (2*e*f*g*p*Log[d + e*x^2])/d
- (f^2*Log[c*(d + e*x^2)^p])/(2*x^4) - (2*f*g*Log[c*(d + e*x^2)^p])/x^2 +
g^2*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + g^2*p*PolyLog[2, 1 + (e*x^2)/
d])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.21

method	result
parts	$-\frac{f^{2}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{4x^{4}}+\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)g^{2}\ln\left(x\right)-\frac{fg\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{x^{2}}-\frac{ep\left(4g^{2}\left(\frac{\ln(x)\left(\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)+\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)\right)}{2e}\right.\right.}{}$
risch	$-\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right)f^{2}}{4x^{4}}+\ln\left(\left(e\,x^{2}+d\right)^{p}\right)g^{2}\ln\left(x\right)-\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right)gf}{x^{2}}-p\,g^{2}\ln\left(x\right)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)-p\,g^{2}\ln$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*f^2*ln(c*(e*x^2+d)^p)/x^4+ln(c*(e*x^2+d)^p)*g^2*ln(x)-f*g*ln(c*(e*x^2
+d)^p)/x^2-1/2*e*p*(4*g^2*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))
+ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d
*e)^(1/2))+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e)-f*(-1/2*(4*d*g-e*f)/
d^2*ln(e*x^2+d)-1/2/d*f/x^2+(4*d*g-e*f)/d^2*ln(x)))
```

Fricas [F]

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^5}\,dx=\int \frac{(gx^2+f)^2\log\left((ex^2+d)^pc\right)}{x^5}\,dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")
```

output
```
integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^5, x)
```

Sympy [F]

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^5} \, dx = \int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^5} \, dx$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**5,x)
```

output
```
Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x**5, x)
```

Maxima [F]

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^5} \, dx = \int \frac{(gx^2+f)^2 \log ((ex^2+d)^p c)}{x^5} \, dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
```

output
```
integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^5, x)
```

Giac [F]

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^5} \, dx = \int \frac{(gx^2+f)^2 \log ((ex^2+d)^p c)}{x^5} \, dx$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
```

output
```
integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,(g\,x^2+f)^2}{x^5}\,dx$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^5,x)
```

output
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^5, x)
```

Reduce [F]

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5}\,dx$$
$$= \frac{4\left(\int \frac{\log\left((e\,x^2+d)^p c\right)}{e\,x^3+dx}dx\right) d^3g^2p\,x^4 + \log\left((e\,x^2+d)^p\,c\right)^2 d^2g^2x^4 - \log\left((e\,x^2+d)^p\,c\right) d^2f^2p - 4\log\left((e\,x^2+d)^p\right.}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x)
```

output
```
(4*int(log((d + e*x**2)**p*c)/(d*x + e*x**3),x)*d**3*g**2*p*x**4 + log((d
+ e*x**2)**p*c)**2*d**2*g**2*x**4 - log((d + e*x**2)**p*c)*d**2*f**2*p - 4
*log((d + e*x**2)**p*c)*d**2*f*g*p*x**2 - 4*log((d + e*x**2)**p*c)*d*e*f*g
*p*x**4 + log((d + e*x**2)**p*c)*e**2*f**2*p*x**4 + 8*log(x)*d*e*f*g*p**2*
x**4 - 2*log(x)*e**2*f**2*p**2*x**4 - d*e*f**2*p**2*x**2)/(4*d**2*p*x**4)
```

3.329 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^7} dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^7} dx = -\frac{ef^2p}{12dx^4} + \frac{ef(ef-3dg)p}{6d^2x^2} + \frac{e(e^2f^2-3defg+3d^2g^2)\,p\log(x)}{3d^3} - \frac{(ef-dg)^3p\log(d+ex^2)}{6d^3f} - \frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6}$$

output
```
-1/12*e*f^2*p/d/x^4+1/6*e*f*(-3*d*g+e*f)*p/d^2/x^2+1/3*e*(3*d^2*g^2-3*d*e*
f*g+e^2*f^2)*p*ln(x)/d^3-1/6*(-d*g+e*f)^3*p*ln(e*x^2+d)/d^3/f-1/6*(g*x^2+f
)^3*ln(c*(e*x^2+d)^p)/f/x^6
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^7}\,dx =$$
$$-\frac{defpx^2(-2efx^2+d(f+6gx^2)) - 4e(e^2f^2-3defg+3d^2g^2)\,px^6\log(x) + 2e(e^2f^2-3defg+3d^2g^2)\,p}{12d^3x^6}$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```

output
```
-1/12*(d*e*f*p*x^2*(-2*e*f*x^2 + d*(f + 6*g*x^2)) - 4*e*(e^2*f^2 - 3*d*e*f
*g + 3*d^2*g^2)*p*x^6*Log[x] + 2*e*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2)*p*x^6
*Log[d + e*x^2] + 2*d^3*(f^2 + 3*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p]
)/(d^3*x^6)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2925, 2861, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^7}\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int \frac{(gx^2+f)^2 \log \left(c(ex^2+d)^p\right)}{x^8}dx^2$$

$$\Big\downarrow \text{2861}$$

$$\frac{1}{2}\left(-ep\int -\frac{(gx^2+f)^3}{3fx^6\,(ex^2+d)}dx^2 - \frac{(f+gx^2)^3 \log \left(c(d+ex^2)^p\right)}{3fx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{ep\int \frac{(gx^2+f)^3}{x^6(ex^2+d)}dx^2}{3f}-\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{3fx^6}\right)$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\left(\frac{ep\int \left(\frac{f^3}{dx^6}+\frac{(3dg-ef)f^2}{d^2x^4}+\frac{(e^2f^2-3degf+3d^2g^2)f}{d^3x^2}+\frac{(dg-ef)^3}{d^3(ex^2+d)}\right)dx^2}{3f}-\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{3fx^6}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{ep\left(-\frac{(ef-dg)^3\log(d+ex^2)}{d^3e}+\frac{f^2(ef-3dg)}{d^2x^2}+\frac{f\log(x^2)(3d^2g^2-3defg+e^2f^2)}{d^3}-\frac{f^3}{2dx^4}\right)}{3f}-\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{3fx^6}\right)$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```

output
```
((e*p*(-1/2*f^3/(d*x^4) + (f^2*(e*f - 3*d*g))/(d^2*x^2) + (f*(e^2*f^2 - 3*
d*e*f*g + 3*d^2*g^2)*Log[x^2])/d^3 - ((e*f - d*g)^3*Log[d + e*x^2])/(d^3*e
)))/(3*f) - ((f + g*x^2)^3*Log[c*(d + e*x^2)^p])/(3*f*x^6))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.22

method	result
parts	$-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)g^{2}}{2x^{2}}-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)gf}{2x^{4}}-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)f^{2}}{6x^{6}}-\frac{ep\left(\frac{\left(3d^{2}g^{2}-3fged+f^{2}e^{2}\right)\ln\left(e\,x^{2}+d\right)}{2d^{3}}+\frac{\left(-3d^{2}g^{2}+3fged\right.}{d^{3}}\right.}{3}$
parallelrisch	$12\ln(x)x^{6}d^{2}e^{2}g^{2}p^{2}-12\ln(x)x^{6}d\,e^{3}fg\,p^{2}+4\ln(x)x^{6}e^{4}f^{2}p^{2}-6x^{6}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)d^{2}e^{2}g^{2}p+6x^{6}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)d\,e^{3}fgp-2x^{6}$
risch	$-\frac{\left(3g^{2}x^{4}+3fg\,x^{2}+f^{2}\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{6x^{6}}+\frac{i\pi\,d^{3}f^{2}\operatorname{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{3}-i\pi\,d^{3}f^{2}\operatorname{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}\operatorname{csgn}(ic)-3i\pi\,d^{3}g^{2}x^{4}\,\mathrm{cs}}{}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(e*x^2+d)^p)*g^2/x^2-1/2*ln(c*(e*x^2+d)^p)*g*f/x^4-1/6*ln(c*(e*x
^2+d)^p)*f^2/x^6-1/3*e*p*(1/2*(3*d^2*g^2-3*d*e*f*g+e^2*f^2)/d^3*ln(e*x^2+d
)+(-3*d^2*g^2+3*d*e*f*g-e^2*f^2)/d^3*ln(x)+1/4/d*f^2/x^4+1/2*f*(3*d*g-e*f)
/d^2/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.41

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^7} dx$$
$$= \frac{4 (e^3f^2 - 3 de^2fg + 3 d^2eg^2)px^6 \log (x) - d^2ef^2px^2 + 2 (de^2f^2 - 3 d^2efg)px^4 - 2 (3 d^3g^2px^4 + 3 d^3fgpx}{12 d^3x^6}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

output
```
1/12*(4*(e^3*f^2 - 3*d*e^2*f*g + 3*d^2*e*g^2)*p*x^6*log(x) - d^2*e*f^2*p*x
^2 + 2*(d*e^2*f^2 - 3*d^2*e*f*g)*p*x^4 - 2*(3*d^3*g^2*p*x^4 + 3*d^3*f*g*p*
x^2 + (e^3*f^2 - 3*d*e^2*f*g + 3*d^2*e*g^2)*p*x^6 + d^3*f^2*p)*log(e*x^2 +
 d) - 2*(3*d^3*g^2*x^4 + 3*d^3*f*g*x^2 + d^3*f^2)*log(c))/(d^3*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^7} dx = \text{Timed out}$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^7} dx =$$
$$-\frac{1}{12} ep \left(\frac{2 (e^2f^2 - 3 defg + 3 d^2g^2) \log (ex^2 + d)}{d^3} - \frac{2 (e^2f^2 - 3 defg + 3 d^2g^2) \log (x^2)}{d^3} + \frac{df^2 - 2 (ef^2}{d^2x^4} \right.$$
$$- \frac{(3 g^2x^4 + 3 fgx^2 + f^2) \log ((ex^2 + d)^pc)}{6 x^6}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

output
```
-1/12*e*p*(2*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2)*log(e*x^2 + d)/d^3 - 2*(e^2
*f^2 - 3*d*e*f*g + 3*d^2*g^2)*log(x^2)/d^3 + (d*f^2 - 2*(e*f^2 - 3*d*f*g)*
x^2)/(d^2*x^4)) - 1/6*(3*g^2*x^4 + 3*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x
^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(120) = 240$.

Time = 0.14 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.57

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{2\left(e^4f^2p+3\,(ex^2+d)e^3fgp-3\,de^3fgp+3\,(ex^2+d)^2e^2g^2p-6\,(ex^2+d)de^2g^2p+3\,d^2e^2g^2p\right)\log(ex^2+d)}{(ex^2+d)^3-3\,(ex^2+d)^2d+3\,(ex^2+d)d^2-d^3} - \frac{2\,(ex^2+d)^2e^4f^2p-5\,(ex^2+d)de^4}{}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

output
```
-1/12*(2*(e^4*f^2*p + 3*(e*x^2 + d)*e^3*f*g*p - 3*d*e^3*f*g*p + 3*(e*x^2 +
 d)^2*e^2*g^2*p - 6*(e*x^2 + d)*d*e^2*g^2*p + 3*d^2*e^2*g^2*p)*log(e*x^2 +
 d)/((e*x^2 + d)^3 - 3*(e*x^2 + d)^2*d + 3*(e*x^2 + d)*d^2 - d^3) - (2*(e*
x^2 + d)^2*e^4*f^2*p - 5*(e*x^2 + d)*d*e^4*f^2*p + 3*d^2*e^4*f^2*p - 6*(e*
x^2 + d)^2*d*e^3*f*g*p + 12*(e*x^2 + d)*d^2*e^3*f*g*p - 6*d^3*e^3*f*g*p -
2*d^2*e^4*f^2*log(c) - 6*(e*x^2 + d)*d^2*e^3*f*g*log(c) + 6*d^3*e^3*f*g*lo
g(c) - 6*(e*x^2 + d)^2*d^2*e^2*g^2*log(c) + 12*(e*x^2 + d)*d^3*e^2*g^2*log
(c) - 6*d^4*e^2*g^2*log(c))/((e*x^2 + d)^3*d^2 - 3*(e*x^2 + d)^2*d^3 + 3*(
e*x^2 + d)*d^4 - d^5) + 2*(e^4*f^2*p - 3*d*e^3*f*g*p + 3*d^2*e^2*g^2*p)*lo
g(e*x^2 + d)/d^3 - 2*(e^4*f^2*p - 3*d*e^3*f*g*p + 3*d^2*e^2*g^2*p)*log(e*x
^2)/d^3)/e
```

Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int \frac{(f+gx^2)^2\log(c(d+ex^2)^p)}{x^7}\,dx = \frac{\ln(x)\,(3\,p\,d^2\,e\,g^2-3\,p\,d\,e^2\,f\,g+p\,e^3\,f^2)}{3\,d^3} - \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(\frac{f^2}{6}+\frac{f\,g\,x^2}{2}+\frac{g^2\,x^4}{2}\right)}{x^6} - \frac{\ln(e\,x^2+d)\,(3\,p\,d^2\,e\,g^2-3\,p\,d\,e^2\,f\,g+p\,e^3\,f^2)}{6\,d^3} - \frac{\frac{e\,f^2\,p}{4\,d}+\frac{e\,f\,p\,x^2\,(3\,d\,g-e\,f)}{2\,d^2}}{3\,x^4}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^7,x)
```

output
```
(log(x)*(e^3*f^2*p + 3*d^2*e*g^2*p - 3*d*e^2*f*g*p))/(3*d^3) - (log(c*(d +
 e*x^2)^p)*(f^2/6 + (g^2*x^4)/2 + (f*g*x^2)/2))/x^6 - (log(d + e*x^2)*(e^3
*f^2*p + 3*d^2*e*g^2*p - 3*d*e^2*f*g*p))/(6*d^3) - ((e*f^2*p)/(4*d) + (e*f
*p*x^2*(3*d*g - e*f))/(2*d^2))/(3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.73

$$\int \frac{(f+gx^2)^2\log(c(d+ex^2)^p)}{x^7}\,dx = \frac{-2\log\left((e\,x^2+d)^p\,c\right)d^3f^2-6\log\left((e\,x^2+d)^p\,c\right)d^3f\,g\,x^2-6\log\left((e\,x^2+d)^p\,c\right)d^3g^2x^4-6\log\left((e\,x^2+d)^p\right.}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x)
```

output
```
( - 2*log((d + e*x**2)**p*c)*d**3*f**2 - 6*log((d + e*x**2)**p*c)*d**3*f*g
*x**2 - 6*log((d + e*x**2)**p*c)*d**3*g**2*x**4 - 6*log((d + e*x**2)**p*c)
*d**2*e*g**2*x**6 + 6*log((d + e*x**2)**p*c)*d*e**2*f*g*x**6 - 2*log((d +
e*x**2)**p*c)*e**3*f**2*x**6 + 12*log(x)*d**2*e*g**2*p*x**6 - 12*log(x)*d*
e**2*f*g*p*x**6 + 4*log(x)*e**3*f**2*p*x**6 - d**2*e*f**2*p*x**2 - 6*d**2*
e*f*g*p*x**4 + 2*d*e**2*f**2*p*x**4)/(12*d**3*x**6)
```

3.330 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 216

$$\begin{aligned}\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9} \, dx = &-\frac{ef^2p}{24dx^6} + \frac{ef(3ef-8dg)p}{48d^2x^4} \\ &- \frac{e(3e^2f^2-8defg+6d^2g^2)\,p}{24d^3x^2} \\ &- \frac{e^2(3e^2f^2-8defg+6d^2g^2)\,p\log(x)}{12d^4} \\ &+ \frac{e^2(3e^2f^2-8defg+6d^2g^2)\,p\log\left(d+ex^2\right)}{24d^4} \\ &- \frac{f^2\log\left(c(d+ex^2)^p\right)}{8x^8} \\ &- \frac{fg\log\left(c(d+ex^2)^p\right)}{3x^6} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{4x^4}\end{aligned}$$

output

```
-1/24*e*f^2*p/d/x^6+1/48*e*f*(-8*d*g+3*e*f)*p/d^2/x^4-1/24*e*(6*d^2*g^2-8*
d*e*f*g+3*e^2*f^2)*p/d^3/x^2-1/12*e^2*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)*p*ln
(x)/d^4+1/24*e^2*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)*p*ln(e*x^2+d)/d^4-1/8*f^2
*ln(c*(e*x^2+d)^p)/x^8-1/3*f*g*ln(c*(e*x^2+d)^p)/x^6-1/4*g^2*ln(c*(e*x^2+d
)^p)/x^4
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^9} \, dx =$$
$$-\frac{depx^2(6e^2f^2x^4 - defx^2(3f+16gx^2) + 2d^2(f^2+4fgx^2+6g^2x^4)) + 4e^2(3e^2f^2-8defg+6d^2g^2)\, px^8}{48d}$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

output
```
-1/48*(d*e*p*x^2*(6*e^2*f^2*x^4 - d*e*f*x^2*(3*f + 16*g*x^2) + 2*d^2*(f^2
+ 4*f*g*x^2 + 6*g^2*x^4)) + 4*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^
8*Log[x] - 2*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^8*Log[d + e*x^2]
+ 2*d^4*(3*f^2 + 8*f*g*x^2 + 6*g^2*x^4)*Log[c*(d + e*x^2)^p])/(d^4*x^8)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2925, 2861, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^9} \, dx$$

$$\Big\downarrow \text{ 2925}$$

$$\frac{1}{2} \int \frac{(gx^2+f)^2 \log \left(c(ex^2+d)^p\right)}{x^{10}} dx^2$$

$$\Big\downarrow \text{ 2861}$$

$$\frac{1}{2} \left(-ep \int -\frac{6g^2x^4+8fgx^2+3f^2}{12x^8\,(ex^2+d)} dx^2 - \frac{f^2 \log \left(c(d+ex^2)^p\right)}{4x^8} - \frac{2fg \log \left(c(d+ex^2)^p\right)}{3x^6} - \frac{g^2 \log \left(c(d+ex^2)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{1}{12}ep\int \frac{6g^2x^4+8fgx^2+3f^2}{x^8\left(ex^2+d\right)}dx^2-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^8}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{2x^4}\right)$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{2}\left(\frac{1}{12}ep\int \left(\frac{\left(3e^2f^2-8degf+6d^2g^2\right)e^2}{d^4\left(ex^2+d\right)}-\frac{\left(3e^2f^2-8degf+6d^2g^2\right)e}{d^4x^2}+\frac{3e^2f^2-8degf+6d^2g^2}{d^3x^4}+\frac{f(8dg-3}{d^2x^6}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^8}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{2x^4}+\frac{1}{12}ep\left(\frac{f(3ef-8dg)}{2d^2x^4}-\frac{e\log\left(x^2\right)\left(\right.}{}\right.\right.$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

output
```
((e*p*(-(f^2/(d*x^6)) + (f*(3*e*f - 8*d*g))/(2*d^2*x^4) - (3*e^2*f^2 - 8*d
*e*f*g + 6*d^2*g^2)/(d^3*x^2) - (e*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*Log
[x^2])/d^4 + (e*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*Log[d + e*x^2])/d^4))/
12 - (f^2*Log[c*(d + e*x^2)^p])/(4*x^8) - (2*f*g*Log[c*(d + e*x^2)^p])/(3*
x^6) - (g^2*Log[c*(d + e*x^2)^p])/(2*x^4))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

method	result
parts	$-\frac{g^{2} \ln \left(c \left(e \,x^{2}+d \right)^{p}\right)}{4 x^{4}}-\frac{f g \ln \left(c \left(e \,x^{2}+d \right)^{p}\right)}{3 x^{6}}-\frac{f^{2} \ln \left(c \left(e \,x^{2}+d \right)^{p}\right)}{8 x^{8}}-\frac{e p \left(-\frac{e \left(6 d^{2} g^{2}-8 f g e d +3 f^{2} e^{2}\right) \ln \left(e \,x^{2}+d \right)}{2 d^{4}}-\frac{-6 d^{2} g^{2}+8}{2 d}\right.}{}$
parallelrisch	$-\frac{24 \ln \left(x \right) x^{8} d^{2} e^{2} g^{2} p^{2}-32 \ln \left(x \right) x^{8} d \,e^{3} f g \,p^{2}+12 \ln \left(x \right) x^{8} e^{4} f^{2} p^{2}-12 x^{8} \ln \left(c \left(e \,x^{2}+d \right)^{p}\right) d^{2} e^{2} g^{2} p +16 x^{8} \ln \left(c \left(e \,x^{2}+d \right)^{p}\right) d \,e^{3} f g p}{}$
risch	$-\frac{\left(6 g^{2} x^{4}+8 f g \,x^{2}+3 f^{2}\right) \ln \left(\left(e \,x^{2}+d \right)^{p}\right)}{24 x^{8}}-\frac{-12 \ln \left(-e \,x^{2}-d \right) d^{2} e^{2} g^{2} p \,x^{8}+24 \ln \left(x \right) d^{2} e^{2} g^{2} p \,x^{8}+12 \ln \left(c \right) d^{4} g^{2} x^{4}-3 i \pi \,d^{4} f^{2} \operatorname{csg}}{}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/4*g^2*ln(c*(e*x^2+d)^p)/x^4-1/3*f*g*ln(c*(e*x^2+d)^p)/x^6-1/8*f^2*ln(c*
(e*x^2+d)^p)/x^8-1/12*e*p*(-1/2*e*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)/d^4*ln(e
*x^2+d)-1/2*(-6*d^2*g^2+8*d*e*f*g-3*e^2*f^2)/d^3/x^2+(6*d^2*g^2-8*d*e*f*g+
3*e^2*f^2)/d^4*e*ln(x)+1/2*f^2/d/x^6+1/4*f*(8*d*g-3*e*f)/d^2/x^4)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9}\,dx =$$
$$-\frac{4\,(3\,e^4f^2-8\,de^3fg+6\,d^2e^2g^2)px^8\log(x)+2\,d^3ef^2px^2+2\,(3\,de^3f^2-8\,d^2e^2fg+6\,d^3eg^2)px^6-(3\,d^2e}{}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")
```

output
```
-1/48*(4*(3*e^4*f^2 - 8*d*e^3*f*g + 6*d^2*e^2*g^2)*p*x^8*log(x) + 2*d^3*e*
f^2*p*x^2 + 2*(3*d*e^3*f^2 - 8*d^2*e^2*f*g + 6*d^3*e*g^2)*p*x^6 - (3*d^2*e
^2*f^2 - 8*d^3*e*f*g)*p*x^4 + 2*(6*d^4*g^2*p*x^4 - (3*e^4*f^2 - 8*d*e^3*f*
g + 6*d^2*e^2*g^2)*p*x^8 + 8*d^4*f*g*p*x^2 + 3*d^4*f^2*p)*log(e*x^2 + d) +
 2*(6*d^4*g^2*x^4 + 8*d^4*f*g*x^2 + 3*d^4*f^2)*log(c))/(d^4*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \text{Timed out}$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**9,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^9}\,dx$$
$$= \frac{1}{48}\,ep\left(\frac{2\,(3\,e^3f^2-8\,de^2fg+6\,d^2eg^2)\log(ex^2+d)}{d^4} - \frac{2\,(3\,e^3f^2-8\,de^2fg+6\,d^2eg^2)\log(x^2)}{d^4} - \frac{2\,(3\,e^2f^2\right.$$
$$- \frac{(6\,g^2x^4+8\,fgx^2+3\,f^2)\log\left((ex^2+d)^pc\right)}{24\,x^8}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

output
```
1/48*e*p*(2*(3*e^3*f^2 - 8*d*e^2*f*g + 6*d^2*e*g^2)*log(e*x^2 + d)/d^4 - 2
*(3*e^3*f^2 - 8*d*e^2*f*g + 6*d^2*e*g^2)*log(x^2)/d^4 - (2*(3*e^2*f^2 - 8*
d*e*f*g + 6*d^2*g^2)*x^4 + 2*d^2*f^2 - (3*d*e*f^2 - 8*d^2*f*g)*x^2)/(d^3*x
^6)) - 1/24*(6*g^2*x^4 + 8*f*g*x^2 + 3*f^2)*log((e*x^2 + d)^p*c)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(200) = 400.

Time = 0.14 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.80

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^9}\,dx =$$
$$-\frac{\frac{2\left(3\,e^5f^2p+8\,(ex^2+d)e^4fgp-8\,de^4fgp+6\,(ex^2+d)^2e^3g^2p-12\,(ex^2+d)de^3g^2p+6\,d^2e^3g^2p\right)\log(ex^2+d)}{(ex^2+d)^4-4\,(ex^2+d)^3d+6\,(ex^2+d)^2d^2-4\,(ex^2+d)d^3+d^4} + \frac{6\,(ex^2+d)^3e^5f^2p-21\,(ex^2+d)}{}}{}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")
```

output
```
-1/48*(2*(3*e^5*f^2*p + 8*(e*x^2 + d)*e^4*f*g*p - 8*d*e^4*f*g*p + 6*(e*x^2
 + d)^2*e^3*g^2*p - 12*(e*x^2 + d)*d*e^3*g^2*p + 6*d^2*e^3*g^2*p)*log(e*x^
2 + d)/((e*x^2 + d)^4 - 4*(e*x^2 + d)^3*d + 6*(e*x^2 + d)^2*d^2 - 4*(e*x^2
 + d)*d^3 + d^4) + (6*(e*x^2 + d)^3*e^5*f^2*p - 21*(e*x^2 + d)^2*d*e^5*f^2
*p + 26*(e*x^2 + d)*d^2*e^5*f^2*p - 11*d^3*e^5*f^2*p - 16*(e*x^2 + d)^3*d*
e^4*f*g*p + 56*(e*x^2 + d)^2*d^2*e^4*f*g*p - 64*(e*x^2 + d)*d^3*e^4*f*g*p
+ 24*d^4*e^4*f*g*p + 12*(e*x^2 + d)^3*d^2*e^3*g^2*p - 36*(e*x^2 + d)^2*d^3
*e^3*g^2*p + 36*(e*x^2 + d)*d^4*e^3*g^2*p - 12*d^5*e^3*g^2*p + 6*d^3*e^5*f
^2*log(c) + 16*(e*x^2 + d)*d^3*e^4*f*g*log(c) - 16*d^4*e^4*f*g*log(c) + 12
*(e*x^2 + d)^2*d^3*e^3*g^2*log(c) - 24*(e*x^2 + d)*d^4*e^3*g^2*log(c) + 12
*d^5*e^3*g^2*log(c))/((e*x^2 + d)^4*d^3 - 4*(e*x^2 + d)^3*d^4 + 6*(e*x^2 +
 d)^2*d^5 - 4*(e*x^2 + d)*d^6 + d^7) - 2*(3*e^5*f^2*p - 8*d*e^4*f*g*p + 6*
d^2*e^3*g^2*p)*log(e*x^2 + d)/d^4 + 2*(3*e^5*f^2*p - 8*d*e^4*f*g*p + 6*d^2
*e^3*g^2*p)*log(e*x^2)/d^4)/e
```

Mupad [B] (verification not implemented)

Time = 25.79 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \frac{\ln\left(e\,x^2+d\right)\,\left(6\,p\,d^2\,e^2\,g^2-8\,p\,d\,e^3\,f\,g+3\,p\,e^4\,f^2\right)}{24\,d^4} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{8}+\frac{f\,g\,x^2}{3}+\frac{g^2\,x^4}{4}\right)}{x^8} - \frac{\frac{e\,f^2\,p}{2\,d}+\frac{e\,p\,x^4\,\left(6\,d^2\,g^2-8\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,d^3}+\frac{e\,f\,p\,x^2\,\left(8\,d\,g-3\,e\,f\right)}{4\,d^2}}{12\,x^6} - \frac{\ln\left(x\right)\,\left(6\,p\,d^2\,e^2\,g^2-8\,p\,d\,e^3\,f\,g+3\,p\,e^4\,f^2\right)}{12\,d^4}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^9,x)
```

output
```
(log(d + e*x^2)*(3*e^4*f^2*p + 6*d^2*e^2*g^2*p - 8*d*e^3*f*g*p))/(24*d^4)
- (log(c*(d + e*x^2)^p)*(f^2/8 + (g^2*x^4)/4 + (f*g*x^2)/3))/x^8 - ((e*f^2
*p)/(2*d) + (e*p*x^4*(6*d^2*g^2 + 3*e^2*f^2 - 8*d*e*f*g))/(2*d^3) + (e*f*p
*x^2*(8*d*g - 3*e*f))/(4*d^2))/(12*x^6) - (log(x)*(3*e^4*f^2*p + 6*d^2*e^2
*g^2*p - 8*d*e^3*f*g*p))/(12*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.25

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^9} dx$$
$$= \frac{-6\log((e\,x^2+d)^p\,c)\,d^4f^2 - 16\log((e\,x^2+d)^p\,c)\,d^4fg\,x^2 - 12\log((e\,x^2+d)^p\,c)\,d^4g^2x^4 + 12\log((e\,x^2+$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x)
```

output
```
( - 6*log((d + e*x**2)**p*c)*d**4*f**2 - 16*log((d + e*x**2)**p*c)*d**4*f*
g*x**2 - 12*log((d + e*x**2)**p*c)*d**4*g**2*x**4 + 12*log((d + e*x**2)**p
*c)*d**2*e**2*g**2*x**8 - 16*log((d + e*x**2)**p*c)*d*e**3*f*g*x**8 + 6*lo
g((d + e*x**2)**p*c)*e**4*f**2*x**8 - 24*log(x)*d**2*e**2*g**2*p*x**8 + 32
*log(x)*d*e**3*f*g*p*x**8 - 12*log(x)*e**4*f**2*p*x**8 - 2*d**3*e*f**2*p*x
**2 - 8*d**3*e*f*g*p*x**4 - 12*d**3*e*g**2*p*x**6 + 3*d**2*e**2*f**2*p*x**
4 + 16*d**2*e**2*f*g*p*x**6 - 6*d*e**3*f**2*p*x**6)/(48*d**4*x**8)
```

3.331 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 253

$$\begin{aligned}\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx = & -\frac{ef^2p}{40dx^8}+\frac{ef(2ef-5dg)p}{60d^2x^6} \\ & -\frac{e(6e^2f^2-15defg+10d^2g^2)\,p}{120d^3x^4} \\ & +\frac{e^2(6e^2f^2-15defg+10d^2g^2)\,p}{60d^4x^2} \\ & +\frac{e^3(6e^2f^2-15defg+10d^2g^2)\,p\log(x)}{30d^5} \\ & -\frac{e^3(6e^2f^2-15defg+10d^2g^2)\,p\log\left(d+ex^2\right)}{60d^5} \\ & -\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}} \\ & -\frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6}\end{aligned}$$

output
```
-1/40*e*f^2*p/d/x^8+1/60*e*f*(-5*d*g+2*e*f)*p/d^2/x^6-1/120*e*(10*d^2*g^2-
15*d*e*f*g+6*e^2*f^2)*p/d^3/x^4+1/60*e^2*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)
*p/d^4/x^2+1/30*e^3*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p*ln(x)/d^5-1/60*e^3
*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p*ln(e*x^2+d)/d^5-1/10*f^2*ln(c*(e*x^2+
d)^p)/x^10-1/4*f*g*ln(c*(e*x^2+d)^p)/x^8-1/6*g^2*ln(c*(e*x^2+d)^p)/x^6
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^{11}} \, dx =$$
$$-\frac{depx^2(-12e^3f^2x^6+6de^2fx^4(f+5gx^2)+d^3(3f^2+10fgx^2+10g^2x^4)-d^2ex^2(4f^2+15fgx^2+20g^2}{}$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```

output
```
-1/120*(d*e*p*x^2*(-12*e^3*f^2*x^6 + 6*d*e^2*f*x^4*(f + 5*g*x^2) + d^3*(3*
f^2 + 10*f*g*x^2 + 10*g^2*x^4) - d^2*e*x^2*(4*f^2 + 15*f*g*x^2 + 20*g^2*x^
4)) - 4*e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[x] + 2*e^3*(6
*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[d + e*x^2] + 2*d^5*(6*f^2 +
 15*f*g*x^2 + 10*g^2*x^4)*Log[c*(d + e*x^2)^p])/(d^5*x^10)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2925, 2861, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^{11}} \, dx$$
$$\Big\downarrow \text{2925}$$
$$\frac{1}{2} \int \frac{(gx^2+f)^2 \log \left(c(ex^2+d)^p\right)}{x^{12}} dx^2$$
$$\Big\downarrow \text{2861}$$
$$\frac{1}{2}\left(-ep\int -\frac{10g^2x^4+15fgx^2+6f^2}{30x^{10}(ex^2+d)}dx^2 - \frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^{10}} - \frac{fg\log\left(c(d+ex^2)^p\right)}{2x^8} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{30}ep\int \frac{10g^2x^4+15fgx^2+6f^2}{x^{10}\left(ex^2+d\right)}dx^2-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{5x^{10}}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{2x^8}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}\right)$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{2}\left(\frac{1}{30}ep\int \left(-\frac{\left(6e^2f^2-15degf+10d^2g^2\right)e^3}{d^5\left(ex^2+d\right)}+\frac{\left(6e^2f^2-15degf+10d^2g^2\right)e^2}{d^5x^2}-\frac{\left(6e^2f^2-15degf+10d^2g^2\right)e}{d^4x^4}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{5x^{10}}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{2x^8}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}+\frac{1}{30}ep\left(\frac{f(2ef-5dg)}{d^2x^6}+\frac{e^2\log\left(x^2\right)\left(\right.}{}\right.\right.$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```

output
```
((e*p*((-3*f^2)/(2*d*x^8) + (f*(2*e*f - 5*d*g))/(d^2*x^6) - (6*e^2*f^2 - 1
5*d*e*f*g + 10*d^2*g^2)/(2*d^3*x^4) + (e*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*
g^2))/(d^4*x^2) + (e^2*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*Log[x^2])/d^5
 - (e^2*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*Log[d + e*x^2])/d^5))/30 - (
f^2*Log[c*(d + e*x^2)^p])/(5*x^10) - (f*g*Log[c*(d + e*x^2)^p])/(2*x^8) -
(g^2*Log[c*(d + e*x^2)^p])/(3*x^6))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2861
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Simp[(a + b*Log[c*(d + e*x)^n])   u, x] - Simp[b*e*n   Int[SimplifyIn
tegrand[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && Integer
Q[r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 5.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.91

method	result
parts	$-\frac{g^{2} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right)}{6 x^{6}}-\frac{f g \ln\left(c \left(e \,x^{2}+d \right)^{p}\right)}{4 x^{8}}-\frac{f^{2} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right)}{10 x^{10}}-\frac{e p \left(\frac{e^{2} \left(10 d^{2} g^{2}-15 f g e d +6 f^{2} e^{2}\right) \ln\left(e \,x^{2}+d \right)}{2 d^{5}}-\frac{-10 d^{2} g^{2}}{}\right.}{}$
parallelrisch	$\frac{40 \ln\left(x \right) x^{10} d^{2} e^{3} g^{2} p^{2}-60 \ln\left(x \right) x^{10} d \,e^{4} f g \,p^{2}+24 \ln\left(x \right) x^{10} e^{5} f^{2} p^{2}-20 x^{10} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right) d^{2} e^{3} g^{2} p +30 x^{10} \ln\left(c \left(e \,x^{2}+d \right)^{p}\right) d \,e^{4} f}{}$
risch	$-\frac{\left(10 g^{2} x^{4}+15 f g \,x^{2}+6 f^{2}\right) \ln\left(\left(e \,x^{2}+d \right)^{p}\right)}{60 x^{10}}-\frac{20 \ln\left(c \right) d^{5} g^{2} x^{4}-24 \ln\left(x \right) e^{5} f^{2} p \,x^{10}+30 \ln\left(c \right) d^{5} f g \,x^{2}-20 d^{3} e^{2} g^{2} p \,x^{8}-12 d \,e^{4} f^{2}}{}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/6*g^2*ln(c*(e*x^2+d)^p)/x^6-1/4*f*g*ln(c*(e*x^2+d)^p)/x^8-1/10*f^2*ln(c
*(e*x^2+d)^p)/x^10-1/30*e*p*(1/2*e^2*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)/d^5
*ln(e*x^2+d)-1/4*(-10*d^2*g^2+15*d*e*f*g-6*e^2*f^2)/d^3/x^4-1/2*(10*d^2*g^
2-15*d*e*f*g+6*e^2*f^2)/d^4*e/x^2+3/4*f^2/d/x^8+1/2*f*(5*d*g-2*e*f)/d^2/x^
6-(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)/d^5*e^2*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx$$
$$= \frac{4\,(6\,e^5f^2-15\,de^4fg+10\,d^2e^3g^2)px^{10}\log(x)-3\,d^4ef^2px^2+2\,(6\,de^4f^2-15\,d^2e^3fg+10\,d^3e^2g^2)px^8-(\cdots}{\cdots}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="fricas")
```

output
```
1/120*(4*(6*e^5*f^2 - 15*d*e^4*f*g + 10*d^2*e^3*g^2)*p*x^10*log(x) - 3*d^4
*e*f^2*p*x^2 + 2*(6*d*e^4*f^2 - 15*d^2*e^3*f*g + 10*d^3*e^2*g^2)*p*x^8 - (
6*d^2*e^3*f^2 - 15*d^3*e^2*f*g + 10*d^4*e*g^2)*p*x^6 + 2*(2*d^3*e^2*f^2 -
5*d^4*e*f*g)*p*x^4 - 2*(10*d^5*g^2*p*x^4 + (6*e^5*f^2 - 15*d*e^4*f*g + 10*
d^2*e^3*g^2)*p*x^10 + 15*d^5*f*g*p*x^2 + 6*d^5*f^2*p)*log(e*x^2 + d) - 2*(
10*d^5*g^2*x^4 + 15*d^5*f*g*x^2 + 6*d^5*f^2)*log(c))/(d^5*x^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx = \text{Timed out}$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**11,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx =$$
$$-\frac{1}{120}\,ep\left(\frac{2\,(6\,e^4f^2-15\,de^3fg+10\,d^2e^2g^2)\log\left(ex^2+d\right)}{d^5}-\frac{2\,(6\,e^4f^2-15\,de^3fg+10\,d^2e^2g^2)\log\left(x^2\right)}{d^5}-\right.$$
$$-\frac{(10\,g^2x^4+15\,fgx^2+6\,f^2)\log\left((ex^2+d)^pc\right)}{60\,x^{10}}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="maxima")
```

output
```
-1/120*e*p*(2*(6*e^4*f^2 - 15*d*e^3*f*g + 10*d^2*e^2*g^2)*log(e*x^2 + d)/d
^5 - 2*(6*e^4*f^2 - 15*d*e^3*f*g + 10*d^2*e^2*g^2)*log(x^2)/d^5 - (2*(6*e^
3*f^2 - 15*d*e^2*f*g + 10*d^2*e*g^2)*x^6 - 3*d^3*f^2 - (6*d*e^2*f^2 - 15*d
^2*e*f*g + 10*d^3*g^2)*x^4 + 2*(2*d^2*e*f^2 - 5*d^3*f*g)*x^2)/(d^4*x^8)) -
 1/60*(10*g^2*x^4 + 15*f*g*x^2 + 6*f^2)*log((e*x^2 + d)^p*c)/x^10
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(235) = 470$.

Time = 0.13 (sec) , antiderivative size = 699, normalized size of antiderivative = 2.76

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="giac")
```

output
```
-1/120*(2*(6*e^6*f^2*p + 15*(e*x^2 + d)*e^5*f*g*p - 15*d*e^5*f*g*p + 10*(e
*x^2 + d)^2*e^4*g^2*p - 20*(e*x^2 + d)*d*e^4*g^2*p + 10*d^2*e^4*g^2*p)*log
(e*x^2 + d)/((e*x^2 + d)^5 - 5*(e*x^2 + d)^4*d + 10*(e*x^2 + d)^3*d^2 - 10
*(e*x^2 + d)^2*d^3 + 5*(e*x^2 + d)*d^4 - d^5) - (12*(e*x^2 + d)^4*e^6*f^2*
p - 54*(e*x^2 + d)^3*d*e^6*f^2*p + 94*(e*x^2 + d)^2*d^2*e^6*f^2*p - 77*(e*
x^2 + d)*d^3*e^6*f^2*p + 25*d^4*e^6*f^2*p - 30*(e*x^2 + d)^4*d*e^5*f*g*p +
 135*(e*x^2 + d)^3*d^2*e^5*f*g*p - 235*(e*x^2 + d)^2*d^3*e^5*f*g*p + 185*(
e*x^2 + d)*d^4*e^5*f*g*p - 55*d^5*e^5*f*g*p + 20*(e*x^2 + d)^4*d^2*e^4*g^2
*p - 90*(e*x^2 + d)^3*d^3*e^4*g^2*p + 150*(e*x^2 + d)^2*d^4*e^4*g^2*p - 11
0*(e*x^2 + d)*d^5*e^4*g^2*p + 30*d^6*e^4*g^2*p - 12*d^4*e^6*f^2*log(c) - 3
0*(e*x^2 + d)*d^4*e^5*f*g*log(c) + 30*d^5*e^5*f*g*log(c) - 20*(e*x^2 + d)^
2*d^4*e^4*g^2*log(c) + 40*(e*x^2 + d)*d^5*e^4*g^2*log(c) - 20*d^6*e^4*g^2*
log(c))/((e*x^2 + d)^5*d^4 - 5*(e*x^2 + d)^4*d^5 + 10*(e*x^2 + d)^3*d^6 -
10*(e*x^2 + d)^2*d^7 + 5*(e*x^2 + d)*d^8 - d^9) + 2*(6*e^6*f^2*p - 15*d*e^
5*f*g*p + 10*d^2*e^4*g^2*p)*log(e*x^2 + d)/d^5 - 2*(6*e^6*f^2*p - 15*d*e^5
*f*g*p + 10*d^2*e^4*g^2*p)*log(e*x^2)/d^5)/e
```

Mupad [B] (verification not implemented)

Time = 25.83 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx = \frac{\ln(x)\,\left(10\,p\,d^2\,e^3\,g^2-15\,p\,d\,e^4\,f\,g+6\,p\,e^5\,f^2\right)}{30\,d^5} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{10}+\frac{f\,g\,x^2}{4}+\frac{g^2\,x^4}{6}\right)}{x^{10}} - \frac{\ln\left(e\,x^2+d\right)\,\left(10\,p\,d^2\,e^3\,g^2-15\,p\,d\,e^4\,f\,g+6\,p\,e^5\,f^2\right)}{60\,d^5} - \frac{\frac{3\,e\,f^2\,p}{4\,d}-\frac{e^2\,p\,x^6\,\left(10\,d^2\,g^2-15\,d\,e\,f\,g+6\,e^2\,f^2\right)}{2\,d^4}+\frac{e\,p\,x^4\,\left(10\,d^2\,g^2-15\,d\,e\,f\,g+6\,e^2\,f^2\right)}{4\,d^3}+\frac{e\,f\,p\,x^2\,\left(5\,d\,g-2\,e\,f\right)}{2\,d^2}}{30\,x^8}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^11,x)
```

output
```
(log(x)*(6*e^5*f^2*p + 10*d^2*e^3*g^2*p - 15*d*e^4*f*g*p))/(30*d^5) - (log
(c*(d + e*x^2)^p)*(f^2/10 + (g^2*x^4)/6 + (f*g*x^2)/4))/x^10 - (log(d + e*
x^2)*(6*e^5*f^2*p + 10*d^2*e^3*g^2*p - 15*d*e^4*f*g*p))/(60*d^5) - ((3*e*f
^2*p)/(4*d) - (e^2*p*x^6*(10*d^2*g^2 + 6*e^2*f^2 - 15*d*e*f*g))/(2*d^4) +
(e*p*x^4*(10*d^2*g^2 + 6*e^2*f^2 - 15*d*e*f*g))/(4*d^3) + (e*f*p*x^2*(5*d*
g - 2*e*f))/(2*d^2))/(30*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.25

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx$$
$$= \frac{-12\log\left((e\,x^2+d)^p\,c\right)d^5f^2 - 30\log\left((e\,x^2+d)^p\,c\right)d^5fg\,x^2 - 20\log\left((e\,x^2+d)^p\,c\right)d^5g^2x^4 - 20\log\left((e\,x^2\right.}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x)
```

output
```
( - 12*log((d + e*x**2)**p*c)*d**5*f**2 - 30*log((d + e*x**2)**p*c)*d**5*f
*g*x**2 - 20*log((d + e*x**2)**p*c)*d**5*g**2*x**4 - 20*log((d + e*x**2)**
p*c)*d**2*e**3*g**2*x**10 + 30*log((d + e*x**2)**p*c)*d*e**4*f*g*x**10 - 1
2*log((d + e*x**2)**p*c)*e**5*f**2*x**10 + 40*log(x)*d**2*e**3*g**2*p*x**1
0 - 60*log(x)*d*e**4*f*g*p*x**10 + 24*log(x)*e**5*f**2*p*x**10 - 3*d**4*e*
f**2*p*x**2 - 10*d**4*e*f*g*p*x**4 - 10*d**4*e*g**2*p*x**6 + 4*d**3*e**2*f
**2*p*x**4 + 15*d**3*e**2*f*g*p*x**6 + 20*d**3*e**2*g**2*p*x**8 - 6*d**2*e
**3*f**2*p*x**6 - 30*d**2*e**3*f*g*p*x**8 + 12*d*e**4*f**2*p*x**8)/(120*d*
*5*x**10)
```

3.332 $\int x^2(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 278

$$\begin{aligned}\int x^2(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx &= \frac{2df^2px}{3e}-\frac{4d^2fgpx}{5e^2}+\frac{2d^3g^2px}{7e^3}-\frac{2}{9}f^2px^3+\frac{4dfgpx^3}{15e}-\frac{2d^2g^2px^3}{21e^2}\\&-\frac{4}{25}fgpx^5+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px^7-\frac{2d^{3/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}\\&+\frac{4d^{5/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}-\frac{2d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}\\&+\frac{1}{3}f^2x^3\log\left(c(d+ex^2)^p\right)+\frac{2}{5}fgx^5\log\left(c(d+ex^2)^p\right)+\frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right)\end{aligned}$$

output
```
2/3*d*f^2*p*x/e-4/5*d^2*f*g*p*x/e^2+2/7*d^3*g^2*p*x/e^3-2/9*f^2*p*x^3+4/15
*d*f*g*p*x^3/e-2/21*d^2*g^2*p*x^3/e^2-4/25*f*g*p*x^5+2/35*d*g^2*p*x^5/e-2/
49*g^2*p*x^7-2/3*d^(3/2)*f^2*p*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)+4/5*d^(5/
2)*f*g*p*arctan(e^(1/2)*x/d^(1/2))/e^(5/2)-2/7*d^(7/2)*g^2*p*arctan(e^(1/2
)*x/d^(1/2))/e^(7/2)+1/3*f^2*x^3*ln(c*(e*x^2+d)^p)+2/5*f*g*x^5*ln(c*(e*x^2
+d)^p)+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

$$\int x^2 (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx$$
$$= \frac{-210d^{3/2}(35e^2f^2-42defg+15d^2g^2)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{e}x(2p(1575d^3g^2-105d^2eg(42f+5gx^2)+10}{}$$

input
```
Integrate[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(-210*d^(3/2)*(35*e^2*f^2 - 42*d*e*f*g + 15*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/
Sqrt[d]] + Sqrt[e]*x*(2*p*(1575*d^3*g^2 - 105*d^2*e*g*(42*f + 5*g*x^2) + 1
05*d*e^2*(35*f^2 + 14*f*g*x^2 + 3*g^2*x^4) - e^3*x^2*(1225*f^2 + 882*f*g*x
^2 + 225*g^2*x^4)) + 105*e^3*x^2*(35*f^2 + 42*f*g*x^2 + 15*g^2*x^4)*Log[c*
(d + e*x^2)^p]))/(11025*e^(7/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(f^2x^2\log\left(c(d+ex^2)^p\right)+2fgx^4\log\left(c(d+ex^2)^p\right)+g^2x^6\log\left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d^{3/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{4d^{5/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}-\frac{2d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}+\frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{2d^3g^2px}{7e^3}-\frac{4d^2fgpx}{5e^2}-\frac{2d^2g^2px^3}{21e^2}+\frac{2df^2px}{3e}+\frac{4dfgpx^3}{15e}+\frac{2dg^2px^5}{35e}-\frac{2}{9}f^2px^3-\frac{4}{25}fgpx^5-\frac{2}{49}g^2px^7$$

input
```
Int[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(2*d*f^2*p*x)/(3*e) - (4*d^2*f*g*p*x)/(5*e^2) + (2*d^3*g^2*p*x)/(7*e^3) -
(2*f^2*p*x^3)/9 + (4*d*f*g*p*x^3)/(15*e) - (2*d^2*g^2*p*x^3)/(21*e^2) - (4
*f*g*p*x^5)/25 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 - (2*d^(3/2)*f^
2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) + (4*d^(5/2)*f*g*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(7*e^(7/2)) + (f^2*x^3*Log[c*(d + e*x^2)^p])/3 + (2*f*g*x^5*Log[c*(d
+ e*x^2)^p])/5 + (g^2*x^7*Log[c*(d + e*x^2)^p])/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g^2x^7\ln(c(e\,x^2+d)^p)}{7}+\frac{2fg\,x^5\ln(c(e\,x^2+d)^p)}{5}+\frac{f^2x^3\ln(c(e\,x^2+d)^p)}{3}-\frac{2ep\left(-\frac{-\frac{15}{7}e^3g^2x^7+3d\,e^2g^2x^5-\frac{42}{5}e^3fg\,x^5-5d^2e\,g^2x^3}{}\right.}{}$
risch	$\frac{2\ln(c)fg\,x^5}{5}-\frac{4d^2fgpx}{5e^2}+\frac{4dfgp\,x^3}{15e}-\frac{4fgp\,x^5}{25}-\frac{i\pi\,f^2x^3\mathrm{csgn}(ic(e\,x^2+d)^p)^3}{6}-\frac{i\pi\,g^2x^7\mathrm{csgn}(ic(e\,x^2+d)^p)^3}{14}-\frac{2\sqrt{-de}\,p}{}$

input
```
int(x^2*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/7*g^2*x^7*ln(c*(e*x^2+d)^p)+2/5*f*g*x^5*ln(c*(e*x^2+d)^p)+1/3*f^2*x^3*ln
(c*(e*x^2+d)^p)-2/105*e*p*(-1/e^4*(-15/7*e^3*g^2*x^7+3*d*e^2*g^2*x^5-42/5*
e^3*f*g*x^5-5*d^2*e*g^2*x^3+14*d*f*g*x^3*e^2-35/3*e^3*f^2*x^3+15*d^3*x*g^2
-42*x*d^2*e*f*g+35*x*d*e^2*f^2)+d^2*(15*d^2*g^2-42*d*e*f*g+35*e^2*f^2)/e^4
/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.77

$$\int x^2(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$

$$=\left[-\frac{450\,e^3g^2px^7+126\,(14\,e^3fg-5\,de^2g^2)px^5+70\,(35\,e^3f^2-42\,de^2fg+15\,d^2eg^2)px^3-105\,(35\,de^2f^2-}{}\right.$$

$$-\frac{450\,e^3g^2px^7+126\,(14\,e^3fg-5\,de^2g^2)px^5+70\,(35\,e^3f^2-42\,de^2fg+15\,d^2eg^2)px^3+210\,(35\,de^2f^2-}{}$$

input
```
integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/11025*(450*e^3*g^2*p*x^7 + 126*(14*e^3*f*g - 5*d*e^2*g^2)*p*x^5 + 70*(
35*e^3*f^2 - 42*d*e^2*f*g + 15*d^2*e*g^2)*p*x^3 - 105*(35*d*e^2*f^2 - 42*d
^2*e*f*g + 15*d^3*g^2)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*
x^2 + d)) - 210*(35*d*e^2*f^2 - 42*d^2*e*f*g + 15*d^3*g^2)*p*x - 105*(15*e
^3*g^2*p*x^7 + 42*e^3*f*g*p*x^5 + 35*e^3*f^2*p*x^3)*log(e*x^2 + d) - 105*(
15*e^3*g^2*x^7 + 42*e^3*f*g*x^5 + 35*e^3*f^2*x^3)*log(c))/e^3, -1/11025*(4
50*e^3*g^2*p*x^7 + 126*(14*e^3*f*g - 5*d*e^2*g^2)*p*x^5 + 70*(35*e^3*f^2 -
 42*d*e^2*f*g + 15*d^2*e*g^2)*p*x^3 + 210*(35*d*e^2*f^2 - 42*d^2*e*f*g + 1
5*d^3*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) - 210*(35*d*e^2*f^2 - 42*d^
2*e*f*g + 15*d^3*g^2)*p*x - 105*(15*e^3*g^2*p*x^7 + 42*e^3*f*g*p*x^5 + 35*
e^3*f^2*p*x^3)*log(e*x^2 + d) - 105*(15*e^3*g^2*x^7 + 42*e^3*f*g*x^5 + 35*
e^3*f^2*x^3)*log(c))/e^3]
```

Sympy [A] (verification not implemented)

Time = 125.21 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.01

$$\int x^2(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$= \begin{cases} \left(\frac{f^2x^3}{3}+\frac{2fgx^5}{5}+\frac{g^2x^7}{7}\right)\log\left(0^pc\right) \\ \left(\frac{f^2x^3}{3}+\frac{2fgx^5}{5}+\frac{g^2x^7}{7}\right)\log\left(cd^p\right) \\ -\frac{2f^2px^3}{9}+\frac{f^2x^3\log\left(c(ex^2)^p\right)}{3}-\frac{4fgpx^5}{25}+\frac{2fgx^5\log\left(c(ex^2)^p\right)}{5}-\frac{2g^2px^7}{49}+\frac{g^2x^7\log\left(c(ex^2)^p\right)}{7} \\ -\frac{2d^4g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{d^4g^2\log\left(c(d+ex^2)^p\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{4d^3fgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{2d^3fg\log\left(c(d+ex^2)^p\right)}{5e^3\sqrt{-\frac{d}{e}}}+\frac{2d^3g^2px}{7e^3}-\frac{2d^2f^2p\log\left(x\right.}{3e^2\sqrt{-}} \end{cases}$$

input
```
integrate(x**2*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise(((f**2*x**3/3 + 2*f*g*x**5/5 + g**2*x**7/7)*log(0**p*c), Eq(d, 0
) & Eq(e, 0)), ((f**2*x**3/3 + 2*f*g*x**5/5 + g**2*x**7/7)*log(c*d**p), Eq
(e, 0)), (-2*f**2*p*x**3/9 + f**2*x**3*log(c*(e*x**2)**p)/3 - 4*f*g*p*x**5
/25 + 2*f*g*x**5*log(c*(e*x**2)**p)/5 - 2*g**2*p*x**7/49 + g**2*x**7*log(c
*(e*x**2)**p)/7, Eq(d, 0)), (-2*d**4*g**2*p*log(x - sqrt(-d/e))/(7*e**4*sq
rt(-d/e)) + d**4*g**2*log(c*(d + e*x**2)**p)/(7*e**4*sqrt(-d/e)) + 4*d**3*
f*g*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - 2*d**3*f*g*log(c*(d + e*x*
*2)**p)/(5*e**3*sqrt(-d/e)) + 2*d**3*g**2*p*x/(7*e**3) - 2*d**2*f**2*p*log
(x - sqrt(-d/e))/(3*e**2*sqrt(-d/e)) + d**2*f**2*log(c*(d + e*x**2)**p)/(3
*e**2*sqrt(-d/e)) - 4*d**2*f*g*p*x/(5*e**2) - 2*d**2*g**2*p*x**3/(21*e**2)
 + 2*d*f**2*p*x/(3*e) + 4*d*f*g*p*x**3/(15*e) + 2*d*g**2*p*x**5/(35*e) - 2
*f**2*p*x**3/9 + f**2*x**3*log(c*(d + e*x**2)**p)/3 - 4*f*g*p*x**5/25 + 2*
f*g*x**5*log(c*(d + e*x**2)**p)/5 - 2*g**2*p*x**7/49 + g**2*x**7*log(c*(d
+ e*x**2)**p)/7, True))
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.77

$$\int x^2 (f+gx^2)^2 \log (c(d+ex^2)^p) \, dx$$
$$= -\frac{1}{49} (2 g^2 p - 7 g^2 \log (c)) x^7 - \frac{2 (14 efgp - 5 dg^2 p - 35 efg \log (c)) x^5}{175 e}$$
$$- \frac{(70 e^2 f^2 p - 84 defgp + 30 d^2 g^2 p - 105 e^2 f^2 \log (c)) x^3}{315 e^2}$$
$$+ \frac{1}{105} (15 g^2 p x^7 + 42 fgpx^5 + 35 f^2 p x^3) \log (ex^2 + d)$$
$$+ \frac{2 (35 de^2 f^2 p - 42 d^2 efgp + 15 d^3 g^2 p) x}{105 e^3}$$
$$- \frac{2 (35 d^2 e^2 f^2 p - 42 d^3 efgp + 15 d^4 g^2 p) \arctan \left(\frac{ex}{\sqrt{de}}\right)}{105 \sqrt{de} e^3}$$

input
```
integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output
```
-1/49*(2*g^2*p - 7*g^2*log(c))*x^7 - 2/175*(14*e*f*g*p - 5*d*g^2*p - 35*e*
f*g*log(c))*x^5/e - 1/315*(70*e^2*f^2*p - 84*d*e*f*g*p + 30*d^2*g^2*p - 10
5*e^2*f^2*log(c))*x^3/e^2 + 1/105*(15*g^2*p*x^7 + 42*f*g*p*x^5 + 35*f^2*p*
x^3)*log(e*x^2 + d) + 2/105*(35*d*e^2*f^2*p - 42*d^2*e*f*g*p + 15*d^3*g^2*
p)*x/e^3 - 2/105*(35*d^2*e^2*f^2*p - 42*d^3*e*f*g*p + 15*d^4*g^2*p)*arctan
(e*x/sqrt(d*e))/(sqrt(d*e)*e^3)
```

Mupad [B] (verification not implemented)

Time = 25.80 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int x^2(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{f^2\,x^3}{3}+\frac{2\,f\,g\,x^5}{5}+\frac{g^2\,x^7}{7}\right)-x^3\,\left(\frac{2\,f^2\,p}{9}-\frac{d\,\left(\frac{4\,f\,g\,p}{5}-\frac{2\,d\,g^2\,p}{7\,e}\right)}{3\,e}\right)-x^5\,\left(\frac{4\,f\,g\,p}{25}-\frac{2\,d\,g^2\,p}{35\,e}\right)-\frac{2\,g^2\,p\,x^7}{49}+\frac{d\,x\,\left(\frac{2\,f^2\,p}{3}-\frac{d\,\left(\frac{4\,f\,g\,p}{5}-\frac{2\,d\,g^2\,p}{7\,e}\right)}{e}\right)}{e}-\frac{2\,d^{3/2}\,p\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,p\,x\,\left(15\,d^2\,g^2-42\,d\,e\,f\,g+35\,e^2\,f^2\right)}{15\,p\,d^4\,g^2-42\,p\,d^3\,e\,f\,g+35\,p\,d^2\,e^2\,f^2}\right)\,\left(15\,d^2\,g^2-42\,d\,e\,f\,g+35\,e^2\,f^2\right)}{105\,e^{7/2}}$$

input
```
int(x^2*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

output
```
log(c*(d + e*x^2)^p)*((f^2*x^3)/3 + (g^2*x^7)/7 + (2*f*g*x^5)/5) - x^3*((2
*f^2*p)/9 - (d*((4*f*g*p)/5 - (2*d*g^2*p)/(7*e)))/(3*e)) - x^5*((4*f*g*p)/
25 - (2*d*g^2*p)/(35*e)) - (2*g^2*p*x^7)/49 + (d*x*((2*f^2*p)/3 - (d*((4*f
*g*p)/5 - (2*d*g^2*p)/(7*e)))/e))/e - (2*d^(3/2)*p*atan((d^(3/2)*e^(1/2)*p
*x*(15*d^2*g^2 + 35*e^2*f^2 - 42*d*e*f*g))/(15*d^4*g^2*p + 35*d^2*e^2*f^2*
p - 42*d^3*e*f*g*p))*(15*d^2*g^2 + 35*e^2*f^2 - 42*d*e*f*g))/(105*e^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.93

$$\int x^2(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\frac{-3150\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)d^3g^2p+8820\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)d^2efgp-7350\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)de^2f^2p}{}$$

input
```
int(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x)
```

output

```
( - 3150*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**3*g**2*p + 8820*
sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*e*f*g*p - 7350*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e**2*f**2*p + 3675*log((d + e*x**2
)**p*c)*e**4*f**2*x**3 + 4410*log((d + e*x**2)**p*c)*e**4*f*g*x**5 + 1575*
log((d + e*x**2)**p*c)*e**4*g**2*x**7 + 3150*d**3*e*g**2*p*x - 8820*d**2*e
**2*f*g*p*x - 1050*d**2*e**2*g**2*p*x**3 + 7350*d*e**3*f**2*p*x + 2940*d*e
**3*f*g*p*x**3 + 630*d*e**3*g**2*p*x**5 - 2450*e**4*f**2*p*x**3 - 1764*e**
4*f*g*p*x**5 - 450*e**4*g**2*p*x**7)/(11025*e**4)
```

3.333 $\int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\begin{aligned}
&\int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx \\
&= -2f^2px + \frac{4dfgpx}{3e} - \frac{2d^2g^2px}{5e^2} - \frac{4}{9}fgpx^3 + \frac{2dg^2px^3}{15e} - \frac{2}{25}g^2px^5 \\
&\quad + \frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{4d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{2d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}} \\
&\quad + f^2x\log\left(c(d+ex^2)^p\right) + \frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right) + \frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)
\end{aligned}$$

output
```
-2*f^2*p*x+4/3*d*f*g*p*x/e-2/5*d^2*g^2*p*x/e^2-4/9*f*g*p*x^3+2/15*d*g^2*p*
x^3/e-2/25*g^2*p*x^5+2*d^(1/2)*f^2*p*arctan(e^(1/2)*x/d^(1/2))/e^(1/2)-4/3
*d^(3/2)*f*g*p*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)+2/5*d^(5/2)*g^2*p*arctan(
e^(1/2)*x/d^(1/2))/e^(5/2)+f^2*x*ln(c*(e*x^2+d)^p)+2/3*f*g*x^3*ln(c*(e*x^2
+d)^p)+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.68

$$\int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) dx$$
$$= \frac{30\sqrt{d}(15e^2f^2-10defg+3d^2g^2)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{e}x\left(-2p(45d^2g^2-15deg(10f+gx^2)+e^2(225f^2+\right.}{225e^{5/2}}$$

input
```
Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
(30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqr
t[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f
^2 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*L
og[c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) dx$$

$$\Big\downarrow \text{2921}$$

$$\int \left(f^2\log \left(c(d+ex^2)^p\right)+2fgx^2\log \left(c(d+ex^2)^p\right)+g^2x^4\log \left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+ f^2x\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)-\frac{2d^2g^2px}{5e^2}+ \frac{4dfgpx}{3e}+\frac{2dg^2px^3}{15e}-2f^2px-\frac{4}{9}fgpx^3-\frac{2}{25}g^2px^5$$

input
```
Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

output
```
-2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)
/9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2))
 + f^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*
x^5*Log[c*(d + e*x^2)^p])/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g^2x^5\ln(c(e\,x^2+d)^p)}{5}+\frac{2fg\,x^3\ln(c(e\,x^2+d)^p)}{3}+f^2x\ln\left(c(e\,x^2+d)^p\right)-\frac{2ep\left(\frac{\frac{3}{5}e^2g^2x^5-de\,g^2x^3+\frac{10}{3}e^2fg\,x^3+3x\,d^2g^2-1}{e^3}\right.}{}$
risch	$-\frac{i\pi fg\,x^3\,\mathrm{csgn}(i(e\,x^2+d)^p)\,\mathrm{csgn}(ic(e\,x^2+d)^p)\,\mathrm{csgn}(ic)}{3}+\frac{2\sqrt{-de}\,p\ln(\sqrt{-de}\,x+d)\,fgd}{3e^2}-\frac{\sqrt{-de}\,p\ln(\sqrt{-de}\,x+d)\,d^2g^2}{5e^3}+\sqrt{-}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

output
```
1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2/3*f*g*x^3*ln(c*(e*x^2+d)^p)+f^2*x*ln(c*(e*
x^2+d)^p)-2/15*e*p*(1/e^3*(3/5*e^2*g^2*x^5-d*e*g^2*x^3+10/3*e^2*f*g*x^3+3*
x*d^2*g^2-10*d*e*f*g*x+15*e^2*f^2*x)-d*(3*d^2*g^2-10*d*e*f*g+15*e^2*f^2)/e
^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.83

$$\int \left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$=\left[-\frac{18\,e^2g^2px^5+10\,(10\,e^2fg-3\,deg^2)px^3-15\,(15\,e^2f^2-10\,defg+3\,d^2g^2)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)}{}\right.$$

$$-\frac{18\,e^2g^2px^5+10\,(10\,e^2fg-3\,deg^2)px^3-30\,(15\,e^2f^2-10\,defg+3\,d^2g^2)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+30\,(1}{}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

output
```
[-1/225*(18*e^2*g^2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 15*(15*e^2
*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e)
- d)/(e*x^2 + d)) + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*x - 15*(3*e
^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*log(e*x^2 + d) - 15*(3*e
^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/e^2, -1/225*(18*e^2*g^
2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 30*(15*e^2*f^2 - 10*d*e*f*g
+ 3*d^2*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 30*(15*e^2*f^2 - 10*d*e
*f*g + 3*d^2*g^2)*p*x - 15*(3*e^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^
2*p*x)*log(e*x^2 + d) - 15*(3*e^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)
*log(c))/e^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(231) = 462$.

Time = 33.37 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.16

$$\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$$
$$= \begin{cases} \left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(0^pc\right) \\ \left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(cd^p\right) \\ -2f^2px+f^2x\log\left(c(ex^2)^p\right)-\frac{4fgpx^3}{9}+\frac{2fgx^3\log\left(c(ex^2)^p\right)}{3}-\frac{2g^2px^5}{25}+\frac{g^2x^5\log\left(c(ex^2)^p\right)}{5} \\ \frac{2d^3g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{d^3g^2\log\left(c(d+ex^2)^p\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{4d^2fgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{2d^2fg\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}-\frac{2d^2g^2px}{5e^2}+\frac{2df^2p\log\left(x-\sqrt{\phantom{-\frac{d}{e}}}\right.}{e\sqrt{-\frac{d}{e}}} \end{cases}$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

output
```
Piecewise(((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(0**p*c), Eq(d, 0) & E
q(e, 0)), ((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(c*d**p), Eq(e, 0)), (
-2*f**2*p*x + f**2*x*log(c*(e*x**2)**p) - 4*f*g*p*x**3/9 + 2*f*g*x**3*log(
c*(e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log(c*(e*x**2)**p)/5, Eq(d
, 0)), (2*d**3*g**2*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - d**3*g**2*
log(c*(d + e*x**2)**p)/(5*e**3*sqrt(-d/e)) - 4*d**2*f*g*p*log(x - sqrt(-d/
e))/(3*e**2*sqrt(-d/e)) + 2*d**2*f*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-
d/e)) - 2*d**2*g**2*p*x/(5*e**2) + 2*d*f**2*p*log(x - sqrt(-d/e))/(e*sqrt(
-d/e)) - d*f**2*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 4*d*f*g*p*x/(3*e)
+ 2*d*g**2*p*x**3/(15*e) - 2*f**2*p*x + f**2*x*log(c*(d + e*x**2)**p) - 4*
f*g*p*x**3/9 + 2*f*g*x**3*log(c*(d + e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g*
*2*x**5*log(c*(d + e*x**2)**p)/5, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.79

$$\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = -\frac{1}{25}\left(2\,g^2p-5\,g^2\log(c)\right)x^5 - \frac{2\,(10\,efgp-3\,dg^2p-15\,efg\log(c))x^3}{45\,e} + \frac{1}{15}\left(3\,g^2px^5+10\,fgpx^3+15\,f^2px\right)\log\left(ex^2+d\right) - \frac{(30\,e^2f^2p-20\,defgp+6\,d^2g^2p-15\,e^2f^2\log(c))x}{15\,e^2} + \frac{2\,(15\,de^2f^2p-10\,d^2efgp+3\,d^3g^2p)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}e^2}$$

input

```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

output

```
-1/25*(2*g^2*p - 5*g^2*log(c))*x^5 - 2/45*(10*e*f*g*p - 3*d*g^2*p - 15*e*f
*g*log(c))*x^3/e + 1/15*(3*g^2*p*x^5 + 10*f*g*p*x^3 + 15*f^2*p*x)*log(e*x^
2 + d) - 1/15*(30*e^2*f^2*p - 20*d*e*f*g*p + 6*d^2*g^2*p - 15*e^2*f^2*log(
c))*x/e^2 + 2/15*(15*d*e^2*f^2*p - 10*d^2*e*f*g*p + 3*d^3*g^2*p)*arctan(e*
x/sqrt(d*e))/(sqrt(d*e)*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.87

$$\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\left(f^2\,x+\frac{2\,f\,g\,x^3}{3}+\frac{g^2\,x^5}{5}\right) - x\left(2\,f^2\,p-\frac{d\left(\frac{4\,f\,g\,p}{3}-\frac{2\,d\,g^2\,p}{5\,e}\right)}{e}\right) - x^3\left(\frac{4\,f\,g\,p}{9}-\frac{2\,d\,g^2\,p}{15\,e}\right) - \frac{2\,g^2\,p\,x^5}{25} + \frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{3\,p\,d^3\,g^2-10\,p\,d^2\,e\,f\,g+15\,p\,d\,e^2\,f^2}\right)\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{15\,e^{5/2}}$$

input
```
int(log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

output
```
log(c*(d + e*x^2)^p)*(f^2*x + (g^2*x^5)/5 + (2*f*g*x^3)/3) - x*(2*f^2*p -
(d*((4*f*g*p)/3 - (2*d*g^2*p)/(5*e)))/e) - x^3*((4*f*g*p)/9 - (2*d*g^2*p)/
(15*e)) - (2*g^2*p*x^5)/25 + (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(3*d^2
*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(3*d^3*g^2*p + 15*d*e^2*f^2*p - 10*d^2*e*
f*g*p))*(3*d^2*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(15*e^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.96

$$\int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) dx$$

$$= \frac{90\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) d^2g^2p - 300\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) defgp + 450\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) e^2f^2p + 225\log}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p),x)
```

output
```
(90*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*g**2*p - 300*sqrt(e
)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*f*g*p + 450*sqrt(e)*sqrt(d)*at
an((e*x)/(sqrt(e)*sqrt(d)))*e**2*f**2*p + 225*log((d + e*x**2)**p*c)*e**3*
f**2*x + 150*log((d + e*x**2)**p*c)*e**3*f*g*x**3 + 45*log((d + e*x**2)**p
*c)*e**3*g**2*x**5 - 90*d**2*e*g**2*p*x + 300*d*e**2*f*g*p*x + 30*d*e**2*g
**2*p*x**3 - 450*e**3*f**2*p*x - 100*e**3*f*g*p*x**3 - 18*e**3*g**2*p*x**5
)/(225*e**3)
```

3.334 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^2}\,dx = -4fgpx + \frac{2dg^2px}{3e} - \frac{2}{9}g^2px^3 + \frac{2\sqrt{e}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{4\sqrt{d}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{3/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{x} + 2fgx\log\left(c(d+ex^2)^p\right) + \frac{1}{3}g^2x^3\log\left(c(d+ex^2)^p\right)$$

output

```
-4*f*g*p*x+2/3*d*g^2*p*x/e-2/9*g^2*p*x^3+2*e^(1/2)*f^2*p*arctan(e^(1/2)*x/
d^(1/2))/d^(1/2)+4*d^(1/2)*f*g*p*arctan(e^(1/2)*x/d^(1/2))/e^(1/2)-2/3*d^(
3/2)*g^2*p*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)-f^2*ln(c*(e*x^2+d)^p)/x+2*f*g
*x*ln(c*(e*x^2+d)^p)+1/3*g^2*x^3*ln(c*(e*x^2+d)^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^2} dx = \frac{1}{9}\left(-\frac{2gpx(18ef-3dg+egx^2)}{e} + \frac{6(3e^2f^2+6defg-d^2g^2)\, p \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}} + \left(-\frac{9f^2}{x}+18fgx+3g^2x^3\right)\log\left(c(d+ex^2)^p\right)\right)$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

output
```
((-2*g*p*x*(18*e*f - 3*d*g + e*g*x^2))/e + (6*(3*e^2*f^2 + 6*d*e*f*g - d^2
*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(3/2)) + ((-9*f^2)/x + 18*
f*g*x + 3*g^2*x^3)*Log[c*(d + e*x^2)^p])/9
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^2} dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(\frac{f^2 \log\left(c(d+ex^2)^p\right)}{x^2} + 2fg\log\left(c(d+ex^2)^p\right) + g^2x^2\log\left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d^{3/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2\sqrt{e}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2dg^2px}{3e}-4fgpx-\frac{2}{9}g^2px^3$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

output
```
-4*f*g*p*x + (2*d*g^2*p*x)/(3*e) - (2*g^2*p*x^3)/9 + (2*Sqrt[e]*f^2*p*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (4*Sqrt[d]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e] - (2*d^(3/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2))
 - (f^2*Log[c*(d + e*x^2)^p])/x + 2*f*g*x*Log[c*(d + e*x^2)^p] + (g^2*x^3*
Log[c*(d + e*x^2)^p])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.71

method	result
parts	$\frac{g^2x^3\ln\left(c\left(e\,x^2+d\right)^p\right)}{3}+2fgx\ln\left(c\left(e\,x^2+d\right)^p\right)-\frac{f^2\ln\left(c\left(e\,x^2+d\right)^p\right)}{x}-\frac{2ep\left(-\frac{g\left(-\frac{1}{3}eg\,x^3+xdg-6efx\right)}{e^2}+\frac{\left(d^2g^2-6fged-3 \cdots\right.}{e \cdots}\right.}{3}$
risch	$-\frac{\left(-g^2x^4-6fg\,x^2+3f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{3x}+\frac{9i\pi e\,f^2\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\right)+18i\pi efg\,x^2\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right) \cdots}{\cdots}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*g^2*x^3*ln(c*(e*x^2+d)^p)+2*f*g*x*ln(c*(e*x^2+d)^p)-f^2*ln(c*(e*x^2+d)
^p)/x-2/3*e*p*(-g/e^2*(-1/3*e*g*x^3+x*d*g-6*e*f*x)+(d^2*g^2-6*d*e*f*g-3*e^
2*f^2)/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.06

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^2}\,dx$$
$$= \left[-\frac{2\,de^2g^2px^4 - 3\,(3\,e^2f^2 + 6\,defg - d^2g^2)\sqrt{-de}px\log\left(\frac{ex^2+2\,\sqrt{-de}x-d}{ex^2+d}\right) + 6\,(6\,de^2fg - d^2eg^2)px^2 - 3\,(c}{9\,de^2x}\right.$$
$$\left. -\frac{2\,de^2g^2px^4 - 6\,(3\,e^2f^2 + 6\,defg - d^2g^2)\sqrt{de}px\arctan\left(\frac{\sqrt{de}x}{d}\right) + 6\,(6\,de^2fg - d^2eg^2)px^2 - 3\,(de^2g^2px^4}{9\,de^2x}\right.$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

output
```
[-1/9*(2*d*e^2*g^2*p*x^4 - 3*(3*e^2*f^2 + 6*d*e*f*g - d^2*g^2)*sqrt(-d*e)*
p*x*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(6*d*e^2*f*g - d^2*e
*g^2)*p*x^2 - 3*(d*e^2*g^2*p*x^4 + 6*d*e^2*f*g*p*x^2 - 3*d*e^2*f^2*p)*log(
e*x^2 + d) - 3*(d*e^2*g^2*x^4 + 6*d*e^2*f*g*x^2 - 3*d*e^2*f^2)*log(c))/(d*
e^2*x), -1/9*(2*d*e^2*g^2*p*x^4 - 6*(3*e^2*f^2 + 6*d*e*f*g - d^2*g^2)*sqrt
(d*e)*p*x*arctan(sqrt(d*e)*x/d) + 6*(6*d*e^2*f*g - d^2*e*g^2)*p*x^2 - 3*(d
*e^2*g^2*p*x^4 + 6*d*e^2*f*g*p*x^2 - 3*d*e^2*f^2*p)*log(e*x^2 + d) - 3*(d*
e^2*g^2*x^4 + 6*d*e^2*f*g*x^2 - 3*d*e^2*f^2)*log(c))/(d*e^2*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(182) = 364$.

Time = 61.97 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.25

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^2}\,dx$$
$$= \begin{cases} \left(-\frac{f^2}{x}+2fgx+\frac{g^2x^3}{3}\right)\log\left(0^pc\right) \\ \left(-\frac{f^2}{x}+2fgx+\frac{g^2x^3}{3}\right)\log\left(cd^p\right) \\ -\frac{2f^2p}{x}-\frac{f^2\log\left(c(ex^2)^p\right)}{x}-4fgpx+2fgx\log\left(c(ex^2)^p\right)-\frac{2g^2px^3}{9}+\frac{g^2x^3\log\left(c(ex^2)^p\right)}{3} \\ -\frac{2d^2g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{d^2g^2\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{4dfgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{2dfg\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}}+\frac{2dg^2px}{3e}+\frac{2f^2p\log\left(x-\sqrt{-\right.}\right.}{\sqrt{-\frac{d}{e}}} \end{cases}$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**2,x)
```

output
```
Piecewise(((-f**2/x + 2*f*g*x + g**2*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e,
 0)), ((-f**2/x + 2*f*g*x + g**2*x**3/3)*log(c*d**p), Eq(e, 0)), (-2*f**2*
p/x - f**2*log(c*(e*x**2)**p)/x - 4*f*g*p*x + 2*f*g*x*log(c*(e*x**2)**p) -
 2*g**2*p*x**3/9 + g**2*x**3*log(c*(e*x**2)**p)/3, Eq(d, 0)), (-2*d**2*g**
2*p*log(x - sqrt(-d/e))/(3*e**2*sqrt(-d/e)) + d**2*g**2*log(c*(d + e*x**2)
**p)/(3*e**2*sqrt(-d/e)) + 4*d*f*g*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) -
2*d*f*g*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 2*d*g**2*p*x/(3*e) + 2*f**
2*p*log(x - sqrt(-d/e))/sqrt(-d/e) - f**2*log(c*(d + e*x**2)**p)/sqrt(-d/e
) - f**2*log(c*(d + e*x**2)**p)/x - 4*f*g*p*x + 2*f*g*x*log(c*(d + e*x**2)
**p) - 2*g**2*p*x**3/9 + g**2*x**3*log(c*(d + e*x**2)**p)/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.76

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^2} \, dx = -\frac{1}{9} \left(2 g^2 p - 3 g^2 \log (c)\right) x^3 + \frac{1}{3} \left(g^2 p x^3 + 6 f g p x - \frac{3 f^2 p}{x}\right) \log (e x^2 + d) - \frac{f^2 \log (c)}{x} - \frac{2 (6 e f g p - d g^2 p - 3 e f g \log (c)) x}{3 e} + \frac{2 (3 e^2 f^2 p + 6 d e f g p - d^2 g^2 p) \arctan \left(\frac{e x}{\sqrt{d e}}\right)}{3 \sqrt{d e} e}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")
```

output
```
-1/9*(2*g^2*p - 3*g^2*log(c))*x^3 + 1/3*(g^2*p*x^3 + 6*f*g*p*x - 3*f^2*p/x
)*log(e*x^2 + d) - f^2*log(c)/x - 2/3*(6*e*f*g*p - d*g^2*p - 3*e*f*g*log(c
))*x/e + 2/3*(3*e^2*f^2*p + 6*d*e*f*g*p - d^2*g^2*p)*arctan(e*x/sqrt(d*e))
/(sqrt(d*e)*e)
```

Mupad [B] (verification not implemented)

Time = 25.71 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^2} dx = \frac{2 p \operatorname{atan}\left(\frac{\sqrt{e}\, p\, x\, (-d^2 g^2+6 d e f g+3 e^2 f^2)}{\sqrt{d}\, (-p d^2 g^2+6 p d e f g+3 p e^2 f^2)}\right) (-d^2 g^2+6 d e f g+3 e^2 f^2)}{3 \sqrt{d}\, e^{3/2}} - x \left(4 f g p - \frac{2 d g^2 p}{3 e}\right) - \frac{2 g^2 p x^3}{9} - \ln\left(c \left(e x^2+d\right)^p\right) \left(\frac{f^2+2 f g x^2+g^2 x^4}{x} - \frac{\frac{4 g^2 x^4}{3}+4 f g x^2}{x}\right)$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^2,x)
```

output
```
(2*p*atan((e^(1/2)*p*x*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(d^(1/2)*(3*e^2*
f^2*p - d^2*g^2*p + 6*d*e*f*g*p)))*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(3*d
^(1/2)*e^(3/2)) - x*(4*f*g*p - (2*d*g^2*p)/(3*e)) - (2*g^2*p*x^3)/9 - log(
c*(d + e*x^2)^p)*((f^2 + g^2*x^4 + 2*f*g*x^2)/x - ((4*g^2*x^4)/3 + 4*f*g*x
^2)/x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.09

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^2} dx = \frac{-6\sqrt{e}\sqrt{d}\, atan\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) d^2 g^2 p x + 36\sqrt{e}\sqrt{d}\, atan\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) d e f g p x + 18\sqrt{e}\sqrt{d}\, atan\left(\frac{ex}{\sqrt{e}\sqrt{d}}\right) e^2 f^2 p x - 9 \log}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x)
```

output
```
( - 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*g**2*p*x + 36*sqr
t(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*f*g*p*x + 18*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**2*f**2*p*x - 9*log((d + e*x**2)**p*c)*d
*e**2*f**2 + 18*log((d + e*x**2)**p*c)*d*e**2*f*g*x**2 + 3*log((d + e*x**2
)**p*c)*d*e**2*g**2*x**4 + 6*d**2*e*g**2*p*x**2 - 36*d*e**2*f*g*p*x**2 - 2
*d*e**2*g**2*p*x**4)/(9*d*e**2*x)
```

3.335 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^4} dx = -\frac{2ef^2p}{3dx} - 2g^2px - \frac{2e^{3/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{4\sqrt{e}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{d}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{2fg\log\left(c(d+ex^2)^p\right)}{x} + g^2x\log\left(c(d+ex^2)^p\right)$$

output
```
-2/3*e*f^2*p/d/x-2*g^2*p*x-2/3*e^(3/2)*f^2*p*arctan(e^(1/2)*x/d^(1/2))/d^(
3/2)+4*e^(1/2)*f*g*p*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)+2*d^(1/2)*g^2*p*arc
tan(e^(1/2)*x/d^(1/2))/e^(1/2)-1/3*f^2*ln(c*(e*x^2+d)^p)/x^3-2*f*g*ln(c*(e
*x^2+d)^p)/x+g^2*x*ln(c*(e*x^2+d)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^4}\,dx = -2g^2px + \frac{2g(2ef+dg)p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - \frac{2ef^2p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} - \frac{(f^2+6fgx^2-3g^2x^4)\log\left(c(d+ex^2)^p\right)}{3x^3}$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]
```

output
```
-2*g^2*p*x + (2*g*(2*e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sq
rt[e]) - (2*e*f^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x)
 - ((f^2 + 6*f*g*x^2 - 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^4}\,dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(\frac{f^2\log\left(c(d+ex^2)^p\right)}{x^4} + \frac{2fg\log\left(c(d+ex^2)^p\right)}{x^2} + g^2\log\left(c(d+ex^2)^p\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2e^{3/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{4\sqrt{e}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{d}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2fg\log\left(c\left(d+ex^2\right)^p\right)}{x}+g^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{2ef^2p}{3dx}-2g^2px$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]
```

output
```
(-2*e*f^2*p)/(3*d*x) - 2*g^2*p*x - (2*e^(3/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(3*d^(3/2)) + (4*Sqrt[e]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d]
 + (2*Sqrt[d]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f^2*Log[c*(d +
 e*x^2)^p])/(3*x^3) - (2*f*g*Log[c*(d + e*x^2)^p])/x + g^2*x*Log[c*(d + e*
x^2)^p]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

method	result
parts	$g^2x\ln\left(c\left(e\,x^2+d\right)^p\right)-\frac{2fg\ln\left(c\left(e\,x^2+d\right)^p\right)}{x}-\frac{f^2\ln\left(c\left(e\,x^2+d\right)^p\right)}{3x^3}-\frac{2ep\left(\frac{3g^2x}{e}+\frac{\left(-3d^2g^2-6fged+f^2e^2\right)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{de\sqrt{de}}+\right.}{3}$
risch	$-\frac{\left(-3g^2x^4+6fg\,x^2+f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{3x^3}+\frac{3i\pi d\,g^2x^4\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2-6i\pi dfg\,x^2\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\right.}{}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^4,x,method=_RETURNVERBOSE)
```

output
```
g^2*x*ln(c*(e*x^2+d)^p)-2*f*g*ln(c*(e*x^2+d)^p)/x-1/3*f^2*ln(c*(e*x^2+d)^p
)/x^3-2/3*e*p*(3*g^2/e*x+1/d/e*(-3*d^2*g^2-6*d*e*f*g+e^2*f^2)/(d*e)^(1/2)*
arctan(x*e/(d*e)^(1/2))+f^2/d/x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.07

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = \left[-\frac{6\,d^2eg^2px^4+2\,de^2f^2px^2-(e^2f^2-6\,defg-3\,d^2g^2)\sqrt{-de}px^3\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right)-(3\,d^2eg^2px^4-6}{3\,d^2ex^3}\right.$$
$$-\frac{6\,d^2eg^2px^4+2\,de^2f^2px^2+2\,(e^2f^2-6\,defg-3\,d^2g^2)\sqrt{de}px^3\arctan\left(\frac{\sqrt{de}x}{d}\right)-(3\,d^2eg^2px^4-6\,d^2efg}{3\,d^2ex^3}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

output
```
[-1/3*(6*d^2*e*g^2*p*x^4 + 2*d*e^2*f^2*p*x^2 - (e^2*f^2 - 6*d*e*f*g - 3*d^
2*g^2)*sqrt(-d*e)*p*x^3*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - (3
*d^2*e*g^2*p*x^4 - 6*d^2*e*f*g*p*x^2 - d^2*e*f^2*p)*log(e*x^2 + d) - (3*d^
2*e*g^2*x^4 - 6*d^2*e*f*g*x^2 - d^2*e*f^2)*log(c))/(d^2*e*x^3), -1/3*(6*d^
2*e*g^2*p*x^4 + 2*d*e^2*f^2*p*x^2 + 2*(e^2*f^2 - 6*d*e*f*g - 3*d^2*g^2)*sq
rt(d*e)*p*x^3*arctan(sqrt(d*e)*x/d) - (3*d^2*e*g^2*p*x^4 - 6*d^2*e*f*g*p*x
^2 - d^2*e*f^2*p)*log(e*x^2 + d) - (3*d^2*e*g^2*x^4 - 6*d^2*e*f*g*x^2 - d^
2*e*f^2)*log(c))/(d^2*e*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(170) = 340$.

Time = 76.24 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.25

$$\int \frac{(f+gx^2)^2\log(c(d+ex^2)^p)}{x^4}\,dx$$
$$= \begin{cases} \left(-\frac{f^2}{3x^3}-\frac{2fg}{x}+g^2x\right)\log(0^pc) \\ -\frac{2f^2p}{9x^3}-\frac{f^2\log(c(ex^2)^p)}{3x^3}-\frac{4fgp}{x}-\frac{2fg\log(c(ex^2)^p)}{x}-2g^2px+g^2x\log(c(ex^2)^p) \\ \left(-\frac{f^2}{3x^3}-\frac{2fg}{x}+g^2x\right)\log(cd^p) \\ \frac{2dg^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{dg^2\log(c(d+ex^2)^p)}{e\sqrt{-\frac{d}{e}}}-\frac{f^2\log(c(d+ex^2)^p)}{3x^3}+\frac{4fgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\sqrt{-\frac{d}{e}}}-\frac{2fg\log(c(d+ex^2)^p)}{\sqrt{-\frac{d}{e}}}-\frac{2fg\log(c(d}{x} \end{cases}$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**4,x)
```

output
```
Piecewise(((-f**2/(3*x**3) - 2*f*g/x + g**2*x)*log(0**p*c), Eq(d, 0) & Eq(
e, 0)), (-2*f**2*p/(9*x**3) - f**2*log(c*(e*x**2)**p)/(3*x**3) - 4*f*g*p/x
 - 2*f*g*log(c*(e*x**2)**p)/x - 2*g**2*p*x + g**2*x*log(c*(e*x**2)**p), Eq
(d, 0)), ((-f**2/(3*x**3) - 2*f*g/x + g**2*x)*log(c*d**p), Eq(e, 0)), (2*d
*g**2*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*g**2*log(c*(d + e*x**2)**p)
/(e*sqrt(-d/e)) - f**2*log(c*(d + e*x**2)**p)/(3*x**3) + 4*f*g*p*log(x - s
qrt(-d/e))/sqrt(-d/e) - 2*f*g*log(c*(d + e*x**2)**p)/sqrt(-d/e) - 2*f*g*lo
g(c*(d + e*x**2)**p)/x - 2*g**2*p*x + g**2*x*log(c*(d + e*x**2)**p) - 2*e*
f**2*p*log(x - sqrt(-d/e))/(3*d*sqrt(-d/e)) - 2*e*f**2*p/(3*d*x) + e*f**2*
log(c*(d + e*x**2)**p)/(3*d*sqrt(-d/e)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)^2\log(c(d+ex^2)^p)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.80

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^4}\,dx = -\left(2\,g^2p-g^2\log\left(c\right)\right)x + \frac{1}{3}\left(3\,g^2px-\frac{6\,fgpx^2+f^2p}{x^3}\right)\log\left(ex^2+d\right) - \frac{2\left(e^2f^2p-6\,defgp-3\,d^2g^2p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{3\sqrt{de}d} - \frac{2\,ef^2px^2+6\,dfgx^2\log\left(c\right)+df^2\log\left(c\right)}{3\,dx^3}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

output
```
-(2*g^2*p - g^2*log(c))*x + 1/3*(3*g^2*p*x - (6*f*g*p*x^2 + f^2*p)/x^3)*lo
g(e*x^2 + d) - 2/3*(e^2*f^2*p - 6*d*e*f*g*p - 3*d^2*g^2*p)*arctan(e*x/sqrt
(d*e))/(sqrt(d*e)*d) - 1/3*(2*e*f^2*p*x^2 + 6*d*f*g*x^2*log(c) + d*f^2*log
(c))/(d*x^3)
```

Mupad [B] (verification not implemented)

Time = 25.81 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^4}\,dx = \ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{8\,g^2\,x}{3}-\frac{\frac{f^2}{3}+2\,f\,g\,x^2+\frac{5\,g^2\,x^4}{3}}{x^3}\right) - 2\,g^2\,p\,x + \frac{2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,d^2\,g^2+6\,d\,e\,f\,g-e^2\,f^2\right)}{3\,d^{3/2}\,\sqrt{e}} - \frac{2\,e\,f^2\,p}{3\,d\,x}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^4,x)
```

output
```
log(c*(d + e*x^2)^p)*((8*g^2*x)/3 - (f^2/3 + (5*g^2*x^4)/3 + 2*f*g*x^2)/x^
3) - 2*g^2*p*x + (2*p*atan((e^(1/2)*x)/d^(1/2))*(3*d^2*g^2 - e^2*f^2 + 6*d
*e*f*g))/(3*d^(3/2)*e^(1/2)) - (2*e*f^2*p)/(3*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^4} \, dx$$
$$= \frac{6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) d^2g^2p\,x^3 + 12\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) defgp\,x^3 - 2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) e^2f^2p\,x^3 - \log(}{3}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x)
```

output
```
(6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*g**2*p*x**3 + 12*sqr
t(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*f*g*p*x**3 - 2*sqrt(e)*sqrt
(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**2*f**2*p*x**3 - log((d + e*x**2)**p*c
)*d**2*e*f**2 - 6*log((d + e*x**2)**p*c)*d**2*e*f*g*x**2 + 3*log((d + e*x*
*2)**p*c)*d**2*e*g**2*x**4 - 6*d**2*e*g**2*p*x**4 - 2*d*e**2*f**2*p*x**2)/
(3*d**2*e*x**3)
```

3.336 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6}\,dx = -\frac{2ef^2p}{15dx^3}+\frac{2e^2f^2p}{5d^2x}-\frac{4efgp}{3dx}+\frac{2e^{5/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{4e^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{x}$$

output
```
-2/15*e*f^2*p/d/x^3+2/5*e^2*f^2*p/d^2/x-4/3*e*f*g*p/d/x+2/5*e^(5/2)*f^2*p*
arctan(e^(1/2)*x/d^(1/2))/d^(5/2)-4/3*e^(3/2)*f*g*p*arctan(e^(1/2)*x/d^(1/
2))/d^(3/2)+2*e^(1/2)*g^2*p*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)-1/5*f^2*ln(c
*(e*x^2+d)^p)/x^5-2/3*f*g*ln(c*(e*x^2+d)^p)/x^3-g^2*ln(c*(e*x^2+d)^p)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^6} \, dx = \frac{2\sqrt{e}g^2p \arctan \left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{2ef^2p \,\text{Hypergeometric2F1} \left(-\frac{3}{2},1,-\frac{1}{2},-\frac{ex^2}{d}\right)}{15dx^3} - \frac{4efgp \,\text{Hypergeometric2F1} \left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} - \frac{f^2 \log (c(d+ex^2)^p)}{5x^5} - \frac{2fg \log (c(d+ex^2)^p)}{3x^3} - \frac{g^2 \log (c(d+ex^2)^p)}{x}$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
```

output
```
(2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f^2*p*Hyperge
ometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (4*e*f*g*p*Hypergeom
etric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[c*(d + e*x^2)^p])
/(5*x^5) - (2*f*g*Log[c*(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p
])/x
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^6} \, dx$$

$$\downarrow \text{2926}$$

$$\int \left(\frac{f^2 \log \left(c \left(d+e x^2\right)^p\right)}{x^6}+\frac{2 f g \log \left(c \left(d+e x^2\right)^p\right)}{x^4}+\frac{g^2 \log \left(c \left(d+e x^2\right)^p\right)}{x^2}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{2 e^{5/2} f^2 p \arctan \left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{5 d^{5/2}}-\frac{4 e^{3/2} f g p \arctan \left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{3 d^{3/2}}+\frac{2 \sqrt{e} g^2 p \arctan \left(\frac{\sqrt{e} x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2 \log \left(c \left(d+e x^2\right)^p\right)}{5 x^5}-\frac{2 f g \log \left(c \left(d+e x^2\right)^p\right)}{3 x^3}-\frac{g^2 \log \left(c \left(d+e x^2\right)^p\right)}{x}+\frac{2 e^2 f^2 p}{5 d^2 x}-\frac{2 e f^2 p}{15 d x^3}-\frac{4 e f g p}{3 d x}$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
```

output
```
(-2*e*f^2*p)/(15*d*x^3) + (2*e^2*f^2*p)/(5*d^2*x) - (4*e*f*g*p)/(3*d*x) +
(2*e^(5/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (4*e^(3/2)*f*g
*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) + (2*Sqrt[e]*g^2*p*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/Sqrt[d] - (f^2*Log[c*(d + e*x^2)^p])/(5*x^5) - (2*f*g*Lo
g[c*(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.67

method	result
parts	$-\frac{g^{2} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{x}-\frac{2 f g \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{3 x^{3}}-\frac{f^{2} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{5 x^{5}}-\frac{2 e p \left(\frac{\left(-15 d^{2} g^{2}+10 f g e d-3 f^{2} e^{2}\right) \arctan\left(\frac{x e}{\sqrt{d e}}\right)}{d^{2} \sqrt{d e}}+\frac{f^{2}}{d\,x^{3}}+\frac{f(10}{}\right.}{15}$
risch	$-\frac{\left(15 g^{2} x^{4}+10 f g\,x^{2}+3 f^{2}\right) \ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{15 x^{5}}+\frac{3 i \pi\,f^{2} \mathrm{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right)^{3} d^{2}-10 i \pi\,d^{2} f g\,x^{2}\,\mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right) \mathrm{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right)^{2}-}{}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-g^2*ln(c*(e*x^2+d)^p)/x-2/3*f*g*ln(c*(e*x^2+d)^p)/x^3-1/5*f^2*ln(c*(e*x^2
+d)^p)/x^5-2/15*e*p*(1/d^2*(-15*d^2*g^2+10*d*e*f*g-3*e^2*f^2)/(d*e)^(1/2)*
arctan(x*e/(d*e)^(1/2))+1/d*f^2/x^3+f*(10*d*g-3*e*f)/d^2/x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.76

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6}\,dx$$
$$= \left[\frac{(3\,e^2f^2-10\,defg+15\,d^2g^2)px^5\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2+2\,dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right)-2\,def^2px^2+2\,(3\,e^2f^2-10\,defg)px^4-(}{15\,d^2x^5}\right.$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")
```

output
```
[1/15*((3*e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2)*p*x^5*sqrt(-e/d)*log((e*x^2 +
 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 2*d*e*f^2*p*x^2 + 2*(3*e^2*f^2 - 10*
d*e*f*g)*p*x^4 - (15*d^2*g^2*p*x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f^2*p)*log(e
*x^2 + d) - (15*d^2*g^2*x^4 + 10*d^2*f*g*x^2 + 3*d^2*f^2)*log(c))/(d^2*x^5
), 1/15*(2*(3*e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2)*p*x^5*sqrt(e/d)*arctan(x*
sqrt(e/d)) - 2*d*e*f^2*p*x^2 + 2*(3*e^2*f^2 - 10*d*e*f*g)*p*x^4 - (15*d^2*
g^2*p*x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f^2*p)*log(e*x^2 + d) - (15*d^2*g^2*x
^4 + 10*d^2*f*g*x^2 + 3*d^2*f^2)*log(c))/(d^2*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(199) = 398$.

Time = 159.42 (sec) , antiderivative size = 1603, normalized size of antiderivative = 8.02

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**6,x)
```

output
```
Piecewise(((-f**2/(5*x**5) - 2*f*g/(3*x**3) - g**2/x)*log(0**p*c), Eq(d, 0
) & Eq(e, 0)), ((-f**2/(5*x**5) - 2*f*g/(3*x**3) - g**2/x)*log(c*d**p), Eq
(e, 0)), (-2*f**2*p/(25*x**5) - f**2*log(c*(e*x**2)**p)/(5*x**5) - 4*f*g*p
/(9*x**3) - 2*f*g*log(c*(e*x**2)**p)/(3*x**3) - 2*g**2*p/x - g**2*log(c*(e
*x**2)**p)/x, Eq(d, 0)), ((-f**2/(5*x**5) - 2*f*g/(3*x**3) - g**2/x)*log(0
**p*c), Eq(d, -e*x**2)), (-3*d**3*f**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(
15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 10*d**3*f*g*x**2*sq
rt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*
sqrt(-d/e)) + 30*d**3*g**2*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-
d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 15*d**3*g**2*x**5*log(c*(d + e*x**2)**
p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 15*d**3*g**2*x*
*4*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*
x**7*sqrt(-d/e)) - 2*d**2*f**2*p*x**2*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/
e + 15*d**2*x**7*sqrt(-d/e)) - 3*d**2*f**2*x**2*sqrt(-d/e)*log(c*(d + e*x*
*2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 20*d**2*f*
g*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqr
t(-d/e)) - 20*d**2*f*g*p*x**4*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d
**2*x**7*sqrt(-d/e)) + 10*d**2*f*g*x**5*log(c*(d + e*x**2)**p)/(15*d**3*x*
*5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 10*d**2*f*g*x**4*sqrt(-d/e)*l
og(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^6} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^6}\,dx = \frac{2\,(3\,e^3f^2p - 10\,de^2fgp + 15\,d^2eg^2p)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}d^2} - \frac{(15\,g^2px^4 + 10\,fgpx^2 + 3\,f^2p)\log(ex^2+d)}{15\,x^5} + \frac{6\,e^2f^2px^4 - 20\,defgpx^4 - 15\,d^2g^2x^4\log(c) - 2\,def^2px^2 - 10\,d^2fgx^2\log(c) - 3\,d^2f^2\log(c)}{15\,d^2x^5}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

output
```
2/15*(3*e^3*f^2*p - 10*d*e^2*f*g*p + 15*d^2*e*g^2*p)*arctan(e*x/sqrt(d*e))
/(sqrt(d*e)*d^2) - 1/15*(15*g^2*p*x^4 + 10*f*g*p*x^2 + 3*f^2*p)*log(e*x^2
+ d)/x^5 + 1/15*(6*e^2*f^2*p*x^4 - 20*d*e*f*g*p*x^4 - 15*d^2*g^2*x^4*log(c
) - 2*d*e*f^2*p*x^2 - 10*d^2*f*g*x^2*log(c) - 3*d^2*f^2*log(c))/(d^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 25.83 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^6}\,dx = \frac{2\,\sqrt{e}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(15\,d^2\,g^2 - 10\,d\,e\,f\,g + 3\,e^2\,f^2)}{15\,d^{5/2}} - \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(\frac{f^2}{5} + \frac{2\,f\,g\,x^2}{3} + g^2\,x^4\right)}{x^5} - \frac{\frac{2\,e\,f^2\,p}{d} + \frac{2\,e\,f\,p\,x^2\,(10\,d\,g - 3\,e\,f)}{d^2}}{15\,x^3}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^6,x)
```

output
```
(2*e^(1/2)*p*atan((e^(1/2)*x)/d^(1/2))*(15*d^2*g^2 + 3*e^2*f^2 - 10*d*e*f*
g))/(15*d^(5/2)) - (log(c*(d + e*x^2)^p)*(f^2/5 + g^2*x^4 + (2*f*g*x^2)/3)
)/x^5 - ((2*e*f^2*p)/d + (2*e*f*p*x^2*(10*d*g - 3*e*f))/d^2)/(15*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx^2)^2 \log (c(d+ex^2)^p)}{x^6} \, dx$$
$$= \frac{30\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) d^2 g^2 p\, x^5 - 20\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) d e f g p\, x^5 + 6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) e^2 f^2 p\, x^5 - 3\,l}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x)
```

output
```
(30*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*g**2*p*x**5 - 20*sq
rt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*f*g*p*x**5 + 6*sqrt(e)*sqr
t(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**2*f**2*p*x**5 - 3*log((d + e*x**2)**
p*c)*d**3*f**2 - 10*log((d + e*x**2)**p*c)*d**3*f*g*x**2 - 15*log((d + e*x
**2)**p*c)*d**3*g**2*x**4 - 2*d**2*e*f**2*p*x**2 - 20*d**2*e*f*g*p*x**4 +
6*d*e**2*f**2*p*x**4)/(15*d**3*x**5)
```

3.337 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^8} dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^8} dx = -\frac{2ef^2p}{35dx^5} + \frac{2e^2f^2p}{21d^2x^3} - \frac{4efgp}{15dx^3} - \frac{2e^3f^2p}{7d^3x} + \frac{4e^2fgp}{5d^2x} - \frac{2eg^2p}{3dx} - \frac{2e^{7/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}} + \frac{4e^{5/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}} - \frac{2e^{3/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7} - \frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}$$

output
```
-2/35*e*f^2*p/d/x^5+2/21*e^2*f^2*p/d^2/x^3-4/15*e*f*g*p/d/x^3-2/7*e^3*f^2*
p/d^3/x+4/5*e^2*f*g*p/d^2/x-2/3*e*g^2*p/d/x-2/7*e^(7/2)*f^2*p*arctan(e^(1/
2)*x/d^(1/2))/d^(7/2)+4/5*e^(5/2)*f*g*p*arctan(e^(1/2)*x/d^(1/2))/d^(5/2)-
2/3*e^(3/2)*g^2*p*arctan(e^(1/2)*x/d^(1/2))/d^(3/2)-1/7*f^2*ln(c*(e*x^2+d)
^p)/x^7-2/5*f*g*ln(c*(e*x^2+d)^p)/x^5-1/3*g^2*ln(c*(e*x^2+d)^p)/x^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^8}\,dx = -\frac{2ef^2p\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\frac{ex^2}{d}\right)}{35dx^5} - \frac{4efgp\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{ex^2}{d}\right)}{15dx^3} - \frac{2eg^2p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7} - \frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}$$

input
```
Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```

output
```
(-2*e*f^2*p*Hypergeometric2F1[-5/2, 1, -3/2, -((e*x^2)/d)])/(35*d*x^5) - (
4*e*f*g*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*
e*g^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[
c*(d + e*x^2)^p])/(7*x^7) - (2*f*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Lo
g[c*(d + e*x^2)^p])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^8}\,dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(\frac{f^2 \log \left(c(d+ex^2)^p\right)}{x^8}+\frac{2fg \log \left(c(d+ex^2)^p\right)}{x^6}+\frac{g^2 \log \left(c(d+ex^2)^p\right)}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2e^{7/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}}+\frac{4e^{5/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{2e^3f^2p}{7d^3x}+\frac{2e^2f^2p}{21d^2x^3}+\frac{4e^2fgp}{5d^2x}-\frac{2ef^2p}{35dx^5}-\frac{4efgp}{15dx^3}-\frac{2eg^2p}{3dx}$$

input
```
Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```

output
```
(-2*e*f^2*p)/(35*d*x^5) + (2*e^2*f^2*p)/(21*d^2*x^3) - (4*e*f*g*p)/(15*d*x
^3) - (2*e^3*f^2*p)/(7*d^3*x) + (4*e^2*f*g*p)/(5*d^2*x) - (2*e*g^2*p)/(3*d
*x) - (2*e^(7/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*d^(7/2)) + (4*e^(5/
2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g^2*p*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f^2*Log[c*(d + e*x^2)^p])/(7*x^7) -
 (2*f*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.66

method	result
parts	$-\frac{g^2 \ln\left(c\left(e\,x^2+d\right)^p\right)}{3x^3}-\frac{2fg\ln\left(c\left(e\,x^2+d\right)^p\right)}{5x^5}-\frac{f^2\ln\left(c\left(e\,x^2+d\right)^p\right)}{7x^7}-\frac{2ep\left(\frac{e\left(35d^2g^2-42fged+15f^2e^2\right)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{d^3\sqrt{de}}-\frac{-35d^2g^2+}{}\right.}{105}$
risch	$-\frac{\left(35g^2x^4+42fg\,x^2+15f^2\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{105x^7}+\frac{-42i\pi\,d^3fg\,x^2\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2\mathrm{csgn}(ic)-15i\pi\,d^3f^2\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2\mathrm{csgn}(ic}{}$

input
```
int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/3*g^2*ln(c*(e*x^2+d)^p)/x^3-2/5*f*g*ln(c*(e*x^2+d)^p)/x^5-1/7*f^2*ln(c*
(e*x^2+d)^p)/x^7-2/105*e*p*(e*(35*d^2*g^2-42*d*e*f*g+15*e^2*f^2)/d^3/(d*e)
^(1/2)*arctan(x*e/(d*e)^(1/2))-(-35*d^2*g^2+42*d*e*f*g-15*e^2*f^2)/d^3/x+3
/d*f^2/x^5+f*(14*d*g-5*e*f)/d^2/x^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.70

$$\int \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x^8}\,dx$$

$$=\left[\frac{(15\,e^3f^2-42\,de^2fg+35\,d^2eg^2)px^7\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2-2\,dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right)-6\,d^2ef^2px^2-2\,(15\,e^3f^2-42\,de^2fg+}{}\right.$$

$$-\frac{2\,(15\,e^3f^2-42\,de^2fg+35\,d^2eg^2)px^7\sqrt{\frac{e}{d}}\arctan\left(x\sqrt{\frac{e}{d}}\right)+6\,d^2ef^2px^2+2\,(15\,e^3f^2-42\,de^2fg+35\,d^2}{}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="fricas")
```

output
```
[1/105*((15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^7*sqrt(-e/d)*log((e
*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 6*d^2*e*f^2*p*x^2 - 2*(15*e^3*
f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^6 + 2*(5*d*e^2*f^2 - 14*d^2*e*f*g)*
p*x^4 - (35*d^3*g^2*p*x^4 + 42*d^3*f*g*p*x^2 + 15*d^3*f^2*p)*log(e*x^2 + d
) - (35*d^3*g^2*x^4 + 42*d^3*f*g*x^2 + 15*d^3*f^2)*log(c))/(d^3*x^7), -1/1
05*(2*(15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^7*sqrt(e/d)*arctan(x*
sqrt(e/d)) + 6*d^2*e*f^2*p*x^2 + 2*(15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g
^2)*p*x^6 - 2*(5*d*e^2*f^2 - 14*d^2*e*f*g)*p*x^4 + (35*d^3*g^2*p*x^4 + 42*
d^3*f*g*p*x^2 + 15*d^3*f^2*p)*log(e*x^2 + d) + (35*d^3*g^2*x^4 + 42*d^3*f*
g*x^2 + 15*d^3*f^2)*log(c))/(d^3*x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^8} dx = \text{Timed out}$$

input
```
integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**8,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^8} dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^8}\,dx$$
$$= -\frac{2\left(15e^4f^2p-42de^3fgp+35d^2e^2g^2p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{105\sqrt{de}d^3} - \frac{(35g^2px^4+42fgpx^2+15f^2p)\log(ex^2+d)}{105x^7} - \frac{30e^3f^2px^6-84de^2fgpx^6+70d^2eg^2px^6-10de^2f^2px^4+28d^2efgpx^4+35d^3g^2x^4\log(c)+6d^2ef^2px}{105d^3x^7}$$

input
```
integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="giac")
```

output
```
-2/105*(15*e^4*f^2*p - 42*d*e^3*f*g*p + 35*d^2*e^2*g^2*p)*arctan(e*x/sqrt(
d*e))/(sqrt(d*e)*d^3) - 1/105*(35*g^2*p*x^4 + 42*f*g*p*x^2 + 15*f^2*p)*log
(e*x^2 + d)/x^7 - 1/105*(30*e^3*f^2*p*x^6 - 84*d*e^2*f*g*p*x^6 + 70*d^2*e*
g^2*p*x^6 - 10*d*e^2*f^2*p*x^4 + 28*d^2*e*f*g*p*x^4 + 35*d^3*g^2*x^4*log(c
) + 6*d^2*e*f^2*p*x^2 + 42*d^3*f*g*x^2*log(c) + 15*d^3*f^2*log(c))/(d^3*x^
7)
```

Mupad [B] (verification not implemented)

Time = 25.71 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.59

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^8}\,dx$$
$$= -\frac{\frac{6\,e\,f^2\,p}{d}+\frac{2\,e\,p\,x^4\,(35\,d^2\,g^2-42\,d\,e\,f\,g+15\,e^2\,f^2)}{d^3}+\frac{2\,e\,f\,p\,x^2\,(14\,d\,g-5\,e\,f)}{d^2}}{105\,x^5} - \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(\frac{f^2}{7}+\frac{2\,f\,g\,x^2}{5}+\frac{g^2\,x^4}{3}\right)}{x^7} - \frac{2\,e^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(35\,d^2\,g^2-42\,d\,e\,f\,g+15\,e^2\,f^2)}{105\,d^{7/2}}$$

input
```
int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^8,x)
```

output
```
- ((6*e*f^2*p)/d + (2*e*p*x^4*(35*d^2*g^2 + 15*e^2*f^2 - 42*d*e*f*g))/d^3
+ (2*e*f*p*x^2*(14*d*g - 5*e*f))/d^2)/(105*x^5) - (log(c*(d + e*x^2)^p)*(f
^2/7 + (g^2*x^4)/3 + (2*f*g*x^2)/5))/x^7 - (2*e^(3/2)*p*atan((e^(1/2)*x)/d
^(1/2))*(35*d^2*g^2 + 15*e^2*f^2 - 42*d*e*f*g))/(105*d^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.95

$$\int \frac{(f + gx^2)^2 \log\left(c(d + ex^2)^p\right)}{x^8}\, dx$$
$$= \frac{-70\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) d^2 e\, g^2 p\, x^7 + 84\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) d\, e^2 f g p\, x^7 - 30\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) e^3 f^2 p\, x^7}{}$$

input
```
int((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x)
```

output
```
( - 70*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*e*g**2*p*x**7 +
84*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e**2*f*g*p*x**7 - 30*sq
rt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e**3*f**2*p*x**7 - 15*log((d +
 e*x**2)**p*c)*d**4*f**2 - 42*log((d + e*x**2)**p*c)*d**4*f*g*x**2 - 35*lo
g((d + e*x**2)**p*c)*d**4*g**2*x**4 - 6*d**3*e*f**2*p*x**2 - 28*d**3*e*f*g
*p*x**4 - 70*d**3*e*g**2*p*x**6 + 10*d**2*e**2*f**2*p*x**4 + 84*d**2*e**2*
f*g*p*x**6 - 30*d*e**3*f**2*p*x**6)/(105*d**4*x**7)
```

3.338 $\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 188

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{f+gx^2} dx = \frac{fpx^2}{2g^2} + \frac{dpx^2}{4eg} - \frac{px^4}{8g} - \frac{d^2p\log\left(d+ex^2\right)}{4e^2g} + \frac{x^4\log\left(c(d+ex^2)^p\right)}{4g} - \frac{f(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg^2} + \frac{f^2\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} + \frac{f^2p\,\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^3}$$

output
```
1/2*f*p*x^2/g^2+1/4*d*p*x^2/e/g-1/8*p*x^4/g-1/4*d^2*p*ln(e*x^2+d)/e^2/g+1/
4*x^4*ln(c*(e*x^2+d)^p)/g-1/2*f*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e/g^2+1/2*f^2*
ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g^3+1/2*f^2*p*polylog(2,-g*(e
*x^2+d)/(-d*g+e*f))/g^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{egpx^2(4ef+2dg-egx^2)-2d^2g^2p\log\left(d+ex^2\right)+e\log\left(c(d+ex^2)^p\right)\left(2g(-2df-2efx^2+egx^4)+4ef\right.}{8e^2g^3}$$

input
```
Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output
```
(e*g*p*x^2*(4*e*f + 2*d*g - e*g*x^2) - 2*d^2*g^2*p*Log[d + e*x^2] + e*Log[
c*(d + e*x^2)^p]*(2*g*(-2*d*f - 2*e*f*x^2 + e*g*x^4) + 4*e*f^2*Log[(e*(f +
 g*x^2))/(e*f - d*g)]) + 4*e^2*f^2*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) +
d*g)])/(8*e^2*g^3)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

$$\Big\downarrow \text{ 2925}$$

$$\frac{1}{2}\int \frac{x^4 \log \left(c(ex^2+d)^p\right)}{gx^2+f}dx^2$$

$$\Big\downarrow \text{ 2863}$$

$$\frac{1}{2}\int \left(\frac{\log\left(c(ex^2+d)^p\right)f^2}{g^2\left(gx^2+f\right)}-\frac{\log\left(c(ex^2+d)^p\right)f}{g^2}+\frac{x^2\log\left(c(ex^2+d)^p\right)}{g}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{g^3}-\frac{f\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{eg^2}+\frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{2g}-\frac{d^2p\log\left(d+e\right.}{2e^2g}\right.$$

input
```
Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output
```
((f*p*x^2)/g^2 + (d*p*x^2)/(2*e*g) - (p*x^4)/(4*g) - (d^2*p*Log[d + e*x^2]
)/(2*e^2*g) + (x^4*Log[c*(d + e*x^2)^p])/(2*g) - (f*(d + e*x^2)*Log[c*(d +
 e*x^2)^p])/(e*g^2) + (f^2*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f -
 d*g)])/g^3 + (f^2*p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/g^3)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.70 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.18

method	result
parts	$\frac{x^4 \ln(c(e\,x^2+d)^p)}{4g} - \frac{\ln(c(e\,x^2+d)^p) f\,x^2}{2g^2} + \frac{\ln(c(e\,x^2+d)^p) f^2 \ln(g\,x^2+f)}{2g^3} - ep \left(\frac{f^2 \left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)} \left(\ln(x-\_\alpha) \ln(\right.\right.}{} \right.$
risch	$\frac{\ln((e\,x^2+d)^p) x^4}{4g} - \frac{\ln((e\,x^2+d)^p) f\,x^2}{2g^2} + \frac{\ln((e\,x^2+d)^p) f^2 \ln(g\,x^2+f)}{2g^3} - \frac{p\,f^2 \left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)} \left(\ln(x-\_\alpha) \ln(g\,x^2+f \right.\right.}{}$

input
```
int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*ln(c*(e*x^2+d)^p)/g-1/2*ln(c*(e*x^2+d)^p)/g^2*f*x^2+1/2*ln(c*(e*x^
2+d)^p)*f^2/g^3*ln(g*x^2+f)-e*p*(1/2*f^2/g^3/e*sum(ln(x-_alpha)*ln(g*x^2+f
)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=
1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/2/g^2*(-1/2/e^2*
(1/2*e*g*x^4-d*g*x^2-2*f*x^2*e)-1/2*d*(d*g+2*e*f)/e^3*ln(e*x^2+d)))
```

Fricas [F]

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^5 \log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

input
```
integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

output
```
Integral(x**5*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{x^5 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{x^5 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^5\,\ln\left(c\,(e\,x^2+d)^p\right)}{g\,x^2+f}\,dx$$

input
```
int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

output
```
int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{8\left(\int \frac{\log\left((e\,x^2+d)^p c\right)x}{eg\,x^4+dg\,x^2+ef\,x^2+df}dx\right)d\,e^2f^2gp-8\left(\int \frac{\log\left((e\,x^2+d)^p c\right)x}{eg\,x^4+dg\,x^2+ef\,x^2+df}dx\right)e^3f^3p+2\log\left((e\,x^2+d)^p c\right)^2e^2f^2-2\log}{}$$

input
```
int(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

output
```
(8*int((log((d + e*x**2)**p*c)*x)/(d*f + d*g*x**2 + e*f*x**2 + e*g*x**4),x
)*d*e**2*f**2*g*p - 8*int((log((d + e*x**2)**p*c)*x)/(d*f + d*g*x**2 + e*f
*x**2 + e*g*x**4),x)*e**3*f**3*p + 2*log((d + e*x**2)**p*c)**2*e**2*f**2 -
 2*log((d + e*x**2)**p*c)*d**2*g**2*p - 4*log((d + e*x**2)**p*c)*d*e*f*g*p
 - 4*log((d + e*x**2)**p*c)*e**2*f*g*p*x**2 + 2*log((d + e*x**2)**p*c)*e**
2*g**2*p*x**4 + 2*d*e*g**2*p**2*x**2 + 4*e**2*f*g*p**2*x**2 - e**2*g**2*p*
*2*x**4)/(8*e**2*g**3*p)
```

3.339 $\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{f+gx^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = -\frac{px^2}{2g} + \frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg} - \frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} - \frac{fp\,\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}$$

output
```
-1/2*p*x^2/g+1/2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e/g-1/2*f*ln(c*(e*x^2+d)^p)*l
n(e*(g*x^2+f)/(-d*g+e*f))/g^2-1/2*f*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g
^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = -\frac{egpx^2 - \log \left(c(d+ex^2)^p\right)\left(dg+egx^2-ef\log \left(\frac{e(f+gx^2)}{ef-dg}\right)\right)+efp\,\text{PolyLog}\left(2,\frac{g(d+ex^2)}{-ef+dg}\right)}{2eg^2}$$

input
```
Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output
```
-1/2*(e*g*p*x^2 - Log[c*(d + e*x^2)^p]*(d*g + e*g*x^2 - e*f*Log[(e*(f + g*
x^2))/(e*f - d*g)]) + e*f*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(e
*g^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

$$\Big\downarrow \ 2925$$

$$\frac{1}{2}\int \frac{x^2 \log \left(c(ex^2+d)^p\right)}{gx^2+f}dx^2$$

$$\Big\downarrow \ 2863$$

$$\frac{1}{2}\int \left(\frac{\log \left(c(ex^2+d)^p\right)}{g}-\frac{f\log \left(c(ex^2+d)^p\right)}{g\,(gx^2+f)}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{g^2}+\frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{eg}-\frac{fp\,\text{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{g^2}-\frac{px^2}{g}\right)$$

input
```
Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output
```
(-((p*x^2)/g) + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/(e*g) - (f*Log[c*(d + e
*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/g^2 - (f*p*PolyLog[2, -((g*(d +
 e*x^2))/(e*f - d*g))])/g^2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.88 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.20

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)x^2}{2g} - \frac{\ln(c(e\,x^2+d)^p) f \ln(g\,x^2+f)}{2g^2} - ep\left(\frac{x^2}{2ge} - \frac{d\ln(e\,x^2+d)}{2g\,e^2} - \frac{f\left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(g\right.\right.}{}\right.$
risch	$\frac{\ln((e\,x^2+d)^p)x^2}{2g} - \frac{\ln((e\,x^2+d)^p) f \ln(g\,x^2+f)}{2g^2} - \frac{p\,x^2}{2g} + \frac{pd\ln(e\,x^2+d)}{2eg} + \frac{pf\left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(g\,x^2+\right.\right.}{}$

input
```
int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*(e*x^2+d)^p)*x^2/g-1/2*ln(c*(e*x^2+d)^p)*f/g^2*ln(g*x^2+f)-e*p*(1
/2/g/e*x^2-1/2/g*d/e^2*ln(e*x^2+d)-1/2*f/g^2/e*sum(ln(x-_alpha)*ln(g*x^2+f
)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=
1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d)))
```

Fricas [F]

$$\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^3 \log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

input
```
integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

output
```
Integral(x**3*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^3\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

input
```
int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

output
```
int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{-4\left(\int \frac{\log\left(\left(e\,x^2+d\right)^p c\right)x}{eg\,x^4+dg\,x^2+ef\,x^2+df}dx\right)defgp+4\left(\int \frac{\log\left(\left(e\,x^2+d\right)^p c\right)x}{eg\,x^4+dg\,x^2+ef\,x^2+df}dx\right)e^2f^2p-\log\left(\left(e\,x^2+d\right)^p c\right)^2ef+2\log\left(\left(e\right.\right.}{4e\,g^2p}$$

input
```
int(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

output
```
( - 4*int((log((d + e*x**2)**p*c)*x)/(d*f + d*g*x**2 + e*f*x**2 + e*g*x**4
),x)*d*e*f*g*p + 4*int((log((d + e*x**2)**p*c)*x)/(d*f + d*g*x**2 + e*f*x*
*2 + e*g*x**4),x)*e**2*f**2*p - log((d + e*x**2)**p*c)**2*e*f + 2*log((d +
 e*x**2)**p*c)*d*g*p + 2*log((d + e*x**2)**p*c)*e*g*p*x**2 - 2*e*g*p**2*x*
*2)/(4*e*g**2*p)
```

3.340 $\int \frac{x \log\left(c(d+ex^2)^p\right)}{f+gx^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{x \log\left(c(d+ex^2)^p\right)}{f+gx^2} dx = \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} + \frac{p\,\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}$$

output
```
1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g+1/2*p*polylog(2,-g*(e*x
^2+d)/(-d*g+e*f))/g
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{x \log\left(c(d+ex^2)^p\right)}{f+gx^2} dx = \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right) + p\,\text{PolyLog}\left(2,\frac{g(d+ex^2)}{-ef+dg}\right)}{2g}$$

input
```
Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output
```
(Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(
d + e*x^2))/(-(e*f) + d*g)])/(2*g)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2925, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log \left(c \left(d+e x^2\right)^p\right)}{f+g x^2} \, dx$$

$$\Big\downarrow \ 2925$$

$$\frac{1}{2} \int \frac{\log \left(c \left(e x^2+d\right)^p\right)}{g x^2+f}dx^2$$

$$\Big\downarrow \ 2841$$

$$\frac{1}{2} \left(\frac{\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{g}-\frac{e p \int \frac{\log \left(\frac{e \left(g x^2+f\right)}{e f-d g}\right)}{e x^2+d}dx^2}{g}\right)$$

$$\Big\downarrow \ 2840$$

$$\frac{1}{2} \left(\frac{\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{g}-\frac{p \int \frac{\log \left(\frac{g \left(e x^2+d\right)}{e f-d g}+1\right)}{x^2}d\left(e x^2+d\right)}{g}\right)$$

$$\Big\downarrow \ 2838$$

$$\frac{1}{2} \left(\frac{\log \left(c \left(d+e x^2\right)^p\right) \log \left(\frac{e \left(f+g x^2\right)}{e f-d g}\right)}{g}+\frac{p \operatorname{PolyLog}\left(2,-\frac{g \left(e x^2+d\right)}{e f-d g}\right)}{g}\right)$$

input
```
Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output
```
((Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/g + (p*PolyLog[2,
 -((g*(d + e*x^2))/(e*f - d*g))])/g)/2
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.60 (sec) , antiderivative size = 301, normalized size of antiderivative = 4.30

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)\ln(g\,x^2+f)}{2g} - \frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(g\,x^2+f)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2 eg+2\_\alpha\_Z ge-\right.}{\mathrm{RootOf}\left(\_Z^2 eg+2\_\alpha\_Z\right.}\right.\right.\right.\right.}{}$
risch	$\frac{\ln((e\,x^2+d)^p)\ln(g\,x^2+f)}{2g} - \frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(g\,x^2+f)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2 eg+2\_\alpha\_Z ge-d\right.}{\mathrm{RootOf}\left(\_Z^2 eg+2\_\alpha\_Z\right.}\right.\right.\right.\right.}{}$

input
```
int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*(e*x^2+d)^p)/g*ln(g*x^2+f)-1/2/g*p*sum(ln(x-_alpha)*ln(g*x^2+f)-l
n(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha
)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*
_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d
*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-
x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))
```

Fricas [F]

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(x*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

input
```
integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

output
```
Integral(x*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(65) = 130$.

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.97

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = \frac{ep\left(\frac{\log(ex^2+d)\log(gx^2+f)}{e} - \frac{\log(gx^2+f)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\text{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)}{e}\right)}{2\,g} - \frac{p\log\left(ex^2+d\right)\log\left(gx^2+f\right)}{2\,g} + \frac{\log\left(gx^2+f\right)\log\left(\left(ex^2+d\right)^p c\right)}{2\,g}$$

input
```
integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*e*p*(log(e*x^2 + d)*log(g*x^2 + f)/e - (log(g*x^2 + f)*log(-(e*g*x^2 +
 e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d*g)))/e)/g - 1/2*p*
log(e*x^2 + d)*log(g*x^2 + f)/g + 1/2*log(g*x^2 + f)*log((e*x^2 + d)^p*c)/
g
```

Giac [F]

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = \int \frac{x \log \left(\left(ex^2+d\right)^p c\right)}{gx^2+f} \, dx$$

input
```
integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(x*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{x\,\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f} \,dx$$

input
```
int((x*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

output
```
int((x*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{x\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$
$$=\frac{4\left(\int \frac{\log \left(\left(e\,x^{2}+d\right)^{p} c\right) x}{eg\,x^{4}+dg\,x^{2}+ef\,x^{2}+df}d x \right) dgp -4\left(\int \frac{\log \left(\left(e\,x^{2}+d\right)^{p} c\right) x}{eg\,x^{4}+dg\,x^{2}+ef\,x^{2}+df}d x \right) efp +\log \left(\left(e\,x^{2}+d\right)^{p} c\right)^{2}}{4gp}$$

input
```
int(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

output
```
(4*int((log((d + e*x**2)**p*c)*x)/(d*f + d*g*x**2 + e*f*x**2 + e*g*x**4),x
)*d*g*p - 4*int((log((d + e*x**2)**p*c)*x)/(d*f + d*g*x**2 + e*f*x**2 + e*
g*x**4),x)*e*f*p + log((d + e*x**2)**p*c)**2)/(4*g*p)
```

3.341 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx = \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f} - \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f} - \frac{p\,\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2f} + \frac{p\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{2f}$$

output
```
1/2*ln(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f-1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/
(-d*g+e*f))/f-1/2*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/f+1/2*p*polylog(2,1
+e*x^2/d)/f
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)}\,dx = \frac{\log \left(c\left(d+ex^2\right)^p\right)\left(\log \left(-\frac{ex^2}{d}\right)-\log \left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)\right)-p\,\text{PolyLog}\left(2,\frac{g\left(d+ex^2\right)}{-ef+dg}\right)+p\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{2f}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

output
```
(Log[c*(d + e*x^2)^p]*(Log[-((e*x^2)/d)] - Log[(e*(f + g*x^2))/(e*f - d*g)
]) - p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^
2)/d])/(2*f)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)}\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int \frac{\log \left(c\left(ex^2+d\right)^p\right)}{x^2\left(gx^2+f\right)}dx^2$$

$$\Big\downarrow \text{2863}$$

$$\frac{1}{2}\int \left(\frac{\log \left(c\left(ex^2+d\right)^p\right)}{fx^2}-\frac{g\log \left(c\left(ex^2+d\right)^p\right)}{f\left(gx^2+f\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{f}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f}-\frac{p\,\text{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{f}+\frac{p\,\text{PolyLo}}{}\right.$$

input
```
Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

output
```
((Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/f - (Log[c*(d + e*x^2)^p]*Log[(e
*(f + g*x^2))/(e*f - d*g)])/f - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g
))])/f + (p*PolyLog[2, 1 + (e*x^2)/d])/f)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.39 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.53

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)\ln(x)}{f}-\frac{\ln(c(e\,x^2+d)^p)\ln(g\,x^2+f)}{2f}-ep\left(\frac{\frac{\ln(x)\left(\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)+\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)\right)}{e}+\frac{\mathrm{dilog}\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)+\mathrm{dilog}}{e}}{f}\right.$
risch	$\frac{\ln((e\,x^2+d)^p)\ln(x)}{f}-\frac{\ln((e\,x^2+d)^p)\ln(g\,x^2+f)}{2f}-\frac{p\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{f}-\frac{p\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{f}-\frac{p\,\mathrm{dilog}\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{f}$

input
```
int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(e*x^2+d)^p)/f*ln(x)-1/2*ln(c*(e*x^2+d)^p)/f*ln(g*x^2+f)-e*p*(2/f*(1/
2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x+(-d*e)^(1/2))/(-d*e)
^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+dilog((e*x+(-d*e)^
(1/2))/(-d*e)^(1/2)))/e)-1/2/f/e*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)
*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z
^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e
*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,inde
x=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/R
ootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*
_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d
*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d)))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x\,(f+gx^2)}\,dx=\int \frac{\log\left((ex^2+d)^pc\right)}{(gx^2+f)x}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g*x^3 + f*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx = -\frac{1}{2}\,ep\left(\frac{2\,\log\left(\frac{ex^2}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)}{ef}-\frac{\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)}{ef}\right) - \frac{1}{2}\left(\frac{\log\left(gx^2+f\right)}{f}-\frac{\log\left(x^2\right)}{f}\right)\log\left(\left(ex^2+d\right)^pc\right)$$

input
```
integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="maxima")
```

output
```
-1/2*e*p*((2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))/(e*f) - (log(g*x^2
 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f -
 d*g)))/(e*f)) - 1/2*(log(g*x^2 + f)/f - log(x^2)/f)*log((e*x^2 + d)^p*c)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{\log \left((ex^2+d)^p c\right)}{(gx^2+f)x}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{\ln \left(c\,(e\,x^2+d)^p\right)}{x\,\left(g\,x^2+f\right)}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx=\int \frac{\log\left((e\,x^2+d)^p\,c\right)}{g\,x^3+fx}dx$$

input
```
int(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x)
```

output
```
int(log((d + e*x**2)**p*c)/(f*x + g*x**3),x)
```

3.342 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)}\,dx = \frac{ep\log(x)}{df}-\frac{ep\log\left(d+ex^2\right)}{2df}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^2}+\frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2}+\frac{gp\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2f^2}-\frac{gp\,\mathrm{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{2f^2}$$

output
```
e*p*ln(x)/d/f-1/2*e*p*ln(e*x^2+d)/d/f-1/2*ln(c*(e*x^2+d)^p)/f/x^2-1/2*g*ln
(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^2+1/2*g*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-
d*g+e*f))/f^2+1/2*g*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/f^2-1/2*g*p*polyl
og(2,1+e*x^2/d)/f^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.95

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x^3 \left(f+gx^2\right)}\,dx = \frac{ep\log(x)}{df}-\frac{ep\log\left(d+ex^2\right)}{2df}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2fx^2} - \frac{g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+p\,\text{PolyLog}\left(2,\frac{d+ex^2}{d}\right)\right)}{2f^2} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)+gp\,\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2f^2}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

output
```
(e*p*Log[x])/(d*f) - (e*p*Log[d + e*x^2])/(2*d*f) - Log[c*(d + e*x^2)^p]/(
2*f*x^2) - (g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d +
e*x^2)/d]))/(2*f^2) + (g*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d
*g)] + g*p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*f^2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x^3 \left(f+gx^2\right)}\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{1}{2}\int \frac{\log\left(c\left(ex^2+d\right)^p\right)}{x^4\left(gx^2+f\right)}dx^2$$

$$\Big\downarrow \text{2863}$$

$$\frac{1}{2}\int \left(\frac{\log\left(c(ex^2+d)^p\right)g^2}{f^2\left(gx^2+f\right)}-\frac{\log\left(c(ex^2+d)^p\right)g}{f^2x^2}+\frac{\log\left(c(ex^2+d)^p\right)}{fx^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{f^2}+\frac{g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{f^2}-\frac{\log\left(c(d+ex^2)^p\right)}{fx^2}+\frac{gp\,\text{PolyLog}\left(2\right.}{f^2}\right.$$

input
```
Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

output
```
((e*p*Log[x^2])/(d*f) - (e*p*Log[d + e*x^2])/(d*f) - Log[c*(d + e*x^2)^p]/
(f*x^2) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/f^2 + (g*Log[c*(d + e
*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/f^2 + (g*p*PolyLog[2, -((g*(d +
 e*x^2))/(e*f - d*g))])/f^2 - (g*p*PolyLog[2, 1 + (e*x^2)/d])/f^2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.08 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.68

method	result
parts	$-\frac{\ln(c(e\,x^2+d)^p)}{2f\,x^2}-\frac{\ln(c(e\,x^2+d)^p)\,g\ln(x)}{f^2}+\frac{\ln(c(e\,x^2+d)^p)\,g\ln(g\,x^2+f)}{2f^2}-ep\left(\frac{g\left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(\right.\right.}{}\right.$
risch	Expression too large to display

input
```
int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(e*x^2+d)^p)/f/x^2-ln(c*(e*x^2+d)^p)*g/f^2*ln(x)+1/2*ln(c*(e*x^2
+d)^p)*g/f^2*ln(g*x^2+f)-e*p*(1/2*g/f^2/e*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(
x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/
RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+
_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z
^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_al
pha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/2/f/d*ln(e*x^2+d)-1/f
/d*ln(x)-2*g/f^2*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x+
(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2)
)+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx=\int \frac{\log\left((ex^2+d)^p c\right)}{(gx^2+f)x^3}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g*x^5 + f*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3 \left(f+gx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3 \left(f+gx^2\right)}\,dx = \frac{1}{2}\,ep\left(\frac{\left(2\,\log\left(\frac{ex^2}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)\right)g}{ef^2} - \frac{\left(\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)\right)g}{ef^2}\right. + \frac{1}{2}\left(\frac{g\log\left(gx^2+f\right)}{f^2} - \frac{g\log\left(x^2\right)}{f^2} - \frac{1}{fx^2}\right)\log\left(\left(ex^2+d\right)^p c\right)$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="maxima")
```

output
```
1/2*e*p*((2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))*g/(e*f^2) - (log(g*
x^2 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*
f - d*g)))*g/(e*f^2) - log(e*x^2 + d)/(d*f) + 2*log(x)/(d*f)) + 1/2*(g*log
(g*x^2 + f)/f^2 - g*log(x^2)/f^2 - 1/(f*x^2))*log((e*x^2 + d)^p*c)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx=\int \frac{\log \left((ex^2+d)^pc\right)}{(gx^2+f)x^3}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx=\int \frac{\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{x^3\,\left(g\,x^2+f\right)}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx=\int \frac{\log \left(\left(e\,x^2+d\right)^p c\right)}{g\,x^5+f\,x^3}dx$$

input
```
int(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x)
```

output
```
int(log((d + e*x**2)**p*c)/(f*x**3 + g*x**5),x)
```

3.343 $\int \frac{x^4 \log \left(c \left(d+e x^2\right)^p\right)}{f+g x^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 667

$$\int \frac{x^4\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}+\frac{2f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}}-\frac{f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}}-\frac{f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}}-\frac{fx\log\left(c(d+ex^2)^p\right)}{g^2}+\frac{x^3\log\left(c(d+ex^2)^p\right)}{3g}+\frac{f^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}-\frac{if^{3/2}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}}+\frac{if^{3/2}p\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}+\frac{if^{3/2}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}$$

output
```
2*f*p*x/g^2+2/3*d*p*x/e/g-2/9*p*x^3/g-2*d^(1/2)*f*p*arctan(e^(1/2)*x/d^(1/
2))/e^(1/2)/g^2-2/3*d^(3/2)*p*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)/g+2*f^(3/2
)*p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/g^(5/2)-
f^(3/2)*p*arctan(g^(1/2)*x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1
/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(5/
2)-f^(3/2)*p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^
(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(
5/2)-f*x*ln(c*(e*x^2+d)^p)/g^2+1/3*x^3*ln(c*(e*x^2+d)^p)/g+f^(3/2)*arctan(
g^(1/2)*x/f^(1/2))*ln(c*(e*x^2+d)^p)/g^(5/2)-I*f^(3/2)*p*polylog(2,1-2*f^(
1/2)/(f^(1/2)-I*g^(1/2)*x))/g^(5/2)+1/2*I*f^(3/2)*p*polylog(2,1+2*f^(1/2)*
g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(
1/2)-I*g^(1/2)*x))/g^(5/2)+1/2*I*f^(3/2)*p*polylog(2,1-2*f^(1/2)*g^(1/2)*(
(-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^
(1/2)*x))/g^(5/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.04

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$
$$= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} - \frac{2d^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}$$
$$- \frac{fx\log\left(c(d+ex^2)^p\right)}{g^2} + \frac{x^3\log\left(c(d+ex^2)^p\right)}{3g} + \frac{f^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}$$
$$- \frac{if^{3/2}p\left(\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right) + \log\left(\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right) - \log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{\ldots}}\right.}{}$$

input
```
Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output
```
(2*f*p*x)/g^2 + (2*d*p*x)/(3*e*g) - (2*p*x^3)/(9*g) - (2*Sqrt[d]*f*p*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) - (2*d^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(3*e^(3/2)*g) - (f*x*Log[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e
*x^2)^p])/(3*g) + (f^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p
])/g^(5/2) - ((I/2)*f^(3/2)*p*(Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqr
t[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[(Sq
rt[g]*(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*L
og[1 - (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)
*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - Log
[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*
Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*
x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + PolyLog[2, (Sqrt[e]*(Sqrt[f]
 - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqr
t[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - Po
lyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*S
qrt[g])]))/g^(5/2)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \log \left(c\left(d+ex^2\right)^p\right)}{f+gx^2} \, dx$$

$$\Big\downarrow \text{ 2926}$$

$$\int \left(\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{g^2 \left(f+gx^2\right)}-\frac{f \log \left(c\left(d+ex^2\right)^p\right)}{g^2}+\frac{x^2 \log \left(c\left(d+ex^2\right)^p\right)}{g}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}-\frac{2d^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}-\frac{f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{g^{5/2}}-\frac{f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{g^{5/2}}-\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}+\frac{2f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}}-\frac{fx\log\left(c(d+ex^2)^p\right)}{g^2}+\frac{x^3\log\left(c(d+ex^2)^p\right)}{3g}+\frac{if^{3/2}p\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2g^{5/2}}+\frac{if^{3/2}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}+\frac{2dpx}{3eg}-\frac{if^{3/2}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}}+\frac{2fpx}{g^2}-\frac{2px^3}{9g}$$

input

```
Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output

```
(2*f*p*x)/g^2 + (2*d*p*x)/(3*e*g) - (2*p*x^3)/(9*g) - (2*Sqrt[d]*f*p*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) - (2*d^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(3*e^(3/2)*g) + (2*f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sq
rt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sqrt[g]*x)/S
qrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f
] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f^(3/2)*p*ArcT
an[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I
*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) -
(f*x*Log[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^2)^p])/(3*g) + (f^(3/
2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(5/2) - (I*f^(3/2)*
p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) + ((I/2)*f^
(3/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt
[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) + ((I/2
)*f^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*
Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.84 (sec) , antiderivative size = 616, normalized size of antiderivative = 0.92

method	result
risch	$\left(\ln\left(\left(e\,x^2+d\right)^p\right)-p\ln\left(e\,x^2+d\right)\right)\left(\frac{\frac{1}{3}g\,x^3-fx}{g^2}+\frac{f^2\arctan\left(\frac{gx}{\sqrt{gf}}\right)}{g^2\sqrt{gf}}\right)+\frac{p\,x^3\ln\left(e\,x^2+d\right)}{3g}-\frac{2p\,x^3}{9g}+\frac{2dpx}{3eg}-$

input
```
int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(1/g^2*(1/3*g*x^3-f*x)+f^2/g^2/(g*f)^(1/2)
*arctan(g*x/(g*f)^(1/2)))+1/3*p/g*x^3*ln(e*x^2+d)-2/9*p*x^3/g+2/3*d*p*x/e/
g-2/3*p/g*d^2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-p*f/g^2*x*ln(e*x^2+d)+
2*f*p*x/g^2-2*p*f/g^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+p*Sum(1/2*(ln(
x-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alp
ha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,
index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/R
ootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^
2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alp
ha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,in
dex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e))*f^
2/g^3/_alpha,_alpha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(
I*c)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csg
n(I*c)+ln(c))*(1/g^2*(1/3*g*x^3-f*x)+f^2/g^2/(g*f)^(1/2)*arctan(g*x/(g*f)^
(1/2)))
```

Fricas [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\, dx = \int \frac{x^4 \log \left((ex^2+d)^p c\right)}{gx^2+f}\, dx$$

input
```
integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\, dx = \int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\, dx$$

input
```
integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

output
```
Integral(x**4*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^4 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^4\ \ln\left(c\,(e\,x^2+d)^p\right)}{g\,x^2+f}\,dx$$

input
```
int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

output
```
int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{x^4 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{-6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)dgp - 18\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)efp + 9\left(\int \frac{\log((e\,x^2+d)^p c)}{g\,x^2+f}dx\right)e^2 f^2 - 9\log((e\,x^2+d)}{9e^2g^2}$$

input
```
int(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

output
```
( - 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*g*p - 18*sqrt(e)*sqr
t(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*e*f*p + 9*int(log((d + e*x**2)**p*c)/(f
 + g*x**2),x)*e**2*f**2 - 9*log((d + e*x**2)**p*c)*e**2*f*x + 3*log((d + e
*x**2)**p*c)*e**2*g*x**3 + 6*d*e*g*p*x + 18*e**2*f*p*x - 2*e**2*g*p*x**3)/
(9*e**2*g**2)
```

3.344 $\int \frac{x^2 \log\left(c(d+ex^2)^p\right)}{f+gx^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 585

$$\begin{aligned}\int \frac{x^2 \log\left(c(d+ex^2)^p\right)}{f+gx^2} dx &= -\frac{2px}{g} + \frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} \\
&- \frac{2\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} \\
&+ \frac{\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}} \\
&+ \frac{\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}} \\
&+ \frac{x\log\left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{3/2}} \\
&+ \frac{i\sqrt{f}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} \\
&- \frac{i\sqrt{f}p\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}} \\
&- \frac{i\sqrt{f}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}}\end{aligned}$$

output
```
-2*p*x/g+2*d^(1/2)*p*arctan(e^(1/2)*x/d^(1/2))/e^(1/2)/g-2*f^(1/2)*p*arcta
n(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/g^(3/2)+f^(1/2)*p
*arctan(g^(1/2)*x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I
*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(3/2)+f^(1/2
)*p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/
(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(3/2)+x*ln
(c*(e*x^2+d)^p)/g-f^(1/2)*arctan(g^(1/2)*x/f^(1/2))*ln(c*(e*x^2+d)^p)/g^(3
/2)+I*f^(1/2)*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/g^(3/2)-1/2*I
*f^(1/2)*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)
*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(3/2)-1/2*I*f^(1/2)*
p*polylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+
(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(3/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.06

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$
$$= \frac{-2\sqrt{g}px + \frac{2\sqrt{d}\sqrt{g}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \sqrt{g}x\log\left(c(d+ex^2)^p\right) - \sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}i\sqrt{f}p\Big(\mathrm{lo}}{}$$

input
```
Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output
```
(-2*Sqrt[g]*p*x + (2*Sqrt[d]*Sqrt[g]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e
] + Sqrt[g]*x*Log[c*(d + e*x^2)^p] - Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*L
og[c*(d + e*x^2)^p] + (I/2)*Sqrt[f]*p*(Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x)
)/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] +
 Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqr
t[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x
))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f
]] - Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sq
rt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*
Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + PolyLog[2, (Sqrt[e]*
(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - PolyLog
[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g
])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sq
rt[-d]*Sqrt[g])]))/g^(3/2)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(c\left(d+ex^2\right)^p\right)}{f+gx^2} \, dx$$

$$\Big\downarrow \text{ 2926}$$

$$\int \left(\frac{\log \left(c\left(d+ex^2\right)^p\right)}{g}-\frac{f \log \left(c\left(d+ex^2\right)^p\right)}{g\left(f+gx^2\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{3/2}}+\frac{\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{g^{3/2}}+\frac{\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{g^{3/2}}+\frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g}-\frac{2\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}}+\frac{x\log\left(c(d+ex^2)^p\right)}{g}-\frac{i\sqrt{f}p\,\text{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2g^{3/2}}-\frac{i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}}+\frac{i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}}-\frac{2px}{g}$$

input
```
Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

output
```
(-2*p*x)/g + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g) - (2*Sq
rt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x
)])/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[
g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f
] - I*Sqrt[g]*x))])/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*S
qrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) + (x*Log[c*(d + e*x^2)^p])/g -
(Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(3/2) + (I*Sq
rt[f]*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(3/2) - ((I
/2)*Sqrt[f]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((
I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) -
 ((I/2)*Sqrt[f]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x)
)/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/
2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.19 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.90

method	result
risch	$\left(\ln\left(\left(e\,x^2+d\right)^p\right)-p\ln\left(e\,x^2+d\right)\right)\left(\frac{x}{g}-\frac{f\arctan\left(\frac{gx}{\sqrt{gf}}\right)}{g\sqrt{gf}}\right)+\frac{px\ln(e\,x^2+d)}{g}-\frac{2px}{g}+\frac{2pd\arctan\left(\frac{xe}{\sqrt{de}}\right)}{g\sqrt{de}}+p$

input
```
int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(x/g-f/g/(g*f)^(1/2)*arctan(g*x/(g*f)^(1/2
)))+p*x/g*ln(e*x^2+d)-2*p*x/g+2*p/g*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+
p*Sum(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((RootOf(_Z^
2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alp
ha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index
=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*(di
log((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^
2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,in
dex=2)))/e))*f/g^2/_alpha,_alpha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2
+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^
2+d)^p)^2*csgn(I*c)+ln(c))*(x/g-f/g/(g*f)^(1/2)*arctan(g*x/(g*f)^(1/2)))
```

Fricas [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^2 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

input
```
integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

output
```
Integral(x**2*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = \int \frac{x^2 \log \left((ex^2+d)^p c\right)}{gx^2+f} \, dx$$

input
```
integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = \int \frac{x^2 \ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f} \, dx$$

input
```
int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

output
```
int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{x^2 \log (c(d+ex^2)^p)}{f+gx^2}\,dx$$
$$= \frac{2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)p - \left(\int \frac{\log((e\,x^2+d)^p c)}{g\,x^2+f}dx\right)ef + \log\left((e\,x^2+d)^p\,c\right)ex - 2epx}{eg}$$

input
```
int(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

output
```
(2*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*p - int(log((d + e*x**2)*
*p*c)/(f + g*x**2),x)*e*f + log((d + e*x**2)**p*c)*e*x - 2*e*p*x)/(e*g)
```

3.345 $\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 533

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx &= \frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} \\
&\quad-\frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad+\frac{ip\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} \\
&\quad+\frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}\end{aligned}$$

output
```
2*p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/
g^(1/2)-p*arctan(g^(1/2)*x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1
/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/
2)/g^(1/2)-p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^
(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(
1/2)/g^(1/2)+arctan(g^(1/2)*x/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)-I
*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+1/2*I*p*po
lylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)
^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1
-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^
(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = -\frac{i\left(p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)+p\log\left(\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\cdots\right)}{\cdots}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

output
```
((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqr
t[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d]
+ Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g
]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[
f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*S
qrt[g]*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p
] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sq
rt[-d]*Sqrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]
*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[
g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sq
rt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*S
qrt[g])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2920, 27, 5463, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}dx$$

$$\Big\downarrow \ 2920$$

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-2ep\int \frac{x\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(ex^2+d\right)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{2ep\int \frac{x\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex^2+d}dx}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 5463$$

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{2ep\int \left(\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{2ep\left(\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2e}+\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2e}-\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right.}{e}\right.}{\sqrt{f}\sqrt{g}}$$

input
```
Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

output
```
(ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (2*
e*p*(-((ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x
)])/e) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] -
Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x
))])/(2*e) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d]
 + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g
]*x))])/(2*e) + ((I/2)*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)]
)/e - ((I/4)*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I
*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e - ((I/4)
*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqr
t[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/e))/(Sqrt[f]*Sqrt[g])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5463
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.57 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.84

method	result
risch	$\frac{\left(\ln\left(\left(e\,x^{2}+d\right)^{p}\right)-p\ln\left(e\,x^{2}+d\right)\right)\arctan\left(\frac{g x}{\sqrt{g f}}\right)}{\sqrt{g f}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(g\_Z^{2}+f\right)}\frac{\ln(x-\_\alpha)\ln\left(e\,x^{2}+d\right)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}eg+\right.}{\mathrm{RootOf}\left(\_Z^{2}\right.}\right.\right.}{}\right.}{}$

input
```
int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
(ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(g*f)^(1/2)*arctan(g*x/(g*f)^(1/2))+1/2*p/
g*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x
+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f
,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_al
pha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^
2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*cs
gn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))/(g
*f)^(1/2)*arctan(g*x/(g*f)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(c\left(d+ex^{2}\right)^{p}\right)}{f+gx^{2}}\,dx=\int \frac{\log\left(\left(ex^{2}+d\right)^{p}c\right)}{gx^{2}+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

input
```
integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

output
```
Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)}{g\,x^2+f}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((e\,x^2+d)^p\,c\right)}{g\,x^2+f}dx$$

input
```
int(log(c*(e*x^2+d)^p)/(g*x^2+f),x)
```

output
```
int(log((d + e*x**2)**p*c)/(f + g*x**2),x)
```

3.346 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 581

$$\begin{aligned}\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)}\,dx &= \frac{2\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}}\\
&+ \frac{\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}}\\
&+ \frac{\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}}\\
&- \frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx} - \frac{\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}}\\
&+ \frac{i\sqrt{g}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}}\\
&- \frac{i\sqrt{g}p\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}}\\
&- \frac{i\sqrt{g}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}}\end{aligned}$$

output
```
2*e^(1/2)*p*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/f-2*g^(1/2)*p*arctan(g^(1/2)
*x/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)+g^(1/2)*p*arctan(g
^(1/2)*x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*
f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)+g^(1/2)*p*arcta
n(g^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2
)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)-ln(c*(e*x^2+d
)^p)/f/x-g^(1/2)*arctan(g^(1/2)*x/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(3/2)+I*g^(
1/2)*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)-1/2*I*g^(1/2)*
p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)-
(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)-1/2*I*g^(1/2)*p*polylog
(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2
)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.06

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 (f+gx^2)}\,dx$$
$$= \frac{\frac{2\sqrt{e}\sqrt{f}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{f}\log\left(c(d+ex^2)^p\right)}{x} - \sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}i\sqrt{g}p\left(\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\right.}{}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]
```

output
```
((2*Sqrt[e]*Sqrt[f]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (Sqrt[f]*Log[
c*(d + e*x^2)^p])/x - Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2
)^p] + (I/2)*Sqrt[g]*p*(Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sq
rt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[(Sqrt[g]*(
Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 -
(I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[
g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 +
 (I*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sq
rt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(S
qrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - PolyLog[2
, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])
]))/f^(3/2)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x^2 \left(f+gx^2\right)} \, dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(\frac{\log \left(c\left(d+ex^2\right)^p\right)}{fx^2}-\frac{g\log \left(c\left(d+ex^2\right)^p\right)}{f\left(f+gx^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}}+\frac{\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{f^{3/2}}+\frac{\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{f^{3/2}}+\frac{2\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f}-\frac{2\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}}-\frac{\log\left(c(d+ex^2)^p\right)}{fx}-\frac{i\sqrt{g}p\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2f^{3/2}}-\frac{i\sqrt{g}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}}+\frac{i\sqrt{g}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}}
$$

input
```
Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]
```

output
```
(2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f) - (2*Sqrt[g]*p*ArcTa
n[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) +
 (Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d]
- Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x))])/f^(3/2) + (Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqr
t[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt
[f] - I*Sqrt[g]*x))])/f^(3/2) - Log[c*(d + e*x^2)^p]/(f*x) - (Sqrt[g]*ArcT
an[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/f^(3/2) + (I*Sqrt[g]*p*PolyL
og[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) - ((I/2)*Sqrt[g]*p
*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqr
t[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2) - ((I/2)*Sqrt[
g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]
*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.10 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.91

method	result
risch	$\left(\ln \left(\left(e\,x^{2}+d\right)^{p}\right)-p\ln \left(e\,x^{2}+d\right)\right)\left(-\frac{1}{fx}-\frac{g\arctan\left(\frac{gx}{\sqrt{gf}}\right)}{f\sqrt{gf}}\right)-\frac{p\ln\left(e\,x^{2}+d\right)}{fx}+\frac{2pe\arctan\left(\frac{xe}{\sqrt{de}}\right)}{f\sqrt{de}}+p\left(\_\alpha\right.$

input
```
int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(-1/f/x-g/f/(g*f)^(1/2)*arctan(g*x/(g*f)^(
1/2)))-p/f/x*ln(e*x^2+d)+2*p/f*e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+p*Sum
(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x
+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*(dilog((
RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g
+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+
d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2
)))/e))/f/_alpha,_alpha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*cs
gn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*c
sgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2
*csgn(I*c)+ln(c))*(-1/f/x-g/f/(g*f)^(1/2)*arctan(g*x/(g*f)^(1/2)))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \int \frac{\log\left((ex^2+d)^pc\right)}{(gx^2+f)x^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g*x^4 + f*x^2), x)
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx$$

input
```
integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f),x)
```

output
```
Integral(log(c*(d + e*x**2)**p)/(x**2*(f + g*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{(gx^2+f)x^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^2\,\left(g\,x^2+f\right)}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 \left(f+gx^2\right)} \, dx = \int \frac{\log \left(\left(e\,x^2+d\right)^p c\right)}{g\,x^4+f\,x^2} dx$$

input

```
int(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x)
```

output

```
int(log((d + e*x**2)**p*c)/(f*x**2 + g*x**4),x)
```

3.347 $\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4(f+gx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 651

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^4 (f+gx^2)}\,dx = -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} + \frac{2g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} - \frac{g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} - \frac{g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{g^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} - \frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} + \frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} + \frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}}$$

output
```
-2/3*e*p/d/f/x-2/3*e^(3/2)*p*arctan(e^(1/2)*x/d^(1/2))/d^(3/2)/f-2*e^(1/2)
*g*p*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/f^2+2*g^(3/2)*p*arctan(g^(1/2)*x/f^
(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(5/2)-g^(3/2)*p*arctan(g^(1/2
)*x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/
2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(5/2)-g^(3/2)*p*arctan(g^(
1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(
1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(5/2)-1/3*ln(c*(e*x^2+d)
^p)/f/x^3+g*ln(c*(e*x^2+d)^p)/f^2/x+g^(3/2)*arctan(g^(1/2)*x/f^(1/2))*ln(c
*(e*x^2+d)^p)/f^(5/2)-I*g^(3/2)*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)
*x))/f^(5/2)+1/2*I*g^(3/2)*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(
1/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(5
/2)+1/2*I*g^(3/2)*p*polylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(
I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.57 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.03

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^4 (f+gx^2)}\,dx = \frac{-\frac{12\sqrt{e}\sqrt{f}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{4ef^{3/2}p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{dx}-\frac{2f^{3/2}\log\left(c(d+ex^2)^p\right)}{x^3}+\frac{6\sqrt{f}g\log\left(c(d+ex^2)^p\right)}{x}+6g^{3/}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

output
```
((-12*Sqrt[e]*Sqrt[f]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (4*e*f^(3
/2)*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(d*x) - (2*f^(3/2)*Lo
g[c*(d + e*x^2)^p])/x^3 + (6*Sqrt[f]*g*Log[c*(d + e*x^2)^p])/x + 6*g^(3/2)
*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] - (3*I)*g^(3/2)*p*(Log[(
Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Lo
g[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((-I)*
Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] - Log[
(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])
]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I
*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + Pol
yLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sq
rt[g])] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] +
I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[
e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[
g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(6*f^(5/2))
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x^4 \left(f+gx^2\right)} \, dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(\frac{g^2 \log \left(c\left(d+ex^2\right)^p\right)}{f^2 \left(f+gx^2\right)}-\frac{g \log \left(c\left(d+ex^2\right)^p\right)}{f^2 x^2}+\frac{\log \left(c\left(d+ex^2\right)^p\right)}{f x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}}-\frac{2e^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f}-\frac{g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{f^{5/2}}-\frac{g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{f^{5/2}}-\frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2}+\frac{2g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}}+\frac{g\log\left(c(d+ex^2)^p\right)}{f^2x}-\frac{\log\left(c(d+ex^2)^p\right)}{3fx^3}+\frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2f^{5/2}}+\frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}}-\frac{2ep}{3dfx}-\frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}}$$

input
```
Int[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

output
```
(-2*e*p)/(3*d*f*x) - (2*e^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)*
f) - (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) + (2*g^(3/2
)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
f^(5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(
Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sq
rt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2) - Log[c*(d + e*x^2)^p]/(3*f*x^3) +
(g*Log[c*(d + e*x^2)^p])/(f^2*x) + (g^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Lo
g[c*(d + e*x^2)^p])/f^(5/2) - (I*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqr
t[f] - I*Sqrt[g]*x)])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 + (2*Sqrt[f]
*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(
Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 - (2*Sqr
t[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g
])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.94 (sec) , antiderivative size = 602, normalized size of antiderivative = 0.92

method	result
risch	$\left(\ln\left(\left(e\,x^{2}+d\right)^{p}\right)-p\ln\left(e\,x^{2}+d\right)\right)\left(-\frac{1}{3f\,x^{3}}+\frac{g}{f^{2}x}+\frac{g^{2}\arctan\left(\frac{gx}{\sqrt{gf}}\right)}{f^{2}\sqrt{gf}}\right)+\frac{pg\ln\left(e\,x^{2}+d\right)}{f^{2}x}-\frac{2pge\arctan\left(\frac{xe}{\sqrt{d}\ldots}\right)}{f^{2}\sqrt{de}}$

input
```
int(ln(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x,method=_RETURNVERBOSE)
```

output
```
(ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(-1/3/f/x^3+g/f^2/x+g^2/f^2/(g*f)^(1/2)*ar
ctan(g*x/(g*f)^(1/2)))+p*g/f^2/x*ln(e*x^2+d)-2*p*g/f^2*e/(d*e)^(1/2)*arcta
n(x*e/(d*e)^(1/2))+p*Sum(1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alph
a)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,in
dex=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+
_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z
^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_al
pha*e*g+d*g-e*f,index=2)))/e))*g/f^2/_alpha,_alpha=RootOf(_Z^2*g+f))-1/3*p
/f/x^3*ln(e*x^2+d)-2/3*e*p/d/f/x-2/3*p/f*e^2/d/(d*e)^(1/2)*arctan(x*e/(d*e
)^(1/2))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*cs
gn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^2
+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))*(-1/3/f/x^3+g/f
^2/x+g^2/f^2/(g*f)^(1/2)*arctan(g*x/(g*f)^(1/2)))
```

Fricas [F]

$$\int \frac{\log \left(c\left(d+e x^2\right)^p\right)}{x^4\left(f+g x^2\right)} \, dx = \int \frac{\log \left((e x^2+d)^p c\right)}{(g x^2+f) x^4} \, dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g*x^6 + f*x^4), x)
```

Sympy [F]

$$\int \frac{\log \left(c\left(d+e x^2\right)^p\right)}{x^4\left(f+g x^2\right)} \, dx = \int \frac{\log \left(c\left(d+e x^2\right)^p\right)}{x^4\left(f+g x^2\right)} \, dx$$

input
```
integrate(ln(c*(e*x**2+d)**p)/x**4/(g*x**2+f),x)
```

output
```
Integral(log(c*(d + e*x**2)**p)/(x**4*(f + g*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4\left(f+gx^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4\left(f+gx^2\right)}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{(gx^2+f)x^4}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4\left(f+gx^2\right)}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)}{x^4\,(g\,x^2+f)}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(x^4*(f + g*x^2)),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(x^4*(f + g*x^2)), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4\,(f+gx^2)}\,dx = \int \frac{\log\left(\left(e\,x^2+d\right)^p c\right)}{g\,x^6+f\,x^4}dx$$

input
```
int(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x)
```

output
```
int(log((d + e*x**2)**p*c)/(f*x**4 + g*x**6),x)
```

3.348 $\int \frac{x^5 \log \left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = -\frac{px^2}{2g^2}+\frac{ef^2p\log\left(d+ex^2\right)}{2g^3(ef-dg)}+\frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg^2} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{ef^2p\log\left(f+gx^2\right)}{2g^3(ef-dg)} - \frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{g^3} - \frac{fp\operatorname{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{g^3}$$

output

```
-1/2*p*x^2/g^2+1/2*e*f^2*p*ln(e*x^2+d)/g^3/(-d*g+e*f)+1/2*(e*x^2+d)*ln(c*(
e*x^2+d)^p)/e/g^2-1/2*f^2*ln(c*(e*x^2+d)^p)/g^3/(g*x^2+f)-1/2*e*f^2*p*ln(g
*x^2+f)/g^3/(-d*g+e*f)-f*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g^3-
f*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.83

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = -\frac{px^2 - \frac{(d+ex^2)\log(c(d+ex^2)^p)}{e} + \frac{f^2\log(c(d+ex^2)^p)}{g(f+gx^2)} + \frac{ef^2p(\log(d+ex^2)-\log(f+gx^2))}{g(-ef+dg)} + \frac{2f\left(\log(c(d+ex^2)^p)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)+\right.}{g}}{2g^2}$$

input
```
Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
-1/2*(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e + (f^2*Log[c*(d + e*x^2
)^p])/(g*(f + g*x^2)) + (e*f^2*p*(Log[d + e*x^2] - Log[f + g*x^2]))/(g*(-(
e*f) + d*g)) + (2*f*(Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)]
 + p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)]))/g)/g^2
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$$

$$\Big\downarrow \; 2925$$

$$\frac{1}{2}\int \frac{x^4 \log\left(c(ex^2+d)^p\right)}{(gx^2+f)^2}dx^2$$

$$\Big\downarrow \; 2863$$

$$\frac{1}{2}\int \left(\frac{\log\left(c(ex^2+d)^p\right)f^2}{g^2(gx^2+f)^2} - \frac{2\log\left(c(ex^2+d)^p\right)f}{g^2(gx^2+f)} + \frac{\log\left(c(ex^2+d)^p\right)}{g^2}\right)dx^2$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{g^3\left(f+gx^2\right)}-\frac{2f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{g^3}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{eg^2}+\frac{ef^2p\log\left(d+\right.}{g^3(ef-d\right.}$$

input
```
Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
(-((p*x^2)/g^2) + (e*f^2*p*Log[d + e*x^2])/(g^3*(e*f - d*g)) + ((d + e*x^2
)*Log[c*(d + e*x^2)^p])/(e*g^2) - (f^2*Log[c*(d + e*x^2)^p])/(g^3*(f + g*x
^2)) - (e*f^2*p*Log[f + g*x^2])/(g^3*(e*f - d*g)) - (2*f*Log[c*(d + e*x^2)
^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/g^3 - (2*f*p*PolyLog[2, -((g*(d + e*
x^2))/(e*f - d*g))])/g^3)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], 
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.12 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.22

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)x^2}{2g^2}-\frac{\ln(c(e\,x^2+d)^p)f\ln(g\,x^2+f)}{g^3}-\frac{f^2\ln(c(e\,x^2+d)^p)}{2g^3(g\,x^2+f)}-ep\left(-\frac{f\left(\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^2+d\right)}\left(\ln(x-\_\alpha)\ln(g\right.\right.}{}\right.$
risch	$\frac{\ln((e\,x^2+d)^p)x^2}{2g^2}-\frac{\ln((e\,x^2+d)^p)f\ln(g\,x^2+f)}{g^3}-\frac{\ln((e\,x^2+d)^p)f^2}{2g^3(g\,x^2+f)}-\frac{p\,x^2}{2g^2}+\frac{p\ln(e\,x^2+d)d^2}{2eg(dg-ef)}-\frac{p\ln(e\,x^2+d)fd}{2g^2(dg-ef)}-\frac{ep\ln(}{2g^3}$

input
```
int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*(e*x^2+d)^p)*x^2/g^2-ln(c*(e*x^2+d)^p)*f/g^3*ln(g*x^2+f)-1/2*f^2*
ln(c*(e*x^2+d)^p)/g^3/(g*x^2+f)-e*p*(-f/g^3/e*sum(ln(x-_alpha)*ln(g*x^2+f)
-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alp
ha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+
2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g
-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1
)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootO
f(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/g^3*(-1/2*g*x^2/e
+1/2*(d^2*g^2-d*e*f*g-e^2*f^2)/(d*g-e*f)/e^2*ln(e*x^2+d)+1/2*f^2/(d*g-e*f)
*ln(g*x^2+f)))
```

Fricas [F]

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^5 \log\left((ex^2+d)^p c\right)}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(x^5*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^5 \log\left((ex^2+d)^p c\right)}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Giac [F]

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^5 \log\left((ex^2+d)^p c\right)}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^5 \ln\left(c\,(e\,x^2+d)^p\right)}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

output
```
int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
( - 4*int((log((d + e*x**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x*
*4 + e*f**2*x**2 + 2*e*f*g*x**4 + e*g**2*x**6),x)*d**3*e*f**2*g**4*p - 4*i
nt((log((d + e*x**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*
f**2*x**2 + 2*e*f*g*x**4 + e*g**2*x**6),x)*d**3*e*f*g**5*p*x**2 + 12*int((
log((d + e*x**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2
*x**2 + 2*e*f*g*x**4 + e*g**2*x**6),x)*d**2*e**2*f**3*g**3*p + 12*int((log
((d + e*x**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x*
*2 + 2*e*f*g*x**4 + e*g**2*x**6),x)*d**2*e**2*f**2*g**4*p*x**2 - 12*int((l
og((d + e*x**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*
x**2 + 2*e*f*g*x**4 + e*g**2*x**6),x)*d*e**3*f**4*g**2*p - 12*int((log((d
+ e*x**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x**2 +
 2*e*f*g*x**4 + e*g**2*x**6),x)*d*e**3*f**3*g**3*p*x**2 + 4*int((log((d +
e*x**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x**2 + 2
*e*f*g*x**4 + e*g**2*x**6),x)*e**4*f**5*g*p + 4*int((log((d + e*x**2)**p*c
)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x**2 + 2*e*f*g*x**4
+ e*g**2*x**6),x)*e**4*f**4*g**2*p*x**2 + log(d + e*x**2)*d**3*f*g**3*p**2
 + log(d + e*x**2)*d**3*g**4*p**2*x**2 - 2*log(d + e*x**2)*d*e**2*f**3*g*p
**2 - 2*log(d + e*x**2)*d*e**2*f**2*g**2*p**2*x**2 - log(f + g*x**2)*d*e**
2*f**3*g*p**2 - log(f + g*x**2)*d*e**2*f**2*g**2*p**2*x**2 + 2*log(f + g*x
**2)*e**3*f**4*p**2 + 2*log(f + g*x**2)*e**3*f**3*g*p**2*x**2 - log((d ...
```

3.349 $\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} dx = -\frac{efp\log(d+ex^2)}{2g^2(ef-dg)} + \frac{f\log\left(c(d+ex^2)^p\right)}{2g^2(f+gx^2)} + \frac{efp\log(f+gx^2)}{2g^2(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} + \frac{p\,\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}$$

output
```
-1/2*e*f*p*ln(e*x^2+d)/g^2/(-d*g+e*f)+1/2*f*ln(c*(e*x^2+d)^p)/g^2/(g*x^2+f
)+1/2*e*f*p*ln(g*x^2+f)/g^2/(-d*g+e*f)+1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f
)/(-d*g+e*f))/g^2+1/2*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \log \left(c\left(d+e x^2\right)^p\right)}{\left(f+g x^2\right)^2} \, dx = \frac{\frac{efp\log(d+ex^2)}{-ef+dg}+\frac{f\log(c(d+ex^2)^p)}{f+gx^2}+\frac{efp\log(f+gx^2)}{ef-dg}+\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)+p\,\mathrm{PolyLog}\left(2,\frac{g(d+ex^2)}{-ef+dg}\right)}{2g^2}$$

input
```
Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x
^2) + (e*f*p*Log[f + g*x^2])/(e*f - d*g) + Log[c*(d + e*x^2)^p]*Log[(e*(f
+ g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(2*
g^2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log \left(c\left(d+e x^2\right)^p\right)}{\left(f+g x^2\right)^2} \, dx$$

$$\Big\downarrow \; 2925$$

$$\frac{1}{2} \int \frac{x^2 \log \left(c\left(e x^2+d\right)^p\right)}{\left(g x^2+f\right)^2}dx^2$$

$$\Big\downarrow \; 2863$$

$$\frac{1}{2} \int \left(\frac{\log \left(c\left(e x^2+d\right)^p\right)}{g\left(g x^2+f\right)}-\frac{f \log \left(c\left(e x^2+d\right)^p\right)}{g\left(g x^2+f\right)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{g^2\left(f+gx^2\right)}+\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{g^2}+\frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{g^2}-\frac{efp\log\left(d+ex^2\right)}{g^2(ef-dg)}+\right.$$

input
```
Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
(-((e*f*p*Log[d + e*x^2])/(g^2*(e*f - d*g))) + (f*Log[c*(d + e*x^2)^p])/(g
^2*(f + g*x^2)) + (e*f*p*Log[f + g*x^2])/(g^2*(e*f - d*g)) + (Log[c*(d + e
*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/g^2 + (p*PolyLog[2, -((g*(d + e
*x^2))/(e*f - d*g))])/g^2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.01 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.46

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)\ln(g\,x^2+f)}{2g^2}+\frac{f\ln(c(e\,x^2+d)^p)}{2g^2(g\,x^2+f)}-ep\left(\frac{\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(g\,x^2+f)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootO}}{\mathrm{R}}\right.\right.\right.}{}\right.$
risch	$\frac{\ln((e\,x^2+d)^p)\ln(g\,x^2+f)}{2g^2}+\frac{\ln((e\,x^2+d)^p)f}{2g^2(g\,x^2+f)}-\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(g\,x^2+f)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}(\_}{\mathrm{RootOf}}\right.\right.\right.\right.}{}$

input
```
int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*(e*x^2+d)^p)/g^2*ln(g*x^2+f)+1/2*f*ln(c*(e*x^2+d)^p)/g^2/(g*x^2+f
)-e*p*(1/2/g^2/e*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf(_Z^
2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alp
ha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index
=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*
g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2))
,_alpha=RootOf(_Z^2*e+d))+f/g^2*(-1/2/(d*g-e*f)*ln(e*x^2+d)+1/2/(d*g-e*f)*
ln(g*x^2+f)))
```

Fricas [F]

$$\int \frac{x^3\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx=\int \frac{x^3\log\left((ex^2+d)^pc\right)}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(x^3*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{(gx^2+f)^2}\,dx$$

input
```
integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^3\ \ln\left(c\,(e\,x^2+d)^p\right)}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

output
```
int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
(4*int((log((d + e*x**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4
+ e*f**2*x**2 + 2*e*f*g*x**4 + e*g**2*x**6),x)*d**3*f*g**4*p + 4*int((log(
(d + e*x**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x**
2 + 2*e*f*g*x**4 + e*g**2*x**6),x)*d**3*g**5*p*x**2 - 12*int((log((d + e*x
**2)**p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x**2 + 2*e*
f*g*x**4 + e*g**2*x**6),x)*d**2*e*f**2*g**3*p - 12*int((log((d + e*x**2)**
p*c)*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x**2 + 2*e*f*g*x*
*4 + e*g**2*x**6),x)*d**2*e*f*g**4*p*x**2 + 12*int((log((d + e*x**2)**p*c)
*x**3)/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x**2 + 2*e*f*g*x**4 +
 e*g**2*x**6),x)*d*e**2*f**3*g**2*p + 12*int((log((d + e*x**2)**p*c)*x**3)
/(d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x**2 + 2*e*f*g*x**4 + e*g**
2*x**6),x)*d*e**2*f**2*g**3*p*x**2 - 4*int((log((d + e*x**2)**p*c)*x**3)/(
d*f**2 + 2*d*f*g*x**2 + d*g**2*x**4 + e*f**2*x**2 + 2*e*f*g*x**4 + e*g**2*
x**6),x)*e**3*f**4*g*p - 4*int((log((d + e*x**2)**p*c)*x**3)/(d*f**2 + 2*d
*f*g*x**2 + d*g**2*x**4 + e*f**2*x**2 + 2*e*f*g*x**4 + e*g**2*x**6),x)*e**
3*f**3*g**2*p*x**2 + 2*log(d + e*x**2)*d*e*f**2*g*p**2 + 2*log(d + e*x**2)
*d*e*f*g**2*p**2*x**2 - 2*log(f + g*x**2)*e**2*f**3*p**2 - 2*log(f + g*x**
2)*e**2*f**2*g*p**2*x**2 + log((d + e*x**2)**p*c)**2*d**2*f*g**2 + log((d
+ e*x**2)**p*c)**2*d**2*g**3*x**2 - log((d + e*x**2)**p*c)**2*d*e*f**2*g -
 log((d + e*x**2)**p*c)**2*d*e*f*g**2*x**2 - 2*log((d + e*x**2)**p*c)*d...
```

3.350 $\int \frac{x \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{x \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \frac{ep \log(d+ex^2)}{2g(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{2g(f+gx^2)} - \frac{ep \log(f+gx^2)}{2g(ef-dg)}$$

output
```
1/2*e*p*ln(e*x^2+d)/g/(-d*g+e*f)-1/2*ln(c*(e*x^2+d)^p)/g/(g*x^2+f)-1/2*e*p
*ln(g*x^2+f)/g/(-d*g+e*f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{x \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \frac{\frac{ep \log(d+ex^2)}{ef-dg} - \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2} + \frac{ep \log(f+gx^2)}{-ef+dg}}{2g}$$

input
```
Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
((e*p*Log[d + e*x^2])/(e*f - d*g) - Log[c*(d + e*x^2)^p]/(f + g*x^2) + (e*
p*Log[f + g*x^2])/(-(e*f) + d*g))/(2*g)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2925, 2842, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} dx$$

$$\Big\downarrow \text{ 2925}$$

$$\frac{1}{2}\int \frac{\log \left(c(ex^2+d)^p\right)}{(gx^2+f)^2}dx^2$$

$$\Big\downarrow \text{ 2842}$$

$$\frac{1}{2}\left(\frac{ep\int \frac{1}{(ex^2+d)(gx^2+f)}dx^2}{g}-\frac{\log \left(c(d+ex^2)^p\right)}{g\left(f+gx^2\right)}\right)$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}\left(\frac{ep\left(\frac{e\int \frac{1}{ex^2+d}dx^2}{ef-dg}-\frac{g\int \frac{1}{gx^2+f}dx^2}{ef-dg}\right)}{g}-\frac{\log \left(c(d+ex^2)^p\right)}{g\left(f+gx^2\right)}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\frac{ep\left(\frac{\log(d+ex^2)}{ef-dg}-\frac{\log(f+gx^2)}{ef-dg}\right)}{g}-\frac{\log \left(c(d+ex^2)^p\right)}{g\left(f+gx^2\right)}\right)$$

input
```
Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
(-(Log[c*(d + e*x^2)^p]/(g*(f + g*x^2))) + (e*p*(Log[d + e*x^2]/(e*f - d*g
) - Log[f + g*x^2]/(e*f - d*g)))/g)/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

method	result
parts	$-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{2g\left(g\,x^{2}+f\right)}+\frac{ep\left(-\frac{\ln\left(e\,x^{2}+d\right)}{2(dg-ef)}+\frac{\ln\left(g\,x^{2}+f\right)}{2dg-2ef}\right)}{g}$
parallelrisch	$-\frac{\ln\left(e\,x^{2}+d\right)x^{2}e^{2}gp-\ln\left(g\,x^{2}+f\right)x^{2}e^{2}gp+\ln\left(e\,x^{2}+d\right)e^{2}fp-\ln\left(g\,x^{2}+f\right)e^{2}fp+\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)deg-\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)e^{2}f}{2(dg-ef)\left(g\,x^{2}+f\right)eg}$
risch	$-\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{2g\left(g\,x^{2}+f\right)}-\frac{i\pi dg\,\mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}-i\pi dg\,\mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\right)-i\pi cs}{}$

input
```
int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(e*x^2+d)^p)/g/(g*x^2+f)+e*p/g*(-1/2/(d*g-e*f)*ln(e*x^2+d)+1/2/(
d*g-e*f)*ln(g*x^2+f))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{(egpx^2+dgp)\log\left(ex^2+d\right)-(egpx^2+efp)\log\left(gx^2+f\right)-(ef-dg)\log\left(c\right)}{2\left(ef^2g-dfg^2+(efg^2-dg^3)x^2\right)}$$

input
```
integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
1/2*((e*g*p*x^2 + d*g*p)*log(e*x^2 + d) - (e*g*p*x^2 + e*f*p)*log(g*x^2 +
f) - (e*f - d*g)*log(c))/(e*f^2*g - d*f*g^2 + (e*f*g^2 - d*g^3)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \frac{ep\left(\frac{\log(ex^2+d)}{ef-dg} - \frac{\log(gx^2+f)}{ef-dg}\right)}{2\,g} - \frac{\log\left((ex^2+d)^p c\right)}{2\,(gx^2+f)g}$$

input
```
integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
1/2*e*p*(log(e*x^2 + d)/(e*f - d*g) - log(g*x^2 + f)/(e*f - d*g))/g - 1/2*
log((e*x^2 + d)^p*c)/((g*x^2 + f)*g)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = -\frac{ep\log(ex^2+d)}{2\,(efg+(ex^2+d)g^2-dg^2)} + \frac{ep\log(ex^2+d)}{2\,(efg-dg^2)} - \frac{ep\log(ef+(ex^2+d)g-dg)}{2\,(efg-dg^2)} - \frac{e\log(c)}{2\,(efg+(ex^2+d)g^2-dg^2)}$$

input
```
integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
-1/2*e*p*log(e*x^2 + d)/(e*f*g + (e*x^2 + d)*g^2 - d*g^2) + 1/2*e*p*log(e*
x^2 + d)/(e*f*g - d*g^2) - 1/2*e*p*log(e*f + (e*x^2 + d)*g - d*g)/(e*f*g -
 d*g^2) - 1/2*e*log(c)/(e*f*g + (e*x^2 + d)*g^2 - d*g^2)
```

Mupad [B] (verification not implemented)

Time = 17.98 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{x\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = -\frac{\ln\left(c\,\left(e\,x^2+d\right)^p\right)}{2\,g\,\left(g\,x^2+f\right)} - \frac{e\,p\,\mathrm{atan}\left(\frac{x^2\,\left(d\,g\,1\mathrm{i}-e\,f\,1\mathrm{i}\right)}{2\,d\,f+d\,g\,x^2+e\,f\,x^2}\right)\,1\mathrm{i}}{d\,g^2-e\,f\,g}$$

input
```
int((x*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

output
```
- log(c*(d + e*x^2)^p)/(2*g*(f + g*x^2)) - (e*p*atan((x^2*(d*g*1i - e*f*1i
))/(2*d*f + d*g*x^2 + e*f*x^2))*1i)/(d*g^2 - e*f*g)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.69

$$\int \frac{x\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{-\log(e\,x^2+d)\,dfgp-\log(e\,x^2+d)\,d\,g^2p\,x^2+\log(g\,x^2+f)\,e\,f^2p+\log(g\,x^2+f)\,efgp\,x^2+\log\left((e\,x^2+\right.}{2fg\left(d\,g^2x^2-efg\,x^2+dfg-e\,f^2\right)}$$

input
```
int(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
( - log(d + e*x**2)*d*f*g*p - log(d + e*x**2)*d*g**2*p*x**2 + log(f + g*x*
*2)*e*f**2*p + log(f + g*x**2)*e*f*g*p*x**2 + log((d + e*x**2)**p*c)*d*g**
2*x**2 - log((d + e*x**2)**p*c)*e*f*g*x**2)/(2*f*g*(d*f*g + d*g**2*x**2 -
e*f**2 - e*f*g*x**2))
```

3.351 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx = & -\frac{ep\log(d+ex^2)}{2f(ef-dg)}+\frac{\log\left(c(d+ex^2)^p\right)}{2f\left(f+gx^2\right)} \\ & +\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2}+\frac{ep\log(f+gx^2)}{2f(ef-dg)} \\ & -\frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} \\ & -\frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2f^2}+\frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{2f^2}\end{aligned}$$

output
```
-1/2*e*p*ln(e*x^2+d)/f/(-d*g+e*f)+1/2*ln(c*(e*x^2+d)^p)/f/(g*x^2+f)+1/2*ln
(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^2+1/2*e*p*ln(g*x^2+f)/f/(-d*g+e*f)-1/2*ln(c
*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/f^2-1/2*p*polylog(2,-g*(e*x^2+d)/
(-d*g+e*f))/f^2+1/2*p*polylog(2,1+e*x^2/d)/f^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.85

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx$$
$$= \frac{\frac{efp\log(d+ex^2)}{-ef+dg}+\frac{f\log(c(d+ex^2)^p)}{f+gx^2}+\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{efp\log(f+gx^2)}{ef-dg}-\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f-}{ef}\right.}{2f^2}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```

output
```
((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x
^2) + Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + (e*f*p*Log[f + g*x^2])/(e*f
 - d*g) - Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] - p*PolyLo
g[2, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2
)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx$$
$$\Big\downarrow \; 2925$$
$$\frac{1}{2}\int \frac{\log \left(c(ex^2+d)^p\right)}{x^2\left(gx^2+f\right)^2}dx^2$$
$$\Big\downarrow \; 2863$$
$$\frac{1}{2}\int \left(-\frac{g\log \left(c(ex^2+d)^p\right)}{f^2\left(gx^2+f\right)}+\frac{\log \left(c(ex^2+d)^p\right)}{f^2x^2}-\frac{g\log \left(c(ex^2+d)^p\right)}{f\left(gx^2+f\right)^2}\right)dx^2$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{f^2}+\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^2}+\frac{\log\left(c\left(d+ex^2\right)^p\right)}{f\left(f+gx^2\right)}-\frac{p\operatorname{PolyLog}\left(2,-\frac{g}{\ }\right.}{f^2}\right.$$

input
```
Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```

output
```
(-((e*p*Log[d + e*x^2])/(f*(e*f - d*g))) + Log[c*(d + e*x^2)^p]/(f*(f + g*
x^2)) + (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/f^2 + (e*p*Log[f + g*x^2]
)/(f*(e*f - d*g)) - (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)]
)/f^2 - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/f^2 + (p*PolyLog[2,
 1 + (e*x^2)/d])/f^2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.88 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.45

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)\ln(x)}{f^2}-\frac{\ln(c(e\,x^2+d)^p)\ln(g\,x^2+f)}{2f^2}+\frac{\ln(c(e\,x^2+d)^p)}{2f(g\,x^2+f)}-ep\left(-\frac{\ln(e\,x^2+d)}{2f(dg-ef)}+\frac{\ln(g\,x^2+f)}{2f(dg-ef)}+\frac{\ln(x)\left(\ln\left(-\right.\right.}{}\right.$
risch	Expression too large to display

input
```
int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(c*(e*x^2+d)^p)/f^2*ln(x)-1/2*ln(c*(e*x^2+d)^p)/f^2*ln(g*x^2+f)+1/2*ln(c
*(e*x^2+d)^p)/f/(g*x^2+f)-e*p*(-1/2/f/(d*g-e*f)*ln(e*x^2+d)+1/2/f/(d*g-e*f
)*ln(g*x^2+f)+2/f^2*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e
*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1
/2))+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e)-1/2/f^2/e*sum(ln(x-_alpha)
*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,ind
ex=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-
dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_
Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d)))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{\log\left((ex^2+d)^pc\right)}{\left(gx^2+f\right)^2x}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx = -\frac{1}{2}\,ep\left(\frac{\log\left(ex^2+d\right)}{ef^2-dfg}-\frac{\log\left(gx^2+f\right)}{ef^2-dfg}+\frac{2\,\log\left(\frac{ex^2}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)}{ef^2}-\frac{\log\left(gx^2+f\right)\log\left(-\frac{eg}{e}\right.}{}\right.$$
$$+\frac{1}{2}\left(\frac{1}{fgx^2+f^2}-\frac{\log\left(gx^2+f\right)}{f^2}+\frac{\log\left(x^2\right)}{f^2}\right)\log\left(\left(ex^2+d\right)^pc\right)$$

input
```
integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*e*p*(log(e*x^2 + d)/(e*f^2 - d*f*g) - log(g*x^2 + f)/(e*f^2 - d*f*g)
+ (2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))/(e*f^2) - (log(g*x^2 + f)*
log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d*g))
)/(e*f^2)) + 1/2*(1/(f*g*x^2 + f^2) - log(g*x^2 + f)/f^2 + log(x^2)/f^2)*l
og((e*x^2 + d)^p*c)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{\log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2 x}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{\ln \left(c\left(e\,x^2+d\right)^p\right)}{x\left(g\,x^2+f\right)^2}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{\log\left((e\,x^2+d)^p\,c\right)}{g^2x^5+2fg\,x^3+f^2x}dx$$

input
```
int(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x)
```

output
```
int(log((d + e*x**2)**p*c)/(f**2*x + 2*f*g*x**3 + g**2*x**5),x)
```

3.352 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 251

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = \frac{ep\log(x)}{df^2}-\frac{ep\log\left(d+ex^2\right)}{2df^2}+\frac{egp\log\left(d+ex^2\right)}{2f^2(ef-dg)}-\frac{\log\left(c(d+ex^2)^p\right)}{2f^2x^2}-\frac{g\log\left(c(d+ex^2)^p\right)}{2f^2\left(f+gx^2\right)}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{f^3}-\frac{egp\log\left(f+gx^2\right)}{2f^2(ef-dg)}+\frac{g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{f^3}+\frac{gp\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{f^3}-\frac{gp\,\mathrm{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{f^3}$$

output
```
e*p*ln(x)/d/f^2-1/2*e*p*ln(e*x^2+d)/d/f^2+1/2*e*g*p*ln(e*x^2+d)/f^2/(-d*g+
e*f)-1/2*ln(c*(e*x^2+d)^p)/f^2/x^2-1/2*g*ln(c*(e*x^2+d)^p)/f^2/(g*x^2+f)-g
*ln(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^3-1/2*e*g*p*ln(g*x^2+f)/f^2/(-d*g+e*f)+g
*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/f^3+g*p*polylog(2,-g*(e*x^2+
d)/(-d*g+e*f))/f^3-g*p*polylog(2,1+e*x^2/d)/f^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.89

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3 \left(f+gx^2\right)^2} dx$$
$$= \frac{\frac{2efp\log(x)}{d} - \frac{efp\log(d+ex^2)}{d} + \frac{efgp\log(d+ex^2)}{ef-dg} - \frac{f\log(c(d+ex^2)^p)}{x^2} - \frac{fg\log(c(d+ex^2)^p)}{f+gx^2} - 2g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex\right.}{}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

output
```
((2*e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/d + (e*f*g*p*Log[d + e*x^2])/
(e*f - d*g) - (f*Log[c*(d + e*x^2)^p])/x^2 - (f*g*Log[c*(d + e*x^2)^p])/(f
 + g*x^2) - 2*g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + (e*f*g*p*Log[f +
g*x^2])/(-(e*f) + d*g) + 2*g*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f
 - d*g)] + 2*g*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)] - 2*g*p*PolyLo
g[2, 1 + (e*x^2)/d])/(2*f^3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3 \left(f+gx^2\right)^2} dx$$
$$\Big\downarrow \text{ 2925}$$
$$\frac{1}{2}\int \frac{\log \left(c(ex^2+d)^p\right)}{x^4 \left(gx^2+f\right)^2}dx^2$$
$$\Big\downarrow \text{ 2863}$$

$$\frac{1}{2}\int\left(\frac{2\log\left(c\left(ex^2+d\right)^p\right)g^2}{f^3\left(gx^2+f\right)}+\frac{\log\left(c\left(ex^2+d\right)^p\right)g^2}{f^2\left(gx^2+f\right)^2}-\frac{2\log\left(c\left(ex^2+d\right)^p\right)g}{f^3x^2}+\frac{\log\left(c\left(ex^2+d\right)^p\right)}{f^2x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3}+\frac{2g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{f^3}-\frac{g\log\left(c\left(d+ex^2\right)^p\right)}{f^2\left(f+gx^2\right)}-\frac{\log\left(c(d+\right.}{f^2x}\right.$$

input
```
Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

output
```
((e*p*Log[x^2])/(d*f^2) - (e*p*Log[d + e*x^2])/(d*f^2) + (e*g*p*Log[d + e*
x^2])/(f^2*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(f^2*x^2) - (g*Log[c*(d + e
*x^2)^p])/(f^2*(f + g*x^2)) - (2*g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])
/f^3 - (e*g*p*Log[f + g*x^2])/(f^2*(e*f - d*g)) + (2*g*Log[c*(d + e*x^2)^p
]*Log[(e*(f + g*x^2))/(e*f - d*g)])/f^3 + (2*g*p*PolyLog[2, -((g*(d + e*x^
2))/(e*f - d*g))])/f^3 - (2*g*p*PolyLog[2, 1 + (e*x^2)/d])/f^3)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.01 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.13

method	result
parts	$-\frac{\ln(c(e\,x^2+d)^p)}{2f^2x^2}-\frac{2\ln(c(e\,x^2+d)^p)\,g\ln(x)}{f^3}+\frac{\ln(c(e\,x^2+d)^p)\,g\ln(g\,x^2+f)}{f^3}-\frac{g\ln(c(e\,x^2+d)^p)}{2f^2(g\,x^2+f)}-ep\Bigg(-\frac{4g\Big(\frac{\ln(x)\big(\ln\big(\cdots}{\cdots}$
risch	Expression too large to display

input
```
int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(c*(e*x^2+d)^p)/f^2/x^2-2*ln(c*(e*x^2+d)^p)/f^3*g*ln(x)+ln(c*(e*x^2
+d)^p)*g/f^3*ln(g*x^2+f)-1/2*g*ln(c*(e*x^2+d)^p)/f^2/(g*x^2+f)-e*p*(-4*g/f
^3*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x+(-d*e)^(1/2))/
(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+dilog((e*x+(
-d*e)^(1/2))/(-d*e)^(1/2)))/e)+g/f^3/e*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_
alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Roo
tOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_a
lpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*
f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/f^2*(-1/2*(2*d*g-e*f)/d/
(d*g-e*f)*ln(e*x^2+d)+1/d*ln(x)+1/2*g/(d*g-e*f)*ln(g*x^2+f)))
```

Fricas [F]

$$\int\frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int\frac{\log\left(\left(ex^2+d\right)^pc\right)}{\left(gx^2+f\right)^2x^3}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g^2*x^7 + 2*f*g*x^5 + f^2*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.18

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = -\frac{1}{2}\left(f\left(\frac{e\log\left(ex^2+d\right)}{def^3-d^2f^2g}-\frac{g\log\left(gx^2+f\right)}{ef^4-df^3g}-\frac{\log\left(x^2\right)}{df^3}\right)-2\,g\left(\frac{\log\left(ex^2+d\right)}{ef^3-df^2g}-\frac{\log\left(gx^2+f\right)}{ef^3-df^2g}\right)-\frac{2\left(2 l\right.}{}\right. \\ -\frac{1}{2}\left(\frac{2\,gx^2+f}{f^2gx^4+f^3x^2}-\frac{2\,g\log\left(gx^2+f\right)}{f^3}+\frac{2\,g\log\left(x^2\right)}{f^3}\right)\log\left(\left(ex^2+d\right)^pc\right)$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
-1/2*(f*(e*log(e*x^2 + d)/(d*e*f^3 - d^2*f^2*g) - g*log(g*x^2 + f)/(e*f^4
- d*f^3*g) - log(x^2)/(d*f^3)) - 2*g*(log(e*x^2 + d)/(e*f^3 - d*f^2*g) - l
og(g*x^2 + f)/(e*f^3 - d*f^2*g)) - 2*(2*log(e*x^2/d + 1)*log(x) + dilog(-e
*x^2/d))*g/(e*f^3) + 2*(log(g*x^2 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) +
1) + dilog((e*g*x^2 + e*f)/(e*f - d*g)))*g/(e*f^3))*e*p - 1/2*((2*g*x^2 +
f)/(f^2*g*x^4 + f^3*x^2) - 2*g*log(g*x^2 + f)/f^3 + 2*g*log(x^2)/f^3)*log(
(e*x^2 + d)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{(gx^2+f)^2 x^3}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = \int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^3\left(g\,x^2+f\right)^2}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left(\left(e\,x^2+d\right)^p c\right)}{g^2x^7+2fg\,x^5+f^2x^3}dx$$

input
```
int(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x)
```

output
```
int(log((d + e*x**2)**p*c)/(f**2*x**3 + 2*f*g*x**5 + g**2*x**7),x)
```

3.353 $\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 802

$$\int \frac{x^4 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = -\frac{2px}{g^2} + \frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} + \frac{\sqrt{d}\sqrt{e}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} - \frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} + \frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} + \frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} + \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2g^{5/2}} + \frac{3i\sqrt{f}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2g^{5/2}} - \frac{3i\sqrt{f}p\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}} - \frac{3i\sqrt{f}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}}$$

output
```
-2*p*x/g^2+2*d^(1/2)*p*arctan(e^(1/2)*x/d^(1/2))/e^(1/2)/g^2+d^(1/2)*e^(1/
2)*f*p*arctan(e^(1/2)*x/d^(1/2))/g^2/(-d*g+e*f)-1/2*e*(-f)^(3/2)*p*ln((-f)
^(1/2)-g^(1/2)*x)/g^(5/2)/(-d*g+e*f)-3*f^(1/2)*p*arctan(g^(1/2)*x/f^(1/2))
*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/g^(5/2)+3/2*f^(1/2)*p*arctan(g^(1/2)*
x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)
-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(5/2)+3/2*f^(1/2)*p*arctan(g
^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f
^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(5/2)+1/2*e*(-f)^(3/2)
*p*ln((-f)^(1/2)+g^(1/2)*x)/g^(5/2)/(-d*g+e*f)+x*ln(c*(e*x^2+d)^p)/g^2-1/4
*f*ln(c*(e*x^2+d)^p)/g^(5/2)/((-f)^(1/2)-g^(1/2)*x)+1/4*f*ln(c*(e*x^2+d)^p
)/g^(5/2)/((-f)^(1/2)+g^(1/2)*x)-3/2*f^(1/2)*arctan(g^(1/2)*x/f^(1/2))*ln(
c*(e*x^2+d)^p)/g^(5/2)+3/2*I*f^(1/2)*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^
(1/2)*x))/g^(5/2)-3/4*I*f^(1/2)*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2
)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))
/g^(5/2)-3/4*I*f^(1/2)*p*polylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)
*x)/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/g^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.07 (sec) , antiderivative size = 915, normalized size of antiderivative = 1.14

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
(-8*Sqrt[g]*p*x + (8*Sqrt[d]*Sqrt[g]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e
] + (2*Sqrt[-d]*Sqrt[e]*f*Sqrt[g]*p*Log[Sqrt[-d] - Sqrt[e]*x])/(-(e*f) + d
*g) + (2*Sqrt[-d]*Sqrt[e]*f*Sqrt[g]*p*Log[Sqrt[-d] + Sqrt[e]*x])/(e*f - d*
g) + (2*e*Sqrt[-f]*f*p*Log[Sqrt[-f] - Sqrt[g]*x])/(e*f - d*g) + (2*e*(-f)^
(3/2)*p*Log[Sqrt[-f] + Sqrt[g]*x])/(e*f - d*g) + (3*I)*Sqrt[f]*p*Log[(Sqrt
[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1
- (I*Sqrt[g]*x)/Sqrt[f]] + (3*I)*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e
]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqr
t[f]] - (3*I)*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]
*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - (3*I)*Sqrt[
f]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sq
rt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + 4*Sqrt[g]*x*Log[c*(d + e*x^2)^p]
- (f*Log[c*(d + e*x^2)^p])/(Sqrt[-f] - Sqrt[g]*x) + (f*Log[c*(d + e*x^2)^p
])/(Sqrt[-f] + Sqrt[g]*x) - 6*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d
 + e*x^2)^p] + (3*I)*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x)
)/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + (3*I)*Sqrt[f]*p*PolyLog[2, (Sq
rt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - (
3*I)*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[
f] - I*Sqrt[-d]*Sqrt[g])] - (3*I)*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] +
 I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/(4*g^(5/2))
```

Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 802, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \log \left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} \, dx$$

$$\Big\downarrow \text{ 2926}$$

$$\int \left(\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{g^2 \left(f+gx^2\right)^2}-\frac{2 f \log \left(c\left(d+ex^2\right)^p\right)}{g^2 \left(f+gx^2\right)}+\frac{\log \left(c\left(d+ex^2\right)^p\right)}{g^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}+\frac{ep\log\left(\sqrt{g}x+\sqrt{-f}\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}-\frac{2px}{g^2}+\frac{\sqrt{d}\sqrt{e}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}+\frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}}+\frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}+\frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}+\frac{x\log\left(c(ex^2+d)^p\right)}{g^2}-\frac{3\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(ex^2+d)^p\right)}{2g^{5/2}}-\frac{f\log\left(c(ex^2+d)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{f\log\left(c(ex^2+d)^p\right)}{4g^{5/2}\left(\sqrt{g}x+\sqrt{-f}\right)}+\frac{3i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2g^{5/2}}-\frac{3i\sqrt{f}p\,\text{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4g^{5/2}}-\frac{3i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}}$$

input
```
Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
(-2*p*x)/g^2 + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) + (
Sqrt[d]*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g^2*(e*f - d*g)) - (e*(-
f)^(3/2)*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) - (3*Sqrt[f]
*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g
^(5/2) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
 I*Sqrt[g]*x))])/(2*g^(5/2)) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Lo
g[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*g^(5/2)) + (e*(-f)^(3/2)*p*Log[Sq
rt[-f] + Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) + (x*Log[c*(d + e*x^2)^p])/g^
2 - (f*Log[c*(d + e*x^2)^p])/(4*g^(5/2)*(Sqrt[-f] - Sqrt[g]*x)) + (f*Log[c
*(d + e*x^2)^p])/(4*g^(5/2)*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[f]*ArcTan[(S
qrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*g^(5/2)) + (((3*I)/2)*Sqrt[f]*
p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (((3*I)/4
)*Sqrt[f]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*
Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (
((3*I)/4)*Sqrt[f]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*
x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(
5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{x^4 \ln \left(c(e\,x^2+d)^p\right)}{\left(g\,x^2+f\right)^2}dx$$

input
```
int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^4 \log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(x^4*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2} \, dx = \text{Timed out}$$

input
```
integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2} \, dx = \int \frac{x^4 \log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2} \, dx$$

input
```
integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

input
```
int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

output
```
int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{too large to display}$$

input
```
int(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output

```
(2*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**3*f*g**4*p + 2*sqrt(e)
*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**3*g**5*p*x**2 - 10*sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*e*f**2*g**3*p - 10*sqrt(e)*sqrt(d)*a
tan((e*x)/(sqrt(e)*sqrt(d)))*d**2*e*f*g**4*p*x**2 + 14*sqrt(e)*sqrt(d)*ata
n((e*x)/(sqrt(e)*sqrt(d)))*d*e**2*f**3*g**2*p + 14*sqrt(e)*sqrt(d)*atan((e
*x)/(sqrt(e)*sqrt(d)))*d*e**2*f**2*g**3*p*x**2 + 4*sqrt(g)*sqrt(f)*atan((g
*x)/(sqrt(g)*sqrt(f)))*d**2*e**2*f**2*g**2*p + 4*sqrt(g)*sqrt(f)*atan((g*x
)/(sqrt(g)*sqrt(f)))*d**2*e**2*f*g**3*p*x**2 - 8*sqrt(g)*sqrt(f)*atan((g*x
)/(sqrt(g)*sqrt(f)))*d*e**3*f**3*g*p - 8*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(
g)*sqrt(f)))*d*e**3*f**2*g**2*p*x**2 - 2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(
g)*sqrt(f)))*e**4*f**4*p - 2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))
*e**4*f**3*g*p*x**2 + 3*int(log((d + e*x**2)**p*c)/(d**2*f**2*g**2 + 2*d**
2*f*g**3*x**2 + d**2*g**4*x**4 - 2*d*e*f**3*g - 4*d*e*f**2*g**2*x**2 - 2*d
*e*f*g**3*x**4 + e**2*f**4 + 2*e**2*f**3*g*x**2 + e**2*f**2*g**2*x**4),x)*
d**4*e**2*f**4*g**5 + 3*int(log((d + e*x**2)**p*c)/(d**2*f**2*g**2 + 2*d**
2*f*g**3*x**2 + d**2*g**4*x**4 - 2*d*e*f**3*g - 4*d*e*f**2*g**2*x**2 - 2*d
*e*f*g**3*x**4 + e**2*f**4 + 2*e**2*f**3*g*x**2 + e**2*f**2*g**2*x**4),x)*
d**4*e**2*f**3*g**6*x**2 - 9*int(log((d + e*x**2)**p*c)/(d**2*f**2*g**2 +
2*d**2*f*g**3*x**2 + d**2*g**4*x**4 - 2*d*e*f**3*g - 4*d*e*f**2*g**2*x**2
- 2*d*e*f*g**3*x**4 + e**2*f**4 + 2*e**2*f**3*g*x**2 + e**2*f**2*g**2*x...
```

3.354 $\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 746

$$\int \frac{x^2 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = -\frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)}$$
$$+ \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}}$$
$$- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}}$$
$$- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}}$$
$$+ \frac{e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$- \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}}$$
$$- \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}g^{3/2}}$$
$$+ \frac{ip\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}}$$
$$+ \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}}$$

output
```
-d^(1/2)*e^(1/2)*p*arctan(e^(1/2)*x/d^(1/2))/g/(-d*g+e*f)-1/2*e*(-f)^(1/2)
*p*ln((-f)^(1/2)-g^(1/2)*x)/g^(3/2)/(-d*g+e*f)+p*arctan(g^(1/2)*x/f^(1/2))
*ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(3/2)-1/2*p*arctan(g^(1/2)*
x/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)
-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(3/2)-1/2*p*arctan(g
^(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f
^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(3/2)+1/2*e*(-
f)^(1/2)*p*ln((-f)^(1/2)+g^(1/2)*x)/g^(3/2)/(-d*g+e*f)+1/4*ln(c*(e*x^2+d)^
p)/g^(3/2)/((-f)^(1/2)-g^(1/2)*x)-1/4*ln(c*(e*x^2+d)^p)/g^(3/2)/((-f)^(1/2
)+g^(1/2)*x)+1/2*arctan(g^(1/2)*x/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(3/
2)-1/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(3/2)+1/
4*I*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1
/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(3/2)+1/4*I*p*pol
ylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(1/2)/g^(3/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.14

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx$$

$$= \frac{\frac{2\sqrt{-d}\sqrt{e}\sqrt{g}p\log\left(\sqrt{-d}-\sqrt{e}x\right)}{ef-dg} + \frac{2\sqrt{-d}\sqrt{e}\sqrt{g}p\log\left(\sqrt{-d}+\sqrt{e}x\right)}{-ef+dg} - \frac{2e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{ef-dg} + \frac{2e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{ef-dg} - \frac{ip\log\left(\frac{\sqrt{g}(\cdots}{i\sqrt{e}\cdots}\right.\cdots}{\cdots}\cdots}{\cdots}$$

input
```
Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
((2*Sqrt[-d]*Sqrt[e]*Sqrt[g]*p*Log[Sqrt[-d] - Sqrt[e]*x])/(e*f - d*g) + (2
*Sqrt[-d]*Sqrt[e]*Sqrt[g]*p*Log[Sqrt[-d] + Sqrt[e]*x])/(-(e*f) + d*g) - (2
*e*Sqrt[-f]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(e*f - d*g) + (2*e*Sqrt[-f]*p*Log
[Sqrt[-f] + Sqrt[g]*x])/(e*f - d*g) - (I*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e
]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f
]])/Sqrt[f] - (I*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt
[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (I*p*Lo
g[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g
])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (I*p*Log[(Sqrt[g]*(Sqrt[-d]
+ Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x
)/Sqrt[f]])/Sqrt[f] + Log[c*(d + e*x^2)^p]/(Sqrt[-f] - Sqrt[g]*x) - Log[c*
(d + e*x^2)^p]/(Sqrt[-f] + Sqrt[g]*x) + (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log
[c*(d + e*x^2)^p])/Sqrt[f] - (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]
*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])])/Sqrt[f] - (I*p*PolyLog[2, (S
qrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/S
qrt[f] + (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f
] - I*Sqrt[-d]*Sqrt[g])])/Sqrt[f] + (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*
Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/Sqrt[f])/(4*g^(3/2))
```

Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log \left(c \left(d+e x^2\right)^p\right)}{\left(f+g x^2\right)^2} \, dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(\frac{\log \left(c \left(d+e x^2\right)^p\right)}{g \left(f+g x^2\right)}-\frac{f \log \left(c \left(d+e x^2\right)^p\right)}{g \left(f+g x^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}}-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2\sqrt{f}g^{3/2}}-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2\sqrt{f}g^{3/2}}-\frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)}+\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}}+\frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)}+\frac{ip\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4\sqrt{f}g^{3/2}}+\frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}}-\frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)}+\frac{e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{3/2}(ef-dg)}-\frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}g^{3/2}}$$

input
```
Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

output
```
-((Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g*(e*f - d*g))) - (e*Sq
rt[-f]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(3/2)*(e*f - d*g)) + (p*ArcTan[(S
qrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3
/2)) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] -
Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x
))])/(2*Sqrt[f]*g^(3/2)) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*S
qrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sq
rt[f] - I*Sqrt[g]*x))])/(2*Sqrt[f]*g^(3/2)) + (e*Sqrt[-f]*p*Log[Sqrt[-f] +
 Sqrt[g]*x])/(2*g^(3/2)*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sq
rt[-f] - Sqrt[g]*x)) - Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] + Sqrt[g]
*x)) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*Sqrt[f]*g^(3/
2)) - ((I/2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[
f]*g^(3/2)) + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[
e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/
(Sqrt[f]*g^(3/2)) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] +
 Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*
x))])/(Sqrt[f]*g^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{x^{2} \ln \left(c {\left(e \,x^{2}+d \right)}^{p}\right)}{\left(g \,x^{2}+f \right)^{2}}d x$$

input
```
int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \int \frac{x^{2} \log \left({\left(e x^{2} + d\right)}^{p} c\right)}{{\left(g x^{2} + f\right)}^{2}} \,d x$$

input
```
integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(x^2*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^2 \log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^2\,\ln\left(c\,(e\,x^2+d)^p\right)}{(g\,x^2+f)^2}\,dx$$

input
```
int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

output
```
int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{too large to display}$$

input
```
int(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
(2*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*f*g**2*p + 2*sqrt(e)
*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*d**2*g**3*p*x**2 - 4*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*d*e*f**2*g*p - 4*sqrt(e)*sqrt(d)*atan((e*x
)/(sqrt(e)*sqrt(d)))*d*e*f*g**2*p*x**2 - 2*sqrt(g)*sqrt(f)*atan((g*x)/(sqr
t(g)*sqrt(f)))*d**2*e*f*g*p - 2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f
)))*d**2*e*g**2*p*x**2 + 4*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*d
*e**2*f**2*p + 4*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*d*e**2*f*g*
p*x**2 - int(log((d + e*x**2)**p*c)/(d**2*f**2*g**2 + 2*d**2*f*g**3*x**2 +
 d**2*g**4*x**4 - 2*d*e*f**3*g - 4*d*e*f**2*g**2*x**2 - 2*d*e*f*g**3*x**4
+ e**2*f**4 + 2*e**2*f**3*g*x**2 + e**2*f**2*g**2*x**4),x)*d**4*e*f**3*g**
4 - int(log((d + e*x**2)**p*c)/(d**2*f**2*g**2 + 2*d**2*f*g**3*x**2 + d**2
*g**4*x**4 - 2*d*e*f**3*g - 4*d*e*f**2*g**2*x**2 - 2*d*e*f*g**3*x**4 + e**
2*f**4 + 2*e**2*f**3*g*x**2 + e**2*f**2*g**2*x**4),x)*d**4*e*f**2*g**5*x**
2 + 3*int(log((d + e*x**2)**p*c)/(d**2*f**2*g**2 + 2*d**2*f*g**3*x**2 + d*
*2*g**4*x**4 - 2*d*e*f**3*g - 4*d*e*f**2*g**2*x**2 - 2*d*e*f*g**3*x**4 + e
**2*f**4 + 2*e**2*f**3*g*x**2 + e**2*f**2*g**2*x**4),x)*d**3*e**2*f**4*g**
3 + 3*int(log((d + e*x**2)**p*c)/(d**2*f**2*g**2 + 2*d**2*f*g**3*x**2 + d*
*2*g**4*x**4 - 2*d*e*f**3*g - 4*d*e*f**2*g**2*x**2 - 2*d*e*f*g**3*x**4 + e
**2*f**4 + 2*e**2*f**3*g*x**2 + e**2*f**2*g**2*x**4),x)*d**3*e**2*f**3*g**
4*x**2 - 3*int(log((d + e*x**2)**p*c)/(d**2*f**2*g**2 + 2*d**2*f*g**3*x...
```

3.355 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 751

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} - \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}} + \frac{ip\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}} + \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}$$

output
```
d^(1/2)*e^(1/2)*p*arctan(e^(1/2)*x/d^(1/2))/f/(-d*g+e*f)-1/2*e*p*ln((-f)^(
1/2)-g^(1/2)*x)/(-f)^(1/2)/g^(1/2)/(-d*g+e*f)+p*arctan(g^(1/2)*x/f^(1/2))*
ln(2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)-1/2*p*arctan(g^(1/2)*x
/f^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)-
(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)-1/2*p*arctan(g^
(1/2)*x/f^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^
(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)+1/2*e*p*l
n((-f)^(1/2)+g^(1/2)*x)/(-f)^(1/2)/g^(1/2)/(-d*g+e*f)-1/4*ln(c*(e*x^2+d)^p
)/f/g^(1/2)/((-f)^(1/2)-g^(1/2)*x)+1/4*ln(c*(e*x^2+d)^p)/f/g^(1/2)/((-f)^(
1/2)+g^(1/2)*x)+1/2*arctan(g^(1/2)*x/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(3/2)/g^
(1/2)-1/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)
+1/4*I*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f
^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)+1/4*I*p*
polylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-
d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(3/2)/g^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.17

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$$

$$= \frac{\frac{2\sqrt{-d}\sqrt{e}\sqrt{f}p\log(\sqrt{-d}-\sqrt{e}x)}{-ef+dg} + \frac{2\sqrt{-d}\sqrt{e}\sqrt{f}p\log(\sqrt{-d}+\sqrt{e}x)}{ef-dg} + \frac{2e\sqrt{-f^2}p\log(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}(ef-dg)} + \frac{2e\sqrt{-f^2}p\log(\sqrt{-f}+\sqrt{g}x)}{\sqrt{g}(-ef+dg)} - \frac{ip\log\left(\cdots\right.}{i\cdots}}{\cdots}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

output
```
((2*Sqrt[-d]*Sqrt[e]*Sqrt[f]*p*Log[Sqrt[-d] - Sqrt[e]*x])/(-(e*f) + d*g) +
 (2*Sqrt[-d]*Sqrt[e]*Sqrt[f]*p*Log[Sqrt[-d] + Sqrt[e]*x])/(e*f - d*g) + (2
*e*Sqrt[-f^2]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(Sqrt[g]*(e*f - d*g)) + (2*e*Sq
rt[-f^2]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(Sqrt[g]*(-(e*f) + d*g)) - (I*p*Log[
(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*L
og[1 - (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] - (I*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqr
t[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/
Sqrt[f]])/Sqrt[g] + (I*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + (
I*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqr
t[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + (Sqrt[f]*Log[c*(d + e*x^2
)^p])/(-(Sqrt[-f]*Sqrt[g]) + g*x) + (Sqrt[f]*Log[c*(d + e*x^2)^p])/(Sqrt[-
f]*Sqrt[g] + g*x) + (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/S
qrt[g] - (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f
] - I*Sqrt[-d]*Sqrt[g])])/Sqrt[g] - (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*
Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/Sqrt[g] + (I*p*PolyLo
g[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[
g])])/Sqrt[g] + (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]
*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/Sqrt[g])/(4*f^(3/2))
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{2921, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2} \, dx$$

$$\Big\downarrow \text{ 2921}$$

$$\int \left(-\frac{g\log \left(c\left(d+ex^2\right)^p\right)}{2f\left(-fg-g^2x^2\right)}-\frac{g\log \left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g\log \left(c\left(d+ex^2\right)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2f^{3/2}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2f^{3/2}\sqrt{g}} + \frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} + \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} - \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{ip\,\mathrm{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4f^{3/2}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}}$$

input

```
Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

output

```
(Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log
[Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt
[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]
) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqr
t[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))]
)/(2*f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt
[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[
f] - I*Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])
/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqr
t[-f] - Sqrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g
]*x)) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt
[g]) - ((I/2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3
/2)*Sqrt[g]) + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt
[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])
/(f^(3/2)*Sqrt[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d]
+ Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x))])/(f^(3/2)*Sqrt[g])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [F]

$$\int \frac{\ln\left(c(e\,x^{2}+d)^{p}\right)}{\left(g\,x^{2}+f\right)^{2}}dx$$

input
```
int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^{2})^{p}\right)}{\left(f+gx^{2}\right)^{2}}\,dx=\int \frac{\log\left(\left(ex^{2}+d\right)^{p}c\right)}{\left(gx^{2}+f\right)^{2}}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \int \frac{\ln \left(c\,(e\,x^2+d)^p\right)}{\left(g\,x^2+f\right)^2} \, dx$$

input
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2,x)
```

output
```
int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \int \frac{\log\left((e\,x^2+d)^p\,c\right)}{g^2x^4+2fg\,x^2+f^2} dx$$

input
```
int(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

output
```
int(log((d + e*x**2)**p*c)/(f**2 + 2*f*g*x**2 + g**2*x**4),x)
```

3.356 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 803

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx = \frac{2\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} + \frac{3\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} + \frac{3\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} + \frac{e\sqrt{g}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{5/2}} + \frac{3i\sqrt{g}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{5/2}} - \frac{3i\sqrt{g}p\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{5/2}} - \frac{3i\sqrt{g}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{5/2}}$$

output
```
2*e^(1/2)*p*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/f^2-d^(1/2)*e^(1/2)*g*p*arct
an(e^(1/2)*x/d^(1/2))/f^2/(-d*g+e*f)-1/2*e*g^(1/2)*p*ln((-f)^(1/2)-g^(1/2)
*x)/(-f)^(3/2)/(-d*g+e*f)-3*g^(1/2)*p*arctan(g^(1/2)*x/f^(1/2))*ln(2*f^(1/
2)/(f^(1/2)-I*g^(1/2)*x))/f^(5/2)+3/2*g^(1/2)*p*arctan(g^(1/2)*x/f^(1/2))*
ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/2)*x)/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)
*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(5/2)+3/2*g^(1/2)*p*arctan(g^(1/2)*x/f^
(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*e^(1/2)*f^(1/2)+(-d)
^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(5/2)+1/2*e*g^(1/2)*p*ln((-f)^(1/
2)+g^(1/2)*x)/(-f)^(3/2)/(-d*g+e*f)-ln(c*(e*x^2+d)^p)/f^2/x+1/4*g^(1/2)*ln
(c*(e*x^2+d)^p)/f^2/((-f)^(1/2)-g^(1/2)*x)-1/4*g^(1/2)*ln(c*(e*x^2+d)^p)/f
^2/((-f)^(1/2)+g^(1/2)*x)-3/2*g^(1/2)*arctan(g^(1/2)*x/f^(1/2))*ln(c*(e*x^
2+d)^p)/f^(5/2)+3/2*I*g^(1/2)*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*g^(1/2)*x
))/f^(5/2)-3/4*I*g^(1/2)*p*polylog(2,1+2*f^(1/2)*g^(1/2)*((-d)^(1/2)-e^(1/
2)*x)/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(5/2
)-3/4*I*g^(1/2)*p*polylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/2)+e^(1/2)*x)/(I*
e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2))/(f^(1/2)-I*g^(1/2)*x))/f^(5/2)
```

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 939, normalized size of antiderivative = 1.17

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 \left(f+gx^2\right)^2} \, dx = \text{Too large to display}$$

input
```
Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```

output
```
((8*Sqrt[e]*Sqrt[f]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[-d]*S
qrt[e]*Sqrt[f]*g*p*Log[Sqrt[-d] - Sqrt[e]*x])/(e*f - d*g) + (2*Sqrt[-d]*Sq
rt[e]*Sqrt[f]*g*p*Log[Sqrt[-d] + Sqrt[e]*x])/(-(e*f) + d*g) - (2*e*Sqrt[-f
^2]*Sqrt[g]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(e*f - d*g) + (2*e*Sqrt[-f^2]*Sqr
t[g]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(e*f - d*g) + (3*I)*Sqrt[g]*p*Log[(Sqrt[
g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 -
 (I*Sqrt[g]*x)/Sqrt[f]] + (3*I)*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]
*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt
[f]] - (3*I)*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*
Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - (3*I)*Sqrt[g
]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqr
t[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - (4*Sqrt[f]*Log[c*(d + e*x^2)^p])/x
 + (Sqrt[f]*Sqrt[g]*Log[c*(d + e*x^2)^p])/(Sqrt[-f] - Sqrt[g]*x) - (Sqrt[f
]*Sqrt[g]*Log[c*(d + e*x^2)^p])/(Sqrt[-f] + Sqrt[g]*x) - 6*Sqrt[g]*ArcTan[
(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + (3*I)*Sqrt[g]*p*PolyLog[2, (Sq
rt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + (
3*I)*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[
f] + I*Sqrt[-d]*Sqrt[g])] - (3*I)*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] +
 I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - (3*I)*Sqrt[g]*p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-...
```

Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x^2 \left(f+gx^2\right)^2} \, dx$$

$$\Big\downarrow \text{2926}$$

$$\int \left(-\frac{g \log \left(c\left(d+ex^2\right)^p\right)}{f^2 \left(f+gx^2\right)}+\frac{\log \left(c\left(d+ex^2\right)^p\right)}{f^2x^2}-\frac{g \log \left(c\left(d+ex^2\right)^p\right)}{f \left(f+gx^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{d}\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)}+\frac{2\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2}-\frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)}-\frac{3\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}}+\frac{3\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}}+\frac{3\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}}+\frac{e\sqrt{g}p\log\left(\sqrt{g}x+\sqrt{-f}\right)}{2(-f)^{3/2}(ef-dg)}-\frac{3\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(ex^2+d)^p\right)}{2f^{5/2}}-\frac{\log\left(c(ex^2+d)^p\right)}{f^2x}+\frac{\sqrt{g}\log\left(c(ex^2+d)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)}-\frac{\sqrt{g}\log\left(c(ex^2+d)^p\right)}{4f^2\left(\sqrt{g}x+\sqrt{-f}\right)}+\frac{3i\sqrt{g}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{5/2}}-\frac{3i\sqrt{g}p\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4f^{5/2}}-\frac{3i\sqrt{g}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{5/2}}$$

input

```
Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```

output

```
(2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) - (Sqrt[d]*Sqrt[e]
*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f^2*(e*f - d*g)) - (e*Sqrt[g]*p*Log[Sqr
t[-f] - Sqrt[g]*x])/(2*(-f)^(3/2)*(e*f - d*g)) - (3*Sqrt[g]*p*ArcTan[(Sqrt
[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) + (3*Sqr
t[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqr
t[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))]
)/(2*f^(5/2)) + (3*Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sq
rt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqr
t[f] - I*Sqrt[g]*x))])/(2*f^(5/2)) + (e*Sqrt[g]*p*Log[Sqrt[-f] + Sqrt[g]*x
])/(2*(-f)^(3/2)*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(f^2*x) + (Sqrt[g]*Lo
g[c*(d + e*x^2)^p])/(4*f^2*(Sqrt[-f] - Sqrt[g]*x)) - (Sqrt[g]*Log[c*(d + e
*x^2)^p])/(4*f^2*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[g]*ArcTan[(Sqrt[g]*x)/S
qrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(5/2)) + (((3*I)/2)*Sqrt[g]*p*PolyLog[2
, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p
*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqr
t[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2) - (((3*I)/4)*S
qrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqr
t[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{\ln \left(c(e\,x^{2}+d)^{p}\right)}{x^{2} \left(g\,x^{2}+f\right)^{2}}dx$$

input
```
int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

output
```
int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 \left(f+gx^2\right)^2}\,dx = \int \frac{\log \left((ex^2+d)^p c\right)}{(gx^2+f)^2 x^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^2 + d)^p*c)/(g^2*x^6 + 2*f*g*x^4 + f^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 \left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 \left(f+gx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 \left(f+gx^2\right)^2}\,dx = \int \frac{\log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2 x^2}\,dx$$

input
```
integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx = \int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^2\left(g\,x^2+f\right)^2}\,dx$$

input
```
int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)^2),x)
```

output
```
int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left(\left(e\,x^2+d\right)^p c\right)}{g^2x^6+2fg\,x^4+f^2x^2}dx$$

input
```
int(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

output
```
int(log((d + e*x**2)**p*c)/(f**2*x**2 + 2*f*g*x**4 + g**2*x**6),x)
```

3.357 $\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 163

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \frac{in\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{in\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
I*n*arctan(b^(1/2)*x/a^(1/2))^2/a^(1/2)/b^(1/2)+2*n*arctan(b^(1/2)*x/a^(1/
2))*ln(2*a^(1/2)/(a^(1/2)+I*b^(1/2)*x))/a^(1/2)/b^(1/2)+arctan(b^(1/2)*x/a
^(1/2))*ln(c*(b*x^2+a)^n)/a^(1/2)/b^(1/2)+I*n*polylog(2,1-2*a^(1/2)/(a^(1/
2)+I*b^(1/2)*x))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.79

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(in\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+2n\log\left(\frac{2i}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\right)+\log\left(c(a+bx^2)^n\right)\right)+in\operatorname{PolyLog}\left(2,\frac{i\sqrt{a}+\sqrt{b}x}{-i\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```

output
```
(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*n*Log[(2
*I)/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^n]) + I*n*PolyLog[2, (I
*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2920, 27, 5455, 27, 5379, 27, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx$$

$$\Big\downarrow \ 2920$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^n\right)}{\sqrt{a}\sqrt{b}}-2bn\int \frac{x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,(bx^2+a)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^n\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}n\int \frac{x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{bx^2+a}dx}{\sqrt{a}}$$

$$\Big\downarrow \ 5455$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}n\left(-\frac{\int\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i\sqrt{a}-\sqrt{b}x}dx}{\sqrt{a}\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}n\left(-\frac{\int\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i\sqrt{a}-\sqrt{b}x}dx}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}$$

$$\Big\downarrow \textbf{5379}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}n\left(-\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}-\frac{\int\frac{a\log\left(\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{bx^2+a}dx}{\sqrt{a}}}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}n\left(-\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}-\sqrt{a}\int\frac{\log\left(\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{bx^2+a}dx}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}$$

$$\Big\downarrow \textbf{2849}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}}-\frac{2\sqrt{b}n\left(-\frac{\frac{i\sqrt{a}\int\frac{\log\left(\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}}d\frac{1}{i\sqrt{b}x+\sqrt{a}}}{\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}$$

$$\Big\downarrow \textbf{2752}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} - \frac{2\sqrt{b}n\left(-\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{2\sqrt{b}}}{\sqrt{b}}-\frac{i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{2b}\right)}{\sqrt{a}}$$

input
```
Int[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```

output
```
(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^n])/(Sqrt[a]*Sqrt[b]) - (2*
Sqrt[b]*n*(((-1/2*I)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/b - ((ArcTan[(Sqrt[b]*
x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] + ((I/2)*Pol
yLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b])/Sqrt[b]))/Sqrt[
a]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.51 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.62

method	result
risch	$\frac{\left(\ln\left(\left(b\,x^{2}+a\right)^{n}\right)-n\ln\left(b\,x^{2}+a\right)\right)\arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}}+\frac{n\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^{2}+a\right)}\frac{2\ln\left(x-\_\alpha\right)\ln\left(b\,x^{2}+a\right)-b\left(\frac{\ln\left(x-\_\alpha\right)^{2}}{\_\alpha b}+\frac{2\_\alpha\ln\left(x-\_\alpha\right)\ln}{a}\right.}{\_\alpha}\right.}{4b}$

input
```
int(ln(c*(b*x^2+a)^n)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(ln((b*x^2+a)^n)-n*ln(b*x^2+a))/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/4*n/
b*sum(1/_alpha*(2*ln(x-_alpha)*ln(b*x^2+a)-b*(1/_alpha/b*ln(x-_alpha)^2+2*
_alpha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/a*dilog(1/2*(x+_a
lpha)/_alpha))),_alpha=RootOf(_Z^2*b+a))+(1/2*I*Pi*csgn(I*(b*x^2+a)^n)*csg
n(I*c*(b*x^2+a)^n)^2-1/2*I*Pi*csgn(I*(b*x^2+a)^n)*csgn(I*c*(b*x^2+a)^n)*cs
gn(I*c)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^n)^3+1/2*I*Pi*csgn(I*c*(b*x^2+a)^n)^2*
csgn(I*c)+ln(c))/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\log\left((bx^2+a)^n c\right)}{bx^2+a}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx$$

input
```
integrate(ln(c*(b*x**2+a)**n)/(b*x**2+a),x)
```

output
```
Integral(log(c*(a + b*x**2)**n)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\log\left((bx^2+a)^n c\right)}{bx^2+a}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\log\left((bx^2+a)^n c\right)}{bx^2+a}\,dx$$

input
```
integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^n\right)}{b\,x^2+a}\,dx$$

input
```
int(log(c*(a + b*x^2)^n)/(a + b*x^2),x)
```

output
```
int(log(c*(a + b*x^2)^n)/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\log\left((b\,x^2+a)^n\,c\right)}{b\,x^2+a}dx$$

input
```
int(log(c*(b*x^2+a)^n)/(b*x^2+a),x)
```

output
```
int(log((a + b*x**2)**n*c)/(a + b*x**2),x)
```

3.358 $\int \frac{\log(1-x^2)}{2-x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = \sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{(2-\sqrt{2})(\sqrt{2}+x)}\right)}{\sqrt{2}} - \frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(1+x)}{(2+\sqrt{2})(\sqrt{2}+x)}\right)}{\sqrt{2}} + \frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} - \frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{\sqrt{2}+x}\right)}{\sqrt{2}} + \frac{\text{PolyLog}\left(2,1+\frac{4(1-x)}{(2-\sqrt{2})(\sqrt{2}+x)}\right)}{2\sqrt{2}} + \frac{\text{PolyLog}\left(2,1-\frac{4(1+x)}{(2+\sqrt{2})(\sqrt{2}+x)}\right)}{2\sqrt{2}}$$

output
```
2^(1/2)*arctanh(1/2*x*2^(1/2))*ln(2*2^(1/2)/(2^(1/2)+x))-1/2*arctanh(1/2*x
*2^(1/2))*ln((-4+4*x)/(2-2^(1/2))/(2^(1/2)+x))*2^(1/2)-1/2*arctanh(1/2*x*2
^(1/2))*ln(4*(1+x)/(2+2^(1/2))/(2^(1/2)+x))*2^(1/2)+1/2*arctanh(1/2*x*2^(1
/2))*ln(-x^2+1)*2^(1/2)-1/2*polylog(2,1-2*2^(1/2)/(2^(1/2)+x))*2^(1/2)+1/4
*polylog(2,1+4*(1-x)/(2-2^(1/2))/(2^(1/2)+x))*2^(1/2)+1/4*polylog(2,1-4*(1
+x)/(2+2^(1/2))/(2^(1/2)+x))*2^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.67

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx$$
$$= \frac{\log\left(-1+\sqrt{2}\right)\log(-1+x)-\log\left(1+\sqrt{2}\right)\log(-1+x)-\log\left(-1+\sqrt{2}\right)\log(1+x)+\log\left(1+\sqrt{2}\right)\log}{}$$

input
```
Integrate[Log[1 - x^2]/(2 - x^2),x]
```

output
```
(Log[-1 + Sqrt[2]]*Log[-1 + x] - Log[1 + Sqrt[2]]*Log[-1 + x] - Log[-1 + S
qrt[2]]*Log[1 + x] + Log[1 + Sqrt[2]]*Log[1 + x] - Log[Sqrt[2] - x]*Log[1
- x^2] + Log[Sqrt[2] + x]*Log[1 - x^2] + PolyLog[2, -((-1 + Sqrt[2])*(-1 +
 x))] - PolyLog[2, (1 + Sqrt[2])*(-1 + x)] - PolyLog[2, (-1 + Sqrt[2])*(1
+ x)] + PolyLog[2, -((1 + Sqrt[2])*(1 + x))])/(2*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2920, 27, 6554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx$$

$$\Big\downarrow \; 2920$$

$$2\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}\,(1-x^2)}dx+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}}$$

$$\Big\downarrow \; 27$$

$$\sqrt{2}\int \frac{x\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}dx+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}}$$

$$\Big\downarrow \; 6554$$

$$\sqrt{2}\int \left(-\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{2(x-1)}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)}{2(x+1)}\right)dx+\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}}$$

$$\Big\downarrow \; 2009$$

$$\sqrt{2}\left(\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{x+\sqrt{2}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{(2-\sqrt{2})\,(x+\sqrt{2})}\right)-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{}{(2}\right.\right.$$
$$\frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}}$$

input
```
Int[Log[1 - x^2]/(2 - x^2),x]
```

output
```
(ArcTanh[x/Sqrt[2]]*Log[1 - x^2])/Sqrt[2] + Sqrt[2]*(ArcTanh[x/Sqrt[2]]*Lo
g[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt[2]]*Log[(-4*(1 - x))/((2 -
Sqrt[2])*(Sqrt[2] + x))])/2 - (ArcTanh[x/Sqrt[2]]*Log[(4*(1 + x))/((2 + Sq
rt[2])*(Sqrt[2] + x))])/2 - PolyLog[2, 1 - (2*Sqrt[2])/(Sqrt[2] + x)]/2 +
PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]/4 + PolyLog[2, 1
 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 6554
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.81

method	result
default	$-\frac{\left(\ln\left(x-\sqrt{2}\right)\ln\left(-x^{2}+1\right)-\operatorname{dilog}\left(\frac{1+x}{1+\sqrt{2}}\right)-\ln\left(x-\sqrt{2}\right)\ln\left(\frac{1+x}{1+\sqrt{2}}\right)-\operatorname{dilog}\left(\frac{x-1}{\sqrt{2}-1}\right)-\ln\left(x-\sqrt{2}\right)\ln\left(\frac{x-1}{\sqrt{2}-1}\right)\right)\sqrt{2}}{4}+\frac{\left(\ln\left(\sqrt{2}+x\right)\ln\right.}{}$
risch	$-\frac{\ln\left(-x^{2}+1\right)\ln\left(x-\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(x-\sqrt{2}\right)\ln\left(\frac{1+x}{1+\sqrt{2}}\right)\sqrt{2}}{4}+\frac{\ln\left(x-\sqrt{2}\right)\ln\left(\frac{x-1}{\sqrt{2}-1}\right)\sqrt{2}}{4}+\frac{\operatorname{dilog}\left(\frac{x-1}{\sqrt{2}-1}\right)\sqrt{2}}{4}+\frac{\operatorname{dilog}\left(\frac{1+x}{1+\sqrt{2}}\right)\sqrt{}}{4}$
parts	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\ln\left(-x^{2}+1\right)\sqrt{2}}{2}+\sqrt{2}\left(-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\ln\left(x^{2}-1\right)}{2}+\frac{\ln\left(\frac{x\sqrt{2}}{2}+1\right)\ln\left(x^{2}-1\right)}{4}-\frac{\ln\left(\frac{x\sqrt{2}}{2}+1\right)\ln\left(\frac{\sqrt{2}-x\sqrt{2}}{2+\sqrt{2}}\right)}{4}-\right.$

input
```
int(ln(-x^2+1)/(-x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(ln(x-2^(1/2))*ln(-x^2+1)-dilog((1+x)/(1+2^(1/2)))-ln(x-2^(1/2))*ln((
1+x)/(1+2^(1/2)))-dilog((x-1)/(2^(1/2)-1))-ln(x-2^(1/2))*ln((x-1)/(2^(1/2)
-1)))*2^(1/2)+1/4*(ln(2^(1/2)+x)*ln(-x^2+1)-dilog((1+x)/(-2^(1/2)+1))-ln(2
^(1/2)+x)*ln((1+x)/(-2^(1/2)+1))-dilog((x-1)/(-1-2^(1/2)))-ln(2^(1/2)+x)*l
n((x-1)/(-1-2^(1/2))))*2^(1/2)
```

Fricas [F]

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = \int -\frac{\log(-x^2+1)}{x^2-2}\,dx$$

input
```
integrate(log(-x^2+1)/(-x^2+2),x, algorithm="fricas")
```

output
```
integral(-log(-x^2 + 1)/(x^2 - 2), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = -\int \frac{\log(1-x^2)}{x^2-2}\,dx$$

input
```
integrate(ln(-x**2+1)/(-x**2+2),x)
```

output
```
-Integral(log(1 - x**2)/(x**2 - 2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.87

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx$$
$$= \frac{1}{4}\sqrt{2}\left(\left(\log\left(2x+2\sqrt{2}\right)-\log\left(2x-2\sqrt{2}\right)\right)\log\left(-x^2+1\right)-\log\left(x+\sqrt{2}\right)\log\left(-\frac{x+\sqrt{2}}{\sqrt{2}+1}+1\right)+\mathrm{lo}\right.$$

input
```
integrate(log(-x^2+1)/(-x^2+2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*((log(2*x + 2*sqrt(2)) - log(2*x - 2*sqrt(2)))*log(-x^2 + 1) -
 log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) + 1) + 1) + log(x - sqrt(2))
*log((x - sqrt(2))/(sqrt(2) + 1) + 1) - log(x + sqrt(2))*log(-(x + sqrt(2)
)/(sqrt(2) - 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt(2) - 1) +
1) - dilog((x + sqrt(2))/(sqrt(2) + 1)) + dilog(-(x - sqrt(2))/(sqrt(2) +
1)) - dilog((x + sqrt(2))/(sqrt(2) - 1)) + dilog(-(x - sqrt(2))/(sqrt(2) -
 1)))
```

Giac [F]

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = \int -\frac{\log(-x^2+1)}{x^2-2}\,dx$$

input
```
integrate(log(-x^2+1)/(-x^2+2),x, algorithm="giac")
```

output
```
integrate(-log(-x^2 + 1)/(x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = -\int \frac{\ln(1-x^2)}{x^2-2}\,dx$$

input
```
int(-log(1 - x^2)/(x^2 - 2),x)
```

output
```
-int(log(1 - x^2)/(x^2 - 2), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = -\left(\int \frac{\log(-x^2+1)}{x^2-2}dx\right)$$

input
```
int(log(-x^2+1)/(-x^2+2),x)
```

output
```
- int(log( - x**2 + 1)/(x**2 - 2),x)
```

3.359 $\int \frac{\log(d+ex^2)}{1-x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 217

$$
\begin{aligned}
\int \frac{\log(d+ex^2)}{1-x^2}\,dx = {} & 2\mathrm{arctanh}(x)\log\left(\frac{2}{1+x}\right) \\
& - \mathrm{arctanh}(x)\log\left(\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(1+x)}\right) \\
& - \mathrm{arctanh}(x)\log\left(\frac{2(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{-d}+\sqrt{e})(1+x)}\right) \\
& + \mathrm{arctanh}(x)\log(d+ex^2) - \mathrm{PolyLog}\left(2,1-\frac{2}{1+x}\right) \\
& + \frac{1}{2}\mathrm{PolyLog}\left(2,1-\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(1+x)}\right) \\
& + \frac{1}{2}\mathrm{PolyLog}\left(2,1-\frac{2(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{-d}+\sqrt{e})(1+x)}\right)
\end{aligned}
$$

output
```
2*arctanh(x)*ln(2/(1+x))-arctanh(x)*ln(2*((-d)^(1/2)-e^(1/2)*x)/((-d)^(1/2
)-e^(1/2))/(1+x))-arctanh(x)*ln(2*((-d)^(1/2)+e^(1/2)*x)/((-d)^(1/2)+e^(1/
2))/(1+x))+arctanh(x)*ln(e*x^2+d)-polylog(2,1-2/(1+x))+1/2*polylog(2,1-2*(
(-d)^(1/2)-e^(1/2)*x)/((-d)^(1/2)-e^(1/2))/(1+x))+1/2*polylog(2,1-2*((-d)^
(1/2)+e^(1/2)*x)/((-d)^(1/2)+e^(1/2))/(1+x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.54

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = \frac{1}{2}\log(1-x)\log\left(\frac{\sqrt{-d}-\sqrt{e}x}{\sqrt{-d}-\sqrt{e}}\right)-\frac{1}{2}\log(1+x)\log\left(\frac{\sqrt{-d}-\sqrt{e}x}{\sqrt{-d}+\sqrt{e}}\right) - \frac{1}{2}\log(1+x)\log\left(\frac{\sqrt{-d}+\sqrt{e}x}{\sqrt{-d}-\sqrt{e}}\right) + \frac{1}{2}\log(1-x)\log\left(\frac{\sqrt{-d}+\sqrt{e}x}{\sqrt{-d}+\sqrt{e}}\right) - \frac{1}{2}\log(1-x)\log(d+ex^2)+\frac{1}{2}\log(1+x)\log(d+ex^2) + \frac{1}{2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(1-x)}{\sqrt{-d}-\sqrt{e}}\right)+\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1-x)}{\sqrt{-d}+\sqrt{e}}\right) - \frac{1}{2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{e}(1+x)}{\sqrt{-d}-\sqrt{e}}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1+x)}{\sqrt{-d}+\sqrt{e}}\right)$$

input
```
Integrate[Log[d + e*x^2]/(1 - x^2),x]
```

output
```
(Log[1 - x]*Log[(Sqrt[-d] - Sqrt[e]*x)/(Sqrt[-d] - Sqrt[e])])/2 - (Log[1 +
 x]*Log[(Sqrt[-d] - Sqrt[e]*x)/(Sqrt[-d] + Sqrt[e])])/2 - (Log[1 + x]*Log[
(Sqrt[-d] + Sqrt[e]*x)/(Sqrt[-d] - Sqrt[e])])/2 + (Log[1 - x]*Log[(Sqrt[-d
] + Sqrt[e]*x)/(Sqrt[-d] + Sqrt[e])])/2 - (Log[1 - x]*Log[d + e*x^2])/2 +
(Log[1 + x]*Log[d + e*x^2])/2 + PolyLog[2, -((Sqrt[e]*(1 - x))/(Sqrt[-d] -
 Sqrt[e]))]/2 + PolyLog[2, (Sqrt[e]*(1 - x))/(Sqrt[-d] + Sqrt[e])]/2 - Pol
yLog[2, -((Sqrt[e]*(1 + x))/(Sqrt[-d] - Sqrt[e]))]/2 - PolyLog[2, (Sqrt[e]
*(1 + x))/(Sqrt[-d] + Sqrt[e])]/2
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2920, 6554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (d+ex^2)}{1-x^2} \, dx$$

$$\Big\downarrow \text{ 2920}$$

$$\text{arctanh}(x) \log (d+ex^2) - 2e \int \frac{x\text{arctanh}(x)}{ex^2+d} dx$$

$$\Big\downarrow \text{ 6554}$$

$$\text{arctanh}(x) \log (d+ex^2) - 2e \int \left(\frac{\text{arctanh}(x)}{2\sqrt{e} (\sqrt{e}x+\sqrt{-d})} - \frac{\text{arctanh}(x)}{2\sqrt{e} (\sqrt{-d}-\sqrt{e}x)} \right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\text{arctanh}(x) \log (d+ex^2) - 2e \left(\frac{\text{arctanh}(x) \log \left(\frac{2(\sqrt{-d}-\sqrt{e}x)}{(x+1)(\sqrt{-d}-\sqrt{e})} \right)}{2e} + \frac{\text{arctanh}(x) \log \left(\frac{2(\sqrt{-d}+\sqrt{e}x)}{(x+1)(\sqrt{-d}+\sqrt{e})} \right)}{2e} - \frac{\text{arctanh}(x) \log \left(\frac{2}{x+1} \right)}{e} - \frac{\text{PolyLog} \left(2 \right.}{} \right.$$

input
```
Int[Log[d + e*x^2]/(1 - x^2),x]
```

output
```
ArcTanh[x]*Log[d + e*x^2] - 2*e*(-((ArcTanh[x]*Log[2/(1 + x)])/e) + (ArcTa
nh[x]*Log[(2*(Sqrt[-d] - Sqrt[e]*x))/((Sqrt[-d] - Sqrt[e])*(1 + x))])/(2*e
) + (ArcTanh[x]*Log[(2*(Sqrt[-d] + Sqrt[e]*x))/((Sqrt[-d] + Sqrt[e])*(1 +
x))])/(2*e) + PolyLog[2, 1 - 2/(1 + x)]/(2*e) - PolyLog[2, 1 - (2*(Sqrt[-d
] - Sqrt[e]*x))/((Sqrt[-d] - Sqrt[e])*(1 + x))]/(4*e) - PolyLog[2, 1 - (2*
(Sqrt[-d] + Sqrt[e]*x))/((Sqrt[-d] + Sqrt[e])*(1 + x))]/(4*e))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 6554
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.30

method	result
risch	$\frac{\ln(1+x)\ln(e\,x^2+d)}{2} - \frac{\ln(1+x)\ln\left(\frac{-e(1+x)+\sqrt{-de}+e}{e+\sqrt{-de}}\right)}{2} - \frac{\ln(1+x)\ln\left(\frac{e(1+x)+\sqrt{-de}-e}{-e+\sqrt{-de}}\right)}{2} - \frac{\operatorname{dilog}\left(\frac{-e(1+x)+\sqrt{-de}+e}{e+\sqrt{-de}}\right)}{2} - \operatorname{dilog}$
default	$\frac{\ln(1+x)\ln(e\,x^2+d)}{2} - e\left(\frac{\ln(1+x)\left(\ln\left(\frac{-e(1+x)+\sqrt{-de}+e}{e+\sqrt{-de}}\right)+\ln\left(\frac{e(1+x)+\sqrt{-de}-e}{-e+\sqrt{-de}}\right)\right)}{2e} + \frac{\operatorname{dilog}\left(\frac{-e(1+x)+\sqrt{-de}+e}{e+\sqrt{-de}}\right)+\operatorname{dilog}\left(\frac{e(1+x)}{-e}\right.}{2e}\right.$
parts	$\operatorname{arctanh}(x)\ln(e\,x^2+d) - 2e\left(\frac{\operatorname{arctanh}(x)\ln(e\,x^2+d)}{2e} - \frac{\frac{\ln(1+x)\ln(e\,x^2+d)}{2} - e\left(\frac{\ln(1+x)\left(\ln\left(\frac{-e(1+x)+\sqrt{-de}+e}{e+\sqrt{-de}}\right)+\ln\left(\right.\right.}{2e}\right.}{}\right.$

input
```
int(ln(e*x^2+d)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1+x)*ln(e*x^2+d)-1/2*ln(1+x)*ln((-e*(1+x)+(-d*e)^(1/2)+e)/(e+(-d*e)
^(1/2)))-1/2*ln(1+x)*ln((e*(1+x)+(-d*e)^(1/2)-e)/(-e+(-d*e)^(1/2)))-1/2*di
log((-e*(1+x)+(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))-1/2*dilog((e*(1+x)+(-d*e)^
(1/2)-e)/(-e+(-d*e)^(1/2)))-1/2*ln(x-1)*ln(e*x^2+d)+1/2*ln(x-1)*ln((-e*(x-
1)+(-d*e)^(1/2)-e)/(-e+(-d*e)^(1/2)))+1/2*ln(x-1)*ln((e*(x-1)+(-d*e)^(1/2)
+e)/(e+(-d*e)^(1/2)))+1/2*dilog((-e*(x-1)+(-d*e)^(1/2)-e)/(-e+(-d*e)^(1/2)
))+1/2*dilog((e*(x-1)+(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))
```

Fricas [F]

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = \int -\frac{\log(ex^2+d)}{x^2-1}\,dx$$

input
```
integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="fricas")
```

output
```
integral(-log(e*x^2 + d)/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = -\int \frac{\log(d+ex^2)}{x^2-1}\,dx$$

input
```
integrate(ln(e*x**2+d)/(-x**2+1),x)
```

output
```
-Integral(log(d + e*x**2)/(x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = \int -\frac{\log(ex^2+d)}{x^2-1}\,dx$$

input
```
integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="maxima")
```

output
```
-integrate(log(e*x^2 + d)/(x^2 - 1), x)
```

Giac [F]

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = \int -\frac{\log(ex^2+d)}{x^2-1}\,dx$$

input
```
integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="giac")
```

output
```
integrate(-log(e*x^2 + d)/(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = -\int \frac{\ln(e\,x^2+d)}{x^2-1}\,dx$$

input
```
int(-log(d + e*x^2)/(x^2 - 1),x)
```

output
```
-int(log(d + e*x^2)/(x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = -\left(\int \frac{\log(e\,x^2+d)}{x^2-1}dx\right)$$

input
```
int(log(e*x^2+d)/(-x^2+1),x)
```

output
```
- int(log(d + e*x**2)/(x**2 - 1),x)
```

3.360 $\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx = -\frac{d^2gpx^n}{3e^2n}+\frac{dgpx^{2n}}{6en}-\frac{gpx^{3n}}{9n}+\frac{d^3gp\log(d+ex^n)}{3e^3n}+\frac{gx^{3n}\log(c(d+ex^n)^p)}{3n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{fp\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
-1/3*d^2*g*p*x^n/e^2/n+1/6*d*g*p*x^(2*n)/e/n-1/9*g*p*x^(3*n)/n+1/3*d^3*g*p
*ln(d+e*x^n)/e^3/n+1/3*g*x^(3*n)*ln(c*(d+e*x^n)^p)/n+f*ln(-e*x^n/d)*ln(c*(
d+e*x^n)^p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{-\frac{gp(ex^n(6d^2-3dex^n+2e^2x^{2n})-6d^3\log(d+ex^n))}{e^3}+6gx^{3n}\log(c(d+ex^n)^p)+18f\left(\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)+p\right.}{18n}$$

input
```
Integrate[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-((g*p*(e*x^n*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n)) - 6*d^3*Log[d + e*x^n])
)/e^3) + 6*g*x^(3*n)*Log[c*(d + e*x^n)^p] + 18*f*(Log[-((e*x^n)/d)]*Log[c*
(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d]))/(18*n)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx$$
$$\Big\downarrow \text{2925}$$
$$\frac{\int x^{-n}(gx^{3n}+f)\log(c(ex^n+d)^p)\,dx^n}{n}$$
$$\Big\downarrow \text{2863}$$
$$\frac{\int \left(f\log(c(ex^n+d)^p)x^{-n}+g\log(c(ex^n+d)^p)x^{2n}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$

$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)+\frac{1}{3}gx^{3n}\log\left(c(d+ex^n)^p\right)+\frac{d^3gp\log(d+ex^n)}{3e^3}-\frac{d^2gpx^n}{3e^2}+fp\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+}{n}$$

input
```
Int[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-1/3*(d^2*g*p*x^n)/e^2 + (d*g*p*x^(2*n))/(6*e) - (g*p*x^(3*n))/9 + (d^3*g
*p*Log[d + e*x^n])/(3*e^3) + (g*x^(3*n)*Log[c*(d + e*x^n)^p])/3 + f*Log[-(
(e*x^n)/d)]*Log[c*(d + e*x^n)^p] + f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.83 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.85

method	result
risch	$\frac{(g\,x^{3n}+3f\ln(x)n)\ln((d+e\,x^n)^p)}{3n}+\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csgn}(\right.$

input
```
int((f+g*x^(3*n))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(g*(x^n)^3+3*f*ln(x)*n)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p
)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^
p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^
p)^2*csgn(I*c)+ln(c))*(f*ln(x)+1/3*g/n*(x^n)^3)-1/9*p/n*g*(x^n)^3+1/6/e*p/
n*g*d*(x^n)^2-1/3*d^2*g*p*x^n/e^2/n+1/3*d^3*g*p*ln(d+e*x^n)/e^3/n-p/n*f*di
log((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx = -\frac{18\,e^3fnp\log(x)\log\left(\frac{ex^n+d}{d}\right)-18\,e^3fn\log(c)\log(x)-3\,de^2gpx^{2\,n}+6\,d^2egpx^n+18\,e^3fp\mathrm{Li}_2\left(-\frac{ex^n+d}{d}\right.}{18\,e^3n}$$

input
```
integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-1/18*(18*e^3*f*n*p*log(x)*log((e*x^n + d)/d) - 18*e^3*f*n*log(c)*log(x) -
 3*d*e^2*g*p*x^(2*n) + 6*d^2*e*g*p*x^n + 18*e^3*f*p*dilog(-(e*x^n + d)/d +
 1) + 2*(e^3*g*p - 3*e^3*g*log(c))*x^(3*n) - 6*(3*e^3*f*n*p*log(x) + e^3*g
*p*x^(3*n) + d^3*g*p)*log(e*x^n + d))/(e^3*n)
```

Sympy [F]

$$\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx$$

input
```
integrate((f+g*x**(3*n))*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f + g*x**(3*n))*log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f + gx^{3n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{(gx^{3\,n} + f) \log ((ex^n + d)^p c)}{x} \, dx$$

input
```
integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/18*(9*e^3*f*n^2*p*log(x)^2 - 3*d*e^2*g*p*x^(2*n) + 6*d^2*e*g*p*x^n + 2*
(e^3*g*p - 3*e^3*g*log(c))*x^(3*n) - 6*(3*e^3*f*n*log(x) + e^3*g*x^(3*n))*
log((e*x^n + d)^p) - 6*(d^3*g*n*p + 3*e^3*f*n*log(c))*log(x))/(e^3*n) + in
tegrate(1/3*(3*d*e^3*f*n*p*log(x) - d^4*g*p)/(e^4*x*x^n + d*e^3*x), x)
```

Giac [F]

$$\int \frac{(f + gx^{3n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{(gx^{3\,n} + f) \log ((ex^n + d)^p c)}{x} \, dx$$

input
```
integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((g*x^(3*n) + f)*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f + gx^{3n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{\ln \left(c \left(d + e\,x^n\right)^p\right) \left(f + g\,x^{3\,n}\right)}{x} \, dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n)))/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n)))/x, x)
```

Reduce [F]

$$\int \frac{(f + gx^{3n}) \log \left(c(d + ex^n)^p\right)}{x}\, dx$$

$$= \frac{6x^{3n}\log((x^n e + d)^p\, c)\, e^3 gp - 2x^{3n} e^3 g\, p^2 + 3x^{2n} d\, e^2 g\, p^2 - 6x^n d^2 eg\, p^2 + 18\left(\int \frac{\log((x^n e+d)^p c)}{x^n ex+dx} dx\right) d\, e^3 fnp +}{18e^3 np}$$

input
```
int((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x)
```

output
```
(6*x**(3*n)*log((x**n*e + d)**p*c)*e**3*g*p - 2*x**(3*n)*e**3*g*p**2 + 3*x
**(2*n)*d*e**2*g*p**2 - 6*x**n*d**2*e*g*p**2 + 18*int(log((x**n*e + d)**p*
c)/(x**n*e*x + d*x),x)*d*e**3*f*n*p + 9*log((x**n*e + d)**p*c)**2*e**3*f +
 6*log((x**n*e + d)**p*c)*d**3*g*p)/(18*e**3*n*p)
```

3.361 $\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{(f+gx^{2n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{dgpx^n}{2en}-\frac{gpx^{2n}}{4n}-\frac{d^2gp\log\left(d+ex^n\right)}{2e^2n} + \frac{gx^{2n}\log\left(c(d+ex^n)^p\right)}{2n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{fp\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
1/2*d*g*p*x^n/e/n-1/4*g*p*x^(2*n)/n-1/2*d^2*g*p*ln(d+e*x^n)/e^2/n+1/2*g*x^
(2*n)*ln(c*(d+e*x^n)^p)/n+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f*p*polylog(2
,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{-egpx^n(-2d+ex^n)-2d^2gp\log(d+ex^n)+2e^2\left(gx^{2n}+2f\log\left(-\frac{ex^n}{d}\right)\right)\log(c(d+ex^n)^p)+4e^2fp\,\mathrm{Polyl}}{4e^2n}$$

input
```
Integrate[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-(e*g*p*x^n*(-2*d + e*x^n)) - 2*d^2*g*p*Log[d + e*x^n] + 2*e^2*(g*x^(2*n)
 + 2*f*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p] + 4*e^2*f*p*PolyLog[2, 1 +
(e*x^n)/d])/(4*e^2*n)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx$$
$$\Big\downarrow \text{2925}$$
$$\frac{\int x^{-n}(gx^{2n}+f)\log(c(ex^n+d)^p)\,dx^n}{n}$$
$$\Big\downarrow \text{2863}$$
$$\frac{\int (f\log(c(ex^n+d)^p)x^{-n}+g\log(c(ex^n+d)^p)x^n)\,dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)+\frac{1}{2}gx^{2n}\log(c(d+ex^n)^p)-\frac{d^2gp\log(d+ex^n)}{2e^2}+fp\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+\frac{dgpx^n}{2e}-\frac{1}{4}}{n}$$

input
```
Int[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
((d*g*p*x^n)/(2*e) - (g*p*x^(2*n))/4 - (d^2*g*p*Log[d + e*x^n])/(2*e^2) +
(g*x^(2*n)*Log[c*(d + e*x^n)^p])/2 + f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)
^p] + f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.44 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.01

method	result
risch	$\frac{(2f\ln(x)n+g\,x^{2n})\ln((d+e\,x^n)^p)}{2n}+\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csgn}(\cdots}{2}\right.$

input
```
int((f+g*x^(2*n))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*f*ln(x)*n+g*(x^n)^2)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p
)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^
p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^
p)^2*csgn(I*c)+ln(c))*(f*ln(x)+1/2*g*(x^n)^2/n)-1/4*p/n*g*(x^n)^2+1/2*d*g*
p*x^n/e/n-1/2*d^2*g*p*ln(d+e*x^n)/e^2/n-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)
*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx = -\frac{4\,e^2 fnp\log(x)\log\left(\frac{ex^n+d}{d}\right) - 4\,e^2 fn\log(c)\log(x) - 2\,degpx^n + 4\,e^2 fp\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right) + (e^2gp - 2\,e^2}{4\,e^2 n}$$

input
```
integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-1/4*(4*e^2*f*n*p*log(x)*log((e*x^n + d)/d) - 4*e^2*f*n*log(c)*log(x) - 2*
d*e*g*p*x^n + 4*e^2*f*p*dilog(-(e*x^n + d)/d + 1) + (e^2*g*p - 2*e^2*g*log
(c))*x^(2*n) - 2*(2*e^2*f*n*p*log(x) + e^2*g*p*x^(2*n) - d^2*g*p)*log(e*x^
n + d))/(e^2*n)
```

Sympy [F]

$$\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx$$

input
```
integrate((f+g*x**(2*n))*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f + g*x**(2*n))*log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f + gx^{2n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{(gx^{2n} + f) \log ((ex^n + d)^p c)}{x} \, dx$$

input
```
integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/4*(2*e^2*f*n^2*p*log(x)^2 - 2*d*e*g*p*x^n + (e^2*g*p - 2*e^2*g*log(c))*
x^(2*n) - 2*(2*e^2*f*n*log(x) + e^2*g*x^(2*n))*log((e*x^n + d)^p) + 2*(d^2
*g*n*p - 2*e^2*f*n*log(c))*log(x))/(e^2*n) + integrate(1/2*(2*d*e^2*f*n*p*
log(x) + d^3*g*p)/(e^3*x*x^n + d*e^2*x), x)
```

Giac [F]

$$\int \frac{(f + gx^{2n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{(gx^{2n} + f) \log ((ex^n + d)^p c)}{x} \, dx$$

input
```
integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((g*x^(2*n) + f)*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f + gx^{2n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{\ln \left(c \left(d + e\, x^n\right)^p\right) \left(f + g\, x^{2\,n}\right)}{x} \, dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n)))/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n)))/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{2x^{2n}\log((x^ne+d)^p c)\,e^2gp - x^{2n}e^2g\,p^2 + 2x^n deg\,p^2 + 4\left(\int \frac{\log((x^ne+d)^p c)}{x^n ex+dx}dx\right)d\,e^2fnp + 2\log((x^ne+d)^p c)^2 \cdots}{4e^2np}$$

input
```
int((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x)
```

output
```
(2*x**(2*n)*log((x**n*e + d)**p*c)*e**2*g*p - x**(2*n)*e**2*g*p**2 + 2*x**
n*d*e*g*p**2 + 4*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d*e**2*f*n
*p + 2*log((x**n*e + d)**p*c)**2*e**2*f - 2*log((x**n*e + d)**p*c)*d**2*g*
p)/(4*e**2*n*p)
```

3.362 $\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx = -\frac{gpx^n}{n}+\frac{g(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fp\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
-g*p*x^n/n+g*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e/n+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)
^p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{-egpx^n+\left(dg+egx^n+ef\log\left(-\frac{ex^n}{d}\right)\right)\log\left(c(d+ex^n)^p\right)+efp\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{en}$$

input
```
Integrate[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-(e*g*p*x^n) + (d*g + e*g*x^n + e*f*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^
p] + e*f*p*PolyLog[2, 1 + (e*x^n)/d])/(e*n)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{\int x^{-n}(gx^n+f)\log(c(ex^n+d)^p)\,dx^n}{n}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\int \left(f\log(c(ex^n+d)^p)\,x^{-n}+g\log(c(ex^n+d)^p)\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)+\frac{g(d+ex^n)\log(c(d+ex^n)^p)}{e}+fp\,\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)-gpx^n}{n}$$

input
```
Int[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-(g*p*x^n) + (g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/e + f*Log[-((e*x^n)/d)]
*Log[c*(d + e*x^n)^p] + f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.69

method	result
risch	$\frac{(f\ln(x)n+g\,x^n)\ln((d+e\,x^n)^p)}{n}+\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csgn}(ic)}{2}\right.$

input
```
int((f+g*x^n)*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
(f*ln(x)*n+g*x^n)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c
*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*
c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(
I*c)+ln(c))*(f*ln(x)+g*x^n/n)-g*p*x^n/n+1/e*p/n*g*d*ln(d+e*x^n)-p/n*f*dilo
g((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{(f + gx^n)\log(c(d + ex^n)^p)}{x} dx = -\frac{efnp\log(x)\log\left(\frac{ex^n+d}{d}\right) - efn\log(c)\log(x) + efp\mathrm{Li}_2\left(-\frac{ex^n+d}{d} + 1\right) + (egp - eg\log(c))x^n - (efnp\log(x) + egpx^n + dgp)\log(ex^n + d)}{en}$$

input
```
integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-(e*f*n*p*log(x)*log((e*x^n + d)/d) - e*f*n*log(c)*log(x) + e*f*p*dilog(-(
e*x^n + d)/d + 1) + (e*g*p - e*g*log(c))*x^n - (e*f*n*p*log(x) + e*g*p*x^n
 + d*g*p)*log(e*x^n + d))/(e*n)
```

Sympy [F]

$$\int \frac{(f + gx^n)\log(c(d + ex^n)^p)}{x} dx = \int \frac{(f + gx^n)\log(c(d + ex^n)^p)}{x} dx$$

input
```
integrate((f+g*x**n)*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f + g*x**n)*log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f + gx^n)\log(c(d + ex^n)^p)}{x} dx = \int \frac{(gx^n + f)\log((ex^n + d)^p c)}{x} dx$$

input
```
integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/2*(e*f*n^2*p*log(x)^2 + 2*(e*g*p - e*g*log(c))*x^n - 2*(e*f*n*log(x) +
e*g*x^n)*log((e*x^n + d)^p) - 2*(d*g*n*p + e*f*n*log(c))*log(x))/(e*n) + i
ntegrate((d*e*f*n*p*log(x) - d^2*g*p)/(e^2*x*x^n + d*e*x), x)
```

Giac [F]

$$\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^n+f)\log((ex^n+d)^pc)}{x}\,dx$$

input
```
integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((g*x^n + f)*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln(c(d+e\,x^n)^p)\,(f+g\,x^n)}{x}\,dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g*x^n))/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g*x^n))/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx = \frac{2x^n\log((x^ne+d)^p\,c)\,egp-2x^neg\,p^2+2\left(\int \frac{\log((x^ne+d)^pc)}{x^nex+dx}dx\right)defnp+\log((x^ne+d)^p\,c)^2ef+2\log((x^ne}{2enp}$$

input
```
int((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x)
```

output
```
(2*x**n*log((x**n*e + d)**p*c)*e*g*p - 2*x**n*e*g*p**2 + 2*int(log((x**n*e
 + d)**p*c)/(x**n*e*x + d*x),x)*d*e*f*n*p + log((x**n*e + d)**p*c)**2*e*f
+ 2*log((x**n*e + d)**p*c)*d*g*p)/(2*e*n*p)
```

3.363 $\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{(f+gx^{-n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{egp\log(x)}{d} - \frac{egp\log\left(d+ex^n\right)}{dn} - \frac{gx^{-n}\log\left(c(d+ex^n)^p\right)}{n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{fp\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
e*g*p*ln(x)/d-e*g*p*ln(d+e*x^n)/d/n-g*ln(c*(d+e*x^n)^p)/n/(x^n)+f*ln(-e*x^
n/d)*ln(c*(d+e*x^n)^p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{(f+gx^{-n})\log (c(d+ex^n)^p)}{x}\,dx$$
$$=\frac{egnp\log(x)-egp\log(d+ex^n)-dgx^{-n}\log(c(d+ex^n)^p)+df\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)+dfp\,\mathrm{PolyL}}{dn}$$

input
```
Integrate[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(e*g*n*p*Log[x] - e*g*p*Log[d + e*x^n] - (d*g*Log[c*(d + e*x^n)^p])/x^n +
d*f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + d*f*p*PolyLog[2, 1 + (e*x^n)/
d])/(d*n)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2005, 2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^{-n})\log (c(d+ex^n)^p)}{x}\,dx$$
$$\Big\downarrow \text{2005}$$
$$\int x^{-n-1}(fx^n+g)\log (c(d+ex^n)^p)\,dx$$
$$\Big\downarrow \text{2925}$$
$$\frac{\int x^{-2n}(fx^n+g)\log (c(ex^n+d)^p)\,dx^n}{n}$$
$$\Big\downarrow \text{2863}$$
$$\frac{\int \left(g\log (c(ex^n+d)^p)\,x^{-2n}+f\log (c(ex^n+d)^p)\,x^{-n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)-gx^{-n}\log\left(c(d+ex^n)^p\right)+fp\operatorname{PolyLog}\left(2,\frac{ex^n}{d}+1\right)+\frac{egp\log(x^n)}{d}-\frac{egp\log(d+ex^n)}{d}}{n}$$

input
```
Int[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
((e*g*p*Log[x^n])/d - (e*g*p*Log[d + e*x^n])/d - (g*Log[c*(d + e*x^n)^p])/
x^n + f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + f*p*PolyLog[2, 1 + (e*x^n
)/d])/n
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
 + n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.84 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.49

method	result
risch	$\frac{(f\ln(x)n\,x^{n}-g)x^{-n}\ln((d+e\,x^{n})^{p})}{n}+\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^{n})^{p})\mathrm{csgn}(ic(d+e\,x^{n})^{p})^{2}}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^{n})^{p})\ \mathrm{csgn}(ic(d+e\,x^{n})^{p})\ \mathrm{csgn}}{2}\right.$

input
```
int((f+g/(x^n))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
(f*ln(x)*n*x^n-g)/n/(x^n)*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*cs
gn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*c
sgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2
*csgn(I*c)+ln(c))*(-1/n*g/(x^n)+1/n*ln(x^n)*f)-p/n*f*dilog((d+e*x^n)/d)-p*
f*ln(x)*ln((d+e*x^n)/d)-e*g*p*ln(d+e*x^n)/d/n+e*p/n*g/d*ln(x^n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{(f+gx^{-n})\log\left(c(d+ex^{n})^{p}\right)}{x}\,dx =$$
$$-\frac{dfnpx^{n}\log(x)\log\left(\frac{ex^{n}+d}{d}\right)+dfpx^{n}\mathrm{Li}_{2}\left(-\frac{ex^{n}+d}{d}+1\right)+dg\log(c)-(egnp+dfn\log(c))x^{n}\log(x)+(d}{dnx^{n}}$$

input
```
integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-(d*f*n*p*x^n*log(x)*log((e*x^n + d)/d) + d*f*p*x^n*dilog(-(e*x^n + d)/d +
 1) + d*g*log(c) - (e*g*n*p + d*f*n*log(c))*x^n*log(x) + (d*g*p - (d*f*n*p
*log(x) - e*g*p)*x^n)*log(e*x^n + d))/(d*n*x^n)
```

Sympy [F]

$$\int \frac{(f + gx^{-n})\log\left(c(d + ex^n)^p\right)}{x}\,dx = \int \frac{x^{-n}(fx^n + g)\log\left(c(d + ex^n)^p\right)}{x}\,dx$$

input
```
integrate((f+g/(x**n))*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f*x**n + g)*log(c*(d + e*x**n)**p)/(x*x**n), x)
```

Maxima [F]

$$\int \frac{(f + gx^{-n})\log\left(c(d + ex^n)^p\right)}{x}\,dx = \int \frac{\left(f + \frac{g}{x^n}\right)\log\left((ex^n + d)^p c\right)}{x}\,dx$$

input
```
integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/2*((f*n^2*p*log(x)^2 - 2*f*n*log(c)*log(x))*x^n - 2*(f*n*x^n*log(x) - g
)*log((e*x^n + d)^p) + 2*g*log(c))/(n*x^n) + integrate((d*f*n*p*log(x) + e
*g*p)/(e*x*x^n + d*x), x)
```

Giac [F]

$$\int \frac{(f + gx^{-n})\log\left(c(d + ex^n)^p\right)}{x}\,dx = \int \frac{\left(f + \frac{g}{x^n}\right)\log\left((ex^n + d)^p c\right)}{x}\,dx$$

input
```
integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((f + g/x^n)*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f + gx^{-n})\log (c(d + ex^n)^p)}{x}\,dx = \int \frac{\ln \left(c\left(d+e\,x^n\right)^p\right)\,\left(f+\frac{g}{x^n}\right)}{x}\,dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g/x^n))/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g/x^n))/x, x)
```

Reduce [F]

$$\int \frac{(f + gx^{-n})\log (c(d + ex^n)^p)}{x}\,dx = \frac{-2x^n\left(\int \frac{\log((x^n e+d)^p c)}{x^{2n}ex+x^n dx}dx\right)d^3 fnp - 2x^n\log(x^n e+d)\,def\,p^2 - 2x^n\log(x^n e+d)\,e^2 g\,p^2 + x^n\log((x^n e+d)^p c}{2x^n de}$$

input
```
int((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x)
```

output
```
( - 2*x**n*int(log((x**n*e + d)**p*c)/(x**(2*n)*e*x + x**n*d*x),x)*d**3*f*
n*p - 2*x**n*log(x**n*e + d)*d*e*f*p**2 - 2*x**n*log(x**n*e + d)*e**2*g*p*
*2 + x**n*log((x**n*e + d)**p*c)**2*d*e*f + 2*x**n*log(x)*d*e*f*n*p**2 + 2
*x**n*log(x)*e**2*g*n*p**2 - 2*log((x**n*e + d)**p*c)*d**2*f*p - 2*log((x*
*n*e + d)**p*c)*d*e*g*p)/(2*x**n*d*e*n*p)
```

3.364 $\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx = -\frac{egpx^{-n}}{2dn} - \frac{e^2gp\log(x)}{2d^2} + \frac{e^2gp\log(d+ex^n)}{2d^2n} - \frac{gx^{-2n}\log(c(d+ex^n)^p)}{2n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{fp\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output

```
-1/2*e*g*p/d/n/(x^n)-1/2*e^2*g*p*ln(x)/d^2+1/2*e^2*g*p*ln(d+e*x^n)/d^2/n-1
/2*g*ln(c*(d+e*x^n)^p)/n/(x^(2*n))+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f*p*
polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(f + gx^{-2n}) \log (c(d + ex^n)^p)}{x} \, dx = -\frac{\frac{egpx^{-n}(d+enx^n \log(x)-ex^n \log(d+ex^n))}{d^2} + gx^{-2n} \log (c(d + ex^n)^p) - 2f\left(\log \left(-\frac{ex^n}{d}\right) \log (c(d + ex^n)^p) + p \operatorname{Poly}\right.}{2n}$$

input
```
Integrate[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
-1/2*((e*g*p*(d + e*n*x^n*Log[x] - e*x^n*Log[d + e*x^n]))/(d^2*x^n) + (g*L
og[c*(d + e*x^n)^p])/x^(2*n) - 2*f*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]
 + p*PolyLog[2, 1 + (e*x^n)/d]))/n
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2005, 2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f + gx^{-2n}) \log (c(d + ex^n)^p)}{x} \, dx$$

$$\Big\downarrow \text{2005}$$

$$\int x^{-2n-1} (fx^{2n} + g) \log (c(d + ex^n)^p) \, dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{\int x^{-3n} (fx^{2n} + g) \log (c(ex^n + d)^p) \, dx^n}{n}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\int (g \log (c(ex^n + d)^p) x^{-3n} + f \log (c(ex^n + d)^p) x^{-n}) \, dx^n}{n}$$

$$\downarrow \text{2009}$$

$$\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)-\frac{1}{2}gx^{-2n}\log\left(c(d+ex^n)^p\right)-\frac{e^2gp\log(x^n)}{2d^2}+\frac{e^2gp\log(d+ex^n)}{2d^2}+fp\,\text{PolyLog}\left(2,\frac{ex^n}{d}+\right.}{n}$$

input
```
Int[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-1/2*(e*g*p)/(d*x^n) - (e^2*g*p*Log[x^n])/(2*d^2) + (e^2*g*p*Log[d + e*x^
n])/(2*d^2) - (g*Log[c*(d + e*x^n)^p])/(2*x^(2*n)) + f*Log[-((e*x^n)/d)]*L
og[c*(d + e*x^n)^p] + f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
 + n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.71 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.12

method	result
risch	$\frac{(2f\ln(x)n\,x^{2n}-g)x^{-2n}\ln((d+e\,x^n)^p)}{2n}+\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\,\mathrm{csgn}(ic(d+e\,x^n)^p)}{2}\right.$

input
```
int((f+g/(x^(2*n)))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*f*ln(x)*n*(x^n)^2-g)/n/(x^n)^2*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+
e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d
+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d
+e*x^n)^p)^2*csgn(I*c)+ln(c))*(1/n*ln(x^n)*f-1/2/n*g/(x^n)^2)+1/2*e^2*g*p*
ln(d+e*x^n)/d^2/n-1/2*e*g*p/d/n/(x^n)-1/2*e^2*p/n*g/d^2*ln(x^n)-p/n*f*dilo
g((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19

$$\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx = -\frac{2\,d^2fnpx^{2\,n}\log(x)\log\left(\frac{ex^n+d}{d}\right)+2\,d^2fpx^{2\,n}\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+degpx^n+d^2g\log(c)+(e^2gnp-2\,d^2f\cdots}{2\,d^2nx^{2\,n}}$$

input
```
integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-1/2*(2*d^2*f*n*p*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*d^2*f*p*x^(2*n)*di
log(-(e*x^n + d)/d + 1) + d*e*g*p*x^n + d^2*g*log(c) + (e^2*g*n*p - 2*d^2*
f*n*log(c))*x^(2*n)*log(x) + (d^2*g*p - (2*d^2*f*n*p*log(x) + e^2*g*p)*x^(
2*n))*log(e*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F]

$$\int \frac{(f + gx^{-2n})\log\left(c(d + ex^n)^p\right)}{x}\,dx = \int \frac{x^{-2n}(fx^{2n} + g)\log\left(c(d + ex^n)^p\right)}{x}\,dx$$

input
```
integrate((f+g/(x**(2*n)))*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f*x**(2*n) + g)*log(c*(d + e*x**n)**p)/(x*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{(f + gx^{-2n})\log\left(c(d + ex^n)^p\right)}{x}\,dx = \int \frac{\left(f + \frac{g}{x^{2n}}\right)\log\left((ex^n + d)^p c\right)}{x}\,dx$$

input
```
integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/2*(e*g*p*x^n + d*g*log(c) + (d*f*n^2*p*log(x)^2 - 2*d*f*n*log(c)*log(x)
)*x^(2*n) - (2*d*f*n*x^(2*n)*log(x) - d*g)*log((e*x^n + d)^p))/(d*n*x^(2*n
)) + integrate(1/2*(2*d^2*f*n*p*log(x) - e^2*g*p)/(d*e*x*x^n + d^2*x), x)
```

Giac [F]

$$\int \frac{(f + gx^{-2n})\log\left(c(d + ex^n)^p\right)}{x}\,dx = \int \frac{\left(f + \frac{g}{x^{2n}}\right)\log\left((ex^n + d)^p c\right)}{x}\,dx$$

input
```
integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((f + g/x^(2*n))*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)\,\left(f+\frac{g}{x^{2\,n}}\right)}{x}\,dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n)))/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n)))/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{2x^{2n}\left(\int \frac{\log((x^ne+d)^pc)}{x^nex+dx}dx\right)d^3fnp - 3x^{2n}\log(x^ne+d)\,d^2f\,p^2 + x^{2n}\log(x^ne+d)\,e^2g\,p^2 + x^{2n}\log((x^ne+d)^p}{2x^{2n}d^2np}$$

input
```
int((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x)
```

output
```
(2*x**(2*n)*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d**3*f*n*p - 3*
x**(2*n)*log(x**n*e + d)*d**2*f*p**2 + x**(2*n)*log(x**n*e + d)*e**2*g*p**
2 + x**(2*n)*log((x**n*e + d)**p*c)**2*d**2*f + 3*x**(2*n)*log((x**n*e + d
)**p*c)*d**2*f*p - x**(2*n)*log(x)*e**2*g*n*p**2 - x**n*d*e*g*p**2 - log((
x**n*e + d)**p*c)*d**2*g*p)/(2*x**(2*n)*d**2*n*p)
```

3.365 $\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 27, antiderivative size = 327

$$\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x} dx = -\frac{2d^2fgpx^n}{3e^2n} + \frac{d^5g^2px^n}{6e^5n} + \frac{dfgpx^{2n}}{3en} - \frac{d^4g^2px^{2n}}{12e^4n} - \frac{2fgpx^{3n}}{9n} + \frac{d^3g^2px^{3n}}{18e^3n} - \frac{d^2g^2px^{4n}}{24e^2n} + \frac{dg^2px^{5n}}{30en} - \frac{g^2px^{6n}}{36n} + \frac{2d^3fgp\log(d+ex^n)}{3e^3n} - \frac{d^6g^2p\log(d+ex^n)}{6e^6n} + \frac{2fgx^{3n}\log(c(d+ex^n)^p)}{3n} + \frac{g^2x^{6n}\log(c(d+ex^n)^p)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{f^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
-2/3*d^2*f*g*p*x^n/e^2/n+1/6*d^5*g^2*p*x^n/e^5/n+1/3*d*f*g*p*x^(2*n)/e/n-1
/12*d^4*g^2*p*x^(2*n)/e^4/n-2/9*f*g*p*x^(3*n)/n+1/18*d^3*g^2*p*x^(3*n)/e^3
/n-1/24*d^2*g^2*p*x^(4*n)/e^2/n+1/30*d*g^2*p*x^(5*n)/e/n-1/36*g^2*p*x^(6*n
)/n+2/3*d^3*f*g*p*ln(d+e*x^n)/e^3/n-1/6*d^6*g^2*p*ln(d+e*x^n)/e^6/n+2/3*f*
g*x^(3*n)*ln(c*(d+e*x^n)^p)/n+1/6*g^2*x^(6*n)*ln(c*(d+e*x^n)^p)/n+f^2*ln(-
e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.64

$$\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{-egpx^n(-60d^5g+30d^4egx^n-20d^3e^2gx^{2n}+10e^5x^{2n}(8f+gx^{3n})-12de^4x^n(10f+gx^{3n})+15d^2e^3(16f}{}$$

input
```
Integrate[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-(e*g*p*x^n*(-60*d^5*g + 30*d^4*e*g*x^n - 20*d^3*e^2*g*x^(2*n) + 10*e^5*x
^(2*n)*(8*f + g*x^(3*n)) - 12*d*e^4*x^n*(10*f + g*x^(3*n)) + 15*d^2*e^3*(1
6*f + g*x^(3*n)))) - 60*d^3*g*(-4*e^3*f + d^3*g)*p*Log[d + e*x^n] + 60*e^6
*(g*x^(3*n)*(4*f + g*x^(3*n)) + 6*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)
^p] + 360*e^6*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(360*e^6*n)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x}\,dx$$
$$\Big\downarrow \text{2925}$$
$$\frac{\int x^{-n}(gx^{3n}+f)^2 \log(c(ex^n+d)^p)\,dx^n}{n}$$
$$\Big\downarrow \text{2863}$$
$$\frac{\int \left(f^2\log(c(ex^n+d)^p)x^{-n}+2fg\log(c(ex^n+d)^p)x^{2n}+g^2\log(c(ex^n+d)^p)x^{5n}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$

$$f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)+\frac{2}{3}fgx^{3n}\log\left(c(d+ex^n)^p\right)+\frac{1}{6}g^2x^{6n}\log\left(c(d+ex^n)^p\right)-\frac{d^6g^2p\log(d+ex^n)}{6e^6}+\frac{d^5g^2px}{6e^5}$$

input
```
Int[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
((-2*d^2*f*g*p*x^n)/(3*e^2) + (d^5*g^2*p*x^n)/(6*e^5) + (d*f*g*p*x^(2*n))/
(3*e) - (d^4*g^2*p*x^(2*n))/(12*e^4) - (2*f*g*p*x^(3*n))/9 + (d^3*g^2*p*x^
(3*n))/(18*e^3) - (d^2*g^2*p*x^(4*n))/(24*e^2) + (d*g^2*p*x^(5*n))/(30*e)
- (g^2*p*x^(6*n))/36 + (2*d^3*f*g*p*Log[d + e*x^n])/(3*e^3) - (d^6*g^2*p*L
og[d + e*x^n])/(6*e^6) + (2*f*g*x^(3*n)*Log[c*(d + e*x^n)^p])/3 + (g^2*x^(
6*n)*Log[c*(d + e*x^n)^p])/6 + f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]
+ f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.54 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.33

method	result
risch	$\frac{(g^2x^{6n}+4fg\,x^{3n}+6f^2\ln(x)n)\ln((d+e\,x^n)^p)}{6n}+\frac{\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csg}}{2}\right.}{}$

input
```
int((f+g*x^(3*n))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(g^2*(x^n)^6+4*f*g*(x^n)^3+6*f^2*ln(x)*n)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*
csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*c
sgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*c
sgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(1/6*g^2*(x^n)^6+2/3*f*g*(x^n)^3
+f^2*ln(x^n))-1/36*p/n*g^2*(x^n)^6+1/30/e*p/n*g^2*d*(x^n)^5-1/24/e^2*p/n*g
^2*d^2*(x^n)^4+1/18/e^3*p/n*g^2*d^3*(x^n)^3-1/12/e^4*p/n*g^2*d^4*(x^n)^2+1
/6*d^5*g^2*p*x^n/e^5/n-1/6*d^6*g^2*p*ln(d+e*x^n)/e^6/n-p/n*f^2*dilog((d+e*
x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)-2/9*p/n*g*f*(x^n)^3+1/3/e*p/n*g*f*d*(x
^n)^2-2/3*d^2*f*g*p*x^n/e^2/n+2/3*d^3*f*g*p*ln(d+e*x^n)/e^3/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx^{3n})^2\log\left(c(d+ex^n)^p\right)}{x}\,dx =$$
$$-\frac{360\,e^6f^2np\log(x)\log\left(\frac{ex^n+d}{d}\right)-360\,e^6f^2n\log(c)\log(x)-12\,de^5g^2px^{5\,n}+15\,d^2e^4g^2px^{4\,n}+360\,e^6f^2p}{}$$

input
```
integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-1/360*(360*e^6*f^2*n*p*log(x)*log((e*x^n + d)/d) - 360*e^6*f^2*n*log(c)*l
og(x) - 12*d*e^5*g^2*p*x^(5*n) + 15*d^2*e^4*g^2*p*x^(4*n) + 360*e^6*f^2*p*
dilog(-(e*x^n + d)/d + 1) - 30*(4*d*e^5*f*g - d^4*e^2*g^2)*p*x^(2*n) + 60*
(4*d^2*e^4*f*g - d^5*e*g^2)*p*x^n + 10*(e^6*g^2*p - 6*e^6*g^2*log(c))*x^(6
*n) - 20*(12*e^6*f*g*log(c) - (4*e^6*f*g - d^3*e^3*g^2)*p)*x^(3*n) - 60*(6
*e^6*f^2*n*p*log(x) + e^6*g^2*p*x^(6*n) + 4*e^6*f*g*p*x^(3*n) + (4*d^3*e^3
*f*g - d^6*g^2)*p)*log(e*x^n + d))/(e^6*n)
```

Sympy [F]

$$\int \frac{(f + gx^{3n})^2 \log(c(d + ex^n)^p)}{x} \, dx = \int \frac{(f + gx^{3n})^2 \log(c(d + ex^n)^p)}{x} \, dx$$

input
```
integrate((f+g*x**(3*n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f + g*x**(3*n))**2*log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f + gx^{3n})^2 \log(c(d + ex^n)^p)}{x} \, dx = \int \frac{(gx^{3n} + f)^2 \log((ex^n + d)^p c)}{x} \, dx$$

input
```
integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/360*(180*e^6*f^2*n^2*p*log(x)^2 - 12*d*e^5*g^2*p*x^(5*n) + 15*d^2*e^4*g
^2*p*x^(4*n) + 10*(e^6*g^2*p - 6*e^6*g^2*log(c))*x^(6*n) + 20*(4*e^6*f*g*p
 - d^3*e^3*g^2*p - 12*e^6*f*g*log(c))*x^(3*n) - 30*(4*d*e^5*f*g*p - d^4*e^
2*g^2*p)*x^(2*n) + 60*(4*d^2*e^4*f*g*p - d^5*e*g^2*p)*x^n - 60*(6*e^6*f^2*
n*log(x) + e^6*g^2*x^(6*n) + 4*e^6*f*g*x^(3*n))*log((e*x^n + d)^p) - 60*(4
*d^3*e^3*f*g*n*p - d^6*g^2*n*p + 6*e^6*f^2*n*log(c))*log(x))/(e^6*n) + int
egrate(1/6*(6*d*e^6*f^2*n*p*log(x) - 4*d^4*e^3*f*g*p + d^7*g^2*p)/(e^7*x*x
^n + d*e^6*x), x)
```

Giac [F]

$$\int \frac{(f+gx^{3n})^2 \log (c(d+ex^n)^p)}{x}\, dx = \int \frac{(gx^{3\,n}+f)^2 \log \left((ex^n+d)^p c\right)}{x}\, dx$$

input
```
integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((g*x^(3*n) + f)^2*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{3n})^2 \log (c(d+ex^n)^p)}{x}\, dx = \int \frac{\ln \left(c\,(d+e\,x^n)^p\right)\,\left(f+g\,x^{3\,n}\right)^2}{x}\, dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n))^2)/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n))^2)/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^{3n})^2 \log (c(d+ex^n)^p)}{x}\, dx = \frac{60x^{6n}\log((x^ne+d)^p\,c)\,e^6g^2p - 10x^{6n}e^6g^2p^2 + 12x^{5n}d\,e^5g^2p^2 - 15x^{4n}d^2e^4g^2p^2 + 240x^{3n}\log((x^ne+d)^p\,c)}{}$$

input
```
int((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x)
```

output
```
(60*x**(6*n)*log((x**n*e + d)**p*c)*e**6*g**2*p - 10*x**(6*n)*e**6*g**2*p*
*2 + 12*x**(5*n)*d*e**5*g**2*p**2 - 15*x**(4*n)*d**2*e**4*g**2*p**2 + 240*
x**(3*n)*log((x**n*e + d)**p*c)*e**6*f*g*p + 20*x**(3*n)*d**3*e**3*g**2*p*
*2 - 80*x**(3*n)*e**6*f*g*p**2 - 30*x**(2*n)*d**4*e**2*g**2*p**2 + 120*x**
(2*n)*d*e**5*f*g*p**2 + 60*x**n*d**5*e*g**2*p**2 - 240*x**n*d**2*e**4*f*g*
p**2 + 360*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d*e**6*f**2*n*p
+ 180*log((x**n*e + d)**p*c)**2*e**6*f**2 - 60*log((x**n*e + d)**p*c)*d**6
*g**2*p + 240*log((x**n*e + d)**p*c)*d**3*e**3*f*g*p)/(360*e**6*n*p)
```

3.366 $\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 254

$$\begin{aligned}\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x} \, dx &= \frac{dfgpx^n}{en} + \frac{d^3g^2px^n}{4e^3n} - \frac{fgpx^{2n}}{2n} - \frac{d^2g^2px^{2n}}{8e^2n} \\ &+ \frac{dg^2px^{3n}}{12en} - \frac{g^2px^{4n}}{16n} - \frac{d^2fgp\log(d+ex^n)}{e^2n} \\ &- \frac{d^4g^2p\log(d+ex^n)}{4e^4n} + \frac{fgx^{2n}\log(c(d+ex^n)^p)}{n} \\ &+ \frac{g^2x^{4n}\log(c(d+ex^n)^p)}{4n} \\ &+ \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} \\ &+ \frac{f^2p\,\text{PolyLog}\left(2, 1+\frac{ex^n}{d}\right)}{n}\end{aligned}$$

output

```
d*f*g*p*x^n/e/n+1/4*d^3*g^2*p*x^n/e^3/n-1/2*f*g*p*x^(2*n)/n-1/8*d^2*g^2*p*
x^(2*n)/e^2/n+1/12*d*g^2*p*x^(3*n)/e/n-1/16*g^2*p*x^(4*n)/n-d^2*f*g*p*ln(d
+e*x^n)/e^2/n-1/4*d^4*g^2*p*ln(d+e*x^n)/e^4/n+f*g*x^(2*n)*ln(c*(d+e*x^n)^p
)/n+1/4*g^2*x^(4*n)*ln(c*(d+e*x^n)^p)/n+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)
/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.67

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{-egpx^n(-12d^3g+6d^2egx^n+3e^3x^n(8f+gx^{2n})-4de^2(12f+gx^{2n}))-12d^2g(4e^2f+d^2g)\,p\log(d+ex}{48e^4n}$$

input
```
Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-(e*g*p*x^n*(-12*d^3*g + 6*d^2*e*g*x^n + 3*e^3*x^n*(8*f + g*x^(2*n)) - 4*
d*e^2*(12*f + g*x^(2*n)))) - 12*d^2*g*(4*e^2*f + d^2*g)*p*Log[d + e*x^n] +
 12*e^4*(g*x^(2*n)*(4*f + g*x^(2*n)) + 4*f^2*Log[-((e*x^n)/d)])*Log[c*(d +
 e*x^n)^p] + 48*e^4*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(48*e^4*n)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x}\,dx$$
$$\Big\downarrow \ 2925$$
$$\frac{\int x^{-n}(gx^{2n}+f)^2\log(c(ex^n+d)^p)\,dx^n}{n}$$
$$\Big\downarrow \ 2863$$
$$\frac{\int \left(f^2\log(c(ex^n+d)^p)\,x^{-n}+2fg\log(c(ex^n+d)^p)\,x^n+g^2\log(c(ex^n+d)^p)\,x^{3n}\right)dx^n}{n}$$
$$\Big\downarrow \ 2009$$

$$\frac{f^2 \log\left(-\frac{ex^n}{d}\right) \log\left(c(d+ex^n)^p\right) + fgx^{2n} \log\left(c(d+ex^n)^p\right) + \frac{1}{4}g^2x^{4n} \log\left(c(d+ex^n)^p\right) - \frac{d^4g^2p\log(d+ex^n)}{4e^4} + \frac{d^3g^2px^n}{4e^3}}{n}$$

input
```
Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
((d*f*g*p*x^n)/e + (d^3*g^2*p*x^n)/(4*e^3) - (f*g*p*x^(2*n))/2 - (d^2*g^2*
p*x^(2*n))/(8*e^2) + (d*g^2*p*x^(3*n))/(12*e) - (g^2*p*x^(4*n))/16 - (d^2*
f*g*p*Log[d + e*x^n])/e^2 - (d^4*g^2*p*Log[d + e*x^n])/(4*e^4) + f*g*x^(2*
n)*Log[c*(d + e*x^n)^p] + (g^2*x^(4*n)*Log[c*(d + e*x^n)^p])/4 + f^2*Log[-
((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.44 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.48

method	result
risch	$\frac{\left(g^{2} x^{4 n}+4 f^{2} \ln \left(x\right) n+4 f g\, x^{2 n}\right) \ln \left(\left(d+e\, x^{n}\right)^{p}\right)}{4 n}+\frac{\left(\frac{i \pi \,\operatorname{csgn}\left(i \left(d+e\, x^{n}\right)^{p}\right) \operatorname{csgn}\left(i c \left(d+e\, x^{n}\right)^{p}\right)^{2}}{2}-\frac{i \pi \,\operatorname{csgn}\left(i \left(d+e\, x^{n}\right)^{p}\right) \operatorname{csgn}\left(i c \left(d+e\, x^{n}\right)^{p}\right) \operatorname{csg}}{2}\right.}{}$

input
```
int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(g^2*(x^n)^4+4*f^2*ln(x)*n+4*f*g*(x^n)^2)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*
csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*c
sgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*c
sgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(1/4*g^2*(x^n)^4+f*g*(x^n)^2+f^2
*ln(x^n))-1/16*p/n*g^2*(x^n)^4+1/12/e*p/n*g^2*d*(x^n)^3-1/8/e^2*p/n*g^2*d^
2*(x^n)^2+1/4*d^3*g^2*p*x^n/e^3/n-1/4*d^4*g^2*p*ln(d+e*x^n)/e^4/n-p/n*f^2*
dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)-1/2*p/n*g*f*(x^n)^2+d*f*g*p
*x^n/e/n-d^2*f*g*p*ln(d+e*x^n)/e^2/n
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.95

$$\int \frac{\left(f+gx^{2n}\right)^2 \log \left(c(d+ex^n)^p\right)}{x}\,dx =$$
$$-\frac{48\,e^4 f^2 np \log\left(x\right) \log\left(\frac{ex^n+d}{d}\right) - 48\,e^4 f^2 n \log\left(c\right) \log\left(x\right) - 4\,de^3 g^2 px^{3\,n} + 48\,e^4 f^2 p \mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right) - 12}{}$$

input
```
integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-1/48*(48*e^4*f^2*n*p*log(x)*log((e*x^n + d)/d) - 48*e^4*f^2*n*log(c)*log(
x) - 4*d*e^3*g^2*p*x^(3*n) + 48*e^4*f^2*p*dilog(-(e*x^n + d)/d + 1) - 12*(
4*d*e^3*f*g + d^3*e*g^2)*p*x^n + 3*(e^4*g^2*p - 4*e^4*g^2*log(c))*x^(4*n)
- 6*(8*e^4*f*g*log(c) - (4*e^4*f*g + d^2*e^2*g^2)*p)*x^(2*n) - 12*(4*e^4*f
^2*n*p*log(x) + e^4*g^2*p*x^(4*n) + 4*e^4*f*g*p*x^(2*n) - (4*d^2*e^2*f*g +
 d^4*g^2)*p)*log(e*x^n + d))/(e^4*n)
```

Sympy [F]

$$\int \frac{(f + gx^{2n})^2 \log(c(d + ex^n)^p)}{x} dx = \int \frac{(f + gx^{2n})^2 \log(c(d + ex^n)^p)}{x} dx$$

input
```
integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f + g*x**(2*n))**2*log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f + gx^{2n})^2 \log(c(d + ex^n)^p)}{x} dx = \int \frac{(gx^{2n} + f)^2 \log((ex^n + d)^p c)}{x} dx$$

input
```
integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/48*(24*e^4*f^2*n^2*p*log(x)^2 - 4*d*e^3*g^2*p*x^(3*n) + 3*(e^4*g^2*p -
4*e^4*g^2*log(c))*x^(4*n) + 6*(4*e^4*f*g*p + d^2*e^2*g^2*p - 8*e^4*f*g*log
(c))*x^(2*n) - 12*(4*d*e^3*f*g*p + d^3*e*g^2*p)*x^n - 12*(4*e^4*f^2*n*log(
x) + e^4*g^2*x^(4*n) + 4*e^4*f*g*x^(2*n))*log((e*x^n + d)^p) + 12*(4*d^2*e
^2*f*g*n*p + d^4*g^2*n*p - 4*e^4*f^2*n*log(c))*log(x))/(e^4*n) + integrate
(1/4*(4*d*e^4*f^2*n*p*log(x) + 4*d^3*e^2*f*g*p + d^5*g^2*p)/(e^5*x*x^n + d
*e^4*x), x)
```

Giac [F]

$$\int \frac{(f+gx^{2n})^2\log(c(d+ex^n)^p)}{x}\,dx=\int \frac{(gx^{2n}+f)^2\log((ex^n+d)^pc)}{x}\,dx$$

input
```
integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((g*x^(2*n) + f)^2*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{2n})^2\log(c(d+ex^n)^p)}{x}\,dx=\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)\,\left(f+g\,x^{2\,n}\right)^2}{x}\,dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n))^2)/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n))^2)/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^{2n})^2\log(c(d+ex^n)^p)}{x}\,dx$$
$$=\frac{12x^{4n}\log((x^ne+d)^p\,c)\,e^4g^2p-3x^{4n}e^4g^2p^2+4x^{3n}d\,e^3g^2p^2+48x^{2n}\log((x^ne+d)^p\,c)\,e^4fgp-6x^{2n}d^2e^2g^2p}{}$$

input
```
int((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x)
```

output

```
(12*x**(4*n)*log((x**n*e + d)**p*c)*e**4*g**2*p - 3*x**(4*n)*e**4*g**2*p**
2 + 4*x**(3*n)*d*e**3*g**2*p**2 + 48*x**(2*n)*log((x**n*e + d)**p*c)*e**4*
f*g*p - 6*x**(2*n)*d**2*e**2*g**2*p**2 - 24*x**(2*n)*e**4*f*g*p**2 + 12*x*
*n*d**3*e*g**2*p**2 + 48*x**n*d*e**3*f*g*p**2 + 48*int(log((x**n*e + d)**p
*c)/(x**n*e*x + d*x),x)*d*e**4*f**2*n*p + 24*log((x**n*e + d)**p*c)**2*e**
4*f**2 - 12*log((x**n*e + d)**p*c)*d**4*g**2*p - 48*log((x**n*e + d)**p*c)
*d**2*e**2*f*g*p)/(48*e**4*n*p)
```

3.367 $\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x} \, dx = -\frac{2fgpx^n}{n} + \frac{dg^2px^n}{2en} - \frac{g^2px^{2n}}{4n} - \frac{d^2g^2p\log(d+ex^n)}{2e^2n} + \frac{g^2x^{2n}\log(c(d+ex^n)^p)}{2n} + \frac{2fg(d+ex^n)\log(c(d+ex^n)^p)}{en} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{f^2p\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
-2*f*g*p*x^n/n+1/2*d*g^2*p*x^n/e/n-1/4*g^2*p*x^(2*n)/n-1/2*d^2*g^2*p*ln(d+
e*x^n)/e^2/n+1/2*g^2*x^(2*n)*ln(c*(d+e*x^n)^p)/n+2*f*g*(d+e*x^n)*ln(c*(d+e
*x^n)^p)/e/n+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/
d)/n
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int \frac{(f+gx^n)^2 \log (c(d+ex^n)^p)}{x} \, dx$$
$$= \frac{-egpx^n(8ef-2dg+egx^n)-2d^2g^2p\log (d+ex^n)+2e\left(4dfg+egx^n(4f+gx^n)+2ef^2\log \left(-\frac{ex^n}{d}\right)\right)\log}{4e^2n}$$

input
```
Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-(e*g*p*x^n*(8*e*f - 2*d*g + e*g*x^n)) - 2*d^2*g^2*p*Log[d + e*x^n] + 2*e
*(4*d*f*g + e*g*x^n*(4*f + g*x^n) + 2*e*f^2*Log[-((e*x^n)/d)])*Log[c*(d +
e*x^n)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(4*e^2*n)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^n)^2 \log (c(d+ex^n)^p)}{x} \, dx$$
$$\Big\downarrow \text{ 2925}$$
$$\frac{\int x^{-n}(gx^n+f)^2 \log (c(ex^n+d)^p) \, dx^n}{n}$$
$$\Big\downarrow \text{ 2863}$$
$$\frac{\int \left(f^2 \log (c(ex^n+d)^p) x^{-n}+g^2 \log (c(ex^n+d)^p) x^n+2fg \log (c(ex^n+d)^p)\right) dx^n}{n}$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{f^2 \log\left(-\frac{ex^n}{d}\right) \log\left(c(d+ex^n)^p\right) + \frac{2fg(d+ex^n)\log(c(d+ex^n)^p)}{e} + \frac{1}{2}g^2x^{2n}\log\left(c(d+ex^n)^p\right) - \frac{d^2g^2p\log(d+ex^n)}{2e^2} + f^2p\,\mathrm{Pol}}{n}$$

input
```
Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-2*f*g*p*x^n + (d*g^2*p*x^n)/(2*e) - (g^2*p*x^(2*n))/4 - (d^2*g^2*p*Log[d
 + e*x^n])/(2*e^2) + (g^2*x^(2*n)*Log[c*(d + e*x^n)^p])/2 + (2*f*g*(d + e*
x^n)*Log[c*(d + e*x^n)^p])/e + f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]
+ f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.93 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.78

method	result
risch	$\frac{(2f^2\ln(x)n+g^2x^{2n}+4fg\,x^n)\ln((d+e\,x^n)^p)}{2n}+\frac{\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csgn}}{2}\right.}{}$

input
```
int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*f^2*ln(x)*n+g^2*(x^n)^2+4*f*g*x^n)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn
(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(
I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(
I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(1/2*g^2*(x^n)^2+2*f*g*x^n+f^2*ln(x^
n))-1/4*p/n*g^2*(x^n)^2+1/2*d*g^2*p*x^n/e/n-1/2*d^2*g^2*p*ln(d+e*x^n)/e^2/
n-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)-2*f*g*p*x^n/n+2/e
*p/n*g*f*d*ln(d+e*x^n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.09

$$\int \frac{(f+gx^n)^2 \log\left(c(d+ex^n)^p\right)}{x}\,dx = -\frac{4\,e^2f^2np\log(x)\log\left(\frac{ex^n+d}{d}\right)-4\,e^2f^2n\log(c)\log(x)+4\,e^2f^2p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+\left(e^2g^2p-2\,e^2g^2\log\left(c\right)\right.}{}$$

input
```
integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-1/4*(4*e^2*f^2*n*p*log(x)*log((e*x^n + d)/d) - 4*e^2*f^2*n*log(c)*log(x)
+ 4*e^2*f^2*p*dilog(-(e*x^n + d)/d + 1) + (e^2*g^2*p - 2*e^2*g^2*log(c))*x
^(2*n) - 2*(4*e^2*f*g*log(c) - (4*e^2*f*g - d*e*g^2)*p)*x^n - 2*(2*e^2*f^2
*n*p*log(x) + e^2*g^2*p*x^(2*n) + 4*e^2*f*g*p*x^n + (4*d*e*f*g - d^2*g^2)*
p)*log(e*x^n + d))/(e^2*n)
```

Sympy [F]

$$\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x}\,dx$$

input
```
integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f + g*x**n)**2*log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^n+f)^2 \log((ex^n+d)^p c)}{x}\,dx$$

input
```
integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/4*(2*e^2*f^2*n^2*p*log(x)^2 + (e^2*g^2*p - 2*e^2*g^2*log(c))*x^(2*n) +
2*(4*e^2*f*g*p - d*e*g^2*p - 4*e^2*f*g*log(c))*x^n - 2*(2*e^2*f^2*n*log(x)
 + e^2*g^2*x^(2*n) + 4*e^2*f*g*x^n)*log((e*x^n + d)^p) - 2*(4*d*e*f*g*n*p
- d^2*g^2*n*p + 2*e^2*f^2*n*log(c))*log(x))/(e^2*n) + integrate(1/2*(2*d*e
^2*f^2*n*p*log(x) - 4*d^2*e*f*g*p + d^3*g^2*p)/(e^3*x*x^n + d*e^2*x), x)
```

Giac [F]

$$\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^n+f)^2 \log((ex^n+d)^p c)}{x}\,dx$$

input
```
integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((g*x^n + f)^2*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln(c\,(d+e\,x^n)^p)\,(f+g\,x^n)^2}{x}\,dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g*x^n)^2)/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g*x^n)^2)/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{2x^{2n}\log((x^ne+d)^p\,c)\,e^2g^2p - x^{2n}e^2g^2p^2 + 8x^n\log((x^ne+d)^p\,c)\,e^2fgp + 2x^nde\,g^2p^2 - 8x^ne^2fg\,p^2 + 4\Big(\int}{4}$$

input
```
int((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x)
```

output
```
(2*x**(2*n)*log((x**n*e + d)**p*c)*e**2*g**2*p - x**(2*n)*e**2*g**2*p**2 +
 8*x**n*log((x**n*e + d)**p*c)*e**2*f*g*p + 2*x**n*d*e*g**2*p**2 - 8*x**n*
e**2*f*g*p**2 + 4*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d*e**2*f*
*2*n*p + 2*log((x**n*e + d)**p*c)**2*e**2*f**2 - 2*log((x**n*e + d)**p*c)*
d**2*g**2*p + 8*log((x**n*e + d)**p*c)*d*e*f*g*p)/(4*e**2*n*p)
```

3.368 $\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 27, antiderivative size = 193

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x} dx = -\frac{eg^2px^{-n}}{2dn} + \frac{2efgp\log(x)}{d} - \frac{e^2g^2p\log(x)}{2d^2} - \frac{2efgp\log(d+ex^n)}{dn} + \frac{e^2g^2p\log(d+ex^n)}{2d^2n} - \frac{g^2x^{-2n}\log(c(d+ex^n)^p)}{2n} - \frac{2fgx^{-n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{f^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
-1/2*e*g^2*p/d/n/(x^n)+2*e*f*g*p*ln(x)/d-1/2*e^2*g^2*p*ln(x)/d^2-2*e*f*g*p
*ln(d+e*x^n)/d/n+1/2*e^2*g^2*p*ln(d+e*x^n)/d^2/n-1/2*g^2*ln(c*(d+e*x^n)^p)
/n/(x^(2*n))-2*f*g*ln(c*(d+e*x^n)^p)/n/(x^n)+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^
n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^{-n})^2\log(c(d+ex^n)^p)}{x}\,dx =$$
$$-\frac{-4defgnp\log(x)+4defgp\log(d+ex^n)+eg^2p(dx^{-n}+en\log(x)-e\log(d+ex^n))+d^2g^2x^{-2n}\log(}{}$$

input
```
Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
-1/2*(-4*d*e*f*g*n*p*Log[x] + 4*d*e*f*g*p*Log[d + e*x^n] + e*g^2*p*(d/x^n
+ e*n*Log[x] - e*Log[d + e*x^n]) + (d^2*g^2*Log[c*(d + e*x^n)^p])/x^(2*n)
+ (4*d^2*f*g*Log[c*(d + e*x^n)^p])/x^n - 2*d^2*f^2*(Log[-((e*x^n)/d)]*Log[
c*(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d]))/(d^2*n)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2005, 2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^{-n})^2\log(c(d+ex^n)^p)}{x}\,dx$$
$$\Big\downarrow \text{ 2005}$$
$$\int x^{-2n-1}(fx^n+g)^2\log(c(d+ex^n)^p)\,dx$$
$$\Big\downarrow \text{ 2925}$$
$$\frac{\int x^{-3n}(fx^n+g)^2\log(c(ex^n+d)^p)\,dx^n}{n}$$
$$\Big\downarrow \text{ 2863}$$

$$\frac{\int \left(g^2 \log \left(c(ex^n+d)^p\right) x^{-3n} + 2fg \log \left(c(ex^n+d)^p\right) x^{-2n} + f^2 \log \left(c(ex^n+d)^p\right) x^{-n}\right) dx^n}{n}$$

$\big\downarrow$ 2009

$$\frac{f^2 \log \left(-\frac{ex^n}{d}\right) \log \left(c(d+ex^n)^p\right) - 2fgx^{-n} \log \left(c(d+ex^n)^p\right) - \frac{1}{2}g^2x^{-2n} \log \left(c(d+ex^n)^p\right) - \frac{e^2g^2p\log(x^n)}{2d^2} + \frac{e^2g^2p\log}{2c}}{n}$$

input
```
Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-1/2*(e*g^2*p)/(d*x^n) + (2*e*f*g*p*Log[x^n])/d - (e^2*g^2*p*Log[x^n])/(2
*d^2) - (2*e*f*g*p*Log[d + e*x^n])/d + (e^2*g^2*p*Log[d + e*x^n])/(2*d^2)
- (g^2*Log[c*(d + e*x^n)^p])/(2*x^(2*n)) - (2*f*g*Log[c*(d + e*x^n)^p])/x^
n + f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + f^2*p*PolyLog[2, 1 + (e*x
^n)/d])/n
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.67 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.73

method	result
risch	$\frac{\left(2 f^{2} \ln \left(x \right) n \,x^{2 n}-4 f g \,x^{n}-g^{2}\right) x^{-2 n} \ln \left(\left(d +e \,x^{n}\right)^{p}\right)}{2 n}+\frac{\left(\frac{i \pi \,\operatorname{csgn}\left(i \left(d +e \,x^{n}\right)^{p}\right) \operatorname{csgn}\left(i c \left(d +e \,x^{n}\right)^{p}\right)^{2}}{2}-\frac{i \pi \,\operatorname{csgn}\left(i \left(d +e \,x^{n}\right)^{p}\right) \operatorname{csgn}\left(i c \left(d +e \,x^{n}\right)^{p}\right)}{2}\right.}{}$

input
```
int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*f^2*ln(x)*n*(x^n)^2-4*f*g*x^n-g^2)/n/(x^n)^2*ln((d+e*x^n)^p)+(1/2*I
*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^
p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*
Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(-2*g*f/(x^n)+f^2*ln(x^n)-1/
2*g^2/(x^n)^2)-2*e*f*g*p*ln(d+e*x^n)/d/n+2*e*p/n*g*f/d*ln(x^n)+1/2*e^2*g^2
*p*ln(d+e*x^n)/d^2/n-1/2*e*g^2*p/d/n/(x^n)-1/2*e^2*p/n*g^2/d^2*ln(x^n)-p/n
*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

$$\int \frac{(f+gx^{-n})^2 \log \left(c(d+ex^n)^p\right)}{x} \, dx = -\frac{2\, d^2 f^2 n p x^{2\,n} \log\left(x\right) \log\left(\frac{ex^n+d}{d}\right) + 2\, d^2 f^2 p x^{2\,n} \mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right) + d^2 g^2 \log\left(c\right) - \left(2\, d^2 f^2 n \log\left(c\right) + (4\, de\right.}{}$$

input
```
integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-1/2*(2*d^2*f^2*n*p*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*d^2*f^2*p*x^(2*n
)*dilog(-(e*x^n + d)/d + 1) + d^2*g^2*log(c) - (2*d^2*f^2*n*log(c) + (4*d*
e*f*g - e^2*g^2)*n*p)*x^(2*n)*log(x) + (d*e*g^2*p + 4*d^2*f*g*log(c))*x^n
+ (4*d^2*f*g*p*x^n + d^2*g^2*p - (2*d^2*f^2*n*p*log(x) - (4*d*e*f*g - e^2*
g^2)*p)*x^(2*n))*log(e*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F]

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{x^{-2n}(fx^n+g)^2 \log(c(d+ex^n)^p)}{x}\,dx$$

input
```
integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f*x**n + g)**2*log(c*(d + e*x**n)**p)/(x*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\left(f+\frac{g}{x^n}\right)^2 \log((ex^n+d)^p c)}{x}\,dx$$

input
```
integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/2*(d*g^2*log(c) + (d*f^2*n^2*p*log(x)^2 - 2*d*f^2*n*log(c)*log(x))*x^(2
*n) + (e*g^2*p + 4*d*f*g*log(c))*x^n - (2*d*f^2*n*x^(2*n)*log(x) - 4*d*f*g
*x^n - d*g^2)*log((e*x^n + d)^p))/(d*n*x^(2*n)) + integrate(1/2*(2*d^2*f^2
*n*p*log(x) + 4*d*e*f*g*p - e^2*g^2*p)/(d*e*x*x^n + d^2*x), x)
```

Giac [F]

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\left(f+\frac{g}{x^n}\right)^2 \log\left((ex^n+d)^p c\right)}{x}\,dx$$

input
```
integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((f + g/x^n)^2*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)\,\left(f+\frac{g}{x^n}\right)^2}{x}\,dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g/x^n)^2)/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g/x^n)^2)/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{2x^{2n}\left(\int \frac{\log((x^n e+d)^p c)}{x^n ex+dx}dx\right) d^3 f^2 np - 3x^{2n}\log(x^n e+d)\, d^2 f^2 p^2 - 4x^{2n}\log(x^n e+d)\, defg\, p^2 + x^{2n}\log(x^n e+ \cdots}{\cdots}$$

input
```
int((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x)
```

output
```
(2*x**(2*n)*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d**3*f**2*n*p -
 3*x**(2*n)*log(x**n*e + d)*d**2*f**2*p**2 - 4*x**(2*n)*log(x**n*e + d)*d*
e*f*g*p**2 + x**(2*n)*log(x**n*e + d)*e**2*g**2*p**2 + x**(2*n)*log((x**n*
e + d)**p*c)**2*d**2*f**2 + 3*x**(2*n)*log((x**n*e + d)**p*c)*d**2*f**2*p
+ 4*x**(2*n)*log(x)*d*e*f*g*n*p**2 - x**(2*n)*log(x)*e**2*g**2*n*p**2 - 4*
x**n*log((x**n*e + d)**p*c)*d**2*f*g*p - x**n*d*e*g**2*p**2 - log((x**n*e
+ d)**p*c)*d**2*g**2*p)/(2*x**(2*n)*d**2*n*p)
```

3.369 $\int \frac{(f+gx^{-2n})^2\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 257

$$\int \frac{(f+gx^{-2n})^2\log(c(d+ex^n)^p)}{x}\,dx = -\frac{eg^2px^{-3n}}{12dn}+\frac{e^2g^2px^{-2n}}{8d^2n}-\frac{efgpx^{-n}}{dn}-\frac{e^3g^2px^{-n}}{4d^3n}-\frac{e^2fgp\log(x)}{d^2}-\frac{e^4g^2p\log(x)}{4d^4}+\frac{e^2fgp\log(d+ex^n)}{d^2n}+\frac{e^4g^2p\log(d+ex^n)}{4d^4n}-\frac{g^2x^{-4n}\log(c(d+ex^n)^p)}{4n}-\frac{fgx^{-2n}\log(c(d+ex^n)^p)}{n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{f^2p\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

output
```
-1/12*e*g^2*p/d/n/(x^(3*n))+1/8*e^2*g^2*p/d^2/n/(x^(2*n))-e*f*g*p/d/n/(x^n
)-1/4*e^3*g^2*p/d^3/n/(x^n)-e^2*f*g*p*ln(x)/d^2-1/4*e^4*g^2*p*ln(x)/d^4+e^
2*f*g*p*ln(d+e*x^n)/d^2/n+1/4*e^4*g^2*p*ln(d+e*x^n)/d^4/n-1/4*g^2*ln(c*(d+
e*x^n)^p)/n/(x^(4*n))-f*g*ln(c*(d+e*x^n)^p)/n/(x^(2*n))+f^2*ln(-e*x^n/d)*l
n(c*(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.73

$$\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = -\frac{\frac{24efgp(dx^{-n}+en\log(x)-e\log(d+ex^n))}{d^2}+\frac{eg^2p(dx^{-3n}(2d^2-3dex^n+6e^2x^{2n})+6e^3n\log(x)-6e^3\log(d+ex^n))}{d^4}+6g^2x^{-4n}\log(c(d}{2}$$

input
```
Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
-1/24*((24*e*f*g*p*(d/x^n + e*n*Log[x] - e*Log[d + e*x^n]))/d^2 + (e*g^2*p
*((d*(2*d^2 - 3*d*e*x^n + 6*e^2*x^(2*n)))/x^(3*n) + 6*e^3*n*Log[x] - 6*e^3
*Log[d + e*x^n]))/d^4 + (6*g^2*Log[c*(d + e*x^n)^p])/x^(4*n) + (24*f*g*Log
[c*(d + e*x^n)^p])/x^(2*n) - 24*f^2*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p
] + p*PolyLog[2, 1 + (e*x^n)/d]))/n
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2005, 2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x}\,dx$$

$$\Big\downarrow \text{ 2005}$$

$$\int x^{-4n-1}(fx^{2n}+g)^2 \log(c(d+ex^n)^p)\,dx$$

$$\Big\downarrow \text{ 2925}$$

$$\frac{\int x^{-5n}(fx^{2n}+g)^2 \log(c(ex^n+d)^p)\,dx^n}{n}$$

$$\Big\downarrow \text{ 2863}$$

$$\frac{\int \left(g^2 \log \left(c(ex^n+d)^p\right) x^{-5n} + 2fg \log \left(c(ex^n+d)^p\right) x^{-3n} + f^2 \log \left(c(ex^n+d)^p\right) x^{-n}\right) dx^n}{n}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{f^2 \log \left(-\frac{ex^n}{d}\right) \log \left(c(d+ex^n)^p\right) - fgx^{-2n} \log \left(c(d+ex^n)^p\right) - \frac{1}{4} g^2 x^{-4n} \log \left(c(d+ex^n)^p\right) - \frac{e^4 g^2 p \log(x^n)}{4d^4} + \frac{e^4 g^2 p \log}{4d}}{n}$$

input
```
Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

output
```
(-1/12*(e*g^2*p)/(d*x^(3*n)) + (e^2*g^2*p)/(8*d^2*x^(2*n)) - (e*f*g*p)/(d*
x^n) - (e^3*g^2*p)/(4*d^3*x^n) - (e^2*f*g*p*Log[x^n])/d^2 - (e^4*g^2*p*Log
[x^n])/(4*d^4) + (e^2*f*g*p*Log[d + e*x^n])/d^2 + (e^4*g^2*p*Log[d + e*x^n
])/(4*d^4) - (g^2*Log[c*(d + e*x^n)^p])/(4*x^(4*n)) - (f*g*Log[c*(d + e*x^
n)^p])/x^(2*n) + f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + f^2*p*PolyLo
g[2, 1 + (e*x^n)/d])/n
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
 + n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.54

method	result
risch	$\frac{\left(4 f^{2} \ln \left(x \right) n \,x^{4 n}-4 f g \,x^{2 n}-g^{2}\right) x^{-4 n} \ln \left(\left(d +e \,x^{n}\right)^{p}\right)}{4 n}+\frac{\left(\frac{i \pi \,\mathrm{csgn}\left(i \left(d +e \,x^{n}\right)^{p}\right) \mathrm{csgn}\left(i c \left(d +e \,x^{n}\right)^{p}\right)^{2}}{2}-\frac{i \pi \,\mathrm{csgn}\left(i \left(d +e \,x^{n}\right)^{p}\right) \mathrm{csgn}\left(i c \left(d +e \,x^{n}\right.\right.}{2}\right.}{}$

input
```
int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*f^2*ln(x)*n*(x^n)^4-4*f*g*(x^n)^2-g^2)/n/(x^n)^4*ln((d+e*x^n)^p)+(1
/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x
^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/
2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(-1/4*g^2/(x^n)^4+f^2*ln
(x^n)-g*f/(x^n)^2)-1/12*e*p/n*g^2/d/(x^n)^3-1/4*e^3*g^2*p/d^3/n/(x^n)+1/8*
e^2*p/n*g^2/d^2/(x^n)^2-1/4*e^4*p/n*g^2/d^4*ln(x^n)+1/4*e^4*g^2*p*ln(d+e*x
^n)/d^4/n-e*f*g*p/d/n/(x^n)-e^2*p/n*g*f/d^2*ln(x^n)+e^2*f*g*p*ln(d+e*x^n)/
d^2/n-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.03

$$\int \frac{\left(f+g x^{-2 n}\right)^2 \log \left(c(d+e x^n)^p\right)}{x}\,dx =$$

$$-\frac{24\,d^4 f^2 n p x^{4\,n} \log\left(x\right) \log\left(\frac{e x^n+d}{d}\right) + 24\,d^4 f^2 p x^{4\,n} \mathrm{Li}_2\left(-\frac{e x^n+d}{d}+1\right) + 2\,d^3 e g^2 p x^n + 6\,d^4 g^2 \log\left(c\right) + 6\,(4\,d}{}$$

input
```
integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

output
```
-1/24*(24*d^4*f^2*n*p*x^(4*n)*log(x)*log((e*x^n + d)/d) + 24*d^4*f^2*p*x^(
4*n)*dilog(-(e*x^n + d)/d + 1) + 2*d^3*e*g^2*p*x^n + 6*d^4*g^2*log(c) + 6*
(4*d^3*e*f*g + d*e^3*g^2)*p*x^(3*n) - 6*(4*d^4*f^2*n*log(c) - (4*d^2*e^2*f
*g + e^4*g^2)*n*p)*x^(4*n)*log(x) - 3*(d^2*e^2*g^2*p - 8*d^4*f*g*log(c))*x
^(2*n) + 6*(4*d^4*f*g*p*x^(2*n) + d^4*g^2*p - (4*d^4*f^2*n*p*log(x) + (4*d
^2*e^2*f*g + e^4*g^2)*p)*x^(4*n))*log(e*x^n + d))/(d^4*n*x^(4*n))
```

Sympy [F]

$$\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{x^{-4n}(fx^{2n}+g)^2 \log(c(d+ex^n)^p)}{x}\,dx$$

input
```
integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)/x,x)
```

output
```
Integral((f*x**(2*n) + g)**2*log(c*(d + e*x**n)**p)/(x*x**(4*n)), x)
```

Maxima [F]

$$\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\left(f+\frac{g}{x^{2n}}\right)^2 \log((ex^n+d)^p c)}{x}\,dx$$

input
```
integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

output
```
-1/24*(2*d^2*e*g^2*p*x^n + 6*d^3*g^2*log(c) + 12*(d^3*f^2*n^2*p*log(x)^2 -
 2*d^3*f^2*n*log(c)*log(x))*x^(4*n) + 6*(4*d^2*e*f*g*p + e^3*g^2*p)*x^(3*n
) - 3*(d*e^2*g^2*p - 8*d^3*f*g*log(c))*x^(2*n) - 6*(4*d^3*f^2*n*x^(4*n)*lo
g(x) - 4*d^3*f*g*x^(2*n) - d^3*g^2)*log((e*x^n + d)^p))/(d^3*n*x^(4*n)) +
integrate(1/4*(4*d^4*f^2*n*p*log(x) - 4*d^2*e^2*f*g*p - e^4*g^2*p)/(d^3*e*
x*x^n + d^4*x), x)
```

Giac [F]

$$\int \frac{(f+gx^{-2n})^2 \log (c(d+ex^n)^p)}{x} \, dx = \int \frac{\left(f+\frac{g}{x^{2n}}\right)^2 \log \left((ex^n+d)^p c\right)}{x} \, dx$$

input
```
integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

output
```
integrate((f + g/x^(2*n))^2*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{-2n})^2 \log (c(d+ex^n)^p)}{x} \, dx = \int \frac{\ln \left(c \left(d+e\,x^n\right)^p\right) \left(f+\frac{g}{x^{2\,n}}\right)^2}{x} \, dx$$

input
```
int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n))^2)/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n))^2)/x, x)
```

Reduce [F]

$$\int \frac{(f+gx^{-2n})^2 \log (c(d+ex^n)^p)}{x} \, dx$$
$$= \frac{24x^{4n} \left(\int \frac{\log((x^n e+d)^p c)}{x^n ex+dx} dx\right) d^5 f^2 np - 50x^{4n} \log(x^n e+d)\, d^4 f^2 p^2 + 24x^{4n} \log(x^n e+d)\, d^2 e^2 f g\, p^2 + 6x^{4n} \log(\cdots}{\cdots}$$

input
```
int((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x)
```

output
```
(24*x**(4*n)*int(log((x**n*e + d)**p*c)/(x**n*e*x + d*x),x)*d**5*f**2*n*p
- 50*x**(4*n)*log(x**n*e + d)*d**4*f**2*p**2 + 24*x**(4*n)*log(x**n*e + d)
*d**2*e**2*f*g*p**2 + 6*x**(4*n)*log(x**n*e + d)*e**4*g**2*p**2 + 12*x**(4
*n)*log((x**n*e + d)**p*c)**2*d**4*f**2 + 50*x**(4*n)*log((x**n*e + d)**p*
c)*d**4*f**2*p - 24*x**(4*n)*log(x)*d**2*e**2*f*g*n*p**2 - 6*x**(4*n)*log(
x)*e**4*g**2*n*p**2 - 24*x**(3*n)*d**3*e*f*g*p**2 - 6*x**(3*n)*d*e**3*g**2
*p**2 - 24*x**(2*n)*log((x**n*e + d)**p*c)*d**4*f*g*p + 3*x**(2*n)*d**2*e*
*2*g**2*p**2 - 2*x**n*d**3*e*g**2*p**2 - 6*log((x**n*e + d)**p*c)*d**4*g**
2*p)/(24*x**(4*n)*d**4*n*p)
```

3.370 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 266

$$\begin{aligned}\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx &= \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn} \\ &\quad - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} \\ &\quad - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn} \\ &\quad - \frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn} \\ &\quad - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} + \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn}\end{aligned}$$

output

```
ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)-
g^(1/2)*x^n)/(e*(-f)^(1/2)+d*g^(1/2)))/f/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f
)^(1/2)+g^(1/2)*x^n)/(e*(-f)^(1/2)-d*g^(1/2)))/f/n-1/2*p*polylog(2,-g^(1/2
)*(d+e*x^n)/(e*(-f)^(1/2)-d*g^(1/2)))/f/n-1/2*p*polylog(2,g^(1/2)*(d+e*x^n
)/(e*(-f)^(1/2)+d*g^(1/2)))/f/n+p*polylog(2,1+e*x^n/d)/f/n
```

Mathematica [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx=\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))), x]
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{\int \frac{x^{-n}\log(c(ex^n+d)^p)}{gx^{2n}+f}dx^n}{n}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\int \left(\frac{x^{-n}\log(c(ex^n+d)^p)}{f}-\frac{gx^n\log(c(ex^n+d)^p)}{f(gx^{2n}+f)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f}-\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f}+\frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{f}-\frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(ex^n+d)}{e\sqrt{-f}-d\sqrt{g}}\right.}{2f}}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]
```

output
```
((Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/f - (Log[c*(d + e*x^n)^p]*Log[(e
*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f) - (Log[c*(d +
e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f
) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*f
) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f) +
 (p*PolyLog[2, 1 + (e*x^n)/d])/f)/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.31 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.80

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(x^n)}{nf}-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^{2n})}{2nf}-\frac{p\,\mathrm{dilog}\left(\frac{d+e\,x^n}{d}\right)}{nf}-\frac{p\ln(x^n)\ln\left(\frac{d+e\,x^n}{d}\right)}{nf}+\frac{p\ln(d+e\,x^n)\ln(f+g\,x^{2n}}{2nf}$

input
```
int(ln(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/n*ln((d+e*x^n)^p)/f*ln(x^n)-1/2/n*ln((d+e*x^n)^p)/f*ln(f+g*(x^n)^2)-1/n*
p/f*dilog((d+e*x^n)/d)-1/n*p/f*ln(x^n)*ln((d+e*x^n)/d)+1/2/n*p/f*ln(d+e*x^
n)*ln(f+g*(x^n)^2)-1/2/n*p/f*ln(d+e*x^n)*ln((e*(-g*f)^(1/2)-(d+e*x^n)*g+d*
g)/(e*(-g*f)^(1/2)+d*g))-1/2/n*p/f*ln(d+e*x^n)*ln((e*(-g*f)^(1/2)+(d+e*x^n
)*g-d*g)/(e*(-g*f)^(1/2)-d*g))-1/2/n*p/f*dilog((e*(-g*f)^(1/2)-(d+e*x^n)*g
+d*g)/(e*(-g*f)^(1/2)+d*g))-1/2/n*p/f*dilog((e*(-g*f)^(1/2)+(d+e*x^n)*g-d*
g)/(e*(-g*f)^(1/2)-d*g))+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^
p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi
*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))
*(1/n/f*ln(x^n)-1/2/n/f*ln(f+g*(x^n)^2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{(gx^{2\,n}+f)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/(g*x*x^(2*n) + f*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx = \int \frac{\log((ex^n+d)^pc)}{(gx^{2n}+f)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

Giac [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx = \int \frac{\log((ex^n+d)^pc)}{(gx^{2n}+f)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx = \int \frac{\ln(c\,(d+e\,x^n)^p)}{x\,(f+g\,x^{2\,n})}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log((x^n e+d)^p\,c)}{x^{2n}gx+fx}dx$$

input
```
int(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x)
```

output
```
int(log((x**n*e + d)**p*c)/(x**(2*n)*g*x + f*x),x)
```

3.371 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn} - \frac{p\,\text{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{fn} + \frac{p\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn}$$

output
```
ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n-ln(c*(d+e*x^n)^p)*ln(e*(f+g*x^n)/(-d*g+
e*f))/f/n-p*polylog(2,-g*(d+e*x^n)/(-d*g+e*f))/f/n+p*polylog(2,1+e*x^n/d)/
f/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)}\,dx = \frac{\log(c(d+ex^n)^p)\left(\log\left(-\frac{ex^n}{d}\right)-\log\left(\frac{e(f+gx^n)}{ef-dg}\right)\right) - p\,\text{PolyLog}\left(2,\frac{g(d+ex^n)}{-ef+dg}\right) + p\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

output
```
(Log[c*(d + e*x^n)^p]*(Log[-((e*x^n)/d)] - Log[(e*(f + g*x^n))/(e*f - d*g)
]) - p*PolyLog[2, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^
n)/d])/(f*n)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{x(f+gx^n)}dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{\int \frac{x^{-n}\log(c(ex^n+d)^p)}{gx^n+f}dx^n}{n}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\int \left(\frac{x^{-n}\log(c(ex^n+d)^p)}{f}-\frac{g\log(c(ex^n+d)^p)}{f(gx^n+f)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f}+\frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{f}-\frac{p\,\text{PolyLog}\left(2,-\frac{g(ex^n+d)}{ef-dg}\right)}{f}+\frac{p\,\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f}}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

output
```
((Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/f - (Log[c*(d + e*x^n)^p]*Log[(e
*(f + g*x^n))/(e*f - d*g)])/f - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f - d*g
))])/f + (p*PolyLog[2, 1 + (e*x^n)/d])/f)/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.61 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.67

method	result
risch	$-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^n)}{nf}+\frac{\ln((d+e\,x^n)^p)\ln(x^n)}{nf}-\frac{p\,\mathrm{dilog}\left(\frac{d+e\,x^n}{d}\right)}{nf}-\frac{p\ln(x^n)\ln\left(\frac{d+e\,x^n}{d}\right)}{nf}+\frac{p\,\mathrm{dilog}\left(\frac{(f+g\,x^n)e+dg-e}{dg-ef}\right)}{nf}$

input
```
int(ln(c*(d+e*x^n)^p)/x/(f+g*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/n*ln((d+e*x^n)^p)/f*ln(f+g*x^n)+1/n*ln((d+e*x^n)^p)/f*ln(x^n)-1/n*p/f*d
ilog((d+e*x^n)/d)-1/n*p/f*ln(x^n)*ln((d+e*x^n)/d)+1/n*p/f*dilog(((f+g*x^n)
*e+d*g-e*f)/(d*g-e*f))+1/n*p/f*ln(f+g*x^n)*ln(((f+g*x^n)*e+d*g-e*f)/(d*g-e
*f))+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I
*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^
p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(-1/n/f*ln(f+g*x^n)
+1/n/f*ln(x^n))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{(gx^n+f)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/(g*x*x^n + f*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.27

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = -enp\left(\frac{\log\left(x^n\right)\log\left(\frac{ex^n}{d}+1\right)+\operatorname{Li}_2\left(-\frac{ex^n}{d}\right)}{efn^2}-\frac{\log\left(gx^n+f\right)\log\left(-\frac{egx^n+ef}{ef-dg}+1\right)+\operatorname{Li}_2\left(\frac{egx^n+ef}{ef-dg}\right)}{efn^2}\right) - \left(\frac{\log\left(gx^n+f\right)}{fn}-\frac{\log\left(x^n\right)}{fn}\right)\log\left((ex^n+d)^pc\right)$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="maxima")
```

output
```
-e*n*p*((log(x^n)*log(e*x^n/d + 1) + dilog(-e*x^n/d))/(e*f*n^2) - (log(g*x
^n + f)*log(-(e*g*x^n + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^n + e*f)/(e*f
 - d*g)))/(e*f*n^2)) - (log(g*x^n + f)/(f*n) - log(x^n)/(f*n))*log((e*x^n
+ d)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)}{(gx^n+f)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/((g*x^n + f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\,(f+gx^n)}\,dx=\int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,\left(f+g\,x^n\right)} \,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\,(f+gx^n)}\,dx=\int \frac{\log\left(\left(x^n e+d\right)^p c\right)}{x^n g x+f x}dx$$

input
```
int(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x)
```

output
```
int(log((x**n*e + d)**p*c)/(x**n*g*x + f*x),x)
```

3.372 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 70

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx = \frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} + \frac{p\,\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{fn}$$

output
```
ln(c*(d+e*x^n)^p)*ln(-e*(g+f*x^n)/(d*f-e*g))/f/n+p*polylog(2,f*(d+e*x^n)/(
d*f-e*g))/f/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx = \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(g+fx^n)}{-df+eg}\right)+p\,\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{fn}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]
```

output
```
(Log[c*(d + e*x^n)^p]*Log[(e*(g + f*x^n))/(-(d*f) + e*g)] + p*PolyLog[2, (
f*(d + e*x^n))/(d*f - e*g)])/(f*n)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2005, 2925, 2841, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{x (f+gx^{-n})} \, dx$$
$$\Big\downarrow \text{2005}$$
$$\int \frac{x^{n-1} \log (c(d+ex^n)^p)}{fx^n+g}dx$$
$$\Big\downarrow \text{2925}$$
$$\frac{\int \frac{\log(c(ex^n+d)^p)}{fx^n+g}dx^n}{n}$$
$$\Big\downarrow \text{2841}$$
$$\frac{\frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{f}-\frac{ep\int \frac{\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{ex^n+d}dx^n}{f}}{n}$$
$$\Big\downarrow \text{2840}$$
$$\frac{\frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{f}-\frac{p\int x^{-n}\log\left(1-\frac{f(ex^n+d)}{df-eg}\right)d(ex^n+d)}{f}}{n}$$
$$\Big\downarrow \text{2838}$$
$$\frac{\frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{f}+\frac{p\,\text{PolyLog}\left(2,\frac{f(ex^n+d)}{df-eg}\right)}{f}}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]
```

output
```
((Log[c*(d + e*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/f + (p*PolyLog
[2, (f*(d + e*x^n))/(d*f - e*g)])/f)/n
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.47

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(g+f\,x^n)}{nf}-\frac{p\,\mathrm{dilog}\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{nf}-\frac{p\ln(g+f\,x^n)\ln\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{nf}+\frac{\left(\frac{i\pi\,\mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(\right.}{2}$

input
```
int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x,method=_RETURNVERBOSE)
```

output
```
1/n*ln((d+e*x^n)^p)*ln(g+f*x^n)/f-1/n/f*p*dilog(((g+f*x^n)*e+d*f-e*g)/(d*f
-e*g))-1/n/f*p*ln(g+f*x^n)*ln(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))+(1/2*I*Pi*c
sgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*cs
gn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*cs
gn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*ln(g+f*x^n)/f
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^n}\right)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="fricas")
```

output
```
integral(x^n*log((e*x^n + d)^p*c)/(f*x*x^n + g*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.60

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \left(\frac{\log \left(f+\frac{g}{x^n}\right)}{fn} - \frac{\log \left(\frac{1}{x^n}\right)}{fn}\right)\log \left((ex^n+d)^p c\right) - \frac{\left(\log \left(fx^n+g\right)\log \left(\frac{efx^n+eg}{df-eg}+1\right)+\mathrm{Li}_2\left(-\frac{efx^n+eg}{df-eg}\right)\right)p}{fn}$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="maxima")
```

output
```
(log(f + g/x^n)/(f*n) - log(1/(x^n))/(f*n))*log((e*x^n + d)^p*c) - (log(f*
x^n + g)*log((e*f*x^n + e*g)/(d*f - e*g) + 1) + dilog(-(e*f*x^n + e*g)/(d*
f - e*g)))*p/(f*n)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^n}\right)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/((f + g/x^n)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,\left(f+\frac{g}{x^n}\right)}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)), x)
```

Reduce [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx$$
$$= \frac{-2\left(\int \frac{x^{2n}\log((x^n e+d)^p c)}{x^{2n}efx+x^n dfx+x^n egx+dgx}dx\right)defnp+2\left(\int \frac{x^{2n}\log((x^n e+d)^p c)}{x^{2n}efx+x^n dfx+x^n egx+dgx}dx\right)e^2gnp+\log((x^n e+d)^p\,c)^2 d}{2egnp}$$

input
```
int(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x)
```

output
```
( - 2*int((x**(2*n)*log((x**n*e + d)**p*c))/(x**(2*n)*e*f*x + x**n*d*f*x +
 x**n*e*g*x + d*g*x),x)*d*e*f*n*p + 2*int((x**(2*n)*log((x**n*e + d)**p*c)
)/(x**(2*n)*e*f*x + x**n*d*f*x + x**n*e*g*x + d*g*x),x)*e**2*g*n*p + log((
x**n*e + d)**p*c)**2*d)/(2*e*g*n*p)
```

3.373 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 221

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{g}-\sqrt{-f}x^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn} + \frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(\sqrt{g}+\sqrt{-f}x^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}$$

output
```
1/2*ln(c*(d+e*x^n)^p)*ln(e*(g^(1/2)-(-f)^(1/2)*x^n)/(d*(-f)^(1/2)+e*g^(1/2
)))/f/n+1/2*ln(c*(d+e*x^n)^p)*ln(-e*(g^(1/2)+(-f)^(1/2)*x^n)/(d*(-f)^(1/2)
-e*g^(1/2)))/f/n+1/2*p*polylog(2,(-f)^(1/2)*(d+e*x^n)/(d*(-f)^(1/2)-e*g^(1
/2)))/f/n+1/2*p*polylog(2,(-f)^(1/2)*(d+e*x^n)/(d*(-f)^(1/2)+e*g^(1/2)))/f
/n
```

Mathematica [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))), x]
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2005, 2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx$$

$$\Big\downarrow \text{2005}$$

$$\int \frac{x^{2n-1}\log(c(d+ex^n)^p)}{fx^{2n}+g}dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{\int \frac{x^n\log(c(ex^n+d)^p)}{fx^{2n}+g}dx^n}{n}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\int \left(\frac{\sqrt{-f}\log(c(ex^n+d)^p)}{2f\left(\sqrt{-f}x^n+\sqrt{g}\right)}-\frac{\sqrt{-f}\log(c(ex^n+d)^p)}{2f\left(\sqrt{g}-\sqrt{-f}x^n\right)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{g}-\sqrt{-f}x^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f}+\frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(\sqrt{-f}x^n+\sqrt{g})}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f}+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{-f}(ex^n+d)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f}+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{-f}(ex^n+d)}{\sqrt{-f}d+e\sqrt{g}}\right)}{2f}}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
```

output
```
((Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] - Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sq
rt[g])])/(2*f) + (Log[c*(d + e*x^n)^p]*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/
(d*Sqrt[-f] - e*Sqrt[g]))])/(2*f) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(
d*Sqrt[-f] - e*Sqrt[g])])/(2*f) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(d*
Sqrt[-f] + e*Sqrt[g])])/(2*f))/n
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.71 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.80

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(g+f\,x^{2n})}{2nf} - \frac{p\ln(d+e\,x^n)\ln(g+f\,x^{2n})}{2nf} + \frac{p\ln(d+e\,x^n)\ln\left(\frac{e\sqrt{-gf}-f(d+e\,x^n)+df}{e\sqrt{-gf}+df}\right)}{2nf} + \frac{p\ln(d+e\,x^n)\ln\left(\frac{e\sqrt{-gf}}{e}\right.}{2nf}$

input
```
int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x,method=_RETURNVERBOSE)
```

output
```
1/2/n*ln((d+e*x^n)^p)/f*ln(g+f*(x^n)^2)-1/2/n/f*p*ln(d+e*x^n)*ln(g+f*(x^n)
^2)+1/2/n/f*p*ln(d+e*x^n)*ln((e*(-g*f)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-g*f)^(1
/2)+d*f))+1/2/n/f*p*ln(d+e*x^n)*ln((e*(-g*f)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-g
*f)^(1/2)-d*f))+1/2/n/f*p*dilog((e*(-g*f)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-g*f)
^(1/2)+d*f))+1/2/n/f*p*dilog((e*(-g*f)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-g*f)^(1
/2)-d*f))+1/2*(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*
Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(
d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n/f*ln(g+f
*(x^n)^2)
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="fricas")
```

output
```
integral(x^(2*n)*log((e*x^n + d)^p*c)/(f*x*x^(2*n) + g*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n))),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="maxima")
```

output
```
integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))*x), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{x\,\left(f+\frac{g}{x^{2\,n}}\right)}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))), x)
```

Reduce [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \frac{2\left(\int \frac{x^{2n}\log((x^n e+d)^p c)}{x^{3n}efx+x^{2n}dfx+x^n egx+dgx}dx\right)dfnp-2\left(\int \frac{x^n\log((x^n e+d)^p c)}{x^{3n}efx+x^{2n}dfx+x^n egx+dgx}dx\right)egnp+\log((x^n e+d)^p c)^2}{2fnp}$$

input
```
int(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x)
```

output
```
(2*int((x**(2*n)*log((x**n*e + d)**p*c))/(x**(3*n)*e*f*x + x**(2*n)*d*f*x
+ x**n*e*g*x + d*g*x),x)*d*f*n*p - 2*int((x**n*log((x**n*e + d)**p*c))/(x*
*(3*n)*e*f*x + x**(2*n)*d*f*x + x**n*e*g*x + d*g*x),x)*e*g*n*p + log((x**n
*e + d)**p*c)**2)/(2*f*n*p)
```

3.374 $\int \frac{\log(c(d+ex^n)^p)}{x\left(f+gx^{2n}\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 419

$$\begin{aligned}\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})^2}\,dx =& -\frac{de\sqrt{g}p\arctan\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}(e^2f+d^2g)\,n}-\frac{e^2p\log(d+ex^n)}{2f(e^2f+d^2g)\,n}\\&+\frac{\log(c(d+ex^n)^p)}{2fn(f+gx^{2n})}+\frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{f^2n}\\&-\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2n}\\&-\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}\\&+\frac{e^2p\log(f+gx^{2n})}{4f(e^2f+d^2g)\,n}-\frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}\\&-\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2n}+\frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{f^2n}\end{aligned}$$

output
```
-1/2*d*e*g^(1/2)*p*arctan(g^(1/2)*x^n/f^(1/2))/f^(3/2)/(d^2*g+e^2*f)/n-1/2
*e^2*p*ln(d+e*x^n)/f/(d^2*g+e^2*f)/n+1/2*ln(c*(d+e*x^n)^p)/f/n/(f+g*x^(2*n
))+ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f^2/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(
1/2)-g^(1/2)*x^n)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2/n-1/2*ln(c*(d+e*x^n)^p)*ln
(e*((-f)^(1/2)+g^(1/2)*x^n)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2/n+1/4*e^2*p*ln(f
+g*x^(2*n))/f/(d^2*g+e^2*f)/n-1/2*p*polylog(2,-g^(1/2)*(d+e*x^n)/(e*(-f)^(
1/2)-d*g^(1/2)))/f^2/n-1/2*p*polylog(2,g^(1/2)*(d+e*x^n)/(e*(-f)^(1/2)+d*g
^(1/2)))/f^2/n+p*polylog(2,1+e*x^n/d)/f^2/n
```

Mathematica [F]

$$\int \frac{\log (c(d+ex^n)^p)}{x (f+gx^{2n})^2} \, dx = \int \frac{\log (c(d+ex^n)^p)}{x (f+gx^{2n})^2} \, dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2), x]
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{x (f+gx^{2n})^2} \, dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{\int \frac{x^{-n}\log (c(ex^n+d)^p)}{(gx^{2n}+f)^2}dx^n}{n}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\int \left(\frac{\log(c(ex^n+d)^p)x^{-n}}{f^2} - \frac{g\log(c(ex^n+d)^p)x^n}{f^2(gx^{2n}+f)} - \frac{g\log(c(ex^n+d)^p)x^n}{f(gx^{2n}+f)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{de\sqrt{g}p\arctan\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}(d^2g+e^2f)} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2} + \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{f^2} +$$

input
```
Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
```

output
```
(-1/2*(d*e*Sqrt[g]*p*ArcTan[(Sqrt[g]*x^n)/Sqrt[f]])/(f^(3/2)*(e^2*f + d^2*
g)) - (e^2*p*Log[d + e*x^n])/(2*f*(e^2*f + d^2*g)) + Log[c*(d + e*x^n)^p]/
(2*f*(f + g*x^(2*n))) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/f^2 - (Lo
g[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqrt[g
])])/(2*f^2) - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/(e*S
qrt[-f] - d*Sqrt[g])])/(2*f^2) + (e^2*p*Log[f + g*x^(2*n)])/(4*f*(e^2*f +
d^2*g)) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt[g]))]
)/(2*f^2) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])
/(2*f^2) + (p*PolyLog[2, 1 + (e*x^n)/d])/f^2)/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 22.05 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.52

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(x^n)}{n\,f^2}-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^{2n})}{2n\,f^2}+\frac{\ln((d+e\,x^n)^p)}{2nf(f+g\,x^{2n})}-\frac{e^2p\ln(d+e\,x^n)}{2f(d^2g+f\,e^2)n}+\frac{e^2p\ln(f+g\,x^{2n})}{4f(d^2g+f\,e^2)n}-\frac{epgd\arctan}{2nf(d^2g+f}$

input
```
int(ln(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/n*ln((d+e*x^n)^p)/f^2*ln(x^n)-1/2/n*ln((d+e*x^n)^p)/f^2*ln(f+g*(x^n)^2)+
1/2/n*ln((d+e*x^n)^p)/f/(f+g*(x^n)^2)-1/2*e^2*p*ln(d+e*x^n)/f/(d^2*g+e^2*f
)/n+1/4/n*e^2*p/f/(d^2*g+e^2*f)*ln(f+g*(x^n)^2)-1/2/n*e*p/f/(d^2*g+e^2*f)*
g*d/(g*f)^(1/2)*arctan(x^n*g/(g*f)^(1/2))-1/n*p/f^2*dilog((d+e*x^n)/d)-1/n
*p/f^2*ln(x^n)*ln((d+e*x^n)/d)+1/2/n*p/f^2*ln(d+e*x^n)*ln(f+g*(x^n)^2)-1/2
/n*p/f^2*ln(d+e*x^n)*ln((e*(-g*f)^(1/2)-(d+e*x^n)*g+d*g)/(e*(-g*f)^(1/2)+d
*g))-1/2/n*p/f^2*ln(d+e*x^n)*ln((e*(-g*f)^(1/2)+(d+e*x^n)*g-d*g)/(e*(-g*f)
^(1/2)-d*g))-1/2/n*p/f^2*dilog((e*(-g*f)^(1/2)-(d+e*x^n)*g+d*g)/(e*(-g*f)^
(1/2)+d*g))-1/2/n*p/f^2*dilog((e*(-g*f)^(1/2)+(d+e*x^n)*g-d*g)/(e*(-g*f)^(
1/2)-d*g))+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*
csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e
*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(1/n/f^2*ln(x
^n)-1/2/n/f^2*ln(f+g*(x^n)^2)+1/2/n/f/(f+g*(x^n)^2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(gx^{2\,n}+f\right)^2 x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/(g^2*x*x^(4*n) + 2*f*g*x*x^(2*n) + f^2*x), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(gx^{2\,n}+f\right)^2 x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="maxima")
```

output
```
integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)^2*x), x)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{\left(gx^{2\,n}+f\right)^2 x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \int \frac{\ln \left(c\left(d+e\,x^n\right)^p\right)}{x\left(f+g\,x^{2\,n}\right)^2}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n)))^2),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))^2), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \int \frac{\log((x^n e+d)^p\,c)}{x^{4n}g^2x+2x^{2n}fgx+f^2x}dx$$

input
```
int(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x)
```

output
```
int(log((x**n*e + d)**p*c)/(x**(4*n)*g**2*x + 2*x**(2*n)*f*g*x + f**2*x),x
)
```

3.375 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)^2}\,dx = -\frac{ep\log(d+ex^n)}{f(ef-dg)n}+\frac{\log(c(d+ex^n)^p)}{fn(f+gx^n)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{f^2n}+\frac{ep\log(f+gx^n)}{f(ef-dg)n} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2n} - \frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{f^2n}+\frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{f^2n}$$

output
```
-e*p*ln(d+e*x^n)/f/(-d*g+e*f)/n+ln(c*(d+e*x^n)^p)/f/n/(f+g*x^n)+ln(-e*x^n/
d)*ln(c*(d+e*x^n)^p)/f^2/n+e*p*ln(f+g*x^n)/f/(-d*g+e*f)/n-ln(c*(d+e*x^n)^p
)*ln(e*(f+g*x^n)/(-d*g+e*f))/f^2/n-p*polylog(2,-g*(d+e*x^n)/(-d*g+e*f))/f^
2/n+p*polylog(2,1+e*x^n/d)/f^2/n
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)^2}\,dx$$
$$= \frac{-\frac{efp\log(d+ex^n)}{ef-dg}+\frac{f\log(c(d+ex^n)^p)}{f+gx^n}+\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)+\frac{efp\log(f+gx^n)}{ef-dg}-\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f}{e}\right.}{f^2n}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```

output
```
(-((e*f*p*Log[d + e*x^n])/(e*f - d*g)) + (f*Log[c*(d + e*x^n)^p])/(f + g*x
^n) + Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + (e*f*p*Log[f + g*x^n])/(e*f
 - d*g) - Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f - d*g)] - p*PolyLo
g[2, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n
)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)^2}\,dx$$
$$\Big\downarrow \; 2925$$
$$\frac{\int \frac{x^{-n}\log(c(ex^n+d)^p)}{(gx^n+f)^2}dx^n}{n}$$
$$\Big\downarrow \; 2863$$
$$\frac{\int \left(\frac{\log(c(ex^n+d)^p)x^{-n}}{f^2}-\frac{g\log(c(ex^n+d)^p)}{f^2(gx^n+f)}-\frac{g\log(c(ex^n+d)^p)}{f(gx^n+f)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2}+\frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{f^2}+\frac{\log(c(d+ex^n)^p)}{f(f+gx^n)}-\frac{p\operatorname{PolyLog}\left(2,-\frac{g(ex^n+d)}{ef-dg}\right)}{f^2}+\frac{p\operatorname{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2}}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```

output
```
(-((e*p*Log[d + e*x^n])/(f*(e*f - d*g))) + Log[c*(d + e*x^n)^p]/(f*(f + g*
x^n)) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/f^2 + (e*p*Log[f + g*x^n]
)/(f*(e*f - d*g)) - (Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f - d*g)]
)/f^2 - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f - d*g))])/f^2 + (p*PolyLog[2,
 1 + (e*x^n)/d])/f^2)/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.14 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^n)}{n\,f^2}+\frac{\ln((d+e\,x^n)^p)}{nf(f+g\,x^n)}+\frac{\ln((d+e\,x^n)^p)\ln(x^n)}{n\,f^2}-\frac{ep\ln(f+g\,x^n)}{nf(dg-ef)}+\frac{ep\ln(d+e\,x^n)}{nf(dg-ef)}-\frac{p\,\mathrm{dilog}\left(\frac{d+e\,x^n}{d}\right.}{n\,f^2}$

input
```
int(ln(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/n*ln((d+e*x^n)^p)/f^2*ln(f+g*x^n)+1/n*ln((d+e*x^n)^p)/f/(f+g*x^n)+1/n*l
n((d+e*x^n)^p)/f^2*ln(x^n)-1/n*e*p/f/(d*g-e*f)*ln(f+g*x^n)+1/n*e*p/f/(d*g-
e*f)*ln(d+e*x^n)-1/n*p/f^2*dilog((d+e*x^n)/d)-1/n*p/f^2*ln(x^n)*ln((d+e*x^
n)/d)+1/n*p/f^2*dilog(((f+g*x^n)*e+d*g-e*f)/(d*g-e*f))+1/n*p/f^2*ln(f+g*x^
n)*ln(((f+g*x^n)*e+d*g-e*f)/(d*g-e*f))+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(
I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn
(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*cs
gn(I*c)+ln(c))/n*(-ln(f+g*x^n)/f^2+1/f/(f+g*x^n)+ln(x^n)/f^2)
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)}{\left(gx^n+f\right)^2x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/(g^2*x*x^(2*n) + 2*f*g*x*x^n + f^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.14

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx =$$

$$-enp\left(\frac{\log\left(\frac{ex^n+d}{e}\right)}{ef^2n^2-dfgn^2}-\frac{\log\left(\frac{gx^n+f}{g}\right)}{ef^2n^2-dfgn^2}+\frac{\log\left(x^n\right)\log\left(\frac{ex^n}{d}+1\right)+\mathrm{Li}_2\left(-\frac{ex^n}{d}\right)}{ef^2n^2}-\frac{\log\left(gx^n+f\right)\log\left(-\frac{e}{}\right.}{}\right.$$
$$+\left(\frac{1}{fgnx^n+f^2n}-\frac{\log\left(gx^n+f\right)}{f^2n}+\frac{\log\left(x^n\right)}{f^2n}\right)\log\left(\left(ex^n+d\right)^pc\right)$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="maxima")
```

output
```
-e*n*p*(log((e*x^n + d)/e)/(e*f^2*n^2 - d*f*g*n^2) - log((g*x^n + f)/g)/(e
*f^2*n^2 - d*f*g*n^2) + (log(x^n)*log(e*x^n/d + 1) + dilog(-e*x^n/d))/(e*f
^2*n^2) - (log(g*x^n + f)*log(-(e*g*x^n + e*f)/(e*f - d*g) + 1) + dilog((e
*g*x^n + e*f)/(e*f - d*g)))/(e*f^2*n^2)) + (1/(f*g*n*x^n + f^2*n) - log(g*
x^n + f)/(f^2*n) + log(x^n)/(f^2*n))*log((e*x^n + d)^p*c)
```

Giac [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)^2}\,dx = \int \frac{\log((ex^n+d)^pc)}{(gx^n+f)^2x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/((g*x^n + f)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)^2}\,dx = \int \frac{\ln(c(d+e\,x^n)^p)}{x\,(f+g\,x^n)^2}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)^2),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)^2), x)
```

Reduce [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)^2}\,dx = \int \frac{\log((x^ne+d)^p\,c)}{x^{2n}g^2x+2x^nfgx+f^2x}dx$$

input
```
int(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x)
```

output
```
int(log((x**n*e + d)**p*c)/(x**(2*n)*g**2*x + 2*x**n*f*g*x + f**2*x),x)
```

3.376 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx=\frac{egp\log\left(d+ex^n\right)}{f^2(df-eg)n}+\frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)}-\frac{egp\log\left(g+fx^n\right)}{f^2(df-eg)n}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{f^2n}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{f^2n}$$

output
```
e*g*p*ln(d+e*x^n)/f^2/(d*f-e*g)/n+g*ln(c*(d+e*x^n)^p)/f^2/n/(g+f*x^n)-e*g*
p*ln(g+f*x^n)/f^2/(d*f-e*g)/n+ln(c*(d+e*x^n)^p)*ln(-e*(g+f*x^n)/(d*f-e*g))
/f^2/n+p*polylog(2,f*(d+e*x^n)/(d*f-e*g))/f^2/n
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 433 vs. $2(156) = 312$.

Time = 1.46 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.78

$$\int \frac{\log (c(d+ex^n)^p)}{x (f+gx^{-n})^2} \, dx$$
$$= \frac{gp\log (f-fx^{-n})+fpx^n\log (f-fx^{-n})-gnp\log(x)\log (f-fx^{-n})-fnpx^n\log(x)\log (f-fx^{-n})-}{}$$
$$-\frac{p\left(-\frac{df\log(e+dx^{-n})}{df-eg}+\frac{fx^n\log(e+dx^{-n})}{g+fx^n}+\log\left(-\frac{dx^{-n}}{e}\right)\log(e+dx^{-n})+\frac{df\log(f+gx^{-n})}{df-eg}-\log(e+dx^{-n})\log\left(\right.}{f^2n}$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

output
```
(g*p*Log[f - f/x^n] + f*p*x^n*Log[f - f/x^n] - g*n*p*Log[x]*Log[f - f/x^n]
 - f*n*p*x^n*Log[x]*Log[f - f/x^n] - p*Log[e + d/x^n]*(-(f*x^n) + (g + f*x
^n)*Log[f - f/x^n]) - f*x^n*Log[c*(d + e*x^n)^p] + g*Log[f - f/x^n]*Log[c*
(d + e*x^n)^p] + f*x^n*Log[f - f/x^n]*Log[c*(d + e*x^n)^p] + g*n*p*Log[x]*
Log[1 + (f*x^n)/g] + f*n*p*x^n*Log[x]*Log[1 + (f*x^n)/g] + p*(g + f*x^n)*P
olyLog[2, -((f*x^n)/g)])/(f^2*n*(g + f*x^n)) - (p*(-((d*f*Log[e + d/x^n])/
(d*f - e*g)) + (f*x^n*Log[e + d/x^n])/(g + f*x^n) + Log[-(d/(e*x^n))]*Log[
e + d/x^n] + (d*f*Log[f + g/x^n])/(d*f - e*g) - Log[e + d/x^n]*Log[(d*(f +
 g/x^n))/(d*f - e*g)] - PolyLog[2, -((g*(e + d/x^n))/(d*f - e*g))] + PolyL
og[2, 1 + d/(e*x^n)]))/(f^2*n)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2005, 2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x \left(f+gx^{-n}\right)^2} \, dx$$

$\Big\downarrow$ 2005

$$\int \frac{x^{2n-1} \log \left(c(d+ex^n)^p\right)}{\left(fx^n+g\right)^2}dx$$

$\Big\downarrow$ 2925

$$\frac{\int \frac{x^n \log(c(ex^n+d)^p)}{(fx^n+g)^2}dx^n}{n}$$

$\Big\downarrow$ 2863

$$\frac{\int \left(\frac{\log(c(ex^n+d)^p)}{f(fx^n+g)}-\frac{g\log(c(ex^n+d)^p)}{f(fx^n+g)^2}\right)dx^n}{n}$$

$\Big\downarrow$ 2009

$$\frac{\frac{g\log(c(d+ex^n)^p)}{f^2(fx^n+g)}+\frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{f^2}+\frac{p\,\text{PolyLog}\left(2,\frac{f(ex^n+d)}{df-eg}\right)}{f^2}+\frac{egp\log(d+ex^n)}{f^2(df-eg)}-\frac{egp\log(fx^n+g)}{f^2(df-eg)}}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

output
```
((e*g*p*Log[d + e*x^n])/(f^2*(d*f - e*g)) + (g*Log[c*(d + e*x^n)^p])/(f^2*
(g + f*x^n)) - (e*g*p*Log[g + f*x^n])/(f^2*(d*f - e*g)) + (Log[c*(d + e*x^
n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/f^2 + (p*PolyLog[2, (f*(d + e*x
^n))/(d*f - e*g)])/f^2)/n
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.21 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.28

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(g+f\,x^n)}{n\,f^2}+\frac{\ln((d+e\,x^n)^p)g}{n\,f^2(g+f\,x^n)}-\frac{p\,\mathrm{dilog}\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{n\,f^2}-\frac{p\ln(g+f\,x^n)\ln\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{n\,f^2}+$

input
```
int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/n*ln((d+e*x^n)^p)/f^2*ln(g+f*x^n)+1/n*ln((d+e*x^n)^p)*g/f^2/(g+f*x^n)-1/
n*p/f^2*dilog(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))-1/n*p/f^2*ln(g+f*x^n)*ln(((
g+f*x^n)*e+d*f-e*g)/(d*f-e*g))+1/n*e*p/f^2*g/(d*f-e*g)*ln((g+f*x^n)*e+d*f-
e*g)-e*g*p*ln(g+f*x^n)/f^2/(d*f-e*g)/n+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(
I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn
(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*cs
gn(I*c)+ln(c))*(1/n/f^2*ln(g+f*x^n)+1/n*g/f^2/(g+f*x^n))
```

Fricas [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})^2}\,dx = \int \frac{\log((ex^n+d)^pc)}{\left(f+\frac{g}{x^n}\right)^2 x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)/(f^2*x + 2*f*g*x*x^n/x^(2*n) + g^2*x/x^(2*n)
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})^2}\,dx$$
$$= enp\left(\frac{d\log\left(\frac{ex^n+d}{e}\right)}{def^2n^2-e^2fgn^2} - \frac{g\log\left(\frac{fx^n+g}{f}\right)}{df^3n^2-ef^2gn^2} - \frac{\log(fx^n+g)\log\left(\frac{efx^n+eg}{df-eg}+1\right)+\mathrm{Li}_2\left(-\frac{efx^n+eg}{df-eg}\right)}{ef^2n^2}\right) - \left(\frac{1}{f^2n+\frac{fgn}{x^n}} - \frac{\log\left(f+\frac{g}{x^n}\right)}{f^2n} + \frac{\log\left(\frac{1}{x^n}\right)}{f^2n}\right)\log((ex^n+d)^pc)$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="maxima")
```

output
```
e*n*p*(d*log((e*x^n + d)/e)/(d*e*f^2*n^2 - e^2*f*g*n^2) - g*log((f*x^n + g
)/f)/(d*f^3*n^2 - e*f^2*g*n^2) - (log(f*x^n + g)*log((e*f*x^n + e*g)/(d*f
- e*g) + 1) + dilog(-(e*f*x^n + e*g)/(d*f - e*g)))/(e*f^2*n^2)) - (1/(f^2*
n + f*g*n/x^n) - log(f + g/x^n)/(f^2*n) + log(1/(x^n))/(f^2*n))*log((e*x^n
 + d)^p*c)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx=\int \frac{\log \left((ex^n+d)^pc\right)}{\left(f+\frac{g}{x^n}\right)^2x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/((f + g/x^n)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx=\int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,{\left(f+\frac{g}{x^n}\right)}^2}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)^2),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)^2), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x)
```

output
```
(2*x**n*int((x**(2*n)*log((x**n*e + d)**p*c))/(x**(3*n)*e*f**2*x + x**(2*n
)*d*f**2*x + 2*x**(2*n)*e*f*g*x + 2*x**n*d*f*g*x + x**n*e*g**2*x + d*g**2*
x),x)*d**3*e*f**5*g*n*p - 6*x**n*int((x**(2*n)*log((x**n*e + d)**p*c))/(x*
*(3*n)*e*f**2*x + x**(2*n)*d*f**2*x + 2*x**(2*n)*e*f*g*x + 2*x**n*d*f*g*x
+ x**n*e*g**2*x + d*g**2*x),x)*d**2*e**2*f**4*g**2*n*p + 6*x**n*int((x**(2
*n)*log((x**n*e + d)**p*c))/(x**(3*n)*e*f**2*x + x**(2*n)*d*f**2*x + 2*x**
(2*n)*e*f*g*x + 2*x**n*d*f*g*x + x**n*e*g**2*x + d*g**2*x),x)*d*e**3*f**3*
g**3*n*p - 2*x**n*int((x**(2*n)*log((x**n*e + d)**p*c))/(x**(3*n)*e*f**2*x
 + x**(2*n)*d*f**2*x + 2*x**(2*n)*e*f*g*x + 2*x**n*d*f*g*x + x**n*e*g**2*x
 + d*g**2*x),x)*e**4*f**2*g**4*n*p + 2*x**n*log(x**n*e + d)*d**3*f**4*p**2
 - 6*x**n*log(x**n*e + d)*d**2*e*f**3*g*p**2 + 6*x**n*log(x**n*e + d)*d*e*
*2*f**2*g**2*p**2 - 2*x**n*log(x**n*f + g)*e**3*f*g**3*p**2 + x**n*log((x*
*n*e + d)**p*c)**2*d**2*e*f**3*g - x**n*log((x**n*e + d)**p*c)**2*d*e**2*f
**2*g**2 - 2*x**n*log((x**n*e + d)**p*c)*d**3*f**4*p + 6*x**n*log((x**n*e
+ d)**p*c)*d**2*e*f**3*g*p - 6*x**n*log((x**n*e + d)**p*c)*d*e**2*f**2*g**
2*p + 2*x**n*log((x**n*e + d)**p*c)*e**3*f*g**3*p + 2*int((x**(2*n)*log((x
**n*e + d)**p*c))/(x**(3*n)*e*f**2*x + x**(2*n)*d*f**2*x + 2*x**(2*n)*e*f*
g*x + 2*x**n*d*f*g*x + x**n*e*g**2*x + d*g**2*x),x)*d**3*e*f**4*g**2*n*p -
 6*int((x**(2*n)*log((x**n*e + d)**p*c))/(x**(3*n)*e*f**2*x + x**(2*n)*d*f
**2*x + 2*x**(2*n)*e*f*g*x + 2*x**n*d*f*g*x + x**n*e*g**2*x + d*g**2*x)...
```

3.377 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 377

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})^2} \, dx = -\frac{de\sqrt{g}p\arctan\left(\frac{\sqrt{f}x^n}{\sqrt{g}}\right)}{2f^{3/2}(d^2f+e^2g)\,n} - \frac{e^2gp\log(d+ex^n)}{2f^2(d^2f+e^2g)\,n} + \frac{g\log(c(d+ex^n)^p)}{2f^2n(g+fx^{2n})} + \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{g}-\sqrt{-f}x^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n} + \frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(\sqrt{g}+\sqrt{-f}x^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n} + \frac{e^2gp\log(g+fx^{2n})}{4f^2(d^2f+e^2g)\,n} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}$$

output
```
-1/2*d*e*g^(1/2)*p*arctan(f^(1/2)*x^n/g^(1/2))/f^(3/2)/(d^2*f+e^2*g)/n-1/2
*e^2*g*p*ln(d+e*x^n)/f^2/(d^2*f+e^2*g)/n+1/2*g*ln(c*(d+e*x^n)^p)/f^2/n/(g+
f*x^(2*n))+1/2*ln(c*(d+e*x^n)^p)*ln(e*(g^(1/2)-(-f)^(1/2)*x^n)/(d*(-f)^(1/
2)+e*g^(1/2)))/f^2/n+1/2*ln(c*(d+e*x^n)^p)*ln(-e*(g^(1/2)+(-f)^(1/2)*x^n)/
(d*(-f)^(1/2)-e*g^(1/2)))/f^2/n+1/4*e^2*g*p*ln(g+f*x^(2*n))/f^2/(d^2*f+e^2
*g)/n+1/2*p*polylog(2,(-f)^(1/2)*(d+e*x^n)/(d*(-f)^(1/2)-e*g^(1/2)))/f^2/n
+1/2*p*polylog(2,(-f)^(1/2)*(d+e*x^n)/(d*(-f)^(1/2)+e*g^(1/2)))/f^2/n
```

Mathematica [F]

$$\int \frac{\log (c(d+ex^n)^p)}{x\left(f+gx^{-2n}\right)^2}\,dx=\int \frac{\log (c(d+ex^n)^p)}{x\left(f+gx^{-2n}\right)^2}\,dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2), x]
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2005, 2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (c(d+ex^n)^p)}{x\left(f+gx^{-2n}\right)^2}\,dx$$

$$\Big\downarrow \text{ 2005}$$

$$\int \frac{x^{4n-1}\log (c(d+ex^n)^p)}{\left(fx^{2n}+g\right)^2}dx$$

$$\Big\downarrow \text{ 2925}$$

$$\frac{\int \frac{x^{3n}\log(c(ex^n+d)^p)}{(fx^{2n}+g)^2}dx^n}{n}$$

$$\Big\downarrow \text{ 2863}$$

$$\frac{\int \left(\frac{x^n\log(c(ex^n+d)^p)}{f(fx^{2n}+g)}-\frac{gx^n\log(c(ex^n+d)^p)}{f(fx^{2n}+g)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{de\sqrt{g}p\arctan\left(\frac{\sqrt{f}x^n}{\sqrt{g}}\right)}{2f^{3/2}(d^2f+e^2g)}+\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{g}-\sqrt{-f}x^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2}+\frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(\sqrt{-f}x^n+\sqrt{g})}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2}+\frac{g\log(c(d+ex^n)^p)}{2f^2(fx^{2n}+g)}+\frac{e^2gp\log(}{4f^2(d^2}}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]
```

output
```
(-1/2*(d*e*Sqrt[g]*p*ArcTan[(Sqrt[f]*x^n)/Sqrt[g]])/(f^(3/2)*(d^2*f + e^2*
g)) - (e^2*g*p*Log[d + e*x^n])/(2*f^2*(d^2*f + e^2*g)) + (g*Log[c*(d + e*x
^n)^p])/(2*f^2*(g + f*x^(2*n))) + (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] -
Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2) + (Log[c*(d + e*x^n)^p]*
Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(d*Sqrt[-f] - e*Sqrt[g]))])/(2*f^2) + (
e^2*g*p*Log[g + f*x^(2*n)])/(4*f^2*(d^2*f + e^2*g)) + (p*PolyLog[2, (Sqrt[
-f]*(d + e*x^n))/(d*Sqrt[-f] - e*Sqrt[g])])/(2*f^2) + (p*PolyLog[2, (Sqrt[
-f]*(d + e*x^n))/(d*Sqrt[-f] + e*Sqrt[g])])/(2*f^2))/n
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 118.10 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.49

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(g+f\,x^{2n})}{2n\,f^2}+\frac{\ln((d+e\,x^n)^p)g}{2n\,f^2(g+f\,x^{2n})}-\frac{p\ln(d+e\,x^n)\ln(g+f\,x^{2n})}{2n\,f^2}+\frac{p\ln(d+e\,x^n)\ln\left(\frac{e\sqrt{-gf}-f(d+e\,x^n)+df}{e\sqrt{-gf}+df}\right)}{2n\,f^2}+\frac{p\ln}{}$

input

```
int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x,method=_RETURNVERBOSE)
```

output

```
1/2/n*ln((d+e*x^n)^p)/f^2*ln(g+f*(x^n)^2)+1/2/n*ln((d+e*x^n)^p)*g/f^2/(g+f
*(x^n)^2)-1/2/n*p/f^2*ln(d+e*x^n)*ln(g+f*(x^n)^2)+1/2/n*p/f^2*ln(d+e*x^n)*
ln((e*(-g*f)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-g*f)^(1/2)+d*f))+1/2/n*p/f^2*ln(d
+e*x^n)*ln((e*(-g*f)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-g*f)^(1/2)-d*f))+1/2/n*p/
f^2*dilog((e*(-g*f)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-g*f)^(1/2)+d*f))+1/2/n*p/f
^2*dilog((e*(-g*f)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-g*f)^(1/2)-d*f))-1/2*e^2*g*
p*ln(d+e*x^n)/f^2/(d^2*f+e^2*g)/n+1/4/n*e^2*p*g/f^2/(d^2*f+e^2*g)*ln(g+f*(
x^n)^2)-1/2/n*e*p*g/f/(d^2*f+e^2*g)*d/(g*f)^(1/2)*arctan(x^n*f/(g*f)^(1/2)
)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d
+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^
3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(1/2/n/f^2*ln(g+f*(x^n
)^2)+1/2/n*g/f^2/(g+f*(x^n)^2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)}{\left(f+\frac{g}{x^{2\,n}}\right)^2x}\,dx$$

input

```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="fricas")
```

output

```
integral(log((e*x^n + d)^p*c)/(f^2*x + 2*f*g*x*x^(2*n)/x^(4*n) + g^2*x/x^(
4*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n)))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)^2 x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="maxima")
```

output
```
integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))^2*x), x)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)^2 x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \int \frac{\ln \left(c\left(d+e\,x^n\right)^p\right)}{x\left(f+\frac{g}{x^{2\,n}}\right)^2}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))^2),x)
```

output
```
int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))^2), x)
```

Reduce [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \text{too large to display}$$

input
```
int(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x)
```

output
```
(6*x**(2*n)*sqrt(g)*sqrt(f)*atan((x**n*f)/(sqrt(g)*sqrt(f)))*d*e**3*f*g*p*
*2 + 6*sqrt(g)*sqrt(f)*atan((x**n*f)/(sqrt(g)*sqrt(f)))*d*e**3*g**2*p**2 +
 2*x**(2*n)*int((x**(5*n)*log((x**n*e + d)**p*c))/(x**(5*n)*e*f**2*x + x**
(4*n)*d*f**2*x + 2*x**(3*n)*e*f*g*x + 2*x**(2*n)*d*f*g*x + x**n*e*g**2*x +
 d*g**2*x),x)*d**4*e*f**5*n*p + 8*x**(2*n)*int((x**(5*n)*log((x**n*e + d)*
*p*c))/(x**(5*n)*e*f**2*x + x**(4*n)*d*f**2*x + 2*x**(3*n)*e*f*g*x + 2*x**
(2*n)*d*f*g*x + x**n*e*g**2*x + d*g**2*x),x)*d**2*e**3*f**4*g*n*p + 6*x**(
2*n)*int((x**(5*n)*log((x**n*e + d)**p*c))/(x**(5*n)*e*f**2*x + x**(4*n)*d
*f**2*x + 2*x**(3*n)*e*f*g*x + 2*x**(2*n)*d*f*g*x + x**n*e*g**2*x + d*g**2
*x),x)*e**5*f**3*g**2*n*p + 6*x**(2*n)*int((x**n*log((x**n*e + d)**p*c))/(
x**(5*n)*e*f**2*x + x**(4*n)*d*f**2*x + 2*x**(3*n)*e*f*g*x + 2*x**(2*n)*d*
f*g*x + x**n*e*g**2*x + d*g**2*x),x)*d**4*e*f**3*g**2*n*p + 8*x**(2*n)*int
((x**n*log((x**n*e + d)**p*c))/(x**(5*n)*e*f**2*x + x**(4*n)*d*f**2*x + 2*
x**(3*n)*e*f*g*x + 2*x**(2*n)*d*f*g*x + x**n*e*g**2*x + d*g**2*x),x)*d**2*
e**3*f**2*g**3*n*p + 2*x**(2*n)*int((x**n*log((x**n*e + d)**p*c))/(x**(5*n
)*e*f**2*x + x**(4*n)*d*f**2*x + 2*x**(3*n)*e*f*g*x + 2*x**(2*n)*d*f*g*x +
 x**n*e*g**2*x + d*g**2*x),x)*e**5*f*g**4*n*p + 2*x**(2*n)*log(x**(2*n)*f
+ g)*d**2*e**2*f**2*g*p**2 - x**(2*n)*log(x**(2*n)*f + g)*e**4*f*g**2*p**2
 - 4*x**(2*n)*log(x**n*e + d)*d**4*f**3*p**2 - 20*x**(2*n)*log(x**n*e + d)
*d**2*e**2*f**2*g*p**2 - 10*x**(2*n)*log(x**n*e + d)*e**4*f*g**2*p**2 -...
```

3.378 $\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})} dx$

Optimal result

Integrand size = 33, antiderivative size = 25

$$\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})} dx = -\frac{\text{PolyLog}(2,1-c(d+ex^n))}{cen}$$

output
```
-polylog(2,1-c*(d+e*x^n))/c/e/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})} dx = -\frac{\text{PolyLog}(2,1-cd-cex^n)}{cen}$$

input
```
Integrate[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]
```

output
```
-(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2005, 2925, 25, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})}dx$$

$$\Big\downarrow \text{ 2005}$$

$$\int \frac{x^{n-1}\log(c(d+ex^n))}{cd+cex^n-1}dx$$

$$\Big\downarrow \text{ 2925}$$

$$\frac{\int -\frac{\log(c(ex^n+d))}{-cex^n-cd+1}dx^n}{n}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\log(c(ex^n+d))}{-cex^n-cd+1}dx^n}{n}$$

$$\Big\downarrow \text{ 2840}$$

$$\frac{\int x^{-n}\log(cex^n+cd)\,d(-cex^n-cd+1)}{cen}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{\text{PolyLog}(2,-cex^n-cd+1)}{cen}$$

input
```
Int[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]
```

output
```
-(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result
derivativedivides	$-\frac{\operatorname{dilog}(ce\,x^{n}+cd)}{nce}$
default	$-\frac{\operatorname{dilog}(ce\,x^{n}+cd)}{nce}$
risch	$\frac{\ln(1-c(d+e\,x^{n}))\ln(d+e\,x^{n})}{nec}-\frac{\ln(1-c(d+e\,x^{n}))\ln(c(d+e\,x^{n}))}{nec}-\frac{\operatorname{dilog}(c(d+e\,x^{n}))}{nec}+\frac{\left(\frac{i\pi\ \operatorname{csgn}(i(d+e\,x^{n}))\operatorname{csgn}(ic}{2}\right.}{}$

input
```
int(ln(c*(d+e*x^n))/x/(c*e-(-c*d+1)/(x^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/n*dilog(c*e*x^n+c*d)/c/e
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(c(d+ex^n)\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx=-\frac{\mathrm{Li}_2(-cex^n-cd+1)}{cen}$$

input
```
integrate(log(c*(d+e*x^n))/x/(c*e-(-c*d+1)/(x^n)),x, algorithm="fricas")
```

output
```
-dilog(-c*e*x^n - c*d + 1)/(c*e*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log \left(c(d+ex^n)\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(ln(c*(d+e*x**n))/x/(c*e-(-c*d+1)/(x**n)),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(24) = 48$.

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.24

$$\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})}\,dx = \left(\frac{\log\left(ce+\frac{cd-1}{x^n}\right)}{cen} - \frac{\log\left(\frac{1}{x^n}\right)}{cen}\right)\log((ex^n+d)c) - \frac{\log(cex^n+cd)\log(cex^n+cd-1)+\mathrm{Li}_2(-cex^n-cd+1)}{cen}$$

input
```
integrate(log(c*(d+e*x^n))/x/(c*e-(-c*d+1)/(x^n)),x, algorithm="maxima")
```

output
```
(log(c*e + (c*d - 1)/x^n)/(c*e*n) - log(1/(x^n))/(c*e*n))*log((e*x^n + d)*
c) - (log(c*e*x^n + c*d)*log(c*e*x^n + c*d - 1) + dilog(-c*e*x^n - c*d + 1
))/(c*e*n)
```

Giac [F]

$$\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})}\,dx = \int \frac{\log((ex^n+d)c)}{\left(ce+\frac{cd-1}{x^n}\right)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n))/x/(c*e-(-c*d+1)/(x^n)),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)*c)/((c*e + (c*d - 1)/x^n)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)\right)}{x\,\left(c\,e+\frac{c\,d-1}{x^n}\right)}\,dx$$

input
```
int(log(c*(d + e*x^n))/(x*(c*e + (c*d - 1)/x^n)),x)
```

output
```
int(log(c*(d + e*x^n))/(x*(c*e + (c*d - 1)/x^n)), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^n)\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx = \frac{-2\left(\int \frac{x^{2n}\log(x^n ce+cd)}{x^{2n}c^2d\,e^2x-x^{2n}c\,e^2x+2x^n c^2d^2ex-3x^n cdex+x^n ex+c^2d^3x-2c\,d^2x+dx}dx\right)cd\,e^2n+2\left(\int \frac{x^{2n}\log(x \cdots}{x^{2n}c^2d\,e^2x-x^{2n}c\,e^2x+2x^n c^2d^2ex- \cdots}\right)\cdots}{2en\,(cd-1)}$$

input
```
int(log(c*(d+e*x^n))/x/(c*e-(-c*d+1)/(x^n)),x)
```

output
```
( - 2*int((x**(2*n)*log(x**n*c*e + c*d))/(x**(2*n)*c**2*d*e**2*x - x**(2*n
)*c*e**2*x + 2*x**n*c**2*d**2*e*x - 3*x**n*c*d*e*x + x**n*e*x + c**2*d**3*
x - 2*c*d**2*x + d*x),x)*c*d*e**2*n + 2*int((x**(2*n)*log(x**n*c*e + c*d))
/(x**(2*n)*c**2*d*e**2*x - x**(2*n)*c*e**2*x + 2*x**n*c**2*d**2*e*x - 3*x*
*n*c*d*e*x + x**n*e*x + c**2*d**3*x - 2*c*d**2*x + d*x),x)*e**2*n + log(x*
*n*c*e + c*d)**2*d)/(2*e*n*(c*d - 1))
```

3.379 $\int \frac{x^{-1+n}\log(c(d+ex^n))}{-1+cd+cex^n}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 25

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = -\frac{\operatorname{PolyLog}\left(2,1-c(d+ex^n)\right)}{cen}$$

output
```
-polylog(2,1-c*(d+e*x^n))/c/e/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = -\frac{\operatorname{PolyLog}\left(2,1-cd-cex^n\right)}{cen}$$

input
```
Integrate[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]
```

output
```
-(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2925, 25, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}\log(c(d+ex^n))}{cd+cex^n-1}dx$$

$$\Big\downarrow \text{ 2925}$$

$$\frac{\int -\frac{\log(c(ex^n+d))}{-cex^n-cd+1}dx^n}{n}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\log(c(ex^n+d))}{-cex^n-cd+1}dx^n}{n}$$

$$\Big\downarrow \text{ 2840}$$

$$\frac{\int x^{-n}\log(cex^n+cd)\,d(-cex^n-cd+1)}{cen}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{\text{PolyLog}(2,-cex^n-cd+1)}{cen}$$

input
```
Int[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]
```

output
```
-(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 6.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\operatorname{dilog}\left(c e\,x^{n}+c d\right)}{n c e}$
risch	$\frac{\ln\left(1-c\left(d+e\,x^{n}\right)\right)\ln\left(d+e\,x^{n}\right)}{n e c}-\frac{\ln\left(1-c\left(d+e\,x^{n}\right)\right)\ln\left(c\left(d+e\,x^{n}\right)\right)}{n e c}-\frac{\operatorname{dilog}\left(c\left(d+e\,x^{n}\right)\right)}{n e c}+\frac{\left(\frac{i\pi\ \operatorname{csgn}\left(i\left(d+e\,x^{n}\right)\right)\operatorname{csgn}\left(ic\left(d+e\,x^{n}\right)\right)^{2}}{2}-\right.}{}$

input
```
int(x^(-1+n)*ln(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/n*dilog(c*e*x^n+c*d)/c/e
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = -\frac{\mathrm{Li}_2(-cex^n-cd+1)}{cen}$$

input
```
integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="fricas"
)
```

output
```
-dilog(-c*e*x^n - c*d + 1)/(c*e*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(-1+n)*ln(c*(d+e*x**n))/(-1+c*d+c*e*x**n),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.36

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = \frac{\log\left(cex^n+cd-1\right)\log\left((ex^n+d)c\right)}{cen} - \frac{\log\left(cex^n+cd-1\right)\log\left(ex^n+d\right)}{cen} + \frac{\log\left(-cex^n-cd+1\right)\log\left(ex^n+d\right)+\mathrm{Li}_2(cex^n+cd)}{cen}$$

input
```
integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="maxima"
)
```

output
```
log(c*e*x^n + c*d - 1)*log((e*x^n + d)*c)/(c*e*n) - log(c*e*x^n + c*d - 1)
*log(e*x^n + d)/(c*e*n) + (log(-c*e*x^n - c*d + 1)*log(e*x^n + d) + dilog(
c*e*x^n + c*d))/(c*e*n)
```

Giac [F]

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx=\int \frac{x^{n-1}\log\left((ex^n+d)c\right)}{cex^n+cd-1}\,dx$$

input
```
integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="giac")
```

output
```
integrate(x^(n - 1)*log((e*x^n + d)*c)/(c*e*x^n + c*d - 1), x)
```

Mupad [B] (verification not implemented)

Time = 14.89 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx=-\frac{\mathrm{Li}_2\left(c\left(d+e\,x^n\right)\right)}{c\,e\,n}$$

input
```
int((x^(n - 1)*log(c*(d + e*x^n)))/(c*d + c*e*x^n - 1),x)
```

output
```
-dilog(c*(d + e*x^n))/(c*e*n)
```

Reduce [F]

$$\int \frac{x^{-1+n} \log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx$$

$$= \frac{-2\left(\int \frac{x^{2n}\log(x^n ce+cd)}{x^{2n}c^2d\,e^2x-x^{2n}c\,e^2x+2x^nc^2d^2ex-3x^ncdex+x^nex+c^2d^3x-2c\,d^2x+dx}dx\right)cd\,e^2n+2\left(\int \frac{x^{2n}\log(x}{x^{2n}c^2d\,e^2x-x^{2n}c\,e^2x+2x^nc^2d^2ex-}\right.}{2en\,(cd-1)}$$

input
```
int(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x)
```

output
```
( - 2*int((x**(2*n)*log(x**n*c*e + c*d))/(x**(2*n)*c**2*d*e**2*x - x**(2*n
)*c*e**2*x + 2*x**n*c**2*d**2*e*x - 3*x**n*c*d*e*x + x**n*e*x + c**2*d**3*
x - 2*c*d**2*x + d*x),x)*c*d*e**2*n + 2*int((x**(2*n)*log(x**n*c*e + c*d))
/(x**(2*n)*c**2*d*e**2*x - x**(2*n)*c*e**2*x + 2*x**n*c**2*d**2*e*x - 3*x*
*n*c*d*e*x + x**n*e*x + c**2*d**3*x - 2*c*d**2*x + d*x),x)*e**2*n + log(x*
*n*c*e + c*d)**2*d)/(2*e*n*(c*d - 1))
```

3.380 $\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 26

$$\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx = \frac{\text{PolyLog}(2,1-c(d+ex^{-n}))}{cen}$$

output
```
polylog(2,1-c*(d+e/(x^n)))/c/e/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx = \frac{\text{PolyLog}(2,-x^{-n}(ce-x^n+cdx^n))}{cen}$$

input
```
Integrate[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]
```

output
```
PolyLog[2, -((c*e - x^n + c*d*x^n)/x^n)]/(c*e*n)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2925, 2005, 2840, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(c(d+ex^{-n})\right)}{x \left(ce-(1-cd)x^n\right)} \, dx$$

$$\Big\downarrow \ 2925$$

$$-\frac{\int \frac{x^n \log \left(c \left(ex^{-n}+d\right)\right)}{ce-(1-cd)x^n}dx^{-n}}{n}$$

$$\Big\downarrow \ 2005$$

$$-\frac{\int \frac{\log \left(c \left(ex^{-n}+d\right)\right)}{cex^{-n}+cd-1}dx^{-n}}{n}$$

$$\Big\downarrow \ 2840$$

$$-\frac{\int x^n \log \left(cex^{-n}+cd\right) d(cex^{-n}+cd-1)}{cen}$$

$$\Big\downarrow \ 2838$$

$$\frac{\text{PolyLog}\left(2,-cex^{-n}-cd+1\right)}{cen}$$

input
```
Int[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]
```

output
```
PolyLog[2, 1 - c*d - (c*e)/x^n]/(c*e*n)
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [A] (verified)

Time = 8.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{dilog}\left(cd+ce\,x^{-n}\right)}{nce}$	24
default	$\frac{\operatorname{dilog}\left(cd+ce\,x^{-n}\right)}{nce}$	24
risch	Expression too large to display	1900

input
```
int(ln(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/n*dilog(c*d+c*e/(x^n))/c/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx = \frac{\mathrm{Li}_2\left(-\frac{cdx^n+ce}{x^n}+1\right)}{cen}$$

input
```
integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="fricas")
```

output
```
dilog(-(c*d*x^n + c*e)/x^n + 1)/(c*e*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e/(x**n)))/x/(c*e-(-c*d+1)*x**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{x^n}\right)\right)}{(ce+(cd-1)x^n)x}\,dx$$

input
```
integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="maxima")
```

output
```
n*integrate(log(x)/(c*d*x*x^n + c*e*x), x) + (log(d*x^n + e)*log(x) + log(
c)*log(x) - log(x)*log(x^n))/(c*e) - log(c)*log((c*e + (c*d - 1)*x^n)/(c*d
 - 1))/(c*e*n) - (log(d*x^n + e)*log((c*d*e + (c*d^2 - d)*x^n - e)/e + 1)
+ dilog(-(c*d*e + (c*d^2 - d)*x^n - e)/e))/(c*e*n) + (log(x^n)*log((c*d -
1)*x^n/(c*e) + 1) + dilog(-(c*d - 1)*x^n/(c*e)))/(c*e*n)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{x^n}\right)\right)}{(ce+(cd-1)x^n)x}\,dx$$

input
```
integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="giac")
```

output
```
integrate(log(c*(d + e/x^n))/((c*e + (c*d - 1)*x^n)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx = \int \frac{\ln\left(c\left(d+\frac{e}{x^n}\right)\right)}{x\left(c\,e+x^n\,\left(c\,d-1\right)\right)}\,dx$$

input
```
int(log(c*(d + e/x^n))/(x*(c*e + x^n*(c*d - 1))),x)
```

output
```
int(log(c*(d + e/x^n))/(x*(c*e + x^n*(c*d - 1))), x)
```

Reduce [F]

$$\int \frac{\log\left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx$$
$$= \frac{-2\left(\int \frac{\log\left(\frac{x^n cd+ce}{x^n}\right)}{x^{2n}c^2d^3x-2x^{2n}c\,d^2x+x^{2n}dx+2x^n c^2d^2ex-3x^n cdex+x^n ex+c^2d\,e^2x-c\,e^2x}dx\right)cd\,e^2n+2\left(\int \frac{}{x^{2n}c^2d^3x-2x^{2n}c\,d^2x+x^{2n}dx+}\right.}{2en\left(cd-1\right)}$$

input
```
int(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x)
```

output
```
( - 2*int(log((x**n*c*d + c*e)/x**n)/(x**(2*n)*c**2*d**3*x - 2*x**(2*n)*c*
d**2*x + x**(2*n)*d*x + 2*x**n*c**2*d**2*e*x - 3*x**n*c*d*e*x + x**n*e*x +
 c**2*d*e**2*x - c*e**2*x),x)*c*d*e**2*n + 2*int(log((x**n*c*d + c*e)/x**n
)/(x**(2*n)*c**2*d**3*x - 2*x**(2*n)*c*d**2*x + x**(2*n)*d*x + 2*x**n*c**2
*d**2*e*x - 3*x**n*c*d*e*x + x**n*e*x + c**2*d*e**2*x - c*e**2*x),x)*e**2*
n - log((x**n*c*d + c*e)/x**n)**2*d)/(2*e*n*(c*d - 1))
```

3.381 $\int \frac{(f+gx^{2n})^2\log^q(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\begin{aligned}
&\int \frac{(f+gx^{2n})^2\log^q(c(d+ex^n)^p)}{x}\,dx \\
&= \frac{4^{-1-q}g^2(d+ex^n)^4\,(c(d+ex^n)^p)^{-4/p}\,\Gamma\left(1+q,-\frac{4\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^4n} \\
&\quad-\frac{3^{-q}dg^2(d+ex^n)^3\,(c(d+ex^n)^p)^{-3/p}\,\Gamma\left(1+q,-\frac{3\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^4n} \\
&\quad+\frac{2^{-q}fg(d+ex^n)^2\,(c(d+ex^n)^p)^{-2/p}\,\Gamma\left(1+q,-\frac{2\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^2n} \\
&\quad+\frac{3\ 2^{-1-q}d^2g^2(d+ex^n)^2\,(c(d+ex^n)^p)^{-2/p}\,\Gamma\left(1+q,-\frac{2\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)}{e^4n} \\
&\quad-\frac{2dfg(d+ex^n)\,(c(d+ex^n)^p)^{-1/p}\,\Gamma\left(1+q,-\frac{\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^2n} \\
&\quad-\frac{d^3g^2(d+ex^n)\,(c(d+ex^n)^p)^{-1/p}\,\Gamma\left(1+q,-\frac{\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^4n} \\
&\quad+f^2\mathrm{Int}\left(\frac{\log^q(c(d+ex^n)^p)}{x},x\right)
\end{aligned}$$

output
```
4^(-1-q)*g^2*(d+e*x^n)^4*GAMMA(1+q,-4*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^
p)^q/e^4/n/((c*(d+e*x^n)^p)^(4/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d*g^2*(d+e*x
^n)^3*GAMMA(1+q,-3*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/(3^q)/e^4/n/((
c*(d+e*x^n)^p)^(3/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+f*g*(d+e*x^n)^2*GAMMA(1+q
,-2*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/(2^q)/e^2/n/((c*(d+e*x^n)^p)^
(2/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+3*2^(-1-q)*d^2*g^2*(d+e*x^n)^2*GAMMA(1+q
,-2*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e^4/n/((c*(d+e*x^n)^p)^(2/p))
/((-ln(c*(d+e*x^n)^p)/p)^q)-2*d*f*g*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)
/p)*ln(c*(d+e*x^n)^p)^q/e^2/n/((c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^p)
/p)^q)-d^3*g^2*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)
^q/e^4/n/((c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+f^2*Defer(Int)
(ln(c*(d+e*x^n)^p)^q/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{2n})^2 \log^q (c(d+ex^n)^p)}{x} \, dx = \int \frac{(f+gx^{2n})^2 \log^q (c(d+ex^n)^p)}{x} \, dx$$

input
```
Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

output
```
Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2929}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} \, dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

input
```
Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(f+g\,x^{2n})^2 \ln(c(d+e\,x^n)^p)^q}{x} dx$$

input
```
int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

output
```
int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(f+gx^{2n})^2 \log^q (c(d+ex^n)^p)}{x} \, dx = \int \frac{(gx^{2n}+f)^2 \log \left(((ex^n+d)^p c)^q\right)}{x} \, dx$$

input
```
integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

output
```
integral((g^2*x^(4*n) + 2*f*g*x^(2*n) + f^2)*log((e*x^n + d)^p*c)^q/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^{2n})^2 \log^q (c(d+ex^n)^p)}{x} \, dx = \text{Timed out}$$

input
```
integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^{2n})^2 \log^q (c(d+ex^n)^p)}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 3.83 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^{2n}+f)^2 \log\left(\left(ex^n+d\right)^p c\right)^q}{x}\,dx$$

input
```
integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

output
```
integrate((g*x^(2*n) + f)^2*log((e*x^n + d)^p*c)^q/x, x)
```

Mupad [N/A]

Not integrable

Time = 14.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)^q \left(f+g\,x^{2\,n}\right)^2}{x}\,dx$$

input
```
int((log(c*(d + e*x^n)^p)^q*(f + g*x^(2*n))^2)/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)^q*(f + g*x^(2*n))^2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 496, normalized size of antiderivative = 17.10

$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{x^{4n}\log((x^ne+d)^p c)^q g^2pq + x^{4n}\log((x^ne+d)^p c)^q g^2p + 4x^{2n}\log((x^ne+d)^p c)^q fgpq + 4x^{2n}\log((x^ne+d)}{}$$

input
```
int((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x)
```

output
```
(x**(4*n)*log((x**n*e + d)**p*c)**q*g**2*p*q + x**(4*n)*log((x**n*e + d)**
p*c)**q*g**2*p + 4*x**(2*n)*log((x**n*e + d)**p*c)**q*f*g*p*q + 4*x**(2*n)
*log((x**n*e + d)**p*c)**q*f*g*p + 4*log((x**n*e + d)**p*c)**q*log((x**n*e
 + d)**p*c)*f**2 + 4*int(log((x**n*e + d)**p*c)**q/(x**n*e*x + d*x),x)*d*f
**2*n*p*q + 4*int(log((x**n*e + d)**p*c)**q/(x**n*e*x + d*x),x)*d*f**2*n*p
 - int((x**(5*n)*log((x**n*e + d)**p*c)**q)/(x**n*log((x**n*e + d)**p*c)*e
*x + log((x**n*e + d)**p*c)*d*x),x)*e*g**2*n*p**2*q**2 - int((x**(5*n)*log
((x**n*e + d)**p*c)**q)/(x**n*log((x**n*e + d)**p*c)*e*x + log((x**n*e + d
)**p*c)*d*x),x)*e*g**2*n*p**2*q - 4*int((x**(3*n)*log((x**n*e + d)**p*c)**
q)/(x**n*log((x**n*e + d)**p*c)*e*x + log((x**n*e + d)**p*c)*d*x),x)*e*f*g
*n*p**2*q**2 - 4*int((x**(3*n)*log((x**n*e + d)**p*c)**q)/(x**n*log((x**n*
e + d)**p*c)*e*x + log((x**n*e + d)**p*c)*d*x),x)*e*f*g*n*p**2*q)/(4*n*p*(
q + 1))
```

3.382 $\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{(f+gx^n)^2 \log^q\left(c(d+ex^n)^p\right)}{x} \, dx = \frac{2^{-1-q} g^2 (d+ex^n)^2 \left(c(d+ex^n)^p\right)^{-2/p} \Gamma\left(1+q,-\frac{2\log(c(d+ex^n)^p)}{p}\right) \log^q\left(c(d+ex^n)^p\right) \left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^2 n} + \frac{2fg(d+ex^n)\left(c(d+ex^n)^p\right)^{-1/p} \Gamma\left(1+q,-\frac{\log(c(d+ex^n)^p)}{p}\right) \log^q\left(c(d+ex^n)^p\right) \left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{en} - \frac{dg^2(d+ex^n)\left(c(d+ex^n)^p\right)^{-1/p} \Gamma\left(1+q,-\frac{\log(c(d+ex^n)^p)}{p}\right) \log^q\left(c(d+ex^n)^p\right) \left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^2 n} + f^2 \text{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x}, x\right)$$

output
```
2^(-1-q)*g^2*(d+e*x^n)^2*GAMMA(1+q,-2*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^
p)^q/e^2/n/((c*(d+e*x^n)^p)^(2/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+2*f*g*(d+e*x
^n)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e/n/((c*(d+e*x^n)^
p)^(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d*g^2*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*
x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e^2/n/((c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*
x^n)^p)/p)^q)+f^2*Defer(Int)(ln(c*(d+e*x^n)^p)^q/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} \, dx = \int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} \, dx$$

input
```
Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

output
```
Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2929}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} \, dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}dx$$

input
```
Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\left(f+g\,x^{n}\right)^{2} \ln \left(c{\left(d+e\,x^{n}\right)}^{p}\right)^{q}}{x}dx$$

input
```
int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

output
```
int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{(f+gx^n)^2\log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{(gx^n+f)^2\log\left((ex^n+d)^pc\right)^q}{x}\,dx$$

input
```
integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

output
```
integral((g^2*x^(2*n) + 2*f*g*x^n + f^2)*log((e*x^n + d)^p*c)^q/x, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^n+f)^2 \log\left((ex^n+d)^p c\right)^q}{x}\,dx$$

input
```
integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

output
```
integrate((g*x^n + f)^2*log((e*x^n + d)^p*c)^q/x, x)
```

Mupad [N/A]

Not integrable

Time = 14.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} dx = \int \frac{\ln(c(d+e x^n)^p)^q (f+g x^n)^2}{x} dx$$

input
```
int((log(c*(d + e*x^n)^p)^q*(f + g*x^n)^2)/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)^q*(f + g*x^n)^2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 492, normalized size of antiderivative = 18.22

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} dx = \frac{x^{2n}\log((x^n e+d)^p c)^q g^2 pq + x^{2n}\log((x^n e+d)^p c)^q g^2 p + 4x^n \log((x^n e+d)^p c)^q fgpq + 4x^n \log((x^n e+d)^p}{}$$

input
```
int((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x)
```

output
```
(x**(2*n)*log((x**n*e + d)**p*c)**q*g**2*p*q + x**(2*n)*log((x**n*e + d)**
p*c)**q*g**2*p + 4*x**n*log((x**n*e + d)**p*c)**q*f*g*p*q + 4*x**n*log((x*
*n*e + d)**p*c)**q*f*g*p + 2*log((x**n*e + d)**p*c)**q*log((x**n*e + d)**p
*c)*f**2 + 2*int(log((x**n*e + d)**p*c)**q/(x**n*e*x + d*x),x)*d*f**2*n*p*
q + 2*int(log((x**n*e + d)**p*c)**q/(x**n*e*x + d*x),x)*d*f**2*n*p - int((
x**(3*n)*log((x**n*e + d)**p*c)**q)/(x**n*log((x**n*e + d)**p*c)*e*x + log
((x**n*e + d)**p*c)*d*x),x)*e*g**2*n*p**2*q**2 - int((x**(3*n)*log((x**n*e
 + d)**p*c)**q)/(x**n*log((x**n*e + d)**p*c)*e*x + log((x**n*e + d)**p*c)*
d*x),x)*e*g**2*n*p**2*q - 4*int((x**(2*n)*log((x**n*e + d)**p*c)**q)/(x**n
*log((x**n*e + d)**p*c)*e*x + log((x**n*e + d)**p*c)*d*x),x)*e*f*g*n*p**2*
q**2 - 4*int((x**(2*n)*log((x**n*e + d)**p*c)**q)/(x**n*log((x**n*e + d)**
p*c)*e*x + log((x**n*e + d)**p*c)*d*x),x)*e*f*g*n*p**2*q)/(2*n*p*(q + 1))
```

3.383 $\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx = \text{Int}\left(x^{-1-2n}(g+fx^n)^2 \log^q(c(d+ex^n)^p), x\right)$$

output
```
Defer(Int)(x^(-1-2*n)*(g+f*x^n)^2*ln(c*(d+e*x^n)^p)^q,x)
```

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx = \int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

input
```
Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

output
```
Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2005, 2929}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} \, dx$$
$$\Big\downarrow \ 2005$$
$$\int x^{-2n-1}(fx^n+g)^2 \log^q(c(d+ex^n)^p) \, dx$$
$$\Big\downarrow \ 2929$$
$$\int x^{-2n-1}(fx^n+g)^2 \log^q(c(d+ex^n)^p) \, dx$$

input
```
Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\left(f+g\,x^{-n}\right)^{2} \ln \left(c\left(d+e\,x^{n}\right)^{p}\right)^{q}}{x}dx$$

input
```
int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

output
```
int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{\left(f+gx^{-n}\right)^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{\left(f+\frac{g}{x^n}\right)^2 \log\left((ex^n+d)^p c\right)^q}{x}\,dx$$

input
```
integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

output
```
integral((f^2*x^(2*n) + 2*f*g*x^n + g^2)*log((e*x^n + d)^p*c)^q/(x*x^(2*n)
), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(f+gx^{-n}\right)^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f + gx^{-n})^2 \log^q (c(d + ex^n)^p)}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f + gx^{-n})^2 \log^q (c(d + ex^n)^p)}{x} \, dx = \int \frac{\left(f + \frac{g}{x^n}\right)^2 \log \left((ex^n + d)^p c\right)^q}{x} \, dx$$

input
```
integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

output
```
integrate((f + g/x^n)^2*log((e*x^n + d)^p*c)^q/x, x)
```

Mupad [N/A]

Not integrable

Time = 14.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f + gx^{-n})^2 \log^q (c(d + ex^n)^p)}{x} \, dx = \int \frac{\ln \left(c \left(d + e\, x^n\right)^p\right)^q \left(f + \frac{g}{x^n}\right)^2}{x} \, dx$$

input
```
int((log(c*(d + e*x^n)^p)^q*(f + g/x^n)^2)/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)^q*(f + g/x^n)^2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 514, normalized size of antiderivative = 17.72

$$\int \frac{(f + gx^{-n})^2 \log^q(c(d + ex^n)^p)}{x} \, dx$$
$$= \frac{2x^{2n}\log((x^ne + d)^p c)^q \log((x^ne + d)^p c) f^2 - 4x^n \log((x^ne + d)^p c)^q fgpq - 4x^n \log((x^ne + d)^p c)^q fgp - \mathrm{lo}}{}$$

input
```
int((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x)
```

output
```
(2*x**(2*n)*log((x**n*e + d)**p*c)**q*log((x**n*e + d)**p*c)*f**2 - 4*x**n
*log((x**n*e + d)**p*c)**q*f*g*p*q - 4*x**n*log((x**n*e + d)**p*c)**q*f*g*
p - log((x**n*e + d)**p*c)**q*g**2*p*q - log((x**n*e + d)**p*c)**q*g**2*p
+ x**(2*n)*int(log((x**n*e + d)**p*c)**q/(x**(2*n)*log((x**n*e + d)**p*c)*
e*x + x**n*log((x**n*e + d)**p*c)*d*x),x)*e*g**2*n*p**2*q**2 + x**(2*n)*in
t(log((x**n*e + d)**p*c)**q/(x**(2*n)*log((x**n*e + d)**p*c)*e*x + x**n*lo
g((x**n*e + d)**p*c)*d*x),x)*e*g**2*n*p**2*q + 4*x**(2*n)*int(log((x**n*e
+ d)**p*c)**q/(x**n*log((x**n*e + d)**p*c)*e*x + log((x**n*e + d)**p*c)*d*
x),x)*e*f*g*n*p**2*q**2 + 4*x**(2*n)*int(log((x**n*e + d)**p*c)**q/(x**n*l
og((x**n*e + d)**p*c)*e*x + log((x**n*e + d)**p*c)*d*x),x)*e*f*g*n*p**2*q
+ 2*x**(2*n)*int(log((x**n*e + d)**p*c)**q/(x**n*e*x + d*x),x)*d*f**2*n*p*
q + 2*x**(2*n)*int(log((x**n*e + d)**p*c)**q/(x**n*e*x + d*x),x)*d*f**2*n*
p)/(2*x**(2*n)*n*p*(q + 1))
```

3.384 $\int \frac{\left(f+gx^{-2n}\right)^2\log^q(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\left(f+gx^{-2n}\right)^2\log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\text{Int}\left(x^{-1-4n}\left(g+fx^{2n}\right)^2\log^q\left(c(d+ex^n)^p\right),x\right)$$

output
```
Defer(Int)(x^(-1-4*n)*(g+f*x^(2*n))^2*ln(c*(d+e*x^n)^p)^q,x)
```

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\left(f+gx^{-2n}\right)^2\log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{\left(f+gx^{-2n}\right)^2\log^q\left(c(d+ex^n)^p\right)}{x}\,dx$$

input
```
Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

output
```
Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2005, 2929}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f+g x^{-2 n}\right)^2 \log ^q\left(c\left(d+e x^n\right)^p\right)}{x} \, dx$$
$$\Big\downarrow \; 2005$$
$$\int x^{-4 n-1} \left(f x^{2 n}+g\right)^2 \log ^q\left(c\left(d+e x^n\right)^p\right) \, dx$$
$$\Big\downarrow \; 2929$$
$$\int x^{-4 n-1} \left(f x^{2 n}+g\right)^2 \log ^q\left(c\left(d+e x^n\right)^p\right) \, dx$$

input
```
Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
 + n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\left(f+g\,x^{-2n}\right)^{2}\ln\left(c\left(d+e\,x^{n}\right)^{p}\right)^{q}}{x}dx$$

input
```
int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

output
```
int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\left(f+gx^{-2n}\right)^{2}\log^{q}\left(c\left(d+ex^{n}\right)^{p}\right)}{x}\,dx=\int \frac{\left(f+\frac{g}{x^{2\,n}}\right)^{2}\log\left(\left(ex^{n}+d\right)^{p}c\right)^{q}}{x}\,dx$$

input
```
integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
```

output
```
integral((f^2*x^(4*n) + 2*f*g*x^(2*n) + g^2)*log((e*x^n + d)^p*c)^q/(x*x^(
4*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(f+gx^{-2n}\right)^{2}\log^{q}\left(c\left(d+ex^{n}\right)^{p}\right)}{x}\,dx=\text{Timed out}$$

input
```
integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)**q/x,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f + gx^{-2n})^2 \log^q (c(d + ex^n)^p)}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{(f + gx^{-2n})^2 \log^q (c(d + ex^n)^p)}{x} \, dx = \int \frac{\left(f + \frac{g}{x^{2n}}\right)^2 \log\left((ex^n + d)^p c\right)^q}{x} \, dx$$

input
```
integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
```

output
```
integrate((f + g/x^(2*n))^2*log((e*x^n + d)^p*c)^q/x, x)
```

Mupad [N/A]

Not integrable

Time = 14.75 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{(f + gx^{-2n})^2 \log^q (c(d + ex^n)^p)}{x} \, dx = \int \frac{\ln\left(c\,(d + e\,x^n)^p\right)^q \left(f + \frac{g}{x^{2\,n}}\right)^2}{x} \, dx$$

input
```
int((log(c*(d + e*x^n)^p)^q*(f + g/x^(2*n))^2)/x,x)
```

output
```
int((log(c*(d + e*x^n)^p)^q*(f + g/x^(2*n))^2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 532, normalized size of antiderivative = 18.34

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx$$
$$= \frac{4x^{4n}\log((x^ne+d)^p c)^q \log((x^ne+d)^p c) f^2 - 4x^{2n}\log((x^ne+d)^p c)^q fgpq - 4x^{2n}\log((x^ne+d)^p c)^q fgp -}{}$$

input
```
int((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x)
```

output
```
(4*x**(4*n)*log((x**n*e + d)**p*c)**q*log((x**n*e + d)**p*c)*f**2 - 4*x**(
2*n)*log((x**n*e + d)**p*c)**q*f*g*p*q - 4*x**(2*n)*log((x**n*e + d)**p*c)
**q*f*g*p - log((x**n*e + d)**p*c)**q*g**2*p*q - log((x**n*e + d)**p*c)**q
*g**2*p + x**(4*n)*int(log((x**n*e + d)**p*c)**q/(x**(4*n)*log((x**n*e + d
)**p*c)*e*x + x**(3*n)*log((x**n*e + d)**p*c)*d*x),x)*e*g**2*n*p**2*q**2 +
 x**(4*n)*int(log((x**n*e + d)**p*c)**q/(x**(4*n)*log((x**n*e + d)**p*c)*e
*x + x**(3*n)*log((x**n*e + d)**p*c)*d*x),x)*e*g**2*n*p**2*q + 4*x**(4*n)*
int(log((x**n*e + d)**p*c)**q/(x**(2*n)*log((x**n*e + d)**p*c)*e*x + x**n*
log((x**n*e + d)**p*c)*d*x),x)*e*f*g*n*p**2*q**2 + 4*x**(4*n)*int(log((x**
n*e + d)**p*c)**q/(x**(2*n)*log((x**n*e + d)**p*c)*e*x + x**n*log((x**n*e
+ d)**p*c)*d*x),x)*e*f*g*n*p**2*q + 4*x**(4*n)*int(log((x**n*e + d)**p*c)*
*q/(x**n*e*x + d*x),x)*d*f**2*n*p*q + 4*x**(4*n)*int(log((x**n*e + d)**p*c
)**q/(x**n*e*x + d*x),x)*d*f**2*n*p)/(4*x**(4*n)*n*p*(q + 1))
```

3.385 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{2n})} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)} \, dx = \operatorname{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}, x\right)$$

output
```
Defer(Int)(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)} \, dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)} \, dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2929\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}dx$$

input
```
Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c(d+e\,x^{n})^{p}\right)^{q}}{x\left(f+g\,x^{2n}\right)}dx$$

input
```
int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

output
```
int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^q}{(gx^{2\,n}+f)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)^q/(g*x*x^(2*n) + f*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**(2*n)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^q \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^q}{(gx^{2n}+f)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)^q/((g*x^(2*n) + f)*x), x)
```

Mupad [N/A]

Not integrable

Time = 15.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q \left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)^q}{x\,\left(f+g\,x^{2\,n}\right)}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^(2*n))),x)
```

output
```
int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^(2*n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log((x^n e+d)^p\,c)^q}{x^{2n}gx+fx}dx$$

input
```
int(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

output
```
int(log((x**n*e + d)**p*c)**q/(x**(2*n)*g*x + f*x),x)
```

3.386 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^n)} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)} \, dx = \operatorname{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}, x\right)$$

output
```
Defer(Int)(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)} \, dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)} \, dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2929}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx$$

$$\Big\downarrow \ 2929$$

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}dx$$

input
```
Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c(d+e\,x^{n})^{p}\right)^{q}}{x\left(f+g\,x^{n}\right)}dx$$

input
```
int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

output
```
int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log^{q}\left(c(d+ex^{n})^{p}\right)}{x\left(f+gx^{n}\right)}\,dx=\int \frac{\log\left((ex^{n}+d)^{p}c\right)^{q}}{(gx^{n}+f)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="fricas")
```

output
```
integral(log((e*x^n + d)^p*c)^q/(g*x*x^n + f*x), x)
```

Sympy [N/A]

Not integrable

Time = 8.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\log^{q}\left(c(d+ex^{n})^{p}\right)}{x\left(f+gx^{n}\right)}\,dx=\int \frac{\log\left(c(d+ex^{n})^{p}\right)^{q}}{x\left(f+gx^{n}\right)}\,dx$$

input
```
integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**n),x)
```

output
```
Integral(log(c*(d + e*x**n)**p)**q/(x*(f + g*x**n)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^n)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^n)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^q}{(gx^n+f)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)^q/((g*x^n + f)*x), x)
```

Mupad [N/A]

Not integrable

Time = 15.70 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log^q \left(c(d+ex^n)^p\right)}{x\,(f+gx^n)}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^q}{x\,\left(f+g\,x^n\right)}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^n)),x)
```

output
```
int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log^q \left(c(d+ex^n)^p\right)}{x\,(f+gx^n)}\,dx = \int \frac{\log((x^n e+d)^p\, c)^q}{x^n g x+f x}dx$$

input
```
int(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

output
```
int(log((x**n*e + d)**p*c)**q/(x**n*g*x + f*x),x)
```

3.387 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-n})} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)} dx = \text{Int}\left(\frac{x^{-1+n}\log^q\left(c(d+ex^n)^p\right)}{g+fx^n}, x\right)$$

output
```
Defer(Int)(x^(-1+n)*ln(c*(d+e*x^n)^p)^q/(g+f*x^n),x)
```

Mathematica [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)} dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)} dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2005, 2925, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}dx$$

$$\Big\downarrow \text{2005}$$

$$\int \frac{x^{n-1}\log^q\left(c(d+ex^n)^p\right)}{fx^n+g}dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{\int \frac{\log^q(c(ex^n+d)^p)}{fx^n+g}dx^n}{n}$$

$$\Big\downarrow \text{2867}$$

$$\frac{\int \frac{\log^q(c(ex^n+d)^p)}{fx^n+g}dx^n}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c(d+e\,x^{n})^{p}\right)^{q}}{x\left(f+g\,x^{-n}\right)}dx$$

input
```
int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

output
```
int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\log^{q}\left(c(d+ex^{n})^{p}\right)}{x\left(f+gx^{-n}\right)}\,dx=\int \frac{\log\left((ex^{n}+d)^{p}c\right)^{q}}{(f+\frac{g}{x^{n}})x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="fricas")
```

output
```
integral(x^n*log((e*x^n + d)^p*c)^q/(f*x*x^n + g*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)^q}{\left(f+\frac{g}{x^n}\right)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)^q/((f + g/x^n)*x), x)
```

Mupad [N/A]

Not integrable

Time = 15.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx=\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^q}{x\,\left(f+\frac{g}{x^n}\right)} \,d x$$

input
```
int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^n)),x)
```

output
```
int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 285, normalized size of antiderivative = 9.83

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \frac{\log((x^ne+d)^p\,c)^q\log((x^ne+d)^p\,c)\,d-\left(\int\frac{x^{2n}\log((x^ne+d)^pc)^q}{x^{2n}efx+x^ndfx+x^negx+dgx}dx\right)defnpq-\left(\int\frac{x^{2n}\log((x^ne+d)^pc)^q}{x^{2n}efx+x^ndfx+x^negx+dg}\right.}{egnp\,(q+1)}$$

input
```
int(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

output
```
(log((x**n*e + d)**p*c)**q*log((x**n*e + d)**p*c)*d - int((x**(2*n)*log((x
**n*e + d)**p*c)**q)/(x**(2*n)*e*f*x + x**n*d*f*x + x**n*e*g*x + d*g*x),x)
*d*e*f*n*p*q - int((x**(2*n)*log((x**n*e + d)**p*c)**q)/(x**(2*n)*e*f*x +
x**n*d*f*x + x**n*e*g*x + d*g*x),x)*d*e*f*n*p + int((x**(2*n)*log((x**n*e
+ d)**p*c)**q)/(x**(2*n)*e*f*x + x**n*d*f*x + x**n*e*g*x + d*g*x),x)*e**2*
g*n*p*q + int((x**(2*n)*log((x**n*e + d)**p*c)**q)/(x**(2*n)*e*f*x + x**n*
d*f*x + x**n*e*g*x + d*g*x),x)*e**2*g*n*p)/(e*g*n*p*(q + 1))
```

3.388 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})} dx = \frac{\text{Int}\left(\frac{x^{-1+n}\log^q(c(d+ex^n)^p)}{\sqrt{g}-\sqrt{-f}x^n},x\right)}{2\sqrt{-f}} - \frac{\text{Int}\left(\frac{x^{-1+n}\log^q(c(d+ex^n)^p)}{\sqrt{g}+\sqrt{-f}x^n},x\right)}{2\sqrt{-f}}$$

output
```
1/2*Defer(Int)(x^(-1+n)*ln(c*(d+e*x^n)^p)^q/(g^(1/2)-(-f)^(1/2)*x^n),x)/(-
f)^(1/2)-1/2*Defer(Int)(x^(-1+n)*ln(c*(d+e*x^n)^p)^q/(g^(1/2)+(-f)^(1/2)*x
^n),x)/(-f)^(1/2)
```

Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})} dx = \int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})} dx$$

input
```
Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
```

output
```
Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

Rubi [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2005, 2925, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})}dx$$

$$\Big\downarrow \text{2005}$$

$$\int \frac{x^{2n-1}\log^q(c(d+ex^n)^p)}{fx^{2n}+g}dx$$

$$\Big\downarrow \text{2925}$$

$$\frac{\int \frac{x^n\log^q(c(ex^n+d)^p)}{fx^{2n}+g}dx^n}{n}$$

$$\Big\downarrow \text{2863}$$

$$\frac{\int \left(\frac{\sqrt{-f}\log^q(c(ex^n+d)^p)}{2f(\sqrt{-f}x^n+\sqrt{g})}-\frac{\sqrt{-f}\log^q(c(ex^n+d)^p)}{2f(\sqrt{g}-\sqrt{-f}x^n)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\int \frac{\log^q(c(ex^n+d)^p)}{\sqrt{g}-\sqrt{-f}x^n}dx^n}{2\sqrt{-f}}-\frac{\int \frac{\log^q(c(ex^n+d)^p)}{\sqrt{-f}x^n+\sqrt{g}}dx^n}{2\sqrt{-f}}}{n}$$

input
```
Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

Maple [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\ln \left(c(d+e\,x^{n})^{p}\right)^{q}}{x\left(f+g\,x^{-2n}\right)}dx$$

input
```
int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

output
```
int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^q}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="fricas")
```

output
```
integral(x^(2*n)*log((e*x^n + d)^p*c)^q/(f*x*x^(2*n) + g*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**(2*n))),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: In function CAR, the value of
the first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)^q}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

input
```
integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="giac")
```

output
```
integrate(log((e*x^n + d)^p*c)^q/((f + g/x^(2*n))*x), x)
```

Mupad [N/A]

Not integrable

Time = 15.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^q}{x\,\left(f+\frac{g}{x^{2\,n}}\right)}\,dx$$

input
```
int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^(2*n))),x)
```

output
```
int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^(2*n))), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 279, normalized size of antiderivative = 9.62

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$
$$= \frac{\log((x^ne+d)^p\,c)^q\log((x^ne+d)^p\,c)+\left(\int\frac{x^{2n}\log((x^ne+d)^pc)^q}{x^{3n}efx+x^{2n}dfx+x^negx+dgx}dx\right)dfnpq+\left(\int\frac{x^{2n}\log((x^ne+d)^pc)^q}{x^{3n}efx+x^{2n}dfx+x^negx+dgx}d\right.}{fnp\,(q+1)}$$

input
```
int(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

output
```
(log((x**n*e + d)**p*c)**q*log((x**n*e + d)**p*c) + int((x**(2*n)*log((x**
n*e + d)**p*c)**q)/(x**(3*n)*e*f*x + x**(2*n)*d*f*x + x**n*e*g*x + d*g*x),
x)*d*f*n*p*q + int((x**(2*n)*log((x**n*e + d)**p*c)**q)/(x**(3*n)*e*f*x +
x**(2*n)*d*f*x + x**n*e*g*x + d*g*x),x)*d*f*n*p - int((x**n*log((x**n*e +
d)**p*c)**q)/(x**(3*n)*e*f*x + x**(2*n)*d*f*x + x**n*e*g*x + d*g*x),x)*e*g
*n*p*q - int((x**n*log((x**n*e + d)**p*c)**q)/(x**(3*n)*e*f*x + x**(2*n)*d
*f*x + x**n*e*g*x + d*g*x),x)*e*g*n*p)/(f*n*p*(q + 1))
```

3.389 $\int \frac{\log(x)\log(d+ex^m)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) - \frac{\log(x)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} + \frac{\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}$$

output
```
1/2*ln(x)^2*ln(d+e*x^m)-1/2*ln(x)^2*ln(1+e*x^m/d)-ln(x)*polylog(2,-e*x^m/d
)/m+polylog(3,-e*x^m/d)/m^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = -\frac{1}{6}\log^2(x)\left(m\log(x)+3\log\left(1+\frac{dx^{-m}}{e}\right) - 3\log(d+ex^m)\right) + \frac{\log(x)\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m} + \frac{\operatorname{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2}$$

input
```
Integrate[(Log[x]*Log[d + e*x^m])/x,x]
```

output
```
-1/6*(Log[x]^2*(m*Log[x] + 3*Log[1 + d/(e*x^m)] - 3*Log[d + e*x^m])) + (Lo
g[x]*PolyLog[2, -(d/(e*x^m))])/m + PolyLog[3, -(d/(e*x^m))]/m^2
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2822, 2775, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x)\log\left(d+ex^m\right)}{x}dx$$

$$\Big\downarrow \text{2822}$$

$$\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}em\int \frac{x^{m-1}\log^2(x)}{ex^m+d}dx$$

$$\Big\downarrow \text{2775}$$

$$\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}em\left(\frac{\log^2(x)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2\int \frac{\log(x)\log\left(\frac{ex^m}{d}+1\right)}{x}dx}{em}\right)$$

$$\Big\downarrow \text{2821}$$

$$\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}em\left(\frac{\log^2(x)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2\left(\frac{\int \frac{\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{x}dx}{m}-\frac{\log(x)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}em\left(\frac{\log^2(x)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{\log(x)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}\right)$$

input
```
Int[(Log[x]*Log[d + e*x^m])/x,x]
```

output
```
(Log[x]^2*Log[d + e*x^m])/2 - (e*m*((Log[x]^2*Log[1 + (e*x^m)/d])/(e*m) -
(2*(-((Log[x]*PolyLog[2, -((e*x^m)/d)])/m) + PolyLog[3, -((e*x^m)/d)]/m^2)
)/(e*m)))/2
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2822
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Simp[f*m*(r/(b*n*(p + 1)))   Int[x^(m
- 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
 e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{\ln(x)^2 \ln(d+e\,x^m)}{2}-\frac{\ln(x)^2 \ln\left(1+\frac{e\,x^m}{d}\right)}{2}-\frac{\ln(x)\operatorname{polylog}\left(2,-\frac{e\,x^m}{d}\right)}{m}+\frac{\operatorname{polylog}\left(3,-\frac{e\,x^m}{d}\right)}{m^2}$	66

input
```
int(ln(x)*ln(d+e*x^m)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)^2*ln(d+e*x^m)-1/2*ln(x)^2*ln(1+e*x^m/d)-ln(x)*polylog(2,-e*x^m/d
)/m+polylog(3,-e*x^m/d)/m^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \frac{m^2\log(ex^m+d)\log(x)^2 - m^2\log(x)^2\log\left(\frac{ex^m+d}{d}\right) - 2\,m\mathrm{Li}_2\left(-\frac{ex^m+d}{d}+1\right)\log(x) + 2\,\operatorname{polylog}\left(3,-\frac{ex^m}{d}\right)}{2\,m^2}$$

input
```
integrate(log(x)*log(d+e*x^m)/x,x, algorithm="fricas")
```

output
```
1/2*(m^2*log(e*x^m + d)*log(x)^2 - m^2*log(x)^2*log((e*x^m + d)/d) - 2*m*d
ilog(-(e*x^m + d)/d + 1)*log(x) + 2*polylog(3, -e*x^m/d))/m^2
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(ln(x)*ln(d+e*x**m)/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \int \frac{\log(ex^m+d)\log(x)}{x}\,dx$$

input
```
integrate(log(x)*log(d+e*x^m)/x,x, algorithm="maxima")
```

output
```
-1/6*m*log(x)^3 + d*m*integrate(1/2*log(x)^2/(e*x*x^m + d*x), x) + 1/2*log
(e*x^m + d)*log(x)^2
```

Giac [F]

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \int \frac{\log(ex^m+d)\log(x)}{x}\,dx$$

input
```
integrate(log(x)*log(d+e*x^m)/x,x, algorithm="giac")
```

output
```
integrate(log(e*x^m + d)*log(x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \int \frac{\ln(d+e\,x^m)\,\ln(x)}{x}\,dx$$

input
```
int((log(d + e*x^m)*log(x))/x,x)
```

output
```
int((log(d + e*x^m)*log(x))/x, x)
```

Reduce [F]

$$\int \frac{\log(x)\log\left(d+ex^m\right)}{x}\,dx = \int \frac{\log(x^m e+d)\log(x)}{x}dx$$

input
```
int(log(x)*log(d+e*x^m)/x,x)
```

output
```
int((log(x**m*e + d)*log(x))/x,x)
```

3.390 $\int \frac{\log\left(\frac{a+x}{x}\right)}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x} \, dx = \text{PolyLog}\left(2, -\frac{a}{x}\right)$$

output
```
polylog(2,-a/x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x} \, dx = -\log\left(-\frac{a}{x}\right)\log\left(\frac{a+x}{x}\right) - \text{PolyLog}\left(2, -\frac{-a-x}{x}\right)$$

input
```
Integrate[Log[(a + x)/x]/x,x]
```

output
```
-(Log[-(a/x)]*Log[(a + x)/x]) - PolyLog[2, -((-a - x)/x)]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx$$

$$\Big\downarrow \text{2897}$$

$$\text{PolyLog}\left(2,1-\frac{a+x}{x}\right)$$

input
```
Int[Log[(a + x)/x]/x,x]
```

output
```
PolyLog[2, 1 - (a + x)/x]
```

Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\operatorname{dilog}\left(1+\frac{a}{x}\right)$	9
default	$\operatorname{dilog}\left(1+\frac{a}{x}\right)$	9
risch	$\operatorname{dilog}\left(1+\frac{a}{x}\right)$	9
parts	$\ln\left(\frac{a+x}{x}\right)\ln\left(x\right)+\frac{\ln\left(x\right)^{2}}{2}-\operatorname{dilog}\left(\frac{a+x}{a}\right)-\ln\left(x\right)\ln\left(\frac{a+x}{a}\right)$	41

input
```
int(ln((a+x)/x)/x,x,method=_RETURNVERBOSE)
```

output
```
dilog(1+a/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx=\operatorname{Li}_2\left(-\frac{a+x}{x}+1\right)$$

input
```
integrate(log((a+x)/x)/x,x, algorithm="fricas")
```

output
```
dilog(-(a + x)/x + 1)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx=\int \frac{\log\left(\frac{a}{x}+1\right)}{x}\,dx$$

input
```
integrate(ln((a+x)/x)/x,x)
```

output
```
Integral(log(a/x + 1)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 7.38

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = -(\log(a+x) - \log(x))\log(x) + \log(a+x)\log(x) - \frac{1}{2}\log(x)^2 + \log(x)\log\left(\frac{a+x}{x}\right) - \log(x)\log\left(\frac{x}{a}+1\right) - \mathrm{Li}_2\left(-\frac{x}{a}\right)$$

input
```
integrate(log((a+x)/x)/x,x, algorithm="maxima")
```

output
```
-(log(a + x) - log(x))*log(x) + log(a + x)*log(x) - 1/2*log(x)^2 + log(x)*
log((a + x)/x) - log(x)*log(x/a + 1) - dilog(-x/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(7) = 14$.

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 8.50

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = -\frac{a^3\left(\frac{1}{\frac{a+x}{x}-1} - \log\left(\frac{|a+x|}{|x|}\right) + \log\left(\left|\frac{a+x}{x}-1\right|\right)\right) + \frac{a^3\log\left(\frac{a+x}{x}\right)}{\left(\frac{a+x}{x}-1\right)^2}}{2\,a^2}$$

input
```
integrate(log((a+x)/x)/x,x, algorithm="giac")
```

output
```
-1/2*(a^3*(1/((a + x)/x - 1) - log(abs(a + x)/abs(x)) + log(abs((a + x)/x
- 1))) + a^3*log((a + x)/x)/((a + x)/x - 1)^2)/a^2
```

Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = \text{polylog}\left(2,-\frac{a}{x}\right)$$

input
```
int(log((a + x)/x)/x,x)
```

output
```
polylog(2, -a/x)
```

Reduce [F]

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = \int \frac{\log\left(\frac{a+x}{x}\right)}{x}dx$$

input
```
int(log((a+x)/x)/x,x)
```

output
```
int(log((a + x)/x)/x,x)
```

3.391 $\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \frac{1}{2}\operatorname{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

output
```
1/2*polylog(2,-a/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \frac{1}{2}\operatorname{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

input
```
Integrate[Log[(a + x^2)/x^2]/x,x]
```

output
```
PolyLog[2, -(a/x^2)]/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2911, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}dx$$
$$\Big\downarrow \text{2911}$$
$$\int \frac{\log\left(\frac{a}{x^2}+1\right)}{x}dx$$
$$\Big\downarrow \text{2838}$$
$$\frac{1}{2}\text{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

input
```
Int[Log[(a + x^2)/x^2]/x,x]
```

output
```
PolyLog[2, -(a/x^2)]/2
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2911
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(10) = 20$.

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 6.33

method	result
risch	$-\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{a}{x^2}\right)+\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{\sqrt{-a}}{x}\right)+\ln\left(\frac{1}{x}\right)\ln\left(1-\frac{\sqrt{-a}}{x}\right)+\operatorname{dilog}\left(1+\frac{\sqrt{-a}}{x}\right)$
derivativedivides	$-\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(1+\frac{\sqrt{-a}}{x}\right)+\ln\left(1-\frac{\sqrt{-a}}{x}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(1+\frac{\sqrt{-a}}{x}\right)+\operatorname{dilog}\left(1-\frac{\sqrt{-a}}{x}\right)}{2a}\right)$
default	$-\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(1+\frac{\sqrt{-a}}{x}\right)+\ln\left(1-\frac{\sqrt{-a}}{x}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(1+\frac{\sqrt{-a}}{x}\right)+\operatorname{dilog}\left(1-\frac{\sqrt{-a}}{x}\right)}{2a}\right)$
parts	$\ln\left(\frac{x^2+a}{x^2}\right)\ln(x)+\ln(x)^2-\ln(x)\ln\left(\frac{\sqrt{-a}-x}{\sqrt{-a}}\right)-\ln(x)\ln\left(\frac{\sqrt{-a}+x}{\sqrt{-a}}\right)-\operatorname{dilog}\left(\frac{\sqrt{-a}-x}{\sqrt{-a}}\right)$

input
```
int(ln((x^2+a)/x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(1/x)*ln(1+a/x^2)+ln(1/x)*ln(1+1/x*(-a)^(1/2))+ln(1/x)*ln(1-1/x*(-a)^(1
/2))+dilog(1+1/x*(-a)^(1/2))+dilog(1-1/x*(-a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \frac{1}{2}\operatorname{Li}_2\left(-\frac{x^2+a}{x^2}+1\right)$$

input
```
integrate(log((x^2+a)/x^2)/x,x, algorithm="fricas")
```

output
```
1/2*dilog(-(x^2 + a)/x^2 + 1)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{a}{x^2}+1\right)}{x}\,dx$$

input
```
integrate(ln((x**2+a)/x**2)/x,x)
```

output
```
Integral(log(a/x**2 + 1)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(9) = 18.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 5.75

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = -\left(\log\left(x^2+a\right)-2\log\left(x\right)\right)\log\left(x\right)+\log\left(x^2+a\right)\log\left(x\right)-\log\left(x\right)^2 - \log\left(x\right)\log\left(\frac{x^2}{a}+1\right)+\log\left(x\right)\log\left(\frac{x^2+a}{x^2}\right)-\frac{1}{2}\,\mathrm{Li}_2\left(-\frac{x^2}{a}\right)$$

input
```
integrate(log((x^2+a)/x^2)/x,x, algorithm="maxima")
```

output
```
-(log(x^2 + a) - 2*log(x))*log(x) + log(x^2 + a)*log(x) - log(x)^2 - log(x
)*log(x^2/a + 1) + log(x)*log((x^2 + a)/x^2) - 1/2*dilog(-x^2/a)
```

Giac [F]

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{x^2+a}{x^2}\right)}{x}\,dx$$

input
```
integrate(log((x^2+a)/x^2)/x,x, algorithm="giac")
```

output
```
integrate(log((x^2 + a)/x^2)/x, x)
```

Mupad [B] (verification not implemented)

Time = 15.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log \left(\frac{a+x^2}{x^2}\right)}{x} \, dx = \frac{\text{polylog}\left(2, -\frac{a}{x^2}\right)}{2}$$

input
```
int(log((a + x^2)/x^2)/x,x)
```

output
```
polylog(2, -a/x^2)/2
```

Reduce [F]

$$\int \frac{\log \left(\frac{a+x^2}{x^2}\right)}{x} \, dx = \int \frac{\log \left(\frac{x^2+a}{x^2}\right)}{x} dx$$

input
```
int(log((x^2+a)/x^2)/x,x)
```

output
```
int(log((a + x**2)/x**2)/x,x)
```

3.392 $\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx = \frac{\text{PolyLog}(2,-ax^{-n})}{n}$$

output
```
polylog(2,-a/(x^n))/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx = \frac{\text{PolyLog}(2,-ax^{-n})}{n}$$

input
```
Integrate[Log[(a + x^n)/x^n]/x,x]
```

output
```
PolyLog[2, -(a/x^n)]/n
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2911, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x^{-n}(a+x^n))}{x}dx$$
$$\Big\downarrow \text{2911}$$
$$\int \frac{\log(ax^{-n}+1)}{x}dx$$
$$\Big\downarrow \text{2838}$$
$$\frac{\text{PolyLog}(2,-ax^{-n})}{n}$$

input
```
Int[Log[(a + x^n)/x^n]/x,x]
```

output
```
PolyLog[2, -(a/x^n)]/n
```

Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2911
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] && !BinomialMatchQ[v, x]
```

Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\operatorname{dilog}(1+a\,x^{-n})}{n}$
default	$\frac{\operatorname{dilog}(1+a\,x^{-n})}{n}$
risch	$-\ln(x)\ln(x^n)+\frac{n\ln(x)^2}{2}+\frac{i\ln(x)\pi\ \operatorname{csgn}(i(a+x^n))\operatorname{csgn}(ix^{-n}(a+x^n))^2}{2}+\frac{i\ln(x)\pi\operatorname{csgn}(ix^{-n}(a+x^n))^2\operatorname{cs}}{2}$

input
```
int(ln((a+x^n)/(x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*dilog(1+a/(x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(13) = 26.

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

$$\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx = \frac{n^2\log(x)^2-2\,n\log(x)\log\left(\frac{a+x^n}{a}\right)+2\,n\log(x)\log\left(\frac{a+x^n}{x^n}\right)-2\,\operatorname{Li}_2\left(-\frac{a+x^n}{a}+1\right)}{2\,n}$$

input
```
integrate(log((a+x^n)/(x^n))/x,x, algorithm="fricas")
```

output
```
1/2*(n^2*log(x)^2 - 2*n*log(x)*log((a + x^n)/a) + 2*n*log(x)*log((a + x^n)
/x^n) - 2*dilog(-(a + x^n)/a + 1))/n
```

Sympy [F]

$$\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx = \int \frac{\log(ax^{-n}+1)}{x}\,dx$$

input
```
integrate(ln((a+x**n)/(x**n))/x,x)
```

output
```
Integral(log(a/x**n + 1)/x, x)
```

Maxima [F]

$$\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx = \int \frac{\log\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

input
```
integrate(log((a+x^n)/(x^n))/x,x, algorithm="maxima")
```

output
```
a*n*integrate(log(x)/(a*x + x*x^n), x) + log(a + x^n)*log(x) - log(x)*log(
x^n)
```

Giac [F]

$$\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx = \int \frac{\log\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

input
```
integrate(log((a+x^n)/(x^n))/x,x, algorithm="giac")
```

output
```
integrate(log((a + x^n)/x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(x^{-n}(a+x^n)\right)}{x}\,dx=\int \frac{\ln\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

input
```
int(log((a + x^n)/x^n)/x,x)
```

output
```
int(log((a + x^n)/x^n)/x, x)
```

Reduce [F]

$$\int \frac{\log\left(x^{-n}(a+x^n)\right)}{x}\,dx=\int \frac{\log\left(\frac{x^n+a}{x^n}\right)}{x}dx$$

input
```
int(log((a+x^n)/(x^n))/x,x)
```

output
```
int(log((x**n + a)/x**n)/x,x)
```

3.393 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right)-\text{PolyLog}\left(2,1+\frac{a}{bx}\right)$$

output
```
-ln(b+a/x)*ln(-a/b/x)-polylog(2,1+a/b/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right)-\text{PolyLog}\left(2,\frac{b+\frac{a}{x}}{b}\right)$$

input
```
Integrate[Log[(a + b*x)/x]/x,x]
```

output
```
-(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, (b + a/x)/b]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2911, 2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x} dx$$

$$\Big\downarrow \text{2911}$$

$$\int \frac{\log\left(\frac{a}{x}+b\right)}{x} dx$$

$$\Big\downarrow \text{2904}$$

$$-\int x \log\left(\frac{a}{x}+b\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2841}$$

$$a\int \frac{\log\left(-\frac{a}{bx}\right)}{\frac{a}{x}+b} d\frac{1}{x} - \log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

$$\Big\downarrow \text{2752}$$

$$-\text{PolyLog}\left(2,\frac{a}{bx}+1\right) - \log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

input
```
Int[Log[(a + b*x)/x]/x,x]
```

output
```
-(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, 1 + a/(b*x)]
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2911
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\operatorname{dilog}\left(-\frac{a}{bx}\right)-\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)$	34
default	$-\operatorname{dilog}\left(-\frac{a}{bx}\right)-\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)$	34
risch	$-\operatorname{dilog}\left(-\frac{a}{bx}\right)-\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)$	34
parts	$\ln\left(\frac{bx+a}{x}\right)\ln\left(x\right)-\left(\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{b}\right)b+\frac{\ln(x)^{2}}{2}$	55

input
```
int(ln((b*x+a)/x)/x,x,method=_RETURNVERBOSE)
```

output
```
-dilog(-a/b/x)-ln(b+a/x)*ln(-a/b/x)
```

Fricas [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = \int \frac{\log\left(\frac{bx+a}{x}\right)}{x}\,dx$$

input
```
integrate(log((b*x+a)/x)/x,x, algorithm="fricas")
```

output
```
integral(log((b*x + a)/x)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = \int \frac{\log\left(\frac{a}{x}+b\right)}{x}\,dx$$

input
```
integrate(ln((b*x+a)/x)/x,x)
```

output
```
Integral(log(a/x + b)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

$$\begin{aligned}\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = &-(\log(bx+a)-\log(x))\log(x)+\log(bx+a)\log(x)\\ &-\log\left(\frac{bx}{a}+1\right)\log(x)-\frac{1}{2}\log(x)^2\\ &+\log(x)\log\left(\frac{bx+a}{x}\right)-\mathrm{Li}_2\left(-\frac{bx}{a}\right)\end{aligned}$$

input
```
integrate(log((b*x+a)/x)/x,x, algorithm="maxima")
```

output
```
-(log(b*x + a) - log(x))*log(x) + log(b*x + a)*log(x) - log(b*x/a + 1)*log
(x) - 1/2*log(x)^2 + log(x)*log((b*x + a)/x) - dilog(-b*x/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(34) = 68$.

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 5.83

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = \frac{a^3\left(\frac{\log\left(\frac{|bx+a|}{|x|}\right)}{b^2} - \frac{\log\left(\left|-b+\frac{bx+a}{x}\right|\right)}{b^2} + \frac{1}{\left(b-\frac{bx+a}{x}\right)b}\right) - \frac{a^3\log\left(-\left(a-\frac{b}{\frac{\left(a-\frac{b}{\frac{b}{a}-\frac{bx+a}{ax}}\right)\left(\frac{b}{a}-\frac{bx+a}{ax}\right)}{a}+\frac{b}{a}}\right)\left(\frac{\left(a-\frac{b}{\frac{b}{a}-\frac{bx+a}{ax}}\right)\left(\frac{b}{a}-\frac{bx+a}{ax}\right)}{a}+\cdots\right)\right)}{\left(b-\frac{bx+a}{x}\right)^2}}{2a^2}$$

input
```
integrate(log((b*x+a)/x)/x,x, algorithm="giac")
```

output
```
1/2*(a^3*(log(abs(b*x + a)/abs(x))/b^2 - log(abs(-b + (b*x + a)/x))/b^2 +
1/((b - (b*x + a)/x)*b)) - a^3*log(-(a - b/((a - b/(b/a - (b*x + a)/(a*x))
)*(b/a - (b*x + a)/(a*x))/a + b/a))*((a - b/(b/a - (b*x + a)/(a*x)))*(b/a
- (b*x + a)/(a*x))/a + b/a))/(b - (b*x + a)/x)^2)/a^2
```

Mupad [B] (verification not implemented)

Time = 15.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = -\mathrm{polylog}\left(2, \frac{a}{b\,x}+1\right) - \ln\left(\frac{a+b\,x}{x}\right)\,\ln\left(-\frac{a}{b\,x}\right)$$

input
```
int(log((a + b*x)/x)/x,x)
```

output
```
- polylog(2, a/(b*x) + 1) - log((a + b*x)/x)*log(-a/(b*x))
```

Reduce [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = \int \frac{\log\left(\frac{bx+a}{x}\right)}{x}dx$$

input
```
int(log((b*x+a)/x)/x,x)
```

output
```
int(log((a + b*x)/x)/x,x)
```

3.394 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)-\frac{1}{2}\text{PolyLog}\left(2,1+\frac{a}{bx^2}\right)$$

output
```
-1/2*ln(b+a/x^2)*ln(-a/b/x^2)-1/2*polylog(2,1+a/b/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{b+\frac{a}{x^2}}{b}\right)$$

input
```
Integrate[Log[(a + b*x^2)/x^2]/x,x]
```

output
```
-1/2*(Log[b + a/x^2]*Log[-(a/(b*x^2))]) - PolyLog[2, (b + a/x^2)/b]/2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2911, 2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(\frac{a+bx^2}{x^2}\right)}{x} \, dx$$

$$\Big\downarrow \text{2911}$$

$$\int \frac{\log \left(\frac{a}{x^2}+b\right)}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{1}{2} \int x^2 \log \left(\frac{a}{x^2}+b\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2841}$$

$$\frac{1}{2} \left(a \int \frac{\log \left(-\frac{a}{bx^2}\right)}{\frac{a}{x^2}+b}d\frac{1}{x^2}-\log \left(\frac{a}{x^2}+b\right) \log \left(-\frac{a}{bx^2}\right)\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2} \left(-\operatorname{PolyLog}\left(2,\frac{a}{bx^2}+1\right)-\log \left(\frac{a}{x^2}+b\right) \log \left(-\frac{a}{bx^2}\right)\right)$$

input
```
Int[Log[(a + b*x^2)/x^2]/x,x]
```

output
```
(-(Log[b + a/x^2]*Log[-(a/(b*x^2))]) - PolyLog[2, 1 + a/(b*x^2)])/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2911
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(35) = 70$.

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.77

method	result
risch	$-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+\ln\left(\frac{1}{x}\right)\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{1}{x}\right)\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)$
parts	$\ln\left(\frac{b\,x^2+a}{x^2}\right)\ln\left(x\right)-2\left(\frac{\ln(x)\left(\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2b}+\frac{\operatorname{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2b}\right)$
derivativedivides	$-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{\frac{a}{x}+\sqrt{}}{\sqrt{}}\right)}{2a}\right.$
default	$-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{\frac{a}{x}+\sqrt{}}{\sqrt{}}\right)}{2a}\right.$

input
```
int(ln((b*x^2+a)/x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(1/x)*ln(b+a/x^2)+ln(1/x)*ln((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln(1/x)*
ln((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2)
)+dilog((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{x}\,dx$$

input
```
integrate(log((b*x^2+a)/x^2)/x,x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)/x^2)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{a}{x^2}+b\right)}{x}\,dx$$

input
```
integrate(ln((b*x**2+a)/x**2)/x,x)
```

output
```
Integral(log(a/x**2 + b)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.97

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = -\left(\log\left(bx^2+a\right) - 2\log\left(x\right)\right)\log\left(x\right) + \log\left(bx^2+a\right)\log\left(x\right) - \log\left(\frac{bx^2}{a}+1\right)\log\left(x\right) - \log\left(x\right)^2 + \log\left(x\right)\log\left(\frac{bx^2+a}{x^2}\right) - \frac{1}{2}\operatorname{Li}_2\left(-\frac{bx^2}{a}\right)$$

input
```
integrate(log((b*x^2+a)/x^2)/x,x, algorithm="maxima")
```

output
```
-(log(b*x^2 + a) - 2*log(x))*log(x) + log(b*x^2 + a)*log(x) - log(b*x^2/a
+ 1)*log(x) - log(x)^2 + log(x)*log((b*x^2 + a)/x^2) - 1/2*dilog(-b*x^2/a)
```

Giac [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{x}\,dx$$

input
```
integrate(log((b*x^2+a)/x^2)/x,x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)/x^2)/x, x)
```

Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = -\frac{\mathrm{Li}_2\left(-\frac{a}{b\,x^2}\right)}{2} - \frac{\ln\left(b+\frac{a}{x^2}\right)\,\ln\left(-\frac{a}{b\,x^2}\right)}{2}$$

input
```
int(log((a + b*x^2)/x^2)/x,x)
```

output
```
- dilog(-a/(b*x^2))/2 - (log(b + a/x^2)*log(-a/(b*x^2)))/2
```

Reduce [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{b\,x^2+a}{x^2}\right)}{x}dx$$

input
```
int(log((b*x^2+a)/x^2)/x,x)
```

output
```
int(log((a + b*x**2)/x**2)/x,x)
```

3.395 $\int \frac{\log(x^{-n}(a+bx^n))}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{\log(x^{-n}(a+bx^n))}{x} \, dx = -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log(b+ax^{-n})}{n} - \frac{\text{PolyLog}\left(2, 1+\frac{ax^{-n}}{b}\right)}{n}$$

output
```
-ln(-a/b/(x^n))*ln(b+a/(x^n))/n-polylog(2,1+a/b/(x^n))/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\log(x^{-n}(a+bx^n))}{x} \, dx = -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log(b+ax^{-n}) + \text{PolyLog}\left(2, \frac{b+ax^{-n}}{b}\right)}{n}$$

input
```
Integrate[Log[(a + b*x^n)/x^n]/x,x]
```

output
```
-((Log[-(a/(b*x^n))]*Log[b + a/x^n] + PolyLog[2, (b + a/x^n)/b])/n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2911, 2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (x^{-n}(a+bx^n))}{x} \, dx$$

$$\Big\downarrow \text{2911}$$

$$\int \frac{\log (ax^{-n}+b)}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{\int x^n \log (ax^{-n}+b)\,dx^{-n}}{n}$$

$$\Big\downarrow \text{2841}$$

$$-\frac{\log \left(-\frac{ax^{-n}}{b}\right) \log (ax^{-n}+b) - a\int \frac{\log\left(-\frac{ax^{-n}}{b}\right)}{ax^{-n}+b}dx^{-n}}{n}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\text{PolyLog}\left(2, \frac{ax^{-n}}{b}+1\right) + \log \left(-\frac{ax^{-n}}{b}\right) \log (ax^{-n}+b)}{n}$$

input
```
Int[Log[(a + b*x^n)/x^n]/x,x]
```

output
```
-((Log[-(a/(b*x^n))]*Log[b + a/x^n] + PolyLog[2, 1 + a/(b*x^n)])/n)
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2911
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Maple [A] (verified)

Time = 6.91 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\operatorname{dilog}\left(-\frac{a\,x^{-n}}{b}\right)-\ln\left(b+a\,x^{-n}\right)\ln\left(-\frac{a\,x^{-n}}{b}\right)}{n}$
default	$\frac{-\operatorname{dilog}\left(-\frac{a\,x^{-n}}{b}\right)-\ln\left(b+a\,x^{-n}\right)\ln\left(-\frac{a\,x^{-n}}{b}\right)}{n}$
risch	$-\ln\left(x\right)\ln\left(x^{n}\right)+\frac{n\ln(x)^{2}}{2}+\frac{i\pi\ln(x)\operatorname{csgn}(i(a+b\,x^{n}))\operatorname{csgn}\left(ix^{-n}(a+b\,x^{n})\right)^{2}}{2}+\frac{i\pi\ln(x)\operatorname{csgn}\left(ix^{-n}(a+b\,x^{n})\right.}{2}$

input
```
int(ln((a+b*x^n)/(x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-dilog(-a/b/(x^n))-ln(b+a/(x^n))*ln(-a/b/(x^n)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx = \frac{n^2\log(x)^2 - 2n\log(x)\log\left(\frac{bx^n+a}{a}\right) + 2n\log(x)\log\left(\frac{bx^n+a}{x^n}\right) - 2\operatorname{Li}_2\left(-\frac{bx^n+a}{a}+1\right)}{2n}$$

input
```
integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="fricas")
```

output
```
1/2*(n^2*log(x)^2 - 2*n*log(x)*log((b*x^n + a)/a) + 2*n*log(x)*log((b*x^n
+ a)/x^n) - 2*dilog(-(b*x^n + a)/a + 1))/n
```

Sympy [F]

$$\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx = \int \frac{\log(ax^{-n}+b)}{x}\,dx$$

input
```
integrate(ln((a+b*x**n)/(x**n))/x,x)
```

output
```
Integral(log(a/x**n + b)/x, x)
```

Maxima [F]

$$\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx = \int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{x}\,dx$$

input
```
integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="maxima")
```

output
```
a*n*integrate(log(x)/(b*x*x^n + a*x), x) + log(b*x^n + a)*log(x) - log(x)*
log(x^n)
```

Giac [F]

$$\int \frac{\log \left(x^{-n}(a+bx^n)\right)}{x}\,dx=\int \frac{\log \left(\frac{bx^n+a}{x^n}\right)}{x}\,dx$$

input
```
integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="giac")
```

output
```
integrate(log((b*x^n + a)/x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(x^{-n}(a+bx^n)\right)}{x}\,dx=\int \frac{\ln \left(\frac{a+b\,x^n}{x^n}\right)}{x}\,dx$$

input
```
int(log((a + b*x^n)/x^n)/x,x)
```

output
```
int(log((a + b*x^n)/x^n)/x, x)
```

Reduce [F]

$$\int \frac{\log \left(x^{-n}(a+bx^n)\right)}{x}\,dx=\int \frac{\log \left(\frac{x^n b+a}{x^n}\right)}{x}dx$$

input
```
int(log((a+b*x^n)/(x^n))/x,x)
```

output
```
int(log((x**n*b + a)/x**n)/x,x)
```

3.396 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} - \frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d} + \frac{\text{PolyLog}\left(2,1+\frac{dx}{c}\right)}{d}$$

output
```
ln(b+a/x)*ln(d*x+c)/d+ln(-d*x/c)*ln(d*x+c)/d-ln(-d*(b*x+a)/(-a*d+b*c))*ln(
d*x+c)/d-polylog(2,b*(d*x+c)/(-a*d+b*c))/d+polylog(2,1+d*x/c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)+\log(x)\log(c+dx)-\log\left(\frac{a}{b}+x\right)\log(c+dx)+\log\left(\frac{a}{b}+x\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-\log(}{d}$$

input
```
Integrate[Log[(a + b*x)/x]/(c + d*x),x]
```

output
```
(Log[b + a/x]*Log[c + d*x] + Log[x]*Log[c + d*x] - Log[a/b + x]*Log[c + d*
x] + Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - Log[x]*Log[1 + (d*x)/c]
 - PolyLog[2, -((d*x)/c)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/d
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2915, 2912, 2005, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}dx$$

$$\Big\downarrow \text{2915}$$

$$\int \frac{\log\left(\frac{a}{x}+b\right)}{c+dx}dx$$

$$\Big\downarrow \text{2912}$$

$$\frac{a\int \frac{\log(c+dx)}{\left(\frac{a}{x}+b\right)x^2}dx}{d}+\frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d}$$

$$\Big\downarrow \text{2005}$$

$$\frac{a\int \frac{\log(c+dx)}{x(a+bx)}dx}{d}+\frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d}$$

$$\Big\downarrow \text{2863}$$

$$\frac{a\int \left(\frac{\log(c+dx)}{ax}-\frac{b\log(c+dx)}{a(a+bx)}\right)dx}{d}+\frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(-\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{a}-\frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{a}+\frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{a}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{a}\right)}{d}+\frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d}$$

input
```
Int[Log[(a + b*x)/x]/(c + d*x),x]
```

output
```
(Log[b + a/x]*Log[c + d*x])/d + (a*((Log[-((d*x)/c)]*Log[c + d*x])/a - (Lo
g[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/a - PolyLog[2, (b*(c + d*x))
/(b*c - a*d)]/a + PolyLog[2, 1 + (d*x)/c]/a))/d
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

rule 2915
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Maple [A] (verified)

Time = 10.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

method	result
risch	$\frac{\operatorname{dilog}\left(\frac{da-bc+c\left(b+\frac{a}{x}\right)}{da-bc}\right)}{d}+\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{da-bc+c\left(b+\frac{a}{x}\right)}{da-bc}\right)}{d}-\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)}{d}-\frac{\operatorname{dilog}\left(-\frac{a}{bx}\right)}{d}$
derivativedivides	$-a\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{da-bc+c\left(b+\frac{a}{x}\right)}{da-bc}\right)}{c}+\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{da-bc+c\left(b+\frac{a}{x}\right)}{da-bc}\right)}{c}\right)c}{da}+\frac{\operatorname{dilog}\left(-\frac{a}{bx}\right)+\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)}{da}\right)$
default	$-a\left(-\frac{\left(\frac{\operatorname{dilog}\left(\frac{da-bc+c\left(b+\frac{a}{x}\right)}{da-bc}\right)}{c}+\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{da-bc+c\left(b+\frac{a}{x}\right)}{da-bc}\right)}{c}\right)c}{da}+\frac{\operatorname{dilog}\left(-\frac{a}{bx}\right)+\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{bx}\right)}{da}\right)$
parts	$\frac{\ln\left(\frac{bx+a}{x}\right)\ln(dx+c)}{d}-\frac{-\left(\operatorname{dilog}\left(-\frac{dx}{c}\right)+\ln(dx+c)\ln\left(-\frac{dx}{c}\right)\right)d^{2}+\left(\frac{\operatorname{dilog}\left(\frac{da-bc+b(dx+c)}{da-bc}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{da-bc+b(dx}{da-bc}\right.}{b}\right.}{d^{3}}$

input
```
int(ln((b*x+a)/x)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*dilog((d*a-b*c+c*(b+a/x))/(a*d-b*c))+1/d*ln(b+a/x)*ln((d*a-b*c+c*(b+a/
x))/(a*d-b*c))-1/d*ln(b+a/x)*ln(-a/b/x)-1/d*dilog(-a/b/x)
```

Fricas [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{bx+a}{x}\right)}{dx+c}\,dx$$

input
```
integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(log((b*x + a)/x)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{a}{x}+b\right)}{c+dx}\,dx$$

input
```
integrate(ln((b*x+a)/x)/(d*x+c),x)
```

output
```
Integral(log(a/x + b)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = & -\frac{(\log(bx+a)-\log(x))\log(dx+c)}{d} \\ & +\frac{\log(dx+c)\log\left(\frac{bx+a}{x}\right)}{d}-\frac{\log\left(\frac{dx}{c}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{dx}{c}\right)}{d} \\ & +\frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{d}\end{aligned}$$

input
```
integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="maxima")
```

output
```
-(log(b*x + a) - log(x))*log(d*x + c)/d + log(d*x + c)*log((b*x + a)/x)/d
- (log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(b*x + a)*log((b*d*x + a
*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/d
```

Giac [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{bx+a}{x}\right)}{dx+c}\,dx$$

input
```
integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log((b*x + a)/x)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \int \frac{\ln\left(\frac{a+b\,x}{x}\right)}{c+d\,x}\,dx$$

input
```
int(log((a + b*x)/x)/(c + d*x),x)
```

output
```
int(log((a + b*x)/x)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{bx+a}{x}\right)}{dx+c}dx$$

input
```
int(log((b*x+a)/x)/(d*x+c),x)
```

output
```
int(log((a + b*x)/x)/(c + d*x),x)
```

3.397 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\begin{aligned}\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx &= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d}+\frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}\\ &\quad-\frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d}\\ &\quad-\frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}c-\sqrt{-a}d}\right)\log(c+dx)}{d}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}\\ &\quad-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{d}+\frac{2\,\text{PolyLog}\left(2,1+\frac{dx}{c}\right)}{d}\end{aligned}$$

output

```
ln(b+a/x^2)*ln(d*x+c)/d+2*ln(-d*x/c)*ln(d*x+c)/d-ln(d*((-a)^(1/2)-b^(1/2)*
x)/(b^(1/2)*c+(-a)^(1/2)*d))*ln(d*x+c)/d-ln(-d*((-a)^(1/2)+b^(1/2)*x)/(b^(
1/2)*c-(-a)^(1/2)*d))*ln(d*x+c)/d-polylog(2,b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a
)^(1/2)*d))/d-polylog(2,b^(1/2)*(d*x+c)/(b^(1/2)*c+(-a)^(1/2)*d))/d+2*poly
log(2,1+d*x/c)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(\sqrt{-a}+\sqrt{b}x)}{\sqrt{b}c-\sqrt{-a}d}\right)\log(c+dx)}{d} + \frac{2\,\text{PolyLog}\left(2,\frac{c+dx}{c}\right)}{d} - \frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d} - \frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{d}$$

input
```
Integrate[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```

output
```
(Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Lo
g[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (
Log[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/
d + (2*PolyLog[2, (c + d*x)/c])/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b
]*c - Sqrt[-a]*d)]/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a
]*d)]/d
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2915, 2912, 2005, 2863, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx$$

$$\Big\downarrow \; 2915$$

$$\int \frac{\log\left(\frac{a}{x^2}+b\right)}{c+dx}dx$$

$$\Big\downarrow \text{2912}$$

$$\frac{2a\int \frac{\log(c+dx)}{\left(\frac{a}{x^2}+b\right)x^3}dx}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}$$

$$\Big\downarrow \text{2005}$$

$$\frac{2a\int \frac{\log(c+dx)}{x(bx^2+a)}dx}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}$$

$$\Big\downarrow \text{2863}$$

$$\frac{2a\int \left(\frac{\log(c+dx)}{ax}-\frac{bx\log(c+dx)}{a(bx^2+a)}\right)dx}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{2a}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{2a}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}d+\sqrt{b}c}\right)}{2a}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}c-\sqrt{-a}d}\right)}{2a}+\text{PolyL}\right.}{d}$$

$$\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}$$

input
```
Int[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```

output
```
(Log[b + a/x^2]*Log[c + d*x])/d + (2*a*((Log[-((d*x)/c)]*Log[c + d*x])/a -
 (Log[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/(
2*a) - (Log[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c
+ d*x])/(2*a) - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c - Sqrt[-a]*d)]/(
2*a) - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]/(2*a) + Po
lyLog[2, 1 + (d*x)/c]/a))/d
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2912
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Simp[b*e*n*(p/g)   Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

rule 2915
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.01

method	result
parts	$\frac{\ln\left(\frac{b x^2+a}{x^2}\right)\ln(dx+c)}{d}-\frac{2\left(-\left(\operatorname{dilog}\left(-\frac{dx}{c}\right)+\ln(dx+c)\ln\left(-\frac{dx}{c}\right)\right)d^3-\left(-\frac{\ln(dx+c)\left(\ln\left(\frac{d\sqrt{-ab}+bc-b(dx+c)}{d\sqrt{-ab}+bc}\right)+\ln\left(\frac{d\sqrt{}}{}\right.\right.}{2b}\right.\right.}{d^4}$
derivativedivides	$-\frac{\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)-2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2a}\right)}{d}+$
default	$-\frac{\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)-2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2a}\right)}{d}+$
risch	$\frac{\ln\left(\frac{1}{x}\right)\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}+\frac{\ln\left(\frac{1}{x}\right)\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}-\frac{\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)}{d}+\frac{\operatorname{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}+\frac{\operatorname{dilog}\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}$

input
```
int(ln((b*x^2+a)/x^2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
ln((b*x^2+a)/x^2)/d*ln(d*x+c)-2/d^4*(-(dilog(-d*x/c)+ln(d*x+c)*ln(-d*x/c))
*d^3-(-1/2*ln(d*x+c)*(ln((d*(-a*b)^(1/2)+b*c-b*(d*x+c))/(d*(-a*b)^(1/2)+b*
c))+ln((d*(-a*b)^(1/2)-b*c+b*(d*x+c))/(d*(-a*b)^(1/2)-b*c)))/b-1/2*(dilog(
(d*(-a*b)^(1/2)+b*c-b*(d*x+c))/(d*(-a*b)^(1/2)+b*c))+dilog((d*(-a*b)^(1/2)
-b*c+b*(d*x+c))/(d*(-a*b)^(1/2)-b*c)))/b)*b*d^3)
```

Fricas [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx=\int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

input
```
integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{a}{x^2}+b\right)}{c+dx}\,dx$$

input
```
integrate(ln((b*x**2+a)/x**2)/(d*x+c),x)
```

output
```
Integral(log(a/x**2 + b)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

input
```
integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

input
```
integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\ln\left(\frac{b\,x^2+a}{x^2}\right)}{c+d\,x}\,dx$$

input
```
int(log((a + b*x^2)/x^2)/(c + d*x),x)
```

output
```
int(log((a + b*x^2)/x^2)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\log \left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{b\,x^2+a}{x^2}\right)}{dx+c}dx$$

input
```
int(log((b*x^2+a)/x^2)/(d*x+c),x)
```

output
```
int(log((a + b*x**2)/x**2)/(c + d*x),x)
```

3.398 $\int \frac{\log(x^{-n}(a+bx^n))}{c+dx} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx} dx = \text{Int}\left(\frac{\log(b+ax^{-n})}{c+dx}, x\right)$$

output
```
Defer(Int)(ln(b+a/(x^n))/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx} dx = \int \frac{\log(x^{-n}(a+bx^n))}{c+dx} dx$$

input
```
Integrate[Log[(a + b*x^n)/x^n]/(c + d*x),x]
```

output
```
Integrate[Log[(a + b*x^n)/x^n]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2915, 2914}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(x^{-n}(a+bx^n)\right)}{c+dx} dx$$
$$\Big\downarrow \text{ 2915}$$
$$\int \frac{\log \left(ax^{-n}+b\right)}{c+dx}dx$$
$$\Big\downarrow \text{ 2914}$$
$$\int \frac{\log \left(ax^{-n}+b\right)}{c+dx}dx$$

input
```
Int[Log[(a + b*x^n)/x^n]/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2914
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)
+ (g_.)*(x_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x)^r*(a + b*Log[c*(d
+ e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r}, x]
```

rule 2915
```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(\left(a+b\,x^{n}\right) x^{-n}\right)}{dx+c}dx$$

input
```
int(ln((a+b*x^n)/(x^n))/(d*x+c),x)
```

output
```
int(ln((a+b*x^n)/(x^n))/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log \left(x^{-n}(a+bx^n)\right)}{c+dx}\,dx=\int \frac{\log \left(\frac{bx^n+a}{x^n}\right)}{dx+c}\,dx$$

input
```
integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="fricas")
```

output
```
integral(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 24.90 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{\log \left(x^{-n}(a+bx^n)\right)}{c+dx}\,dx=\int \frac{\log \left(ax^{-n}+b\right)}{c+dx}\,dx$$

input
```
integrate(ln((a+b*x**n)/(x**n))/(d*x+c),x)
```

output
```
Integral(log(a/x**n + b)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log (x^{-n}(a+bx^n))}{c+dx} \, dx = \int \frac{\log \left(\frac{bx^n+a}{x^n}\right)}{dx+c} \, dx$$

input
```
integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log (x^{-n}(a+bx^n))}{c+dx} \, dx = \int \frac{\log \left(\frac{bx^n+a}{x^n}\right)}{dx+c} \, dx$$

input
```
integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 15.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log \left(x^{-n}(a+bx^n)\right)}{c+dx} \, dx = \int \frac{\ln \left(\frac{a+b\,x^n}{x^n}\right)}{c+d\,x} \, dx$$

input
```
int(log((a + b*x^n)/x^n)/(c + d*x),x)
```

output
```
int(log((a + b*x^n)/x^n)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log \left(x^{-n}(a+bx^n)\right)}{c+dx} \, dx = \int \frac{\log \left(\frac{x^n b+a}{x^n}\right)}{dx+c} dx$$

input
```
int(log((a+b*x^n)/(x^n))/(d*x+c),x)
```

output
```
int(log((x**n*b + a)/x**n)/(c + d*x),x)
```

3.399 $\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\begin{aligned}\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx &= -\frac{bemnx^{1+m}(fx)^q\,\text{Hypergeometric2F1}\left(1,\frac{1+m+q}{m},\frac{1+2m+q}{m},-\frac{ex^m}{d}\right)}{d(1+q)(1+m+q)}\\&\quad+\frac{(fx)^{1+q}(a+b\log(c(d+ex^m)^n))}{f(1+q)}\end{aligned}$$

output
```
-b*e*m*n*x^(1+m)*(f*x)^q*hypergeom([1, (1+m+q)/m],[(1+2*m+q)/m],-e*x^m/d)/
d/(1+q)/(1+m+q)+(f*x)^(1+q)*(a+b*ln(c*(d+e*x^m)^n))/f/(1+q)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx = \frac{x(fx)^q\left(-bemnx^m\,\text{Hypergeometric2F1}\left(1,\frac{1+m+q}{m},\frac{1+2m+q}{m},-\frac{ex^m}{d}\right)+d(1+m+q)(a+b\log(c(d+ex^m)\right.}{d(1+q)(1+m+q)}$$

input
```
Integrate[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```

output
```
(x*(f*x)^q*(-(b*e*m*n*x^m*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m + q
)/m, -((e*x^m)/d)]) + d*(1 + m + q)*(a + b*Log[c*(d + e*x^m)^n])))/(d*(1 +
 q)*(1 + m + q))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2905, 30, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^q (a+b\log (c(d+ex^m)^n)) \, dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{(fx)^{q+1} (a+b\log (c(d+ex^m)^n))}{f(q+1)}-\frac{bemn\int \frac{x^{m-1}(fx)^{q+1}}{ex^m+d}dx}{f(q+1)}$$

$$\Big\downarrow \text{ 30}$$

$$\frac{(fx)^{q+1} (a+b\log (c(d+ex^m)^n))}{f(q+1)}-\frac{bemnx^{-q}(fx)^q\int \frac{x^{m+q}}{ex^m+d}dx}{q+1}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(fx)^{q+1} (a+b\log (c(d+ex^m)^n))}{f(q+1)}-\frac{bemnx^{m+1}(fx)^q \text{Hypergeometric2F1}\left(1,\frac{m+q+1}{m},\frac{2m+q+1}{m},-\frac{ex^m}{d}\right)}{d(q+1)(m+q+1)}$$

input
```
Int[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```

output
```
-((b*e*m*n*x^(1 + m)*(f*x)^q*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m
+ q)/m, -((e*x^m)/d)])/(d*(1 + q)*(1 + m + q))) + ((f*x)^(1 + q)*(a + b*Lo
g[c*(d + e*x^m)^n]))/(f*(1 + q))
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int (fx)^q (a+b\ln (c(d+e\,x^m)^n))\,dx$$

input
```
int((f*x)^q*(a+b*ln(c*(d+e*x^m)^n)),x)
```

output
```
int((f*x)^q*(a+b*ln(c*(d+e*x^m)^n)),x)
```

Fricas [F]

$$\int (fx)^q (a+b\log (c(d+ex^m)^n))\ dx=\int (b\log ((ex^m+d)^n c)+a)(fx)^q\ dx$$

input
```
integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="fricas")
```

output
```
integral((f*x)^q*b*log((e*x^m + d)^n*c) + (f*x)^q*a, x)
```

Sympy [F]

$$\int (fx)^q (a+b\log (c(d+ex^m)^n))\,dx=\int (fx)^q (a+b\log (c(d+ex^m)^n))\,dx$$

input
```
integrate((f*x)**q*(a+b*ln(c*(d+e*x**m)**n)),x)
```

output
```
Integral((f*x)**q*(a + b*log(c*(d + e*x**m)**n)), x)
```

Maxima [F]

$$\int (fx)^q (a+b\log (c(d+ex^m)^n))\,dx=\int (b\log ((ex^m+d)^n c)+a)(fx)^q\,dx$$

input
```
integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="maxima")
```

output
```
(d^2*f^q*m^2*n*integrate(x^q/((m*(q + 1) - q^2 - 2*q - 1)*e^2*x^(2*m) + 2*
(m*(q + 1) - q^2 - 2*q - 1)*d*e*x^m + (m*(q + 1) - q^2 - 2*q - 1)*d^2), x)
 - (((m*(q + 1) - q^2 - 2*q - 1)*e*f^q*x*x^m + (m*(q + 1) - q^2 - 2*q - 1)
*d*f^q*x)*x^q*log((e*x^m + d)^n) + (((m*(q + 1) - q^2 - 2*q - 1)*e*f^q*log
(c) - (m^2*n - m*n*(q + 1))*e*f^q)*x*x^m - (d*f^q*m^2*n - (m*(q + 1) - q^2
 - 2*q - 1)*d*f^q*log(c))*x)*x^q)/((q^3 - (q^2 + 2*q + 1)*m + 3*q^2 + 3*q
+ 1)*e*x^m + (q^3 - (q^2 + 2*q + 1)*m + 3*q^2 + 3*q + 1)*d))*b + (f*x)^(q
+ 1)*a/(f*(q + 1))
```

Giac [F]

$$\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx = \int (b\log((ex^m+d)^n c)+a)(fx)^q\,dx$$

input
```
integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="giac")
```

output
```
integrate((b*log((e*x^m + d)^n*c) + a)*(f*x)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx = \int (f\,x)^q\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)\right)\,dx$$

input
```
int((f*x)^q*(a + b*log(c*(d + e*x^m)^n)),x)
```

output
```
int((f*x)^q*(a + b*log(c*(d + e*x^m)^n)), x)
```

Reduce [F]

$$\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx = \frac{f^q\left(x^q\log((x^me+d)^n c)\,bqx+x^q\log((x^me+d)^n c)\,bx+x^qaqx+x^qax-x^qbmnx+\left(\int \frac{x^q}{x^meq+x^me+dq+d}dx\right.\right.}{q^2+2q+1}$$

input
```
int((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x)
```

output

```
(f**q*(x**q*log((x**m*e + d)**n*c)*b*q*x + x**q*log((x**m*e + d)**n*c)*b*x
 + x**q*a*q*x + x**q*a*x - x**q*b*m*n*x + int(x**q/(x**m*e*q + x**m*e + d*
q + d),x)*b*d*m*n*q**2 + 2*int(x**q/(x**m*e*q + x**m*e + d*q + d),x)*b*d*m
*n*q + int(x**q/(x**m*e*q + x**m*e + d*q + d),x)*b*d*m*n))/(q**2 + 2*q + 1
)
```

3.400 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 166

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{bd^7n\sqrt{x}}{4e^7}-\frac{bd^6nx}{8e^6}+\frac{bd^5nx^{3/2}}{12e^5}-\frac{bd^4nx^2}{16e^4}+\frac{bd^3nx^{5/2}}{20e^3}-\frac{bd^2nx^3}{24e^2}+\frac{bdnx^{7/2}}{28e}-\frac{1}{32}bnx^4-\frac{bd^8n\log\left(d+e\sqrt{x}\right)}{4e^8}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

output
```
1/4*b*d^7*n*x^(1/2)/e^7-1/8*b*d^6*n*x/e^6+1/12*b*d^5*n*x^(3/2)/e^5-1/16*b*
d^4*n*x^2/e^4+1/20*b*d^3*n*x^(5/2)/e^3-1/24*b*d^2*n*x^3/e^2+1/28*b*d*n*x^(
7/2)/e-1/32*b*n*x^4-1/4*b*d^8*n*ln(d+e*x^(1/2))/e^8+1/4*x^4*(a+b*ln(c*(d+e
*x^(1/2))^n))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.95

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{ax^4}{4}-\frac{1}{8}ben\left(-\frac{2d^7\sqrt{x}}{e^8}+\frac{d^6x}{e^7}-\frac{2d^5x^{3/2}}{3e^6}+\frac{d^4x^2}{2e^5}-\frac{2d^3x^{5/2}}{5e^4}+\frac{d^2x^3}{3e^3}-\frac{2dx^{7/2}}{7e^2}+\frac{x^4}{4e}+\frac{2d^8\log\left(d+e\sqrt{x}\right)}{e^9}\right)+\frac{1}{4}bx^4\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

output
```
(a*x^4)/4 - (b*e*n*((-2*d^7*Sqrt[x])/e^8 + (d^6*x)/e^7 - (2*d^5*x^(3/2))/(
3*e^6) + (d^4*x^2)/(2*e^5) - (2*d^3*x^(5/2))/(5*e^4) + (d^2*x^3)/(3*e^3) -
 (2*d*x^(7/2))/(7*e^2) + x^4/(4*e) + (2*d^8*Log[d + e*Sqrt[x]])/e^9))/8 +
(b*x^4*Log[c*(d + e*Sqrt[x])^n])/4
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int x^{7/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2842}$$

$$2\left(\frac{1}{8}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{8}ben\int\frac{x^4}{d+e\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{49}$$

$$2\left(\frac{1}{8}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{8}ben\int\left(\frac{d^8}{e^8\left(d+e\sqrt{x}\right)}-\frac{d^7}{e^8}+\frac{\sqrt{x}d^6}{e^7}-\frac{xd^5}{e^6}+\frac{x^{3/2}d^4}{e^5}-\frac{x^2d^3}{e^4}+\frac{x^{5/2}d^2}{e^3}-\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{8}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{8}ben\left(\frac{d^8\log\left(d+e\sqrt{x}\right)}{e^9}-\frac{d^7\sqrt{x}}{e^8}+\frac{d^6x}{2e^7}-\frac{d^5x^{3/2}}{3e^6}+\frac{d^4x^2}{4e^5}-\frac{d^3x^{5/2}}{5e^4}+\frac{d^2x^3}{6e^3}\right.\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

output
```
2*(-1/8*(b*e*n*(-((d^7*Sqrt[x])/e^8) + (d^6*x)/(2*e^7) - (d^5*x^(3/2))/(3*
e^6) + (d^4*x^2)/(4*e^5) - (d^3*x^(5/2))/(5*e^4) + (d^2*x^3)/(6*e^3) - (d*
x^(7/2))/(7*e^2) + x^4/(8*e) + (d^8*Log[d + e*Sqrt[x]])/e^9)) + (x^4*(a +
b*Log[c*(d + e*Sqrt[x])^n]))/8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = \frac{840\,be^8x^4\log\left(c\right)-140\,bd^2e^6nx^3-210\,bd^4e^4nx^2-420\,bd^6e^2nx-105\,(be^8n-8\,ae^8)x^4+840\,(be^8nx^4-\ldots}{3360\,e^8}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

output
```
1/3360*(840*b*e^8*x^4*log(c) - 140*b*d^2*e^6*n*x^3 - 210*b*d^4*e^4*n*x^2 -
 420*b*d^6*e^2*n*x - 105*(b*e^8*n - 8*a*e^8)*x^4 + 840*(b*e^8*n*x^4 - b*d^
8*n)*log(e*sqrt(x) + d) + 8*(15*b*d*e^7*n*x^3 + 21*b*d^3*e^5*n*x^2 + 35*b*
d^5*e^3*n*x + 105*b*d^7*e*n)*sqrt(x))/e^8
```

Sympy [A] (verification not implemented)

Time = 6.95 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.93

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{ax^4}{4}+b\left(-\frac{en\left(\frac{2d^8\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^8}-\frac{2d^7\sqrt{x}}{e^8}+\frac{d^6x}{e^7}-\frac{2d^5x^{\frac{3}{2}}}{3e^6}+\frac{d^4x^2}{2e^5}-\frac{2d^3x^{\frac{5}{2}}}{5e^4}+\frac{d^2x^3}{3e^3}-\frac{2dx^{\frac{7}{2}}}{7e^2}+\frac{x^4}{4e}\right)}{8}+\frac{x^4\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{4}\right)$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

output
```
a*x**4/4 + b*(-e*n*(2*d**8*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqr
t(x))/e, True))/e**8 - 2*d**7*sqrt(x)/e**8 + d**6*x/e**7 - 2*d**5*x**(3/2)
/(3*e**6) + d**4*x**2/(2*e**5) - 2*d**3*x**(5/2)/(5*e**4) + d**2*x**3/(3*e
**3) - 2*d*x**(7/2)/(7*e**2) + x**4/(4*e))/8 + x**4*log(c*(d + e*sqrt(x))*
*n)/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.77

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{4}\,ax^4$$
$$-\frac{1}{3360}\,ben\left(\frac{840\,d^8\log\left(e\sqrt{x}+d\right)}{e^9}+\frac{105\,e^7x^4-120\,de^6x^{\frac{7}{2}}+140\,d^2e^5x^3-168\,d^3e^4x^{\frac{5}{2}}+210\,d^4e^3x^2-28}{e^8}\right.$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

output
```
1/4*b*x^4*log((e*sqrt(x) + d)^n*c) + 1/4*a*x^4 - 1/3360*b*e*n*(840*d^8*log
(e*sqrt(x) + d)/e^9 + (105*e^7*x^4 - 120*d*e^6*x^(7/2) + 140*d^2*e^5*x^3 -
 168*d^3*e^4*x^(5/2) + 210*d^4*e^3*x^2 - 280*d^5*e^2*x^(3/2) + 420*d^6*e*x
 - 840*d^7*sqrt(x))/e^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(134) = 268.

Time = 0.12 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.10

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=\frac{840\,bex^4\log\left(c\right)+840\,aex^4+\left(\frac{840\,\left(e\sqrt{x}+d\right)^8\log\left(e\sqrt{x}+d\right)}{e^7}-\frac{6720\,\left(e\sqrt{x}+d\right)^7d\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{23520\,\left(e\sqrt{x}+d\right)^6d^2\log\left(e\sqrt{x}+d\right)}{e^7}\right.}{}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

output
```
1/3360*(840*b*e*x^4*log(c) + 840*a*e*x^4 + (840*(e*sqrt(x) + d)^8*log(e*sq
rt(x) + d)/e^7 - 6720*(e*sqrt(x) + d)^7*d*log(e*sqrt(x) + d)/e^7 + 23520*(
e*sqrt(x) + d)^6*d^2*log(e*sqrt(x) + d)/e^7 - 47040*(e*sqrt(x) + d)^5*d^3*
log(e*sqrt(x) + d)/e^7 + 58800*(e*sqrt(x) + d)^4*d^4*log(e*sqrt(x) + d)/e^
7 - 47040*(e*sqrt(x) + d)^3*d^5*log(e*sqrt(x) + d)/e^7 + 23520*(e*sqrt(x)
+ d)^2*d^6*log(e*sqrt(x) + d)/e^7 - 6720*(e*sqrt(x) + d)*d^7*log(e*sqrt(x)
 + d)/e^7 - 105*(e*sqrt(x) + d)^8/e^7 + 960*(e*sqrt(x) + d)^7*d/e^7 - 3920
*(e*sqrt(x) + d)^6*d^2/e^7 + 9408*(e*sqrt(x) + d)^5*d^3/e^7 - 14700*(e*sqr
t(x) + d)^4*d^4/e^7 + 15680*(e*sqrt(x) + d)^3*d^5/e^7 - 11760*(e*sqrt(x) +
 d)^2*d^6/e^7 + 6720*(e*sqrt(x) + d)*d^7/e^7)*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{a\,x^4}{4}-\frac{b\,n\,x^4}{32}+\frac{b\,x^4\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{4}+\frac{b\,d\,n\,x^{7/2}}{28\,e}-\frac{b\,d^6\,n\,x}{8\,e^6}-\frac{b\,d^8\,n\,\ln\left(d+e\,\sqrt{x}\right)}{4\,e^8}-\frac{b\,d^2\,n\,x^3}{24\,e^2}-\frac{b\,d^4\,n\,x^2}{16\,e^4}+\frac{b\,d^3\,n\,x^{5/2}}{20\,e^3}+\frac{b\,d^5\,n\,x^{3/2}}{12\,e^5}+\frac{b\,d^7\,n\,\sqrt{x}}{4\,e^7}$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

output
```
(a*x^4)/4 - (b*n*x^4)/32 + (b*x^4*log(c*(d + e*x^(1/2))^n))/4 + (b*d*n*x^(
7/2))/(28*e) - (b*d^6*n*x)/(8*e^6) - (b*d^8*n*log(d + e*x^(1/2)))/(4*e^8)
- (b*d^2*n*x^3)/(24*e^2) - (b*d^4*n*x^2)/(16*e^4) + (b*d^3*n*x^(5/2))/(20*
e^3) + (b*d^5*n*x^(3/2))/(12*e^5) + (b*d^7*n*x^(1/2))/(4*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=\frac{840\sqrt{x}\,b\,d^7en+280\sqrt{x}\,b\,d^5e^3nx+168\sqrt{x}\,b\,d^3e^5n\,x^2+120\sqrt{x}\,bd\,e^7n\,x^3-840\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,d^8+}{336}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x)
```

output
```
(840*sqrt(x)*b*d**7*e*n + 280*sqrt(x)*b*d**5*e**3*n*x + 168*sqrt(x)*b*d**3
*e**5*n*x**2 + 120*sqrt(x)*b*d*e**7*n*x**3 - 840*log((sqrt(x)*e + d)**n*c)
*b*d**8 + 840*log((sqrt(x)*e + d)**n*c)*b*e**8*x**4 + 840*a*e**8*x**4 - 42
0*b*d**6*e**2*n*x - 210*b*d**4*e**4*n*x**2 - 140*b*d**2*e**6*n*x**3 - 105*
b*e**8*n*x**4)/(3360*e**8)
```

3.401 $\int x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx = \frac{b d^5 n \sqrt{x}}{3 e^5}-\frac{b d^4 n x}{6 e^4}+\frac{b d^3 n x^{3/2}}{9 e^3}-\frac{b d^2 n x^2}{12 e^2}+\frac{b d n x^{5/2}}{15 e}-\frac{1}{18} b n x^3-\frac{b d^6 n \log \left(d+e \sqrt{x}\right)}{3 e^6}+\frac{1}{3} x^3 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)$$

output
```
1/3*b*d^5*n*x^(1/2)/e^5-1/6*b*d^4*n*x/e^4+1/9*b*d^3*n*x^(3/2)/e^3-1/12*b*d
^2*n*x^2/e^2+1/15*b*d*n*x^(5/2)/e-1/18*b*n*x^3-1/3*b*d^6*n*ln(d+e*x^(1/2))
/e^6+1/3*x^3*(a+b*ln(c*(d+e*x^(1/2))^n))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.96

$$\int x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx = \frac{a x^3}{3}-\frac{1}{6} b e n \left(-\frac{2 d^5 \sqrt{x}}{e^6}+\frac{d^4 x}{e^5}-\frac{2 d^3 x^{3/2}}{3 e^4}+\frac{d^2 x^2}{2 e^3}-\frac{2 d x^{5/2}}{5 e^2}+\frac{x^3}{3 e}+\frac{2 d^6 \log \left(d+e \sqrt{x}\right)}{e^7}\right)+\frac{1}{3} b x^3 \log \left(c \left(d+e \sqrt{x}\right)^n\right)$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

output
```
(a*x^3)/3 - (b*e*n*((-2*d^5*Sqrt[x])/e^6 + (d^4*x)/e^5 - (2*d^3*x^(3/2))/(
3*e^4) + (d^2*x^2)/(2*e^3) - (2*d*x^(5/2))/(5*e^2) + x^3/(3*e) + (2*d^6*Lo
g[d + e*Sqrt[x]])/e^7))/6 + (b*x^3*Log[c*(d + e*Sqrt[x])^n])/3
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx$$

$$\Big\downarrow \text{2904}$$

$$2 \int x^{5/2} \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) d\sqrt{x}$$

$$\Big\downarrow \text{2842}$$

$$2 \left(\frac{1}{6} x^3 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)-\frac{1}{6} b e n \int \frac{x^3}{d+e \sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{49}$$

$$2 \left(\frac{1}{6} x^3 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)-\frac{1}{6} b e n \int \left(\frac{d^6}{e^6 \left(d+e \sqrt{x}\right)}-\frac{d^5}{e^6}+\frac{\sqrt{x} d^4}{e^5}-\frac{x d^3}{e^4}+\frac{x^{3/2} d^2}{e^3}-\frac{x^2 d}{e^2}+\frac{x^{5/2}}{e}\right)d\sqrt{x}\right)$$

$$\Big\downarrow \text{2009}$$

$$2 \left(\frac{1}{6} x^3 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)-\frac{1}{6} b e n \left(\frac{d^6 \log \left(d+e \sqrt{x}\right)}{e^7}-\frac{d^5 \sqrt{x}}{e^6}+\frac{d^4 x}{2 e^5}-\frac{d^3 x^{3/2}}{3 e^4}+\frac{d^2 x^2}{4 e^3}-\frac{d x^{5/2}}{5 e^2}+\frac{x^3}{6 e}\right)\right)$$

input
```
Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

output
```
2*(-1/6*(b*e*n*(-((d^5*Sqrt[x])/e^6) + (d^4*x)/(2*e^5) - (d^3*x^(3/2))/(3*
e^4) + (d^2*x^2)/(4*e^3) - (d*x^(5/2))/(5*e^2) + x^3/(6*e) + (d^6*Log[d +
e*Sqrt[x]])/e^7)) + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = \frac{60\,be^6x^3\log\left(c\right)-15\,bd^2e^4nx^2-30\,bd^4e^2nx-10\,(be^6n-6\,ae^6)x^3+60\,(be^6nx^3-bd^6n)\log\left(e\sqrt{x}+d\right)+}{180\,e^6}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

output
```
1/180*(60*b*e^6*x^3*log(c) - 15*b*d^2*e^4*n*x^2 - 30*b*d^4*e^2*n*x - 10*(b
*e^6*n - 6*a*e^6)*x^3 + 60*(b*e^6*n*x^3 - b*d^6*n)*log(e*sqrt(x) + d) + 4*
(3*b*d*e^5*n*x^2 + 5*b*d^3*e^3*n*x + 15*b*d^5*e*n)*sqrt(x))/e^6
```

Sympy [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{ax^3}{3}+b\left(-\frac{en\left(\frac{2d^6\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^6}-\frac{2d^5\sqrt{x}}{e^6}+\frac{d^4x}{e^5}-\frac{2d^3x^{\frac{3}{2}}}{3e^4}+\frac{d^2x^2}{2e^3}-\frac{2dx^{\frac{5}{2}}}{5e^2}+\frac{x^3}{3e}\right)}{6}+\frac{x^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3}\right)$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

output
```
a*x**3/3 + b*(-e*n*(2*d**6*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqr
t(x))/e, True))/e**6 - 2*d**5*sqrt(x)/e**6 + d**4*x/e**5 - 2*d**3*x**(3/2)
/(3*e**4) + d**2*x**2/(2*e**3) - 2*d*x**(5/2)/(5*e**2) + x**3/(3*e))/6 + x
**3*log(c*(d + e*sqrt(x))**n)/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = \frac{1}{3}\,bx^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{3}\,ax^3 - \frac{1}{180}\,ben\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{10\,e^5x^3-12\,de^4x^{\frac{5}{2}}+15\,d^2e^3x^2-20\,d^3e^2x^{\frac{3}{2}}+30\,d^4ex-60\,d^5\sqrt{x}}{e^6}\right)$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

output
```
1/3*b*x^3*log((e*sqrt(x) + d)^n*c) + 1/3*a*x^3 - 1/180*b*e*n*(60*d^6*log(e
*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d
^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(108) = 216$.

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.97

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = \frac{60\,bex^3\log\left(c\right)+60\,aex^3+\left(\frac{60\,\left(e\sqrt{x}+d\right)^6\log\left(e\sqrt{x}+d\right)}{e^5}-\frac{360\,\left(e\sqrt{x}+d\right)^5d\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{900\,\left(e\sqrt{x}+d\right)^4d^2\log\left(e\sqrt{x}+d\right)}{e^5}-120}{}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

output
```
1/180*(60*b*e*x^3*log(c) + 60*a*e*x^3 + (60*(e*sqrt(x) + d)^6*log(e*sqrt(x
) + d)/e^5 - 360*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 + 900*(e*sqrt(
x) + d)^4*d^2*log(e*sqrt(x) + d)/e^5 - 1200*(e*sqrt(x) + d)^3*d^3*log(e*sq
rt(x) + d)/e^5 + 900*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 - 360*(e
*sqrt(x) + d)*d^5*log(e*sqrt(x) + d)/e^5 - 10*(e*sqrt(x) + d)^6/e^5 + 72*(
e*sqrt(x) + d)^5*d/e^5 - 225*(e*sqrt(x) + d)^4*d^2/e^5 + 400*(e*sqrt(x) +
d)^3*d^3/e^5 - 450*(e*sqrt(x) + d)^2*d^4/e^5 + 360*(e*sqrt(x) + d)*d^5/e^5
)*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{a\,x^3}{3}-\frac{b\,n\,x^3}{18}+\frac{b\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3}+\frac{b\,d\,n\,x^{5/2}}{15\,e}-\frac{b\,d^4\,n\,x}{6\,e^4}-\frac{b\,d^6\,n\,\ln\left(d+e\,\sqrt{x}\right)}{3\,e^6}-\frac{b\,d^2\,n\,x^2}{12\,e^2}+\frac{b\,d^3\,n\,x^{3/2}}{9\,e^3}+\frac{b\,d^5\,n\,\sqrt{x}}{3\,e^5}$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

output
```
(a*x^3)/3 - (b*n*x^3)/18 + (b*x^3*log(c*(d + e*x^(1/2))^n))/3 + (b*d*n*x^(
5/2))/(15*e) - (b*d^4*n*x)/(6*e^4) - (b*d^6*n*log(d + e*x^(1/2)))/(3*e^6)
- (b*d^2*n*x^2)/(12*e^2) + (b*d^3*n*x^(3/2))/(9*e^3) + (b*d^5*n*x^(1/2))/(
3*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{60\sqrt{x}\,b\,d^5en+20\sqrt{x}\,b\,d^3e^3nx+12\sqrt{x}\,bd\,e^5n\,x^2-60\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,d^6+60\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,e}{180e^6}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x)
```

output
```
(60*sqrt(x)*b*d**5*e*n + 20*sqrt(x)*b*d**3*e**3*n*x + 12*sqrt(x)*b*d*e**5*
n*x**2 - 60*log((sqrt(x)*e + d)**n*c)*b*d**6 + 60*log((sqrt(x)*e + d)**n*c
)*b*e**6*x**3 + 60*a*e**6*x**3 - 30*b*d**4*e**2*n*x - 15*b*d**2*e**4*n*x**
2 - 10*b*e**6*n*x**3)/(180*e**6)
```

3.402 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

output
```
1/2*b*d^3*n*x^(1/2)/e^3-1/4*b*d^2*n*x/e^2+1/6*b*d*n*x^(3/2)/e-1/8*b*n*x^2-
1/2*b*d^4*n*ln(d+e*x^(1/2))/e^4+1/2*x^2*(a+b*ln(c*(d+e*x^(1/2))^n))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{ax^2}{2}-\frac{1}{4}ben\left(-\frac{2d^3\sqrt{x}}{e^4}+\frac{d^2x}{e^3}-\frac{2dx^{3/2}}{3e^2}+\frac{x^2}{2e}+\frac{2d^4\log\left(d+e\sqrt{x}\right)}{e^5}\right)+\frac{1}{2}bx^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

input
```
Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

output
```
(a*x^2)/2 - (b*e*n*((-2*d^3*Sqrt[x])/e^4 + (d^2*x)/e^3 - (2*d*x^(3/2))/(3*
e^2) + x^2/(2*e) + (2*d^4*Log[d + e*Sqrt[x]])/e^5))/4 + (b*x^2*Log[c*(d +
e*Sqrt[x])^n])/2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx$$

$$\Big\downarrow \text{ 2904}$$

$$2 \int x^{3/2} \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2842}$$

$$2 \left(\frac{1}{4} x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)-\frac{1}{4} b e n \int \frac{x^2}{d+e \sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 49}$$

$$2 \left(\frac{1}{4} x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)-\frac{1}{4} b e n \int \left(\frac{d^4}{e^4 \left(d+e \sqrt{x}\right)}-\frac{d^3}{e^4}+\frac{\sqrt{x} d^2}{e^3}-\frac{x d}{e^2}+\frac{x^{3/2}}{e}\right)d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2 \left(\frac{1}{4} x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)-\frac{1}{4} b e n \left(\frac{d^4 \log \left(d+e \sqrt{x}\right)}{e^5}-\frac{d^3 \sqrt{x}}{e^4}+\frac{d^2 x}{2 e^3}-\frac{d x^{3/2}}{3 e^2}+\frac{x^2}{4 e}\right)\right)$$

input
```
Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

output
```
2*(-1/4*(b*e*n*(-((d^3*Sqrt[x])/e^4) + (d^2*x)/(2*e^3) - (d*x^(3/2))/(3*e^
2) + x^2/(4*e) + (d^4*Log[d + e*Sqrt[x]])/e^5)) + (x^2*(a + b*Log[c*(d + e
*Sqrt[x])^n]))/4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=\frac{12\,be^4x^2\log\left(c\right)-6\,bd^2e^2nx-3\,(be^4n-4\,ae^4)x^2+12\,(be^4nx^2-bd^4n)\log\left(e\sqrt{x}+d\right)+4\,(bde^3nx+3\,bd}{24\,e^4}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

output
```
1/24*(12*b*e^4*x^2*log(c) - 6*b*d^2*e^2*n*x - 3*(b*e^4*n - 4*a*e^4)*x^2 +
12*(b*e^4*n*x^2 - b*d^4*n)*log(e*sqrt(x) + d) + 4*(b*d*e^3*n*x + 3*b*d^3*e
*n)*sqrt(x))/e^4
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.98

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=\frac{ax^2}{2}+b\left(-\frac{en\left(\frac{2d^4\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log(d+e\sqrt{x})}{e} & \text{otherwise}\end{cases}\right)}{e^4}-\frac{2d^3\sqrt{x}}{e^4}+\frac{d^2x}{e^3}-\frac{2dx^{\frac{3}{2}}}{3e^2}+\frac{x^2}{2e}\right)}{4}+\frac{x^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2}\right)$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

output
```
a*x**2/2 + b*(-e*n*(2*d**4*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqr
t(x))/e, True))/e**4 - 2*d**3*sqrt(x)/e**4 + d**2*x/e**3 - 2*d*x**(3/2)/(3
*e**2) + x**2/(2*e))/4 + x**2*log(c*(d + e*sqrt(x))**n)/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=-\frac{1}{24}\,ben\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)+\frac{1}{2}\,bx^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

output
```
-1/24*b*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2)
+ 6*d^2*e*x - 12*d^3*sqrt(x))/e^4) + 1/2*b*x^2*log((e*sqrt(x) + d)^n*c) +
1/2*a*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(82) = 164.

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.76

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=\frac{12\,bex^2\log\left(c\right)+12\,aex^2+\left(\frac{12\,\left(e\sqrt{x}+d\right)^4\log\left(e\sqrt{x}+d\right)}{e^3}-\frac{48\,\left(e\sqrt{x}+d\right)^3d\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{72\,\left(e\sqrt{x}+d\right)^2d^2\log\left(e\sqrt{x}+d\right)}{e^3}-\frac{48\,(e\sqrt{}}{}}{24\,e}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

output
```
1/24*(12*b*e*x^2*log(c) + 12*a*e*x^2 + (12*(e*sqrt(x) + d)^4*log(e*sqrt(x)
 + d)/e^3 - 48*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 + 72*(e*sqrt(x)
+ d)^2*d^2*log(e*sqrt(x) + d)/e^3 - 48*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) +
 d)/e^3 - 3*(e*sqrt(x) + d)^4/e^3 + 16*(e*sqrt(x) + d)^3*d/e^3 - 36*(e*sqr
t(x) + d)^2*d^2/e^3 + 48*(e*sqrt(x) + d)*d^3/e^3)*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{8}+\frac{b\,x^2\,\ln\left(c\left(d+e\,\sqrt{x}\right)^n\right)}{2}-\frac{b\,d^2\,n\,x}{4\,e^2}+\frac{b\,d\,n\,x^{3/2}}{6\,e}-\frac{b\,d^4\,n\,\ln\left(d+e\,\sqrt{x}\right)}{2\,e^4}+\frac{b\,d^3\,n\,\sqrt{x}}{2\,e^3}$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

output
```
(a*x^2)/2 - (b*n*x^2)/8 + (b*x^2*log(c*(d + e*x^(1/2))^n))/2 - (b*d^2*n*x)
/(4*e^2) + (b*d*n*x^(3/2))/(6*e) - (b*d^4*n*log(d + e*x^(1/2)))/(2*e^4) +
(b*d^3*n*x^(1/2))/(2*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{12\sqrt{x}\,b\,d^3en+4\sqrt{x}\,bd\,e^3nx-12\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,d^4+12\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,e^4x^2+12a\,e^4x^2-6b}{24e^4}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/2))^n)),x)
```

output
```
(12*sqrt(x)*b*d**3*e*n + 4*sqrt(x)*b*d*e**3*n*x - 12*log((sqrt(x)*e + d)**
n*c)*b*d**4 + 12*log((sqrt(x)*e + d)**n*c)*b*e**4*x**2 + 12*a*e**4*x**2 -
6*b*d**2*e**2*n*x - 3*b*e**4*n*x**2)/(24*e**4)
```

3.403 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

output
```
b*d*n*x^(1/2)/e+a*x-1/2*b*n*x-b*d^2*n*ln(d+e*x^(1/2))/e^2+b*x*ln(c*(d+e*x^
(1/2))^n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

input
```
Integrate[a + b*Log[c*(d + e*Sqrt[x])^n],x]
```

output
```
(b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b
*x*Log[c*(d + e*Sqrt[x])^n]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}-\frac{bnx}{2}$$

input
```
Int[a + b*Log[c*(d + e*Sqrt[x])^n],x]
```

output
```
(b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b
*x*Log[c*(d + e*Sqrt[x])^n]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{b\,d^2n\ln\left(d+e\sqrt{x}\right)}{e^2}+bx\ln\left(c\left(d+e\sqrt{x}\right)^n\right)$	53
parts	$\frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{b\,d^2n\ln\left(d+e\sqrt{x}\right)}{e^2}+bx\ln\left(c\left(d+e\sqrt{x}\right)^n\right)$	53
derivativedivides	$ax+bx\ln\left(c\,\mathrm{e}^{n\ln\left(d+e\sqrt{x}\right)}\right)-\frac{bnx}{2}-\frac{b\,d^2n\ln\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}$	55

input
```
int(a+b*ln(c*(d+e*x^(1/2))^n),x,method=_RETURNVERBOSE)
```

output
```
b*d*n*x^(1/2)/e+a*x-1/2*b*n*x-b*d^2*n*ln(d+e*x^(1/2))/e^2+b*x*ln(c*(d+e*x^
(1/2))^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = \frac{2\,be^2x\log\left(c\right)+2\,bden\sqrt{x}-\left(be^2n-2\,ae^2\right)x+2\,\left(be^2nx-bd^2n\right)\log\left(e\sqrt{x}+d\right)}{2\,e^2}$$

input
```
integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="fricas")
```

output
```
1/2*(2*b*e^2*x*log(c) + 2*b*d*e*n*sqrt(x) - (b*e^2*n - 2*a*e^2)*x + 2*(b*e
^2*n*x - b*d^2*n)*log(e*sqrt(x) + d))/e^2
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = ax+b\left(-\frac{en\left(\frac{2d^2\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^2}-\frac{2d\sqrt{x}}{e^2}+\frac{x}{e}\right)}{2}+x\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

input
```
integrate(a+b*ln(c*(d+e*x**(1/2))**n),x)
```

output
```
a*x + b*(-e*n*(2*d**2*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt(x))
/e, True))/e**2 - 2*d*sqrt(x)/e**2 + x/e)/2 + x*log(c*(d + e*sqrt(x))**n))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = -\frac{1}{2}\left(en\left(\frac{2\,d^2\log \left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)-2\,x\log \left(\left(e\sqrt{x}+d\right)^nc\right)\right)b+ax$$

input
```
integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="maxima")
```

output
```
-1/2*(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*l
og((e*sqrt(x) + d)^n*c))*b + a*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.70

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = ax + \frac{b\left(\frac{\left(2\left(e\sqrt{x}+d\right)^2\log\left(e\sqrt{x}+d\right)-4\left(e\sqrt{x}+d\right)d\log\left(e\sqrt{x}+d\right)-\left(e\sqrt{x}+d\right)^2+4\left(e\sqrt{x}+d\right)d\right)n}{e}+\frac{2\left(\left(e\sqrt{x}+d\right)^2-2\left(e\sqrt{x}+d\right)d\right)\log(c)}{e}\right)}{2\,e}$$

input
```
integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="giac")
```

output
```
a*x + 1/2*b*((2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(x) + d)*d
*log(e*sqrt(x) + d) - (e*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*n/e + 2*((e
*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*log(c)/e)/e
```

Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = a\,x+b\,x\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)-\frac{b\,n\,\left(e^2\,x+2\,d^2\,\ln\left(d+e\,\sqrt{x}\right)-2\,d\,e\,\sqrt{x}\right)}{2\,e^2}$$

input
```
int(a + b*log(c*(d + e*x^(1/2))^n),x)
```

output
```
a*x + b*x*log(c*(d + e*x^(1/2))^n) - (b*n*(e^2*x + 2*d^2*log(d + e*x^(1/2)
) - 2*d*e*x^(1/2)))/(2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = \frac{2\sqrt{x}\,bden-2\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,d^2+2\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,e^2x+2a\,e^2x-b\,e^2nx}{2e^2}$$

input
```
int(a+b*log(c*(d+e*x^(1/2))^n),x)
```

output
```
(2*sqrt(x)*b*d*e*n - 2*log((sqrt(x)*e + d)**n*c)*b*d**2 + 2*log((sqrt(x)*e
 + d)**n*c)*b*e**2*x + 2*a*e**2*x - b*e**2*n*x)/(2*e**2)
```

3.404 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx = 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right) + 2bn\,\text{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)$$

output
```
2*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(-e*x^(1/2)/d)+2*b*n*polylog(2,1+e*x^(1/2)
/d)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx = 2b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\log\left(-\frac{e\sqrt{x}}{d}\right) + a\log(x) + 2bn\,\text{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

output
```
2*b*Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)] + a*Log[x] + 2*b*n*Poly
Log[2, (d + e*Sqrt[x])/d]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$2\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{2841}$$

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-ben\int \frac{\log\left(-\frac{e\sqrt{x}}{d}\right)}{d+e\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{2752}$$

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{\sqrt{x}e}{d}+1\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

output
```
2*((a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)] + b*n*PolyLog[2,
 1 + (e*Sqrt[x])/d])
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))/x,x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x}\,dx=\int \frac{b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="fricas")
```

output
```
integral((b*log((e*sqrt(x) + d)^n*c) + a)/x, x)
```

Sympy [F]

$$\int \frac{a + b\log\left(c\left(d + e\sqrt{x}\right)^n\right)}{x}\,dx = \int \frac{a + b\log\left(c\left(d + e\sqrt{x}\right)^n\right)}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(44) = 88.

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.12

$$\begin{aligned}&\int \frac{a + b\log\left(c\left(d + e\sqrt{x}\right)^n\right)}{x}\,dx\\&= -2\left(\log\left(\frac{e\sqrt{x}}{d} + 1\right)\log\left(\sqrt{x}\right) + \mathrm{Li}_2\left(-\frac{e\sqrt{x}}{d}\right)\right)bn + \frac{2\left(ben\sqrt{x}\log\left(\sqrt{x}\right) - ben\sqrt{x}\right)}{d}\\&\quad + \frac{bd\log\left(\left(e\sqrt{x} + d\right)^n\right)\log\left(x\right) + \left(bd\log\left(c\right) + ad\right)\log\left(x\right) - \frac{benx\log(x) - 2\,benx}{\sqrt{x}}}{d}\end{aligned}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="maxima")
```

output
```
-2*(log(e*sqrt(x)/d + 1)*log(sqrt(x)) + dilog(-e*sqrt(x)/d))*b*n + 2*(b*e*
n*sqrt(x)*log(sqrt(x)) - b*e*n*sqrt(x))/d + (b*d*log((e*sqrt(x) + d)^n)*lo
g(x) + (b*d*log(c) + a*d)*log(x) - (b*e*n*x*log(x) - 2*b*e*n*x)/sqrt(x))/d
```

Giac [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^n*c) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx=\int \frac{a+b\,\ln\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^n))/x, x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx$$
$$=\frac{\left(\int \frac{\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)}{-e^2x^2+d^2x}dx\right)b\,d^2n-\left(\int \frac{\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)}{-e^2x^2+d^2x}dx\right)bden+\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)^2 b+\log(x)\,an}{n}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))/x,x)
```

output
```
(int(log((sqrt(x)*e + d)**n*c)/(d**2*x - e**2*x**2),x)*b*d**2*n - int((sqr
t(x)*log((sqrt(x)*e + d)**n*c))/(d**2*x - e**2*x**2),x)*b*d*e*n + log((sqr
t(x)*e + d)**n*c)**2*b + log(x)*a*n)/n
```

3.405 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = -\frac{ben}{d\sqrt{x}}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2} - \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}-\frac{be^2n\log(x)}{2d^2}$$

output
```
-b*e*n/d/x^(1/2)+b*e^2*n*ln(d+e*x^(1/2))/d^2-(a+b*ln(c*(d+e*x^(1/2))^n))/x
-1/2*b*e^2*n*ln(x)/d^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = -\frac{a}{x}-\frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + \frac{1}{2}ben\left(-\frac{2}{d\sqrt{x}}+\frac{2e\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{e\log(x)}{d^2}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
```

output
```
-(a/x) - (b*Log[c*(d + e*Sqrt[x])^n])/x + (b*e*n*(-2/(d*Sqrt[x]) + (2*e*Lo
g[d + e*Sqrt[x]])/d^2 - (e*Log[x])/d^2))/2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2842}$$

$$2\left(\frac{1}{2}ben\int \frac{1}{\left(d+e\sqrt{x}\right)x}d\sqrt{x}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x}\right)$$

$$\Big\downarrow \text{ 54}$$

$$2\left(\frac{1}{2}ben\int \left(\frac{e^2}{d^2\left(d+e\sqrt{x}\right)}-\frac{e}{d^2\sqrt{x}}+\frac{1}{dx}\right)d\sqrt{x}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{1}{2}ben\left(\frac{e\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{e\log\left(\sqrt{x}\right)}{d^2}-\frac{1}{d\sqrt{x}}\right)-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x}\right)$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
```

output
```
2*(-1/2*(a + b*Log[c*(d + e*Sqrt[x])^n])/x + (b*e*n*(-(1/(d*Sqrt[x])) + (e
*Log[d + e*Sqrt[x]])/d^2 - (e*Log[Sqrt[x]])/d^2))/2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a + b\ln\left(c\left(d + e\sqrt{x}\right)^{n}\right)}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = -\frac{be^2nx\log\left(\sqrt{x}\right)+bden\sqrt{x}+bd^2\log\left(c\right)+ad^2-\left(be^2nx-bd^2n\right)\log\left(e\sqrt{x}+d\right)}{d^2x}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="fricas")
```

output
```
-(b*e^2*n*x*log(sqrt(x)) + b*d*e*n*sqrt(x) + b*d^2*log(c) + a*d^2 - (b*e^2
*n*x - b*d^2*n)*log(e*sqrt(x) + d))/(d^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(65) = 130.

Time = 13.69 (sec) , antiderivative size = 442, normalized size of antiderivative = 6.31

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = \begin{cases} -\frac{a+b\log\left(0^nc\right)}{x} \\ -\frac{a}{x}-\frac{bn}{2x}-\frac{b\log\left(c\left(e\sqrt{x}\right)^n\right)}{x} \\ -\frac{a+b\log\left(0^nc\right)}{x} \\ -\frac{2ad^3\sqrt{x}}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2ad^2ex}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2bd^3\sqrt{x}\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2bd^2enx}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{2bd^2ex\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2d^3x^{\frac{3}{2}}+2d^2ex^2}-\frac{bde^2nx^{\frac{3}{2}}\log\left(x\right)}{2d^3x^{\frac{3}{2}}+2d^2ex^2} \end{cases}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**2,x)
```

output
```
Piecewise((-(a + b*log(0**n*c))/x, Eq(d, 0) & Eq(e, 0)), (-a/x - b*n/(2*x)
 - b*log(c*(e*sqrt(x))**n)/x, Eq(d, 0)), (-(a + b*log(0**n*c))/x, Eq(d, -e
*sqrt(x))), (-2*a*d**3*sqrt(x)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*a*d**
2*e*x/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d**3*sqrt(x)*log(c*(d + e*sq
rt(x))**n)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d**2*e*n*x/(2*d**3*x**(
3/2) + 2*d**2*e*x**2) - 2*b*d**2*e*x*log(c*(d + e*sqrt(x))**n)/(2*d**3*x**
(3/2) + 2*d**2*e*x**2) - b*d*e**2*n*x**(3/2)*log(x)/(2*d**3*x**(3/2) + 2*d
**2*e*x**2) - 2*b*d*e**2*n*x**(3/2)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) + 2*
b*d*e**2*x**(3/2)*log(c*(d + e*sqrt(x))**n)/(2*d**3*x**(3/2) + 2*d**2*e*x*
*2) - b*e**3*n*x**2*log(x)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) + 2*b*e**3*x*
*2*log(c*(d + e*sqrt(x))**n)/(2*d**3*x**(3/2) + 2*d**2*e*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = \frac{1}{2}ben\left(\frac{2\,e\log\left(e\sqrt{x}+d\right)}{d^2}-\frac{e\log\left(x\right)}{d^2}-\frac{2}{d\sqrt{x}}\right) - \frac{b\log\left(\left(e\sqrt{x}+d\right)^nc\right)}{x}-\frac{a}{x}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="maxima")
```

output
```
1/2*b*e*n*(2*e*log(e*sqrt(x) + d)/d^2 - e*log(x)/d^2 - 2/(d*sqrt(x))) - b*
log((e*sqrt(x) + d)^n*c)/x - a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(62) = 124$.

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = -\frac{\frac{be^3n\log\left(e\sqrt{x}+d\right)}{\left(e\sqrt{x}+d\right)^2-2\left(e\sqrt{x}+d\right)d+d^2}-\frac{be^3n\log\left(e\sqrt{x}+d\right)}{d^2}+\frac{be^3n\log\left(e\sqrt{x}\right)}{d^2}+\frac{\left(e\sqrt{x}+d\right)be^3n-bde^3n+bde^3\log\left(c\right)+ade^3}{\left(e\sqrt{x}+d\right)^2d-2\left(e\sqrt{x}+d\right)d^2+d^3}}{e}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="giac")
```

output
```
-(b*e^3*n*log(e*sqrt(x) + d)/((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d + d^
2) - b*e^3*n*log(e*sqrt(x) + d)/d^2 + b*e^3*n*log(e*sqrt(x))/d^2 + ((e*sqr
t(x) + d)*b*e^3*n - b*d*e^3*n + b*d*e^3*log(c) + a*d*e^3)/((e*sqrt(x) + d)
^2*d - 2*(e*sqrt(x) + d)*d^2 + d^3))/e
```

Mupad [B] (verification not implemented)

Time = 15.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = \frac{2\,b\,e^2\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{d^2} - \frac{b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{x} - \frac{b\,e\,n}{d\,\sqrt{x}} - \frac{a}{x}$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))/x^2,x)
```

output
```
(2*b*e^2*n*atanh((2*e*x^(1/2))/d + 1))/d^2 - (b*log(c*(d + e*x^(1/2))^n))/
x - (b*e*n)/(d*x^(1/2)) - a/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = \frac{-\sqrt{x}\,bden-\log\left(\sqrt{x}\right)b\,e^2nx-\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,d^2+\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,e^2x-a\,d^2}{d^2x}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x)
```

output
```
( - sqrt(x)*b*d*e*n - log(sqrt(x))*b*e**2*n*x - log((sqrt(x)*e + d)**n*c)*
b*d**2 + log((sqrt(x)*e + d)**n*c)*b*e**2*x - a*d**2)/(d**2*x)
```

3.406 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = -\frac{ben}{6dx^{3/2}}+\frac{be^2n}{4d^2x}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}-\frac{be^4n\log(x)}{4d^4}$$

output
```
-1/6*b*e*n/d/x^(3/2)+1/4*b*e^2*n/d^2/x-1/2*b*e^3*n/d^3/x^(1/2)+1/2*b*e^4*n
*ln(d+e*x^(1/2))/d^4-1/2*(a+b*ln(c*(d+e*x^(1/2))^n))/x^2-1/4*b*e^4*n*ln(x)
/d^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = -\frac{a}{2x^2}-\frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}+\frac{1}{4}ben\left(-\frac{2}{3dx^{3/2}}+\frac{e}{d^2x} - \frac{2e^2}{d^3\sqrt{x}}+\frac{2e^3\log\left(d+e\sqrt{x}\right)}{d^4}-\frac{e^3\log(x)}{d^4}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*Log[c*(d + e*Sqrt[x])^n])/(2*x^2) + (b*e*n*(-2/(3*d*x^(3/2
)) + e/(d^2*x) - (2*e^2)/(d^3*Sqrt[x]) + (2*e^3*Log[d + e*Sqrt[x]])/d^4 -
(e^3*Log[x])/d^4))/4
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2842}$$

$$2\left(\frac{1}{4}ben\int \frac{1}{\left(d+e\sqrt{x}\right)x^2}d\sqrt{x}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{4x^2}\right)$$

$$\Big\downarrow \text{ 54}$$

$$2\left(\frac{1}{4}ben\int \left(\frac{e^4}{d^4\left(d+e\sqrt{x}\right)}-\frac{e^3}{d^4\sqrt{x}}+\frac{e^2}{d^3x}-\frac{e}{d^2x^{3/2}}+\frac{1}{dx^2}\right)d\sqrt{x}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{4x^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{1}{4}ben\left(\frac{e^3\log\left(d+e\sqrt{x}\right)}{d^4}-\frac{e^3\log\left(\sqrt{x}\right)}{d^4}-\frac{e^2}{d^3\sqrt{x}}+\frac{e}{2d^2x}-\frac{1}{3dx^{3/2}}\right)-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{4x^2}\right)$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
```

output
```
2*(-1/4*(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2 + (b*e*n*(-1/3*1/(d*x^(3/2))
+ e/(2*d^2*x) - e^2/(d^3*Sqrt[x]) + (e^3*Log[d + e*Sqrt[x]])/d^4 - (e^3*Lo
g[Sqrt[x]])/d^4))/4)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = -\frac{6\,be^4nx^2\log\left(\sqrt{x}\right)-3\,bd^2e^2nx+6\,bd^4\log\left(c\right)+6\,ad^4-6\,\left(be^4nx^2-bd^4n\right)\log\left(e\sqrt{x}+d\right)+2\,\left(3\,bde^3nx\right.}{12\,d^4x^2}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="fricas")
```

output
```
-1/12*(6*b*e^4*n*x^2*log(sqrt(x)) - 3*b*d^2*e^2*n*x + 6*b*d^4*log(c) + 6*a
*d^4 - 6*(b*e^4*n*x^2 - b*d^4*n)*log(e*sqrt(x) + d) + 2*(3*b*d*e^3*n*x + b
*d^3*e*n)*sqrt(x))/(d^4*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(104) = 208.

Time = 101.14 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.88

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = \begin{cases} -\frac{a+b\log\left(0^nc\right)}{2x^2} \\ -\frac{a}{2x^2}-\frac{bn}{8x^2}-\frac{b\log\left(c\left(e\sqrt{x}\right)^n\right)}{2x^2} \\ -\frac{a+b\log\left(0^nc\right)}{2x^2} \\ -\frac{6ad^5\sqrt{x}}{12d^5x^{\frac{5}{2}}+12d^4ex^3}-\frac{6ad^4ex}{12d^5x^{\frac{5}{2}}+12d^4ex^3}-\frac{6bd^5\sqrt{x}\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{12d^5x^{\frac{5}{2}}+12d^4ex^3}-\frac{2bd^4enx}{12d^5x^{\frac{5}{2}}+12d^4ex^3}-\frac{6bd^4ex\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{12d^5x^{\frac{5}{2}}+12d^4ex^3}+\frac{bd^3e^{2}}{12d^5x^{\frac{5}{2}}+}\end{cases}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**3,x)
```

output
```
Piecewise((-(a + b*log(0**n*c))/(2*x**2), Eq(d, 0) & Eq(e, 0)), (-a/(2*x**
2) - b*n/(8*x**2) - b*log(c*(e*sqrt(x))**n)/(2*x**2), Eq(d, 0)), (-(a + b*
log(0**n*c))/(2*x**2), Eq(d, -e*sqrt(x))), (-6*a*d**5*sqrt(x)/(12*d**5*x**
(5/2) + 12*d**4*e*x**3) - 6*a*d**4*e*x/(12*d**5*x**(5/2) + 12*d**4*e*x**3)
 - 6*b*d**5*sqrt(x)*log(c*(d + e*sqrt(x))**n)/(12*d**5*x**(5/2) + 12*d**4*
e*x**3) - 2*b*d**4*e*n*x/(12*d**5*x**(5/2) + 12*d**4*e*x**3) - 6*b*d**4*e*
x*log(c*(d + e*sqrt(x))**n)/(12*d**5*x**(5/2) + 12*d**4*e*x**3) + b*d**3*e
**2*n*x**(3/2)/(12*d**5*x**(5/2) + 12*d**4*e*x**3) - 3*b*d**2*e**3*n*x**2/
(12*d**5*x**(5/2) + 12*d**4*e*x**3) - 3*b*d*e**4*n*x**(5/2)*log(x)/(12*d**
5*x**(5/2) + 12*d**4*e*x**3) - 6*b*d*e**4*n*x**(5/2)/(12*d**5*x**(5/2) + 1
2*d**4*e*x**3) + 6*b*d*e**4*x**(5/2)*log(c*(d + e*sqrt(x))**n)/(12*d**5*x*
*(5/2) + 12*d**4*e*x**3) - 3*b*e**5*n*x**3*log(x)/(12*d**5*x**(5/2) + 12*d
**4*e*x**3) + 6*b*e**5*x**3*log(c*(d + e*sqrt(x))**n)/(12*d**5*x**(5/2) +
12*d**4*e*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.77

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx$$
$$=\frac{1}{12}\,ben\left(\frac{6\,e^3\log\left(e\sqrt{x}+d\right)}{d^4}-\frac{3\,e^3\log\left(x\right)}{d^4}-\frac{6\,e^2x-3\,de\sqrt{x}+2\,d^2}{d^3x^{\frac{3}{2}}}\right)$$
$$-\frac{b\log\left(\left(e\sqrt{x}+d\right)^nc\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="maxima")
```

output
```
1/12*b*e*n*(6*e^3*log(e*sqrt(x) + d)/d^4 - 3*e^3*log(x)/d^4 - (6*e^2*x - 3
*d*e*sqrt(x) + 2*d^2)/(d^3*x^(3/2))) - 1/2*b*log((e*sqrt(x) + d)^n*c)/x^2
 - 1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(89) = 178$.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.28

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = -\frac{\frac{6\,be^5n\log\left(e\sqrt{x}+d\right)}{\left(e\sqrt{x}+d\right)^4-4\left(e\sqrt{x}+d\right)^3d+6\left(e\sqrt{x}+d\right)^2d^2-4\left(e\sqrt{x}+d\right)d^3+d^4}-\frac{6\,be^5n\log\left(e\sqrt{x}+d\right)}{d^4}+\frac{6\,be^5n\log\left(e\sqrt{x}\right)}{d^4}+\frac{6\left(e\sqrt{x}+d\right)^3be^5n-21\left(e\sqrt{x}\cdots\right.}{\left(e\sqrt{x}+d\right)^4d^3\cdots}}{12\,e}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="giac")
```

output
```
-1/12*(6*b*e^5*n*log(e*sqrt(x) + d)/((e*sqrt(x) + d)^4 - 4*(e*sqrt(x) + d)
^3*d + 6*(e*sqrt(x) + d)^2*d^2 - 4*(e*sqrt(x) + d)*d^3 + d^4) - 6*b*e^5*n*
log(e*sqrt(x) + d)/d^4 + 6*b*e^5*n*log(e*sqrt(x))/d^4 + (6*(e*sqrt(x) + d)
^3*b*e^5*n - 21*(e*sqrt(x) + d)^2*b*d*e^5*n + 26*(e*sqrt(x) + d)*b*d^2*e^5
*n - 11*b*d^3*e^5*n + 6*b*d^3*e^5*log(c) + 6*a*d^3*e^5)/((e*sqrt(x) + d)^4
*d^3 - 4*(e*sqrt(x) + d)^3*d^4 + 6*(e*sqrt(x) + d)^2*d^5 - 4*(e*sqrt(x) +
d)*d^6 + d^7))/e
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = \frac{b\,e^4\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{d^4} - \frac{\frac{b\,e\,n}{3\,d}+\frac{b\,e^3\,n\,x}{d^3}-\frac{b\,e^2\,n\,\sqrt{x}}{2\,d^2}}{2\,x^{3/2}} - \frac{b\,\ln\left(c\left(d+e\,\sqrt{x}\right)^n\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))/x^3,x)
```

output
```
(b*e^4*n*atanh((2*e*x^(1/2))/d + 1))/d^4 - ((b*e*n)/(3*d) + (b*e^3*n*x)/d^
3 - (b*e^2*n*x^(1/2))/(2*d^2))/(2*x^(3/2)) - (b*log(c*(d + e*x^(1/2))^n))/
(2*x^2) - a/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx$$
$$= \frac{-2\sqrt{x}\,b\,d^3en-6\sqrt{x}\,bd\,e^3nx-6\log\left(\sqrt{x}\right)b\,e^4n\,x^2-6\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,d^4+6\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,e^4}{12d^4x^2}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x)
```

output
```
( - 2*sqrt(x)*b*d**3*e*n - 6*sqrt(x)*b*d*e**3*n*x - 6*log(sqrt(x))*b*e**4*
n*x**2 - 6*log((sqrt(x)*e + d)**n*c)*b*d**4 + 6*log((sqrt(x)*e + d)**n*c)*
b*e**4*x**2 - 6*a*d**4 + 3*b*d**2*e**2*n*x)/(12*d**4*x**2)
```

3.407 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = -\frac{ben}{15dx^{5/2}}+\frac{be^2n}{12d^2x^2}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^4n}{6d^4x}-\frac{be^5n}{3d^5\sqrt{x}}+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}-\frac{be^6n\log(x)}{6d^6}$$

output

```
-1/15*b*e*n/d/x^(5/2)+1/12*b*e^2*n/d^2/x^2-1/9*b*e^3*n/d^3/x^(3/2)+1/6*b*e
^4*n/d^4/x-1/3*b*e^5*n/d^5/x^(1/2)+1/3*b*e^6*n*ln(d+e*x^(1/2))/d^6-1/3*(a+
b*ln(c*(d+e*x^(1/2))^n))/x^3-1/6*b*e^6*n*ln(x)/d^6
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}+\frac{1}{6}ben\left(-\frac{2}{5dx^{5/2}}+\frac{e}{2d^2x^2}-\frac{2e^2}{3d^3x^{3/2}}+\frac{e^3}{d^4x}-\frac{2e^4}{d^5\sqrt{x}}+\frac{2e^5\log\left(d+e\sqrt{x}\right)}{d^6}-\frac{e^5\log(x)}{d^6}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) + (b*e*n*(-2/(5*d*x^(5/2
)) + e/(2*d^2*x^2) - (2*e^2)/(3*d^3*x^(3/2)) + e^3/(d^4*x) - (2*e^4)/(d^5*
Sqrt[x]) + (2*e^5*Log[d + e*Sqrt[x]])/d^6 - (e^5*Log[x])/d^6))/6
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2842}$$

$$2\left(\frac{1}{6}ben\int \frac{1}{\left(d+e\sqrt{x}\right)x^3}d\sqrt{x}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{6x^3}\right)$$

$$\Big\downarrow \text{54}$$

$$2\left(\frac{1}{6}ben\int\left(\frac{e^6}{d^6\left(d+e\sqrt{x}\right)}-\frac{e^5}{d^6\sqrt{x}}+\frac{e^4}{d^5x}-\frac{e^3}{d^4x^{3/2}}+\frac{e^2}{d^3x^2}-\frac{e}{d^2x^{5/2}}+\frac{1}{dx^3}\right)d\sqrt{x}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{6x^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{6}ben\left(\frac{e^5\log\left(d+e\sqrt{x}\right)}{d^6}-\frac{e^5\log\left(\sqrt{x}\right)}{d^6}-\frac{e^4}{d^5\sqrt{x}}+\frac{e^3}{2d^4x}-\frac{e^2}{3d^3x^{3/2}}+\frac{e}{4d^2x^2}-\frac{1}{5dx^{5/2}}\right)-\frac{a+b\log\left(c\left(d+e\right.\right.}{6x^3}\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

output
```
2*(-1/6*(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3 + (b*e*n*(-1/5*1/(d*x^(5/2))
+ e/(4*d^2*x^2) - e^2/(3*d^3*x^(3/2)) + e^3/(2*d^4*x) - e^4/(d^5*Sqrt[x])
+ (e^5*Log[d + e*Sqrt[x]])/d^6 - (e^5*Log[Sqrt[x]])/d^6))/6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))/x^4,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.88

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = -\frac{60\,be^6nx^3\log\left(\sqrt{x}\right) - 30\,bd^2e^4nx^2 - 15\,bd^4e^2nx + 60\,bd^6\log\left(c\right) + 60\,ad^6 - 60\left(be^6nx^3 - bd^6n\right)\log\left(e\right.}{180\,d^6x^3}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="fricas")
```

output
```
-1/180*(60*b*e^6*n*x^3*log(sqrt(x)) - 30*b*d^2*e^4*n*x^2 - 15*b*d^4*e^2*n*
x + 60*b*d^6*log(c) + 60*a*d^6 - 60*(b*e^6*n*x^3 - b*d^6*n)*log(e*sqrt(x)
+ d) + 4*(15*b*d*e^5*n*x^2 + 5*b*d^3*e^3*n*x + 3*b*d^5*e*n)*sqrt(x))/(d^6*
x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$$
$$= \frac{1}{180}ben\left(\frac{60\,e^5\log\left(e\sqrt{x}+d\right)}{d^6} - \frac{30\,e^5\log\left(x\right)}{d^6} - \frac{60\,e^4x^2 - 30\,de^3x^{\frac{3}{2}} + 20\,d^2e^2x - 15\,d^3e\sqrt{x} + 12\,d^4}{d^5x^{\frac{5}{2}}}\right) - \frac{b\log\left(\left(e\sqrt{x}+d\right)^nc\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="maxima")
```

output
```
1/180*b*e*n*(60*e^5*log(e*sqrt(x) + d)/d^6 - 30*e^5*log(x)/d^6 - (60*e^4*x
^2 - 30*d*e^3*x^(3/2) + 20*d^2*e^2*x - 15*d^3*e*sqrt(x) + 12*d^4)/(d^5*x^(
5/2))) - 1/3*b*log((e*sqrt(x) + d)^n*c)/x^3 - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(115) = 230$.

Time = 0.13 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.43

$$\int \frac{a + b \log \left(c \left(d + e\sqrt{x}\right)^n\right)}{x^4} \, dx = -\frac{\frac{60 \, b e^7 n \log\left(e\sqrt{x} + d\right)}{\left(e\sqrt{x}+d\right)^6 - 6 \left(e\sqrt{x}+d\right)^5 d + 15 \left(e\sqrt{x}+d\right)^4 d^2 - 20 \left(e\sqrt{x}+d\right)^3 d^3 + 15 \left(e\sqrt{x}+d\right)^2 d^4 - 6 \left(e\sqrt{x}+d\right) d^5 + d^6} - \frac{60 \, b e^7 n \log\left(e\sqrt{x}+d\right)}{d^6} + \frac{60 \, b e^7 n \log}{d^6}}{}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="giac")
```

output
```
-1/180*(60*b*e^7*n*log(e*sqrt(x) + d)/((e*sqrt(x) + d)^6 - 6*(e*sqrt(x) +
d)^5*d + 15*(e*sqrt(x) + d)^4*d^2 - 20*(e*sqrt(x) + d)^3*d^3 + 15*(e*sqrt(
x) + d)^2*d^4 - 6*(e*sqrt(x) + d)*d^5 + d^6) - 60*b*e^7*n*log(e*sqrt(x) +
d)/d^6 + 60*b*e^7*n*log(e*sqrt(x))/d^6 + (60*(e*sqrt(x) + d)^5*b*e^7*n - 3
30*(e*sqrt(x) + d)^4*b*d*e^7*n + 740*(e*sqrt(x) + d)^3*b*d^2*e^7*n - 855*(
e*sqrt(x) + d)^2*b*d^3*e^7*n + 522*(e*sqrt(x) + d)*b*d^4*e^7*n - 137*b*d^5
*e^7*n + 60*b*d^5*e^7*log(c) + 60*a*d^5*e^7)/((e*sqrt(x) + d)^6*d^5 - 6*(e
*sqrt(x) + d)^5*d^6 + 15*(e*sqrt(x) + d)^4*d^7 - 20*(e*sqrt(x) + d)^3*d^8
+ 15*(e*sqrt(x) + d)^2*d^9 - 6*(e*sqrt(x) + d)*d^10 + d^11))/e
```

Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.78

$$\int \frac{a + b \log \left(c \left(d + e\sqrt{x}\right)^n\right)}{x^4} \, dx = \frac{2 \, b \, e^6 \, n \operatorname{atanh}\left(\frac{2 \, e \sqrt{x}}{d} + 1\right)}{3 \, d^6} - \frac{\frac{b \, e \, n}{5 \, d} + \frac{b \, e^3 \, n \, x}{3 \, d^3} - \frac{b \, e^2 \, n \sqrt{x}}{4 \, d^2} + \frac{b \, e^5 \, n \, x^2}{d^5} - \frac{b \, e^4 \, n \, x^{3/2}}{2 \, d^4}}{3 \, x^{5/2}} - \frac{b \ln\left(c \left(d + e \sqrt{x}\right)^n\right)}{3 \, x^3} - \frac{a}{3 \, x^3}$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))/x^4,x)
```

output
```
(2*b*e^6*n*atanh((2*e*x^(1/2))/d + 1))/(3*d^6) - ((b*e*n)/(5*d) + (b*e^3*n
*x)/(3*d^3) - (b*e^2*n*x^(1/2))/(4*d^2) + (b*e^5*n*x^2)/d^5 - (b*e^4*n*x^(
3/2))/(2*d^4))/(3*x^(5/2)) - (b*log(c*(d + e*x^(1/2))^n))/(3*x^3) - a/(3*x
^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$$
$$= \frac{-12\sqrt{x}\,b\,d^5en-20\sqrt{x}\,b\,d^3e^3nx-60\sqrt{x}\,bd\,e^5n\,x^2-60\log\left(\sqrt{x}\right)b\,e^6n\,x^3-60\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b\,d^6+}{180d^6x^3}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x)
```

output
```
( - 12*sqrt(x)*b*d**5*e*n - 20*sqrt(x)*b*d**3*e**3*n*x - 60*sqrt(x)*b*d*e*
*5*n*x**2 - 60*log(sqrt(x))*b*e**6*n*x**3 - 60*log((sqrt(x)*e + d)**n*c)*b
*d**6 + 60*log((sqrt(x)*e + d)**n*c)*b*e**6*x**3 - 60*a*d**6 + 15*b*d**4*e
**2*n*x + 30*b*d**2*e**4*n*x**2)/(180*d**6*x**3)
```

3.408 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 480

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=\frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6}-\frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6}+\frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^5}{25e^6}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^6}{54e^6}-\frac{4b^2d^5n^2\sqrt{x}}{e^5}+\frac{b^2d^6n^2\log^2\left(d+e\sqrt{x}\right)}{3e^6}+\frac{4bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}-\frac{5bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}+\frac{40bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}-\frac{5bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6}+\frac{4bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{5e^6}-\frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}-\frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^6}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2$$

output

```
5/2*b^2*d^4*n^2*(d+e*x^(1/2))^2/e^6-40/27*b^2*d^3*n^2*(d+e*x^(1/2))^3/e^6+
5/8*b^2*d^2*n^2*(d+e*x^(1/2))^4/e^6-4/25*b^2*d*n^2*(d+e*x^(1/2))^5/e^6+1/5
4*b^2*n^2*(d+e*x^(1/2))^6/e^6-4*b^2*d^5*n^2*x^(1/2)/e^5+1/3*b^2*d^6*n^2*ln
(d+e*x^(1/2))^2/e^6+4*b*d^5*n*(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))/e^
6-5*b*d^4*n*(d+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6+40/9*b*d^3*n*(
d+e*x^(1/2))^3*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6-5/2*b*d^2*n*(d+e*x^(1/2))^4
*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6+4/5*b*d*n*(d+e*x^(1/2))^5*(a+b*ln(c*(d+e*
x^(1/2))^n))/e^6-1/9*b*n*(d+e*x^(1/2))^6*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6-2
/3*b*d^6*n*ln(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6+1/3*x^3*(a+b*ln
(c*(d+e*x^(1/2))^n))^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.65

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{e\sqrt{x}\left(1800a^2e^5x^{5/2}+60abn\left(60d^5-30d^4e\sqrt{x}+20d^3e^2x-15d^2e^3x^{3/2}+12de^4x^2-10e^5x^{5/2}\right)+b^2n^2\left(-8\right.\right.}{}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

output
```
(e*Sqrt[x]*(1800*a^2*e^5*x^(5/2) + 60*a*b*n*(60*d^5 - 30*d^4*e*Sqrt[x] + 2
0*d^3*e^2*x - 15*d^2*e^3*x^(3/2) + 12*d*e^4*x^2 - 10*e^5*x^(5/2)) + b^2*n^
2*(-8820*d^5 + 2610*d^4*e*Sqrt[x] - 1140*d^3*e^2*x + 555*d^2*e^3*x^(3/2) -
 264*d*e^4*x^2 + 100*e^5*x^(5/2))) + 180*b*d^6*n*(-20*a + 49*b*n)*Log[d +
e*Sqrt[x]] - 60*b*e*Sqrt[x]*(-60*a*e^5*x^(5/2) + b*n*(-60*d^5 + 30*d^4*e*S
qrt[x] - 20*d^3*e^2*x + 15*d^2*e^3*x^(3/2) - 12*d*e^4*x^2 + 10*e^5*x^(5/2)
))*Log[c*(d + e*Sqrt[x])^n] - 1800*b^2*(d^6 - e^6*x^3)*Log[c*(d + e*Sqrt[x
])^n]^2)/(5400*e^6)
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.62, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{2904}$$

$$2\int x^{5/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,d\sqrt{x}$$

$$\Big\downarrow \text{2845}$$

$$2\left(\frac{1}{6}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{1}{3}ben\int\frac{x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d+e\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \; 2858$$

$$2\left(\frac{1}{6}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{1}{3}bn\int x^{5/2}\left(a+b\log\left(cx^{n/2}\right)\right)d\left(d+e\sqrt{x}\right)\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{1}{6}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{bn\int e^6x^{5/2}\left(a+b\log\left(cx^{n/2}\right)\right)d\left(d+e\sqrt{x}\right)}{3e^6}\right)$$

$$\Big\downarrow \; 2772$$

$$2\left(\frac{1}{6}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{bn\left(-bn\int\left(\frac{\log\left(d+e\sqrt{x}\right)d^6}{\sqrt{x}}-6d^5+\frac{15}{2}\left(d+e\sqrt{x}\right)d^4-\frac{20xd^3}{3}+\frac{15}{4}x^{3/2}d^2-\frac{6x^2}{5}\right.\right.}{}\right.$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{1}{6}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{bn\left(d^6\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(cx^{n/2}\right)\right)-6d^5\left(d+e\sqrt{x}\right)\left(a+b\log\left(cx^{n/2}\right)\right.\right.}{}\right.$$

input

```
Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

output

```
2*((x^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/6 - (b*n*(-(b*n*(-6*d^5*(d + e
*Sqrt[x]) + (15*d^4*x)/4 - (20*d^3*x^(3/2))/9 + (15*d^2*x^2)/16 - (6*d*x^(
5/2))/25 + x^3/36 + (d^6*Log[d + e*Sqrt[x]]^2)/2)) - 6*d^5*(d + e*Sqrt[x])
*(a + b*Log[c*x^(n/2)]) + (15*d^4*x*(a + b*Log[c*x^(n/2)]))/2 - (20*d^3*x^
(3/2)*(a + b*Log[c*x^(n/2)]))/3 + (15*d^2*x^2*(a + b*Log[c*x^(n/2)]))/4 -
(6*d*x^(5/2)*(a + b*Log[c*x^(n/2)]))/5 + (x^3*(a + b*Log[c*x^(n/2)]))/6 +
d^6*Log[d + e*Sqrt[x]]*(a + b*Log[c*x^(n/2)])))/(3*e^6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2 \left(a + b\ln\left(c\left(d + e\sqrt{x}\right)^n\right)\right)^2 dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.01

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt{x}\right)^n\right)\right)^2 \, dx = \frac{1800\, b^2e^6x^3\log\left(c\right)^2 + 100\,(b^2e^6n^2 - 6\,abe^6n + 18\,a^2e^6)x^3 + 15\,(37\,b^2d^2e^4n^2 - 60\,abd^2e^4n)x^2 + 1800\,(b^2e^6}{}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

output
```
1/5400*(1800*b^2*e^6*x^3*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^
2*e^6)*x^3 + 15*(37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n)*x^2 + 1800*(b^2*e^
6*n^2*x^3 - b^2*d^6*n^2)*log(e*sqrt(x) + d)^2 + 90*(29*b^2*d^4*e^2*n^2 - 2
0*a*b*d^4*e^2*n)*x - 60*(15*b^2*d^2*e^4*n^2*x^2 + 30*b^2*d^4*e^2*n^2*x - 1
47*b^2*d^6*n^2 + 60*a*b*d^6*n + 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^3 - 60*(b
^2*e^6*n*x^3 - b^2*d^6*n)*log(c) - 4*(3*b^2*d*e^5*n^2*x^2 + 5*b^2*d^3*e^3*
n^2*x + 15*b^2*d^5*e*n^2)*sqrt(x))*log(e*sqrt(x) + d) - 300*(3*b^2*d^2*e^4
*n*x^2 + 6*b^2*d^4*e^2*n*x + 2*(b^2*e^6*n - 6*a*b*e^6)*x^3)*log(c) - 12*(7
35*b^2*d^5*e*n^2 - 300*a*b*d^5*e*n + 2*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)
*x^2 + 5*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x - 20*(3*b^2*d*e^5*n*x^2
 + 5*b^2*d^3*e^3*n*x + 15*b^2*d^5*e*n)*log(c))*sqrt(x))/e^6
```

Sympy [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

output
```
Integral(x**2*(a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.68

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{1}{3}\,b^2x^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2+\frac{2}{3}\,abx^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{3}\,a^2x^3$$
$$-\frac{1}{90}\,aben\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{10\,e^5x^3-12\,de^4x^{\frac{5}{2}}+15\,d^2e^3x^2-20\,d^3e^2x^{\frac{3}{2}}+30\,d^4ex-60\,d^5\sqrt{x}}{e^6}\right)$$
$$-\frac{1}{5400}\left(60\,en\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{10\,e^5x^3-12\,de^4x^{\frac{5}{2}}+15\,d^2e^3x^2-20\,d^3e^2x^{\frac{3}{2}}+30\,d^4ex-60\,d^5\sqrt{}}{e^6}\right.\right.$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3*log((e*sqrt(x) + d)^n*c)^2 + 2/3*a*b*x^3*log((e*sqrt(x) + d)^n
*c) + 1/3*a^2*x^3 - 1/90*a*b*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*
x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x
- 60*d^5*sqrt(x))/e^6) - 1/5400*(60*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (
10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d
^4*e*x - 60*d^5*sqrt(x))/e^6)*log((e*sqrt(x) + d)^n*c) - (100*e^6*x^3 - 26
4*d*e^5*x^(5/2) + 555*d^2*e^4*x^2 + 1800*d^6*log(e*sqrt(x) + d)^2 - 1140*d
^3*e^3*x^(3/2) + 2610*d^4*e^2*x + 8820*d^6*log(e*sqrt(x) + d) - 8820*d^5*e
*sqrt(x))*n^2/e^6)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(412) = 824$.

Time = 0.13 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.94

$$\int x^2 \left(a + b \log \left(c \left(d + e \sqrt{x}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

output
```
1/5400*(1800*b^2*e*x^3*log(c)^2 + 3600*a*b*e*x^3*log(c) + 1800*a^2*e*x^3 +
 (1800*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)^2/e^5 - 10800*(e*sqrt(x) + d)^
5*d*log(e*sqrt(x) + d)^2/e^5 + 27000*(e*sqrt(x) + d)^4*d^2*log(e*sqrt(x) +
 d)^2/e^5 - 36000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)^2/e^5 + 27000*(
e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)^2/e^5 - 10800*(e*sqrt(x) + d)*d^5*
log(e*sqrt(x) + d)^2/e^5 - 600*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5 +
4320*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 - 13500*(e*sqrt(x) + d)^4*
d^2*log(e*sqrt(x) + d)/e^5 + 24000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d
)/e^5 - 27000*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 + 21600*(e*sqrt
(x) + d)*d^5*log(e*sqrt(x) + d)/e^5 + 100*(e*sqrt(x) + d)^6/e^5 - 864*(e*s
qrt(x) + d)^5*d/e^5 + 3375*(e*sqrt(x) + d)^4*d^2/e^5 - 8000*(e*sqrt(x) + d
)^3*d^3/e^5 + 13500*(e*sqrt(x) + d)^2*d^4/e^5 - 21600*(e*sqrt(x) + d)*d^5/
e^5)*b^2*n^2 + 60*(60*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5 - 360*(e*sq
rt(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 + 900*(e*sqrt(x) + d)^4*d^2*log(e*sq
rt(x) + d)/e^5 - 1200*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)/e^5 + 900*(
e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 - 360*(e*sqrt(x) + d)*d^5*log(
e*sqrt(x) + d)/e^5 - 10*(e*sqrt(x) + d)^6/e^5 + 72*(e*sqrt(x) + d)^5*d/e^5
 - 225*(e*sqrt(x) + d)^4*d^2/e^5 + 400*(e*sqrt(x) + d)^3*d^3/e^5 - 450*(e*
sqrt(x) + d)^2*d^4/e^5 + 360*(e*sqrt(x) + d)*d^5/e^5)*b^2*n*log(c) + 60*(6
0*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5 - 360*(e*sqrt(x) + d)^5*d*lo...
```

Mupad [B] (verification not implemented)

Time = 16.24 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.90

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{3}+\frac{b^2\,n^2\,x^3}{54}+\frac{2\,a\,b\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3}-\frac{b^2\,d^6\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{3\,e^6}-\frac{a\,b\,n\,x^3}{9}-\frac{b^2\,n\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{9}+\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+e\,\sqrt{x}\right)}{30\,e^6}+\frac{37\,b^2\,d^2\,n^2\,x^2}{360\,e^2}-\frac{19\,b^2\,d^3\,n^2\,x^{3/2}}{90\,e^3}-\frac{49\,b^2\,d^5\,n^2\,\sqrt{x}}{30\,e^5}-\frac{11\,b^2\,d\,n^2\,x^{5/2}}{225\,e}+\frac{29\,b^2\,d^4\,n^2\,x}{60\,e^4}-\frac{b^2\,d^2\,n\,x^2\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{6\,e^2}+\frac{2\,b^2\,d^3\,n\,x^{3/2}\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{9\,e^3}+\frac{2\,b^2\,d^5\,n\,\sqrt{x}\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3\,e^5}+\frac{2\,a\,b\,d\,n\,x^{5/2}}{15\,e}-\frac{a\,b\,d^4\,n\,x}{3\,e^4}-\frac{2\,a\,b\,d^6\,n\,\ln\left(d+e\,\sqrt{x}\right)}{3\,e^6}+\frac{2\,b^2\,d\,n\,x^{5/2}\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{15\,e}-\frac{b^2\,d^4\,n\,x\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3\,e^4}-\frac{a\,b\,d^2\,n\,x^2}{6\,e^2}+\frac{2\,a\,b\,d^3\,n\,x^{3/2}}{9\,e^3}+\frac{2\,a\,b\,d^5\,n\,\sqrt{x}}{3\,e^5}$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^3*log(c*(d + e*x^(1/2))^n)^2)/3 + (b^2*n^2*x^3)/54 +
(2*a*b*x^3*log(c*(d + e*x^(1/2))^n))/3 - (b^2*d^6*log(c*(d + e*x^(1/2))^n)
^2)/(3*e^6) - (a*b*n*x^3)/9 - (b^2*n*x^3*log(c*(d + e*x^(1/2))^n))/9 + (49
*b^2*d^6*n^2*log(d + e*x^(1/2)))/(30*e^6) + (37*b^2*d^2*n^2*x^2)/(360*e^2)
 - (19*b^2*d^3*n^2*x^(3/2))/(90*e^3) - (49*b^2*d^5*n^2*x^(1/2))/(30*e^5) -
 (11*b^2*d*n^2*x^(5/2))/(225*e) + (29*b^2*d^4*n^2*x)/(60*e^4) - (b^2*d^2*n
*x^2*log(c*(d + e*x^(1/2))^n))/(6*e^2) + (2*b^2*d^3*n*x^(3/2)*log(c*(d + e
*x^(1/2))^n))/(9*e^3) + (2*b^2*d^5*n*x^(1/2)*log(c*(d + e*x^(1/2))^n))/(3*
e^5) + (2*a*b*d*n*x^(5/2))/(15*e) - (a*b*d^4*n*x)/(3*e^4) - (2*a*b*d^6*n*l
og(d + e*x^(1/2)))/(3*e^6) + (2*b^2*d*n*x^(5/2)*log(c*(d + e*x^(1/2))^n))/
(15*e) - (b^2*d^4*n*x*log(c*(d + e*x^(1/2))^n))/(3*e^4) - (a*b*d^2*n*x^2)/
(6*e^2) + (2*a*b*d^3*n*x^(3/2))/(9*e^3) + (2*a*b*d^5*n*x^(1/2))/(3*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.91

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{3600\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2d^5en+1200\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2d^3e^3nx+720\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)}{}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x)
```

output
```
(3600*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**2*d**5*e*n + 1200*sqrt(x)*log((
sqrt(x)*e + d)**n*c)*b**2*d**3*e**3*n*x + 720*sqrt(x)*log((sqrt(x)*e + d)*
*n*c)*b**2*d*e**5*n*x**2 + 3600*sqrt(x)*a*b*d**5*e*n + 1200*sqrt(x)*a*b*d*
*3*e**3*n*x + 720*sqrt(x)*a*b*d*e**5*n*x**2 - 8820*sqrt(x)*b**2*d**5*e*n**
2 - 1140*sqrt(x)*b**2*d**3*e**3*n**2*x - 264*sqrt(x)*b**2*d*e**5*n**2*x**2
 - 1800*log((sqrt(x)*e + d)**n*c)**2*b**2*d**6 + 1800*log((sqrt(x)*e + d)*
*n*c)**2*b**2*e**6*x**3 - 3600*log((sqrt(x)*e + d)**n*c)*a*b*d**6 + 3600*l
og((sqrt(x)*e + d)**n*c)*a*b*e**6*x**3 + 8820*log((sqrt(x)*e + d)**n*c)*b*
*2*d**6*n - 1800*log((sqrt(x)*e + d)**n*c)*b**2*d**4*e**2*n*x - 900*log((s
qrt(x)*e + d)**n*c)*b**2*d**2*e**4*n*x**2 - 600*log((sqrt(x)*e + d)**n*c)*
b**2*e**6*n*x**3 + 1800*a**2*e**6*x**3 - 1800*a*b*d**4*e**2*n*x - 900*a*b*
d**2*e**4*n*x**2 - 600*a*b*e**6*n*x**3 + 2610*b**2*d**4*e**2*n**2*x + 555*
b**2*d**2*e**4*n**2*x**2 + 100*b**2*e**6*n**2*x**3)/(5400*e**6)
```

3.409 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 342

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx &= \frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4}\\
&+\frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4}-\frac{4b^2d^3n^2\sqrt{x}}{e^3}\\
&+\frac{b^2d^4n^2\log^2\left(d+e\sqrt{x}\right)}{2e^4}\\
&+\frac{4bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}\\
&-\frac{3bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}\\
&+\frac{4bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}\\
&-\frac{bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{4e^4}\\
&-\frac{bd^4n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}\\
&+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\end{aligned}$$

output
```
3/2*b^2*d^2*n^2*(d+e*x^(1/2))^2/e^4-4/9*b^2*d*n^2*(d+e*x^(1/2))^3/e^4+1/16
*b^2*n^2*(d+e*x^(1/2))^4/e^4-4*b^2*d^3*n^2*x^(1/2)/e^3+1/2*b^2*d^4*n^2*ln(
d+e*x^(1/2))^2/e^4+4*b*d^3*n*(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))/e^4
-3*b*d^2*n*(d+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^(1/2))^n))/e^4+4/3*b*d*n*(d+e*
x^(1/2))^3*(a+b*ln(c*(d+e*x^(1/2))^n))/e^4-1/4*b*n*(d+e*x^(1/2))^4*(a+b*ln
(c*(d+e*x^(1/2))^n))/e^4-b*d^4*n*ln(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n
))/e^4+1/2*x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.65

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{e\sqrt{x}\left(72a^2e^3x^{3/2}+12abn\left(12d^3-6d^2e\sqrt{x}+4de^2x-3e^3x^{3/2}\right)+b^2n^2\left(-300d^3+78d^2e\sqrt{x}-28de^2x+9\right.\right.}{}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

output
```
(e*Sqrt[x]*(72*a^2*e^3*x^(3/2) + 12*a*b*n*(12*d^3 - 6*d^2*e*Sqrt[x] + 4*d*
e^2*x - 3*e^3*x^(3/2)) + b^2*n^2*(-300*d^3 + 78*d^2*e*Sqrt[x] - 28*d*e^2*x
 + 9*e^3*x^(3/2))) - 12*b*(12*a*(d^4 - e^4*x^2) + b*n*(-25*d^4 - 12*d^3*e*
Sqrt[x] + 6*d^2*e^2*x - 4*d*e^3*x^(3/2) + 3*e^4*x^2))*Log[c*(d + e*Sqrt[x]
)^n] - 72*b^2*(d^4 - e^4*x^2)*Log[c*(d + e*Sqrt[x])^n]^2)/(144*e^4)
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2904, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{2904}$$

$$2\int x^{3/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{2845}$$

$$2\left(\frac{1}{4}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{1}{2}ben\int\frac{x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d+e\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{2858}$$

$$2\left(\frac{1}{4}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{1}{2}bn\int x^{3/2}\left(a+b\log\left(cx^{n/2}\right)\right)d\left(d+e\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{4}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{bn\int e^4x^{3/2}\left(a+b\log\left(cx^{n/2}\right)\right)d\left(d+e\sqrt{x}\right)}{2e^4}\right)$$

$$\Big\downarrow \text{2772}$$

$$2\left(\frac{1}{4}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{bn\left(-bn\int\left(\frac{\log\left(d+e\sqrt{x}\right)d^4}{\sqrt{x}}-4d^3+3\left(d+e\sqrt{x}\right)d^2-\frac{4xd}{3}+\frac{x^{3/2}}{4}\right)d\left(d+e\sqrt{x}\right)\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{4}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{bn\left(d^4\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(cx^{n/2}\right)\right)-4d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(cx^{n/2}\right)\right.\right.}{}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

output
```
2*((x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/4 - (b*n*(-(b*n*(-4*d^3*(d + e
*Sqrt[x]) + (3*d^2*x)/2 - (4*d*x^(3/2))/9 + x^2/16 + (d^4*Log[d + e*Sqrt[x
]]^2)/2)) - 4*d^3*(d + e*Sqrt[x])*(a + b*Log[c*x^(n/2)]) + 3*d^2*x*(a + b*
Log[c*x^(n/2)]) - (4*d*x^(3/2)*(a + b*Log[c*x^(n/2)]))/3 + (x^2*(a + b*Log
[c*x^(n/2)]))/4 + d^4*Log[d + e*Sqrt[x]]*(a + b*Log[c*x^(n/2)])))/(2*e^4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.04

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx = \frac{72\,b^{2}e^{4}x^{2}\log\left(c\right)^{2}+9\,(b^{2}e^{4}n^{2}-4\,abe^{4}n+8\,a^{2}e^{4})x^{2}+72\,(b^{2}e^{4}n^{2}x^{2}-b^{2}d^{4}n^{2})\log\left(e\sqrt{x}+d\right)^{2}+6\,(13\,b^{2}d^{2}\cdots}{}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

output
```
1/144*(72*b^2*e^4*x^2*log(c)^2 + 9*(b^2*e^4*n^2 - 4*a*b*e^4*n + 8*a^2*e^4)
*x^2 + 72*(b^2*e^4*n^2*x^2 - b^2*d^4*n^2)*log(e*sqrt(x) + d)^2 + 6*(13*b^2
*d^2*e^2*n^2 - 12*a*b*d^2*e^2*n)*x - 12*(6*b^2*d^2*e^2*n^2*x - 25*b^2*d^4*
n^2 + 12*a*b*d^4*n + 3*(b^2*e^4*n^2 - 4*a*b*e^4*n)*x^2 - 12*(b^2*e^4*n*x^2
 - b^2*d^4*n)*log(c) - 4*(b^2*d*e^3*n^2*x + 3*b^2*d^3*e*n^2)*sqrt(x))*log(
e*sqrt(x) + d) - 36*(2*b^2*d^2*e^2*n*x + (b^2*e^4*n - 4*a*b*e^4)*x^2)*log(
c) - 4*(75*b^2*d^3*e*n^2 - 36*a*b*d^3*e*n + (7*b^2*d*e^3*n^2 - 12*a*b*d*e^
3*n)*x - 12*(b^2*d*e^3*n*x + 3*b^2*d^3*e*n)*log(c))*sqrt(x))/e^4
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

output
```
Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.75

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=\frac{1}{2}b^2x^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2-\frac{1}{12}aben\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)+abx^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{2}a^2x^2-\frac{1}{144}\left(12\,en\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)\log\left(\left(e\sqrt{x}+d\right)^nc\right)-\frac{(9\,e\ldots}{}\right.$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*log((e*sqrt(x) + d)^n*c)^2 - 1/12*a*b*e*n*(12*d^4*log(e*sqrt(x
) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^
4) + a*b*x^2*log((e*sqrt(x) + d)^n*c) + 1/2*a^2*x^2 - 1/144*(12*e*n*(12*d^
4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d
^3*sqrt(x))/e^4)*log((e*sqrt(x) + d)^n*c) - (9*e^4*x^2 + 72*d^4*log(e*sqrt
(x) + d)^2 - 28*d*e^3*x^(3/2) + 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d)
- 300*d^3*e*sqrt(x))*n^2/e^4)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(296) = 592$.

Time = 0.13 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.82

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

output
```
1/144*(72*b^2*e*x^2*log(c)^2 + 144*a*b*e*x^2*log(c) + (72*(e*sqrt(x) + d)^
4*log(e*sqrt(x) + d)^2/e^3 - 288*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)^2/
e^3 + 432*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)^2/e^3 - 288*(e*sqrt(x)
+ d)*d^3*log(e*sqrt(x) + d)^2/e^3 - 36*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d
)/e^3 + 192*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 - 432*(e*sqrt(x) +
d)^2*d^2*log(e*sqrt(x) + d)/e^3 + 576*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) +
d)/e^3 + 9*(e*sqrt(x) + d)^4/e^3 - 64*(e*sqrt(x) + d)^3*d/e^3 + 216*(e*sqr
t(x) + d)^2*d^2/e^3 - 576*(e*sqrt(x) + d)*d^3/e^3)*b^2*n^2 + 72*a^2*e*x^2
+ 12*(12*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e^3 - 48*(e*sqrt(x) + d)^3*d
*log(e*sqrt(x) + d)/e^3 + 72*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)/e^3
- 48*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/e^3 - 3*(e*sqrt(x) + d)^4/e^3
+ 16*(e*sqrt(x) + d)^3*d/e^3 - 36*(e*sqrt(x) + d)^2*d^2/e^3 + 48*(e*sqrt(x
) + d)*d^3/e^3)*b^2*n*log(c) + 12*(12*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)
/e^3 - 48*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 + 72*(e*sqrt(x) + d)^
2*d^2*log(e*sqrt(x) + d)/e^3 - 48*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/e
^3 - 3*(e*sqrt(x) + d)^4/e^3 + 16*(e*sqrt(x) + d)^3*d/e^3 - 36*(e*sqrt(x)
+ d)^2*d^2/e^3 + 48*(e*sqrt(x) + d)*d^3/e^3)*a*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.23

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = x\left(\frac{d\left(\frac{d\left(2a^2-abn+\frac{b^2n^2}{4}\right)}{e}-\frac{d\left(6a^2-b^2n^2\right)}{3e}\right)}{2e}+\frac{b^2d^2n^2}{4e^2}\right)$$
$$-x^{3/2}\left(\frac{d\left(2a^2-abn+\frac{b^2n^2}{4}\right)}{3e}-\frac{d\left(6a^2-b^2n^2\right)}{9e}\right)$$
$$+\ln\left(c\left(d+e\sqrt{x}\right)^n\right)^2\left(\frac{b^2x^2}{2}-\frac{b^2d^4}{2e^4}\right)+x^2\left(\frac{a^2}{2}-\frac{abn}{4}+\frac{b^2n^2}{16}\right)$$
$$-\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\left(x^{3/2}\left(\frac{bd\left(4a-bn\right)}{3e}-\frac{4abd}{3e}\right)-\frac{bx^2\left(4a-bn\right)}{4}+\frac{d^2\sqrt{x}\left(\frac{bd\left(4a-bn\right)}{e}-\frac{4abd}{e}\right)}{e^2}-\frac{d}{}\right.$$
$$-\sqrt{x}\left(\frac{d\left(\frac{d\left(\frac{d\left(2a^2-abn+\frac{b^2n^2}{4}\right)}{e}-\frac{d\left(6a^2-b^2n^2\right)}{3e}\right)}{e}+\frac{b^2d^2n^2}{2e^2}\right)}{e}+\frac{b^2d^3n^2}{e^3}\right)$$
$$+\frac{\ln\left(d+e\sqrt{x}\right)\left(25b^2d^4n^2-12abd^4n\right)}{12e^4}$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

output
```
x*((d*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))
/(2*e) + (b^2*d^2*n^2)/(4*e^2)) - x^(3/2)*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n
))/(3*e) - (d*(6*a^2 - b^2*n^2))/(9*e)) + log(c*(d + e*x^(1/2))^n)^2*((b^2
*x^2)/2 - (b^2*d^4)/(2*e^4)) + x^2*(a^2/2 + (b^2*n^2)/16 - (a*b*n)/4) - lo
g(c*(d + e*x^(1/2))^n)*(x^(3/2)*((b*d*(4*a - b*n))/(3*e) - (4*a*b*d)/(3*e)
) - (b*x^2*(4*a - b*n))/4 + (d^2*x^(1/2)*((b*d*(4*a - b*n))/e - (4*a*b*d)/
e))/e^2 - (d*x*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(2*e)) - x^(1/2)*((d*(
(d*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/e
+ (b^2*d^2*n^2)/(2*e^2)))/e + (b^2*d^3*n^2)/e^3) + (log(d + e*x^(1/2))*(25
*b^2*d^4*n^2 - 12*a*b*d^4*n))/(12*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.93

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{144\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2d^3en+48\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2d\,e^3nx+144\sqrt{x}\,ab\,d^3en+48\sqrt{x}\,abd\,e^3n}{}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x)
```

output
```
(144*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**2*d**3*e*n + 48*sqrt(x)*log((sqr
t(x)*e + d)**n*c)*b**2*d*e**3*n*x + 144*sqrt(x)*a*b*d**3*e*n + 48*sqrt(x)*
a*b*d*e**3*n*x - 300*sqrt(x)*b**2*d**3*e*n**2 - 28*sqrt(x)*b**2*d*e**3*n**
2*x - 72*log((sqrt(x)*e + d)**n*c)**2*b**2*d**4 + 72*log((sqrt(x)*e + d)**
n*c)**2*b**2*e**4*x**2 - 144*log((sqrt(x)*e + d)**n*c)*a*b*d**4 + 144*log(
(sqrt(x)*e + d)**n*c)*a*b*e**4*x**2 + 300*log((sqrt(x)*e + d)**n*c)*b**2*d
**4*n - 72*log((sqrt(x)*e + d)**n*c)*b**2*d**2*e**2*n*x - 36*log((sqrt(x)*
e + d)**n*c)*b**2*e**4*n*x**2 + 72*a**2*e**4*x**2 - 72*a*b*d**2*e**2*n*x -
 36*a*b*e**4*n*x**2 + 78*b**2*d**2*e**2*n**2*x + 9*b**2*e**4*n**2*x**2)/(1
44*e**4)
```

3.410 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=\frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2}+\frac{4abdn\sqrt{x}}{e}-\frac{4b^2dn^2\sqrt{x}}{e}+\frac{4b^2dn\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2}-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2}-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}$$

output
```
1/2*b^2*n^2*(d+e*x^(1/2))^2/e^2+4*a*b*d*n*x^(1/2)/e-4*b^2*d*n^2*x^(1/2)/e+
4*b^2*d*n*(d+e*x^(1/2))*ln(c*(d+e*x^(1/2))^n)/e^2-b*n*(d+e*x^(1/2))^2*(a+b
*ln(c*(d+e*x^(1/2))^n))/e^2-2*d*(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))^
2/e^2+(d+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2/e^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{-2abn\left(d-e\sqrt{x}\right)^2+b^2en^2\left(-6d+e\sqrt{x}\right)\sqrt{x}-2a^2(d^2-e^2x)+2b\left(d+e\sqrt{x}\right)\left(-2ad+3bdn+2ae\sqrt{x}-\right.}{2e^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

output
```
(-2*a*b*n*(d - e*Sqrt[x])^2 + b^2*e*n^2*(-6*d + e*Sqrt[x])*Sqrt[x] - 2*a^2
*(d^2 - e^2*x) + 2*b*(d + e*Sqrt[x])*(-2*a*d + 3*b*d*n + 2*a*e*Sqrt[x] - b
*e*n*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n] - 2*b^2*(d^2 - e^2*x)*Log[c*(d + e*
Sqrt[x])^n]^2)/(2*e^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2901, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$\Big\downarrow \text{2901}$$
$$2\int \sqrt{x}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2d\sqrt{x}$$
$$\Big\downarrow \text{2848}$$
$$2\int \left(\frac{\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e}-\frac{d\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e}\right)d\sqrt{x}$$
$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2}-\frac{d\left(d+e\sqrt{x}\right)\left(a+b\log\right.}{e^2}\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

output
```
2*((b^2*n^2*(d + e*Sqrt[x])^2)/(4*e^2) + (2*a*b*d*n*Sqrt[x])/e - (2*b^2*d*
n^2*Sqrt[x])/e + (2*b^2*d*n*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^2
- (b*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*e^2) - (d*(d
 + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 + ((d + e*Sqrt[x])^2
*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx = \frac{2\,b^{2}e^{2}x\log\left(c\right)^{2}+2\,(b^{2}e^{2}n^{2}x-b^{2}d^{2}n^{2})\log\left(e\sqrt{x}+d\right)^{2}-2\,(b^{2}e^{2}n-2\,abe^{2})x\log\left(c\right)+(b^{2}e^{2}n^{2}-2\,abe^{2}n}{}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*e^2*x*log(c)^2 + 2*(b^2*e^2*n^2*x - b^2*d^2*n^2)*log(e*sqrt(x)
+ d)^2 - 2*(b^2*e^2*n - 2*a*b*e^2)*x*log(c) + (b^2*e^2*n^2 - 2*a*b*e^2*n +
 2*a^2*e^2)*x + 2*(2*b^2*d*e*n^2*sqrt(x) + 3*b^2*d^2*n^2 - 2*a*b*d^2*n - (
b^2*e^2*n^2 - 2*a*b*e^2*n)*x + 2*(b^2*e^2*n*x - b^2*d^2*n)*log(c))*log(e*s
qrt(x) + d) - 2*(3*b^2*d*e*n^2 - 2*b^2*d*e*n*log(c) - 2*a*b*d*e*n)*sqrt(x)
)/e^2
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx = \int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=-\left(en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^n c\right)\right)ab$$
$$-\frac{1}{2}\left(2\,en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)\log\left(\left(e\sqrt{x}+d\right)^n c\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^n c\right)^2-\frac{\left(2\,d^2\log\right.}{}\right.$$
$$+a^2x$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

output
```
-(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*log((
e*sqrt(x) + d)^n*c))*a*b - 1/2*(2*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x
 - 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c) - 2*x*log((e*sqrt(x) + d)^n*
c)^2 - (2*d^2*log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) - 6*
d*e*sqrt(x))*n^2/e^2)*b^2 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.74

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{\left(2\,\left(e\sqrt{x}+d\right)^2\log\left(e\sqrt{x}+d\right)^2-4\,\left(e\sqrt{x}+d\right)d\log\left(e\sqrt{x}+d\right)^2-2\,\left(e\sqrt{x}+d\right)^2\log\left(e\sqrt{x}+d\right)+8\,\left(e\sqrt{x}+d\right)d\log\left(e\sqrt{x}+d\right)+\left(e\sqrt{x}+d\right)^2-8\,\left(e\sqrt{x}+d\right)d\right)b^2n}{e}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

output
```
1/2*((2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^2 - 4*(e*sqrt(x) + d)*d*log(e
*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) + 8*(e*sqrt(x) +
d)*d*log(e*sqrt(x) + d) + (e*sqrt(x) + d)^2 - 8*(e*sqrt(x) + d)*d)*b^2*n^2
/e + 2*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(x) + d)*d*log(e
*sqrt(x) + d) - (e*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*b^2*n*log(c)/e +
2*((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*b^2*log(c)^2/e + 2*(2*(e*sqrt(
x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(x) + d)*d*log(e*sqrt(x) + d) - (e
*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*a*b*n/e + 4*((e*sqrt(x) + d)^2 - 2*
(e*sqrt(x) + d)*d)*a*b*log(c)/e + 2*((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)
*d)*a^2/e)/e
```

Mupad [B] (verification not implemented)

Time = 15.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.95

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 dx = x\left(a^2-a\,b\,n+\frac{b^2\,n^2}{2}\right) - \sqrt{x}\left(\frac{d\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{e}-\frac{2\,d\,(a^2-b^2\,n^2)}{e}\right) + \ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)^2\left(b^2\,x-\frac{b^2\,d^2}{e^2}\right) - \ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\,\left(\sqrt{x}\left(\frac{2\,b\,d\,(2\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right)-b\,x\,(2\,a-b\,n)\right) + \frac{\ln\left(d+e\,\sqrt{x}\right)\,\left(3\,b^2\,d^2\,n^2-2\,a\,b\,d^2\,n\right)}{e^2}$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

output
```
x*(a^2 + (b^2*n^2)/2 - a*b*n) - x^(1/2)*((d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e
 - (2*d*(a^2 - b^2*n^2))/e) + log(c*(d + e*x^(1/2))^n)^2*(b^2*x - (b^2*d^2
)/e^2) - log(c*(d + e*x^(1/2))^n)*(x^(1/2)*((2*b*d*(2*a - b*n))/e - (4*a*b
*d)/e) - b*x*(2*a - b*n)) + (log(d + e*x^(1/2))*(3*b^2*d^2*n^2 - 2*a*b*d^2
*n))/e^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.02

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$

$$= \frac{4\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2den+4\sqrt{x}\,abden-6\sqrt{x}\,b^2de\,n^2-2\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)^2b^2d^2+2\log\left(\left(\sqrt{x}\,e+\right.\right.}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))^2,x)
```

output
```
(4*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**2*d*e*n + 4*sqrt(x)*a*b*d*e*n - 6*
sqrt(x)*b**2*d*e*n**2 - 2*log((sqrt(x)*e + d)**n*c)**2*b**2*d**2 + 2*log((
sqrt(x)*e + d)**n*c)**2*b**2*e**2*x - 4*log((sqrt(x)*e + d)**n*c)*a*b*d**2
 + 4*log((sqrt(x)*e + d)**n*c)*a*b*e**2*x + 6*log((sqrt(x)*e + d)**n*c)*b*
*2*d**2*n - 2*log((sqrt(x)*e + d)**n*c)*b**2*e**2*n*x + 2*a**2*e**2*x - 2*
a*b*e**2*n*x + b**2*e**2*n**2*x)/(2*e**2)
```

3.411 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx = 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) + 4bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right) - 4b^2n^2\operatorname{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right)$$

output
```
2*(a+b*ln(c*(d+e*x^(1/2))^n))^2*ln(-e*x^(1/2)/d)+4*b*n*(a+b*ln(c*(d+e*x^(1
/2))^n))*polylog(2,1+e*x^(1/2)/d)-4*b^2*n^2*polylog(3,1+e*x^(1/2)/d)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(93) = 186.

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx = \left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log(x) + 2bn\left(a-bn\log\left(d+e\sqrt{x}\right) + b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(\left(\log\left(d+e\sqrt{x}\right) - \log\left(1+\frac{e\sqrt{x}}{d}\right)\right)\log(x) - 2\,\text{PolyLog}\left(2,-\frac{e\sqrt{x}}{d}\right)\right) + 2b^2n^2\left(\log^2\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right) + 2\log\left(d+e\sqrt{x}\right)\text{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right) - 2\,\text{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right)\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

output
```
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] + 2*b*n
*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((Log[d + e*Sqr
t[x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)]) +
 2*b^2*n^2*(Log[d + e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[
x]]*PolyLog[2, 1 + (e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d])
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx$$

$\Big\downarrow$ 2904

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{\sqrt{x}}d\sqrt{x}$$

$\Big\downarrow$ 2843

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-2ben\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d+e\sqrt{x}}d\sqrt{x}\right)$$

$\Big\downarrow$ 2881

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-2bn\int \frac{\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{\sqrt{x}}d\left(d+e\sqrt{x}\right)\right)$$

$\Big\downarrow$ 2821

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-2bn\left(bn\int \frac{\operatorname{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)}{\sqrt{x}}d\left(d+e\sqrt{x}\right)-\operatorname{PolyLog}\left(2,\frac{d+e}{d}\right.\right.\right.$$

$\Big\downarrow$ 7143

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-2bn\left(bn\operatorname{PolyLog}\left(3,\frac{d+e\sqrt{x}}{d}\right)-\operatorname{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)\left(a+b\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

output
```
2*((a + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt[x])/d)] - 2*b*n*(-((a
+ b*Log[c*x^(n/2)])*PolyLog[2, (d + e*Sqrt[x])/d]) + b*n*PolyLog[3, (d + e
*Sqrt[x])/d]))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{2}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c)
+ a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="maxima")
```

output
```
b^2*log((e*sqrt(x) + d)^n)^2*log(x) + integrate(((b^2*e*log(c)^2 + 2*a*b*e
*log(c) + a^2*e)*x - (b^2*e*n*x*log(x) - 2*(b^2*e*log(c) + a*b*e)*x - 2*(b
^2*d*log(c) + a*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + (b^2*d*log(c)^2 + 2
*a*b*d*log(c) + a^2*d)*sqrt(x))/(e*x^2 + d*x^(3/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\right)^2}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))^2/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^n))^2/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx$$
$$=\frac{3\left(\int \frac{\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)^2}{-e^2x^2+d^2x}dx\right)b^2d^2n+6\left(\int \frac{\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)}{-e^2x^2+d^2x}dx\right)ab\,d^2n-3\left(\int \frac{\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)^2}{-e^2x^2+d^2x}dx\right)b^2den-6\left(\int \frac{\sqrt{}}{}\right.}{3n}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x)
```

output
```
(3*int(log((sqrt(x)*e + d)**n*c)**2/(d**2*x - e**2*x**2),x)*b**2*d**2*n +
6*int(log((sqrt(x)*e + d)**n*c)/(d**2*x - e**2*x**2),x)*a*b*d**2*n - 3*int
((sqrt(x)*log((sqrt(x)*e + d)**n*c)**2)/(d**2*x - e**2*x**2),x)*b**2*d*e*n
 - 6*int((sqrt(x)*log((sqrt(x)*e + d)**n*c))/(d**2*x - e**2*x**2),x)*a*b*d
*e*n + 2*log((sqrt(x)*e + d)**n*c)**3*b**2 + 6*log((sqrt(x)*e + d)**n*c)**
2*a*b + 3*log(x)*a**2*n)/(3*n)
```

3.412 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx = & -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} \\ & -\frac{2be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2} \\ & -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}+\frac{b^2e^2n^2\log(x)}{d^2} \\ & +\frac{2b^2e^2n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^2}\end{aligned}$$

output
```
-2*b*e*n*(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))/d^2/x^(1/2)-2*b*e^2*n*l
n(1-d/(d+e*x^(1/2)))*(a+b*ln(c*(d+e*x^(1/2))^n))/d^2-(a+b*ln(c*(d+e*x^(1/2
))^n))^2/x+b^2*e^2*n^2*ln(x)/d^2+2*b^2*e^2*n^2*polylog(2,d/(d+e*x^(1/2)))/
d^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx = 2\left(-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x} + ben\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{d\sqrt{x}} + \frac{e\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2bd^2n} - \frac{e\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^2} + \frac{bn\left(-\frac{e\log(d+e\sqrt{x})}{d}+\frac{e\log(x)}{2d}\right)}{d} - \frac{ben\,\mathrm{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)}{d^2}\right)\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

output
```
2*(-1/2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x + b*e*n*(-((a + b*Log[c*(d +
e*Sqrt[x])^n])/(d*Sqrt[x])) + (e*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*b*
d^2*n) - (e*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/d^2 +
(b*n*(-((e*Log[d + e*Sqrt[x]])/d) + (e*Log[x])/(2*d)))/d - (b*e*n*PolyLog[
2, (d + e*Sqrt[x])/d])/d^2))
```

Rubi [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2904, 2845, 2858, 27, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$$

$$\Big\downarrow \ 2904$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \ 2845$$

$$2\left(ben\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{\left(d+e\sqrt{x}\right)x}d\sqrt{x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x}\right)$$

$$\Big\downarrow \ 2858$$

$$2\left(bn\int \frac{a+b\log\left(cx^{n/2}\right)}{x^{3/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(be^2n\int \frac{a+b\log\left(cx^{n/2}\right)}{e^2x^{3/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x}\right)$$

$$\Big\downarrow \ 2789$$

$$2\left(be^2n\left(\frac{\int \frac{a+b\log\left(cx^{n/2}\right)}{e^2x}d\left(d+e\sqrt{x}\right)}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{ex}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x}\right)$$

$$\Big\downarrow \ 2751$$

$$2\left(be^2n\left(\frac{-\frac{bn\int -\frac{1}{e\sqrt{x}}d\left(d+e\sqrt{x}\right)}{d}-\frac{\left(d+e\sqrt{x}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{ex}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.}{2x}\right.$$

$$\Big\downarrow \ 16$$

$$2\left(be^2n\left(\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{ex}d\left(d+e\sqrt{x}\right)}{d}+\frac{\frac{bn\log\left(-e\sqrt{x}\right)}{d}-\frac{\left(d+e\sqrt{x}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x}\right)$$

$$\Big\downarrow \ 2779$$

$$2\left(be^2n\left(\frac{\frac{bn\int\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)}{\sqrt{x}}d(d+e\sqrt{x})}{d}-\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)(a+b\log(cx^{n/2}))}{d}}{d}+\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})(a+b\log(cx^{n/2}))}{de\sqrt{x}}}{d}\right)-\frac{(a+b\log\ldots}{}\right.$$

$$\Big\downarrow \text{2838}$$

$$2\left(be^2n\left(\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})(a+b\log(cx^{n/2}))}{de\sqrt{x}}}{d}+\frac{\frac{bn\operatorname{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)}{d}-\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)(a+b\log(cx^{n/2}))}{d}}{d}\right)-\frac{(a+b\log(c(d\ldots}{2x\ldots}\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

output
```
2*(-1/2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x + b*e^2*n*(((b*n*Log[-(e*Sqrt
[x])])/d - ((d + e*Sqrt[x])*(a + b*Log[c*x^(n/2)]))/(d*e*Sqrt[x]))/d + (-(
(Log[1 - d/Sqrt[x]]*(a + b*Log[c*x^(n/2)]))/d) + (b*n*PolyLog[2, d/Sqrt[x]
])/d)/d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx = \int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c)
+ a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**2,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="maxima")
```

output
```
2*(log(e*sqrt(x)/d + 1)*log(sqrt(x)) + dilog(-e*sqrt(x)/d))*b^2*e^2*n^2/d^
2 + 2*(a*b*e^2*n - (e^2*n^2 - e^2*n*log(c))*b^2)*log(e*sqrt(x) + d)/d^2 -
2*(b^2*e^2*n*log(c) + a*b*e^2*n)*log(sqrt(x))/d^2 + integrate((b^2*e^4*n^2
*x + b^2*d^2*e^2*n^2)/x, x)/d^4 + 1/3*(2*b^2*e^5*n^2*x^(3/2) - 6*b^2*d^2*e
^3*n^2*sqrt(x)*log(sqrt(x)) - 3*b^2*d*e^4*n^2*x + 12*b^2*d^2*e^3*n^2*sqrt(
x))/d^5 - 1/3*(3*b^2*d^3*e^2*n^2*x^(3/2)*log(e*sqrt(x) + d)^2 + 2*b^2*e^5*
n^2*x^3 - 3*b^2*d^2*e^3*n^2*x^2*log(x) + 12*b^2*d^2*e^3*n^2*x^2 + 3*b^2*d^
5*sqrt(x)*log((e*sqrt(x) + d)^n)^2 + 6*(b^2*d^4*e*n*log(c) + a*b*d^4*e*n)*
x - 3*(2*b^2*d^3*e^2*n*x^(3/2)*log(e*sqrt(x) + d) - 2*b^2*d^4*e*n*x - (b^2
*d^3*e^2*n*x*log(x) + 2*b^2*d^5*log(c) + 2*a*b*d^5)*sqrt(x))*log((e*sqrt(x
) + d)^n))/(d^5*x^(3/2))
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$$
$$=\frac{-2\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2den-2\sqrt{x}\,abden-\left(\int\frac{\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)}{-e^2x^2+d^2x}dx\right)b^2d^2e^2nx+\left(\int\frac{\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)}{-e^2x^2+d^2x}dx\right.}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x)
```

output
```
( - 2*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**2*d*e*n - 2*sqrt(x)*a*b*d*e*n -
 int(log((sqrt(x)*e + d)**n*c)/(d**2*x - e**2*x**2),x)*b**2*d**2*e**2*n*x
+ int((sqrt(x)*log((sqrt(x)*e + d)**n*c))/(d**2*x - e**2*x**2),x)*b**2*d*e
**3*n*x - 2*log(sqrt(x))*a*b*e**2*n*x + 2*log(sqrt(x))*b**2*e**2*n**2*x -
log((sqrt(x)*e + d)**n*c)**2*b**2*d**2 - 2*log((sqrt(x)*e + d)**n*c)*a*b*d
**2 + 2*log((sqrt(x)*e + d)**n*c)*a*b*e**2*x - 2*log((sqrt(x)*e + d)**n*c)
*b**2*e**2*n*x - a**2*d**2)/(d**2*x)
```

3.413 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 293

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx = & -\frac{b^2e^2n^2}{6d^2x}+\frac{5b^2e^3n^2}{6d^3\sqrt{x}}-\frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4} \\ & -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} \\ & +\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} \\ & -\frac{be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4\sqrt{x}} \\ & -\frac{be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4} \\ & -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2}+\frac{11b^2e^4n^2\log(x)}{12d^4} \\ & +\frac{b^2e^4n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^4}\end{aligned}$$

output
```
-1/6*b^2*e^2*n^2/d^2/x+5/6*b^2*e^3*n^2/d^3/x^(1/2)-5/6*b^2*e^4*n^2*ln(d+e*
x^(1/2))/d^4-1/3*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d/x^(3/2)+1/2*b*e^2*n*(
a+b*ln(c*(d+e*x^(1/2))^n))/d^2/x-b*e^3*n*(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1
/2))^n))/d^4/x^(1/2)-b*e^4*n*ln(1-d/(d+e*x^(1/2)))*(a+b*ln(c*(d+e*x^(1/2))
^n))/d^4-1/2*(a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2+11/12*b^2*e^4*n^2*ln(x)/d^4
+b^2*e^4*n^2*polylog(2,d/(d+e*x^(1/2)))/d^4
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx =$$

$$-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+\frac{e\sqrt{x}\left(4bd^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-6bd^2en\sqrt{x}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+12bde^2nx\left(a+b\log\left(c(d\right.\right.\right.}{}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

output
```
-1/12*(6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2 + (e*Sqrt[x]*(4*b*d^3*n*(a + b
*Log[c*(d + e*Sqrt[x])^n]) - 6*b*d^2*e*n*Sqrt[x]*(a + b*Log[c*(d + e*Sqrt[
x])^n]) + 12*b*d*e^2*n*x*(a + b*Log[c*(d + e*Sqrt[x])^n]) - 6*e^3*x^(3/2)*
(a + b*Log[c*(d + e*Sqrt[x])^n])^2 + 12*b*e^3*n*x^(3/2)*(a + b*Log[c*(d +
e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)] + 6*b^2*e^3*n^2*x^(3/2)*(2*Log[d + e*
Sqrt[x]] - Log[x]) - 3*b^2*e^2*n^2*x*(2*d - 2*e*Sqrt[x]*Log[d + e*Sqrt[x]]
 + e*Sqrt[x]*Log[x]) + 2*b^2*e*n^2*Sqrt[x]*(d*(d - 2*e*Sqrt[x]) + 2*e^2*x*
Log[d + e*Sqrt[x]] - e^2*x*Log[x]) + 12*b^2*e^3*n^2*x^(3/2)*PolyLog[2, 1 +
 (e*Sqrt[x])/d]))/d^4)/x^2
```

Rubi [A] (warning: unable to verify)

Time = 2.32 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2845}$$

$$2\left(\frac{1}{2}ben\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{\left(d+e\sqrt{x}\right)x^2}d\sqrt{x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4x^2}\right)$$

$$\Big\downarrow \text{ 2858}$$

$$2\left(\frac{1}{2}bn\int \frac{a+b\log\left(cx^{n/2}\right)}{x^{5/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{1}{2}be^4n\int \frac{a+b\log\left(cx^{n/2}\right)}{e^4x^{5/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4x^2}\right)$$

$$\Big\downarrow \text{ 2789}$$

$$2\left(\frac{1}{2}be^4n\left(\frac{\int \frac{a+b\log\left(cx^{n/2}\right)}{e^4x^2}d\left(d+e\sqrt{x}\right)}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^2}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4x^2}\right)$$

$$\Big\downarrow \text{ 2756}$$

$$2\left(\frac{1}{2}be^4n\left(\frac{-\frac{1}{3}bn\int -\frac{1}{e^3x^2}d\left(d+e\sqrt{x}\right)-\frac{a+b\log\left(cx^{n/2}\right)}{3e^3x^{3/2}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^2}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.}{4x^2}\right.$$

$$\Big\downarrow \text{ 54}$$

$$2\left(\frac{1}{2}be^4n\left(\frac{-\frac{1}{3}bn\int \left(-\frac{1}{d^3e\sqrt{x}}+\frac{1}{d^3\sqrt{x}}+\frac{1}{d^2e^2x}-\frac{1}{de^3x^{3/2}}\right)d\left(d+e\sqrt{x}\right)-\frac{a+b\log\left(cx^{n/2}\right)}{3e^3x^{3/2}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^2}d\left(d+e\surd\right.}{d}\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{1}{2}be^4n\left(\frac{\int -\frac{a+b\log(cx^{n/2})}{e^3x^2}d(d+e\sqrt{x})}{d}+\frac{-\frac{a+b\log(cx^{n/2})}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\frac{\log(d+e\sqrt{x})}{d^3}-\frac{\log(-e\sqrt{x})}{d^3}-\frac{1}{d^2e\sqrt{x}}+\frac{1}{2de^2x}\right)}{d}\right)-\right.$$

$\downarrow$ 2789

$$2\left(\frac{1}{2}be^4n\left(\frac{\frac{\int -\frac{a+b\log(cx^{n/2})}{e^3x^{3/2}}d(d+e\sqrt{x})}{d}+\frac{\int \frac{a+b\log(cx^{n/2})}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}}{d}+\frac{-\frac{a+b\log(cx^{n/2})}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\frac{\log(d+e\sqrt{x})}{d^3}-\frac{\log(-e\sqrt{x})}{d^3}-\right.}{d}\right.\right.$$

$\downarrow$ 2756

$$2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{a+b\log(cx^{n/2})}{2e^2x}-\frac{1}{2}bn\int \frac{1}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}+\frac{\int \frac{a+b\log(cx^{n/2})}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}}{d}+\frac{-\frac{a+b\log(cx^{n/2})}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\frac{\log(d+e\sqrt{x})}{d^3}-\right.}{d}\right.\right.$$

$\downarrow$ 54

$$2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{a+b\log(cx^{n/2})}{2e^2x}-\frac{1}{2}bn\int\left(-\frac{1}{d^2e\sqrt{x}}+\frac{1}{d^2\sqrt{x}}+\frac{1}{de^2x}\right)d(d+e\sqrt{x})}{d}+\frac{\int \frac{a+b\log(cx^{n/2})}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}}{d}+\frac{-\frac{a+b\log(cx^{n/2})}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\right.}{}\right.\right.$$

$\downarrow$ 2009

$$2\left(\frac{1}{2}be^4n\left(\frac{\frac{\int \frac{a+b\log(cx^{n/2})}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}+\frac{\frac{a+b\log(cx^{n/2})}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{a+b\log(cx^{n/2})}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\frac{\log(d+e}{d^3}\right.}{}\right.\right.$$

$\downarrow$ 2789

$$2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{\int \frac{a+b\log(cx^{n/2})}{e^2x}d(d+e\sqrt{x})}{d}+\frac{\int -\frac{a+b\log(cx^{n/2})}{ex}d(d+e\sqrt{x})}{d}}{d}+\frac{\frac{a+b\log(cx^{n/2})}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{a+b\log}{3e}}{}\right.\right.$$

$\downarrow$ 2751

$$2\left(\frac{1}{2}be^4n\left(\frac{\frac{-\frac{bn\int -\frac{1}{e\sqrt{x}}d(d+e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{ex}d(d+e\sqrt{x})}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x}}{d^2}\right.}{d}\right.\right.$$

$\downarrow$ 16

$$2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{ex}d(d+e\sqrt{x})}{d}+\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\frac{1}{de\sqrt{x}}\right.}{d}\right.\right.$$

$\downarrow$ 2779

$$2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{\frac{bn\int \frac{\log\left(1-\frac{d}{\sqrt{x}}\right)}{\sqrt{x}}d(d+e\sqrt{x})}{d}-\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{d}}{d}+\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log}{}\right.}{}\right.\right.$$

$\downarrow$ 2838

$$2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}+\frac{\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}+\frac{\frac{bn\operatorname{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)}{d}-\frac{\log\left(1-\right.}{d}}{d}}{d}}{d}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

output
```
2*(-1/4*(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2 + (b*e^4*n*((-1/3*(b*n*(1/(
2*d*e^2*x) - 1/(d^2*e*Sqrt[x]) + Log[d + e*Sqrt[x]]/d^3 - Log[-(e*Sqrt[x])
]/d^3)) - (a + b*Log[c*x^(n/2)])/(3*e^3*x^(3/2)))/d + ((-1/2*(b*n*(-(1/(d*
e*Sqrt[x])) + Log[d + e*Sqrt[x]]/d^2 - Log[-(e*Sqrt[x])]/d^2)) + (a + b*Lo
g[c*x^(n/2)])/(2*e^2*x))/d + (((b*n*Log[-(e*Sqrt[x])])/d - ((d + e*Sqrt[x]
)*(a + b*Log[c*x^(n/2)]))/(d*e*Sqrt[x]))/d + (-((Log[1 - d/Sqrt[x]]*(a + b
*Log[c*x^(n/2)]))/d) + (b*n*PolyLog[2, d/Sqrt[x]])/d)/d)/d)/d))/2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c)
+ a^2)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**3,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*b^2*log((e*sqrt(x) + d)^n)^2/x^2 + integrate(1/2*(2*(b^2*e*log(c)^2 +
 2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 4*(b^2*e*log(c) + a*b*e)*x + 4*(
b^2*d*log(c) + a*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + 2*(b^2*d*log(c)^2
+ 2*a*b*d*log(c) + a^2*d)*sqrt(x))/(e*x^4 + d*x^(7/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)^2}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^3,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$$
$$= \frac{-2\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2d^3en - 6\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2d\,e^3nx - 2\sqrt{x}\,ab\,d^3en - 6\sqrt{x}\,abd\,e^3nx + 5}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x)
```

output
```
( - 2*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**2*d**3*e*n - 6*sqrt(x)*log((sqr
t(x)*e + d)**n*c)*b**2*d*e**3*n*x - 2*sqrt(x)*a*b*d**3*e*n - 6*sqrt(x)*a*b
*d*e**3*n*x + 5*sqrt(x)*b**2*d*e**3*n**2*x - 3*int(log((sqrt(x)*e + d)**n*
c)/(d**2*x - e**2*x**2),x)*b**2*d**2*e**4*n*x**2 + 3*int((sqrt(x)*log((sqr
t(x)*e + d)**n*c))/(d**2*x - e**2*x**2),x)*b**2*d*e**5*n*x**2 - 6*log(sqrt
(x))*a*b*e**4*n*x**2 + 11*log(sqrt(x))*b**2*e**4*n**2*x**2 - 3*log((sqrt(x
)*e + d)**n*c)**2*b**2*d**4 - 6*log((sqrt(x)*e + d)**n*c)*a*b*d**4 + 6*log
((sqrt(x)*e + d)**n*c)*a*b*e**4*x**2 + 3*log((sqrt(x)*e + d)**n*c)*b**2*d*
*2*e**2*n*x - 11*log((sqrt(x)*e + d)**n*c)*b**2*e**4*n*x**2 - 3*a**2*d**4
+ 3*a*b*d**2*e**2*n*x - b**2*d**2*e**2*n**2*x)/(6*d**4*x**2)
```

3.414 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 408

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx &= -\frac{b^2e^2n^2}{30d^2x^2}+\frac{b^2e^3n^2}{10d^3x^{3/2}}-\frac{47b^2e^4n^2}{180d^4x} \\ &\quad+\frac{77b^2e^5n^2}{90d^5\sqrt{x}}-\frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6} \\ &\quad-\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} \\ &\quad+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} \\ &\quad-\frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}} \\ &\quad+\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^4x} \\ &\quad-\frac{2be^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6\sqrt{x}} \\ &\quad-\frac{2be^6n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6} \\ &\quad-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3}+\frac{137b^2e^6n^2\log(x)}{180d^6} \\ &\quad+\frac{2b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{3d^6}\end{aligned}$$

output
```
-1/30*b^2*e^2*n^2/d^2/x^2+1/10*b^2*e^3*n^2/d^3/x^(3/2)-47/180*b^2*e^4*n^2/
d^4/x+77/90*b^2*e^5*n^2/d^5/x^(1/2)-77/90*b^2*e^6*n^2*ln(d+e*x^(1/2))/d^6-
2/15*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d/x^(5/2)+1/6*b*e^2*n*(a+b*ln(c*(d+
e*x^(1/2))^n))/d^2/x^2-2/9*b*e^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d^3/x^(3/2)
+1/3*b*e^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d^4/x-2/3*b*e^5*n*(d+e*x^(1/2))*(
a+b*ln(c*(d+e*x^(1/2))^n))/d^6/x^(1/2)-2/3*b*e^6*n*ln(1-d/(d+e*x^(1/2)))*(
a+b*ln(c*(d+e*x^(1/2))^n))/d^6-1/3*(a+b*ln(c*(d+e*x^(1/2))^n))^2/x^3+137/1
80*b^2*e^6*n^2*ln(x)/d^6+2/3*b^2*e^6*n^2*polylog(2,d/(d+e*x^(1/2)))/d^6
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx = -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3}$$
$$-\frac{be\Big(24ad^5n-30ad^4en\sqrt{x}+6bd^4en^2\sqrt{x}+40ad^3e^2nx-18bd^3e^2n^2x-60ad^2e^3nx^{3/2}+47bd^2e^3n^2x^{3/2}\ \cdots}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]
```

output
```
-1/3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3 - (b*e*(24*a*d^5*n - 30*a*d^4*
e*n*Sqrt[x] + 6*b*d^4*e*n^2*Sqrt[x] + 40*a*d^3*e^2*n*x - 18*b*d^3*e^2*n^2*
x - 60*a*d^2*e^3*n*x^(3/2) + 47*b*d^2*e^3*n^2*x^(3/2) + 120*a*d*e^4*n*x^2
- 154*b*d*e^4*n^2*x^2 + 2*e^5*n*(-60*a + 137*b*n)*x^(5/2)*Log[d + e*Sqrt[x
]] + 24*b*d^5*n*Log[c*(d + e*Sqrt[x])^n] - 30*b*d^4*e*n*Sqrt[x]*Log[c*(d +
 e*Sqrt[x])^n] + 40*b*d^3*e^2*n*x*Log[c*(d + e*Sqrt[x])^n] - 60*b*d^2*e^3*
n*x^(3/2)*Log[c*(d + e*Sqrt[x])^n] + 120*b*d*e^4*n*x^2*Log[c*(d + e*Sqrt[x
])^n] - 60*b*e^5*x^(5/2)*Log[c*(d + e*Sqrt[x])^n]^2 + 120*b*e^5*n*x^(5/2)*
Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)] + 60*a*e^5*n*x^(5/2)*Log[x]
 - 137*b*e^5*n^2*x^(5/2)*Log[x] + 120*b*e^5*n^2*x^(5/2)*PolyLog[2, 1 + (e*
Sqrt[x])/d]))/(180*d^6*x^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 4.06 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.37, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.042, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2845}$$

$$2\left(\frac{1}{3}ben\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{\left(d+e\sqrt{x}\right)x^3}d\sqrt{x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{6x^3}\right)$$

$$\Big\downarrow \text{ 2858}$$

$$2\left(\frac{1}{3}bn\int \frac{a+b\log\left(cx^{n/2}\right)}{x^{7/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{6x^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{1}{3}be^6n\int \frac{a+b\log\left(cx^{n/2}\right)}{e^6x^{7/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{6x^3}\right)$$

$$\Big\downarrow \text{ 2789}$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\int \frac{a+b\log\left(cx^{n/2}\right)}{e^6x^3}d\left(d+e\sqrt{x}\right)}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^5x^3}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{6x^3}\right)$$

$$\Big\downarrow \text{ 2756}$$

$$2\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int -\frac{1}{e^5x^3}d\left(d+e\sqrt{x}\right)-\frac{a+b\log\left(cx^{n/2}\right)}{5e^5x^{5/2}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^5x^3}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right.\right.}{6x^3}\right.$$

$$\Big\downarrow \; 54$$

$$2\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int\left(-\frac{1}{d^5e\sqrt{x}}+\frac{1}{d^5\sqrt{x}}+\frac{1}{d^4e^2x}-\frac{1}{d^3e^3x^{3/2}}+\frac{1}{d^2e^4x^2}-\frac{1}{de^5x^{5/2}}\right)d\left(d+e\sqrt{x}\right)-\frac{a+b\log\left(cx^{n/2}\right)}{5e^5x^{5/2}}}{d}+\frac{\int-\frac{a+\cdots}{\cdots}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\int-\frac{a+b\log\left(cx^{n/2}\right)}{e^5x^3}d\left(d+e\sqrt{x}\right)}{d}+\frac{-\frac{a+b\log\left(cx^{n/2}\right)}{5e^5x^{5/2}}-\frac{1}{5}bn\left(\frac{\log\left(d+e\sqrt{x}\right)}{d^5}-\frac{\log\left(-e\sqrt{x}\right)}{d^5}-\frac{1}{d^4e\sqrt{x}}+\frac{1}{2d^3e^2x}-\frac{1}{3d^2\cdots}\cdots\right)}{d}\right.\right.$$

$$\Big\downarrow \; 2789$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\int-\frac{a+b\log\left(cx^{n/2}\right)}{e^5x^{5/2}}d\left(d+e\sqrt{x}\right)}{d}+\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^4x^{5/2}}d\left(d+e\sqrt{x}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/2}\right)}{5e^5x^{5/2}}-\frac{1}{5}bn\left(\frac{\log\left(d+e\sqrt{x}\right)}{d^5}-\frac{\log\left(-e\sqrt{x}\right)}{d^5}-\cdots\right)}{d}\right.\right.$$

$$\Big\downarrow \; 2756$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^4x^{5/2}}d\left(d+e\sqrt{x}\right)}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{4e^4x^2}-\frac{1}{4}bn\int\frac{1}{e^4x^{5/2}}d\left(d+e\sqrt{x}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/2}\right)}{5e^5x^{5/2}}-\frac{1}{5}bn\left(\frac{\log\left(d+e\sqrt{x}\right)}{d^5}-\frac{\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \; 54$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{a+b\log\left(cx^{n/2}\right)}{4e^4x^2}-\frac{1}{4}bn\int\left(-\frac{1}{d^4e\sqrt{x}}+\frac{1}{d^4\sqrt{x}}+\frac{1}{d^3e^2x}-\frac{1}{d^2e^3x^{3/2}}+\frac{1}{de^4x^2}\right)d\left(d+e\sqrt{x}\right)}{d}+\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^4x^{5/2}}d\left(d+e\sqrt{x}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(c\cdots\right.}{5e^5x^{5\cdots}}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^4x^{5/2}}d\left(d+e\sqrt{x}\right)}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{4e^4x^2}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt{x}\right)}{d^4}-\frac{\log\left(-e\sqrt{x}\right)}{d^4}-\frac{1}{d^3e\sqrt{x}}+\frac{1}{2d^2e^2x}-\frac{1}{3de^3x^{3/2}}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/\cdots}\right.}{5e^5x^{5/2}}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2789$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int \frac{a+b\log\left(cx^{n/2}\right)}{e^4x^2}d(d+e\sqrt{x})}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^2}d(d+e\sqrt{x})}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{4e^4x^2}-\frac{1}{4}bn\left(\frac{\log(d+e\sqrt{x})}{d^4}-\frac{\log(-e\sqrt{x})}{d^4}-\frac{1}{d^3e\sqrt{x}}+\frac{1}{2d^2e^2x}-\frac{}{3d}\right.}{d}}{d}\right.\right.$$

$$\downarrow 2756$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int -\frac{1}{e^3x^2}d(d+e\sqrt{x})-\frac{a+b\log\left(cx^{n/2}\right)}{3e^3x^{3/2}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^2}d(d+e\sqrt{x})}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{4e^4x^2}-\frac{1}{4}bn\left(\frac{\log(d+e\sqrt{x})}{d^4}-\frac{\log(-e\sqrt{x})}{d^4}-\frac{1}{d^3e\sqrt{}}\right.}{d}}{d}\right.\right.$$

$$\downarrow 54$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int\left(-\frac{1}{d^3e\sqrt{x}}+\frac{1}{d^3\sqrt{x}}+\frac{1}{d^2e^2x}-\frac{1}{de^3x^{3/2}}\right)d(d+e\sqrt{x})-\frac{a+b\log\left(cx^{n/2}\right)}{3e^3x^{3/2}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^2}d(d+e\sqrt{x})}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{4e^4x^2}-\frac{1}{4}bn\left(\frac{\log}{}\right.}{}}{d}\right.\right.$$

$$\downarrow 2009$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^2}d(d+e\sqrt{x})}{d}+\frac{-\frac{a+b\log\left(cx^{n/2}\right)}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\frac{\log(d+e\sqrt{x})}{d^3}-\frac{\log(-e\sqrt{x})}{d^3}-\frac{1}{d^2e\sqrt{x}}+\frac{1}{2de^2x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{4e^4x^2}-\frac{1}{4}bn\left(\frac{\log(d+e}{d^4}\right.}{}}{d}\right.\right.$$

$$\downarrow 2789$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^{3/2}}d(d+e\sqrt{x})}{d}+\frac{\int \frac{a+b\log\left(cx^{n/2}\right)}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/2}\right)}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\frac{\log(d+e\sqrt{x})}{d^3}-\frac{\log(-e\sqrt{x})}{d^3}-\frac{1}{d^2e\sqrt{x}}+\frac{1}{2de^2x}\right)}{d}}{d}+}{d}\right.\right.$$

$$\downarrow 2756$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\int\frac{1}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}+\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/2}\right)}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\frac{\log(d+e\sqrt{x})}{d^3}-\frac{\log(-e\sqrt{x})}{d^3}-\frac{1}{d^2e\sqrt{x}}+\cdots\right.}{d}\cdots}{d}\cdots}{d}\cdots\right.\right.$$

$$\Big\downarrow \ 54$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\int\left(-\frac{1}{d^2e\sqrt{x}}+\frac{1}{d^2\sqrt{x}}+\frac{1}{de^2x}\right)d(d+e\sqrt{x})}{d}+\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/2}\right)}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\frac{\log(d+e\sqrt{x})}{d^3}-\cdots\right.}{d}\cdots}{d}\cdots}{d}\cdots\right.\right.$$

$$\Big\downarrow \ 2009$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^2x^{3/2}}d(d+e\sqrt{x})}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/2}\right)}{3e^3x^{3/2}}-\frac{1}{3}bn\left(\frac{\log(d+e\sqrt{x})}{d^3}-\frac{\log(-e\cdots}{d^3}\right.}{d}\cdots}{d}\cdots}{d}\cdots\right.\right.$$

$$\Big\downarrow \ 2789$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^2x}d(d+e\sqrt{x})}{d}+\frac{\int-\frac{a+b\log\left(cx^{n/2}\right)}{ex}d(d+e\sqrt{x})}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n\cdots}\right.}{3e^3x^{3/2}}\cdots}{\cdots}}{d}\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \ 2751$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{-\frac{bn\int-\frac{1}{e\sqrt{x}}d(d+e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}+\frac{\int-\frac{a+b\log\left(cx^{n/2}\right)}{ex}d(d+e\sqrt{x})}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\cdots\right.}{d}\cdots}{d}\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \ 16$$

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{ex}d(d+e\sqrt{x})}{d}+\frac{bn\log\left(-e\sqrt{x}\right)}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}}{d}-\cdots}{d}\cdots\right.\right.$$

$\downarrow$ 2779

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{bn\int\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)}{\sqrt{x}}d(d+e\sqrt{x})}{d}-\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{d}}{d}+\frac{bn\log\left(-e\sqrt{x}\right)}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(\cdots}{}\cdots\right.}{}\cdots}{d}\cdots\right.\right.$$

$\downarrow$ 2838

$$2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}+\frac{\frac{bn\log\left(-e\sqrt{x}\right)}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}+\frac{\frac{bn\,\text{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)}{d}-\frac{\log\left(1-\frac{d}{\sqrt{\cdots}}\cdots\right.}{d}\cdots}{d}}{d}}{d}\cdots}{d}\cdots\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]
```

output
```
2*(-1/6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3 + (b*e^6*n*((-1/5*(b*n*(1/(
4*d*e^4*x^2) - 1/(3*d^2*e^3*x^(3/2)) + 1/(2*d^3*e^2*x) - 1/(d^4*e*Sqrt[x])
 + Log[d + e*Sqrt[x]]/d^5 - Log[-(e*Sqrt[x])]/d^5)) - (a + b*Log[c*x^(n/2)
])/(5*e^5*x^(5/2)))/d + ((-1/4*(b*n*(-1/3*1/(d*e^3*x^(3/2)) + 1/(2*d^2*e^2
*x) - 1/(d^3*e*Sqrt[x]) + Log[d + e*Sqrt[x]]/d^4 - Log[-(e*Sqrt[x])]/d^4))
 + (a + b*Log[c*x^(n/2)])/(4*e^4*x^2))/d + ((-1/3*(b*n*(1/(2*d*e^2*x) - 1/
(d^2*e*Sqrt[x]) + Log[d + e*Sqrt[x]]/d^3 - Log[-(e*Sqrt[x])]/d^3)) - (a +
b*Log[c*x^(n/2)])/(3*e^3*x^(3/2)))/d + ((-1/2*(b*n*(-(1/(d*e*Sqrt[x])) + L
og[d + e*Sqrt[x]]/d^2 - Log[-(e*Sqrt[x])]/d^2)) + (a + b*Log[c*x^(n/2)])/(
2*e^2*x))/d + (((b*n*Log[-(e*Sqrt[x])])/d - ((d + e*Sqrt[x])*(a + b*Log[c*
x^(n/2)]))/(d*e*Sqrt[x]))/d + (-((Log[1 - d/Sqrt[x]]*(a + b*Log[c*x^(n/2)]
))/d) + (b*n*PolyLog[2, d/Sqrt[x]])/d)/d)/d)/d)/d)/d))/3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^4,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{4}}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{2}}{x^{4}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c)
+ a^2)/x^4, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**4,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*b^2*log((e*sqrt(x) + d)^n)^2/x^3 + integrate(1/3*(3*(b^2*e*log(c)^2 +
 2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(
b^2*d*log(c) + a*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + 3*(b^2*d*log(c)^2
+ 2*a*b*d*log(c) + a^2*d)*sqrt(x))/(e*x^5 + d*x^(9/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)^2}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^4,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx = \frac{-24\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2d^5en-40\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2d^3e^3nx-120\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)b^2d}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x)
```

output
```
( - 24*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**2*d**5*e*n - 40*sqrt(x)*log((s
qrt(x)*e + d)**n*c)*b**2*d**3*e**3*n*x - 120*sqrt(x)*log((sqrt(x)*e + d)**
n*c)*b**2*d*e**5*n*x**2 - 24*sqrt(x)*a*b*d**5*e*n - 40*sqrt(x)*a*b*d**3*e*
*3*n*x - 120*sqrt(x)*a*b*d*e**5*n*x**2 + 18*sqrt(x)*b**2*d**3*e**3*n**2*x
 + 154*sqrt(x)*b**2*d*e**5*n**2*x**2 - 60*int(log((sqrt(x)*e + d)**n*c)/(d*
*2*x - e**2*x**2),x)*b**2*d**2*e**6*n*x**3 + 60*int((sqrt(x)*log((sqrt(x)*
e + d)**n*c))/(d**2*x - e**2*x**2),x)*b**2*d*e**7*n*x**3 - 120*log(sqrt(x)
)*a*b*e**6*n*x**3 + 274*log(sqrt(x))*b**2*e**6*n**2*x**3 - 60*log((sqrt(x)
*e + d)**n*c)**2*b**2*d**6 - 120*log((sqrt(x)*e + d)**n*c)*a*b*d**6 + 120*
log((sqrt(x)*e + d)**n*c)*a*b*e**6*x**3 + 30*log((sqrt(x)*e + d)**n*c)*b**
2*d**4*e**2*n*x + 60*log((sqrt(x)*e + d)**n*c)*b**2*d**2*e**4*n*x**2 - 274
*log((sqrt(x)*e + d)**n*c)*b**2*e**6*n*x**3 - 60*a**2*d**6 + 30*a*b*d**4*e
**2*n*x + 60*a*b*d**2*e**4*n*x**2 - 6*b**2*d**4*e**2*n**2*x - 47*b**2*d**2
*e**4*n**2*x**2)/(180*d**6*x**3)
```

3.415 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 907

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

output
```
1/18*b^2*n^2*(d+e*x^(1/2))^6*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6-1/6*b*n*(d+e*
x^(1/2))^6*(a+b*ln(c*(d+e*x^(1/2))^n))^2/e^6-1/108*b^3*n^3*(d+e*x^(1/2))^6
/e^6-2*d^5*(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))^3/e^6+5*d^4*(d+e*x^(1
/2))^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3/e^6-20/3*d^3*(d+e*x^(1/2))^3*(a+b*ln(
c*(d+e*x^(1/2))^n))^3/e^6+5*d^2*(d+e*x^(1/2))^4*(a+b*ln(c*(d+e*x^(1/2))^n)
)^3/e^6-2*d*(d+e*x^(1/2))^5*(a+b*ln(c*(d+e*x^(1/2))^n))^3/e^6-15/4*b^3*d^4
*n^3*(d+e*x^(1/2))^2/e^6+40/27*b^3*d^3*n^3*(d+e*x^(1/2))^3/e^6-15/32*b^3*d
^2*n^3*(d+e*x^(1/2))^4/e^6+12/125*b^3*d*n^3*(d+e*x^(1/2))^5/e^6+12*b^3*d^5
*n^3*x^(1/2)/e^5+1/3*(d+e*x^(1/2))^6*(a+b*ln(c*(d+e*x^(1/2))^n))^3/e^6-12*
a*b^2*d^5*n^2*x^(1/2)/e^5-12*b^3*d^5*n^2*(d+e*x^(1/2))*ln(c*(d+e*x^(1/2))^
n)/e^6+15/2*b^2*d^4*n^2*(d+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6-40
/9*b^2*d^3*n^2*(d+e*x^(1/2))^3*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6+15/8*b^2*d^
2*n^2*(d+e*x^(1/2))^4*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6-12/25*b^2*d*n^2*(d+e
*x^(1/2))^5*(a+b*ln(c*(d+e*x^(1/2))^n))/e^6+6*b*d^5*n*(d+e*x^(1/2))*(a+b*l
n(c*(d+e*x^(1/2))^n))^2/e^6-15/2*b*d^4*n*(d+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^
(1/2))^n))^2/e^6+20/3*b*d^3*n*(d+e*x^(1/2))^3*(a+b*ln(c*(d+e*x^(1/2))^n))^
2/e^6-15/4*b*d^2*n*(d+e*x^(1/2))^4*(a+b*ln(c*(d+e*x^(1/2))^n))^2/e^6+6/5*b
*d*n*(d+e*x^(1/2))^5*(a+b*ln(c*(d+e*x^(1/2))^n))^2/e^6
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.64

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt{x}\right)^n\right)\right)^3 dx$$
$$= \frac{b^3en^3\sqrt{x}\left(809340d^5 - 140070d^4e\sqrt{x} + 41180d^3e^2x - 13785d^2e^3x^{3/2} + 4368de^4x^2 - 1000e^5x^{5/2}\right) + 1800a}{}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

output
```
(b^3*e*n^3*Sqrt[x]*(809340*d^5 - 140070*d^4*e*Sqrt[x] + 41180*d^3*e^2*x -
13785*d^2*e^3*x^(3/2) + 4368*d*e^4*x^2 - 1000*e^5*x^(5/2)) + 1800*a^2*b*n*
(147*d^6 + 60*d^5*e*Sqrt[x] - 30*d^4*e^2*x + 20*d^3*e^3*x^(3/2) - 15*d^2*e
^4*x^2 + 12*d*e^5*x^(5/2) - 10*e^6*x^3) - 36000*a^3*(d^6 - e^6*x^3) + 60*a
*b^2*n^2*(8111*d^6 - 8820*d^5*e*Sqrt[x] + 2610*d^4*e^2*x - 1140*d^3*e^3*x^
(3/2) + 555*d^2*e^4*x^2 - 264*d*e^5*x^(5/2) + 100*e^6*x^3) - 60*b*(b^2*n^2
*(13489*d^6 + 8820*d^5*e*Sqrt[x] - 2610*d^4*e^2*x + 1140*d^3*e^3*x^(3/2) -
 555*d^2*e^4*x^2 + 264*d*e^5*x^(5/2) - 100*e^6*x^3) - 60*a*b*n*(147*d^6 +
60*d^5*e*Sqrt[x] - 30*d^4*e^2*x + 20*d^3*e^3*x^(3/2) - 15*d^2*e^4*x^2 + 12
*d*e^5*x^(5/2) - 10*e^6*x^3) + 1800*a^2*(d^6 - e^6*x^3))*Log[c*(d + e*Sqrt
[x])^n] - 1800*b^2*(60*a*(d^6 - e^6*x^3) + b*n*(-147*d^6 - 60*d^5*e*Sqrt[x
] + 30*d^4*e^2*x - 20*d^3*e^3*x^(3/2) + 15*d^2*e^4*x^2 - 12*d*e^5*x^(5/2)
+ 10*e^6*x^3))*Log[c*(d + e*Sqrt[x])^n]^2 - 36000*b^3*(d^6 - e^6*x^3)*Log[
c*(d + e*Sqrt[x])^n]^3)/(108000*e^6)
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt{x}\right)^n\right)\right)^3 dx$$

$$\Big\downarrow \text{2904}$$

$$2\int x^{5/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3d\sqrt{x}$$

$$\Big\downarrow \text{2848}$$

$$2\int\left(-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3d^5}{e^5}+\frac{5\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3d^4}{e^5}-\frac{10\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(\right.\right.}{e^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{b^3n^3\left(d+e\sqrt{x}\right)^6}{216e^6}+\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)^6}{6e^6}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)^6}{12e^6}+\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

output
```
2*((-15*b^3*d^4*n^3*(d + e*Sqrt[x])^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e*Sq
rt[x])^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e*Sqrt[x])^4)/(64*e^6) + (6*b^3*
d*n^3*(d + e*Sqrt[x])^5)/(125*e^6) - (b^3*n^3*(d + e*Sqrt[x])^6)/(216*e^6)
 - (6*a*b^2*d^5*n^2*Sqrt[x])/e^5 + (6*b^3*d^5*n^3*Sqrt[x])/e^5 - (6*b^3*d^
5*n^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^6 + (15*b^2*d^4*n^2*(d +
 e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(4*e^6) - (20*b^2*d^3*n^2*
(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) + (15*b^2*d^2*
n^2*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(16*e^6) - (6*b^2*
d*n^2*(d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(25*e^6) + (b^2*
n^2*(d + e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(36*e^6) + (3*b*d^
5*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^6 - (15*b*d^4*n*
(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*e^6) + (10*b*d^3*
n*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(3*e^6) - (15*b*d^
2*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(8*e^6) + (3*b*d
*n*(d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(5*e^6) - (b*n*(d
 + e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(12*e^6) - (d^5*(d + e
*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^6 + (5*d^4*(d + e*Sqrt[x])
^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*e^6) - (10*d^3*(d + e*Sqrt[x])^3
*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(3*e^6) + (5*d^2*(d + e*Sqrt[x])^4*(a
 + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*e^6) - (d*(d + e*Sqrt[x])^5*(a + b...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1197, normalized size of antiderivative = 1.32

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

output
```
1/108000*(36000*b^3*e^6*x^3*log(c)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2
 + 18*a^2*b*e^6*n - 36*a^3*e^6)*x^3 + 36000*(b^3*e^6*n^3*x^3 - b^3*d^6*n^3
)*log(e*sqrt(x) + d)^3 - 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2
+ 1800*a^2*b*d^2*e^4*n)*x^2 - 1800*(15*b^3*d^2*e^4*n^3*x^2 + 30*b^3*d^4*e^
2*n^3*x - 147*b^3*d^6*n^3 + 60*a*b^2*d^6*n^2 + 10*(b^3*e^6*n^3 - 6*a*b^2*e
^6*n^2)*x^3 - 60*(b^3*e^6*n^2*x^3 - b^3*d^6*n^2)*log(c) - 4*(3*b^3*d*e^5*n
^3*x^2 + 5*b^3*d^3*e^3*n^3*x + 15*b^3*d^5*e*n^3)*sqrt(x))*log(e*sqrt(x) +
d)^2 - 9000*(3*b^3*d^2*e^4*n*x^2 + 6*b^3*d^4*e^2*n*x + 2*(b^3*e^6*n - 6*a*
b^2*e^6)*x^3)*log(c)^2 - 30*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n^2
 + 1800*a^2*b*d^4*e^2*n)*x - 60*(13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 +
1800*a^2*b*d^6*n - 100*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^
3 - 15*(37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x^2 - 1800*(b^3*e^6*n*x
^3 - b^3*d^6*n)*log(c)^2 - 90*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*
x + 60*(15*b^3*d^2*e^4*n^2*x^2 + 30*b^3*d^4*e^2*n^2*x - 147*b^3*d^6*n^2 +
60*a*b^2*d^6*n + 10*(b^3*e^6*n^2 - 6*a*b^2*e^6*n)*x^3)*log(c) + 12*(735*b^
3*d^5*e*n^3 - 300*a*b^2*d^5*e*n^2 + 2*(11*b^3*d*e^5*n^3 - 30*a*b^2*d*e^5*n
^2)*x^2 + 5*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 20*(3*b^3*d*e^
5*n^2*x^2 + 5*b^3*d^3*e^3*n^2*x + 15*b^3*d^5*e*n^2)*log(c))*sqrt(x))*log(e
*sqrt(x) + d) + 300*(20*(b^3*e^6*n^2 - 6*a*b^2*e^6*n + 18*a^2*b*e^6)*x^3 +
 3*(37*b^3*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n)*x^2 + 18*(29*b^3*d^4*e^2*n...
```

Sympy [F]

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt{x}\right)^n\right)\right)^3 \, dx = \int x^2 \left(a + b\log\left(c\left(d + e\sqrt{x}\right)^n\right)\right)^3 \, dx$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

output
```
Integral(x**2*(a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 666, normalized size of antiderivative = 0.73

$$\int x^2 \left(a + b \log \left(c \left(d + e\sqrt{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3*log((e*sqrt(x) + d)^n*c)^3 + a*b^2*x^3*log((e*sqrt(x) + d)^n*c
)^2 + a^2*b*x^3*log((e*sqrt(x) + d)^n*c) + 1/3*a^3*x^3 - 1/60*a^2*b*e*n*(6
0*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3
*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6) - 1/1800*(60
*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*
d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)*log((
e*sqrt(x) + d)^n*c) - (100*e^6*x^3 - 264*d*e^5*x^(5/2) + 555*d^2*e^4*x^2 +
 1800*d^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*x^(3/2) + 2610*d^4*e^2*x + 8
820*d^6*log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x))*n^2/e^6)*a*b^2 - 1/108000
*(1800*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2)
 + 15*d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)
*log((e*sqrt(x) + d)^n*c)^2 + e*n*((1000*e^6*x^3 + 36000*d^6*log(e*sqrt(x)
 + d)^3 - 4368*d*e^5*x^(5/2) + 13785*d^2*e^4*x^2 + 264600*d^6*log(e*sqrt(x
) + d)^2 - 41180*d^3*e^3*x^(3/2) + 140070*d^4*e^2*x + 809340*d^6*log(e*sqr
t(x) + d) - 809340*d^5*e*sqrt(x))*n^2/e^7 - 60*(100*e^6*x^3 - 264*d*e^5*x^
(5/2) + 555*d^2*e^4*x^2 + 1800*d^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*x^(
3/2) + 2610*d^4*e^2*x + 8820*d^6*log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x))*
n*log((e*sqrt(x) + d)^n*c)/e^7))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2160 vs. 2(787) = 1574.

Time = 0.15 (sec) , antiderivative size = 2160, normalized size of antiderivative = 2.38

$$\int x^2 \left(a + b \log \left(c \left(d + e\sqrt{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

output
```
1/108000*(36000*b^3*e*x^3*log(c)^3 + 108000*a*b^2*e*x^3*log(c)^2 + 108000*
a^2*b*e*x^3*log(c) + (36000*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)^3/e^5 - 2
16000*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)^3/e^5 + 540000*(e*sqrt(x) + d
)^4*d^2*log(e*sqrt(x) + d)^3/e^5 - 720000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt
(x) + d)^3/e^5 + 540000*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)^3/e^5 - 2
16000*(e*sqrt(x) + d)*d^5*log(e*sqrt(x) + d)^3/e^5 - 18000*(e*sqrt(x) + d)
^6*log(e*sqrt(x) + d)^2/e^5 + 129600*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d
)^2/e^5 - 405000*(e*sqrt(x) + d)^4*d^2*log(e*sqrt(x) + d)^2/e^5 + 720000*(
e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)^2/e^5 - 810000*(e*sqrt(x) + d)^2*d
^4*log(e*sqrt(x) + d)^2/e^5 + 648000*(e*sqrt(x) + d)*d^5*log(e*sqrt(x) + d
)^2/e^5 + 6000*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5 - 51840*(e*sqrt(x)
 + d)^5*d*log(e*sqrt(x) + d)/e^5 + 202500*(e*sqrt(x) + d)^4*d^2*log(e*sqrt
(x) + d)/e^5 - 480000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)/e^5 + 81000
0*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 - 1296000*(e*sqrt(x) + d)*d
^5*log(e*sqrt(x) + d)/e^5 - 1000*(e*sqrt(x) + d)^6/e^5 + 10368*(e*sqrt(x)
+ d)^5*d/e^5 - 50625*(e*sqrt(x) + d)^4*d^2/e^5 + 160000*(e*sqrt(x) + d)^3*
d^3/e^5 - 405000*(e*sqrt(x) + d)^2*d^4/e^5 + 1296000*(e*sqrt(x) + d)*d^5/e
^5)*b^3*n^3 + 36000*a^3*e*x^3 + 60*(1800*(e*sqrt(x) + d)^6*log(e*sqrt(x) +
 d)^2/e^5 - 10800*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)^2/e^5 + 27000*(e*
sqrt(x) + d)^4*d^2*log(e*sqrt(x) + d)^2/e^5 - 36000*(e*sqrt(x) + d)^3*d...
```

Mupad [B] (verification not implemented)

Time = 21.30 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.08

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

output
```
(a^3*x^3)/3 + (b^3*x^3*log(c*(d + e*x^(1/2))^n)^3)/3 - (b^3*n^3*x^3)/108 +
 a*b^2*x^3*log(c*(d + e*x^(1/2))^n)^2 - (b^3*n*x^3*log(c*(d + e*x^(1/2))^n
)^2)/6 + (b^3*n^2*x^3*log(c*(d + e*x^(1/2))^n))/18 + (a*b^2*n^2*x^3)/18 -
(b^3*d^6*log(c*(d + e*x^(1/2))^n)^3)/(3*e^6) + a^2*b*x^3*log(c*(d + e*x^(1
/2))^n) - (a^2*b*n*x^3)/6 - (a*b^2*n*x^3*log(c*(d + e*x^(1/2))^n))/3 - (13
489*b^3*d^6*n^3*log(d + e*x^(1/2)))/(1800*e^6) - (919*b^3*d^2*n^3*x^2)/(72
00*e^2) + (2059*b^3*d^3*n^3*x^(3/2))/(5400*e^3) + (13489*b^3*d^5*n^3*x^(1/
2))/(1800*e^5) - (a*b^2*d^6*log(c*(d + e*x^(1/2))^n)^2)/e^6 + (49*b^3*d^6*
n*log(c*(d + e*x^(1/2))^n)^2)/(20*e^6) + (91*b^3*d*n^3*x^(5/2))/(2250*e) -
 (4669*b^3*d^4*n^3*x)/(3600*e^4) - (a^2*b*d^6*n*log(d + e*x^(1/2)))/e^6 +
(b^3*d*n*x^(5/2)*log(c*(d + e*x^(1/2))^n)^2)/(5*e) - (11*b^3*d*n^2*x^(5/2)
*log(c*(d + e*x^(1/2))^n))/(75*e) - (b^3*d^4*n*x*log(c*(d + e*x^(1/2))^n)^
2)/(2*e^4) + (29*b^3*d^4*n^2*x*log(c*(d + e*x^(1/2))^n))/(20*e^4) - (a^2*b
*d^2*n*x^2)/(4*e^2) - (11*a*b^2*d*n^2*x^(5/2))/(75*e) + (29*a*b^2*d^4*n^2*
x)/(20*e^4) + (a^2*b*d^3*n*x^(3/2))/(3*e^3) + (a^2*b*d^5*n*x^(1/2))/e^5 +
(49*a*b^2*d^6*n^2*log(d + e*x^(1/2)))/(10*e^6) - (b^3*d^2*n*x^2*log(c*(d +
 e*x^(1/2))^n)^2)/(4*e^2) + (37*b^3*d^2*n^2*x^2*log(c*(d + e*x^(1/2))^n))/
(120*e^2) + (b^3*d^3*n*x^(3/2)*log(c*(d + e*x^(1/2))^n)^2)/(3*e^3) - (19*b
^3*d^3*n^2*x^(3/2)*log(c*(d + e*x^(1/2))^n))/(30*e^3) + (b^3*d^5*n*x^(1/2)
*log(c*(d + e*x^(1/2))^n)^2)/e^5 - (49*b^3*d^5*n^2*x^(1/2)*log(c*(d + e...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 973, normalized size of antiderivative = 1.07

$$\int x^2 \left(a + b \log\left(c\left(d + e\sqrt{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x)
```

output
```
(108000*sqrt(x)*log((sqrt(x)*e + d)**n*c)**2*b**3*d**5*e*n + 36000*sqrt(x)
*log((sqrt(x)*e + d)**n*c)**2*b**3*d**3*e**3*n*x + 21600*sqrt(x)*log((sqrt
(x)*e + d)**n*c)**2*b**3*d*e**5*n*x**2 + 216000*sqrt(x)*log((sqrt(x)*e + d
)**n*c)*a*b**2*d**5*e*n + 72000*sqrt(x)*log((sqrt(x)*e + d)**n*c)*a*b**2*d
**3*e**3*n*x + 43200*sqrt(x)*log((sqrt(x)*e + d)**n*c)*a*b**2*d*e**5*n*x**
2 - 529200*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**3*d**5*e*n**2 - 68400*sqrt
(x)*log((sqrt(x)*e + d)**n*c)*b**3*d**3*e**3*n**2*x - 15840*sqrt(x)*log((s
qrt(x)*e + d)**n*c)*b**3*d*e**5*n**2*x**2 + 108000*sqrt(x)*a**2*b*d**5*e*n
 + 36000*sqrt(x)*a**2*b*d**3*e**3*n*x + 21600*sqrt(x)*a**2*b*d*e**5*n*x**2
 - 529200*sqrt(x)*a*b**2*d**5*e*n**2 - 68400*sqrt(x)*a*b**2*d**3*e**3*n**2
*x - 15840*sqrt(x)*a*b**2*d*e**5*n**2*x**2 + 809340*sqrt(x)*b**3*d**5*e*n*
*3 + 41180*sqrt(x)*b**3*d**3*e**3*n**3*x + 4368*sqrt(x)*b**3*d*e**5*n**3*x
**2 - 36000*log((sqrt(x)*e + d)**n*c)**3*b**3*d**6 + 36000*log((sqrt(x)*e
+ d)**n*c)**3*b**3*e**6*x**3 - 108000*log((sqrt(x)*e + d)**n*c)**2*a*b**2*
d**6 + 108000*log((sqrt(x)*e + d)**n*c)**2*a*b**2*e**6*x**3 + 264600*log((
sqrt(x)*e + d)**n*c)**2*b**3*d**6*n - 54000*log((sqrt(x)*e + d)**n*c)**2*b
**3*d**4*e**2*n*x - 27000*log((sqrt(x)*e + d)**n*c)**2*b**3*d**2*e**4*n*x*
*2 - 18000*log((sqrt(x)*e + d)**n*c)**2*b**3*e**6*n*x**3 - 108000*log((sqr
t(x)*e + d)**n*c)*a**2*b*d**6 + 108000*log((sqrt(x)*e + d)**n*c)*a**2*b*e*
*6*x**3 + 529200*log((sqrt(x)*e + d)**n*c)*a*b**2*d**6*n - 108000*log((...
```

3.416 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 595

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=-\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}-\frac{12ab^2d^3n^2\sqrt{x}}{e^3}+\frac{12b^3d^3n^3\sqrt{x}}{e^3}-\frac{12b^3d^3n^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^4}+\frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{16e^4}+\frac{6bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}-\frac{9bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^4}+\frac{2bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}-\frac{3bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{8e^4}-\frac{2d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}+\frac{3d^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}-\frac{2d\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}+\frac{\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2e^4}$$

output
```
-9/4*b^3*d^2*n^3*(d+e*x^(1/2))^2/e^4+4/9*b^3*d*n^3*(d+e*x^(1/2))^3/e^4-3/6
4*b^3*n^3*(d+e*x^(1/2))^4/e^4-12*a*b^2*d^3*n^2*x^(1/2)/e^3+12*b^3*d^3*n^3*
x^(1/2)/e^3-12*b^3*d^3*n^2*(d+e*x^(1/2))*ln(c*(d+e*x^(1/2))^n)/e^4+9/2*b^2
*d^2*n^2*(d+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^(1/2))^n))/e^4-4/3*b^2*d*n^2*(d+
e*x^(1/2))^3*(a+b*ln(c*(d+e*x^(1/2))^n))/e^4+3/16*b^2*n^2*(d+e*x^(1/2))^4*
(a+b*ln(c*(d+e*x^(1/2))^n))/e^4+6*b*d^3*n*(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(
1/2))^n))^2/e^4-9/2*b*d^2*n*(d+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2/
e^4+2*b*d*n*(d+e*x^(1/2))^3*(a+b*ln(c*(d+e*x^(1/2))^n))^2/e^4-3/8*b*n*(d+e
*x^(1/2))^4*(a+b*ln(c*(d+e*x^(1/2))^n))^2/e^4-2*d^3*(d+e*x^(1/2))*(a+b*ln(
c*(d+e*x^(1/2))^n))^3/e^4+3*d^2*(d+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^(1/2))^n)
)^3/e^4-2*d*(d+e*x^(1/2))^3*(a+b*ln(c*(d+e*x^(1/2))^n))^3/e^4+1/2*(d+e*x^(
1/2))^4*(a+b*ln(c*(d+e*x^(1/2))^n))^3/e^4
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.73

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{b^3en^3\sqrt{x}\left(4980d^3-690d^2e\sqrt{x}+148de^2x-27e^3x^{3/2}\right)+72a^2bn\left(25d^4+12d^3e\sqrt{x}-6d^2e^2x+4de^3x^{3/2}-\right.}{}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

output
```
(b^3*e*n^3*Sqrt[x]*(4980*d^3 - 690*d^2*e*Sqrt[x] + 148*d*e^2*x - 27*e^3*x^
(3/2)) + 72*a^2*b*n*(25*d^4 + 12*d^3*e*Sqrt[x] - 6*d^2*e^2*x + 4*d*e^3*x^(
3/2) - 3*e^4*x^2) - 288*a^3*(d^4 - e^4*x^2) + 12*a*b^2*n^2*(161*d^4 - 300*
d^3*e*Sqrt[x] + 78*d^2*e^2*x - 28*d*e^3*x^(3/2) + 9*e^4*x^2) - 12*b*(b^2*n
^2*(415*d^4 + 300*d^3*e*Sqrt[x] - 78*d^2*e^2*x + 28*d*e^3*x^(3/2) - 9*e^4*
x^2) - 12*a*b*n*(25*d^4 + 12*d^3*e*Sqrt[x] - 6*d^2*e^2*x + 4*d*e^3*x^(3/2)
 - 3*e^4*x^2) + 72*a^2*(d^4 - e^4*x^2))*Log[c*(d + e*Sqrt[x])^n] - 72*b^2*
(12*a*(d^4 - e^4*x^2) + b*n*(-25*d^4 - 12*d^3*e*Sqrt[x] + 6*d^2*e^2*x - 4*
d*e^3*x^(3/2) + 3*e^4*x^2))*Log[c*(d + e*Sqrt[x])^n]^2 - 288*b^3*(d^4 - e^
4*x^2)*Log[c*(d + e*Sqrt[x])^n]^3)/(576*e^4)
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3dx$$

$$\Big\downarrow \text{2904}$$

$$2\int x^{3/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3d\sqrt{x}$$

$$\Big\downarrow \text{2848}$$

$$2\int\left(\frac{\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^3}-\frac{3d\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^3}+\frac{3d^2\left(d+e\sqrt{x}\right)\left(a-\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{4e^4}+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{32e^4}-\frac{2b^2dn^2\left(d+\right.}{}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

output
```
2*((-9*b^3*d^2*n^3*(d + e*Sqrt[x])^2)/(8*e^4) + (2*b^3*d*n^3*(d + e*Sqrt[x
])^3)/(9*e^4) - (3*b^3*n^3*(d + e*Sqrt[x])^4)/(128*e^4) - (6*a*b^2*d^3*n^2
*Sqrt[x])/e^3 + (6*b^3*d^3*n^3*Sqrt[x])/e^3 - (6*b^3*d^3*n^2*(d + e*Sqrt[x
])*Log[c*(d + e*Sqrt[x])^n])/e^4 + (9*b^2*d^2*n^2*(d + e*Sqrt[x])^2*(a + b
*Log[c*(d + e*Sqrt[x])^n]))/(4*e^4) - (2*b^2*d*n^2*(d + e*Sqrt[x])^3*(a +
b*Log[c*(d + e*Sqrt[x])^n]))/(3*e^4) + (3*b^2*n^2*(d + e*Sqrt[x])^4*(a + b
*Log[c*(d + e*Sqrt[x])^n]))/(32*e^4) + (3*b*d^3*n*(d + e*Sqrt[x])*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2)/e^4 - (9*b*d^2*n*(d + e*Sqrt[x])^2*(a + b*Log[
c*(d + e*Sqrt[x])^n])^2)/(4*e^4) + (b*d*n*(d + e*Sqrt[x])^3*(a + b*Log[c*(
d + e*Sqrt[x])^n])^2)/e^4 - (3*b*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*S
qrt[x])^n])^2)/(16*e^4) - (d^3*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x]
)^n])^3)/e^4 + (3*d^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3
)/(2*e^4) - (d*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 +
 ((d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(4*e^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.45

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

output
```
1/576*(288*b^3*e^4*x^2*log(c)^3 + 288*(b^3*e^4*n^3*x^2 - b^3*d^4*n^3)*log(
e*sqrt(x) + d)^3 - 9*(3*b^3*e^4*n^3 - 12*a*b^2*e^4*n^2 + 24*a^2*b*e^4*n -
32*a^3*e^4)*x^2 - 72*(6*b^3*d^2*e^2*n^3*x - 25*b^3*d^4*n^3 + 12*a*b^2*d^4*
n^2 + 3*(b^3*e^4*n^3 - 4*a*b^2*e^4*n^2)*x^2 - 12*(b^3*e^4*n^2*x^2 - b^3*d^
4*n^2)*log(c) - 4*(b^3*d*e^3*n^3*x + 3*b^3*d^3*e*n^3)*sqrt(x))*log(e*sqrt(
x) + d)^2 - 216*(2*b^3*d^2*e^2*n*x + (b^3*e^4*n - 4*a*b^2*e^4)*x^2)*log(c)
^2 - 6*(115*b^3*d^2*e^2*n^3 - 156*a*b^2*d^2*e^2*n^2 + 72*a^2*b*d^2*e^2*n)*
x - 12*(415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n - 9*(b^3*e^4*
n^3 - 4*a*b^2*e^4*n^2 + 8*a^2*b*e^4*n)*x^2 - 72*(b^3*e^4*n*x^2 - b^3*d^4*n
)*log(c)^2 - 6*(13*b^3*d^2*e^2*n^3 - 12*a*b^2*d^2*e^2*n^2)*x + 12*(6*b^3*d
^2*e^2*n^2*x - 25*b^3*d^4*n^2 + 12*a*b^2*d^4*n + 3*(b^3*e^4*n^2 - 4*a*b^2*
e^4*n)*x^2)*log(c) + 4*(75*b^3*d^3*e*n^3 - 36*a*b^2*d^3*e*n^2 + (7*b^3*d*e
^3*n^3 - 12*a*b^2*d*e^3*n^2)*x - 12*(b^3*d*e^3*n^2*x + 3*b^3*d^3*e*n^2)*lo
g(c))*sqrt(x))*log(e*sqrt(x) + d) + 36*(3*(b^3*e^4*n^2 - 4*a*b^2*e^4*n + 8
*a^2*b*e^4)*x^2 + 2*(13*b^3*d^2*e^2*n^2 - 12*a*b^2*d^2*e^2*n)*x)*log(c) +
4*(1245*b^3*d^3*e*n^3 - 900*a*b^2*d^3*e*n^2 + 216*a^2*b*d^3*e*n + 72*(b^3*
d*e^3*n*x + 3*b^3*d^3*e*n)*log(c)^2 + (37*b^3*d*e^3*n^3 - 84*a*b^2*d*e^3*n
^2 + 72*a^2*b*d*e^3*n)*x - 12*(75*b^3*d^3*e*n^2 - 36*a*b^2*d^3*e*n + (7*b^
3*d*e^3*n^2 - 12*a*b^2*d*e^3*n)*x)*log(c))*sqrt(x))/e^4
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

output
```
Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx\\
&=\frac{1}{2}\,b^3x^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^3+\frac{3}{2}\,ab^2x^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2\\
&\quad-\frac{1}{8}\,a^2ben\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)\\
&\quad+\frac{3}{2}\,a^2bx^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{2}\,a^3x^2\\
&\quad-\frac{1}{48}\left(12\,en\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)\log\left(\left(e\sqrt{x}+d\right)^nc\right)-\frac{\left(9\,e^4\right.}{}\right.\\
&\quad-\frac{1}{576}\left(72\,en\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2+en\right.
\end{aligned}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2*log((e*sqrt(x) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*sqrt(x) + d)
^n*c)^2 - 1/8*a^2*b*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*
e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4) + 3/2*a^2*b*x^2*log((e*sqrt
(x) + d)^n*c) + 1/2*a^3*x^2 - 1/48*(12*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5
+ (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)*log((e*s
qrt(x) + d)^n*c) - (9*e^4*x^2 + 72*d^4*log(e*sqrt(x) + d)^2 - 28*d*e^3*x^(
3/2) + 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*d^3*e*sqrt(x))*n^2/
e^4)*a*b^2 - 1/576*(72*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4
*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)*log((e*sqrt(x) + d)^n*c)
^2 + e*n*((288*d^4*log(e*sqrt(x) + d)^3 + 27*e^4*x^2 + 1800*d^4*log(e*sqrt
(x) + d)^2 - 148*d*e^3*x^(3/2) + 690*d^2*e^2*x + 4980*d^4*log(e*sqrt(x) +
d) - 4980*d^3*e*sqrt(x))*n^2/e^5 - 12*(9*e^4*x^2 + 72*d^4*log(e*sqrt(x) +
d)^2 - 28*d*e^3*x^(3/2) + 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*
d^3*e*sqrt(x))*n*log((e*sqrt(x) + d)^n*c)/e^5))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. $2(519) = 1038$.

Time = 0.15 (sec) , antiderivative size = 1440, normalized size of antiderivative = 2.42

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

output
```
1/576*(288*b^3*e*x^2*log(c)^3 + 864*a*b^2*e*x^2*log(c)^2 + (288*(e*sqrt(x)
 + d)^4*log(e*sqrt(x) + d)^3/e^3 - 1152*(e*sqrt(x) + d)^3*d*log(e*sqrt(x)
+ d)^3/e^3 + 1728*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)^3/e^3 - 1152*(e
*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)^3/e^3 - 216*(e*sqrt(x) + d)^4*log(e*s
qrt(x) + d)^2/e^3 + 1152*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)^2/e^3 - 25
92*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)^2/e^3 + 3456*(e*sqrt(x) + d)*d
^3*log(e*sqrt(x) + d)^2/e^3 + 108*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e^3
 - 768*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 + 2592*(e*sqrt(x) + d)^2
*d^2*log(e*sqrt(x) + d)/e^3 - 6912*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/
e^3 - 27*(e*sqrt(x) + d)^4/e^3 + 256*(e*sqrt(x) + d)^3*d/e^3 - 1296*(e*sqr
t(x) + d)^2*d^2/e^3 + 6912*(e*sqrt(x) + d)*d^3/e^3)*b^3*n^3 + 12*(72*(e*sq
rt(x) + d)^4*log(e*sqrt(x) + d)^2/e^3 - 288*(e*sqrt(x) + d)^3*d*log(e*sqrt
(x) + d)^2/e^3 + 432*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)^2/e^3 - 288*
(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)^2/e^3 - 36*(e*sqrt(x) + d)^4*log(e*
sqrt(x) + d)/e^3 + 192*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 - 432*(e
*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)/e^3 + 576*(e*sqrt(x) + d)*d^3*log(e
*sqrt(x) + d)/e^3 + 9*(e*sqrt(x) + d)^4/e^3 - 64*(e*sqrt(x) + d)^3*d/e^3 +
 216*(e*sqrt(x) + d)^2*d^2/e^3 - 576*(e*sqrt(x) + d)*d^3/e^3)*b^3*n^2*log(
c) + 864*a^2*b*e*x^2*log(c) + 72*(12*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/
e^3 - 48*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 + 72*(e*sqrt(x) + d...
```

Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.41

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

output
```
log(c*(d + e*x^(1/2))^n)^3*((b^3*x^2)/2 - (b^3*d^4)/(2*e^4)) - x^(3/2)*((d
*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/(3*e) - (d*(2
4*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(36*e)) - log(c*(d + e*x^(1/2))^n)^2*((
x^(3/2)*((b^2*d*(4*a - b*n))/e - (4*a*b^2*d)/e))/2 - (3*b^2*x^2*(4*a - b*n
))/8 + (d*(12*a*b^2*d^3 - 25*b^3*d^3*n))/(8*e^4) + (d^2*x^(1/2)*((6*b^2*d*
(4*a - b*n))/e - (24*a*b^2*d)/e))/(4*e^2) - (d*x*((6*b^2*d*(4*a - b*n))/e
- (24*a*b^2*d)/e))/(8*e)) + x*((d*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n
^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)
))/(2*e) + (b^2*d^2*n^2*(12*a - 13*b*n))/(16*e^2)) - x^(1/2)*((d*((d*((d*(
2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3
+ 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)))/e + (b^2*d^2*n^2*(12*a - 13*b*n))/(8
*e^2)))/e + (b^2*d^3*n^2*(12*a - 25*b*n))/(4*e^3)) + x^2*(a^3/2 - (3*b^3*n
^3)/64 + (3*a*b^2*n^2)/16 - (3*a^2*b*n)/8) + (log(c*(d + e*x^(1/2))^n)*((x
^(3/2)*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b
*n)))/(12*e^2) - (x*((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2
+ b^2*n^2 - 4*a*b*n)))/e - 24*b^3*d^2*e^2*n^2))/(8*e^2) + (x^(1/2)*((d*((d
*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))/
e - 24*b^3*d^2*e^2*n^2))/e - 48*b^3*d^3*e*n^2))/(4*e^2) + (3*b*e^2*x^2*(8*
a^2 + b^2*n^2 - 4*a*b*n))/4))/(4*e^2) - (log(d + e*x^(1/2))*(415*b^3*d^4*n
^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n))/(48*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.16

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x)
```

output
```
(864*sqrt(x)*log((sqrt(x)*e + d)**n*c)**2*b**3*d**3*e*n + 288*sqrt(x)*log(
(sqrt(x)*e + d)**n*c)**2*b**3*d*e**3*n*x + 1728*sqrt(x)*log((sqrt(x)*e + d
)**n*c)*a*b**2*d**3*e*n + 576*sqrt(x)*log((sqrt(x)*e + d)**n*c)*a*b**2*d*e
**3*n*x - 3600*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**3*d**3*e*n**2 - 336*sq
rt(x)*log((sqrt(x)*e + d)**n*c)*b**3*d*e**3*n**2*x + 864*sqrt(x)*a**2*b*d*
*3*e*n + 288*sqrt(x)*a**2*b*d*e**3*n*x - 3600*sqrt(x)*a*b**2*d**3*e*n**2 -
 336*sqrt(x)*a*b**2*d*e**3*n**2*x + 4980*sqrt(x)*b**3*d**3*e*n**3 + 148*sq
rt(x)*b**3*d*e**3*n**3*x - 288*log((sqrt(x)*e + d)**n*c)**3*b**3*d**4 + 28
8*log((sqrt(x)*e + d)**n*c)**3*b**3*e**4*x**2 - 864*log((sqrt(x)*e + d)**n
*c)**2*a*b**2*d**4 + 864*log((sqrt(x)*e + d)**n*c)**2*a*b**2*e**4*x**2 + 1
800*log((sqrt(x)*e + d)**n*c)**2*b**3*d**4*n - 432*log((sqrt(x)*e + d)**n*
c)**2*b**3*d**2*e**2*n*x - 216*log((sqrt(x)*e + d)**n*c)**2*b**3*e**4*n*x*
*2 - 864*log((sqrt(x)*e + d)**n*c)*a**2*b*d**4 + 864*log((sqrt(x)*e + d)**
n*c)*a**2*b*e**4*x**2 + 3600*log((sqrt(x)*e + d)**n*c)*a*b**2*d**4*n - 864
*log((sqrt(x)*e + d)**n*c)*a*b**2*d**2*e**2*n*x - 432*log((sqrt(x)*e + d)*
*n*c)*a*b**2*e**4*n*x**2 - 4980*log((sqrt(x)*e + d)**n*c)*b**3*d**4*n**2 +
 936*log((sqrt(x)*e + d)**n*c)*b**3*d**2*e**2*n**2*x + 108*log((sqrt(x)*e
+ d)**n*c)*b**3*e**4*n**2*x**2 + 288*a**3*e**4*x**2 - 432*a**2*b*d**2*e**2
*n*x - 216*a**2*b*e**4*n*x**2 + 936*a*b**2*d**2*e**2*n**2*x + 108*a*b**2*e
**4*n**2*x**2 - 690*b**3*d**2*e**2*n**3*x - 27*b**3*e**4*n**3*x**2)/(57...
```

3.417 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 284

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=&-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}+\frac{12b^3dn^3\sqrt{x}}{e}\\&-\frac{12b^3dn^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2}\\&+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2}\\&+\frac{6bdn\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}\\&-\frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2}\\&-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2}\\&+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2}\end{aligned}$$

output
```
-3/4*b^3*n^3*(d+e*x^(1/2))^2/e^2-12*a*b^2*d*n^2*x^(1/2)/e+12*b^3*d*n^3*x^(
1/2)/e-12*b^3*d*n^2*(d+e*x^(1/2))*ln(c*(d+e*x^(1/2))^n)/e^2+3/2*b^2*n^2*(d
+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^(1/2))^n))/e^2+6*b*d*n*(d+e*x^(1/2))*(a+b*l
n(c*(d+e*x^(1/2))^n))^2/e^2-3/2*b*n*(d+e*x^(1/2))^2*(a+b*ln(c*(d+e*x^(1/2)
)^n))^2/e^2-2*d*(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))^3/e^2+(d+e*x^(1/
2))^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3/e^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.85

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$
$$= \frac{-8d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3+4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3+24bdn\Big(\left(d+e\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

output
```
(-8*d*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 4*(d + e*Sqrt[x
])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 24*b*d*n*((d + e*Sqrt[x])*(a + b
*Log[c*(d + e*Sqrt[x])^n])^2 - 2*b*n*(e*(a - b*n)*Sqrt[x] + b*(d + e*Sqrt[
x])*Log[c*(d + e*Sqrt[x])^n])) - 3*b*n*(2*(d + e*Sqrt[x])^2*(a + b*Log[c*(
d + e*Sqrt[x])^n])^2 + b*n*(b*e*n*(2*d*Sqrt[x] + e*x) - 2*(d + e*Sqrt[x])^
2*(a + b*Log[c*(d + e*Sqrt[x])^n]))))/(4*e^2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2901, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{2901}$$

$$2\int \sqrt{x}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 d\sqrt{x}$$

$$\Big\downarrow \text{2848}$$

$$2\int \left(\frac{\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e}-\frac{d\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{4e^2}-\frac{6ab^2dn^2\sqrt{x}}{e}-\frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4e^2}+\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

output
```
2*((-3*b^3*n^3*(d + e*Sqrt[x])^2)/(8*e^2) - (6*a*b^2*d*n^2*Sqrt[x])/e + (6
*b^3*d*n^3*Sqrt[x])/e - (6*b^3*d*n^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])
^n])/e^2 + (3*b^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/
(4*e^2) + (3*b*d*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2
 - (3*b*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*e^2) -
(d*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2 + ((d + e*Sqrt[
x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(248) = 496$.

Time = 0.10 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.86

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}\,dx$$
$$=\frac{4\,b^3e^2x\log\left(c\right)^3+4\left(b^3e^2n^3x-b^3d^2n^3\right)\log\left(e\sqrt{x}+d\right)^3-6\left(b^3e^2n-2\,ab^2e^2\right)x\log\left(c\right)^2+6\left(2\,b^3den^3\sqrt{x}+\right.}{}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

output
```
1/4*(4*b^3*e^2*x*log(c)^3 + 4*(b^3*e^2*n^3*x - b^3*d^2*n^3)*log(e*sqrt(x)
+ d)^3 - 6*(b^3*e^2*n - 2*a*b^2*e^2)*x*log(c)^2 + 6*(2*b^3*d*e*n^3*sqrt(x)
 + 3*b^3*d^2*n^3 - 2*a*b^2*d^2*n^2 - (b^3*e^2*n^3 - 2*a*b^2*e^2*n^2)*x + 2
*(b^3*e^2*n^2*x - b^3*d^2*n^2)*log(c))*log(e*sqrt(x) + d)^2 + 6*(b^3*e^2*n
^2 - 2*a*b^2*e^2*n + 2*a^2*b*e^2)*x*log(c) - (3*b^3*e^2*n^3 - 6*a*b^2*e^2*
n^2 + 6*a^2*b*e^2*n - 4*a^3*e^2)*x - 6*(7*b^3*d^2*n^3 - 6*a*b^2*d^2*n^2 +
2*a^2*b*d^2*n - 2*(b^3*e^2*n*x - b^3*d^2*n)*log(c)^2 - (b^3*e^2*n^3 - 2*a*
b^2*e^2*n^2 + 2*a^2*b*e^2*n)*x - 2*(3*b^3*d^2*n^2 - 2*a*b^2*d^2*n - (b^3*e
^2*n^2 - 2*a*b^2*e^2*n)*x)*log(c) + 2*(3*b^3*d*e*n^3 - 2*b^3*d*e*n^2*log(c
) - 2*a*b^2*d*e*n^2)*sqrt(x))*log(e*sqrt(x) + d) + 6*(7*b^3*d*e*n^3 + 2*b^
3*d*e*n*log(c)^2 - 6*a*b^2*d*e*n^2 + 2*a^2*b*d*e*n - 2*(3*b^3*d*e*n^2 - 2*
a*b^2*d*e*n)*log(c))*sqrt(x))/e^2
```

Sympy [F]

$$\int \left(a + b \log \left(c \left(d + e \sqrt{x}\right)^n\right)\right)^3 \, dx = \int \left(a + b \log \left(c \left(d + e \sqrt{x}\right)^n\right)\right)^3 \, dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.34

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$
$$= -\frac{3}{2}\left(en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)\right)a^2b$$
$$-\frac{3}{2}\left(2\,en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)\log\left(\left(e\sqrt{x}+d\right)^nc\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2-\frac{\left(2\,d^2\log\right.}{}\right.$$
$$-\frac{1}{4}\left(6\,en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2-4\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)^3+en\left(\frac{\left(4\right.}{}\right.\right.$$
$$+a^3x$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

output
```
-3/2*(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*l
og((e*sqrt(x) + d)^n*c))*a^2*b - 3/2*(2*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3
+ (e*x - 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c) - 2*x*log((e*sqrt(x) +
 d)^n*c)^2 - (2*d^2*log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d
) - 6*d*e*sqrt(x))*n^2/e^2)*a*b^2 - 1/4*(6*e*n*(2*d^2*log(e*sqrt(x) + d)/e
^3 + (e*x - 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c)^2 - 4*x*log((e*sqrt
(x) + d)^n*c)^3 + e*n*((4*d^2*log(e*sqrt(x) + d)^3 + 18*d^2*log(e*sqrt(x)
+ d)^2 + 3*e^2*x + 42*d^2*log(e*sqrt(x) + d) - 42*d*e*sqrt(x))*n^2/e^3 - 6
*(2*d^2*log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) - 6*d*e*sq
rt(x))*n*log((e*sqrt(x) + d)^n*c)/e^3))*b^3 + a^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. 2(248) = 496.

Time = 0.13 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.51

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

output
```
1/4*((4*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^3 - 8*(e*sqrt(x) + d)*d*log(e
*sqrt(x) + d)^3 - 6*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^2 + 24*(e*sqrt(x)
 + d)*d*log(e*sqrt(x) + d)^2 + 6*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 48
*(e*sqrt(x) + d)*d*log(e*sqrt(x) + d) - 3*(e*sqrt(x) + d)^2 + 48*(e*sqrt(x
) + d)*d)*b^3*n^3/e + 6*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^2 - 4*(e*s
qrt(x) + d)*d*log(e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d
) + 8*(e*sqrt(x) + d)*d*log(e*sqrt(x) + d) + (e*sqrt(x) + d)^2 - 8*(e*sqrt
(x) + d)*d)*b^3*n^2*log(c)/e + 6*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) -
 4*(e*sqrt(x) + d)*d*log(e*sqrt(x) + d) - (e*sqrt(x) + d)^2 + 4*(e*sqrt(x)
 + d)*d)*b^3*n*log(c)^2/e + 4*((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*b^
3*log(c)^3/e + 6*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^2 - 4*(e*sqrt(x)
+ d)*d*log(e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) + 8*(
e*sqrt(x) + d)*d*log(e*sqrt(x) + d) + (e*sqrt(x) + d)^2 - 8*(e*sqrt(x) + d
)*d)*a*b^2*n^2/e + 12*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(
x) + d)*d*log(e*sqrt(x) + d) - (e*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*a*
b^2*n*log(c)/e + 12*((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*a*b^2*log(c)
^2/e + 6*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(x) + d)*d*log
(e*sqrt(x) + d) - (e*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*a^2*b*n/e + 12*
((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*a^2*b*log(c)/e + 4*((e*sqrt(x) +
 d)^2 - 2*(e*sqrt(x) + d)*d)*a^3/e)/e
```

Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.23

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$
$$= x\left(a^3-\frac{3a^2bn}{2}+\frac{3ab^2n^2}{2}-\frac{3b^3n^3}{4}\right)$$
$$-\sqrt{x}\left(\frac{d\left(2a^3-3a^2bn+3ab^2n^2-\frac{3b^3n^3}{2}\right)}{e}-\frac{d\left(2a^3-6ab^2n^2+9b^3n^3\right)}{e}\right)$$
$$+\ln\left(c\left(d+e\sqrt{x}\right)^n\right)^3\left(b^3x-\frac{b^3d^2}{e^2}\right)$$
$$-\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\left(\sqrt{x}\left(\frac{3bd\left(2a^2-2abn+b^2n^2\right)}{e}-\frac{6bd\left(a^2-b^2n^2\right)}{e}\right)-\frac{3bx\left(2a^2-2abn+b^2n^2\right)}{2}\right)$$
$$-\ln\left(c\left(d+e\sqrt{x}\right)^n\right)^2\left(\sqrt{x}\left(\frac{3b^2d\left(2a-bn\right)}{e}-\frac{6ab^2d}{e}\right)+\frac{3d\left(2ab^2d-3b^3dn\right)}{2e^2}-\frac{3b^2x\left(2a-bn\right)}{2}\right)-\frac{\ln\left(d+e\sqrt{x}\right)\left(6a^2bd^2n-18ab^2d^2n^2+21b^3d^2n^3\right)}{2e^2}$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

output
```
x*(a^3 - (3*b^3*n^3)/4 + (3*a*b^2*n^2)/2 - (3*a^2*b*n)/2) - x^(1/2)*((d*(2
*a^3 - (3*b^3*n^3)/2 + 3*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(2*a^3 + 9*b^3*n^3
 - 6*a*b^2*n^2))/e) + log(c*(d + e*x^(1/2))^n)^3*(b^3*x - (b^3*d^2)/e^2) -
 log(c*(d + e*x^(1/2))^n)*(x^(1/2)*((3*b*d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e
- (6*b*d*(a^2 - b^2*n^2))/e) - (3*b*x*(2*a^2 + b^2*n^2 - 2*a*b*n))/2) - lo
g(c*(d + e*x^(1/2))^n)^2*(x^(1/2)*((3*b^2*d*(2*a - b*n))/e - (6*a*b^2*d)/e
) + (3*d*(2*a*b^2*d - 3*b^3*d*n))/(2*e^2) - (3*b^2*x*(2*a - b*n))/2) - (lo
g(d + e*x^(1/2))*(21*b^3*d^2*n^3 - 18*a*b^2*d^2*n^2 + 6*a^2*b*d^2*n))/(2*e
^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.47

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

$$= \frac{12\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)^2 b^3 den + 24\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right) a\,b^2 den - 36\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right) b^3 de\,n^2}{}$$

input

```
int((a+b*log(c*(d+e*x^(1/2))^n))^3,x)
```

output

```
(12*sqrt(x)*log((sqrt(x)*e + d)**n*c)**2*b**3*d*e*n + 24*sqrt(x)*log((sqrt
(x)*e + d)**n*c)*a*b**2*d*e*n - 36*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**3*
d*e*n**2 + 12*sqrt(x)*a**2*b*d*e*n - 36*sqrt(x)*a*b**2*d*e*n**2 + 42*sqrt(
x)*b**3*d*e*n**3 - 4*log((sqrt(x)*e + d)**n*c)**3*b**3*d**2 + 4*log((sqrt(
x)*e + d)**n*c)**3*b**3*e**2*x - 12*log((sqrt(x)*e + d)**n*c)**2*a*b**2*d*
*2 + 12*log((sqrt(x)*e + d)**n*c)**2*a*b**2*e**2*x + 18*log((sqrt(x)*e + d
)**n*c)**2*b**3*d**2*n - 6*log((sqrt(x)*e + d)**n*c)**2*b**3*e**2*n*x - 12
*log((sqrt(x)*e + d)**n*c)*a**2*b*d**2 + 12*log((sqrt(x)*e + d)**n*c)*a**2
*b*e**2*x + 36*log((sqrt(x)*e + d)**n*c)*a*b**2*d**2*n - 12*log((sqrt(x)*e
 + d)**n*c)*a*b**2*e**2*n*x - 42*log((sqrt(x)*e + d)**n*c)*b**3*d**2*n**2
+ 6*log((sqrt(x)*e + d)**n*c)*b**3*e**2*n**2*x + 4*a**3*e**2*x - 6*a**2*b*
e**2*n*x + 6*a*b**2*e**2*n**2*x - 3*b**3*e**2*n**3*x)/(4*e**2)
```

3.418 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx = 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right)+6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)-12b^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right)+12b^3n^3\operatorname{PolyLog}\left(4,1+\frac{e\sqrt{x}}{d}\right)$$

output
```
2*(a+b*ln(c*(d+e*x^(1/2))^n))^3*ln(-e*x^(1/2)/d)+6*b*n*(a+b*ln(c*(d+e*x^(1
/2))^n))^2*polylog(2,1+e*x^(1/2)/d)-12*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))
*polylog(3,1+e*x^(1/2)/d)+12*b^3*n^3*polylog(4,1+e*x^(1/2)/d)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 333 vs. $2(135) = 270$.

Time = 0.18 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.47

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx = \left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log(x) + 3bn\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(\left(\log\left(d+e\sqrt{x}\right)-\log\left(1+\frac{e\sqrt{x}}{d}\right)\right)\log(x)-2\,\text{PolyLog}\left(2,-\frac{e\sqrt{x}}{d}\right)\right)+6b^2n^2\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(\log^2\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+2\log\left(d+e\sqrt{x}\right)\text{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)-2\,\text{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right)\right)+2b^3n^3\left(\log^3\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+3\log^2\left(d+e\sqrt{x}\right)\text{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)-6\log\left(d+e\sqrt{x}\right)\text{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right)+6\,\text{PolyLog}\left(4,1+\frac{e\sqrt{x}}{d}\right)\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```

output
```
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[x] + 3*b*n
*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*((Log[d + e*S
qrt[x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)])
 + 6*b^2*n^2*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(Lo
g[d + e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[x]]*PolyLog[2,
 1 + (e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d]) + 2*b^3*n^3*(Log[d
+ e*Sqrt[x]]^3*Log[-((e*Sqrt[x])/d)] + 3*Log[d + e*Sqrt[x]]^2*PolyLog[2, 1
 + (e*Sqrt[x])/d] - 6*Log[d + e*Sqrt[x]]*PolyLog[3, 1 + (e*Sqrt[x])/d] + 6
*PolyLog[4, 1 + (e*Sqrt[x])/d])
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{2843}$$

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3-3ben\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right)}{d+e\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{2881}$$

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3-3bn\int \frac{\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(cx^{n/2}\right)\right)^2}{\sqrt{x}}d\left(d+e\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{2821}$$

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3-3bn\left(2bn\int\frac{\left(a+b\log\left(cx^{n/2}\right)\right)\text{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)}{\sqrt{x}}d(d+e\sqrt{x})\right.\right.$$

$\downarrow$ 2830

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+e\sqrt{x}}{d}\right)\left(a+b\log\left(cx^{n/2}\right)\right)-bn\int\right.\right.\right.$$

$\downarrow$ 7143

$$2\left(\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+e\sqrt{x}}{d}\right)\left(a+b\log\left(cx^{n/2}\right)\right)-bn\,\text{Poly}\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```

output
```
2*((a + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[-((e*Sqrt[x])/d)] - 3*b*n*(-((a
+ b*Log[c*x^(n/2)])^2*PolyLog[2, (d + e*Sqrt[x])/d]) + 2*b*n*((a + b*Log[c
*x^(n/2)])*PolyLog[3, (d + e*Sqrt[x])/d] - b*n*PolyLog[4, (d + e*Sqrt[x])/
d])))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c
)^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="maxima")
```

output
```
b^3*log((e*sqrt(x) + d)^n)^3*log(x) + integrate(-1/2*(3*(b^3*e*n*x*log(x)
- 2*(b^3*e*log(c) + a*b^2*e)*x - 2*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*log((
e*sqrt(x) + d)^n)^2 - 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*l
og(c) + a^3*e)*x - 6*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b
^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n
) - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqr
t(x))/(e*x^2 + d*x^(3/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)^3}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))^3/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^n))^3/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx$$
$$= \frac{2\left(\int \frac{\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)^3}{-e^2x^2+d^2x}dx\right) b^3d^2n + 6\left(\int \frac{\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)^2}{-e^2x^2+d^2x}dx\right) a\,b^2d^2n + 6\left(\int \frac{\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)}{-e^2x^2+d^2x}dx\right) a^2b\,d^2n - 2\left(\int \frac{\sqrt{}}{}\right.}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x)
```

output
```
(2*int(log((sqrt(x)*e + d)**n*c)**3/(d**2*x - e**2*x**2),x)*b**3*d**2*n +
6*int(log((sqrt(x)*e + d)**n*c)**2/(d**2*x - e**2*x**2),x)*a*b**2*d**2*n +
 6*int(log((sqrt(x)*e + d)**n*c)/(d**2*x - e**2*x**2),x)*a**2*b*d**2*n - 2
*int((sqrt(x)*log((sqrt(x)*e + d)**n*c)**3)/(d**2*x - e**2*x**2),x)*b**3*d
*e*n - 6*int((sqrt(x)*log((sqrt(x)*e + d)**n*c)**2)/(d**2*x - e**2*x**2),x
)*a*b**2*d*e*n - 6*int((sqrt(x)*log((sqrt(x)*e + d)**n*c))/(d**2*x - e**2*
x**2),x)*a**2*b*d*e*n + log((sqrt(x)*e + d)**n*c)**4*b**3 + 4*log((sqrt(x)
*e + d)**n*c)**3*a*b**2 + 6*log((sqrt(x)*e + d)**n*c)**2*a**2*b + 2*log(x)
*a**3*n)/(2*n)
```

3.419 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 263

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx = -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} - \frac{3be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + \frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^2} + \frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^2} + \frac{6b^3e^2n^3\operatorname{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)}{d^2} + \frac{6b^3e^2n^3\operatorname{PolyLog}\left(3,\frac{d}{d+e\sqrt{x}}\right)}{d^2}$$

output
```
-3*b*e*n*(d+e*x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d^2/x^(1/2)-3*b*e^2*n
*ln(1-d/(d+e*x^(1/2)))*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d^2-(a+b*ln(c*(d+e*x^
(1/2))^n))^3/x+6*b^2*e^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(-e*x^(1/2)/d)/
d^2+6*b^2*e^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*polylog(2,d/(d+e*x^(1/2)))/d
^2+6*b^3*e^2*n^3*polylog(2,1+e*x^(1/2)/d)/d^2+6*b^3*e^2*n^3*polylog(3,d/(d
+e*x^(1/2)))/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 536 vs. $2(263) = 526$.

Time = 0.84 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.04

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= \frac{-3bden\sqrt{x}\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-3bd^2n\log\left(d+e\sqrt{x}\right)\left(a-bn\log\left(d+e\sqrt{x}\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

output
```
(-3*b*d*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^
n])^2 - 3*b*d^2*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c
*(d + e*Sqrt[x])^n])^2 + 3*b*e^2*n*x*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e
*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 - d^2*(a - b*n*Log[d + e*Sqrt[x]
] + b*Log[c*(d + e*Sqrt[x])^n])^3 - (3*b*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x
]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x])/2 + 3*b^2*n^2*(a - b*n*Log[d +
e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((d + e*Sqrt[x])*Log[d + e*Sqrt[x
]]*(-2*e*Sqrt[x] + (-d + e*Sqrt[x])*Log[d + e*Sqrt[x]]) - 2*e^2*x*(-1 + Lo
g[d + e*Sqrt[x]])*Log[-((e*Sqrt[x])/d)] - 2*e^2*x*PolyLog[2, 1 + (e*Sqrt[x
])/d]) + b^3*n^3*((d + e*Sqrt[x])*Log[d + e*Sqrt[x]]^2*(-3*e*Sqrt[x] + (-d
 + e*Sqrt[x])*Log[d + e*Sqrt[x]]) - 3*e^2*x*(-2 + Log[d + e*Sqrt[x]])*Log[
d + e*Sqrt[x]]*Log[-((e*Sqrt[x])/d)] - 6*e^2*x*(-1 + Log[d + e*Sqrt[x]])*P
olyLog[2, 1 + (e*Sqrt[x])/d] + 6*e^2*x*PolyLog[3, 1 + (e*Sqrt[x])/d]))/(d^
2*x)
```

Rubi [A] (warning: unable to verify)

Time = 1.96 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {2904, 2845, 2858, 27, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}dx$$

$$\Big\downarrow \text{2904}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{2845}$$

$$2\left(\frac{3}{2}ben\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{\left(d+e\sqrt{x}\right)x}d\sqrt{x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x}\right)$$

$$\Big\downarrow \text{2858}$$

$$2\left(\frac{3}{2}bn\int \frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{x^{3/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{3}{2}be^2n\int \frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^2x^{3/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x}\right)$$

$$\Big\downarrow \text{2789}$$

$$2\left(\frac{3}{2}be^2n\left(\frac{\int \frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^2x}d\left(d+e\sqrt{x}\right)}{d}+\frac{\int -\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{ex}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x}\right)$$

$$\Big\downarrow \text{2755}$$

$$2\left(\frac{3}{2}be^2n\left(\frac{-\frac{2bn\int -\frac{a+b\log\left(cx^{n/2}\right)}{e\sqrt{x}}d\left(d+e\sqrt{x}\right)}{d}-\frac{\left(d+e\sqrt{x}\right)\left(a+b\log\left(cx^{n/2}\right)\right)^2}{de\sqrt{x}}}{d}+\frac{\int -\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{ex}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{(a+b

$$\Big\downarrow 2754$$

$$2\left(\frac{3}{2}be^2n\left(\frac{-\frac{2bn\left(bn\int\frac{\log\left(1-\frac{d+e\sqrt{x}}{d}\right)}{\sqrt{x}}d(d+e\sqrt{x})-\log\left(1-\frac{d+e\sqrt{x}}{d}\right)(a+b\log(cx^{n/2}))\right)}{d}-\frac{(d+e\sqrt{x})(a+b\log(cx^{n/2}))^2}{de\sqrt{x}}}{d}+\int-\frac{(a+b\log(}{e}\right.\right.$$

$$\Big\downarrow 2779$$

$$2\left(\frac{3}{2}be^2n\left(\frac{-\frac{2bn\left(bn\int\frac{\log\left(1-\frac{d+e\sqrt{x}}{d}\right)}{\sqrt{x}}d(d+e\sqrt{x})-\log\left(1-\frac{d+e\sqrt{x}}{d}\right)(a+b\log(cx^{n/2}))\right)}{d}-\frac{(d+e\sqrt{x})(a+b\log(cx^{n/2}))^2}{de\sqrt{x}}}{d}+\frac{2bn\int\frac{\log\left(1-\right.}{\sqrt{}}}{}\right.\right.$$

$$\Big\downarrow 2821$$

$$2\left(\frac{3}{2}be^2n\left(\frac{\frac{2bn\left(\operatorname{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)(a+b\log(cx^{n/2}))-bn\int\frac{\operatorname{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)}{\sqrt{x}}d(d+e\sqrt{x})\right)}{d}-\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)(a+b\log(cx^{n/2}))^2}{d}}{d}+\frac{-\frac{2bn\left(bn\int\right.}{}}{}\right.\right.$$

$$\Big\downarrow 2838$$

$$2\left(\frac{3}{2}be^2n\left(\frac{\frac{2bn\left(\operatorname{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)(a+b\log(cx^{n/2}))-bn\int\frac{\operatorname{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)}{\sqrt{x}}d(d+e\sqrt{x})\right)}{d}-\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)(a+b\log(cx^{n/2}))^2}{d}}{d}+\frac{-\frac{2bn\left(-\log\right.}{}}{}\right.\right.$$

$$\Big\downarrow 7143$$

$$2\left(\frac{3}{2}be^2n\left(\frac{-\frac{2bn\left(-\log\left(1-\frac{d+e\sqrt{x}}{d}\right)(a+b\log(cx^{n/2}))-bn\operatorname{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)\right)}{d}-\frac{(d+e\sqrt{x})(a+b\log(cx^{n/2}))^2}{de\sqrt{x}}}{d}+\frac{2bn\left(\operatorname{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)\right.}{}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

output
```
2*(-1/2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x + (3*b*e^2*n*((-(((d + e*Sqrt
[x])*(a + b*Log[c*x^(n/2)])^2)/(d*e*Sqrt[x])) - (2*b*n*(-(Log[1 - (d + e*S
qrt[x])/d]*(a + b*Log[c*x^(n/2)])) - b*n*PolyLog[2, (d + e*Sqrt[x])/d]))/d
)/d + (-((Log[1 - d/Sqrt[x]]*(a + b*Log[c*x^(n/2)])^2)/d) + (2*b*n*((a + b
*Log[c*x^(n/2)])*PolyLog[2, d/Sqrt[x]] + b*n*PolyLog[3, d/Sqrt[x]]))/d)/d)
)/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx = \int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c
)^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx = \int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**2,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b^3*d^2*sqrt(x)*log((e*sqrt(x) + d)^n)^3 - 3*(2*b^3*e^2*n*x^(3/2)*
log(e*sqrt(x) + d) - 2*b^3*d*e*n*x - (b^3*e^2*n*x*log(x) + 2*b^3*d^2*log(c
) + 2*a*b^2*d^2)*sqrt(x))*log((e*sqrt(x) + d)^n)^2)/(d^2*x^(3/2)) - integr
ate(-1/2*(2*(b^3*d^2*e*log(c)^3 + 3*a*b^2*d^2*e*log(c)^2 + 3*a^2*b*d^2*e*l
og(c) + a^3*d^2*e)*x^(3/2) + 2*(b^3*d^3*log(c)^3 + 3*a*b^2*d^3*log(c)^2 +
3*a^2*b*d^3*log(c) + a^3*d^3)*x - 3*(2*b^3*e^3*n^2*x^(5/2)*log(e*sqrt(x) +
 d) - 2*b^3*d*e^2*n^2*x^2 - 2*(b^3*d^2*e*log(c)^2 + 2*a*b^2*d^2*e*log(c) +
 a^2*b*d^2*e)*x^(3/2) - 2*(b^3*d^3*log(c)^2 + 2*a*b^2*d^3*log(c) + a^2*b*d
^3)*x - (b^3*e^3*n^2*x^2*log(x) + 2*(b^3*d^2*e*n*log(c) + a*b^2*d^2*e*n)*x
)*sqrt(x))*log((e*sqrt(x) + d)^n))/(d^2*e*x^(7/2) + d^3*x^3), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)^3}{x^2}\,dx$$

input

```
int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^2,x)
```

output

```
int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$$
$$=\frac{-6\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)^2 b^3den-12\sqrt{x}\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)a\,b^2den-6\sqrt{x}\,a^2bden-3\left(\int\frac{\log\left(\left(\sqrt{x}\,e+d\right)^n c\right)}{-e^2x^2+d^2x}\right.}{}$$

input

```
int((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x)
```

output

```
( - 6*sqrt(x)*log((sqrt(x)*e + d)**n*c)**2*b**3*d*e*n - 12*sqrt(x)*log((sq
rt(x)*e + d)**n*c)*a*b**2*d*e*n - 6*sqrt(x)*a**2*b*d*e*n - 3*int(log((sqrt
(x)*e + d)**n*c)**2/(d**2*x - e**2*x**2),x)*b**3*d**2*e**2*n*x - 6*int(log
((sqrt(x)*e + d)**n*c)/(d**2*x - e**2*x**2),x)*a*b**2*d**2*e**2*n*x + 6*in
t(log((sqrt(x)*e + d)**n*c)/(d**2*x - e**2*x**2),x)*b**3*d**2*e**2*n**2*x
 + 3*int((sqrt(x)*log((sqrt(x)*e + d)**n*c)**2)/(d**2*x - e**2*x**2),x)*b**
3*d*e**3*n*x + 6*int((sqrt(x)*log((sqrt(x)*e + d)**n*c))/(d**2*x - e**2*x*
*2),x)*a*b**2*d*e**3*n*x - 6*int((sqrt(x)*log((sqrt(x)*e + d)**n*c))/(d**2
*x - e**2*x**2),x)*b**3*d*e**3*n**2*x - 6*log(sqrt(x))*a**2*b*e**2*n*x + 1
2*log(sqrt(x))*a*b**2*e**2*n**2*x - 2*log((sqrt(x)*e + d)**n*c)**3*b**3*d*
*2 - 6*log((sqrt(x)*e + d)**n*c)**2*a*b**2*d**2 - 6*log((sqrt(x)*e + d)**n
*c)*a**2*b*d**2 + 6*log((sqrt(x)*e + d)**n*c)*a**2*b*e**2*x - 12*log((sqrt
(x)*e + d)**n*c)*a*b**2*e**2*n*x - 2*a**3*d**2)/(2*d**2*x)
```

3.420 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 573

$$\begin{aligned}
&\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx \\
&= -\frac{b^3e^3n^3}{2d^3\sqrt{x}}+\frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4}-\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} \\
&\quad+\frac{5b^2e^3n^2\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4\sqrt{x}} \\
&\quad+\frac{5b^2e^4n^2\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4} \\
&\quad-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}}+\frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4d^2x} \\
&\quad-\frac{3be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4\sqrt{x}} \\
&\quad-\frac{3be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4} \\
&\quad-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2}+\frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^4} \\
&\quad-\frac{3b^3e^4n^3\log(x)}{2d^4}-\frac{5b^3e^4n^3\,\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{2d^4} \\
&\quad+\frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^4} \\
&\quad+\frac{3b^3e^4n^3\,\text{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)}{d^4}+\frac{3b^3e^4n^3\,\text{PolyLog}\left(3,\frac{d}{d+e\sqrt{x}}\right)}{d^4}
\end{aligned}$$

output
```
-1/2*b^3*e^3*n^3/d^3/x^(1/2)+1/2*b^3*e^4*n^3*ln(d+e*x^(1/2))/d^4-1/2*b^2*e
^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))/d^2/x+5/2*b^2*e^3*n^2*(d+e*x^(1/2))*(a+
b*ln(c*(d+e*x^(1/2))^n))/d^4/x^(1/2)+5/2*b^2*e^4*n^2*ln(1-d/(d+e*x^(1/2)))
*(a+b*ln(c*(d+e*x^(1/2))^n))/d^4-1/2*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d
/x^(3/2)+3/4*b*e^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d^2/x-3/2*b*e^3*n*(d+e*
x^(1/2))*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d^4/x^(1/2)-3/2*b*e^4*n*ln(1-d/(d+e
*x^(1/2)))*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d^4-1/2*(a+b*ln(c*(d+e*x^(1/2))^n
))^3/x^2+3*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(-e*x^(1/2)/d)/d^4-3/
2*b^3*e^4*n^3*ln(x)/d^4-5/2*b^3*e^4*n^3*polylog(2,d/(d+e*x^(1/2)))/d^4+3*b
^2*e^4*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*polylog(2,d/(d+e*x^(1/2)))/d^4+3*b^
3*e^4*n^3*polylog(2,1+e*x^(1/2)/d)/d^4+3*b^3*e^4*n^3*polylog(3,d/(d+e*x^(1
/2)))/d^4
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 841, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

output
```
-1/4*(2*b*d^3*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqr
t[x])^n])^2 - 3*b*d^2*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e
*Sqrt[x])^n])^2 + 6*b*d*e^3*n*x^(3/2)*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[
c*(d + e*Sqrt[x])^n])^2 + 6*b*d^4*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*
Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 - 6*b*e^4*n*x^2*Log[d + e*Sqrt[x]
]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 + 2*d^4*(a -
 b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3 + 3*b*e^4*n*x^2*(a
 - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] - 2*b^2*n
^2*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(-3*(d^4 - e^
4*x^2)*Log[d + e*Sqrt[x]]^2 + e^2*x*(-d^2 + 5*d*e*Sqrt[x] + 11*e^2*x*Log[-
((e*Sqrt[x])/d)]) - Log[d + e*Sqrt[x]]*(2*d^3*e*Sqrt[x] - 3*d^2*e^2*x + 6*
d*e^3*x^(3/2) + 11*e^4*x^2 + 6*e^4*x^2*Log[-((e*Sqrt[x])/d)]) - 6*e^4*x^2*
PolyLog[2, 1 + (e*Sqrt[x])/d]) + b^3*n^3*(d^2*e^2*x*(2 - 3*Log[d + e*Sqrt[
x]])*Log[d + e*Sqrt[x]] + 2*d^3*e*Sqrt[x]*Log[d + e*Sqrt[x]]^2 + 2*d^4*Log
[d + e*Sqrt[x]]^3 + 2*d*e^3*x^(3/2)*(1 - 5*Log[d + e*Sqrt[x]] + 3*Log[d +
e*Sqrt[x]]^2) + 12*e^4*x^2*(-Log[d + e*Sqrt[x]] + Log[-((e*Sqrt[x])/d)]) +
 11*e^4*x^2*(Log[d + e*Sqrt[x]]*(Log[d + e*Sqrt[x]] - 2*Log[-((e*Sqrt[x])/
d)]) - 2*PolyLog[2, 1 + (e*Sqrt[x])/d]) - 2*e^4*x^2*(Log[d + e*Sqrt[x]]^2*
(Log[d + e*Sqrt[x]] - 3*Log[-((e*Sqrt[x])/d)]) - 6*Log[d + e*Sqrt[x]]*Poly
Log[2, 1 + (e*Sqrt[x])/d] + 6*PolyLog[3, 1 + (e*Sqrt[x])/d])))/(d^4*x^2...
```

Rubi [A] (warning: unable to verify)

Time = 5.23 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.05, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx$$

$$\Big\downarrow \text{2904}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{2845}$$

$$2\left(\frac{3}{4}ben\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{\left(d+e\sqrt{x}\right)x^2}d\sqrt{x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{4x^2}\right)$$

$\big\downarrow$ 2858

$$2\left(\frac{3}{4}bn\int \frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{x^{5/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{4x^2}\right)$$

$\big\downarrow$ 27

$$2\left(\frac{3}{4}be^4n\int \frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^4x^{5/2}}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{4x^2}\right)$$

$\big\downarrow$ 2789

$$2\left(\frac{3}{4}be^4n\left(\frac{\int \frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^4x^2}d\left(d+e\sqrt{x}\right)}{d}+\frac{\int -\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^3x^2}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{4x^2}\right)$$

$\big\downarrow$ 2756

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^2}d\left(d+e\sqrt{x}\right)-\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{3e^3x^{3/2}}}{d}+\frac{\int -\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^3x^2}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{(a+b\log}{}\right.$$

$\big\downarrow$ 2789

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{e^3x^{3/2}}d\left(d+e\sqrt{x}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{n/2}\right)}{e^2x^{3/2}}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{3e^3x^{3/2}}}{d}+\frac{\int -\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^3x^{3/2}}d(d}{d}\right.\right.$$

$\big\downarrow$ 2756

$$2\left(\frac{3}{4}be^4n\left(\frac{\frac{\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{2e^2x}-bn\int \frac{a+b\log\left(cx^{n/2}\right)}{e^2x^{3/2}}d\left(d+e\sqrt{x}\right)}{d}+\frac{\int \frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^2x^{3/2}}d\left(d+e\sqrt{x}\right)}{d}}{d}+\frac{-\frac{2}{3}bn\left(\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\int \frac{1}{e^2x}}{d}\right.}{}\right.\right.$$

$\big\downarrow$ 54

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\int\left(-\frac{1}{d^2e\sqrt{x}}+\frac{1}{d^2\sqrt{x}}+\frac{1}{de^2x}\right)d\left(d+e\sqrt{x}\right)}{d}+\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^2x^{3/2}}d\left(d+e\sqrt{x}\right)}{d}\right)-\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{3e^3x^{3/2}}}{d}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^2x^{3/2}}d\left(d+e\sqrt{x}\right)}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{\log\left(-e\sqrt{x}\right)}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}\right)-\frac{\left(a+b\log\left(cx^{n/2}\right)\right)^2}{3e^3x^{3/2}}}{d}+\right.\right.$$

$$\Big\downarrow \text{2789}$$

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\int\frac{a+b\log\left(cx^{n/2}\right)}{e^2x}d\left(d+e\sqrt{x}\right)}{d}+\frac{\int-\frac{a+b\log\left(cx^{n/2}\right)}{ex}d\left(d+e\sqrt{x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{\log\left(-e\sqrt{x}\right)}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}\right)}{d}\right.\right.$$

$$\Big\downarrow \text{2751}$$

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{-\frac{bn\int-\frac{1}{e\sqrt{x}}d\left(d+e\sqrt{x}\right)}{d}-\frac{\left(d+e\sqrt{x}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}+\frac{\int-\frac{a+b\log\left(cx^{n/2}\right)}{ex}d\left(d+e\sqrt{x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log\left(d+e\sqrt{x}\right)}{d^2}-\right.}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \text{16}$$

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{ex}d(d+e\sqrt{x})}{d}+\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots\right.\right.$$

$\downarrow$ 2755

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{ex}d(d+e\sqrt{x})}{d}+\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots\right.\right.$$

$\downarrow$ 2754

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\int -\frac{a+b\log\left(cx^{n/2}\right)}{ex}d(d+e\sqrt{x})}{d}+\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{\cdots}}{d^2}\cdots\right.}{d}\cdots\right.}{d}\cdots\right.\right.$$

$\downarrow$ 2779

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\frac{bn\int\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)}{\sqrt{x}}d(d+e\sqrt{x})}{d}-\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{d}}{d}+\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}\cdots}{\cdots}\cdots\right.}{d}\cdots\right.\right.$$

$\downarrow$ 2821

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\frac{bn\int\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)}{\sqrt{x}}d(d+e\sqrt{x})}{d}-\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{d}}{d}+\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}}{d}+\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}\right.}{d}\right.\right.$$

$\downarrow$ 2838

$$2\left(\frac{3}{4}be^4n\left(\frac{\frac{\frac{\frac{2bn\left(\mathrm{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)-bn\int\frac{\mathrm{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right)}{\sqrt{x}}d(d+e\sqrt{x})\right)}{d}}{d}-\frac{\log\left(1-\frac{d}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)^2}{d}}{d}+\frac{-\frac{2bn\left(-\log\left(1-\frac{d+e\sqrt{x}}{d}\right)\right.}{}}{}}{}\right.\right.$$

$\downarrow$ 7143

$$2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{a+b\log\left(cx^{n/2}\right)}{2e^2x}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt{x})}{d^2}-\frac{\log(-e\sqrt{x})}{d^2}-\frac{1}{de\sqrt{x}}\right)}{d}+\frac{\frac{bn\log(-e\sqrt{x})}{d}-\frac{(d+e\sqrt{x})\left(a+b\log\left(cx^{n/2}\right)\right)}{de\sqrt{x}}}{d}+\frac{\frac{bn\,\mathrm{PolyLog}\left(2,\frac{d}{\sqrt{x}}\right.}{d}}{d}\right.}{d}\right.\right.$$

input

```
Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

output
```
2*(-1/4*(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2 + (3*b*e^4*n*((-1/3*(a + b*
Log[c*x^(n/2)])^2/(e^3*x^(3/2)) - (2*b*n*((-1/2*(b*n*(-(1/(d*e*Sqrt[x])) +
 Log[d + e*Sqrt[x]]/d^2 - Log[-(e*Sqrt[x])]/d^2)) + (a + b*Log[c*x^(n/2)])
/(2*e^2*x))/d + (((b*n*Log[-(e*Sqrt[x])])/d - ((d + e*Sqrt[x])*(a + b*Log[
c*x^(n/2)]))/(d*e*Sqrt[x]))/d + (-((Log[1 - d/Sqrt[x]]*(a + b*Log[c*x^(n/2
)]))/d) + (b*n*PolyLog[2, d/Sqrt[x]])/d)/d)/d))/3)/d + (((a + b*Log[c*x^(n
/2)])^2/(2*e^2*x) - b*n*(((b*n*Log[-(e*Sqrt[x])])/d - ((d + e*Sqrt[x])*(a
+ b*Log[c*x^(n/2)]))/(d*e*Sqrt[x]))/d + (-((Log[1 - d/Sqrt[x]]*(a + b*Log[
c*x^(n/2)]))/d) + (b*n*PolyLog[2, d/Sqrt[x]])/d)/d))/d + ((-(((d + e*Sqrt[
x])*(a + b*Log[c*x^(n/2)])^2)/(d*e*Sqrt[x])) - (2*b*n*(-(Log[1 - (d + e*Sq
rt[x])/d]*(a + b*Log[c*x^(n/2)])) - b*n*PolyLog[2, (d + e*Sqrt[x])/d]))/d)
/d + (-((Log[1 - d/Sqrt[x]]*(a + b*Log[c*x^(n/2)])^2)/d) + (2*b*n*((a + b*
Log[c*x^(n/2)])*PolyLog[2, d/Sqrt[x]] + b*n*PolyLog[3, d/Sqrt[x]]))/d)/d)/
d)/d))/4)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c
)^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**3,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="maxima")
```

output
```
-1/2*b^3*log((e*sqrt(x) + d)^n)^3/x^2 + integrate(1/4*(3*(b^3*e*n*x + 4*(b
^3*e*log(c) + a*b^2*e)*x + 4*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*log((e*sqrt
(x) + d)^n)^2 + 4*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c)
+ a^3*e)*x + 12*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*
log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + 4
*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqrt(x))
/(e*x^4 + d*x^(7/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^3,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x)
```

output
```
( - 2*sqrt(x)*log((sqrt(x)*e + d)**n*c)**2*b**3*d**3*e*n - 6*sqrt(x)*log((
sqrt(x)*e + d)**n*c)**2*b**3*d*e**3*n*x - 4*sqrt(x)*log((sqrt(x)*e + d)**n
*c)*a*b**2*d**3*e*n - 12*sqrt(x)*log((sqrt(x)*e + d)**n*c)*a*b**2*d*e**3*n
*x + 10*sqrt(x)*log((sqrt(x)*e + d)**n*c)*b**3*d*e**3*n**2*x - 2*sqrt(x)*a
**2*b*d**3*e*n - 6*sqrt(x)*a**2*b*d*e**3*n*x + 10*sqrt(x)*a*b**2*d*e**3*n*
*2*x - 2*sqrt(x)*b**3*d*e**3*n**3*x - 3*int(log((sqrt(x)*e + d)**n*c)**2/(
d**2*x - e**2*x**2),x)*b**3*d**2*e**4*n*x**2 - 6*int(log((sqrt(x)*e + d)**
n*c)/(d**2*x - e**2*x**2),x)*a*b**2*d**2*e**4*n*x**2 + 11*int(log((sqrt(x)
*e + d)**n*c)/(d**2*x - e**2*x**2),x)*b**3*d**2*e**4*n**2*x**2 + 3*int((sq
rt(x)*log((sqrt(x)*e + d)**n*c)**2)/(d**2*x - e**2*x**2),x)*b**3*d*e**5*n*
x**2 + 6*int((sqrt(x)*log((sqrt(x)*e + d)**n*c))/(d**2*x - e**2*x**2),x)*a
*b**2*d*e**5*n*x**2 - 11*int((sqrt(x)*log((sqrt(x)*e + d)**n*c))/(d**2*x -
 e**2*x**2),x)*b**3*d*e**5*n**2*x**2 - 6*log(sqrt(x))*a**2*b*e**4*n*x**2 +
 22*log(sqrt(x))*a*b**2*e**4*n**2*x**2 - 12*log(sqrt(x))*b**3*e**4*n**3*x*
*2 - 2*log((sqrt(x)*e + d)**n*c)**3*b**3*d**4 - 6*log((sqrt(x)*e + d)**n*c
)**2*a*b**2*d**4 + 3*log((sqrt(x)*e + d)**n*c)**2*b**3*d**2*e**2*n*x - 6*l
og((sqrt(x)*e + d)**n*c)*a**2*b*d**4 + 6*log((sqrt(x)*e + d)**n*c)*a**2*b*
e**4*x**2 + 6*log((sqrt(x)*e + d)**n*c)*a*b**2*d**2*e**2*n*x - 22*log((sqr
t(x)*e + d)**n*c)*a*b**2*e**4*n*x**2 - 2*log((sqrt(x)*e + d)**n*c)*b**3*d*
*2*e**2*n**2*x + 12*log((sqrt(x)*e + d)**n*c)*b**3*e**4*n**2*x**2 - 2*a...
```

3.421 $\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right) dx$

Optimal result

Integrand size = 22, antiderivative size = 171

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) dx = \frac{be^7 n\sqrt{x}}{4d^7} - \frac{be^6 nx}{8d^6} + \frac{be^5 nx^{3/2}}{12d^5} - \frac{be^4 nx^2}{16d^4} + \frac{be^3 nx^{5/2}}{20d^3} - \frac{be^2 nx^3}{24d^2} + \frac{benx^{7/2}}{28d} - \frac{be^8 n \log \left(d+\frac{e}{\sqrt{x}}\right)}{4d^8} + \frac{1}{4} x^4 \left(a+b \log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) - \frac{be^8 n \log(x)}{8d^8}$$

output
```
1/4*b*e^7*n*x^(1/2)/d^7-1/8*b*e^6*n*x/d^6+1/12*b*e^5*n*x^(3/2)/d^5-1/16*b*
e^4*n*x^2/d^4+1/20*b*e^3*n*x^(5/2)/d^3-1/24*b*e^2*n*x^3/d^2+1/28*b*e*n*x^(
7/2)/d-1/4*b*e^8*n*ln(d+e/x^(1/2))/d^8+1/4*x^4*(a+b*ln(c*(d+e/x^(1/2))^n))
-1/8*b*e^8*n*ln(x)/d^8
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx=\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{8}ben\left(\frac{2e^6\sqrt{x}}{d^7}-\frac{e^5x}{d^6}+\frac{2e^4x^{3/2}}{3d^5}-\frac{e^3x^2}{2d^4}+\frac{2e^2x^{5/2}}{5d^3}-\frac{ex^3}{3d^2}+\frac{2x^{7/2}}{7d}-\frac{2e^7\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^8}-\frac{e^7\log(x)}{d^8}\right)$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

output
```
(a*x^4)/4 + (b*x^4*Log[c*(d + e/Sqrt[x])^n])/4 + (b*e*n*((2*e^6*Sqrt[x])/d
^7 - (e^5*x)/d^6 + (2*e^4*x^(3/2))/(3*d^5) - (e^3*x^2)/(2*d^4) + (2*e^2*x^
(5/2))/(5*d^3) - (e*x^3)/(3*d^2) + (2*x^(7/2))/(7*d) - (2*e^7*Log[d + e/Sq
rt[x]])/d^8 - (e^7*Log[x])/d^8))/8
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int x^{9/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2842}$$

$$-2\left(\frac{1}{8}ben\int\frac{x^4}{d+\frac{e}{\sqrt{x}}}d\frac{1}{\sqrt{x}}-\frac{1}{8}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\right)$$

$\big\downarrow$ 54

$$-2\left(\frac{1}{8}ben\int\left(\frac{e^8}{d^8\left(d+\frac{e}{\sqrt{x}}\right)}-\frac{\sqrt{x}e^7}{d^8}+\frac{xe^6}{d^7}-\frac{x^{3/2}e^5}{d^6}+\frac{x^2e^4}{d^5}-\frac{x^{5/2}e^3}{d^4}+\frac{x^3e^2}{d^3}-\frac{x^{7/2}e}{d^2}+\frac{x^4}{d}\right)d\frac{1}{\sqrt{x}}-\frac{1}{8}x^4\left(a\right.\right.$$

$\big\downarrow$ 2009

$$-2\left(\frac{1}{8}ben\left(\frac{e^7\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^8}-\frac{e^7\log\left(\frac{1}{\sqrt{x}}\right)}{d^8}-\frac{e^6\sqrt{x}}{d^7}+\frac{e^5x}{2d^6}-\frac{e^4x^{3/2}}{3d^5}+\frac{e^3x^2}{4d^4}-\frac{e^2x^{5/2}}{5d^3}+\frac{ex^3}{6d^2}-\frac{x^{7/2}}{7d}\right)-\frac{1}{8}x^4\left(\right.\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

output
```
-2*(-1/8*(x^4*(a + b*Log[c*(d + e/Sqrt[x])^n])) + (b*e*n*(-((e^6*Sqrt[x])/
d^7) + (e^5*x)/(2*d^6) - (e^4*x^(3/2))/(3*d^5) + (e^3*x^2)/(4*d^4) - (e^2*
x^(5/2))/(5*d^3) + (e*x^3)/(6*d^2) - x^(7/2)/(7*d) + (e^7*Log[d + e/Sqrt[x
]])/d^8 - (e^7*Log[1/Sqrt[x]])/d^8))/8)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

input
```
int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

output
```
int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx = \frac{420\,bd^8x^4\log\left(c\right)-70\,bd^6e^2nx^3+420\,ad^8x^4-105\,bd^4e^4nx^2-210\,bd^2e^6nx-420\,bd^8n\log\left(\sqrt{x}\right)+420\,(b}{}$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

output
```
1/1680*(420*b*d^8*x^4*log(c) - 70*b*d^6*e^2*n*x^3 + 420*a*d^8*x^4 - 105*b*
d^4*e^4*n*x^2 - 210*b*d^2*e^6*n*x - 420*b*d^8*n*log(sqrt(x)) + 420*(b*d^8
- b*e^8)*n*log(d*sqrt(x) + e) + 420*(b*d^8*n*x^4 - b*d^8*n)*log((d*x + e*s
qrt(x))/x) + 4*(15*b*d^7*e*n*x^3 + 21*b*d^5*e^3*n*x^2 + 35*b*d^3*e^5*n*x +
 105*b*d*e^7*n)*sqrt(x))/d^8
```

Sympy [A] (verification not implemented)

Time = 46.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{ax^4}{4}+b\left(\frac{en\left(\frac{2x^{\frac{7}{2}}}{7d}-\frac{ex^3}{3d^2}+\frac{2e^2x^{\frac{5}{2}}}{5d^3}-\frac{e^3x^2}{2d^4}+\frac{2e^4x^{\frac{3}{2}}}{3d^5}-\frac{e^5x}{d^6}-\frac{2e^7\left(\begin{cases}\frac{\sqrt{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^7}+\frac{2e^6\sqrt{x}}{d^7}\right)}{8}+\frac{x^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{4}\right)$$

input

```
integrate(x**3*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

output

```
a*x**4/4 + b*(e*n*(2*x**(7/2)/(7*d) - e*x**3/(3*d**2) + 2*e**2*x**(5/2)/(5
*d**3) - e**3*x**2/(2*d**4) + 2*e**4*x**(3/2)/(3*d**5) - e**5*x/d**6 - 2*e
**7*Piecewise((sqrt(x)/e, Eq(d, 0)), (log(d*sqrt(x) + e)/d, True))/d**7 +
2*e**6*sqrt(x)/d**7)/8 + x**4*log(c*(d + e/sqrt(x))**n)/4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{4}\,ax^4$$
$$-\frac{1}{1680}\,ben\left(\frac{420\,e^7\log\left(d\sqrt{x}+e\right)}{d^8}-\frac{60\,d^6x^{\frac{7}{2}}-70\,d^5ex^3+84\,d^4e^2x^{\frac{5}{2}}-105\,d^3e^3x^2+140\,d^2e^4x^{\frac{3}{2}}-210\,d}{d^7}\right.$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

output
```
1/4*b*x^4*log(c*(d + e/sqrt(x))^n) + 1/4*a*x^4 - 1/1680*b*e*n*(420*e^7*log
(d*sqrt(x) + e)/d^8 - (60*d^6*x^(7/2) - 70*d^5*e*x^3 + 84*d^4*e^2*x^(5/2)
- 105*d^3*e^3*x^2 + 140*d^2*e^4*x^(3/2) - 210*d*e^5*x + 420*e^6*sqrt(x))/d
^7)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.56

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(c\right)+\frac{1}{4}\,ax^4$$
$$-\frac{\left(e^9\left(\frac{420\,\log\left(\frac{|d\sqrt{x}+e|}{\sqrt{|x|}}\right)}{d^8}-\frac{420\,\log\left(\left|-d+\frac{d\sqrt{x}+e}{\sqrt{x}}\right|\right)}{d^8}+\frac{1089\,d^7-\frac{4683\,(d\sqrt{x}+e)d^6}{\sqrt{x}}+\frac{9639\,(d\sqrt{x}+e)^2d^5}{x}-\frac{11165\,(d\sqrt{x}+e)^3d^4}{x^{\frac{3}{2}}}+\frac{7490\,(d\sqrt{x}+e)^4}{x^2}}{\left(d-\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^7d^8}\right.\right.}{1680\,e}$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

output
```
1/4*b*x^4*log(c) + 1/4*a*x^4 - 1/1680*(e^9*(420*log(abs(d*sqrt(x) + e)/sqr
t(abs(x)))/d^8 - 420*log(abs(-d + (d*sqrt(x) + e)/sqrt(x)))/d^8 + (1089*d^
7 - 4683*(d*sqrt(x) + e)*d^6/sqrt(x) + 9639*(d*sqrt(x) + e)^2*d^5/x - 1116
5*(d*sqrt(x) + e)^3*d^4/x^(3/2) + 7490*(d*sqrt(x) + e)^4*d^3/x^2 - 2730*(d
*sqrt(x) + e)^5*d^2/x^(5/2) + 420*(d*sqrt(x) + e)^6*d/x^3)/((d - (d*sqrt(x
) + e)/sqrt(x))^7*d^8)) - 420*e^9*log(-(e - d/(d/e - (d*sqrt(x) + e)/(e*sq
rt(x))))*(d/e - (d*sqrt(x) + e)/(e*sqrt(x))))/(d - (d*sqrt(x) + e)/sqrt(x)
)^8)*b*n/e
```

Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=\frac{\frac{b\,d\,e^7\,n\,\sqrt{x}}{4}-\frac{b\,d^2\,e^6\,n\,x}{8}+\frac{b\,d^7\,e\,n\,x^{7/2}}{28}-\frac{b\,d^4\,e^4\,n\,x^2}{16}-\frac{b\,d^6\,e^2\,n\,x^3}{24}+\frac{b\,d^3\,e^5\,n\,x^{3/2}}{12}+\frac{b\,d^5\,e^3\,n\,x^{5/2}}{20}+\frac{b\,e^8\,n\,\mathrm{atan}\left(\frac{d\,1\mathrm{i}+\frac{e\,2\mathrm{i}}{\sqrt{x}}}{d}\right)\,1\mathrm{i}}{2}}{d^8}+\frac{a\,x^4}{4}+\frac{b\,x^4\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{4}$$

input
```
int(x^3*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

output
```
((b*e^8*n*atan((d*1i + (e*2i)/x^(1/2))/d)*1i)/2 - (b*d^2*e^6*n*x)/8 + (b*d
*e^7*n*x^(1/2))/4 + (b*d^7*e*n*x^(7/2))/28 - (b*d^4*e^4*n*x^2)/16 - (b*d^6
*e^2*n*x^3)/24 + (b*d^3*e^5*n*x^(3/2))/12 + (b*d^5*e^3*n*x^(5/2))/20)/d^8
+ (a*x^4)/4 + (b*x^4*log(c*(d + e/x^(1/2))^n))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.96

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx = \frac{60\sqrt{x}\,b\,d^7en\,x^3+84\sqrt{x}\,b\,d^5e^3n\,x^2+140\sqrt{x}\,b\,d^3e^5nx+420\sqrt{x}\,bd\,e^7n+420\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b\,d^8x^4-420}{1680d}$$

input
```
int(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x)
```

output
```
(60*sqrt(x)*b*d**7*e*n*x**3 + 84*sqrt(x)*b*d**5*e**3*n*x**2 + 140*sqrt(x)*
b*d**3*e**5*n*x + 420*sqrt(x)*b*d*e**7*n + 420*log(((sqrt(x)*d + e)**n*c)/
x**(n/2))*b*d**8*x**4 - 420*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b*e**8 -
420*log(sqrt(x))*b*e**8*n + 420*a*d**8*x**4 - 70*b*d**6*e**2*n*x**3 - 105*
b*d**4*e**4*n*x**2 - 210*b*d**2*e**6*n*x)/(1680*d**8)
```

3.422 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^6n\log(x)}{6d^6}$$

output
```
1/3*b*e^5*n*x^(1/2)/d^5-1/6*b*e^4*n*x/d^4+1/9*b*e^3*n*x^(3/2)/d^3-1/12*b*e
^2*n*x^2/d^2+1/15*b*e*n*x^(5/2)/d-1/3*b*e^6*n*ln(d+e/x^(1/2))/d^6+1/3*x^3*
(a+b*ln(c*(d+e/x^(1/2))^n))-1/6*b*e^6*n*ln(x)/d^6
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{6}ben\left(\frac{2e^4\sqrt{x}}{d^5}-\frac{e^3x}{d^4}+\frac{2e^2x^{3/2}}{3d^3}-\frac{ex^2}{2d^2}+\frac{2x^{5/2}}{5d}-\frac{2e^5\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^6}-\frac{e^5\log(x)}{d^6}\right)$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

output
```
(a*x^3)/3 + (b*x^3*Log[c*(d + e/Sqrt[x])^n])/3 + (b*e*n*((2*e^4*Sqrt[x])/d
^5 - (e^3*x)/d^4 + (2*e^2*x^(3/2))/(3*d^3) - (e*x^2)/(2*d^2) + (2*x^(5/2))
/(5*d) - (2*e^5*Log[d + e/Sqrt[x]])/d^6 - (e^5*Log[x])/d^6))/6
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int x^{7/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2842}$$

$$-2\left(\frac{1}{6}ben\int \frac{x^3}{d+\frac{e}{\sqrt{x}}}d\frac{1}{\sqrt{x}}-\frac{1}{6}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\right)$$

$\big\downarrow$ 54

$$-2\left(\frac{1}{6}ben\int \left(\frac{e^6}{d^6\left(d+\frac{e}{\sqrt{x}}\right)}-\frac{\sqrt{x}e^5}{d^6}+\frac{xe^4}{d^5}-\frac{x^{3/2}e^3}{d^4}+\frac{x^2e^2}{d^3}-\frac{x^{5/2}e}{d^2}+\frac{x^3}{d}\right)d\frac{1}{\sqrt{x}}-\frac{1}{6}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.\right.$$

$\big\downarrow$ 2009

$$-2\left(\frac{1}{6}ben\left(\frac{e^5\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^6}-\frac{e^5\log\left(\frac{1}{\sqrt{x}}\right)}{d^6}-\frac{e^4\sqrt{x}}{d^5}+\frac{e^3x}{2d^4}-\frac{e^2x^{3/2}}{3d^3}+\frac{ex^2}{4d^2}-\frac{x^{5/2}}{5d}\right)-\frac{1}{6}x^3\left(a+b\log\left(c\left(d+\right.\right.\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

output
```
-2*(-1/6*(x^3*(a + b*Log[c*(d + e/Sqrt[x])^n])) + (b*e*n*(-((e^4*Sqrt[x])/
d^5) + (e^3*x)/(2*d^4) - (e^2*x^(3/2))/(3*d^3) + (e*x^2)/(4*d^2) - x^(5/2)
/(5*d) + (e^5*Log[d + e/Sqrt[x]])/d^6 - (e^5*Log[1/Sqrt[x]])/d^6))/6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

input
```
int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

output
```
int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx = \frac{60\,bd^6x^3\log(c)-15\,bd^4e^2nx^2+60\,ad^6x^3-30\,bd^2e^4nx-60\,bd^6n\log\left(\sqrt{x}\right)+60\,(bd^6-be^6)n\log\left(d\sqrt{x}+\right.}{180\,d^6}$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

output
```
1/180*(60*b*d^6*x^3*log(c) - 15*b*d^4*e^2*n*x^2 + 60*a*d^6*x^3 - 30*b*d^2*
e^4*n*x - 60*b*d^6*n*log(sqrt(x)) + 60*(b*d^6 - b*e^6)*n*log(d*sqrt(x) + e
) + 60*(b*d^6*n*x^3 - b*d^6*n)*log((d*x + e*sqrt(x))/x) + 4*(3*b*d^5*e*n*x
^2 + 5*b*d^3*e^3*n*x + 15*b*d*e^5*n)*sqrt(x))/d^6
```

Sympy [A] (verification not implemented)

Time = 16.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$

$$=\frac{ax^3}{3}+b\left(\frac{en\left(\frac{2x^{\frac{5}{2}}}{5d}-\frac{ex^2}{2d^2}+\frac{2e^2x^{\frac{3}{2}}}{3d^3}-\frac{e^3x}{d^4}-\frac{2e^5\left(\begin{cases}\frac{\sqrt{x}}{e} & \text{for } d=0\\ \frac{\log(d\sqrt{x}+e)}{d} & \text{otherwise}\end{cases}\right)}{d^5}+\frac{2e^4\sqrt{x}}{d^5}\right)}{6}+\frac{x^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3}\right)$$

input
```
integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

output
```
a*x**3/3 + b*(e*n*(2*x**(5/2)/(5*d) - e*x**2/(2*d**2) + 2*e**2*x**(3/2)/(3
*d**3) - e**3*x/d**4 - 2*e**5*Piecewise((sqrt(x)/e, Eq(d, 0)), (log(d*sqrt
(x) + e)/d, True))/d**5 + 2*e**4*sqrt(x)/d**5)/6 + x**3*log(c*(d + e/sqrt(
x))**n)/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right) dx = \frac{1}{3} b x^3 \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right) + \frac{1}{3} a x^3 - \frac{1}{180} b e n \left(\frac{60 e^5 \log \left(d\sqrt{x} + e\right)}{d^6} - \frac{12 d^4 x^{\frac{5}{2}} - 15 d^3 e x^2 + 20 d^2 e^2 x^{\frac{3}{2}} - 30 d e^3 x + 60 e^4 \sqrt{x}}{d^5}\right)$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

output
```
1/3*b*x^3*log(c*(d + e/sqrt(x))^n) + 1/3*a*x^3 - 1/180*b*e*n*(60*e^5*log(d
*sqrt(x) + e)/d^6 - (12*d^4*x^(5/2) - 15*d^3*e*x^2 + 20*d^2*e^2*x^(3/2) -
30*d*e^3*x + 60*e^4*sqrt(x))/d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(113) = 226.

Time = 0.17 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.67

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right) dx = \frac{1}{3} b x^3 \log (c) + \frac{1}{3} a x^3 - \frac{\left(e^7 \left(\frac{60 \log \left(\frac{|d\sqrt{x}+e|}{\sqrt{|x|}}\right)}{d^6} - \frac{60 \log \left(\left|-d + \frac{d\sqrt{x}+e}{\sqrt{x}}\right|\right)}{d^6} + \frac{137 d^5 - \frac{385 (d\sqrt{x}+e) d^4}{\sqrt{x}} + \frac{470 (d\sqrt{x}+e)^2 d^3}{x} - \frac{270 (d\sqrt{x}+e)^3 d^2}{x^{\frac{3}{2}}} + \frac{60 (d\sqrt{x}+e)^4 d}{x^2}}{\left(d - \frac{d\sqrt{x}+e}{\sqrt{x}}\right)^5 d^6}\right) - \right.}{180 e}$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

output
```
1/3*b*x^3*log(c) + 1/3*a*x^3 - 1/180*(e^7*(60*log(abs(d*sqrt(x) + e)/sqrt(
abs(x)))/d^6 - 60*log(abs(-d + (d*sqrt(x) + e)/sqrt(x)))/d^6 + (137*d^5 -
385*(d*sqrt(x) + e)*d^4/sqrt(x) + 470*(d*sqrt(x) + e)^2*d^3/x - 270*(d*sqr
t(x) + e)^3*d^2/x^(3/2) + 60*(d*sqrt(x) + e)^4*d/x^2)/((d - (d*sqrt(x) + e
)/sqrt(x))^5*d^6)) - 60*e^7*log(-(e - d/(d/e - (d*sqrt(x) + e)/(e*sqrt(x))
))*(d/e - (d*sqrt(x) + e)/(e*sqrt(x))))/(d - (d*sqrt(x) + e)/sqrt(x))^6)*b
*n/e
```

Mupad [B] (verification not implemented)

Time = 15.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{a\,x^3}{3}+\frac{b\left(60\,d^6\,x^3\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-120\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,\sqrt{x}}+1\right)-15\,d^4\,e^2\,n\,x^2+20\,d^3\,e^3\,n\,x^{3/2}-30\,d^2\,e^4\,n\,x\right.}{180\,d^6}$$

input
```
int(x^2*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

output
```
(a*x^3)/3 + (b*(60*d^6*x^3*log(c*(d + e/x^(1/2))^n) - 120*e^6*n*atanh((2*e
)/(d*x^(1/2)) + 1) - 15*d^4*e^2*n*x^2 + 20*d^3*e^3*n*x^(3/2) - 30*d^2*e^4*
n*x + 60*d*e^5*n*x^(1/2) + 12*d^5*e*n*x^(5/2)))/(180*d^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{12\sqrt{x}\,b\,d^5en\,x^2+20\sqrt{x}\,b\,d^3e^3nx+60\sqrt{x}\,bd\,e^5n+60\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b\,d^6x^3-60\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b\,e^6-6}{180d^6}$$

input
```
int(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x)
```

output
```
(12*sqrt(x)*b*d**5*e*n*x**2 + 20*sqrt(x)*b*d**3*e**3*n*x + 60*sqrt(x)*b*d*
e**5*n + 60*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b*d**6*x**3 - 60*log(((sq
rt(x)*d + e)**n*c)/x**(n/2))*b*e**6 - 60*log(sqrt(x))*b*e**6*n + 60*a*d**6
*x**3 - 15*b*d**4*e**2*n*x**2 - 30*b*d**2*e**4*n*x)/(180*d**6)
```

3.423 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^4n\log(x)}{4d^4}$$

output
```
1/2*b*e^3*n*x^(1/2)/d^3-1/4*b*e^2*n*x/d^2+1/6*b*e*n*x^(3/2)/d-1/2*b*e^4*n*
ln(d+e/x^(1/2))/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))-1/4*b*e^4*n*ln(x)/
d^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{4}ben\left(\frac{2e^2\sqrt{x}}{d^3}-\frac{ex}{d^2}+\frac{2x^{3/2}}{3d}-\frac{2e^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^4}-\frac{e^3\log(x)}{d^4}\right)$$

input
```
Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

output
```
(a*x^2)/2 + (b*x^2*Log[c*(d + e/Sqrt[x])^n])/2 + (b*e*n*((2*e^2*Sqrt[x])/d
^3 - (e*x)/d^2 + (2*x^(3/2))/(3*d) - (2*e^3*Log[d + e/Sqrt[x]])/d^4 - (e^3
*Log[x])/d^4))/4
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{ 2904}$$

$$-2\int x^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2842}$$

$$-2\left(\frac{1}{4}ben\int\frac{x^2}{d+\frac{e}{\sqrt{x}}}d\frac{1}{\sqrt{x}}-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\right)$$

$$\Big\downarrow \text{ 54}$$

$$-2\left(\frac{1}{4}ben\int\left(\frac{e^4}{d^4\left(d+\frac{e}{\sqrt{x}}\right)}-\frac{\sqrt{x}e^3}{d^4}+\frac{xe^2}{d^3}-\frac{x^{3/2}e}{d^2}+\frac{x^2}{d}\right)d\frac{1}{\sqrt{x}}-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-2\left(\frac{1}{4}ben\left(\frac{e^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^4}-\frac{e^3\log\left(\frac{1}{\sqrt{x}}\right)}{d^4}-\frac{e^2\sqrt{x}}{d^3}+\frac{ex}{2d^2}-\frac{x^{3/2}}{3d}\right)-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\right)$$

input
```
Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

output
```
-2*(-1/4*(x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])) + (b*e*n*(-((e^2*Sqrt[x])/
d^3) + (e*x)/(2*d^2) - x^(3/2)/(3*d) + (e^3*Log[d + e/Sqrt[x]])/d^4 - (e^3
*Log[1/Sqrt[x]])/d^4))/4)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)\,dx = \frac{6\,bd^4x^2\log\left(c\right)-3\,bd^2e^2nx+6\,ad^4x^2-6\,bd^4n\log\left(\sqrt{x}\right)+6\left(bd^4-be^4\right)n\log\left(d\sqrt{x}+e\right)+6\left(bd^4nx^2-bd\right.}{12\,d^4}$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

output
```
1/12*(6*b*d^4*x^2*log(c) - 3*b*d^2*e^2*n*x + 6*a*d^4*x^2 - 6*b*d^4*n*log(s
qrt(x)) + 6*(b*d^4 - b*e^4)*n*log(d*sqrt(x) + e) + 6*(b*d^4*n*x^2 - b*d^4*
n)*log((d*x + e*sqrt(x))/x) + 2*(b*d^3*e*n*x + 3*b*d*e^3*n)*sqrt(x))/d^4
```

Sympy [A] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=\frac{ax^2}{2}+b\left(\frac{en\left(\frac{2x^{\frac{3}{2}}}{3d}-\frac{ex}{d^2}-\frac{2e^3\left(\begin{cases}\frac{\sqrt{x}}{e} & \text{for } d=0\\ \frac{\log(d\sqrt{x}+e)}{d} & \text{otherwise}\end{cases}\right)}{d^3}+\frac{2e^2\sqrt{x}}{d^3}\right)}{4}+\frac{x^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2}\right)$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

output
```
a*x**2/2 + b*(e*n*(2*x**(3/2)/(3*d) - e*x/d**2 - 2*e**3*Piecewise((sqrt(x)
/e, Eq(d, 0)), (log(d*sqrt(x) + e)/d, True))/d**3 + 2*e**2*sqrt(x)/d**3)/4
 + x**2*log(c*(d + e/sqrt(x))**n)/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx = -\frac{1}{12}\,ben\left(\frac{6\,e^3\log\left(d\sqrt{x}+e\right)}{d^4}-\frac{2\,d^2x^{\frac{3}{2}}-3\,dex+6\,e^2\sqrt{x}}{d^3}\right) + \frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

output
```
-1/12*b*e*n*(6*e^3*log(d*sqrt(x) + e)/d^4 - (2*d^2*x^(3/2) - 3*d*e*x + 6*e
^2*sqrt(x))/d^3) + 1/2*b*x^2*log(c*(d + e/sqrt(x))^n) + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx = \frac{1}{2}\,bx^2\log\left(c\right) + \frac{1}{12}\left(6\,x^2\log\left(d+\frac{e}{\sqrt{x}}\right)-e\left(\frac{6\,e^3\log\left(\left|d\sqrt{x}+e\right|\right)}{d^4}-\frac{2\,d^2x^{\frac{3}{2}}-3\,dex+6\,e^2\sqrt{x}}{d^3}\right)\right)bn + \frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

output
```
1/2*b*x^2*log(c) + 1/12*(6*x^2*log(d + e/sqrt(x)) - e*(6*e^3*log(abs(d*sqr
t(x) + e))/d^4 - (2*d^2*x^(3/2) - 3*d*e*x + 6*e^2*sqrt(x))/d^3))*b*n + 1/2
*a*x^2
```

Mupad [B] (verification not implemented)

Time = 15.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{x^{3/2}\left(\frac{b\,e\,n}{3\,d}-\frac{b\,e^2\,n}{2\,d^2\,\sqrt{x}}+\frac{b\,e^3\,n}{d^3\,x}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2}-\frac{b\,e^4\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,\sqrt{x}}+1\right)}{d^4}$$

input

```
int(x*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

output

```
(x^(3/2)*((b*e*n)/(3*d) - (b*e^2*n)/(2*d^2*x^(1/2)) + (b*e^3*n)/(d^3*x)))/
2 + (a*x^2)/2 + (b*x^2*log(c*(d + e/x^(1/2))^n))/2 - (b*e^4*n*atanh((2*e)/
(d*x^(1/2)) + 1))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{2\sqrt{x}\,b\,d^3enx+6\sqrt{x}\,bd\,e^3n+6\log\left(\frac{\left(\sqrt{x}\,d+e\right)^nc}{x^{\frac{n}{2}}}\right)b\,d^4x^2-6\log\left(\frac{\left(\sqrt{x}\,d+e\right)^nc}{x^{\frac{n}{2}}}\right)b\,e^4-6\log\left(\sqrt{x}\right)b\,e^4n+6a\,d^4x}{12d^4}$$

input

```
int(x*(a+b*log(c*(d+e/x^(1/2))^n)),x)
```

output

```
(2*sqrt(x)*b*d**3*e*n*x + 6*sqrt(x)*b*d*e**3*n + 6*log(((sqrt(x)*d + e)**n
*c)/x**(n/2))*b*d**4*x**2 - 6*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b*e**4
- 6*log(sqrt(x))*b*e**4*n + 6*a*d**4*x**2 - 3*b*d**2*e**2*n*x)/(12*d**4)
```

3.424 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{ben\sqrt{x}}{d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(e+d\sqrt{x}\right)}{d^2}$$

output
```
b*e*n*x^(1/2)/d+a*x+b*x*ln(c*(d+e/x^(1/2))^n)-b*e^2*n*ln(e+d*x^(1/2))/d^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}ben\left(\frac{2\sqrt{x}}{d}-\frac{2e\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{e\log(x)}{d^2}\right)$$

input
```
Integrate[a + b*Log[c*(d + e/Sqrt[x])^n],x]
```

output
```
a*x + b*x*Log[c*(d + e/Sqrt[x])^n] + (b*e*n*((2*Sqrt[x])/d - (2*e*Log[d +
e/Sqrt[x]])/d^2 - (e*Log[x])/d^2))/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ax+bx\log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log \left(d\sqrt{x}+e\right)}{d^2}+\frac{ben\sqrt{x}}{d}$$

input
```
Int[a + b*Log[c*(d + e/Sqrt[x])^n],x]
```

output
```
(b*e*n*Sqrt[x])/d + a*x + b*x*Log[c*(d + e/Sqrt[x])^n] - (b*e^2*n*Log[e +
d*Sqrt[x]])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

method	result	size
default	$ax+b\left(x\ln\left(c\left(\frac{e+d\sqrt{x}}{\sqrt{x}}\right)^{n}\right)+\frac{en\left(\frac{2\sqrt{x}}{d}+\frac{e\ln(d\sqrt{x}-e)}{d^{2}}-\frac{e\ln(e+d\sqrt{x})}{d^{2}}-\frac{e\ln\left(d^{2}x-e^{2}\right)}{d^{2}}\right)}{2}\right)$	86
parts	$ax+b\left(x\ln\left(c\left(\frac{e+d\sqrt{x}}{\sqrt{x}}\right)^{n}\right)+\frac{en\left(\frac{2\sqrt{x}}{d}+\frac{e\ln(d\sqrt{x}-e)}{d^{2}}-\frac{e\ln(e+d\sqrt{x})}{d^{2}}-\frac{e\ln\left(d^{2}x-e^{2}\right)}{d^{2}}\right)}{2}\right)$	86

input
```
int(a+b*ln(c*(d+e/x^(1/2))^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*ln(c*((e+d*x^(1/2))/x^(1/2))^n)+1/2*e*n*(2*x^(1/2)/d+1/d^2*e*ln(d
*x^(1/2)-e)-1/d^2*e*ln(e+d*x^(1/2))-e*ln(d^2*x-e^2)/d^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)dx$$
$$=\frac{bd^{2}x\log(c)-bd^{2}n\log\left(\sqrt{x}\right)+bden\sqrt{x}+ad^{2}x+(bd^{2}-be^{2})n\log\left(d\sqrt{x}+e\right)+(bd^{2}nx-bd^{2}n)\log\left(\frac{dx+\cdots}{\cdots}\right.}{d^{2}}$$

input
```
integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="fricas")
```

output
```
(b*d^2*x*log(c) - b*d^2*n*log(sqrt(x)) + b*d*e*n*sqrt(x) + a*d^2*x + (b*d^
2 - b*e^2)*n*log(d*sqrt(x) + e) + (b*d^2*n*x - b*d^2*n)*log((d*x + e*sqrt(
x))/x))/d^2
```

Sympy [A] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx = ax + b\left(\frac{en\left(-\frac{2e\left(\begin{cases}\frac{\sqrt{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d}+\frac{2\sqrt{x}}{d}\right)}{2} + x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)$$

input
```
integrate(a+b*ln(c*(d+e/x**(1/2))**n),x)
```

output
```
a*x + b*(e*n*(-2*e*Piecewise((sqrt(x)/e, Eq(d, 0)), (log(d*sqrt(x) + e)/d,
 True))/d + 2*sqrt(x)/d)/2 + x*log(c*(d + e/sqrt(x))**n))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=-\left(en\left(\frac{e\log\left(d\sqrt{x}+e\right)}{d^2}-\frac{\sqrt{x}}{d}\right)-x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)b+ax$$

input
```
integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="maxima")
```

output
```
-(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d) - x*log(c*(d + e/sqrt(x))^n))
*b + a*x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=-\left(\left(e\left(\frac{e\log\left(\left|d\sqrt{x}+e\right|\right)}{d^2}-\frac{\sqrt{x}}{d}\right)-x\log\left(d+\frac{e}{\sqrt{x}}\right)\right)n-x\log\left(c\right)\right)b+ax$$

input
```
integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="giac")
```

output
```
-((e*(e*log(abs(d*sqrt(x) + e))/d^2 - sqrt(x)/d) - x*log(d + e/sqrt(x)))*n
 - x*log(c))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx = a\,x+b\,x\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^n\right) - \frac{b\,e\,n\,\left(e\,\ln\left(e+d\,\sqrt{x}\right)-d\,\sqrt{x}\right)}{d^2}$$

input
```
int(a + b*log(c*(d + e/x^(1/2))^n),x)
```

output
```
a*x + b*x*log(c*(d + e/x^(1/2))^n) - (b*e*n*(e*log(e + d*x^(1/2)) - d*x^(1
/2)))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx = \frac{\sqrt{x}\,bden+\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b\,d^2x-\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b\,e^2-\log\left(\sqrt{x}\right)b\,e^2n+a\,d^2x}{d^2}$$

input
```
int(a+b*log(c*(d+e/x^(1/2))^n),x)
```

output
```
(sqrt(x)*b*d*e*n + log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b*d**2*x - log(((s
qrt(x)*d + e)**n*c)/x**(n/2))*b*e**2 - log(sqrt(x))*b*e**2*n + a*d**2*x)/d
**2
```

3.425 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=-2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)-2bn\,\text{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right)$$

output
```
-2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^(1/2))-2*b*n*polylog(2,1+e/d/x^(1
/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=-2b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\log\left(-\frac{e}{d\sqrt{x}}\right)+a\log(x)-2bn\,\text{PolyLog}\left(2,\frac{d+\frac{e}{\sqrt{x}}}{d}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

output
```
-2*b*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] + a*Log[x] - 2*b*n*Pol
yLog[2, (d + e/Sqrt[x])/d]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2841}$$

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-ben\int \frac{\log\left(-\frac{e}{d\sqrt{x}}\right)}{d+\frac{e}{\sqrt{x}}}d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{2752}$$

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

output
```
-2*((a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] + b*n*PolyLog[2
, 1 + e/(d*Sqrt[x])])
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))/x,x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x}\,dx=\int \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*((d*x + e*sqrt(x))/x)^n) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x,x)
```

output
```
Integral((a + b*log(c*(d + e/sqrt(x))**n))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(44) = 88.

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.43

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=-2\left(\log\left(\frac{d\sqrt{x}}{e}+1\right)\log\left(\sqrt{x}\right)+\mathrm{Li}_2\left(-\frac{d\sqrt{x}}{e}\right)\right)bn + \frac{ben\log(x)^2+4\,bdn\sqrt{x}\log(x)+4\,be\log\left(\left(d\sqrt{x}+e\right)^n\right)\log(x)-4\,be\log(x)\log\left(x^{\frac{1}{2}n}\right)-8\,bdn\sqrt{x}+4}{4\,e}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="maxima")
```

output
```
-2*(log(d*sqrt(x)/e + 1)*log(sqrt(x)) + dilog(-d*sqrt(x)/e))*b*n + 1/4*(b*
e*n*log(x)^2 + 4*b*d*n*sqrt(x)*log(x) + 4*b*e*log((d*sqrt(x) + e)^n)*log(x
) - 4*b*e*log(x)*log(x^(1/2*n)) - 8*b*d*n*sqrt(x) + 4*(b*e*log(c) + a*e)*l
og(x) - 4*(b*d*n*x*log(x) - 2*b*d*n*x)/sqrt(x))/e
```

Giac [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=\int \frac{a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^n))/x, x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=\left(\int \frac{\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))/x,x)
```

output
```
int(log(((sqrt(x)*d + e)**n*c)/x**(n/2))/x,x)*b + log(x)*a
```

3.426 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx=\frac{bn}{2x}-\frac{bdn}{e\sqrt{x}}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}$$

output
```
1/2*b*n/x-b*d*n/e/x^(1/2)+b*d^2*n*ln(d+e/x^(1/2))/e^2-(a+b*ln(c*(d+e/x^(1/
2))^n))/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx=-\frac{a}{x}+\frac{bn}{2x}-\frac{bdn}{e\sqrt{x}}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

output
```
-(a/x) + (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/
e^2 - (b*Log[c*(d + e/Sqrt[x])^n])/x
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{\sqrt{x}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2842}$$

$$-2\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x}-\frac{1}{2}ben\int \frac{1}{\left(d+\frac{e}{\sqrt{x}}\right)x}d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{49}$$

$$-2\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x}-\frac{1}{2}ben\int \left(\frac{d^2}{e^2\left(d+\frac{e}{\sqrt{x}}\right)}-\frac{d}{e^2}+\frac{1}{e\sqrt{x}}\right)d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x}-\frac{1}{2}ben\left(\frac{d^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^3}-\frac{d}{e^2\sqrt{x}}+\frac{1}{2ex}\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

output
```
-2*(-1/2*(b*e*n*(1/(2*e*x) - d/(e^2*Sqrt[x]) + (d^2*Log[d + e/Sqrt[x]])/e^
3)) + (a + b*Log[c*(d + e/Sqrt[x])^n])/(2*x))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{a}{x}-\frac{b\ln\left(c\,\mathrm{e}^{n\ln\left(d+\frac{e}{\sqrt{x}}\right)}\right)}{x}+\frac{bn}{2x}+\frac{b\,d^{2}n\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^{2}}-\frac{bdn}{e\sqrt{x}}$	63
default	$-\frac{a}{x}-\frac{b\ln\left(c\,\mathrm{e}^{n\ln\left(d+\frac{e}{\sqrt{x}}\right)}\right)}{x}+\frac{bn}{2x}+\frac{b\,d^{2}n\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^{2}}-\frac{bdn}{e\sqrt{x}}$	63

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*ln(c*exp(n*ln(d+e/x^(1/2))))+1/2*b*n/x+b*d^2*n*ln(d+e/x^(1/2))/e^
2-b*d*n/e/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x^{2}}\,dx$$
$$=-\frac{2\,bden\sqrt{x}-be^{2}n+2\,be^{2}\log\left(c\right)+2\,ae^{2}-2\left(bd^{2}nx-be^{2}n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)}{2\,e^{2}x}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*e*n*sqrt(x) - b*e^2*n + 2*b*e^2*log(c) + 2*a*e^2 - 2*(b*d^2*n*
x - b*e^2*n)*log((d*x + e*sqrt(x))/x))/(e^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(56) = 112$.

Time = 47.49 (sec) , antiderivative size = 391, normalized size of antiderivative = 6.02

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx$$
$$= \begin{cases} -\frac{a+b\log(0^n c)}{x} \\ -\frac{a+b\log(cd^n)}{x} \\ -\frac{2ade^2x^3}{2de^2x^4+2e^3x^{\frac{7}{2}}} - \frac{2ae^3x^{\frac{5}{2}}}{2de^2x^4+2e^3x^{\frac{7}{2}}} + \frac{2bd^3x^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2de^2x^4+2e^3x^{\frac{7}{2}}} - \frac{2bd^2enx^{\frac{7}{2}}}{2de^2x^4+2e^3x^{\frac{7}{2}}} + \frac{2bd^2ex^{\frac{7}{2}}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2de^2x^4+2e^3x^{\frac{7}{2}}} - \frac{bde^2nx^3}{2de^2x^4+2e^3x^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**2,x)
```

output
```
Piecewise((-(a + b*log(0**n*c))/x, (Eq(d, 0) | Eq(d, -e/sqrt(x))) & (Eq(e,
 0) | Eq(d, -e/sqrt(x)))), (-(a + b*log(c*d**n))/x, Eq(e, 0)), (-2*a*d*e**
2*x**3/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) - 2*a*e**3*x**(5/2)/(2*d*e**2*x**
4 + 2*e**3*x**(7/2)) + 2*b*d**3*x**4*log(c*(d + e/sqrt(x))**n)/(2*d*e**2*x
**4 + 2*e**3*x**(7/2)) - 2*b*d**2*e*n*x**(7/2)/(2*d*e**2*x**4 + 2*e**3*x**
(7/2)) + 2*b*d**2*e*x**(7/2)*log(c*(d + e/sqrt(x))**n)/(2*d*e**2*x**4 + 2*
e**3*x**(7/2)) - b*d*e**2*n*x**3/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) - 2*b*d
*e**2*x**3*log(c*(d + e/sqrt(x))**n)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) + b
*e**3*n*x**(5/2)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) - 2*b*e**3*x**(5/2)*log
(c*(d + e/sqrt(x))**n)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = \frac{1}{2}ben\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3} - \frac{d^2\log(x)}{e^3} - \frac{2\,d\sqrt{x}-e}{e^2x}\right) - \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} - \frac{a}{x}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="maxima")
```

output
```
1/2*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) -
e)/(e^2*x)) - b*log(c*(d + e/sqrt(x))^n)/x - a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(57) = 114.

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = \frac{2\left(\frac{2\left(d\sqrt{x}+e\right)bdn}{e\sqrt{x}}-\frac{\left(d\sqrt{x}+e\right)^2bn}{ex}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)+\frac{\left(bn-2\,b\log\left(c\right)-2\,a\right)\left(d\sqrt{x}+e\right)^2}{ex}-\frac{4\left(bdn-bd\log\left(c\right)-ad\right)\left(d\sqrt{x}+e\right)}{e\sqrt{x}}}{2\,e}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="giac")
```

output
```
1/2*(2*(2*(d*sqrt(x) + e)*b*d*n/(e*sqrt(x)) - (d*sqrt(x) + e)^2*b*n/(e*x))
*log((d*sqrt(x) + e)/sqrt(x)) + (b*n - 2*b*log(c) - 2*a)*(d*sqrt(x) + e)^2
/(e*x) - 4*(b*d*n - b*d*log(c) - a*d)*(d*sqrt(x) + e)/(e*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = \frac{b\,n}{2\,x}-\frac{a}{x}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)}{x}-\frac{b\,d\,n}{e\,\sqrt{x}}+\frac{b\,d^2\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))/x^2,x)
```

output
```
(b*n)/(2*x) - a/x - (b*log(c*(d + e/x^(1/2))^n))/x - (b*d*n)/(e*x^(1/2)) +
 (b*d^2*n*log(d + e/x^(1/2)))/e^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx$$
$$= \frac{-2\sqrt{x}\,bden+2\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b\,d^2x-2\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b\,e^2-2a\,e^2+b\,e^2n}{2e^2x}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x)
```

output
```
( - 2*sqrt(x)*b*d*e*n + 2*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b*d**2*x -
2*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b*e**2 - 2*a*e**2 + b*e**2*n)/(2*e*
*2*x)
```

3.427 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx=\frac{bn}{8x^2}-\frac{bdn}{6ex^{3/2}}+\frac{bd^2n}{4e^2x}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}$$

output
```
1/8*b*n/x^2-1/6*b*d*n/e/x^(3/2)+1/4*b*d^2*n/e^2/x-1/2*b*d^3*n/e^3/x^(1/2)+
1/2*b*d^4*n*ln(d+e/x^(1/2))/e^4-1/2*(a+b*ln(c*(d+e/x^(1/2))^n))/x^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{1}{4}ben\left(-\frac{1}{2ex^2}+\frac{2d}{3e^2x^{3/2}}-\frac{d^2}{e^3x}+\frac{2d^3}{e^4\sqrt{x}}-\frac{2d^4\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^5}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*e*n*(-1/2*1/(e*x^2) + (2*d)/(3*e^2*x^(3/2)) - d^2/(e^3*x)
+ (2*d^3)/(e^4*Sqrt[x]) - (2*d^4*Log[d + e/Sqrt[x]])/e^5))/4 - (b*Log[c*(d
 + e/Sqrt[x])^n])/(2*x^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}dx$$

$$\Big\downarrow \text{ 2904}$$

$$-2\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^{3/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2842}$$

$$-2\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{4x^2}-\frac{1}{4}ben\int \frac{1}{\left(d+\frac{e}{\sqrt{x}}\right)x^2}d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 49}$$

$$-2\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{4x^2}-\frac{1}{4}ben\int \left(\frac{d^4}{e^4\left(d+\frac{e}{\sqrt{x}}\right)}-\frac{d^3}{e^4}+\frac{d^2}{e^3\sqrt{x}}-\frac{d}{e^2x}+\frac{1}{ex^{3/2}}\right)d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-2\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{4x^2}-\frac{1}{4}ben\left(\frac{d^4\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^5}-\frac{d^3}{e^4\sqrt{x}}+\frac{d^2}{2e^3x}-\frac{d}{3e^2x^{3/2}}+\frac{1}{4ex^2}\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```

output
```
-2*(-1/4*(b*e*n*(1/(4*e*x^2) - d/(3*e^2*x^(3/2)) + d^2/(2*e^3*x) - d^3/(e^
4*Sqrt[x]) + (d^4*Log[d + e/Sqrt[x]])/e^5)) + (a + b*Log[c*(d + e/Sqrt[x])
^n])/(4*x^2))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = \frac{6\,bd^2e^2nx+3\,be^4n-12\,be^4\log\left(c\right)-12\,ae^4+12\left(bd^4nx^2-be^4n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)-4\left(3\,bd^3enx+bde^3n\right)\sqrt{}}{24\,e^4x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="fricas")
```

output
```
1/24*(6*b*d^2*e^2*n*x + 3*b*e^4*n - 12*b*e^4*log(c) - 12*a*e^4 + 12*(b*d^4
*n*x^2 - b*e^4*n)*log((d*x + e*sqrt(x))/x) - 4*(3*b*d^3*e*n*x + b*d*e^3*n)
*sqrt(x))/(e^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$$
$$= \frac{1}{24}ben\left(\frac{12\,d^4\log\left(d\sqrt{x}+e\right)}{e^5} - \frac{6\,d^4\log\left(x\right)}{e^5} - \frac{12\,d^3x^{\frac{3}{2}} - 6\,d^2ex + 4\,de^2\sqrt{x} - 3\,e^3}{e^4x^2}\right) - \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="maxima")
```

output
```
1/24*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(
3/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) - 1/2*b*log(c*(d +
e/sqrt(x))^n)/x^2 - 1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$$
$$= \frac{12\left(\frac{4\,(d\sqrt{x}+e)bd^3n}{e^3\sqrt{x}} - \frac{6\,(d\sqrt{x}+e)^2bd^2n}{e^3x} + \frac{4\,(d\sqrt{x}+e)^3bdn}{e^3x^{\frac{3}{2}}} - \frac{(d\sqrt{x}+e)^4bn}{e^3x^2}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right) + \frac{3\,(bn-4\,b\log(c)-4\,a)(d\sqrt{x}+e)^4}{e^3x^2} - }{24\,e}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="giac")
```

output
```
1/24*(12*(4*(d*sqrt(x) + e)*b*d^3*n/(e^3*sqrt(x)) - 6*(d*sqrt(x) + e)^2*b*
d^2*n/(e^3*x) + 4*(d*sqrt(x) + e)^3*b*d*n/(e^3*x^(3/2)) - (d*sqrt(x) + e)^
4*b*n/(e^3*x^2))*log((d*sqrt(x) + e)/sqrt(x)) + 3*(b*n - 4*b*log(c) - 4*a)
*(d*sqrt(x) + e)^4/(e^3*x^2) - 16*(b*d*n - 3*b*d*log(c) - 3*a*d)*(d*sqrt(x
) + e)^3/(e^3*x^(3/2)) + 36*(b*d^2*n - 2*b*d^2*log(c) - 2*a*d^2)*(d*sqrt(x
) + e)^2/(e^3*x) - 48*(b*d^3*n - b*d^3*log(c) - a*d^3)*(d*sqrt(x) + e)/(e^
3*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = \frac{b\,n}{8\,x^2}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2\,x^2}-\frac{b\,d\,n}{6\,e\,x^{3/2}}+\frac{b\,d^4\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{2\,e^4}+\frac{b\,d^2\,n}{4\,e^2\,x}-\frac{b\,d^3\,n}{2\,e^3\,\sqrt{x}}$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))/x^3,x)
```

output
```
(b*n)/(8*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(1/2))^n))/(2*x^2) - (b*d*n)
/(6*e*x^(3/2)) + (b*d^4*n*log(d + e/x^(1/2)))/(2*e^4) + (b*d^2*n)/(4*e^2*x
) - (b*d^3*n)/(2*e^3*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = \frac{-12\sqrt{x}\,b\,d^3enx-4\sqrt{x}\,bd\,e^3n+12\log\left(\frac{\left(\sqrt{x}\,d+e\right)^nc}{x^{\frac{n}{2}}}\right)b\,d^4x^2-12\log\left(\frac{\left(\sqrt{x}\,d+e\right)^nc}{x^{\frac{n}{2}}}\right)b\,e^4-12a\,e^4+6b\,d^2e^2nx}{24e^4x^2}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x)
```

output
```
( - 12*sqrt(x)*b*d**3*e*n*x - 4*sqrt(x)*b*d*e**3*n + 12*log(((sqrt(x)*d +
e)**n*c)/x**(n/2))*b*d**4*x**2 - 12*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b
*e**4 - 12*a*e**4 + 6*b*d**2*e**2*n*x + 3*b*e**4*n)/(24*e**4*x**2)
```

3.428 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx=\frac{bn}{18x^3}-\frac{bdn}{15ex^{5/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^4n}{6e^4x}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}$$

output
```
1/18*b*n/x^3-1/15*b*d*n/e/x^(5/2)+1/12*b*d^2*n/e^2/x^2-1/9*b*d^3*n/e^3/x^(
3/2)+1/6*b*d^4*n/e^4/x-1/3*b*d^5*n/e^5/x^(1/2)+1/3*b*d^6*n*ln(d+e/x^(1/2))
/e^6-1/3*(a+b*ln(c*(d+e/x^(1/2))^n))/x^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx=-\frac{a}{3x^3}-\frac{1}{6}ben\left(-\frac{1}{3ex^3}+\frac{2d}{5e^2x^{5/2}}-\frac{d^2}{2e^3x^2}+\frac{2d^3}{3e^4x^{3/2}}-\frac{d^4}{e^5x}+\frac{2d^5}{e^6\sqrt{x}}-\frac{2d^6\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^7}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*e*n*(-1/3*1/(e*x^3) + (2*d)/(5*e^2*x^(5/2)) - d^2/(2*e^3*x
^2) + (2*d^3)/(3*e^4*x^(3/2)) - d^4/(e^5*x) + (2*d^5)/(e^6*Sqrt[x]) - (2*d
^6*Log[d + e/Sqrt[x]])/e^7))/6 - (b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^{5/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2842}$$

$$-2\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{6x^3}-\frac{1}{6}ben\int\frac{1}{\left(d+\frac{e}{\sqrt{x}}\right)x^3}d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{49}$$

$$-2\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{6x^3}-\frac{1}{6}ben\int\left(\frac{d^6}{e^6\left(d+\frac{e}{\sqrt{x}}\right)}-\frac{d^5}{e^6}+\frac{d^4}{e^5\sqrt{x}}-\frac{d^3}{e^4x}+\frac{d^2}{e^3x^{3/2}}-\frac{d}{e^2x^2}+\frac{1}{ex^{5/2}}\right)d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{6x^3}-\frac{1}{6}ben\left(\frac{d^6\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^7}-\frac{d^5}{e^6\sqrt{x}}+\frac{d^4}{2e^5x}-\frac{d^3}{3e^4x^{3/2}}+\frac{d^2}{4e^3x^2}-\frac{d}{5e^2x^{5/2}}+\frac{1}{6ex^3}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

output
```
-2*(-1/6*(b*e*n*(1/(6*e*x^3) - d/(5*e^2*x^(5/2)) + d^2/(4*e^3*x^2) - d^3/(
3*e^4*x^(3/2)) + d^4/(2*e^5*x) - d^5/(e^6*Sqrt[x]) + (d^6*Log[d + e/Sqrt[x
]])/e^7)) + (a + b*Log[c*(d + e/Sqrt[x])^n])/(6*x^3))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x^{4}}\,dx = \frac{30\,bd^{4}e^{2}nx^{2}+15\,bd^{2}e^{4}nx+10\,be^{6}n-60\,be^{6}\log\left(c\right)-60\,ae^{6}+60\,(bd^{6}nx^{3}-be^{6}n)\log\left(\frac{dx+e\sqrt{x}}{x}\right)-4\,(15}{180\,e^{6}x^{3}}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="fricas")
```

output
```
1/180*(30*b*d^4*e^2*n*x^2 + 15*b*d^2*e^4*n*x + 10*b*e^6*n - 60*b*e^6*log(c
) - 60*a*e^6 + 60*(b*d^6*n*x^3 - b*e^6*n)*log((d*x + e*sqrt(x))/x) - 4*(15
*b*d^5*e*n*x^2 + 5*b*d^3*e^3*n*x + 3*b*d*e^5*n)*sqrt(x))/(e^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$$
$$= \frac{1}{180}ben\left(\frac{60\,d^6\log\left(d\sqrt{x}+e\right)}{e^7} - \frac{30\,d^6\log\left(x\right)}{e^7} - \frac{60\,d^5x^{\frac{5}{2}} - 30\,d^4ex^2 + 20\,d^3e^2x^{\frac{3}{2}} - 15\,d^2e^3x + 12\,de^4\sqrt{x}}{e^6x^3}\right.$$
$$- \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="maxima")
```

output
```
1/180*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x
^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(
x) - 10*e^5)/(e^6*x^3)) - 1/3*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(110) = 220$.

Time = 0.14 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.62

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$$
$$= \frac{60\left(\frac{6\,(d\sqrt{x}+e)bd^5n}{e^5\sqrt{x}} - \frac{15\,(d\sqrt{x}+e)^2bd^4n}{e^5x} + \frac{20\,(d\sqrt{x}+e)^3bd^3n}{e^5x^{\frac{3}{2}}} - \frac{15\,(d\sqrt{x}+e)^4bd^2n}{e^5x^2} + \frac{6\,(d\sqrt{x}+e)^5bdn}{e^5x^{\frac{5}{2}}} - \frac{(d\sqrt{x}+e)^6bn}{e^5x^3}\right)\log\left(\frac{d\sqrt{}}{\sqrt{}}\right.}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="giac")
```

output
```
1/180*(60*(6*(d*sqrt(x) + e)*b*d^5*n/(e^5*sqrt(x)) - 15*(d*sqrt(x) + e)^2*
b*d^4*n/(e^5*x) + 20*(d*sqrt(x) + e)^3*b*d^3*n/(e^5*x^(3/2)) - 15*(d*sqrt(
x) + e)^4*b*d^2*n/(e^5*x^2) + 6*(d*sqrt(x) + e)^5*b*d*n/(e^5*x^(5/2)) - (d
*sqrt(x) + e)^6*b*n/(e^5*x^3))*log((d*sqrt(x) + e)/sqrt(x)) + 10*(b*n - 6*
b*log(c) - 6*a)*(d*sqrt(x) + e)^6/(e^5*x^3) - 72*(b*d*n - 5*b*d*log(c) - 5
*a*d)*(d*sqrt(x) + e)^5/(e^5*x^(5/2)) + 225*(b*d^2*n - 4*b*d^2*log(c) - 4*
a*d^2)*(d*sqrt(x) + e)^4/(e^5*x^2) - 400*(b*d^3*n - 3*b*d^3*log(c) - 3*a*d
^3)*(d*sqrt(x) + e)^3/(e^5*x^(3/2)) + 450*(b*d^4*n - 2*b*d^4*log(c) - 2*a*
d^4)*(d*sqrt(x) + e)^2/(e^5*x) - 360*(b*d^5*n - b*d^5*log(c) - a*d^5)*(d*s
qrt(x) + e)/(e^5*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.83

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx = \frac{b\,n}{18\,x^3} - \frac{a}{3\,x^3} - \frac{b\,\ln\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)}{3\,x^3} - \frac{b\,d\,n}{15\,e\,x^{5/2}} + \frac{b\,d^6\,n\,\ln\left(d + \frac{e}{\sqrt{x}}\right)}{3\,e^6} + \frac{b\,d^2\,n}{12\,e^2\,x^2} + \frac{b\,d^4\,n}{6\,e^4\,x} - \frac{b\,d^3\,n}{9\,e^3\,x^{3/2}} - \frac{b\,d^5\,n}{3\,e^5\,\sqrt{x}}$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))/x^4,x)
```

output
```
(b*n)/(18*x^3) - a/(3*x^3) - (b*log(c*(d + e/x^(1/2))^n))/(3*x^3) - (b*d*n
)/(15*e*x^(5/2)) + (b*d^6*n*log(d + e/x^(1/2)))/(3*e^6) + (b*d^2*n)/(12*e^
2*x^2) + (b*d^4*n)/(6*e^4*x) - (b*d^3*n)/(9*e^3*x^(3/2)) - (b*d^5*n)/(3*e^
5*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$$
$$= \frac{-60\sqrt{x}\,b\,d^5en\,x^2 - 20\sqrt{x}\,b\,d^3e^3nx - 12\sqrt{x}\,bd\,e^5n + 60\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right) b\,d^6x^3 - 60\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right) b\,e^6 -}{180e^6x^3}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x)
```

output
```
( - 60*sqrt(x)*b*d**5*e*n*x**2 - 20*sqrt(x)*b*d**3*e**3*n*x - 12*sqrt(x)*b
*d*e**5*n + 60*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b*d**6*x**3 - 60*log((
(sqrt(x)*d + e)**n*c)/x**(n/2))*b*e**6 - 60*a*e**6 + 30*b*d**4*e**2*n*x**2
 + 15*b*d**2*e**4*n*x + 10*b*e**6*n)/(180*e**6*x**3)
```

3.429 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 24, antiderivative size = 404

$$\begin{aligned}
&\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx \\
&= -\frac{77b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2} \\
&\quad+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6}+\frac{2be^5n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6} \\
&\quad-\frac{be^4nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^4}+\frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3} \\
&\quad-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}+\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d} \\
&\quad+\frac{2be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6} \\
&\quad+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{2b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{3d^6}
\end{aligned}$$

output
```
-77/90*b^2*e^5*n^2*x^(1/2)/d^5+47/180*b^2*e^4*n^2*x/d^4-1/10*b^2*e^3*n^2*x
^(3/2)/d^3+1/30*b^2*e^2*n^2*x^2/d^2+77/90*b^2*e^6*n^2*ln(d+e/x^(1/2))/d^6+
2/3*b*e^5*n*(d+e/x^(1/2))*x^(1/2)*(a+b*ln(c*(d+e/x^(1/2))^n))/d^6-1/3*b*e^
4*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4+2/9*b*e^3*n*x^(3/2)*(a+b*ln(c*(d+e/x
^(1/2))^n))/d^3-1/6*b*e^2*n*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+2/15*b*e*n
*x^(5/2)*(a+b*ln(c*(d+e/x^(1/2))^n))/d+2/3*b*e^6*n*ln(1-d/(d+e/x^(1/2)))*(
a+b*ln(c*(d+e/x^(1/2))^n))/d^6+1/3*x^3*(a+b*ln(c*(d+e/x^(1/2))^n))^2+137/1
80*b^2*e^6*n^2*ln(x)/d^6-2/3*b^2*e^6*n^2*polylog(2,d/(d+e/x^(1/2)))/d^6
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.07

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\frac{1}{3}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{ben\left(120ade^4\sqrt{x}-154bde^4n\sqrt{x}-60ad^2e^3x+47bd^2e^3nx+40ad^3e^2x^{3/2}-18bd^3e^2nx^{3/2}-30ad^4ex^2\right.}{}\right.$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

output
```
(x^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(120*a*d*e^4*Sqrt[x] - 15
4*b*d*e^4*n*Sqrt[x] - 60*a*d^2*e^3*x + 47*b*d^2*e^3*n*x + 40*a*d^3*e^2*x^(
3/2) - 18*b*d^3*e^2*n*x^(3/2) - 30*a*d^4*e*x^2 + 6*b*d^4*e*n*x^2 + 24*a*d^
5*x^(5/2) + 214*b*e^5*n*Log[d + e/Sqrt[x]] + 120*b*d*e^4*Sqrt[x]*Log[c*(d
+ e/Sqrt[x])^n] - 60*b*d^2*e^3*x*Log[c*(d + e/Sqrt[x])^n] + 40*b*d^3*e^2*x
^(3/2)*Log[c*(d + e/Sqrt[x])^n] - 30*b*d^4*e*x^2*Log[c*(d + e/Sqrt[x])^n]
+ 24*b*d^5*x^(5/2)*Log[c*(d + e/Sqrt[x])^n] - 120*a*e^5*Log[e + d*Sqrt[x]]
 + 60*b*e^5*n*Log[e + d*Sqrt[x]] - 120*b*e^5*Log[c*(d + e/Sqrt[x])^n]*Log[
e + d*Sqrt[x]] + 60*b*e^5*n*Log[e + d*Sqrt[x]]^2 - 120*b*e^5*n*Log[e + d*S
qrt[x]]*Log[-((d*Sqrt[x])/e)] + 107*b*e^5*n*Log[x] - 120*b*e^5*n*PolyLog[2
, 1 + (d*Sqrt[x])/e]))/(60*d^6))/3
```

Rubi [A] (warning: unable to verify)

Time = 3.91 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.37, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.042, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int x^{7/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2845}$$

$$-2\left(\frac{1}{3}ben\int \frac{x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d+\frac{e}{\sqrt{x}}}d\frac{1}{\sqrt{x}}-\frac{1}{6}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2858}$$

$$-2\left(\frac{1}{3}bn\int x^{7/2}\left(a+b\log\left(cx^{-n/2}\right)\right)d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{1}{6}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{1}{3}be^6n\int \frac{x^{7/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^6}d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{1}{6}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2789}$$

$$-2\left(\frac{1}{3}be^6n\left(\frac{\int \frac{x^3\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^6}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int -\frac{x^3\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{1}{6}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2756}$$

$$-2\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int -\frac{x^3}{e^5}d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{x^{5/2}(a+b\log(cx^{-n/2}))}{5e^5}}{d}+\frac{\int -\frac{x^3(a+b\log(cx^{-n/2}))}{e^5}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{1}{6}x^3\left(a+b\log\right.\right.$$

$\downarrow$ 54

$$-2\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int\left(-\frac{x^{5/2}}{de^5}+\frac{x^2}{d^2e^4}-\frac{x^{3/2}}{d^3e^3}+\frac{x}{d^4e^2}-\frac{\sqrt{x}}{d^5e}+\frac{\sqrt{x}}{d^5}\right)d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{x^{5/2}(a+b\log(cx^{-n/2}))}{5e^5}}{d}+\frac{\int -\frac{x^3(a+b\log}{e}}{}\right.\right.$$

$\downarrow$ 2009

$$-2\left(\frac{1}{3}be^6n\left(\frac{\int -\frac{x^3(a+b\log(cx^{-n/2}))}{e^5}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{-\frac{x^{5/2}(a+b\log(cx^{-n/2}))}{5e^5}-\frac{1}{5}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^5}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^5}-\frac{\sqrt{x}}{d^4e}+\frac{}{2d}\right.}{d}\right.\right.$$

$\downarrow$ 2789

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\int -\frac{x^{5/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int \frac{x^{5/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{x^{5/2}(a+b\log(cx^{-n/2}))}{5e^5}-\frac{1}{5}bn\left(\frac{\log\left(d\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 2756

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\int \frac{x^{5/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)}{4e^4}-\frac{1}{4}bn\int\frac{x^{5/2}}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{x^{5/2}(a+b\log(cx^{-n/2}))}{5e^5}-\frac{1}{5}bn\left(\right.}{}\right.\right.$$

$\downarrow$ 54

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)}{4e^4}-\frac{1}{4}bn\int\left(\frac{x^2}{de^4}-\frac{x^{3/2}}{d^2e^3}+\frac{x}{d^3e^2}-\frac{\sqrt{x}}{d^4e}+\frac{\sqrt{x}}{d^4}\right)d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int \frac{x^{5/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{x^{5/2}(a}{}}{}\right.\right.$$

$\downarrow$ 2009

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\int \frac{x^{5/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^4}-\frac{\sqrt{x}}{d^3e}+\frac{x}{2d^2e^2}-\frac{x^{3/2}}{3de^3}\right)}{d}}{d}+\frac{-\frac{x^5}{\cdots}}{\cdots}\right.\right.$$

$\Big\downarrow$ 2789

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int \frac{x^{2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int -\frac{x^{2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^4}\cdots\right.}{d}}{d}\right.\right.$$

$\Big\downarrow$ 2756

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int -\frac{x^2}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{3e^3}}{d}+\frac{\int -\frac{x^{2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\cdots\right.}{d^4}\cdots\right.}{\cdots}}{d}\right.\right.$$

$\Big\downarrow$ 54

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int\left(-\frac{x^{3/2}}{de^3}+\frac{x}{d^2e^2}-\frac{\sqrt{x}}{d^3e}+\frac{\sqrt{x}}{d^3}\right)d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{3e^3}}{d}+\frac{\int -\frac{x^{2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{\frac{x^2\left(a+b\log\left(cx^{-\cdots}\right.\right.}{4e^4}\cdots}{\cdots}}{d}\right.\right.$$

$\Big\downarrow$ 2009

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int -\frac{x^{2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^3}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^3}-\frac{\sqrt{x}}{d^2e}+\frac{x}{2de^2}\right)}{d}}{d}+\frac{\frac{x^2\left(a+b\log\left(cx\cdots\right.\right.}{4e^4}\cdots}{\cdots}}{d}\right.\right.$$

$\Big\downarrow$ 2789

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int -\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int \frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^3}-\frac{\log \cdots}{\cdots}\right.}{d}}{d}\cdots}{d}\right.\right.$$

$$\Big\downarrow \; 2756$$

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\int \frac{x^{3/2}}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int \frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^3}\cdots\right.}{d}}{d}\cdots}{d}\right.\right.$$

$$\Big\downarrow \; 54$$

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\int \left(\frac{x}{de^2}-\frac{\sqrt{x}}{d^2e}+\frac{\sqrt{x}}{d^2}\right)d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int \frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\cdots\right.}{\cdots}}{d}\cdots}{d}\right.\right.$$

$$\Big\downarrow \; 2009$$

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int \frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\sqrt{x}}{de}\right)}{d}}{d}+\frac{-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{3e^3}-\cdots}{\cdots}}{d}\cdots}{d}\right.\right.$$

$$\Big\downarrow \; 2789$$

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int \frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^2}-\cdots\right)}{d}}{d}}{d}\cdots\right)\right)$$

$\Big\downarrow$ 2751

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{-\frac{bn\int -\frac{\sqrt{x}}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}}{d}+\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\cdots\right)}{d^2}\cdots\right)}{d}}{d}\cdots\right)\right)$$

$\Big\downarrow$ 16

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\cdots\right)}{d}\cdots\right)\right)$$

$\Big\downarrow$ 2779

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{bn\int \sqrt{x}\log\left(1-d\sqrt{x}\right)d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\log\left(1-d\sqrt{x}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}}{d}}{d}+\frac{x\left(a+b\log\left(cx^{-\cdots}\right)\cdots\right)}{2e^2}\cdots}{d}\cdots\right)\right)$$

$\Big\downarrow$ 2838

$$-2\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\sqrt{x}}{de}\right)}{d}+\frac{\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}+\frac{\frac{bn\operatorname{PolyLog}(2,d\sqrt{x}}{d}}{d}}{d}}{d}}{d}}{}\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

output
```
-2*(-1/6*(x^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2) + (b*e^6*n*((-1/5*(b*n*(
-(Sqrt[x]/(d^4*e)) + x/(2*d^3*e^2) - x^(3/2)/(3*d^2*e^3) + x^2/(4*d*e^4) +
 Log[d + e/Sqrt[x]]/d^5 - Log[-(e/Sqrt[x])]/d^5)) - (x^(5/2)*(a + b*Log[c/
x^(n/2)]))/(5*e^5))/d + ((-1/4*(b*n*(-(Sqrt[x]/(d^3*e)) + x/(2*d^2*e^2) -
x^(3/2)/(3*d*e^3) + Log[d + e/Sqrt[x]]/d^4 - Log[-(e/Sqrt[x])]/d^4)) + (x^
2*(a + b*Log[c/x^(n/2)]))/(4*e^4))/d + ((-1/3*(b*n*(-(Sqrt[x]/(d^2*e)) + x
/(2*d*e^2) + Log[d + e/Sqrt[x]]/d^3 - Log[-(e/Sqrt[x])]/d^3)) - (x^(3/2)*(
a + b*Log[c/x^(n/2)]))/(3*e^3))/d + ((-1/2*(b*n*(-(Sqrt[x]/(d*e)) + Log[d
+ e/Sqrt[x]]/d^2 - Log[-(e/Sqrt[x])]/d^2)) + (x*(a + b*Log[c/x^(n/2)]))/(2
*e^2))/d + (((b*n*Log[-(e/Sqrt[x])])/d - ((d + e/Sqrt[x])*Sqrt[x]*(a + b*L
og[c/x^(n/2)]))/(d*e))/d + (-((Log[1 - d*Sqrt[x]]*(a + b*Log[c/x^(n/2)]))/
d) + (b*n*PolyLog[2, d*Sqrt[x]])/d)/d)/d)/d)/d)/d))/3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx$$

input
```
int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Fricas [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x
+ e*sqrt(x))/x)^n) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

input
```
integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

output
```
Integral(x**2*(a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

Maxima [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3*log((d*sqrt(x) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
 2*a*b*d*log(c) + a^2*d)*x^3 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)
*x^(5/2) + 3*(b^2*d*x^3 + b^2*e*x^(5/2))*log(x^(1/2*n))^2 - (b^2*d*n*x^3 -
 6*(b^2*d*log(c) + a*b*d)*x^3 - 6*(b^2*e*log(c) + a*b*e)*x^(5/2) + 6*(b^2*
d*x^3 + b^2*e*x^(5/2))*log(x^(1/2*n)))*log((d*sqrt(x) + e)^n) - 6*((b^2*d*
log(c) + a*b*d)*x^3 + (b^2*e*log(c) + a*b*e)*x^(5/2))*log(x^(1/2*n)))/(d*x
 + e*sqrt(x)), x)
```

Giac [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int x^2\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e/x^(1/2))^n))^2,x)
```

output
```
int(x^2*(a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

Reduce [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

$$=\frac{60\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)^2 b^2 d\,e^5+24\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right) b^2 d^5 en\,x^2+40\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right) b^2 d^3 e^3 nx+120}{}$$

input
```
int(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x)
```

output
```
(60*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**2*d*e**5 + 24*sqrt(
x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**5*e*n*x**2 + 40*sqrt(x)*lo
g(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**3*e**3*n*x + 120*sqrt(x)*log(((
sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d*e**5*n + 24*sqrt(x)*a*b*d**5*e*n*x**
2 + 40*sqrt(x)*a*b*d**3*e**3*n*x + 120*sqrt(x)*a*b*d*e**5*n - 18*sqrt(x)*b
**2*d**3*e**3*n**2*x - 154*sqrt(x)*b**2*d*e**5*n**2 - 30*int((sqrt(x)*log(
((sqrt(x)*d + e)**n*c)/x**(n/2))**2)/x,x)*b**2*d*e**5 + 60*log(((sqrt(x)*d
 + e)**n*c)/x**(n/2))**2*b**2*d**6*x**3 + 120*log(((sqrt(x)*d + e)**n*c)/x
**(n/2))*a*b*d**6*x**3 - 120*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b*e**6
 - 30*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**4*e**2*n*x**2 - 60*log(
((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**2*e**4*n*x + 274*log(((sqrt(x)*d
+ e)**n*c)/x**(n/2))*b**2*e**6*n - 120*log(sqrt(x))*a*b*e**6*n + 274*log(s
qrt(x))*b**2*e**6*n**2 + 60*a**2*d**6*x**3 - 30*a*b*d**4*e**2*n*x**2 - 60*
a*b*d**2*e**4*n*x + 6*b**2*d**4*e**2*n**2*x**2 + 47*b**2*d**2*e**4*n**2*x)
/(180*d**6)
```

3.430 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 288

$$\begin{aligned}
&\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx\\
&=-\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}\\
&\quad+\frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}\\
&\quad-\frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}\\
&\quad+\frac{be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}\\
&\quad+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{11b^2e^4n^2\log(x)}{12d^4}-\frac{b^2e^4n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}
\end{aligned}$$

output
```
-5/6*b^2*e^3*n^2*x^(1/2)/d^3+1/6*b^2*e^2*n^2*x/d^2+5/6*b^2*e^4*n^2*ln(d+e/
x^(1/2))/d^4+b*e^3*n*(d+e/x^(1/2))*x^(1/2)*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4
-1/2*b*e^2*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+1/3*b*e*n*x^(3/2)*(a+b*ln(c
*(d+e/x^(1/2))^n))/d+b*e^4*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))
^n))/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2+11/12*b^2*e^4*n^2*ln(x)/d^4
-b^2*e^4*n^2*polylog(2,d/(d+e/x^(1/2)))/d^4
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.11

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = \frac{1}{6}\left(3x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{ben\left(6ade^2\sqrt{x}-5bde^2n\sqrt{x}-3ad^2ex+bd^2enx+2ad^3x^{3/2}+8be^3n\log\left(d+\frac{e}{\sqrt{x}}\right)+6bde^2\sqrt{x}\log\left(c\left(d\right.\right.\right.}{}\right.$$

input
```
Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

output
```
(3*x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(6*a*d*e^2*Sqrt[x] - 5*
b*d*e^2*n*Sqrt[x] - 3*a*d^2*e*x + b*d^2*e*n*x + 2*a*d^3*x^(3/2) + 8*b*e^3*
n*Log[d + e/Sqrt[x]] + 6*b*d*e^2*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 3*b*d^
2*e*x*Log[c*(d + e/Sqrt[x])^n] + 2*b*d^3*x^(3/2)*Log[c*(d + e/Sqrt[x])^n]
 - 6*a*e^3*Log[e + d*Sqrt[x]] + 3*b*e^3*n*Log[e + d*Sqrt[x]] - 6*b*e^3*Log[
c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] + 3*b*e^3*n*Log[e + d*Sqrt[x]]^2 -
 6*b*e^3*n*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] + 4*b*e^3*n*Log[x] - 6
*b*e^3*n*PolyLog[2, 1 + (d*Sqrt[x])/e]))/d^4)/6
```

Rubi [A] (warning: unable to verify)

Time = 2.18 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int x^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2845}$$

$$-2\left(\frac{1}{2}ben\int \frac{x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d+\frac{e}{\sqrt{x}}}d\frac{1}{\sqrt{x}}-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2858}$$

$$-2\left(\frac{1}{2}bn\int x^{5/2}\left(a+b\log\left(cx^{-n/2}\right)\right)d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{1}{2}be^4n\int \frac{x^{5/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2789}$$

$$-2\left(\frac{1}{2}be^4n\left(\frac{\int \frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2756}$$

$$-2\left(\frac{1}{2}be^4n\left(\frac{-\frac{1}{3}bn\int -\frac{x^2}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{x^{3/2}(a+b\log(cx^{-n/2}))}{3e^3}}{d}+\frac{\int -\frac{x^2(a+b\log(cx^{-n/2}))}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{1}{4}x^2\left(a+b\log\right.\right.$$

$\downarrow$ 54

$$-2\left(\frac{1}{2}be^4n\left(\frac{-\frac{1}{3}bn\int\left(-\frac{x^{3/2}}{de^3}+\frac{x}{d^2e^2}-\frac{\sqrt{x}}{d^3e}+\frac{\sqrt{x}}{d^3}\right)d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{x^{3/2}(a+b\log(cx^{-n/2}))}{3e^3}}{d}+\frac{\int -\frac{x^2(a+b\log(cx^{-n/2}))}{e^3}d\left(d+\right.}{d}\right.\right.$$

$\downarrow$ 2009

$$-2\left(\frac{1}{2}be^4n\left(\frac{\int -\frac{x^2(a+b\log(cx^{-n/2}))}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{-\frac{x^{3/2}(a+b\log(cx^{-n/2}))}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^3}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^3}-\frac{\sqrt{x}}{d^2e}+\frac{x}{2d}\right.}{d}\right.\right.$$

$\downarrow$ 2789

$$-2\left(\frac{1}{2}be^4n\left(\frac{\frac{\int -\frac{x^{3/2}(a+b\log(cx^{-n/2}))}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int \frac{x^{3/2}(a+b\log(cx^{-n/2}))}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{x^{3/2}(a+b\log(cx^{-n/2}))}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d\right.}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2756

$$-2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{x(a+b\log(cx^{-n/2}))}{2e^2}-\frac{1}{2}bn\int\frac{x^{3/2}}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int \frac{x^{3/2}(a+b\log(cx^{-n/2}))}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{x^{3/2}(a+b\log(cx^{-n/2}))}{3e^3}-\frac{1}{3}bn\left(\right.}{}\right.\right.$$

$\downarrow$ 54

$$-2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{x(a+b\log(cx^{-n/2}))}{2e^2}-\frac{1}{2}bn\int\left(\frac{x}{de^2}-\frac{\sqrt{x}}{d^2e}+\frac{\sqrt{x}}{d^2}\right)d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int \frac{x^{3/2}(a+b\log(cx^{-n/2}))}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{x^{3/2}(a+b\log(cx^{-n/2})}{3e^3}}{}\right.\right.$$

$\downarrow$ 2009

$$-2\left(\frac{1}{2}be^4n\left(\frac{\frac{\int \frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\sqrt{x}}{de}\right)}{d}}{d}+\frac{-\frac{x^{3/2}\left(a+b\log\left(cx^{-\cdots\right.\right.}{3e^3}\cdots}{\cdots}\right.\right.$$

$\big\downarrow$ 2789

$$-2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{\int \frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^2}\cdots\right.}{d}}{d}\cdots\right.\right.$$

$\big\downarrow$ 2751

$$-2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{-\frac{bn\int -\frac{\sqrt{x}}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}+\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(\cdots\right.}{\cdots}\right.}{d}}{d}\cdots\right.\right.$$

$\big\downarrow$ 16

$$-2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}\cdots\right.}{d}}{d}\cdots\right.\right.$$

$\big\downarrow$ 2779

$$-2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{\frac{bn\int \sqrt{x}\log(1-d\sqrt{x})d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\log(1-d\sqrt{x})\left(a+b\log\left(cx^{-n/2}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx\cdots\right.\right.}{2e^2}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$\big\downarrow$ 2838

$$-2\left(\frac{1}{2}be^4n\left(\frac{\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\sqrt{x}}{de}\right)}{d}+\frac{\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}+\frac{bn\operatorname{PolyLog}(2,}{d}}{d}}{d}\right.\right.$$

input
```
Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

output
```
-2*(-1/4*(x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2) + (b*e^4*n*((-1/3*(b*n*(
-(Sqrt[x]/(d^2*e)) + x/(2*d*e^2) + Log[d + e/Sqrt[x]]/d^3 - Log[-(e/Sqrt[x
])]/d^3)) - (x^(3/2)*(a + b*Log[c/x^(n/2)]))/(3*e^3))/d + ((-1/2*(b*n*(-(S
qrt[x]/(d*e)) + Log[d + e/Sqrt[x]]/d^2 - Log[-(e/Sqrt[x])]/d^2)) + (x*(a +
 b*Log[c/x^(n/2)]))/(2*e^2))/d + (((b*n*Log[-(e/Sqrt[x])])/d - ((d + e/Sqr
t[x])*Sqrt[x]*(a + b*Log[c/x^(n/2)]))/(d*e))/d + (-((Log[1 - d*Sqrt[x]]*(a
 + b*Log[c/x^(n/2)]))/d) + (b*n*PolyLog[2, d*Sqrt[x]])/d)/d)/d)/d))/2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*
sqrt(x))/x)^n) + a^2*x, x)
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

output
```
Integral(x*(a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*log((d*sqrt(x) + e)^n)^2 - integrate(-1/2*(2*(b^2*d*log(c)^2 +
 2*a*b*d*log(c) + a^2*d)*x^2 + 2*(b^2*d*x^2 + b^2*e*x^(3/2))*log(x^(1/2*n)
)^2 + 2*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(3/2) - (b^2*d*n*x^2 -
 4*(b^2*d*log(c) + a*b*d)*x^2 - 4*(b^2*e*log(c) + a*b*e)*x^(3/2) + 4*(b^2*
d*x^2 + b^2*e*x^(3/2))*log(x^(1/2*n)))*log((d*sqrt(x) + e)^n) - 4*((b^2*d*
log(c) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(3/2))*log(x^(1/2*n)))/(d*x
 + e*sqrt(x)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(1/2))^n))^2,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx$$

$$=\frac{6\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)^2b^2d\,e^3+4\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b^2d^3enx+12\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b^2d\,e^3n+4\sqrt{x}\,ab\,d^3}{}$$

input
```
int(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x)
```

output
```
(6*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**2*d*e**3 + 4*sqrt(x)
*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**3*e*n*x + 12*sqrt(x)*log(((s
qrt(x)*d + e)**n*c)/x**(n/2))*b**2*d*e**3*n + 4*sqrt(x)*a*b*d**3*e*n*x + 1
2*sqrt(x)*a*b*d*e**3*n - 10*sqrt(x)*b**2*d*e**3*n**2 - 3*int((sqrt(x)*log(
((sqrt(x)*d + e)**n*c)/x**(n/2))**2)/x,x)*b**2*d*e**3 + 6*log(((sqrt(x)*d
+ e)**n*c)/x**(n/2))**2*b**2*d**4*x**2 + 12*log(((sqrt(x)*d + e)**n*c)/x**
(n/2))*a*b*d**4*x**2 - 12*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b*e**4 -
6*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**2*e**2*n*x + 22*log(((sqrt(
x)*d + e)**n*c)/x**(n/2))*b**2*e**4*n - 12*log(sqrt(x))*a*b*e**4*n + 22*lo
g(sqrt(x))*b**2*e**4*n**2 + 6*a**2*d**4*x**2 - 6*a*b*d**2*e**2*n*x + 2*b**
2*d**2*e**2*n**2*x)/(12*d**4)
```

3.431 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + \frac{2be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{b^2e^2n^2\log(x)}{d^2} - \frac{2b^2e^2n^2\,\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}$$

output
```
2*b*e*n*(d+e/x^(1/2))*x^(1/2)*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+2*b*e^2*n*ln
(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+x*(a+b*ln(c*(d+e/x^(1/
2))^n))^2+b^2*e^2*n^2*ln(x)/d^2-2*b^2*e^2*n^2*polylog(2,d/(d+e/x^(1/2)))/d
^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.13

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{ben\left(2ad\sqrt{x}+2ben\log\left(d+\frac{e}{\sqrt{x}}\right)+2bd\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-2e\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log(e+\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

output
```
x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(2*a*d*Sqrt[x] + 2*b*e*n*Log
[d + e/Sqrt[x]] + 2*b*d*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 2*e*(a + b*Log[
c*(d + e/Sqrt[x])^n])*Log[e + d*Sqrt[x]] + b*e*n*Log[x] + b*e*n*(Log[e + d
*Sqrt[x]]*(Log[e + d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1
+ (d*Sqrt[x])/e])))/d^2
```

Rubi [A] (warning: unable to verify)

Time = 1.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2901, 2904, 2845, 2858, 27, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$$

$$\Big\downarrow \text{ 2901}$$

$$2\int \sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 d\sqrt{x}$$

$$\Big\downarrow \text{ 2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^{3/2}}d\frac{1}{\sqrt{x}}$$

$$\downarrow \text{2845}$$

$$-2\left(ben\int\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{\left(d+\frac{e}{\sqrt{x}}\right)x}d\frac{1}{\sqrt{x}}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x}\right)$$

$$\downarrow \text{2858}$$

$$-2\left(bn\int\left(d+\frac{e}{\sqrt{x}}\right)x\left(a+b\log\left(cx^{n/2}\right)\right)d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x}\right)$$

$$\downarrow \text{27}$$

$$-2\left(be^2n\int\frac{\left(d+\frac{e}{\sqrt{x}}\right)x\left(a+b\log\left(cx^{n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x}\right)$$

$$\downarrow \text{2789}$$

$$-2\left(be^2n\left(\frac{\int\frac{x\left(a+b\log\left(cx^{n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int-\frac{\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(cx^{n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right.\right.}{2x}$$

$$\downarrow \text{2751}$$

$$-2\left(be^2n\left(\frac{-\frac{bn\int-\frac{\sqrt{x}}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{de}}{d}+\frac{\int-\frac{\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(cx^{n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{\left(a+b\right.}{}$$

$$\downarrow \text{16}$$

$$-2\left(be^2n\left(\frac{\int-\frac{\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(cx^{n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{de}}{d}\right)-\frac{\left(a+b\log\left(c\right.\right.}{2}$$

$$\downarrow \text{2779}$$

$$-2\left(be^2n\left(\frac{\frac{bn\int\left(d+\frac{e}{\sqrt{x}}\right)\log\left(1-d\left(d+\frac{e}{\sqrt{x}}\right)\right)d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\log\left(1-d\left(d+\frac{e}{\sqrt{x}}\right)\right)\left(a+b\log\left(cx^{n/2}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\right.}{de}}{d}\right.\right.$$

$$\Big\downarrow \text{2838}$$

$$-2\left(be^2n\left(\frac{\frac{bn\operatorname{PolyLog}\left(2,d\left(d+\frac{e}{\sqrt{x}}\right)\right)}{d}-\frac{\log\left(1-d\left(d+\frac{e}{\sqrt{x}}\right)\right)(a+b\log(cx^{n/2}))}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)(a+b\log(cx^{n/2}))}{de}}{d}\right)-\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

output
```
-2*(-1/2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x + b*e^2*n*(((b*n*Log[-(e/Sqr
t[x])])/d - ((d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*x^(n/2)]))/(d*e))/d + (-
((Log[1 - d*(d + e/Sqrt[x])]*(a + b*Log[c*x^(n/2)]))/d) + (b*n*PolyLog[2,
d*(d + e/Sqrt[x])])/d)/d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2 dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*log(c*((d*x + e*sqrt
(x))/x)^n) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = \int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
```

output
```
-2*(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d) - x*log(c*(d + e/sqrt(x))^n
))*a*b + (x*log((d*sqrt(x) + e)^n)^2 - integrate(-(d*x*log(c)^2 + e*sqrt(x
)*log(c)^2 + (d*x + e*sqrt(x))*log(x^(1/2*n))^2 - (d*n*x - 2*d*x*log(c) -
2*e*sqrt(x)*log(c) + 2*(d*x + e*sqrt(x))*log(x^(1/2*n)))*log((d*sqrt(x) +
e)^n) - 2*(d*x*log(c) + e*sqrt(x)*log(c))*log(x^(1/2*n)))/(d*x + e*sqrt(x)
), x))*b^2 + a^2*x
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int \left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$$

$$= \frac{2\sqrt{x}\,\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)^2 b^2de + 4\sqrt{x}\,\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right) b^2den + 4\sqrt{x}\,abden - \left(\int \frac{\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)^2}{x}dx\right) b^2de + \cdots}{\cdots}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^2,x)
```

output
```
(2*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**2*d*e + 4*sqrt(x)*lo
g(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d*e*n + 4*sqrt(x)*a*b*d*e*n - int(
(sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2)/x,x)*b**2*d*e + 2*log(((
sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**2*d**2*x + 4*log(((sqrt(x)*d + e)**n*
c)/x**(n/2))*a*b*d**2*x - 4*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b*e**2
+ 4*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*e**2*n - 4*log(sqrt(x))*a*b*
e**2*n + 4*log(sqrt(x))*b**2*e**2*n**2 + 2*a**2*d**2*x)/(2*d**2)
```

3.432 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx = -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) - 4bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right) + 4b^2n^2\operatorname{PolyLog}\left(3,1+\frac{e}{d\sqrt{x}}\right)$$

output
```
-2*(a+b*ln(c*(d+e/x^(1/2))^n))^2*ln(-e/d/x^(1/2))-4*b*n*(a+b*ln(c*(d+e/x^(
1/2))^n))*polylog(2,1+e/d/x^(1/2))+4*b^2*n^2*polylog(3,1+e/d/x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 386 vs. $2(93) = 186$.

Time = 0.37 (sec) , antiderivative size = 386, normalized size of antiderivative = 4.15

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx$$
$$=\left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log(x)+2bn\left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\left(\left(\log\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(1+\frac{e}{d\sqrt{x}}\right)\right)\log(x)+2\,\text{PolyLog}\left(2,-\frac{e}{d\sqrt{x}}\right)\right)$$
$$+\frac{1}{12}b^2n^2\left(24\log^2\left(\frac{e}{d}+\sqrt{x}\right)\log\left(-\frac{d\sqrt{x}}{e}\right)+12\log^2\left(d+\frac{e}{\sqrt{x}}\right)\log(x)-12\log^2\left(\frac{e}{d}+\sqrt{x}\right)\log(x)-24\log\left(d+\frac{e}{\sqrt{x}}\right)\log\left(1+\frac{d\sqrt{x}}{e}\right)\log(x)+24\log\left(\frac{e}{d}+\sqrt{x}\right)\log\left(1+\frac{d\sqrt{x}}{e}\right)\log(x)+6\log\left(d+\frac{e}{\sqrt{x}}\right)\log^2(x)-6\log\left(1+\frac{d\sqrt{x}}{e}\right)\log^2(x)+\log^3(x)+48\log\left(\frac{e}{d}+\sqrt{x}\right)\text{PolyLog}\left(2,1+\frac{d\sqrt{x}}{e}\right)-48\left(\log\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(\frac{e}{d}+\sqrt{x}\right)\right)\text{PolyLog}\left(2,-\frac{d\sqrt{x}}{e}\right)-48\,\text{PolyLog}\left(3,1+\frac{d\sqrt{x}}{e}\right)-48\,\text{PolyLog}\left(3,-\frac{d\sqrt{x}}{e}\right)\right)$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

output
```
(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[x] + 2*b*n
*(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*((Log[d + e/Sqr
t[x]] - Log[1 + e/(d*Sqrt[x])])*Log[x] + 2*PolyLog[2, -(e/(d*Sqrt[x]))]) +
 (b^2*n^2*(24*Log[e/d + Sqrt[x]]^2*Log[-((d*Sqrt[x])/e)] + 12*Log[d + e/Sq
rt[x]]^2*Log[x] - 12*Log[e/d + Sqrt[x]]^2*Log[x] - 24*Log[d + e/Sqrt[x]]*L
og[1 + (d*Sqrt[x])/e]*Log[x] + 24*Log[e/d + Sqrt[x]]*Log[1 + (d*Sqrt[x])/e
]*Log[x] + 6*Log[d + e/Sqrt[x]]*Log[x]^2 - 6*Log[1 + (d*Sqrt[x])/e]*Log[x]
^2 + Log[x]^3 + 48*Log[e/d + Sqrt[x]]*PolyLog[2, 1 + (d*Sqrt[x])/e] - 48*(
Log[d + e/Sqrt[x]] - Log[e/d + Sqrt[x]])*PolyLog[2, -((d*Sqrt[x])/e)] - 48
*PolyLog[3, 1 + (d*Sqrt[x])/e] - 48*PolyLog[3, -((d*Sqrt[x])/e)]))/12
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2843}$$

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-2ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)}{d+\frac{e}{\sqrt{x}}}d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{2881}$$

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-2bn\int \sqrt{x}\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)d\left(d+\frac{e}{\sqrt{x}}\right)\right.$$

$$\Big\downarrow \text{2821}$$

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-2bn\left(bn\int \sqrt{x}\,\text{PolyLog}\left(2,\frac{d+\frac{e}{\sqrt{x}}}{d}\right)d\left(d+\frac{e}{\sqrt{x}}\right)-\text{PolyLo}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-2bn\left(bn\,\text{PolyLog}\left(3,\frac{d+\frac{e}{\sqrt{x}}}{d}\right)-\text{PolyLog}\left(2,\frac{d+\frac{e}{\sqrt{x}}}{d}\right)\left(a\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

output
```
-2*((a + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[x]))] - 2*b*n*(-((a
 + b*Log[c/x^(n/2)])*PolyLog[2, (d + e/Sqrt[x])/d]) + b*n*PolyLog[3, (d +
e/Sqrt[x])/d]))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*log(c*((d*x + e*sqr
t(x))/x)^n) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x,x)
```

output
```
Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="maxima")
```

output
```
b^2*log((d*sqrt(x) + e)^n)^2*log(x) - integrate(-((b^2*d*x + b^2*e*sqrt(x)
)*log(x^(1/2*n))^2 + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x - (b^2*d*
n*x*log(x) - 2*(b^2*d*log(c) + a*b*d)*x + 2*(b^2*d*x + b^2*e*sqrt(x))*log(
x^(1/2*n)) - 2*(b^2*e*log(c) + a*b*e)*sqrt(x))*log((d*sqrt(x) + e)^n) - 2*
((b^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*sqrt(x))*log(x^(1/2*n))
 + (b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*sqrt(x))/(d*x^2 + e*x^(3/2)),
 x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^2/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^n))^2/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx=\left(\int \frac{\log\left(\frac{\left(\sqrt{x}\,d+e\right)^{n}c}{x^{\frac{n}{2}}}\right)^{2}}{x}dx\right)b^{2}+2\left(\int \frac{\log\left(\frac{\left(\sqrt{x}\,d+e\right)^{n}c}{x^{\frac{n}{2}}}\right)}{x}dx\right)ab+\log(x)\,a^{2}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x)
```

output
```
int(log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2/x,x)*b**2 + 2*int(log(((sqrt(x
)*d + e)**n*c)/x**(n/2))/x,x)*a*b + log(x)*a**2
```

3.433 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 195

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2} - \frac{4abdn}{e\sqrt{x}} + \frac{4b^2dn^2}{e\sqrt{x}} - \frac{4b^2dn\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}$$

output

```
-1/2*b^2*n^2*(d+e/x^(1/2))^2/e^2-4*a*b*d*n/e/x^(1/2)+4*b^2*d*n^2/e/x^(1/2)
-4*b^2*d*n*(d+e/x^(1/2))*ln(c*(d+e/x^(1/2))^n)/e^2+b*n*(d+e/x^(1/2))^2*(a+
b*ln(c*(d+e/x^(1/2))^n))/e^2+2*d*(d+e/x^(1/2))*(a+b*ln(c*(d+e/x^(1/2))^n))
^2/e^2-(d+e/x^(1/2))^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2/e^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx =$$

$$-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{bn\left(4ade\sqrt{x}-4bden\sqrt{x}+bn\left(e\left(e-2d\sqrt{x}\right)+2d^2x\log\left(d+\frac{e}{\sqrt{x}}\right)\right)+4bd\left(e+d\sqrt{x}\right)\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right.\right.}{}}{}$$

input

```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

output

```
-1/2*(2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*n*(4*a*d*e*Sqrt[x] - 4*b*d
*e*n*Sqrt[x] + b*n*(e*(e - 2*d*Sqrt[x]) + 2*d^2*x*Log[d + e/Sqrt[x]]) + 4*
b*d*(e + d*Sqrt[x])*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 2*e^2*(a + b*Log[c*
(d + e/Sqrt[x])^n]) - 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[e + d*S
qrt[x]] - 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] -
 4*b*d^2*n*x*PolyLog[2, 1 + e/(d*Sqrt[x])] + 2*b*d^2*n*x*(Log[e + d*Sqrt[x
]]*(Log[e + d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1 + (d*Sq
rt[x])/e])))/e^2)/x
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{\sqrt{x}}d\frac{1}{\sqrt{x}}$$

$\Big\downarrow$ 2848

$$-2\int \left(\frac{\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e}-\frac{d\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e}\right)d\frac{1}{\sqrt{x}}$$

$\Big\downarrow$ 2009

$$-2\left(-\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2}+\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2}-\frac{d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

output
```
-2*((b^2*n^2*(d + e/Sqrt[x])^2)/(4*e^2) + (2*a*b*d*n)/(e*Sqrt[x]) - (2*b^2
*d*n^2)/(e*Sqrt[x]) + (2*b^2*d*n*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])
/e^2 - (b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*e^2) -
(d*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2 + ((d + e/Sqrt[
x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx = -\frac{b^2e^2n^2+2\,b^2e^2\log\left(c\right)^2-2\,abe^2n+2\,a^2e^2-2\left(b^2d^2n^2x-b^2e^2n^2\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^2-2\left(b^2e^2n-2\,abe^2\right)\mathrm{l}}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*e^2*n^2 + 2*b^2*e^2*log(c)^2 - 2*a*b*e^2*n + 2*a^2*e^2 - 2*(b^2*
d^2*n^2*x - b^2*e^2*n^2)*log((d*x + e*sqrt(x))/x)^2 - 2*(b^2*e^2*n - 2*a*b
*e^2)*log(c) + 2*(2*b^2*d*e*n^2*sqrt(x) - b^2*e^2*n^2 + 2*a*b*e^2*n + (3*b
^2*d^2*n^2 - 2*a*b*d^2*n)*x - 2*(b^2*d^2*n*x - b^2*e^2*n)*log(c))*log((d*x
 + e*sqrt(x))/x) - 2*(3*b^2*d*e*n^2 - 2*b^2*d*e*n*log(c) - 2*a*b*d*e*n)*sq
rt(x))/(e^2*x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**2,x)
```

output
```
Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.27

$$\begin{aligned}&\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx\\&=aben\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log\left(x\right)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)\\&\quad+\frac{1}{4}\left(4\,en\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log\left(x\right)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{\left(4\,d^2x\log\left(d\sqrt{x}+e\right.\right.}{}\right.\\&\quad-\frac{b^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{x}-\frac{2\,ab\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}-\frac{a^2}{x}\end{aligned}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="maxima")
```

output
```
a*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e)
/(e^2*x)) + 1/4*(4*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2
*d*sqrt(x) - e)/(e^2*x))*log(c*(d + e/sqrt(x))^n) - (4*d^2*x*log(d*sqrt(x)
 + e)^2 + d^2*x*log(x)^2 - 6*d^2*x*log(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^
2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) + e))*n^2/(e^2*x))*b^2 - b^2*log(c*(d
+ e/sqrt(x))^n)^2/x - 2*a*b*log(c*(d + e/sqrt(x))^n)/x - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$= \frac{2\left(\frac{2\,(d\sqrt{x}+e)b^2dn^2}{e\sqrt{x}} - \frac{(d\sqrt{x}+e)^2b^2n^2}{ex}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^2 + 2\left(\frac{(b^2n^2-2\,b^2n\log(c)-2\,abn)\,(d\sqrt{x}+e)^2}{ex} - \frac{4\,(b^2dn^2-b^2dn\log(c)-abdn)}{e\sqrt{x}}\right.}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="giac")
```

output
```
1/2*(2*(2*(d*sqrt(x) + e)*b^2*d*n^2/(e*sqrt(x)) - (d*sqrt(x) + e)^2*b^2*n^
2/(e*x))*log((d*sqrt(x) + e)/sqrt(x))^2 + 2*((b^2*n^2 - 2*b^2*n*log(c) - 2
*a*b*n)*(d*sqrt(x) + e)^2/(e*x) - 4*(b^2*d*n^2 - b^2*d*n*log(c) - a*b*d*n)
*(d*sqrt(x) + e)/(e*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x)) - (b^2*n^2 - 2*
b^2*n*log(c) + 2*b^2*log(c)^2 - 2*a*b*n + 4*a*b*log(c) + 2*a^2)*(d*sqrt(x)
 + e)^2/(e*x) + 4*(2*b^2*d*n^2 - 2*b^2*d*n*log(c) + b^2*d*log(c)^2 - 2*a*b
*d*n + 2*a*b*d*log(c) + a^2*d)*(d*sqrt(x) + e)/(e*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx = \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{2\,b\,d\,(2\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}}{\sqrt{x}} - \frac{b\,(2\,a-b\,n)}{x}\right) - \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{b^2}{x}-\frac{b^2\,d^2}{e^2}\right) + \frac{\frac{d\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{e}-\frac{2\,d\,(a^2-b^2\,n^2)}{e}}{\sqrt{x}} - \frac{a^2-a\,b\,n+\frac{b^2\,n^2}{2}}{x} - \frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)\,(3\,b^2\,d^2\,n^2-2\,a\,b\,d^2\,n)}{e^2}$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^2,x)
```

output
```
log(c*(d + e/x^(1/2))^n)*(((2*b*d*(2*a - b*n))/e - (4*a*b*d)/e)/x^(1/2) -
(b*(2*a - b*n))/x) - log(c*(d + e/x^(1/2))^n)^2*(b^2/x - (b^2*d^2)/e^2) +
((d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e - (2*d*(a^2 - b^2*n^2))/e)/x^(1/2) - (a
^2 + (b^2*n^2)/2 - a*b*n)/x - (log(d + e/x^(1/2))*(3*b^2*d^2*n^2 - 2*a*b*d
^2*n))/e^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$= \frac{-4\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b^2den - 4\sqrt{x}\,abden + 6\sqrt{x}\,b^2de\,n^2 + 2\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)^2 b^2d^2x - 2\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)^2}{}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x)
```

output
```
( - 4*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d*e*n - 4*sqrt(x)*
a*b*d*e*n + 6*sqrt(x)*b**2*d*e*n**2 + 2*log(((sqrt(x)*d + e)**n*c)/x**(n/2
))**2*b**2*d**2*x - 2*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**2*e**2 +
4*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b*d**2*x - 4*log(((sqrt(x)*d + e)
**n*c)/x**(n/2))*a*b*e**2 - 6*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d*
*2*n*x + 2*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*e**2*n - 2*a**2*e**2
+ 2*a*b*e**2*n - b**2*e**2*n**2)/(2*e**2*x)
```

3.434 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 341

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx = -\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4}+\frac{4b^2d^3n^2}{e^3\sqrt{x}} - \frac{b^2d^4n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4} - \frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} + \frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4} + \frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2}$$

output

```
-3/2*b^2*d^2*n^2*(d+e/x^(1/2))^2/e^4+4/9*b^2*d*n^2*(d+e/x^(1/2))^3/e^4-1/1
6*b^2*n^2*(d+e/x^(1/2))^4/e^4+4*b^2*d^3*n^2/e^3/x^(1/2)-1/2*b^2*d^4*n^2*ln
(d+e/x^(1/2))^2/e^4-4*b*d^3*n*(d+e/x^(1/2))*(a+b*ln(c*(d+e/x^(1/2))^n))/e^
4+3*b*d^2*n*(d+e/x^(1/2))^2*(a+b*ln(c*(d+e/x^(1/2))^n))/e^4-4/3*b*d*n*(d+e
/x^(1/2))^3*(a+b*ln(c*(d+e/x^(1/2))^n))/e^4+1/4*b*n*(d+e/x^(1/2))^4*(a+b*l
n(c*(d+e/x^(1/2))^n))/e^4+b*d^4*n*ln(d+e/x^(1/2))*(a+b*ln(c*(d+e/x^(1/2))^
n))/e^4-1/2*(a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$
$$= \frac{-72e^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+bn\left(36ae^4-9be^4n-48ade^3\sqrt{x}+28bde^3n\sqrt{x}+72ad^2e^2x-78bd^2e\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

output
```
(-72*e^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + b*n*(36*a*e^4 - 9*b*e^4*n -
48*a*d*e^3*Sqrt[x] + 28*b*d*e^3*n*Sqrt[x] + 72*a*d^2*e^2*x - 78*b*d^2*e^2*
n*x - 144*a*d^3*e*x^(3/2) + 300*b*d^3*e*n*x^(3/2) - 300*b*d^4*n*x^2*Log[d
+ e/Sqrt[x]] + 36*b*e^4*Log[c*(d + e/Sqrt[x])^n] - 48*b*d*e^3*Sqrt[x]*Log[
c*(d + e/Sqrt[x])^n] + 72*b*d^2*e^2*x*Log[c*(d + e/Sqrt[x])^n] - 144*b*d^3
*e*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] + 144*a*d^4*x^2*Log[e + d*Sqrt[x]] + 1
44*b*d^4*x^2*Log[c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] - 72*b*d^4*n*x^2*
Log[e + d*Sqrt[x]]^2 + 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*S
qrt[x]))] + 144*b*d^4*n*x^2*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] - 72*
a*d^4*x^2*Log[x] + 144*b*d^4*n*x^2*PolyLog[2, 1 + e/(d*Sqrt[x])] + 144*b*d
^4*n*x^2*PolyLog[2, 1 + (d*Sqrt[x])/e]))/(144*e^4*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.67, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$

$\downarrow$ 2904

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^{3/2}}d\frac{1}{\sqrt{x}}$$

$\downarrow$ 2845

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4x^2}-\frac{1}{2}ben\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{\left(d+\frac{e}{\sqrt{x}}\right)x^2}d\frac{1}{\sqrt{x}}\right)$$

$\downarrow$ 2858

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4x^2}-\frac{1}{2}bn\int \frac{a+b\log\left(cx^{-n/2}\right)}{x^{3/2}}d\left(d+\frac{e}{\sqrt{x}}\right)\right)$$

$\downarrow$ 27

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4x^2}-\frac{bn\int \frac{e^4\left(a+b\log\left(cx^{-n/2}\right)\right)}{x^{3/2}}d\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}\right)$$

$\downarrow$ 2772

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4x^2}-\frac{bn\left(-bn\int\left(\sqrt{x}\log\left(d+\frac{e}{\sqrt{x}}\right)d^4-4d^3+3\left(d+\frac{e}{\sqrt{x}}\right)d^2-\frac{4d}{3x}+\frac{1}{4x^{3/2}}\right)d\left(d+\right.\right.}{}\right.$$

$\downarrow$ 2009

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4x^2}-\frac{bn\left(d^4\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)-4d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right.\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

output
```
-2*((a + b*Log[c*(d + e/Sqrt[x])^n])^2/(4*x^2) - (b*n*(-(b*n*(-4*d^3*(d +
e/Sqrt[x]) + 1/(16*x^2) - (4*d)/(9*x^(3/2)) + (3*d^2)/(2*x) + (d^4*Log[d +
 e/Sqrt[x]]^2)/2)) - 4*d^3*(d + e/Sqrt[x])*(a + b*Log[c/x^(n/2)]) + (a + b
*Log[c/x^(n/2)])/(4*x^2) - (4*d*(a + b*Log[c/x^(n/2)]))/(3*x^(3/2)) + (3*d
^2*(a + b*Log[c/x^(n/2)]))/x + d^4*Log[d + e/Sqrt[x]]*(a + b*Log[c/x^(n/2)
])))/(2*e^4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx =$$
$$-\frac{9\,b^{2}e^{4}n^{2}+72\,b^{2}e^{4}\log\left(c\right)^{2}-36\,abe^{4}n+72\,a^{2}e^{4}-72\left(b^{2}d^{4}n^{2}x^{2}-b^{2}e^{4}n^{2}\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^{2}+6\left(13\,b^{2}d^{2}e^{2}\right.}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

output
```
-1/144*(9*b^2*e^4*n^2 + 72*b^2*e^4*log(c)^2 - 36*a*b*e^4*n + 72*a^2*e^4 -
72*(b^2*d^4*n^2*x^2 - b^2*e^4*n^2)*log((d*x + e*sqrt(x))/x)^2 + 6*(13*b^2*
d^2*e^2*n^2 - 12*a*b*d^2*e^2*n)*x - 36*(2*b^2*d^2*e^2*n*x + b^2*e^4*n - 4*
a*b*e^4)*log(c) - 12*(6*b^2*d^2*e^2*n^2*x + 3*b^2*e^4*n^2 - 12*a*b*e^4*n -
 (25*b^2*d^4*n^2 - 12*a*b*d^4*n)*x^2 + 12*(b^2*d^4*n*x^2 - b^2*e^4*n)*log(
c) - 4*(3*b^2*d^3*e*n^2*x + b^2*d*e^3*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/
x) - 4*(7*b^2*d*e^3*n^2 - 12*a*b*d*e^3*n + 3*(25*b^2*d^3*e*n^2 - 12*a*b*d^
3*e*n)*x - 12*(3*b^2*d^3*e*n*x + b^2*d*e^3*n)*log(c))*sqrt(x))/(e^4*x^2)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**3,x)
```

output
```
Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$
$$=\frac{1}{12}aben\left(\frac{12\,d^4\log\left(d\sqrt{x}+e\right)}{e^5}-\frac{6\,d^4\log\left(x\right)}{e^5}-\frac{12\,d^3x^{\frac{3}{2}}-6\,d^2ex+4\,de^2\sqrt{x}-3\,e^3}{e^4x^2}\right)$$
$$+\frac{1}{144}\left(12\,en\left(\frac{12\,d^4\log\left(d\sqrt{x}+e\right)}{e^5}-\frac{6\,d^4\log\left(x\right)}{e^5}-\frac{12\,d^3x^{\frac{3}{2}}-6\,d^2ex+4\,de^2\sqrt{x}-3\,e^3}{e^4x^2}\right)\log\left(c\left(d+\right.\right.\right.$$
$$-\frac{b^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{2\,x^2}-\frac{ab\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}-\frac{a^2}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="maxima")
```

output
```
1/12*a*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x
^(3/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) + 1/144*(12*e*n*(
12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2
*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n) - (72*
d^4*x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) -
300*d^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*sqrt(x) + 9*e^4 - 12*(6*d^4*x^
2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n^2/(e^4*x^2))*b^2 - 1/2*b^2*lo
g(c*(d + e/sqrt(x))^n)^2/x^2 - a*b*log(c*(d + e/sqrt(x))^n)/x^2 - 1/2*a^2/
x^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$

$$= \frac{72\left(\frac{4\,(d\sqrt{x}+e)b^2d^3n^2}{e^3\sqrt{x}}-\frac{6\,(d\sqrt{x}+e)^2b^2d^2n^2}{e^3x}+\frac{4\,(d\sqrt{x}+e)^3b^2dn^2}{e^3x^{\frac{3}{2}}}-\frac{(d\sqrt{x}+e)^4b^2n^2}{e^3x^2}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^2+12\left(\frac{3\,(b^2n^2-4\,b^2n\log(c}{e^3}\right.}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="giac")
```

output
```
1/144*(72*(4*(d*sqrt(x) + e)*b^2*d^3*n^2/(e^3*sqrt(x)) - 6*(d*sqrt(x) + e)
^2*b^2*d^2*n^2/(e^3*x) + 4*(d*sqrt(x) + e)^3*b^2*d*n^2/(e^3*x^(3/2)) - (d*
sqrt(x) + e)^4*b^2*n^2/(e^3*x^2))*log((d*sqrt(x) + e)/sqrt(x))^2 + 12*(3*(
b^2*n^2 - 4*b^2*n*log(c) - 4*a*b*n)*(d*sqrt(x) + e)^4/(e^3*x^2) - 16*(b^2*
d*n^2 - 3*b^2*d*n*log(c) - 3*a*b*d*n)*(d*sqrt(x) + e)^3/(e^3*x^(3/2)) + 36
*(b^2*d^2*n^2 - 2*b^2*d^2*n*log(c) - 2*a*b*d^2*n)*(d*sqrt(x) + e)^2/(e^3*x
) - 48*(b^2*d^3*n^2 - b^2*d^3*n*log(c) - a*b*d^3*n)*(d*sqrt(x) + e)/(e^3*s
qrt(x)))*log((d*sqrt(x) + e)/sqrt(x)) - 9*(b^2*n^2 - 4*b^2*n*log(c) + 8*b^
2*log(c)^2 - 4*a*b*n + 16*a*b*log(c) + 8*a^2)*(d*sqrt(x) + e)^4/(e^3*x^2)
+ 32*(2*b^2*d*n^2 - 6*b^2*d*n*log(c) + 9*b^2*d*log(c)^2 - 6*a*b*d*n + 18*a
*b*d*log(c) + 9*a^2*d)*(d*sqrt(x) + e)^3/(e^3*x^(3/2)) - 216*(b^2*d^2*n^2
- 2*b^2*d^2*n*log(c) + 2*b^2*d^2*log(c)^2 - 2*a*b*d^2*n + 4*a*b*d^2*log(c)
 + 2*a^2*d^2)*(d*sqrt(x) + e)^2/(e^3*x) + 288*(2*b^2*d^3*n^2 - 2*b^2*d^3*n
*log(c) + b^2*d^3*log(c)^2 - 2*a*b*d^3*n + 2*a*b*d^3*log(c) + a^2*d^3)*(d*
sqrt(x) + e)/(e^3*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 14.80 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$

$$=\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{b\,d\,(4\,a-b\,n)}{3\,e}-\frac{4\,a\,b\,d}{3\,e}}{x^{3/2}}-\frac{b\,(4\,a-b\,n)}{4\,x^2}-\frac{d\left(\frac{b\,d\,(4\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right)}{2\,e\,x}+\frac{d^2\left(\frac{b\,d\,(4\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right)}{e^2\,\sqrt{x}}\right)$$
$$+\frac{\frac{d\left(2\,a^2-a\,b\,n+\frac{b^2\,n^2}{4}\right)}{3\,e}-\frac{d\,(6\,a^2-b^2\,n^2)}{9\,e}}{x^{3/2}}-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{b^2}{2\,x^2}-\frac{b^2\,d^4}{2\,e^4}\right)$$
$$-\frac{\frac{a^2}{2}-\frac{a\,b\,n}{4}+\frac{b^2\,n^2}{16}}{x^2}-\frac{\frac{d\left(\frac{d\left(2\,a^2-a\,b\,n+\frac{b^2\,n^2}{4}\right)}{e}-\frac{d\,(6\,a^2-b^2\,n^2)}{3\,e}\right)}{2\,e}+\frac{b^2\,d^2\,n^2}{4\,e^2}}{x}$$
$$+\frac{\frac{d\left(\frac{d\left(\frac{d\left(2\,a^2-a\,b\,n+\frac{b^2\,n^2}{4}\right)}{e}-\frac{d\,(6\,a^2-b^2\,n^2)}{3\,e}\right)}{e}+\frac{b^2\,d^2\,n^2}{2\,e^2}\right)}{e}+\frac{b^2\,d^3\,n^2}{e^3}}{\sqrt{x}}$$
$$-\frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)\,(25\,b^2\,d^4\,n^2-12\,a\,b\,d^4\,n)}{12\,e^4}$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^3,x)
```

output
```
log(c*(d + e/x^(1/2))^n)*(((b*d*(4*a - b*n))/(3*e) - (4*a*b*d)/(3*e))/x^(3
/2) - (b*(4*a - b*n))/(4*x^2) - (d*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(2
*e*x) + (d^2*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(e^2*x^(1/2))) + ((d*(2*
a^2 + (b^2*n^2)/4 - a*b*n))/(3*e) - (d*(6*a^2 - b^2*n^2))/(9*e))/x^(3/2) -
 log(c*(d + e/x^(1/2))^n)^2*(b^2/(2*x^2) - (b^2*d^4)/(2*e^4)) - (a^2/2 + (
b^2*n^2)/16 - (a*b*n)/4)/x^2 - ((d*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e -
(d*(6*a^2 - b^2*n^2))/(3*e)))/(2*e) + (b^2*d^2*n^2)/(4*e^2))/x + ((d*((d*(
(d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/e + (b
^2*d^2*n^2)/(2*e^2)))/e + (b^2*d^3*n^2)/e^3)/x^(1/2) - (log(d + e/x^(1/2))
*(25*b^2*d^4*n^2 - 12*a*b*d^4*n))/(12*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$
$$= \frac{-144\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b^2d^3enx-48\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b^2d\,e^3n-144\sqrt{x}\,ab\,d^3enx-48\sqrt{x}\,abd\,e^3n+3}{}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x)
```

output
```
( - 144*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**3*e*n*x - 48*
sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d*e**3*n - 144*sqrt(x)*a
*b*d**3*e*n*x - 48*sqrt(x)*a*b*d*e**3*n + 300*sqrt(x)*b**2*d**3*e*n**2*x +
 28*sqrt(x)*b**2*d*e**3*n**2 + 72*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*
b**2*d**4*x**2 - 72*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**2*e**4 + 14
4*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b*d**4*x**2 - 144*log(((sqrt(x)*d
 + e)**n*c)/x**(n/2))*a*b*e**4 - 300*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*
b**2*d**4*n*x**2 + 72*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**2*e**2*
n*x + 36*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*e**4*n - 72*a**2*e**4 +
 72*a*b*d**2*e**2*n*x + 36*a*b*e**4*n - 78*b**2*d**2*e**2*n**2*x - 9*b**2*
e**4*n**2)/(144*e**4*x**2)
```

3.435 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 480

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6}+\frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{54e^6}+\frac{4b^2d^5n^2}{e^5\sqrt{x}} - \frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6} - \frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} + \frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} - \frac{40bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} + \frac{5bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6} - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} + \frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3}$$

output
```
-5/2*b^2*d^4*n^2*(d+e/x^(1/2))^2/e^6+40/27*b^2*d^3*n^2*(d+e/x^(1/2))^3/e^6
-5/8*b^2*d^2*n^2*(d+e/x^(1/2))^4/e^6+4/25*b^2*d*n^2*(d+e/x^(1/2))^5/e^6-1/
54*b^2*n^2*(d+e/x^(1/2))^6/e^6+4*b^2*d^5*n^2/e^5/x^(1/2)-1/3*b^2*d^6*n^2*l
n(d+e/x^(1/2))^2/e^6-4*b*d^5*n*(d+e/x^(1/2))*(a+b*ln(c*(d+e/x^(1/2))^n))/e
^6+5*b*d^4*n*(d+e/x^(1/2))^2*(a+b*ln(c*(d+e/x^(1/2))^n))/e^6-40/9*b*d^3*n*
(d+e/x^(1/2))^3*(a+b*ln(c*(d+e/x^(1/2))^n))/e^6+5/2*b*d^2*n*(d+e/x^(1/2))^
4*(a+b*ln(c*(d+e/x^(1/2))^n))/e^6-4/5*b*d*n*(d+e/x^(1/2))^5*(a+b*ln(c*(d+e
/x^(1/2))^n))/e^6+1/9*b*n*(d+e/x^(1/2))^6*(a+b*ln(c*(d+e/x^(1/2))^n))/e^6+
2/3*b*d^6*n*ln(d+e/x^(1/2))*(a+b*ln(c*(d+e/x^(1/2))^n))/e^6-1/3*(a+b*ln(c*
(d+e/x^(1/2))^n))^2/x^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.57 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$$
$$= \frac{-1800\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{bn\left(600ae^6-100be^6n-720ade^5\sqrt{x}+264bde^5n\sqrt{x}+900ad^2e^4x-555bd^2e^4nx-1200ad^3e^3x^{3/}\right.}{}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

output
```
(-1800*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*n*(600*a*e^6 - 100*b*e^6*n
- 720*a*d*e^5*Sqrt[x] + 264*b*d*e^5*n*Sqrt[x] + 900*a*d^2*e^4*x - 555*b*d^
2*e^4*n*x - 1200*a*d^3*e^3*x^(3/2) + 1140*b*d^3*e^3*n*x^(3/2) + 1800*a*d^4
*e^2*x^2 - 2610*b*d^4*e^2*n*x^2 - 3600*a*d^5*e*x^(5/2) + 8820*b*d^5*e*n*x^
(5/2) - 8820*b*d^6*n*x^3*Log[d + e/Sqrt[x]] + 600*b*e^6*Log[c*(d + e/Sqrt[
x])^n] - 720*b*d*e^5*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] + 900*b*d^2*e^4*x*Lo
g[c*(d + e/Sqrt[x])^n] - 1200*b*d^3*e^3*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] +
 1800*b*d^4*e^2*x^2*Log[c*(d + e/Sqrt[x])^n] - 3600*b*d^5*e*x^(5/2)*Log[c*
(d + e/Sqrt[x])^n] + 3600*a*d^6*x^3*Log[e + d*Sqrt[x]] + 3600*b*d^6*x^3*Lo
g[c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] - 1800*b*d^6*n*x^3*Log[e + d*Sqr
t[x]]^2 + 3600*b*d^6*x^3*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] +
3600*b*d^6*n*x^3*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] - 1800*a*d^6*x^3
*Log[x] + 3600*b*d^6*n*x^3*PolyLog[2, 1 + e/(d*Sqrt[x])] + 3600*b*d^6*n*x^
3*PolyLog[2, 1 + (d*Sqrt[x])/e]))/e^6)/(5400*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.63, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^{5/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2845}$$

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{6x^3}-\frac{1}{3}ben\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{\left(d+\frac{e}{\sqrt{x}}\right)x^3}d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{2858}$$

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{6x^3}-\frac{1}{3}bn\int\frac{a+b\log\left(cx^{-n/2}\right)}{x^{5/2}}d\left(d+\frac{e}{\sqrt{x}}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{6x^3}-\frac{bn\int\frac{e^6\left(a+b\log\left(cx^{-n/2}\right)\right)}{x^{5/2}}d\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}\right)$$

$$\Big\downarrow \text{2772}$$

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{6x^3}-\frac{bn\left(-bn\int\left(\sqrt{x}\log\left(d+\frac{e}{\sqrt{x}}\right)d^6-6d^5+\frac{15}{2}\left(d+\frac{e}{\sqrt{x}}\right)d^4-\frac{20d^3}{3x}+\frac{15d^2}{4x^{3/2}}-\frac{6d}{5x}\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{6x^3}-\frac{bn\left(d^6\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)-6d^5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

output
```
-2*((a + b*Log[c*(d + e/Sqrt[x])^n])^2/(6*x^3) - (b*n*(-(b*n*(-6*d^5*(d +
e/Sqrt[x]) + 1/(36*x^3) - (6*d)/(25*x^(5/2)) + (15*d^2)/(16*x^2) - (20*d^3
)/(9*x^(3/2)) + (15*d^4)/(4*x) + (d^6*Log[d + e/Sqrt[x]]^2)/2)) - 6*d^5*(d
 + e/Sqrt[x])*(a + b*Log[c/x^(n/2)]) + (a + b*Log[c/x^(n/2)])/(6*x^3) - (6
*d*(a + b*Log[c/x^(n/2)]))/(5*x^(5/2)) + (15*d^2*(a + b*Log[c/x^(n/2)]))/(
4*x^2) - (20*d^3*(a + b*Log[c/x^(n/2)]))/(3*x^(3/2)) + (15*d^4*(a + b*Log[
c/x^(n/2)]))/(2*x) + d^6*Log[d + e/Sqrt[x]]*(a + b*Log[c/x^(n/2)])))/(3*e^
6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx =$$
$$-\frac{100\,b^2e^6n^2+1800\,b^2e^6\log\left(c\right)^2-600\,abe^6n+1800\,a^2e^6+90\,(29\,b^2d^4e^2n^2-20\,abd^4e^2n)x^2-1800\,(b^2c}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="fricas")
```

output
```
-1/5400*(100*b^2*e^6*n^2 + 1800*b^2*e^6*log(c)^2 - 600*a*b*e^6*n + 1800*a^
2*e^6 + 90*(29*b^2*d^4*e^2*n^2 - 20*a*b*d^4*e^2*n)*x^2 - 1800*(b^2*d^6*n^2
*x^3 - b^2*e^6*n^2)*log((d*x + e*sqrt(x))/x)^2 + 15*(37*b^2*d^2*e^4*n^2 -
60*a*b*d^2*e^4*n)*x - 300*(6*b^2*d^4*e^2*n*x^2 + 3*b^2*d^2*e^4*n*x + 2*b^2
*e^6*n - 12*a*b*e^6)*log(c) - 60*(30*b^2*d^4*e^2*n^2*x^2 + 15*b^2*d^2*e^4*
n^2*x + 10*b^2*e^6*n^2 - 60*a*b*e^6*n - 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*
x^3 + 60*(b^2*d^6*n*x^3 - b^2*e^6*n)*log(c) - 4*(15*b^2*d^5*e*n^2*x^2 + 5*
b^2*d^3*e^3*n^2*x + 3*b^2*d*e^5*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 1
2*(22*b^2*d*e^5*n^2 - 60*a*b*d*e^5*n + 15*(49*b^2*d^5*e*n^2 - 20*a*b*d^5*e
*n)*x^2 + 5*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x - 20*(15*b^2*d^5*e*n
*x^2 + 5*b^2*d^3*e^3*n*x + 3*b^2*d*e^5*n)*log(c))*sqrt(x))/(e^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$$
$$= \frac{1}{90}aben\left(\frac{60\,d^6\log\left(d\sqrt{x}+e\right)}{e^7} - \frac{30\,d^6\log\left(x\right)}{e^7} - \frac{60\,d^5x^{\frac{5}{2}} - 30\,d^4ex^2 + 20\,d^3e^2x^{\frac{3}{2}} - 15\,d^2e^3x + 12\,de^4\sqrt{x}}{e^6x^3}\right.$$
$$+\frac{1}{5400}\left(60\,en\left(\frac{60\,d^6\log\left(d\sqrt{x}+e\right)}{e^7} - \frac{30\,d^6\log\left(x\right)}{e^7} - \frac{60\,d^5x^{\frac{5}{2}} - 30\,d^4ex^2 + 20\,d^3e^2x^{\frac{3}{2}} - 15\,d^2e^3x + 12}{e^6x^3}\right.\right.$$
$$-\frac{b^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{3\,x^3} - \frac{2\,ab\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,x^3} - \frac{a^2}{3\,x^3}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="maxima")
```

output
```
1/90*a*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*
x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt
(x) - 10*e^5)/(e^6*x^3)) + 1/5400*(60*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 -
 30*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) -
 15*d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^3))*log(c*(d + e/sqrt(x)
)^n) - (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^
6*x^3*log(x) - 8820*d^5*e*x^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2
) + 555*d^2*e^4*x - 264*d*e^5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) -
 49*d^6*x^3)*log(d*sqrt(x) + e))*n^2/(e^6*x^3))*b^2 - 1/3*b^2*log(c*(d + e
/sqrt(x))^n)^2/x^3 - 2/3*a*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(412) = 824$.

Time = 0.17 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="giac")
```

output
```
1/5400*(1800*(6*(d*sqrt(x) + e)*b^2*d^5*n^2/(e^5*sqrt(x)) - 15*(d*sqrt(x)
+ e)^2*b^2*d^4*n^2/(e^5*x) + 20*(d*sqrt(x) + e)^3*b^2*d^3*n^2/(e^5*x^(3/2)
) - 15*(d*sqrt(x) + e)^4*b^2*d^2*n^2/(e^5*x^2) + 6*(d*sqrt(x) + e)^5*b^2*d
*n^2/(e^5*x^(5/2)) - (d*sqrt(x) + e)^6*b^2*n^2/(e^5*x^3))*log((d*sqrt(x) +
 e)/sqrt(x))^2 + 60*(10*(b^2*n^2 - 6*b^2*n*log(c) - 6*a*b*n)*(d*sqrt(x) +
e)^6/(e^5*x^3) - 72*(b^2*d*n^2 - 5*b^2*d*n*log(c) - 5*a*b*d*n)*(d*sqrt(x)
+ e)^5/(e^5*x^(5/2)) + 225*(b^2*d^2*n^2 - 4*b^2*d^2*n*log(c) - 4*a*b*d^2*n
)*(d*sqrt(x) + e)^4/(e^5*x^2) - 400*(b^2*d^3*n^2 - 3*b^2*d^3*n*log(c) - 3*
a*b*d^3*n)*(d*sqrt(x) + e)^3/(e^5*x^(3/2)) + 450*(b^2*d^4*n^2 - 2*b^2*d^4*
n*log(c) - 2*a*b*d^4*n)*(d*sqrt(x) + e)^2/(e^5*x) - 360*(b^2*d^5*n^2 - b^2
*d^5*n*log(c) - a*b*d^5*n)*(d*sqrt(x) + e)/(e^5*sqrt(x)))*log((d*sqrt(x) +
 e)/sqrt(x)) - 100*(b^2*n^2 - 6*b^2*n*log(c) + 18*b^2*log(c)^2 - 6*a*b*n +
 36*a*b*log(c) + 18*a^2)*(d*sqrt(x) + e)^6/(e^5*x^3) + 432*(2*b^2*d*n^2 -
10*b^2*d*n*log(c) + 25*b^2*d*log(c)^2 - 10*a*b*d*n + 50*a*b*d*log(c) + 25*
a^2*d)*(d*sqrt(x) + e)^5/(e^5*x^(5/2)) - 3375*(b^2*d^2*n^2 - 4*b^2*d^2*n*l
og(c) + 8*b^2*d^2*log(c)^2 - 4*a*b*d^2*n + 16*a*b*d^2*log(c) + 8*a^2*d^2)*
(d*sqrt(x) + e)^4/(e^5*x^2) + 4000*(2*b^2*d^3*n^2 - 6*b^2*d^3*n*log(c) + 9
*b^2*d^3*log(c)^2 - 6*a*b*d^3*n + 18*a*b*d^3*log(c) + 9*a^2*d^3)*(d*sqrt(x
) + e)^3/(e^5*x^(3/2)) - 13500*(b^2*d^4*n^2 - 2*b^2*d^4*n*log(c) + 2*b^2*d
^4*log(c)^2 - 2*a*b*d^4*n + 4*a*b*d^4*log(c) + 2*a^2*d^4)*(d*sqrt(x) + ...
```

Mupad [B] (verification not implemented)

Time = 15.72 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = \frac{b^2\,d^6\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{3\,e^6}-\frac{b^2\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{3\,x^3}-\frac{b^2\,n^2}{54\,x^3}-\frac{2\,a\,b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,x^3}-\frac{a^2}{3\,x^3}+\frac{a\,b\,n}{9\,x^3}+\frac{b^2\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{9\,x^3}-\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{30\,e^6}-\frac{37\,b^2\,d^2\,n^2}{360\,e^2\,x^2}-\frac{29\,b^2\,d^4\,n^2}{60\,e^4\,x}+\frac{19\,b^2\,d^3\,n^2}{90\,e^3\,x^{3/2}}+\frac{49\,b^2\,d^5\,n^2}{30\,e^5\,\sqrt{x}}+\frac{11\,b^2\,d\,n^2}{225\,e\,x^{5/2}}+\frac{b^2\,d^2\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{6\,e^2\,x^2}+\frac{b^2\,d^4\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,e^4\,x}-\frac{2\,b^2\,d^3\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{9\,e^3\,x^{3/2}}-\frac{2\,b^2\,d^5\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,e^5\,\sqrt{x}}-\frac{2\,a\,b\,d\,n}{15\,e\,x^{5/2}}+\frac{2\,a\,b\,d^6\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{3\,e^6}-\frac{2\,b^2\,d\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{15\,e\,x^{5/2}}+\frac{a\,b\,d^2\,n}{6\,e^2\,x^2}+\frac{a\,b\,d^4\,n}{3\,e^4\,x}-\frac{2\,a\,b\,d^3\,n}{9\,e^3\,x^{3/2}}-\frac{2\,a\,b\,d^5\,n}{3\,e^5\,\sqrt{x}}$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^4,x)
```

output
```
(b^2*d^6*log(c*(d + e/x^(1/2))^n)^2)/(3*e^6) - (b^2*log(c*(d + e/x^(1/2))^
n)^2)/(3*x^3) - (b^2*n^2)/(54*x^3) - (2*a*b*log(c*(d + e/x^(1/2))^n))/(3*x
^3) - a^2/(3*x^3) + (a*b*n)/(9*x^3) + (b^2*n*log(c*(d + e/x^(1/2))^n))/(9*
x^3) - (49*b^2*d^6*n^2*log(d + e/x^(1/2)))/(30*e^6) - (37*b^2*d^2*n^2)/(36
0*e^2*x^2) - (29*b^2*d^4*n^2)/(60*e^4*x) + (19*b^2*d^3*n^2)/(90*e^3*x^(3/2
)) + (49*b^2*d^5*n^2)/(30*e^5*x^(1/2)) + (11*b^2*d*n^2)/(225*e*x^(5/2)) +
(b^2*d^2*n*log(c*(d + e/x^(1/2))^n))/(6*e^2*x^2) + (b^2*d^4*n*log(c*(d + e
/x^(1/2))^n))/(3*e^4*x) - (2*b^2*d^3*n*log(c*(d + e/x^(1/2))^n))/(9*e^3*x^
(3/2)) - (2*b^2*d^5*n*log(c*(d + e/x^(1/2))^n))/(3*e^5*x^(1/2)) - (2*a*b*d
*n)/(15*e*x^(5/2)) + (2*a*b*d^6*n*log(d + e/x^(1/2)))/(3*e^6) - (2*b^2*d*n
*log(c*(d + e/x^(1/2))^n))/(15*e*x^(5/2)) + (a*b*d^2*n)/(6*e^2*x^2) + (a*b
*d^4*n)/(3*e^4*x) - (2*a*b*d^3*n)/(9*e^3*x^(3/2)) - (2*a*b*d^5*n)/(3*e^5*x
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$$

$$= \frac{-3600\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b^2d^5en\,x^2-1200\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b^2d^3e^3nx-720\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)b^2d\,e^{5}\cdots}{\cdots}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x)
```

output
```
( - 3600*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**5*e*n*x**2 -
 1200*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d**3*e**3*n*x - 72
0*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d*e**5*n - 3600*sqrt(x
)*a*b*d**5*e*n*x**2 - 1200*sqrt(x)*a*b*d**3*e**3*n*x - 720*sqrt(x)*a*b*d*e
**5*n + 8820*sqrt(x)*b**2*d**5*e*n**2*x**2 + 1140*sqrt(x)*b**2*d**3*e**3*n
**2*x + 264*sqrt(x)*b**2*d*e**5*n**2 + 1800*log(((sqrt(x)*d + e)**n*c)/x**
(n/2))**2*b**2*d**6*x**3 - 1800*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b*
*2*e**6 + 3600*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b*d**6*x**3 - 3600*l
og(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b*e**6 - 8820*log(((sqrt(x)*d + e)**
n*c)/x**(n/2))*b**2*d**6*n*x**3 + 1800*log(((sqrt(x)*d + e)**n*c)/x**(n/2)
)*b**2*d**4*e**2*n*x**2 + 900*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*d*
*2*e**4*n*x + 600*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**2*e**6*n - 1800*
a**2*e**6 + 1800*a*b*d**4*e**2*n*x**2 + 900*a*b*d**2*e**4*n*x + 600*a*b*e*
*6*n - 2610*b**2*d**4*e**2*n**2*x**2 - 555*b**2*d**2*e**4*n**2*x - 100*b**
2*e**6*n**2)/(5400*e**6*x**3)
```

3.436 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 569

$$
\begin{aligned}
&\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx \\
&=\frac{b^3e^3n^3\sqrt{x}}{2d^3}-\frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4} \\
&+\frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2} \\
&-\frac{5b^2e^4n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4} \\
&+\frac{3be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4} \\
&-\frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d} \\
&+\frac{3be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4} \\
&+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)}{d^4} \\
&-\frac{3b^3e^4n^3\log(x)}{2d^4}+\frac{5b^3e^4n^3\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{2d^4} \\
&-\frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4} \\
&-\frac{3b^3e^4n^3\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right)}{d^4}-\frac{3b^3e^4n^3\operatorname{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}
\end{aligned}
$$

output
```
1/2*b^3*e^3*n^3*x^(1/2)/d^3-1/2*b^3*e^4*n^3*ln(d+e/x^(1/2))/d^4-5/2*b^2*e^
3*n^2*(d+e/x^(1/2))*x^(1/2)*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4+1/2*b^2*e^2*n^
2*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2-5/2*b^2*e^4*n^2*ln(1-d/(d+e/x^(1/2)))*
(a+b*ln(c*(d+e/x^(1/2))^n))/d^4+3/2*b*e^3*n*(d+e/x^(1/2))*x^(1/2)*(a+b*ln(
c*(d+e/x^(1/2))^n))^2/d^4-3/4*b*e^2*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^2+
1/2*b*e*n*x^(3/2)*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d+3/2*b*e^4*n*ln(1-d/(d+e/
x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(1/2)
)^n))^3-3*b^2*e^4*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^(1/2))/d^4-3/2
*b^3*e^4*n^3*ln(x)/d^4+5/2*b^3*e^4*n^3*polylog(2,d/(d+e/x^(1/2)))/d^4-3*b^
2*e^4*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*polylog(2,d/(d+e/x^(1/2)))/d^4-3*b^3
*e^4*n^3*polylog(2,1+e/d/x^(1/2))/d^4-3*b^3*e^4*n^3*polylog(3,d/(d+e/x^(1/
2)))/d^4
```

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 5.04 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.05, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.864$, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int x^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2845}$$

$$-2\left(\frac{3}{4}ben\int\frac{x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d+\frac{e}{\sqrt{x}}}d\frac{1}{\sqrt{x}}-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2858}$$

$$-2\left(\frac{3}{4}bn\int x^{5/2}\left(a+b\log\left(cx^{-n/2}\right)\right)^2 d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{3}{4}be^4n\int\frac{x^{5/2}\left(a+b\log\left(cx^{-n/2}\right)\right)^2}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2789}$$

$$-2\left(\frac{3}{4}be^4n\left(\frac{\int\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)^2}{e^4}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int-\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)^2}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{1}{4}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2756}$$

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\int-\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)^2}{3e^3}}{d}+\frac{\int-\frac{x^2\left(a+b\log\left(cx^{-n/2}\right)\right)^2}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right.\right.$$

$$\Big\downarrow \text{2789}$$

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\int-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)^2}{3e^3}}{d}+\frac{\int}{-}\right.\right.$$

$$\Big\downarrow \text{2756}$$

$$-2\left(\frac{3}{4}be^4n\left(\frac{\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)^2}{2e^2}-bn\int\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)^2}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{-\frac{2}{3}bn\left(\frac{x\left(a+b\log\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 54

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\int\left(\frac{x}{de^2}-\frac{\sqrt{x}}{d^2e}+\frac{\sqrt{x}}{d^2}\right)d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{x^{3/2}(a+b\log}{3e}}{d}\right.\right.$$

$\downarrow$ 2009

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\int\frac{x^{3/2}\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\sqrt{x}}{de}\right)}{d}\right)-\frac{x^{3/2}(a+}{}}{d}\right.\right.$$

$\downarrow$ 2789

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\int\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int-\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\mathrm{lo}}{}\right.}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2751

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{-\frac{bn\int -\frac{\sqrt{x}}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}}{d}+\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\cdots}{\cdots}\right.\cdots}{d}\cdots\right.\right.$$

$\big\downarrow$ 16

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\cdots\right.}{\cdots}\right.\cdots}{d}\cdots\right.\right.$$

$\big\downarrow$ 2755

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}}{d}+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\cdots\right.}{\cdots}\right.\cdots}{d}\cdots\right.\right.$$

$\big\downarrow$ 2754

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}\right)+\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\ldots\right.}{\ldots}}{d}\right.\right.$$

$\downarrow$ 2779

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\frac{bn\int\sqrt{x}\log(1-d\sqrt{x})d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\log(1-d\sqrt{x})\left(a+b\log\left(cx^{-n/2}\right)\right)}{d}}{d}+\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}\right)+\ldots}{d}\right.\right.$$

$\downarrow$ 2821

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{\frac{bn\int\sqrt{x}\log(1-d\sqrt{x})d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\log(1-d\sqrt{x})\left(a+b\log\left(cx^{-n/2}\right)\right)}{d}}{d}+\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}\right)+\ldots}{d}\right.\right.$$

$\downarrow$ 2838

$$-2\left(\frac{3}{4}be^4n\left(\frac{\frac{\frac{2bn\left(\text{PolyLog}(2,d\sqrt{x})\left(a+b\log\left(cx^{-n/2}\right)\right)-bn\int\sqrt{x}\,\text{PolyLog}(2,d\sqrt{x})d\left(d+\frac{e}{\sqrt{x}}\right)\right)}{d}-\frac{\log(1-d\sqrt{x})\left(a+b\log\left(cx^{-n/2}\right)\right)^2}{d}}{d}+\frac{-\frac{2bn\left(-\log\left(1-\frac{d+\ldots}{\ldots}\right.\right.}{\ldots}}{\ldots}}{d}\right.\right.$$

$\downarrow$ 7143

$$-2\left(\frac{3}{4}be^4n\left(\frac{-\frac{2}{3}bn\left(\frac{\frac{x\left(a+b\log\left(cx^{-n/2}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{\sqrt{x}}{de}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)}{de}}{d}+\frac{bn\ldots}{d}\right.}{d}\right.\right.$$

input
```
Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

output
```
-2*(-1/4*(x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3) + (3*b*e^4*n*((-1/3*(x^(
3/2)*(a + b*Log[c/x^(n/2)])^2)/e^3 - (2*b*n*((-1/2*(b*n*(-(Sqrt[x]/(d*e))
+ Log[d + e/Sqrt[x]]/d^2 - Log[-(e/Sqrt[x])]/d^2)) + (x*(a + b*Log[c/x^(n/
2)]))/(2*e^2))/d + (((b*n*Log[-(e/Sqrt[x])])/d - ((d + e/Sqrt[x])*Sqrt[x]*
(a + b*Log[c/x^(n/2)]))/(d*e))/d + (-((Log[1 - d*Sqrt[x]]*(a + b*Log[c/x^(
n/2)]))/d) + (b*n*PolyLog[2, d*Sqrt[x]])/d)/d)/d))/3)/d + (((x*(a + b*Log[
c/x^(n/2)])^2)/(2*e^2) - b*n*(((b*n*Log[-(e/Sqrt[x])])/d - ((d + e/Sqrt[x]
)*Sqrt[x]*(a + b*Log[c/x^(n/2)]))/(d*e))/d + (-((Log[1 - d*Sqrt[x]]*(a + b
*Log[c/x^(n/2)]))/d) + (b*n*PolyLog[2, d*Sqrt[x]])/d)/d))/d + ((-(((d + e/
Sqrt[x])*Sqrt[x]*(a + b*Log[c/x^(n/2)])^2)/(d*e)) - (2*b*n*(-(Log[1 - (d +
 e/Sqrt[x])/d]*(a + b*Log[c/x^(n/2)])) - b*n*PolyLog[2, (d + e/Sqrt[x])/d]
))/d)/d + (-((Log[1 - d*Sqrt[x]]*(a + b*Log[c/x^(n/2)])^2)/d) + (2*b*n*((a
 + b*Log[c/x^(n/2)])*PolyLog[2, d*Sqrt[x]] + b*n*PolyLog[3, d*Sqrt[x]]))/d
)/d)/d)/d))/4)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{3}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x +
e*sqrt(x))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*sqrt(x))/x)^n) + a^3*x, x)
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
```

output
```
Integral(x*(a + b*log(c*(d + e/sqrt(x))**n))**3, x)
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2*log((d*sqrt(x) + e)^n)^3 - integrate(1/4*(4*(b^3*d*x^2 + b^3*e
*x^(3/2))*log(x^(1/2*n))^3 - 4*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^
2*b*d*log(c) + a^3*d)*x^2 + 3*(b^3*d*n*x^2 - 4*(b^3*d*log(c) + a*b^2*d)*x^
2 - 4*(b^3*e*log(c) + a*b^2*e)*x^(3/2) + 4*(b^3*d*x^2 + b^3*e*x^(3/2))*log
(x^(1/2*n)))*log((d*sqrt(x) + e)^n)^2 - 12*((b^3*d*log(c) + a*b^2*d)*x^2 +
 (b^3*e*log(c) + a*b^2*e)*x^(3/2))*log(x^(1/2*n))^2 - 4*(b^3*e*log(c)^3 +
3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(3/2) - 12*((b^3*d*log(c)
^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*d*x^2 + b^3*e*x^(3/2))*log(x^(
1/2*n))^2 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(3/2) - 2*((b^
3*d*log(c) + a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(3/2))*log(x^(1/2*n
)))*log((d*sqrt(x) + e)^n) + 12*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*
b*d)*x^2 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(3/2))*log(x^(1
/2*n)))/(d*x + e*sqrt(x)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(1/2))^n))^3,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(1/2))^n))^3, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x)
```

output
```
(4*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3*b**3*d*e**3*n + 4*sqrt(
x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d**3*e*n**2*x + 12*sqrt(x)
*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d*e**3*n**2 + 8*sqrt(x)*log(
((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*d**3*e*n**2*x + 24*sqrt(x)*log(((s
qrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*d*e**3*n**2 - 20*sqrt(x)*log(((sqrt(x
)*d + e)**n*c)/x**(n/2))*b**3*d*e**3*n**3 + 4*sqrt(x)*a**2*b*d**3*e*n**2*x
 + 12*sqrt(x)*a**2*b*d*e**3*n**2 - 20*sqrt(x)*a*b**2*d*e**3*n**3 + 4*sqrt(
x)*b**3*d*e**3*n**4 + 2*int(log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3/(d**2*
x**2 - e**2*x),x)*b**3*e**6*n + 12*int(log(((sqrt(x)*d + e)**n*c)/x**(n/2)
)/(d**2*x**2 - e**2*x),x)*a*b**2*e**6*n**2 - 22*int(log(((sqrt(x)*d + e)**
n*c)/x**(n/2))/(d**2*x**2 - e**2*x),x)*b**3*e**6*n**3 - 12*int(log(((sqrt(
x)*d + e)**n*c)/x**(n/2))/(d**2*x - e**2),x)*a*b**2*d**2*e**4*n**2 + 22*in
t(log(((sqrt(x)*d + e)**n*c)/x**(n/2))/(d**2*x - e**2),x)*b**3*d**2*e**4*n
**3 - 2*int((sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3)/(d**2*x - e*
*2),x)*b**3*d**3*e**3*n - log(((sqrt(x)*d + e)**n*c)/x**(n/2))**4*b**3*e**
4 + 4*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3*b**3*d**4*n*x**2 + 12*log(((
sqrt(x)*d + e)**n*c)/x**(n/2))**2*a*b**2*d**4*n*x**2 - 12*log(((sqrt(x)*d
+ e)**n*c)/x**(n/2))**2*a*b**2*e**4*n - 6*log(((sqrt(x)*d + e)**n*c)/x**(n
/2))**2*b**3*d**2*e**2*n**2*x + 22*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2
*b**3*e**4*n**2 + 12*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a**2*b*d**4*n...
```

3.437 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 20, antiderivative size = 260

$$\begin{aligned}
&\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx \\
&= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} \\
&\quad + \frac{3be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} \\
&\quad + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)}{d^2} \\
&\quad - \frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2} \\
&\quad - \frac{6b^3e^2n^3\,\text{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right)}{d^2} - \frac{6b^3e^2n^3\,\text{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}
\end{aligned}$$

output
```
3*b*e*n*(d+e/x^(1/2))*x^(1/2)*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^2+3*b*e^2*n*
ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^2+x*(a+b*ln(c*(d+e/x
^(1/2))^n))^3-6*b^2*e^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^(1/2))/d
^2-6*b^2*e^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*polylog(2,d/(d+e/x^(1/2)))/d^
2-6*b^3*e^2*n^3*polylog(2,1+e/d/x^(1/2))/d^2-6*b^3*e^2*n^3*polylog(3,d/(d+
e/x^(1/2)))/d^2
```

Mathematica [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 2.02 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2901, 2904, 2845, 2858, 27, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$$

$$\Big\downarrow \text{2901}$$

$$2\int \sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 d\sqrt{x}$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^{3/2}}d\frac{1}{\sqrt{x}}$$

$\Big\downarrow$ 2845

$$-2\left(\frac{3}{2}ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{\left(d+\frac{e}{\sqrt{x}}\right)x}d\frac{1}{\sqrt{x}}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2x}\right)$$

$\Big\downarrow$ 2858

$$-2\left(\frac{3}{2}bn\int \left(d+\frac{e}{\sqrt{x}}\right)x\left(a+b\log\left(cx^{n/2}\right)\right)^2d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2x}\right)$$

$\Big\downarrow$ 27

$$-2\left(\frac{3}{2}be^2n\int \frac{\left(d+\frac{e}{\sqrt{x}}\right)x\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2x}\right)$$

$\Big\downarrow$ 2789

$$-2\left(\frac{3}{2}be^2n\left(\frac{\int \frac{x\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e^2}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}+\frac{\int -\frac{\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.}{2x}\right.$$

$\Big\downarrow$ 2755

$$-2\left(\frac{3}{2}be^2n\left(\frac{-\frac{2bn\int -\frac{\sqrt{x}\left(a+b\log\left(cx^{n/2}\right)\right)}{e}d\left(d+\frac{e}{\sqrt{x}}\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)^2}{de}}{d}+\frac{\int -\frac{\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(cx^{n/2}\right)\right)^2}{e}d\left(d+\right.}{d}\right.\right.$$

$\Big\downarrow$ 2754

$$-2\left(\frac{3}{2}be^2n\left(\frac{-\frac{2bn\left(bn\int\left(d+\frac{e}{\sqrt{x}}\right)\log\left(1-\frac{d+\frac{e}{\sqrt{x}}}{d}\right)d\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(1-\frac{d+\frac{e}{\sqrt{x}}}{d}\right)\left(a+b\log\left(cx^{n/2}\right)\right)\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(cx^{n/2}\right)\right)^2}{de}}{d}+\right.\right.$$

$\Big\downarrow$ 2779

$$-2\left(\frac{3}{2}be^2n\left(\frac{-\frac{2bn\left(bn\int\left(d+\frac{e}{\sqrt{x}}\right)\log\left(1-\frac{d+\frac{e}{\sqrt{x}}}{d}\right)d\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(1-\frac{d+\frac{e}{\sqrt{x}}}{d}\right)(a+b\log(cx^{n/2}))\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)(a+b\log(cx^{n/2}))^2}{de}}{d}+\right.\right.$$

$\downarrow$ 2821

$$-2\left(\frac{3}{2}be^2n\left(\frac{\frac{2bn\left(\text{PolyLog}\left(2,d\left(d+\frac{e}{\sqrt{x}}\right)\right)(a+b\log(cx^{n/2}))-bn\int\left(d+\frac{e}{\sqrt{x}}\right)\text{PolyLog}\left(2,d\left(d+\frac{e}{\sqrt{x}}\right)\right)d\left(d+\frac{e}{\sqrt{x}}\right)\right)}{d}-\frac{\log\left(1-d\left(d+\frac{e}{\sqrt{x}}\right)\right)(a+b}{d}}{d}\right.\right.$$

$\downarrow$ 2838

$$-2\left(\frac{3}{2}be^2n\left(\frac{\frac{2bn\left(\text{PolyLog}\left(2,d\left(d+\frac{e}{\sqrt{x}}\right)\right)(a+b\log(cx^{n/2}))-bn\int\left(d+\frac{e}{\sqrt{x}}\right)\text{PolyLog}\left(2,d\left(d+\frac{e}{\sqrt{x}}\right)\right)d\left(d+\frac{e}{\sqrt{x}}\right)\right)}{d}-\frac{\log\left(1-d\left(d+\frac{e}{\sqrt{x}}\right)\right)(a+b}{d}}{d}\right.\right.$$

$\downarrow$ 7143

$$-2\left(\frac{3}{2}be^2n\left(\frac{-\frac{2bn\left(-\log\left(1-\frac{d+\frac{e}{\sqrt{x}}}{d}\right)(a+b\log(cx^{n/2}))-bn\,\text{PolyLog}\left(2,\frac{d+\frac{e}{\sqrt{x}}}{d}\right)\right)}{d}-\frac{\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)(a+b\log(cx^{n/2}))^2}{de}}{d}+\frac{2bn\left(\text{PolyLog}\right.}{}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

output
```
-2*(-1/2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x + (3*b*e^2*n*((-(((d + e/Sqr
t[x])*Sqrt[x]*(a + b*Log[c*x^(n/2)])^2)/(d*e)) - (2*b*n*(-(Log[1 - (d + e/
Sqrt[x])/d]*(a + b*Log[c*x^(n/2)])) - b*n*PolyLog[2, (d + e/Sqrt[x])/d]))/
d)/d + (-((Log[1 - d*(d + e/Sqrt[x])]*(a + b*Log[c*x^(n/2)])^2)/d) + (2*b*
n*((a + b*Log[c*x^(n/2)])*PolyLog[2, d*(d + e/Sqrt[x])] + b*n*PolyLog[3, d
*(d + e/Sqrt[x])]))/d)/d))/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3} dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{3} dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*sq
rt(x))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*sqrt(x))/x)^n) + a^3, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3} dx = \int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3} dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
```

output
```
Integral((a + b*log(c*(d + e/sqrt(x))**n))**3, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3 dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")
```

output
```
b^3*x*log((d*sqrt(x) + e)^n)^3 - 3*(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x
)/d) - x*log(c*(d + e/sqrt(x))^n))*a^2*b + a^3*x - integrate(1/2*(2*(b^3*d
*x + b^3*e*sqrt(x))*log(x^(1/2*n))^3 + 3*(b^3*d*n*x - 2*(b^3*d*log(c) + a*
b^2*d)*x + 2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c) +
a*b^2*e)*sqrt(x))*log((d*sqrt(x) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x
 + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n))^2 - 2*(b^3*d*log(c)^3
+ 3*a*b^2*d*log(c)^2)*x - 6*((b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n))^2 +
(b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 2*((b^3*d*log(c) + a*b^2*d)*x + (b
^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n)) + (b^3*e*log(c)^2 + 2*a*b^2
*e*log(c))*sqrt(x))*log((d*sqrt(x) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*
d*log(c))*x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*sqrt(x))*log(x^(1/2*n))
- 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2)*sqrt(x))/(d*x + e*sqrt(x)), x)
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3 dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\int \left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^3,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^n))^3, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^3,x)
```

output
```
(4*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3*b**3*d*e*n + 12*sqrt(x)
*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d*e*n**2 + 24*sqrt(x)*log(((
sqrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*d*e*n**2 + 12*sqrt(x)*a**2*b*d*e*n**
2 + 2*int(log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3/(d**2*x**2 - e**2*x),x)*
b**3*e**4*n + 12*int(log(((sqrt(x)*d + e)**n*c)/x**(n/2))/(d**2*x**2 - e**
2*x),x)*a*b**2*e**4*n**2 - 12*int(log(((sqrt(x)*d + e)**n*c)/x**(n/2))/(d*
*2*x**2 - e**2*x),x)*b**3*e**4*n**3 - 12*int(log(((sqrt(x)*d + e)**n*c)/x*
*(n/2))/(d**2*x - e**2),x)*a*b**2*d**2*e**2*n**2 + 12*int(log(((sqrt(x)*d
+ e)**n*c)/x**(n/2))/(d**2*x - e**2),x)*b**3*d**2*e**2*n**3 - 2*int((sqrt(
x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3)/(d**2*x - e**2),x)*b**3*d**3*e
*n - log(((sqrt(x)*d + e)**n*c)/x**(n/2))**4*b**3*e**2 + 4*log(((sqrt(x)*d
 + e)**n*c)/x**(n/2))**3*b**3*d**2*n*x + 12*log(((sqrt(x)*d + e)**n*c)/x**
(n/2))**2*a*b**2*d**2*n*x - 12*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*a*b
**2*e**2*n + 12*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*e**2*n**2 + 1
2*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a**2*b*d**2*n*x - 12*log(((sqrt(x)*
d + e)**n*c)/x**(n/2))*a**2*b*e**2*n + 24*log(((sqrt(x)*d + e)**n*c)/x**(n
/2))*a*b**2*e**2*n**2 - 12*log(sqrt(x))*a**2*b*e**2*n**2 + 24*log(sqrt(x))
*a*b**2*e**2*n**3 + 4*a**3*d**2*n*x)/(4*d**2*n)
```

3.438 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx = -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right) + 12b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(3,1+\frac{e}{d\sqrt{x}}\right) - 12b^3n^3\operatorname{PolyLog}\left(4,1+\frac{e}{d\sqrt{x}}\right)$$

output
```
-2*(a+b*ln(c*(d+e/x^(1/2))^n))^3*ln(-e/d/x^(1/2))-6*b*n*(a+b*ln(c*(d+e/x^(
1/2))^n))^2*polylog(2,1+e/d/x^(1/2))+12*b^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n)
)*polylog(3,1+e/d/x^(1/2))-12*b^3*n^3*polylog(4,1+e/d/x^(1/2))
```

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {2904, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2843}$$

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-3ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right)}{d+\frac{e}{\sqrt{x}}}d\frac{1}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{2881}$$

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-3bn\int \sqrt{x}\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)^2d\left(d+\frac{e}{\sqrt{x}}\right)\right)$$

$\downarrow$ 2821

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-3bn\left(2bn\int\sqrt{x}\left(a+b\log\left(cx^{-n/2}\right)\right)\text{PolyLog}\left(2,\frac{d+\frac{e}{\sqrt{x}}}{d}\right)\right.\right.$$

$\downarrow$ 2830

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+\frac{e}{\sqrt{x}}}{d}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)-bn\right.\right.\right.$$

$\downarrow$ 7143

$$-2\left(\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+\frac{e}{\sqrt{x}}}{d}\right)\left(a+b\log\left(cx^{-n/2}\right)\right)-bn\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
```

output
```
-2*((a + b*Log[c*(d + e/Sqrt[x])^n])^3*Log[-(e/(d*Sqrt[x]))] - 3*b*n*(-((a
 + b*Log[c/x^(n/2)])^2*PolyLog[2, (d + e/Sqrt[x])/d]) + 2*b*n*((a + b*Log[
c/x^(n/2)])*PolyLog[3, (d + e/Sqrt[x])/d] - b*n*PolyLog[4, (d + e/Sqrt[x])
/d])))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*s
qrt(x))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*sqrt(x))/x)^n) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x,x)
```

output
```
Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="maxima")
```

output
```
b^3*log((d*sqrt(x) + e)^n)^3*log(x) - integrate(1/2*(2*(b^3*d*x + b^3*e*sq
rt(x))*log(x^(1/2*n))^3 + 3*(b^3*d*n*x*log(x) - 2*(b^3*d*log(c) + a*b^2*d)
*x + 2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c) + a*b^2*
e)*sqrt(x))*log((d*sqrt(x) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x + (b^
3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n))^2 - 2*(b^3*d*log(c)^3 + 3*a*
b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 6*((b^3*d*x + b^3*e*sqrt(x)
)*log(x^(1/2*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 2*(
(b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*
n)) + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*sqrt(x))*log((d*sqrt(x
) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*lo
g(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*lo
g(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*sqrt(x))/(d*x^2 +
e*x^(3/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)\right)^3}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^3/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^n))^3/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx = \left(\int \frac{\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)^3}{x}dx\right)b^3 + 3\left(\int \frac{\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)^2}{x}dx\right)a\,b^2 + 3\left(\int \frac{\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)}{x}dx\right)a^2b + \log(x)\,a^3$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x)
```

output
```
int(log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3/x,x)*b**3 + 3*int(log(((sqrt(x
)*d + e)**n*c)/x**(n/2))**2/x,x)*a*b**2 + 3*int(log(((sqrt(x)*d + e)**n*c)
/x**(n/2))/x,x)*a**2*b + log(x)*a**3
```

3.439 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 285

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx &= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}-\frac{12b^3dn^3}{e\sqrt{x}}\\
&+\frac{12b^3dn^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2}\\
&-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2}\\
&-\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}\\
&+\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2}\\
&+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2}\\
&-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2}\end{aligned}$$

output
```
3/4*b^3*n^3*(d+e/x^(1/2))^2/e^2+12*a*b^2*d*n^2/e/x^(1/2)-12*b^3*d*n^3/e/x^
(1/2)+12*b^3*d*n^2*(d+e/x^(1/2))*ln(c*(d+e/x^(1/2))^n)/e^2-3/2*b^2*n^2*(d+
e/x^(1/2))^2*(a+b*ln(c*(d+e/x^(1/2))^n))/e^2-6*b*d*n*(d+e/x^(1/2))*(a+b*ln
(c*(d+e/x^(1/2))^n))^2/e^2+3/2*b*n*(d+e/x^(1/2))^2*(a+b*ln(c*(d+e/x^(1/2))
^n))^2/e^2+2*d*(d+e/x^(1/2))*(a+b*ln(c*(d+e/x^(1/2))^n))^3/e^2-(d+e/x^(1/2
))^2*(a+b*ln(c*(d+e/x^(1/2))^n))^3/e^2
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.96

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= \frac{-4a^3e^2+6a^2be^2n-6ab^2e^2n^2+3b^3e^2n^3-12a^2bden\sqrt{x}+36ab^2den^2\sqrt{x}-42b^3den^3\sqrt{x}-8b^3d^2n^3x\log^3}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```

output
```
(-4*a^3*e^2 + 6*a^2*b*e^2*n - 6*a*b^2*e^2*n^2 + 3*b^3*e^2*n^3 - 12*a^2*b*d
*e*n*Sqrt[x] + 36*a*b^2*d*e*n^2*Sqrt[x] - 42*b^3*d*e*n^3*Sqrt[x] - 8*b^3*d
^2*n^3*x*Log[d + e/Sqrt[x]]^3 - 4*b^3*e^2*Log[c*(d + e/Sqrt[x])^n]^3 + 12*
a^2*b*d^2*n*x*Log[e + d*Sqrt[x]] - 36*a*b^2*d^2*n^2*x*Log[e + d*Sqrt[x]] +
 42*b^3*d^2*n^3*x*Log[e + d*Sqrt[x]] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]*
(-2*a + 3*b*n - 2*b*Log[c*(d + e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[
x]) - 6*a^2*b*d^2*n*x*Log[x] + 18*a*b^2*d^2*n^2*x*Log[x] - 21*b^3*d^2*n^3*
x*Log[x] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]^2*(2*a - 3*b*n + 2*b*Log[c*(
d + e/Sqrt[x])^n] + 2*b*n*Log[e + d*Sqrt[x]] - b*n*Log[x]) + 6*b^2*Log[c*(
d + e/Sqrt[x])^n]^2*(e*(-2*a*e + b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*x*Log[
e + d*Sqrt[x]] - b*d^2*n*x*Log[x]) - 6*b*Log[c*(d + e/Sqrt[x])^n]*(e*(2*a^
2*e + b^2*n^2*(e - 6*d*Sqrt[x]) - 2*a*b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*(
-2*a + 3*b*n)*x*Log[e + d*Sqrt[x]] + b*d^2*n*(2*a - 3*b*n)*x*Log[x]))/(4*e
^2*x)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{\sqrt{x}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2848}$$

$$-2\int \left(\frac{\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e}-\frac{d\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e}\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^2}-\frac{6ab^2dn^2}{e\sqrt{x}}-\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4e^2}+\frac{3b\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```

output
```
-2*((-3*b^3*n^3*(d + e/Sqrt[x])^2)/(8*e^2) - (6*a*b^2*d*n^2)/(e*Sqrt[x]) +
 (6*b^3*d*n^3)/(e*Sqrt[x]) - (6*b^3*d*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqr
t[x])^n])/e^2 + (3*b^2*n^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^
n]))/(4*e^2) + (3*b*d*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2
)/e^2 - (3*b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*e^
2) - (d*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2 + ((d + e/
Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(249) = 498.

Time = 0.09 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.90

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx$$
$$= \frac{3\,b^{3}e^{2}n^{3}-4\,b^{3}e^{2}\log\left(c\right)^{3}-6\,ab^{2}e^{2}n^{2}+6\,a^{2}be^{2}n-4\,a^{3}e^{2}+4\left(b^{3}d^{2}n^{3}x-b^{3}e^{2}n^{3}\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^{3}+6\left(b^{3}e^{2}\right.}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="fricas")
```

output
```
1/4*(3*b^3*e^2*n^3 - 4*b^3*e^2*log(c)^3 - 6*a*b^2*e^2*n^2 + 6*a^2*b*e^2*n
- 4*a^3*e^2 + 4*(b^3*d^2*n^3*x - b^3*e^2*n^3)*log((d*x + e*sqrt(x))/x)^3 +
 6*(b^3*e^2*n - 2*a*b^2*e^2)*log(c)^2 - 6*(2*b^3*d*e*n^3*sqrt(x) - b^3*e^2
*n^3 + 2*a*b^2*e^2*n^2 + (3*b^3*d^2*n^3 - 2*a*b^2*d^2*n^2)*x - 2*(b^3*d^2*
n^2*x - b^3*e^2*n^2)*log(c))*log((d*x + e*sqrt(x))/x)^2 - 6*(b^3*e^2*n^2 -
 2*a*b^2*e^2*n + 2*a^2*b*e^2)*log(c) - 6*(b^3*e^2*n^3 - 2*a*b^2*e^2*n^2 +
2*a^2*b*e^2*n - 2*(b^3*d^2*n*x - b^3*e^2*n)*log(c)^2 - (7*b^3*d^2*n^3 - 6*
a*b^2*d^2*n^2 + 2*a^2*b*d^2*n)*x - 2*(b^3*e^2*n^2 - 2*a*b^2*e^2*n - (3*b^3
*d^2*n^2 - 2*a*b^2*d^2*n)*x)*log(c) - 2*(3*b^3*d*e*n^3 - 2*b^3*d*e*n^2*log
(c) - 2*a*b^2*d*e*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 6*(7*b^3*d*e*n^
3 + 2*b^3*d*e*n*log(c)^2 - 6*a*b^2*d*e*n^2 + 2*a^2*b*d*e*n - 2*(3*b^3*d*e*
n^2 - 2*a*b^2*d*e*n)*log(c))*sqrt(x))/(e^2*x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**2,x)
```

output
```
Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(249) = 498$.

Time = 0.08 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.99

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="maxima")
```

output
```
3/2*a^2*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x
) - e)/(e^2*x)) - b^3*log(c*(d + e/sqrt(x))^n)^3/x + 3/4*(4*e*n*(2*d^2*log
(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e)/(e^2*x))*log(c*(d
 + e/sqrt(x))^n) - (4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*log(x)^2 - 6*d^2*
x*log(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(
x) + e))*n^2/(e^2*x))*a*b^2 + 1/8*(12*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 -
d^2*log(x)/e^3 - (2*d*sqrt(x) - e)/(e^2*x))*log(c*(d + e/sqrt(x))^n)^2 + e
*n*((8*d^2*x*log(d*sqrt(x) + e)^3 - d^2*x*log(x)^3 + 9*d^2*x*log(x)^2 - 42
*d^2*x*log(x) - 12*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) + e)^2 - 84*d*e*
sqrt(x) + 6*e^2 + 6*(d^2*x*log(x)^2 - 6*d^2*x*log(x) + 14*d^2*x)*log(d*sqr
t(x) + e))*n^2/(e^3*x) - 6*(4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*log(x)^2
- 6*d^2*x*log(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*log(x) - 3*d^2*x)*log
(d*sqrt(x) + e))*n*log(c*(d + e/sqrt(x))^n)/(e^3*x)))*b^3 - 3*a*b^2*log(c*
(d + e/sqrt(x))^n)^2/x - 3*a^2*b*log(c*(d + e/sqrt(x))^n)/x - a^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(249) = 498$.

Time = 0.20 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.91

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx = \frac{4\left(\frac{2\,(d\sqrt{x}+e)b^3dn^3}{e\sqrt{x}} - \frac{(d\sqrt{x}+e)^2b^3n^3}{ex}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^3 + 6\left(\frac{(b^3n^3-2\,b^3n^2\log(c)-2\,ab^2n^2)(d\sqrt{x}+e)^2}{ex} - \frac{4\,(b^3dn^3-b^3dn^2\log(c)-a}{e\sqrt{x}}\right.}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="giac")
```

output
```
1/4*(4*(2*(d*sqrt(x) + e)*b^3*d*n^3/(e*sqrt(x)) - (d*sqrt(x) + e)^2*b^3*n^
3/(e*x))*log((d*sqrt(x) + e)/sqrt(x))^3 + 6*((b^3*n^3 - 2*b^3*n^2*log(c) -
 2*a*b^2*n^2)*(d*sqrt(x) + e)^2/(e*x) - 4*(b^3*d*n^3 - b^3*d*n^2*log(c) -
a*b^2*d*n^2)*(d*sqrt(x) + e)/(e*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x))^2 -
 6*((b^3*n^3 - 2*b^3*n^2*log(c) + 2*b^3*n*log(c)^2 - 2*a*b^2*n^2 + 4*a*b^2
*n*log(c) + 2*a^2*b*n)*(d*sqrt(x) + e)^2/(e*x) - 4*(2*b^3*d*n^3 - 2*b^3*d*
n^2*log(c) + b^3*d*n*log(c)^2 - 2*a*b^2*d*n^2 + 2*a*b^2*d*n*log(c) + a^2*b
*d*n)*(d*sqrt(x) + e)/(e*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x)) + (3*b^3*n
^3 - 6*b^3*n^2*log(c) + 6*b^3*n*log(c)^2 - 4*b^3*log(c)^3 - 6*a*b^2*n^2 +
12*a*b^2*n*log(c) - 12*a*b^2*log(c)^2 + 6*a^2*b*n - 12*a^2*b*log(c) - 4*a^
3)*(d*sqrt(x) + e)^2/(e*x) - 8*(6*b^3*d*n^3 - 6*b^3*d*n^2*log(c) + 3*b^3*d
*n*log(c)^2 - b^3*d*log(c)^3 - 6*a*b^2*d*n^2 + 6*a*b^2*d*n*log(c) - 3*a*b^
2*d*log(c)^2 + 3*a^2*b*d*n - 3*a^2*b*d*log(c) - a^3*d)*(d*sqrt(x) + e)/(e*
sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= \frac{\frac{d\left(2a^3-3a^2bn+3ab^2n^2-\frac{3b^3n^3}{2}\right)}{e}-\frac{d\left(2a^3-6ab^2n^2+9b^3n^3\right)}{e}}{\sqrt{x}}$$
$$-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^3\left(\frac{b^3}{x}-\frac{b^3d^2}{e^2}\right)$$
$$+\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{3bd\left(2a^2-2abn+b^2n^2\right)}{e}-\frac{6bd\left(a^2-b^2n^2\right)}{e}}{\sqrt{x}}\right.$$
$$\left.-\frac{3b\left(2a^2-2abn+b^2n^2\right)}{2x}\right)+\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{\frac{3b^2d(2a-bn)}{e}-\frac{6ab^2d}{e}}{\sqrt{x}}\right.$$
$$\left.-\frac{3b^2\left(2a-bn\right)}{2x}+\frac{3d\left(2ab^2d-3b^3dn\right)}{2e^2}\right)-\frac{a^3-\frac{3a^2bn}{2}+\frac{3ab^2n^2}{2}-\frac{3b^3n^3}{4}}{x}$$
$$+\frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)\left(6a^2bd^2n-18ab^2d^2n^2+21b^3d^2n^3\right)}{2e^2}$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^2,x)
```

output
```
((d*(2*a^3 - (3*b^3*n^3)/2 + 3*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(2*a^3 + 9*b
^3*n^3 - 6*a*b^2*n^2))/e)/x^(1/2) - log(c*(d + e/x^(1/2))^n)^3*(b^3/x - (b
^3*d^2)/e^2) + log(c*(d + e/x^(1/2))^n)*(((3*b*d*(2*a^2 + b^2*n^2 - 2*a*b*
n))/e - (6*b*d*(a^2 - b^2*n^2))/e)/x^(1/2) - (3*b*(2*a^2 + b^2*n^2 - 2*a*b
*n))/(2*x)) + log(c*(d + e/x^(1/2))^n)^2*(((3*b^2*d*(2*a - b*n))/e - (6*a*
b^2*d)/e)/x^(1/2) - (3*b^2*(2*a - b*n))/(2*x) + (3*d*(2*a*b^2*d - 3*b^3*d*
n))/(2*e^2)) - (a^3 - (3*b^3*n^3)/4 + (3*a*b^2*n^2)/2 - (3*a^2*b*n)/2)/x +
 (log(d + e/x^(1/2))*(21*b^3*d^2*n^3 - 18*a*b^2*d^2*n^2 + 6*a^2*b*d^2*n))/
(2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx = \frac{-12\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right)^2 b^3 den - 24\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right) a\,b^2 den + 36\sqrt{x}\log\left(\frac{(\sqrt{x}\,d+e)^n c}{x^{\frac{n}{2}}}\right) b^3 de\,n^2 - 12\sqrt{x}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x)
```

output
```
( - 12*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d*e*n - 24*sqr
t(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*d*e*n + 36*sqrt(x)*log(((
sqrt(x)*d + e)**n*c)/x**(n/2))*b**3*d*e*n**2 - 12*sqrt(x)*a**2*b*d*e*n + 3
6*sqrt(x)*a*b**2*d*e*n**2 - 42*sqrt(x)*b**3*d*e*n**3 + 4*log(((sqrt(x)*d +
 e)**n*c)/x**(n/2))**3*b**3*d**2*x - 4*log(((sqrt(x)*d + e)**n*c)/x**(n/2)
)**3*b**3*e**2 + 12*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*a*b**2*d**2*x
- 12*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*a*b**2*e**2 - 18*log(((sqrt(x
)*d + e)**n*c)/x**(n/2))**2*b**3*d**2*n*x + 6*log(((sqrt(x)*d + e)**n*c)/x
**(n/2))**2*b**3*e**2*n + 12*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a**2*b*d
**2*x - 12*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a**2*b*e**2 - 36*log(((sqr
t(x)*d + e)**n*c)/x**(n/2))*a*b**2*d**2*n*x + 12*log(((sqrt(x)*d + e)**n*c
)/x**(n/2))*a*b**2*e**2*n + 42*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**3*d
**2*n**2*x - 6*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**3*e**2*n**2 - 4*a**
3*e**2 + 6*a**2*b*e**2*n - 6*a*b**2*e**2*n**2 + 3*b**3*e**2*n**3)/(4*e**2*
x)
```

3.440 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 595

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4} - \frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} + \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4} + \frac{12ab^2d^3n^2}{e^3\sqrt{x}} - \frac{12b^3d^3n^3}{e^3\sqrt{x}} + \frac{12b^3d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^4} - \frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} - \frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{16e^4} - \frac{6bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} + \frac{9bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^4} - \frac{2bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{8e^4} + \frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} - \frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2e^4}$$

output
```
9/4*b^3*d^2*n^3*(d+e/x^(1/2))^2/e^4-4/9*b^3*d*n^3*(d+e/x^(1/2))^3/e^4+3/64
*b^3*n^3*(d+e/x^(1/2))^4/e^4+12*a*b^2*d^3*n^2/e^3/x^(1/2)-12*b^3*d^3*n^3/e
^3/x^(1/2)+12*b^3*d^3*n^2*(d+e/x^(1/2))*ln(c*(d+e/x^(1/2))^n)/e^4-9/2*b^2*
d^2*n^2*(d+e/x^(1/2))^2*(a+b*ln(c*(d+e/x^(1/2))^n))/e^4+4/3*b^2*d*n^2*(d+e
/x^(1/2))^3*(a+b*ln(c*(d+e/x^(1/2))^n))/e^4-3/16*b^2*n^2*(d+e/x^(1/2))^4*(
a+b*ln(c*(d+e/x^(1/2))^n))/e^4-6*b*d^3*n*(d+e/x^(1/2))*(a+b*ln(c*(d+e/x^(1
/2))^n))^2/e^4+9/2*b*d^2*n*(d+e/x^(1/2))^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2/e
^4-2*b*d*n*(d+e/x^(1/2))^3*(a+b*ln(c*(d+e/x^(1/2))^n))^2/e^4+3/8*b*n*(d+e/
x^(1/2))^4*(a+b*ln(c*(d+e/x^(1/2))^n))^2/e^4+2*d^3*(d+e/x^(1/2))*(a+b*ln(c
*(d+e/x^(1/2))^n))^3/e^4-3*d^2*(d+e/x^(1/2))^2*(a+b*ln(c*(d+e/x^(1/2))^n))
^3/e^4+2*d*(d+e/x^(1/2))^3*(a+b*ln(c*(d+e/x^(1/2))^n))^3/e^4-1/2*(d+e/x^(1
/2))^4*(a+b*ln(c*(d+e/x^(1/2))^n))^3/e^4
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = \frac{-288a^3e^4+216a^2be^4n-108ab^2e^4n^2+27b^3e^4n^3-288a^2bde^3n\sqrt{x}+336ab^2de^3n^2\sqrt{x}-148b^3de^3n^3\sqrt{x}-}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```

output
```
(-288*a^3*e^4 + 216*a^2*b*e^4*n - 108*a*b^2*e^4*n^2 + 27*b^3*e^4*n^3 - 288
*a^2*b*d*e^3*n*Sqrt[x] + 336*a*b^2*d*e^3*n^2*Sqrt[x] - 148*b^3*d*e^3*n^3*S
qrt[x] + 432*a^2*b*d^2*e^2*n*x - 936*a*b^2*d^2*e^2*n^2*x + 690*b^3*d^2*e^2
*n^3*x - 864*a^2*b*d^3*e*n*x^(3/2) + 3600*a*b^2*d^3*e*n^2*x^(3/2) - 4980*b
^3*d^3*e*n^3*x^(3/2) - 576*b^3*d^4*n^3*x^2*Log[d + e/Sqrt[x]]^3 - 288*b^3*
e^4*Log[c*(d + e/Sqrt[x])^n]^3 + 864*a^2*b*d^4*n*x^2*Log[e + d*Sqrt[x]] -
3600*a*b^2*d^4*n^2*x^2*Log[e + d*Sqrt[x]] + 4980*b^3*d^4*n^3*x^2*Log[e + d
*Sqrt[x]] + 72*b^2*d^4*n^2*x^2*Log[d + e/Sqrt[x]]*(-12*a + 25*b*n - 12*b*L
og[c*(d + e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 432*a^2*b*d^4*n
*x^2*Log[x] + 1800*a*b^2*d^4*n^2*x^2*Log[x] - 2490*b^3*d^4*n^3*x^2*Log[x]
+ 72*b^2*d^4*n^2*x^2*Log[d + e/Sqrt[x]]^2*(12*a - 25*b*n + 12*b*Log[c*(d +
 e/Sqrt[x])^n] + 12*b*n*Log[e + d*Sqrt[x]] - 6*b*n*Log[x]) + 72*b^2*Log[c*
(d + e/Sqrt[x])^n]^2*(e*(-12*a*e^3 + 3*b*e^3*n - 4*b*d*e^2*n*Sqrt[x] + 6*b
*d^2*e*n*x - 12*b*d^3*n*x^(3/2)) + 12*b*d^4*n*x^2*Log[e + d*Sqrt[x]] - 6*b
*d^4*n*x^2*Log[x]) - 12*b*Log[c*(d + e/Sqrt[x])^n]*(72*a^2*e^4 + b^2*e*n^2
*(9*e^3 - 28*d*e^2*Sqrt[x] + 78*d^2*e*x - 300*d^3*x^(3/2)) - 12*a*b*e*n*(3
*e^3 - 4*d*e^2*Sqrt[x] + 6*d^2*e*x - 12*d^3*x^(3/2)) + 12*b*d^4*n*(-12*a +
 25*b*n)*x^2*Log[e + d*Sqrt[x]] + 6*b*d^4*n*(12*a - 25*b*n)*x^2*Log[x]))/(
576*e^4*x^2)
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^{3/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2848}$$

$$-2\int \left(\frac{\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^3}-\frac{3d\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^3}+\frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)\left(a\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4}+\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{32e^4}-\frac{2b^2dn^2\left(d\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```

output
```
-2*((-9*b^3*d^2*n^3*(d + e/Sqrt[x])^2)/(8*e^4) + (2*b^3*d*n^3*(d + e/Sqrt[
x])^3)/(9*e^4) - (3*b^3*n^3*(d + e/Sqrt[x])^4)/(128*e^4) - (6*a*b^2*d^3*n^
2)/(e^3*Sqrt[x]) + (6*b^3*d^3*n^3)/(e^3*Sqrt[x]) - (6*b^3*d^3*n^2*(d + e/S
qrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^4 + (9*b^2*d^2*n^2*(d + e/Sqrt[x])^2*(
a + b*Log[c*(d + e/Sqrt[x])^n]))/(4*e^4) - (2*b^2*d*n^2*(d + e/Sqrt[x])^3*
(a + b*Log[c*(d + e/Sqrt[x])^n]))/(3*e^4) + (3*b^2*n^2*(d + e/Sqrt[x])^4*(
a + b*Log[c*(d + e/Sqrt[x])^n]))/(32*e^4) + (3*b*d^3*n*(d + e/Sqrt[x])*(a
+ b*Log[c*(d + e/Sqrt[x])^n])^2)/e^4 - (9*b*d^2*n*(d + e/Sqrt[x])^2*(a + b
*Log[c*(d + e/Sqrt[x])^n])^2)/(4*e^4) + (b*d*n*(d + e/Sqrt[x])^3*(a + b*Lo
g[c*(d + e/Sqrt[x])^n])^2)/e^4 - (3*b*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d
+ e/Sqrt[x])^n])^2)/(16*e^4) - (d^3*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sq
rt[x])^n])^3)/e^4 + (3*d^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^
n])^3)/(2*e^4) - (d*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/
e^4 + ((d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(4*e^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="fricas")
```

output
```
1/576*(27*b^3*e^4*n^3 - 288*b^3*e^4*log(c)^3 - 108*a*b^2*e^4*n^2 + 216*a^2
*b*e^4*n - 288*a^3*e^4 + 288*(b^3*d^4*n^3*x^2 - b^3*e^4*n^3)*log((d*x + e*
sqrt(x))/x)^3 + 216*(2*b^3*d^2*e^2*n*x + b^3*e^4*n - 4*a*b^2*e^4)*log(c)^2
 + 72*(6*b^3*d^2*e^2*n^3*x + 3*b^3*e^4*n^3 - 12*a*b^2*e^4*n^2 - (25*b^3*d^
4*n^3 - 12*a*b^2*d^4*n^2)*x^2 + 12*(b^3*d^4*n^2*x^2 - b^3*e^4*n^2)*log(c)
- 4*(3*b^3*d^3*e*n^3*x + b^3*d*e^3*n^3)*sqrt(x))*log((d*x + e*sqrt(x))/x)^
2 + 6*(115*b^3*d^2*e^2*n^3 - 156*a*b^2*d^2*e^2*n^2 + 72*a^2*b*d^2*e^2*n)*x
 - 36*(3*b^3*e^4*n^2 - 12*a*b^2*e^4*n + 24*a^2*b*e^4 + 2*(13*b^3*d^2*e^2*n
^2 - 12*a*b^2*d^2*e^2*n)*x)*log(c) - 12*(9*b^3*e^4*n^3 - 36*a*b^2*e^4*n^2
+ 72*a^2*b*e^4*n - (415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n)*
x^2 - 72*(b^3*d^4*n*x^2 - b^3*e^4*n)*log(c)^2 + 6*(13*b^3*d^2*e^2*n^3 - 12
*a*b^2*d^2*e^2*n^2)*x - 12*(6*b^3*d^2*e^2*n^2*x + 3*b^3*e^4*n^2 - 12*a*b^2
*e^4*n - (25*b^3*d^4*n^2 - 12*a*b^2*d^4*n)*x^2)*log(c) - 4*(7*b^3*d*e^3*n^
3 - 12*a*b^2*d*e^3*n^2 + 3*(25*b^3*d^3*e*n^3 - 12*a*b^2*d^3*e*n^2)*x - 12*
(3*b^3*d^3*e*n^2*x + b^3*d*e^3*n^2)*log(c))*sqrt(x))*log((d*x + e*sqrt(x))
/x) - 4*(37*b^3*d*e^3*n^3 - 84*a*b^2*d*e^3*n^2 + 72*a^2*b*d*e^3*n + 72*(3*
b^3*d^3*e*n*x + b^3*d*e^3*n)*log(c)^2 + 3*(415*b^3*d^3*e*n^3 - 300*a*b^2*d
^3*e*n^2 + 72*a^2*b*d^3*e*n)*x - 12*(7*b^3*d*e^3*n^2 - 12*a*b^2*d*e^3*n +
3*(25*b^3*d^3*e*n^2 - 12*a*b^2*d^3*e*n)*x)*log(c))*sqrt(x))/(e^4*x^2)
```

Sympy [F]

$$\int \frac{\left(a + b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{3}}\, dx = \int \frac{\left(a + b \log \left(c \left(d + \frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{3}}\, dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**3,x)
```

output
```
Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="maxima")
```

output
```
1/8*a^2*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*
x^(3/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) + 1/48*(12*e*n*(
12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2
*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n) - (72*
d^4*x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) -
300*d^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*sqrt(x) + 9*e^4 - 12*(6*d^4*x^
2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n^2/(e^4*x^2))*a*b^2 + 1/576*(7
2*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2)
- 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n)
^2 + e*n*((288*d^4*x^2*log(d*sqrt(x) + e)^3 - 36*d^4*x^2*log(x)^3 + 450*d^
4*x^2*log(x)^2 - 2490*d^4*x^2*log(x) - 4980*d^3*e*x^(3/2) + 690*d^2*e^2*x
- 148*d*e^3*sqrt(x) + 27*e^4 - 72*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*sq
rt(x) + e)^2 + 12*(18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) + 415*d^4*x^2)
*log(d*sqrt(x) + e))*n^2/(e^5*x^2) - 12*(72*d^4*x^2*log(d*sqrt(x) + e)^2 +
 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) - 300*d^3*e*x^(3/2) + 78*d^2*e^2
*x - 28*d*e^3*sqrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*s
qrt(x) + e))*n*log(c*(d + e/sqrt(x))^n)/(e^5*x^2)))*b^3 - 1/2*b^3*log(c*(d
 + e/sqrt(x))^n)^3/x^2 - 3/2*a*b^2*log(c*(d + e/sqrt(x))^n)^2/x^2 - 3/2*a^
2*b*log(c*(d + e/sqrt(x))^n)/x^2 - 1/2*a^3/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1147 vs. 2(519) = 1038.

Time = 0.21 (sec) , antiderivative size = 1147, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="giac")
```

output
```
1/576*(288*(4*(d*sqrt(x) + e)*b^3*d^3*n^3/(e^3*sqrt(x)) - 6*(d*sqrt(x) + e
)^2*b^3*d^2*n^3/(e^3*x) + 4*(d*sqrt(x) + e)^3*b^3*d*n^3/(e^3*x^(3/2)) - (d
*sqrt(x) + e)^4*b^3*n^3/(e^3*x^2))*log((d*sqrt(x) + e)/sqrt(x))^3 + 72*(3*
(b^3*n^3 - 4*b^3*n^2*log(c) - 4*a*b^2*n^2)*(d*sqrt(x) + e)^4/(e^3*x^2) - 1
6*(b^3*d*n^3 - 3*b^3*d*n^2*log(c) - 3*a*b^2*d*n^2)*(d*sqrt(x) + e)^3/(e^3*
x^(3/2)) + 36*(b^3*d^2*n^3 - 2*b^3*d^2*n^2*log(c) - 2*a*b^2*d^2*n^2)*(d*sq
rt(x) + e)^2/(e^3*x) - 48*(b^3*d^3*n^3 - b^3*d^3*n^2*log(c) - a*b^2*d^3*n^
2)*(d*sqrt(x) + e)/(e^3*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x))^2 - 12*(9*(
b^3*n^3 - 4*b^3*n^2*log(c) + 8*b^3*n*log(c)^2 - 4*a*b^2*n^2 + 16*a*b^2*n*l
og(c) + 8*a^2*b*n)*(d*sqrt(x) + e)^4/(e^3*x^2) - 32*(2*b^3*d*n^3 - 6*b^3*d
*n^2*log(c) + 9*b^3*d*n*log(c)^2 - 6*a*b^2*d*n^2 + 18*a*b^2*d*n*log(c) + 9
*a^2*b*d*n)*(d*sqrt(x) + e)^3/(e^3*x^(3/2)) + 216*(b^3*d^2*n^3 - 2*b^3*d^2
*n^2*log(c) + 2*b^3*d^2*n*log(c)^2 - 2*a*b^2*d^2*n^2 + 4*a*b^2*d^2*n*log(c
) + 2*a^2*b*d^2*n)*(d*sqrt(x) + e)^2/(e^3*x) - 288*(2*b^3*d^3*n^3 - 2*b^3*
d^3*n^2*log(c) + b^3*d^3*n*log(c)^2 - 2*a*b^2*d^3*n^2 + 2*a*b^2*d^3*n*log(
c) + a^2*b*d^3*n)*(d*sqrt(x) + e)/(e^3*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(
x)) + 9*(3*b^3*n^3 - 12*b^3*n^2*log(c) + 24*b^3*n*log(c)^2 - 32*b^3*log(c)
^3 - 12*a*b^2*n^2 + 48*a*b^2*n*log(c) - 96*a*b^2*log(c)^2 + 24*a^2*b*n - 9
6*a^2*b*log(c) - 32*a^3)*(d*sqrt(x) + e)^4/(e^3*x^2) - 128*(2*b^3*d*n^3 -
6*b^3*d*n^2*log(c) + 9*b^3*d*n*log(c)^2 - 9*b^3*d*log(c)^3 - 6*a*b^2*d*...
```

Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.42

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^3,x)
```

output
```
((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/(3*e) - (d
*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(36*e))/x^(3/2) - log(c*(d + e/x^(1/
2))^n)^3*(b^3/(2*x^2) - (b^3*d^4)/(2*e^4)) + ((d*((d*((d*(2*a^3 - (3*b^3*n
^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12
*a*b^2*n^2))/(12*e)))/e + (b^2*d^2*n^2*(12*a - 13*b*n))/(8*e^2)))/e + (b^2
*d^3*n^2*(12*a - 25*b*n))/(4*e^3))/x^(1/2) + log(c*(d + e/x^(1/2))^n)^2*((
(b^2*d*(4*a - b*n))/e - (4*a*b^2*d)/e)/(2*x^(3/2)) - (3*b^2*(4*a - b*n))/(
8*x^2) + (d*(12*a*b^2*d^3 - 25*b^3*d^3*n))/(8*e^4) - (d*((6*b^2*d*(4*a - b
*n))/e - (24*a*b^2*d)/e))/(8*e*x) + (d^2*((6*b^2*d*(4*a - b*n))/e - (24*a*
b^2*d)/e))/(4*e^2*x^(1/2))) - ((d*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n
^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)
))/(2*e) + (b^2*d^2*n^2*(12*a - 13*b*n))/(16*e^2))/x - (a^3/2 - (3*b^3*n^3
)/64 + (3*a*b^2*n^2)/16 - (3*a^2*b*n)/8)/x^2 - (log(c*(d + e/x^(1/2))^n)*(
(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n))/(1
2*e^2*x^(3/2)) + ((d*((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2
 + b^2*n^2 - 4*a*b*n)))/e - 24*b^3*d^2*e^2*n^2))/e - 48*b^3*d^3*e*n^2)/(4*
e^2*x^(1/2)) - ((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2
*n^2 - 4*a*b*n)))/e - 24*b^3*d^2*e^2*n^2)/(8*e^2*x) + (3*b*e^2*(8*a^2 + b^
2*n^2 - 4*a*b*n))/(4*x^2)))/(4*e^2) + (log(d + e/x^(1/2))*(415*b^3*d^4*n^3
 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n))/(48*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x)
```

output

```
( - 864*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d**3*e*n*x -
288*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d*e**3*n - 1728*s
qrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*d**3*e*n*x - 576*sqrt(x
)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*d*e**3*n + 3600*sqrt(x)*log(
((sqrt(x)*d + e)**n*c)/x**(n/2))*b**3*d**3*e*n**2*x + 336*sqrt(x)*log(((sq
rt(x)*d + e)**n*c)/x**(n/2))*b**3*d*e**3*n**2 - 864*sqrt(x)*a**2*b*d**3*e*
n*x - 288*sqrt(x)*a**2*b*d*e**3*n + 3600*sqrt(x)*a*b**2*d**3*e*n**2*x + 33
6*sqrt(x)*a*b**2*d*e**3*n**2 - 4980*sqrt(x)*b**3*d**3*e*n**3*x - 148*sqrt(
x)*b**3*d*e**3*n**3 + 288*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3*b**3*d**
4*x**2 - 288*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3*b**3*e**4 + 864*log((
(sqrt(x)*d + e)**n*c)/x**(n/2))**2*a*b**2*d**4*x**2 - 864*log(((sqrt(x)*d
+ e)**n*c)/x**(n/2))**2*a*b**2*e**4 - 1800*log(((sqrt(x)*d + e)**n*c)/x**(
n/2))**2*b**3*d**4*n*x**2 + 432*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b*
*3*d**2*e**2*n*x + 216*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*e**4*n
 + 864*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a**2*b*d**4*x**2 - 864*log(((s
qrt(x)*d + e)**n*c)/x**(n/2))*a**2*b*e**4 - 3600*log(((sqrt(x)*d + e)**n*c
)/x**(n/2))*a*b**2*d**4*n*x**2 + 864*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*
a*b**2*d**2*e**2*n*x + 432*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*e**
4*n + 4980*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**3*d**4*n**2*x**2 - 936*
log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**3*d**2*e**2*n**2*x - 108*log(((...
```

3.441 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 907

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

output
```
-1/18*b^2*n^2*(d+e/x^(1/2))^6*(a+b*ln(c*(d+e/x^(1/2))^n))/e^6+1/6*b*n*(d+e
/x^(1/2))^6*(a+b*ln(c*(d+e/x^(1/2))^n))^2/e^6+15/4*b^3*d^4*n^3*(d+e/x^(1/2
))^2/e^6-40/27*b^3*d^3*n^3*(d+e/x^(1/2))^3/e^6+15/32*b^3*d^2*n^3*(d+e/x^(1
/2))^4/e^6-12/125*b^3*d*n^3*(d+e/x^(1/2))^5/e^6-12*b^3*d^5*n^3/e^5/x^(1/2)
+12*a*b^2*d^5*n^2/e^5/x^(1/2)+12*b^3*d^5*n^2*(d+e/x^(1/2))*ln(c*(d+e/x^(1/
2))^n)/e^6-15/2*b^2*d^4*n^2*(d+e/x^(1/2))^2*(a+b*ln(c*(d+e/x^(1/2))^n))/e^
6+40/9*b^2*d^3*n^2*(d+e/x^(1/2))^3*(a+b*ln(c*(d+e/x^(1/2))^n))/e^6-15/8*b^
2*d^2*n^2*(d+e/x^(1/2))^4*(a+b*ln(c*(d+e/x^(1/2))^n))/e^6+12/25*b^2*d*n^2*
(d+e/x^(1/2))^5*(a+b*ln(c*(d+e/x^(1/2))^n))/e^6-6*b*d^5*n*(d+e/x^(1/2))*(a
+b*ln(c*(d+e/x^(1/2))^n))^2/e^6+15/2*b*d^4*n*(d+e/x^(1/2))^2*(a+b*ln(c*(d+
e/x^(1/2))^n))^2/e^6-20/3*b*d^3*n*(d+e/x^(1/2))^3*(a+b*ln(c*(d+e/x^(1/2))^
n))^2/e^6+15/4*b*d^2*n*(d+e/x^(1/2))^4*(a+b*ln(c*(d+e/x^(1/2))^n))^2/e^6-6
/5*b*d*n*(d+e/x^(1/2))^5*(a+b*ln(c*(d+e/x^(1/2))^n))^2/e^6-1/3*(d+e/x^(1/2
))^6*(a+b*ln(c*(d+e/x^(1/2))^n))^3/e^6+20/3*d^3*(d+e/x^(1/2))^3*(a+b*ln(c*
(d+e/x^(1/2))^n))^3/e^6-5*d^2*(d+e/x^(1/2))^4*(a+b*ln(c*(d+e/x^(1/2))^n))^
3/e^6+2*d*(d+e/x^(1/2))^5*(a+b*ln(c*(d+e/x^(1/2))^n))^3/e^6+2*d^5*(d+e/x^(
1/2))*(a+b*ln(c*(d+e/x^(1/2))^n))^3/e^6-5*d^4*(d+e/x^(1/2))^2*(a+b*ln(c*(d
+e/x^(1/2))^n))^3/e^6+1/108*b^3*n^3*(d+e/x^(1/2))^6/e^6
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]
```

output
```
(-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^
3 - 21600*a^2*b*d*e^5*n*Sqrt[x] + 15840*a*b^2*d*e^5*n^2*Sqrt[x] - 4368*b^3
*d*e^5*n^3*Sqrt[x] + 27000*a^2*b*d^2*e^4*n*x - 33300*a*b^2*d^2*e^4*n^2*x +
 13785*b^3*d^2*e^4*n^3*x - 36000*a^2*b*d^3*e^3*n*x^(3/2) + 68400*a*b^2*d^3
*e^3*n^2*x^(3/2) - 41180*b^3*d^3*e^3*n^3*x^(3/2) + 54000*a^2*b*d^4*e^2*n*x
^2 - 156600*a*b^2*d^4*e^2*n^2*x^2 + 140070*b^3*d^4*e^2*n^3*x^2 - 108000*a^
2*b*d^5*e*n*x^(5/2) + 529200*a*b^2*d^5*e*n^2*x^(5/2) - 809340*b^3*d^5*e*n^
3*x^(5/2) - 72000*b^3*d^6*n^3*x^3*Log[d + e/Sqrt[x]]^3 - 36000*b^3*e^6*Log
[c*(d + e/Sqrt[x])^n]^3 + 108000*a^2*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 5292
00*a*b^2*d^6*n^2*x^3*Log[e + d*Sqrt[x]] + 809340*b^3*d^6*n^3*x^3*Log[e + d
*Sqrt[x]] + 5400*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]*(-20*a + 49*b*n - 20*b
*Log[c*(d + e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 54000*a^2*b*d
^6*n*x^3*Log[x] + 264600*a*b^2*d^6*n^2*x^3*Log[x] - 404670*b^3*d^6*n^3*x^3
*Log[x] + 5400*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]^2*(20*a - 49*b*n + 20*b*
Log[c*(d + e/Sqrt[x])^n] + 20*b*n*Log[e + d*Sqrt[x]] - 10*b*n*Log[x]) + 18
00*b^2*Log[c*(d + e/Sqrt[x])^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n - 12*b*d*e^4*
n*Sqrt[x] + 15*b*d^2*e^3*n*x - 20*b*d^3*e^2*n*x^(3/2) + 30*b*d^4*e*n*x^2 -
 60*b*d^5*n*x^(5/2)) + 60*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 30*b*d^6*n*x^3*
Log[x]) - 60*b*Log[c*(d + e/Sqrt[x])^n]*(1800*a^2*e^6 + b^2*e*n^2*(100*e^5
 - 264*d*e^4*Sqrt[x] + 555*d^2*e^3*x - 1140*d^3*e^2*x^(3/2) + 2610*d^4*...
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^{5/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2848}$$

$$-2\int \left(-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 d^5}{e^5}+\frac{5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 d^4}{e^5}-\frac{10\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\right.}{e^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{216e^6}+\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{6e^6}-\frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{12e^6}+\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]
```

output
```
-2*((-15*b^3*d^4*n^3*(d + e/Sqrt[x])^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e/S
qrt[x])^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e/Sqrt[x])^4)/(64*e^6) + (6*b^3
*d*n^3*(d + e/Sqrt[x])^5)/(125*e^6) - (b^3*n^3*(d + e/Sqrt[x])^6)/(216*e^6
) - (6*a*b^2*d^5*n^2)/(e^5*Sqrt[x]) + (6*b^3*d^5*n^3)/(e^5*Sqrt[x]) - (6*b
^3*d^5*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^6 + (15*b^2*d^4*n^2
*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(4*e^6) - (20*b^2*d^3
*n^2*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) + (15*b^2
*d^2*n^2*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(16*e^6) - (6
*b^2*d*n^2*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(25*e^6) +
(b^2*n^2*(d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(36*e^6) + (3
*b*d^5*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^6 - (15*b*d
^4*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*e^6) + (10*b
*d^3*n*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(3*e^6) - (15
*b*d^2*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(8*e^6) + (
3*b*d*n*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(5*e^6) - (b
*n*(d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(12*e^6) - (d^5*(
d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^6 + (5*d^4*(d + e/Sqr
t[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*e^6) - (10*d^3*(d + e/Sqrt[
x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(3*e^6) + (5*d^2*(d + e/Sqrt[x])
^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*e^6) - (d*(d + e/Sqrt[x])^5*(...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1203, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{4}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="fricas")
```

output
```
1/108000*(1000*b^3*e^6*n^3 - 36000*b^3*e^6*log(c)^3 - 6000*a*b^2*e^6*n^2 +
 18000*a^2*b*e^6*n - 36000*a^3*e^6 + 36000*(b^3*d^6*n^3*x^3 - b^3*e^6*n^3)
*log((d*x + e*sqrt(x))/x)^3 + 30*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^
2*n^2 + 1800*a^2*b*d^4*e^2*n)*x^2 + 9000*(6*b^3*d^4*e^2*n*x^2 + 3*b^3*d^2*
e^4*n*x + 2*b^3*e^6*n - 12*a*b^2*e^6)*log(c)^2 + 1800*(30*b^3*d^4*e^2*n^3*
x^2 + 15*b^3*d^2*e^4*n^3*x + 10*b^3*e^6*n^3 - 60*a*b^2*e^6*n^2 - 3*(49*b^3
*d^6*n^3 - 20*a*b^2*d^6*n^2)*x^3 + 60*(b^3*d^6*n^2*x^3 - b^3*e^6*n^2)*log(
c) - 4*(15*b^3*d^5*e*n^3*x^2 + 5*b^3*d^3*e^3*n^3*x + 3*b^3*d*e^5*n^3)*sqrt
(x))*log((d*x + e*sqrt(x))/x)^2 + 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2
*e^4*n^2 + 1800*a^2*b*d^2*e^4*n)*x - 300*(20*b^3*e^6*n^2 - 120*a*b^2*e^6*n
 + 360*a^2*b*e^6 + 18*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n)*x^2 + 3*(3
7*b^3*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n)*x)*log(c) - 60*(100*b^3*e^6*n^3 -
600*a*b^2*e^6*n^2 + 1800*a^2*b*e^6*n - (13489*b^3*d^6*n^3 - 8820*a*b^2*d^6
*n^2 + 1800*a^2*b*d^6*n)*x^3 + 90*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n
^2)*x^2 - 1800*(b^3*d^6*n*x^3 - b^3*e^6*n)*log(c)^2 + 15*(37*b^3*d^2*e^4*n
^3 - 60*a*b^2*d^2*e^4*n^2)*x - 60*(30*b^3*d^4*e^2*n^2*x^2 + 15*b^3*d^2*e^4
*n^2*x + 10*b^3*e^6*n^2 - 60*a*b^2*e^6*n - 3*(49*b^3*d^6*n^2 - 20*a*b^2*d^
6*n)*x^3)*log(c) - 12*(22*b^3*d*e^5*n^3 - 60*a*b^2*d*e^5*n^2 + 15*(49*b^3*
d^5*e*n^3 - 20*a*b^2*d^5*e*n^2)*x^2 + 5*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3
*e^3*n^2)*x - 20*(15*b^3*d^5*e*n^2*x^2 + 5*b^3*d^3*e^3*n^2*x + 3*b^3*d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 864, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="maxima")
```

output
```
1/60*a^2*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^
5*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sq
rt(x) - 10*e^5)/(e^6*x^3)) + 1/1800*(60*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7
 - 30*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2)
 - 15*d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^3))*log(c*(d + e/sqrt(
x))^n) - (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*
d^6*x^3*log(x) - 8820*d^5*e*x^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3
/2) + 555*d^2*e^4*x - 264*d*e^5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x)
 - 49*d^6*x^3)*log(d*sqrt(x) + e))*n^2/(e^6*x^3))*a*b^2 + 1/108000*(1800*e
*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x^(5/2) -
30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e
^5)/(e^6*x^3))*log(c*(d + e/sqrt(x))^n)^2 + e*n*((36000*d^6*x^3*log(d*sqrt
(x) + e)^3 - 4500*d^6*x^3*log(x)^3 + 66150*d^6*x^3*log(x)^2 - 404670*d^6*x
^3*log(x) - 809340*d^5*e*x^(5/2) + 140070*d^4*e^2*x^2 - 41180*d^3*e^3*x^(3
/2) + 13785*d^2*e^4*x - 4368*d*e^5*sqrt(x) + 1000*e^6 - 5400*(10*d^6*x^3*l
og(x) - 49*d^6*x^3)*log(d*sqrt(x) + e)^2 + 60*(450*d^6*x^3*log(x)^2 - 4410
*d^6*x^3*log(x) + 13489*d^6*x^3)*log(d*sqrt(x) + e))*n^2/(e^7*x^3) - 60*(1
800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*log
(x) - 8820*d^5*e*x^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 555*d
^2*e^4*x - 264*d*e^5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1747 vs. 2(787) = 1574.

Time = 0.23 (sec) , antiderivative size = 1747, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="giac")
```

output
```
1/108000*(36000*(6*(d*sqrt(x) + e)*b^3*d^5*n^3/(e^5*sqrt(x)) - 15*(d*sqrt(
x) + e)^2*b^3*d^4*n^3/(e^5*x) + 20*(d*sqrt(x) + e)^3*b^3*d^3*n^3/(e^5*x^(3
/2)) - 15*(d*sqrt(x) + e)^4*b^3*d^2*n^3/(e^5*x^2) + 6*(d*sqrt(x) + e)^5*b^
3*d*n^3/(e^5*x^(5/2)) - (d*sqrt(x) + e)^6*b^3*n^3/(e^5*x^3))*log((d*sqrt(x
) + e)/sqrt(x))^3 + 1800*(10*(b^3*n^3 - 6*b^3*n^2*log(c) - 6*a*b^2*n^2)*(d
*sqrt(x) + e)^6/(e^5*x^3) - 72*(b^3*d*n^3 - 5*b^3*d*n^2*log(c) - 5*a*b^2*d
*n^2)*(d*sqrt(x) + e)^5/(e^5*x^(5/2)) + 225*(b^3*d^2*n^3 - 4*b^3*d^2*n^2*l
og(c) - 4*a*b^2*d^2*n^2)*(d*sqrt(x) + e)^4/(e^5*x^2) - 400*(b^3*d^3*n^3 -
3*b^3*d^3*n^2*log(c) - 3*a*b^2*d^3*n^2)*(d*sqrt(x) + e)^3/(e^5*x^(3/2)) +
450*(b^3*d^4*n^3 - 2*b^3*d^4*n^2*log(c) - 2*a*b^2*d^4*n^2)*(d*sqrt(x) + e)
^2/(e^5*x) - 360*(b^3*d^5*n^3 - b^3*d^5*n^2*log(c) - a*b^2*d^5*n^2)*(d*sqr
t(x) + e)/(e^5*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x))^2 - 60*(100*(b^3*n^3
 - 6*b^3*n^2*log(c) + 18*b^3*n*log(c)^2 - 6*a*b^2*n^2 + 36*a*b^2*n*log(c)
+ 18*a^2*b*n)*(d*sqrt(x) + e)^6/(e^5*x^3) - 432*(2*b^3*d*n^3 - 10*b^3*d*n^
2*log(c) + 25*b^3*d*n*log(c)^2 - 10*a*b^2*d*n^2 + 50*a*b^2*d*n*log(c) + 25
*a^2*b*d*n)*(d*sqrt(x) + e)^5/(e^5*x^(5/2)) + 3375*(b^3*d^2*n^3 - 4*b^3*d^
2*n^2*log(c) + 8*b^3*d^2*n*log(c)^2 - 4*a*b^2*d^2*n^2 + 16*a*b^2*d^2*n*log
(c) + 8*a^2*b*d^2*n)*(d*sqrt(x) + e)^4/(e^5*x^2) - 4000*(2*b^3*d^3*n^3 - 6
*b^3*d^3*n^2*log(c) + 9*b^3*d^3*n*log(c)^2 - 6*a*b^2*d^3*n^2 + 18*a*b^2*d^
3*n*log(c) + 9*a^2*b*d^3*n)*(d*sqrt(x) + e)^3/(e^5*x^(3/2)) + 13500*(b^...
```

Mupad [B] (verification not implemented)

Time = 21.01 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^4,x)
```

output
```
(b^3*n^3)/(108*x^3) - (b^3*log(c*(d + e/x^(1/2))^n)^3)/(3*x^3) - a^3/(3*x^
3) - (a*b^2*log(c*(d + e/x^(1/2))^n)^2)/x^3 + (b^3*n*log(c*(d + e/x^(1/2))
^n)^2)/(6*x^3) - (b^3*n^2*log(c*(d + e/x^(1/2))^n))/(18*x^3) - (a*b^2*n^2)
/(18*x^3) + (b^3*d^6*log(c*(d + e/x^(1/2))^n)^3)/(3*e^6) - (a^2*b*log(c*(d
 + e/x^(1/2))^n))/x^3 + (a^2*b*n)/(6*x^3) + (a*b^2*n*log(c*(d + e/x^(1/2))
^n))/(3*x^3) + (13489*b^3*d^6*n^3*log(d + e/x^(1/2)))/(1800*e^6) + (919*b^
3*d^2*n^3)/(7200*e^2*x^2) + (4669*b^3*d^4*n^3)/(3600*e^4*x) - (2059*b^3*d^
3*n^3)/(5400*e^3*x^(3/2)) - (13489*b^3*d^5*n^3)/(1800*e^5*x^(1/2)) + (a*b^
2*d^6*log(c*(d + e/x^(1/2))^n)^2)/e^6 - (49*b^3*d^6*n*log(c*(d + e/x^(1/2)
)^n)^2)/(20*e^6) - (91*b^3*d*n^3)/(2250*e*x^(5/2)) + (a^2*b*d^6*n*log(d +
e/x^(1/2)))/e^6 - (b^3*d*n*log(c*(d + e/x^(1/2))^n)^2)/(5*e*x^(5/2)) + (11
*b^3*d*n^2*log(c*(d + e/x^(1/2))^n))/(75*e*x^(5/2)) + (a^2*b*d^2*n)/(4*e^2
*x^2) + (a^2*b*d^4*n)/(2*e^4*x) + (11*a*b^2*d*n^2)/(75*e*x^(5/2)) - (a^2*b
*d^3*n)/(3*e^3*x^(3/2)) - (a^2*b*d^5*n)/(e^5*x^(1/2)) - (49*a*b^2*d^6*n^2*
log(d + e/x^(1/2)))/(10*e^6) + (b^3*d^2*n*log(c*(d + e/x^(1/2))^n)^2)/(4*e
^2*x^2) - (37*b^3*d^2*n^2*log(c*(d + e/x^(1/2))^n))/(120*e^2*x^2) + (b^3*d
^4*n*log(c*(d + e/x^(1/2))^n)^2)/(2*e^4*x) - (29*b^3*d^4*n^2*log(c*(d + e/
x^(1/2))^n))/(20*e^4*x) - (b^3*d^3*n*log(c*(d + e/x^(1/2))^n)^2)/(3*e^3*x^
(3/2)) + (19*b^3*d^3*n^2*log(c*(d + e/x^(1/2))^n))/(30*e^3*x^(3/2)) - (b^3
*d^5*n*log(c*(d + e/x^(1/2))^n)^2)/(e^5*x^(1/2)) + (49*b^3*d^5*n^2*log(...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1153, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x)
```

output
```
( - 108000*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d**5*e*n*x
**2 - 36000*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d**3*e**3
*n*x - 21600*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d*e**5*n
 - 216000*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*d**5*e*n*x**
2 - 72000*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*d**3*e**3*n*
x - 43200*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*a*b**2*d*e**5*n + 5
29200*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**3*d**5*e*n**2*x**2 +
 68400*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**3*d**3*e**3*n**2*x
+ 15840*sqrt(x)*log(((sqrt(x)*d + e)**n*c)/x**(n/2))*b**3*d*e**5*n**2 - 10
8000*sqrt(x)*a**2*b*d**5*e*n*x**2 - 36000*sqrt(x)*a**2*b*d**3*e**3*n*x - 2
1600*sqrt(x)*a**2*b*d*e**5*n + 529200*sqrt(x)*a*b**2*d**5*e*n**2*x**2 + 68
400*sqrt(x)*a*b**2*d**3*e**3*n**2*x + 15840*sqrt(x)*a*b**2*d*e**5*n**2 - 8
09340*sqrt(x)*b**3*d**5*e*n**3*x**2 - 41180*sqrt(x)*b**3*d**3*e**3*n**3*x
- 4368*sqrt(x)*b**3*d*e**5*n**3 + 36000*log(((sqrt(x)*d + e)**n*c)/x**(n/2
))**3*b**3*d**6*x**3 - 36000*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**3*b**3*
e**6 + 108000*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*a*b**2*d**6*x**3 - 1
08000*log(((sqrt(x)*d + e)**n*c)/x**(n/2))**2*a*b**2*e**6 - 264600*log(((s
qrt(x)*d + e)**n*c)/x**(n/2))**2*b**3*d**6*n*x**3 + 54000*log(((sqrt(x)*d
+ e)**n*c)/x**(n/2))**2*b**3*d**4*e**2*n*x**2 + 27000*log(((sqrt(x)*d + e)
**n*c)/x**(n/2))**2*b**3*d**2*e**4*n*x + 18000*log(((sqrt(x)*d + e)**n*...
```

3.442 $\int x^3 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 234

$$\int x^3 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^n\right)\right) \, dx = \frac{b d^{11} n \sqrt[3]{x}}{4 e^{11}}-\frac{b d^{10} n x^{2/3}}{8 e^{10}}+\frac{b d^9 n x}{12 e^9}-\frac{b d^8 n x^{4/3}}{16 e^8}+\frac{b d^7 n x^{5/3}}{20 e^7}-\frac{b d^6 n x^2}{24 e^6}+\frac{b d^5 n x^{7/3}}{28 e^5}-\frac{b d^4 n x^{8/3}}{32 e^4}+\frac{b d^3 n x^3}{36 e^3}-\frac{b d^2 n x^{10/3}}{40 e^2}+\frac{b d n x^{11/3}}{44 e}-\frac{1}{48} b n x^4-\frac{b d^{12} n \log \left(d+e \sqrt[3]{x}\right)}{4 e^{12}}+\frac{1}{4} x^4 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^n\right)\right)$$

output

```
1/4*b*d^11*n*x^(1/3)/e^11-1/8*b*d^10*n*x^(2/3)/e^10+1/12*b*d^9*n*x/e^9-1/1
6*b*d^8*n*x^(4/3)/e^8+1/20*b*d^7*n*x^(5/3)/e^7-1/24*b*d^6*n*x^2/e^6+1/28*b
*d^5*n*x^(7/3)/e^5-1/32*b*d^4*n*x^(8/3)/e^4+1/36*b*d^3*n*x^3/e^3-1/40*b*d^
2*n*x^(10/3)/e^2+1/44*b*d*n*x^(11/3)/e-1/48*b*n*x^4-1/4*b*d^12*n*ln(d+e*x^
(1/3))/e^12+1/4*x^4*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^4}{4}-\frac{1}{12}ben\left(-\frac{3d^{11}\sqrt[3]{x}}{e^{12}}+\frac{3d^{10}x^{2/3}}{2e^{11}}-\frac{d^9x}{e^{10}}+\frac{3d^8x^{4/3}}{4e^9}-\frac{3d^7x^{5/3}}{5e^8}+\frac{d^6x^2}{2e^7}-\frac{3d^5x^{7/3}}{7e^6}+\frac{3d^4x^{8/3}}{8e^5}-\frac{d^3x^3}{3e^4}+\frac{3d^2x^{10/3}}{10e^3}-\frac{3dx^{11/3}}{11e^2}+\frac{x^4}{4e}+\frac{3d^{12}\log\left(d+e\sqrt[3]{x}\right)}{e^{13}}\right)+\frac{1}{4}bx^4\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

output
```
(a*x^4)/4 - (b*e*n*((-3*d^11*x^(1/3))/e^12 + (3*d^10*x^(2/3))/(2*e^11) - (
d^9*x)/e^10 + (3*d^8*x^(4/3))/(4*e^9) - (3*d^7*x^(5/3))/(5*e^8) + (d^6*x^2
)/(2*e^7) - (3*d^5*x^(7/3))/(7*e^6) + (3*d^4*x^(8/3))/(8*e^5) - (d^3*x^3)/
(3*e^4) + (3*d^2*x^(10/3))/(10*e^3) - (3*d*x^(11/3))/(11*e^2) + x^4/(4*e)
+ (3*d^12*Log[d + e*x^(1/3)])/e^13))/12 + (b*x^4*Log[c*(d + e*x^(1/3))^n])
/4
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{11/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)d\sqrt[3]{x}$$

$$\downarrow 2842$$

$$3\left(\frac{1}{12}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{12}ben\int\frac{x^4}{d+e\sqrt[3]{x}}d\sqrt[3]{x}\right)$$

$$\downarrow 49$$

$$3\left(\frac{1}{12}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{12}ben\int\left(\frac{d^{12}}{e^{12}\left(d+e\sqrt[3]{x}\right)}-\frac{d^{11}}{e^{12}}+\frac{\sqrt[3]{x}d^{10}}{e^{11}}-\frac{x^{2/3}d^9}{e^{10}}+\frac{xd^8}{e^9}-\frac{x^{4/3}d^7}{e^8}+\frac{x^5}{}\right.\right.$$

$$\downarrow 2009$$

$$3\left(\frac{1}{12}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{12}ben\left(\frac{d^{12}\log\left(d+e\sqrt[3]{x}\right)}{e^{13}}-\frac{d^{11}\sqrt[3]{x}}{e^{12}}+\frac{d^{10}x^{2/3}}{2e^{11}}-\frac{d^9x}{3e^{10}}+\frac{d^8x^{4/3}}{4e^9}-\frac{d^7x^{5/3}}{5e^8}\right.\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

output
```
3*(-1/12*(b*e*n*(-((d^11*x^(1/3))/e^12) + (d^10*x^(2/3))/(2*e^11) - (d^9*x
)/(3*e^10) + (d^8*x^(4/3))/(4*e^9) - (d^7*x^(5/3))/(5*e^8) + (d^6*x^2)/(6*
e^7) - (d^5*x^(7/3))/(7*e^6) + (d^4*x^(8/3))/(8*e^5) - (d^3*x^3)/(9*e^4) +
 (d^2*x^(10/3))/(10*e^3) - (d*x^(11/3))/(11*e^2) + x^4/(12*e) + (d^12*Log[
d + e*x^(1/3)])/e^13)) + (x^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3 \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right) dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.86

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$= \frac{27720\,be^{12}x^4\log\left(c\right)+3080\,bd^3e^9nx^3-4620\,bd^6e^6nx^2+9240\,bd^9e^3nx-2310\left(be^{12}n-12\,ae^{12}\right)x^4+2772}{}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

output
```
1/110880*(27720*b*e^12*x^4*log(c) + 3080*b*d^3*e^9*n*x^3 - 4620*b*d^6*e^6*
n*x^2 + 9240*b*d^9*e^3*n*x - 2310*(b*e^12*n - 12*a*e^12)*x^4 + 27720*(b*e^
12*n*x^4 - b*d^12*n)*log(e*x^(1/3) + d) + 63*(40*b*d*e^11*n*x^3 - 55*b*d^4
*e^8*n*x^2 + 88*b*d^7*e^5*n*x - 220*b*d^10*e^2*n)*x^(2/3) - 198*(14*b*d^2*
e^10*n*x^3 - 20*b*d^5*e^7*n*x^2 + 35*b*d^8*e^4*n*x - 140*b*d^11*e*n)*x^(1/
3))/e^12
```

Sympy [A] (verification not implemented)

Time = 34.07 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.92

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^4}{4}+b\left(-\frac{en\left(\frac{3d^{12}\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^{12}}-\frac{3d^{11}\sqrt[3]{x}}{e^{12}}+\frac{3d^{10}x^{\frac{2}{3}}}{2e^{11}}-\frac{d^9x}{e^{10}}+\frac{3d^8x^{\frac{4}{3}}}{4e^9}-\frac{3d^7x^{\frac{5}{3}}}{5e^8}+\frac{d^6x^2}{2e^7}-\frac{3d^5x^{\frac{7}{3}}}{7e^6}+\cdots\right)}{12}+\frac{x^4\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{4}\right)$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

output
```
a*x**4/4 + b*(-e*n*(3*d**12*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x
**(1/3))/e, True))/e**12 - 3*d**11*x**(1/3)/e**12 + 3*d**10*x**(2/3)/(2*e*
*11) - d**9*x/e**10 + 3*d**8*x**(4/3)/(4*e**9) - 3*d**7*x**(5/3)/(5*e**8)
+ d**6*x**2/(2*e**7) - 3*d**5*x**(7/3)/(7*e**6) + 3*d**4*x**(8/3)/(8*e**5)
 - d**3*x**3/(3*e**4) + 3*d**2*x**(10/3)/(10*e**3) - 3*d*x**(11/3)/(11*e**
2) + x**4/(4*e))/12 + x**4*log(c*(d + e*x**(1/3))**n)/4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{4}\,ax^4$$
$$-\frac{1}{110880}\,ben\left(\frac{27720\,d^{12}\log\left(ex^{\frac{1}{3}}+d\right)}{e^{13}}+\frac{2310\,e^{11}x^4-2520\,de^{10}x^{\frac{11}{3}}+2772\,d^2e^9x^{\frac{10}{3}}-3080\,d^3e^8x^3+3\right.$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

output
```
1/4*b*x^4*log((e*x^(1/3) + d)^n*c) + 1/4*a*x^4 - 1/110880*b*e*n*(27720*d^1
2*log(e*x^(1/3) + d)/e^13 + (2310*e^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d
^2*e^9*x^(10/3) - 3080*d^3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x
^(7/3) + 4620*d^6*e^5*x^2 - 5544*d^7*e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) -
9240*d^9*e^2*x + 13860*d^10*e*x^(2/3) - 27720*d^11*x^(1/3))/e^12)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(186) = 372$.

Time = 0.13 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.21

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

output
```
1/110880*(27720*b*e*x^4*log(c) + 27720*a*e*x^4 + (27720*(e*x^(1/3) + d)^12
*log(e*x^(1/3) + d)/e^11 - 332640*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)/
e^11 + 1829520*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)/e^11 - 6098400*(e
*x^(1/3) + d)^9*d^3*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) + d)^8*d
^4*log(e*x^(1/3) + d)/e^11 - 21954240*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3)
+ d)/e^11 + 25613280*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)/e^11 - 21954
240*(e*x^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) +
d)^4*d^8*log(e*x^(1/3) + d)/e^11 - 6098400*(e*x^(1/3) + d)^3*d^9*log(e*x^(
1/3) + d)/e^11 + 1829520*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)/e^11 -
332640*(e*x^(1/3) + d)*d^11*log(e*x^(1/3) + d)/e^11 - 2310*(e*x^(1/3) + d)
^12/e^11 + 30240*(e*x^(1/3) + d)^11*d/e^11 - 182952*(e*x^(1/3) + d)^10*d^2
/e^11 + 677600*(e*x^(1/3) + d)^9*d^3/e^11 - 1715175*(e*x^(1/3) + d)^8*d^4/
e^11 + 3136320*(e*x^(1/3) + d)^7*d^5/e^11 - 4268880*(e*x^(1/3) + d)^6*d^6/
e^11 + 4390848*(e*x^(1/3) + d)^5*d^7/e^11 - 3430350*(e*x^(1/3) + d)^4*d^8/
e^11 + 2032800*(e*x^(1/3) + d)^3*d^9/e^11 - 914760*(e*x^(1/3) + d)^2*d^10/
e^11 + 332640*(e*x^(1/3) + d)*d^11/e^11)*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 14.81 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.81

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{a\,x^4}{4}-\frac{b\,n\,x^4}{48}+\frac{b\,x^4\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{4}+\frac{b\,d\,n\,x^{11/3}}{44\,e}+\frac{b\,d^9\,n\,x}{12\,e^9}-\frac{b\,d^{12}\,n\,\ln\left(d+e\,x^{1/3}\right)}{4\,e^{12}}+\frac{b\,d^3\,n\,x^3}{36\,e^3}-\frac{b\,d^6\,n\,x^2}{24\,e^6}-\frac{b\,d^2\,n\,x^{10/3}}{40\,e^2}-\frac{b\,d^4\,n\,x^{8/3}}{32\,e^4}+\frac{b\,d^5\,n\,x^{7/3}}{28\,e^5}+\frac{b\,d^7\,n\,x^{5/3}}{20\,e^7}-\frac{b\,d^8\,n\,x^{4/3}}{16\,e^8}-\frac{b\,d^{10}\,n\,x^{2/3}}{8\,e^{10}}+\frac{b\,d^{11}\,n\,x^{1/3}}{4\,e^{11}}$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

output
```
(a*x^4)/4 - (b*n*x^4)/48 + (b*x^4*log(c*(d + e*x^(1/3))^n))/4 + (b*d*n*x^(
11/3))/(44*e) + (b*d^9*n*x)/(12*e^9) - (b*d^12*n*log(d + e*x^(1/3)))/(4*e^
12) + (b*d^3*n*x^3)/(36*e^3) - (b*d^6*n*x^2)/(24*e^6) - (b*d^2*n*x^(10/3))
/(40*e^2) - (b*d^4*n*x^(8/3))/(32*e^4) + (b*d^5*n*x^(7/3))/(28*e^5) + (b*d
^7*n*x^(5/3))/(20*e^7) - (b*d^8*n*x^(4/3))/(16*e^8) - (b*d^10*n*x^(2/3))/(
8*e^10) + (b*d^11*n*x^(1/3))/(4*e^11)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.86

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$=\frac{-13860x^{\frac{2}{3}}b\,d^{10}e^2n+5544x^{\frac{5}{3}}b\,d^7e^5n-3465x^{\frac{8}{3}}b\,d^4e^8n+2520x^{\frac{11}{3}}bd\,e^{11}n+27720x^{\frac{1}{3}}b\,d^{11}en-6930x^{\frac{4}{3}}b\,d^{8}\cdots}{}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x)
```

output
```
( - 13860*x**(2/3)*b*d**10*e**2*n + 5544*x**(2/3)*b*d**7*e**5*n*x - 3465*x
**(2/3)*b*d**4*e**8*n*x**2 + 2520*x**(2/3)*b*d*e**11*n*x**3 + 27720*x**(1/
3)*b*d**11*e*n - 6930*x**(1/3)*b*d**8*e**4*n*x + 3960*x**(1/3)*b*d**5*e**7
*n*x**2 - 2772*x**(1/3)*b*d**2*e**10*n*x**3 - 27720*log((x**(1/3)*e + d)**
n*c)*b*d**12 + 27720*log((x**(1/3)*e + d)**n*c)*b*e**12*x**4 + 27720*a*e**
12*x**4 + 9240*b*d**9*e**3*n*x - 4620*b*d**6*e**6*n*x**2 + 3080*b*d**3*e**
9*n*x**3 - 2310*b*e**12*n*x**4)/(110880*e**12)
```

3.443 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 185

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=-\frac{bd^8n\sqrt[3]{x}}{3e^8}+\frac{bd^7nx^{2/3}}{6e^7}-\frac{bd^6nx}{9e^6}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}+\frac{bdnx^{8/3}}{24e}-\frac{1}{27}bnx^3+\frac{bd^9n\log\left(d+e\sqrt[3]{x}\right)}{3e^9}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

output
```
-1/3*b*d^8*n*x^(1/3)/e^8+1/6*b*d^7*n*x^(2/3)/e^7-1/9*b*d^6*n*x/e^6+1/12*b*
d^5*n*x^(4/3)/e^5-1/15*b*d^4*n*x^(5/3)/e^4+1/18*b*d^3*n*x^2/e^3-1/21*b*d^2
*n*x^(7/3)/e^2+1/24*b*d*n*x^(8/3)/e-1/27*b*n*x^3+1/3*b*d^9*n*ln(d+e*x^(1/3
))/e^9+1/3*x^3*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^3}{3}-\frac{1}{9}ben\left(\frac{3d^8\sqrt[3]{x}}{e^9}-\frac{3d^7x^{2/3}}{2e^8}+\frac{d^6x}{e^7}-\frac{3d^5x^{4/3}}{4e^6}+\frac{3d^4x^{5/3}}{5e^5}-\frac{d^3x^2}{2e^4}+\frac{3d^2x^{7/3}}{7e^3}-\frac{3dx^{8/3}}{8e^2}+\frac{x^3}{3e}-\frac{3d^9\log\left(d+e\sqrt[3]{x}\right)}{e^{10}}\right)+\frac{1}{3}bx^3\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

output
```
(a*x^3)/3 - (b*e*n*((3*d^8*x^(1/3))/e^9 - (3*d^7*x^(2/3))/(2*e^8) + (d^6*x
)/e^7 - (3*d^5*x^(4/3))/(4*e^6) + (3*d^4*x^(5/3))/(5*e^5) - (d^3*x^2)/(2*e
^4) + (3*d^2*x^(7/3))/(7*e^3) - (3*d*x^(8/3))/(8*e^2) + x^3/(3*e) - (3*d^9
*Log[d + e*x^(1/3)])/e^10))/9 + (b*x^3*Log[c*(d + e*x^(1/3))^n])/3
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2842}$$

$$3\left(\frac{1}{9}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{9}ben\int\frac{x^3}{d+e\sqrt[3]{x}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{49}$$

$$3\left(\frac{1}{9}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{9}ben\int\left(-\frac{d^9}{e^9\left(d+e\sqrt[3]{x}\right)}+\frac{d^8}{e^9}-\frac{\sqrt[3]{x}d^7}{e^8}+\frac{x^{2/3}d^6}{e^7}-\frac{xd^5}{e^6}+\frac{x^{4/3}d^4}{e^5}-\frac{x^{5/3}d}{e^4}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{9}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{9}ben\left(-\frac{d^9\log\left(d+e\sqrt[3]{x}\right)}{e^{10}}+\frac{d^8\sqrt[3]{x}}{e^9}-\frac{d^7x^{2/3}}{2e^8}+\frac{d^6x}{3e^7}-\frac{d^5x^{4/3}}{4e^6}+\frac{d^4x^{5/3}}{5e^5}-\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

output
```
3*(-1/9*(b*e*n*((d^8*x^(1/3))/e^9 - (d^7*x^(2/3))/(2*e^8) + (d^6*x)/(3*e^7
) - (d^5*x^(4/3))/(4*e^6) + (d^4*x^(5/3))/(5*e^5) - (d^3*x^2)/(6*e^4) + (d
^2*x^(7/3))/(7*e^3) - (d*x^(8/3))/(8*e^2) + x^3/(9*e) - (d^9*Log[d + e*x^(
1/3)])/e^10)) + (x^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeqQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.87

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = \frac{2520\,be^9x^3\log\left(c\right)+420\,bd^3e^6nx^2-840\,bd^6e^3nx-280\,(be^9n-9\,ae^9)x^3+2520\,(be^9nx^3+bd^9n)\log\left(ex^{\frac{1}{3}}\right.}{7560\,e^9}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

output
```
1/7560*(2520*b*e^9*x^3*log(c) + 420*b*d^3*e^6*n*x^2 - 840*b*d^6*e^3*n*x -
280*(b*e^9*n - 9*a*e^9)*x^3 + 2520*(b*e^9*n*x^3 + b*d^9*n)*log(e*x^(1/3) +
 d) + 63*(5*b*d*e^8*n*x^2 - 8*b*d^4*e^5*n*x + 20*b*d^7*e^2*n)*x^(2/3) - 90
*(4*b*d^2*e^7*n*x^2 - 7*b*d^5*e^4*n*x + 28*b*d^8*e*n)*x^(1/3))/e^9
```

Sympy [A] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^3}{3}+b\left(-\frac{en\left(-\frac{3d^9\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^9}+\frac{3d^8\sqrt[3]{x}}{e^9}-\frac{3d^7x^{\frac{2}{3}}}{2e^8}+\frac{d^6x}{e^7}-\frac{3d^5x^{\frac{4}{3}}}{4e^6}+\frac{3d^4x^{\frac{5}{3}}}{5e^5}-\frac{d^3x^2}{2e^4}+\frac{3d^2x^{\frac{7}{3}}}{7e^3}-\right.}{9}+\frac{x^3\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3}\right)$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

output
```
a*x**3/3 + b*(-e*n*(-3*d**9*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x
**(1/3))/e, True))/e**9 + 3*d**8*x**(1/3)/e**9 - 3*d**7*x**(2/3)/(2*e**8)
+ d**6*x/e**7 - 3*d**5*x**(4/3)/(4*e**6) + 3*d**4*x**(5/3)/(5*e**5) - d**3
*x**2/(2*e**4) + 3*d**2*x**(7/3)/(7*e**3) - 3*d*x**(8/3)/(8*e**2) + x**3/(
3*e))/9 + x**3*log(c*(d + e*x**(1/3))**n)/3)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.76

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{1}{3}\,bx^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{3}\,ax^3$$
$$+\frac{1}{7560}\,ben\left(\frac{2520\,d^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^8x^3-315\,de^7x^{\frac{8}{3}}+360\,d^2e^6x^{\frac{7}{3}}-420\,d^3e^5x^2+504\,d^4e^4x^{\frac{5}{3}}-\cdots}{e^9}\right.$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

output
```
1/3*b*x^3*log((e*x^(1/3) + d)^n*c) + 1/3*a*x^3 + 1/7560*b*e*n*(2520*d^9*lo
g(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(
7/3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d
^6*e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(147) = 294.

Time = 0.13 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.11

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$=\frac{2520\,bex^3\log\left(c\right)+2520\,aex^3+\left(\frac{2520\left(ex^{\frac{1}{3}}+d\right)^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^8}-\frac{22680\left(ex^{\frac{1}{3}}+d\right)^8d\log\left(ex^{\frac{1}{3}}+d\right)}{e^8}+\frac{90720\left(ex^{\frac{1}{3}}+d\right)^7d^2\log\left(\cdots\right.}{e^8}\cdots\right.}{\cdots}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

output
```
1/7560*(2520*b*e*x^3*log(c) + 2520*a*e*x^3 + (2520*(e*x^(1/3) + d)^9*log(e
*x^(1/3) + d)/e^8 - 22680*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)/e^8 + 907
20*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)/e^8 - 211680*(e*x^(1/3) + d)^6
*d^3*log(e*x^(1/3) + d)/e^8 + 317520*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) +
 d)/e^8 - 317520*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 + 211680*(e*
x^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)/e^8 - 90720*(e*x^(1/3) + d)^2*d^7*lo
g(e*x^(1/3) + d)/e^8 + 22680*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 -
280*(e*x^(1/3) + d)^9/e^8 + 2835*(e*x^(1/3) + d)^8*d/e^8 - 12960*(e*x^(1/3
) + d)^7*d^2/e^8 + 35280*(e*x^(1/3) + d)^6*d^3/e^8 - 63504*(e*x^(1/3) + d)
^5*d^4/e^8 + 79380*(e*x^(1/3) + d)^4*d^5/e^8 - 70560*(e*x^(1/3) + d)^3*d^6
/e^8 + 45360*(e*x^(1/3) + d)^2*d^7/e^8 - 22680*(e*x^(1/3) + d)*d^8/e^8)*b*
n)/e
```

Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{a\,x^3}{3}-\frac{b\,n\,x^3}{27}+\frac{b\,x^3\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3}+\frac{b\,d\,n\,x^{8/3}}{24\,e}-\frac{b\,d^6\,n\,x}{9\,e^6}+\frac{b\,d^9\,n\,\ln\left(d+e\,x^{1/3}\right)}{3\,e^9}+\frac{b\,d^3\,n\,x^2}{18\,e^3}-\frac{b\,d^2\,n\,x^{7/3}}{21\,e^2}-\frac{b\,d^4\,n\,x^{5/3}}{15\,e^4}+\frac{b\,d^5\,n\,x^{4/3}}{12\,e^5}+\frac{b\,d^7\,n\,x^{2/3}}{6\,e^7}-\frac{b\,d^8\,n\,x^{1/3}}{3\,e^8}$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

output
```
(a*x^3)/3 - (b*n*x^3)/27 + (b*x^3*log(c*(d + e*x^(1/3))^n))/3 + (b*d*n*x^(
8/3))/(24*e) - (b*d^6*n*x)/(9*e^6) + (b*d^9*n*log(d + e*x^(1/3)))/(3*e^9)
+ (b*d^3*n*x^2)/(18*e^3) - (b*d^2*n*x^(7/3))/(21*e^2) - (b*d^4*n*x^(5/3))/
(15*e^4) + (b*d^5*n*x^(4/3))/(12*e^5) + (b*d^7*n*x^(2/3))/(6*e^7) - (b*d^8
*n*x^(1/3))/(3*e^8)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.88

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$=\frac{1260x^{\frac{2}{3}}b\,d^7e^2n-504x^{\frac{5}{3}}b\,d^4e^5n+315x^{\frac{8}{3}}bd\,e^8n-2520x^{\frac{1}{3}}b\,d^8en+630x^{\frac{4}{3}}b\,d^5e^4n-360x^{\frac{7}{3}}b\,d^2e^7n+2520\,\mathrm{l}}{}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x)
```

output
```
(1260*x**(2/3)*b*d**7*e**2*n - 504*x**(2/3)*b*d**4*e**5*n*x + 315*x**(2/3)
*b*d*e**8*n*x**2 - 2520*x**(1/3)*b*d**8*e*n + 630*x**(1/3)*b*d**5*e**4*n*x
 - 360*x**(1/3)*b*d**2*e**7*n*x**2 + 2520*log((x**(1/3)*e + d)**n*c)*b*d**
9 + 2520*log((x**(1/3)*e + d)**n*c)*b*e**9*x**3 + 2520*a*e**9*x**3 - 840*b
*d**6*e**3*n*x + 420*b*d**3*e**6*n*x**2 - 280*b*e**9*n*x**3)/(7560*e**9)
```

3.444 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 136

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{bd^5n\sqrt[3]{x}}{2e^5}-\frac{bd^4nx^{2/3}}{4e^4}+\frac{bd^3nx}{6e^3}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

output
```
1/2*b*d^5*n*x^(1/3)/e^5-1/4*b*d^4*n*x^(2/3)/e^4+1/6*b*d^3*n*x/e^3-1/8*b*d^
2*n*x^(4/3)/e^2+1/10*b*d*n*x^(5/3)/e-1/12*b*n*x^2-1/2*b*d^6*n*ln(d+e*x^(1/
3))/e^6+1/2*x^2*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^2}{2}-\frac{1}{6}ben\left(-\frac{3d^5\sqrt[3]{x}}{e^6}+\frac{3d^4x^{2/3}}{2e^5}-\frac{d^3x}{e^4}+\frac{3d^2x^{4/3}}{4e^3}-\frac{3dx^{5/3}}{5e^2}+\frac{x^2}{2e}+\frac{3d^6\log\left(d+e\sqrt[3]{x}\right)}{e^7}\right)+\frac{1}{2}bx^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

output
```
(a*x^2)/2 - (b*e*n*((-3*d^5*x^(1/3))/e^6 + (3*d^4*x^(2/3))/(2*e^5) - (d^3*
x)/e^4 + (3*d^2*x^(4/3))/(4*e^3) - (3*d*x^(5/3))/(5*e^2) + x^2/(2*e) + (3*
d^6*Log[d + e*x^(1/3)])/e^7))/6 + (b*x^2*Log[c*(d + e*x^(1/3))^n])/2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2842}$$

$$3\left(\frac{1}{6}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{6}ben\int\frac{x^2}{d+e\sqrt[3]{x}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{ 49}$$

$$3\left(\frac{1}{6}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{6}ben\int\left(\frac{d^6}{e^6\left(d+e\sqrt[3]{x}\right)}-\frac{d^5}{e^6}+\frac{\sqrt[3]{x}d^4}{e^5}-\frac{x^{2/3}d^3}{e^4}+\frac{xd^2}{e^3}-\frac{x^{4/3}d}{e^2}+\frac{x^{5/3}}{e}\right)d$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(\frac{1}{6}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{6}ben\left(\frac{d^6\log\left(d+e\sqrt[3]{x}\right)}{e^7}-\frac{d^5\sqrt[3]{x}}{e^6}+\frac{d^4x^{2/3}}{2e^5}-\frac{d^3x}{3e^4}+\frac{d^2x^{4/3}}{4e^3}-\frac{dx^{5/3}}{5e^2}+\frac{x^2}{6e}\right)$$

input
```
Int[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

output
```
3*(-1/6*(b*e*n*(-((d^5*x^(1/3))/e^6) + (d^4*x^(2/3))/(2*e^5) - (d^3*x)/(3*
e^4) + (d^2*x^(4/3))/(4*e^3) - (d*x^(5/3))/(5*e^2) + x^2/(6*e) + (d^6*Log[
d + e*x^(1/3)])/e^7)) + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$=\frac{60\,be^6x^2\log\left(c\right)+20\,bd^3e^3nx-10\,(be^6n-6\,ae^6)x^2+60\,(be^6nx^2-bd^6n)\log\left(ex^{\frac{1}{3}}+d\right)+6\,(2\,bde^5nx-}{120\,e^6}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

output
```
1/120*(60*b*e^6*x^2*log(c) + 20*b*d^3*e^3*n*x - 10*(b*e^6*n - 6*a*e^6)*x^2
 + 60*(b*e^6*n*x^2 - b*d^6*n)*log(e*x^(1/3) + d) + 6*(2*b*d*e^5*n*x - 5*b*
d^4*e^2*n)*x^(2/3) - 15*(b*d^2*e^4*n*x - 4*b*d^5*e*n)*x^(1/3))/e^6
```

Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)\,dx=\frac{ax^{2}}{2}+b\left(-\frac{en\left(\frac{3d^{6}\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^{6}}-\frac{3d^{5}\sqrt[3]{x}}{e^{6}}+\frac{3d^{4}x^{\frac{2}{3}}}{2e^{5}}-\frac{d^{3}x}{e^{4}}+\frac{3d^{2}x^{\frac{4}{3}}}{4e^{3}}-\frac{3dx^{\frac{5}{3}}}{5e^{2}}+\frac{x^{2}}{2e}\right)}{6}+\frac{x^{2}\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)}{2}\right)$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

output
```
a*x**2/2 + b*(-e*n*(3*d**6*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x*
*(1/3))/e, True))/e**6 - 3*d**5*x**(1/3)/e**6 + 3*d**4*x**(2/3)/(2*e**5) -
 d**3*x/e**4 + 3*d**2*x**(4/3)/(4*e**3) - 3*d*x**(5/3)/(5*e**2) + x**2/(2*
e))/6 + x**2*log(c*(d + e*x**(1/3))**n)/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = -\frac{1}{120}\,ben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x^{\frac{1}{3}}}{e^6}\right) + \frac{1}{2}\,bx^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

output
```
-1/120*b*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/
3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3)
)/e^6) + 1/2*b*x^2*log((e*x^(1/3) + d)^n*c) + 1/2*a*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(108) = 216.

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.94

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = \frac{60\,bex^2\log\left(c\right)+60\,aex^2+\left(\frac{60\left(ex^{\frac{1}{3}}+d\right)^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^5}-\frac{360\left(ex^{\frac{1}{3}}+d\right)^5d\log\left(ex^{\frac{1}{3}}+d\right)}{e^5}+\frac{900\left(ex^{\frac{1}{3}}+d\right)^4d^2\log\left(ex^{\frac{1}{3}}+d\right)}{e^5}-\right.}{}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

output
```
1/120*(60*b*e*x^2*log(c) + 60*a*e*x^2 + (60*(e*x^(1/3) + d)^6*log(e*x^(1/3
) + d)/e^5 - 360*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/
3) + d)^4*d^2*log(e*x^(1/3) + d)/e^5 - 1200*(e*x^(1/3) + d)^3*d^3*log(e*x^
(1/3) + d)/e^5 + 900*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^5 - 360*(e
*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)/e^5 - 10*(e*x^(1/3) + d)^6/e^5 + 72*(
e*x^(1/3) + d)^5*d/e^5 - 225*(e*x^(1/3) + d)^4*d^2/e^5 + 400*(e*x^(1/3) +
d)^3*d^3/e^5 - 450*(e*x^(1/3) + d)^2*d^4/e^5 + 360*(e*x^(1/3) + d)*d^5/e^5
)*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 14.59 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{12}+\frac{b\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{2}+\frac{b\,d^3\,n\,x}{6\,e^3}+\frac{b\,d\,n\,x^{5/3}}{10\,e}-\frac{b\,d^6\,n\,\ln\left(d+e\,x^{1/3}\right)}{2\,e^6}-\frac{b\,d^2\,n\,x^{4/3}}{8\,e^2}-\frac{b\,d^4\,n\,x^{2/3}}{4\,e^4}+\frac{b\,d^5\,n\,x^{1/3}}{2\,e^5}$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

output
```
(a*x^2)/2 - (b*n*x^2)/12 + (b*x^2*log(c*(d + e*x^(1/3))^n))/2 + (b*d^3*n*x
)/(6*e^3) + (b*d*n*x^(5/3))/(10*e) - (b*d^6*n*log(d + e*x^(1/3)))/(2*e^6)
- (b*d^2*n*x^(4/3))/(8*e^2) - (b*d^4*n*x^(2/3))/(4*e^4) + (b*d^5*n*x^(1/3)
)/(2*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{-30x^{\frac{2}{3}}b\,d^4e^2n+12x^{\frac{5}{3}}bd\,e^5n+60x^{\frac{1}{3}}b\,d^5en-15x^{\frac{4}{3}}b\,d^2e^4n-60\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b\,d^6+60\log\left(\left(x^{\frac{1}{3}}e+\right.\right.}{120e^6}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/3))^n)),x)
```

output
```
( - 30*x**(2/3)*b*d**4*e**2*n + 12*x**(2/3)*b*d*e**5*n*x + 60*x**(1/3)*b*d
**5*e*n - 15*x**(1/3)*b*d**2*e**4*n*x - 60*log((x**(1/3)*e + d)**n*c)*b*d*
*6 + 60*log((x**(1/3)*e + d)**n*c)*b*e**6*x**2 + 60*a*e**6*x**2 + 20*b*d**
3*e**3*n*x - 10*b*e**6*n*x**2)/(120*e**6)
```

3.445 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}+ax-\frac{bnx}{3}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

output
```
-b*d^2*n*x^(1/3)/e^2+1/2*b*d*n*x^(2/3)/e+a*x-1/3*b*n*x+b*d^3*n*ln(d+e*x^(1
/3))/e^3+b*x*ln(c*(d+e*x^(1/3))^n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}+ax-\frac{bnx}{3}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

input
```
Integrate[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

output
```
-((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^
3*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}-\frac{bnx}{3}$$

input
```
Int[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

output
```
-((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^
3*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{b\,d^{2} n\,x^{\frac{1}{3}}}{e^{2}}+\frac{b d n\,x^{\frac{2}{3}}}{2 e}+a x-\frac{b n x}{3}+\frac{b\,d^{3} n \ln \left(d+e\,x^{\frac{1}{3}}\right)}{e^{3}}+b x \ln \left(c \left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)$	66
parts	$-\frac{b\,d^{2} n\,x^{\frac{1}{3}}}{e^{2}}+\frac{b d n\,x^{\frac{2}{3}}}{2 e}+a x-\frac{b n x}{3}+\frac{b\,d^{3} n \ln \left(d+e\,x^{\frac{1}{3}}\right)}{e^{3}}+b x \ln \left(c \left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)$	66

input
```
int(a+b*ln(c*(d+e*x^(1/3))^n),x,method=_RETURNVERBOSE)
```

output
```
-b*d^2*n*x^(1/3)/e^2+1/2*b*d*n*x^(2/3)/e+a*x-1/3*b*n*x+b*d^3*n*ln(d+e*x^(1
/3))/e^3+b*x*ln(c*(d+e*x^(1/3))^n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)\,dx = \frac{6\,be^{3}x\log \left(c\right)+3\,bde^{2}nx^{\frac{2}{3}}-6\,bd^{2}enx^{\frac{1}{3}}-2\,(be^{3}n-3\,ae^{3})x+6\,(be^{3}nx+bd^{3}n)\log \left(ex^{\frac{1}{3}}+d\right)}{6\,e^{3}}$$

input
```
integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="fricas")
```

output
```
1/6*(6*b*e^3*x*log(c) + 3*b*d*e^2*n*x^(2/3) - 6*b*d^2*e*n*x^(1/3) - 2*(b*e
^3*n - 3*a*e^3)*x + 6*(b*e^3*n*x + b*d^3*n)*log(e*x^(1/3) + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$

$$= ax+b\left(-\frac{en\left(-\frac{3d^3\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^3}+\frac{3d^2\sqrt[3]{x}}{e^3}-\frac{3dx^{\frac{2}{3}}}{2e^2}+\frac{x}{e}\right)}{3}+x\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

input
```
integrate(a+b*ln(c*(d+e*x**(1/3))**n),x)
```

output
```
a*x + b*(-e*n*(-3*d**3*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**(1/
3))/e, True))/e**3 + 3*d**2*x**(1/3)/e**3 - 3*d*x**(2/3)/(2*e**2) + x/e)/3
 + x*log(c*(d + e*x**(1/3))**n))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx =\frac{1}{6}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)\right)b + ax$$

input
```
integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="maxima")
```

output
```
1/6*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*
x^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*b + a*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.71

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = ax + \frac{\left(6\,ex\log\left(c\right)+\left(\frac{6\left(ex^{\frac{1}{3}}+d\right)^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^2}-\frac{18\left(ex^{\frac{1}{3}}+d\right)^2d\log\left(ex^{\frac{1}{3}}+d\right)}{e^2}+\frac{18\left(ex^{\frac{1}{3}}+d\right)d^2\log\left(ex^{\frac{1}{3}}+d\right)}{e^2}-\frac{2\left(ex^{\frac{1}{3}}+d\right)^3}{e^2}+\right.\right.}{6\,e}$$

input
```
integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="giac")
```

output
```
a*x + 1/6*(6*e*x*log(c) + (6*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2 - 18
*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 + 18*(e*x^(1/3) + d)*d^2*log(e
*x^(1/3) + d)/e^2 - 2*(e*x^(1/3) + d)^3/e^2 + 9*(e*x^(1/3) + d)^2*d/e^2 -
18*(e*x^(1/3) + d)*d^2/e^2)*n)*b/e
```

Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = a\,x+b\,x\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)-\frac{b\,n\,x}{3}+\frac{b\,d\,n\,x^{2/3}}{2\,e}+\frac{b\,d^3\,n\,\ln\left(d+e\,x^{1/3}\right)}{e^3}-\frac{b\,d^2\,n\,x^{1/3}}{e^2}$$

input
```
int(a + b*log(c*(d + e*x^(1/3))^n),x)
```

output
```
a*x + b*x*log(c*(d + e*x^(1/3))^n) - (b*n*x)/3 + (b*d*n*x^(2/3))/(2*e) + (
b*d^3*n*log(d + e*x^(1/3)))/e^3 - (b*d^2*n*x^(1/3))/e^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = \frac{3x^{\frac{2}{3}}bd\,e^2n-6x^{\frac{1}{3}}b\,d^2en+6\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b\,d^3+6\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b\,e^3x+6a\,e^3x-2b\,e^3nx}{6e^3}$$

input
```
int(a+b*log(c*(d+e*x^(1/3))^n),x)
```

output
```
(3*x**(2/3)*b*d*e**2*n - 6*x**(1/3)*b*d**2*e*n + 6*log((x**(1/3)*e + d)**n
*c)*b*d**3 + 6*log((x**(1/3)*e + d)**n*c)*b*e**3*x + 6*a*e**3*x - 2*b*e**3
*n*x)/(6*e**3)
```

3.446 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx = 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 3bn\,\text{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right)$$

output
```
3*(a+b*ln(c*(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)+3*b*n*polylog(2,1+e*x^(1/3)
/d)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx = 3b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + a\log(x) + 3bn\,\text{PolyLog}\left(2,\frac{d+e\sqrt[3]{x}}{d}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
```

output
```
3*b*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/d)] + a*Log[x] + 3*b*n*Poly
Log[2, (d + e*x^(1/3))/d]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$3\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2841}$$

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-ben\int \frac{\log\left(-\frac{e\sqrt[3]{x}}{d}\right)}{d+e\sqrt[3]{x}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{2752}$$

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{\sqrt[3]{x}e}{d}+1\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
```

output
```
3*((a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)] + b*n*PolyLog[2,
 1 + (e*x^(1/3))/d])
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))/x,x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="fricas")
```

output
```
integral((b*log((e*x^(1/3) + d)^n*c) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx=\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))**n))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(44) = 88$.

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.25

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx=-3\left(\log\left(\frac{ex^{\frac{1}{3}}}{d}+1\right)\log\left(x^{\frac{1}{3}}\right)+\mathrm{Li}_2\left(-\frac{ex^{\frac{1}{3}}}{d}\right)\right)bn+\frac{4\,bd^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right)\log(x)+4\left(bd^2\log(c)+ad^2\right)\log(x)+\frac{2\,be^2nx\log(x)-3\,be^2nx}{x^{\frac{1}{3}}}-\frac{4\,(bdenx\log(x)-3\,bdenx}{x^{\frac{2}{3}}}}{4\,d^2}+\frac{3\left(be^2nx^{\frac{2}{3}}-4\,bdenx^{\frac{1}{3}}-2\left(be^2nx^{\frac{2}{3}}-2\,bdenx^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}}\right)\right)}{4\,d^2}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="maxima")
```

output
```
-3*(log(e*x^(1/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(1/3)/d))*b*n + 1/4*(4*
b*d^2*log((e*x^(1/3) + d)^n)*log(x) + 4*(b*d^2*log(c) + a*d^2)*log(x) + (2
*b*e^2*n*x*log(x) - 3*b*e^2*n*x)/x^(1/3) - 4*(b*d*e*n*x*log(x) - 3*b*d*e*n
*x)/x^(2/3))/d^2 + 3/4*(b*e^2*n*x^(2/3) - 4*b*d*e*n*x^(1/3) - 2*(b*e^2*n*x
^(2/3) - 2*b*d*e*n*x^(1/3))*log(x^(1/3)))/d^2
```

Giac [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^n*c) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^n))/x, x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx=\frac{2\left(\int \frac{\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)}{x^{\frac{4}{3}}e+dx}dx\right)bdn+3\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)^2 b+2\log(x)\,an}{2n}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))/x,x)
```

output
```
(2*int(log((x**(1/3)*e + d)**n*c)/(x**(1/3)*e*x + d*x),x)*b*d*n + 3*log((x
**(1/3)*e + d)**n*c)**2*b + 2*log(x)*a*n)/(2*n)
```

3.447 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = -\frac{ben}{2dx^{2/3}}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+\frac{be^3n\log(x)}{3d^3}$$

output
```
-1/2*b*e*n/d/x^(2/3)+b*e^2*n/d^2/x^(1/3)-b*e^3*n*ln(d+e*x^(1/3))/d^3-(a+b*
ln(c*(d+e*x^(1/3))^n))/x+1/3*b*e^3*n*ln(x)/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = -\frac{a}{x}-\frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+\frac{1}{3}ben\left(-\frac{3}{2dx^{2/3}}+\frac{3e}{d^2\sqrt[3]{x}}-\frac{3e^2\log\left(d+e\sqrt[3]{x}\right)}{d^3}+\frac{e^2\log(x)}{d^3}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

output
```
-(a/x) - (b*Log[c*(d + e*x^(1/3))^n])/x + (b*e*n*(-3/(2*d*x^(2/3)) + (3*e)
/(d^2*x^(1/3)) - (3*e^2*Log[d + e*x^(1/3)])/d^3 + (e^2*Log[x])/d^3))/3
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}dx$$

$$\Big\downarrow \text{ 2904}$$

$$3\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2842}$$

$$3\left(\frac{1}{3}ben\int \frac{1}{\left(d+e\sqrt[3]{x}\right)x}d\sqrt[3]{x}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x}\right)$$

$$\Big\downarrow \text{ 54}$$

$$3\left(\frac{1}{3}ben\int \left(-\frac{e^3}{d^3\left(d+e\sqrt[3]{x}\right)}+\frac{e^2}{d^3\sqrt[3]{x}}-\frac{e}{d^2x^{2/3}}+\frac{1}{dx}\right)d\sqrt[3]{x}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(\frac{1}{3}ben\left(-\frac{e^2\log\left(d+e\sqrt[3]{x}\right)}{d^3}+\frac{e^2\log\left(\sqrt[3]{x}\right)}{d^3}+\frac{e}{d^2\sqrt[3]{x}}-\frac{1}{2dx^{2/3}}\right)-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x}\right)$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

output
```
3*(-1/3*(a + b*Log[c*(d + e*x^(1/3))^n])/x + (b*e*n*(-1/2*1/(d*x^(2/3)) +
e/(d^2*x^(1/3)) - (e^2*Log[d + e*x^(1/3)])/d^3 + (e^2*Log[x^(1/3)])/d^3))/
3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a + b\ln\left(c\left(d + e\,x^{\frac{1}{3}}\right)^{n}\right)}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = \frac{2\,be^3nx\log\left(x^{\frac{1}{3}}\right)+2\,bde^2nx^{\frac{2}{3}}-bd^2enx^{\frac{1}{3}}-2\,bd^3\log\left(c\right)-2\,ad^3-2\,(be^3nx+bd^3n)\log\left(ex^{\frac{1}{3}}+d\right)}{2\,d^3x}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*e^3*n*x*log(x^(1/3)) + 2*b*d*e^2*n*x^(2/3) - b*d^2*e*n*x^(1/3) -
2*b*d^3*log(c) - 2*a*d^3 - 2*(b*e^3*n*x + b*d^3*n)*log(e*x^(1/3) + d))/(d^
3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(82) = 164.

Time = 82.30 (sec) , antiderivative size = 483, normalized size of antiderivative = 5.55

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = \begin{cases} -\frac{a+b\log\left(0^nc\right)}{x} \\ -\frac{a}{x}-\frac{bn}{3x}-\frac{b\log\left(c\left(e\sqrt[3]{x}\right)^n\right)}{x} \\ -\frac{a+b\log\left(0^nc\right)}{x} \\ -\frac{6ad^4x^{\frac{2}{3}}}{6d^4x^{\frac{5}{3}}+6d^3ex^2}-\frac{6ad^3ex}{6d^4x^{\frac{5}{3}}+6d^3ex^2}-\frac{6bd^4x^{\frac{2}{3}}\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{6d^4x^{\frac{5}{3}}+6d^3ex^2}-\frac{3bd^3enx}{6d^4x^{\frac{5}{3}}+6d^3ex^2}-\frac{6bd^3ex\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{6d^4x^{\frac{5}{3}}+6d^3ex^2}+\frac{3bd^2e^2n}{6d^4x^{\frac{5}{3}}+6d^3ex^2} \end{cases}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**2,x)
```

output
```
Piecewise((-(a + b*log(0**n*c))/x, Eq(d, 0) & Eq(e, 0)), (-a/x - b*n/(3*x)
 - b*log(c*(e*x**(1/3))**n)/x, Eq(d, 0)), (-(a + b*log(0**n*c))/x, Eq(d, -
e*x**(1/3))), (-6*a*d**4*x**(2/3)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 6*a*
d**3*e*x/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 6*b*d**4*x**(2/3)*log(c*(d +
e*x**(1/3))**n)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 3*b*d**3*e*n*x/(6*d**4
*x**(5/3) + 6*d**3*e*x**2) - 6*b*d**3*e*x*log(c*(d + e*x**(1/3))**n)/(6*d*
*4*x**(5/3) + 6*d**3*e*x**2) + 3*b*d**2*e**2*n*x**(4/3)/(6*d**4*x**(5/3) +
 6*d**3*e*x**2) + 2*b*d*e**3*n*x**(5/3)*log(x)/(6*d**4*x**(5/3) + 6*d**3*e
*x**2) + 6*b*d*e**3*n*x**(5/3)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 6*b*d*e
**3*x**(5/3)*log(c*(d + e*x**(1/3))**n)/(6*d**4*x**(5/3) + 6*d**3*e*x**2)
+ 2*b*e**4*n*x**2*log(x)/(6*d**4*x**(5/3) + 6*d**3*e*x**2) - 6*b*e**4*x**2
*log(c*(d + e*x**(1/3))**n)/(6*d**4*x**(5/3) + 6*d**3*e*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = -\frac{1}{6}\,ben\left(\frac{6\,e^2\log\left(ex^{\frac{1}{3}}+d\right)}{d^3}-\frac{2\,e^2\log\left(x\right)}{d^3}-\frac{3\left(2\,ex^{\frac{1}{3}}-d\right)}{d^2x^{\frac{2}{3}}}\right)-\frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)}{x}-\frac{a}{x}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="maxima")
```

output
```
-1/6*b*e*n*(6*e^2*log(e*x^(1/3) + d)/d^3 - 2*e^2*log(x)/d^3 - 3*(2*e*x^(1/
3) - d)/(d^2*x^(2/3))) - b*log((e*x^(1/3) + d)^n*c)/x - a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(75) = 150$.

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.38

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = -\frac{\frac{2\,be^4n\log\left(ex^{\frac{1}{3}}+d\right)}{\left(ex^{\frac{1}{3}}+d\right)^3-3\left(ex^{\frac{1}{3}}+d\right)^2d+3\left(ex^{\frac{1}{3}}+d\right)d^2-d^3}+\frac{2\,be^4n\log\left(ex^{\frac{1}{3}}+d\right)}{d^3}-\frac{2\,be^4n\log\left(ex^{\frac{1}{3}}\right)}{d^3}-\frac{2\left(ex^{\frac{1}{3}}+d\right)^2be^4n-5\left(ex^{\frac{1}{3}}+d\right)bde^4n+3\,b}{\left(ex^{\frac{1}{3}}+d\right)^3d^2-3\left(ex^{\frac{1}{3}}+d\right)^2d^3}}{2\,e}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*e^4*n*log(e*x^(1/3) + d)/((e*x^(1/3) + d)^3 - 3*(e*x^(1/3) + d)^
2*d + 3*(e*x^(1/3) + d)*d^2 - d^3) + 2*b*e^4*n*log(e*x^(1/3) + d)/d^3 - 2*
b*e^4*n*log(e*x^(1/3))/d^3 - (2*(e*x^(1/3) + d)^2*b*e^4*n - 5*(e*x^(1/3) +
 d)*b*d*e^4*n + 3*b*d^2*e^4*n - 2*b*d^2*e^4*log(c) - 2*a*d^2*e^4)/((e*x^(1
/3) + d)^3*d^2 - 3*(e*x^(1/3) + d)^2*d^3 + 3*(e*x^(1/3) + d)*d^4 - d^5))/e
```

Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = -\frac{\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n\,x^{1/3}}{d^2}}{x^{2/3}}-\frac{a}{x}-\frac{b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{x}-\frac{2\,b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{d^3}$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))/x^2,x)
```

output
```
- ((b*e*n)/(2*d) - (b*e^2*n*x^(1/3))/d^2)/x^(2/3) - a/x - (b*log(c*(d + e*
x^(1/3))^n))/x - (2*b*e^3*n*atanh((2*e*x^(1/3))/d + 1))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx$$
$$= \frac{2x^{\frac{2}{3}}bd\,e^2n - x^{\frac{1}{3}}b\,d^2en + 2\log\left(x^{\frac{1}{3}}\right)b\,e^3nx - 2\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b\,d^3 - 2\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b\,e^3x - 2a}{2d^3x}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x)
```

output
```
(2*x**(2/3)*b*d*e**2*n - x**(1/3)*b*d**2*e*n + 2*log(x**(1/3))*b*e**3*n*x
- 2*log((x**(1/3)*e + d)**n*c)*b*d**3 - 2*log((x**(1/3)*e + d)**n*c)*b*e**
3*x - 2*a*d**3)/(2*d**3*x)
```

3.448 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = -\frac{ben}{10dx^{5/3}}+\frac{be^2n}{8d^2x^{4/3}}-\frac{be^3n}{6d^3x}+\frac{be^4n}{4d^4x^{2/3}}-\frac{be^5n}{2d^5\sqrt[3]{x}}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}-\frac{be^6n\log(x)}{6d^6}$$

output

```
-1/10*b*e*n/d/x^(5/3)+1/8*b*e^2*n/d^2/x^(4/3)-1/6*b*e^3*n/d^3/x+1/4*b*e^4*
n/d^4/x^(2/3)-1/2*b*e^5*n/d^5/x^(1/3)+1/2*b*e^6*n*ln(d+e*x^(1/3))/d^6-1/2*
(a+b*ln(c*(d+e*x^(1/3))^n))/x^2-1/6*b*e^6*n*ln(x)/d^6
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = -\frac{a}{2x^2}-\frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{6}ben\left(-\frac{3}{5dx^{5/3}}+\frac{3e}{4d^2x^{4/3}}-\frac{e^2}{d^3x}+\frac{3e^3}{2d^4x^{2/3}}-\frac{3e^4}{d^5\sqrt[3]{x}}+\frac{3e^5\log\left(d+e\sqrt[3]{x}\right)}{d^6}-\frac{e^5\log(x)}{d^6}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) + (b*e*n*(-3/(5*d*x^(5/3
)) + (3*e)/(4*d^2*x^(4/3)) - e^2/(d^3*x) + (3*e^3)/(2*d^4*x^(2/3)) - (3*e^
4)/(d^5*x^(1/3)) + (3*e^5*Log[d + e*x^(1/3)])/d^6 - (e^5*Log[x])/d^6))/6
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^{7/3}}\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2842}$$

$$3\left(\frac{1}{6}ben\int \frac{1}{\left(d+e\sqrt[3]{x}\right)x^2}\,d\sqrt[3]{x}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{6x^2}\right)$$

$$\Big\downarrow \text{54}$$

$$3\left(\frac{1}{6}ben\int\left(\frac{e^6}{d^6\left(d+e\sqrt[3]{x}\right)}-\frac{e^5}{d^6\sqrt[3]{x}}+\frac{e^4}{d^5x^{2/3}}-\frac{e^3}{d^4x}+\frac{e^2}{d^3x^{4/3}}-\frac{e}{d^2x^{5/3}}+\frac{1}{dx^2}\right)d\sqrt[3]{x}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{6x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{6}ben\left(\frac{e^5\log\left(d+e\sqrt[3]{x}\right)}{d^6}-\frac{e^5\log\left(\sqrt[3]{x}\right)}{d^6}-\frac{e^4}{d^5\sqrt[3]{x}}+\frac{e^3}{2d^4x^{2/3}}-\frac{e^2}{3d^3x}+\frac{e}{4d^2x^{4/3}}-\frac{1}{5dx^{5/3}}\right)-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{6x^2}\right)$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

output
```
3*(-1/6*(a + b*Log[c*(d + e*x^(1/3))^n])/x^2 + (b*e*n*(-1/5*1/(d*x^(5/3))
+ e/(4*d^2*x^(4/3)) - e^2/(3*d^3*x) + e^3/(2*d^4*x^(2/3)) - e^4/(d^5*x^(1/
3)) + (e^5*Log[d + e*x^(1/3)])/d^6 - (e^5*Log[x^(1/3)])/d^6))/6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = -\frac{60\,be^{6}nx^{2}\log\left(x^{\frac{1}{3}}\right)+20\,bd^{3}e^{3}nx+60\,bd^{6}\log\left(c\right)+60\,ad^{6}-60\left(be^{6}nx^{2}-bd^{6}n\right)\log\left(ex^{\frac{1}{3}}+d\right)+15\,(4}{120\,d^{6}x^{2}}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="fricas")
```

output
```
-1/120*(60*b*e^6*n*x^2*log(x^(1/3)) + 20*b*d^3*e^3*n*x + 60*b*d^6*log(c) +
 60*a*d^6 - 60*(b*e^6*n*x^2 - b*d^6*n)*log(e*x^(1/3) + d) + 15*(4*b*d*e^5*
n*x - b*d^4*e^2*n)*x^(2/3) - 6*(5*b*d^2*e^4*n*x - 2*b*d^5*e*n)*x^(1/3))/(d
^6*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = \frac{1}{120}\,ben\left(\frac{60\,e^5\log\left(ex^{\frac{1}{3}}+d\right)}{d^6}-\frac{20\,e^5\log\left(x\right)}{d^6}-\frac{60\,e^4x^{\frac{4}{3}}-30\,de^3x+20\,d^2e^2x^{\frac{2}{3}}-15\,d^3ex^{\frac{1}{3}}+12\,d^4}{d^5x^{\frac{5}{3}}}\right) - \frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="maxima")
```

output
```
1/120*b*e*n*(60*e^5*log(e*x^(1/3) + d)/d^6 - 20*e^5*log(x)/d^6 - (60*e^4*x
^(4/3) - 30*d*e^3*x + 20*d^2*e^2*x^(2/3) - 15*d^3*e*x^(1/3) + 12*d^4)/(d^5
*x^(5/3))) - 1/2*b*log((e*x^(1/3) + d)^n*c)/x^2 - 1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(115) = 230$.

Time = 0.13 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.40

$$\int \frac{a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx =$$

$$-\frac{\frac{60\,be^7n\log\left(ex^{\frac{1}{3}}+d\right)}{\left(ex^{\frac{1}{3}}+d\right)^6-6\left(ex^{\frac{1}{3}}+d\right)^5d+15\left(ex^{\frac{1}{3}}+d\right)^4d^2-20\left(ex^{\frac{1}{3}}+d\right)^3d^3+15\left(ex^{\frac{1}{3}}+d\right)^2d^4-6\left(ex^{\frac{1}{3}}+d\right)d^5+d^6} - \frac{60\,be^7n\log\left(ex^{\frac{1}{3}}+d\right)}{d^6} + \frac{60\,be^7n\cdots}{\cdots}}{\cdots}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="giac")
```

output
```
-1/120*(60*b*e^7*n*log(e*x^(1/3) + d)/((e*x^(1/3) + d)^6 - 6*(e*x^(1/3) +
d)^5*d + 15*(e*x^(1/3) + d)^4*d^2 - 20*(e*x^(1/3) + d)^3*d^3 + 15*(e*x^(1/
3) + d)^2*d^4 - 6*(e*x^(1/3) + d)*d^5 + d^6) - 60*b*e^7*n*log(e*x^(1/3) +
d)/d^6 + 60*b*e^7*n*log(e*x^(1/3))/d^6 + (60*(e*x^(1/3) + d)^5*b*e^7*n - 3
30*(e*x^(1/3) + d)^4*b*d*e^7*n + 740*(e*x^(1/3) + d)^3*b*d^2*e^7*n - 855*(
e*x^(1/3) + d)^2*b*d^3*e^7*n + 522*(e*x^(1/3) + d)*b*d^4*e^7*n - 137*b*d^5
*e^7*n + 60*b*d^5*e^7*log(c) + 60*a*d^5*e^7)/((e*x^(1/3) + d)^6*d^5 - 6*(e
*x^(1/3) + d)^5*d^6 + 15*(e*x^(1/3) + d)^4*d^7 - 20*(e*x^(1/3) + d)^3*d^8
+ 15*(e*x^(1/3) + d)^2*d^9 - 6*(e*x^(1/3) + d)*d^10 + d^11))/e
```

Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = \frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{d^6} - \frac{\frac{b\,e\,n}{5\,d} - \frac{b\,e^4\,n\,x}{2\,d^4} - \frac{b\,e^2\,n\,x^{1/3}}{4\,d^2} + \frac{b\,e^3\,n\,x^{2/3}}{3\,d^3} + \frac{b\,e^5\,n\,x^{4/3}}{d^5}}{2\,x^{5/3}} - \frac{b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))/x^3,x)
```

output
```
(b*e^6*n*atanh((2*e*x^(1/3))/d + 1))/d^6 - ((b*e*n)/(5*d) - (b*e^4*n*x)/(2
*d^4) - (b*e^2*n*x^(1/3))/(4*d^2) + (b*e^3*n*x^(2/3))/(3*d^3) + (b*e^5*n*x
^(4/3))/d^5)/(2*x^(5/3)) - (b*log(c*(d + e*x^(1/3))^n))/(2*x^2) - a/(2*x^2
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = \frac{15x^{\frac{2}{3}}b\,d^4e^2n-60x^{\frac{5}{3}}bd\,e^5n-12x^{\frac{1}{3}}b\,d^5en+30x^{\frac{4}{3}}b\,d^2e^4n-60\log\left(x^{\frac{1}{3}}\right)b\,e^6n\,x^2-60\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)b}{120d^6x^2}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x)
```

output
```
(15*x**(2/3)*b*d**4*e**2*n - 60*x**(2/3)*b*d*e**5*n*x - 12*x**(1/3)*b*d**5
*e*n + 30*x**(1/3)*b*d**2*e**4*n*x - 60*log(x**(1/3))*b*e**6*n*x**2 - 60*l
og((x**(1/3)*e + d)**n*c)*b*d**6 + 60*log((x**(1/3)*e + d)**n*c)*b*e**6*x*
*2 - 60*a*d**6 - 20*b*d**3*e**3*n*x)/(120*d**6*x**2)
```

3.449 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 192

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx = -\frac{ben}{24dx^{8/3}}+\frac{be^2n}{21d^2x^{7/3}}-\frac{be^3n}{18d^3x^2}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^6n}{9d^6x}-\frac{be^7n}{6d^7x^{2/3}}+\frac{be^8n}{3d^8\sqrt[3]{x}}-\frac{be^9n\log\left(d+e\sqrt[3]{x}\right)}{3d^9}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}+\frac{be^9n\log(x)}{9d^9}$$

output
```
-1/24*b*e*n/d/x^(8/3)+1/21*b*e^2*n/d^2/x^(7/3)-1/18*b*e^3*n/d^3/x^2+1/15*b
*e^4*n/d^4/x^(5/3)-1/12*b*e^5*n/d^5/x^(4/3)+1/9*b*e^6*n/d^6/x-1/6*b*e^7*n/
d^7/x^(2/3)+1/3*b*e^8*n/d^8/x^(1/3)-1/3*b*e^9*n*ln(d+e*x^(1/3))/d^9-1/3*(a
+b*ln(c*(d+e*x^(1/3))^n))/x^3+1/9*b*e^9*n*ln(x)/d^9
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}+\frac{1}{9}ben\left(-\frac{3}{8dx^{8/3}}+\frac{3e}{7d^2x^{7/3}}-\frac{e^2}{2d^3x^2}+\frac{3e^3}{5d^4x^{5/3}}-\frac{3e^4}{4d^5x^{4/3}}+\frac{e^5}{d^6x}-\frac{3e^6}{2d^7x^{2/3}}+\frac{3e^7}{d^8\sqrt[3]{x}}-\frac{3e^8\log\left(d+e\sqrt[3]{x}\right)}{d^9}+\frac{e^8\log(x)}{d^9}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) + (b*e*n*(-3/(8*d*x^(8/3
)) + (3*e)/(7*d^2*x^(7/3)) - e^2/(2*d^3*x^2) + (3*e^3)/(5*d^4*x^(5/3)) - (
3*e^4)/(4*d^5*x^(4/3)) + e^5/(d^6*x) - (3*e^6)/(2*d^7*x^(2/3)) + (3*e^7)/(
d^8*x^(1/3)) - (3*e^8*Log[d + e*x^(1/3)])/d^9 + (e^8*Log[x])/d^9))/9
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2842}$$

$$3\left(\frac{1}{9}ben\int \frac{1}{\left(d+e\sqrt[3]{x}\right)x^3}d\sqrt[3]{x}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{9x^3}\right)$$

$$\Big\downarrow \text{54}$$

$$3\left(\frac{1}{9}ben\int\left(-\frac{e^9}{d^9\left(d+e\sqrt[3]{x}\right)}+\frac{e^8}{d^9\sqrt[3]{x}}-\frac{e^7}{d^8x^{2/3}}+\frac{e^6}{d^7x}-\frac{e^5}{d^6x^{4/3}}+\frac{e^4}{d^5x^{5/3}}-\frac{e^3}{d^4x^2}+\frac{e^2}{d^3x^{7/3}}-\frac{e}{d^2x^{8/3}}+\frac{1}{dx^3}\right)d\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{9}ben\left(-\frac{e^8\log\left(d+e\sqrt[3]{x}\right)}{d^9}+\frac{e^8\log\left(\sqrt[3]{x}\right)}{d^9}+\frac{e^7}{d^8\sqrt[3]{x}}-\frac{e^6}{2d^7x^{2/3}}+\frac{e^5}{3d^6x}-\frac{e^4}{4d^5x^{4/3}}+\frac{e^3}{5d^4x^{5/3}}-\frac{e^2}{6d^3x^2}+\frac{e}{7d^2x^7}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

output
```
3*(-1/9*(a + b*Log[c*(d + e*x^(1/3))^n])/x^3 + (b*e*n*(-1/8*1/(d*x^(8/3))
+ e/(7*d^2*x^(7/3)) - e^2/(6*d^3*x^2) + e^3/(5*d^4*x^(5/3)) - e^4/(4*d^5*x
^(4/3)) + e^5/(3*d^6*x) - e^6/(2*d^7*x^(2/3)) + e^7/(d^8*x^(1/3)) - (e^8*L
og[d + e*x^(1/3)])/d^9 + (e^8*Log[x^(1/3)])/d^9))/9)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))/x^4,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)}{x^{4}}\,dx = \frac{840\,be^{9}nx^{3}\log\left(x^{\frac{1}{3}}\right)+280\,bd^{3}e^{6}nx^{2}-140\,bd^{6}e^{3}nx-840\,bd^{9}\log\left(c\right)-840\,ad^{9}-840\left(be^{9}nx^{3}+bd^{9}n\right)\log}{2520\,d}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="fricas")
```

output
```
1/2520*(840*b*e^9*n*x^3*log(x^(1/3)) + 280*b*d^3*e^6*n*x^2 - 140*b*d^6*e^3
*n*x - 840*b*d^9*log(c) - 840*a*d^9 - 840*(b*e^9*n*x^3 + b*d^9*n)*log(e*x^
(1/3) + d) + 30*(28*b*d*e^8*n*x^2 - 7*b*d^4*e^5*n*x + 4*b*d^7*e^2*n)*x^(2/
3) - 21*(20*b*d^2*e^7*n*x^2 - 8*b*d^5*e^4*n*x + 5*b*d^8*e*n)*x^(1/3))/(d^9
*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.72

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx=$$
$$-\frac{1}{2520}\,ben\left(\frac{840\,e^8\log\left(ex^{\frac{1}{3}}+d\right)}{d^9}-\frac{280\,e^8\log\left(x\right)}{d^9}-\frac{840\,e^7x^{\frac{7}{3}}-420\,de^6x^2+280\,d^2e^5x^{\frac{5}{3}}-210\,d^3e^4x^{\frac{4}{3}}}{d^8x}\right.$$
$$-\frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="maxima")
```

output
```
-1/2520*b*e*n*(840*e^8*log(e*x^(1/3) + d)/d^9 - 280*e^8*log(x)/d^9 - (840*
e^7*x^(7/3) - 420*d*e^6*x^2 + 280*d^2*e^5*x^(5/3) - 210*d^3*e^4*x^(4/3) +
168*d^4*e^3*x - 140*d^5*e^2*x^(2/3) + 120*d^6*e*x^(1/3) - 105*d^7)/(d^8*x^
(8/3))) - 1/3*b*log((e*x^(1/3) + d)^n*c)/x^3 - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(154) = 308$.

Time = 0.14 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.55

$$\int \frac{a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx =$$

$$-\frac{840\,be^{10}n\log\left(ex^{\frac{1}{3}}+d\right)}{\left(ex^{\frac{1}{3}}+d\right)^9-9\left(ex^{\frac{1}{3}}+d\right)^8d+36\left(ex^{\frac{1}{3}}+d\right)^7d^2-84\left(ex^{\frac{1}{3}}+d\right)^6d^3+126\left(ex^{\frac{1}{3}}+d\right)^5d^4-126\left(ex^{\frac{1}{3}}+d\right)^4d^5+84\left(ex^{\frac{1}{3}}+d\right)^3d^6-36\left(ex^{\frac{1}{3}}+d\right)^2d^7\cdots}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="giac")
```

output
```
-1/2520*(840*b*e^10*n*log(e*x^(1/3) + d)/((e*x^(1/3) + d)^9 - 9*(e*x^(1/3)
 + d)^8*d + 36*(e*x^(1/3) + d)^7*d^2 - 84*(e*x^(1/3) + d)^6*d^3 + 126*(e*x
^(1/3) + d)^5*d^4 - 126*(e*x^(1/3) + d)^4*d^5 + 84*(e*x^(1/3) + d)^3*d^6 -
 36*(e*x^(1/3) + d)^2*d^7 + 9*(e*x^(1/3) + d)*d^8 - d^9) + 840*b*e^10*n*lo
g(e*x^(1/3) + d)/d^9 - 840*b*e^10*n*log(e*x^(1/3))/d^9 - (840*(e*x^(1/3) +
 d)^8*b*e^10*n - 7140*(e*x^(1/3) + d)^7*b*d*e^10*n + 26740*(e*x^(1/3) + d)
^6*b*d^2*e^10*n - 57750*(e*x^(1/3) + d)^5*b*d^3*e^10*n + 78918*(e*x^(1/3)
+ d)^4*b*d^4*e^10*n - 70252*(e*x^(1/3) + d)^3*b*d^5*e^10*n + 40188*(e*x^(1
/3) + d)^2*b*d^6*e^10*n - 13827*(e*x^(1/3) + d)*b*d^7*e^10*n + 2283*b*d^8*
e^10*n - 840*b*d^8*e^10*log(c) - 840*a*d^8*e^10)/((e*x^(1/3) + d)^9*d^8 -
9*(e*x^(1/3) + d)^8*d^9 + 36*(e*x^(1/3) + d)^7*d^10 - 84*(e*x^(1/3) + d)^6
*d^11 + 126*(e*x^(1/3) + d)^5*d^12 - 126*(e*x^(1/3) + d)^4*d^13 + 84*(e*x^
(1/3) + d)^3*d^14 - 36*(e*x^(1/3) + d)^2*d^15 + 9*(e*x^(1/3) + d)*d^16 - d
^17))/e
```

Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.80

$$\int \frac{a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx =$$

$$-\frac{\frac{a\,d^9}{3}+\frac{b\,d^9\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3}+\frac{b\,d^6\,e^3\,n\,x}{18}+\frac{b\,d^8\,e\,n\,x^{1/3}}{24}-\frac{b\,d\,e^8\,n\,x^{8/3}}{3}-\frac{b\,d^3\,e^6\,n\,x^2}{9}-\frac{b\,d^7\,e^2\,n\,x^{2/3}}{21}-\frac{b\,d^5\,e^4\,n\,x^{4/3}}{15}+b\,d\cdots}{d^9\,x^3}$$

$$-\frac{2\,b\,e^9\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{3\,d^9}$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))/x^4,x)
```

output
```
- ((a*d^9)/3 + (b*d^9*log(c*(d + e*x^(1/3))^n))/3 + (b*d^6*e^3*n*x)/18 + (
b*d^8*e*n*x^(1/3))/24 - (b*d*e^8*n*x^(8/3))/3 - (b*d^3*e^6*n*x^2)/9 - (b*d
^7*e^2*n*x^(2/3))/21 - (b*d^5*e^4*n*x^(4/3))/15 + (b*d^4*e^5*n*x^(5/3))/12
 + (b*d^2*e^7*n*x^(7/3))/6)/(d^9*x^3) - (2*b*e^9*n*atanh((2*e*x^(1/3))/d +
 1))/(3*d^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx$$
$$= \frac{120x^{\frac{2}{3}}b\,d^7e^2n-210x^{\frac{5}{3}}b\,d^4e^5n+840x^{\frac{8}{3}}bd\,e^8n-105x^{\frac{1}{3}}b\,d^8en+168x^{\frac{4}{3}}b\,d^5e^4n-420x^{\frac{7}{3}}b\,d^2e^7n+840\log\Big(}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x)
```

output
```
(120*x**(2/3)*b*d**7*e**2*n - 210*x**(2/3)*b*d**4*e**5*n*x + 840*x**(2/3)*
b*d*e**8*n*x**2 - 105*x**(1/3)*b*d**8*e*n + 168*x**(1/3)*b*d**5*e**4*n*x -
 420*x**(1/3)*b*d**2*e**7*n*x**2 + 840*log(x**(1/3))*b*e**9*n*x**3 - 840*l
og((x**(1/3)*e + d)**n*c)*b*d**9 - 840*log((x**(1/3)*e + d)**n*c)*b*e**9*x
**3 - 840*a*d**9 - 140*b*d**6*e**3*n*x + 280*b*d**3*e**6*n*x**2)/(2520*d**
9*x**3)
```

3.450 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 680

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx = -\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9} - \frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9}+\frac{84b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^5}{25e^9} - \frac{14b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^6}{9e^9}+\frac{24b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^7}{49e^9} - \frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^8}{32e^9}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^9}{243e^9} + \frac{6b^2d^8n^2\sqrt[3]{x}}{e^8}-\frac{b^2d^9n^2\log^2\left(d+e\sqrt[3]{x}\right)}{3e^9} - \frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} + \frac{12bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} - \frac{56bd^6n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} + \frac{21bd^5n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} - \frac{84bd^4n\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^9} + \frac{28bd^3n\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} - \frac{24bd^2n\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{7e^9} + \frac{3bdn\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^9} - \frac{2bn\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{27e^9} + \frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2$$

output
```
-6*b^2*d^7*n^2*(d+e*x^(1/3))^2/e^9+56/9*b^2*d^6*n^2*(d+e*x^(1/3))^3/e^9-21
/4*b^2*d^5*n^2*(d+e*x^(1/3))^4/e^9+84/25*b^2*d^4*n^2*(d+e*x^(1/3))^5/e^9-1
4/9*b^2*d^3*n^2*(d+e*x^(1/3))^6/e^9+24/49*b^2*d^2*n^2*(d+e*x^(1/3))^7/e^9-
3/32*b^2*d*n^2*(d+e*x^(1/3))^8/e^9+2/243*b^2*n^2*(d+e*x^(1/3))^9/e^9+6*b^2
*d^8*n^2*x^(1/3)/e^8-1/3*b^2*d^9*n^2*ln(d+e*x^(1/3))^2/e^9-6*b*d^8*n*(d+e*
x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+12*b*d^7*n*(d+e*x^(1/3))^2*(a+b*l
n(c*(d+e*x^(1/3))^n))/e^9-56/3*b*d^6*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1
/3))^n))/e^9+21*b*d^5*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-84
/5*b*d^4*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+28/3*b*d^3*n*(d
+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-24/7*b*d^2*n*(d+e*x^(1/3))^7
*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+3/4*b*d*n*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*
x^(1/3))^n))/e^9-2/27*b*n*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+
2/3*b*d^9*n*ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+1/3*x^3*(a+b*l
n(c*(d+e*x^(1/3))^n))^2
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.63

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx = \frac{e\sqrt[3]{x}\left(3175200a^2e^8x^{8/3}-2520abn\left(2520d^8-1260d^7e\sqrt[3]{x}+840d^6e^2x^{2/3}-630d^5e^3x+504d^4e^4x^{4/3}-420\right.\right.}{}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

output
```
(e*x^(1/3)*(3175200*a^2*e^8*x^(8/3) - 2520*a*b*n*(2520*d^8 - 1260*d^7*e*x^
(1/3) + 840*d^6*e^2*x^(2/3) - 630*d^5*e^3*x + 504*d^4*e^4*x^(4/3) - 420*d^
3*e^5*x^(5/3) + 360*d^2*e^6*x^2 - 315*d*e^7*x^(7/3) + 280*e^8*x^(8/3)) + b
^2*n^2*(17965080*d^8 - 5807340*d^7*e*x^(1/3) + 2813160*d^6*e^2*x^(2/3) - 1
580670*d^5*e^3*x + 947016*d^4*e^4*x^(4/3) - 577500*d^3*e^5*x^(5/3) + 34380
0*d^2*e^6*x^2 - 187425*d*e^7*x^(7/3) + 78400*e^8*x^(8/3))) + 2520*b*d^9*n*
(2520*a - 7129*b*n)*Log[d + e*x^(1/3)] - 2520*b*e*x^(1/3)*(-2520*a*e^8*x^(
8/3) + b*n*(2520*d^8 - 1260*d^7*e*x^(1/3) + 840*d^6*e^2*x^(2/3) - 630*d^5*
e^3*x + 504*d^4*e^4*x^(4/3) - 420*d^3*e^5*x^(5/3) + 360*d^2*e^6*x^2 - 315*
d*e^7*x^(7/3) + 280*e^8*x^(8/3)))*Log[c*(d + e*x^(1/3))^n] + 3175200*b^2*(
d^9 + e^9*x^3)*Log[c*(d + e*x^(1/3))^n]^2)/(9525600*e^9)
```

Rubi [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.59, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2904, 2845, 2858, 25, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2d\sqrt[3]{x}$$

$$\Big\downarrow \text{2845}$$

$$3\left(\frac{1}{9}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{2}{9}ben\int\frac{x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d+e\sqrt[3]{x}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{2858}$$

$$3\left(\frac{1}{9}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{2}{9}bn\int x^{8/3}\left(a+b\log\left(cx^{n/3}\right)\right)d\left(d+e\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$3\left(\frac{2}{9}bn\int -x^{8/3}\left(a+b\log\left(cx^{n/3}\right)\right)d\left(d+e\sqrt[3]{x}\right)+\frac{1}{9}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{2bn\int -e^9x^{8/3}\left(a+b\log\left(cx^{n/3}\right)\right)d\left(d+e\sqrt[3]{x}\right)}{9e^9}+\frac{1}{9}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2772}$$

$$3\left(\frac{2bn\left(-bn\int\left(\frac{\log\left(d+e\sqrt[3]{x}\right)d^9}{\sqrt[3]{x}}-9d^8+18\left(d+e\sqrt[3]{x}\right)d^7-28x^{2/3}d^6+\frac{63xd^5}{2}-\frac{126}{5}x^{4/3}d^4+14x^{5/3}d^3-\frac{36x^2d^2}{7}+\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{2bn\left(d^9\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)-9d^8\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)+18d^7x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)\right.}{}\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

output
```
3*((x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/9 + (2*b*n*(-(b*n*(-9*d^8*(d +
 e*x^(1/3)) + 9*d^7*x^(2/3) - (28*d^6*x)/3 + (63*d^5*x^(4/3))/8 - (126*d^4
*x^(5/3))/25 + (7*d^3*x^2)/3 - (36*d^2*x^(7/3))/49 + (9*d*x^(8/3))/64 - x^
3/81 + (d^9*Log[d + e*x^(1/3)]^2)/2)) - 9*d^8*(d + e*x^(1/3))*(a + b*Log[c
*x^(n/3)]) + 18*d^7*x^(2/3)*(a + b*Log[c*x^(n/3)]) - 28*d^6*x*(a + b*Log[c
*x^(n/3)]) + (63*d^5*x^(4/3)*(a + b*Log[c*x^(n/3)]))/2 - (126*d^4*x^(5/3)*
(a + b*Log[c*x^(n/3)]))/5 + 14*d^3*x^2*(a + b*Log[c*x^(n/3)]) - (36*d^2*x^
(7/3)*(a + b*Log[c*x^(n/3)]))/7 + (9*d*x^(8/3)*(a + b*Log[c*x^(n/3)]))/8 -
 (x^3*(a + b*Log[c*x^(n/3)]))/9 + d^9*Log[d + e*x^(1/3)]*(a + b*Log[c*x^(n
/3)])))/(9*e^9))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2 \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)^2 dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 674, normalized size of antiderivative = 0.99

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

output
```
1/9525600*(3175200*b^2*e^9*x^3*log(c)^2 + 39200*(2*b^2*e^9*n^2 - 18*a*b*e^
9*n + 81*a^2*e^9)*x^3 - 2100*(275*b^2*d^3*e^6*n^2 - 504*a*b*d^3*e^6*n)*x^2
 + 3175200*(b^2*e^9*n^2*x^3 + b^2*d^9*n^2)*log(e*x^(1/3) + d)^2 + 840*(334
9*b^2*d^6*e^3*n^2 - 2520*a*b*d^6*e^3*n)*x + 2520*(420*b^2*d^3*e^6*n^2*x^2
- 840*b^2*d^6*e^3*n^2*x - 7129*b^2*d^9*n^2 + 2520*a*b*d^9*n - 280*(b^2*e^9
*n^2 - 9*a*b*e^9*n)*x^3 + 2520*(b^2*e^9*n*x^3 + b^2*d^9*n)*log(c) + 63*(5*
b^2*d*e^8*n^2*x^2 - 8*b^2*d^4*e^5*n^2*x + 20*b^2*d^7*e^2*n^2)*x^(2/3) - 90
*(4*b^2*d^2*e^7*n^2*x^2 - 7*b^2*d^5*e^4*n^2*x + 28*b^2*d^8*e*n^2)*x^(1/3))
*log(e*x^(1/3) + d) + 352800*(3*b^2*d^3*e^6*n*x^2 - 6*b^2*d^6*e^3*n*x - 2*
(b^2*e^9*n - 9*a*b*e^9)*x^3)*log(c) - 63*(92180*b^2*d^7*e^2*n^2 - 50400*a*
b*d^7*e^2*n + 175*(17*b^2*d*e^8*n^2 - 72*a*b*d*e^8*n)*x^2 - 8*(1879*b^2*d^
4*e^5*n^2 - 2520*a*b*d^4*e^5*n)*x - 2520*(5*b^2*d*e^8*n*x^2 - 8*b^2*d^4*e^
5*n*x + 20*b^2*d^7*e^2*n)*log(c))*x^(2/3) + 90*(199612*b^2*d^8*e*n^2 - 705
60*a*b*d^8*e*n + 20*(191*b^2*d^2*e^7*n^2 - 504*a*b*d^2*e^7*n)*x^2 - 7*(250
9*b^2*d^5*e^4*n^2 - 2520*a*b*d^5*e^4*n)*x - 2520*(4*b^2*d^2*e^7*n*x^2 - 7*
b^2*d^5*e^4*n*x + 28*b^2*d^8*e*n)*log(c))*x^(1/3))/e^9
```

Sympy [F]

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 \, dx = \int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 \, dx$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

output
```
Integral(x**2*(a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.62

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{1}{3}\,b^2x^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^2+\frac{2}{3}\,abx^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{3}\,a^2x^3$$
$$+\frac{1}{3780}\,aben\left(\frac{2520\,d^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^8x^3-315\,de^7x^{\frac{8}{3}}+360\,d^2e^6x^{\frac{7}{3}}-420\,d^3e^5x^2+504\,d^4e^4x^{\frac{5}{3}}-}{e^9}\right.$$
$$+\frac{1}{9525600}\left(2520\,en\left(\frac{2520\,d^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^8x^3-315\,de^7x^{\frac{8}{3}}+360\,d^2e^6x^{\frac{7}{3}}-420\,d^3e^5x^2+504\right.\right.$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3*log((e*x^(1/3) + d)^n*c)^2 + 2/3*a*b*x^3*log((e*x^(1/3) + d)^n
*c) + 1/3*a^2*x^3 + 1/3780*a*b*e*n*(2520*d^9*log(e*x^(1/3) + d)/e^10 - (28
0*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 50
4*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7*e*x^(2/
3) + 2520*d^8*x^(1/3))/e^9) + 1/9525600*(2520*e*n*(2520*d^9*log(e*x^(1/3)
+ d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d
^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1
260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*log((e*x^(1/3) + d)^n*c) + (784
00*e^9*x^3 - 187425*d*e^8*x^(8/3) + 343800*d^2*e^7*x^(7/3) - 577500*d^3*e^
6*x^2 - 3175200*d^9*log(e*x^(1/3) + d)^2 + 947016*d^4*e^5*x^(5/3) - 158067
0*d^5*e^4*x^(4/3) + 2813160*d^6*e^3*x - 17965080*d^9*log(e*x^(1/3) + d) -
5807340*d^7*e^2*x^(2/3) + 17965080*d^8*e*x^(1/3))*n^2/e^9)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. $2(586) = 1172$.

Time = 0.14 (sec) , antiderivative size = 1389, normalized size of antiderivative = 2.04

$$\int x^2 \left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^n\right)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

output
```
1/9525600*(3175200*b^2*e*x^3*log(c)^2 + 6350400*a*b*e*x^3*log(c) + 3175200
*a^2*e*x^3 + (3175200*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)^2/e^8 - 2857680
0*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)^2/e^8 + 114307200*(e*x^(1/3) + d)
^7*d^2*log(e*x^(1/3) + d)^2/e^8 - 266716800*(e*x^(1/3) + d)^6*d^3*log(e*x^
(1/3) + d)^2/e^8 + 400075200*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)^2/e^
8 - 400075200*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)^2/e^8 + 266716800*(
e*x^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)^2/e^8 - 114307200*(e*x^(1/3) + d)^
2*d^7*log(e*x^(1/3) + d)^2/e^8 + 28576800*(e*x^(1/3) + d)*d^8*log(e*x^(1/3
) + d)^2/e^8 - 705600*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)/e^8 + 7144200*(
e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)/e^8 - 32659200*(e*x^(1/3) + d)^7*d^2
*log(e*x^(1/3) + d)/e^8 + 88905600*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d
)/e^8 - 160030080*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)/e^8 + 200037600
*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 - 177811200*(e*x^(1/3) + d)^
3*d^6*log(e*x^(1/3) + d)/e^8 + 114307200*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/
3) + d)/e^8 - 57153600*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 + 78400*
(e*x^(1/3) + d)^9/e^8 - 893025*(e*x^(1/3) + d)^8*d/e^8 + 4665600*(e*x^(1/3
) + d)^7*d^2/e^8 - 14817600*(e*x^(1/3) + d)^6*d^3/e^8 + 32006016*(e*x^(1/3
) + d)^5*d^4/e^8 - 50009400*(e*x^(1/3) + d)^4*d^5/e^8 + 59270400*(e*x^(1/3
) + d)^3*d^6/e^8 - 57153600*(e*x^(1/3) + d)^2*d^7/e^8 + 57153600*(e*x^(1/3
) + d)*d^8/e^8)*b^2*n^2 + 2520*(2520*(e*x^(1/3) + d)^9*log(e*x^(1/3) + ...
```

Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 608, normalized size of antiderivative = 0.89

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^2}{3}+\frac{2\,b^2\,n^2\,x^3}{243}+\frac{2\,a\,b\,x^3\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3}+\frac{b^2\,d^9\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^2}{3\,e^9}-\frac{2\,a\,b\,n\,x^3}{27}-\frac{2\,b^2\,n\,x^3\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{27}-\frac{7129\,b^2\,d^9\,n^2\,\ln\left(d+e\,x^{1/3}\right)}{3780\,e^9}-\frac{275\,b^2\,d^3\,n^2\,x^2}{4536\,e^3}+\frac{191\,b^2\,d^2\,n^2\,x^{7/3}}{5292\,e^2}+\frac{1879\,b^2\,d^4\,n^2\,x^{5/3}}{18900\,e^4}-\frac{2509\,b^2\,d^5\,n^2\,x^{4/3}}{15120\,e^5}-\frac{4609\,b^2\,d^7\,n^2\,x^{2/3}}{7560\,e^7}+\frac{7129\,b^2\,d^8\,n^2\,x^{1/3}}{3780\,e^8}-\frac{17\,b^2\,d\,n^2\,x^{8/3}}{864\,e}+\frac{3349\,b^2\,d^6\,n^2\,x}{11340\,e^6}+\frac{b^2\,d^3\,n\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{9\,e^3}-\frac{2\,b^2\,d^2\,n\,x^{7/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{21\,e^2}-\frac{2\,b^2\,d^4\,n\,x^{5/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{15\,e^4}+\frac{b^2\,d^5\,n\,x^{4/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{6\,e^5}+\frac{b^2\,d^7\,n\,x^{2/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3\,e^7}-\frac{2\,b^2\,d^8\,n\,x^{1/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3\,e^8}+\frac{a\,b\,d\,n\,x^{8/3}}{12\,e}-\frac{2\,a\,b\,d^6\,n\,x}{9\,e^6}+\frac{2\,a\,b\,d^9\,n\,\ln\left(d+e\,x^{1/3}\right)}{3\,e^9}+\frac{b^2\,d\,n\,x^{8/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{12\,e}-\frac{2\,b^2\,d^6\,n\,x\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{9\,e^6}+\frac{a\,b\,d^3\,n\,x^2}{9\,e^3}-\frac{2\,a\,b\,d^2\,n\,x^{7/3}}{21\,e^2}-\frac{2\,a\,b\,d^4\,n\,x^{5/3}}{15\,e^4}+\frac{a\,b\,d^5\,n\,x^{4/3}}{6\,e^5}+\frac{a\,b\,d^7\,n\,x^{2/3}}{3\,e^7}-\frac{2\,a\,b\,d^8\,n\,x^{1/3}}{3\,e^8}$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^3*log(c*(d + e*x^(1/3))^n)^2)/3 + (2*b^2*n^2*x^3)/243
 + (2*a*b*x^3*log(c*(d + e*x^(1/3))^n))/3 + (b^2*d^9*log(c*(d + e*x^(1/3))
^n)^2)/(3*e^9) - (2*a*b*n*x^3)/27 - (2*b^2*n*x^3*log(c*(d + e*x^(1/3))^n))
/27 - (7129*b^2*d^9*n^2*log(d + e*x^(1/3)))/(3780*e^9) - (275*b^2*d^3*n^2*
x^2)/(4536*e^3) + (191*b^2*d^2*n^2*x^(7/3))/(5292*e^2) + (1879*b^2*d^4*n^2
*x^(5/3))/(18900*e^4) - (2509*b^2*d^5*n^2*x^(4/3))/(15120*e^5) - (4609*b^2
*d^7*n^2*x^(2/3))/(7560*e^7) + (7129*b^2*d^8*n^2*x^(1/3))/(3780*e^8) - (17
*b^2*d*n^2*x^(8/3))/(864*e) + (3349*b^2*d^6*n^2*x)/(11340*e^6) + (b^2*d^3*
n*x^2*log(c*(d + e*x^(1/3))^n))/(9*e^3) - (2*b^2*d^2*n*x^(7/3)*log(c*(d +
e*x^(1/3))^n))/(21*e^2) - (2*b^2*d^4*n*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(
15*e^4) + (b^2*d^5*n*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(6*e^5) + (b^2*d^7*
n*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(3*e^7) - (2*b^2*d^8*n*x^(1/3)*log(c*(
d + e*x^(1/3))^n))/(3*e^8) + (a*b*d*n*x^(8/3))/(12*e) - (2*a*b*d^6*n*x)/(9
*e^6) + (2*a*b*d^9*n*log(d + e*x^(1/3)))/(3*e^9) + (b^2*d*n*x^(8/3)*log(c*
(d + e*x^(1/3))^n))/(12*e) - (2*b^2*d^6*n*x*log(c*(d + e*x^(1/3))^n))/(9*e
^6) + (a*b*d^3*n*x^2)/(9*e^3) - (2*a*b*d^2*n*x^(7/3))/(21*e^2) - (2*a*b*d^
4*n*x^(5/3))/(15*e^4) + (a*b*d^5*n*x^(4/3))/(6*e^5) + (a*b*d^7*n*x^(2/3))/
(3*e^7) - (2*a*b*d^8*n*x^(1/3))/(3*e^8)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 621, normalized size of antiderivative = 0.91

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x)
```

output

```
(3175200*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**2*d**7*e**2*n - 1270080*x*
*(2/3)*log((x**(1/3)*e + d)**n*c)*b**2*d**4*e**5*n*x + 793800*x**(2/3)*log
((x**(1/3)*e + d)**n*c)*b**2*d*e**8*n*x**2 + 3175200*x**(2/3)*a*b*d**7*e**
2*n - 1270080*x**(2/3)*a*b*d**4*e**5*n*x + 793800*x**(2/3)*a*b*d*e**8*n*x*
*2 - 5807340*x**(2/3)*b**2*d**7*e**2*n**2 + 947016*x**(2/3)*b**2*d**4*e**5
*n**2*x - 187425*x**(2/3)*b**2*d*e**8*n**2*x**2 - 6350400*x**(1/3)*log((x*
*(1/3)*e + d)**n*c)*b**2*d**8*e*n + 1587600*x**(1/3)*log((x**(1/3)*e + d)*
*n*c)*b**2*d**5*e**4*n*x - 907200*x**(1/3)*log((x**(1/3)*e + d)**n*c)*b**2
*d**2*e**7*n*x**2 - 6350400*x**(1/3)*a*b*d**8*e*n + 1587600*x**(1/3)*a*b*d
**5*e**4*n*x - 907200*x**(1/3)*a*b*d**2*e**7*n*x**2 + 17965080*x**(1/3)*b*
*2*d**8*e*n**2 - 1580670*x**(1/3)*b**2*d**5*e**4*n**2*x + 343800*x**(1/3)*
b**2*d**2*e**7*n**2*x**2 + 3175200*log((x**(1/3)*e + d)**n*c)**2*b**2*d**9
 + 3175200*log((x**(1/3)*e + d)**n*c)**2*b**2*e**9*x**3 + 6350400*log((x**
(1/3)*e + d)**n*c)*a*b*d**9 + 6350400*log((x**(1/3)*e + d)**n*c)*a*b*e**9*
x**3 - 17965080*log((x**(1/3)*e + d)**n*c)*b**2*d**9*n - 2116800*log((x**(
1/3)*e + d)**n*c)*b**2*d**6*e**3*n*x + 1058400*log((x**(1/3)*e + d)**n*c)*
b**2*d**3*e**6*n*x**2 - 705600*log((x**(1/3)*e + d)**n*c)*b**2*e**9*n*x**3
 + 3175200*a**2*e**9*x**3 - 2116800*a*b*d**6*e**3*n*x + 1058400*a*b*d**3*e
**6*n*x**2 - 705600*a*b*e**9*n*x**3 + 2813160*b**2*d**6*e**3*n**2*x - 5775
00*b**2*d**3*e**6*n**2*x**2 + 78400*b**2*e**9*n**2*x**3)/(9525600*e**9)
```

3.451 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 480

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6}-\frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6}-\frac{6b^2dn^2\left(d+e\sqrt[3]{x}\right)^5}{25e^6}+\frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^6}{36e^6}-\frac{6b^2d^5n^2\sqrt[3]{x}}{e^5}+\frac{b^2d^6n^2\log^2\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}-\frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^6}+\frac{20bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6}-\frac{15bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6}+\frac{6bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^6}-\frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{6e^6}-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2$$

output

```
15/4*b^2*d^4*n^2*(d+e*x^(1/3))^2/e^6-20/9*b^2*d^3*n^2*(d+e*x^(1/3))^3/e^6+
15/16*b^2*d^2*n^2*(d+e*x^(1/3))^4/e^6-6/25*b^2*d*n^2*(d+e*x^(1/3))^5/e^6+1
/36*b^2*n^2*(d+e*x^(1/3))^6/e^6-6*b^2*d^5*n^2*x^(1/3)/e^5+1/2*b^2*d^6*n^2*
ln(d+e*x^(1/3))^2/e^6+6*b*d^5*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/
e^6-15/2*b*d^4*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+20/3*b*d^
3*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-15/4*b*d^2*n*(d+e*x^(1
/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+6/5*b*d*n*(d+e*x^(1/3))^5*(a+b*ln(c
*(d+e*x^(1/3))^n))/e^6-1/6*b*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))
/e^6-b*d^6*n*ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+1/2*x^2*(a+b*
ln(c*(d+e*x^(1/3))^n))^2
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.66

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{e\sqrt[3]{x}\left(1800a^2e^5x^{5/3}+60abn\left(60d^5-30d^4e\sqrt[3]{x}+20d^3e^2x^{2/3}-15d^2e^3x+12de^4x^{4/3}-10e^5x^{5/3}\right)+b^2n^2\left(-\right.\right.}{}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

output
```
(e*x^(1/3)*(1800*a^2*e^5*x^(5/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(1/3) + 2
0*d^3*e^2*x^(2/3) - 15*d^2*e^3*x + 12*d*e^4*x^(4/3) - 10*e^5*x^(5/3)) + b^
2*n^2*(-8820*d^5 + 2610*d^4*e*x^(1/3) - 1140*d^3*e^2*x^(2/3) + 555*d^2*e^3
*x - 264*d*e^4*x^(4/3) + 100*e^5*x^(5/3))) + 180*b*d^6*n*(-20*a + 49*b*n)*
Log[d + e*x^(1/3)] - 60*b*e*x^(1/3)*(-60*a*e^5*x^(5/3) + b*n*(-60*d^5 + 30
*d^4*e*x^(1/3) - 20*d^3*e^2*x^(2/3) + 15*d^2*e^3*x - 12*d*e^4*x^(4/3) + 10
*e^5*x^(5/3)))*Log[c*(d + e*x^(1/3))^n] - 1800*b^2*(d^6 - e^6*x^2)*Log[c*(
d + e*x^(1/3))^n]^2)/(3600*e^6)
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.63, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2904, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2845}$$

$$3\left(\frac{1}{6}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{1}{3}ben\int\frac{x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d+e\sqrt[3]{x}}d\sqrt[3]{x}\right)$$

$\Big\downarrow$ 2858

$$3\left(\frac{1}{6}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{1}{3}bn\int x^{5/3}\left(a+b\log\left(cx^{n/3}\right)\right)d\left(d+e\sqrt[3]{x}\right)\right)$$

$\Big\downarrow$ 27

$$3\left(\frac{1}{6}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{bn\int e^6x^{5/3}\left(a+b\log\left(cx^{n/3}\right)\right)d\left(d+e\sqrt[3]{x}\right)}{3e^6}\right)$$

$\Big\downarrow$ 2772

$$3\left(\frac{1}{6}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{bn\left(-bn\int\left(\frac{\log\left(d+e\sqrt[3]{x}\right)d^6}{\sqrt[3]{x}}-6d^5+\frac{15}{2}\left(d+e\sqrt[3]{x}\right)d^4-\frac{20}{3}x^{2/3}d^3+\frac{15xd^2}{4}-\right.}{}\right.$$

$\Big\downarrow$ 2009

$$3\left(\frac{1}{6}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{bn\left(d^6\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)-6d^5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right.\right.}{}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

output
```
3*((x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/6 - (b*n*(-(b*n*(-6*d^5*(d + e
*x^(1/3)) + (15*d^4*x^(2/3))/4 - (20*d^3*x)/9 + (15*d^2*x^(4/3))/16 - (6*d
*x^(5/3))/25 + x^2/36 + (d^6*Log[d + e*x^(1/3)]^2)/2)) - 6*d^5*(d + e*x^(1
/3))*(a + b*Log[c*x^(n/3)]) + (15*d^4*x^(2/3)*(a + b*Log[c*x^(n/3)]))/2 -
(20*d^3*x*(a + b*Log[c*x^(n/3)]))/3 + (15*d^2*x^(4/3)*(a + b*Log[c*x^(n/3)
]))/4 - (6*d*x^(5/3)*(a + b*Log[c*x^(n/3)]))/5 + (x^2*(a + b*Log[c*x^(n/3)
]))/6 + d^6*Log[d + e*x^(1/3)]*(a + b*Log[c*x^(n/3)])))/(3*e^6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.01

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}\,dx = \frac{1800\,b^2e^6x^2\log\left(c\right)^2+100\,(b^2e^6n^2-6\,abe^6n+18\,a^2e^6)x^2+1800\,(b^2e^6n^2x^2-b^2d^6n^2)\log\left(ex^{\frac{1}{3}}+d\right)^2-6}{}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

output
```
1/3600*(1800*b^2*e^6*x^2*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^
2*e^6)*x^2 + 1800*(b^2*e^6*n^2*x^2 - b^2*d^6*n^2)*log(e*x^(1/3) + d)^2 - 6
0*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x + 60*(20*b^2*d^3*e^3*n^2*x + 1
47*b^2*d^6*n^2 - 60*a*b*d^6*n - 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^2 + 60*(b
^2*e^6*n*x^2 - b^2*d^6*n)*log(c) + 6*(2*b^2*d*e^5*n^2*x - 5*b^2*d^4*e^2*n^
2)*x^(2/3) - 15*(b^2*d^2*e^4*n^2*x - 4*b^2*d^5*e*n^2)*x^(1/3))*log(e*x^(1/
3) + d) + 600*(2*b^2*d^3*e^3*n*x - (b^2*e^6*n - 6*a*b*e^6)*x^2)*log(c) + 6
*(435*b^2*d^4*e^2*n^2 - 300*a*b*d^4*e^2*n - 4*(11*b^2*d*e^5*n^2 - 30*a*b*d
*e^5*n)*x + 60*(2*b^2*d*e^5*n*x - 5*b^2*d^4*e^2*n)*log(c))*x^(2/3) - 15*(5
88*b^2*d^5*e*n^2 - 240*a*b*d^5*e*n - (37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*
n)*x + 60*(b^2*d^2*e^4*n*x - 4*b^2*d^5*e*n)*log(c))*x^(1/3))/e^6
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

output
```
Integral(x*(a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.67

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\frac{1}{2}b^2x^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^2-\frac{1}{60}aben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x^{\frac{1}{3}}}{e^6}\right)+abx^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{2}a^2x^2-\frac{1}{3600}\left(60\,en\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x\right.}{e^6}\right.\right.$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*log((e*x^(1/3) + d)^n*c)^2 - 1/60*a*b*e*n*(60*d^6*log(e*x^(1/3
) + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*
e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/e^6) + a*b*x^2*log((e*x^(1/3) +
 d)^n*c) + 1/2*a^2*x^2 - 1/3600*(60*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (
10*e^5*x^2 - 12*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4
*e*x^(2/3) - 60*d^5*x^(1/3))/e^6)*log((e*x^(1/3) + d)^n*c) - (100*e^6*x^2
+ 1800*d^6*log(e*x^(1/3) + d)^2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3)
- 1140*d^3*e^3*x + 8820*d^6*log(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 88
20*d^5*e*x^(1/3))*n^2/e^6)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(412) = 824$.

Time = 0.14 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.94

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

output
```
1/3600*(1800*b^2*e*x^2*log(c)^2 + 3600*a*b*e*x^2*log(c) + (1800*(e*x^(1/3)
 + d)^6*log(e*x^(1/3) + d)^2/e^5 - 10800*(e*x^(1/3) + d)^5*d*log(e*x^(1/3)
 + d)^2/e^5 + 27000*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)^2/e^5 - 36000
*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)^2/e^5 + 27000*(e*x^(1/3) + d)^2*
d^4*log(e*x^(1/3) + d)^2/e^5 - 10800*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d
)^2/e^5 - 600*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 + 4320*(e*x^(1/3) +
 d)^5*d*log(e*x^(1/3) + d)/e^5 - 13500*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3)
 + d)/e^5 + 24000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 - 27000*(e*
x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^5 + 21600*(e*x^(1/3) + d)*d^5*log(
e*x^(1/3) + d)/e^5 + 100*(e*x^(1/3) + d)^6/e^5 - 864*(e*x^(1/3) + d)^5*d/e
^5 + 3375*(e*x^(1/3) + d)^4*d^2/e^5 - 8000*(e*x^(1/3) + d)^3*d^3/e^5 + 135
00*(e*x^(1/3) + d)^2*d^4/e^5 - 21600*(e*x^(1/3) + d)*d^5/e^5)*b^2*n^2 + 18
00*a^2*e*x^2 + 60*(60*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 - 360*(e*x^
(1/3) + d)^5*d*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/3) + d)^4*d^2*log(e*x^
(1/3) + d)/e^5 - 1200*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 + 900*(
e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)*d^5*log(
e*x^(1/3) + d)/e^5 - 10*(e*x^(1/3) + d)^6/e^5 + 72*(e*x^(1/3) + d)^5*d/e^5
 - 225*(e*x^(1/3) + d)^4*d^2/e^5 + 400*(e*x^(1/3) + d)^3*d^3/e^5 - 450*(e*
x^(1/3) + d)^2*d^4/e^5 + 360*(e*x^(1/3) + d)*d^5/e^5)*b^2*n*log(c) + 60*(6
0*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)^5*d*lo...
```

Mupad [B] (verification not implemented)

Time = 15.74 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.90

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\frac{a^2x^2}{2}+\frac{b^2x^2\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{2}+\frac{b^2n^2x^2}{36}+a\,b\,x^2\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)-\frac{b^2d^6\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{2\,e^6}-\frac{a\,b\,n\,x^2}{6}-\frac{b^2n\,x^2\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{6}+\frac{49\,b^2d^6n^2\,\ln\left(d+e\,x^{1/3}\right)}{20\,e^6}+\frac{37\,b^2d^2n^2x^{4/3}}{240\,e^2}+\frac{29\,b^2d^4n^2x^{2/3}}{40\,e^4}-\frac{49\,b^2d^5n^2x^{1/3}}{20\,e^5}-\frac{19\,b^2d^3n^2x}{60\,e^3}-\frac{11\,b^2d\,n^2x^{5/3}}{150\,e}-\frac{b^2d^2n\,x^{4/3}\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{4\,e^2}-\frac{b^2d^4n\,x^{2/3}\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{2\,e^4}+\frac{b^2d^5n\,x^{1/3}\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{e^5}+\frac{a\,b\,d^3n\,x}{3\,e^3}+\frac{a\,b\,d\,n\,x^{5/3}}{5\,e}-\frac{a\,b\,d^6n\,\ln\left(d+e\,x^{1/3}\right)}{e^6}+\frac{b^2d^3n\,x\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{3\,e^3}+\frac{b^2d\,n\,x^{5/3}\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{5\,e}-\frac{a\,b\,d^2n\,x^{4/3}}{4\,e^2}-\frac{a\,b\,d^4n\,x^{2/3}}{2\,e^4}+\frac{a\,b\,d^5n\,x^{1/3}}{e^5}$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

output
```
(a^2*x^2)/2 + (b^2*x^2*log(c*(d + e*x^(1/3))^n)^2)/2 + (b^2*n^2*x^2)/36 +
a*b*x^2*log(c*(d + e*x^(1/3))^n) - (b^2*d^6*log(c*(d + e*x^(1/3))^n)^2)/(2
*e^6) - (a*b*n*x^2)/6 - (b^2*n*x^2*log(c*(d + e*x^(1/3))^n))/6 + (49*b^2*d
^6*n^2*log(d + e*x^(1/3)))/(20*e^6) + (37*b^2*d^2*n^2*x^(4/3))/(240*e^2) +
 (29*b^2*d^4*n^2*x^(2/3))/(40*e^4) - (49*b^2*d^5*n^2*x^(1/3))/(20*e^5) - (
19*b^2*d^3*n^2*x)/(60*e^3) - (11*b^2*d*n^2*x^(5/3))/(150*e) - (b^2*d^2*n*x
^(4/3)*log(c*(d + e*x^(1/3))^n))/(4*e^2) - (b^2*d^4*n*x^(2/3)*log(c*(d + e
*x^(1/3))^n))/(2*e^4) + (b^2*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^5 +
 (a*b*d^3*n*x)/(3*e^3) + (a*b*d*n*x^(5/3))/(5*e) - (a*b*d^6*n*log(d + e*x^
(1/3)))/e^6 + (b^2*d^3*n*x*log(c*(d + e*x^(1/3))^n))/(3*e^3) + (b^2*d*n*x^
(5/3)*log(c*(d + e*x^(1/3))^n))/(5*e) - (a*b*d^2*n*x^(4/3))/(4*e^2) - (a*b
*d^4*n*x^(2/3))/(2*e^4) + (a*b*d^5*n*x^(1/3))/e^5
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.93

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

$$= \frac{-1800x^{\frac{2}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b^2d^4e^2n+720x^{\frac{5}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b^2d\,e^5n-1800x^{\frac{2}{3}}ab\,d^4e^2n+720x^{\frac{5}{3}}abd}{}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x)
```

output
```
( - 1800*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**2*d**4*e**2*n + 720*x**(2/
3)*log((x**(1/3)*e + d)**n*c)*b**2*d*e**5*n*x - 1800*x**(2/3)*a*b*d**4*e**
2*n + 720*x**(2/3)*a*b*d*e**5*n*x + 2610*x**(2/3)*b**2*d**4*e**2*n**2 - 26
4*x**(2/3)*b**2*d*e**5*n**2*x + 3600*x**(1/3)*log((x**(1/3)*e + d)**n*c)*b
**2*d**5*e*n - 900*x**(1/3)*log((x**(1/3)*e + d)**n*c)*b**2*d**2*e**4*n*x
+ 3600*x**(1/3)*a*b*d**5*e*n - 900*x**(1/3)*a*b*d**2*e**4*n*x - 8820*x**(1
/3)*b**2*d**5*e*n**2 + 555*x**(1/3)*b**2*d**2*e**4*n**2*x - 1800*log((x**(
1/3)*e + d)**n*c)**2*b**2*d**6 + 1800*log((x**(1/3)*e + d)**n*c)**2*b**2*e
**6*x**2 - 3600*log((x**(1/3)*e + d)**n*c)*a*b*d**6 + 3600*log((x**(1/3)*e
 + d)**n*c)*a*b*e**6*x**2 + 8820*log((x**(1/3)*e + d)**n*c)*b**2*d**6*n +
1200*log((x**(1/3)*e + d)**n*c)*b**2*d**3*e**3*n*x - 600*log((x**(1/3)*e +
 d)**n*c)*b**2*e**6*n*x**2 + 1800*a**2*e**6*x**2 + 1200*a*b*d**3*e**3*n*x
- 600*a*b*e**6*n*x**2 - 1140*b**2*d**3*e**3*n**2*x + 100*b**2*e**6*n**2*x*
*2)/(3600*e**6)
```

3.452 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 267

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=-\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3}+\frac{6b^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{b^2d^3n^2\log^2\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{2bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}+\frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2$$

output
```
-3/2*b^2*d*n^2*(d+e*x^(1/3))^2/e^3+2/9*b^2*n^2*(d+e*x^(1/3))^3/e^3+6*b^2*d
^2*n^2*x^(1/3)/e^2-b^2*d^3*n^2*ln(d+e*x^(1/3))^2/e^3-6*b*d^2*n*(d+e*x^(1/3
))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+3*b*d*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*
x^(1/3))^n))/e^3-2/3*b*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+2
*b*d^3*n*ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+x*(a+b*ln(c*(d+e*
x^(1/3))^n))^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.74

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{b^2en^2\left(66d^2-15de\sqrt[3]{x}+4e^2x^{2/3}\right)\sqrt[3]{x}+6abn\left(7d^3-6d^2e\sqrt[3]{x}+3de^2x^{2/3}-2e^3x\right)+18a^2\left(d^3+e^3x\right)+6b(\ldots}{\ldots}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

output
```
(b^2*e*n^2*(66*d^2 - 15*d*e*x^(1/3) + 4*e^2*x^(2/3))*x^(1/3) + 6*a*b*n*(7*
d^3 - 6*d^2*e*x^(1/3) + 3*d*e^2*x^(2/3) - 2*e^3*x) + 18*a^2*(d^3 + e^3*x)
+ 6*b*(6*a*(d^3 + e^3*x) - b*n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3)
 + 2*e^3*x))*Log[c*(d + e*x^(1/3))^n] + 18*b^2*(d^3 + e^3*x)*Log[c*(d + e*
x^(1/3))^n]^2)/(18*e^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.71, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2901, 2845, 2858, 25, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{2901}$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2845}$$

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{2}{3}ben\int\frac{x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d+e\sqrt[3]{x}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{2858}$$

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{2}{3}bn\int x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)d\left(d+e\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$3\left(\frac{2}{3}bn\int -x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)d\left(d+e\sqrt[3]{x}\right)+\frac{1}{3}x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{2bn\int -e^3x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)d\left(d+e\sqrt[3]{x}\right)}{3e^3}+\frac{1}{3}x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2772}$$

$$3\left(\frac{2bn\left(-bn\int\left(\frac{\log\left(d+e\sqrt[3]{x}\right)d^3}{\sqrt[3]{x}}-3d^2+\frac{3}{2}\left(d+e\sqrt[3]{x}\right)d-\frac{x^{2/3}}{3}\right)d\left(d+e\sqrt[3]{x}\right)+d^3\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right.\right.\right.}{3e^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{2bn\left(d^3\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)-3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)+\frac{3}{2}dx^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)-\frac{1}{3}\right.}{3e^3}\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

output
```
3*((x*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/3 + (2*b*n*(-(b*n*(-3*d^2*(d + e
*x^(1/3)) + (3*d*x^(2/3))/4 - x/9 + (d^3*Log[d + e*x^(1/3)]^2)/2)) - 3*d^2
*(d + e*x^(1/3))*(a + b*Log[c*x^(n/3)]) + (3*d*x^(2/3)*(a + b*Log[c*x^(n/3
)]))/2 - (x*(a + b*Log[c*x^(n/3)]))/3 + d^3*Log[d + e*x^(1/3)]*(a + b*Log[
c*x^(n/3)])))/(3*e^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.07

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}\,dx = \frac{18\,b^{2}e^{3}x\log\left(c\right)^{2}+18\,(b^{2}e^{3}n^{2}x+b^{2}d^{3}n^{2})\log\left(ex^{\frac{1}{3}}+d\right)^{2}-12\,(b^{2}e^{3}n-3\,abe^{3})x\log\left(c\right)+2\,(2\,b^{2}e^{3}n^{2}-6}{}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

output
```
1/18*(18*b^2*e^3*x*log(c)^2 + 18*(b^2*e^3*n^2*x + b^2*d^3*n^2)*log(e*x^(1/
3) + d)^2 - 12*(b^2*e^3*n - 3*a*b*e^3)*x*log(c) + 2*(2*b^2*e^3*n^2 - 6*a*b
*e^3*n + 9*a^2*e^3)*x + 6*(3*b^2*d*e^2*n^2*x^(2/3) - 6*b^2*d^2*e*n^2*x^(1/
3) - 11*b^2*d^3*n^2 + 6*a*b*d^3*n - 2*(b^2*e^3*n^2 - 3*a*b*e^3*n)*x + 6*(b
^2*e^3*n*x + b^2*d^3*n)*log(c))*log(e*x^(1/3) + d) - 3*(5*b^2*d*e^2*n^2 -
6*b^2*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(2/3) + 6*(11*b^2*d^2*e*n^2 - 6*b^
2*d^2*e*n*log(c) - 6*a*b*d^2*e*n)*x^(1/3))/e^3
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.81

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{1}{3}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)\right)ab$$
$$+\frac{1}{18}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+18\,x\log\left(\left(ex^{\frac{1}{3}}+d\right.\right.\right.$$
$$+a^2x$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

output
```
1/3*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*
x^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*a*b + 1/18*(6*e*n*(6*d^3*log
(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((
e*x^(1/3) + d)^n*c) + 18*x*log((e*x^(1/3) + d)^n*c)^2 - (18*d^3*log(e*x^(1
/3) + d)^2 - 4*e^3*x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d
^2*e*x^(1/3))*n^2/e^3)*b^2 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.74

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

output
```
1/18*(18*b^2*e*x*log(c)^2 + (18*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)^2/e^2
 - 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^2/e^2 + 54*(e*x^(1/3) + d)*d^
2*log(e*x^(1/3) + d)^2/e^2 - 12*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2 +
 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 - 108*(e*x^(1/3) + d)*d^2*l
og(e*x^(1/3) + d)/e^2 + 4*(e*x^(1/3) + d)^3/e^2 - 27*(e*x^(1/3) + d)^2*d/e
^2 + 108*(e*x^(1/3) + d)*d^2/e^2)*b^2*n^2 + 6*(6*(e*x^(1/3) + d)^3*log(e*x
^(1/3) + d)/e^2 - 18*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 + 18*(e*x^
(1/3) + d)*d^2*log(e*x^(1/3) + d)/e^2 - 2*(e*x^(1/3) + d)^3/e^2 + 9*(e*x^(
1/3) + d)^2*d/e^2 - 18*(e*x^(1/3) + d)*d^2/e^2)*b^2*n*log(c) + 36*a*b*e*x*
log(c) + 6*(6*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2 - 18*(e*x^(1/3) + d
)^2*d*log(e*x^(1/3) + d)/e^2 + 18*(e*x^(1/3) + d)*d^2*log(e*x^(1/3) + d)/e
^2 - 2*(e*x^(1/3) + d)^3/e^2 + 9*(e*x^(1/3) + d)^2*d/e^2 - 18*(e*x^(1/3) +
 d)*d^2/e^2)*a*b*n + 18*a^2*e*x)/e
```

Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.09

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)\left(\frac{2\,b\,x\,(3\,a-b\,n)}{3}-x^{2/3}\left(\frac{b\,d\,(3\,a-b\,n)}{e}-\frac{3\,a\,b\,d}{e}\right)+\frac{d\,x^{1/3}\left(\frac{2\,b\,d\,(3\,a-b\,n)}{e}-\frac{6\,a\,b\,d}{e}\right)}{e}\right)-x^{2/3}\left(\frac{d\left(3\,a^2-2\,a\,b\,n+\frac{2\,b^2\,n^2}{3}\right)}{2\,e}-\frac{d}{}\right.$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

output
```
log(c*(d + e*x^(1/3))^n)*((2*b*x*(3*a - b*n))/3 - x^(2/3)*((b*d*(3*a - b*n
))/e - (3*a*b*d)/e) + (d*x^(1/3)*((2*b*d*(3*a - b*n))/e - (6*a*b*d)/e))/e)
 - x^(2/3)*((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/(2*e) - (d*(3*a^2 - b^2*
n^2))/(2*e)) + x^(1/3)*((d*((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/e - (d*(
3*a^2 - b^2*n^2))/e))/e + (2*b^2*d^2*n^2)/e^2) + x*(a^2 + (2*b^2*n^2)/9 -
(2*a*b*n)/3) + log(c*(d + e*x^(1/3))^n)^2*(b^2*x + (b^2*d^3)/e^3) - (log(d
 + e*x^(1/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(3*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

$$=\frac{18x^{\frac{2}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^{n}c\right)b^2d\,e^2n+18x^{\frac{2}{3}}abd\,e^2n-15x^{\frac{2}{3}}b^2d\,e^2n^2-36x^{\frac{1}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^{n}c\right)b^2d^2en-36x}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))^2,x)
```

output
```
(18*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**2*d*e**2*n + 18*x**(2/3)*a*b*d*
e**2*n - 15*x**(2/3)*b**2*d*e**2*n**2 - 36*x**(1/3)*log((x**(1/3)*e + d)**
n*c)*b**2*d**2*e*n - 36*x**(1/3)*a*b*d**2*e*n + 66*x**(1/3)*b**2*d**2*e*n*
*2 + 18*log((x**(1/3)*e + d)**n*c)**2*b**2*d**3 + 18*log((x**(1/3)*e + d)*
*n*c)**2*b**2*e**3*x + 36*log((x**(1/3)*e + d)**n*c)*a*b*d**3 + 36*log((x*
*(1/3)*e + d)**n*c)*a*b*e**3*x - 66*log((x**(1/3)*e + d)**n*c)*b**2*d**3*n
 - 12*log((x**(1/3)*e + d)**n*c)*b**2*e**3*n*x + 18*a**2*e**3*x - 12*a*b*e
**3*n*x + 4*b**2*e**3*n**2*x)/(18*e**3)
```

3.453 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx = 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right) - 6b^2n^2\operatorname{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right)$$

output
```
3*(a+b*ln(c*(d+e*x^(1/3))^n))^2*ln(-e*x^(1/3)/d)+6*b*n*(a+b*ln(c*(d+e*x^(1
/3))^n))*polylog(2,1+e*x^(1/3)/d)-6*b^2*n^2*polylog(3,1+e*x^(1/3)/d)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. $2(93) = 186$.

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx = \left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log(x) + 2bn\left(a-bn\log\left(d+e\sqrt[3]{x}\right) + b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(\left(\log\left(d+e\sqrt[3]{x}\right) - \log\left(1+\frac{e\sqrt[3]{x}}{d}\right)\right)\log(x) - 3\,\text{PolyLog}\left(2,-\frac{e\sqrt[3]{x}}{d}\right)\right) + 3b^2n^2\left(\log^2\left(d+e\sqrt[3]{x}\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 2\log\left(d+e\sqrt[3]{x}\right)\text{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right) - 2\,\text{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right)\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```

output
```
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] + 2*b*n
*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*((Log[d + e*x^(
1/3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)]) +
 3*b^2*n^2*(Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/
3)]*PolyLog[2, 1 + (e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d])
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx$$

$\Big\downarrow$ 2904

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$\Big\downarrow$ 2843

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-2ben\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)}{d+e\sqrt[3]{x}}d\sqrt[3]{x}\right)$$

$\Big\downarrow$ 2881

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-2bn\int \frac{\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{\sqrt[3]{x}}d\left(d+e\sqrt[3]{x}\right)\right)$$

$\Big\downarrow$ 2821

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-2bn\left(bn\int \frac{\text{PolyLog}\left(2,\frac{d+e\sqrt[3]{x}}{d}\right)}{\sqrt[3]{x}}d\left(d+e\sqrt[3]{x}\right)-\text{PolyLog}\left(2,\frac{d+\right.\right.$$

$\Big\downarrow$ 7143

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-2bn\left(bn\,\text{PolyLog}\left(3,\frac{d+e\sqrt[3]{x}}{d}\right)-\text{PolyLog}\left(2,\frac{d+e\sqrt[3]{x}}{d}\right)\left(a+b\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```

output
```
3*((a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-((e*x^(1/3))/d)] - 2*b*n*(-((a
+ b*Log[c*x^(n/3)])*PolyLog[2, (d + e*x^(1/3))/d]) + b*n*PolyLog[3, (d + e
*x^(1/3))/d]))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c)
+ a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="maxima")
```

output
```
b^2*log((e*x^(1/3) + d)^n)^2*log(x) + integrate(1/3*(3*(b^2*e*log(c)^2 + 2
*a*b*e*log(c) + a^2*e)*x - 2*(b^2*e*n*x*log(x) - 3*(b^2*e*log(c) + a*b*e)*
x - 3*(b^2*d*log(c) + a*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 3*(b^2*d*lo
g(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^(2/3))/(e*x^2 + d*x^(5/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)\right)^2}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))^2/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^n))^2/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx$$
$$=\frac{\left(\int \frac{\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)^2}{x^{\frac{4}{3}}e+dx}dx\right)b^2dn+2\left(\int \frac{\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)}{x^{\frac{4}{3}}e+dx}dx\right)abdn+\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)^3b^2+3\log\left(\left(x^{\frac{1}{3}}e+d\right.\right.}{n}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x)
```

output
```
(int(log((x**(1/3)*e + d)**n*c)**2/(x**(1/3)*e*x + d*x),x)*b**2*d*n + 2*in
t(log((x**(1/3)*e + d)**n*c)/(x**(1/3)*e*x + d*x),x)*a*b*d*n + log((x**(1/
3)*e + d)**n*c)**3*b**2 + 3*log((x**(1/3)*e + d)**n*c)**2*a*b + log(x)*a**
2*n)/n
```

3.454 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 231

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx &= -\frac{b^2e^2n^2}{d^2\sqrt[3]{x}}+\frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}}\\&\quad+\frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}\\&\quad+\frac{2be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}\\&\quad-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}-\frac{b^2e^3n^2\log(x)}{d^3}-\frac{2b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}\end{aligned}$$

output

```
-b^2*e^2*n^2/d^2/x^(1/3)+b^2*e^3*n^2*ln(d+e*x^(1/3))/d^3-b*e*n*(a+b*ln(c*(
d+e*x^(1/3))^n))/d/x^(2/3)+2*b*e^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))
^n))/d^3/x^(1/3)+2*b*e^3*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n
))/d^3-(a+b*ln(c*(d+e*x^(1/3))^n))^2/x-b^2*e^3*n^2*ln(x)/d^3-2*b^2*e^3*n^2
*polylog(2,d/(d+e*x^(1/3)))/d^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx=-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}$$
$$-\frac{e\left(3bd^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-6bden\sqrt[3]{x}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+3e^2x^{2/3}\left(a+b\log\left(c\left(d+\right.\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

output
```
-((a + b*Log[c*(d + e*x^(1/3))^n])^2/x) - (e*(3*b*d^2*n*(a + b*Log[c*(d +
e*x^(1/3))^n]) - 6*b*d*e*n*x^(1/3)*(a + b*Log[c*(d + e*x^(1/3))^n]) + 3*e^
2*x^(2/3)*(a + b*Log[c*(d + e*x^(1/3))^n])^2 - 6*b*e^2*n*x^(2/3)*(a + b*Lo
g[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)] - 2*b^2*e^2*n^2*x^(2/3)*(3*L
og[d + e*x^(1/3)] - Log[x]) + b^2*e*n^2*x^(1/3)*(3*d - 3*e*x^(1/3)*Log[d +
 e*x^(1/3)] + e*x^(1/3)*Log[x]) - 6*b^2*e^2*n^2*x^(2/3)*PolyLog[2, 1 + (e*
x^(1/3))/d]))/(3*d^3*x^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 1.62 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2904, 2845, 2858, 25, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}dx$$

$$\Big\downarrow \text{ 2904}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2845}$$

$$3\left(\frac{2}{3}ben\int\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{\left(d+e\sqrt[3]{x}\right)x}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{3x}\right)$$

$$\Big\downarrow \text{2858}$$

$$3\left(\frac{2}{3}bn\int\frac{a+b\log\left(cx^{n/3}\right)}{x^{4/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{3x}\right)$$

$$\Big\downarrow \text{25}$$

$$3\left(-\frac{2}{3}bn\int-\frac{a+b\log\left(cx^{n/3}\right)}{x^{4/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{3x}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(-\frac{2}{3}be^3n\int-\frac{a+b\log\left(cx^{n/3}\right)}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{3x}\right)$$

$$\Big\downarrow \text{2789}$$

$$3\left(-\frac{2}{3}be^3n\left(\frac{\int-\frac{a+b\log\left(cx^{n/3}\right)}{e^3x}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{3x}\right)$$

$$\Big\downarrow \text{2756}$$

$$3\left(-\frac{2}{3}be^3n\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\int\frac{1}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3x}\right.$$

$$\Big\downarrow \text{54}$$

$$3\left(-\frac{2}{3}be^3n\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\int\left(-\frac{1}{d^2e\sqrt[3]{x}}+\frac{1}{d^2\sqrt[3]{x}}+\frac{1}{de^2x^{2/3}}\right)d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{2}{3}be^3n\left(\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^2}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^2}-\frac{1}{de\sqrt[3]{x}}\right)}{d}\right)-\frac{(a+b}{}\right.$$

$\Big\downarrow$ 2789

$$3\Bigg(-\frac{2}{3}be^3n\Bigg(\frac{\frac{\int \frac{a+b\log(cx^{n/3})}{e^2x^{2/3}}d(d+e\sqrt[3]{x})}{d}+\frac{\int -\frac{a+b\log(cx^{n/3})}{ex^{2/3}}d(d+e\sqrt[3]{x})}{d}}{d}+\frac{\frac{a+b\log(cx^{n/3})}{2e^2x^{2/3}}-\frac{1}{2}bn\Big(\frac{\log(d+e\sqrt[3]{x})}{d^2}-\frac{\log(-e\sqrt[3]{\cdots}}{d^2}\cdots}{d}$$

$\Big\downarrow$ 2751

$$3\Bigg(-\frac{2}{3}be^3n\Bigg(\frac{\frac{-\frac{bn\int -\frac{1}{e\sqrt[3]{x}}d(d+e\sqrt[3]{x})}{d}-\frac{(d+e\sqrt[3]{x})(a+b\log(cx^{n/3}))}{de\sqrt[3]{x}}}{d}+\frac{\int -\frac{a+b\log(cx^{n/3})}{ex^{2/3}}d(d+e\sqrt[3]{x})}{d}}{d}+\frac{\frac{a+b\log(cx^{n/3})}{2e^2x^{2/3}}-\frac{1}{2}bn\Big(\cdots}{\cdots}$$

$\Big\downarrow$ 16

$$3\Bigg(-\frac{2}{3}be^3n\Bigg(\frac{\frac{\int -\frac{a+b\log(cx^{n/3})}{ex^{2/3}}d(d+e\sqrt[3]{x})}{d}+\frac{\frac{bn\log(-e\sqrt[3]{x})}{d}-\frac{(d+e\sqrt[3]{x})(a+b\log(cx^{n/3}))}{de\sqrt[3]{x}}}{d}}{d}+\frac{\frac{a+b\log(cx^{n/3})}{2e^2x^{2/3}}-\frac{1}{2}bn\Big(\frac{\log(d+e\sqrt[3]{\cdots}}{d^2}\cdots}{d}$$

$\Big\downarrow$ 2779

$$3\Bigg(-\frac{2}{3}be^3n\Bigg(\frac{\frac{\frac{bn\int \frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)}{\sqrt[3]{x}}d(d+e\sqrt[3]{x})}{d}-\frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)(a+b\log(cx^{n/3}))}{d}}{d}+\frac{\frac{bn\log(-e\sqrt[3]{x})}{d}-\frac{(d+e\sqrt[3]{x})(a+b\log(cx^{n/3}))}{de\sqrt[3]{x}}}{d}}{d}+\frac{\frac{a+b\log\cdots}{2e\cdots}\cdots}{\cdots}$$

$\Big\downarrow$ 2838

$$3\Bigg(-\frac{2}{3}be^3n\Bigg(\frac{\frac{a+b\log(cx^{n/3})}{2e^2x^{2/3}}-\frac{1}{2}bn\left(\frac{\log(d+e\sqrt[3]{x})}{d^2}-\frac{\log(-e\sqrt[3]{x})}{d^2}-\frac{1}{de\sqrt[3]{x}}\right)}{d}+\frac{\frac{bn\log(-e\sqrt[3]{x})}{d}-\frac{(d+e\sqrt[3]{x})(a+b\log(cx^{n/3}))}{de\sqrt[3]{x}}}{d}-\cdots$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

output
```
3*(-1/3*(a + b*Log[c*(d + e*x^(1/3))^n])^2/x - (2*b*e^3*n*((-1/2*(b*n*(-(1
/(d*e*x^(1/3))) + Log[d + e*x^(1/3)]/d^2 - Log[-(e*x^(1/3))]/d^2)) + (a +
b*Log[c*x^(n/3)])/(2*e^2*x^(2/3)))/d + (((b*n*Log[-(e*x^(1/3))])/d - ((d +
 e*x^(1/3))*(a + b*Log[c*x^(n/3)]))/(d*e*x^(1/3)))/d + (-((Log[1 - d/x^(1/
3)]*(a + b*Log[c*x^(n/3)]))/d) + (b*n*PolyLog[2, d/x^(1/3)])/d)/d)/d))/3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c)
+ a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="maxima")
```

output
```
-2*(log(e*x^(1/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(1/3)/d))*b^2*e^3*n^2/d
^3 - (2*a*b*e^3*n - (3*e^3*n^2 - 2*e^3*n*log(c))*b^2)*log(e*x^(1/3) + d)/d
^3 + 2*(b^2*e^3*n*log(c) + a*b*e^3*n)*log(x^(1/3))/d^3 + integrate((b^2*e^
6*n^2*x - b^2*d^3*e^3*n^2)/x, x)/d^6 - 1/20*(12*b^2*e^8*n^2*x^(5/3) - 15*b
^2*d*e^7*n^2*x^(4/3) + 20*b^2*d^2*e^6*n^2*x - 40*b^2*d^3*e^5*n^2*x^(2/3) +
 100*b^2*d^4*e^4*n^2*x^(1/3) + 20*(b^2*d^3*e^5*n^2*x^(2/3) - 2*b^2*d^4*e^4
*n^2*x^(1/3))*log(x^(1/3)))/d^8 + 1/60*(60*b^2*d^5*e^3*n^2*x^(5/3)*log(e*x
^(1/3) + d)^2 - 45*b^2*d*e^7*n^2*x^3 - 40*b^2*d^4*e^4*n^2*x^2*log(x) + 300
*b^2*d^4*e^4*n^2*x^2 - 60*b^2*d^8*x^(2/3)*log((e*x^(1/3) + d)^n)^2 - 60*(b
^2*d^7*e*n*log(c) + a*b*d^7*e*n)*x - 20*(6*b^2*d^5*e^3*n*x^(5/3)*log(e*x^(
1/3) + d) - 6*b^2*d^6*e^2*n*x^(4/3) + 3*b^2*d^7*e*n*x - 2*(b^2*d^5*e^3*n*x
*log(x) - 3*b^2*d^8*log(c) - 3*a*b*d^8)*x^(2/3))*log((e*x^(1/3) + d)^n) -
60*(b^2*d^8*log(c)^2 + 2*a*b*d^8*log(c) + a^2*d^8)*x^(2/3) + 4*(9*b^2*e^8*
n^2*x^3 + 5*b^2*d^3*e^5*n^2*x^2*log(x) - 15*b^2*d^3*e^5*n^2*x^2 + 30*(b^2*
d^6*e^2*n*log(c) + a*b*d^6*e^2*n)*x)*x^(1/3) - 60*(b^2*d^3*e^5*n^2*x^3 + b
^2*d^6*e^2*n^2*x^2)/x^(2/3))/(d^8*x^(5/3))
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx$$

$$= \frac{6x^{\frac{2}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)b^2de^2n+6x^{\frac{2}{3}}abde^2n-3x^{\frac{2}{3}}b^2de^2n^2-3x^{\frac{1}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^nc\right)b^2d^2en-3x^{\frac{1}{3}}abd^2}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x)
```

output
```
(6*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**2*d*e**2*n + 6*x**(2/3)*a*b*d*e*
*2*n - 3*x**(2/3)*b**2*d*e**2*n**2 - 3*x**(1/3)*log((x**(1/3)*e + d)**n*c)
*b**2*d**2*e*n - 3*x**(1/3)*a*b*d**2*e*n + 2*int(log((x**(1/3)*e + d)**n*c
)/(x**(1/3)*e*x + d*x),x)*b**2*d*e**3*n*x + 6*log(x**(1/3))*a*b*e**3*n*x -
 9*log(x**(1/3))*b**2*e**3*n**2*x - 3*log((x**(1/3)*e + d)**n*c)**2*b**2*d
**3 - 6*log((x**(1/3)*e + d)**n*c)*a*b*d**3 - 6*log((x**(1/3)*e + d)**n*c)
*a*b*e**3*x + 9*log((x**(1/3)*e + d)**n*c)*b**2*e**3*n*x - 3*a**2*d**3)/(3
*d**3*x)
```

3.455 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 405

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx = -\frac{b^2e^2n^2}{20d^2x^{4/3}}+\frac{3b^2e^3n^2}{20d^3x}-\frac{47b^2e^4n^2}{120d^4x^{2/3}}+\frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}}-\frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}}-\frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3d^3x}+\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}}-\frac{be^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6\sqrt[3]{x}}-\frac{be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2}+\frac{137b^2e^6n^2\log(x)}{180d^6}+\frac{b^2e^6n^2\,\text{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^6}$$

output
```
-1/20*b^2*e^2*n^2/d^2/x^(4/3)+3/20*b^2*e^3*n^2/d^3/x-47/120*b^2*e^4*n^2/d^
4/x^(2/3)+77/60*b^2*e^5*n^2/d^5/x^(1/3)-77/60*b^2*e^6*n^2*ln(d+e*x^(1/3))/
d^6-1/5*b*e*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d/x^(5/3)+1/4*b*e^2*n*(a+b*ln(c*
(d+e*x^(1/3))^n))/d^2/x^(4/3)-1/3*b*e^3*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3/
x+1/2*b*e^4*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d^4/x^(2/3)-b*e^5*n*(d+e*x^(1/3)
)*(a+b*ln(c*(d+e*x^(1/3))^n))/d^6/x^(1/3)-b*e^6*n*ln(1-d/(d+e*x^(1/3)))*(a
+b*ln(c*(d+e*x^(1/3))^n))/d^6-1/2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2+137/18
0*b^2*e^6*n^2*ln(x)/d^6+b^2*e^6*n^2*polylog(2,d/(d+e*x^(1/3)))/d^6
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx = -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2}$$
$$-\frac{be\left(72ad^5n-90ad^4en\sqrt[3]{x}+18bd^4en^2\sqrt[3]{x}+120ad^3e^2nx^{2/3}-54bd^3e^2n^2x^{2/3}-180ad^2e^3nx+141bd^2e^3\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

output
```
-1/2*(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2 - (b*e*(72*a*d^5*n - 90*a*d^4*
e*n*x^(1/3) + 18*b*d^4*e*n^2*x^(1/3) + 120*a*d^3*e^2*n*x^(2/3) - 54*b*d^3*
e^2*n^2*x^(2/3) - 180*a*d^2*e^3*n*x + 141*b*d^2*e^3*n^2*x + 360*a*d*e^4*n*
x^(4/3) - 462*b*d*e^4*n^2*x^(4/3) + 6*e^5*n*(-60*a + 137*b*n)*x^(5/3)*Log[
d + e*x^(1/3)] + 72*b*d^5*n*Log[c*(d + e*x^(1/3))^n] - 90*b*d^4*e*n*x^(1/3
)*Log[c*(d + e*x^(1/3))^n] + 120*b*d^3*e^2*n*x^(2/3)*Log[c*(d + e*x^(1/3))
^n] - 180*b*d^2*e^3*n*x*Log[c*(d + e*x^(1/3))^n] + 360*b*d*e^4*n*x^(4/3)*L
og[c*(d + e*x^(1/3))^n] - 180*b*e^5*x^(5/3)*Log[c*(d + e*x^(1/3))^n]^2 + 3
60*b*e^5*n*x^(5/3)*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/d)] + 120*a*
e^5*n*x^(5/3)*Log[x] - 274*b*e^5*n^2*x^(5/3)*Log[x] + 360*b*e^5*n^2*x^(5/3
)*PolyLog[2, 1 + (e*x^(1/3))/d]))/(360*d^6*x^(5/3))
```

Rubi [A] (warning: unable to verify)

Time = 4.00 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.40, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.042$, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2845}$$

$$3\left(\frac{1}{3}ben\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{\left(d+e\sqrt[3]{x}\right)x^2}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{6x^2}\right)$$

$$\Big\downarrow \text{2858}$$

$$3\left(\frac{1}{3}bn\int \frac{a+b\log\left(cx^{n/3}\right)}{x^{7/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{6x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{1}{3}be^6n\int \frac{a+b\log\left(cx^{n/3}\right)}{e^6x^{7/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{6x^2}\right)$$

$$\Big\downarrow \text{2789}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^6x^2}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^5x^2}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{6x^2}\right)$$

$$\Big\downarrow \text{2756}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int -\frac{1}{e^5x^2}d\left(d+e\sqrt[3]{x}\right)-\frac{a+b\log\left(cx^{n/3}\right)}{5e^5x^{5/3}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^5x^2}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{6x^2}\right.$$

$$\Big\downarrow \text{54}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int\left(-\frac{1}{d^5e\sqrt[3]{x}}+\frac{1}{d^5\sqrt[3]{x}}+\frac{1}{d^4e^2x^{2/3}}-\frac{1}{d^3e^3x}+\frac{1}{d^2e^4x^{4/3}}-\frac{1}{de^5x^{5/3}}\right)d\left(d+e\sqrt[3]{x}\right)-\frac{a+b\log\left(cx^{n/3}\right)}{5e^5x^{5/3}}}{d}+\frac{\int}{}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^5x^2}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{5e^5x^{5/3}}-\frac{1}{5}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^5}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^5}-\frac{1}{d^4e\sqrt[3]{x}}+\frac{1}{2d^3e^2x^{2/}}\right.}{d}\right.\right.$$

$$\Big\downarrow \text{2789}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^5x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{5e^5x^{5/3}}-\frac{1}{5}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^5}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^5}\right.}{d}\right.\right.$$

$$\Big\downarrow \text{2756}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\int\frac{1}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{5e^5x^{5/3}}-\frac{1}{5}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^5}-\right.}{}\right.\right.$$

$$\Big\downarrow \text{54}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\int\left(-\frac{1}{d^4e\sqrt[3]{x}}+\frac{1}{d^4\sqrt[3]{x}}+\frac{1}{d^3e^2x^{2/3}}-\frac{1}{d^2e^3x}+\frac{1}{de^4x^{4/3}}\right)d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}}{d}+\right.\right.$$

$$\Big\downarrow \text{2789}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{4/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}\right.}{d}}{d}\right.\right.$$

$$\Big\downarrow \text{2756}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int -\frac{1}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\cdots\right)}{d\cdots}\cdots\right)}{d}\cdots}{d}\cdots\right.\right.$$

$$\Big\downarrow \text{ 54}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int\left(-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{d^3\sqrt[3]{x}}+\frac{1}{d^2e^2x^{2/3}}-\frac{1}{de^3x}\right)d\left(d+e\sqrt[3]{x}\right)-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}-\frac{1}{3}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^3}-\frac{1}{d^2e\sqrt[3]{x}}+\frac{1}{2de^2x^{2/3}}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$$\Big\downarrow \text{ 2789}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^3x}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}-\frac{1}{3}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^3}-\frac{1}{d^2e\sqrt[3]{x}}\cdots\right)}{d}}{d}\cdots}{d}\cdots\right.\right.$$

$$\Big\downarrow \text{ 2756}$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\int\frac{1}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}-\frac{1}{3}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^3}-\cdots\right)}{d}}{d}\cdots}{d}\cdots\right.\right.$$

$\Big\downarrow$ 54

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\int\left(-\frac{1}{d^2e\sqrt[3]{x}}+\frac{1}{d^2\sqrt[3]{x}}+\frac{1}{de^2x^{2/3}}\right)d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}-\frac{1}{3}bn\left(\frac{\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.$$

$\Big\downarrow$ 2009

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^2}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^2}-\frac{1}{de\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}-\frac{1}{3}bn\left(\frac{\log\left(d+e\cdots\right.}{d^3}\right.}{\cdots}}{d}\right.\right.$$

$\Big\downarrow$ 2789

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^2x^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int-\frac{a+b\log\left(cx^{n/3}\right)}{ex^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^2}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^2}-\frac{1}{de\sqrt[3]{x}}\right)}{d}}{d}+\cdots}{d}\right.\right.$$

$\Big\downarrow$ 2751

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{-\frac{bn\int-\frac{1}{e\sqrt[3]{x}}d\left(d+e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de\sqrt[3]{x}}}{d}}{d}+\frac{\int-\frac{a+b\log\left(cx^{n/3}\right)}{ex^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^2}\cdots\right.}{d}}{d}\right.\right.$$

$\Big\downarrow$ 16

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{ex^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\frac{bn\log\left(-e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de\sqrt[3]{x}}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^2}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^2}\cdots\right.}{d}}{d}}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2779$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{bn\int\frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)}{\sqrt[3]{x}}d\left(d+e\sqrt[3]{x}\right)}{d}-\frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de\sqrt[3]{x}}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2838$$

$$3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{a+b\log\left(cx^{n/3}\right)}{2e^2x^{2/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^2}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^2}-\frac{1}{de\sqrt[3]{x}}\right)}{d}+\frac{\frac{\frac{bn\log\left(-e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de\sqrt[3]{x}}}{d}+\frac{bn\,\mathrm{PolyLog}\left(2,\cdots\right)}{d}\cdots}{d}}{d}\cdots\right.\right.$$

input

```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

output
```
3*(-1/6*(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2 + (b*e^6*n*((-1/5*(b*n*(1/(
4*d*e^4*x^(4/3)) - 1/(3*d^2*e^3*x) + 1/(2*d^3*e^2*x^(2/3)) - 1/(d^4*e*x^(1
/3)) + Log[d + e*x^(1/3)]/d^5 - Log[-(e*x^(1/3))]/d^5)) - (a + b*Log[c*x^(
n/3)])/(5*e^5*x^(5/3)))/d + ((-1/4*(b*n*(-1/3*1/(d*e^3*x) + 1/(2*d^2*e^2*x
^(2/3)) - 1/(d^3*e*x^(1/3)) + Log[d + e*x^(1/3)]/d^4 - Log[-(e*x^(1/3))]/d
^4)) + (a + b*Log[c*x^(n/3)])/(4*e^4*x^(4/3)))/d + ((-1/3*(b*n*(1/(2*d*e^2
*x^(2/3)) - 1/(d^2*e*x^(1/3)) + Log[d + e*x^(1/3)]/d^3 - Log[-(e*x^(1/3))]
/d^3)) - (a + b*Log[c*x^(n/3)])/(3*e^3*x))/d + ((-1/2*(b*n*(-(1/(d*e*x^(1/
3))) + Log[d + e*x^(1/3)]/d^2 - Log[-(e*x^(1/3))]/d^2)) + (a + b*Log[c*x^(
n/3)])/(2*e^2*x^(2/3)))/d + (((b*n*Log[-(e*x^(1/3))])/d - ((d + e*x^(1/3))
*(a + b*Log[c*x^(n/3)]))/(d*e*x^(1/3)))/d + (-((Log[1 - d/x^(1/3)]*(a + b*
Log[c*x^(n/3)]))/d) + (b*n*PolyLog[2, d/x^(1/3)])/d)/d)/d)/d)/d)/d))/3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{3}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c)
+ a^2)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**3,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*b^2*log((e*x^(1/3) + d)^n)^2/x^2 + integrate(1/3*(3*(b^2*e*log(c)^2 +
 2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(
b^2*d*log(c) + a*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 3*(b^2*d*log(c)^2
+ 2*a*b*d*log(c) + a^2*d)*x^(2/3))/(e*x^4 + d*x^(11/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)\right)^2}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^3,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx$$
$$=\frac{30x^{\frac{2}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b^2d^4e^2n-120x^{\frac{5}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b^2d\,e^5n+30x^{\frac{2}{3}}ab\,d^4e^2n-120x^{\frac{5}{3}}abd\,e^5n-}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x)
```

output
```
(30*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**2*d**4*e**2*n - 120*x**(2/3)*lo
g((x**(1/3)*e + d)**n*c)*b**2*d*e**5*n*x + 30*x**(2/3)*a*b*d**4*e**2*n - 1
20*x**(2/3)*a*b*d*e**5*n*x - 6*x**(2/3)*b**2*d**4*e**2*n**2 + 154*x**(2/3)
*b**2*d*e**5*n**2*x - 24*x**(1/3)*log((x**(1/3)*e + d)**n*c)*b**2*d**5*e*n
 + 60*x**(1/3)*log((x**(1/3)*e + d)**n*c)*b**2*d**2*e**4*n*x - 24*x**(1/3)
*a*b*d**5*e*n + 60*x**(1/3)*a*b*d**2*e**4*n*x - 47*x**(1/3)*b**2*d**2*e**4
*n**2*x - 40*int(log((x**(1/3)*e + d)**n*c)/(x**(1/3)*e*x + d*x),x)*b**2*d
*e**6*n*x**2 - 120*log(x**(1/3))*a*b*e**6*n*x**2 + 274*log(x**(1/3))*b**2*
e**6*n**2*x**2 - 60*log((x**(1/3)*e + d)**n*c)**2*b**2*d**6 - 120*log((x**
(1/3)*e + d)**n*c)*a*b*d**6 + 120*log((x**(1/3)*e + d)**n*c)*a*b*e**6*x**2
 - 40*log((x**(1/3)*e + d)**n*c)*b**2*d**3*e**3*n*x - 274*log((x**(1/3)*e
+ d)**n*c)*b**2*e**6*n*x**2 - 60*a**2*d**6 - 40*a*b*d**3*e**3*n*x + 18*b**
2*d**3*e**3*n**2*x)/(120*d**6*x**2)
```

3.456 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 1835

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

output
```
-1188/49*b^2*d^5*n^2*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+1485
/128*b^2*d^4*n^2*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-110/27*b
^2*d^3*n^2*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+99/100*b^2*d^2
*n^2*(d+e*x^(1/3))^10*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-18/121*b^2*d*n^2*(d
+e*x^(1/3))^11*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+9*b*d^11*n*(d+e*x^(1/3))*(
a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-99/4*b*d^10*n*(d+e*x^(1/3))^2*(a+b*ln(c*
(d+e*x^(1/3))^n))^2/e^12+55*b*d^9*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3)
)^n))^2/e^12-1485/16*b*d^8*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2
/e^12+594/5*b*d^7*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-231
/2*b*d^6*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+594/7*b*d^5*
n*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-1485/32*b*d^4*n*(d+e*
x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+55/3*b*d^3*n*(d+e*x^(1/3))^9
*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-99/20*b*d^2*n*(d+e*x^(1/3))^10*(a+b*ln
(c*(d+e*x^(1/3))^n))^2/e^12+9/11*b*d*n*(d+e*x^(1/3))^11*(a+b*ln(c*(d+e*x^(
1/3))^n))^2/e^12-18*b^3*d^11*n^2*(d+e*x^(1/3))*ln(c*(d+e*x^(1/3))^n)/e^12-
18*a*b^2*d^11*n^2*x^(1/3)/e^11+99/4*b^2*d^10*n^2*(d+e*x^(1/3))^2*(a+b*ln(c
*(d+e*x^(1/3))^n))/e^12-110/3*b^2*d^9*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x
^(1/3))^n))/e^12+1485/32*b^2*d^8*n^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3
))^n))/e^12-1188/25*b^2*d^7*n^2*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n)
)/e^12+77/2*b^2*d^6*n^2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^1...
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 1025, normalized size of antiderivative = 0.56

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

output
```
(e*x^(1/3)*(3550000608000*a^3*e^11*x^(11/3) + b^3*n^3*(119225632485960*d^1
1 - 26563616859780*d^10*e*x^(1/3) + 10242678720120*d^9*e^2*x^(2/3) - 48363
09598890*d^8*e^3*x + 2516628075192*d^7*e^4*x^(4/3) - 1373077023780*d^6*e^5
*x^(5/3) + 761128152840*d^5*e^6*x^2 - 417533743935*d^4*e^7*x^(7/3) + 22016
1492320*d^3*e^8*x^(8/3) - 106944990768*d^2*e^9*x^3 + 44119404000*d*e^10*x^
(10/3) - 12326391000*e^11*x^(11/3)) - 27720*a*b^2*n^2*(2384502120*d^11 - 8
08051860*d^10*e*x^(1/3) + 410634840*d^9*e^2*x^(2/3) - 243942930*d^8*e^3*x
+ 156734424*d^7*e^4*x^(4/3) - 104998740*d^6*e^5*x^(5/3) + 71703720*d^5*e^6
*x^2 - 49019355*d^4*e^7*x^(7/3) + 32900560*d^3*e^8*x^(8/3) - 21072744*d^2*
e^9*x^3 + 12171600*d*e^10*x^(10/3) - 5336100*e^11*x^(11/3)) + 384199200*a^
2*b*n*(27720*d^11 - 13860*d^10*e*x^(1/3) + 9240*d^9*e^2*x^(2/3) - 6930*d^8
*e^3*x + 5544*d^7*e^4*x^(4/3) - 4620*d^6*e^5*x^(5/3) + 3960*d^5*e^6*x^2 -
3465*d^4*e^7*x^(7/3) + 3080*d^3*e^8*x^(8/3) - 2772*d^2*e^9*x^3 + 2520*d*e^
10*x^(10/3) - 2310*e^11*x^(11/3))) - 27720*b*d^12*n*(384199200*a^2 - 23845
02120*a*b*n + 4301068993*b^2*n^2)*Log[d + e*x^(1/3)] + 27720*b*e*x^(1/3)*(
384199200*a^2*e^11*x^(11/3) + 27720*a*b*n*(27720*d^11 - 13860*d^10*e*x^(1/
3) + 9240*d^9*e^2*x^(2/3) - 6930*d^8*e^3*x + 5544*d^7*e^4*x^(4/3) - 4620*d
^6*e^5*x^(5/3) + 3960*d^5*e^6*x^2 - 3465*d^4*e^7*x^(7/3) + 3080*d^3*e^8*x^
(8/3) - 2772*d^2*e^9*x^3 + 2520*d*e^10*x^(10/3) - 2310*e^11*x^(11/3)) + b^
2*n^2*(-2384502120*d^11 + 808051860*d^10*e*x^(1/3) - 410634840*d^9*e^2*...
```

Rubi [A] (verified)

Time = 4.64 (sec) , antiderivative size = 1843, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{11/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int\left(-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3d^{11}}{e^{11}}+\frac{11\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3d^{10}}{e^{11}}-\frac{55\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\right.}{e}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^{12}}{3456e^{12}}+\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^{12}}{12e^{12}}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^{12}}{48e^{12}}\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

output
```
3*((-33*b^3*d^10*n^3*(d + e*x^(1/3))^2)/(8*e^12) + (110*b^3*d^9*n^3*(d + e
*x^(1/3))^3)/(27*e^12) - (495*b^3*d^8*n^3*(d + e*x^(1/3))^4)/(128*e^12) +
(396*b^3*d^7*n^3*(d + e*x^(1/3))^5)/(125*e^12) - (77*b^3*d^6*n^3*(d + e*x^
(1/3))^6)/(36*e^12) + (396*b^3*d^5*n^3*(d + e*x^(1/3))^7)/(343*e^12) - (49
5*b^3*d^4*n^3*(d + e*x^(1/3))^8)/(1024*e^12) + (110*b^3*d^3*n^3*(d + e*x^(
1/3))^9)/(729*e^12) - (33*b^3*d^2*n^3*(d + e*x^(1/3))^10)/(1000*e^12) + (6
*b^3*d*n^3*(d + e*x^(1/3))^11)/(1331*e^12) - (b^3*n^3*(d + e*x^(1/3))^12)/
(3456*e^12) - (6*a*b^2*d^11*n^2*x^(1/3))/e^11 + (6*b^3*d^11*n^3*x^(1/3))/e
^11 - (6*b^3*d^11*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^12 + (33
*b^2*d^10*n^2*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^12)
 - (110*b^2*d^9*n^2*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(9
*e^12) + (495*b^2*d^8*n^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n
]))/(32*e^12) - (396*b^2*d^7*n^2*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(
1/3))^n]))/(25*e^12) + (77*b^2*d^6*n^2*(d + e*x^(1/3))^6*(a + b*Log[c*(d +
 e*x^(1/3))^n]))/(6*e^12) - (396*b^2*d^5*n^2*(d + e*x^(1/3))^7*(a + b*Log[
c*(d + e*x^(1/3))^n]))/(49*e^12) + (495*b^2*d^4*n^2*(d + e*x^(1/3))^8*(a +
 b*Log[c*(d + e*x^(1/3))^n]))/(128*e^12) - (110*b^2*d^3*n^2*(d + e*x^(1/3)
)^9*(a + b*Log[c*(d + e*x^(1/3))^n]))/(81*e^12) + (33*b^2*d^2*n^2*(d + e*x
^(1/3))^10*(a + b*Log[c*(d + e*x^(1/3))^n]))/(100*e^12) - (6*b^2*d*n^2*(d
+ e*x^(1/3))^11*(a + b*Log[c*(d + e*x^(1/3))^n]))/(121*e^12) + (b^2*n^2...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 2183, normalized size of antiderivative = 1.19

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

output
```
1/14200002432000*(3550000608000*b^3*e^12*x^4*log(c)^3 - 12326391000*(b^3*e
^12*n^3 - 12*a*b^2*e^12*n^2 + 72*a^2*b*e^12*n - 288*a^3*e^12)*x^4 + 603680
*(364699*b^3*d^3*e^9*n^3 - 1510740*a*b^2*d^3*e^9*n^2 + 1960200*a^2*b*d^3*e
^9*n)*x^3 + 3550000608000*(b^3*e^12*n^3*x^4 - b^3*d^12*n^3)*log(e*x^(1/3)
+ d)^3 - 4620*(297202819*b^3*d^6*e^6*n^3 - 629992440*a*b^2*d^6*e^6*n^2 + 3
84199200*a^2*b*d^6*e^6*n)*x^2 + 384199200*(3080*b^3*d^3*e^9*n^3*x^3 - 4620
*b^3*d^6*e^6*n^3*x^2 + 9240*b^3*d^9*e^3*n^3*x + 86021*b^3*d^12*n^3 - 27720
*a*b^2*d^12*n^2 - 2310*(b^3*e^12*n^3 - 12*a*b^2*e^12*n^2)*x^4 + 27720*(b^3
*e^12*n^2*x^4 - b^3*d^12*n^2)*log(c) + 63*(40*b^3*d*e^11*n^3*x^3 - 55*b^3*
d^4*e^8*n^3*x^2 + 88*b^3*d^7*e^5*n^3*x - 220*b^3*d^10*e^2*n^3)*x^(2/3) - 1
98*(14*b^3*d^2*e^10*n^3*x^3 - 20*b^3*d^5*e^7*n^3*x^2 + 35*b^3*d^8*e^4*n^3*
x - 140*b^3*d^11*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 295833384000*(4*b^
3*d^3*e^9*n*x^3 - 6*b^3*d^6*e^6*n*x^2 + 12*b^3*d^9*e^3*n*x - 3*(b^3*e^12*n
 - 12*a*b^2*e^12)*x^4)*log(c)^2 + 9240*(1108515013*b^3*d^9*e^3*n^3 - 12319
04520*a*b^2*d^9*e^3*n^2 + 384199200*a^2*b*d^9*e^3*n)*x - 27720*(4301068993
*b^3*d^12*n^3 - 2384502120*a*b^2*d^12*n^2 + 384199200*a^2*b*d^12*n - 53361
00*(b^3*e^12*n^3 - 12*a*b^2*e^12*n^2 + 72*a^2*b*e^12*n)*x^4 + 43120*(763*b
^3*d^3*e^9*n^3 - 1980*a*b^2*d^3*e^9*n^2)*x^3 - 4620*(22727*b^3*d^6*e^6*n^3
 - 27720*a*b^2*d^6*e^6*n^2)*x^2 - 384199200*(b^3*e^12*n*x^4 - b^3*d^12*n)*
log(c)^2 + 9240*(44441*b^3*d^9*e^3*n^3 - 27720*a*b^2*d^9*e^3*n^2)*x - 2...
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1064, normalized size of antiderivative = 0.58

$$\int x^3 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4*log((e*x^(1/3) + d)^n*c)^3 + 3/4*a*b^2*x^4*log((e*x^(1/3) + d)
^n*c)^2 + 3/4*a^2*b*x^4*log((e*x^(1/3) + d)^n*c) + 1/4*a^3*x^4 - 1/36960*a
^2*b*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^13 + (2310*e^11*x^4 - 2520*d*e^1
0*x^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^3*e^8*x^3 + 3465*d^4*e^7*x^(8/
3) - 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2 - 5544*d^7*e^4*x^(5/3) + 6930
*d^8*e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10*e*x^(2/3) - 27720*d^11*x^(1
/3))/e^12) - 1/512265600*(27720*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^13 +
(2310*e^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^3*e
^8*x^3 + 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2 -
5544*d^7*e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10*
e*x^(2/3) - 27720*d^11*x^(1/3))/e^12)*log((e*x^(1/3) + d)^n*c) - (5336100*
e^12*x^4 - 12171600*d*e^11*x^(11/3) + 21072744*d^2*e^10*x^(10/3) - 3290056
0*d^3*e^9*x^3 + 49019355*d^4*e^8*x^(8/3) - 71703720*d^5*e^7*x^(7/3) + 1049
98740*d^6*e^6*x^2 + 384199200*d^12*log(e*x^(1/3) + d)^2 - 156734424*d^7*e^
5*x^(5/3) + 243942930*d^8*e^4*x^(4/3) - 410634840*d^9*e^3*x + 2384502120*d
^12*log(e*x^(1/3) + d) + 808051860*d^10*e^2*x^(2/3) - 2384502120*d^11*e*x^
(1/3))*n^2/e^12)*a*b^2 - 1/14200002432000*(384199200*e*n*(27720*d^12*log(e
*x^(1/3) + d)/e^13 + (2310*e^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2*e^9*
x^(10/3) - 3080*d^3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7/3)
+ 4620*d^6*e^5*x^2 - 5544*d^7*e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4320 vs. 2(1591) = 3182.

Time = 0.19 (sec) , antiderivative size = 4320, normalized size of antiderivative = 2.35

$$\int x^3 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

output
```
1/14200002432000*(3550000608000*b^3*e*x^4*log(c)^3 + 10650001824000*a*b^2*
e*x^4*log(c)^2 + 10650001824000*a^2*b*e*x^4*log(c) + 3550000608000*a^3*e*x
^4 + (3550000608000*(e*x^(1/3) + d)^12*log(e*x^(1/3) + d)^3/e^11 - 4260000
7296000*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)^3/e^11 + 234300040128000*(
e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)^3/e^11 - 781000133760000*(e*x^(1/
3) + d)^9*d^3*log(e*x^(1/3) + d)^3/e^11 + 1757250300960000*(e*x^(1/3) + d)
^8*d^4*log(e*x^(1/3) + d)^3/e^11 - 2811600481536000*(e*x^(1/3) + d)^7*d^5*
log(e*x^(1/3) + d)^3/e^11 + 3280200561792000*(e*x^(1/3) + d)^6*d^6*log(e*x
^(1/3) + d)^3/e^11 - 2811600481536000*(e*x^(1/3) + d)^5*d^7*log(e*x^(1/3)
+ d)^3/e^11 + 1757250300960000*(e*x^(1/3) + d)^4*d^8*log(e*x^(1/3) + d)^3/
e^11 - 781000133760000*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)^3/e^11 + 2
34300040128000*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)^3/e^11 - 42600007
296000*(e*x^(1/3) + d)*d^11*log(e*x^(1/3) + d)^3/e^11 - 887500152000*(e*x^
(1/3) + d)^12*log(e*x^(1/3) + d)^2/e^11 + 11618183808000*(e*x^(1/3) + d)^1
1*d*log(e*x^(1/3) + d)^2/e^11 - 70290012038400*(e*x^(1/3) + d)^10*d^2*log(
e*x^(1/3) + d)^2/e^11 + 260333377920000*(e*x^(1/3) + d)^9*d^3*log(e*x^(1/3
) + d)^2/e^11 - 658968862860000*(e*x^(1/3) + d)^8*d^4*log(e*x^(1/3) + d)^2
/e^11 + 1204971634944000*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)^2/e^11 -
 1640100280896000*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)^2/e^11 + 168696
0288921600*(e*x^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)^2/e^11 - 1317937725...
```

Mupad [B] (verification not implemented)

Time = 21.28 (sec) , antiderivative size = 1802, normalized size of antiderivative = 0.98

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

output
```
(a^3*x^4)/4 + (b^3*x^4*log(c*(d + e*x^(1/3))^n)^3)/4 - (b^3*n^3*x^4)/1152
+ (3*a*b^2*x^4*log(c*(d + e*x^(1/3))^n)^2)/4 - (b^3*n*x^4*log(c*(d + e*x^(
1/3))^n)^2)/16 + (b^3*n^2*x^4*log(c*(d + e*x^(1/3))^n))/96 + (a*b^2*n^2*x^
4)/96 - (b^3*d^12*log(c*(d + e*x^(1/3))^n)^3)/(4*e^12) + (3*a^2*b*x^4*log(
c*(d + e*x^(1/3))^n))/4 - (a^2*b*n*x^4)/16 - (a*b^2*n*x^4*log(c*(d + e*x^(
1/3))^n))/8 - (4301068993*b^3*d^12*n^3*log(d + e*x^(1/3)))/(512265600*e^12
) + (364699*b^3*d^3*n^3*x^3)/(23522400*e^3) - (297202819*b^3*d^6*n^3*x^2)/
(3073593600*e^6) - (21871*b^3*d^2*n^3*x^(10/3))/(2904000*e^2) - (2459191*b
^3*d^4*n^3*x^(8/3))/(83635200*e^4) + (192204079*b^3*d^5*n^3*x^(7/3))/(3585
859200*e^5) + (453937243*b^3*d^7*n^3*x^(5/3))/(2561328000*e^7) - (69788017
3*b^3*d^8*n^3*x^(4/3))/(2049062400*e^8) - (1916566873*b^3*d^10*n^3*x^(2/3)
)/(1024531200*e^10) + (4301068993*b^3*d^11*n^3*x^(1/3))/(512265600*e^11) -
 (3*a*b^2*d^12*log(c*(d + e*x^(1/3))^n)^2)/(4*e^12) + (86021*b^3*d^12*n*lo
g(c*(d + e*x^(1/3))^n)^2)/(36960*e^12) + (397*b^3*d*n^3*x^(11/3))/(127776*
e) + (1108515013*b^3*d^9*n^3*x)/(1536796800*e^9) - (3*a^2*b*d^12*n*log(d +
 e*x^(1/3)))/(4*e^12) + (3*b^3*d*n*x^(11/3)*log(c*(d + e*x^(1/3))^n)^2)/(4
4*e) - (23*b^3*d*n^2*x^(11/3)*log(c*(d + e*x^(1/3))^n))/(968*e) + (b^3*d^9
*n*x*log(c*(d + e*x^(1/3))^n)^2)/(4*e^9) - (44441*b^3*d^9*n^2*x*log(c*(d +
 e*x^(1/3))^n))/(55440*e^9) + (a^2*b*d^3*n*x^3)/(12*e^3) - (a^2*b*d^6*n*x^
2)/(8*e^6) - (23*a*b^2*d*n^2*x^(11/3))/(968*e) - (3*a^2*b*d^2*n*x^(10/3...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1816, normalized size of antiderivative = 0.99

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x)
```

output
```
( - 5325000912000*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**10*e**2*n
 + 2130000364800*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**7*e**5*n*x
 - 1331250228000*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**4*e**8*n*x
**2 + 968181984000*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d*e**11*n*x
**3 - 10650001824000*x**(2/3)*log((x**(1/3)*e + d)**n*c)*a*b**2*d**10*e**2
*n + 4260000729600*x**(2/3)*log((x**(1/3)*e + d)**n*c)*a*b**2*d**7*e**5*n*
x - 2662500456000*x**(2/3)*log((x**(1/3)*e + d)**n*c)*a*b**2*d**4*e**8*n*x
**2 + 1936363968000*x**(2/3)*log((x**(1/3)*e + d)**n*c)*a*b**2*d*e**11*n*x
**3 + 22399197559200*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**3*d**10*e**2*n
**2 - 4344678233280*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**3*d**7*e**5*n**
2*x + 1358816520600*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**3*d**4*e**8*n**
2*x**2 - 337396752000*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**3*d*e**11*n**
2*x**3 - 5325000912000*x**(2/3)*a**2*b*d**10*e**2*n + 2130000364800*x**(2/
3)*a**2*b*d**7*e**5*n*x - 1331250228000*x**(2/3)*a**2*b*d**4*e**8*n*x**2 +
 968181984000*x**(2/3)*a**2*b*d*e**11*n*x**3 + 22399197559200*x**(2/3)*a*b
**2*d**10*e**2*n**2 - 4344678233280*x**(2/3)*a*b**2*d**7*e**5*n**2*x + 135
8816520600*x**(2/3)*a*b**2*d**4*e**8*n**2*x**2 - 337396752000*x**(2/3)*a*b
**2*d*e**11*n**2*x**3 - 26563616859780*x**(2/3)*b**3*d**10*e**2*n**3 + 251
6628075192*x**(2/3)*b**3*d**7*e**5*n**3*x - 417533743935*x**(2/3)*b**3*d**
4*e**8*n**3*x**2 + 44119404000*x**(2/3)*b**3*d*e**11*n**3*x**3 + 106500...
```

3.457 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1357

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

output
```
18*a*b^2*d^8*n^2*x^(1/3)/e^8+18*b^3*d^8*n^2*(d+e*x^(1/3))*ln(c*(d+e*x^(1/3
))^n)/e^9-18*b^2*d^7*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+5
6/3*b^2*d^6*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-63/4*b^2*d
^5*n^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+252/25*b^2*d^4*n^2*
(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-14/3*b^2*d^3*n^2*(d+e*x^(1
/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+72/49*b^2*d^2*n^2*(d+e*x^(1/3))^7*(
a+b*ln(c*(d+e*x^(1/3))^n))/e^9-9/32*b^2*d*n^2*(d+e*x^(1/3))^8*(a+b*ln(c*(d
+e*x^(1/3))^n))/e^9-9*b*d^8*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/
e^9+18*b*d^7*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9-28*b*d^6*
n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+63/2*b*d^5*n*(d+e*x^(1
/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9-126/5*b*d^4*n*(d+e*x^(1/3))^5*(a+
b*ln(c*(d+e*x^(1/3))^n))^2/e^9+14*b*d^3*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x
^(1/3))^n))^2/e^9-36/7*b*d^2*n*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))
^2/e^9+9/8*b*d*n*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+1/3*(d+
e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+9*b^3*d^7*n^3*(d+e*x^(1/3))
^2/e^9-56/9*b^3*d^6*n^3*(d+e*x^(1/3))^3/e^9+63/16*b^3*d^5*n^3*(d+e*x^(1/3)
)^4/e^9-252/125*b^3*d^4*n^3*(d+e*x^(1/3))^5/e^9+7/9*b^3*d^3*n^3*(d+e*x^(1/
3))^6/e^9-72/343*b^3*d^2*n^3*(d+e*x^(1/3))^7/e^9+9/256*b^3*d*n^3*(d+e*x^(1
/3))^8/e^9-18*b^3*d^8*n^3*x^(1/3)/e^8+2/81*b^2*n^2*(d+e*x^(1/3))^9*(a+b*ln
(c*(d+e*x^(1/3))^n))/e^9-1/9*b*n*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3)...
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 808, normalized size of antiderivative = 0.60

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = \frac{b^3en^3\sqrt[3]{x}\left(-76356985320d^8+15542491860d^7e\sqrt[3]{x}-5483495640d^6e^2x^{2/3}+2340330930d^5e^3x-1075607\right.}{}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

output
```
(b^3*e*n^3*x^(1/3)*(-76356985320*d^8 + 15542491860*d^7*e*x^(1/3) - 5483495
640*d^6*e^2*x^(2/3) + 2340330930*d^5*e^3*x - 1075607064*d^4*e^4*x^(4/3) +
498592500*d^3*e^5*x^(5/3) - 219465000*d^2*e^6*x^2 + 83734875*d*e^7*x^(7/3)
 - 21952000*e^8*x^(8/3)) - 2520*a*b^2*n^2*(26853209*d^9 - 17965080*d^8*e*x
^(1/3) + 5807340*d^7*e^2*x^(2/3) - 2813160*d^6*e^3*x + 1580670*d^5*e^4*x^(
4/3) - 947016*d^4*e^5*x^(5/3) + 577500*d^3*e^6*x^2 - 343800*d^2*e^7*x^(7/3
) + 187425*d*e^8*x^(8/3) - 78400*e^9*x^3) + 2667168000*a^3*(d^9 + e^9*x^3)
 - 3175200*a^2*b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) +
 840*d^6*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x
^2 + 360*d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3) + 2520*b*(3175
200*a^2*(d^9 + e^9*x^3) - 2520*a*b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260
*d^7*e^2*x^(2/3) + 840*d^6*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/
3) - 420*d^3*e^6*x^2 + 360*d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x
^3) + b^2*n^2*(30300391*d^9 + 17965080*d^8*e*x^(1/3) - 5807340*d^7*e^2*x^(
2/3) + 2813160*d^6*e^3*x - 1580670*d^5*e^4*x^(4/3) + 947016*d^4*e^5*x^(5/3
) - 577500*d^3*e^6*x^2 + 343800*d^2*e^7*x^(7/3) - 187425*d*e^8*x^(8/3) + 7
8400*e^9*x^3))*Log[c*(d + e*x^(1/3))^n] + 3175200*b^2*(2520*a*(d^9 + e^9*x
^3) - b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) + 840*d^6*
e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x^2 + 360*
d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3))*Log[c*(d + e*x^(1/3...
```

Rubi [A] (verified)

Time = 3.20 (sec) , antiderivative size = 1366, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 d^8}{e^8}-\frac{8\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 d^7}{e^8}+\frac{28\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\right.\right.}{e^8}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^9}{2187e^9}+\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^9}{9e^9}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^9}{27e^9}\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

output
```
3*((3*b^3*d^7*n^3*(d + e*x^(1/3))^2)/e^9 - (56*b^3*d^6*n^3*(d + e*x^(1/3))
^3)/(27*e^9) + (21*b^3*d^5*n^3*(d + e*x^(1/3))^4)/(16*e^9) - (84*b^3*d^4*n
^3*(d + e*x^(1/3))^5)/(125*e^9) + (7*b^3*d^3*n^3*(d + e*x^(1/3))^6)/(27*e^
9) - (24*b^3*d^2*n^3*(d + e*x^(1/3))^7)/(343*e^9) + (3*b^3*d*n^3*(d + e*x^
(1/3))^8)/(256*e^9) - (2*b^3*n^3*(d + e*x^(1/3))^9)/(2187*e^9) + (6*a*b^2*
d^8*n^2*x^(1/3))/e^8 - (6*b^3*d^8*n^3*x^(1/3))/e^8 + (6*b^3*d^8*n^2*(d + e
*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^9 - (6*b^2*d^7*n^2*(d + e*x^(1/3))^2
*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3
*(a + b*Log[c*(d + e*x^(1/3))^n]))/(9*e^9) - (21*b^2*d^5*n^2*(d + e*x^(1/3
))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^9) + (84*b^2*d^4*n^2*(d + e*x^
(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^9) - (14*b^2*d^3*n^2*(d +
 e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(9*e^9) + (24*b^2*d^2*n^2*
(d + e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n]))/(49*e^9) - (3*b^2*d*n^
2*(d + e*x^(1/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32*e^9) + (2*b^2*n^
2*(d + e*x^(1/3))^9*(a + b*Log[c*(d + e*x^(1/3))^n]))/(243*e^9) - (3*b*d^8
*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (6*b*d^7*n*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (28*b*d^6*n*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(3*e^9) + (21*b*d^5*n*(d
 + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^9) - (42*b*d^4*n*
(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^9) + (14*b*d...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)^3dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1688, normalized size of antiderivative = 1.24

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

output
```
1/8001504000*(2667168000*b^3*e^9*x^3*log(c)^3 - 10976000*(2*b^3*e^9*n^3 -
18*a*b^2*e^9*n^2 + 81*a^2*b*e^9*n - 243*a^3*e^9)*x^3 + 2667168000*(b^3*e^9
*n^3*x^3 + b^3*d^9*n^3)*log(e*x^(1/3) + d)^3 + 10500*(47485*b^3*d^3*e^6*n^
3 - 138600*a*b^2*d^3*e^6*n^2 + 127008*a^2*b*d^3*e^6*n)*x^2 + 3175200*(420*
b^3*d^3*e^6*n^3*x^2 - 840*b^3*d^6*e^3*n^3*x - 7129*b^3*d^9*n^3 + 2520*a*b^
2*d^9*n^2 - 280*(b^3*e^9*n^3 - 9*a*b^2*e^9*n^2)*x^3 + 2520*(b^3*e^9*n^2*x^
3 + b^3*d^9*n^2)*log(c) + 63*(5*b^3*d*e^8*n^3*x^2 - 8*b^3*d^4*e^5*n^3*x +
20*b^3*d^7*e^2*n^3)*x^(2/3) - 90*(4*b^3*d^2*e^7*n^3*x^2 - 7*b^3*d^5*e^4*n^
3*x + 28*b^3*d^8*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 444528000*(3*b^3*d
^3*e^6*n*x^2 - 6*b^3*d^6*e^3*n*x - 2*(b^3*e^9*n - 9*a*b^2*e^9)*x^3)*log(c)
^2 - 840*(6527971*b^3*d^6*e^3*n^3 - 8439480*a*b^2*d^6*e^3*n^2 + 3175200*a^
2*b*d^6*e^3*n)*x + 2520*(30300391*b^3*d^9*n^3 - 17965080*a*b^2*d^9*n^2 + 3
175200*a^2*b*d^9*n + 39200*(2*b^3*e^9*n^3 - 18*a*b^2*e^9*n^2 + 81*a^2*b*e^
9*n)*x^3 - 2100*(275*b^3*d^3*e^6*n^3 - 504*a*b^2*d^3*e^6*n^2)*x^2 + 317520
0*(b^3*e^9*n*x^3 + b^3*d^9*n)*log(c)^2 + 840*(3349*b^3*d^6*e^3*n^3 - 2520*
a*b^2*d^6*e^3*n^2)*x + 2520*(420*b^3*d^3*e^6*n^2*x^2 - 840*b^3*d^6*e^3*n^2
*x - 7129*b^3*d^9*n^2 + 2520*a*b^2*d^9*n - 280*(b^3*e^9*n^2 - 9*a*b^2*e^9*
n)*x^3)*log(c) - 63*(92180*b^3*d^7*e^2*n^3 - 50400*a*b^2*d^7*e^2*n^2 + 175
*(17*b^3*d*e^8*n^3 - 72*a*b^2*d*e^8*n^2)*x^2 - 8*(1879*b^3*d^4*e^5*n^3 - 2
520*a*b^2*d^4*e^5*n^2)*x - 2520*(5*b^3*d*e^8*n^2*x^2 - 8*b^3*d^4*e^5*n^...
```

Sympy [F]

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

output
```
Integral(x**2*(a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 867, normalized size of antiderivative = 0.64

$$\int x^2 \left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3*log((e*x^(1/3) + d)^n*c)^3 + a*b^2*x^3*log((e*x^(1/3) + d)^n*c
)^2 + a^2*b*x^3*log((e*x^(1/3) + d)^n*c) + 1/3*a^3*x^3 + 1/2520*a^2*b*e*n*
(2520*d^9*log(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360
*d^2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(
4/3) + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9) + 1/317
5200*(2520*e*n*(2520*d^9*log(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^
7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) -
630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3
))/e^9)*log((e*x^(1/3) + d)^n*c) + (78400*e^9*x^3 - 187425*d*e^8*x^(8/3) +
 343800*d^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3175200*d^9*log(e*x^(1/3) +
 d)^2 + 947016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*x^(4/3) + 2813160*d^6*e^3
*x - 17965080*d^9*log(e*x^(1/3) + d) - 5807340*d^7*e^2*x^(2/3) + 17965080*
d^8*e*x^(1/3))*n^2/e^9)*a*b^2 + 1/8001504000*(3175200*e*n*(2520*d^9*log(e*
x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3)
 - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e
^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*log((e*x^(1/3) + d)^n*c
)^2 - e*n*((21952000*e^9*x^3 - 2667168000*d^9*log(e*x^(1/3) + d)^3 - 83734
875*d*e^8*x^(8/3) + 219465000*d^2*e^7*x^(7/3) - 498592500*d^3*e^6*x^2 - 22
636000800*d^9*log(e*x^(1/3) + d)^2 + 1075607064*d^4*e^5*x^(5/3) - 23403309
30*d^5*e^4*x^(4/3) + 5483495640*d^6*e^3*x - 76356985320*d^9*log(e*x^(1/...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3240 vs. 2(1189) = 2378.

Time = 0.17 (sec) , antiderivative size = 3240, normalized size of antiderivative = 2.39

$$\int x^2 \left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

output
```
1/8001504000*(2667168000*b^3*e*x^3*log(c)^3 + 8001504000*a*b^2*e*x^3*log(c
)^2 + 8001504000*a^2*b*e*x^3*log(c) + (2667168000*(e*x^(1/3) + d)^9*log(e*
x^(1/3) + d)^3/e^8 - 24004512000*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)^3/
e^8 + 96018048000*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)^3/e^8 - 2240421
12000*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)^3/e^8 + 336063168000*(e*x^(
1/3) + d)^5*d^4*log(e*x^(1/3) + d)^3/e^8 - 336063168000*(e*x^(1/3) + d)^4*
d^5*log(e*x^(1/3) + d)^3/e^8 + 224042112000*(e*x^(1/3) + d)^3*d^6*log(e*x^
(1/3) + d)^3/e^8 - 96018048000*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)^3/
e^8 + 24004512000*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)^3/e^8 - 889056000
*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)^2/e^8 + 9001692000*(e*x^(1/3) + d)^8
*d*log(e*x^(1/3) + d)^2/e^8 - 41150592000*(e*x^(1/3) + d)^7*d^2*log(e*x^(1
/3) + d)^2/e^8 + 112021056000*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)^2/e
^8 - 201637900800*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)^2/e^8 + 2520473
76000*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)^2/e^8 - 224042112000*(e*x^(
1/3) + d)^3*d^6*log(e*x^(1/3) + d)^2/e^8 + 144027072000*(e*x^(1/3) + d)^2*
d^7*log(e*x^(1/3) + d)^2/e^8 - 72013536000*(e*x^(1/3) + d)*d^8*log(e*x^(1/
3) + d)^2/e^8 + 197568000*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)/e^8 - 22504
23000*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)/e^8 + 11757312000*(e*x^(1/3)
+ d)^7*d^2*log(e*x^(1/3) + d)/e^8 - 37340352000*(e*x^(1/3) + d)^6*d^3*log(
e*x^(1/3) + d)/e^8 + 80655160320*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + ...
```

Mupad [B] (verification not implemented)

Time = 21.04 (sec) , antiderivative size = 1386, normalized size of antiderivative = 1.02

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

output
```
(a^3*x^3)/3 + (b^3*x^3*log(c*(d + e*x^(1/3))^n)^3)/3 - (2*b^3*n^3*x^3)/729
 + a*b^2*x^3*log(c*(d + e*x^(1/3))^n)^2 - (b^3*n*x^3*log(c*(d + e*x^(1/3))
^n)^2)/9 + (2*b^3*n^2*x^3*log(c*(d + e*x^(1/3))^n))/81 + (2*a*b^2*n^2*x^3)
/81 + (b^3*d^9*log(c*(d + e*x^(1/3))^n)^3)/(3*e^9) + a^2*b*x^3*log(c*(d +
e*x^(1/3))^n) - (a^2*b*n*x^3)/9 - (2*a*b^2*n*x^3*log(c*(d + e*x^(1/3))^n))
/9 + (30300391*b^3*d^9*n^3*log(d + e*x^(1/3)))/(3175200*e^9) + (47485*b^3*
d^3*n^3*x^2)/(762048*e^3) - (24385*b^3*d^2*n^3*x^(7/3))/(889056*e^2) - (21
34141*b^3*d^4*n^3*x^(5/3))/(15876000*e^4) + (3714811*b^3*d^5*n^3*x^(4/3))/
(12700800*e^5) + (12335311*b^3*d^7*n^3*x^(2/3))/(6350400*e^7) - (30300391*
b^3*d^8*n^3*x^(1/3))/(3175200*e^8) + (a*b^2*d^9*log(c*(d + e*x^(1/3))^n)^2
)/e^9 - (7129*b^3*d^9*n*log(c*(d + e*x^(1/3))^n)^2)/(2520*e^9) + (217*b^3*
d*n^3*x^(8/3))/(20736*e) - (6527971*b^3*d^6*n^3*x)/(9525600*e^6) + (a^2*b*
d^9*n*log(d + e*x^(1/3)))/e^9 + (b^3*d*n*x^(8/3)*log(c*(d + e*x^(1/3))^n)^
2)/(8*e) - (17*b^3*d*n^2*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(288*e) - (b^3*
d^6*n*x*log(c*(d + e*x^(1/3))^n)^2)/(3*e^6) + (3349*b^3*d^6*n^2*x*log(c*(d
 + e*x^(1/3))^n))/(3780*e^6) + (a^2*b*d^3*n*x^2)/(6*e^3) - (17*a*b^2*d*n^2
*x^(8/3))/(288*e) + (3349*a*b^2*d^6*n^2*x)/(3780*e^6) - (a^2*b*d^2*n*x^(7/
3))/(7*e^2) - (a^2*b*d^4*n*x^(5/3))/(5*e^4) + (a^2*b*d^5*n*x^(4/3))/(4*e^5
) + (a^2*b*d^7*n*x^(2/3))/(2*e^7) - (a^2*b*d^8*n*x^(1/3))/e^8 - (7129*a*b^
2*d^9*n^2*log(d + e*x^(1/3)))/(1260*e^9) + (b^3*d^3*n*x^2*log(c*(d + e*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1405, normalized size of antiderivative = 1.04

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x)
```

output

```
(4000752000*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**7*e**2*n - 1600
300800*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**4*e**5*n*x + 1000188
000*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d*e**8*n*x**2 + 8001504000
*x**(2/3)*log((x**(1/3)*e + d)**n*c)*a*b**2*d**7*e**2*n - 3200601600*x**(2
/3)*log((x**(1/3)*e + d)**n*c)*a*b**2*d**4*e**5*n*x + 2000376000*x**(2/3)*
log((x**(1/3)*e + d)**n*c)*a*b**2*d*e**8*n*x**2 - 14634496800*x**(2/3)*log
((x**(1/3)*e + d)**n*c)*b**3*d**7*e**2*n**2 + 2386480320*x**(2/3)*log((x**
(1/3)*e + d)**n*c)*b**3*d**4*e**5*n**2*x - 472311000*x**(2/3)*log((x**(1/3
)*e + d)**n*c)*b**3*d*e**8*n**2*x**2 + 4000752000*x**(2/3)*a**2*b*d**7*e**
2*n - 1600300800*x**(2/3)*a**2*b*d**4*e**5*n*x + 1000188000*x**(2/3)*a**2*
b*d*e**8*n*x**2 - 14634496800*x**(2/3)*a*b**2*d**7*e**2*n**2 + 2386480320*
x**(2/3)*a*b**2*d**4*e**5*n**2*x - 472311000*x**(2/3)*a*b**2*d*e**8*n**2*x
**2 + 15542491860*x**(2/3)*b**3*d**7*e**2*n**3 - 1075607064*x**(2/3)*b**3*
d**4*e**5*n**3*x + 83734875*x**(2/3)*b**3*d*e**8*n**3*x**2 - 8001504000*x*
*(1/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**8*e*n + 2000376000*x**(1/3)*l
og((x**(1/3)*e + d)**n*c)**2*b**3*d**5*e**4*n*x - 1143072000*x**(1/3)*log(
(x**(1/3)*e + d)**n*c)**2*b**3*d**2*e**7*n*x**2 - 16003008000*x**(1/3)*log
((x**(1/3)*e + d)**n*c)*a*b**2*d**8*e*n + 4000752000*x**(1/3)*log((x**(1/3
)*e + d)**n*c)*a*b**2*d**5*e**4*n*x - 2286144000*x**(1/3)*log((x**(1/3)*e
+ d)**n*c)*a*b**2*d**2*e**7*n*x**2 + 45272001600*x**(1/3)*log((x**(1/3)...
```

3.458 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 907

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

output
```
10*b*d^3*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6-45/8*b*d^2*n*
(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6+9/5*b*d*n*(d+e*x^(1/3))^
5*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6-18*a*b^2*d^5*n^2*x^(1/3)/e^5-18*b^3*d^
5*n^2*(d+e*x^(1/3))*ln(c*(d+e*x^(1/3))^n)/e^6+45/4*b^2*d^4*n^2*(d+e*x^(1/3
))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-20/3*b^2*d^3*n^2*(d+e*x^(1/3))^3*(a+b
*ln(c*(d+e*x^(1/3))^n))/e^6+45/16*b^2*d^2*n^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d
+e*x^(1/3))^n))/e^6-18/25*b^2*d*n^2*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3)
)^n))/e^6+9*b*d^5*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6-45/4*b
*d^4*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6+1/2*(d+e*x^(1/3))
^6*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-45/8*b^3*d^4*n^3*(d+e*x^(1/3))^2/e^6+
20/9*b^3*d^3*n^3*(d+e*x^(1/3))^3/e^6-45/64*b^3*d^2*n^3*(d+e*x^(1/3))^4/e^6
+18/125*b^3*d*n^3*(d+e*x^(1/3))^5/e^6+18*b^3*d^5*n^3*x^(1/3)/e^5+1/12*b^2*
n^2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-1/4*b*n*(d+e*x^(1/3))^
6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6+15/2*d^4*(d+e*x^(1/3))^2*(a+b*ln(c*(d+
e*x^(1/3))^n))^3/e^6-10*d^3*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/
e^6+15/2*d^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-3*d*(d+e*x^
(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-1/72*b^3*n^3*(d+e*x^(1/3))^6/e^
6-3*d^5*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 589, normalized size of antiderivative = 0.65

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{b^3en^3\sqrt[3]{x}\left(809340d^5-140070d^4e\sqrt[3]{x}+41180d^3e^2x^{2/3}-13785d^2e^3x+4368de^4x^{4/3}-1000e^5x^{5/3}\right)+180}{}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

output
```
(b^3*e*n^3*x^(1/3)*(809340*d^5 - 140070*d^4*e*x^(1/3) + 41180*d^3*e^2*x^(2
/3) - 13785*d^2*e^3*x + 4368*d*e^4*x^(4/3) - 1000*e^5*x^(5/3)) + 1800*a^2*
b*n*(147*d^6 + 60*d^5*e*x^(1/3) - 30*d^4*e^2*x^(2/3) + 20*d^3*e^3*x - 15*d
^2*e^4*x^(4/3) + 12*d*e^5*x^(5/3) - 10*e^6*x^2) - 36000*a^3*(d^6 - e^6*x^2
) + 60*a*b^2*n^2*(8111*d^6 - 8820*d^5*e*x^(1/3) + 2610*d^4*e^2*x^(2/3) - 1
140*d^3*e^3*x + 555*d^2*e^4*x^(4/3) - 264*d*e^5*x^(5/3) + 100*e^6*x^2) - 6
0*b*(b^2*n^2*(13489*d^6 + 8820*d^5*e*x^(1/3) - 2610*d^4*e^2*x^(2/3) + 1140
*d^3*e^3*x - 555*d^2*e^4*x^(4/3) + 264*d*e^5*x^(5/3) - 100*e^6*x^2) - 60*a
*b*n*(147*d^6 + 60*d^5*e*x^(1/3) - 30*d^4*e^2*x^(2/3) + 20*d^3*e^3*x - 15*
d^2*e^4*x^(4/3) + 12*d*e^5*x^(5/3) - 10*e^6*x^2) + 1800*a^2*(d^6 - e^6*x^2
))*Log[c*(d + e*x^(1/3))^n] - 1800*b^2*(60*a*(d^6 - e^6*x^2) + b*n*(-147*d
^6 - 60*d^5*e*x^(1/3) + 30*d^4*e^2*x^(2/3) - 20*d^3*e^3*x + 15*d^2*e^4*x^(
4/3) - 12*d*e^5*x^(5/3) + 10*e^6*x^2))*Log[c*(d + e*x^(1/3))^n]^2 - 36000*
b^3*(d^6 - e^6*x^2)*Log[c*(d + e*x^(1/3))^n]^3)/(72000*e^6)
```

Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int\left(-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3d^5}{e^5}+\frac{5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3d^4}{e^5}-\frac{10\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\right.}{e^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^6}{216e^6}+\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^6}{6e^6}-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^6}{12e^6}+\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

output
```
3*((-15*b^3*d^4*n^3*(d + e*x^(1/3))^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e*x^
(1/3))^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e*x^(1/3))^4)/(64*e^6) + (6*b^3*
d*n^3*(d + e*x^(1/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(1/3))^6)/(216*e^6)
 - (6*a*b^2*d^5*n^2*x^(1/3))/e^5 + (6*b^3*d^5*n^3*x^(1/3))/e^5 - (6*b^3*d^
5*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^6 + (15*b^2*d^4*n^2*(d +
 e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^6) - (20*b^2*d^3*n^2*
(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(9*e^6) + (15*b^2*d^2*
n^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(16*e^6) - (6*b^2*
d*n^2*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^6) + (b^2*
n^2*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(36*e^6) + (3*b*d^
5*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (15*b*d^4*n*
(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) + (10*b*d^3*
n*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(3*e^6) - (15*b*d^
2*n*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(8*e^6) + (3*b*d
*n*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^6) - (b*n*(d
 + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(12*e^6) - (d^5*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^6 + (5*d^4*(d + e*x^(1/3))
^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^6) - (10*d^3*(d + e*x^(1/3))^3
*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(3*e^6) + (5*d^2*(d + e*x^(1/3))^4*(a
 + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^6) - (d*(d + e*x^(1/3))^5*(a + b...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1190, normalized size of antiderivative = 1.31

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

output
```
1/72000*(36000*b^3*e^6*x^2*log(c)^3 + 36000*(b^3*e^6*n^3*x^2 - b^3*d^6*n^3
)*log(e*x^(1/3) + d)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^
6*n - 36*a^3*e^6)*x^2 + 1800*(20*b^3*d^3*e^3*n^3*x + 147*b^3*d^6*n^3 - 60*
a*b^2*d^6*n^2 - 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2)*x^2 + 60*(b^3*e^6*n^2*x
^2 - b^3*d^6*n^2)*log(c) + 6*(2*b^3*d*e^5*n^3*x - 5*b^3*d^4*e^2*n^3)*x^(2/
3) - 15*(b^3*d^2*e^4*n^3*x - 4*b^3*d^5*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^
2 + 18000*(2*b^3*d^3*e^3*n*x - (b^3*e^6*n - 6*a*b^2*e^6)*x^2)*log(c)^2 + 2
0*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*n)*x
 - 60*(13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n - 100*(b^
3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^2 - 1800*(b^3*e^6*n*x^2 -
b^3*d^6*n)*log(c)^2 + 60*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 6
0*(20*b^3*d^3*e^3*n^2*x + 147*b^3*d^6*n^2 - 60*a*b^2*d^6*n - 10*(b^3*e^6*n
^2 - 6*a*b^2*e^6*n)*x^2)*log(c) - 6*(435*b^3*d^4*e^2*n^3 - 300*a*b^2*d^4*e
^2*n^2 - 4*(11*b^3*d*e^5*n^3 - 30*a*b^2*d*e^5*n^2)*x + 60*(2*b^3*d*e^5*n^2
*x - 5*b^3*d^4*e^2*n^2)*log(c))*x^(2/3) + 15*(588*b^3*d^5*e*n^3 - 240*a*b^
2*d^5*e*n^2 - (37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x + 60*(b^3*d^2*
e^4*n^2*x - 4*b^3*d^5*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 1200*(5
*(b^3*e^6*n^2 - 6*a*b^2*e^6*n + 18*a^2*b*e^6)*x^2 - 3*(19*b^3*d^3*e^3*n^2
- 20*a*b^2*d^3*e^3*n)*x)*log(c) - 6*(23345*b^3*d^4*e^2*n^3 - 26100*a*b^2*d
^4*e^2*n^2 + 9000*a^2*b*d^4*e^2*n - 1800*(2*b^3*d*e^5*n*x - 5*b^3*d^4*e...
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

output
```
Integral(x*(a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 668, normalized size of antiderivative = 0.74

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2*log((e*x^(1/3) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*x^(1/3) + d)
^n*c)^2 - 1/40*a^2*b*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12
*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60
*d^5*x^(1/3))/e^6) + 3/2*a^2*b*x^2*log((e*x^(1/3) + d)^n*c) + 1/2*a^3*x^2
- 1/1200*(60*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x
^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(
1/3))/e^6)*log((e*x^(1/3) + d)^n*c) - (100*e^6*x^2 + 1800*d^6*log(e*x^(1/3
) + d)^2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1140*d^3*e^3*x + 8820
*d^6*log(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d^5*e*x^(1/3))*n^2/e
^6)*a*b^2 - 1/72000*(1800*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2
 - 12*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3)
 - 60*d^5*x^(1/3))/e^6)*log((e*x^(1/3) + d)^n*c)^2 + e*n*((36000*d^6*log(e
*x^(1/3) + d)^3 + 1000*e^6*x^2 + 264600*d^6*log(e*x^(1/3) + d)^2 - 4368*d*
e^5*x^(5/3) + 13785*d^2*e^4*x^(4/3) - 41180*d^3*e^3*x + 809340*d^6*log(e*x
^(1/3) + d) + 140070*d^4*e^2*x^(2/3) - 809340*d^5*e*x^(1/3))*n^2/e^7 - 60*
(100*e^6*x^2 + 1800*d^6*log(e*x^(1/3) + d)^2 - 264*d*e^5*x^(5/3) + 555*d^2
*e^4*x^(4/3) - 1140*d^3*e^3*x + 8820*d^6*log(e*x^(1/3) + d) + 2610*d^4*e^2
*x^(2/3) - 8820*d^5*e*x^(1/3))*n*log((e*x^(1/3) + d)^n*c)/e^7))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2160 vs. 2(787) = 1574.

Time = 0.15 (sec) , antiderivative size = 2160, normalized size of antiderivative = 2.38

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

output
```
1/72000*(36000*b^3*e*x^2*log(c)^3 + 108000*a*b^2*e*x^2*log(c)^2 + (36000*(
e*x^(1/3) + d)^6*log(e*x^(1/3) + d)^3/e^5 - 216000*(e*x^(1/3) + d)^5*d*log
(e*x^(1/3) + d)^3/e^5 + 540000*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)^3/
e^5 - 720000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)^3/e^5 + 540000*(e*x^
(1/3) + d)^2*d^4*log(e*x^(1/3) + d)^3/e^5 - 216000*(e*x^(1/3) + d)*d^5*log
(e*x^(1/3) + d)^3/e^5 - 18000*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)^2/e^5 +
 129600*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)^2/e^5 - 405000*(e*x^(1/3) +
 d)^4*d^2*log(e*x^(1/3) + d)^2/e^5 + 720000*(e*x^(1/3) + d)^3*d^3*log(e*x^
(1/3) + d)^2/e^5 - 810000*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)^2/e^5 +
 648000*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)^2/e^5 + 6000*(e*x^(1/3) + d
)^6*log(e*x^(1/3) + d)/e^5 - 51840*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)/
e^5 + 202500*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)/e^5 - 480000*(e*x^(1
/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 + 810000*(e*x^(1/3) + d)^2*d^4*log(e
*x^(1/3) + d)/e^5 - 1296000*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)/e^5 - 1
000*(e*x^(1/3) + d)^6/e^5 + 10368*(e*x^(1/3) + d)^5*d/e^5 - 50625*(e*x^(1/
3) + d)^4*d^2/e^5 + 160000*(e*x^(1/3) + d)^3*d^3/e^5 - 405000*(e*x^(1/3) +
 d)^2*d^4/e^5 + 1296000*(e*x^(1/3) + d)*d^5/e^5)*b^3*n^3 + 60*(1800*(e*x^(
1/3) + d)^6*log(e*x^(1/3) + d)^2/e^5 - 10800*(e*x^(1/3) + d)^5*d*log(e*x^(
1/3) + d)^2/e^5 + 27000*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)^2/e^5 - 3
6000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)^2/e^5 + 27000*(e*x^(1/3) ...
```

Mupad [B] (verification not implemented)

Time = 20.96 (sec) , antiderivative size = 979, normalized size of antiderivative = 1.08

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

output
```
(a^3*x^2)/2 + (b^3*x^2*log(c*(d + e*x^(1/3))^n)^3)/2 - (b^3*n^3*x^2)/72 +
(3*a*b^2*x^2*log(c*(d + e*x^(1/3))^n)^2)/2 - (b^3*n*x^2*log(c*(d + e*x^(1/
3))^n)^2)/4 + (b^3*n^2*x^2*log(c*(d + e*x^(1/3))^n))/12 + (a*b^2*n^2*x^2)/
12 - (b^3*d^6*log(c*(d + e*x^(1/3))^n)^3)/(2*e^6) + (3*a^2*b*x^2*log(c*(d
+ e*x^(1/3))^n))/2 - (a^2*b*n*x^2)/4 - (a*b^2*n*x^2*log(c*(d + e*x^(1/3))^
n))/2 - (13489*b^3*d^6*n^3*log(d + e*x^(1/3)))/(1200*e^6) - (919*b^3*d^2*n
^3*x^(4/3))/(4800*e^2) - (4669*b^3*d^4*n^3*x^(2/3))/(2400*e^4) + (13489*b^
3*d^5*n^3*x^(1/3))/(1200*e^5) - (3*a*b^2*d^6*log(c*(d + e*x^(1/3))^n)^2)/(
2*e^6) + (147*b^3*d^6*n*log(c*(d + e*x^(1/3))^n)^2)/(40*e^6) + (2059*b^3*d
^3*n^3*x)/(3600*e^3) + (91*b^3*d*n^3*x^(5/3))/(1500*e) - (3*a^2*b*d^6*n*lo
g(d + e*x^(1/3)))/(2*e^6) + (b^3*d^3*n*x*log(c*(d + e*x^(1/3))^n)^2)/(2*e^
3) - (19*b^3*d^3*n^2*x*log(c*(d + e*x^(1/3))^n))/(20*e^3) + (3*b^3*d*n*x^(
5/3)*log(c*(d + e*x^(1/3))^n)^2)/(10*e) - (11*b^3*d*n^2*x^(5/3)*log(c*(d +
 e*x^(1/3))^n))/(50*e) - (19*a*b^2*d^3*n^2*x)/(20*e^3) - (11*a*b^2*d*n^2*x
^(5/3))/(50*e) - (3*a^2*b*d^2*n*x^(4/3))/(8*e^2) - (3*a^2*b*d^4*n*x^(2/3))
/(4*e^4) + (3*a^2*b*d^5*n*x^(1/3))/(2*e^5) + (147*a*b^2*d^6*n^2*log(d + e*
x^(1/3)))/(20*e^6) - (3*b^3*d^2*n*x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(8*e
^2) + (37*b^3*d^2*n^2*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(80*e^2) - (3*b^3*
d^4*n*x^(2/3)*log(c*(d + e*x^(1/3))^n)^2)/(4*e^4) + (87*b^3*d^4*n^2*x^(2/3
)*log(c*(d + e*x^(1/3))^n))/(40*e^4) + (3*b^3*d^5*n*x^(1/3)*log(c*(d + ...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 994, normalized size of antiderivative = 1.10

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x)
```

output
```
( - 54000*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**4*e**2*n + 21600*
x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d*e**5*n*x - 108000*x**(2/3)*l
og((x**(1/3)*e + d)**n*c)*a*b**2*d**4*e**2*n + 43200*x**(2/3)*log((x**(1/3
)*e + d)**n*c)*a*b**2*d*e**5*n*x + 156600*x**(2/3)*log((x**(1/3)*e + d)**n
*c)*b**3*d**4*e**2*n**2 - 15840*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**3*d
*e**5*n**2*x - 54000*x**(2/3)*a**2*b*d**4*e**2*n + 21600*x**(2/3)*a**2*b*d
*e**5*n*x + 156600*x**(2/3)*a*b**2*d**4*e**2*n**2 - 15840*x**(2/3)*a*b**2*
d*e**5*n**2*x - 140070*x**(2/3)*b**3*d**4*e**2*n**3 + 4368*x**(2/3)*b**3*d
*e**5*n**3*x + 108000*x**(1/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**5*e*n
 - 27000*x**(1/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**2*e**4*n*x + 21600
0*x**(1/3)*log((x**(1/3)*e + d)**n*c)*a*b**2*d**5*e*n - 54000*x**(1/3)*log
((x**(1/3)*e + d)**n*c)*a*b**2*d**2*e**4*n*x - 529200*x**(1/3)*log((x**(1/
3)*e + d)**n*c)*b**3*d**5*e*n**2 + 33300*x**(1/3)*log((x**(1/3)*e + d)**n*
c)*b**3*d**2*e**4*n**2*x + 108000*x**(1/3)*a**2*b*d**5*e*n - 27000*x**(1/3
)*a**2*b*d**2*e**4*n*x - 529200*x**(1/3)*a*b**2*d**5*e*n**2 + 33300*x**(1/
3)*a*b**2*d**2*e**4*n**2*x + 809340*x**(1/3)*b**3*d**5*e*n**3 - 13785*x**(
1/3)*b**3*d**2*e**4*n**3*x - 36000*log((x**(1/3)*e + d)**n*c)**3*b**3*d**6
 + 36000*log((x**(1/3)*e + d)**n*c)**3*b**3*e**6*x**2 - 108000*log((x**(1/
3)*e + d)**n*c)**2*a*b**2*d**6 + 108000*log((x**(1/3)*e + d)**n*c)**2*a*b*
*2*e**6*x**2 + 264600*log((x**(1/3)*e + d)**n*c)**2*b**3*d**6*n + 36000...
```

3.459 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 438

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3}-\frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3}+\frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{18b^3d^2n^3\sqrt[3]{x}}{e^2}+\frac{18b^3d^2n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{e^3}-\frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}-\frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3}+\frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2e^3}-\frac{bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3}+\frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}-\frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}+\frac{\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}$$

output
```
9/4*b^3*d*n^3*(d+e*x^(1/3))^2/e^3-2/9*b^3*n^3*(d+e*x^(1/3))^3/e^3+18*a*b^2
*d^2*n^2*x^(1/3)/e^2-18*b^3*d^2*n^3*x^(1/3)/e^2+18*b^3*d^2*n^2*(d+e*x^(1/3
))*ln(c*(d+e*x^(1/3))^n)/e^3-9/2*b^2*d*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*
x^(1/3))^n))/e^3+2/3*b^2*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e
^3-9*b*d^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^3+9/2*b*d*n*(d+
e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^3-b*n*(d+e*x^(1/3))^3*(a+b*ln
(c*(d+e*x^(1/3))^n))^2/e^3+3*d^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))
^3/e^3-3*d*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^3+(d+e*x^(1/3))
^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.83

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{b^3en^3\left(-510d^2+57de\sqrt[3]{x}-8e^2x^{2/3}\right)\sqrt[3]{x}-6ab^2n^2\left(23d^3-66d^2e\sqrt[3]{x}+15de^2x^{2/3}-4e^3x\right)+36a^3\left(d^3+e\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

output
```
(b^3*e*n^3*(-510*d^2 + 57*d*e*x^(1/3) - 8*e^2*x^(2/3))*x^(1/3) - 6*a*b^2*n
^2*(23*d^3 - 66*d^2*e*x^(1/3) + 15*d*e^2*x^(2/3) - 4*e^3*x) + 36*a^3*(d^3
+ e^3*x) - 18*a^2*b*n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) + 2*e^3*
x) + 6*b*(18*a^2*(d^2 - d*e*x^(1/3) + e^2*x^(2/3)) - 6*a*b*n*(11*d^2 - 5*d
*e*x^(1/3) + 2*e^2*x^(2/3)) + b^2*n^2*(85*d^2 - 19*d*e*x^(1/3) + 4*e^2*x^(
2/3)))*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n] + 18*b^2*(6*a*(d^3 + e^3*x
) - b*n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) + 2*e^3*x))*Log[c*(d +
 e*x^(1/3))^n]^2 + 36*b^3*(d^3 + e^3*x)*Log[c*(d + e*x^(1/3))^n]^3)/(36*e^
3)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2901, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{ 2901}$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2848}$$

$$3\int \left(\frac{\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^2}-\frac{2d\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^2}+\frac{d^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{e^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{9e^3}-\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3}+\frac{6ab^2d^2n^2\sqrt[3]{x}}{e^2}\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

output
```
3*((3*b^3*d*n^3*(d + e*x^(1/3))^2)/(4*e^3) - (2*b^3*n^3*(d + e*x^(1/3))^3)
/(27*e^3) + (6*a*b^2*d^2*n^2*x^(1/3))/e^2 - (6*b^3*d^2*n^3*x^(1/3))/e^2 +
(6*b^3*d^2*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^3 - (3*b^2*d*n^
2*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(2*e^3) + (2*b^2*n^2
*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(9*e^3) - (3*b*d^2*n*
(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (3*b*d*n*(d + e*
x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^3) - (b*n*(d + e*x^(1/
3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(3*e^3) + (d^2*(d + e*x^(1/3))*(
a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^3 - (d*(d + e*x^(1/3))^2*(a + b*Log[c
*(d + e*x^(1/3))^n])^3)/e^3 + ((d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/
3))^n])^3)/(3*e^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.58

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

output
```
1/36*(36*b^3*e^3*x*log(c)^3 + 36*(b^3*e^3*n^3*x + b^3*d^3*n^3)*log(e*x^(1/
3) + d)^3 - 36*(b^3*e^3*n - 3*a*b^2*e^3)*x*log(c)^2 + 18*(3*b^3*d*e^2*n^3*
x^(2/3) - 6*b^3*d^2*e*n^3*x^(1/3) - 11*b^3*d^3*n^3 + 6*a*b^2*d^3*n^2 - 2*(
b^3*e^3*n^3 - 3*a*b^2*e^3*n^2)*x + 6*(b^3*e^3*n^2*x + b^3*d^3*n^2)*log(c))
*log(e*x^(1/3) + d)^2 + 12*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*x
*log(c) - 4*(2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*
x + 6*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n + 18*(b^3*e^3*n*
x + b^3*d^3*n)*log(c)^2 + 2*(2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3
*n)*x - 6*(11*b^3*d^3*n^2 - 6*a*b^2*d^3*n + 2*(b^3*e^3*n^2 - 3*a*b^2*e^3*n
)*x)*log(c) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*d*e^2*
n^2)*x^(2/3) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a*b^2*d^2*
e*n^2)*x^(1/3))*log(e*x^(1/3) + d) + 3*(19*b^3*d*e^2*n^3 + 18*b^3*d*e^2*n*
log(c)^2 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*d*e^2*n^2 - 6*
a*b^2*d*e^2*n)*log(c))*x^(2/3) - 6*(85*b^3*d^2*e*n^3 + 18*b^3*d^2*e*n*log(
c)^2 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*d^2*e*n^2 - 6*a*b
^2*d^2*e*n)*log(c))*x^(1/3))/e^3
```

Sympy [F]

$$\int \left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^{n}\right)\right)^{3}\, dx = \int \left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^{n}\right)\right)^{3}\, dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.04

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{1}{2}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)\right)a^2b$$
$$+\frac{1}{6}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+18\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)\right.\right.$$
$$+\frac{1}{36}\left(18\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^2+36\,x\log\left(\left(ex^{\frac{1}{3}}+\right.\right.\right.$$
$$+a^3x$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

output
```
1/2*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*
x^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*a^2*b + 1/6*(6*e*n*(6*d^3*lo
g(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log(
(e*x^(1/3) + d)^n*c) + 18*x*log((e*x^(1/3) + d)^n*c)^2 - (18*d^3*log(e*x^(
1/3) + d)^2 - 4*e^3*x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*
d^2*e*x^(1/3))*n^2/e^3)*a*b^2 + 1/36*(18*e*n*(6*d^3*log(e*x^(1/3) + d)/e^4
 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*x^(1/3) + d)^n*c)
^2 + 36*x*log((e*x^(1/3) + d)^n*c)^3 + e*n*((36*d^3*log(e*x^(1/3) + d)^3 +
 198*d^3*log(e*x^(1/3) + d)^2 - 8*e^3*x + 510*d^3*log(e*x^(1/3) + d) + 57*
d*e^2*x^(2/3) - 510*d^2*e*x^(1/3))*n^2/e^4 - 6*(18*d^3*log(e*x^(1/3) + d)^
2 - 4*e^3*x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e*x^(1
/3))*n*log((e*x^(1/3) + d)^n*c)/e^4))*b^3 + a^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1072 vs. $2(384) = 768$.

Time = 0.15 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.45

$$\int \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

output
```
1/36*(36*b^3*e*x*log(c)^3 + (36*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)^3/e^2
 - 108*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^3/e^2 + 108*(e*x^(1/3) + d)*
d^2*log(e*x^(1/3) + d)^3/e^2 - 36*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)^2/e
^2 + 162*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^2/e^2 - 324*(e*x^(1/3) + d
)*d^2*log(e*x^(1/3) + d)^2/e^2 + 24*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e
^2 - 162*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 + 648*(e*x^(1/3) + d)*
d^2*log(e*x^(1/3) + d)/e^2 - 8*(e*x^(1/3) + d)^3/e^2 + 81*(e*x^(1/3) + d)^
2*d/e^2 - 648*(e*x^(1/3) + d)*d^2/e^2)*b^3*n^3 + 6*(18*(e*x^(1/3) + d)^3*l
og(e*x^(1/3) + d)^2/e^2 - 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^2/e^2
+ 54*(e*x^(1/3) + d)*d^2*log(e*x^(1/3) + d)^2/e^2 - 12*(e*x^(1/3) + d)^3*l
og(e*x^(1/3) + d)/e^2 + 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 - 10
8*(e*x^(1/3) + d)*d^2*log(e*x^(1/3) + d)/e^2 + 4*(e*x^(1/3) + d)^3/e^2 - 2
7*(e*x^(1/3) + d)^2*d/e^2 + 108*(e*x^(1/3) + d)*d^2/e^2)*b^3*n^2*log(c) +
18*(6*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2 - 18*(e*x^(1/3) + d)^2*d*lo
g(e*x^(1/3) + d)/e^2 + 18*(e*x^(1/3) + d)*d^2*log(e*x^(1/3) + d)/e^2 - 2*(
e*x^(1/3) + d)^3/e^2 + 9*(e*x^(1/3) + d)^2*d/e^2 - 18*(e*x^(1/3) + d)*d^2/
e^2)*b^3*n*log(c)^2 + 108*a*b^2*e*x*log(c)^2 + 6*(18*(e*x^(1/3) + d)^3*log
(e*x^(1/3) + d)^2/e^2 - 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^2/e^2 +
54*(e*x^(1/3) + d)*d^2*log(e*x^(1/3) + d)^2/e^2 - 12*(e*x^(1/3) + d)^3*log
(e*x^(1/3) + d)/e^2 + 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 - 1...
```

Mupad [B] (verification not implemented)

Time = 14.84 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.27

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = x\left(a^3-a^2\,b\,n+\frac{2\,a\,b^2\,n^2}{3}-\frac{2\,b^3\,n^3}{9}\right)$$
$$-x^{2/3}\left(\frac{d\left(3\,a^3-3\,a^2\,b\,n+2\,a\,b^2\,n^2-\frac{2\,b^3\,n^3}{3}\right)}{2\,e}-\frac{d\,(6\,a^3-6\,a\,b^2\,n^2+5\,b^3\,n^3)}{4\,e}\right)$$
$$+\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^3\left(b^3\,x+\frac{b^3\,d^3}{e^3}\right)$$

$$+\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2\left(\frac{d\,(6\,a\,b^2\,d^2-11\,b^3\,d^2\,n)}{2\,e^3}-x^{2/3}\left(\frac{3\,b^2\,d\,(3\,a-b\,n)}{2\,e}-\frac{9\,a\,b^2\,d}{2\,e}\right)+b^2\,x\,(3\,a-b\,n)+\right.$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

output
```
x*(a^3 - (2*b^3*n^3)/9 + (2*a*b^2*n^2)/3 - a^2*b*n) - x^(2/3)*((d*(3*a^3 -
 (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/(2*e) - (d*(6*a^3 + 5*b^3*n^3 -
 6*a*b^2*n^2))/(4*e)) + log(c*(d + e*x^(1/3))^n)^3*(b^3*x + (b^3*d^3)/e^3)
 + log(c*(d + e*x^(1/3))^n)^2*((d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(2*e^3) -
x^(2/3)*((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e)) + b^2*x*(3*a - b
*n) + (d*x^(1/3)*((3*b^2*d*(3*a - b*n))/e - (9*a*b^2*d)/e))/e) + x^(1/3)*(
(d*((d*(3*a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(6*a^3 +
5*b^3*n^3 - 6*a*b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/e^2) +
(log(d + e*x^(1/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n))/
(6*e^3) + (log(c*(d + e*x^(1/3))^n)*((x^(1/3)*((d*(b*d*e*(9*a^2 + 2*b^2*n^
2 - 6*a*b*n) - 3*b*d*e*(3*a^2 - b^2*n^2)))/e + 6*b^3*d^2*n^2))/e - (x^(2/3
)*(b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 3*b*d*e*(3*a^2 - b^2*n^2)))/(2*e)
 + (b*e*x*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/3))/e
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.31

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))^3,x)
```

output
```
(54*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d*e**2*n + 108*x**(2/3)*lo
g((x**(1/3)*e + d)**n*c)*a*b**2*d*e**2*n - 90*x**(2/3)*log((x**(1/3)*e + d
)**n*c)*b**3*d*e**2*n**2 + 54*x**(2/3)*a**2*b*d*e**2*n - 90*x**(2/3)*a*b**
2*d*e**2*n**2 + 57*x**(2/3)*b**3*d*e**2*n**3 - 108*x**(1/3)*log((x**(1/3)*
e + d)**n*c)**2*b**3*d**2*e*n - 216*x**(1/3)*log((x**(1/3)*e + d)**n*c)*a*
b**2*d**2*e*n + 396*x**(1/3)*log((x**(1/3)*e + d)**n*c)*b**3*d**2*e*n**2 -
 108*x**(1/3)*a**2*b*d**2*e*n + 396*x**(1/3)*a*b**2*d**2*e*n**2 - 510*x**(
1/3)*b**3*d**2*e*n**3 + 36*log((x**(1/3)*e + d)**n*c)**3*b**3*d**3 + 36*lo
g((x**(1/3)*e + d)**n*c)**3*b**3*e**3*x + 108*log((x**(1/3)*e + d)**n*c)**
2*a*b**2*d**3 + 108*log((x**(1/3)*e + d)**n*c)**2*a*b**2*e**3*x - 198*log(
(x**(1/3)*e + d)**n*c)**2*b**3*d**3*n - 36*log((x**(1/3)*e + d)**n*c)**2*b
**3*e**3*n*x + 108*log((x**(1/3)*e + d)**n*c)*a**2*b*d**3 + 108*log((x**(1
/3)*e + d)**n*c)*a**2*b*e**3*x - 396*log((x**(1/3)*e + d)**n*c)*a*b**2*d**
3*n - 72*log((x**(1/3)*e + d)**n*c)*a*b**2*e**3*n*x + 510*log((x**(1/3)*e
+ d)**n*c)*b**3*d**3*n**2 + 24*log((x**(1/3)*e + d)**n*c)*b**3*e**3*n**2*x
 + 36*a**3*e**3*x - 36*a**2*b*e**3*n*x + 24*a*b**2*e**3*n**2*x - 8*b**3*e*
*3*n**3*x)/(36*e**3)
```

3.460 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx = 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right)-18b^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\operatorname{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right)+18b^3n^3\operatorname{PolyLog}\left(4,1+\frac{e\sqrt[3]{x}}{d}\right)$$

output
```
3*(a+b*ln(c*(d+e*x^(1/3))^n))^3*ln(-e*x^(1/3)/d)+9*b*n*(a+b*ln(c*(d+e*x^(1
/3))^n))^2*polylog(2,1+e*x^(1/3)/d)-18*b^2*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))
*polylog(3,1+e*x^(1/3)/d)+18*b^3*n^3*polylog(4,1+e*x^(1/3)/d)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 333 vs. $2(135) = 270$.

Time = 0.20 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.47

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx = \left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log(x) + 3bn\Big(a-bn\log\left(d+e\sqrt[3]{x}\right) + b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\Big)^2\left(\left(\log\left(d+e\sqrt[3]{x}\right) - \log\left(1+\frac{e\sqrt[3]{x}}{d}\right)\right)\log(x) - 3\,\text{PolyLog}\left(2,-\frac{e\sqrt[3]{x}}{d}\right)\right) + 9b^2n^2\Big(a - bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\Big)\left(\log^2\left(d + e\sqrt[3]{x}\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 2\log\left(d+e\sqrt[3]{x}\right)\text{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right) - 2\,\text{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right)\right) + 3b^3n^3\left(\log^3\left(d+e\sqrt[3]{x}\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 3\log^2\left(d+e\sqrt[3]{x}\right)\text{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right) - 6\log\left(d+e\sqrt[3]{x}\right)\text{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right) + 6\,\text{PolyLog}\left(4,1+\frac{e\sqrt[3]{x}}{d}\right)\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```

output
```
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3*Log[x] + 3*b*n
*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*((Log[d + e*x
^(1/3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)])
 + 9*b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(Lo
g[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*PolyLog[2,
 1 + (e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d]) + 3*b^3*n^3*(Log[d
+ e*x^(1/3)]^3*Log[-((e*x^(1/3))/d)] + 3*Log[d + e*x^(1/3)]^2*PolyLog[2, 1
 + (e*x^(1/3))/d] - 6*Log[d + e*x^(1/3)]*PolyLog[3, 1 + (e*x^(1/3))/d] + 6
*PolyLog[4, 1 + (e*x^(1/3))/d])
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2843}$$

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3-3ben\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right)}{d+e\sqrt[3]{x}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{2881}$$

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3-3bn\int \frac{\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(cx^{n/3}\right)\right)^2}{\sqrt[3]{x}}d\left(d+e\sqrt[3]{x}\right)\right)$$

$\big\downarrow$ 2821

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c(d+e\sqrt[3]{x})^n\right)\right)^3-3bn\left(2bn\int\frac{\left(a+b\log\left(cx^{n/3}\right)\right)\text{PolyLog}\left(2,\frac{d+e\sqrt[3]{x}}{d}\right)}{\sqrt[3]{x}}d(d+e\sqrt[3]{x}\right.\right.$$

$\big\downarrow$ 2830

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c(d+e\sqrt[3]{x})^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(cx^{n/3}\right)\right)-bn\int\right.\right.\right.$$

$\big\downarrow$ 7143

$$3\left(\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c(d+e\sqrt[3]{x})^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(cx^{n/3}\right)\right)-bn\,\text{Poly}\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```

output
```
3*((a + b*Log[c*(d + e*x^(1/3))^n])^3*Log[-((e*x^(1/3))/d)] - 3*b*n*(-((a
+ b*Log[c*x^(n/3)])^2*PolyLog[2, (d + e*x^(1/3))/d]) + 2*b*n*((a + b*Log[c
*x^(n/3)])*PolyLog[3, (d + e*x^(1/3))/d] - b*n*PolyLog[4, (d + e*x^(1/3))/
d])))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c
)^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="maxima")
```

output
```
b^3*log((e*x^(1/3) + d)^n)^3*log(x) + integrate(-((b^3*e*n*x*log(x) - 3*(b
^3*e*log(c) + a*b^2*e)*x - 3*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((e*x^(1
/3) + d)^n)^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) +
a^3*e)*x - 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log
(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) - (b^3
*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(e*x
^2 + d*x^(5/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^3}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))^3/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^n))^3/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx$$
$$= \frac{4\left(\int \frac{\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)^3}{x^{\frac{4}{3}}e+dx}dx\right)b^3dn+12\left(\int \frac{\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)^2}{x^{\frac{4}{3}}e+dx}dx\right)a\,b^2dn+12\left(\int \frac{\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)}{x^{\frac{4}{3}}e+dx}dx\right)a^2bdn+3}{4n}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x)
```

output
```
(4*int(log((x**(1/3)*e + d)**n*c)**3/(x**(1/3)*e*x + d*x),x)*b**3*d*n + 12
*int(log((x**(1/3)*e + d)**n*c)**2/(x**(1/3)*e*x + d*x),x)*a*b**2*d*n + 12
*int(log((x**(1/3)*e + d)**n*c)/(x**(1/3)*e*x + d*x),x)*a**2*b*d*n + 3*log
((x**(1/3)*e + d)**n*c)**4*b**3 + 12*log((x**(1/3)*e + d)**n*c)**3*a*b**2
+ 18*log((x**(1/3)*e + d)**n*c)**2*a**2*b + 4*log(x)*a**3*n)/(4*n)
```

3.461 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 439

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}$$
$$-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}$$
$$-\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}}+\frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3\sqrt[3]{x}}$$
$$+\frac{3be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3}$$
$$-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}-\frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)}{d^3}$$
$$+\frac{b^3e^3n^3\log(x)}{d^3}+\frac{3b^3e^3n^3\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}$$
$$-\frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}$$
$$-\frac{6b^3e^3n^3\operatorname{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right)}{d^3}-\frac{6b^3e^3n^3\operatorname{PolyLog}\left(3,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}$$

output

```
-3*b^2*e^2*n^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3/x^(1/3)-3*b^2
*e^3*n^2*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3-3/2*b*e*n*(
a+b*ln(c*(d+e*x^(1/3))^n))^2/d/x^(2/3)+3*b*e^2*n*(d+e*x^(1/3))*(a+b*ln(c*(
d+e*x^(1/3))^n))^2/d^3/x^(1/3)+3*b*e^3*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(
d+e*x^(1/3))^n))^2/d^3-(a+b*ln(c*(d+e*x^(1/3))^n))^3/x-6*b^2*e^3*n^2*(a+b*
ln(c*(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)/d^3+b^3*e^3*n^3*ln(x)/d^3+3*b^3*e^
3*n^3*polylog(2,d/(d+e*x^(1/3)))/d^3-6*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/3))
^n))*polylog(2,d/(d+e*x^(1/3)))/d^3-6*b^3*e^3*n^3*polylog(2,1+e*x^(1/3)/d)
/d^3-6*b^3*e^3*n^3*polylog(3,d/(d+e*x^(1/3)))/d^3
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx=\text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

output
```
(-3*b*d^2*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3)
)^n])^2 + 6*b*d*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e
*x^(1/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)]
 + b*Log[c*(d + e*x^(1/3))^n])^2 - 6*b*e^3*n*x*Log[d + e*x^(1/3)]*(a - b*n
*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 2*d^3*(a - b*n*Log[d
 + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3 + 2*b*e^3*n*x*(a - b*n*Log[d
 + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] - 6*b^2*n^2*(a - b*n*
Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(d*e^2*x^(2/3) + (d^3 + e
^3*x)*Log[d + e*x^(1/3)]^2 + 3*e^3*x*Log[-((e*x^(1/3))/d)] + Log[d + e*x^(
1/3)]*(d^2*e*x^(1/3) - 2*d*e^2*x^(2/3) - 3*e^3*x - 2*e^3*x*Log[-((e*x^(1/3
))/d)]) - 2*e^3*x*PolyLog[2, 1 + (e*x^(1/3))/d]) + b^3*n^3*(-6*d*e^2*x^(2/
3)*Log[d + e*x^(1/3)] - 6*e^3*x*Log[d + e*x^(1/3)] - 3*d^2*e*x^(1/3)*Log[d
 + e*x^(1/3)]^2 + 6*d*e^2*x^(2/3)*Log[d + e*x^(1/3)]^2 + 9*e^3*x*Log[d + e
*x^(1/3)]^2 - 2*d^3*Log[d + e*x^(1/3)]^3 - 2*e^3*x*Log[d + e*x^(1/3)]^3 +
6*e^3*x*Log[-((e*x^(1/3))/d)] - 18*e^3*x*Log[d + e*x^(1/3)]*Log[-((e*x^(1/
3))/d)] + 6*e^3*x*Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 6*e^3*x*(-3
 + 2*Log[d + e*x^(1/3)])*PolyLog[2, 1 + (e*x^(1/3))/d] - 12*e^3*x*PolyLog[
3, 1 + (e*x^(1/3))/d]))/(2*d^3*x)
```

Rubi [A] (warning: unable to verify)

Time = 3.39 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.85, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2904, 2845, 2858, 25, 27, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$$

$\Big\downarrow$ 2904

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^{4/3}}d\sqrt[3]{x}$$

$\Big\downarrow$ 2845

$$3\left(ben\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{\left(d+e\sqrt[3]{x}\right)x}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{3x}\right)$$

$\Big\downarrow$ 2858

$$3\left(bn\int \frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{x^{4/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{3x}\right)$$

$\Big\downarrow$ 25

$$3\left(-bn\int -\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{x^{4/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{3x}\right)$$

$\Big\downarrow$ 27

$$3\left(-be^3n\int -\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{3x}\right)$$

$\Big\downarrow$ 2789

$$3\left(-be^3n\left(\frac{\int -\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^3x}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int \frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{3x}\right)$$

$\Big\downarrow$ 2756

$$3\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2x^{2/3}}-bn\int \frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int \frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d\right.\right.\right.}{3x}\right.$$

$\Big\downarrow$ 2789

$$3\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2x^{2/3}}-bn\left(\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^2x^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{ex^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}\right)}{d}+\frac{\frac{\int \frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^2x^{2/3}}d\left(d+\right.}{d}}{}\right.\right.$$

$\Big\downarrow$ 2751

$$3\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2x^{2/3}}-bn\left(\frac{-\frac{bn\int -\frac{1}{e\sqrt[3]{x}}d\left(d+e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de\sqrt[3]{x}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{ex^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}\right)}{d}+\right.\right.$$

$\Big\downarrow$ 16

$$3\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2x^{2/3}}-bn\left(\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{ex^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\frac{bn\log\left(-e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de\sqrt[3]{x}}}{d}\right)}{d}+\frac{\int \frac{\left(a+b\log\right.}{e^2}}{}\right.\right.$$

$\Big\downarrow$ 2755

$$3\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2x^{2/3}}-bn\left(\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{ex^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\frac{bn\log\left(-e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de\sqrt[3]{x}}}{d}\right)}{d}+\frac{-\frac{2bn\int -}{}}{}\right.\right.$$

$\Big\downarrow$ 2754

$$3\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2x^{2/3}}-bn\left(\frac{\int-\frac{a+b\log\left(cx^{n/3}\right)}{ex^{2/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\frac{bn\log\left(-e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de\sqrt[3]{x}}}{d}\right)}{d}+\frac{-\frac{2bn\left(bn\cdots\right.}{\cdots}}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \text{ 2779}$$

$$3\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2x^{2/3}}-bn\left(\frac{\frac{bn\int\frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)}{\sqrt[3]{x}}d\left(d+e\sqrt[3]{x}\right)}{d}-\frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(\cdots\right.}{de\cdots}}{d}\cdots\right.}{d}\cdots\right.\right.$$

$$\Big\downarrow \text{ 2821}$$

$$3\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2x^{2/3}}-bn\left(\frac{\frac{bn\int\frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)}{\sqrt[3]{x}}d\left(d+e\sqrt[3]{x}\right)}{d}-\frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(\cdots\right.}{de\cdots}}{d}\cdots\right.}{d}\cdots\right.\right.$$

$$\Big\downarrow \text{ 2838}$$

$$3\left(-be^3n\left(\frac{\frac{\frac{2bn\left(\text{PolyLog}\left(2,\frac{d}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{n/3}\right)\right)-bn\int\frac{\text{PolyLog}\left(2,\frac{d}{\sqrt[3]{x}}\right)}{\sqrt[3]{x}}d\left(d+e\sqrt[3]{x}\right)\right)}{d}-\frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{n/3}\right)\right)^2}{d}}{d}+\frac{-\frac{2bn\left(-\log\left(\right.\right.}{}}{}}{d}\right.\right.$$

$\Big\downarrow$ 7143

$$3\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2x^{2/3}}-bn\left(\frac{\frac{bn\log\left(-e\sqrt[3]{x}\right)}{d}-\frac{\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de\sqrt[3]{x}}}{d}+\frac{\frac{bn\,\text{PolyLog}\left(2,\frac{d}{\sqrt[3]{x}}\right)}{d}-\frac{\log\left(1-\frac{d}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{n}\right.\right.}{d}}{d}\right.}{d}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

output
```
3*(-1/3*(a + b*Log[c*(d + e*x^(1/3))^n])^3/x - b*e^3*n*(((a + b*Log[c*x^(n
/3)])^2/(2*e^2*x^(2/3)) - b*n*(((b*n*Log[-(e*x^(1/3))])/d - ((d + e*x^(1/3
))*(a + b*Log[c*x^(n/3)]))/(d*e*x^(1/3)))/d + (-((Log[1 - d/x^(1/3)]*(a +
b*Log[c*x^(n/3)]))/d) + (b*n*PolyLog[2, d/x^(1/3)])/d)/d))/d + ((-(((d + e
*x^(1/3))*(a + b*Log[c*x^(n/3)])^2)/(d*e*x^(1/3))) - (2*b*n*(-(Log[1 - (d
+ e*x^(1/3))/d]*(a + b*Log[c*x^(n/3)])) - b*n*PolyLog[2, (d + e*x^(1/3))/d
]))/d)/d + (-((Log[1 - d/x^(1/3)]*(a + b*Log[c*x^(n/3)])^2)/d) + (2*b*n*((
a + b*Log[c*x^(n/3)])*PolyLog[2, d/x^(1/3)] + b*n*PolyLog[3, d/x^(1/3)]))/
d)/d)/d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c
)^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b^3*d^3*x^(2/3)*log((e*x^(1/3) + d)^n)^3 + (6*b^3*e^3*n*x^(5/3)*lo
g(e*x^(1/3) + d) - 6*b^3*d*e^2*n*x^(4/3) + 3*b^3*d^2*e*n*x - 2*(b^3*e^3*n*
x*log(x) - 3*b^3*d^3*log(c) - 3*a*b^2*d^3)*x^(2/3))*log((e*x^(1/3) + d)^n)
^2)/(d^3*x^(5/3)) + integrate(1/3*(3*(b^3*d^3*e*log(c)^3 + 3*a*b^2*d^3*e*l
og(c)^2 + 3*a^2*b*d^3*e*log(c) + a^3*d^3*e)*x^(5/3) + 3*(b^3*d^4*log(c)^3
+ 3*a*b^2*d^4*log(c)^2 + 3*a^2*b*d^4*log(c) + a^3*d^4)*x^(4/3) + (6*b^3*e^
4*n^2*x^(8/3)*log(e*x^(1/3) + d) - 6*b^3*d*e^3*n^2*x^(7/3) + 3*b^3*d^2*e^2
*n^2*x^2 + 9*(b^3*d^3*e*log(c)^2 + 2*a*b^2*d^3*e*log(c) + a^2*b*d^3*e)*x^(
5/3) + 9*(b^3*d^4*log(c)^2 + 2*a*b^2*d^4*log(c) + a^2*b*d^4)*x^(4/3) - 2*(
b^3*e^4*n^2*x^2*log(x) - 3*(b^3*d^3*e*n*log(c) + a*b^2*d^3*e*n)*x)*x^(2/3)
)*log((e*x^(1/3) + d)^n))/(d^3*e*x^(11/3) + d^4*x^(10/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)\right)^3}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$$

$$=\frac{6x^{\frac{2}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)^2 b^3 d\,e^2 n+12x^{\frac{2}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)a\,b^2 d\,e^2 n-6x^{\frac{2}{3}}\log\left(\left(x^{\frac{1}{3}}e+d\right)^n c\right)b^3 d\,e^2 n^2+}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x)
```

output
```
(6*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d*e**2*n + 12*x**(2/3)*log(
(x**(1/3)*e + d)**n*c)*a*b**2*d*e**2*n - 6*x**(2/3)*log((x**(1/3)*e + d)**
n*c)*b**3*d*e**2*n**2 + 6*x**(2/3)*a**2*b*d*e**2*n - 6*x**(2/3)*a*b**2*d*e
**2*n**2 - 3*x**(1/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**2*e*n - 6*x**(
1/3)*log((x**(1/3)*e + d)**n*c)*a*b**2*d**2*e*n - 3*x**(1/3)*a**2*b*d**2*e
*n + 2*int(log((x**(1/3)*e + d)**n*c)**2/(x**(1/3)*e*x + d*x),x)*b**3*d*e*
*3*n*x + 4*int(log((x**(1/3)*e + d)**n*c)/(x**(1/3)*e*x + d*x),x)*a*b**2*d
*e**3*n*x - 6*int(log((x**(1/3)*e + d)**n*c)/(x**(1/3)*e*x + d*x),x)*b**3*
d*e**3*n**2*x + 6*log(x**(1/3))*a**2*b*e**3*n*x - 18*log(x**(1/3))*a*b**2*
e**3*n**2*x + 6*log(x**(1/3))*b**3*e**3*n**3*x - 2*log((x**(1/3)*e + d)**n
*c)**3*b**3*d**3 - 6*log((x**(1/3)*e + d)**n*c)**2*a*b**2*d**3 - 6*log((x*
*(1/3)*e + d)**n*c)*a**2*b*d**3 - 6*log((x**(1/3)*e + d)**n*c)*a**2*b*e**3
*x + 18*log((x**(1/3)*e + d)**n*c)*a*b**2*e**3*n*x - 6*log((x**(1/3)*e + d
)**n*c)*b**3*e**3*n**2*x - 2*a**3*d**3)/(2*d**3*x)
```

3.462 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 765

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

output
```
-1/20*b^3*e^3*n^3/d^3/x+3/10*b^3*e^4*n^3/d^4/x^(2/3)-71/40*b^3*e^5*n^3/d^5
/x^(1/3)+71/40*b^3*e^6*n^3*ln(d+e*x^(1/3))/d^6-3/20*b^2*e^2*n^2*(a+b*ln(c*
(d+e*x^(1/3))^n))/d^2/x^(4/3)+9/20*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))
/d^3/x-47/40*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))/d^4/x^(2/3)+77/20*b^2
*e^5*n^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^6/x^(1/3)+77/20*b^2*e
^6*n^2*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^6-3/10*b*e*n*(a
+b*ln(c*(d+e*x^(1/3))^n))^2/d/x^(5/3)+3/8*b*e^2*n*(a+b*ln(c*(d+e*x^(1/3))^
n))^2/d^2/x^(4/3)-1/2*b*e^3*n*(a+b*ln(c*(d+e*x^(1/3))^n))^2/d^3/x+3/4*b*e^
4*n*(a+b*ln(c*(d+e*x^(1/3))^n))^2/d^4/x^(2/3)-3/2*b*e^5*n*(d+e*x^(1/3))*(a
+b*ln(c*(d+e*x^(1/3))^n))^2/d^6/x^(1/3)-3/2*b*e^6*n*ln(1-d/(d+e*x^(1/3)))*
(a+b*ln(c*(d+e*x^(1/3))^n))^2/d^6-1/2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2+3*
b^2*e^6*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)/d^6-15/8*b^3*e^6*
n^3*ln(x)/d^6-77/20*b^3*e^6*n^3*polylog(2,d/(d+e*x^(1/3)))/d^6+3*b^2*e^6*n
^2*(a+b*ln(c*(d+e*x^(1/3))^n))*polylog(2,d/(d+e*x^(1/3)))/d^6+3*b^3*e^6*n^
3*polylog(2,1+e*x^(1/3)/d)/d^6+3*b^3*e^6*n^3*polylog(3,d/(d+e*x^(1/3)))/d^
6
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 1074, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

output
```
-1/40*(12*b*d^5*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x
^(1/3))^n])^2 - 15*b*d^4*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log
[c*(d + e*x^(1/3))^n])^2 + 20*b*d^3*e^3*n*x*(a - b*n*Log[d + e*x^(1/3)] +
b*Log[c*(d + e*x^(1/3))^n])^2 - 30*b*d^2*e^4*n*x^(4/3)*(a - b*n*Log[d + e*
x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d*e^5*n*x^(5/3)*(a - b*n*L
og[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d^6*n*Log[d + e*x
^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 60*b
*e^6*n*x^2*Log[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e
*x^(1/3))^n])^2 + 20*d^6*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1
/3))^n])^3 + 20*b*e^6*n*x^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x
^(1/3))^n])^2*Log[x] + b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d +
e*x^(1/3))^n])*(6*d^4*e^2*x^(2/3) - 18*d^3*e^3*x + 47*d^2*e^4*x^(4/3) - 15
4*d*e^5*x^(5/3) + 60*(d^6 - e^6*x^2)*Log[d + e*x^(1/3)]^2 - 274*e^6*x^2*Lo
g[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*(12*d^5*e*x^(1/3) - 15*d^4*e^2*
x^(2/3) + 20*d^3*e^3*x - 30*d^2*e^4*x^(4/3) + 60*d*e^5*x^(5/3) + 137*e^6*x
^2 + 60*e^6*x^2*Log[-((e*x^(1/3))/d)]) + 120*e^6*x^2*PolyLog[2, 1 + (e*x^(
1/3))/d]) + b^3*n^3*(3*d^4*e^2*x^(2/3)*(2 - 5*Log[d + e*x^(1/3)])*Log[d +
e*x^(1/3)] + 12*d^5*e*x^(1/3)*Log[d + e*x^(1/3)]^2 + 20*d^6*Log[d + e*x^(1
/3)]^3 + 2*d^3*e^3*x*(1 - 9*Log[d + e*x^(1/3)] + 10*Log[d + e*x^(1/3)]^2)
- d^2*e^4*x^(4/3)*(12 - 47*Log[d + e*x^(1/3)] + 30*Log[d + e*x^(1/3)]^2...
```

Rubi [A] (warning: unable to verify)

Time = 11.00 (sec) , antiderivative size = 1382, normalized size of antiderivative = 1.81, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2845}$$

$$3\left(\frac{1}{2}ben\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{\left(d+e\sqrt[3]{x}\right)x^2}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{6x^2}\right)$$

$$\Big\downarrow \text{2858}$$

$$3\left(\frac{1}{2}bn\int \frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{x^{7/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{6x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$3\left(\frac{1}{2}be^6n\int \frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^6x^{7/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{6x^2}\right)$$

$$\Big\downarrow \text{2789}$$

$$3\left(\frac{1}{2}be^6n\left(\frac{\int \frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^6x^2}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int -\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^5x^2}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{6x^2}\right)$$

$$\Big\downarrow \text{2756}$$

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^5x^2}d\left(d+e\sqrt[3]{x}\right)-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}}{d}+\frac{\int -\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^5x^2}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\frac{\left(a+b\log\right.}{}\right.$$

$$\Big\downarrow \text{2789}$$

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^5x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}\right)-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}}{d}+\frac{\int -\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^5x^{5/3}}}{d}\right.\right.$$

$\downarrow$ 2756

$$3\left(\frac{1}{2}be^6n\left(\frac{\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{4e^4x^{4/3}}-\frac{1}{2}bn\int\frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn \cdots}{d \cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 54

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\int\left(-\frac{1}{d^4e\sqrt[3]{x}}+\frac{1}{d^4\sqrt[3]{x}}+\frac{1}{d^3e^2x^{2/3}}-\frac{1}{d^2e^3x}+\frac{1}{de^4x^{4/3}}\right)d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x \cdots}\right.}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2009

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{5/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right) \cdots}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2789

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\frac{\int\frac{a+b\log\left(cx^{n/3}\right)}{e^4x^{4/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int-\frac{a+b\log\left(cx^{n/3}\right)}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4} \cdots\right.}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2756

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\frac{-\frac{1}{3}bn\int -\frac{1}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}\cdots\right.}{\cdots}\cdots\right.}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 54$$

$$3\left(\frac{1}{2}be^6n\left(\frac{\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{4e^4x^{4/3}}-\frac{1}{2}bn\left(\frac{-\frac{1}{3}bn\int\left(-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{d^3\sqrt[3]{x}}+\frac{1}{d^2e^2x^{2/3}}-\frac{1}{de^3x}\right)d\left(d+e\sqrt[3]{x}\right)-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}}{d}+\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^3x^{4/3}}d\left(d+\cdots\right.}{d}\cdots\right.}{d}\cdots}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2009$$

$$3\left(\frac{1}{2}be^6n\left(\frac{\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{4e^4x^{4/3}}-\frac{1}{2}bn\left(\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^3x^{4/3}}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}-\frac{1}{3}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^3}-\frac{1}{d^2e\sqrt[3]{x}}+\frac{\cdots}{2de^2\cdots}\cdots\right.}{d}\cdots\right.}{d}\cdots}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2789$$

$$3\left(\frac{1}{2}be^6n\left(\frac{\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{4e^4x^{4/3}}-\frac{1}{2}bn\left(\frac{\frac{\int -\frac{a+b\log\left(cx^{n/3}\right)}{e^3x}d\left(d+e\sqrt[3]{x}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{n/3}\right)}{e^2x}d\left(d+e\sqrt[3]{x}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{n/3}\right)}{3e^3x}-\frac{1}{3}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^3}\right.}{\cdots}\right.}{d}\right.\right.$$

$\downarrow$ 2756

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 54

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 2009

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 2789

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 2751

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn\ \cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$$\downarrow 16$$

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn\ \cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$$\downarrow 2755$$

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{(a+b\log(cx^{n/3}))^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log(cx^{n/3})}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log(d+e\sqrt[3]{x})}{d^4}-\frac{\log(-e\sqrt[3]{x})}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn}{}\right.}{}\right.\right.$$

$\downarrow$ 2754

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{(a+b\log(cx^{n/3}))^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log(cx^{n/3})}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log(d+e\sqrt[3]{x})}{d^4}-\frac{\log(-e\sqrt[3]{x})}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn}{}\right.}{}\right.\right.$$

$\downarrow$ 2779

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn}{\quad}\right.}{\quad}\right.\right.$$

$\Big\downarrow$ 2821

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn}{\quad}\right.}{\quad}\right.\right.$$

$\Big\downarrow$ 2838

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn}{}\right.}{}\right.\right.$$

$\downarrow$ 7143

$$3\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{n/3}\right)\right)^2}{5e^5x^{5/3}}-\frac{2}{5}bn\left(\frac{\frac{a+b\log\left(cx^{n/3}\right)}{4e^4x^{4/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+e\sqrt[3]{x}\right)}{d^4}-\frac{\log\left(-e\sqrt[3]{x}\right)}{d^4}-\frac{1}{d^3e\sqrt[3]{x}}+\frac{1}{2d^2e^2x^{2/3}}-\frac{1}{3de^3x}\right)}{d}+\frac{-\frac{1}{3}bn}{}\right.}{}\right.\right.$$

input

```
Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

output
```
3*(-1/6*(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2 + (b*e^6*n*((-1/5*(a + b*Lo
g[c*x^(n/3)])^2/(e^5*x^(5/3)) - (2*b*n*((-1/4*(b*n*(-1/3*1/(d*e^3*x) + 1/(
2*d^2*e^2*x^(2/3)) - 1/(d^3*e*x^(1/3)) + Log[d + e*x^(1/3)]/d^4 - Log[-(e*
x^(1/3))]/d^4)) + (a + b*Log[c*x^(n/3)])/(4*e^4*x^(4/3)))/d + ((-1/3*(b*n*
(1/(2*d*e^2*x^(2/3)) - 1/(d^2*e*x^(1/3)) + Log[d + e*x^(1/3)]/d^3 - Log[-(
e*x^(1/3))]/d^3)) - (a + b*Log[c*x^(n/3)])/(3*e^3*x))/d + ((-1/2*(b*n*(-(1
/(d*e*x^(1/3))) + Log[d + e*x^(1/3)]/d^2 - Log[-(e*x^(1/3))]/d^2)) + (a +
b*Log[c*x^(n/3)])/(2*e^2*x^(2/3)))/d + (((b*n*Log[-(e*x^(1/3))])/d - ((d +
 e*x^(1/3))*(a + b*Log[c*x^(n/3)]))/(d*e*x^(1/3)))/d + (-((Log[1 - d/x^(1/
3)]*(a + b*Log[c*x^(n/3)]))/d) + (b*n*PolyLog[2, d/x^(1/3)])/d)/d)/d)/d)/d
))/5)/d + (((a + b*Log[c*x^(n/3)])^2/(4*e^4*x^(4/3)) - (b*n*((-1/3*(b*n*(1
/(2*d*e^2*x^(2/3)) - 1/(d^2*e*x^(1/3)) + Log[d + e*x^(1/3)]/d^3 - Log[-(e*
x^(1/3))]/d^3)) - (a + b*Log[c*x^(n/3)])/(3*e^3*x))/d + ((-1/2*(b*n*(-(1/(
d*e*x^(1/3))) + Log[d + e*x^(1/3)]/d^2 - Log[-(e*x^(1/3))]/d^2)) + (a + b*
Log[c*x^(n/3)])/(2*e^2*x^(2/3)))/d + (((b*n*Log[-(e*x^(1/3))])/d - ((d + e
*x^(1/3))*(a + b*Log[c*x^(n/3)]))/(d*e*x^(1/3)))/d + (-((Log[1 - d/x^(1/3)
]*(a + b*Log[c*x^(n/3)]))/d) + (b*n*PolyLog[2, d/x^(1/3)])/d)/d)/d)/d))/2)
/d + ((-1/3*(a + b*Log[c*x^(n/3)])^2/(e^3*x) - (2*b*n*((-1/2*(b*n*(-(1/(d*
e*x^(1/3))) + Log[d + e*x^(1/3)]/d^2 - Log[-(e*x^(1/3))]/d^2)) + (a + b*Lo
g[c*x^(n/3)])/(2*e^2*x^(2/3)))/d + (((b*n*Log[-(e*x^(1/3))])/d - ((d + ...
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{3}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c
)^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**3,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="maxima")
```

output
```
-1/2*b^3*log((e*x^(1/3) + d)^n)^3/x^2 + integrate(1/2*((b^3*e*n*x + 6*(b^3
*e*log(c) + a*b^2*e)*x + 6*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((e*x^(1/3
) + d)^n)^2 + 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) +
a^3*e)*x + 6*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log
(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 2*(b
^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(e
*x^4 + d*x^(11/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^3,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x)
```

output
```
(45*x**(2/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**4*e**2*n - 180*x**(2/3)
*log((x**(1/3)*e + d)**n*c)**2*b**3*d*e**5*n*x + 90*x**(2/3)*log((x**(1/3)
*e + d)**n*c)*a*b**2*d**4*e**2*n - 360*x**(2/3)*log((x**(1/3)*e + d)**n*c)
*a*b**2*d*e**5*n*x - 18*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**3*d**4*e**2
*n**2 + 462*x**(2/3)*log((x**(1/3)*e + d)**n*c)*b**3*d*e**5*n**2*x + 45*x*
*(2/3)*a**2*b*d**4*e**2*n - 180*x**(2/3)*a**2*b*d*e**5*n*x - 18*x**(2/3)*a
*b**2*d**4*e**2*n**2 + 462*x**(2/3)*a*b**2*d*e**5*n**2*x - 213*x**(2/3)*b*
*3*d*e**5*n**3*x - 36*x**(1/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**5*e*n
 + 90*x**(1/3)*log((x**(1/3)*e + d)**n*c)**2*b**3*d**2*e**4*n*x - 72*x**(1
/3)*log((x**(1/3)*e + d)**n*c)*a*b**2*d**5*e*n + 180*x**(1/3)*log((x**(1/3
)*e + d)**n*c)*a*b**2*d**2*e**4*n*x - 141*x**(1/3)*log((x**(1/3)*e + d)**n
*c)*b**3*d**2*e**4*n**2*x - 36*x**(1/3)*a**2*b*d**5*e*n + 90*x**(1/3)*a**2
*b*d**2*e**4*n*x - 141*x**(1/3)*a*b**2*d**2*e**4*n**2*x + 36*x**(1/3)*b**3
*d**2*e**4*n**3*x - 60*int(log((x**(1/3)*e + d)**n*c)**2/(x**(1/3)*e*x + d
*x),x)*b**3*d*e**6*n*x**2 - 120*int(log((x**(1/3)*e + d)**n*c)/(x**(1/3)*e
*x + d*x),x)*a*b**2*d*e**6*n*x**2 + 274*int(log((x**(1/3)*e + d)**n*c)/(x*
*(1/3)*e*x + d*x),x)*b**3*d*e**6*n**2*x**2 - 180*log(x**(1/3))*a**2*b*e**6
*n*x**2 + 822*log(x**(1/3))*a*b**2*e**6*n**2*x**2 - 675*log(x**(1/3))*b**3
*e**6*n**3*x**2 - 60*log((x**(1/3)*e + d)**n*c)**3*b**3*d**6 - 180*log((x*
*(1/3)*e + d)**n*c)**2*a*b**2*d**6 - 60*log((x**(1/3)*e + d)**n*c)**2*b...
```

3.463 $\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 138

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

output
```
1/4*b*d^5*n*x^(2/3)/e^5-1/8*b*d^4*n*x^(4/3)/e^4+1/12*b*d^3*n*x^2/e^3-1/16*
b*d^2*n*x^(8/3)/e^2+1/20*b*d*n*x^(10/3)/e-1/24*b*n*x^4-1/4*b*d^6*n*ln(d+e*
x^(2/3))/e^6+1/4*x^4*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{ax^4}{4}-\frac{1}{4}ben\left(-\frac{d^5x^{2/3}}{e^6}+\frac{d^4x^{4/3}}{2e^5}-\frac{d^3x^2}{3e^4}+\frac{d^2x^{8/3}}{4e^3}-\frac{dx^{10/3}}{5e^2}+\frac{x^4}{6e}+\frac{d^6\log\left(d+ex^{2/3}\right)}{e^7}\right)+\frac{1}{4}bx^4\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

output
```
(a*x^4)/4 - (b*e*n*(-((d^5*x^(2/3))/e^6) + (d^4*x^(4/3))/(2*e^5) - (d^3*x^
2)/(3*e^4) + (d^2*x^(8/3))/(4*e^3) - (d*x^(10/3))/(5*e^2) + x^4/(6*e) + (d
^6*Log[d + e*x^(2/3)])/e^7))/4 + (b*x^4*Log[c*(d + e*x^(2/3))^n])/4
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int x^{10/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx^{2/3}$$

$$\Big\downarrow \text{2842}$$

$$\frac{3}{2}\left(\frac{1}{6}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{6}ben\int\frac{x^4}{d+ex^{2/3}}dx^{2/3}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{3}{2}\left(\frac{1}{6}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{6}ben\int\left(\frac{d^6}{e^6\left(d+ex^{2/3}\right)}-\frac{d^5}{e^6}+\frac{x^{2/3}d^4}{e^5}-\frac{x^{4/3}d^3}{e^4}+\frac{x^2d^2}{e^3}-\frac{x^{8/3}d}{e^2}+\frac{x^{10}}{e}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\left(\frac{1}{6}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{6}ben\left(\frac{d^6\log\left(d+ex^{2/3}\right)}{e^7}-\frac{d^5x^{2/3}}{e^6}+\frac{d^4x^{4/3}}{2e^5}-\frac{d^3x^2}{3e^4}+\frac{d^2x^{8/3}}{4e^3}-\frac{dx^{10/3}}{5e^2}\right.\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

output
```
(3*(-1/6*(b*e*n*(-((d^5*x^(2/3))/e^6) + (d^4*x^(4/3))/(2*e^5) - (d^3*x^2)/
(3*e^4) + (d^2*x^(8/3))/(4*e^3) - (d*x^(10/3))/(5*e^2) + x^4/(6*e) + (d^6*
Log[d + e*x^(2/3)])/e^7)) + (x^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/6))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3 \left(a + b \ln \left(c \left(d + e\, x^{\frac{2}{3}}\right)^{n}\right)\right) dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right) \, dx = \frac{60 \, b e^6 x^4 \log \left(c\right) + 20 \, b d^3 e^3 n x^2 - 10 \, \left(b e^6 n - 6 \, a e^6\right) x^4 + 60 \, \left(b e^6 n x^4 - b d^6 n\right) \log}{240 \, e}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

output
```
1/240*(60*b*e^6*x^4*log(c) + 20*b*d^3*e^3*n*x^2 - 10*(b*e^6*n - 6*a*e^6)*x
^4 + 60*(b*e^6*n*x^4 - b*d^6*n)*log(e*x^(2/3) + d) - 15*(b*d^2*e^4*n*x^2 -
 4*b*d^5*e*n)*x^(2/3) + 6*(2*b*d*e^5*n*x^3 - 5*b*d^4*e^2*n*x)*x^(1/3))/e^6
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right) \, dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right) \, dx = \frac{1}{4} \, b x^4 \log \left(\left(e x^{\frac{2}{3}} + d\right)^n c\right) + \frac{1}{4} \, a x^4 - \frac{1}{240} \, b e n \left(\frac{60 \, d^6 \log \left(e x^{\frac{2}{3}} + d\right)}{e^7} + \frac{10 \, e^5 x^4 - 12 \, d e^4 x^{\frac{10}{3}} + 15 \, d^2 e^3 x^{\frac{8}{3}} - 20 \, d^3 e^2 x^2 + 30 \, d^4 e x^{\frac{4}{3}} - 60 \, d^5 x^{\frac{2}{3}}}{e^6}\right)$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

output
```
1/4*b*x^4*log((e*x^(2/3) + d)^n*c) + 1/4*a*x^4 - 1/240*b*e*n*(60*d^6*log(e
*x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) -
 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(110) = 220$.

Time = 0.18 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.84

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(c\right)+\frac{1}{4}\,ax^4$$
$$+\frac{1}{240}\,bn\left(\frac{60\left(ex^{\frac{2}{3}}+d\right)^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^6}-\frac{360\left(ex^{\frac{2}{3}}+d\right)^5d\log\left(ex^{\frac{2}{3}}+d\right)}{e^6}+\frac{900\left(ex^{\frac{2}{3}}+d\right)^4d^2\log\left(ex^{\frac{2}{3}}\right.}{e^6}\right.$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

output
```
1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*b*n*(60*(e*x^(2/3) + d)^6*log(e*x^(2/
3) + d)/e^6 - 360*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 + 900*(e*x^(2
/3) + d)^4*d^2*log(e*x^(2/3) + d)/e^6 - 1200*(e*x^(2/3) + d)^3*d^3*log(e*x
^(2/3) + d)/e^6 + 900*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 - 10*(e
*x^(2/3) + d)^6/e^6 + 72*(e*x^(2/3) + d)^5*d/e^6 - 225*(e*x^(2/3) + d)^4*d
^2/e^6 + 400*(e*x^(2/3) + d)^3*d^3/e^6 - 450*(e*x^(2/3) + d)^2*d^4/e^6 - 3
60*((e*x^(2/3) + d)*log(e*x^(2/3) + d) - e*x^(2/3) - d)*d^5/e^6)
```

Mupad [B] (verification not implemented)

Time = 14.59 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{a\,x^4}{4}-\frac{b\,n\,x^4}{24}+\frac{b\,x^4\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{4}+\frac{b\,d\,n\,x^{10/3}}{20\,e}-\frac{b\,d^6\,n\,\ln\left(d+e\,x^{2/3}\right)}{4\,e^6}+\frac{b\,d^3\,n\,x^2}{12\,e^3}-\frac{b\,d^2\,n\,x^{8/3}}{16\,e^2}-\frac{b\,d^4\,n\,x^{4/3}}{8\,e^4}+\frac{b\,d^5\,n\,x^{2/3}}{4\,e^5}$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

output
```
(a*x^4)/4 - (b*n*x^4)/24 + (b*x^4*log(c*(d + e*x^(2/3))^n))/4 + (b*d*n*x^(
10/3))/(20*e) - (b*d^6*n*log(d + e*x^(2/3)))/(4*e^6) + (b*d^3*n*x^2)/(12*e
^3) - (b*d^2*n*x^(8/3))/(16*e^2) - (b*d^4*n*x^(4/3))/(8*e^4) + (b*d^5*n*x^
(2/3))/(4*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{60x^{\frac{2}{3}}b\,d^5en-15x^{\frac{8}{3}}b\,d^2e^4n-30x^{\frac{4}{3}}b\,d^4e^2n+12x^{\frac{10}{3}}bd\,e^5n-60\log\left(\left(x^{\frac{2}{3}}e+d\right)\right.}{24}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x)
```

output
```
(60*x**(2/3)*b*d**5*e*n - 15*x**(2/3)*b*d**2*e**4*n*x**2 - 30*x**(1/3)*b*d
**4*e**2*n*x + 12*x**(1/3)*b*d*e**5*n*x**3 - 60*log((x**(2/3)*e + d)**n*c)
*b*d**6 + 60*log((x**(2/3)*e + d)**n*c)*b*e**6*x**4 + 60*a*e**6*x**4 + 20*
b*d**3*e**3*n*x**2 - 10*b*e**6*n*x**4)/(240*e**6)
```

3.464 $\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right) \, dx = -\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3+\frac{2bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

output

```
-2/3*b*d^4*n*x^(1/3)/e^4+2/9*b*d^3*n*x/e^3-2/15*b*d^2*n*x^(5/3)/e^2+2/21*b
*d*n*x^(7/3)/e-2/27*b*n*x^3+2/3*b*d^(9/2)*n*arctan(e^(1/2)*x^(1/3)/d^(1/2)
)/e^(9/2)+1/3*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right) \, dx = \frac{ax^3}{3}-\frac{2}{9}ben\left(\frac{3d^4\sqrt[3]{x}}{e^5}-\frac{d^3x}{e^4}+\frac{3d^2x^{5/3}}{5e^3}-\frac{3dx^{7/3}}{7e^2}+\frac{x^3}{3e}-\frac{3d^{9/2}\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{11/2}}\right)+\frac{1}{3}bx^3\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

output
```
(a*x^3)/3 - (2*b*e*n*((3*d^4*x^(1/3))/e^5 - (d^3*x)/e^4 + (3*d^2*x^(5/3))/
(5*e^3) - (3*d*x^(7/3))/(7*e^2) + x^3/(3*e) - (3*d^(9/2)*ArcTan[(Sqrt[e]*x
^(1/3))/Sqrt[d]])/e^(11/2)))/9 + (b*x^3*Log[c*(d + e*x^(2/3))^n])/3
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2905, 864, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{ 2905}$$

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{2}{9}ben\int \frac{x^{8/3}}{d+ex^{2/3}}dx$$

$$\Big\downarrow \text{ 864}$$

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{2}{3}ben\int \frac{x^{10/3}}{d+ex^{2/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{2}{3}ben\int \left(-\frac{d^5}{e^5\left(d+ex^{2/3}\right)}+\frac{d^4}{e^5}-\frac{x^{2/3}d^3}{e^4}+\frac{x^{4/3}d^2}{e^3}-\frac{x^2d}{e^2}+\frac{x^{8/3}}{e}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{2}{3}ben\left(-\frac{d^{9/2}\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{11/2}}+\frac{d^4\sqrt[3]{x}}{e^5}-\frac{d^3x}{3e^4}+\frac{d^2x^{5/3}}{5e^3}-\frac{dx^{7/3}}{7e^2}+\frac{x^3}{9e}\right)$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

output
```
(-2*b*e*n*((d^4*x^(1/3))/e^5 - (d^3*x)/(3*e^4) + (d^2*x^(5/3))/(5*e^3) - (
d*x^(7/3))/(7*e^2) + x^3/(9*e) - (d^(9/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]
])/e^(11/2)))/3 + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/3
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.59

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx = \left[\frac{315\,be^4nx^3\log\left(ex^{\frac{2}{3}}+d\right)+315\,be^4x^3\log\left(c\right)-126\,bd^2e^2nx^{\frac{5}{3}}+315\,bd^4n\sqrt{\cdots}}{\cdots}\right.$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

output
```
[1/945*(315*b*e^4*n*x^3*log(e*x^(2/3) + d) + 315*b*e^4*x^3*log(c) - 126*b*
d^2*e^2*n*x^(5/3) + 315*b*d^4*n*sqrt(-d/e)*log((e^3*x^2 - 2*d*e^2*x*sqrt(-
d/e) - d^3 + 2*(e^3*x*sqrt(-d/e) + d^2*e)*x^(2/3) - 2*(d*e^2*x - d^2*e*sqr
t(-d/e))*x^(1/3))/(e^3*x^2 + d^3)) + 210*b*d^3*e*n*x - 35*(2*b*e^4*n - 9*a
*e^4)*x^3 + 90*(b*d*e^3*n*x^2 - 7*b*d^4*n)*x^(1/3))/e^4, 1/945*(315*b*e^4*
n*x^3*log(e*x^(2/3) + d) + 315*b*e^4*x^3*log(c) - 126*b*d^2*e^2*n*x^(5/3)
+ 630*b*d^4*n*sqrt(d/e)*arctan(e*x^(1/3)*sqrt(d/e)/d) + 210*b*d^3*e*n*x -
35*(2*b*e^4*n - 9*a*e^4)*x^3 + 90*(b*d*e^3*n*x^2 - 7*b*d^4*n)*x^(1/3))/e^4
]
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3+\frac{1}{945}\left(315\,x^3\log\left(ex^{\frac{2}{3}}+d\right)+2\,e\left(\frac{315\,d^5\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}e^5}-\frac{35\,e^8x^3-45\,de^7x^{\frac{7}{3}}+63\,d^2e^6x^{\frac{5}{3}}-105\,d^3e^5x+}{e^9}\right.\right.$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

output
```
1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/945*(315*x^3*log(e*x^(2/3) + d) + 2*e*(31
5*d^5*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^5) - (35*e^8*x^3 - 45*d*e^7
*x^(7/3) + 63*d^2*e^6*x^(5/3) - 105*d^3*e^5*x + 315*d^4*e^4*x^(1/3))/e^9))
*b*n
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int x^2\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)\,dx=\frac{630\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)b\,d^4n-126x^{\frac{5}{3}}b\,d^2e^3n-630x^{\frac{1}{3}}b\,d^4en+90x^{\frac{7}{3}}bd\,e^4n+}{945e^5}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x)
```

output
```
(630*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b*d**4*n - 126*x
**(2/3)*b*d**2*e**3*n*x - 630*x**(1/3)*b*d**4*e*n + 90*x**(1/3)*b*d*e**4*n
*x**2 + 315*log((x**(2/3)*e + d)**n*c)*b*e**5*x**3 + 315*a*e**5*x**3 + 210
*b*d**3*e**2*n*x - 70*b*e**5*n*x**3)/(945*e**5)
```

3.465 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

output
```
-1/2*b*d^2*n*x^(2/3)/e^2+1/4*b*d*n*x^(4/3)/e-1/6*b*n*x^2+1/2*b*d^3*n*ln(d+
e*x^(2/3))/e^3+1/2*x^2*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}+\frac{ax^2}{2}-\frac{1}{6}bnx^2+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{1}{2}bx^2\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

output
```
-1/2*(b*d^2*n*x^(2/3))/e^2 + (b*d*n*x^(4/3))/(4*e) + (a*x^2)/2 - (b*n*x^2)
/6 + (b*d^3*n*Log[d + e*x^(2/3)])/(2*e^3) + (b*x^2*Log[c*(d + e*x^(2/3))^n
])/2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)dx^{2/3}$$

$$\Big\downarrow \text{2842}$$

$$\frac{3}{2}\left(\frac{1}{3}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}ben\int\frac{x^2}{d+ex^{2/3}}dx^{2/3}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{3}{2}\left(\frac{1}{3}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}ben\int\left(-\frac{d^3}{e^3\left(d+ex^{2/3}\right)}+\frac{d^2}{e^3}-\frac{x^{2/3}d}{e^2}+\frac{x^{4/3}}{e}\right)dx^{2/3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\left(\frac{1}{3}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}ben\left(-\frac{d^3\log\left(d+ex^{2/3}\right)}{e^4}+\frac{d^2x^{2/3}}{e^3}-\frac{dx^{4/3}}{2e^2}+\frac{x^2}{3e}\right)\right)$$

input
```
Int[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

output
```
(3*(-1/3*(b*e*n*((d^2*x^(2/3))/e^3 - (d*x^(4/3))/(2*e^2) + x^2/(3*e) - (d^
3*Log[d + e*x^(2/3)])/e^4)) + (x^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/3))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{6\,be^3x^2\log\left(c\right)+3\,bde^2nx^{\frac{4}{3}}-6\,bd^2enx^{\frac{2}{3}}-2\,(be^3n-3\,ae^3)x^2+6\,(be^3nx^2+b}{12\,e^3}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

output
```
1/12*(6*b*e^3*x^2*log(c) + 3*b*d*e^2*n*x^(4/3) - 6*b*d^2*e*n*x^(2/3) - 2*(
b*e^3*n - 3*a*e^3)*x^2 + 6*(b*e^3*n*x^2 + b*d^3*n)*log(e*x^(2/3) + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 70.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{ax^2}{2}+b\left(-\frac{en\left(-\frac{3d^3\left(\begin{cases}\frac{x^{\frac{2}{3}}}{d} & \text{for}\ e=0\\ \frac{\log\left(d+ex^{\frac{2}{3}}\right)}{e} & \text{otherwise}\end{cases}\right)}{2e^3}+\frac{3d^2x^{\frac{2}{3}}}{2e^3}-\frac{3dx^{\frac{4}{3}}}{4e^2}+\frac{x^2}{2e}\right)}{3}+\frac{x^2\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)}{2}\right)$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

output
```
a*x**2/2 + b*(-e*n*(-3*d**3*Piecewise((x**(2/3)/d, Eq(e, 0)), (log(d + e*x
**(2/3))/e, True))/(2*e**3) + 3*d**2*x**(2/3)/(2*e**3) - 3*d*x**(4/3)/(4*e
**2) + x**2/(2*e))/3 + x**2*log(c*(d + e*x**(2/3))**n)/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int x\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)\,dx=\frac{1}{12}\,ben\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)+\frac{1}{2}\,bx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

output
```
1/12*b*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*
d^2*x^(2/3))/e^3) + 1/2*b*x^2*log((e*x^(2/3) + d)^n*c) + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int x\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)\,dx=\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{12}\left(6\,x^2\log\left(ex^{\frac{2}{3}}+d\right)+e\left(\frac{6\,d^3\log\left(\left|ex^{\frac{2}{3}}+d\right|\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\right)bn+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

output
```
1/2*b*x^2*log(c) + 1/12*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(
2/3) + d))/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*b*n + 1
/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{6}+\frac{b\,x^2\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{2}+\frac{b\,d\,n\,x^{4/3}}{4\,e}+\frac{b\,d^3\,n\,\ln\left(d+e\,x^{2/3}\right)}{2\,e^3}-\frac{b\,d^2\,n\,x^{2/3}}{2\,e^2}$$

input
```
int(x*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

output
```
(a*x^2)/2 - (b*n*x^2)/6 + (b*x^2*log(c*(d + e*x^(2/3))^n))/2 + (b*d*n*x^(4
/3))/(4*e) + (b*d^3*n*log(d + e*x^(2/3)))/(2*e^3) - (b*d^2*n*x^(2/3))/(2*e
^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{-6x^{\frac{2}{3}}b\,d^2en+3x^{\frac{4}{3}}bd\,e^2n+6\log\left(\left(x^{\frac{2}{3}}e+d\right)^nc\right)b\,d^3+6\log\left(\left(x^{\frac{2}{3}}e+d\right)^nc\right)}{12e^3}$$

input
```
int(x*(a+b*log(c*(d+e*x^(2/3))^n)),x)
```

output
```
( - 6*x**(2/3)*b*d**2*e*n + 3*x**(1/3)*b*d*e**2*n*x + 6*log((x**(2/3)*e +
d)**n*c)*b*d**3 + 6*log((x**(2/3)*e + d)**n*c)*b*e**3*x**2 + 6*a*e**3*x**2
 - 2*b*e**3*n*x**2)/(12*e**3)
```

3.466 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) dx = \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3} - \frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

output
```
2*b*d*n*x^(1/3)/e+a*x-2/3*b*n*x-2*b*d^(3/2)*n*arctan(e^(1/2)*x^(1/3)/d^(1/
2))/e^(3/2)+b*x*ln(c*(d+e*x^(2/3))^n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) dx = \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3} - \frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

input
```
Integrate[a + b*Log[c*(d + e*x^(2/3))^n],x]
```

output
```
(2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x
^(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax-\frac{2bd^{3/2}n\arctan \left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log \left(c\left(d+ex^{2/3}\right)^n\right)+\frac{2bdn\sqrt[3]{x}}{e}-\frac{2bnx}{3}$$

input
```
Int[a + b*Log[c*(d + e*x^(2/3))^n],x]
```

output
```
(2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x
^(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

method	result	size
default	$ax+bx\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)-\frac{2bnx}{3}+\frac{2bdn\,x^{\frac{1}{3}}}{e}-\frac{2bn\,d^{2}\arctan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{de}}\right)}{e\sqrt{de}}$	62
parts	$ax+bx\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)-\frac{2bnx}{3}+\frac{2bdn\,x^{\frac{1}{3}}}{e}-\frac{2bn\,d^{2}\arctan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{de}}\right)}{e\sqrt{de}}$	62

input
```
int(a+b*ln(c*(d+e*x^(2/3))^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*ln(c*(d+e*x^(2/3))^n)-2/3*b*n*x+2*b*d*n*x^(1/3)/e-2*b/e*n*d^2/(d*e
)^(1/2)*arctan(x^(1/3)*e/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.21

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)\,dx=\left[\frac{3\,benx\log\left(ex^{\frac{2}{3}}+d\right)+3\,bdn\sqrt{-\frac{d}{e}}\log\left(\frac{e^{3}x^{2}+2\,de^{2}x\sqrt{-\frac{d}{e}}-d^{3}-2\left(e^{3}x\sqrt{-\frac{d}{e}}-d^{2}e\right)x^{\frac{2}{3}}}{e^{3}x^{2}+d^{3}}\right.}{3\,e}\right.$$

input
```
integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="fricas")
```

output
```
[1/3*(3*b*e*n*x*log(e*x^(2/3) + d) + 3*b*d*n*sqrt(-d/e)*log((e^3*x^2 + 2*d
*e^2*x*sqrt(-d/e) - d^3 - 2*(e^3*x*sqrt(-d/e) - d^2*e)*x^(2/3) - 2*(d*e^2*
x + d^2*e*sqrt(-d/e))*x^(1/3))/(e^3*x^2 + d^3)) + 3*b*e*x*log(c) + 6*b*d*n
*x^(1/3) - (2*b*e*n - 3*a*e)*x)/e, 1/3*(3*b*e*n*x*log(e*x^(2/3) + d) - 6*b
*d*n*sqrt(d/e)*arctan(e*x^(1/3)*sqrt(d/e)/d) + 3*b*e*x*log(c) + 6*b*d*n*x^
(1/3) - (2*b*e*n - 3*a*e)*x)/e]
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=ax+b\left(-\frac{2en\left(\frac{3d^2\operatorname{atan}\left(\frac{\sqrt[3]{x}}{\sqrt{\frac{d}{e}}}\right)}{e^3\sqrt{\frac{d}{e}}}-\frac{3d\sqrt[3]{x}}{e^2}+\frac{x}{e}\right)}{3}+x\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)$$

input
```
integrate(a+b*ln(c*(d+e*x**(2/3))**n),x)
```

output
```
a*x + b*(-2*e*n*(3*d**2*atan(x**(1/3)/sqrt(d/e))/(e**3*sqrt(d/e)) - 3*d*x*
*(1/3)/e**2 + x/e)/3 + x*log(c*(d + e*x**(2/3))**n))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx = -\frac{1}{3}\left(\left(2\,e\left(\frac{3\,d^2\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}e^2}+\frac{e^2x-3\,dex^{\frac{1}{3}}}{e^3}\right)-3\,x\log\left(ex^{\frac{2}{3}}+d\right)\right)n-3\,x\log\left(c\right)\right)b + ax$$

input
```
integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="giac")
```

output
```
-1/3*((2*e*(3*d^2*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^2) + (e^2*x - 3
*d*e*x^(1/3))/e^3) - 3*x*log(e*x^(2/3) + d))*n - 3*x*log(c))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx = a\,x + b\,x\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)-\frac{2\,b\,n\,x}{3}+\frac{2\,b\,d\,n\,x^{1/3}}{e}-\frac{2\,b\,d^{3/2}\,n\,\mathrm{atan}\left(\frac{\sqrt{e}\,x^{1/3}}{\sqrt{d}}\right)}{e^{3/2}}$$

input
```
int(a + b*log(c*(d + e*x^(2/3))^n),x)
```

output
```
a*x + b*x*log(c*(d + e*x^(2/3))^n) - (2*b*n*x)/3 + (2*b*d*n*x^(1/3))/e - (
2*b*d^(3/2)*n*atan((e^(1/2)*x^(1/3))/d^(1/2)))/e^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)\, dx = \frac{-6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right) bdn + 6x^{\frac{1}{3}}bden + 3\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right) b\,e^2x + 3a\,e^2x}{3e^2}$$

input
```
int(a+b*log(c*(d+e*x^(2/3))^n),x)
```

output
```
( - 6*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b*d*n + 6*x**(1
/3)*b*d*e*n + 3*log((x**(2/3)*e + d)**n*c)*b*e**2*x + 3*a*e**2*x - 2*b*e**
2*n*x)/(3*e**2)
```

3.467 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx = \frac{3}{2}\left(a + b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{3}{2}bn\,\text{PolyLog}\left(2,1+\frac{ex^{2/3}}{d}\right)$$

output
```
3/2*(a+b*ln(c*(d+e*x^(2/3))^n))*ln(-e*x^(2/3)/d)+3/2*b*n*polylog(2,1+e*x^(
2/3)/d)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx = a\log(x) + \frac{3}{2}b\left(\log\left(c\left(d+ex^{2/3}\right)^n\right)\log\left(-\frac{ex^{2/3}}{d}\right)+n\,\text{PolyLog}\left(2,\frac{d+ex^{2/3}}{d}\right)\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
```

output
```
a*Log[x] + (3*b*(Log[c*(d + e*x^(2/3))^n]*Log[-((e*x^(2/3))/d)] + n*PolyLo
g[2, (d + e*x^(2/3))/d]))/2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^{2/3}}dx^{2/3}$$

$$\Big\downarrow \text{2841}$$

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-ben\int \frac{\log\left(-\frac{ex^{2/3}}{d}\right)}{d+ex^{2/3}}dx^{2/3}\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{x^{2/3}e}{d}+1\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
```

output
```
(3*((a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)] + b*n*PolyLog[2
, 1 + (e*x^(2/3))/d]))/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))/x,x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="fricas")
```

output
```
integral((b*log((e*x^(2/3) + d)^n*c) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx=\int \frac{a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(2/3))**n))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(44) = 88$.

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx=&-\frac{3}{2}\left(2\,\log\left(\frac{ex^{\frac{2}{3}}}{d}+1\right)\log\left(x^{\frac{1}{3}}\right)+\mathrm{Li}_2\left(-\frac{ex^{\frac{2}{3}}}{d}\right)\right)bn\\&+\frac{3\left(2\,benx^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}\right)-benx^{\frac{2}{3}}\right)}{2\,d}\\&+\frac{2\,bd\log\left(\left(ex^{\frac{2}{3}}+d\right)^n\right)\log\left(x\right)+2\left(bd\log\left(c\right)+ad\right)\log\left(x\right)-\frac{2\,benx\log(x)-3\,benx}{x^{\frac{1}{3}}}}{2\,d}\end{aligned}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="maxima")
```

output
```
-3/2*(2*log(e*x^(2/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(2/3)/d))*b*n + 3/2
*(2*b*e*n*x^(2/3)*log(x^(1/3)) - b*e*n*x^(2/3))/d + 1/2*(2*b*d*log((e*x^(2
/3) + d)^n)*log(x) + 2*(b*d*log(c) + a*d)*log(x) - (2*b*e*n*x*log(x) - 3*b
*e*n*x)/x^(1/3))/d
```

Giac [F]

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))/x, x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx=\frac{4\left(\int \frac{\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)}{x^{\frac{5}{3}}e+dx}dx\right)bdn+3\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)^2 b+4\log(x)\,an}{4n}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))/x,x)
```

output
```
(4*int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*e*x + d*x),x)*b*d*n + 3*log((x
**(2/3)*e + d)**n*c)**2*b + 4*log(x)*a*n)/(4*n)
```

3.468 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = -\frac{2ben}{d\sqrt[3]{x}} - \frac{2be^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

output
```
-2*b*e*n/d/x^(1/3)-2*b*e^(3/2)*n*arctan(e^(1/2)*x^(1/3)/d^(1/2))/d^(3/2)-(
a+b*ln(c*(d+e*x^(2/3))^n))/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = -\frac{a}{x} - \frac{2ben\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^{2/3}}{d}\right)}{d\sqrt[3]{x}} - \frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
```

output
```
-(a/x) - (2*b*e*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(d*x^
(1/3)) - (b*Log[c*(d + e*x^(2/3))^n])/x
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2905, 864, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{2}{3}ben\int \frac{1}{\left(d+ex^{2/3}\right)x^{4/3}}dx-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

$$\Big\downarrow \text{864}$$

$$2ben\int \frac{1}{\left(d+ex^{2/3}\right)x^{2/3}}d\sqrt[3]{x}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

$$\Big\downarrow \text{264}$$

$$2ben\left(-\frac{e\int \frac{1}{d+ex^{2/3}}d\sqrt[3]{x}}{d}-\frac{1}{d\sqrt[3]{x}}\right)-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

$$\Big\downarrow \text{218}$$

$$2ben\left(-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{1}{d\sqrt[3]{x}}\right)-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
```

output
```
2*b*e*n*(-(1/(d*x^(1/3))) - (Sqrt[e]*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^
(3/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])/x
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.06

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = \left[\frac{benx\sqrt{-\frac{e}{d}}\log\left(\frac{e^3x^2+2\,d^2ex\sqrt{-\frac{e}{d}}-d^3-2\left(de^2x\sqrt{-\frac{e}{d}}-d^2e\right)x^{\frac{2}{3}}-2\left(de^2x+d^3\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{e^3x^2+d^3}\right)}{dx}\right.$$
$$\left.-\frac{2\,benx\sqrt{\frac{e}{d}}\arctan\left(x^{\frac{1}{3}}\sqrt{\frac{e}{d}}\right)+bdn\log\left(ex^{\frac{2}{3}}+d\right)+2\,benx^{\frac{2}{3}}+bd\log\left(c\right)+ad}{dx}\right]$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="fricas")
```

output
```
[(b*e*n*x*sqrt(-e/d)*log((e^3*x^2 + 2*d^2*e*x*sqrt(-e/d) - d^3 - 2*(d*e^2*
x*sqrt(-e/d) - d^2*e)*x^(2/3) - 2*(d*e^2*x + d^3*sqrt(-e/d))*x^(1/3))/(e^3
*x^2 + d^3)) - b*d*n*log(e*x^(2/3) + d) - 2*b*e*n*x^(2/3) - b*d*log(c) - a
*d)/(d*x), -(2*b*e*n*x*sqrt(e/d)*arctan(x^(1/3)*sqrt(e/d)) + b*d*n*log(e*x
^(2/3) + d) + 2*b*e*n*x^(2/3) + b*d*log(c) + a*d)/(d*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = -\left(2\,e\left(\frac{e\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}d}+\frac{1}{dx^{\frac{1}{3}}}\right)+\frac{\log\left(ex^{\frac{2}{3}}+d\right)}{x}\right)bn-\frac{b\log\left(c\right)}{x}-\frac{a}{x}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="giac")
```

output
```
-(2*e*(e*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d) + 1/(d*x^(1/3))) + log(
e*x^(2/3) + d)/x)*b*n - b*log(c)/x - a/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx=\int \frac{a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx=\frac{-2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)benx-2x^{\frac{2}{3}}bden-\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)b\,d^2-a\,d^2}{d^2x}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x)
```

output
```
( - 2*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b*e*n*x - 2*x**
(2/3)*b*d*e*n - log((x**(2/3)*e + d)**n*c)*b*d**2 - a*d**2)/(d**2*x)
```

3.469 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx=-\frac{ben}{4dx^{4/3}}+\frac{be^2n}{2d^2x^{2/3}}-\frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{be^3n\log(x)}{3d^3}$$

output
```
-1/4*b*e*n/d/x^(4/3)+1/2*b*e^2*n/d^2/x^(2/3)-1/2*b*e^3*n*ln(d+e*x^(2/3))/d
^3-1/2*(a+b*ln(c*(d+e*x^(2/3))^n))/x^2+1/3*b*e^3*n*ln(x)/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{1}{3}ben\left(-\frac{3}{4dx^{4/3}}+\frac{3e}{2d^2x^{2/3}}-\frac{3e^2\log\left(d+ex^{2/3}\right)}{2d^3}+\frac{e^2\log(x)}{d^3}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e*n*(-3/(4*d*x^(4/3
)) + (3*e)/(2*d^2*x^(2/3)) - (3*e^2*Log[d + e*x^(2/3)])/(2*d^3) + (e^2*Log
[x])/d^3))/3
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^{8/3}}dx^{2/3}$$

$$\Big\downarrow \text{2842}$$

$$\frac{3}{2}\left(\frac{1}{3}ben\int \frac{1}{\left(d+ex^{2/3}\right)x^2}dx^{2/3}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^2}\right)$$

$$\Big\downarrow \text{54}$$

$$\frac{3}{2}\left(\frac{1}{3}ben\int \left(-\frac{e^3}{d^3\left(d+ex^{2/3}\right)}+\frac{e^2}{d^3x^{2/3}}-\frac{e}{d^2x^{4/3}}+\frac{1}{dx^2}\right)dx^{2/3}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\left(\frac{1}{3}ben\left(-\frac{e^2\log\left(d+ex^{2/3}\right)}{d^3}+\frac{e^2\log\left(x^{2/3}\right)}{d^3}+\frac{e}{d^2x^{2/3}}-\frac{1}{2dx^{4/3}}\right)-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^2}\right)$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
```

output
```
(3*(-1/3*(a + b*Log[c*(d + e*x^(2/3))^n])/x^2 + (b*e*n*(-1/2*1/(d*x^(4/3))
 + e/(d^2*x^(2/3)) - (e^2*Log[d + e*x^(2/3)])/d^3 + (e^2*Log[x^(2/3)])/d^3
))/3))/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = \frac{4\,be^3nx^2\log\left(x^{\frac{1}{3}}\right)+2\,bde^2nx^{\frac{4}{3}}-bd^2enx^{\frac{2}{3}}-2\,bd^3\log\left(c\right)-2\,ad^3-2\,(be^3}{4\,d^3x^2}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="fricas")
```

output
```
1/4*(4*b*e^3*n*x^2*log(x^(1/3)) + 2*b*d*e^2*n*x^(4/3) - b*d^2*e*n*x^(2/3)
- 2*b*d^3*log(c) - 2*a*d^3 - 2*(b*e^3*n*x^2 + b*d^3*n)*log(e*x^(2/3) + d))
/(d^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = -\frac{1}{4}\,ben\left(\frac{2\,e^2\log\left(ex^{\frac{2}{3}}+d\right)}{d^3}-\frac{2\,e^2\log\left(x^{\frac{2}{3}}\right)}{d^3}-\frac{2\,ex^{\frac{2}{3}}-d}{d^2x^{\frac{4}{3}}}\right) - \frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="maxima")
```

output
```
-1/4*b*e*n*(2*e^2*log(e*x^(2/3) + d)/d^3 - 2*e^2*log(x^(2/3))/d^3 - (2*e*x
^(2/3) - d)/(d^2*x^(4/3))) - 1/2*b*log((e*x^(2/3) + d)^n*c)/x^2 - 1/2*a/x^
2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = -\frac{\left(e^4\left(\frac{2\log\left(\left|ex^{\frac{2}{3}}+d\right|\right)}{d^3}-\frac{2\log\left(\left|ex^{\frac{2}{3}}\right|\right)}{d^3}-\frac{2\left(ex^{\frac{2}{3}}+d\right)d-3d^2}{d^3e^2x^{\frac{4}{3}}}\right)+\frac{2e\log\left(ex^{\frac{2}{3}}+d\right)}{x^2}\right)bn}{4e} - \frac{b\log(c)}{2x^2} - \frac{a}{2x^2}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="giac")
```

output
```
-1/4*(e^4*(2*log(abs(e*x^(2/3) + d))/d^3 - 2*log(abs(e*x^(2/3)))/d^3 - (2*
(e*x^(2/3) + d)*d - 3*d^2)/(d^3*e^2*x^(4/3))) + 2*e*log(e*x^(2/3) + d)/x^2
)*b*n/e - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

Mupad [B] (verification not implemented)

Time = 14.91 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = -\frac{\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n\,x^{2/3}}{d^2}}{2\,x^{4/3}} - \frac{a}{2\,x^2} - \frac{b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{2\,x^2} - \frac{b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{2/3}}{d}+1\right)}{d^3}$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))/x^3,x)
```

output
```
- ((b*e*n)/(2*d) - (b*e^2*n*x^(2/3))/d^2)/(2*x^(4/3)) - a/(2*x^2) - (b*log
(c*(d + e*x^(2/3))^n))/(2*x^2) - (b*e^3*n*atanh((2*e*x^(2/3))/d + 1))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = \frac{-x^{\frac{2}{3}}b\,d^2en+2x^{\frac{4}{3}}bd\,e^2n+4\log\left(x^{\frac{1}{3}}\right)b\,e^3n\,x^2-2\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)b\,d^3}{4d^3x^2}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x)
```

output
```
( - x**(2/3)*b*d**2*e*n + 2*x**(1/3)*b*d*e**2*n*x + 4*log(x**(1/3))*b*e**3
*n*x**2 - 2*log((x**(2/3)*e + d)**n*c)*b*d**3 - 2*log((x**(2/3)*e + d)**n*
c)*b*e**3*x**2 - 2*a*d**3)/(4*d**3*x**2)
```

3.470 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}+\frac{2be^4n}{3d^4\sqrt[3]{x}}+\frac{2be^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

output
```
-2/21*b*e*n/d/x^(7/3)+2/15*b*e^2*n/d^2/x^(5/3)-2/9*b*e^3*n/d^3/x+2/3*b*e^4
*n/d^4/x^(1/3)+2/3*b*e^(9/2)*n*arctan(e^(1/2)*x^(1/3)/d^(1/2))/d^(9/2)-1/3
*(a+b*ln(c*(d+e*x^(2/3))^n))/x^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.53

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{2ben\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\frac{ex^{2/3}}{d}\right)}{21dx^{7/3}}-\frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
```

output
```
-1/3*a/x^3 - (2*b*e*n*Hypergeometric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/
(21*d*x^(7/3)) - (b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2905, 864, 264, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{2}{9}ben\int \frac{1}{\left(d+ex^{2/3}\right)x^{10/3}}dx-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{864}$$

$$\frac{2}{3}ben\int \frac{1}{\left(d+ex^{2/3}\right)x^{8/3}}d\sqrt[3]{x}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{3}ben\left(-\frac{e\int \frac{1}{\left(d+ex^{2/3}\right)x^2}d\sqrt[3]{x}}{d}-\frac{1}{7dx^{7/3}}\right)-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{3}ben\left(-\frac{e\left(-\frac{e\int \frac{1}{\left(d+ex^{2/3}\right)x^{4/3}}d\sqrt[3]{x}}{d}-\frac{1}{5dx^{5/3}}\right)}{d}-\frac{1}{7dx^{7/3}}\right)-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{3}ben\left(-\frac{e\left(-\frac{e\left(-\frac{e\int\frac{1}{\left(d+ex^{2/3}\right)x^{2/3}}d\sqrt[3]{x}}{d}-\frac{1}{3dx}\right)}{d}-\frac{1}{5dx^{5/3}}\right)}{d}-\frac{1}{7dx^{7/3}}\right)-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{3}ben\left(-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{e\int\frac{1}{d+ex^{2/3}}d\sqrt[3]{x}}{d}-\frac{1}{d\sqrt[3]{x}}\right)}{d}-\frac{1}{3dx}\right)}{d}-\frac{1}{5dx^{5/3}}\right)}{d}-\frac{1}{7dx^{7/3}}\right)-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{2}{3}ben\left(-\frac{e\left(-\frac{e\left(-\frac{e\left(-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{1}{d\sqrt[3]{x}}\right)}{d}-\frac{1}{3dx}\right)}{d}-\frac{1}{5dx^{5/3}}\right)}{d}-\frac{1}{7dx^{7/3}}\right)-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
```

output
```
(2*b*e*n*(-1/7*1/(d*x^(7/3)) - (e*(-1/5*1/(d*x^(5/3)) - (e*(-1/3*1/(d*x) -
 (e*(-(1/(d*x^(1/3))) - (Sqrt[e]*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(3/2
)))/d))/d))/d))/3 - (a + b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{a + b\ln\left(c\left(d + e\,x^{\frac{2}{3}}\right)^{n}\right)}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))/x^4,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.54

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = \left[\frac{105\,be^4nx^3\sqrt{-\frac{e}{d}}\log\left(\frac{e^3x^2-2\,d^2ex\sqrt{-\frac{e}{d}}-d^3+2\left(de^2x\sqrt{-\frac{e}{d}}+d^2e\right)x^{\frac{2}{3}}-2\left(de^2x-d^3\sqrt{-\right.}{e^3x^2+d^3}\right.}{}\right.$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="fricas")
```

output
```
[1/315*(105*b*e^4*n*x^3*sqrt(-e/d)*log((e^3*x^2 - 2*d^2*e*x*sqrt(-e/d) - d
^3 + 2*(d*e^2*x*sqrt(-e/d) + d^2*e)*x^(2/3) - 2*(d*e^2*x - d^3*sqrt(-e/d))
*x^(1/3))/(e^3*x^2 + d^3)) - 70*b*d*e^3*n*x^2 + 42*b*d^2*e^2*n*x^(4/3) - 1
05*b*d^4*n*log(e*x^(2/3) + d) - 105*b*d^4*log(c) - 105*a*d^4 + 30*(7*b*e^4
*n*x^2 - b*d^3*e*n)*x^(2/3))/(d^4*x^3), 1/315*(210*b*e^4*n*x^3*sqrt(e/d)*a
rctan(x^(1/3)*sqrt(e/d)) - 70*b*d*e^3*n*x^2 + 42*b*d^2*e^2*n*x^(4/3) - 105
*b*d^4*n*log(e*x^(2/3) + d) - 105*b*d^4*log(c) - 105*a*d^4 + 30*(7*b*e^4*n
*x^2 - b*d^3*e*n)*x^(2/3))/(d^4*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)}{x^4}\,dx = \frac{1}{315}\left(2\,e\left(\frac{105\,e^4\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}d^4} + \frac{105\,e^3x^2 - 35\,de^2x^{\frac{4}{3}} + 21\,d^2ex^{\frac{2}{3}} - 15\,d^3}{d^4x^{\frac{7}{3}}}\right) - \frac{105\log\left(ex^{\frac{2}{3}} + d\right)}{x^3}\right)bn - \frac{b\log\left(c\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="giac")
```

output
```
1/315*(2*e*(105*e^4*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d^4) + (105*e^3
*x^2 - 35*d*e^2*x^(4/3) + 21*d^2*e*x^(2/3) - 15*d^3)/(d^4*x^(7/3))) - 105*
log(e*x^(2/3) + d)/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))/x^4,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx=\frac{210\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)b\,e^4n\,x^3-30x^{\frac{2}{3}}b\,d^4en+210x^{\frac{8}{3}}bd\,e^4n+42x^{\frac{4}{3}}b\,d^3}{315d^5x^3}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x)
```

output
```
(210*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b*e**4*n*x**3 -
30*x**(2/3)*b*d**4*e*n + 210*x**(2/3)*b*d*e**4*n*x**2 + 42*x**(1/3)*b*d**3
*e**2*n*x - 105*log((x**(2/3)*e + d)**n*c)*b*d**5 - 105*a*d**5 - 70*b*d**2
*e**3*n*x**2)/(315*d**5*x**3)
```

3.471 $\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 482

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6}-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+ex^{2/3}\right)^4}{32e^6}-\frac{3b^2dn^2\left(d+ex^{2/3}\right)^5}{25e^6}+\frac{b^2n^2\left(d+ex^{2/3}\right)^6}{72e^6}-\frac{3b^2d^5n^2x^{2/3}}{e^5}+\frac{b^2d^6n^2\log^2\left(d+ex^{2/3}\right)}{4e^6}+\frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6}-\frac{15bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^6}+\frac{10bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6}-\frac{15bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6}+\frac{3bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5e^6}-\frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{12e^6}-\frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2$$

output

```
15/8*b^2*d^4*n^2*(d+e*x^(2/3))^2/e^6-10/9*b^2*d^3*n^2*(d+e*x^(2/3))^3/e^6+
15/32*b^2*d^2*n^2*(d+e*x^(2/3))^4/e^6-3/25*b^2*d*n^2*(d+e*x^(2/3))^5/e^6+1
/72*b^2*n^2*(d+e*x^(2/3))^6/e^6-3*b^2*d^5*n^2*x^(2/3)/e^5+1/4*b^2*d^6*n^2*
ln(d+e*x^(2/3))^2/e^6+3*b*d^5*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/
e^6-15/4*b*d^4*n*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+10/3*b*d^
3*n*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-15/8*b*d^2*n*(d+e*x^(2
/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+3/5*b*d*n*(d+e*x^(2/3))^5*(a+b*ln(c
*(d+e*x^(2/3))^n))/e^6-1/12*b*n*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2/3))^n)
)/e^6-1/2*b*d^6*n*ln(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+1/4*x^4*
(a+b*ln(c*(d+e*x^(2/3))^n))^2
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.67

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \frac{ex^{2/3}\left(1800a^2e^5x^{10/3}+60abn\left(60d^5-30d^4ex^{2/3}+20d^3e^2x^{4/3}-15d^2e^3x^2+\right.\right.}{}$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

output
```
(e*x^(2/3)*(1800*a^2*e^5*x^(10/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(2/3) +
20*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2 + 12*d*e^4*x^(8/3) - 10*e^5*x^(10/3))
+ b^2*n^2*(-8820*d^5 + 2610*d^4*e*x^(2/3) - 1140*d^3*e^2*x^(4/3) + 555*d^2
*e^3*x^2 - 264*d*e^4*x^(8/3) + 100*e^5*x^(10/3))) + 180*b*d^6*n*(-20*a + 4
9*b*n)*Log[d + e*x^(2/3)] - 60*b*e*x^(2/3)*(-60*a*e^5*x^(10/3) + b*n*(-60*
d^5 + 30*d^4*e*x^(2/3) - 20*d^3*e^2*x^(4/3) + 15*d^2*e^3*x^2 - 12*d*e^4*x^
(8/3) + 10*e^5*x^(10/3)))*Log[c*(d + e*x^(2/3))^n] - 1800*b^2*(d^6 - e^6*x
^4)*Log[c*(d + e*x^(2/3))^n]^2)/(7200*e^6)
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int x^{10/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2dx^{2/3}$$

$$\Big\downarrow \text{2845}$$

$$\frac{3}{2}\left(\frac{1}{6}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{3}ben\int\frac{x^4\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)}{d+ex^{2/3}}dx^{2/3}\right)$$

$\Big\downarrow$ 2858

$$\frac{3}{2}\left(\frac{1}{6}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{3}bn\int x^{10/3}\left(a+b\log\left(cx^{2n/3}\right)\right)d\left(d+ex^{2/3}\right)\right)$$

$\Big\downarrow$ 27

$$\frac{3}{2}\left(\frac{1}{6}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{bn\int e^6x^{10/3}(a+b\log(cx^{2n/3}))\,d(d+ex^{2/3})}{3e^6}\right)$$

$\Big\downarrow$ 2772

$$\frac{3}{2}\left(\frac{1}{6}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{bn\left(-bn\int\left(\frac{\log(d+ex^{2/3})d^6}{x^{2/3}}-6d^5+\frac{15}{2}\left(d+ex^{2/3}\right)d^4-\frac{20}{3}x^{4/3}d^3+\frac{15x^2d^2}{4}\right.\right.}{}\right.$$

$\Big\downarrow$ 2009

$$\frac{3}{2}\left(\frac{1}{6}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{bn\left(d^6\log\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)-6d^5\left(d+ex^{2/3}\right)\left(a+b\log\left(c\right.\right.\right.}{}\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

output
```
(3*((x^4*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/6 - (b*n*(-(b*n*(-6*d^5*(d +
e*x^(2/3)) + (15*d^4*x^(4/3))/4 - (20*d^3*x^2)/9 + (15*d^2*x^(8/3))/16 - (
6*d*x^(10/3))/25 + x^4/36 + (d^6*Log[d + e*x^(2/3)]^2)/2)) - 6*d^5*(d + e*
x^(2/3))*(a + b*Log[c*x^((2*n)/3)]) + (15*d^4*x^(4/3)*(a + b*Log[c*x^((2*n
)/3)]))/2 - (20*d^3*x^2*(a + b*Log[c*x^((2*n)/3)]))/3 + (15*d^2*x^(8/3)*(a
 + b*Log[c*x^((2*n)/3)]))/4 - (6*d*x^(10/3)*(a + b*Log[c*x^((2*n)/3)]))/5
+ (x^4*(a + b*Log[c*x^((2*n)/3)]))/6 + d^6*Log[d + e*x^(2/3)]*(a + b*Log[c
*x^((2*n)/3)])))/(3*e^6)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3 \left(a + b \ln \left(c \left(d + e\, x^{\frac{2}{3}}\right)^n\right)\right)^2 dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.05

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^2 dx = \frac{1800\, b^2 e^6 x^4 \log(c)^2 + 100\, (b^2 e^6 n^2 - 6\, abe^6 n + 18\, a^2 e^6) x^4 - 60\, (19\, b^2 d^3 e^3 n^2 \cdots}{\cdots}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

output
```
1/7200*(1800*b^2*e^6*x^4*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^
2*e^6)*x^4 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x^2 + 1800*(b^2*e^
6*n^2*x^4 - b^2*d^6*n^2)*log(e*x^(2/3) + d)^2 + 60*(20*b^2*d^3*e^3*n^2*x^2
 + 147*b^2*d^6*n^2 - 60*a*b*d^6*n - 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^4 + 6
0*(b^2*e^6*n*x^4 - b^2*d^6*n)*log(c) - 15*(b^2*d^2*e^4*n^2*x^2 - 4*b^2*d^5
*e*n^2)*x^(2/3) + 6*(2*b^2*d*e^5*n^2*x^3 - 5*b^2*d^4*e^2*n^2*x)*x^(1/3))*l
og(e*x^(2/3) + d) + 600*(2*b^2*d^3*e^3*n*x^2 - (b^2*e^6*n - 6*a*b*e^6)*x^4
)*log(c) - 15*(588*b^2*d^5*e*n^2 - 240*a*b*d^5*e*n - (37*b^2*d^2*e^4*n^2 -
 60*a*b*d^2*e^4*n)*x^2 + 60*(b^2*d^2*e^4*n*x^2 - 4*b^2*d^5*e*n)*log(c))*x^
(2/3) - 6*(4*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*x^3 - 15*(29*b^2*d^4*e^2*
n^2 - 20*a*b*d^4*e^2*n)*x - 60*(2*b^2*d*e^5*n*x^3 - 5*b^2*d^4*e^2*n*x)*log
(c))*x^(1/3))/e^6
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.68

$$\int x^3 \left(a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)\right)^2 dx = \frac{1}{4}b^2x^4\log\left(\left(ex^{\frac{2}{3}} + d\right)^n c\right)^2 + \frac{1}{2}abx^4\log\left(\left(ex^{\frac{2}{3}} + d\right)^n c\right) + \frac{1}{4}a^2x^4 - \frac{1}{120}aben\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}} + d\right)}{e^7} + \frac{10\,e^5x^4 - 12\,de^4x^{\frac{10}{3}} + 15\,d^2e^3x^{\frac{8}{3}} - 20\,d^3e^2x^2 + 30\,d^4ex^{\frac{4}{3}} - 60\,d^5x^{\frac{2}{3}}}{e^6}\right) - \frac{1}{7200}\left(60\,en\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}} + d\right)}{e^7} + \frac{10\,e^5x^4 - 12\,de^4x^{\frac{10}{3}} + 15\,d^2e^3x^{\frac{8}{3}} - 20\,d^3e^2x^2 + 30\,d^4ex^{\frac{4}{3}} - 60\,d^5x^{\frac{2}{3}}}{e^6}\right.\right.$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*log((e*x^(2/3) + d)^n*c)^2 + 1/2*a*b*x^4*log((e*x^(2/3) + d)^n
*c) + 1/4*a^2*x^4 - 1/120*a*b*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5
*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*
x^(4/3) - 60*d^5*x^(2/3))/e^6) - 1/7200*(60*e*n*(60*d^6*log(e*x^(2/3) + d)
/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x
^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6)*log((e*x^(2/3) + d)^n*c) - (1
00*e^6*x^4 - 264*d*e^5*x^(10/3) + 555*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2 +
 1800*d^6*log(e*x^(2/3) + d)^2 + 2610*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^(
2/3) + d) - 8820*d^5*e*x^(2/3))*n^2/e^6)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(412) = 824$.

Time = 0.34 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.88

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input

```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

output

```
1/4*b^2*x^4*log(c)^2 + 1/2*a*b*x^4*log(c) + 1/4*a^2*x^4 + 1/7200*(1800*(e*
x^(2/3) + d)^6*log(e*x^(2/3) + d)^2/e^6 - 10800*(e*x^(2/3) + d)^5*d*log(e*
x^(2/3) + d)^2/e^6 + 27000*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)^2/e^6
- 36000*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)^2/e^6 + 27000*(e*x^(2/3)
+ d)^2*d^4*log(e*x^(2/3) + d)^2/e^6 - 600*(e*x^(2/3) + d)^6*log(e*x^(2/3)
+ d)/e^6 + 4320*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 - 13500*(e*x^(2
/3) + d)^4*d^2*log(e*x^(2/3) + d)/e^6 + 24000*(e*x^(2/3) + d)^3*d^3*log(e*
x^(2/3) + d)/e^6 - 27000*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 + 10
0*(e*x^(2/3) + d)^6/e^6 - 864*(e*x^(2/3) + d)^5*d/e^6 + 3375*(e*x^(2/3) +
d)^4*d^2/e^6 - 8000*(e*x^(2/3) + d)^3*d^3/e^6 + 13500*(e*x^(2/3) + d)^2*d^
4/e^6 - 10800*((e*x^(2/3) + d)*log(e*x^(2/3) + d)^2 - 2*(e*x^(2/3) + d)*lo
g(e*x^(2/3) + d) + 2*e*x^(2/3) + 2*d)*d^5/e^6)*b^2*n^2 + 1/120*b^2*n*(60*(
e*x^(2/3) + d)^6*log(e*x^(2/3) + d)/e^6 - 360*(e*x^(2/3) + d)^5*d*log(e*x^
(2/3) + d)/e^6 + 900*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)/e^6 - 1200*(
e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)/e^6 + 900*(e*x^(2/3) + d)^2*d^4*lo
g(e*x^(2/3) + d)/e^6 - 10*(e*x^(2/3) + d)^6/e^6 + 72*(e*x^(2/3) + d)^5*d/e
^6 - 225*(e*x^(2/3) + d)^4*d^2/e^6 + 400*(e*x^(2/3) + d)^3*d^3/e^6 - 450*(
e*x^(2/3) + d)^2*d^4/e^6 - 360*((e*x^(2/3) + d)*log(e*x^(2/3) + d) - e*x^(
2/3) - d)*d^5/e^6)*log(c) + 1/120*a*b*n*(60*(e*x^(2/3) + d)^6*log(e*x^(2/3
) + d)/e^6 - 360*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 + 900*(e*x^...
```

Mupad [B] (verification not implemented)

Time = 15.88 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{a^2\,x^4}{4}+\frac{b^2\,x^4\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)^2}{4}+\frac{b^2\,n^2\,x^4}{72}+\frac{a\,b\,x^4\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{2}-\frac{b^2\,d^6\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)^2}{4\,e^6}-\frac{a\,b\,n\,x^4}{12}-\frac{b^2\,n\,x^4\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{12}+\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+e\,x^{2/3}\right)}{40\,e^6}-\frac{19\,b^2\,d^3\,n^2\,x^2}{120\,e^3}+\frac{37\,b^2\,d^2\,n^2\,x^{8/3}}{480\,e^2}+\frac{29\,b^2\,d^4\,n^2\,x^{4/3}}{80\,e^4}-\frac{49\,b^2\,d^5\,n^2\,x^{2/3}}{40\,e^5}-\frac{11\,b^2\,d\,n^2\,x^{10/3}}{300\,e}+\frac{b^2\,d^3\,n\,x^2\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{6\,e^3}-\frac{b^2\,d^2\,n\,x^{8/3}\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{8\,e^2}-\frac{b^2\,d^4\,n\,x^{4/3}\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{4\,e^4}+\frac{b^2\,d^5\,n\,x^{2/3}\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{2\,e^5}+\frac{a\,b\,d\,n\,x^{10/3}}{10\,e}-\frac{a\,b\,d^6\,n\,\ln\left(d+e\,x^{2/3}\right)}{2\,e^6}+\frac{b^2\,d\,n\,x^{10/3}\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{10\,e}+\frac{a\,b\,d^3\,n\,x^2}{6\,e^3}-\frac{a\,b\,d^2\,n\,x^{8/3}}{8\,e^2}-\frac{a\,b\,d^4\,n\,x^{4/3}}{4\,e^4}+\frac{a\,b\,d^5\,n\,x^{2/3}}{2\,e^5}$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^4*log(c*(d + e*x^(2/3))^n)^2)/4 + (b^2*n^2*x^4)/72 +
(a*b*x^4*log(c*(d + e*x^(2/3))^n))/2 - (b^2*d^6*log(c*(d + e*x^(2/3))^n)^2
)/(4*e^6) - (a*b*n*x^4)/12 - (b^2*n*x^4*log(c*(d + e*x^(2/3))^n))/12 + (49
*b^2*d^6*n^2*log(d + e*x^(2/3)))/(40*e^6) - (19*b^2*d^3*n^2*x^2)/(120*e^3)
 + (37*b^2*d^2*n^2*x^(8/3))/(480*e^2) + (29*b^2*d^4*n^2*x^(4/3))/(80*e^4)
- (49*b^2*d^5*n^2*x^(2/3))/(40*e^5) - (11*b^2*d*n^2*x^(10/3))/(300*e) + (b
^2*d^3*n*x^2*log(c*(d + e*x^(2/3))^n))/(6*e^3) - (b^2*d^2*n*x^(8/3)*log(c*
(d + e*x^(2/3))^n))/(8*e^2) - (b^2*d^4*n*x^(4/3)*log(c*(d + e*x^(2/3))^n))
/(4*e^4) + (b^2*d^5*n*x^(2/3)*log(c*(d + e*x^(2/3))^n))/(2*e^5) + (a*b*d*n
*x^(10/3))/(10*e) - (a*b*d^6*n*log(d + e*x^(2/3)))/(2*e^6) + (b^2*d*n*x^(1
0/3)*log(c*(d + e*x^(2/3))^n))/(10*e) + (a*b*d^3*n*x^2)/(6*e^3) - (a*b*d^2
*n*x^(8/3))/(8*e^2) - (a*b*d^4*n*x^(4/3))/(4*e^4) + (a*b*d^5*n*x^(2/3))/(2
*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.94

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{3600x^{\frac{2}{3}}\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)b^2d^5en-900x^{\frac{8}{3}}\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)b^2d^2e^4n+36$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x)
```

output
```
(3600*x**(2/3)*log((x**(2/3)*e + d)**n*c)*b**2*d**5*e*n - 900*x**(2/3)*log
((x**(2/3)*e + d)**n*c)*b**2*d**2*e**4*n*x**2 + 3600*x**(2/3)*a*b*d**5*e*n
 - 900*x**(2/3)*a*b*d**2*e**4*n*x**2 - 8820*x**(2/3)*b**2*d**5*e*n**2 + 55
5*x**(2/3)*b**2*d**2*e**4*n**2*x**2 - 1800*x**(1/3)*log((x**(2/3)*e + d)**
n*c)*b**2*d**4*e**2*n*x + 720*x**(1/3)*log((x**(2/3)*e + d)**n*c)*b**2*d*e
**5*n*x**3 - 1800*x**(1/3)*a*b*d**4*e**2*n*x + 720*x**(1/3)*a*b*d*e**5*n*x
**3 + 2610*x**(1/3)*b**2*d**4*e**2*n**2*x - 264*x**(1/3)*b**2*d*e**5*n**2*
x**3 - 1800*log((x**(2/3)*e + d)**n*c)**2*b**2*d**6 + 1800*log((x**(2/3)*e
 + d)**n*c)**2*b**2*e**6*x**4 - 3600*log((x**(2/3)*e + d)**n*c)*a*b*d**6 +
 3600*log((x**(2/3)*e + d)**n*c)*a*b*e**6*x**4 + 8820*log((x**(2/3)*e + d)
**n*c)*b**2*d**6*n + 1200*log((x**(2/3)*e + d)**n*c)*b**2*d**3*e**3*n*x**2
 - 600*log((x**(2/3)*e + d)**n*c)*b**2*e**6*n*x**4 + 1800*a**2*e**6*x**4 +
 1200*a*b*d**3*e**3*n*x**2 - 600*a*b*e**6*n*x**4 - 1140*b**2*d**3*e**3*n**
2*x**2 + 100*b**2*e**6*n**2*x**4)/(7200*e**6)
```

3.472 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 22, antiderivative size = 275

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx = & -\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3} \\ & +\frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{3b^2d^2n^2x^{2/3}}{e^2}-\frac{b^2d^3n^2\log^2\left(d+ex^{2/3}\right)}{2e^3} \\ & -\frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} \\ & +\frac{3bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3} \\ & -\frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3} \\ & +\frac{bd^3n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} \\ & +\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\end{aligned}$$

output

```
-3/4*b^2*d*n^2*(d+e*x^(2/3))^2/e^3+1/9*b^2*n^2*(d+e*x^(2/3))^3/e^3+3*b^2*d
^2*n^2*x^(2/3)/e^2-1/2*b^2*d^3*n^2*ln(d+e*x^(2/3))^2/e^3-3*b*d^2*n*(d+e*x^
(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3+3/2*b*d*n*(d+e*x^(2/3))^2*(a+b*ln(c
*(d+e*x^(2/3))^n))/e^3-1/3*b*n*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))
/e^3+b*d^3*n*ln(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3+1/2*x^2*(a+b*
ln(c*(d+e*x^(2/3))^n))^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.87

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{18a^2d^3-36abd^2enx^{2/3}+66b^2d^2en^2x^{2/3}+18abde^2nx^{4/3}-15b^2de^2n^2x^{4/3}+\cdots}{\cdots}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

output
```
(18*a^2*d^3 - 36*a*b*d^2*e*n*x^(2/3) + 66*b^2*d^2*e*n^2*x^(2/3) + 18*a*b*d
*e^2*n*x^(4/3) - 15*b^2*d*e^2*n^2*x^(4/3) + 18*a^2*e^3*x^2 - 12*a*b*e^3*n*
x^2 + 4*b^2*e^3*n^2*x^2 - 30*b^2*d^3*n^2*Log[d + e*x^(2/3)] + 6*b*(6*a*(d^
3 + e^3*x^2) - b*n*(6*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2)
)*Log[c*(d + e*x^(2/3))^n] + 18*b^2*(d^3 + e^3*x^2)*Log[c*(d + e*x^(2/3))^
n]^2)/(36*e^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.318$, Rules used = {2904, 2845, 2858, 25, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx^{2/3}$$

$$\Big\downarrow \text{2845}$$

$$\frac{3}{2}\left(\frac{1}{3}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{2}{3}ben\int\frac{x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d+ex^{2/3}}dx^{2/3}\right)$$

$$\Big\downarrow \text{2858}$$

$$\frac{3}{2}\left(\frac{1}{3}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{2}{3}bn\int x^{4/3}\left(a+b\log\left(cx^{2n/3}\right)\right)d\left(d+ex^{2/3}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}\left(\frac{2}{3}bn\int -x^{4/3}\left(a+b\log\left(cx^{2n/3}\right)\right)d\left(d+ex^{2/3}\right)+\frac{1}{3}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\left(\frac{2bn\int -e^3x^{4/3}\left(a+b\log\left(cx^{2n/3}\right)\right)d\left(d+ex^{2/3}\right)}{3e^3}+\frac{1}{3}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2772}$$

$$\frac{3}{2}\left(\frac{2bn\left(-bn\int\left(\frac{\log\left(d+ex^{2/3}\right)d^3}{x^{2/3}}-3d^2+\frac{3}{2}\left(d+ex^{2/3}\right)d-\frac{x^{4/3}}{3}\right)d\left(d+ex^{2/3}\right)+d^3\log\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)\right.}{3e^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\left(\frac{2bn\left(d^3\log\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)-3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)+\frac{3}{2}dx^{4/3}\left(a+b\log\left(cx^{2n/3}\right)\right)\right.}{3e^3}\right.$$

input

```
Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

output

```
(3*((x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/3 + (2*b*n*(-(b*n*(-3*d^2*(d
+ e*x^(2/3)) + (3*d*x^(4/3))/4 - x^2/9 + (d^3*Log[d + e*x^(2/3)]^2)/2)) -
3*d^2*(d + e*x^(2/3))*(a + b*Log[c*x^((2*n)/3)]) + (3*d*x^(4/3)*(a + b*Log
[c*x^((2*n)/3)]))/2 - (x^2*(a + b*Log[c*x^((2*n)/3)]))/3 + d^3*Log[d + e*x
^(2/3)]*(a + b*Log[c*x^((2*n)/3)])))/(3*e^3)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.11

$$\int x\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\Big)^{2}\,dx = \frac{18\,b^{2}e^{3}x^{2}\log\left(c\right)^{2}-12\,(b^{2}e^{3}n-3\,abe^{3})x^{2}\log\left(c\right)+2\,(2\,b^{2}e^{3}n^{2}-6\,abe^{3}n+9}{}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

output
```
1/36*(18*b^2*e^3*x^2*log(c)^2 - 12*(b^2*e^3*n - 3*a*b*e^3)*x^2*log(c) + 2*
(2*b^2*e^3*n^2 - 6*a*b*e^3*n + 9*a^2*e^3)*x^2 + 18*(b^2*e^3*n^2*x^2 + b^2*
d^3*n^2)*log(e*x^(2/3) + d)^2 + 6*(3*b^2*d*e^2*n^2*x^(4/3) - 6*b^2*d^2*e*n
^2*x^(2/3) - 11*b^2*d^3*n^2 + 6*a*b*d^3*n - 2*(b^2*e^3*n^2 - 3*a*b*e^3*n)*
x^2 + 6*(b^2*e^3*n*x^2 + b^2*d^3*n)*log(c))*log(e*x^(2/3) + d) + 6*(11*b^2
*d^2*e*n^2 - 6*b^2*d^2*e*n*log(c) - 6*a*b*d^2*e*n)*x^(2/3) + 3*(6*b^2*d*e^
2*n*x*log(c) - (5*b^2*d*e^2*n^2 - 6*a*b*d*e^2*n)*x)*x^(1/3))/e^3
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\int x\left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2\,dx$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

output
```
Integral(x*(a + b*log(c*(d + e*x**(2/3))**n))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.84

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{1}{2}\,b^2x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2$$
$$+\frac{1}{6}\,aben\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)$$
$$+abx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{2}\,a^2x^2$$
$$+\frac{1}{36}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{\left(4\,e^3x^2-18\,d^3\log\left(\right.\right.}{}\right.$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*log((e*x^(2/3) + d)^n*c)^2 + 1/6*a*b*e*n*(6*d^3*log(e*x^(2/3)
+ d)/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3) + a*b*x^2*log(
(e*x^(2/3) + d)^n*c) + 1/2*a^2*x^2 + 1/36*(6*e*n*(6*d^3*log(e*x^(2/3) + d)
/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d
)^n*c) + (4*e^3*x^2 - 18*d^3*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 66*
d^3*log(e*x^(2/3) + d) + 66*d^2*e*x^(2/3))*n^2/e^3)*b^2
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.14

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{1}{2}\,b^2x^2\log(c)^2$$
$$+\frac{1}{36}\left(18\,x^2\log\left(ex^{\frac{2}{3}}+d\right)^2-\left(6\left(\frac{2\left(ex^{\frac{2}{3}}+d\right)^3}{e^4}-\frac{9\left(ex^{\frac{2}{3}}+d\right)^2d}{e^4}+\frac{18\left(ex^{\frac{2}{3}}+d\right)d^2}{e^4}\right)\log\left(ex^{\frac{2}{3}}+d\right)-\right.\right.$$
$$+\frac{1}{6}\left(6\,x^2\log\left(ex^{\frac{2}{3}}+d\right)+e\left(\frac{6\,d^3\log\left(\left|ex^{\frac{2}{3}}+d\right|\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\right)b^2n\log(c)$$
$$+abx^2\log(c)$$
$$+\frac{1}{6}\left(6\,x^2\log\left(ex^{\frac{2}{3}}+d\right)+e\left(\frac{6\,d^3\log\left(\left|ex^{\frac{2}{3}}+d\right|\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\right)abn$$
$$+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

output
```
1/2*b^2*x^2*log(c)^2 + 1/36*(18*x^2*log(e*x^(2/3) + d)^2 - (6*(2*(e*x^(2/3
) + d)^3/e^4 - 9*(e*x^(2/3) + d)^2*d/e^4 + 18*(e*x^(2/3) + d)*d^2/e^4)*log
(e*x^(2/3) + d) - 18*d^3*log(e*x^(2/3) + d)^2/e^4 - 4*(e*x^(2/3) + d)^3/e^
4 + 27*(e*x^(2/3) + d)^2*d/e^4 - 108*(e*x^(2/3) + d)*d^2/e^4)*e)*b^2*n^2 +
 1/6*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(2/3) + d))/e^4 - (2
*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*b^2*n*log(c) + a*b*x^2*log
(c) + 1/6*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(2/3) + d))/e^4
 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*a*b*n + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^2\left(\frac{b^2\,x^2}{2}+\frac{b^2\,d^3}{2\,e^3}\right)$$

$$-x^{4/3}\left(\frac{d\left(\frac{3\,a^2}{2}-a\,b\,n+\frac{b^2\,n^2}{3}\right)}{2\,e}-\frac{d\,(3\,a^2-b^2\,n^2)}{4\,e}\right)+x^2\left(\frac{a^2}{2}-\frac{a\,b\,n}{3}+\frac{b^2\,n^2}{9}\right)+\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\left(\frac{b\,x^2\,(\right.$$

input
```
int(x*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

output
```
log(c*(d + e*x^(2/3))^n)^2*((b^2*x^2)/2 + (b^2*d^3)/(2*e^3)) - x^(4/3)*((d
*((3*a^2)/2 + (b^2*n^2)/3 - a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(4*e)) +
 x^2*(a^2/2 + (b^2*n^2)/9 - (a*b*n)/3) + log(c*(d + e*x^(2/3))^n)*((b*x^2*
(3*a - b*n))/3 - x^(4/3)*((b*d*(3*a - b*n))/(2*e) - (3*a*b*d)/(2*e)) + (d*
x^(2/3)*((b*d*(3*a - b*n))/e - (3*a*b*d)/e))/e) + x^(2/3)*((d*((d*((3*a^2)
/2 + (b^2*n^2)/3 - a*b*n))/e - (d*(3*a^2 - b^2*n^2))/(2*e)))/e + (b^2*d^2*
n^2)/e^2) - (log(d + e*x^(2/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(6*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.01

$$\int x\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^2\,dx=\frac{-36x^{\frac{2}{3}}\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)b^2d^2en-36x^{\frac{2}{3}}ab\,d^2en+66x^{\frac{2}{3}}b^2d^2e\,n^2+18x^{\frac{4}{3}}\log}{}$$

input
```
int(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x)
```

output
```
( - 36*x**(2/3)*log((x**(2/3)*e + d)**n*c)*b**2*d**2*e*n - 36*x**(2/3)*a*b
*d**2*e*n + 66*x**(2/3)*b**2*d**2*e*n**2 + 18*x**(1/3)*log((x**(2/3)*e + d
)**n*c)*b**2*d*e**2*n*x + 18*x**(1/3)*a*b*d*e**2*n*x - 15*x**(1/3)*b**2*d*
e**2*n**2*x + 18*log((x**(2/3)*e + d)**n*c)**2*b**2*d**3 + 18*log((x**(2/3
)*e + d)**n*c)**2*b**2*e**3*x**2 + 36*log((x**(2/3)*e + d)**n*c)*a*b*d**3
+ 36*log((x**(2/3)*e + d)**n*c)*a*b*e**3*x**2 - 66*log((x**(2/3)*e + d)**n
*c)*b**2*d**3*n - 12*log((x**(2/3)*e + d)**n*c)*b**2*e**3*n*x**2 + 18*a**2
*e**3*x**2 - 12*a*b*e**3*n*x**2 + 4*b**2*e**3*n**2*x**2)/(36*e**3)
```

3.473 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx = \frac{3}{2}\Big(a +b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^2\log\left(-\frac{ex^{2/3}}{d}\right)+3bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{ex^{2/3}}{d}\right)-3b^2n^2\operatorname{Po}$$

output
```
3/2*(a+b*ln(c*(d+e*x^(2/3))^n))^2*ln(-e*x^(2/3)/d)+3*b*n*(a+b*ln(c*(d+e*x^
(2/3))^n))*polylog(2,1+e*x^(2/3)/d)-3*b^2*n^2*polylog(3,1+e*x^(2/3)/d)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(95) = 190.

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx = \Big(a -bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^2\log(x)+2bn\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\Big(\Big(\cdots$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

output
```
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] + 2*b*n
*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((Log[d + e*x^(
2/3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)])/
2) + (3*b^2*n^2*(Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*
x^(2/3)]*PolyLog[2, 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))
/2
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{2/3}}dx^{2/3}$$

$$\Big\downarrow \text{2843}$$

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-2ben\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)}{d+ex^{2/3}}dx^{2/3}\right)$$

$$\Big\downarrow \text{2881}$$

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-2bn\int \frac{\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{x^{2/3}}d\left(d+ex^{2/3}\right)\right)$$

$$\Big\downarrow \text{2821}$$

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-2bn\left(bn\int\frac{\text{PolyLog}\left(2,\frac{d+ex^{2/3}}{d}\right)}{x^{2/3}}d\left(d+ex^{2/3}\right)-\text{PolyLog}\left(\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-2bn\left(bn\,\text{PolyLog}\left(3,\frac{d+ex^{2/3}}{d}\right)-\text{PolyLog}\left(2,\frac{d+ex^{2/3}}{d}\right)\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

output
```
(3*((a + b*Log[c*(d + e*x^(2/3))^n])^2*Log[-((e*x^(2/3))/d)] - 2*b*n*(-((a
 + b*Log[c*x^((2*n)/3)])*PolyLog[2, (d + e*x^(2/3))/d]) + b*n*PolyLog[3, (
d + e*x^(2/3))/d])))/2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c)
+ a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(2/3))**n))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="maxima")
```

output
```
b^2*log((e*x^(2/3) + d)^n)^2*log(x) + integrate(1/3*(3*(b^2*e*log(c)^2 + 2
*a*b*e*log(c) + a^2*e)*x - 2*(2*b^2*e*n*x*log(x) - 3*(b^2*e*log(c) + a*b*e
)*x - 3*(b^2*d*log(c) + a*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + 3*(b^2*d*
log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^2 + d*x^(4/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx=\frac{2\left(\int \frac{\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)^2}{x^{\frac{5}{3}}e+dx}dx\right)b^2dn+4\left(\int \frac{\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)}{x^{\frac{5}{3}}e+dx}dx\right)abdn+\log\left(\right.}{2n}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x)
```

output
```
(2*int(log((x**(2/3)*e + d)**n*c)**2/(x**(2/3)*e*x + d*x),x)*b**2*d*n + 4*
int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*e*x + d*x),x)*a*b*d*n + log((x**(
2/3)*e + d)**n*c)**3*b**2 + 3*log((x**(2/3)*e + d)**n*c)**2*a*b + 2*log(x)
*a**2*n)/(2*n)
```

3.474 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 238

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx &= -\frac{b^2e^2n^2}{2d^2x^{2/3}}+\frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3}\\ &-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}}+\frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3x^{2/3}}\\ &+\frac{be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}\\ &-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2}-\frac{b^2e^3n^2\log(x)}{d^3}-\frac{b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{d^3}\end{aligned}$$

output
```
-1/2*b^2*e^2*n^2/d^2/x^(2/3)+1/2*b^2*e^3*n^2*ln(d+e*x^(2/3))/d^3-1/2*b*e*n
*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(4/3)+b*e^2*n*(d+e*x^(2/3))*(a+b*ln(c*(d+
e*x^(2/3))^n))/d^3/x^(2/3)+b*e^3*n*ln(1-d/(d+e*x^(2/3)))*(a+b*ln(c*(d+e*x^
(2/3))^n))/d^3-1/2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2-b^2*e^3*n^2*ln(x)/d^3
-b^2*e^3*n^2*polylog(2,d/(d+e*x^(2/3)))/d^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx =$$

$$-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{ex^{2/3}\left(3bd^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-6bdenx^{2/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+3e^2x^{4/3}\left(a+b\log\left(\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

output
```
-1/6*(3*(a + b*Log[c*(d + e*x^(2/3))^n])^2 + (e*x^(2/3)*(3*b*d^2*n*(a + b*
Log[c*(d + e*x^(2/3))^n]) - 6*b*d*e*n*x^(2/3)*(a + b*Log[c*(d + e*x^(2/3))
^n]) + 3*e^2*x^(4/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*b^2*e^2*n^2*x^
(4/3)*(3*Log[d + e*x^(2/3)] - 2*Log[x]) + b^2*e*n^2*x^(2/3)*(3*d - 3*e*x^(
2/3)*Log[d + e*x^(2/3)] + 2*e*x^(2/3)*Log[x]) - 6*b*e^2*n*x^(4/3)*((a + b*
Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)] + b*n*PolyLog[2, 1 + (e*x^
(2/3))/d])))/d^3)/x^2
```

Rubi [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2904, 2845, 2858, 25, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{8/3}}dx^{2/3}$$

$$\Big\downarrow \text{2845}$$

$$\frac{3}{2}\left(\frac{2}{3}ben\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{\left(d+ex^{2/3}\right)x^2}dx^{2/3}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^2}\right)$$

$\Big\downarrow$ 2858

$$\frac{3}{2}\left(\frac{2}{3}bn\int \frac{a+b\log\left(cx^{2n/3}\right)}{x^{8/3}}d\left(d+ex^{2/3}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^2}\right)$$

$\Big\downarrow$ 25

$$\frac{3}{2}\left(-\frac{2}{3}bn\int -\frac{a+b\log\left(cx^{2n/3}\right)}{x^{8/3}}d\left(d+ex^{2/3}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^2}\right)$$

$\Big\downarrow$ 27

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^3x^{8/3}}d\left(d+ex^{2/3}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^2}\right)$$

$\Big\downarrow$ 2789

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^3x^2}d\left(d+ex^{2/3}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^2}\right)$$

$\Big\downarrow$ 2756

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{2e^2x^{4/3}}-\frac{1}{2}bn\int \frac{1}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right.\right.\right.}{3x^2}\right.$$

$\Big\downarrow$ 54

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{2e^2x^{4/3}}-\frac{1}{2}bn\int \left(-\frac{1}{d^2ex^{2/3}}+\frac{1}{d^2x^{2/3}}+\frac{1}{de^2x^{4/3}}\right)d\left(d+ex^{2/3}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}\right)\right.$$

$\Big\downarrow$ 2009

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\int \frac{a+b\log(cx^{2n/3})}{e^2x^2}d(d+ex^{2/3})}{d}+\frac{\frac{a+b\log(cx^{2n/3})}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{\log(d+ex^{2/3})}{d^2}-\frac{\log(-ex^{2/3})}{d^2}-\frac{1}{dex^{2/3}}\right)}{d}\right)-\frac{(a+$$

$\downarrow$ 2789

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^{4/3}}d(d+ex^{2/3})}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d(d+ex^{2/3})}{d}}{d}+\frac{\frac{a+b\log(cx^{2n/3})}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{\log(d+ex^{2/3})}{d^2}-\frac{\log(-ex}{d^2}\right.}{d}\right.\right.$$

$\downarrow$ 2751

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{-\frac{bn\int -\frac{1}{ex^{2/3}}d\left(d+ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d(d+ex^{2/3})}{d}}{d}+\frac{\frac{a+b\log(cx^{2n/3})}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{1}{\right.}}{\right.}\right.\right.$$

$\downarrow$ 16

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d(d+ex^{2/3})}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}}{d}+\frac{\frac{a+b\log(cx^{2n/3})}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{\log(d+ex^{2/}}{d^2}\right.}{d}\right.\right.$$

$\downarrow$ 2779

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{\frac{bn\int \frac{\log\left(1-\frac{d}{x^{2/3}}\right)}{x^{2/3}}d\left(d+ex^{2/3}\right)}{d}-\frac{\log\left(1-\frac{d}{x^{2/3}}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}}{d}+\frac{a+b\log}{2e^2}\right.\right.$$

$\downarrow$ 2838

$$\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{a+b\log(cx^{2n/3})}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{\log(d+ex^{2/3})}{d^2}-\frac{\log(-ex^{2/3})}{d^2}-\frac{1}{dex^{2/3}}\right)}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}+\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

output
```
(3*(-1/3*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2 - (2*b*e^3*n*((-1/2*(b*n*(
-(1/(d*e*x^(2/3))) + Log[d + e*x^(2/3)]/d^2 - Log[-(e*x^(2/3))]/d^2)) + (a
 + b*Log[c*x^((2*n)/3)])/(2*e^2*x^(4/3)))/d + (((b*n*Log[-(e*x^(2/3))])/d
- ((d + e*x^(2/3))*(a + b*Log[c*x^((2*n)/3)]))/(d*e*x^(2/3)))/d + (-((Log[
1 - d/x^(2/3)]*(a + b*Log[c*x^((2*n)/3)]))/d) + (b*n*PolyLog[2, d/x^(2/3)]
)/d)/d)/d))/3))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{3}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c)
+ a^2)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*b^2*log((e*x^(2/3) + d)^n)^2/x^2 + integrate(1/3*(3*(b^2*e*log(c)^2 +
 2*a*b*e*log(c) + a^2*e)*x + 2*(b^2*e*n*x + 3*(b^2*e*log(c) + a*b*e)*x + 3
*(b^2*d*log(c) + a*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + 3*(b^2*d*log(c)^
2 + 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^4 + d*x^(10/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^3,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx=\frac{-3x^{\frac{2}{3}}\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)b^2d^2en-3x^{\frac{2}{3}}ab\,d^2en+6x^{\frac{4}{3}}\log\left(\left(x^{\frac{2}{3}}e+d\right)^n\right.}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x)
```

output
```
( - 3*x**(2/3)*log((x**(2/3)*e + d)**n*c)*b**2*d**2*e*n - 3*x**(2/3)*a*b*d
**2*e*n + 6*x**(1/3)*log((x**(2/3)*e + d)**n*c)*b**2*d*e**2*n*x + 6*x**(1/
3)*a*b*d*e**2*n*x - 3*x**(1/3)*b**2*d*e**2*n**2*x + 4*int(log((x**(2/3)*e
 + d)**n*c)/(x**(2/3)*e*x + d*x),x)*b**2*d*e**3*n*x**2 + 12*log(x**(1/3))*a
*b*e**3*n*x**2 - 18*log(x**(1/3))*b**2*e**3*n**2*x**2 - 3*log((x**(2/3)*e
 + d)**n*c)**2*b**2*d**3 - 6*log((x**(2/3)*e + d)**n*c)*a*b*d**3 - 6*log((x
**(2/3)*e + d)**n*c)*a*b*e**3*x**2 + 9*log((x**(2/3)*e + d)**n*c)*b**2*e**
3*n*x**2 - 3*a**2*d**3)/(6*d**3*x**2)
```

3.475 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 412

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = &-\frac{b^2e^2n^2}{40d^2x^{8/3}}+\frac{3b^2e^3n^2}{40d^3x^2} \\ &-\frac{47b^2e^4n^2}{240d^4x^{4/3}}+\frac{77b^2e^5n^2}{120d^5x^{2/3}}-\frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6} \\ &-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}}+\frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}} \\ &-\frac{be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{6d^3x^2}+\frac{be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4d^4x^{4/3}} \\ &-\frac{be^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6x^{2/3}} \\ &-\frac{be^6n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6} \\ &-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} \\ &+\frac{137b^2e^6n^2\log(x)}{180d^6}+\frac{b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^6}\end{aligned}$$

output
```
-1/40*b^2*e^2*n^2/d^2/x^(8/3)+3/40*b^2*e^3*n^2/d^3/x^2-47/240*b^2*e^4*n^2/
d^4/x^(4/3)+77/120*b^2*e^5*n^2/d^5/x^(2/3)-77/120*b^2*e^6*n^2*ln(d+e*x^(2/
3))/d^6-1/10*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(10/3)+1/8*b*e^2*n*(a+b
*ln(c*(d+e*x^(2/3))^n))/d^2/x^(8/3)-1/6*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n)
)/d^3/x^2+1/4*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(4/3)-1/2*b*e^5*n*
(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6/x^(2/3)-1/2*b*e^6*n*ln(1-d/(
d+e*x^(2/3)))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6-1/4*(a+b*ln(c*(d+e*x^(2/3))^
n))^2/x^4+137/180*b^2*e^6*n^2*ln(x)/d^6+1/2*b^2*e^6*n^2*polylog(2,d/(d+e*x
^(2/3)))/d^6
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} - \frac{be\Big(72ad^5n - 90ad^4enx^{2/3} + 18bd^4en^2x^{2/3} + 120ad^3e^2nx^{4/3} - 54bd^3e^2n^2x^{4/3} - 180ad^2e^3nx^2 + 141bd^2e^3 \cdots}{\cdots}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

output
```
-1/4*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4 - (b*e*(72*a*d^5*n - 90*a*d^4*
e*n*x^(2/3) + 18*b*d^4*e*n^2*x^(2/3) + 120*a*d^3*e^2*n*x^(4/3) - 54*b*d^3*
e^2*n^2*x^(4/3) - 180*a*d^2*e^3*n*x^2 + 141*b*d^2*e^3*n^2*x^2 + 360*a*d*e^
4*n*x^(8/3) - 462*b*d*e^4*n^2*x^(8/3) + 6*e^5*n*(-60*a + 137*b*n)*x^(10/3)
*Log[d + e*x^(2/3)] + 72*b*d^5*n*Log[c*(d + e*x^(2/3))^n] - 90*b*d^4*e*n*x
^(2/3)*Log[c*(d + e*x^(2/3))^n] + 120*b*d^3*e^2*n*x^(4/3)*Log[c*(d + e*x^(
2/3))^n] - 180*b*d^2*e^3*n*x^2*Log[c*(d + e*x^(2/3))^n] + 360*b*d*e^4*n*x^
(8/3)*Log[c*(d + e*x^(2/3))^n] - 180*b*e^5*x^(10/3)*Log[c*(d + e*x^(2/3))^
n]^2 + 360*b*e^5*n*x^(10/3)*Log[c*(d + e*x^(2/3))^n]*Log[-((e*x^(2/3))/d)]
 + 240*a*e^5*n*x^(10/3)*Log[x] - 548*b*e^5*n^2*x^(10/3)*Log[x] + 360*b*e^5
*n^2*x^(10/3)*PolyLog[2, 1 + (e*x^(2/3))/d]))/(720*d^6*x^(10/3))
```

Rubi [A] (warning: unable to verify)

Time = 3.94 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.38, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.042, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{14/3}}dx^{2/3}$$

$$\Big\downarrow \text{2845}$$

$$\frac{3}{2}\left(\frac{1}{3}ben\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{\left(d+ex^{2/3}\right)x^4}dx^{2/3}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{6x^4}\right)$$

$$\Big\downarrow \text{2858}$$

$$\frac{3}{2}\left(\frac{1}{3}bn\int \frac{a+b\log\left(cx^{2n/3}\right)}{x^{14/3}}d\left(d+ex^{2/3}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{6x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^6x^{14/3}}d\left(d+ex^{2/3}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{6x^4}\right)$$

$$\Big\downarrow \text{2789}$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^6x^4}d\left(d+ex^{2/3}\right)}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^5x^4}d\left(d+ex^{2/3}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{6x^4}\right)$$

$$\Big\downarrow \text{2756}$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int -\frac{1}{e^5x^4}d\left(d+ex^{2/3}\right)-\frac{a+b\log\left(cx^{2n/3}\right)}{5e^5x^{10/3}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^5x^4}d\left(d+ex^{2/3}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+\right.\right.\right.}{6x^4}\right.$$

$\Big\downarrow$ 54

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int\left(-\frac{1}{d^5ex^{2/3}}+\frac{1}{d^5x^{2/3}}+\frac{1}{d^4e^2x^{4/3}}-\frac{1}{d^3e^3x^2}+\frac{1}{d^2e^4x^{8/3}}-\frac{1}{de^5x^{10/3}}\right)d\left(d+ex^{2/3}\right)-\frac{a+b\log\left(cx^{2n/3}\right)}{5e^5x^{10/3}}}{d}+\right.\right.$$

$\Big\downarrow$ 2009

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^5x^4}d\left(d+ex^{2/3}\right)}{d}+\frac{-\frac{a+b\log\left(cx^{2n/3}\right)}{5e^5x^{10/3}}-\frac{1}{5}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^5}-\frac{\log\left(-ex^{2/3}\right)}{d^5}-\frac{1}{d^4ex^{2/3}}+\frac{1}{2d^3e^2x^{4}}\right.}{d}\right.\right.$$

$\Big\downarrow$ 2789

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^5x^{10/3}}d\left(d+ex^{2/3}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^4x^{10/3}}d\left(d+ex^{2/3}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{2n/3}\right)}{5e^5x^{10/3}}-\frac{1}{5}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^5}-\frac{\log\left(-ex\right.}{d^5}\right.}{}\right.\right.$$

$\Big\downarrow$ 2756

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{4e^4x^{8/3}}-\frac{1}{4}bn\int\frac{1}{e^4x^{10/3}}d\left(d+ex^{2/3}\right)}{d}+\frac{\int\frac{a+b\log\left(cx^{2n/3}\right)}{e^4x^{10/3}}d\left(d+ex^{2/3}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{2n/3}\right)}{5e^5x^{10/3}}-\frac{1}{5}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^5}\right.}{}\right.\right.$$

$\Big\downarrow$ 54

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{4e^4x^{8/3}}-\frac{1}{4}bn\int\left(-\frac{1}{d^4ex^{2/3}}+\frac{1}{d^4x^{2/3}}+\frac{1}{d^3e^2x^{4/3}}-\frac{1}{d^2e^3x^2}+\frac{1}{de^4x^{8/3}}\right)d\left(d+ex^{2/3}\right)}{d}+\frac{\int\frac{a+b\log\left(cx^{2n/3}\right)}{e^4x^{10/3}}d\left(d+ex^{2/3}\right)}{d}}{d}+\right.\right.$$

$\Big\downarrow$ 2009

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^4x^{10/3}}d\left(d+ex^{2/3}\right)}{d}+\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{4e^4x^{8/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^4}-\frac{\log\left(-ex^{2/3}\right)}{d^4}-\frac{1}{d^3ex^{2/3}}+\frac{1}{2d^2e^2x^{4/3}}-\frac{1}{3de^3x^2}\right)}{d}}{d}+\frac{-\frac{a}{\cdots}}{\cdots}\cdots\right.\right.$$

$\big\downarrow$ 2789

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^4x^{8/3}}d\left(d+ex^{2/3}\right)}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^3x^{8/3}}d\left(d+ex^{2/3}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{4e^4x^{8/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^4}-\frac{\log\left(-ex^{2/3}\right)}{d^4}-\frac{1}{d^3ex^{2/3}}\cdots\right.}{d}}{d}\cdots\right.\right.$$

$\big\downarrow$ 2756

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^3x^{8/3}}d\left(d+ex^{2/3}\right)}{d}+\frac{-\frac{1}{3}bn\int -\frac{1}{e^3x^{8/3}}d\left(d+ex^{2/3}\right)-\frac{a+b\log\left(cx^{2n/3}\right)}{3e^3x^2}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{4e^4x^{8/3}}-\frac{1}{4}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^4}-\frac{\log\left(-\cdots\right.}{\cdots}\right.}{d}}{d}\cdots\right.\right.$$

$\big\downarrow$ 54

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int\left(-\frac{1}{d^3ex^{2/3}}+\frac{1}{d^3x^{2/3}}+\frac{1}{d^2e^2x^{4/3}}-\frac{1}{de^3x^2}\right)d\left(d+ex^{2/3}\right)-\frac{a+b\log\left(cx^{2n/3}\right)}{3e^3x^2}}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^3x^{8/3}}d\left(d+ex^{2/3}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{2n/\cdots}\right.}{4e^4x^{8/3}}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$\big\downarrow$ 2009

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^3x^{8/3}}d\left(d+ex^{2/3}\right)}{d}+\frac{-\frac{a+b\log\left(cx^{2n/3}\right)}{3e^3x^2}-\frac{1}{3}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^3}-\frac{\log\left(-ex^{2/3}\right)}{d^3}-\frac{1}{d^2ex^{2/3}}+\frac{1}{2de^2x^{4/3}}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{4e^4x^{8/3}}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$\big\downarrow$ 2789

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{e^3x^2}d\left(d+ex^{2/3}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{2n/3}\right)}{3e^3x^2}-\frac{1}{3}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^3}-\frac{\log\left(-ex^{2/3}\right)}{d^3}-\frac{1}{d^2ex^{2/3}}\right.}{d}}{d}}{d}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{2756}$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{2e^2x^{4/3}}-\frac{1}{2}bn\int \frac{1}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{2n/3}\right)}{3e^3x^2}-\frac{1}{3}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^3}-\frac{\log\left(-ex^{2/3}\right)}{d^3}\right.}{d}}{d}}{d}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{54}$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{2e^2x^{4/3}}-\frac{1}{2}bn\int \left(-\frac{1}{d^2ex^{2/3}}+\frac{1}{d^2x^{2/3}}+\frac{1}{de^2x^{4/3}}\right)d\left(d+ex^{2/3}\right)}{d}+\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{2n/3}\right)}{3e^3x^2}-\frac{1}{3}bn\left(\log\right.}{}}{d}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}+\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^2}-\frac{\log\left(-ex^{2/3}\right)}{d^2}-\frac{1}{dex^{2/3}}\right)}{d}}{d}+\frac{-\frac{a+b\log\left(cx^{2n/3}\right)}{3e^3x^2}-\frac{1}{3}bn\left(\frac{\log\left(d+e\right.}{d^3}\right.}{}}{d}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{2789}$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^{4/3}}d\left(d+ex^{2/3}\right)}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d\left(d+ex^{2/3}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^2}-\frac{\log\left(-ex^{2/3}\right)}{d^2}-\frac{1}{dex^{2/3}}\right)}{d}}{d}}{d}\right.\right.$$

$$\Big\downarrow \textcolor{blue}{2751}$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{-\frac{bn\int -\frac{1}{ex^{2/3}}d\left(d+ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d\left(d+ex^{2/3}\right)}{d}}{d}+\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^2}\right.}{d}}{d}}{d}\right.\right.$$

$$\Big\downarrow \ 16$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d\left(d+ex^{2/3}\right)}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}}{d}+\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^2}-\frac{\log\left(-ex\right.}{d^2}\right.}{d}}{d}}{d}\right.\right.$$

$$\Big\downarrow \ 2779$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{bn\int \frac{\log\left(1-\frac{d}{x^{2/3}}\right)}{x^{2/3}}d\left(d+ex^{2/3}\right)}{d}-\frac{\log\left(1-\frac{d}{x^{2/3}}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}}{d}+\frac{a+b\log\left(cx^{2n/3}\right.}{2e^2x^{4/3}}}{d}}{d}\right.\right.$$

$$\Big\downarrow \ 2838$$

$$\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{a+b\log\left(cx^{2n/3}\right)}{2e^2x^{4/3}}-\frac{1}{2}bn\left(\frac{\log\left(d+ex^{2/3}\right)}{d^2}-\frac{\log\left(-ex^{2/3}\right)}{d^2}-\frac{1}{dex^{2/3}}\right)}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}+\frac{bn\,\mathrm{PolyLog}\left(2,\frac{d}{x^{2/}}\right.}{d}}{d}}{d}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

output
```
(3*(-1/6*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4 + (b*e^6*n*((-1/5*(b*n*(1/
(4*d*e^4*x^(8/3)) - 1/(3*d^2*e^3*x^2) + 1/(2*d^3*e^2*x^(4/3)) - 1/(d^4*e*x
^(2/3)) + Log[d + e*x^(2/3)]/d^5 - Log[-(e*x^(2/3))]/d^5)) - (a + b*Log[c*
x^((2*n)/3)])/(5*e^5*x^(10/3)))/d + ((-1/4*(b*n*(-1/3*1/(d*e^3*x^2) + 1/(2
*d^2*e^2*x^(4/3)) - 1/(d^3*e*x^(2/3)) + Log[d + e*x^(2/3)]/d^4 - Log[-(e*x
^(2/3))]/d^4)) + (a + b*Log[c*x^((2*n)/3)])/(4*e^4*x^(8/3)))/d + ((-1/3*(b
*n*(1/(2*d*e^2*x^(4/3)) - 1/(d^2*e*x^(2/3)) + Log[d + e*x^(2/3)]/d^3 - Log
[-(e*x^(2/3))]/d^3)) - (a + b*Log[c*x^((2*n)/3)])/(3*e^3*x^2))/d + ((-1/2*
(b*n*(-(1/(d*e*x^(2/3))) + Log[d + e*x^(2/3)]/d^2 - Log[-(e*x^(2/3))]/d^2)
) + (a + b*Log[c*x^((2*n)/3)])/(2*e^2*x^(4/3)))/d + (((b*n*Log[-(e*x^(2/3)
)])/d - ((d + e*x^(2/3))*(a + b*Log[c*x^((2*n)/3)]))/(d*e*x^(2/3)))/d + (-
((Log[1 - d/x^(2/3)]*(a + b*Log[c*x^((2*n)/3)]))/d) + (b*n*PolyLog[2, d/x^
(2/3)])/d)/d)/d)/d)/d)/d))/3))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{5}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{5}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{5}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c)
+ a^2)/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="maxima")
```

output
```
-1/4*b^2*log((e*x^(2/3) + d)^n)^2/x^4 + integrate(1/3*(3*(b^2*e*log(c)^2 +
 2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(
b^2*d*log(c) + a*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + 3*(b^2*d*log(c)^2
+ 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^6 + d*x^(16/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^2}{x^5}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^5,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^5, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx=\frac{-24x^{\frac{2}{3}}\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)b^2d^5en+60x^{\frac{8}{3}}\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)b^2d^2e^4n}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x)
```

output
```
( - 24*x**(2/3)*log((x**(2/3)*e + d)**n*c)*b**2*d**5*e*n + 60*x**(2/3)*log
((x**(2/3)*e + d)**n*c)*b**2*d**2*e**4*n*x**2 - 24*x**(2/3)*a*b*d**5*e*n +
 60*x**(2/3)*a*b*d**2*e**4*n*x**2 - 47*x**(2/3)*b**2*d**2*e**4*n**2*x**2 +
 30*x**(1/3)*log((x**(2/3)*e + d)**n*c)*b**2*d**4*e**2*n*x - 120*x**(1/3)*
log((x**(2/3)*e + d)**n*c)*b**2*d*e**5*n*x**3 + 30*x**(1/3)*a*b*d**4*e**2*
n*x - 120*x**(1/3)*a*b*d*e**5*n*x**3 - 6*x**(1/3)*b**2*d**4*e**2*n**2*x +
154*x**(1/3)*b**2*d*e**5*n**2*x**3 - 80*int(log((x**(2/3)*e + d)**n*c)/(x*
*(2/3)*e*x + d*x),x)*b**2*d*e**6*n*x**4 - 240*log(x**(1/3))*a*b*e**6*n*x**
4 + 548*log(x**(1/3))*b**2*e**6*n**2*x**4 - 60*log((x**(2/3)*e + d)**n*c)*
*2*b**2*d**6 - 120*log((x**(2/3)*e + d)**n*c)*a*b*d**6 + 120*log((x**(2/3)
*e + d)**n*c)*a*b*e**6*x**4 - 40*log((x**(2/3)*e + d)**n*c)*b**2*d**3*e**3
*n*x**2 - 274*log((x**(2/3)*e + d)**n*c)*b**2*e**6*n*x**4 - 60*a**2*d**6 -
 40*a*b*d**3*e**3*n*x**2 + 18*b**2*d**3*e**3*n**2*x**2)/(240*d**6*x**4)
```

3.476 $\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 547

$$\begin{aligned}\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx &= -\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4}-\frac{1984b^2d^3n^2x}{2835e^3}\\ &+\frac{1144b^2d^2n^2x^{5/3}}{4725e^2}-\frac{128b^2dn^2x^{7/3}}{1323e}+\frac{8}{243}b^2n^2x^3-\frac{4504b^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{945e^{9/2}}\\ &+\frac{4ib^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3e^{9/2}}+\frac{8b^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}}\\ &-\frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}\\ &-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2}+\frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e}\\ &-\frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{4bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}}+\frac{1}{3}x^3\left(a+b\log\right.\end{aligned}$$

output
```
-4/3*a*b*d^4*n*x^(1/3)/e^4+4504/945*b^2*d^4*n^2*x^(1/3)/e^4-1984/2835*b^2*
d^3*n^2*x/e^3+1144/4725*b^2*d^2*n^2*x^(5/3)/e^2-128/1323*b^2*d*n^2*x^(7/3)
/e+8/243*b^2*n^2*x^3-4504/945*b^2*d^(9/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/
2))/e^(9/2)+4/3*I*b^2*d^(9/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))^2/e^(9/2
)+8/3*b^2*d^(9/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2
)+I*e^(1/2)*x^(1/3)))/e^(9/2)-4/3*b^2*d^4*n*x^(1/3)*ln(c*(d+e*x^(2/3))^n)/
e^4+4/9*b*d^3*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3-4/15*b*d^2*n*x^(5/3)*(a+
b*ln(c*(d+e*x^(2/3))^n))/e^2+4/21*b*d*n*x^(7/3)*(a+b*ln(c*(d+e*x^(2/3))^n)
)/e-4/27*b*n*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))+4/3*b*d^(9/2)*n*arctan(e^(1/2
)*x^(1/3)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(9/2)+1/3*x^3*(a+b*ln(c*(
d+e*x^(2/3))^n))^2+4/3*I*b^2*d^(9/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*
e^(1/2)*x^(1/3)))/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.80

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{396900ib^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2+1260bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(315a-\right.}{}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

output
```
((396900*I)*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 + 1260*b*d
^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(315*a - 1126*b*n + 630*b*n*Log
[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 315*b*Log[c*(d + e*x^(2/3))^
n]) + Sqrt[e]*x^(1/3)*(99225*a^2*e^4*x^(8/3) - 1260*a*b*n*(315*d^4 - 105*d
^3*e*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)) + 8*b^2
*n^2*(177345*d^4 - 26040*d^3*e*x^(2/3) + 9009*d^2*e^2*x^(4/3) - 3600*d*e^3
*x^2 + 1225*e^4*x^(8/3)) - 630*b*(-315*a*e^4*x^(8/3) + 2*b*n*(315*d^4 - 10
5*d^3*e*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)))*Log
[c*(d + e*x^(2/3))^n] + 99225*b^2*e^4*x^(8/3)*Log[c*(d + e*x^(2/3))^n]^2)
+ (396900*I)*b^2*d^(9/2)*n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I
)*Sqrt[d] + Sqrt[e]*x^(1/3))])/(297675*e^(9/2))
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2908, 2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)^2 \, dx$$

$$\Big\downarrow \text{2908}$$

$$3 \int x^{8/3} \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)^2 d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3 \left(\frac{1}{9} x^3 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)^2-\frac{4}{9} b e n \int \frac{x^{10/3} \left(a+b \log \left(c (d+e x^{2/3})^n\right)\right)}{d+e x^{2/3}} d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{2926}$$

$$3 \left(\frac{1}{9} x^3 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)^2-\frac{4}{9} b e n \int \left(-\frac{\left(a+b \log \left(c (d+e x^{2/3})^n\right)\right) d^5}{e^5 \left(d+e x^{2/3}\right)}+\frac{\left(a+b \log \left(c (d+e x^{2/3}\right)^n}{e^5}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3 \left(\frac{1}{9} x^3 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^n\right)\right)^2-\frac{4}{9} b e n \left(-\frac{d^{9/2} \arctan \left(\frac{\sqrt{e} \sqrt[3]{x}}{\sqrt{d}}\right) \left(a+b \log \left(c (d+e x^{2/3})^n\right)\right)}{e^{11/2}}-\frac{d^3 x \left(a+\right.}{}\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

output
```
3*((x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/9 - (4*b*e*n*((a*d^4*x^(1/3))/
e^5 - (1126*b*d^4*n*x^(1/3))/(315*e^5) + (496*b*d^3*n*x)/(945*e^4) - (286*
b*d^2*n*x^(5/3))/(1575*e^3) + (32*b*d*n*x^(7/3))/(441*e^2) - (2*b*n*x^3)/(
81*e) + (1126*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(315*e^(11/2)
) - (I*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(11/2) - (2*b*d^
(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqr
t[e]*x^(1/3))])/e^(11/2) + (b*d^4*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/e^5 -
(d^3*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^4) + (d^2*x^(5/3)*(a + b*Log
[c*(d + e*x^(2/3))^n]))/(5*e^3) - (d*x^(7/3)*(a + b*Log[c*(d + e*x^(2/3))^
n]))/(7*e^2) + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*e) - (d^(9/2)*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^(11/2)
- (I*b*d^(9/2)*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))]
)/e^(11/2)))/9)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int x^2 \left(a + b \ln \left(c \left(d + e\, x^{\frac{2}{3}}\right)^n\right)\right)^2 dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Fricas [F]

$$\int x^2 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^2 dx = \int \left(b \log \left(\left(e x^{\frac{2}{3}} + d\right)^n c\right) + a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*x^2*log((e*x^(2/3) + d
)^n*c) + a^2*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^2 \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 \, dx = \int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 \, dx = \int x^2 \left(a+b\,\ln \left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^2 \, dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^2, x)
```

Reduce [F]

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx = \frac{396900\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)ab\,d^4n - 1418760\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)b^2d^4n^2 - 7}{}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x)
```

output
```
(396900*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a*b*d**4*n -
1418760*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b**2*d**4*n**
2 - 79380*x**(2/3)*log((x**(2/3)*e + d)**n*c)*b**2*d**2*e**3*n*x - 79380*x
**(2/3)*a*b*d**2*e**3*n*x + 72072*x**(2/3)*b**2*d**2*e**3*n**2*x - 396900*
x**(1/3)*log((x**(2/3)*e + d)**n*c)*b**2*d**4*e*n + 56700*x**(1/3)*log((x*
*(2/3)*e + d)**n*c)*b**2*d*e**4*n*x**2 - 396900*x**(1/3)*a*b*d**4*e*n + 56
700*x**(1/3)*a*b*d*e**4*n*x**2 + 1418760*x**(1/3)*b**2*d**4*e*n**2 - 28800
*x**(1/3)*b**2*d*e**4*n**2*x**2 + 132300*int(log((x**(2/3)*e + d)**n*c)/(x
**(2/3)*d + x**(1/3)*e*x),x)*b**2*d**5*e*n + 99225*log((x**(2/3)*e + d)**n
*c)**2*b**2*e**5*x**3 + 198450*log((x**(2/3)*e + d)**n*c)*a*b*e**5*x**3 +
132300*log((x**(2/3)*e + d)**n*c)*b**2*d**3*e**2*n*x - 44100*log((x**(2/3)
*e + d)**n*c)*b**2*e**5*n*x**3 + 99225*a**2*e**5*x**3 + 132300*a*b*d**3*e*
*2*n*x - 44100*a*b*e**5*n*x**3 - 208320*b**2*d**3*e**2*n**2*x + 9800*b**2*
e**5*n**2*x**3)/(297675*e**5)
```

3.477 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 364

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx = \frac{4abdn\sqrt[3]{x}}{e}-\frac{32b^2dn^2\sqrt[3]{x}}{3e}+\frac{8}{9}b^2n^2x+\frac{32b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{4ib^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}-\frac{8b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}+\frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{4bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+x\left(a+b\log\left(c(\right.\right.$$

output
```
4*a*b*d*n*x^(1/3)/e-32/3*b^2*d*n^2*x^(1/3)/e+8/9*b^2*n^2*x+32/3*b^2*d^(3/2
)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))/e^(3/2)-4*I*b^2*d^(3/2)*n^2*arctan(e
^(1/2)*x^(1/3)/d^(1/2))^2/e^(3/2)-8*b^2*d^(3/2)*n^2*arctan(e^(1/2)*x^(1/3)
/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/e^(3/2)+4*b^2*d*n*x^(1
/3)*ln(c*(d+e*x^(2/3))^n)/e-4/3*b*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))-4*b*d^(3
/2)*n*arctan(e^(1/2)*x^(1/3)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(3/2)+
x*(a+b*ln(c*(d+e*x^(2/3))^n))^2-4*I*b^2*d^(3/2)*n^2*polylog(2,1-2*d^(1/2)/
(d^(1/2)+I*e^(1/2)*x^(1/3)))/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.88

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx = \frac{-36ib^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2 - 12bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(3a-8bn+6\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

output
```
((-36*I)*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 12*b*d^(3/2
)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(3*a - 8*b*n + 6*b*n*Log[(2*Sqrt[d])
/(Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 3*b*Log[c*(d + e*x^(2/3))^n]) + Sqrt[e]*
x^(1/3)*(12*a*b*n*(3*d - e*x^(2/3)) + 8*b^2*n^2*(-12*d + e*x^(2/3)) + 9*a^
2*e*x^(2/3) + 6*b*(6*b*d*n + 3*a*e*x^(2/3) - 2*b*e*n*x^(2/3))*Log[c*(d + e
*x^(2/3))^n] + 9*b^2*e*x^(2/3)*Log[c*(d + e*x^(2/3))^n]^2) - (36*I)*b^2*d^
(3/2)*n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]
*x^(1/3))])/(9*e^(3/2))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2901, 2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx$$

$$\Big\downarrow \text{2901}$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{4}{3}ben\int\frac{x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d+ex^{2/3}}d\sqrt[3]{x}\right)$$

$\Big\downarrow$ 2926

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{4}{3}ben\int\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)d^2}{e^2\left(d+ex^{2/3}\right)}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^2}\right.\right.$$

$\Big\downarrow$ 2009

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{4}{3}ben\left(\frac{d^{3/2}\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{5/2}}+\frac{x\left(a+b\log\right.}{}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

output
```
3*((x*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/3 - (4*b*e*n*(-((a*d*x^(1/3))/e^
2) + (8*b*d*n*x^(1/3))/(3*e^2) - (2*b*n*x)/(9*e) - (8*b*d^(3/2)*n*ArcTan[(
Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(5/2)) + (I*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]]^2)/e^(5/2) + (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[
d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/e^(5/2) - (b*d*x^(1/3)
*Log[c*(d + e*x^(2/3))^n])/e^2 + (x*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e
) + (d^(3/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3)
)^n]))/e^(5/2) + (I*b*d^(3/2)*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x^(1/3))])/e^(5/2)))/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \left(a+b\ln \left(c{\left(d+e\,x^{\frac{2}{3}}\right)}^{n}\right)\right)^{2}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Fricas [F]

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\int \left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^2\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(2/3))**n))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^2\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx=\int \left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^2 \,d x$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx=\frac{-36\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)abdn+96\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)b^2d\,n^2+36x^{\frac{1}{3}}\log\left(\left(x\right.\right.}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^2,x)
```

output
```
( - 36*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a*b*d*n + 96*s
qrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b**2*d*n**2 + 36*x**(1
/3)*log((x**(2/3)*e + d)**n*c)*b**2*d*e*n + 36*x**(1/3)*a*b*d*e*n - 96*x**
(1/3)*b**2*d*e*n**2 - 12*int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*d + x**(
1/3)*e*x),x)*b**2*d**2*e*n + 9*log((x**(2/3)*e + d)**n*c)**2*b**2*e**2*x +
 18*log((x**(2/3)*e + d)**n*c)*a*b*e**2*x - 12*log((x**(2/3)*e + d)**n*c)*
b**2*e**2*n*x + 9*a**2*e**2*x - 12*a*b*e**2*n*x + 8*b**2*e**2*n**2*x)/(9*e
**2)
```

3.478 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 298

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx = \frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} - \frac{4ib^2e^{3/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}$$

output
```
8*b^2*e^(3/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))/d^(3/2)-4*I*b^2*e^(3/2)*
n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))^2/d^(3/2)-8*b^2*e^(3/2)*n^2*arctan(e^(
1/2)*x^(1/3)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/d^(3/2)-4*
b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(1/3)-4*b*e^(3/2)*n*arctan(e^(1/2)*x
^(1/3)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(3/2)-(a+b*ln(c*(d+e*x^(2/3)
)^n))^2/x-4*I*b^2*e^(3/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(
1/3)))/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx = \frac{-4ib^2e^{3/2}n^2x\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2-4be^{3/2}nx\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a-2b\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```

output
```
((-4*I)*b^2*e^(3/2)*n^2*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 4*b*e^(3/2
)*n*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - 2*b*n + 2*b*n*Log[(2*Sqrt[d])
/(Sqrt[d] + I*Sqrt[e]*x^(1/3))] + b*Log[c*(d + e*x^(2/3))^n]) - Sqrt[d]*(a
 + b*Log[c*(d + e*x^(2/3))^n])*(a*d + 4*b*e*n*x^(2/3) + b*d*Log[c*(d + e*x
^(2/3))^n]) - (4*I)*b^2*e^(3/2)*n^2*x*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1
/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))])/(d^(3/2)*x)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2908, 2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(\frac{4}{3}ben\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{\left(d+ex^{2/3}\right)x^{2/3}}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x}\right)$$

$$\Big\downarrow \text{2926}$$

$$3\left(\frac{4}{3}ben\int \left(\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{dx^{2/3}}-\frac{e\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\left(d+ex^{2/3}\right)}\right)d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x}+\frac{4}{3}ben\left(-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{a+b\log\left(c(\right.}{d\sqrt[3]{}}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```

output
```
3*(-1/3*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x + (4*b*e*n*((2*b*Sqrt[e]*n*Ar
cTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(3/2) - (I*b*Sqrt[e]*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]^2)/d^(3/2) - (2*b*Sqrt[e]*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(3/2) - (a + b*Log
[c*(d + e*x^(2/3))^n])/(d*x^(1/3)) - (Sqrt[e]*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(3/2) - (I*b*Sqrt[e]*n*PolyLog[2
, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(3/2)))/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c)
+ a^2)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx=\frac{-12\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)abenx-8\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)b^2e\,n^2x-12x}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x)
```

output
```
( - 12*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a*b*e*n*x - 8*
sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b**2*e*n**2*x - 12*x*
*(2/3)*a*b*d*e*n - 8*x**(2/3)*b**2*d*e*n**2 - 4*int(log((x**(2/3)*e + d)**
n*c)/(x**(2/3)*e*x**2 + d*x**2),x)*b**2*d**3*n*x - 3*log((x**(2/3)*e + d)*
*n*c)**2*b**2*d**2 - 6*log((x**(2/3)*e + d)**n*c)*a*b*d**2 - 4*log((x**(2/
3)*e + d)**n*c)*b**2*d**2*n - 3*a**2*d**2)/(3*d**2*x)
```

3.479 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 476

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx &= -\frac{8b^2e^2n^2}{105d^2x^{5/3}}+\frac{32b^2e^3n^2}{105d^3x}-\frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}}\\
&-\frac{1408b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}}+\frac{4ib^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3d^{9/2}}\\
&+\frac{8b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}}\\
&-\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}}+\frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}}\\
&-\frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x}+\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}}\\
&+\frac{4be^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}}\\
&-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3}+\frac{4ib^2e^{9/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}}\end{aligned}$$

output
```
-8/105*b^2*e^2*n^2/d^2/x^(5/3)+32/105*b^2*e^3*n^2/d^3/x-568/315*b^2*e^4*n^
2/d^4/x^(1/3)-1408/315*b^2*e^(9/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))/d^(
9/2)+4/3*I*b^2*e^(9/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))^2/d^(9/2)+8/3*b
^2*e^(9/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(
1/2)*x^(1/3)))/d^(9/2)-4/21*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(7/3)+4/
15*b*e^2*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^2/x^(5/3)-4/9*b*e^3*n*(a+b*ln(c*(
d+e*x^(2/3))^n))/d^3/x+4/3*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(1/3)
+4/3*b*e^(9/2)*n*arctan(e^(1/2)*x^(1/3)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n
))/d^(9/2)-1/3*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x^3+4/3*I*b^2*e^(9/2)*n^2*pol
ylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/d^(9/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3}$$

$$+\frac{4}{3}ben\left(-\frac{2be^{7/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{9/2}}-\frac{2ben\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\frac{ex^{2/3}}{d}\right)}{35d^2x^{5/3}}+\frac{2be^2n\,\text{Hypergeomet}}{}\right.$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```

output
```
-1/3*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3 + (4*b*e*n*((-2*b*e^(7/2)*n*Ar
cTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(9/2) - (2*b*e*n*Hypergeometric2F1[-5/2
, 1, -3/2, -((e*x^(2/3))/d)])/(35*d^2*x^(5/3)) + (2*b*e^2*n*Hypergeometric
2F1[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^3*x) - (2*b*e^3*n*Hypergeometr
ic2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^4*x^(1/3)) - (a + b*Log[c*(d +
 e*x^(2/3))^n])/(7*d*x^(7/3)) + (e*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^
2*x^(5/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^3*x) + (e^3*(a +
b*Log[c*(d + e*x^(2/3))^n]))/(d^4*x^(1/3)) + (e^(7/2)*ArcTan[(Sqrt[e]*x^(1
/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(9/2) + (I*b*e^(7/2)*n*(
ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*
I)*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))]) + PolyLog[2, (I*Sqrt[d]
 + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))]))/d^(9/2)))/3
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2908, 2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(\frac{4}{9}ben\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{\left(d+ex^{2/3}\right)x^{8/3}}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{9x^3}\right)$$

$$\Big\downarrow \text{2926}$$

$$3\left(\frac{4}{9}ben\int \left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)e^4}{d^4\left(d+ex^{2/3}\right)}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)e^3}{d^4x^{2/3}}+\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)e}{d^3x^{4/3}}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{9x^3}+\frac{4}{9}ben\left(\frac{e^{7/2}\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}}+\frac{e^3\left(a+b\log\left(\right.\right.}{d}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```

output
```
3*(-1/9*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3 + (4*b*e*n*((-2*b*e*n)/(35*
d^2*x^(5/3)) + (8*b*e^2*n)/(35*d^3*x) - (142*b*e^3*n)/(105*d^4*x^(1/3)) -
(352*b*e^(7/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(105*d^(9/2)) + (I*b*e
^(7/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(9/2) + (2*b*e^(7/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3
))])/d^(9/2) - (a + b*Log[c*(d + e*x^(2/3))^n])/(7*d*x^(7/3)) + (e*(a + b*
Log[c*(d + e*x^(2/3))^n]))/(5*d^2*x^(5/3)) - (e^2*(a + b*Log[c*(d + e*x^(2
/3))^n]))/(3*d^3*x) + (e^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^4*x^(1/3))
 + (e^(7/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))
^n]))/d^(9/2) + (I*b*e^(7/2)*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqr
t[e]*x^(1/3))])/d^(9/2)))/9)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^4,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{4}}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{4}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c)
+ a^2)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{4}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^4,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = \frac{3780\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)ab\,e^4n\,x^3+840\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)b^2e^4n^2x^3}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x)
```

output
```
(3780*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a*b*e**4*n*x**3
 + 840*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b**2*e**4*n**2
*x**3 - 540*x**(2/3)*a*b*d**4*e*n + 3780*x**(2/3)*a*b*d*e**4*n*x**2 - 120*
x**(2/3)*b**2*d**4*e*n**2 + 840*x**(2/3)*b**2*d*e**4*n**2*x**2 + 756*x**(1
/3)*a*b*d**3*e**2*n*x + 168*x**(1/3)*b**2*d**3*e**2*n**2*x - 1260*int(log(
(x**(2/3)*e + d)**n*c)/(x**(2/3)*e*x**4 + d*x**4),x)*b**2*d**6*n*x**3 - 94
5*log((x**(2/3)*e + d)**n*c)**2*b**2*d**5 - 1890*log((x**(2/3)*e + d)**n*c
)*a*b*d**5 - 420*log((x**(2/3)*e + d)**n*c)*b**2*d**5*n - 945*a**2*d**5 -
1260*a*b*d**2*e**3*n*x**2 - 280*b**2*d**2*e**3*n**2*x**2)/(2835*d**5*x**3)
```

3.480 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 640

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx = -\frac{8b^2e^2n^2}{715d^2x^{11/3}}+\frac{64b^2e^3n^2}{2145d^3x^3} -\frac{2872b^2e^4n^2}{45045d^4x^{7/3}}+\frac{1216b^2e^5n^2}{9009d^5x^{5/3}}-\frac{224072b^2e^6n^2}{675675d^6x}+\frac{344192b^2e^7n^2}{225225d^7\sqrt[3]{x}} +\frac{704552b^2e^{15/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{225225d^{15/2}}-\frac{4ib^2e^{15/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}} -\frac{8b^2e^{15/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}} -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}}+\frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} -\frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3} +\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}}-\frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}} +\frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x}-\frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}} -\frac{4be^{15/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5}-\frac{4ib^2e^{15/2}n^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}}$$

output
```
-8/715*b^2*e^2*n^2/d^2/x^(11/3)+64/2145*b^2*e^3*n^2/d^3/x^3-2872/45045*b^2
*e^4*n^2/d^4/x^(7/3)+1216/9009*b^2*e^5*n^2/d^5/x^(5/3)-224072/675675*b^2*e
^6*n^2/d^6/x+344192/225225*b^2*e^7*n^2/d^7/x^(1/3)+704552/225225*b^2*e^(15
/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))/d^(15/2)-4/5*I*b^2*e^(15/2)*n^2*ar
ctan(e^(1/2)*x^(1/3)/d^(1/2))^2/d^(15/2)-8/5*b^2*e^(15/2)*n^2*arctan(e^(1/
2)*x^(1/3)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/d^(15/2)-4/6
5*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(13/3)+4/55*b*e^2*n*(a+b*ln(c*(d+e
*x^(2/3))^n))/d^2/x^(11/3)-4/45*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x^
3+4/35*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(7/3)-4/25*b*e^5*n*(a+b*l
n(c*(d+e*x^(2/3))^n))/d^5/x^(5/3)+4/15*b*e^6*n*(a+b*ln(c*(d+e*x^(2/3))^n))
/d^6/x-4/5*b*e^7*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^7/x^(1/3)-4/5*b*e^(15/2)*
n*arctan(e^(1/2)*x^(1/3)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(15/2)-1/5
*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5-4/5*I*b^2*e^(15/2)*n^2*polylog(2,1-2*d^
(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/d^(15/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.24 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.06

$$\int \frac{\left(a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```

output
```
-1/5*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5 + (4*b*e*n*((2*b*e^(13/2)*n*Ar
cTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(15/2) - (2*b*e*n*Hypergeometric2F1[-11
/2, 1, -9/2, -((e*x^(2/3))/d)])/(143*d^2*x^(11/3)) + (2*b*e^2*n*Hypergeome
tric2F1[-9/2, 1, -7/2, -((e*x^(2/3))/d)])/(99*d^3*x^3) - (2*b*e^3*n*Hyperg
eometric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/(63*d^4*x^(7/3)) + (2*b*e^4*
n*Hypergeometric2F1[-5/2, 1, -3/2, -((e*x^(2/3))/d)])/(35*d^5*x^(5/3)) - (
2*b*e^5*n*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^6*x) +
 (2*b*e^6*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^7*x^(1
/3)) - (a + b*Log[c*(d + e*x^(2/3))^n])/(13*d*x^(13/3)) + (e*(a + b*Log[c*
(d + e*x^(2/3))^n]))/(11*d^2*x^(11/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))
^n]))/(9*d^3*x^3) + (e^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(7*d^4*x^(7/3))
 - (e^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^5*x^(5/3)) + (e^5*(a + b*Lo
g[c*(d + e*x^(2/3))^n]))/(3*d^6*x) - (e^6*(a + b*Log[c*(d + e*x^(2/3))^n])
)/(d^7*x^(1/3)) - (e^(13/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c
*(d + e*x^(2/3))^n]))/d^(15/2) - (I*b*e^(13/2)*n*(ArcTan[(Sqrt[e]*x^(1/3))
/Sqrt[d]]*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*I)*Log[(2*Sqrt[d])/(Sqrt
[d] + I*Sqrt[e]*x^(1/3))]) + PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I
)*Sqrt[d] + Sqrt[e]*x^(1/3))]))/d^(15/2)))/5
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2908, 2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^{16/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(\frac{4}{15}ben\int\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{\left(d+ex^{2/3}\right)x^{14/3}}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{15x^5}\right)$$

$$\Big\downarrow \text{2926}$$

$$3\left(\frac{4}{15}ben\int\left(-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)e^7}{d^7\left(d+ex^{2/3}\right)}+\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)e^6}{d^7x^{2/3}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right.}{d^6x^{4/3}}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{15x^5}+\frac{4}{15}ben\left(-\frac{e^{13/2}\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{15/2}}-\frac{e^6\left(a+b\right.}{}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```

output
```
3*(-1/15*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5 + (4*b*e*n*((-2*b*e*n)/(14
3*d^2*x^(11/3)) + (16*b*e^2*n)/(429*d^3*x^3) - (718*b*e^3*n)/(9009*d^4*x^(
7/3)) + (1520*b*e^4*n)/(9009*d^5*x^(5/3)) - (56018*b*e^5*n)/(135135*d^6*x)
 + (86048*b*e^6*n)/(45045*d^7*x^(1/3)) + (176138*b*e^(13/2)*n*ArcTan[(Sqrt
[e]*x^(1/3))/Sqrt[d]])/(45045*d^(15/2)) - (I*b*e^(13/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]^2)/d^(15/2) - (2*b*e^(13/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/S
qrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(15/2) - (a + b*
Log[c*(d + e*x^(2/3))^n])/(13*d*x^(13/3)) + (e*(a + b*Log[c*(d + e*x^(2/3)
)^n]))/(11*d^2*x^(11/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*d^3*x
^3) + (e^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(7*d^4*x^(7/3)) - (e^4*(a + b
*Log[c*(d + e*x^(2/3))^n]))/(5*d^5*x^(5/3)) + (e^5*(a + b*Log[c*(d + e*x^(
2/3))^n]))/(3*d^6*x) - (e^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^7*x^(1/3)
) - (e^(13/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3
))^n]))/d^(15/2) - (I*b*e^(13/2)*n*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I
*Sqrt[e]*x^(1/3))])/d^(15/2)))/15)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{6}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^6,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^6,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^2}{x^6}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="fricas")
```

output
```
integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c)
+ a^2)/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^2}{x^6}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^6}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^6,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^6, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx=\frac{-2702700\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)ab\,e^7n\,x^5-360360\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x)
```

output

```
( - 2702700*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a*b*e**7*
n*x**5 - 360360*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b**2*
e**7*n**2*x**5 - 207900*x**(2/3)*a*b*d**7*e*n + 386100*x**(2/3)*a*b*d**4*e
**4*n*x**2 - 2702700*x**(2/3)*a*b*d*e**7*n*x**4 - 27720*x**(2/3)*b**2*d**7
*e*n**2 + 51480*x**(2/3)*b**2*d**4*e**4*n**2*x**2 - 360360*x**(2/3)*b**2*d
*e**7*n**2*x**4 + 245700*x**(1/3)*a*b*d**6*e**2*n*x - 540540*x**(1/3)*a*b*
d**3*e**5*n*x**3 + 32760*x**(1/3)*b**2*d**6*e**2*n**2*x - 72072*x**(1/3)*b
**2*d**3*e**5*n**2*x**3 - 900900*int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*
e*x**6 + d*x**6),x)*b**2*d**9*n*x**5 - 675675*log((x**(2/3)*e + d)**n*c)**
2*b**2*d**8 - 1351350*log((x**(2/3)*e + d)**n*c)*a*b*d**8 - 180180*log((x*
*(2/3)*e + d)**n*c)*b**2*d**8*n - 675675*a**2*d**8 - 300300*a*b*d**5*e**3*
n*x**2 + 900900*a*b*d**2*e**6*n*x**4 - 40040*b**2*d**5*e**3*n**2*x**2 + 12
0120*b**2*d**2*e**6*n**2*x**4)/(3378375*d**8*x**5)
```

3.481 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 913

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

output

```
-9*a*b^2*d^5*n^2*x^(2/3)/e^5-9*b^3*d^5*n^2*(d+e*x^(2/3))*ln(c*(d+e*x^(2/3)
)^n)/e^6+45/8*b^2*d^4*n^2*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-
10/3*b^2*d^3*n^2*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+45/32*b^2
*d^2*n^2*(d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-9/25*b^2*d*n^2*(d
+e*x^(2/3))^5*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+9/2*b*d^5*n*(d+e*x^(2/3))*(a
+b*ln(c*(d+e*x^(2/3))^n))^2/e^6-45/8*b*d^4*n*(d+e*x^(2/3))^2*(a+b*ln(c*(d+
e*x^(2/3))^n))^2/e^6+5*b*d^3*n*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))
^2/e^6-45/16*b*d^2*n*(d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6+9/1
0*b*d*n*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6+1/4*(d+e*x^(2/3)
)^6*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6+9*b^3*d^5*n^3*x^(2/3)/e^5+1/24*b^2*n
^2*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-1/8*b*n*(d+e*x^(2/3))^6
*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6-45/16*b^3*d^4*n^3*(d+e*x^(2/3))^2/e^6+1
0/9*b^3*d^3*n^3*(d+e*x^(2/3))^3/e^6-45/128*b^3*d^2*n^3*(d+e*x^(2/3))^4/e^6
+9/125*b^3*d*n^3*(d+e*x^(2/3))^5/e^6-3/2*d^5*(d+e*x^(2/3))*(a+b*ln(c*(d+e*
x^(2/3))^n))^3/e^6+15/4*d^4*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/
e^6-5*d^3*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6+15/4*d^2*(d+e*
x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6-3/2*d*(d+e*x^(2/3))^5*(a+b*ln
(c*(d+e*x^(2/3))^n))^3/e^6-1/144*b^3*n^3*(d+e*x^(2/3))^6/e^6
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.65

$$\int x^3\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^3\,dx=\frac{ex^{2/3}\left(36000a^3e^5x^{10/3}+b^3n^3\left(809340d^5-140070d^4ex^{2/3}+41180d^3e^2x^{4/3}-\right.\right.}{}$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

output
```
(e*x^(2/3)*(36000*a^3*e^5*x^(10/3) + b^3*n^3*(809340*d^5 - 140070*d^4*e*x^
(2/3) + 41180*d^3*e^2*x^(4/3) - 13785*d^2*e^3*x^2 + 4368*d*e^4*x^(8/3) - 1
000*e^5*x^(10/3)) - 60*a*b^2*n^2*(8820*d^5 - 2610*d^4*e*x^(2/3) + 1140*d^3
*e^2*x^(4/3) - 555*d^2*e^3*x^2 + 264*d*e^4*x^(8/3) - 100*e^5*x^(10/3)) + 1
800*a^2*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 20*d^3*e^2*x^(4/3) - 15*d^2*e^3*x
^2 + 12*d*e^4*x^(8/3) - 10*e^5*x^(10/3))) - 60*b*d^6*n*(1800*a^2 - 8820*a*
b*n + 13489*b^2*n^2)*Log[d + e*x^(2/3)] + 60*b*e*x^(2/3)*(1800*a^2*e^5*x^(
10/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 20*d^3*e^2*x^(4/3) - 15*d^2*
e^3*x^2 + 12*d*e^4*x^(8/3) - 10*e^5*x^(10/3)) + b^2*n^2*(-8820*d^5 + 2610*
d^4*e*x^(2/3) - 1140*d^3*e^2*x^(4/3) + 555*d^2*e^3*x^2 - 264*d*e^4*x^(8/3)
 + 100*e^5*x^(10/3)))*Log[c*(d + e*x^(2/3))^n] + 1800*b^2*(b*n*(147*d^6 +
60*d^5*e*x^(2/3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x^2 - 15*d^2*e^4*x^(8/3
) + 12*d*e^5*x^(10/3) - 10*e^6*x^4) - 60*a*(d^6 - e^6*x^4))*Log[c*(d + e*x
^(2/3))^n]^2 - 36000*b^3*(d^6 - e^6*x^4)*Log[c*(d + e*x^(2/3))^n]^3)/(1440
00*e^6)
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 915, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int x^{10/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3dx^{2/3}$$

$$\Big\downarrow \text{2848}$$

$$\frac{3}{2}\int\left(-\frac{\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^3d^5}{e^5}+\frac{5(d+ex^{2/3})\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^3d^4}{e^5}-\frac{10(d+ex^{2/3})^2\left(a\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\left(-\frac{b^3n^3(d+ex^{2/3})^6}{216e^6}+\frac{\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^3(d+ex^{2/3})^6}{6e^6}-\frac{bn\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^2(d+ex^{2}}{12e^6}\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

output
```
(3*((-15*b^3*d^4*n^3*(d + e*x^(2/3))^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e*x
^(2/3))^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e*x^(2/3))^4)/(64*e^6) + (6*b^3
*d*n^3*(d + e*x^(2/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(2/3))^6)/(216*e^6
) - (6*a*b^2*d^5*n^2*x^(2/3))/e^5 + (6*b^3*d^5*n^3*x^(2/3))/e^5 - (6*b^3*d
^5*n^2*(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^6 + (15*b^2*d^4*n^2*(d
+ e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*e^6) - (20*b^2*d^3*n^2
*(d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*e^6) + (15*b^2*d^2
*n^2*(d + e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(16*e^6) - (6*b^2
*d*n^2*(d + e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*e^6) + (b^2
*n^2*(d + e*x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(36*e^6) + (3*b*d
^5*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e^6 - (15*b*d^4*n
*(d + e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*e^6) + (10*b*d^3
*n*(d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*e^6) - (15*b*d
^2*n*(d + e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) + (3*b*
d*n*(d + e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(5*e^6) - (b*n*(
d + e*x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(12*e^6) - (d^5*(d +
e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/e^6 + (5*d^4*(d + e*x^(2/3)
)^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/(2*e^6) - (10*d^3*(d + e*x^(2/3))^
3*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/(3*e^6) + (5*d^2*(d + e*x^(2/3))^4*(
a + b*Log[c*(d + e*x^(2/3))^n])^3)/(2*e^6) - (d*(d + e*x^(2/3))^5*(a + ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^{3}\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1241, normalized size of antiderivative = 1.36

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

output
```
1/144000*(36000*b^3*e^6*x^4*log(c)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2
 + 18*a^2*b*e^6*n - 36*a^3*e^6)*x^4 + 36000*(b^3*e^6*n^3*x^4 - b^3*d^6*n^3
)*log(e*x^(2/3) + d)^3 + 20*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2
 + 1800*a^2*b*d^3*e^3*n)*x^2 + 1800*(20*b^3*d^3*e^3*n^3*x^2 + 147*b^3*d^6*
n^3 - 60*a*b^2*d^6*n^2 - 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2)*x^4 + 60*(b^3*
e^6*n^2*x^4 - b^3*d^6*n^2)*log(c) - 15*(b^3*d^2*e^4*n^3*x^2 - 4*b^3*d^5*e*
n^3)*x^(2/3) + 6*(2*b^3*d*e^5*n^3*x^3 - 5*b^3*d^4*e^2*n^3*x)*x^(1/3))*log(
e*x^(2/3) + d)^2 + 18000*(2*b^3*d^3*e^3*n*x^2 - (b^3*e^6*n - 6*a*b^2*e^6)*
x^4)*log(c)^2 - 60*(13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^
6*n - 100*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^4 + 60*(19*b^
3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x^2 - 1800*(b^3*e^6*n*x^4 - b^3*d^6*
n)*log(c)^2 - 60*(20*b^3*d^3*e^3*n^2*x^2 + 147*b^3*d^6*n^2 - 60*a*b^2*d^6*
n - 10*(b^3*e^6*n^2 - 6*a*b^2*e^6*n)*x^4)*log(c) + 15*(588*b^3*d^5*e*n^3 -
 240*a*b^2*d^5*e*n^2 - (37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x^2 + 6
0*(b^3*d^2*e^4*n^2*x^2 - 4*b^3*d^5*e*n^2)*log(c))*x^(2/3) + 6*(4*(11*b^3*d
*e^5*n^3 - 30*a*b^2*d*e^5*n^2)*x^3 - 15*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4
*e^2*n^2)*x - 60*(2*b^3*d*e^5*n^2*x^3 - 5*b^3*d^4*e^2*n^2*x)*log(c))*x^(1/
3))*log(e*x^(2/3) + d) + 1200*(5*(b^3*e^6*n^2 - 6*a*b^2*e^6*n + 18*a^2*b*e
^6)*x^4 - 3*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3*n)*x^2)*log(c) + 15*(53
956*b^3*d^5*e*n^3 - 35280*a*b^2*d^5*e*n^2 + 7200*a^2*b*d^5*e*n - (919*b...
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)\right)^3 dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 680, normalized size of antiderivative = 0.74

$$\int x^3 \left(a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4*log((e*x^(2/3) + d)^n*c)^3 + 3/4*a*b^2*x^4*log((e*x^(2/3) + d)
^n*c)^2 + 3/4*a^2*b*x^4*log((e*x^(2/3) + d)^n*c) + 1/4*a^3*x^4 - 1/80*a^2*
b*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 1
5*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^
6) - 1/2400*(60*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^
4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d
^5*x^(2/3))/e^6)*log((e*x^(2/3) + d)^n*c) - (100*e^6*x^4 - 264*d*e^5*x^(10
/3) + 555*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2 + 1800*d^6*log(e*x^(2/3) + d)
^2 + 2610*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^(2/3) + d) - 8820*d^5*e*x^(2/
3))*n^2/e^6)*a*b^2 - 1/144000*(1800*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (
10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*
d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6)*log((e*x^(2/3) + d)^n*c)^2 + e*n*((10
00*e^6*x^4 - 4368*d*e^5*x^(10/3) + 36000*d^6*log(e*x^(2/3) + d)^3 + 13785*
d^2*e^4*x^(8/3) - 41180*d^3*e^3*x^2 + 264600*d^6*log(e*x^(2/3) + d)^2 + 14
0070*d^4*e^2*x^(4/3) + 809340*d^6*log(e*x^(2/3) + d) - 809340*d^5*e*x^(2/3
))*n^2/e^7 - 60*(100*e^6*x^4 - 264*d*e^5*x^(10/3) + 555*d^2*e^4*x^(8/3) -
1140*d^3*e^3*x^2 + 1800*d^6*log(e*x^(2/3) + d)^2 + 2610*d^4*e^2*x^(4/3) +
8820*d^6*log(e*x^(2/3) + d) - 8820*d^5*e*x^(2/3))*n*log((e*x^(2/3) + d)^n*
c)/e^7))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2104 vs. 2(787) = 1574.

Time = 0.59 (sec) , antiderivative size = 2104, normalized size of antiderivative = 2.30

$$\int x^3 \left(a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

output
```
1/4*b^3*x^4*log(c)^3 + 3/4*a*b^2*x^4*log(c)^2 + 3/4*a^2*b*x^4*log(c) + 1/1
44000*(36000*(e*x^(2/3) + d)^6*log(e*x^(2/3) + d)^3/e^6 - 216000*(e*x^(2/3
) + d)^5*d*log(e*x^(2/3) + d)^3/e^6 + 540000*(e*x^(2/3) + d)^4*d^2*log(e*x
^(2/3) + d)^3/e^6 - 720000*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)^3/e^6
+ 540000*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)^3/e^6 - 18000*(e*x^(2/3)
 + d)^6*log(e*x^(2/3) + d)^2/e^6 + 129600*(e*x^(2/3) + d)^5*d*log(e*x^(2/3
) + d)^2/e^6 - 405000*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)^2/e^6 + 720
000*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)^2/e^6 - 810000*(e*x^(2/3) + d
)^2*d^4*log(e*x^(2/3) + d)^2/e^6 + 6000*(e*x^(2/3) + d)^6*log(e*x^(2/3) +
d)/e^6 - 51840*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 + 202500*(e*x^(2
/3) + d)^4*d^2*log(e*x^(2/3) + d)/e^6 - 480000*(e*x^(2/3) + d)^3*d^3*log(e
*x^(2/3) + d)/e^6 + 810000*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 -
1000*(e*x^(2/3) + d)^6/e^6 + 10368*(e*x^(2/3) + d)^5*d/e^6 - 50625*(e*x^(2
/3) + d)^4*d^2/e^6 + 160000*(e*x^(2/3) + d)^3*d^3/e^6 - 405000*(e*x^(2/3)
+ d)^2*d^4/e^6 - 216000*((e*x^(2/3) + d)*log(e*x^(2/3) + d)^3 - 3*(e*x^(2/
3) + d)*log(e*x^(2/3) + d)^2 + 6*(e*x^(2/3) + d)*log(e*x^(2/3) + d) - 6*e*
x^(2/3) - 6*d)*d^5/e^6)*b^3*n^3 + 1/4*a^3*x^4 + 1/2400*(1800*(e*x^(2/3) +
d)^6*log(e*x^(2/3) + d)^2/e^6 - 10800*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) +
d)^2/e^6 + 27000*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)^2/e^6 - 36000*(e
*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)^2/e^6 + 27000*(e*x^(2/3) + d)^2*...
```

Mupad [B] (verification not implemented)

Time = 35.14 (sec) , antiderivative size = 992, normalized size of antiderivative = 1.09

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(2/3)))^n))^3,x)
```

output
```
(a^3*x^4)/4 + (b^3*x^4*log(c*(d + e*x^(2/3))^n)^3)/4 - (b^3*n^3*x^4)/144 +
 (3*a*b^2*x^4*log(c*(d + e*x^(2/3))^n)^2)/4 - (b^3*n*x^4*log(c*(d + e*x^(2
/3))^n)^2)/8 + (b^3*n^2*x^4*log(c*(d + e*x^(2/3))^n))/24 + (a*b^2*n^2*x^4)
/24 - (b^3*d^6*log(c*(d + e*x^(2/3))^n)^3)/(4*e^6) + (3*a^2*b*x^4*log(c*(d
 + e*x^(2/3))^n))/4 - (a^2*b*n*x^4)/8 - (a*b^2*n*x^4*log(c*(d + e*x^(2/3))
^n))/4 - (13489*b^3*d^6*n^3*log(d + e*x^(2/3)))/(2400*e^6) + (2059*b^3*d^3
*n^3*x^2)/(7200*e^3) - (919*b^3*d^2*n^3*x^(8/3))/(9600*e^2) - (4669*b^3*d^
4*n^3*x^(4/3))/(4800*e^4) + (13489*b^3*d^5*n^3*x^(2/3))/(2400*e^5) - (3*a*
b^2*d^6*log(c*(d + e*x^(2/3))^n)^2)/(4*e^6) + (147*b^3*d^6*n*log(c*(d + e*
x^(2/3))^n)^2)/(80*e^6) + (91*b^3*d*n^3*x^(10/3))/(3000*e) - (3*a^2*b*d^6*
n*log(d + e*x^(2/3)))/(4*e^6) + (3*b^3*d*n*x^(10/3)*log(c*(d + e*x^(2/3))^
n)^2)/(20*e) - (11*b^3*d*n^2*x^(10/3)*log(c*(d + e*x^(2/3))^n))/(100*e) +
(a^2*b*d^3*n*x^2)/(4*e^3) - (3*a^2*b*d^2*n*x^(8/3))/(16*e^2) - (3*a^2*b*d^
4*n*x^(4/3))/(8*e^4) + (3*a^2*b*d^5*n*x^(2/3))/(4*e^5) - (11*a*b^2*d*n^2*x
^(10/3))/(100*e) + (147*a*b^2*d^6*n^2*log(d + e*x^(2/3)))/(40*e^6) + (b^3*
d^3*n*x^2*log(c*(d + e*x^(2/3))^n)^2)/(4*e^3) - (19*b^3*d^3*n^2*x^2*log(c*
(d + e*x^(2/3))^n))/(40*e^3) - (3*b^3*d^2*n*x^(8/3)*log(c*(d + e*x^(2/3))^
n)^2)/(16*e^2) + (37*b^3*d^2*n^2*x^(8/3)*log(c*(d + e*x^(2/3))^n))/(160*e^
2) - (3*b^3*d^4*n*x^(4/3)*log(c*(d + e*x^(2/3))^n)^2)/(8*e^4) + (87*b^3*d^
4*n^2*x^(4/3)*log(c*(d + e*x^(2/3))^n))/(80*e^4) + (3*b^3*d^5*n*x^(2/3)...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.10

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x)
```

output

```
(108000*x**(2/3)*log((x**(2/3)*e + d)**n*c)**2*b**3*d**5*e*n - 27000*x**(2
/3)*log((x**(2/3)*e + d)**n*c)**2*b**3*d**2*e**4*n*x**2 + 216000*x**(2/3)*
log((x**(2/3)*e + d)**n*c)*a*b**2*d**5*e*n - 54000*x**(2/3)*log((x**(2/3)*
e + d)**n*c)*a*b**2*d**2*e**4*n*x**2 - 529200*x**(2/3)*log((x**(2/3)*e + d
)**n*c)*b**3*d**5*e*n**2 + 33300*x**(2/3)*log((x**(2/3)*e + d)**n*c)*b**3*
d**2*e**4*n**2*x**2 + 108000*x**(2/3)*a**2*b*d**5*e*n - 27000*x**(2/3)*a**
2*b*d**2*e**4*n*x**2 - 529200*x**(2/3)*a*b**2*d**5*e*n**2 + 33300*x**(2/3)
*a*b**2*d**2*e**4*n**2*x**2 + 809340*x**(2/3)*b**3*d**5*e*n**3 - 13785*x**
(2/3)*b**3*d**2*e**4*n**3*x**2 - 54000*x**(1/3)*log((x**(2/3)*e + d)**n*c)
**2*b**3*d**4*e**2*n*x + 21600*x**(1/3)*log((x**(2/3)*e + d)**n*c)**2*b**3
*d*e**5*n*x**3 - 108000*x**(1/3)*log((x**(2/3)*e + d)**n*c)*a*b**2*d**4*e*
*2*n*x + 43200*x**(1/3)*log((x**(2/3)*e + d)**n*c)*a*b**2*d*e**5*n*x**3 +
156600*x**(1/3)*log((x**(2/3)*e + d)**n*c)*b**3*d**4*e**2*n**2*x - 15840*x
**(1/3)*log((x**(2/3)*e + d)**n*c)*b**3*d*e**5*n**2*x**3 - 54000*x**(1/3)*
a**2*b*d**4*e**2*n*x + 21600*x**(1/3)*a**2*b*d*e**5*n*x**3 + 156600*x**(1/
3)*a*b**2*d**4*e**2*n**2*x - 15840*x**(1/3)*a*b**2*d*e**5*n**2*x**3 - 1400
70*x**(1/3)*b**3*d**4*e**2*n**3*x + 4368*x**(1/3)*b**3*d*e**5*n**3*x**3 -
36000*log((x**(2/3)*e + d)**n*c)**3*b**3*d**6 + 36000*log((x**(2/3)*e + d)
**n*c)**3*b**3*e**6*x**4 - 108000*log((x**(2/3)*e + d)**n*c)**2*a*b**2*d**
6 + 108000*log((x**(2/3)*e + d)**n*c)**2*a*b**2*e**6*x**4 + 264600*log(...
```

3.482 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 449

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3}-\frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9b^3d^2n^3x^{2/3}}{e^2}+\frac{9b^3d^2n^2\left(d+ex^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e^3}-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3}+\frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}-\frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}+\frac{9bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4e^3}-\frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}+\frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}-\frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}+\frac{\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}$$

output
```
9/8*b^3*d*n^3*(d+e*x^(2/3))^2/e^3-1/9*b^3*n^3*(d+e*x^(2/3))^3/e^3+9*a*b^2*
d^2*n^2*x^(2/3)/e^2-9*b^3*d^2*n^3*x^(2/3)/e^2+9*b^3*d^2*n^2*(d+e*x^(2/3))*
ln(c*(d+e*x^(2/3))^n)/e^3-9/4*b^2*d*n^2*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(
2/3))^n))/e^3+1/3*b^2*n^2*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3-
9/2*b*d^2*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3+9/4*b*d*n*(d+e
*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3-1/2*b*n*(d+e*x^(2/3))^3*(a+b
*ln(c*(d+e*x^(2/3))^n))^2/e^3+3/2*d^2*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3)
)^n))^3/e^3-3/2*d*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^3+1/2*(d
+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^3
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.95

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\frac{36a^3d^3-198a^2bd^3n-108a^2bd^2enx^{2/3}+396ab^2d^2en^2x^{2/3}-510b^3d^2en^3x^{2/3}}{}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

output
```
(36*a^3*d^3 - 198*a^2*b*d^3*n - 108*a^2*b*d^2*e*n*x^(2/3) + 396*a*b^2*d^2*
e*n^2*x^(2/3) - 510*b^3*d^2*e*n^3*x^(2/3) + 54*a^2*b*d*e^2*n*x^(4/3) - 90*
a*b^2*d*e^2*n^2*x^(4/3) + 57*b^3*d*e^2*n^3*x^(4/3) + 36*a^3*e^3*x^2 - 36*a
^2*b*e^3*n*x^2 + 24*a*b^2*e^3*n^2*x^2 - 8*b^3*e^3*n^3*x^2 + 114*b^3*d^3*n^
3*Log[d + e*x^(2/3)] + 6*b*(18*a^2*(d^3 + e^3*x^2) - 6*a*b*n*(11*d^3 + 6*d
^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2) + b^2*n^2*(66*d^3 + 66*d^2*e*x
^(2/3) - 15*d*e^2*x^(4/3) + 4*e^3*x^2))*Log[c*(d + e*x^(2/3))^n] + 18*b^2*
(6*a*(d^3 + e^3*x^2) - b*n*(11*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2
*e^3*x^2))*Log[c*(d + e*x^(2/3))^n]^2 + 36*b^3*(d^3 + e^3*x^2)*Log[c*(d +
e*x^(2/3))^n]^3)/(72*e^3)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3dx^{2/3}$$

$$\Big\downarrow \text{2848}$$

$$\frac{3}{2}\int\left(\frac{\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{e^2}-\frac{2d\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{e^2}+\frac{d^2\left(a+b\log\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\left(\frac{2b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}-\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3}+\frac{6ab^2d^2}{e}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

output
```
(3*((3*b^3*d*n^3*(d + e*x^(2/3))^2)/(4*e^3) - (2*b^3*n^3*(d + e*x^(2/3))^3
)/(27*e^3) + (6*a*b^2*d^2*n^2*x^(2/3))/e^2 - (6*b^3*d^2*n^3*x^(2/3))/e^2 +
 (6*b^3*d^2*n^2*(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^3 - (3*b^2*d*n
^2*(d + e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*e^3) + (2*b^2*n^
2*(d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*e^3) - (3*b*d^2*n
*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e^3 + (3*b*d*n*(d + e
*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) - (b*n*(d + e*x^(2
/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*e^3) + (d^2*(d + e*x^(2/3))*
(a + b*Log[c*(d + e*x^(2/3))^n])^3)/e^3 - (d*(d + e*x^(2/3))^2*(a + b*Log[
c*(d + e*x^(2/3))^n])^3)/e^3 + ((d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2
/3))^n])^3)/(3*e^3)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.60

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

output
```
1/72*(36*b^3*e^3*x^2*log(c)^3 - 36*(b^3*e^3*n - 3*a*b^2*e^3)*x^2*log(c)^2
+ 36*(b^3*e^3*n^3*x^2 + b^3*d^3*n^3)*log(e*x^(2/3) + d)^3 + 12*(2*b^3*e^3*
n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*x^2*log(c) - 4*(2*b^3*e^3*n^3 - 6*a*b^2
*e^3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x^2 + 18*(3*b^3*d*e^2*n^3*x^(4/3) -
6*b^3*d^2*e*n^3*x^(2/3) - 11*b^3*d^3*n^3 + 6*a*b^2*d^3*n^2 - 2*(b^3*e^3*n^
3 - 3*a*b^2*e^3*n^2)*x^2 + 6*(b^3*e^3*n^2*x^2 + b^3*d^3*n^2)*log(c))*log(e
*x^(2/3) + d)^2 + 6*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n +
2*(2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n)*x^2 + 18*(b^3*e^3*n*x^
2 + b^3*d^3*n)*log(c)^2 - 6*(11*b^3*d^3*n^2 - 6*a*b^2*d^3*n + 2*(b^3*e^3*n
^2 - 3*a*b^2*e^3*n)*x^2)*log(c) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*lo
g(c) - 6*a*b^2*d^2*e*n^2)*x^(2/3) + 3*(6*b^3*d*e^2*n^2*x*log(c) - (5*b^3*d
*e^2*n^3 - 6*a*b^2*d*e^2*n^2)*x)*x^(1/3))*log(e*x^(2/3) + d) - 6*(85*b^3*d
^2*e*n^3 + 18*b^3*d^2*e*n*log(c)^2 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n
 - 6*(11*b^3*d^2*e*n^2 - 6*a*b^2*d^2*e*n)*log(c))*x^(2/3) + 3*(18*b^3*d*e^
2*n*x*log(c)^2 - 6*(5*b^3*d*e^2*n^2 - 6*a*b^2*d*e^2*n)*x*log(c) + (19*b^3*
d*e^2*n^3 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n)*x)*x^(1/3))/e^3
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.08

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\frac{1}{2}\,b^3x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^3$$
$$+\frac{3}{2}\,ab^2x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2$$
$$+\frac{1}{4}\,a^2ben\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)$$
$$+\frac{3}{2}\,a^2bx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{2}\,a^3x^2$$
$$+\frac{1}{12}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{\left(4\,e^3x^2-18\,d^3\log\left(\right.\right.}{}\right.$$
$$+\frac{1}{72}\left(18\,en\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2+en\left(\frac{\left(36\,d^3\log\left(ex\right.\right.}{}\right.\right.$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2*log((e*x^(2/3) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*x^(2/3) + d)
^n*c)^2 + 1/4*a^2*b*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e
*x^(4/3) + 6*d^2*x^(2/3))/e^3) + 3/2*a^2*b*x^2*log((e*x^(2/3) + d)^n*c) +
1/2*a^3*x^2 + 1/12*(6*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d
*e*x^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*c) + (4*e^3*x^2 - 1
8*d^3*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 66*d^3*log(e*x^(2/3) + d)
+ 66*d^2*e*x^(2/3))*n^2/e^3)*a*b^2 + 1/72*(18*e*n*(6*d^3*log(e*x^(2/3) + d
)/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) +
d)^n*c)^2 + e*n*((36*d^3*log(e*x^(2/3) + d)^3 - 8*e^3*x^2 + 198*d^3*log(e*
x^(2/3) + d)^2 + 57*d*e^2*x^(4/3) + 510*d^3*log(e*x^(2/3) + d) - 510*d^2*e
*x^(2/3))*n^2/e^4 + 6*(4*e^3*x^2 - 18*d^3*log(e*x^(2/3) + d)^2 - 15*d*e^2*
x^(4/3) - 66*d^3*log(e*x^(2/3) + d) + 66*d^2*e*x^(2/3))*n*log((e*x^(2/3) +
 d)^n*c)/e^4))*b^3
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.68

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

output
```
1/2*b^3*x^2*log(c)^3 + 1/72*(36*x^2*log(e*x^(2/3) + d)^3 - (18*(2*(e*x^(2/
3) + d)^3/e^4 - 9*(e*x^(2/3) + d)^2*d/e^4 + 18*(e*x^(2/3) + d)*d^2/e^4)*lo
g(e*x^(2/3) + d)^2 - 6*(4*(e*x^(2/3) + d)^3/e^4 - 27*(e*x^(2/3) + d)^2*d/e
^4 + 108*(e*x^(2/3) + d)*d^2/e^4)*log(e*x^(2/3) + d) - 36*d^3*log(e*x^(2/3
) + d)^3/e^4 + 8*(e*x^(2/3) + d)^3/e^4 - 81*(e*x^(2/3) + d)^2*d/e^4 + 648*
(e*x^(2/3) + d)*d^2/e^4)*e)*b^3*n^3 + 1/12*(18*x^2*log(e*x^(2/3) + d)^2 -
(6*(2*(e*x^(2/3) + d)^3/e^4 - 9*(e*x^(2/3) + d)^2*d/e^4 + 18*(e*x^(2/3) +
d)*d^2/e^4)*log(e*x^(2/3) + d) - 18*d^3*log(e*x^(2/3) + d)^2/e^4 - 4*(e*x^
(2/3) + d)^3/e^4 + 27*(e*x^(2/3) + d)^2*d/e^4 - 108*(e*x^(2/3) + d)*d^2/e^
4)*e)*b^3*n^2*log(c) + 1/4*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*
x^(2/3) + d))/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*b^3*
n*log(c)^2 + 3/2*a*b^2*x^2*log(c)^2 + 1/12*(18*x^2*log(e*x^(2/3) + d)^2 -
(6*(2*(e*x^(2/3) + d)^3/e^4 - 9*(e*x^(2/3) + d)^2*d/e^4 + 18*(e*x^(2/3) +
d)*d^2/e^4)*log(e*x^(2/3) + d) - 18*d^3*log(e*x^(2/3) + d)^2/e^4 - 4*(e*x^
(2/3) + d)^3/e^4 + 27*(e*x^(2/3) + d)^2*d/e^4 - 108*(e*x^(2/3) + d)*d^2/e^
4)*e)*a*b^2*n^2 + 1/2*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(2/
3) + d))/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*a*b^2*n*l
og(c) + 3/2*a^2*b*x^2*log(c) + 1/4*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*lo
g(abs(e*x^(2/3) + d))/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^
3))*a^2*b*n + 1/2*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 25.85 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.28

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^3\left(\frac{b^3\,x^2}{2}+\frac{b^3\,d^3}{2\,e^3}\right)$$

$$-x^{4/3}\left(\frac{d\left(\frac{3\,a^3}{2}-\frac{3\,a^2\,b\,n}{2}+a\,b^2\,n^2-\frac{b^3\,n^3}{3}\right)}{2\,e}-\frac{d\left(6\,a^3-6\,a\,b^2\,n^2+5\,b^3\,n^3\right)}{8\,e}\right)+\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^2\left(\frac{b^2\,x^2\,(3\right.$$

input
```
int(x*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

output
```
log(c*(d + e*x^(2/3))^n)^3*((b^3*x^2)/2 + (b^3*d^3)/(2*e^3)) - x^(4/3)*((d
*((3*a^3)/2 - (b^3*n^3)/3 + a*b^2*n^2 - (3*a^2*b*n)/2))/(2*e) - (d*(6*a^3
+ 5*b^3*n^3 - 6*a*b^2*n^2))/(8*e)) + log(c*(d + e*x^(2/3))^n)^2*((b^2*x^2*
(3*a - b*n))/2 - (x^(4/3)*((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e)
))/2 + (d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(4*e^3) + (d*x^(2/3)*((6*b^2*d*(3*
a - b*n))/e - (18*a*b^2*d)/e))/(4*e)) + x^(2/3)*((d*((d*((3*a^3)/2 - (b^3*
n^3)/3 + a*b^2*n^2 - (3*a^2*b*n)/2))/e - (d*(6*a^3 + 5*b^3*n^3 - 6*a*b^2*n
^2))/(4*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/(2*e^2)) + x^2*(a^3/2 - (b^3
*n^3)/9 + (a*b^2*n^2)/3 - (a^2*b*n)/2) + (log(c*(d + e*x^(2/3))^n)*((x^(2/
3)*((d*(2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^2 - b^2*n^2))
)/e + 12*b^3*d^2*n^2))/(2*e) - (x^(4/3)*(2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*
b*n) - 6*b*d*e*(3*a^2 - b^2*n^2)))/(4*e) + (b*e*x^2*(9*a^2 + 2*b^2*n^2 - 6
*a*b*n))/3))/(2*e) + (log(d + e*x^(2/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^
2 + 18*a^2*b*d^3*n))/(12*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.33

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x)
```

output
```
( - 108*x**(2/3)*log((x**(2/3)*e + d)**n*c)**2*b**3*d**2*e*n - 216*x**(2/3
)*log((x**(2/3)*e + d)**n*c)*a*b**2*d**2*e*n + 396*x**(2/3)*log((x**(2/3)*
e + d)**n*c)*b**3*d**2*e*n**2 - 108*x**(2/3)*a**2*b*d**2*e*n + 396*x**(2/3
)*a*b**2*d**2*e*n**2 - 510*x**(2/3)*b**3*d**2*e*n**3 + 54*x**(1/3)*log((x*
*(2/3)*e + d)**n*c)**2*b**3*d*e**2*n*x + 108*x**(1/3)*log((x**(2/3)*e + d)
**n*c)*a*b**2*d*e**2*n*x - 90*x**(1/3)*log((x**(2/3)*e + d)**n*c)*b**3*d*e
**2*n**2*x + 54*x**(1/3)*a**2*b*d*e**2*n*x - 90*x**(1/3)*a*b**2*d*e**2*n**
2*x + 57*x**(1/3)*b**3*d*e**2*n**3*x + 36*log((x**(2/3)*e + d)**n*c)**3*b*
*3*d**3 + 36*log((x**(2/3)*e + d)**n*c)**3*b**3*e**3*x**2 + 108*log((x**(2
/3)*e + d)**n*c)**2*a*b**2*d**3 + 108*log((x**(2/3)*e + d)**n*c)**2*a*b**2
*e**3*x**2 - 198*log((x**(2/3)*e + d)**n*c)**2*b**3*d**3*n - 36*log((x**(2
/3)*e + d)**n*c)**2*b**3*e**3*n*x**2 + 108*log((x**(2/3)*e + d)**n*c)*a**2
*b*d**3 + 108*log((x**(2/3)*e + d)**n*c)*a**2*b*e**3*x**2 - 396*log((x**(2
/3)*e + d)**n*c)*a*b**2*d**3*n - 72*log((x**(2/3)*e + d)**n*c)*a*b**2*e**3
*n*x**2 + 510*log((x**(2/3)*e + d)**n*c)*b**3*d**3*n**2 + 24*log((x**(2/3)
*e + d)**n*c)*b**3*e**3*n**2*x**2 + 36*a**3*e**3*x**2 - 36*a**2*b*e**3*n*x
**2 + 24*a*b**2*e**3*n**2*x**2 - 8*b**3*e**3*n**3*x**2)/(72*e**3)
```

3.483 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \frac{3}{2}\Big(a$$
$$+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^3\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\text{PolyLog}\left(2,1+\frac{ex^{2/3}}{d}\right)-9b^2n^2\Big($$

output
```
3/2*(a+b*ln(c*(d+e*x^(2/3))^n))^3*ln(-e*x^(2/3)/d)+9/2*b*n*(a+b*ln(c*(d+e*
x^(2/3))^n))^2*polylog(2,1+e*x^(2/3)/d)-9*b^2*n^2*(a+b*ln(c*(d+e*x^(2/3))^
n))*polylog(3,1+e*x^(2/3)/d)+9*b^3*n^3*polylog(4,1+e*x^(2/3)/d)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(139) = 278.

Time = 0.22 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.44

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \Big(a$$
$$-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^3\log(x)+3bn\Big(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^2\Big(\Big($$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

output
```
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3*Log[x] + 3*b*n
*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*((Log[d + e*x
^(2/3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)]
)/2) + (9*b^2*n^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n]
)*(Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*x^(2/3)]*PolyL
og[2, 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))/2 + (3*b^3*n^
3*(Log[d + e*x^(2/3)]^3*Log[-((e*x^(2/3))/d)] + 3*Log[d + e*x^(2/3)]^2*Pol
yLog[2, 1 + (e*x^(2/3))/d] - 6*Log[d + e*x^(2/3)]*PolyLog[3, 1 + (e*x^(2/3
))/d] + 6*PolyLog[4, 1 + (e*x^(2/3))/d]))/2
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{3}{2}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^{2/3}}dx^{2/3}$$

$$\Big\downarrow \text{ 2843}$$

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-3ben\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right)}{d+ex^{2/3}}dx^{2/3}\right)$$

$$\Big\downarrow \text{ 2881}$$

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-3bn\int\frac{\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{x^{2/3}}d\left(d+ex^{2/3}\right)\right)$$

$\Big\downarrow$ 2821

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-3bn\left(2bn\int\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)\text{PolyLog}\left(2,\frac{d+ex^{2/3}}{d}\right)}{x^{2/3}}d\left(d+\right.\right.\right.$$

$\Big\downarrow$ 2830

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+ex^{2/3}}{d}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)-\right.\right.\right.$$

$\Big\downarrow$ 7143

$$\frac{3}{2}\left(\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+ex^{2/3}}{d}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)-b\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

output
```
(3*((a + b*Log[c*(d + e*x^(2/3))^n])^3*Log[-((e*x^(2/3))/d)] - 3*b*n*(-((a
 + b*Log[c*x^((2*n)/3)])^2*PolyLog[2, (d + e*x^(2/3))/d]) + 2*b*n*((a + b*
Log[c*x^((2*n)/3)])*PolyLog[3, (d + e*x^(2/3))/d] - b*n*PolyLog[4, (d + e*
x^(2/3))/d]))))/2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c
)^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(2/3))**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="maxima")
```

output
```
b^3*log((e*x^(2/3) + d)^n)^3*log(x) + integrate(-((2*b^3*e*n*x*log(x) - 3*
(b^3*e*log(c) + a*b^2*e)*x - 3*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*log((e*x^
(2/3) + d)^n)^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c)
+ a^3*e)*x - 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*l
og(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) - (b
^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(e
*x^2 + d*x^(4/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \frac{8\left(\int \frac{\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)^3}{x^{\frac{5}{3}}e+dx}dx\right)b^3dn + 24\left(\int \frac{\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)^2}{x^{\frac{5}{3}}e+dx}dx\right)a\,b^2dn +}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x)
```

output
```
(8*int(log((x**(2/3)*e + d)**n*c)**3/(x**(2/3)*e*x + d*x),x)*b**3*d*n + 24
*int(log((x**(2/3)*e + d)**n*c)**2/(x**(2/3)*e*x + d*x),x)*a*b**2*d*n + 24
*int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*e*x + d*x),x)*a**2*b*d*n + 3*log
((x**(2/3)*e + d)**n*c)**4*b**3 + 12*log((x**(2/3)*e + d)**n*c)**3*a*b**2
+ 18*log((x**(2/3)*e + d)**n*c)**2*a**2*b + 8*log(x)*a**3*n)/(8*n)
```

3.484 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 451

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} - \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3} - \frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} + \frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3x^{2/3}} + \frac{3be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} - \frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)}{d^3} + \frac{b^3e^3n^3\log(x)}{d^3} + \frac{3b^3e^3n^3\,\mathrm{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^3} - \frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{d^3} - \frac{3b^3e^3n^3\,\mathrm{PolyLog}\left(2,1+\frac{ex^{2/3}}{d}\right)}{d^3} - \frac{3b^3e^3n^3\,\mathrm{PolyLog}\left(3,\frac{d}{d+ex^{2/3}}\right)}{d^3}$$

output

```
-3/2*b^2*e^2*n^2*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x^(2/3)-3/2
*b^2*e^3*n^2*ln(1-d/(d+e*x^(2/3)))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3-3/4*b*e
*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(4/3)+3/2*b*e^2*n*(d+e*x^(2/3))*(a+b*
ln(c*(d+e*x^(2/3))^n))^2/d^3/x^(2/3)+3/2*b*e^3*n*ln(1-d/(d+e*x^(2/3)))*(a+
b*ln(c*(d+e*x^(2/3))^n))^2/d^3-1/2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2-3*b^2
*e^3*n^2*(a+b*ln(c*(d+e*x^(2/3))^n))*ln(-e*x^(2/3)/d)/d^3+b^3*e^3*n^3*ln(x
)/d^3+3/2*b^3*e^3*n^3*polylog(2,d/(d+e*x^(2/3)))/d^3-3*b^2*e^3*n^2*(a+b*ln
(c*(d+e*x^(2/3))^n))*polylog(2,d/(d+e*x^(2/3)))/d^3-3*b^3*e^3*n^3*polylog(
2,1+e*x^(2/3)/d)/d^3-3*b^3*e^3*n^3*polylog(3,d/(d+e*x^(2/3)))/d^3
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.69

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = \frac{-3bd^2enx^{2/3}\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+6bde}{}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

output
```
(-3*b*d^2*e*n*x^(2/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3)
)^n])^2 + 6*b*d*e^2*n*x^(4/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e
*x^(2/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/3)]
 + b*Log[c*(d + e*x^(2/3))^n])^2 - 6*b*e^3*n*x^2*Log[d + e*x^(2/3)]*(a - b
*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*d^3*(a - b*n*Log
[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3 + 4*b*e^3*n*x^2*(a - b*n*L
og[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] - 6*b^2*n^2*(a -
b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((d^3 + e^3*x^2)*Log[
d + e*x^(2/3)]^2 + e^2*x^(4/3)*(d + 3*e*x^(2/3)*Log[-((e*x^(2/3))/d)]) + L
og[d + e*x^(2/3)]*(d^2*e*x^(2/3) - 2*d*e^2*x^(4/3) - 3*e^3*x^2 - 2*e^3*x^2
*Log[-((e*x^(2/3))/d)]) - 2*e^3*x^2*PolyLog[2, 1 + (e*x^(2/3))/d]) + b^3*n
^3*(-6*d*e^2*x^(4/3)*Log[d + e*x^(2/3)] - 6*e^3*x^2*Log[d + e*x^(2/3)] - 3
*d^2*e*x^(2/3)*Log[d + e*x^(2/3)]^2 + 6*d*e^2*x^(4/3)*Log[d + e*x^(2/3)]^2
 + 9*e^3*x^2*Log[d + e*x^(2/3)]^2 - 2*d^3*Log[d + e*x^(2/3)]^3 - 2*e^3*x^2
*Log[d + e*x^(2/3)]^3 + 6*e^3*x^2*Log[-((e*x^(2/3))/d)] - 18*e^3*x^2*Log[d
 + e*x^(2/3)]*Log[-((e*x^(2/3))/d)] + 6*e^3*x^2*Log[d + e*x^(2/3)]^2*Log[-
((e*x^(2/3))/d)] + 6*e^3*x^2*(-3 + 2*Log[d + e*x^(2/3)])*PolyLog[2, 1 + (e
*x^(2/3))/d] - 12*e^3*x^2*PolyLog[3, 1 + (e*x^(2/3))/d]))/(4*d^3*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 3.42 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.83, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2904, 2845, 2858, 25, 27, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^{8/3}}dx^{2/3}$$

$$\Big\downarrow \text{2845}$$

$$\frac{3}{2}\left(ben\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{\left(d+ex^{2/3}\right)x^2}dx^{2/3}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^2}\right)$$

$$\Big\downarrow \text{2858}$$

$$\frac{3}{2}\left(bn\int \frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{x^{8/3}}d\left(d+ex^{2/3}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}\left(-bn\int -\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{x^{8/3}}d\left(d+ex^{2/3}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\left(-be^3n\int -\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{e^3x^{8/3}}d\left(d+ex^{2/3}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^2}\right)$$

$$\Big\downarrow \text{2789}$$

$$\frac{3}{2}\left(-be^3n\left(\frac{\int -\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{e^3x^2}d\left(d+ex^{2/3}\right)}{d}+\frac{\int \frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}\right)-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3x^2}\right.$$

$$\Big\downarrow \text{2756}$$

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{2e^2x^{4/3}}-bn\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}+\frac{\int \frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{e^2x^2}d\left(d+ex^{2/3}\right)}{d}\right)-\frac{\left(a+b\log\right.}{}\right.$$

$$\Big\downarrow \text{2789}$$

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{2e^2x^{4/3}}-bn\left(\frac{\int \frac{a+b\log\left(cx^{2n/3}\right)}{e^2x^{4/3}}d(d+ex^{2/3})}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d(d+ex^{2/3})}{d}\right)}{d}+\frac{\int \frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{e^2x^{4/3}}d(}{d}\right.\right.$$

$\Big\downarrow$ 2751

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{2e^2x^{4/3}}-bn\left(\frac{-\frac{bn\int -\frac{1}{ex^{2/3}}d\left(d+ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}+\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d(d+ex^{2/3})}{d}\right)}{d}+\right.\right.$$

$\Big\downarrow$ 16

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{2e^2x^{4/3}}-bn\left(\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d(d+ex^{2/3})}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}\right)}{d}+\frac{\int \frac{\left(a+b\log\right.}{}}{}\right.\right.$$

$\Big\downarrow$ 2755

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{2e^2x^{4/3}}-bn\left(\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d(d+ex^{2/3})}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}\right)}{d}+\frac{-\frac{2bn\int -}{}}{}\right.\right.$$

$\Big\downarrow$ 2754

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{2e^2x^{4/3}}-bn\left(\frac{\int -\frac{a+b\log\left(cx^{2n/3}\right)}{ex^{4/3}}d(d+ex^{2/3})}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}\right)}{d}+\frac{-\frac{2bn\left(bn\right.}{}}{}\right.\right.$$

$\downarrow$ 2779

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{2e^2x^{4/3}}-bn\left(\frac{\frac{bn\int\frac{\log\left(1-\frac{d}{x^{2/3}}\right)}{x^{2/3}}d\left(d+ex^{2/3}\right)}{d}-\frac{\log\left(1-\frac{d}{x^{2/3}}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a-\right.}{dex}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2821

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{2e^2x^{4/3}}-bn\left(\frac{\frac{bn\int\frac{\log\left(1-\frac{d}{x^{2/3}}\right)}{x^{2/3}}d\left(d+ex^{2/3}\right)}{d}-\frac{\log\left(1-\frac{d}{x^{2/3}}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a-\right.}{dex}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2838

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\frac{2bn\left(\text{PolyLog}\left(2,\frac{d}{x^{2/3}}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)-bn\int\frac{\text{PolyLog}\left(2,\frac{d}{x^{2/3}}\right)}{x^{2/3}}d\left(d+ex^{2/3}\right)\right)}{d}-\frac{\log\left(1-\frac{d}{x^{2/3}}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{d}}{d}+\frac{-\frac{2bn\left(-\log\left(\right.\right.}{}}{}}{d}\right.\right.$$

$\downarrow$ 7143

$$\frac{3}{2}\left(-be^3n\left(\frac{\frac{\left(a+b\log\left(cx^{2n/3}\right)\right)^2}{2e^2x^{4/3}}-bn\left(\frac{\frac{bn\log\left(-ex^{2/3}\right)}{d}-\frac{\left(d+ex^{2/3}\right)\left(a+b\log\left(cx^{2n/3}\right)\right)}{dex^{2/3}}}{d}+\frac{\frac{bn\,\text{PolyLog}\left(2,\frac{d}{x^{2/3}}\right)}{d}-\frac{\log\left(1-\frac{d}{x^{2/3}}\right)\left(a+b\log\left(cx^{2n}\right.\right.}{d}}{d}\right.}{d}\right.\right.$$

input

```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

output
```
(3*(-1/3*(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2 - b*e^3*n*(((a + b*Log[c*x
^((2*n)/3)])^2/(2*e^2*x^(4/3)) - b*n*(((b*n*Log[-(e*x^(2/3))])/d - ((d + e
*x^(2/3))*(a + b*Log[c*x^((2*n)/3)]))/(d*e*x^(2/3)))/d + (-((Log[1 - d/x^(
2/3)]*(a + b*Log[c*x^((2*n)/3)]))/d) + (b*n*PolyLog[2, d/x^(2/3)])/d)/d))/
d + ((-(((d + e*x^(2/3))*(a + b*Log[c*x^((2*n)/3)])^2)/(d*e*x^(2/3))) - (2
*b*n*(-(Log[1 - (d + e*x^(2/3))/d]*(a + b*Log[c*x^((2*n)/3)])) - b*n*PolyL
og[2, (d + e*x^(2/3))/d]))/d)/d + (-((Log[1 - d/x^(2/3)]*(a + b*Log[c*x^((
2*n)/3)])^2)/d) + (2*b*n*((a + b*Log[c*x^((2*n)/3)])*PolyLog[2, d/x^(2/3)]
 + b*n*PolyLog[3, d/x^(2/3)]))/d)/d)/d))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c
)^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

output
```
-1/2*b^3*log((e*x^(2/3) + d)^n)^3/x^2 + integrate(((b^3*e*n*x + 3*(b^3*e*l
og(c) + a*b^2*e)*x + 3*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*log((e*x^(2/3) +
d)^n)^2 + (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)
*x + 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^2
+ 2*a*b^2*d*log(c) + a^2*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + (b^3*d*log
(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(e*x^4 + d
*x^(10/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^3,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = \frac{-3x^{\frac{2}{3}}\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)^2 b^3d^2en-6x^{\frac{2}{3}}\log\left(\left(x^{\frac{2}{3}}e+d\right)^n c\right)ab^2d^2en-}{}$$

input

```
int((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x)
```

output

```
( - 3*x**(2/3)*log((x**(2/3)*e + d)**n*c)**2*b**3*d**2*e*n - 6*x**(2/3)*lo
g((x**(2/3)*e + d)**n*c)*a*b**2*d**2*e*n - 3*x**(2/3)*a**2*b*d**2*e*n + 6*
x**(1/3)*log((x**(2/3)*e + d)**n*c)**2*b**3*d*e**2*n*x + 12*x**(1/3)*log((
x**(2/3)*e + d)**n*c)*a*b**2*d*e**2*n*x - 6*x**(1/3)*log((x**(2/3)*e + d)*
*n*c)*b**3*d*e**2*n**2*x + 6*x**(1/3)*a**2*b*d*e**2*n*x - 6*x**(1/3)*a*b**
2*d*e**2*n**2*x + 4*int(log((x**(2/3)*e + d)**n*c)**2/(x**(2/3)*e*x + d*x)
,x)*b**3*d*e**3*n*x**2 + 8*int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*e*x +
d*x),x)*a*b**2*d*e**3*n*x**2 - 12*int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)
*e*x + d*x),x)*b**3*d*e**3*n**2*x**2 + 12*log(x**(1/3))*a**2*b*e**3*n*x**2
 - 36*log(x**(1/3))*a*b**2*e**3*n**2*x**2 + 12*log(x**(1/3))*b**3*e**3*n**
3*x**2 - 2*log((x**(2/3)*e + d)**n*c)**3*b**3*d**3 - 6*log((x**(2/3)*e + d
)**n*c)**2*a*b**2*d**3 - 6*log((x**(2/3)*e + d)**n*c)*a**2*b*d**3 - 6*log(
(x**(2/3)*e + d)**n*c)*a**2*b*e**3*x**2 + 18*log((x**(2/3)*e + d)**n*c)*a*
b**2*e**3*n*x**2 - 6*log((x**(2/3)*e + d)**n*c)*b**3*e**3*n**2*x**2 - 2*a*
*3*d**3)/(4*d**3*x**2)
```

3.485 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

output
```
4504/315*a*b^2*d^4*n^2*x^(1/3)/e^4-3475504/99225*b^3*d^4*n^3*x^(1/3)/e^4+6
37984/297675*b^3*d^3*n^3*x/e^3-221344/496125*b^3*d^2*n^3*x^(5/3)/e^2+3088/
27783*b^3*d*n^3*x^(7/3)/e-16/729*b^3*n^3*x^3+3475504/99225*b^3*d^(9/2)*n^3
*arctan(e^(1/2)*x^(1/3)/d^(1/2))/e^(9/2)-4504/315*I*b^3*d^(9/2)*n^3*arctan
(e^(1/2)*x^(1/3)/d^(1/2))^2/e^(9/2)-9008/315*b^3*d^(9/2)*n^3*arctan(e^(1/2
)*x^(1/3)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/e^(9/2)+4504/
315*b^3*d^4*n^2*x^(1/3)*ln(c*(d+e*x^(2/3))^n)/e^4-1984/945*b^2*d^3*n^2*x*(
a+b*ln(c*(d+e*x^(2/3))^n))/e^3+1144/1575*b^2*d^2*n^2*x^(5/3)*(a+b*ln(c*(d+
e*x^(2/3))^n))/e^2-128/441*b^2*d*n^2*x^(7/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e
+8/81*b^2*n^2*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))-4504/315*b^2*d^(9/2)*n^2*arc
tan(e^(1/2)*x^(1/3)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(9/2)-2*b*d^4*n
*x^(1/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^4+2/3*b*d^3*n*x*(a+b*ln(c*(d+e*x^
(2/3))^n))^2/e^3-2/5*b*d^2*n*x^(5/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^2+2/7
*b*d*n*x^(7/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e-2/9*b*n*x^3*(a+b*ln(c*(d+e*
x^(2/3))^n))^2+1/3*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3-4504/315*I*b^3*d^(9/2
)*n^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/e^(9/2)+2/3*b*d^5
*n*Defer(Int)((a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/e^4
```

Mathematica [A] (verified)

Time = 12.55 (sec) , antiderivative size = 1552, normalized size of antiderivative = 64.67

$$\int x^2 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

output
```
(-2*b*d^4*n*x^(1/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^
n])^2)/e^4 + (2*b*d^3*n*x*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(
2/3))^n])^2)/(3*e^3) - (2*b*d^2*n*x^(5/3)*(a - b*n*Log[d + e*x^(2/3)] + b*
Log[c*(d + e*x^(2/3))^n])^2)/(5*e^2) + (2*b*d*n*x^(7/3)*(a - b*n*Log[d + e
*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/(7*e) + (2*b*d^(9/2)*n*ArcTan[(
Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(
2/3))^n])^2)/e^(9/2) + b*n*x^3*Log[d + e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/
3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + (x^3*(a - b*n*Log[d + e*x^(2/3)] + b
*Log[c*(d + e*x^(2/3))^n])^2*(3*a - 2*b*n - 3*b*n*Log[d + e*x^(2/3)] + 3*b
*Log[c*(d + e*x^(2/3))^n]))/9 - (b^3*n^3*(1094783760*d^(9/2)*Sqrt[d + e*x^
(2/3)]*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*ArcSin[Sqrt[d]/Sqrt[d + e*x^(2/3)
]] - e*x^(2/3)*(-16*(68423985*d^4 - 4186770*d^3*e*x^(2/3) + 871542*d^2*e^2
*x^(4/3) - 217125*d*e^3*x^2 + 42875*e^4*x^(8/3)) + 2520*(177345*d^4 - 2604
0*d^3*e*x^(2/3) + 9009*d^2*e^2*x^(4/3) - 3600*d*e^3*x^2 + 1225*e^4*x^(8/3)
)*Log[d + e*x^(2/3)] - 198450*(315*d^4 - 105*d^3*e*x^(2/3) + 63*d^2*e^2*x^
(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3))*Log[d + e*x^(2/3)]^2 + 10418625*e^4
*x^(8/3)*Log[d + e*x^(2/3)]^3) + 62511750*d^(9/2)*Sqrt[(e*x^(2/3))/(d + e*
x^(2/3))]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/
2}, d/(d + e*x^(2/3))] + Log[d + e*x^(2/3)]*(4*Sqrt[d]*HypergeometricPFQ[{
1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^(2/3))] + Sqrt[d + e*x^(2/3)]*Ar...
```

Rubi [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3dx$$

$$\Big\downarrow \text{ 2908}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2907}$$

$$3\left(\frac{1}{9}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-\frac{2}{3}ben\int\frac{x^{10/3}\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^2}{d+ex^{2/3}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{ 2926}$$

$$3\left(\frac{1}{9}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-\frac{2}{3}ben\int\left(-\frac{\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^2d^5}{e^5\left(d+ex^{2/3}\right)}+\frac{\left(a+b\log\left(c(d+ex^{2/3}\right)}{e^5}\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(\frac{1}{9}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-\frac{2}{3}ben\left(-\frac{d^5\int\frac{\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^2}{d+ex^{2/3}}d\sqrt[3]{x}}{e^5}+\frac{2252bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315}\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*x^2*log((e*x^(2/3) +
 d)^n*c)^2 + 3*a^2*b*x^2*log((e*x^(2/3) + d)^n*c) + a^3*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^3 \, dx = \int \left(b \log \left(\left(e x^{\frac{2}{3}} + d\right)^n c\right) + a\right)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^3 \, dx = \int x^2 \left(a + b \ln \left(c \left(d + e x^{2/3}\right)^n\right)\right)^3 \, dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 929, normalized size of antiderivative = 38.71

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x)
```

output
```
(62511750*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a**2*b*d**4
*n - 446909400*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a*b**2
*d**4*n**2 + 1094783760*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)
))*b**3*d**4*n**3 - 12502350*x**(2/3)*log((x**(2/3)*e + d)**n*c)**2*b**3*d
**2*e**3*n*x - 25004700*x**(2/3)*log((x**(2/3)*e + d)**n*c)*a*b**2*d**2*e*
*3*n*x + 22702680*x**(2/3)*log((x**(2/3)*e + d)**n*c)*b**3*d**2*e**3*n**2*
x - 12502350*x**(2/3)*a**2*b*d**2*e**3*n*x + 22702680*x**(2/3)*a*b**2*d**2
*e**3*n**2*x - 13944672*x**(2/3)*b**3*d**2*e**3*n**3*x - 62511750*x**(1/3)
*log((x**(2/3)*e + d)**n*c)**2*b**3*d**4*e*n + 8930250*x**(1/3)*log((x**(2
/3)*e + d)**n*c)**2*b**3*d*e**4*n*x**2 - 125023500*x**(1/3)*log((x**(2/3)*
e + d)**n*c)*a*b**2*d**4*e*n + 17860500*x**(1/3)*log((x**(2/3)*e + d)**n*c
)*a*b**2*d*e**4*n*x**2 + 446909400*x**(1/3)*log((x**(2/3)*e + d)**n*c)*b**
3*d**4*e*n**2 - 9072000*x**(1/3)*log((x**(2/3)*e + d)**n*c)*b**3*d*e**4*n*
*2*x**2 - 62511750*x**(1/3)*a**2*b*d**4*e*n + 8930250*x**(1/3)*a**2*b*d*e*
*4*n*x**2 + 446909400*x**(1/3)*a*b**2*d**4*e*n**2 - 9072000*x**(1/3)*a*b**
2*d*e**4*n**2*x**2 - 1094783760*x**(1/3)*b**3*d**4*e*n**3 + 3474000*x**(1/
3)*b**3*d*e**4*n**3*x**2 + 20837250*int(log((x**(2/3)*e + d)**n*c)**2/(x**
(2/3)*d + x**(1/3)*e*x),x)*b**3*d**5*e*n + 41674500*int(log((x**(2/3)*e +
d)**n*c)/(x**(2/3)*d + x**(1/3)*e*x),x)*a*b**2*d**5*e*n - 148969800*int(lo
g((x**(2/3)*e + d)**n*c)/(x**(2/3)*d + x**(1/3)*e*x),x)*b**3*d**5*e*n**...
```

3.486 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=-\frac{32ab^2dn^2\sqrt[3]{x}}{e}+\frac{208b^3dn^3\sqrt[3]{x}}{3e}$$
$$-\frac{16}{9}b^3n^3x-\frac{208b^3d^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{32ib^3d^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}$$
$$+\frac{64b^3d^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{32b^3dn^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}$$
$$+\frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{32b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+\frac{6bdn\sqrt[3]{x}(a}{}$$

output
```
-32*a*b^2*d*n^2*x^(1/3)/e+208/3*b^3*d*n^3*x^(1/3)/e-16/9*b^3*n^3*x-208/3*b
^3*d^(3/2)*n^3*arctan(e^(1/2)*x^(1/3)/d^(1/2))/e^(3/2)+32*I*b^3*d^(3/2)*n^
3*arctan(e^(1/2)*x^(1/3)/d^(1/2))^2/e^(3/2)+64*b^3*d^(3/2)*n^3*arctan(e^(1
/2)*x^(1/3)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/e^(3/2)-32*
b^3*d*n^2*x^(1/3)*ln(c*(d+e*x^(2/3))^n)/e+8/3*b^2*n^2*x*(a+b*ln(c*(d+e*x^(
2/3))^n))+32*b^2*d^(3/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))*(a+b*ln(c*(d+
e*x^(2/3))^n))/e^(3/2)+6*b*d*n*x^(1/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e-2*b
*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))^2+x*(a+b*ln(c*(d+e*x^(2/3))^n))^3+32*I*b^
3*d^(3/2)*n^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/e^(3/2)-2
*b*d^2*n*Defer(Int)((a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)
/e
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1299 vs. $2(486) = 972$.

Time = 9.40 (sec) , antiderivative size = 1299, normalized size of antiderivative = 64.95

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

output
```
(6*b*d*n*x^(1/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])
^2)/e - (6*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d +
e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/e^(3/2) + 3*b*n*x*Log[d + e*x^
(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a
- b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*(a - 2*b*n - b*n*
Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n]) + (b^2*n^2*x^(1/3)*(a - b
*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((-96*d^(3/2)*ArcSin[S
qrt[d]/Sqrt[d + e*x^(2/3)]])/(Sqrt[d + e*x^(2/3)]*Sqrt[(e*x^(2/3))/(d + e*
x^(2/3))]) - d*(104 - 48*Log[d + e*x^(2/3)] + 9*Log[d + e*x^(2/3)]^2) + (d
 + e*x^(2/3))*(8 - 12*Log[d + e*x^(2/3)] + 9*Log[d + e*x^(2/3)]^2) + (36*(
-d)^(3/2)*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*(Log[d + e*x^(2/3)] - Log[1 +
(e*x^(2/3))/d]))/Sqrt[e*x^(2/3)] + (9*d*(2*Log[(1 + Sqrt[-((e*x^(2/3))/d)]
)/2]^2 - 4*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]*Log[1 + (e*x^(2/3))/d] + Lo
g[1 + (e*x^(2/3))/d]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((e*x^(2/3))/d)]/2]))/Sq
rt[-((e*x^(2/3))/d)]))/(3*e) + (b^3*n^3*(624*d*e*x^(2/3) - 16*e^2*x^(4/3)
+ 624*d^(3/2)*Sqrt[d + e*x^(2/3)]*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*ArcSin
[Sqrt[d]/Sqrt[d + e*x^(2/3)]] + 432*d^2*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*
HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^(2/3))
] + 144*d^2*Sqrt[-((e*x^(2/3))/d)]*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 -
 288*d*e*x^(2/3)*Log[d + e*x^(2/3)] + 24*e^2*x^(4/3)*Log[d + e*x^(2/3)]...
```

Rubi [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2901, 2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx$$

$$\Big\downarrow \text{ 2901}$$

$$3\int x^{2/3}\left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2907}$$

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-2ben\int\frac{x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d+ex^{2/3}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{2926}$$

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-2ben\int\left(\frac{x^{2/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e}+\frac{d^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right.\right.}{e^2\left(d+ex^{2/3}\right)}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-2ben\left(\frac{d^2\int\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d+ex^{2/3}}d\sqrt[3]{x}}{e^2}-\frac{16bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+\right.}{3e^{5/2}}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln \left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.10

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{3}\,dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3, x)
```

Sympy [N/A]

Not integrable

Time = 61.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx=\int \left(a+b\log \left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^3 dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(2/3))**n))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3 dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\int \left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 504, normalized size of antiderivative = 25.20

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\frac{-54\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)a^2bdn+288\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)a\,b^2d\,n^2-624\sqrt{e}\cdots}{\cdots}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^3,x)
```

output

```
( - 54*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a**2*b*d*n + 2
88*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a*b**2*d*n**2 - 62
4*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b**3*d*n**3 + 54*x*
*(1/3)*log((x**(2/3)*e + d)**n*c)**2*b**3*d*e*n + 108*x**(1/3)*log((x**(2/
3)*e + d)**n*c)*a*b**2*d*e*n - 288*x**(1/3)*log((x**(2/3)*e + d)**n*c)*b**
3*d*e*n**2 + 54*x**(1/3)*a**2*b*d*e*n - 288*x**(1/3)*a*b**2*d*e*n**2 + 624
*x**(1/3)*b**3*d*e*n**3 - 18*int(log((x**(2/3)*e + d)**n*c)**2/(x**(2/3)*d
 + x**(1/3)*e*x),x)*b**3*d**2*e*n - 36*int(log((x**(2/3)*e + d)**n*c)/(x**
(2/3)*d + x**(1/3)*e*x),x)*a*b**2*d**2*e*n + 96*int(log((x**(2/3)*e + d)**
n*c)/(x**(2/3)*d + x**(1/3)*e*x),x)*b**3*d**2*e*n**2 + 9*log((x**(2/3)*e +
 d)**n*c)**3*b**3*e**2*x + 27*log((x**(2/3)*e + d)**n*c)**2*a*b**2*e**2*x
- 18*log((x**(2/3)*e + d)**n*c)**2*b**3*e**2*n*x + 27*log((x**(2/3)*e + d)
**n*c)*a**2*b*e**2*x - 36*log((x**(2/3)*e + d)**n*c)*a*b**2*e**2*n*x + 24*
log((x**(2/3)*e + d)**n*c)*b**3*e**2*n**2*x + 9*a**3*e**2*x - 18*a**2*b*e*
*2*n*x + 24*a*b**2*e**2*n**2*x - 16*b**3*e**2*n**3*x)/(9*e**2)
```

3.487 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$
\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx &= \frac{24ib^3e^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} \\
&+ \frac{48b^3e^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} \\
&+ \frac{24b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} \\
&- \frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} \\
&+ \frac{24ib^3e^{3/2}n^3\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} \\
&- \frac{2be^2n\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{\left(d+ex^{2/3}\right)x^{2/3}},x\right)}{d}
\end{aligned}
$$

output
```
24*I*b^3*e^(3/2)*n^3*arctan(e^(1/2)*x^(1/3)/d^(1/2))^2/d^(3/2)+48*b^3*e^(3
/2)*n^3*arctan(e^(1/2)*x^(1/3)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^
(1/3)))/d^(3/2)+24*b^2*e^(3/2)*n^2*arctan(e^(1/2)*x^(1/3)/d^(1/2))*(a+b*ln
(c*(d+e*x^(2/3))^n))/d^(3/2)-6*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(1/
3)-(a+b*ln(c*(d+e*x^(2/3))^n))^3/x+24*I*b^3*e^(3/2)*n^3*polylog(2,1-2*d^(1
/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/d^(3/2)-2*b*e^2*n*Defer(Int)((a+b*ln(c*(d
+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1158 vs. $2(319) = 638$.

Time = 10.31 (sec) , antiderivative size = 1158, normalized size of antiderivative = 48.25

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

output
```
(-6*b*e*n*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/(d*
x^(1/3)) - (6*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d
 + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/d^(3/2) - (3*b*n*Log[d + e*
x^(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/x -
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3/x + (3*b^2*e*n
^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((-16*Sqrt[d
+ e*x^(2/3)]*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*ArcSin[Sqrt[d]/Sqrt[d + e*x
^(2/3)]])/d^(3/2) - (8*Log[d + e*x^(2/3)])/d - (2*Log[d + e*x^(2/3)]^2)/(e
*x^(2/3)) - (8*Sqrt[e*x^(2/3)]*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*(Log[d +
e*x^(2/3)] - Log[1 + (e*x^(2/3))/d]))/(-d)^(3/2) - (2*Sqrt[-((e*x^(2/3))/d
)]*(2*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 - 4*Log[(1 + Sqrt[-((e*x^(2/3)
)/d)])/2]*Log[1 + (e*x^(2/3))/d] + Log[1 + (e*x^(2/3))/d]^2 - 4*PolyLog[2,
 1/2 - Sqrt[-((e*x^(2/3))/d)]/2]))/d))/(2*x^(1/3)) + (b^3*n^3*(48*Sqrt[-d^
2]*e*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*x^(2/3)*HypergeometricPFQ[{1/2, 1/2
, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^(2/3))] - 12*d*Sqrt[-d^2]*(-((e*x
^(2/3))/d))^(3/2)*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 - 24*Sqrt[d]*(e*x^
(2/3))^(3/2)*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*Log[d + e*x^(2/3)] + 24*Sqr
t[-d^2]*e*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*x^(2/3)*HypergeometricPFQ[{1/2
, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^(2/3))]*Log[d + e*x^(2/3)] - 6*Sqrt[-d
^2]*e*x^(2/3)*Log[d + e*x^(2/3)]^2 + 6*Sqrt[-d]*(d + e*x^(2/3))^(3/2)*(...
```

Rubi [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$$

$$\Big\downarrow \text{ 2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(2ben\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{\left(d+ex^{2/3}\right)x^{2/3}}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x}\right)$$

$$\Big\downarrow \text{2926}$$

$$3\left(2ben\int \left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{dx^{2/3}}-\frac{e\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\left(d+ex^{2/3}\right)}\right)d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right.\right.}{3x}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x}+2ben\left(-\frac{e\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d+ex^{2/3}}d\sqrt[3]{x}}{d}+\frac{4b\sqrt{e}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\right.}{d^{3/2}}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c
)^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 393, normalized size of antiderivative = 16.38

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\frac{-18\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)a^2benx-24\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{e}\,\sqrt{d}}\right)a\,b^2e\,n^2x-}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x)
```

output
```
( - 18*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a**2*b*e*n*x -
 24*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a*b**2*e*n**2*x -
 16*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b**3*e*n**3*x - 1
8*x**(2/3)*a**2*b*d*e*n - 24*x**(2/3)*a*b**2*d*e*n**2 - 16*x**(2/3)*b**3*d
*e*n**3 - 6*int(log((x**(2/3)*e + d)**n*c)**2/(x**(2/3)*e*x**2 + d*x**2),x
)*b**3*d**3*n*x - 12*int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*e*x**2 + d*x
**2),x)*a*b**2*d**3*n*x - 8*int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*e*x**
2 + d*x**2),x)*b**3*d**3*n**2*x - 3*log((x**(2/3)*e + d)**n*c)**3*b**3*d**
2 - 9*log((x**(2/3)*e + d)**n*c)**2*a*b**2*d**2 - 6*log((x**(2/3)*e + d)**
n*c)**2*b**3*d**2*n - 9*log((x**(2/3)*e + d)**n*c)*a**2*b*d**2 - 12*log((x
**(2/3)*e + d)**n*c)*a*b**2*d**2*n - 8*log((x**(2/3)*e + d)**n*c)*b**3*d**
2*n**2 - 3*a**3*d**2)/(3*d**2*x)
```

3.488 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx = -\frac{16b^3e^3n^3}{105d^3x}+\frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}}+\frac{1376b^3e^{9/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}-\frac{1408ib^3e^{9/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{105d^{9/2}}-\frac{2816b^3e^{9/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{105d^{9/2}}-\frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}}+\frac{32b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^3x}-\frac{568b^2e^4n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^4\sqrt[3]{x}}-\frac{1408b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^{9/2}}-\frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}}+\frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}-\frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x}+\frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3}-\frac{1408ib^3e^{9/2}n^3\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{105d^{9/2}}+\frac{2be^5n\,\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{\left(d+ex^{2/3}\right)x^{2/3}},x\right)}{3d^4}$$

output
```
-16/105*b^3*e^3*n^3/d^3/x+16/7*b^3*e^4*n^3/d^4/x^(1/3)+1376/105*b^3*e^(9/2
)*n^3*arctan(e^(1/2)*x^(1/3)/d^(1/2))/d^(9/2)-1408/105*I*b^3*e^(9/2)*n^3*p
olylog(2,1-2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x^(1/3)))/d^(9/2)-2816/105*b^3*e^(
9/2)*n^3*arctan(e^(1/2)*x^(1/3)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*e^(1/2)*x
^(1/3)))/d^(9/2)-8/35*b^2*e^2*n^2*(a+b*ln(c*(d+e*x^(2/3))^n))/d^2/x^(5/3)+
32/35*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x-568/105*b^2*e^4*n^2*(a
+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(1/3)-1408/105*b^2*e^(9/2)*n^2*arctan(e^(1
/2)*x^(1/3)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(9/2)-2/7*b*e*n*(a+b*ln
(c*(d+e*x^(2/3))^n))^2/d/x^(7/3)+2/5*b*e^2*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2
/d^2/x^(5/3)-2/3*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d^3/x+2*b*e^4*n*(a+
b*ln(c*(d+e*x^(2/3))^n))^2/d^4/x^(1/3)-1/3*(a+b*ln(c*(d+e*x^(2/3))^n))^3/x
^3-1408/105*I*b^3*e^(9/2)*n^3*arctan(e^(1/2)*x^(1/3)/d^(1/2))^2/d^(9/2)+2/
3*b*e^5*n*Defer(Int)((a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x
)/d^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1385 vs. 2(632) = 1264.

Time = 11.66 (sec) , antiderivative size = 1385, normalized size of antiderivative = 57.71

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

output
```
((-60*b*e*n*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/(
d*x^(7/3)) + (84*b*e^2*n*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2
/3))^n])^2)/(d^2*x^(5/3)) - (140*b*e^3*n*(a - b*n*Log[d + e*x^(2/3)] + b*L
og[c*(d + e*x^(2/3))^n])^2)/(d^3*x) + (420*b*e^4*n*(a - b*n*Log[d + e*x^(2
/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/(d^4*x^(1/3)) + (420*b*e^(9/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d +
e*x^(2/3))^n])^2)/d^(9/2) - (210*b*n*Log[d + e*x^(2/3)]*(a - b*n*Log[d + e
*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/x^3 - (70*(a - b*n*Log[d + e*x^
(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3)/x^3 - (2*b^3*n^3*(1376*e^3*(d + e*
x^(2/3))^(3/2)*((e*x^(2/3))/(d + e*x^(2/3)))^(3/2)*x^2*ArcSin[Sqrt[d]/Sqrt
[d + e*x^(2/3)]] + Sqrt[d]*(16*e^3*(d - 15*e*x^(2/3))*x^2 + 8*(3*d^2*e^2*x
^(4/3) - 12*d*e^3*x^2 + 71*e^4*x^(8/3))*Log[d + e*x^(2/3)] + (30*d^3*e*x^(
2/3) - 42*d^2*e^2*x^(4/3) + 70*d*e^3*x^2 - 210*e^4*x^(8/3))*Log[d + e*x^(2
/3)]^2 + 35*d^4*Log[d + e*x^(2/3)]^3) + 210*e^4*Sqrt[(e*x^(2/3))/(d + e*x^
(2/3))]*x^(8/3)*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3
/2, 3/2}, d/(d + e*x^(2/3))] + Log[d + e*x^(2/3)]*(4*Sqrt[d]*Hypergeometri
cPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^(2/3))] + Sqrt[d + e*x^(2/3)]
*ArcSin[Sqrt[d]/Sqrt[d + e*x^(2/3)]]*Log[d + e*x^(2/3)])) + (352*d^(3/2)*e
^4*x^(8/3)*(4*Sqrt[e*x^(2/3)]*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*(Log[d + e
*x^(2/3)] - Log[1 + (e*x^(2/3))/d]) - Sqrt[-d]*Sqrt[-((e*x^(2/3))/d)]*(...
```

Rubi [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2907, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(\frac{2}{3}ben\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{\left(d+ex^{2/3}\right)x^{8/3}}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{9x^3}\right)$$

$$\Big\downarrow \text{2926}$$

$$3\left(\frac{2}{3}ben\int \left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2e^4}{d^4\left(d+ex^{2/3}\right)}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2e^3}{d^4x^{2/3}}+\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right.}{d^3x^{4/3}}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{9x^3}+\frac{2}{3}ben\left(\frac{e^4\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d+ex^{2/3}}d\sqrt[3]{x}}{d^4}-\frac{704be^{7/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+\right.}{105d^{9/2}}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^4,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{3}}{x^{4}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{4}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="fricas")
```

output
```
integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c
)^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 25.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^4,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 564, normalized size of antiderivative = 23.50

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\frac{-2430x^{\frac{2}{3}}a^2b\,d^4en+17010x^{\frac{8}{3}}a^2bd\,e^4n-1080x^{\frac{2}{3}}a\,b^2d^4e\,n^2+7560x^{\frac{8}{3}}a}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x)
```

output
```
(17010*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a**2*b*e**4*n*
x**3 + 7560*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*a*b**2*e*
*4*n**2*x**3 + 1680*sqrt(e)*sqrt(d)*atan((x**(1/3)*e)/(sqrt(e)*sqrt(d)))*b
**3*e**4*n**3*x**3 - 2430*x**(2/3)*a**2*b*d**4*e*n + 17010*x**(2/3)*a**2*b
*d*e**4*n*x**2 - 1080*x**(2/3)*a*b**2*d**4*e*n**2 + 7560*x**(2/3)*a*b**2*d
*e**4*n**2*x**2 - 240*x**(2/3)*b**3*d**4*e*n**3 + 1680*x**(2/3)*b**3*d*e**
4*n**3*x**2 + 3402*x**(1/3)*a**2*b*d**3*e**2*n*x + 1512*x**(1/3)*a*b**2*d*
*3*e**2*n**2*x + 336*x**(1/3)*b**3*d**3*e**2*n**3*x - 5670*int(log((x**(2/
3)*e + d)**n*c)**2/(x**(2/3)*e*x**4 + d*x**4),x)*b**3*d**6*n*x**3 - 11340*
int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*e*x**4 + d*x**4),x)*a*b**2*d**6*n
*x**3 - 2520*int(log((x**(2/3)*e + d)**n*c)/(x**(2/3)*e*x**4 + d*x**4),x)*
b**3*d**6*n**2*x**3 - 2835*log((x**(2/3)*e + d)**n*c)**3*b**3*d**5 - 8505*
log((x**(2/3)*e + d)**n*c)**2*a*b**2*d**5 - 1890*log((x**(2/3)*e + d)**n*c
)**2*b**3*d**5*n - 8505*log((x**(2/3)*e + d)**n*c)*a**2*b*d**5 - 3780*log(
(x**(2/3)*e + d)**n*c)*a*b**2*d**5*n - 840*log((x**(2/3)*e + d)**n*c)*b**3
*d**5*n**2 - 2835*a**3*d**5 - 5670*a**2*b*d**2*e**3*n*x**2 - 2520*a*b**2*d
**2*e**3*n**2*x**2 - 560*b**3*d**2*e**3*n**3*x**2)/(8505*d**5*x**3)
```

3.489 $\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 239

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) \, dx = \frac{b e^{11} n \sqrt[3]{x}}{4 d^{11}}-\frac{b e^{10} n x^{2/3}}{8 d^{10}}+\frac{b e^9 n x}{12 d^9}-\frac{b e^8 n x^{4/3}}{16 d^8}+\frac{b e^7 n x^{5/3}}{20 d^7}-\frac{b e^6 n x^2}{24 d^6}+\frac{b e^5 n x^{7/3}}{28 d^5}-\frac{b e^4 n x^{8/3}}{32 d^4}+\frac{b e^3 n x^3}{36 d^3}-\frac{b e^2 n x^{10/3}}{40 d^2}+\frac{b e n x^{11/3}}{44 d}-\frac{b e^{12} n \log \left(d+\frac{e}{\sqrt[3]{x}}\right)}{4 d^{12}}+\frac{1}{4} x^4 \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{b e^{12} n \log (x)}{12 d^{12}}$$

output

```
1/4*b*e^11*n*x^(1/3)/d^11-1/8*b*e^10*n*x^(2/3)/d^10+1/12*b*e^9*n*x/d^9-1/1
6*b*e^8*n*x^(4/3)/d^8+1/20*b*e^7*n*x^(5/3)/d^7-1/24*b*e^6*n*x^2/d^6+1/28*b
*e^5*n*x^(7/3)/d^5-1/32*b*e^4*n*x^(8/3)/d^4+1/36*b*e^3*n*x^3/d^3-1/40*b*e^
2*n*x^(10/3)/d^2+1/44*b*e*n*x^(11/3)/d-1/4*b*e^12*n*ln(d+e/x^(1/3))/d^12+1
/4*x^4*(a+b*ln(c*(d+e/x^(1/3))^n))-1/12*b*e^12*n*ln(x)/d^12
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{12}ben\left(\frac{3e^{10}\sqrt[3]{x}}{d^{11}}-\frac{3e^9x^{2/3}}{2d^{10}}+\frac{e^8x}{d^9}-\frac{3e^7x^{4/3}}{4d^8}+\frac{3e^6x^{5/3}}{5d^7}-\frac{e^5x^2}{2d^6}+\frac{3e^4x^{7/3}}{7d^5}-\frac{3e^3x^{8/3}}{8d^4}+\frac{e^2x^3}{3d^3}-\frac{3ex^{10/3}}{10d^2}+\frac{3x^{11/3}}{11d}-\frac{3e^{11}\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^{12}}-\frac{e^{11}\log(x)}{d^{12}}\right)$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

output
```
(a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(1/3))^n])/4 + (b*e*n*((3*e^10*x^(1/3))/
d^11 - (3*e^9*x^(2/3))/(2*d^10) + (e^8*x)/d^9 - (3*e^7*x^(4/3))/(4*d^8) +
(3*e^6*x^(5/3))/(5*d^7) - (e^5*x^2)/(2*d^6) + (3*e^4*x^(7/3))/(7*d^5) - (3
*e^3*x^(8/3))/(8*d^4) + (e^2*x^3)/(3*d^3) - (3*e*x^(10/3))/(10*d^2) + (3*x
^(11/3))/(11*d) - (3*e^11*Log[d + e/x^(1/3)])/d^12 - (e^11*Log[x])/d^12))/
12
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int x^{13/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2842}$$

$$-3\left(\frac{1}{12}ben\int\frac{x^4}{d+\frac{e}{\sqrt[3]{x}}}d\frac{1}{\sqrt[3]{x}}-\frac{1}{12}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\right)$$

$$\Big\downarrow \text{54}$$

$$-3\left(\frac{1}{12}ben\int\left(\frac{e^{12}}{d^{12}\left(d+\frac{e}{\sqrt[3]{x}}\right)}-\frac{\sqrt[3]{x}e^{11}}{d^{12}}+\frac{x^{2/3}e^{10}}{d^{11}}-\frac{xe^9}{d^{10}}+\frac{x^{4/3}e^8}{d^9}-\frac{x^{5/3}e^7}{d^8}+\frac{x^2e^6}{d^7}-\frac{x^{7/3}e^5}{d^6}+\frac{x^{8/3}e^4}{d^5}-\frac{x^3}{d}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-3\left(\frac{1}{12}ben\left(\frac{e^{11}\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^{12}}-\frac{e^{11}\log\left(\frac{1}{\sqrt[3]{x}}\right)}{d^{12}}-\frac{e^{10}\sqrt[3]{x}}{d^{11}}+\frac{e^9x^{2/3}}{2d^{10}}-\frac{e^8x}{3d^9}+\frac{e^7x^{4/3}}{4d^8}-\frac{e^6x^{5/3}}{5d^7}+\frac{e^5x^2}{6d^6}-\frac{e^4x^{7/}}{7d^5}\right.\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

output
```
-3*(-1/12*(x^4*(a + b*Log[c*(d + e/x^(1/3))^n])) + (b*e*n*(-((e^10*x^(1/3)
)/d^11) + (e^9*x^(2/3))/(2*d^10) - (e^8*x)/(3*d^9) + (e^7*x^(4/3))/(4*d^8)
 - (e^6*x^(5/3))/(5*d^7) + (e^5*x^2)/(6*d^6) - (e^4*x^(7/3))/(7*d^5) + (e^
3*x^(8/3))/(8*d^4) - (e^2*x^3)/(9*d^3) + (e*x^(10/3))/(10*d^2) - x^(11/3)/
(11*d) + (e^11*Log[d + e/x^(1/3)])/d^12 - (e^11*Log[x^(-1/3)])/d^12))/12)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3 \left(a + b\ln\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right) dx$$

input
```
int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

output
```
int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{27720\,bd^{12}x^4\log(c)+3080\,bd^9e^3nx^3+27720\,ad^{12}x^4-4620\,bd^6e^6nx^2+9240\,bd^3e^9nx-27720\,bd^{12}n\log}{}$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

output
```
1/110880*(27720*b*d^12*x^4*log(c) + 3080*b*d^9*e^3*n*x^3 + 27720*a*d^12*x^
4 - 4620*b*d^6*e^6*n*x^2 + 9240*b*d^3*e^9*n*x - 27720*b*d^12*n*log(x^(1/3)
) + 27720*(b*d^12 - b*e^12)*n*log(d*x^(1/3) + e) + 27720*(b*d^12*n*x^4 - b
*d^12*n)*log((d*x + e*x^(2/3))/x) + 63*(40*b*d^11*e*n*x^3 - 55*b*d^8*e^4*n
*x^2 + 88*b*d^5*e^7*n*x - 220*b*d^2*e^10*n)*x^(2/3) - 198*(14*b*d^10*e^2*n
*x^3 - 20*b*d^7*e^5*n*x^2 + 35*b*d^4*e^8*n*x - 140*b*d*e^11*n)*x^(1/3))/d^
12
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.68

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx=\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{4}ax^4-\frac{1}{110880}ben\left(\frac{27720e^{11}\log\left(dx^{\frac{1}{3}}+e\right)}{d^{12}}-\frac{2520d^{10}x^{\frac{11}{3}}-2772d^9ex^{\frac{10}{3}}+3080d^8e^2x^3-3465d^7e^3x^{\frac{8}{3}}+39\right.$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

output
```
1/4*b*x^4*log(c*(d + e/x^(1/3))^n) + 1/4*a*x^4 - 1/110880*b*e*n*(27720*e^1
1*log(d*x^(1/3) + e)/d^12 - (2520*d^10*x^(11/3) - 2772*d^9*e*x^(10/3) + 30
80*d^8*e^2*x^3 - 3465*d^7*e^3*x^(8/3) + 3960*d^6*e^4*x^(7/3) - 4620*d^5*e^
5*x^2 + 5544*d^4*e^6*x^(5/3) - 6930*d^3*e^7*x^(4/3) + 9240*d^2*e^8*x - 138
60*d*e^9*x^(2/3) + 27720*e^10*x^(1/3))/d^11)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.71

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx=\frac{1}{4}bx^4\log(c)+\frac{1}{4}ax^4+\frac{1}{110880}\left(27720x^4\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)-e\left(\frac{27720e^{11}\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^{12}}-\frac{2520d^{10}x^{\frac{11}{3}}-2772d^9ex^{\frac{10}{3}}+308\right.\right.$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

output
```
1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/110880*(27720*x^4*log(d + e/x^(1/3)) - e*
(27720*e^11*log(abs(d*x^(1/3) + e))/d^12 - (2520*d^10*x^(11/3) - 2772*d^9*
e*x^(10/3) + 3080*d^8*e^2*x^3 - 3465*d^7*e^3*x^(8/3) + 3960*d^6*e^4*x^(7/3
) - 4620*d^5*e^5*x^2 + 5544*d^4*e^6*x^(5/3) - 6930*d^3*e^7*x^(4/3) + 9240*
d^2*e^8*x - 13860*d*e^9*x^(2/3) + 27720*e^10*x^(1/3))/d^11))*b*n
```

Mupad [B] (verification not implemented)

Time = 25.72 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.80

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=\frac{\frac{a\,d^{12}\,x^4}{4}-\frac{b\,e^{12}\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)}{2}+\frac{b\,d^{12}\,x^4\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{4}+\frac{b\,d^3\,e^9\,n\,x}{12}+\frac{b\,d\,e^{11}\,n\,x^{1/3}}{4}+\frac{b\,d^{11}\,e\,n\,x^{11/3}}{44}-\frac{b\,d^6\,e^6\,n\,x^2}{24}+b}{d^{12}}$$

input
```
int(x^3*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

output
```
((a*d^12*x^4)/4 - (b*e^12*n*atanh((2*e)/(d*x^(1/3)) + 1))/2 + (b*d^12*x^4*
log(c*(d + e/x^(1/3))^n))/4 + (b*d^3*e^9*n*x)/12 + (b*d*e^11*n*x^(1/3))/4
+ (b*d^11*e*n*x^(11/3))/44 - (b*d^6*e^6*n*x^2)/24 + (b*d^9*e^3*n*x^3)/36 -
 (b*d^2*e^10*n*x^(2/3))/8 - (b*d^4*e^8*n*x^(4/3))/16 + (b*d^5*e^7*n*x^(5/3
))/20 + (b*d^7*e^5*n*x^(7/3))/28 - (b*d^8*e^4*n*x^(8/3))/32 - (b*d^10*e^2*
n*x^(10/3))/40)/d^12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=\frac{2520x^{\frac{11}{3}}b\,d^{11}en-3465x^{\frac{8}{3}}b\,d^8e^4n+5544x^{\frac{5}{3}}b\,d^5e^7n-13860x^{\frac{2}{3}}b\,d^2e^{10}n-2772x^{\frac{10}{3}}b\,d^{10}e^2n+3960x^{\frac{7}{3}}b\,d^7e^5}{}$$

input
```
int(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x)
```

output
```
(2520*x**(2/3)*b*d**11*e*n*x**3 - 3465*x**(2/3)*b*d**8*e**4*n*x**2 + 5544*
x**(2/3)*b*d**5*e**7*n*x - 13860*x**(2/3)*b*d**2*e**10*n - 2772*x**(1/3)*b
*d**10*e**2*n*x**3 + 3960*x**(1/3)*b*d**7*e**5*n*x**2 - 6930*x**(1/3)*b*d*
*4*e**8*n*x + 27720*x**(1/3)*b*d*e**11*n - 27720*log(x**(1/3))*b*e**12*n +
 27720*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b*d**12*x**4 - 27720*log(((x*
*(1/3)*d + e)**n*c)/x**(n/3))*b*e**12 + 27720*a*d**12*x**4 + 3080*b*d**9*e
**3*n*x**3 - 4620*b*d**6*e**6*n*x**2 + 9240*b*d**3*e**9*n*x)/(110880*d**12
)
```

3.490 $\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 190

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right) \, dx = -\frac{be^8 n \sqrt[3]{x}}{3d^8} + \frac{be^7 n x^{2/3}}{6d^7} - \frac{be^6 n x}{9d^6} + \frac{be^5 n x^{4/3}}{12d^5} - \frac{be^4 n x^{5/3}}{15d^4} + \frac{be^3 n x^2}{18d^3} - \frac{be^2 n x^{7/3}}{21d^2} + \frac{ben x^{8/3}}{24d} + \frac{be^9 n \log \left(d + \frac{e}{\sqrt[3]{x}}\right)}{3d^9} + \frac{1}{3} x^3 \left(a + b \log \left(c \left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right) + \frac{be^9 n \log(x)}{9d^9}$$

output

```
-1/3*b*e^8*n*x^(1/3)/d^8+1/6*b*e^7*n*x^(2/3)/d^7-1/9*b*e^6*n*x/d^6+1/12*b*
e^5*n*x^(4/3)/d^5-1/15*b*e^4*n*x^(5/3)/d^4+1/18*b*e^3*n*x^2/d^3-1/21*b*e^2
*n*x^(7/3)/d^2+1/24*b*e*n*x^(8/3)/d+1/3*b*e^9*n*ln(d+e/x^(1/3))/d^9+1/3*x^
3*(a+b*ln(c*(d+e/x^(1/3))^n))+1/9*b*e^9*n*ln(x)/d^9
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.90

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx=\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{9}ben\left(-\frac{3e^7\sqrt[3]{x}}{d^8}+\frac{3e^6x^{2/3}}{2d^7}-\frac{e^5x}{d^6}+\frac{3e^4x^{4/3}}{4d^5}-\frac{3e^3x^{5/3}}{5d^4}+\frac{e^2x^2}{2d^3}-\frac{3ex^{7/3}}{7d^2}+\frac{3x^{8/3}}{8d}+\frac{3e^8\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^9}+\frac{e^8\log(x)}{d^9}\right)$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

output
```
(a*x^3)/3 + (b*x^3*Log[c*(d + e/x^(1/3))^n])/3 + (b*e*n*((-3*e^7*x^(1/3))/
d^8 + (3*e^6*x^(2/3))/(2*d^7) - (e^5*x)/d^6 + (3*e^4*x^(4/3))/(4*d^5) - (3
*e^3*x^(5/3))/(5*d^4) + (e^2*x^2)/(2*d^3) - (3*e*x^(7/3))/(7*d^2) + (3*x^(
8/3))/(8*d) + (3*e^8*Log[d + e/x^(1/3)])/d^9 + (e^8*Log[x])/d^9))/9
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

$$\Big\downarrow \ 2904$$

$$-3\int x^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2842}$$

$$-3\left(\frac{1}{9}ben\int\frac{x^3}{d+\frac{e}{\sqrt[3]{x}}}d\frac{1}{\sqrt[3]{x}}-\frac{1}{9}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\right)$$

$$\Big\downarrow \text{54}$$

$$-3\left(\frac{1}{9}ben\int\left(-\frac{e^9}{d^9\left(d+\frac{e}{\sqrt[3]{x}}\right)}+\frac{\sqrt[3]{x}e^8}{d^9}-\frac{x^{2/3}e^7}{d^8}+\frac{xe^6}{d^7}-\frac{x^{4/3}e^5}{d^6}+\frac{x^{5/3}e^4}{d^5}-\frac{x^2e^3}{d^4}+\frac{x^{7/3}e^2}{d^3}-\frac{x^{8/3}e}{d^2}+\frac{x^3}{d}\right)d\right.$$

$$\Big\downarrow \text{2009}$$

$$-3\left(\frac{1}{9}ben\left(-\frac{e^8\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^9}+\frac{e^8\log\left(\frac{1}{\sqrt[3]{x}}\right)}{d^9}+\frac{e^7\sqrt[3]{x}}{d^8}-\frac{e^6x^{2/3}}{2d^7}+\frac{e^5x}{3d^6}-\frac{e^4x^{4/3}}{4d^5}+\frac{e^3x^{5/3}}{5d^4}-\frac{e^2x^2}{6d^3}+\frac{ex^{7/3}}{7d^2}-\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

output
```
-3*(-1/9*(x^3*(a + b*Log[c*(d + e/x^(1/3))^n])) + (b*e*n*((e^7*x^(1/3))/d^
8 - (e^6*x^(2/3))/(2*d^7) + (e^5*x)/(3*d^6) - (e^4*x^(4/3))/(4*d^5) + (e^3
*x^(5/3))/(5*d^4) - (e^2*x^2)/(6*d^3) + (e*x^(7/3))/(7*d^2) - x^(8/3)/(8*d
) - (e^8*Log[d + e/x^(1/3)])/d^9 + (e^8*Log[x^(-1/3)])/d^9))/9)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)dx$$

input
```
int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

output
```
int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{840\,bd^9x^3\log\left(c\right)+140\,bd^6e^3nx^2+840\,ad^9x^3-280\,bd^3e^6nx-840\,bd^9n\log\left(x^{\frac{1}{3}}\right)+840\,(bd^9+be^9)n\log}{}$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

output
```
1/2520*(840*b*d^9*x^3*log(c) + 140*b*d^6*e^3*n*x^2 + 840*a*d^9*x^3 - 280*b
*d^3*e^6*n*x - 840*b*d^9*n*log(x^(1/3)) + 840*(b*d^9 + b*e^9)*n*log(d*x^(1
/3) + e) + 840*(b*d^9*n*x^3 - b*d^9*n)*log((d*x + e*x^(2/3))/x) + 21*(5*b*
d^8*e*n*x^2 - 8*b*d^5*e^4*n*x + 20*b*d^2*e^7*n)*x^(2/3) - 30*(4*b*d^7*e^2*
n*x^2 - 7*b*d^4*e^5*n*x + 28*b*d*e^8*n)*x^(1/3))/d^9
```

Sympy [A] (verification not implemented)

Time = 46.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.85

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{ax^3}{3}+b\left(\frac{en\left(\frac{3x^{\frac{8}{3}}}{8d}-\frac{3ex^{\frac{7}{3}}}{7d^2}+\frac{e^2x^2}{2d^3}-\frac{3e^3x^{\frac{5}{3}}}{5d^4}+\frac{3e^4x^{\frac{4}{3}}}{4d^5}-\frac{e^5x}{d^6}+\frac{3e^6x^{\frac{2}{3}}}{2d^7}+\frac{3e^8\left(\begin{cases}\frac{\sqrt[3]{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt[3]{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^8}-\frac{3e^7\sqrt[3]{x}}{d^8}\right)}{9}+\frac{x^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3}\right)$$

input
```
integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

output
```
a*x**3/3 + b*(e*n*(3*x**(8/3)/(8*d) - 3*e*x**(7/3)/(7*d**2) + e**2*x**2/(2
*d**3) - 3*e**3*x**(5/3)/(5*d**4) + 3*e**4*x**(4/3)/(4*d**5) - e**5*x/d**6
 + 3*e**6*x**(2/3)/(2*d**7) + 3*e**8*Piecewise((x**(1/3)/e, Eq(d, 0)), (lo
g(d*x**(1/3) + e)/d, True))/d**8 - 3*e**7*x**(1/3)/d**8)/9 + x**3*log(c*(d
 + e/x**(1/3))**n)/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.67

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{1}{3}\,bx^3\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{3}\,ax^3+\frac{1}{2520}\,ben\left(\frac{840\,e^8\log\left(dx^{\frac{1}{3}}+e\right)}{d^9}+\frac{105\,d^7x^{\frac{8}{3}}-120\,d^6ex^{\frac{7}{3}}+140\,d^5e^2x^2-168\,d^4e^3x^{\frac{5}{3}}+210\,d^3e^4x^{\frac{4}{3}}-2}{d^8}\right.$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

output
```
1/3*b*x^3*log(c*(d + e/x^(1/3))^n) + 1/3*a*x^3 + 1/2520*b*e*n*(840*e^8*log
(d*x^(1/3) + e)/d^9 + (105*d^7*x^(8/3) - 120*d^6*e*x^(7/3) + 140*d^5*e^2*x
^2 - 168*d^4*e^3*x^(5/3) + 210*d^3*e^4*x^(4/3) - 280*d^2*e^5*x + 420*d*e^6
*x^(2/3) - 840*e^7*x^(1/3))/d^8)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3+\frac{1}{2520}\left(840\,x^3\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)+e\left(\frac{840\,e^8\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^9}+\frac{105\,d^7x^{\frac{8}{3}}-120\,d^6ex^{\frac{7}{3}}+140\,d^5e^2x^2-168}{}\right.\right.$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

output
```
1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/2520*(840*x^3*log(d + e/x^(1/3)) + e*(840
*e^8*log(abs(d*x^(1/3) + e))/d^9 + (105*d^7*x^(8/3) - 120*d^6*e*x^(7/3) +
140*d^5*e^2*x^2 - 168*d^4*e^3*x^(5/3) + 210*d^3*e^4*x^(4/3) - 280*d^2*e^5*
x + 420*d*e^6*x^(2/3) - 840*e^7*x^(1/3))/d^8))*b*n
```

Mupad [B] (verification not implemented)

Time = 25.90 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=\frac{840\,a\,d^9\,x^3+1680\,b\,e^9\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)+840\,b\,d^9\,x^3\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)^n\right)-280\,b\,d^3\,e^6\,n\,x-840\,b\,d\,e^8\,n\,x}{}$$

input
```
int(x^2*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

output
```
(840*a*d^9*x^3 + 1680*b*e^9*n*atanh((2*e)/(d*x^(1/3)) + 1) + 840*b*d^9*x^3
*log(c*(d + e/x^(1/3))^n) - 280*b*d^3*e^6*n*x - 840*b*d*e^8*n*x^(1/3) + 10
5*b*d^8*e*n*x^(8/3) + 140*b*d^6*e^3*n*x^2 + 420*b*d^2*e^7*n*x^(2/3) + 210*
b*d^4*e^5*n*x^(4/3) - 168*b*d^5*e^4*n*x^(5/3) - 120*b*d^7*e^2*n*x^(7/3))/(
2520*d^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=\frac{105x^{\frac{8}{3}}b\,d^8en-168x^{\frac{5}{3}}b\,d^5e^4n+420x^{\frac{2}{3}}b\,d^2e^7n-120x^{\frac{7}{3}}b\,d^7e^2n+210x^{\frac{4}{3}}b\,d^4e^5n-840x^{\frac{1}{3}}bd\,e^8n+840\log\Big(}{2}$$

input
```
int(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x)
```

output
```
(105*x**(2/3)*b*d**8*e*n*x**2 - 168*x**(2/3)*b*d**5*e**4*n*x + 420*x**(2/3
)*b*d**2*e**7*n - 120*x**(1/3)*b*d**7*e**2*n*x**2 + 210*x**(1/3)*b*d**4*e*
*5*n*x - 840*x**(1/3)*b*d*e**8*n + 840*log(x**(1/3))*b*e**9*n + 840*log(((
x**(1/3)*d + e)**n*c)/x**(n/3))*b*d**9*x**3 + 840*log(((x**(1/3)*d + e)**n
*c)/x**(n/3))*b*e**9 + 840*a*d**9*x**3 + 140*b*d**6*e**3*n*x**2 - 280*b*d*
*3*e**6*n*x)/(2520*d**9)
```

3.491 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{be^5n\sqrt[3]{x}}{2d^5}-\frac{be^4nx^{2/3}}{4d^4}+\frac{be^3nx}{6d^3}-\frac{be^2nx^{4/3}}{8d^2}+\frac{benx^{5/3}}{10d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^6n\log(x)}{6d^6}$$

output
```
1/2*b*e^5*n*x^(1/3)/d^5-1/4*b*e^4*n*x^(2/3)/d^4+1/6*b*e^3*n*x/d^3-1/8*b*e^
2*n*x^(4/3)/d^2+1/10*b*e*n*x^(5/3)/d-1/2*b*e^6*n*ln(d+e/x^(1/3))/d^6+1/2*x
^2*(a+b*ln(c*(d+e/x^(1/3))^n))-1/6*b*e^6*n*ln(x)/d^6
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{6}ben\left(\frac{3e^4\sqrt[3]{x}}{d^5}-\frac{3e^3x^{2/3}}{2d^4}+\frac{e^2x}{d^3}-\frac{3ex^{4/3}}{4d^2}+\frac{3x^{5/3}}{5d}-\frac{3e^5\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^6}-\frac{e^5\log(x)}{d^6}\right)$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

output
```
(a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(1/3))^n])/2 + (b*e*n*((3*e^4*x^(1/3))/d
^5 - (3*e^3*x^(2/3))/(2*d^4) + (e^2*x)/d^3 - (3*e*x^(4/3))/(4*d^2) + (3*x^
(5/3))/(5*d) - (3*e^5*Log[d + e/x^(1/3)])/d^6 - (e^5*Log[x])/d^6))/6
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int x^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2842}$$

$$-3\left(\frac{1}{6}ben\int \frac{x^2}{d+\frac{e}{\sqrt[3]{x}}}d\frac{1}{\sqrt[3]{x}}-\frac{1}{6}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\right)$$

$$\Big\downarrow \textbf{54}$$

$$-3\left(\frac{1}{6}ben\int \left(\frac{e^6}{d^6\left(d+\frac{e}{\sqrt[3]{x}}\right)}-\frac{\sqrt[3]{x}e^5}{d^6}+\frac{x^{2/3}e^4}{d^5}-\frac{xe^3}{d^4}+\frac{x^{4/3}e^2}{d^3}-\frac{x^{5/3}e}{d^2}+\frac{x^2}{d}\right)d\frac{1}{\sqrt[3]{x}}-\frac{1}{6}x^2\left(a+b\log\left(c\left(d+\right.\right.\right.\right.$$

$$\Big\downarrow \textbf{2009}$$

$$-3\left(\frac{1}{6}ben\left(\frac{e^5\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^6}-\frac{e^5\log\left(\frac{1}{\sqrt[3]{x}}\right)}{d^6}-\frac{e^4\sqrt[3]{x}}{d^5}+\frac{e^3x^{2/3}}{2d^4}-\frac{e^2x}{3d^3}+\frac{ex^{4/3}}{4d^2}-\frac{x^{5/3}}{5d}\right)-\frac{1}{6}x^2\left(a+b\log\left(c\right.\right.\right.$$

input
```
Int[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

output
```
-3*(-1/6*(x^2*(a + b*Log[c*(d + e/x^(1/3))^n])) + (b*e*n*(-((e^4*x^(1/3))/
d^5) + (e^3*x^(2/3))/(2*d^4) - (e^2*x)/(3*d^3) + (e*x^(4/3))/(4*d^2) - x^(
5/3)/(5*d) + (e^5*Log[d + e/x^(1/3)])/d^6 - (e^5*Log[x^(-1/3)])/d^6))/6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)dx$$
$$=\frac{60\,bd^{6}x^{2}\log\left(c\right)+20\,bd^{3}e^{3}nx+60\,ad^{6}x^{2}-60\,bd^{6}n\log\left(x^{\frac{1}{3}}\right)+60\,(bd^{6}-be^{6})n\log\left(dx^{\frac{1}{3}}+e\right)+60\,(bd^{6}n}{120\,d^{6}}$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
```

output
```
1/120*(60*b*d^6*x^2*log(c) + 20*b*d^3*e^3*n*x + 60*a*d^6*x^2 - 60*b*d^6*n*
log(x^(1/3)) + 60*(b*d^6 - b*e^6)*n*log(d*x^(1/3) + e) + 60*(b*d^6*n*x^2 -
 b*d^6*n)*log((d*x + e*x^(2/3))/x) + 6*(2*b*d^5*e*n*x - 5*b*d^2*e^4*n)*x^(
2/3) - 15*(b*d^4*e^2*n*x - 4*b*d*e^5*n)*x^(1/3))/d^6
```

Sympy [A] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=\frac{ax^2}{2}+b\left(\frac{en\left(\frac{3x^{\frac{5}{3}}}{5d}-\frac{3ex^{\frac{4}{3}}}{4d^2}+\frac{e^2x}{d^3}-\frac{3e^3x^{\frac{2}{3}}}{2d^4}-\frac{3e^5\left(\begin{cases}\frac{\sqrt[3]{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt[3]{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^5}+\frac{3e^4\sqrt[3]{x}}{d^5}\right)}{6}+\frac{x^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2}\right)$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

output
```
a*x**2/2 + b*(e*n*(3*x**(5/3)/(5*d) - 3*e*x**(4/3)/(4*d**2) + e**2*x/d**3
- 3*e**3*x**(2/3)/(2*d**4) - 3*e**5*Piecewise((x**(1/3)/e, Eq(d, 0)), (log
(d*x**(1/3) + e)/d, True))/d**5 + 3*e**4*x**(1/3)/d**5)/6 + x**2*log(c*(d
+ e/x**(1/3))**n)/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = -\frac{1}{120}\,ben\left(\frac{60\,e^5\log\left(dx^{\frac{1}{3}}+e\right)}{d^6}-\frac{12\,d^4x^{\frac{5}{3}}-15\,d^3ex^{\frac{4}{3}}+20\,d^2e^2x-30\,de^3x^{\frac{2}{3}}+60\,e^4x^{\frac{1}{3}}}{d^5}\right) + \frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

output
```
-1/120*b*e*n*(60*e^5*log(d*x^(1/3) + e)/d^6 - (12*d^4*x^(5/3) - 15*d^3*e*x
^(4/3) + 20*d^2*e^2*x - 30*d*e^3*x^(2/3) + 60*e^4*x^(1/3))/d^5) + 1/2*b*x^
2*log(c*(d + e/x^(1/3))^n) + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{1}{2}\,bx^2\log\left(c\right) + \frac{1}{120}\left(60\,x^2\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)-e\left(\frac{60\,e^5\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^6}-\frac{12\,d^4x^{\frac{5}{3}}-15\,d^3ex^{\frac{4}{3}}+20\,d^2e^2x-30\,de^3x^{\frac{2}{3}}+}{d^5}\right.\right. + \frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

output
```
1/2*b*x^2*log(c) + 1/120*(60*x^2*log(d + e/x^(1/3)) - e*(60*e^5*log(abs(d*
x^(1/3) + e))/d^6 - (12*d^4*x^(5/3) - 15*d^3*e*x^(4/3) + 20*d^2*e^2*x - 30
*d*e^3*x^(2/3) + 60*e^4*x^(1/3))/d^5))*b*n + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 26.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{x^{5/3}\left(\frac{b\,e\,n}{5\,d}-\frac{b\,e^2\,n}{4\,d^2\,x^{1/3}}-\frac{b\,e^4\,n}{2\,d^4\,x}+\frac{b\,e^3\,n}{3\,d^3\,x^{2/3}}+\frac{b\,e^5\,n}{d^5\,x^{4/3}}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{2}-\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)}{d^6}$$

input
```
int(x*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

output
```
(x^(5/3)*((b*e*n)/(5*d) - (b*e^2*n)/(4*d^2*x^(1/3)) - (b*e^4*n)/(2*d^4*x)
+ (b*e^3*n)/(3*d^3*x^(2/3)) + (b*e^5*n)/(d^5*x^(4/3))))/2 + (a*x^2)/2 + (b
*x^2*log(c*(d + e/x^(1/3))^n))/2 - (b*e^6*n*atanh((2*e)/(d*x^(1/3)) + 1))/
d^6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx =\frac{12x^{\frac{5}{3}}b\,d^5en-30x^{\frac{2}{3}}b\,d^2e^4n-15x^{\frac{4}{3}}b\,d^4e^2n+60x^{\frac{1}{3}}bd\,e^5n-60\log\left(x^{\frac{1}{3}}\right)b\,e^6n+60\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b\,d^6x^2}{120d^6}$$

input
```
int(x*(a+b*log(c*(d+e/x^(1/3))^n)),x)
```

output

```
(12*x**(2/3)*b*d**5*e*n*x - 30*x**(2/3)*b*d**2*e**4*n - 15*x**(1/3)*b*d**4
*e**2*n*x + 60*x**(1/3)*b*d*e**5*n - 60*log(x**(1/3))*b*e**6*n + 60*log(((
x**(1/3)*d + e)**n*c)/x**(n/3))*b*d**6*x**2 - 60*log(((x**(1/3)*d + e)**n*
c)/x**(n/3))*b*e**6 + 60*a*d**6*x**2 + 20*b*d**3*e**3*n*x)/(120*d**6)
```

3.492 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 70

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=-\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log\left(e+d\sqrt[3]{x}\right)}{d^3}$$

output
```
-b*e^2*n*x^(1/3)/d^2+1/2*b*e*n*x^(2/3)/d+a*x+b*x*ln(c*(d+e/x^(1/3))^n)+b*e
^3*n*ln(e+d*x^(1/3))/d^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{3}ben\left(-\frac{3e\sqrt[3]{x}}{d^2}+\frac{3x^{2/3}}{2d}+\frac{3e^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}+\frac{e^2\log(x)}{d^3}\right)$$

input
```
Integrate[a + b*Log[c*(d + e/x^(1/3))^n],x]
```

output
```
a*x + b*x*Log[c*(d + e/x^(1/3))^n] + (b*e*n*((-3*e*x^(1/3))/d^2 + (3*x^(2/
3))/(2*d) + (3*e^2*Log[d + e/x^(1/3)])/d^3 + (e^2*Log[x])/d^3))/3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx$$

$$\Big\downarrow \; 2009$$

$$ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log\left(d\sqrt[3]{x}+e\right)}{d^3}-\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}$$

input
```
Int[a + b*Log[c*(d + e/x^(1/3))^n],x]
```

output
```
-((b*e^2*n*x^(1/3))/d^2) + (b*e*n*x^(2/3))/(2*d) + a*x + b*x*Log[c*(d + e/
x^(1/3))^n] + (b*e^3*n*Log[e + d*x^(1/3)])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.54

method	result	size
default	$ax+b\left(x\ln\left(c\left(\frac{e+d\,x^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right)^{n}\right)+\frac{en\left(\frac{e^{2}\ln\left(d^{3}x+e^{3}\right)}{d^{3}}+\frac{3x^{\frac{2}{3}}}{2d}+\frac{2e^{2}\ln\left(e+d\,x^{\frac{1}{3}}\right)}{d^{3}}-\frac{e^{2}\ln\left(d^{2}x^{\frac{2}{3}}-ed\,x^{\frac{1}{3}}+e^{2}\right)}{d^{3}}-\frac{3e\,x^{\frac{1}{3}}}{d^{2}}\right)}{3}\right)$	108
parts	$ax+b\left(x\ln\left(c\left(\frac{e+d\,x^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right)^{n}\right)+\frac{en\left(\frac{e^{2}\ln\left(d^{3}x+e^{3}\right)}{d^{3}}+\frac{3x^{\frac{2}{3}}}{2d}+\frac{2e^{2}\ln\left(e+d\,x^{\frac{1}{3}}\right)}{d^{3}}-\frac{e^{2}\ln\left(d^{2}x^{\frac{2}{3}}-ed\,x^{\frac{1}{3}}+e^{2}\right)}{d^{3}}-\frac{3e\,x^{\frac{1}{3}}}{d^{2}}\right)}{3}\right)$	108

input
```
int(a+b*ln(c*(d+e/x^(1/3))^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*ln(c*((e+d*x^(1/3))/x^(1/3))^n)+1/3*e*n*(e^2*ln(d^3*x+e^3)/d^3+3/
2/d*x^(2/3)+2/d^3*e^2*ln(e+d*x^(1/3))-1/d^3*e^2*ln(d^2*x^(2/3)-e*d*x^(1/3)
+e^2)-3/d^2*e*x^(1/3)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.53

$$\int\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)dx$$
$$=\frac{2\,bd^{3}x\log\left(c\right)-2\,bd^{3}n\log\left(x^{\frac{1}{3}}\right)+bd^{2}enx^{\frac{2}{3}}-2\,bde^{2}nx^{\frac{1}{3}}+2\,ad^{3}x+2\,(bd^{3}+be^{3})n\log\left(dx^{\frac{1}{3}}+e\right)+2\,(bd}{2\,d^{3}}$$

input
```
integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="fricas")
```

output
```
1/2*(2*b*d^3*x*log(c) - 2*b*d^3*n*log(x^(1/3)) + b*d^2*e*n*x^(2/3) - 2*b*d
*e^2*n*x^(1/3) + 2*a*d^3*x + 2*(b*d^3 + b*e^3)*n*log(d*x^(1/3) + e) + 2*(b
*d^3*n*x - b*d^3*n)*log((d*x + e*x^(2/3))/x))/d^3
```

Sympy [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = ax+b\left(\frac{en\left(\frac{3x^{\frac{2}{3}}}{2d}+\frac{3e^2\left(\begin{cases}\frac{\sqrt[3]{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt[3]{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^2}-\frac{3e\sqrt[3]{x}}{d^2}\right)}{3}+x\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)$$

input
```
integrate(a+b*ln(c*(d+e/x**(1/3))**n),x)
```

output
```
a*x + b*(e*n*(3*x**(2/3)/(2*d) + 3*e**2*Piecewise((x**(1/3)/e, Eq(d, 0)),
(log(d*x**(1/3) + e)/d, True))/d**2 - 3*e*x**(1/3)/d**2)/3 + x*log(c*(d +
e/x**(1/3))**n))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{1}{2}\left(en\left(\frac{2\,e^2\log\left(dx^{\frac{1}{3}}+e\right)}{d^3}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^2}\right)+2\,x\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)b+ax$$

input
```
integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="maxima")
```

output
```
1/2*(e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2) +
2*x*log(c*(d + e/x^(1/3))^n))*b + a*x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{1}{2}\left(\left(e\left(\frac{2\,e^2\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^3}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^2}\right)+2\,x\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)n+2\,x\log\left(c\right)\right)b + ax$$

input
```
integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="giac")
```

output
```
1/2*((e*(2*e^2*log(abs(d*x^(1/3) + e))/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2
) + 2*x*log(d + e/x^(1/3)))*n + 2*x*log(c))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 25.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$= a\,x+b\,x\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)+\frac{b\left(2\,e^3\,n\,\ln\left(e+d\,x^{1/3}\right)-2\,d\,e^2\,n\,x^{1/3}+d^2\,e\,n\,x^{2/3}\right)}{2\,d^3}$$

input
```
int(a + b*log(c*(d + e/x^(1/3))^n),x)
```

output
```
a*x + b*x*log(c*(d + e/x^(1/3))^n) + (b*(2*e^3*n*log(e + d*x^(1/3)) - 2*d*
e^2*n*x^(1/3) + d^2*e*n*x^(2/3)))/(2*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$= \frac{x^{\frac{2}{3}}b\,d^2en-2x^{\frac{1}{3}}bd\,e^2n+2\log\left(x^{\frac{1}{3}}\right)b\,e^3n+2\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^nc}{x^{\frac{n}{3}}}\right)b\,d^3x+2\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^nc}{x^{\frac{n}{3}}}\right)b\,e^3+2a\,d^3x}{2d^3}$$

input
```
int(a+b*log(c*(d+e/x^(1/3))^n),x)
```

output
```
(x**(2/3)*b*d**2*e*n - 2*x**(1/3)*b*d*e**2*n + 2*log(x**(1/3))*b*e**3*n +
2*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b*d**3*x + 2*log(((x**(1/3)*d + e)
**n*c)/x**(n/3))*b*e**3 + 2*a*d**3*x)/(2*d**3)
```

3.493 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx = -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 3bn\,\text{PolyLog}\left(2,1+\frac{e}{d\sqrt[3]{x}}\right)$$

output
```
-3*(a+b*ln(c*(d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))-3*b*n*polylog(2,1+e/d/x^(1
/3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx = -3b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + a\log(x) - 3bn\,\text{PolyLog}\left(2,\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

output
```
-3*b*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x^(1/3)))] + a*Log[x] - 3*b*n*Pol
yLog[2, (d + e/x^(1/3))/d]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2841}$$

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-ben\int \frac{\log\left(-\frac{e}{d\sqrt[3]{x}}\right)}{d+\frac{e}{\sqrt[3]{x}}}d\frac{1}{\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{2752}$$

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

output
```
-3*((a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] + b*n*PolyLog[2
, 1 + e/(d*x^(1/3))])
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))/x,x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x}\,dx=\int \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*((d*x + e*x^(2/3))/x)^n) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx=\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x,x)
```

output
```
Integral((a + b*log(c*(d + e/x**(1/3))**n))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(44) = 88$.

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.63

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx=-3\left(\log\left(\frac{dx^{\frac{1}{3}}}{e}+1\right)\log\left(x^{\frac{1}{3}}\right)+\mathrm{Li}_2\left(-\frac{dx^{\frac{1}{3}}}{e}\right)\right)bn+\frac{2\,be^2n\log(x)^2+12\,be^2\log\left(\left(dx^{\frac{1}{3}}+e\right)^n\right)\log(x)-12\,be^2\log(x)\log\left(x^{\frac{1}{3}n}\right)+9\,bd^2nx^{\frac{2}{3}}-36\,bdenx^{\frac{1}{3}}-\cdots}{\cdots}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="maxima")
```

output
```
-3*(log(d*x^(1/3)/e + 1)*log(x^(1/3)) + dilog(-d*x^(1/3)/e))*b*n + 1/12*(2
*b*e^2*n*log(x)^2 + 12*b*e^2*log((d*x^(1/3) + e)^n)*log(x) - 12*b*e^2*log(
x)*log(x^(1/3*n)) + 9*b*d^2*n*x^(2/3) - 36*b*d*e*n*x^(1/3) - 6*(b*d^2*n*x^
(2/3) - 2*b*d*e*n*x^(1/3))*log(x) + 12*(b*e^2*log(c) + a*e^2)*log(x) + 3*(
2*b*d^2*n*x*log(x) - 3*b*d^2*n*x)/x^(1/3) - 12*(b*d*e*n*x*log(x) - 3*b*d*e
*n*x)/x^(2/3))/e^2
```

Giac [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx=\int \frac{a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^n))/x, x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx=\left(\int \frac{\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))/x,x)
```

output
```
int(log(((x**(1/3)*d + e)**n*c)/x**(n/3))/x,x)*b + log(x)*a
```

3.494 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx=\frac{bn}{3x}-\frac{bdn}{2ex^{2/3}}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}$$

output
```
1/3*b*n/x-1/2*b*d*n/e/x^(2/3)+b*d^2*n/e^2/x^(1/3)-b*d^3*n*ln(d+e/x^(1/3))/
e^3-(a+b*ln(c*(d+e/x^(1/3))^n))/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx=-\frac{a}{x}+\frac{bn}{3x}-\frac{bdn}{2ex^{2/3}}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```

output
```
-(a/x) + (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (
b*d^3*n*Log[d + e/x^(1/3)])/e^3 - (b*Log[c*(d + e/x^(1/3))^n])/x
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^{2/3}}d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2842}$$

$$-3\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x}-\frac{1}{3}ben\int \frac{1}{\left(d+\frac{e}{\sqrt[3]{x}}\right)x}d\frac{1}{\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{49}$$

$$-3\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x}-\frac{1}{3}ben\int \left(-\frac{d^3}{e^3\left(d+\frac{e}{\sqrt[3]{x}}\right)}+\frac{d^2}{e^3}-\frac{d}{e^2\sqrt[3]{x}}+\frac{1}{ex^{2/3}}\right)d\frac{1}{\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{2009}$$

$$-3\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x}-\frac{1}{3}ben\left(-\frac{d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^4}+\frac{d^2}{e^3\sqrt[3]{x}}-\frac{d}{2e^2x^{2/3}}+\frac{1}{3ex}\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```

output
```
-3*(-1/3*(b*e*n*(1/(3*e*x) - d/(2*e^2*x^(2/3)) + d^2/(e^3*x^(1/3)) - (d^3*
Log[d + e/x^(1/3)])/e^4)) + (a + b*Log[c*(d + e/x^(1/3))^n])/(3*x))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x^{2}}\,dx = \frac{6\,bd^{2}enx^{\frac{2}{3}}-3\,bde^{2}nx^{\frac{1}{3}}+2\,be^{3}n-6\,ae^{3}-2\,(be^{3}n-3\,ae^{3})x+6\,(be^{3}x-be^{3})\log\left(c\right)-6\,(bd^{3}nx+be^{3}n)\,\mathrm{l}}{6\,e^{3}x}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="fricas")
```

output
```
1/6*(6*b*d^2*e*n*x^(2/3) - 3*b*d*e^2*n*x^(1/3) + 2*b*e^3*n - 6*a*e^3 - 2*(
b*e^3*n - 3*a*e^3)*x + 6*(b*e^3*x - b*e^3)*log(c) - 6*(b*d^3*n*x + b*e^3*n
)*log((d*x + e*x^(2/3))/x))/(e^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x^{2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx$$
$$= -\frac{1}{6}\,ben\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4} - \frac{2\,d^3\log\left(x\right)}{e^4} - \frac{6\,d^2x^{\frac{2}{3}} - 3\,dex^{\frac{1}{3}} + 2\,e^2}{e^3x}\right)$$
$$- \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x} - \frac{a}{x}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="maxima")
```

output
```
-1/6*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2/
3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - b*log(c*(d + e/x^(1/3))^n)/x - a/x
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx =$$
$$-\frac{1}{6}\left(e\left(\frac{6\,d^3\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{e^4} - \frac{2\,d^3\log\left(|x|\right)}{e^4} - \frac{6\,d^2ex^{\frac{2}{3}} - 3\,de^2x^{\frac{1}{3}} + 2\,e^3}{e^4x}\right) + \frac{6\,\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)}{x}\right)bn$$
$$- \frac{b\log\left(c\right)}{x} - \frac{a}{x}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="giac")
```

output
```
-1/6*(e*(6*d^3*log(abs(d*x^(1/3) + e))/e^4 - 2*d^3*log(abs(x))/e^4 - (6*d^
2*e*x^(2/3) - 3*d*e^2*x^(1/3) + 2*e^3)/(e^4*x)) + 6*log(d + e/x^(1/3))/x)*
b*n - b*log(c)/x - a/x
```

Mupad [B] (verification not implemented)

Time = 25.92 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx = \frac{b\,n}{3\,x}-\frac{a}{x}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{x}-\frac{b\,d\,n}{2\,e\,x^{2/3}}-\frac{b\,d^3\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{e^3}+\frac{b\,d^2\,n}{e^2\,x^{1/3}}$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))/x^2,x)
```

output
```
(b*n)/(3*x) - a/x - (b*log(c*(d + e/x^(1/3))^n))/x - (b*d*n)/(2*e*x^(2/3))
 - (b*d^3*n*log(d + e/x^(1/3)))/e^3 + (b*d^2*n)/(e^2*x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx = \frac{6x^{\frac{2}{3}}b\,d^2en-3x^{\frac{1}{3}}bd\,e^2n-6\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b\,d^3x-6\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b\,e^3-6a\,e^3+2b\,e^3n}{6e^3x}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x)
```

output
```
(6*x**(2/3)*b*d**2*e*n - 3*x**(1/3)*b*d*e**2*n - 6*log(((x**(1/3)*d + e)**
n*c)/x**(n/3))*b*d**3*x - 6*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b*e**3 -
 6*a*e**3 + 2*b*e**3*n)/(6*e**3*x)
```

3.495 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx=\frac{bn}{12x^2}-\frac{bdn}{10ex^{5/3}}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bd^3n}{6e^3x}+\frac{bd^4n}{4e^4x^{2/3}}-\frac{bd^5n}{2e^5\sqrt[3]{x}}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}$$

output

```
1/12*b*n/x^2-1/10*b*d*n/e/x^(5/3)+1/8*b*d^2*n/e^2/x^(4/3)-1/6*b*d^3*n/e^3/
x+1/4*b*d^4*n/e^4/x^(2/3)-1/2*b*d^5*n/e^5/x^(1/3)+1/2*b*d^6*n*ln(d+e/x^(1/
3))/e^6-1/2*(a+b*ln(c*(d+e/x^(1/3))^n))/x^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = -\frac{a}{2x^2}-\frac{1}{6}ben\left(-\frac{1}{2ex^2}+\frac{3d}{5e^2x^{5/3}}-\frac{3d^2}{4e^3x^{4/3}}+\frac{d^3}{e^4x}-\frac{3d^4}{2e^5x^{2/3}}+\frac{3d^5}{e^6\sqrt[3]{x}}-\frac{3d^6\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^7}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*e*n*(-1/2*1/(e*x^2) + (3*d)/(5*e^2*x^(5/3)) - (3*d^2)/(4*e
^3*x^(4/3)) + d^3/(e^4*x) - (3*d^4)/(2*e^5*x^(2/3)) + (3*d^5)/(e^6*x^(1/3)
) - (3*d^6*Log[d + e/x^(1/3)])/e^7))/6 - (b*Log[c*(d + e/x^(1/3))^n])/(2*x
^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$-3\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^{5/3}}d\frac{1}{\sqrt[3]{x}}$$

$\Big\downarrow$ 2842

$$-3\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{6x^2}-\frac{1}{6}ben\int \frac{1}{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^2}d\frac{1}{\sqrt[3]{x}}\right)$$

$\Big\downarrow$ 49

$$-3\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{6x^2}-\frac{1}{6}ben\int \left(\frac{d^6}{e^6\left(d+\frac{e}{\sqrt[3]{x}}\right)}-\frac{d^5}{e^6}+\frac{d^4}{e^5\sqrt[3]{x}}-\frac{d^3}{e^4x^{2/3}}+\frac{d^2}{e^3x}-\frac{d}{e^2x^{4/3}}+\frac{1}{ex^{5/3}}\right)$$

$\Big\downarrow$ 2009

$$-3\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{6x^2}-\frac{1}{6}ben\left(\frac{d^6\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^7}-\frac{d^5}{e^6\sqrt[3]{x}}+\frac{d^4}{2e^5x^{2/3}}-\frac{d^3}{3e^4x}+\frac{d^2}{4e^3x^{4/3}}-\frac{d}{5e^2x^{5/3}}+\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
```

output
```
-3*(-1/6*(b*e*n*(1/(6*e*x^2) - d/(5*e^2*x^(5/3)) + d^2/(4*e^3*x^(4/3)) - d
^3/(3*e^4*x) + d^4/(2*e^5*x^(2/3)) - d^5/(e^6*x^(1/3)) + (d^6*Log[d + e/x^
(1/3)])/e^7)) + (a + b*Log[c*(d + e/x^(1/3))^n])/(6*x^2))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x^{3}}\,dx = -\frac{20\,bd^{3}e^{3}nx-10\,be^{6}n+60\,ae^{6}-10\,(6\,ae^{6}+(2\,bd^{3}e^{3}-be^{6})n)x^{2}-60\,(be^{6}x^{2}-be^{6})\log(c)-60\,(bd^{6}nx}{120\,e^{6}x^{2}}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="fricas")
```

output
```
-1/120*(20*b*d^3*e^3*n*x - 10*b*e^6*n + 60*a*e^6 - 10*(6*a*e^6 + (2*b*d^3*
e^3 - b*e^6)*n)*x^2 - 60*(b*e^6*x^2 - b*e^6)*log(c) - 60*(b*d^6*n*x^2 - b*
e^6*n)*log((d*x + e*x^(2/3))/x) + 15*(4*b*d^5*e*n*x - b*d^2*e^4*n)*x^(2/3)
 - 6*(5*b*d^4*e^2*n*x - 2*b*d*e^5*n)*x^(1/3))/(e^6*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = \frac{1}{120}ben\left(\frac{60\,d^6\log\left(dx^{\frac{1}{3}}+e\right)}{e^7}-\frac{20\,d^6\log\left(x\right)}{e^7}-\frac{60\,d^5x^{\frac{5}{3}}-30\,d^4ex^{\frac{4}{3}}+20\,d^3e^2x-15\,d^2e^3x^{\frac{2}{3}}+12\,de^4x^{\frac{1}{3}}}{e^6x^2}\right.$$
$$-\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="maxima")
```

output
```
1/120*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x
^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x
^(1/3) - 10*e^5)/(e^6*x^2)) - 1/2*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a/x
^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx$$
$$=\frac{1}{120}\left(e\left(\frac{60\,d^6\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{e^7}-\frac{20\,d^6\log\left(|x|\right)}{e^7}-\frac{60\,d^5ex^{\frac{5}{3}}-30\,d^4e^2x^{\frac{4}{3}}+20\,d^3e^3x-15\,d^2e^4x^{\frac{2}{3}}+12\,de}{e^7x^2}\right.\right.$$
$$-\frac{b\log\left(c\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="giac")
```

output
```
1/120*(e*(60*d^6*log(abs(d*x^(1/3) + e))/e^7 - 20*d^6*log(abs(x))/e^7 - (6
0*d^5*e*x^(5/3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x - 15*d^2*e^4*x^(2/3) +
 12*d*e^5*x^(1/3) - 10*e^6)/(e^7*x^2)) - 60*log(d + e/x^(1/3))/x^2)*b*n -
1/2*b*log(c)/x^2 - 1/2*a/x^2
```

Mupad [B] (verification not implemented)

Time = 25.53 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx=\frac{b\,n}{12\,x^2}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{2\,x^2}-\frac{b\,d\,n}{10\,e\,x^{5/3}}+\frac{b\,d^6\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{2\,e^6}-\frac{b\,d^3\,n}{6\,e^3\,x}+\frac{b\,d^2\,n}{8\,e^2\,x^{4/3}}+\frac{b\,d^4\,n}{4\,e^4\,x^{2/3}}-\frac{b\,d^5\,n}{2\,e^5\,x^{1/3}}$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))/x^3,x)
```

output
```
(b*n)/(12*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(1/3))^n))/(2*x^2) - (b*d*n
)/(10*e*x^(5/3)) + (b*d^6*n*log(d + e/x^(1/3)))/(2*e^6) - (b*d^3*n)/(6*e^3
*x) + (b*d^2*n)/(8*e^2*x^(4/3)) + (b*d^4*n)/(4*e^4*x^(2/3)) - (b*d^5*n)/(2
*e^5*x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.97

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = \frac{-60x^{\frac{5}{3}}b\,d^5en+15x^{\frac{2}{3}}b\,d^2e^4n+30x^{\frac{4}{3}}b\,d^4e^2n-12x^{\frac{1}{3}}bd\,e^5n+60\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^nc}{x^{\frac{n}{3}}}\right)b\,d^6x^2-60\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n}{x^{\frac{n}{3}}}\right.}{120e^6x^2}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x)
```

output
```
( - 60*x**(2/3)*b*d**5*e*n*x + 15*x**(2/3)*b*d**2*e**4*n + 30*x**(1/3)*b*d
**4*e**2*n*x - 12*x**(1/3)*b*d*e**5*n + 60*log(((x**(1/3)*d + e)**n*c)/x**
(n/3))*b*d**6*x**2 - 60*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b*e**6 - 60*
a*e**6 - 20*b*d**3*e**3*n*x + 10*b*e**6*n)/(120*e**6*x**2)
```

3.496 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 187

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = \frac{bn}{27x^3}-\frac{bdn}{24ex^{8/3}}+\frac{bd^2n}{21e^2x^{7/3}}-\frac{bd^3n}{18e^3x^2}+\frac{bd^4n}{15e^4x^{5/3}}-\frac{bd^5n}{12e^5x^{4/3}}+\frac{bd^6n}{9e^6x}-\frac{bd^7n}{6e^7x^{2/3}}+\frac{bd^8n}{3e^8\sqrt[3]{x}}-\frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}$$

output
```
1/27*b*n/x^3-1/24*b*d*n/e/x^(8/3)+1/21*b*d^2*n/e^2/x^(7/3)-1/18*b*d^3*n/e^
3/x^2+1/15*b*d^4*n/e^4/x^(5/3)-1/12*b*d^5*n/e^5/x^(4/3)+1/9*b*d^6*n/e^6/x-
1/6*b*d^7*n/e^7/x^(2/3)+1/3*b*d^8*n/e^8/x^(1/3)-1/3*b*d^9*n*ln(d+e/x^(1/3)
)/e^9-1/3*(a+b*ln(c*(d+e/x^(1/3))^n))/x^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{1}{9}ben\left(-\frac{1}{3ex^3}+\frac{3d}{8e^2x^{8/3}}-\frac{3d^2}{7e^3x^{7/3}}+\frac{d^3}{2e^4x^2}-\frac{3d^4}{5e^5x^{5/3}}+\frac{3d^5}{4e^6x^{4/3}}-\frac{d^6}{e^7x}+\frac{3d^7}{2e^8x^{2/3}}-\frac{3d^8}{e^9\sqrt[3]{x}}+\frac{3d^9\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^{10}}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*e*n*(-1/3*1/(e*x^3) + (3*d)/(8*e^2*x^(8/3)) - (3*d^2)/(7*e
^3*x^(7/3)) + d^3/(2*e^4*x^2) - (3*d^4)/(5*e^5*x^(5/3)) + (3*d^5)/(4*e^6*x
^(4/3)) - d^6/(e^7*x) + (3*d^7)/(2*e^8*x^(2/3)) - (3*d^8)/(e^9*x^(1/3)) +
(3*d^9*Log[d + e/x^(1/3)])/e^10))/9 - (b*Log[c*(d + e/x^(1/3))^n])/(3*x^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^{8/3}}d\frac{1}{\sqrt[3]{x}}$$

$\Big\downarrow$ 2842

$$-3\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{9x^3}-\frac{1}{9}ben\int \frac{1}{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^3}d\frac{1}{\sqrt[3]{x}}\right)$$

$\Big\downarrow$ 49

$$-3\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{9x^3}-\frac{1}{9}ben\int \left(-\frac{d^9}{e^9\left(d+\frac{e}{\sqrt[3]{x}}\right)}+\frac{d^8}{e^9}-\frac{d^7}{e^8\sqrt[3]{x}}+\frac{d^6}{e^7x^{2/3}}-\frac{d^5}{e^6x}+\frac{d^4}{e^5x^{4/3}}-\frac{d^3}{e^4x^{5/3}}\right.\right.$$

$\Big\downarrow$ 2009

$$-3\left(\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{9x^3}-\frac{1}{9}ben\left(-\frac{d^9\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^{10}}+\frac{d^8}{e^9\sqrt[3]{x}}-\frac{d^7}{2e^8x^{2/3}}+\frac{d^6}{3e^7x}-\frac{d^5}{4e^6x^{4/3}}+\frac{d^4}{5e^5x^{5/3}}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
```

output
```
-3*(-1/9*(b*e*n*(1/(9*e*x^3) - d/(8*e^2*x^(8/3)) + d^2/(7*e^3*x^(7/3)) - d
^3/(6*e^4*x^2) + d^4/(5*e^5*x^(5/3)) - d^5/(4*e^6*x^(4/3)) + d^6/(3*e^7*x)
 - d^7/(2*e^8*x^(2/3)) + d^8/(e^9*x^(1/3)) - (d^9*Log[d + e/x^(1/3)])/e^10
)) + (a + b*Log[c*(d + e/x^(1/3))^n])/(9*x^3))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.14

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x^{4}}\,dx = \frac{840\,bd^{6}e^{3}nx^{2}-420\,bd^{3}e^{6}nx+280\,be^{9}n-2520\,ae^{9}+140\,(18\,ae^{9}-(6\,bd^{6}e^{3}-3\,bd^{3}e^{6}+2\,be^{9})n)x^{3}+252}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="fricas")
```

output
```
1/7560*(840*b*d^6*e^3*n*x^2 - 420*b*d^3*e^6*n*x + 280*b*e^9*n - 2520*a*e^9
 + 140*(18*a*e^9 - (6*b*d^6*e^3 - 3*b*d^3*e^6 + 2*b*e^9)*n)*x^3 + 2520*(b*
e^9*x^3 - b*e^9)*log(c) - 2520*(b*d^9*n*x^3 + b*e^9*n)*log((d*x + e*x^(2/3
))/x) + 90*(28*b*d^8*e*n*x^2 - 7*b*d^5*e^4*n*x + 4*b*d^2*e^7*n)*x^(2/3) -
63*(20*b*d^7*e^2*n*x^2 - 8*b*d^4*e^5*n*x + 5*b*d*e^8*n)*x^(1/3))/(e^9*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.80

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx= -\frac{1}{7560}\,ben\left(\frac{2520\,d^9\log\left(dx^{\frac{1}{3}}+e\right)}{e^{10}}-\frac{840\,d^9\log\left(x\right)}{e^{10}}-\frac{2520\,d^8x^{\frac{8}{3}}-1260\,d^7ex^{\frac{7}{3}}+840\,d^6e^2x^2-630\,d^5e^3\right. - \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="maxima")
```

output
```
-1/7560*b*e*n*(2520*d^9*log(d*x^(1/3) + e)/e^10 - 840*d^9*log(x)/e^10 - (2
520*d^8*x^(8/3) - 1260*d^7*e*x^(7/3) + 840*d^6*e^2*x^2 - 630*d^5*e^3*x^(5/
3) + 504*d^4*e^4*x^(4/3) - 420*d^3*e^5*x + 360*d^2*e^6*x^(2/3) - 315*d*e^7
*x^(1/3) + 280*e^8)/(e^9*x^3)) - 1/3*b*log(c*(d + e/x^(1/3))^n)/x^3 - 1/3*
a/x^3
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx =$$
$$-\frac{1}{7560}\left(e\left(\frac{2520\,d^9\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{e^{10}}-\frac{840\,d^9\log\left(|x|\right)}{e^{10}}-\frac{2520\,d^8ex^{\frac{8}{3}}-1260\,d^7e^2x^{\frac{7}{3}}+840\,d^6e^3x^2-630\right.\right.$$
$$-\frac{b\log\left(c\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="giac")
```

output
```
-1/7560*(e*(2520*d^9*log(abs(d*x^(1/3) + e))/e^10 - 840*d^9*log(abs(x))/e^
10 - (2520*d^8*e*x^(8/3) - 1260*d^7*e^2*x^(7/3) + 840*d^6*e^3*x^2 - 630*d^
5*e^4*x^(5/3) + 504*d^4*e^5*x^(4/3) - 420*d^3*e^6*x + 360*d^2*e^7*x^(2/3)
- 315*d*e^8*x^(1/3) + 280*e^9)/(e^10*x^3)) + 2520*log(d + e/x^(1/3))/x^3)*
b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

Mupad [B] (verification not implemented)

Time = 25.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.81

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = \frac{b\,n}{27\,x^3}-\frac{a}{3\,x^3}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{3\,x^3}-\frac{b\,d\,n}{24\,e\,x^{8/3}}-\frac{b\,d^9\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{3\,e^9}-\frac{b\,d^3\,n}{18\,e^3\,x^2}+\frac{b\,d^6\,n}{9\,e^6\,x}+\frac{b\,d^2\,n}{21\,e^2\,x^{7/3}}+\frac{b\,d^4\,n}{15\,e^4\,x^{5/3}}-\frac{b\,d^5\,n}{12\,e^5\,x^{4/3}}-\frac{b\,d^7\,n}{6\,e^7\,x^{2/3}}+\frac{b\,d^8\,n}{3\,e^8\,x^{1/3}}$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))/x^4,x)
```

output
```
(b*n)/(27*x^3) - a/(3*x^3) - (b*log(c*(d + e/x^(1/3))^n))/(3*x^3) - (b*d*n
)/(24*e*x^(8/3)) - (b*d^9*n*log(d + e/x^(1/3)))/(3*e^9) - (b*d^3*n)/(18*e^
3*x^2) + (b*d^6*n)/(9*e^6*x) + (b*d^2*n)/(21*e^2*x^(7/3)) + (b*d^4*n)/(15*
e^4*x^(5/3)) - (b*d^5*n)/(12*e^5*x^(4/3)) - (b*d^7*n)/(6*e^7*x^(2/3)) + (b
*d^8*n)/(3*e^8*x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx$$

$$= \frac{2520x^{\frac{8}{3}}b\,d^8en-630x^{\frac{5}{3}}b\,d^5e^4n+360x^{\frac{2}{3}}b\,d^2e^7n-1260x^{\frac{7}{3}}b\,d^7e^2n+504x^{\frac{4}{3}}b\,d^4e^5n-315x^{\frac{1}{3}}bd\,e^8n-2520\,\mathrm{lo}}{7560}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x)
```

output
```
(2520*x**(2/3)*b*d**8*e*n*x**2 - 630*x**(2/3)*b*d**5*e**4*n*x + 360*x**(2/
3)*b*d**2*e**7*n - 1260*x**(1/3)*b*d**7*e**2*n*x**2 + 504*x**(1/3)*b*d**4*
e**5*n*x - 315*x**(1/3)*b*d*e**8*n - 2520*log(((x**(1/3)*d + e)**n*c)/x**(
n/3))*b*d**9*x**3 - 2520*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b*e**9 - 25
20*a*e**9 + 840*b*d**6*e**3*n*x**2 - 420*b*d**3*e**6*n*x + 280*b*e**9*n)/(
7560*e**9*x**3)
```

3.497 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 572

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = \frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8} - \frac{341b^2e^7n^2x^{2/3}}{840d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} + \frac{73b^2e^4n^2x^{5/3}}{1260d^4} - \frac{5b^2e^3n^2x^2}{168d^3} + \frac{b^2e^2n^2x^{7/3}}{84d^2} - \frac{481b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{420d^9} - \frac{2be^8n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9} + \frac{be^7nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^7} - \frac{2be^6nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^6} + \frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5} - \frac{2be^4nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{15d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3} - \frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} - \frac{2be^9n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{761b^2e^9n^2\log(x)}{1260d^9} + \frac{2b^2e^9n^2\,\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{3d^9}$$

output
```
481/420*b^2*e^8*n^2*x^(1/3)/d^8-341/840*b^2*e^7*n^2*x^(2/3)/d^7+743/3780*b
^2*e^6*n^2*x/d^6-533/5040*b^2*e^5*n^2*x^(4/3)/d^5+73/1260*b^2*e^4*n^2*x^(5
/3)/d^4-5/168*b^2*e^3*n^2*x^2/d^3+1/84*b^2*e^2*n^2*x^(7/3)/d^2-481/420*b^2
*e^9*n^2*ln(d+e/x^(1/3))/d^9-2/3*b*e^8*n*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(
d+e/x^(1/3))^n))/d^9+1/3*b*e^7*n*x^(2/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^7-2
/9*b*e^6*n*x*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+1/6*b*e^5*n*x^(4/3)*(a+b*ln(c
*(d+e/x^(1/3))^n))/d^5-2/15*b*e^4*n*x^(5/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^
4+1/9*b*e^3*n*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3-2/21*b*e^2*n*x^(7/3)*(a+
b*ln(c*(d+e/x^(1/3))^n))/d^2+1/12*b*e*n*x^(8/3)*(a+b*ln(c*(d+e/x^(1/3))^n)
)/d-2/3*b*e^9*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^9+1/3*
x^3*(a+b*ln(c*(d+e/x^(1/3))^n))^2-761/1260*b^2*e^9*n^2*ln(x)/d^9+2/3*b^2*e
^9*n^2*polylog(2,d/(d+e/x^(1/3)))/d^9
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.05

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\frac{1}{3}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\right.$$
$$+\frac{ben\left(-10080ade^7\sqrt[3]{x}+17316bde^7n\sqrt[3]{x}+5040ad^2e^6x^{2/3}-6138bd^2e^6nx^{2/3}-3360ad^3e^5x+2972bd^3e^5\cdots\right.}{\cdots}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

output
```
(x^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2 + (b*e*n*(-10080*a*d*e^7*x^(1/3) +
 17316*b*d*e^7*n*x^(1/3) + 5040*a*d^2*e^6*x^(2/3) - 6138*b*d^2*e^6*n*x^(2/
3) - 3360*a*d^3*e^5*x + 2972*b*d^3*e^5*n*x + 2520*a*d^4*e^4*x^(4/3) - 1599
*b*d^4*e^4*n*x^(4/3) - 2016*a*d^5*e^3*x^(5/3) + 876*b*d^5*e^3*n*x^(5/3) +
1680*a*d^6*e^2*x^2 - 450*b*d^6*e^2*n*x^2 - 1440*a*d^7*e*x^(7/3) + 180*b*d^
7*e*n*x^(7/3) + 1260*a*d^8*x^(8/3) - 22356*b*e^8*n*Log[d + e/x^(1/3)] - 10
080*b*d*e^7*x^(1/3)*Log[c*(d + e/x^(1/3))^n] + 5040*b*d^2*e^6*x^(2/3)*Log[
c*(d + e/x^(1/3))^n] - 3360*b*d^3*e^5*x*Log[c*(d + e/x^(1/3))^n] + 2520*b*
d^4*e^4*x^(4/3)*Log[c*(d + e/x^(1/3))^n] - 2016*b*d^5*e^3*x^(5/3)*Log[c*(d
 + e/x^(1/3))^n] + 1680*b*d^6*e^2*x^2*Log[c*(d + e/x^(1/3))^n] - 1440*b*d^
7*e*x^(7/3)*Log[c*(d + e/x^(1/3))^n] + 1260*b*d^8*x^(8/3)*Log[c*(d + e/x^(
1/3))^n] + 10080*a*e^8*Log[e + d*x^(1/3)] - 5040*b*e^8*n*Log[e + d*x^(1/3)
] + 10080*b*e^8*Log[c*(d + e/x^(1/3))^n]*Log[e + d*x^(1/3)] - 5040*b*e^8*n
*Log[e + d*x^(1/3)]^2 + 10080*b*e^8*n*Log[e + d*x^(1/3)]*Log[-((d*x^(1/3))
/e)] - 7452*b*e^8*n*Log[x] + 10080*b*e^8*n*PolyLog[2, 1 + (d*x^(1/3))/e]))
/(5040*d^9))/3
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int x^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2845}$$

$$-3\left(\frac{2}{9}ben\int \frac{x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d+\frac{e}{\sqrt[3]{x}}}d\frac{1}{\sqrt[3]{x}}-\frac{1}{9}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2858}$$

$$-3\left(\frac{2}{9}bn\int x^{10/3}\left(a+b\log\left(cx^{-n/3}\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{1}{9}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \; 25$$

$$-3\left(-\frac{2}{9}bn\int -x^{10/3}\left(a+b\log\left(cx^{-n/3}\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{1}{9}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \; 27$$

$$-3\left(-\frac{2}{9}be^9n\int -\frac{x^{10/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^9}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{1}{9}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \; 2789$$

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\int -\frac{x^3\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^9}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^3\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^8}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}\right)-\frac{1}{9}x^3\left(a+b\log\left(c\left(d-\right.\right.\right.\right.$$

$$\Big\downarrow \; 2756$$

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{8e^8}-\frac{1}{8}bn\int \frac{x^3}{e^8}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^3\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^8}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}\right)-\frac{1}{9}x^3\left(a+b\,\mathrm{lo}\right.\right.$$

$$\Big\downarrow \; 54$$

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{8e^8}-\frac{1}{8}bn\int \left(\frac{x^{8/3}}{de^8}-\frac{x^{7/3}}{d^2e^7}+\frac{x^2}{d^3e^6}-\frac{x^{5/3}}{d^4e^5}+\frac{x^{4/3}}{d^5e^4}-\frac{x}{d^6e^3}+\frac{x^{2/3}}{d^7e^2}-\frac{\sqrt[3]{x}}{d^8e}+\frac{\sqrt[3]{x}}{d^8}\right)d\left(d\right.}{d}\right.\right.$$

$$\Big\downarrow \; 2009$$

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\int \frac{x^3\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^8}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{8e^8}-\frac{1}{8}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^8}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^8}-\frac{\sqrt[3]{x}}{d^7e}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 2789$$

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\int \frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^8}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^7}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\frac{x^{8/3}(a+b\log(cx^{-n/3}))}{8e^8}-\frac{1}{8}bn\Bigg(\right.\right.$$

$\downarrow$ 2756

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{-\frac{1}{7}bn\int -\frac{x^{8/3}}{e^7}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{7e^7}}{d}+\frac{\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^7}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\frac{x^{8/3}(a+b\log(cx^{-n/}}{8e^8}\right.\right.$$

$\downarrow$ 54

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{-\frac{1}{7}bn\int\left(-\frac{x^{7/3}}{de^7}+\frac{x^2}{d^2e^6}-\frac{x^{5/3}}{d^3e^5}+\frac{x^{4/3}}{d^4e^4}-\frac{x}{d^5e^3}+\frac{x^{2/3}}{d^6e^2}-\frac{\sqrt[3]{x}}{d^7e}+\frac{\sqrt[3]{x}}{d^7}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{7e^7}}{d}+\frac{\int -\frac{x^{8/3}\left(a+b\right.}{}}{}}{d}\right.\right.$$

$\downarrow$ 2009

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^7}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{-\frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{7e^7}-\frac{1}{7}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^7}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^7}-\frac{\sqrt[3]{x}}{d^6e}+\frac{x^{2/3}}{2d^5e^2}-\right.}{d}}{d}\right.\right.$$

$\downarrow$ 2789

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\int -\frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^7}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^6}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{7e^7}-\frac{1}{7}bn\left(\frac{\log\left(d+\right.}{d}\right.}{d}\right.\right.$$

$$\Big\downarrow \text{ 2756}$$

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\int \frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^6}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{6e^6}-\frac{1}{6}bn\int \frac{x^{7/3}}{e^6}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{7e^7}-\frac{1}{7}bn\left(\right.}{d}\right.\right.$$

$$\Big\downarrow \text{ 54}$$

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{6e^6}-\frac{1}{6}bn\int \left(\frac{x^2}{de^6}-\frac{x^{5/3}}{d^2e^5}+\frac{x^{4/3}}{d^3e^4}-\frac{x}{d^4e^3}+\frac{x^{2/3}}{d^5e^2}-\frac{\sqrt[3]{x}}{d^6e}+\frac{\sqrt[3]{x}}{d^6}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^6}d\left(d+\frac{e}{\sqrt[3]{\right.}}\right)}{d}}{d}\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\int \frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^6}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{6e^6}-\frac{1}{6}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^6}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^6}-\frac{\sqrt[3]{x}}{d^5e}+\frac{x^{2/3}}{2d^4e^2}-\frac{x}{3d^3e^3}+\frac{x}{4d}\right.}{d}}{d}\right.\right.$$

$$\Big\downarrow \text{ 2789}$$

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\frac{\int \frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^6}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{6e^6}-\frac{1}{6}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^6}-\ldots\right.}{\ldots}}{d}\right.\right.$$

$\downarrow$ 2756

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\frac{-\frac{1}{5}bn\int -\frac{x^2}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}}{d}}{d}+\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{6e^6}-\frac{1}{6}bn\left(\log\left(\ldots\right.\right.}{\ldots}}{d}\right.\right.$$

$\downarrow$ 54

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\frac{-\frac{1}{5}bn\int\left(-\frac{x^{5/3}}{de^5}+\frac{x^{4/3}}{d^2e^4}-\frac{x}{d^3e^3}+\frac{x^{2/3}}{d^4e^2}-\frac{\sqrt[3]{x}}{d^5e}+\frac{\sqrt[3]{x}}{d^5}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}}{d}}{d}+\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{\ldots}}\right.}{d}}{d}\right.\right.$$

$\downarrow$ 2009

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}-\frac{1}{5}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^5}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^5}-\frac{\sqrt[3]{x}}{d^4e}+\frac{x^{2/3}}{2d^3e^2}-\frac{x}{3d^2e^3}\right.}{d}}{d}}{d}\right.\right.$$

$\Big\downarrow$ 2789

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\frac{\frac{\int -\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}-\frac{1}{5}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^5}\right.}{d}}{}}{}\right.\right.$$

$\Big\downarrow$ 2756

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\frac{\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\int \frac{x^{5/3}}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}-\frac{1}{5}bn\left(\right.}{d}}{}}{}\right.\right.$$

$\Big\downarrow$ 54

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\frac{\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\int\left(\frac{x^{4/3}}{de^4}-\frac{x}{d^2e^3}+\frac{x^{2/3}}{d^3e^2}-\frac{\sqrt[3]{x}}{d^4e}+\frac{\sqrt[3]{x}}{d^4}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\cdots}{d}}{\cdots}\right.\right.$$

$\downarrow$ 2009

$$-3\left(-\frac{2}{9}be^9n\left(\frac{\frac{\frac{\frac{\int\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\sqrt[3]{x}}{d^3e}+\frac{x^{2/3}}{2d^2e^2}-\frac{x}{3de^3}\right)}{d}}{d}\cdots}{d}}{\cdots}\right.\right.$$

$\downarrow$ 2789

$$-3\left(-\frac{2}{9}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{8e^8}-\frac{1}{8}bn\left(-\frac{x^{7/3}}{7de^7}+\frac{x^2}{6d^2e^6}-\frac{x^{5/3}}{5d^3e^5}+\frac{x^{4/3}}{4d^4e^4}-\frac{x}{3d^5e^3}+\frac{x^{2/3}}{2d^6e^2}-\frac{\sqrt[3]{x}}{d^7e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^8}-\cdots\right.}{d}\right.\right.$$

$\downarrow$ 2756

$$-3\left(-\frac{2}{9}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{8e^8}-\frac{1}{8}bn\left(-\frac{x^{7/3}}{7de^7}+\frac{x^2}{6d^2e^6}-\frac{x^{5/3}}{5d^3e^5}+\frac{x^{4/3}}{4d^4e^4}-\frac{x}{3d^5e^3}+\frac{x^{2/3}}{2d^6e^2}-\frac{\sqrt[3]{x}}{d^7e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^8}-\right.}{d}\right.\right.$$

$\downarrow$ 54

$$-3\left(-\frac{2}{9}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{8e^8}-\frac{1}{8}bn\left(-\frac{x^{7/3}}{7de^7}+\frac{x^2}{6d^2e^6}-\frac{x^{5/3}}{5d^3e^5}+\frac{x^{4/3}}{4d^4e^4}-\frac{x}{3d^5e^3}+\frac{x^{2/3}}{2d^6e^2}-\frac{\sqrt[3]{x}}{d^7e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^8}-\right.}{d}\right.\right.$$

$\downarrow$ 2009

$$-3\left(-\frac{2}{9}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{8e^8}-\frac{1}{8}bn\left(-\frac{x^{7/3}}{7de^7}+\frac{x^2}{6d^2e^6}-\frac{x^{5/3}}{5d^3e^5}+\frac{x^{4/3}}{4d^4e^4}-\frac{x}{3d^5e^3}+\frac{x^{2/3}}{2d^6e^2}-\frac{\sqrt[3]{x}}{d^7e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^8}-\right.}{d}\right.\right.$$

$\downarrow$ 2789

$$-3\left(-\frac{2}{9}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{8e^8}-\frac{1}{8}bn\left(-\frac{x^{7/3}}{7de^7}+\frac{x^2}{6d^2e^6}-\frac{x^{5/3}}{5d^3e^5}+\frac{x^{4/3}}{4d^4e^4}-\frac{x}{3d^5e^3}+\frac{x^{2/3}}{2d^6e^2}-\frac{\sqrt[3]{x}}{d^7e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^8}-\cdots\right.}{d}\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)^2dx$$

input
```
int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Fricas [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x
+ e*x^(2/3))/x)^n) + a^2*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3*log((d*x^(1/3) + e)^n)^2 - integrate(-1/9*(9*(b^2*d*log(c)^2 +
 2*a*b*d*log(c) + a^2*d)*x^3 + 9*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)
*x^(8/3) + 9*(b^2*d*x^3 + b^2*e*x^(8/3))*log(x^(1/3*n))^2 - 2*(b^2*d*n*x^3
 - 9*(b^2*d*log(c) + a*b*d)*x^3 - 9*(b^2*e*log(c) + a*b*e)*x^(8/3) + 9*(b^
2*d*x^3 + b^2*e*x^(8/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 18*((b^2
*d*log(c) + a*b*d)*x^3 + (b^2*e*log(c) + a*b*e)*x^(8/3))*log(x^(1/3*n)))/(
d*x + e*x^(2/3)), x)
```

Giac [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int x^2\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e/x^(1/3))^n))^2,x)
```

output
```
int(x^2*(a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

Reduce [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x)
```

output
```
(1260*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**8*e*n*x**2 -
2016*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**5*e**4*n*x + 5
040*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**2*e**7*n + 1260
*x**(2/3)*a*b*d**8*e*n*x**2 - 2016*x**(2/3)*a*b*d**5*e**4*n*x + 5040*x**(2
/3)*a*b*d**2*e**7*n + 876*x**(2/3)*b**2*d**5*e**4*n**2*x - 6138*x**(2/3)*b
**2*d**2*e**7*n**2 - 5040*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**
2*b**2*d*e**8 - 1440*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d
**7*e**2*n*x**2 + 2520*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2
*d**4*e**5*n*x - 10080*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2
*d*e**8*n - 1440*x**(1/3)*a*b*d**7*e**2*n*x**2 + 2520*x**(1/3)*a*b*d**4*e*
*5*n*x - 10080*x**(1/3)*a*b*d*e**8*n + 180*x**(1/3)*b**2*d**7*e**2*n**2*x*
*2 - 1599*x**(1/3)*b**2*d**4*e**5*n**2*x + 17316*x**(1/3)*b**2*d*e**8*n**2
 + 1680*int(log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2/x**(2/3),x)*b**2*d*e*
*8 + 10080*log(x**(1/3))*a*b*e**9*n - 27396*log(x**(1/3))*b**2*e**9*n**2 +
 5040*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**2*d**9*x**3 + 10080*log(
((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b*d**9*x**3 + 10080*log(((x**(1/3)*d +
 e)**n*c)/x**(n/3))*a*b*e**9 + 1680*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*
b**2*d**6*e**3*n*x**2 - 3360*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d*
*3*e**6*n*x - 27396*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*e**9*n + 50
40*a**2*d**9*x**3 + 1680*a*b*d**6*e**3*n*x**2 - 3360*a*b*d**3*e**6*n*x ...
```

3.498 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 22, antiderivative size = 400

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = -\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6}+\frac{be^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}-\frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4}+\frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^3}-\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2}+\frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d}+\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^6}$$

output
```
-77/60*b^2*e^5*n^2*x^(1/3)/d^5+47/120*b^2*e^4*n^2*x^(2/3)/d^4-3/20*b^2*e^3
*n^2*x/d^3+1/20*b^2*e^2*n^2*x^(4/3)/d^2+77/60*b^2*e^6*n^2*ln(d+e/x^(1/3))/
d^6+b*e^5*n*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6-1/2*b*e^
4*n*x^(2/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^4+1/3*b*e^3*n*x*(a+b*ln(c*(d+e/x
^(1/3))^n))/d^3-1/4*b*e^2*n*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^2+1/5*b*
e*n*x^(5/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d+b*e^6*n*ln(1-d/(d+e/x^(1/3)))*(a
+b*ln(c*(d+e/x^(1/3))^n))/d^6+1/2*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2+137/18
0*b^2*e^6*n^2*ln(x)/d^6-b^2*e^6*n^2*polylog(2,d/(d+e/x^(1/3)))/d^6
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2$$
$$+\frac{ben\left(360ade^4\sqrt[3]{x}-462bde^4n\sqrt[3]{x}-180ad^2e^3x^{2/3}+141bd^2e^3nx^{2/3}+120ad^3e^2x-54bd^3e^2nx-90ad^4e\right.}{}$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

output
```
(x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/2 + (b*e*n*(360*a*d*e^4*x^(1/3) -
 462*b*d*e^4*n*x^(1/3) - 180*a*d^2*e^3*x^(2/3) + 141*b*d^2*e^3*n*x^(2/3) +
 120*a*d^3*e^2*x - 54*b*d^3*e^2*n*x - 90*a*d^4*e*x^(4/3) + 18*b*d^4*e*n*x^
(4/3) + 72*a*d^5*x^(5/3) + 642*b*e^5*n*Log[d + e/x^(1/3)] + 360*b*d*e^4*x^
(1/3)*Log[c*(d + e/x^(1/3))^n] - 180*b*d^2*e^3*x^(2/3)*Log[c*(d + e/x^(1/3
))^n] + 120*b*d^3*e^2*x*Log[c*(d + e/x^(1/3))^n] - 90*b*d^4*e*x^(4/3)*Log[
c*(d + e/x^(1/3))^n] + 72*b*d^5*x^(5/3)*Log[c*(d + e/x^(1/3))^n] - 360*a*e
^5*Log[e + d*x^(1/3)] + 180*b*e^5*n*Log[e + d*x^(1/3)] - 360*b*e^5*Log[c*(
d + e/x^(1/3))^n]*Log[e + d*x^(1/3)] + 180*b*e^5*n*Log[e + d*x^(1/3)]^2 -
360*b*e^5*n*Log[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] + 214*b*e^5*n*Log[x]
- 360*b*e^5*n*PolyLog[2, 1 + (d*x^(1/3))/e]))/(360*d^6)
```

Rubi [A] (warning: unable to verify)

Time = 3.75 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.40, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.136, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx$$

$$\Big\downarrow \text{ 2904}$$

$$-3\int x^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{ 2845}$$

$$-3\left(\frac{1}{3}ben\int\frac{x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d+\frac{e}{\sqrt[3]{x}}}d\frac{1}{\sqrt[3]{x}}-\frac{1}{6}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{ 2858}$$

$$-3\left(\frac{1}{3}bn\int x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{1}{6}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$-3\left(\frac{1}{3}be^6n\int\frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^6}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{1}{6}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{ 2789}$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\int\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^6}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int-\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}\right)-\frac{1}{6}x^2\left(a+b\log\left(c\left(d+\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 2756}$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int -\frac{x^2}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}}{d}+\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}\right)-\frac{1}{6}x^2\left(a+\right.\right.$$

$\big\downarrow$ 54

$$-3\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int\left(-\frac{x^{5/3}}{de^5}+\frac{x^{4/3}}{d^2e^4}-\frac{x}{d^3e^3}+\frac{x^{2/3}}{d^4e^2}-\frac{\sqrt[3]{x}}{d^5e}+\frac{\sqrt[3]{x}}{d^5}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}}{d}+\frac{\int -\frac{x^2(a}{}}{}\right.\right.$$

$\big\downarrow$ 2009

$$-3\left(\frac{1}{3}be^6n\left(\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}-\frac{1}{5}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^5}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^5}-\frac{\sqrt[3]{}}{d^4}\right.}{d}\right.\right.$$

$\big\downarrow$ 2789

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\int -\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}-\frac{1}{5}bn\left(\right.}{}\right.\right.$$

$\big\downarrow$ 2756

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\int\frac{x^{5/3}}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{5e^5}-}{}\right.\right.$$

$\big\downarrow$ 54

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\int\left(\frac{x^{4/3}}{de^4}-\frac{x}{d^2e^3}+\frac{x^{2/3}}{d^3e^2}-\frac{\sqrt[3]{x}}{d^4e}+\frac{\sqrt[3]{x}}{d^4}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}\right.\right.$$

$\downarrow$ 2009

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\int\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\sqrt[3]{x}}{d^3e}+\frac{x^{2/3}}{2d^2e^2}-\frac{x}{3de^3}\right.}{d}}{d}\right.\right.$$

$\downarrow$ 2789

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int-\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right.}{d}}{d}\right.\right.$$

$\downarrow$ 2756

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int-\frac{x^{4/3}}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}}{d}+\frac{\int-\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\right.}{}}{d}\right.\right.$$

$\downarrow$ 54

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int\left(-\frac{x}{de^3}+\frac{x^{2/3}}{d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\sqrt[3]{x}}{d^3}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}}{d}+\frac{\int-\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{x^{4/3}\left(a+b\cdots\right.}{\cdots}}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2009$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int-\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{\sqrt[3]{x}}{d^2e}+\frac{x^{2/3}}{2de^2}\right)}{d}}{d}+\frac{x^{4/3}\left(\cdots\right.}{\cdots}}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2789$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{\log\left(-\cdots\right.}{\cdots}\right.}{d}}{d}}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2756$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\int\frac{x}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\int\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}\cdots\right.}{d}}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 54$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\int\left(\frac{x^{2/3}}{de^2}-\frac{\sqrt[3]{x}}{d^2e}+\frac{\sqrt[3]{x}}{d^2}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2009$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^2}-\frac{\sqrt[3]{x}}{de}\right)}{d}}{d}+\frac{-\frac{x\left(a+b\log\left(cx^{-n/\cdots}\right)\right)}{3e^3}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2789$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int-\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}}{d}+\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^2}\cdots\right.}{d}\cdots}{d}\cdots\right.\right.$$

$$\Big\downarrow \; 2751$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{-\frac{bn\int -\frac{\sqrt[3]{x}}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)}{de}}{d}+\frac{\int -\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{-n/\cdots}\right)\right)}{2e^2}\cdots}{\cdots}}{d}\cdots}{\cdots}}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \; 16$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{\int -\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)}{de}}{d}}{d}+\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\cdots}{\cdots}}{d}\cdots}{d}\cdots}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \; 2779$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{\frac{bn\int \sqrt[3]{x}\log\left(1-d\sqrt[3]{x}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\log\left(1-d\sqrt[3]{x}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)}{de}}{d}}{d}\cdots}{d}\cdots}{\cdots}}{\cdots}\cdots\right.\right.$$

$$\Big\downarrow \; 2838$$

$$-3\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^2}-\frac{\sqrt[3]{x}}{de}\right)}{d}+\frac{\frac{\frac{bn\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)}{de}}{d}+}{d}}{}}{}\right.\right.$$

input
```
Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

output
```
-3*(-1/6*(x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2) + (b*e^6*n*((-1/5*(b*n*(
-(x^(1/3)/(d^4*e)) + x^(2/3)/(2*d^3*e^2) - x/(3*d^2*e^3) + x^(4/3)/(4*d*e^
4) + Log[d + e/x^(1/3)]/d^5 - Log[-(e/x^(1/3))]/d^5)) - (x^(5/3)*(a + b*Lo
g[c/x^(n/3)]))/(5*e^5))/d + ((-1/4*(b*n*(-(x^(1/3)/(d^3*e)) + x^(2/3)/(2*d
^2*e^2) - x/(3*d*e^3) + Log[d + e/x^(1/3)]/d^4 - Log[-(e/x^(1/3))]/d^4)) +
 (x^(4/3)*(a + b*Log[c/x^(n/3)]))/(4*e^4))/d + ((-1/3*(b*n*(-(x^(1/3)/(d^2
*e)) + x^(2/3)/(2*d*e^2) + Log[d + e/x^(1/3)]/d^3 - Log[-(e/x^(1/3))]/d^3)
) - (x*(a + b*Log[c/x^(n/3)]))/(3*e^3))/d + ((-1/2*(b*n*(-(x^(1/3)/(d*e))
+ Log[d + e/x^(1/3)]/d^2 - Log[-(e/x^(1/3))]/d^2)) + (x^(2/3)*(a + b*Log[c
/x^(n/3)]))/(2*e^2))/d + (((b*n*Log[-(e/x^(1/3))])/d - ((d + e/x^(1/3))*x^
(1/3)*(a + b*Log[c/x^(n/3)]))/(d*e))/d + (-((Log[1 - d*x^(1/3)]*(a + b*Log
[c/x^(n/3)]))/d) + (b*n*PolyLog[2, d*x^(1/3)])/d)/d)/d)/d)/d)/d))/3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*
x^(2/3))/x)^n) + a^2*x, x)
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

output
```
Integral(x*(a + b*log(c*(d + e/x**(1/3))**n))**2, x)
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*log((d*x^(1/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
 2*a*b*d*log(c) + a^2*d)*x^2 + 3*(b^2*d*x^2 + b^2*e*x^(5/3))*log(x^(1/3*n)
)^2 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(5/3) - (b^2*d*n*x^2 -
 6*(b^2*d*log(c) + a*b*d)*x^2 - 6*(b^2*e*log(c) + a*b*e)*x^(5/3) + 6*(b^2*
d*x^2 + b^2*e*x^(5/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*((b^2*d*
log(c) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(5/3))*log(x^(1/3*n)))/(d*x
 + e*x^(2/3)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(1/3))^n))^2,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$$

$$=\frac{24x^{\frac{5}{3}}\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b^2d^5en-60x^{\frac{2}{3}}\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b^2d^2e^4n+24x^{\frac{5}{3}}ab\,d^5en-60x^{\frac{2}{3}}ab\,d^2e^4n+47x^{\frac{2}{3}}b^2d^2}{}$$

input
```
int(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x)
```

output
```
(24*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**5*e*n*x - 60*x*
*(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**2*e**4*n + 24*x**(2/3
)*a*b*d**5*e*n*x - 60*x**(2/3)*a*b*d**2*e**4*n + 47*x**(2/3)*b**2*d**2*e**
4*n**2 + 60*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**2*d*e**5
- 30*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**4*e**2*n*x + 1
20*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d*e**5*n - 30*x**(1
/3)*a*b*d**4*e**2*n*x + 120*x**(1/3)*a*b*d*e**5*n + 6*x**(1/3)*b**2*d**4*e
**2*n**2*x - 154*x**(1/3)*b**2*d*e**5*n**2 - 20*int(log(((x**(1/3)*d + e)*
*n*c)/x**(n/3))**2/x**(2/3),x)*b**2*d*e**5 - 120*log(x**(1/3))*a*b*e**6*n
+ 274*log(x**(1/3))*b**2*e**6*n**2 + 60*log(((x**(1/3)*d + e)**n*c)/x**(n/
3))**2*b**2*d**6*x**2 + 120*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b*d**6
*x**2 - 120*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b*e**6 + 40*log(((x**(
1/3)*d + e)**n*c)/x**(n/3))*b**2*d**3*e**3*n*x + 274*log(((x**(1/3)*d + e)
**n*c)/x**(n/3))*b**2*e**6*n + 60*a**2*d**6*x**2 + 40*a*b*d**3*e**3*n*x -
18*b**2*d**3*e**3*n**2*x)/(120*d**6)
```

3.499 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx &= \frac{b^2e^2n^2\sqrt[3]{x}}{d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}\\ &\quad-\frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}\\ &\quad+\frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d}\\ &\quad-\frac{2be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}\\ &\quad+x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{b^2e^3n^2\log(x)}{d^3}+\frac{2b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}\end{aligned}$$

output
```
b^2*e^2*n^2*x^(1/3)/d^2-b^2*e^3*n^2*ln(d+e/x^(1/3))/d^3-2*b*e^2*n*(d+e/x^(
1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3+b*e*n*x^(2/3)*(a+b*ln(c*(d+e
/x^(1/3))^n))/d-2*b*e^3*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n)
)/d^3+x*(a+b*ln(c*(d+e/x^(1/3))^n))^2-b^2*e^3*n^2*ln(x)/d^3+2*b^2*e^3*n^2*
polylog(2,d/(d+e/x^(1/3)))/d^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.06

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{ben\left(6ade\sqrt[3]{x}+6be^2n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+6bde\sqrt[3]{x}\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-3d^2x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

output
```
x*(a + b*Log[c*(d + e/x^(1/3))^n])^2 - (b*e*n*(6*a*d*e*x^(1/3) + 6*b*e^2*n
*Log[d + e/x^(1/3)] + 6*b*d*e*x^(1/3)*Log[c*(d + e/x^(1/3))^n] - 3*d^2*x^(
2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]) - 6*e^2*(a + b*Log[c*(d + e/x^(1/3))
^n])*Log[e + d*x^(1/3)] + 3*b*e*n*(-(d*x^(1/3)) + e*Log[e + d*x^(1/3)]) +
2*b*e^2*n*Log[x] + 3*b*e^2*n*(Log[e + d*x^(1/3)]*(Log[e + d*x^(1/3)] - 2*L
og[-((d*x^(1/3))/e)]) - 2*PolyLog[2, 1 + (d*x^(1/3))/e])))/(3*d^3)
```

Rubi [A] (warning: unable to verify)

Time = 1.69 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {2901, 2904, 2845, 2858, 25, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$$

$\Big\downarrow$ 2901

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 d\sqrt[3]{x}$$

$\Big\downarrow$ 2904

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^{4/3}}d\frac{1}{\sqrt[3]{x}}$$

$\Big\downarrow$ 2845

$$-3\left(\frac{2}{3}ben\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{\left(d+\frac{e}{\sqrt[3]{x}}\right)x}d\frac{1}{\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{3x}\right)$$

$\Big\downarrow$ 2858

$$-3\left(\frac{2}{3}bn\int \left(d+\frac{e}{\sqrt[3]{x}}\right)x\left(a+b\log\left(cx^{n/3}\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{3x}\right)$$

$\Big\downarrow$ 25

$$-3\left(-\frac{2}{3}bn\int -\left(\left(d+\frac{e}{\sqrt[3]{x}}\right)x\left(a+b\log\left(cx^{n/3}\right)\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{3x}\right)$$

$\Big\downarrow$ 27

$$-3\left(-\frac{2}{3}be^3n\int -\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x\left(a+b\log\left(cx^{n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{3x}\right)$$

$\Big\downarrow$ 2789

$$-3\left(-\frac{2}{3}be^3n\left(\frac{\int -\frac{x\left(a+b\log\left(cx^{n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}\right)-\frac{\left(a+b\log\left(c\right.\right.}{}$$

$\downarrow$ 2756

$$-3\left(-\frac{2}{3}be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\int \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^{2/3}}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right.}{d}\right.\right.$$

$\downarrow$ 54

$$-3\left(-\frac{2}{3}be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\int \left(\frac{d+\frac{e}{\sqrt[3]{x}}}{d^2}+\frac{x^{2/3}}{de^2}-\frac{\sqrt[3]{x}}{d^2e}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{e^2}}{d}\right.\right.$$

$\downarrow$ 2009

$$-3\left(-\frac{2}{3}be^3n\left(\frac{\int \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right.}{d^2}\right.}{d}\right.\right.$$

$\downarrow$ 2789

$$-3\left(-\frac{2}{3}be^3n\left(\frac{\frac{\int \frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int -\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(cx^{n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{2e^2}-\frac{1}{2}}{}\right.\right.$$

$\Big\downarrow$ 2751

$$-3\left(-\frac{2}{3}be^3n\left(\frac{\frac{-\frac{bn\int -\frac{\sqrt[3]{x}}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de}}{d}+\frac{\int -\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(cx^{n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{x^{2/}}{}\right.\right.$$

$\Big\downarrow$ 16

$$-3\left(-\frac{2}{3}be^3n\left(\frac{\frac{\int -\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(cx^{n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de}}{d}}{d}+\frac{\frac{x^{2/3}(a+b\log}{2e^2}}{}\right.\right.$$

$\Big\downarrow$ 2779

$$-3\left(-\frac{2}{3}be^3n\left(\frac{\frac{\frac{bn\int\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(1-d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\log\left(1-d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{}}\right.}{}}{d}}{d}\right.\right.$$

$\Big\downarrow$ 2838

$$-3\left(-\frac{2}{3}be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^2}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^2}-\frac{\sqrt[3]{x}}{de}\right)}{d}+\frac{\frac{bn\,\mathrm{PolyLog}\left(2,d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{d}-\frac{\log\left(1-d\left(\right.\right.}{}}{d}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

output
```
-3*(-1/3*(a + b*Log[c*(d + e/x^(1/3))^n])^2/x - (2*b*e^3*n*((-1/2*(b*n*(-(
x^(1/3)/(d*e)) + Log[d + e/x^(1/3)]/d^2 - Log[-(e/x^(1/3))]/d^2)) + (x^(2/
3)*(a + b*Log[c*x^(n/3)]))/(2*e^2))/d + (((b*n*Log[-(e/x^(1/3))])/d - ((d
+ e/x^(1/3))*x^(1/3)*(a + b*Log[c*x^(n/3)]))/(d*e))/d + (-((Log[1 - d*(d +
 e/x^(1/3))]*(a + b*Log[c*x^(n/3)]))/d) + (b*n*PolyLog[2, d*(d + e/x^(1/3)
)])/d)/d)/d))/3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(2
/3))/x)^n) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e/x**(1/3))**n))**2, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2 dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

output
```
(e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2*x*
log(c*(d + e/x^(1/3))^n))*a*b + (x*log((d*x^(1/3) + e)^n)^2 - integrate(-1
/3*(3*d*x*log(c)^2 + 3*e*x^(2/3)*log(c)^2 + 3*(d*x + e*x^(2/3))*log(x^(1/3
*n))^2 - 2*(d*n*x - 3*d*x*log(c) - 3*e*x^(2/3)*log(c) + 3*(d*x + e*x^(2/3)
)*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*(d*x*log(c) + e*x^(2/3)*log(c
))*log(x^(1/3*n)))/(d*x + e*x^(2/3)), x))*b^2 + a^2*x
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2 dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = \int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2 dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$$

$$= \frac{3x^{\frac{2}{3}}\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b^2d^2en+3x^{\frac{2}{3}}ab\,d^2en-3x^{\frac{1}{3}}\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)^2 b^2d\,e^2-6x^{\frac{1}{3}}\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b^2d\,e^2n-}{}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))^2,x)
```

output
```
(3*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**2*e*n + 3*x**(2/
3)*a*b*d**2*e*n - 3*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**2
*d*e**2 - 6*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d*e**2*n -
 6*x**(1/3)*a*b*d*e**2*n + 3*x**(1/3)*b**2*d*e**2*n**2 + int(log(((x**(1/3
)*d + e)**n*c)/x**(n/3))**2/x**(2/3),x)*b**2*d*e**2 + 6*log(x**(1/3))*a*b*
e**3*n - 9*log(x**(1/3))*b**2*e**3*n**2 + 3*log(((x**(1/3)*d + e)**n*c)/x*
*(n/3))**2*b**2*d**3*x + 6*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b*d**3*
x + 6*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b*e**3 - 9*log(((x**(1/3)*d
+ e)**n*c)/x**(n/3))*b**2*e**3*n + 3*a**2*d**3*x)/(3*d**3)
```

3.500 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt[3]{x}}\right) + 6b^2n^2\operatorname{PolyLog}\left(3,1+\frac{e}{d\sqrt[3]{x}}\right)$$

output
```
-3*(a+b*ln(c*(d+e/x^(1/3))^n))^2*ln(-e/d/x^(1/3))-6*b*n*(a+b*ln(c*(d+e/x^(
1/3))^n))*polylog(2,1+e/d/x^(1/3))+6*b^2*n^2*polylog(3,1+e/d/x^(1/3))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 389 vs. $2(93) = 186$.

Time = 0.24 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.18

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx$$
$$= \left(a-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log(x)+2bn\left(a-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\right.$$
$$\left.+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\left(\left(\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-\log\left(1+\frac{e}{d\sqrt[3]{x}}\right)\right)\log(x)\right.$$
$$\left.+3\operatorname{PolyLog}\left(2,-\frac{e}{d\sqrt[3]{x}}\right)\right)+3b^2n^2\left(2\log\left(\frac{e}{d}+\sqrt[3]{x}\right)\operatorname{PolyLog}\left(2,1+\frac{d\sqrt[3]{x}}{e}\right)\right.$$
$$-2\left(\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-\log\left(\frac{e}{d}+\sqrt[3]{x}\right)\right)\operatorname{PolyLog}\left(2,-\frac{d\sqrt[3]{x}}{e}\right)$$
$$+\frac{1}{81}\left(81\log^2\left(\frac{e}{d}+\sqrt[3]{x}\right)\log\left(-\frac{d\sqrt[3]{x}}{e}\right)+27\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log(x)\right.$$
$$-27\log^2\left(\frac{e}{d}+\sqrt[3]{x}\right)\log(x)-54\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(1+\frac{d\sqrt[3]{x}}{e}\right)\log(x)$$
$$+54\log\left(\frac{e}{d}+\sqrt[3]{x}\right)\log\left(1+\frac{d\sqrt[3]{x}}{e}\right)\log(x)+9\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\log^2(x)$$
$$-9\log\left(1+\frac{d\sqrt[3]{x}}{e}\right)\log^2(x)+\log^3(x)-162\operatorname{PolyLog}\left(3,1+\frac{d\sqrt[3]{x}}{e}\right)$$
$$\left.\left.-162\operatorname{PolyLog}\left(3,-\frac{d\sqrt[3]{x}}{e}\right)\right)\right)$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

output
```
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*Log[x] + 2*b*n
*(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*((Log[d + e/x^(
1/3)] - Log[1 + e/(d*x^(1/3))])*Log[x] + 3*PolyLog[2, -(e/(d*x^(1/3)))]) +
 3*b^2*n^2*(2*Log[e/d + x^(1/3)]*PolyLog[2, 1 + (d*x^(1/3))/e] - 2*(Log[d
+ e/x^(1/3)] - Log[e/d + x^(1/3)])*PolyLog[2, -((d*x^(1/3))/e)] + (81*Log[
e/d + x^(1/3)]^2*Log[-((d*x^(1/3))/e)] + 27*Log[d + e/x^(1/3)]^2*Log[x] -
27*Log[e/d + x^(1/3)]^2*Log[x] - 54*Log[d + e/x^(1/3)]*Log[1 + (d*x^(1/3))
/e]*Log[x] + 54*Log[e/d + x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 9*Log[d
 + e/x^(1/3)]*Log[x]^2 - 9*Log[1 + (d*x^(1/3))/e]*Log[x]^2 + Log[x]^3 - 16
2*PolyLog[3, 1 + (d*x^(1/3))/e] - 162*PolyLog[3, -((d*x^(1/3))/e)])/81)
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2843}$$

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-2ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)}{d+\frac{e}{\sqrt[3]{x}}}d\frac{1}{\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{2881}$$

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-2bn\int \sqrt[3]{x}\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)$$

$$\Big\downarrow \text{2821}$$

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-2bn\left(bn\int\sqrt[3]{x}\,\text{PolyLog}\left(2,\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\text{Poly}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-2bn\left(bn\,\text{PolyLog}\left(3,\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)-\text{PolyLog}\left(2,\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

output
```
-3*((a + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(e/(d*x^(1/3)))] - 2*b*n*(-((a
 + b*Log[c/x^(n/3)])*PolyLog[2, (d + e/x^(1/3))/d]) + b*n*PolyLog[3, (d +
e/x^(1/3))/d]))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(
2/3))/x)^n) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = \int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x,x)
```

output
```
Integral((a + b*log(c*(d + e/x**(1/3))**n))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)^n\right) + a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="maxima")
```

output
```
b^2*log((d*x^(1/3) + e)^n)^2*log(x) - integrate(-1/3*(3*(b^2*d*x + b^2*e*x
^(2/3))*log(x^(1/3*n))^2 + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x -
 2*(b^2*d*n*x*log(x) - 3*(b^2*d*log(c) + a*b*d)*x + 3*(b^2*d*x + b^2*e*x^(
2/3))*log(x^(1/3*n)) - 3*(b^2*e*log(c) + a*b*e)*x^(2/3))*log((d*x^(1/3) +
e)^n) - 6*((b^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*x^(2/3))*log(
x^(1/3*n)) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(2/3))/(d*x^2 +
 e*x^(5/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))^2/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^n))^2/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx=\left(\int \frac{\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)^2}{x}dx\right)b^2+2\left(\int \frac{\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)}{x}dx\right)ab+\log(x)\,a^2$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x)
```

output
```
int(log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2/x,x)*b**2 + 2*int(log(((x**(1
/3)*d + e)**n*c)/x**(n/3))/x,x)*a*b + log(x)*a**2
```

3.501 $\displaystyle \int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 269

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$=\frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3}-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}-\frac{6b^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{b^2d^3n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}+\frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}-\frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}+\frac{2bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}-\frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}$$

output
```
3/2*b^2*d*n^2*(d+e/x^(1/3))^2/e^3-2/9*b^2*n^2*(d+e/x^(1/3))^3/e^3-6*b^2*d^
2*n^2/e^2/x^(1/3)+b^2*d^3*n^2*ln(d+e/x^(1/3))^2/e^3+6*b*d^2*n*(d+e/x^(1/3)
)*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-3*b*d*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x
^(1/3))^n))/e^3+2/3*b*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-2*
b*d^3*n*ln(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-(a+b*ln(c*(d+e/x^(
1/3))^n))^2/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$= \frac{-18\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+\frac{bn\left(-2ben\left(2e^2-3de\sqrt[3]{x}+6d^2x^{2/3}\right)+9bden\left(e-2d\sqrt[3]{x}\right)\sqrt[3]{x}+36ad^2ex^{2/3}-36bd^2enx^2}{}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

output
```
(-18*(a + b*Log[c*(d + e/x^(1/3))^n])^2 + (b*n*(-2*b*e*n*(2*e^2 - 3*d*e*x^
(1/3) + 6*d^2*x^(2/3)) + 9*b*d*e*n*(e - 2*d*x^(1/3))*x^(1/3) + 36*a*d^2*e*
x^(2/3) - 36*b*d^2*e*n*x^(2/3) + 30*b*d^3*n*x*Log[d + e/x^(1/3)] + 36*b*d^
2*(e + d*x^(1/3))*x^(2/3)*Log[c*(d + e/x^(1/3))^n] + 12*e^3*(a + b*Log[c*(
d + e/x^(1/3))^n]) - 18*d*e^2*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]) - 3
6*d^3*x*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[e + d*x^(1/3)] - 36*d^3*x*(a
+ b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] + 18*b*d^3*n*x*Log[e +
 d*x^(1/3)]*(Log[e + d*x^(1/3)] - 2*Log[-((d*x^(1/3))/e)]) - 36*b*d^3*n*x*
PolyLog[2, 1 + e/(d*x^(1/3))] - 36*b*d^3*n*x*PolyLog[2, 1 + (d*x^(1/3))/e]
))/e^3)/(18*x)
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.73, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2904, 2845, 2858, 25, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$

$\Big\downarrow$ 2904

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^{2/3}}d\frac{1}{\sqrt[3]{x}}$$

$\Big\downarrow$ 2845

$$-3\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{3x}-\frac{2}{3}ben\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{\left(d+\frac{e}{\sqrt[3]{x}}\right)x}d\frac{1}{\sqrt[3]{x}}\right)$$

$\Big\downarrow$ 2858

$$-3\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{3x}-\frac{2}{3}bn\int \frac{a+b\log\left(cx^{-n/3}\right)}{x^{2/3}}d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)$$

$\Big\downarrow$ 25

$$-3\left(\frac{2}{3}bn\int -\frac{a+b\log\left(cx^{-n/3}\right)}{x^{2/3}}d\left(d+\frac{e}{\sqrt[3]{x}}\right)+\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{3x}\right)$$

$\Big\downarrow$ 27

$$-3\left(\frac{2bn\int -\frac{e^3\left(a+b\log\left(cx^{-n/3}\right)\right)}{x^{2/3}}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^3}+\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{3x}\right)$$

$\Big\downarrow$ 2772

$$-3\left(\frac{2bn\left(-bn\int \left(\sqrt[3]{x}\log\left(d+\frac{e}{\sqrt[3]{x}}\right)d^3-3d^2+\frac{3}{2}\left(d+\frac{e}{\sqrt[3]{x}}\right)d-\frac{1}{3x^{2/3}}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)+d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)(a+\right.}{3e^3}\right.$$

$\Big\downarrow$ 2009

$$-3\left(\frac{2bn\left(d^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)-3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)+\frac{3d\left(a+b\log\left(cx^{-n/3}\right)\right)}{2x^{2/3}}-\frac{a\cdot}{}\right.}{3e^3}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

output
```
-3*((a + b*Log[c*(d + e/x^(1/3))^n])^2/(3*x) + (2*b*n*(-(b*n*(-3*d^2*(d +
e/x^(1/3)) - 1/(9*x) + (3*d)/(4*x^(2/3)) + (d^3*Log[d + e/x^(1/3)]^2)/2))
- 3*d^2*(d + e/x^(1/3))*(a + b*Log[c/x^(n/3)]) - (a + b*Log[c/x^(n/3)])/(3
*x) + (3*d*(a + b*Log[c/x^(n/3)]))/(2*x^(2/3)) + d^3*Log[d + e/x^(1/3)]*(a
 + b*Log[c/x^(n/3)])))/(3*e^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{4\,b^2e^3n^2-12\,abe^3n+18\,a^2e^3-18\,(b^2e^3x-b^2e^3)\log\left(c\right)^2+18\,(b^2d^3n^2x+b^2e^3n^2)\log\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^2-2\,(}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="fricas")
```

output
```
-1/18*(4*b^2*e^3*n^2 - 12*a*b*e^3*n + 18*a^2*e^3 - 18*(b^2*e^3*x - b^2*e^3
)*log(c)^2 + 18*(b^2*d^3*n^2*x + b^2*e^3*n^2)*log((d*x + e*x^(2/3))/x)^2 -
 2*(2*b^2*e^3*n^2 - 6*a*b*e^3*n + 9*a^2*e^3)*x - 12*(b^2*e^3*n - 3*a*b*e^3
 - (b^2*e^3*n - 3*a*b*e^3)*x)*log(c) - 6*(6*b^2*d^2*e*n^2*x^(2/3) - 3*b^2*
d*e^2*n^2*x^(1/3) + 2*b^2*e^3*n^2 - 6*a*b*e^3*n + (11*b^2*d^3*n^2 - 6*a*b*
d^3*n)*x - 6*(b^2*d^3*n*x + b^2*e^3*n)*log(c))*log((d*x + e*x^(2/3))/x) +
6*(11*b^2*d^2*e*n^2 - 6*b^2*d^2*e*n*log(c) - 6*a*b*d^2*e*n)*x^(2/3) - 3*(5
*b^2*d*e^2*n^2 - 6*b^2*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(1/3))/(e^3*x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**2,x)
```

output
```
Integral((a + b*log(c*(d + e/x**(1/3))**n))**2/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$= -\frac{1}{3}\,aben\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2\,d^3\log\left(x\right)}{e^4}-\frac{6\,d^2x^{\frac{2}{3}}-3\,dex^{\frac{1}{3}}+2\,e^2}{e^3x}\right)$$
$$-\frac{1}{18}\left(6\,en\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2\,d^3\log\left(x\right)}{e^4}-\frac{6\,d^2x^{\frac{2}{3}}-3\,dex^{\frac{1}{3}}+2\,e^2}{e^3x}\right)\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)-\frac{(18\,d}{}\right.$$
$$-\frac{b^2\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)^2}{x}-\frac{2\,ab\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x}-\frac{a^2}{x}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="maxima")
```

output
```
-1/3*a*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(
2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - 1/18*(6*e*n*(6*d^3*log(d*x^(1/3)
+ e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3
*x))*log(c*(d + e/x^(1/3))^n) - (18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*l
og(x)^2 - 22*d^3*x*log(x) - 66*d^2*e*x^(2/3) + 15*d*e^2*x^(1/3) - 4*e^3 -
6*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) + e))*n^2/(e^3*x))*b^2 - b^2*l
og(c*(d + e/x^(1/3))^n)^2/x - 2*a*b*log(c*(d + e/x^(1/3))^n)/x - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx =$$
$$-\frac{18\left(\frac{3\left(dx^{\frac{1}{3}}+e\right)b^2d^2n^2}{e^2x^{\frac{1}{3}}}-\frac{3\left(dx^{\frac{1}{3}}+e\right)^2b^2dn^2}{e^2x^{\frac{2}{3}}}+\frac{\left(dx^{\frac{1}{3}}+e\right)^3b^2n^2}{e^2x}\right)\log\left(\frac{dx^{\frac{1}{3}}+e}{x^{\frac{1}{3}}}\right)^2-6\left(\frac{2\left(b^2n^2-3\,b^2n\log(c)-3\,abn\right)\left(dx^{\frac{1}{3}}+e\right)}{e^2x}\right.}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="giac")
```

output
```
-1/18*(18*(3*(d*x^(1/3) + e)*b^2*d^2*n^2/(e^2*x^(1/3)) - 3*(d*x^(1/3) + e)
^2*b^2*d*n^2/(e^2*x^(2/3)) + (d*x^(1/3) + e)^3*b^2*n^2/(e^2*x))*log((d*x^(
1/3) + e)/x^(1/3))^2 - 6*(2*(b^2*n^2 - 3*b^2*n*log(c) - 3*a*b*n)*(d*x^(1/3
) + e)^3/(e^2*x) - 9*(b^2*d*n^2 - 2*b^2*d*n*log(c) - 2*a*b*d*n)*(d*x^(1/3)
 + e)^2/(e^2*x^(2/3)) + 18*(b^2*d^2*n^2 - b^2*d^2*n*log(c) - a*b*d^2*n)*(d
*x^(1/3) + e)/(e^2*x^(1/3)))*log((d*x^(1/3) + e)/x^(1/3)) + 2*(2*b^2*n^2 -
 6*b^2*n*log(c) + 9*b^2*log(c)^2 - 6*a*b*n + 18*a*b*log(c) + 9*a^2)*(d*x^(
1/3) + e)^3/(e^2*x) - 27*(b^2*d*n^2 - 2*b^2*d*n*log(c) + 2*b^2*d*log(c)^2
- 2*a*b*d*n + 4*a*b*d*log(c) + 2*a^2*d)*(d*x^(1/3) + e)^2/(e^2*x^(2/3)) +
54*(2*b^2*d^2*n^2 - 2*b^2*d^2*n*log(c) + b^2*d^2*log(c)^2 - 2*a*b*d^2*n +
2*a*b*d^2*log(c) + a^2*d^2)*(d*x^(1/3) + e)/(e^2*x^(1/3)))/e
```

Mupad [B] (verification not implemented)

Time = 25.50 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$= \frac{\frac{d\left(3a^2-2abn+\frac{2b^2n^2}{3}\right)}{2e}-\frac{d\left(3a^2-b^2n^2\right)}{2e}}{x^{2/3}} - \ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2\left(\frac{b^2}{x}+\frac{b^2d^3}{e^3}\right)$$
$$-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\left(\frac{2b(3a-bn)}{3x}-\frac{\frac{bd(3a-bn)}{e}-\frac{3abd}{e}}{x^{2/3}}+\frac{d\left(\frac{2bd(3a-bn)}{e}-\frac{6abd}{e}\right)}{e\,x^{1/3}}\right)-\frac{d\left(\frac{d\left(3a^2-2abn+\frac{2b}{}\right.}{e}\right.}{}$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))^2/x^2,x)
```

output
```
((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(2*e)
)/x^(2/3) - log(c*(d + e/x^(1/3))^n)^2*(b^2/x + (b^2*d^3)/e^3) - log(c*(d
+ e/x^(1/3))^n)*((2*b*(3*a - b*n))/(3*x) - ((b*d*(3*a - b*n))/e - (3*a*b*d
)/e)/x^(2/3) + (d*((2*b*d*(3*a - b*n))/e - (6*a*b*d)/e))/(e*x^(1/3))) - ((
d*((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/e - (d*(3*a^2 - b^2*n^2))/e))/e +
 (2*b^2*d^2*n^2)/e^2)/x^(1/3) - (a^2 + (2*b^2*n^2)/9 - (2*a*b*n)/3)/x + (l
og(d + e/x^(1/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(3*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$

$$= \frac{36x^{\frac{2}{3}}\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b^2d^2en + 36x^{\frac{2}{3}}ab\,d^2en - 66x^{\frac{2}{3}}b^2d^2e\,n^2 - 18x^{\frac{1}{3}}\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b^2d\,e^2n - 18x^{\frac{1}{3}}abd\,e}{}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x)
```

output
```
(36*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**2*e*n + 36*x**(
2/3)*a*b*d**2*e*n - 66*x**(2/3)*b**2*d**2*e*n**2 - 18*x**(1/3)*log(((x**(1
/3)*d + e)**n*c)/x**(n/3))*b**2*d*e**2*n - 18*x**(1/3)*a*b*d*e**2*n + 15*x
**(1/3)*b**2*d*e**2*n**2 - 18*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**
2*d**3*x - 18*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**2*e**3 - 36*log(
((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b*d**3*x - 36*log(((x**(1/3)*d + e)**n
*c)/x**(n/3))*a*b*e**3 + 66*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**
3*n*x + 12*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*e**3*n - 18*a**2*e**
3 + 12*a*b*e**3*n - 4*b**2*e**3*n**2)/(18*e**3*x)
```

3.502 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 479

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$$
$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6}+\frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6}-\frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6}$$
$$+\frac{6b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{25e^6}-\frac{b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{36e^6}+\frac{6b^2d^5n^2}{e^5\sqrt[3]{x}}$$
$$-\frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}$$
$$+\frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^6}$$
$$-\frac{20bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6}$$
$$+\frac{15bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6}$$
$$-\frac{6bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5e^6}$$
$$+\frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6e^6}$$
$$+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}$$
$$-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2}$$

output
```
-15/4*b^2*d^4*n^2*(d+e/x^(1/3))^2/e^6+20/9*b^2*d^3*n^2*(d+e/x^(1/3))^3/e^6
-15/16*b^2*d^2*n^2*(d+e/x^(1/3))^4/e^6+6/25*b^2*d*n^2*(d+e/x^(1/3))^5/e^6-
1/36*b^2*n^2*(d+e/x^(1/3))^6/e^6+6*b^2*d^5*n^2/e^5/x^(1/3)-1/2*b^2*d^6*n^2
*ln(d+e/x^(1/3))^2/e^6-6*b*d^5*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))
/e^6+15/2*b*d^4*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-20/3*b*d
^3*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+15/4*b*d^2*n*(d+e/x^(
1/3))^4*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-6/5*b*d*n*(d+e/x^(1/3))^5*(a+b*ln(
c*(d+e/x^(1/3))^n))/e^6+1/6*b*n*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n)
)/e^6+b*d^6*n*ln(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-1/2*(a+b*ln(
c*(d+e/x^(1/3))^n))^2/x^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$$

$$=\frac{-1800\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+\frac{bn\left(600ae^6-100be^6n-720ade^5\sqrt[3]{x}+264bde^5n\sqrt[3]{x}+900ad^2e^4x^{2/3}-555bd^2e^4nx^{2/3}-\right.}{}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]
```

output
```
(-1800*(a + b*Log[c*(d + e/x^(1/3))^n])^2 + (b*n*(600*a*e^6 - 100*b*e^6*n
- 720*a*d*e^5*x^(1/3) + 264*b*d*e^5*n*x^(1/3) + 900*a*d^2*e^4*x^(2/3) - 55
5*b*d^2*e^4*n*x^(2/3) - 1200*a*d^3*e^3*x + 1140*b*d^3*e^3*n*x + 1800*a*d^4
*e^2*x^(4/3) - 2610*b*d^4*e^2*n*x^(4/3) - 3600*a*d^5*e*x^(5/3) + 8820*b*d^
5*e*n*x^(5/3) - 8820*b*d^6*n*x^2*Log[d + e/x^(1/3)] + 600*b*e^6*Log[c*(d +
 e/x^(1/3))^n] - 720*b*d*e^5*x^(1/3)*Log[c*(d + e/x^(1/3))^n] + 900*b*d^2*
e^4*x^(2/3)*Log[c*(d + e/x^(1/3))^n] - 1200*b*d^3*e^3*x*Log[c*(d + e/x^(1/
3))^n] + 1800*b*d^4*e^2*x^(4/3)*Log[c*(d + e/x^(1/3))^n] - 3600*b*d^5*e*x^
(5/3)*Log[c*(d + e/x^(1/3))^n] + 3600*a*d^6*x^2*Log[e + d*x^(1/3)] + 3600*
b*d^6*x^2*Log[c*(d + e/x^(1/3))^n]*Log[e + d*x^(1/3)] - 1800*b*d^6*n*x^2*L
og[e + d*x^(1/3)]^2 + 3600*b*d^6*x^2*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x
^(1/3)))] + 3600*b*d^6*n*x^2*Log[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] - 12
00*a*d^6*x^2*Log[x] + 3600*b*d^6*n*x^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 360
0*b*d^6*n*x^2*PolyLog[2, 1 + (d*x^(1/3))/e]))/e^6)/(3600*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^{5/3}}d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2845}$$

$$-3\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{6x^2}-\frac{1}{3}ben\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^2}d\frac{1}{\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{2858}$$

$$-3\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{6x^2}-\frac{1}{3}bn\int\frac{a+b\log\left(cx^{-n/3}\right)}{x^{5/3}}d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-3\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{6x^2}-\frac{bn\int\frac{e^6\left(a+b\log\left(cx^{-n/3}\right)\right)}{x^{5/3}}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^6}\right)$$

$$\Big\downarrow \text{2772}$$

$$-3\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{6x^2}-\frac{bn\left(-bn\int\left(\sqrt[3]{x}\log\left(d+\frac{e}{\sqrt[3]{x}}\right)d^6-6d^5+\frac{15}{2}\left(d+\frac{e}{\sqrt[3]{x}}\right)d^4-\frac{20d^3}{3x^{2/3}}+\frac{15}{4}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-3\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{6x^2}-\frac{bn\left(d^6\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)-6d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\right.\right.\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]
```

output
```
-3*((a + b*Log[c*(d + e/x^(1/3))^n])^2/(6*x^2) - (b*n*(-(b*n*(-6*d^5*(d +
e/x^(1/3)) + 1/(36*x^2) - (6*d)/(25*x^(5/3)) + (15*d^2)/(16*x^(4/3)) - (20
*d^3)/(9*x) + (15*d^4)/(4*x^(2/3)) + (d^6*Log[d + e/x^(1/3)]^2)/2)) - 6*d^
5*(d + e/x^(1/3))*(a + b*Log[c/x^(n/3)]) + (a + b*Log[c/x^(n/3)])/(6*x^2)
- (6*d*(a + b*Log[c/x^(n/3)]))/(5*x^(5/3)) + (15*d^2*(a + b*Log[c/x^(n/3)]
))/(4*x^(4/3)) - (20*d^3*(a + b*Log[c/x^(n/3)]))/(3*x) + (15*d^4*(a + b*Lo
g[c/x^(n/3)]))/(2*x^(2/3)) + d^6*Log[d + e/x^(1/3)]*(a + b*Log[c/x^(n/3)])
))/(3*e^6))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx =$$
$$-\frac{100\,b^2e^6n^2-600\,abe^6n+1800\,a^2e^6-20\,(90\,a^2e^6-(57\,b^2d^3e^3-5\,b^2e^6)n^2+30\,(2\,abd^3e^3-abe^6)n)x^2}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="fricas")
```

output
```
-1/3600*(100*b^2*e^6*n^2 - 600*a*b*e^6*n + 1800*a^2*e^6 - 20*(90*a^2*e^6 -
 (57*b^2*d^3*e^3 - 5*b^2*e^6)*n^2 + 30*(2*a*b*d^3*e^3 - a*b*e^6)*n)*x^2 -
1800*(b^2*e^6*x^2 - b^2*e^6)*log(c)^2 - 1800*(b^2*d^6*n^2*x^2 - b^2*e^6*n^
2)*log((d*x + e*x^(2/3))/x)^2 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)
*x + 600*(2*b^2*d^3*e^3*n*x - b^2*e^6*n + 6*a*b*e^6 - (6*a*b*e^6 + (2*b^2*
d^3*e^3 - b^2*e^6)*n)*x^2)*log(c) + 60*(20*b^2*d^3*e^3*n^2*x - 10*b^2*e^6*
n^2 + 60*a*b*e^6*n + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^2 - 60*(b^2*d^6*n
*x^2 - b^2*e^6*n)*log(c) + 15*(4*b^2*d^5*e*n^2*x - b^2*d^2*e^4*n^2)*x^(2/3
) - 6*(5*b^2*d^4*e^2*n^2*x - 2*b^2*d*e^5*n^2)*x^(1/3))*log((d*x + e*x^(2/3
))/x) + 15*(37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n - 12*(49*b^2*d^5*e*n^2 -
 20*a*b*d^5*e*n)*x + 60*(4*b^2*d^5*e*n*x - b^2*d^2*e^4*n)*log(c))*x^(2/3)
- 6*(44*b^2*d*e^5*n^2 - 120*a*b*d*e^5*n - 15*(29*b^2*d^4*e^2*n^2 - 20*a*b*
d^4*e^2*n)*x + 60*(5*b^2*d^4*e^2*n*x - 2*b^2*d*e^5*n)*log(c))*x^(1/3))/(e^
6*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$$
$$= \frac{1}{60}aben\left(\frac{60\,d^6\log\left(dx^{\frac{1}{3}}+e\right)}{e^7} - \frac{20\,d^6\log(x)}{e^7} - \frac{60\,d^5x^{\frac{5}{3}} - 30\,d^4ex^{\frac{4}{3}} + 20\,d^3e^2x - 15\,d^2e^3x^{\frac{2}{3}} + 12\,de^4x^{\frac{1}{3}}}{e^6x^2}\right.$$
$$+\frac{1}{3600}\left(60\,en\left(\frac{60\,d^6\log\left(dx^{\frac{1}{3}}+e\right)}{e^7} - \frac{20\,d^6\log(x)}{e^7} - \frac{60\,d^5x^{\frac{5}{3}} - 30\,d^4ex^{\frac{4}{3}} + 20\,d^3e^2x - 15\,d^2e^3x^{\frac{2}{3}} + 12}{e^6x^2}\right.\right.$$
$$-\frac{b^2\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)^2}{2\,x^2} - \frac{ab\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x^2} - \frac{a^2}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="maxima")
```

output
```
1/60*a*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*
x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*
x^(1/3) - 10*e^5)/(e^6*x^2)) + 1/3600*(60*e*n*(60*d^6*log(d*x^(1/3) + e)/e
^7 - 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x
 - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) - 10*e^5)/(e^6*x^2))*log(c*(d + e
/x^(1/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 -
 2940*d^6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^
3*e^3*x + 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x
^2*log(x) - 147*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^6*x^2))*b^2 - 1/2*b^2*
log(c*(d + e/x^(1/3))^n)^2/x^2 - a*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a^
2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(411) = 822$.

Time = 0.17 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="giac")
```

output
```
1/3600*(1800*(6*(d*x^(1/3) + e)*b^2*d^5*n^2/(e^5*x^(1/3)) - 15*(d*x^(1/3)
+ e)^2*b^2*d^4*n^2/(e^5*x^(2/3)) + 20*(d*x^(1/3) + e)^3*b^2*d^3*n^2/(e^5*x
) - 15*(d*x^(1/3) + e)^4*b^2*d^2*n^2/(e^5*x^(4/3)) + 6*(d*x^(1/3) + e)^5*b
^2*d*n^2/(e^5*x^(5/3)) - (d*x^(1/3) + e)^6*b^2*n^2/(e^5*x^2))*log((d*x^(1/
3) + e)/x^(1/3))^2 + 60*(10*(b^2*n^2 - 6*b^2*n*log(c) - 6*a*b*n)*(d*x^(1/3
) + e)^6/(e^5*x^2) - 72*(b^2*d*n^2 - 5*b^2*d*n*log(c) - 5*a*b*d*n)*(d*x^(1
/3) + e)^5/(e^5*x^(5/3)) + 225*(b^2*d^2*n^2 - 4*b^2*d^2*n*log(c) - 4*a*b*d
^2*n)*(d*x^(1/3) + e)^4/(e^5*x^(4/3)) - 400*(b^2*d^3*n^2 - 3*b^2*d^3*n*log
(c) - 3*a*b*d^3*n)*(d*x^(1/3) + e)^3/(e^5*x) + 450*(b^2*d^4*n^2 - 2*b^2*d^
4*n*log(c) - 2*a*b*d^4*n)*(d*x^(1/3) + e)^2/(e^5*x^(2/3)) - 360*(b^2*d^5*n
^2 - b^2*d^5*n*log(c) - a*b*d^5*n)*(d*x^(1/3) + e)/(e^5*x^(1/3)))*log((d*x
^(1/3) + e)/x^(1/3)) - 100*(b^2*n^2 - 6*b^2*n*log(c) + 18*b^2*log(c)^2 - 6
*a*b*n + 36*a*b*log(c) + 18*a^2)*(d*x^(1/3) + e)^6/(e^5*x^2) + 432*(2*b^2*
d*n^2 - 10*b^2*d*n*log(c) + 25*b^2*d*log(c)^2 - 10*a*b*d*n + 50*a*b*d*log(
c) + 25*a^2*d)*(d*x^(1/3) + e)^5/(e^5*x^(5/3)) - 3375*(b^2*d^2*n^2 - 4*b^2
*d^2*n*log(c) + 8*b^2*d^2*log(c)^2 - 4*a*b*d^2*n + 16*a*b*d^2*log(c) + 8*a
^2*d^2)*(d*x^(1/3) + e)^4/(e^5*x^(4/3)) + 4000*(2*b^2*d^3*n^2 - 6*b^2*d^3*
n*log(c) + 9*b^2*d^3*log(c)^2 - 6*a*b*d^3*n + 18*a*b*d^3*log(c) + 9*a^2*d^
3)*(d*x^(1/3) + e)^3/(e^5*x) - 13500*(b^2*d^4*n^2 - 2*b^2*d^4*n*log(c) + 2
*b^2*d^4*log(c)^2 - 2*a*b*d^4*n + 4*a*b*d^4*log(c) + 2*a^2*d^4)*(d*x^(1...
```

Mupad [B] (verification not implemented)

Time = 27.05 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = \frac{b^2\,d^6\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2}{2\,e^6} - \frac{b^2\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2}{2\,x^2} - \frac{b^2\,n^2}{36\,x^2} - \frac{a\,b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{x^2} - \frac{a^2}{2\,x^2} + \frac{a\,b\,n}{6\,x^2} + \frac{b^2\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{6\,x^2} - \frac{49\,b^2\,d^6\,n^2\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{20\,e^6} + \frac{19\,b^2\,d^3\,n^2}{60\,e^3\,x} - \frac{37\,b^2\,d^2\,n^2}{240\,e^2\,x^{4/3}} - \frac{29\,b^2\,d^4\,n^2}{40\,e^4\,x^{2/3}} + \frac{49\,b^2\,d^5\,n^2}{20\,e^5\,x^{1/3}} + \frac{11\,b^2\,d\,n^2}{150\,e\,x^{5/3}} - \frac{b^2\,d^3\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{3\,e^3\,x} + \frac{b^2\,d^2\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{4\,e^2\,x^{4/3}} + \frac{b^2\,d^4\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{2\,e^4\,x^{2/3}} - \frac{b^2\,d^5\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{e^5\,x^{1/3}} - \frac{a\,b\,d\,n}{5\,e\,x^{5/3}} + \frac{a\,b\,d^6\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{e^6} - \frac{b^2\,d\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{5\,e\,x^{5/3}} - \frac{a\,b\,d^3\,n}{3\,e^3\,x} + \frac{a\,b\,d^2\,n}{4\,e^2\,x^{4/3}} + \frac{a\,b\,d^4\,n}{2\,e^4\,x^{2/3}} - \frac{a\,b\,d^5\,n}{e^5\,x^{1/3}}$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))^2/x^3,x)
```

output
```
(b^2*d^6*log(c*(d + e/x^(1/3))^n)^2)/(2*e^6) - (b^2*log(c*(d + e/x^(1/3))^
n)^2)/(2*x^2) - (b^2*n^2)/(36*x^2) - (a*b*log(c*(d + e/x^(1/3))^n))/x^2 -
a^2/(2*x^2) + (a*b*n)/(6*x^2) + (b^2*n*log(c*(d + e/x^(1/3))^n))/(6*x^2) -
 (49*b^2*d^6*n^2*log(d + e/x^(1/3)))/(20*e^6) + (19*b^2*d^3*n^2)/(60*e^3*x
) - (37*b^2*d^2*n^2)/(240*e^2*x^(4/3)) - (29*b^2*d^4*n^2)/(40*e^4*x^(2/3))
 + (49*b^2*d^5*n^2)/(20*e^5*x^(1/3)) + (11*b^2*d*n^2)/(150*e*x^(5/3)) - (b
^2*d^3*n*log(c*(d + e/x^(1/3))^n))/(3*e^3*x) + (b^2*d^2*n*log(c*(d + e/x^(
1/3))^n))/(4*e^2*x^(4/3)) + (b^2*d^4*n*log(c*(d + e/x^(1/3))^n))/(2*e^4*x^
(2/3)) - (b^2*d^5*n*log(c*(d + e/x^(1/3))^n))/(e^5*x^(1/3)) - (a*b*d*n)/(5
*e*x^(5/3)) + (a*b*d^6*n*log(d + e/x^(1/3)))/e^6 - (b^2*d*n*log(c*(d + e/x
^(1/3))^n))/(5*e*x^(5/3)) - (a*b*d^3*n)/(3*e^3*x) + (a*b*d^2*n)/(4*e^2*x^(
4/3)) + (a*b*d^4*n)/(2*e^4*x^(2/3)) - (a*b*d^5*n)/(e^5*x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$$

$$= \frac{-3600x^{\frac{5}{3}}\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b^2d^5en+900x^{\frac{2}{3}}\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)b^2d^2e^4n-3600x^{\frac{5}{3}}ab\,d^5en+900x^{\frac{2}{3}}ab\,d^2e^4n+}{}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x)
```

output

```
( - 3600*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**5*e*n*x +
900*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**2*e**4*n - 3600
*x**(2/3)*a*b*d**5*e*n*x + 900*x**(2/3)*a*b*d**2*e**4*n + 8820*x**(2/3)*b*
*2*d**5*e*n**2*x - 555*x**(2/3)*b**2*d**2*e**4*n**2 + 1800*x**(1/3)*log(((
x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*d**4*e**2*n*x - 720*x**(1/3)*log(((x*
*(1/3)*d + e)**n*c)/x**(n/3))*b**2*d*e**5*n + 1800*x**(1/3)*a*b*d**4*e**2*
n*x - 720*x**(1/3)*a*b*d*e**5*n - 2610*x**(1/3)*b**2*d**4*e**2*n**2*x + 26
4*x**(1/3)*b**2*d*e**5*n**2 + 1800*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**
2*b**2*d**6*x**2 - 1800*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**2*e**6
 + 3600*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b*d**6*x**2 - 3600*log(((x
**(1/3)*d + e)**n*c)/x**(n/3))*a*b*e**6 - 8820*log(((x**(1/3)*d + e)**n*c)
/x**(n/3))*b**2*d**6*n*x**2 - 1200*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b
**2*d**3*e**3*n*x + 600*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**2*e**6*n
- 1800*a**2*e**6 - 1200*a*b*d**3*e**3*n*x + 600*a*b*e**6*n + 1140*b**2*d**
3*e**3*n**2*x - 100*b**2*e**6*n**2)/(3600*e**6*x**2)
```

3.503 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 22, antiderivative size = 759

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

output
```
71/40*b^3*e^5*n^3*x^(1/3)/d^5-3/10*b^3*e^4*n^3*x^(2/3)/d^4+1/20*b^3*e^3*n^
3*x/d^3-71/40*b^3*e^6*n^3*ln(d+e/x^(1/3))/d^6-77/20*b^2*e^5*n^2*(d+e/x^(1/
3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+47/40*b^2*e^4*n^2*x^(2/3)*(a+b
*ln(c*(d+e/x^(1/3))^n))/d^4-9/20*b^2*e^3*n^2*x*(a+b*ln(c*(d+e/x^(1/3))^n))
/d^3+3/20*b^2*e^2*n^2*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^2-77/20*b^2*e^
6*n^2*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+3/2*b*e^5*n*(d
+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^6-3/4*b*e^4*n*x^(2/3)*
(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^4+1/2*b*e^3*n*x*(a+b*ln(c*(d+e/x^(1/3))^n)
)^2/d^3-3/8*b*e^2*n*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^2+3/10*b*e*n*x
^(5/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d+3/2*b*e^6*n*ln(1-d/(d+e/x^(1/3)))*(
a+b*ln(c*(d+e/x^(1/3))^n))^2/d^6+1/2*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3-3*b
^2*e^6*n^2*(a+b*ln(c*(d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))/d^6-15/8*b^3*e^6*n
^3*ln(x)/d^6+77/20*b^3*e^6*n^3*polylog(2,d/(d+e/x^(1/3)))/d^6-3*b^2*e^6*n^
2*(a+b*ln(c*(d+e/x^(1/3))^n))*polylog(2,d/(d+e/x^(1/3)))/d^6-3*b^3*e^6*n^3
*polylog(2,1+e/d/x^(1/3))/d^6-3*b^3*e^6*n^3*polylog(3,d/(d+e/x^(1/3)))/d^6
```

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 10.85 (sec) , antiderivative size = 1374, normalized size of antiderivative = 1.81, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.227, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int x^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2845}$$

$$-3\left(\frac{1}{2}ben\int\frac{x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d+\frac{e}{\sqrt[3]{x}}}d\frac{1}{\sqrt[3]{x}}-\frac{1}{6}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2858}$$

$$-3\left(\frac{1}{2}bn\int x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{1}{6}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\right)$$

$\downarrow$ 27

$$-3\left(\frac{1}{2}be^6n\int \frac{x^{7/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{e^6}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{1}{6}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\right)$$

$\downarrow$ 2789

$$-3\left(\frac{1}{2}be^6n\left(\frac{\int \frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{e^6}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}\right)-\frac{1}{6}x^2\left(a+b\log\left(c\left(d-\right.\right.\right.\right.$$

$\downarrow$ 2756

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\int -\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}}{d}+\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{e^5}d\left(d+\frac{e}{\sqrt[3]{\right.}}\right.}{d}\right.\right.$$

$\downarrow$ 2789

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\int -\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^5}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}\right)-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}}{d}\right.\right.$$

$\downarrow$ 2756

$$-3\left(\frac{1}{2}be^6n\left(\frac{\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{4e^4}-\frac{1}{2}bn\int \frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{2}{5}bn}{}\right.\right.$$

$\downarrow$ 54

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\int\left(\frac{x^{4/3}}{de^4}-\frac{x}{d^2e^3}+\frac{x^{2/3}}{d^3e^2}-\frac{\sqrt[3]{x}}{d^4e}+\frac{\sqrt[3]{x}}{d^4}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\right.}{d}\right.}{d}\right.\right.$$

$\Big\downarrow$ 2009

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\int\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\sqrt[3]{x}}{d^3e}+\frac{x^2}{2d^2}\right.}{d}\right.}{d}\right.\right.$$

$\Big\downarrow$ 2789

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\frac{\int\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^4}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int-\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\right.}{}\right.}{d}\right.\right.$$

$\Big\downarrow$ 2756

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\frac{-\frac{1}{3}bn\int -\frac{x^{4/3}}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}}{d}+\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right.}{4e^4}}{\cdots}\right.}{d}\right.\right.$$

$$\Big\downarrow \; 54$$

$$-3\left(\frac{1}{2}be^6n\left(\frac{\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{4e^4}-\frac{1}{2}bn\left(\frac{-\frac{1}{3}bn\int\left(-\frac{x}{de^3}+\frac{x^{2/3}}{d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\sqrt[3]{x}}{d^3}\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}}{d}+\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-\cdots}\right.\right.}{e^3}}{d}\right.}{d}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2009$$

$$-3\left(\frac{1}{2}be^6n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{4e^4}-\frac{1}{2}bn\left(\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{\log\left(-\right.}{d^3}\right.}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2789

$$-3\left(\frac{1}{2}be^6n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{4e^4}-\frac{1}{2}bn\left(\frac{\frac{\int -\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}}{d}+\frac{-\frac{x\left(a+b\log\left(cx^{-n/3}\right)\right)}{3e^3}}{}\right.}{d}\right.\right.$$

$\downarrow$ 2756

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 54

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 2009

$$-3\left(\frac{1}{2}be^6n\,\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{}\right.$$

$\downarrow$ 2789

$$-3\left(\frac{1}{2}be^6n\,\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{}\right.$$

$\downarrow$ 2751

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{}\right.\right.$$

$\downarrow$ 16

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{}\right.\right.$$

$\downarrow$ 2755

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{\;}\right.\right.\ldots$$

$\downarrow$ 2754

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{\;}\right.\right.\ldots$$

$\downarrow$ 2779

$$-3\left(\frac{1}{2}be^6n\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{}\right.$$

$\downarrow$ 2821

$$-3\left(\frac{1}{2}be^6n\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{}\right.$$

$\downarrow$ 2838

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{}\right.\right.$$

$\downarrow$ 7143

$$-3\left(\frac{1}{2}be^6n\left(\frac{-\frac{x^{5/3}\left(a+b\log\left(cx^{-n/3}\right)\right)^2}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x}{3de^3}+\frac{x^{2/3}}{2d^2e^2}-\frac{\sqrt[3]{x}}{d^3e}+\frac{\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^4}-\frac{\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d^4}\right)}{d}\right.}{}\right.\right.$$

input
```
Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

output
```
-3*(-1/6*(x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^3) + (b*e^6*n*((-1/5*(x^(5/
3)*(a + b*Log[c/x^(n/3)])^2)/e^5 - (2*b*n*((-1/4*(b*n*(-(x^(1/3)/(d^3*e))
+ x^(2/3)/(2*d^2*e^2) - x/(3*d*e^3) + Log[d + e/x^(1/3)]/d^4 - Log[-(e/x^(
1/3))]/d^4)) + (x^(4/3)*(a + b*Log[c/x^(n/3)]))/(4*e^4))/d + ((-1/3*(b*n*(
-(x^(1/3)/(d^2*e)) + x^(2/3)/(2*d*e^2) + Log[d + e/x^(1/3)]/d^3 - Log[-(e/
x^(1/3))]/d^3)) - (x*(a + b*Log[c/x^(n/3)]))/(3*e^3))/d + ((-1/2*(b*n*(-(x
^(1/3)/(d*e)) + Log[d + e/x^(1/3)]/d^2 - Log[-(e/x^(1/3))]/d^2)) + (x^(2/3
)*(a + b*Log[c/x^(n/3)]))/(2*e^2))/d + (((b*n*Log[-(e/x^(1/3))])/d - ((d +
 e/x^(1/3))*x^(1/3)*(a + b*Log[c/x^(n/3)]))/(d*e))/d + (-((Log[1 - d*x^(1/
3)]*(a + b*Log[c/x^(n/3)]))/d) + (b*n*PolyLog[2, d*x^(1/3)])/d)/d)/d)/d)/d
))/5)/d + (((x^(4/3)*(a + b*Log[c/x^(n/3)])^2)/(4*e^4) - (b*n*((-1/3*(b*n*
(-(x^(1/3)/(d^2*e)) + x^(2/3)/(2*d*e^2) + Log[d + e/x^(1/3)]/d^3 - Log[-(e
/x^(1/3))]/d^3)) - (x*(a + b*Log[c/x^(n/3)]))/(3*e^3))/d + ((-1/2*(b*n*(-(
x^(1/3)/(d*e)) + Log[d + e/x^(1/3)]/d^2 - Log[-(e/x^(1/3))]/d^2)) + (x^(2/
3)*(a + b*Log[c/x^(n/3)]))/(2*e^2))/d + (((b*n*Log[-(e/x^(1/3))])/d - ((d
+ e/x^(1/3))*x^(1/3)*(a + b*Log[c/x^(n/3)]))/(d*e))/d + (-((Log[1 - d*x^(1
/3)]*(a + b*Log[c/x^(n/3)]))/d) + (b*n*PolyLog[2, d*x^(1/3)])/d)/d)/d)/d))
/2)/d + ((-1/3*(x*(a + b*Log[c/x^(n/3)])^2)/e^3 - (2*b*n*((-1/2*(b*n*(-(x^
(1/3)/(d*e)) + Log[d + e/x^(1/3)]/d^2 - Log[-(e/x^(1/3))]/d^2)) + (x^(2/3)
*(a + b*Log[c/x^(n/3)]))/(2*e^2))/d + (((b*n*Log[-(e/x^(1/3))])/d - ((d...
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x +
e*x^(2/3))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*x^(2/3))/x)^n) + a^3*x, x)
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**3,x)
```

output
```
Integral(x*(a + b*log(c*(d + e/x**(1/3))**n))**3, x)
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2*log((d*x^(1/3) + e)^n)^3 - integrate(1/2*(2*(b^3*d*x^2 + b^3*e
*x^(5/3))*log(x^(1/3*n))^3 - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^
2*b*d*log(c) + a^3*d)*x^2 + (b^3*d*n*x^2 - 6*(b^3*d*log(c) + a*b^2*d)*x^2
- 6*(b^3*e*log(c) + a*b^2*e)*x^(5/3) + 6*(b^3*d*x^2 + b^3*e*x^(5/3))*log(x
^(1/3*n)))*log((d*x^(1/3) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x^2 + (b
^3*e*log(c) + a*b^2*e)*x^(5/3))*log(x^(1/3*n))^2 - 2*(b^3*e*log(c)^3 + 3*a
*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(5/3) - 6*((b^3*d*log(c)^2 +
 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*d*x^2 + b^3*e*x^(5/3))*log(x^(1/3*
n))^2 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(5/3) - 2*((b^3*d*
log(c) + a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(5/3))*log(x^(1/3*n)))*
log((d*x^(1/3) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*
x^2 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(5/3))*log(x^(1/3*n)
))/(d*x + e*x^(2/3)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx=\int x\,\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)\right)^3\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(1/3))^n))^3,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(1/3))^n))^3, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x)
```

output
```
(36*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*d**5*e*n*x - 90
*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*d**2*e**4*n + 72*x
**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*d**5*e*n*x - 180*x**(
2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*d**2*e**4*n + 141*x**(2/
3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*d**2*e**4*n**2 + 36*x**(2/3)
*a**2*b*d**5*e*n*x - 90*x**(2/3)*a**2*b*d**2*e**4*n + 141*x**(2/3)*a*b**2*
d**2*e**4*n**2 - 36*x**(2/3)*b**3*d**2*e**4*n**3 + 60*x**(1/3)*log(((x**(1
/3)*d + e)**n*c)/x**(n/3))**3*b**3*d*e**5 - 45*x**(1/3)*log(((x**(1/3)*d +
 e)**n*c)/x**(n/3))**2*b**3*d**4*e**2*n*x + 180*x**(1/3)*log(((x**(1/3)*d
+ e)**n*c)/x**(n/3))**2*b**3*d*e**5*n - 90*x**(1/3)*log(((x**(1/3)*d + e)*
*n*c)/x**(n/3))*a*b**2*d**4*e**2*n*x + 360*x**(1/3)*log(((x**(1/3)*d + e)*
*n*c)/x**(n/3))*a*b**2*d*e**5*n + 18*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/
x**(n/3))*b**3*d**4*e**2*n**2*x - 462*x**(1/3)*log(((x**(1/3)*d + e)**n*c)
/x**(n/3))*b**3*d*e**5*n**2 - 45*x**(1/3)*a**2*b*d**4*e**2*n*x + 180*x**(1
/3)*a**2*b*d*e**5*n + 18*x**(1/3)*a*b**2*d**4*e**2*n**2*x - 462*x**(1/3)*a
*b**2*d*e**5*n**2 + 213*x**(1/3)*b**3*d*e**5*n**3 - 20*int(log(((x**(1/3)*
d + e)**n*c)/x**(n/3))**3/(x**(2/3)*e + d*x),x)*b**3*d*e**6 - 120*int(log(
((x**(1/3)*d + e)**n*c)/x**(n/3))/(x**(2/3)*e + d*x),x)*a*b**2*d*e**6*n +
274*int(log(((x**(1/3)*d + e)**n*c)/x**(n/3))/(x**(2/3)*e + d*x),x)*b**3*d
*e**6*n**2 - 20*int((x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**3)...
```

3.504 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 20, antiderivative size = 436

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx$$
$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$
$$+ \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$
$$- \frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}$$
$$+ \frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d}$$
$$- \frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}$$
$$+ x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3$$
$$+ \frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)}{d^3}$$
$$+ \frac{b^3e^3n^3\log(x)}{d^3} - \frac{3b^3e^3n^3\,\mathrm{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$
$$+ \frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$
$$+ \frac{6b^3e^3n^3\,\mathrm{PolyLog}\left(2,1+\frac{e}{d\sqrt[3]{x}}\right)}{d^3} + \frac{6b^3e^3n^3\,\mathrm{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$

output
```
3*b^2*e^2*n^2*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3+3*b^2*
e^3*n^2*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3-3*b*e^2*n*(d
+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^3+3/2*b*e*n*x^(2/3)*(a
+b*ln(c*(d+e/x^(1/3))^n))^2/d-3*b*e^3*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d
+e/x^(1/3))^n))^2/d^3+x*(a+b*ln(c*(d+e/x^(1/3))^n))^3+6*b^2*e^3*n^2*(a+b*l
n(c*(d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))/d^3+b^3*e^3*n^3*ln(x)/d^3-3*b^3*e^3
*n^3*polylog(2,d/(d+e/x^(1/3)))/d^3+6*b^2*e^3*n^2*(a+b*ln(c*(d+e/x^(1/3))^
n))*polylog(2,d/(d+e/x^(1/3)))/d^3+6*b^3*e^3*n^3*polylog(2,1+e/d/x^(1/3))/
d^3+6*b^3*e^3*n^3*polylog(3,d/(d+e/x^(1/3)))/d^3
```

Mathematica [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 3.38 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.90, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.850$, Rules used = {2901, 2904, 2845, 2858, 25, 27, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{2901}$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^{4/3}}d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2845}$$

$$-3\left(ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{\left(d+\frac{e}{\sqrt[3]{x}}\right)x}d\frac{1}{\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{3x}\right)$$

$$\Big\downarrow \text{2858}$$

$$-3\left(bn\int \left(d+\frac{e}{\sqrt[3]{x}}\right)x\left(a+b\log\left(cx^{n/3}\right)\right)^2d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{3x}\right)$$

$$\Big\downarrow \text{25}$$

$$-3\left(-bn\int -\left(\left(d+\frac{e}{\sqrt[3]{x}}\right)x\left(a+b\log\left(cx^{n/3}\right)\right)^2\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{3x}\right)$$

$$\Big\downarrow \text{27}$$

$$-3\left(-be^3n\int -\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{3x}\right)$$

$$\Big\downarrow \text{2789}$$

$$-3\left(-be^3n\left(\frac{\int -\frac{x\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^3}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}\right)-\frac{\left(a+b\log\left(c\right.\right.}{}\right.$$

$$\Big\downarrow \text{2756}$$

$$-3\left(-be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2}-bn\int\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{e^2}}{d}\right.\right.$$

$\downarrow$ 2789

$$-3\left(-be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\int\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)}{e^2}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\int-\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(cx^{n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}\right)}{d}\right.\right.$$

$\downarrow$ 2751

$$-3\left(-be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2}-bn\left(\frac{-\frac{bn\int-\frac{\sqrt[3]{x}}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{de}}{d}+\frac{\int-\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\right.}{e}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 16

$$-3\left(-be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\int -\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(cx^{n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+\right.}{de}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2755

$$-3\left(-be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\int -\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(cx^{n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+\right.}{de}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2754

$$-3\left(-be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\int -\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(cx^{n/3}\right)\right)}{e}d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+\right.}{de}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2779

$$-3\left(-be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\frac{bn\int\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(1-d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\log\left(1-d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{d}}{d}+\cdots\right.}{d}\right.\right.$$

$\Big\downarrow$ 2821

$$-3\left(-be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\frac{bn\int\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(1-d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d}-\frac{\log\left(1-d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{d}}{d}+\cdots\right.}{d}\right.\right.$$

$\Big\downarrow$ 2838

$$-3\left(-be^3n\left(\frac{\frac{\frac{2bn\left(\text{PolyLog}\left(2,d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\left(a+b\log\left(cx^{n/3}\right)\right)-bn\int\left(d+\frac{e}{\sqrt[3]{x}}\right)\text{PolyLog}\left(2,d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{d}-\frac{\log\left(1-d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\left(\cdots\right.}{d}}{d}\cdots}{\cdots}\right.\right.$$

$\Big\downarrow$ 7143

$$-3\left(-be^3n\left(\frac{\frac{x^{2/3}\left(a+b\log\left(cx^{n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\frac{bn\,\text{PolyLog}\left(2,d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{d}-\frac{\log\left(1-d\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\left(a+b\log\left(cx^{n/3}\right)\right)}{d}}{d}+\frac{bn\log\left(-\frac{e}{\sqrt[3]{x}}\right)}{d}-\cdots\right.}{d}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

output
```
-3*(-1/3*(a + b*Log[c*(d + e/x^(1/3))^n])^3/x - b*e^3*n*(((x^(2/3)*(a + b*
Log[c*x^(n/3)])^2)/(2*e^2) - b*n*(((b*n*Log[-(e/x^(1/3))])/d - ((d + e/x^(
1/3))*x^(1/3)*(a + b*Log[c*x^(n/3)]))/(d*e))/d + (-((Log[1 - d*(d + e/x^(1
/3))]*(a + b*Log[c*x^(n/3)]))/d) + (b*n*PolyLog[2, d*(d + e/x^(1/3))])/d)/
d))/d + ((-(((d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*x^(n/3)])^2)/(d*e)) - (2
*b*n*(-(Log[1 - (d + e/x^(1/3))/d]*(a + b*Log[c*x^(n/3)])) - b*n*PolyLog[2
, (d + e/x^(1/3))/d]))/d)/d + (-((Log[1 - d*(d + e/x^(1/3))]*(a + b*Log[c*
x^(n/3)])^2)/d) + (2*b*n*((a + b*Log[c*x^(n/3)])*PolyLog[2, d*(d + e/x^(1/
3))] + b*n*PolyLog[3, d*(d + e/x^(1/3))]))/d)/d)/d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^3,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(2/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(2/3))/x)^n) + a^3, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3,x)
```

output
```
Integral((a + b*log(c*(d + e/x**(1/3))**n))**3, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3 dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")
```

output
```
b^3*x*log((d*x^(1/3) + e)^n)^3 + 3/2*(e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 +
(d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2*x*log(c*(d + e/x^(1/3))^n))*a^2*b + a^3
*x - integrate(((b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n))^3 + (b^3*d*n*x -
3*(b^3*d*log(c) + a*b^2*d)*x + 3*(b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n))
- 3*(b^3*e*log(c) + a*b^2*e)*x^(2/3))*log((d*x^(1/3) + e)^n)^2 - 3*((b^3*d
*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n))^2
- (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2)*x - 3*((b^3*d*x + b^3*e*x^(2/3))*l
og(x^(1/3*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 2*((b^3*d*log(c)
 + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n)) + (b^3*e*
log(c)^2 + 2*a*b^2*e*log(c))*x^(2/3))*log((d*x^(1/3) + e)^n) + 3*((b^3*d*l
og(c)^2 + 2*a*b^2*d*log(c))*x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*x^(2/3
))*log(x^(1/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2)*x^(2/3))/(d*x +
e*x^(2/3)), x)
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3 dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = \int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^3 dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))^3,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^n))^3, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))^3,x)
```

output
```
(9*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*d**2*e*n + 18*x*
*(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*d**2*e*n + 9*x**(2/3)*
a**2*b*d**2*e*n - 6*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**3*b**3
*d*e**2 - 18*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*d*e**2
*n - 36*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*d*e**2*n + 1
8*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*d*e**2*n**2 - 18*x**
(1/3)*a**2*b*d*e**2*n + 18*x**(1/3)*a*b**2*d*e**2*n**2 + 2*int(log(((x**(1
/3)*d + e)**n*c)/x**(n/3))**3/(x**(2/3)*e + d*x),x)*b**3*d*e**3 + 12*int(l
og(((x**(1/3)*d + e)**n*c)/x**(n/3))/(x**(2/3)*e + d*x),x)*a*b**2*d*e**3*n
 - 18*int(log(((x**(1/3)*d + e)**n*c)/x**(n/3))/(x**(2/3)*e + d*x),x)*b**3
*d*e**3*n**2 + 2*int((x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**3)/(
x**(2/3)*e + d*x),x)*b**3*d**2*e**2 + 18*log(x**(1/3))*a**2*b*e**3*n - 54*
log(x**(1/3))*a*b**2*e**3*n**2 + 18*log(x**(1/3))*b**3*e**3*n**3 + 6*log((
(x**(1/3)*d + e)**n*c)/x**(n/3))**3*b**3*d**3*x + 18*log(((x**(1/3)*d + e)
**n*c)/x**(n/3))**2*a*b**2*d**3*x + 18*log(((x**(1/3)*d + e)**n*c)/x**(n/3
))*a**2*b*d**3*x + 18*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a**2*b*e**3 -
54*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*e**3*n + 18*log(((x**(1/3)
*d + e)**n*c)/x**(n/3))*b**3*e**3*n**2 + 6*a**3*d**3*x)/(6*d**3)
```

3.505 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx &= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) \\ &\quad - 9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt[3]{x}}\right) \\ &\quad + 18b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(3,1+\frac{e}{d\sqrt[3]{x}}\right) \\ &\quad - 18b^3n^3\operatorname{PolyLog}\left(4,1+\frac{e}{d\sqrt[3]{x}}\right)\end{aligned}$$

output

```
-3*(a+b*ln(c*(d+e/x^(1/3))^n))^3*ln(-e/d/x^(1/3))-9*b*n*(a+b*ln(c*(d+e/x^(
1/3))^n))^2*polylog(2,1+e/d/x^(1/3))+18*b^2*n^2*(a+b*ln(c*(d+e/x^(1/3))^n)
)*polylog(3,1+e/d/x^(1/3))-18*b^3*n^3*polylog(4,1+e/d/x^(1/3))
```

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2843}$$

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-3ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)}{d+\frac{e}{\sqrt[3]{x}}}d\frac{1}{\sqrt[3]{x}}\right)$$

$$\Big\downarrow \text{2881}$$

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-3bn\int\sqrt[3]{x}\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)^2d\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.$$

$\big\downarrow$ 2821

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-3bn\left(2bn\int\sqrt[3]{x}\left(a+b\log\left(cx^{-n/3}\right)\right)\text{PolyLog}\left(2,\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right.\right.\right.$$

$\big\downarrow$ 2830

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)-\right.\right.\right.$$

$\big\downarrow$ 7143

$$-3\left(\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)\left(a+b\log\left(cx^{-n/3}\right)\right)-\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
```

output
```
-3*((a + b*Log[c*(d + e/x^(1/3))^n])^3*Log[-(e/(d*x^(1/3)))] - 3*b*n*(-((a
 + b*Log[c/x^(n/3)])^2*PolyLog[2, (d + e/x^(1/3))/d]) + 2*b*n*((a + b*Log[
c/x^(n/3)])*PolyLog[3, (d + e/x^(1/3))/d] - b*n*PolyLog[4, (d + e/x^(1/3))
/d])))
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x
^(2/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(2/3))/x)^n) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x,x)
```

output
```
Integral((a + b*log(c*(d + e/x**(1/3))**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="maxima")
```

output
```
b^3*log((d*x^(1/3) + e)^n)^3*log(x) - integrate(((b^3*d*x + b^3*e*x^(2/3))
*log(x^(1/3*n))^3 + (b^3*d*n*x*log(x) - 3*(b^3*d*log(c) + a*b^2*d)*x + 3*(
b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n)) - 3*(b^3*e*log(c) + a*b^2*e)*x^(2/
3))*log((d*x^(1/3) + e)^n)^2 - 3*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(
c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n))^2 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(
c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 3*((b^3*d*x + b^3*e*x^(2/3))*log(x^(1
/3*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 2*((b^3*d*log
(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n)) + (b^3
*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(2/3))*log((d*x^(1/3) + e)^n)
+ 3*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2
*a*b^2*e*log(c) + a^2*b*e)*x^(2/3))*log(x^(1/3*n)) - (b^3*e*log(c)^3 + 3*a
*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(2/3))/(d*x^2 + e*x^(5/3)),
x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)\right)^3}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))^3/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^n))^3/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx = \left(\int \frac{\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)^3}{x}dx\right)b^3 + 3\left(\int \frac{\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)^2}{x}dx\right)a\,b^2 + 3\left(\int \frac{\log\left(\frac{\left(x^{\frac{1}{3}}d+e\right)^n c}{x^{\frac{n}{3}}}\right)}{x}dx\right)a^2b + \log(x)\,a^3$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x)
```

output
```
int(log(((x**(1/3)*d + e)**n*c)/x**(n/3))**3/x,x)*b**3 + 3*int(log(((x**(1
/3)*d + e)**n*c)/x**(n/3))**2/x,x)*a*b**2 + 3*int(log(((x**(1/3)*d + e)**n
*c)/x**(n/3))/x,x)*a**2*b + log(x)*a**3
```

3.506 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 438

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= -\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3}+\frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}$$
$$+\frac{18b^3d^2n^3}{e^2\sqrt[3]{x}}-\frac{18b^3d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{e^3}$$
$$+\frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3}$$
$$-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}$$
$$+\frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}$$
$$-\frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2e^3}$$
$$+\frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}$$
$$-\frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$
$$+\frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$
$$-\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$

output
```
-9/4*b^3*d*n^3*(d+e/x^(1/3))^2/e^3+2/9*b^3*n^3*(d+e/x^(1/3))^3/e^3-18*a*b^
2*d^2*n^2/e^2/x^(1/3)+18*b^3*d^2*n^3/e^2/x^(1/3)-18*b^3*d^2*n^2*(d+e/x^(1/
3))*ln(c*(d+e/x^(1/3))^n)/e^3+9/2*b^2*d*n^2*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e
/x^(1/3))^n))/e^3-2/3*b^2*n^2*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/
e^3+9*b*d^2*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^3-9/2*b*d*n*(d
+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^3+b*n*(d+e/x^(1/3))^3*(a+b*l
n(c*(d+e/x^(1/3))^n))^2/e^3-3*d^2*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n)
)^3/e^3+3*d*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^3-(d+e/x^(1/3)
)^3*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^3
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.52

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$$

$$= \frac{-36a^3e^3+36a^2be^3n-24ab^2e^3n^2+8b^3e^3n^3-54a^2bde^2n\sqrt[3]{x}+90ab^2de^2n^2\sqrt[3]{x}-57b^3de^2n^3\sqrt[3]{x}+108a^2b}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```

output
```
(-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*
b*d*e^2*n*x^(1/3) + 90*a*b^2*d*e^2*n^2*x^(1/3) - 57*b^3*d*e^2*n^3*x^(1/3)
+ 108*a^2*b*d^2*e*n*x^(2/3) - 396*a*b^2*d^2*e*n^2*x^(2/3) + 510*b^3*d^2*e*
n^3*x^(2/3) + 72*b^3*d^3*n^3*x*Log[d + e/x^(1/3)]^3 - 36*b^3*e^3*Log[c*(d
+ e/x^(1/3))^n]^3 - 108*a^2*b*d^3*n*x*Log[e + d*x^(1/3)] + 396*a*b^2*d^3*n
^2*x*Log[e + d*x^(1/3)] - 510*b^3*d^3*n^3*x*Log[e + d*x^(1/3)] + 12*b^2*d^
3*n^2*x*Log[d + e/x^(1/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n])*(
3*Log[e + d*x^(1/3)] - Log[x]) + 36*a^2*b*d^3*n*x*Log[x] - 132*a*b^2*d^3*n
^2*x*Log[x] + 170*b^3*d^3*n^3*x*Log[x] - 18*b^2*d^3*n^2*x*Log[d + e/x^(1/3
)]^2*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n] + 6*b*n*Log[e + d*x^(1/3
)] - 2*b*n*Log[x]) + 18*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-6*a*e^2 + 2*b*
e^2*n - 3*b*d*e*n*x^(1/3) + 6*b*d^2*n*x^(2/3)) - 6*b*d^3*n*x*Log[e + d*x^(
1/3)] + 2*b*d^3*n*x*Log[x]) - 6*b*Log[c*(d + e/x^(1/3))^n]*(18*a^2*e^3 - 6
*a*b*e*n*(2*e^2 - 3*d*e*x^(1/3) + 6*d^2*x^(2/3)) + b^2*e*n^2*(4*e^2 - 15*d
*e*x^(1/3) + 66*d^2*x^(2/3)) + 6*b*d^3*n*(6*a - 11*b*n)*x*Log[e + d*x^(1/3
)] + 2*b*d^3*n*(-6*a + 11*b*n)*x*Log[x]))/(36*e^3*x)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^{2/3}}d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2848}$$

$$-3\int\left(\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^2}-\frac{2d\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^2}+\frac{d^2\left(a+\right.}{}\right.$$

$$\downarrow \text{2009}$$

$$-3\left(\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9e^3}-\frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3}+\frac{6c}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```

output
```
-3*((3*b^3*d*n^3*(d + e/x^(1/3))^2)/(4*e^3) - (2*b^3*n^3*(d + e/x^(1/3))^3
)/(27*e^3) + (6*a*b^2*d^2*n^2)/(e^2*x^(1/3)) - (6*b^3*d^2*n^3)/(e^2*x^(1/3
)) + (6*b^3*d^2*n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^3 - (3*b^2
*d*n^2*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(2*e^3) + (2*b^
2*n^2*(d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*e^3) - (3*b*d
^2*n*(d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 + (3*b*d*n*(d
 + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*e^3) - (b*n*(d + e/
x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(3*e^3) + (d^2*(d + e/x^(1/
3))*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/e^3 - (d*(d + e/x^(1/3))^2*(a + b*
Log[c*(d + e/x^(1/3))^n])^3)/e^3 + ((d + e/x^(1/3))^3*(a + b*Log[c*(d + e/
x^(1/3))^n])^3)/(3*e^3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(384) = 768$.

Time = 0.16 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.86

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="fricas")
```

output
```
1/36*(8*b^3*e^3*n^3 - 24*a*b^2*e^3*n^2 + 36*a^2*b*e^3*n - 36*a^3*e^3 + 36*
(b^3*e^3*x - b^3*e^3)*log(c)^3 - 36*(b^3*d^3*n^3*x + b^3*e^3*n^3)*log((d*x
 + e*x^(2/3))/x)^3 + 36*(b^3*e^3*n - 3*a*b^2*e^3 - (b^3*e^3*n - 3*a*b^2*e^
3)*x)*log(c)^2 + 18*(6*b^3*d^2*e*n^3*x^(2/3) - 3*b^3*d*e^2*n^3*x^(1/3) + 2
*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + (11*b^3*d^3*n^3 - 6*a*b^2*d^3*n^2)*x - 6*
(b^3*d^3*n^2*x + b^3*e^3*n^2)*log(c))*log((d*x + e*x^(2/3))/x)^2 - 4*(2*b^
3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x - 12*(2*b^3*e^3
*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3 - (2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^
2*b*e^3)*x)*log(c) - 6*(4*b^3*e^3*n^3 - 12*a*b^2*e^3*n^2 + 18*a^2*b*e^3*n
+ 18*(b^3*d^3*n*x + b^3*e^3*n)*log(c)^2 + (85*b^3*d^3*n^3 - 66*a*b^2*d^3*n
^2 + 18*a^2*b*d^3*n)*x - 6*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + (11*b^3*d^3*n^
2 - 6*a*b^2*d^3*n)*x)*log(c) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c
) - 6*a*b^2*d^2*e*n^2)*x^(2/3) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(
c) - 6*a*b^2*d*e^2*n^2)*x^(1/3))*log((d*x + e*x^(2/3))/x) + 6*(85*b^3*d^2*
e*n^3 + 18*b^3*d^2*e*n*log(c)^2 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n -
6*(11*b^3*d^2*e*n^2 - 6*a*b^2*d^2*e*n)*log(c))*x^(2/3) - 3*(19*b^3*d*e^2*n
^3 + 18*b^3*d*e^2*n*log(c)^2 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(
5*b^3*d*e^2*n^2 - 6*a*b^2*d*e^2*n)*log(c))*x^(1/3))/(e^3*x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**2,x)
```

output
```
Integral((a + b*log(c*(d + e/x**(1/3))**n))**3/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x
^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - b^3*log(c*(d + e/x^(1/3))^n)^3/
x - 1/6*(6*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x
^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x))*log(c*(d + e/x^(1/3))^n) - (18*d^
3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)^2 - 22*d^3*x*log(x) - 66*d^2*e*x
^(2/3) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^
(1/3) + e))*n^2/(e^3*x))*a*b^2 - 1/108*(54*e*n*(6*d^3*log(d*x^(1/3) + e)/e
^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x))*l
og(c*(d + e/x^(1/3))^n)^2 + e*n*((108*d^3*x*log(d*x^(1/3) + e)^3 - 4*d^3*x
*log(x)^3 + 66*d^3*x*log(x)^2 - 510*d^3*x*log(x) - 1530*d^2*e*x^(2/3) + 17
1*d*e^2*x^(1/3) - 24*e^3 - 54*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) +
e)^2 + 18*(2*d^3*x*log(x)^2 - 22*d^3*x*log(x) + 85*d^3*x)*log(d*x^(1/3) +
e))*n^2/(e^4*x) - 18*(18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)^2 - 2
2*d^3*x*log(x) - 66*d^2*e*x^(2/3) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*d^3*x*
log(x) - 11*d^3*x)*log(d*x^(1/3) + e))*n*log(c*(d + e/x^(1/3))^n)/(e^4*x))
)*b^3 - 3*a*b^2*log(c*(d + e/x^(1/3))^n)^2/x - 3*a^2*b*log(c*(d + e/x^(1/3
))^n)/x - a^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(384) = 768.

Time = 0.22 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="giac")
```

output
```
-1/36*(36*(3*(d*x^(1/3) + e)*b^3*d^2*n^3/(e^2*x^(1/3)) - 3*(d*x^(1/3) + e)
^2*b^3*d*n^3/(e^2*x^(2/3)) + (d*x^(1/3) + e)^3*b^3*n^3/(e^2*x))*log((d*x^(
1/3) + e)/x^(1/3))^3 - 18*(2*(b^3*n^3 - 3*b^3*n^2*log(c) - 3*a*b^2*n^2)*(d
*x^(1/3) + e)^3/(e^2*x) - 9*(b^3*d*n^3 - 2*b^3*d*n^2*log(c) - 2*a*b^2*d*n^
2)*(d*x^(1/3) + e)^2/(e^2*x^(2/3)) + 18*(b^3*d^2*n^3 - b^3*d^2*n^2*log(c)
- a*b^2*d^2*n^2)*(d*x^(1/3) + e)/(e^2*x^(1/3)))*log((d*x^(1/3) + e)/x^(1/3
))^2 + 6*(2*(2*b^3*n^3 - 6*b^3*n^2*log(c) + 9*b^3*n*log(c)^2 - 6*a*b^2*n^2
 + 18*a*b^2*n*log(c) + 9*a^2*b*n)*(d*x^(1/3) + e)^3/(e^2*x) - 27*(b^3*d*n^
3 - 2*b^3*d*n^2*log(c) + 2*b^3*d*n*log(c)^2 - 2*a*b^2*d*n^2 + 4*a*b^2*d*n*
log(c) + 2*a^2*b*d*n)*(d*x^(1/3) + e)^2/(e^2*x^(2/3)) + 54*(2*b^3*d^2*n^3
- 2*b^3*d^2*n^2*log(c) + b^3*d^2*n*log(c)^2 - 2*a*b^2*d^2*n^2 + 2*a*b^2*d^
2*n*log(c) + a^2*b*d^2*n)*(d*x^(1/3) + e)/(e^2*x^(1/3)))*log((d*x^(1/3) +
e)/x^(1/3)) - 4*(2*b^3*n^3 - 6*b^3*n^2*log(c) + 9*b^3*n*log(c)^2 - 9*b^3*l
og(c)^3 - 6*a*b^2*n^2 + 18*a*b^2*n*log(c) - 27*a*b^2*log(c)^2 + 9*a^2*b*n
- 27*a^2*b*log(c) - 9*a^3)*(d*x^(1/3) + e)^3/(e^2*x) + 27*(3*b^3*d*n^3 - 6
*b^3*d*n^2*log(c) + 6*b^3*d*n*log(c)^2 - 4*b^3*d*log(c)^3 - 6*a*b^2*d*n^2
+ 12*a*b^2*d*n*log(c) - 12*a*b^2*d*log(c)^2 + 6*a^2*b*d*n - 12*a^2*b*d*log
(c) - 4*a^3*d)*(d*x^(1/3) + e)^2/(e^2*x^(2/3)) - 108*(6*b^3*d^2*n^3 - 6*b^
3*d^2*n^2*log(c) + 3*b^3*d^2*n*log(c)^2 - b^3*d^2*log(c)^3 - 6*a*b^2*d^2*n
^2 + 6*a*b^2*d^2*n*log(c) - 3*a*b^2*d^2*log(c)^2 + 3*a^2*b*d^2*n - 3*a^...
```

Mupad [B] (verification not implemented)

Time = 25.58 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.30

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= \frac{\frac{d\left(3a^3-3a^2bn+2ab^2n^2-\frac{2b^3n^3}{3}\right)}{2e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{4e}}{x^{2/3}} - \ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^3\left(\frac{b^3}{x}+\frac{b^3d^3}{e^3}\right)$$
$$- \ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2\left(\frac{b^2(3a-bn)}{x} - \frac{\frac{3b^2d(3a-bn)}{2e}-\frac{9ab^2d}{2e}}{x^{2/3}} + \frac{d\left(6ab^2d^2-11b^3d^2n\right)}{2e^3} + \frac{d\left(\frac{3b^2d(3a-bn)}{e}\right.}{e\,x^{1/3}}\right.$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))^3/x^2,x)
```

output
```
((d*(3*a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/(2*e) - (d*(6*a^3 +
 5*b^3*n^3 - 6*a*b^2*n^2))/(4*e))/x^(2/3) - log(c*(d + e/x^(1/3))^n)^3*(b^
3/x + (b^3*d^3)/e^3) - log(c*(d + e/x^(1/3))^n)^2*((b^2*(3*a - b*n))/x - (
(3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e))/x^(2/3) + (d*(6*a*b^2*d^2
 - 11*b^3*d^2*n))/(2*e^3) + (d*((3*b^2*d*(3*a - b*n))/e - (9*a*b^2*d)/e))/
(e*x^(1/3))) - (a^3 - (2*b^3*n^3)/9 + (2*a*b^2*n^2)/3 - a^2*b*n)/x - ((d*(
(d*(3*a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(6*a^3 + 5*b^
3*n^3 - 6*a*b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/e^2)/x^(1/3
) - (log(c*(d + e/x^(1/3))^n)*(((d*(b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) -
3*b*d*e*(3*a^2 - b^2*n^2)))/e + 6*b^3*d^2*n^2)/(e*x^(1/3)) - (b*d*e*(9*a^2
 + 2*b^2*n^2 - 6*a*b*n) - 3*b*d*e*(3*a^2 - b^2*n^2))/(2*e*x^(2/3)) + (b*e*
(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/(3*x)))/e - (log(d + e/x^(1/3))*(85*b^3*d^3
*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n))/(6*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.60

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x)
```

output

```
(108*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*d**2*e*n + 216
*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*d**2*e*n - 396*x**(
2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*d**2*e*n**2 + 108*x**(2/3)
*a**2*b*d**2*e*n - 396*x**(2/3)*a*b**2*d**2*e*n**2 + 510*x**(2/3)*b**3*d**
2*e*n**3 - 54*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*d*e**
2*n - 108*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*d*e**2*n +
 90*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*d*e**2*n**2 - 54*x
**(1/3)*a**2*b*d*e**2*n + 90*x**(1/3)*a*b**2*d*e**2*n**2 - 57*x**(1/3)*b**
3*d*e**2*n**3 - 36*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**3*b**3*d**3*x -
36*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**3*b**3*e**3 - 108*log(((x**(1/3)
*d + e)**n*c)/x**(n/3))**2*a*b**2*d**3*x - 108*log(((x**(1/3)*d + e)**n*c)
/x**(n/3))**2*a*b**2*e**3 + 198*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b
**3*d**3*n*x + 36*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*e**3*n - 1
08*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a**2*b*d**3*x - 108*log(((x**(1/3
)*d + e)**n*c)/x**(n/3))*a**2*b*e**3 + 396*log(((x**(1/3)*d + e)**n*c)/x**
(n/3))*a*b**2*d**3*n*x + 72*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*e
**3*n - 510*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*d**3*n**2*x - 24*lo
g(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*e**3*n**2 - 36*a**3*e**3 + 36*a**
2*b*e**3*n - 24*a*b**2*e**3*n**2 + 8*b**3*e**3*n**3)/(36*e**3*x)
```

3.507 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 907

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

output
```
-1/2*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6+18*a*b^2*d^5*n^2/e^
5/x^(1/3)+18*b^3*d^5*n^2*(d+e/x^(1/3))*ln(c*(d+e/x^(1/3))^n)/e^6-45/4*b^2*
d^4*n^2*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+20/3*b^2*d^3*n^2*(
d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-45/16*b^2*d^2*n^2*(d+e/x^(1
/3))^4*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+18/25*b^2*d*n^2*(d+e/x^(1/3))^5*(a+
b*ln(c*(d+e/x^(1/3))^n))/e^6-9*b*d^5*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3
))^n))^2/e^6+45/4*b*d^4*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^
6-10*b*d^3*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+45/8*b*d^2*
n*(d+e/x^(1/3))^4*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6-9/5*b*d*n*(d+e/x^(1/3)
)^5*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+45/8*b^3*d^4*n^3*(d+e/x^(1/3))^2/e^6
-20/9*b^3*d^3*n^3*(d+e/x^(1/3))^3/e^6+45/64*b^3*d^2*n^3*(d+e/x^(1/3))^4/e^
6-18/125*b^3*d*n^3*(d+e/x^(1/3))^5/e^6-18*b^3*d^5*n^3/e^5/x^(1/3)-1/12*b^2
*n^2*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+1/4*b*n*(d+e/x^(1/3))
^6*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+1/72*b^3*n^3*(d+e/x^(1/3))^6/e^6+3*d^
5*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6-15/2*d^4*(d+e/x^(1/3))^2
*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6+10*d^3*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x
^(1/3))^n))^3/e^6-15/2*d^2*(d+e/x^(1/3))^4*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e
^6+3*d*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

output
```
(-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^
3 - 21600*a^2*b*d*e^5*n*x^(1/3) + 15840*a*b^2*d*e^5*n^2*x^(1/3) - 4368*b^3
*d*e^5*n^3*x^(1/3) + 27000*a^2*b*d^2*e^4*n*x^(2/3) - 33300*a*b^2*d^2*e^4*n
^2*x^(2/3) + 13785*b^3*d^2*e^4*n^3*x^(2/3) - 36000*a^2*b*d^3*e^3*n*x + 684
00*a*b^2*d^3*e^3*n^2*x - 41180*b^3*d^3*e^3*n^3*x + 54000*a^2*b*d^4*e^2*n*x
^(4/3) - 156600*a*b^2*d^4*e^2*n^2*x^(4/3) + 140070*b^3*d^4*e^2*n^3*x^(4/3)
 - 108000*a^2*b*d^5*e*n*x^(5/3) + 529200*a*b^2*d^5*e*n^2*x^(5/3) - 809340*
b^3*d^5*e*n^3*x^(5/3) - 72000*b^3*d^6*n^3*x^2*Log[d + e/x^(1/3)]^3 - 36000
*b^3*e^6*Log[c*(d + e/x^(1/3))^n]^3 + 108000*a^2*b*d^6*n*x^2*Log[e + d*x^(
1/3)] - 529200*a*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)] + 809340*b^3*d^6*n^3*x
^2*Log[e + d*x^(1/3)] + 3600*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]*(-20*a + 4
9*b*n - 20*b*Log[c*(d + e/x^(1/3))^n])*(3*Log[e + d*x^(1/3)] - Log[x]) - 3
6000*a^2*b*d^6*n*x^2*Log[x] + 176400*a*b^2*d^6*n^2*x^2*Log[x] - 269780*b^3
*d^6*n^3*x^2*Log[x] + 1800*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]^2*(60*a - 14
7*b*n + 60*b*Log[c*(d + e/x^(1/3))^n] + 60*b*n*Log[e + d*x^(1/3)] - 20*b*n
*Log[x]) + 1800*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n
- 12*b*d*e^4*n*x^(1/3) + 15*b*d^2*e^3*n*x^(2/3) - 20*b*d^3*e^2*n*x + 30*b*
d^4*e*n*x^(4/3) - 60*b*d^5*n*x^(5/3)) + 60*b*d^6*n*x^2*Log[e + d*x^(1/3)]
- 20*b*d^6*n*x^2*Log[x]) - 60*b*Log[c*(d + e/x^(1/3))^n]*(1800*a^2*e^6 + b
^2*e*n^2*(100*e^5 - 264*d*e^4*x^(1/3) + 555*d^2*e^3*x^(2/3) - 1140*d^3*...
```

Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^{5/3}}d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \; 2848$$

$$-3\int\left(-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 d^5}{e^5}+\frac{5\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 d^4}{e^5}-\frac{10\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right.$$

$$\Big\downarrow \; 2009$$

$$-3\left(-\frac{b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{216e^6}+\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{6e^6}-\frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\left(d\right.}{12e^6}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

output
```
-3*((-15*b^3*d^4*n^3*(d + e/x^(1/3))^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e/x
^(1/3))^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e/x^(1/3))^4)/(64*e^6) + (6*b^3
*d*n^3*(d + e/x^(1/3))^5)/(125*e^6) - (b^3*n^3*(d + e/x^(1/3))^6)/(216*e^6
) - (6*a*b^2*d^5*n^2)/(e^5*x^(1/3)) + (6*b^3*d^5*n^3)/(e^5*x^(1/3)) - (6*b
^3*d^5*n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^6 + (15*b^2*d^4*n^2
*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*e^6) - (20*b^2*d^3
*n^2*(d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*e^6) + (15*b^2
*d^2*n^2*(d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n]))/(16*e^6) - (6
*b^2*d*n^2*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n]))/(25*e^6) +
(b^2*n^2*(d + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n]))/(36*e^6) + (3
*b*d^5*n*(d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 - (15*b*d
^4*n*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) + (10*b
*d^3*n*(d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(3*e^6) - (15
*b*d^2*n*(d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(8*e^6) + (
3*b*d*n*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(5*e^6) - (b
*n*(d + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(12*e^6) - (d^5*(
d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/e^6 + (5*d^4*(d + e/x^(
1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/(2*e^6) - (10*d^3*(d + e/x^(1/
3))^3*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/(3*e^6) + (5*d^2*(d + e/x^(1/3))
^4*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/(2*e^6) - (d*(d + e/x^(1/3))^5*(...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1404, normalized size of antiderivative = 1.55

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="fricas")
```

output
```
1/72000*(1000*b^3*e^6*n^3 - 6000*a*b^2*e^6*n^2 + 18000*a^2*b*e^6*n - 36000
*a^3*e^6 + 36000*(b^3*e^6*x^2 - b^3*e^6)*log(c)^3 + 36000*(b^3*d^6*n^3*x^2
 - b^3*e^6*n^3)*log((d*x + e*x^(2/3))/x)^3 + 20*(1800*a^3*e^6 + (2059*b^3*
d^3*e^3 - 50*b^3*e^6)*n^3 - 60*(57*a*b^2*d^3*e^3 - 5*a*b^2*e^6)*n^2 + 900*
(2*a^2*b*d^3*e^3 - a^2*b*e^6)*n)*x^2 - 18000*(2*b^3*d^3*e^3*n*x - b^3*e^6*
n + 6*a*b^2*e^6 - (6*a*b^2*e^6 + (2*b^3*d^3*e^3 - b^3*e^6)*n)*x^2)*log(c)^
2 - 1800*(20*b^3*d^3*e^3*n^3*x - 10*b^3*e^6*n^3 + 60*a*b^2*e^6*n^2 + 3*(49
*b^3*d^6*n^3 - 20*a*b^2*d^6*n^2)*x^2 - 60*(b^3*d^6*n^2*x^2 - b^3*e^6*n^2)*
log(c) + 15*(4*b^3*d^5*e*n^3*x - b^3*d^2*e^4*n^3)*x^(2/3) - 6*(5*b^3*d^4*e
^2*n^3*x - 2*b^3*d*e^5*n^3)*x^(1/3))*log((d*x + e*x^(2/3))/x)^2 - 20*(2059
*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*n)*x - 1200
*(5*b^3*e^6*n^2 - 30*a*b^2*e^6*n + 90*a^2*b*e^6 - (90*a^2*b*e^6 - (57*b^3*
d^3*e^3 - 5*b^3*e^6)*n^2 + 30*(2*a*b^2*d^3*e^3 - a*b^2*e^6)*n)*x^2 - 3*(19
*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3*n)*x)*log(c) - 60*(100*b^3*e^6*n^3 - 6
00*a*b^2*e^6*n^2 + 1800*a^2*b*e^6*n - (13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*
n^2 + 1800*a^2*b*d^6*n)*x^2 - 1800*(b^3*d^6*n*x^2 - b^3*e^6*n)*log(c)^2 -
60*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x + 60*(20*b^3*d^3*e^3*n^2*
x - 10*b^3*e^6*n^2 + 60*a*b^2*e^6*n + 3*(49*b^3*d^6*n^2 - 20*a*b^2*d^6*n)*
x^2)*log(c) + 15*(37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2 - 12*(49*b^3*d
^5*e*n^3 - 20*a*b^2*d^5*e*n^2)*x + 60*(4*b^3*d^5*e*n^2*x - b^3*d^2*e^4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 864, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="maxima")
```

output
```
1/40*a^2*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^
5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^
4*x^(1/3) - 10*e^5)/(e^6*x^2)) + 1/1200*(60*e*n*(60*d^6*log(d*x^(1/3) + e)
/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2
*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) - 10*e^5)/(e^6*x^2))*log(c*(d +
 e/x^(1/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2
 - 2940*d^6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*
d^3*e^3*x + 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6
*x^2*log(x) - 147*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^6*x^2))*a*b^2 + 1/21
6000*(5400*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^
5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^
4*x^(1/3) - 10*e^5)/(e^6*x^2))*log(c*(d + e/x^(1/3))^n)^2 + e*n*((108000*d
^6*x^2*log(d*x^(1/3) + e)^3 - 4000*d^6*x^2*log(x)^3 + 88200*d^6*x^2*log(x)
^2 - 809340*d^6*x^2*log(x) - 2428020*d^5*e*x^(5/3) + 420210*d^4*e^2*x^(4/3
) - 123540*d^3*e^3*x + 41355*d^2*e^4*x^(2/3) - 13104*d*e^5*x^(1/3) + 3000*
e^6 - 5400*(20*d^6*x^2*log(x) - 147*d^6*x^2)*log(d*x^(1/3) + e)^2 + 180*(2
00*d^6*x^2*log(x)^2 - 2940*d^6*x^2*log(x) + 13489*d^6*x^2)*log(d*x^(1/3) +
 e))*n^2/(e^7*x^2) - 180*(1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*
log(x)^2 - 2940*d^6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3)
 - 1140*d^3*e^3*x + 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 -...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1747 vs. $2(787) = 1574$.

Time = 0.23 (sec) , antiderivative size = 1747, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="giac")
```

output
```
1/72000*(36000*(6*(d*x^(1/3) + e)*b^3*d^5*n^3/(e^5*x^(1/3)) - 15*(d*x^(1/3
) + e)^2*b^3*d^4*n^3/(e^5*x^(2/3)) + 20*(d*x^(1/3) + e)^3*b^3*d^3*n^3/(e^5
*x) - 15*(d*x^(1/3) + e)^4*b^3*d^2*n^3/(e^5*x^(4/3)) + 6*(d*x^(1/3) + e)^5
*b^3*d*n^3/(e^5*x^(5/3)) - (d*x^(1/3) + e)^6*b^3*n^3/(e^5*x^2))*log((d*x^(
1/3) + e)/x^(1/3))^3 + 1800*(10*(b^3*n^3 - 6*b^3*n^2*log(c) - 6*a*b^2*n^2)
*(d*x^(1/3) + e)^6/(e^5*x^2) - 72*(b^3*d*n^3 - 5*b^3*d*n^2*log(c) - 5*a*b^
2*d*n^2)*(d*x^(1/3) + e)^5/(e^5*x^(5/3)) + 225*(b^3*d^2*n^3 - 4*b^3*d^2*n^
2*log(c) - 4*a*b^2*d^2*n^2)*(d*x^(1/3) + e)^4/(e^5*x^(4/3)) - 400*(b^3*d^3
*n^3 - 3*b^3*d^3*n^2*log(c) - 3*a*b^2*d^3*n^2)*(d*x^(1/3) + e)^3/(e^5*x) +
 450*(b^3*d^4*n^3 - 2*b^3*d^4*n^2*log(c) - 2*a*b^2*d^4*n^2)*(d*x^(1/3) + e
)^2/(e^5*x^(2/3)) - 360*(b^3*d^5*n^3 - b^3*d^5*n^2*log(c) - a*b^2*d^5*n^2)
*(d*x^(1/3) + e)/(e^5*x^(1/3)))*log((d*x^(1/3) + e)/x^(1/3))^2 - 60*(100*(
b^3*n^3 - 6*b^3*n^2*log(c) + 18*b^3*n*log(c)^2 - 6*a*b^2*n^2 + 36*a*b^2*n*
log(c) + 18*a^2*b*n)*(d*x^(1/3) + e)^6/(e^5*x^2) - 432*(2*b^3*d*n^3 - 10*b
^3*d*n^2*log(c) + 25*b^3*d*n*log(c)^2 - 10*a*b^2*d*n^2 + 50*a*b^2*d*n*log(
c) + 25*a^2*b*d*n)*(d*x^(1/3) + e)^5/(e^5*x^(5/3)) + 3375*(b^3*d^2*n^3 - 4
*b^3*d^2*n^2*log(c) + 8*b^3*d^2*n*log(c)^2 - 4*a*b^2*d^2*n^2 + 16*a*b^2*d^
2*n*log(c) + 8*a^2*b*d^2*n)*(d*x^(1/3) + e)^4/(e^5*x^(4/3)) - 4000*(2*b^3*
d^3*n^3 - 6*b^3*d^3*n^2*log(c) + 9*b^3*d^3*n*log(c)^2 - 6*a*b^2*d^3*n^2 +
18*a*b^2*d^3*n*log(c) + 9*a^2*b*d^3*n)*(d*x^(1/3) + e)^3/(e^5*x) + 1350...
```

Mupad [B] (verification not implemented)

Time = 35.06 (sec) , antiderivative size = 992, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^n))^3/x^3,x)
```

output
```
(b^3*n^3)/(72*x^2) - (b^3*log(c*(d + e/x^(1/3))^n)^3)/(2*x^2) - a^3/(2*x^2
) - (3*a*b^2*log(c*(d + e/x^(1/3))^n)^2)/(2*x^2) + (b^3*n*log(c*(d + e/x^(
1/3))^n)^2)/(4*x^2) - (b^3*n^2*log(c*(d + e/x^(1/3))^n))/(12*x^2) - (a*b^2
*n^2)/(12*x^2) + (b^3*d^6*log(c*(d + e/x^(1/3))^n)^3)/(2*e^6) - (3*a^2*b*l
og(c*(d + e/x^(1/3))^n))/(2*x^2) + (a^2*b*n)/(4*x^2) + (a*b^2*n*log(c*(d +
 e/x^(1/3))^n))/(2*x^2) + (13489*b^3*d^6*n^3*log(d + e/x^(1/3)))/(1200*e^6
) - (2059*b^3*d^3*n^3)/(3600*e^3*x) + (919*b^3*d^2*n^3)/(4800*e^2*x^(4/3))
 + (4669*b^3*d^4*n^3)/(2400*e^4*x^(2/3)) - (13489*b^3*d^5*n^3)/(1200*e^5*x
^(1/3)) + (3*a*b^2*d^6*log(c*(d + e/x^(1/3))^n)^2)/(2*e^6) - (147*b^3*d^6*
n*log(c*(d + e/x^(1/3))^n)^2)/(40*e^6) - (91*b^3*d*n^3)/(1500*e*x^(5/3)) +
 (3*a^2*b*d^6*n*log(d + e/x^(1/3)))/(2*e^6) - (3*b^3*d*n*log(c*(d + e/x^(1
/3))^n)^2)/(10*e*x^(5/3)) + (11*b^3*d*n^2*log(c*(d + e/x^(1/3))^n))/(50*e*
x^(5/3)) - (a^2*b*d^3*n)/(2*e^3*x) + (11*a*b^2*d*n^2)/(50*e*x^(5/3)) + (3*
a^2*b*d^2*n)/(8*e^2*x^(4/3)) + (3*a^2*b*d^4*n)/(4*e^4*x^(2/3)) - (3*a^2*b*
d^5*n)/(2*e^5*x^(1/3)) - (147*a*b^2*d^6*n^2*log(d + e/x^(1/3)))/(20*e^6) -
 (b^3*d^3*n*log(c*(d + e/x^(1/3))^n)^2)/(2*e^3*x) + (19*b^3*d^3*n^2*log(c*
(d + e/x^(1/3))^n))/(20*e^3*x) + (3*b^3*d^2*n*log(c*(d + e/x^(1/3))^n)^2)/
(8*e^2*x^(4/3)) - (37*b^3*d^2*n^2*log(c*(d + e/x^(1/3))^n))/(80*e^2*x^(4/3
)) + (3*b^3*d^4*n*log(c*(d + e/x^(1/3))^n)^2)/(4*e^4*x^(2/3)) - (87*b^3*d^
4*n^2*log(c*(d + e/x^(1/3))^n))/(40*e^4*x^(2/3)) - (3*b^3*d^5*n*log(c*(...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1174, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x)
```

output
```
( - 108000*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*d**5*e*n
*x + 27000*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*d**2*e**
4*n - 216000*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*d**5*e*
n*x + 54000*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*d**2*e**
4*n + 529200*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*d**5*e*n*
*2*x - 33300*x**(2/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*d**2*e**4
*n**2 - 108000*x**(2/3)*a**2*b*d**5*e*n*x + 27000*x**(2/3)*a**2*b*d**2*e**
4*n + 529200*x**(2/3)*a*b**2*d**5*e*n**2*x - 33300*x**(2/3)*a*b**2*d**2*e*
*4*n**2 - 809340*x**(2/3)*b**3*d**5*e*n**3*x + 13785*x**(2/3)*b**3*d**2*e*
*4*n**3 + 54000*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*d**
4*e**2*n*x - 21600*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**2*b**3*
d*e**5*n + 108000*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*d*
*4*e**2*n*x - 43200*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*a*b**2*
d*e**5*n - 156600*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*d**4
*e**2*n**2*x + 15840*x**(1/3)*log(((x**(1/3)*d + e)**n*c)/x**(n/3))*b**3*d
*e**5*n**2 + 54000*x**(1/3)*a**2*b*d**4*e**2*n*x - 21600*x**(1/3)*a**2*b*d
*e**5*n - 156600*x**(1/3)*a*b**2*d**4*e**2*n**2*x + 15840*x**(1/3)*a*b**2*
d*e**5*n**2 + 140070*x**(1/3)*b**3*d**4*e**2*n**3*x - 4368*x**(1/3)*b**3*d
*e**5*n**3 + 36000*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**3*b**3*d**6*x**2
 - 36000*log(((x**(1/3)*d + e)**n*c)/x**(n/3))**3*b**3*e**6 + 108000*lo...
```

3.508 $\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) \, dx = \frac{b e^5 n x^{2/3}}{4 d^5}-\frac{b e^4 n x^{4/3}}{8 d^4}+\frac{b e^3 n x^2}{12 d^3}-\frac{b e^2 n x^{8/3}}{16 d^2}+\frac{b e n x^{10/3}}{20 d}-\frac{b e^6 n \log \left(d+\frac{e}{x^{2/3}}\right)}{4 d^6}+\frac{1}{4} x^4 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{b e^6 n \log (x)}{6 d^6}$$

output
```
1/4*b*e^5*n*x^(2/3)/d^5-1/8*b*e^4*n*x^(4/3)/d^4+1/12*b*e^3*n*x^2/d^3-1/16*
b*e^2*n*x^(8/3)/d^2+1/20*b*e*n*x^(10/3)/d-1/4*b*e^6*n*ln(d+e/x^(2/3))/d^6+
1/4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))-1/6*b*e^6*n*ln(x)/d^6
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) \, dx = \frac{a x^4}{4}+\frac{1}{4} b x^4 \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{6} b e n \left(\frac{3 e^4 x^{2/3}}{2 d^5}-\frac{3 e^3 x^{4/3}}{4 d^4}+\frac{e^2 x^2}{2 d^3}-\frac{3 e x^{8/3}}{8 d^2}+\frac{3 x^{10/3}}{10 d}-\frac{3 e^5 \log \left(d+\frac{e}{x^{2/3}}\right)}{2 d^6}-\frac{e^5 \log (x)}{d^6}\right)$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

output
```
(a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(2/3))^n])/4 + (b*e*n*((3*e^4*x^(2/3))/(
2*d^5) - (3*e^3*x^(4/3))/(4*d^4) + (e^2*x^2)/(2*d^3) - (3*e*x^(8/3))/(8*d^
2) + (3*x^(10/3))/(10*d) - (3*e^5*Log[d + e/x^(2/3)])/(2*d^6) - (e^5*Log[x
])/d^6))/6
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2}\int x^{14/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2842}$$

$$-\frac{3}{2}\left(\frac{1}{6}ben\int \frac{x^4}{d+\frac{e}{x^{2/3}}}d\frac{1}{x^{2/3}}-\frac{1}{6}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\right)$$

$$\Big\downarrow \text{54}$$

$$-\frac{3}{2}\left(\frac{1}{6}ben\int \left(\frac{e^6}{d^6\left(d+\frac{e}{x^{2/3}}\right)}-\frac{x^{2/3}e^5}{d^6}+\frac{x^{4/3}e^4}{d^5}-\frac{x^2e^3}{d^4}+\frac{x^{8/3}e^2}{d^3}-\frac{x^{10/3}e}{d^2}+\frac{x^4}{d}\right)d\frac{1}{x^{2/3}}-\frac{1}{6}x^4\left(a+b\log\left(c\left(\right.\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}\left(\frac{1}{6}ben\left(\frac{e^5\log\left(d+\frac{e}{x^{2/3}}\right)}{d^6}-\frac{e^5\log\left(\frac{1}{x^{2/3}}\right)}{d^6}-\frac{e^4x^{2/3}}{d^5}+\frac{e^3x^{4/3}}{2d^4}-\frac{e^2x^2}{3d^3}+\frac{ex^{8/3}}{4d^2}-\frac{x^{10/3}}{5d}\right)-\frac{1}{6}x^4\left(a+b\log\left(\right.\right.\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

output
```
(-3*(-1/6*(x^4*(a + b*Log[c*(d + e/x^(2/3))^n])) + (b*e*n*(-((e^4*x^(2/3))
/d^5) + (e^3*x^(4/3))/(2*d^4) - (e^2*x^2)/(3*d^3) + (e*x^(8/3))/(4*d^2) -
x^(10/3)/(5*d) + (e^5*Log[d + e/x^(2/3)])/d^6 - (e^5*Log[x^(-2/3)])/d^6))/
6))/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3 \left(a + b\ln\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right) dx$$

input
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

output
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{60\,bd^6x^4\log(c)+60\,ad^6x^4+20\,bd^3e^3nx^2-120\,bd^6n\log\left(x^{\frac{1}{3}}\right)+60\,(bd^6-be$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

output
```
1/240*(60*b*d^6*x^4*log(c) + 60*a*d^6*x^4 + 20*b*d^3*e^3*n*x^2 - 120*b*d^6
*n*log(x^(1/3)) + 60*(b*d^6 - b*e^6)*n*log(d*x^(2/3) + e) + 60*(b*d^6*n*x^
4 - b*d^6*n)*log((d*x + e*x^(1/3))/x) - 15*(b*d^4*e^2*n*x^2 - 4*b*d*e^5*n)
*x^(2/3) + 6*(2*b*d^5*e*n*x^3 - 5*b*d^2*e^4*n*x)*x^(1/3))/d^6
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{4}\,ax^4-\frac{1}{240}\,ben\left(\frac{60\,e^5\log\left(dx^{\frac{2}{3}}+e\right)}{d^6}-\frac{12\,d^4x^{\frac{10}{3}}-15\,d^3ex^{\frac{8}{3}}+20\,d^2e^2x^2-30\,de^3x^{\frac{4}{3}}+60\,e^4x^{\frac{2}{3}}}{d^5}\right)$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

output
```
1/4*b*x^4*log(c*(d + e/x^(2/3))^n) + 1/4*a*x^4 - 1/240*b*e*n*(60*e^5*log(d
*x^(2/3) + e)/d^6 - (12*d^4*x^(10/3) - 15*d^3*e*x^(8/3) + 20*d^2*e^2*x^2 -
 30*d*e^3*x^(4/3) + 60*e^4*x^(2/3))/d^5)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.73

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{4}bx^4\log(c)+\frac{1}{4}ax^4$$
$$+\frac{1}{240}\left(60\,x^4\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)-e\left(\frac{60\,e^5\log\left(\left|dx^{\frac{2}{3}}+e\right|\right)}{d^6}-\frac{12\,d^4x^{\frac{10}{3}}-15\,d^3ex^{\frac{8}{3}}+20\,d^2e^2x^2-30\,de^3x^{\frac{4}{3}}+6}{d^5}\right.\right.$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

output
```
1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*(60*x^4*log(d + e/x^(2/3)) - e*(60*e^
5*log(abs(d*x^(2/3) + e))/d^6 - (12*d^4*x^(10/3) - 15*d^3*e*x^(8/3) + 20*d
^2*e^2*x^2 - 30*d*e^3*x^(4/3) + 60*e^4*x^(2/3))/d^5))*b*n
```

Mupad [B] (verification not implemented)

Time = 25.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int x^3\Bigg(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\Bigg)\,dx=\frac{x^{10/3}\left(\frac{b\,e\,n}{5\,d}-\frac{b\,e^2\,n}{4\,d^2\,x^{2/3}}-\frac{b\,e^4\,n}{2\,d^4\,x^2}+\frac{b\,e^3\,n}{3\,d^3\,x^{4/3}}+\frac{b\,e^5\,n}{d^5\,x^{8/3}}\right)}{4}$$
$$+\frac{a\,x^4}{4}+\frac{b\,x^4\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{4}-\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{2/3}}+1\right)}{2\,d^6}$$

input
```
int(x^3*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

output
```
(x^(10/3)*((b*e*n)/(5*d) - (b*e^2*n)/(4*d^2*x^(2/3)) - (b*e^4*n)/(2*d^4*x^
2) + (b*e^3*n)/(3*d^3*x^(4/3)) + (b*e^5*n)/(d^5*x^(8/3))))/4 + (a*x^4)/4 +
 (b*x^4*log(c*(d + e/x^(2/3))^n))/4 - (b*e^6*n*atanh((2*e)/(d*x^(2/3)) + 1
))/(2*d^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{-15x^{\frac{8}{3}}b\,d^4e^2n+60x^{\frac{2}{3}}bd\,e^5n+12x^{\frac{10}{3}}b\,d^5en-30x^{\frac{4}{3}}b\,d^2e^4n-120\log\left(x^{\frac{1}{3}}\right)b\,e^{6}$$

input
```
int(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x)
```

output
```
( - 15*x**(2/3)*b*d**4*e**2*n*x**2 + 60*x**(2/3)*b*d*e**5*n + 12*x**(1/3)*
b*d**5*e*n*x**3 - 30*x**(1/3)*b*d**2*e**4*n*x - 120*log(x**(1/3))*b*e**6*n
 + 60*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b*d**6*x**4 - 60*log(((x**
(2/3)*d + e)**n*c)/x**((2*n)/3))*b*e**6 + 60*a*d**6*x**4 + 20*b*d**3*e**3*
n*x**2)/(240*d**6)
```

3.509 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}+\frac{2be^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

output
```
-2/3*b*e^4*n*x^(1/3)/d^4+2/9*b*e^3*n*x/d^3-2/15*b*e^2*n*x^(5/3)/d^2+2/21*b
*e*n*x^(7/3)/d+2/3*b*e^(9/2)*n*arctan(d^(1/2)*x^(1/3)/e^(1/2))/d^(9/2)+1/3
*x^3*(a+b*ln(c*(d+e/x^(2/3))^n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.54

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{ax^3}{3}+\frac{2benx^{7/3}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\frac{e}{dx^{2/3}}\right)}{21d}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

output
```
(a*x^3)/3 + (2*b*e*n*x^(7/3)*Hypergeometric2F1[-7/2, 1, -5/2, -(e/(d*x^(2/
3)))])/(21*d) + (b*x^3*Log[c*(d + e/x^(2/3))^n])/3
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2905, 795, 864, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx$$

$$\Big\downarrow \text{2905}$$

$$\frac{2}{9}ben\int \frac{x^{4/3}}{d+\frac{e}{x^{2/3}}}dx+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$\Big\downarrow \text{795}$$

$$\frac{2}{9}ben\int \frac{x^2}{x^{2/3}d+e}dx+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$\Big\downarrow \text{864}$$

$$\frac{2}{3}ben\int \frac{x^{8/3}}{x^{2/3}d+e}d\sqrt[3]{x}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{2}{3}ben\int \left(\frac{e^4}{d^4\left(x^{2/3}d+e\right)}-\frac{e^3}{d^4}+\frac{x^{2/3}e^2}{d^3}-\frac{x^{4/3}e}{d^2}+\frac{x^2}{d}\right)d\sqrt[3]{x}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{2}{3}ben\left(\frac{e^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{9/2}}-\frac{e^3\sqrt[3]{x}}{d^4}+\frac{e^2x}{3d^3}-\frac{ex^{5/3}}{5d^2}+\frac{x^{7/3}}{7d}\right)$$

input
```
Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

output
```
(2*b*e*n*(-((e^3*x^(1/3))/d^4) + (e^2*x)/(3*d^3) - (e*x^(5/3))/(5*d^2) + x
^(7/3)/(7*d) + (e^(7/2)*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/d^(9/2)))/3 + (
x^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/3
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int x^2 \left(a + b \ln \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right) dx$$

input
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

output
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.30

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right) dx = \left[\frac{105\, bd^4 x^3 \log (c) + 105\, ad^4 x^3 - 42\, bd^2 e^2 n x^{\frac{5}{3}} + 105\, be^4 n \sqrt{-\frac{e}{d}} \log \left(\frac{d^3x^2 - 2\, d^2 e \cdots}{\cdots} \right.}{\cdots} \right.$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

output
```
[1/315*(105*b*d^4*x^3*log(c) + 105*a*d^4*x^3 - 42*b*d^2*e^2*n*x^(5/3) + 10
5*b*e^4*n*sqrt(-e/d)*log((d^3*x^2 - 2*d^2*e*x*sqrt(-e/d) - e^3 + 2*(d^3*x*
sqrt(-e/d) + d*e^2)*x^(2/3) - 2*(d^2*e*x - d*e^2*sqrt(-e/d))*x^(1/3))/(d^3
*x^2 + e^3)) + 70*b*d*e^3*n*x + 105*b*d^4*n*log(d*x^(2/3) + e) - 210*b*d^4
*n*log(x^(1/3)) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + e*x^(1/3))/x) + 3
0*(b*d^3*e*n*x^2 - 7*b*e^4*n)*x^(1/3))/d^4, 1/315*(105*b*d^4*x^3*log(c) +
105*a*d^4*x^3 - 42*b*d^2*e^2*n*x^(5/3) + 210*b*e^4*n*sqrt(e/d)*arctan(d*x^
(1/3)*sqrt(e/d)/e) + 70*b*d*e^3*n*x + 105*b*d^4*n*log(d*x^(2/3) + e) - 210
*b*d^4*n*log(x^(1/3)) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + e*x^(1/3))/
x) + 30*(b*d^3*e*n*x^2 - 7*b*e^4*n)*x^(1/3))/d^4]
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3$$
$$+\frac{1}{315}\left(105\,x^3\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+2\,e\left(\frac{105\,e^4\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}d^4}+\frac{15\,d^6x^{\frac{7}{3}}-21\,d^5ex^{\frac{5}{3}}+35\,d^4e^2x-105\,d^3e^3x^{\frac{1}{3}}}{d^7}\right)\right)$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

output
```
1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/315*(105*x^3*log(d + e/x^(2/3)) + 2*e*(10
5*e^4*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d^4) + (15*d^6*x^(7/3) - 21*d
^5*e*x^(5/3) + 35*d^4*e^2*x - 105*d^3*e^3*x^(1/3))/d^7))*b*n
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\int x^2\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

output
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int x^2\Big(a+b\log\left(c\Big(d+\frac{e}{x^{2/3}}\Big)^n\right)\Big)\,dx=\frac{210\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)b\,e^4n-42x^{\frac{5}{3}}b\,d^3e^2n+30x^{\frac{7}{3}}b\,d^4en-210x^{\frac{1}{3}}bd\,e^4n+}{315d^5}$$

input
```
int(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x)
```

output
```
(210*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*b*e**4*n - 42*x**
*(2/3)*b*d**3*e**2*n*x + 30*x**(1/3)*b*d**4*e*n*x**2 - 210*x**(1/3)*b*d*e*
*4*n + 105*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b*d**5*x**3 + 105*a*d
**5*x**3 + 70*b*d**2*e**3*n*x)/(315*d**5)
```

3.510 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=-\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^3n\log(x)}{3d^3}$$

output
```
-1/2*b*e^2*n*x^(2/3)/d^2+1/4*b*e*n*x^(4/3)/d+1/2*b*e^3*n*ln(d+e/x^(2/3))/d
^3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))+1/3*b*e^3*n*ln(x)/d^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{3}ben\left(-\frac{3ex^{2/3}}{2d^2}+\frac{3x^{4/3}}{4d}+\frac{3e^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{e^2\log(x)}{d^3}\right)$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

output
```
(a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(2/3))^n])/2 + (b*e*n*((-3*e*x^(2/3))/(2
*d^2) + (3*x^(4/3))/(4*d) + (3*e^2*Log[d + e/x^(2/3)])/(2*d^3) + (e^2*Log[
x])/d^3))/3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2904, 2842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) \, dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2} \int x^{8/3} \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2842}$$

$$-\frac{3}{2} \left(\frac{1}{3} b e n \int \frac{x^2}{d+\frac{e}{x^{2/3}}}d\frac{1}{x^{2/3}}-\frac{1}{3} x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\right)$$

$$\Big\downarrow \text{54}$$

$$-\frac{3}{2} \left(\frac{1}{3} b e n \int \left(-\frac{e^3}{d^3 \left(d+\frac{e}{x^{2/3}}\right)}+\frac{x^{2/3} e^2}{d^3}-\frac{x^{4/3} e}{d^2}+\frac{x^2}{d}\right)d\frac{1}{x^{2/3}}-\frac{1}{3} x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2} \left(\frac{1}{3} b e n \left(-\frac{e^2 \log \left(d+\frac{e}{x^{2/3}}\right)}{d^3}+\frac{e^2 \log \left(\frac{1}{x^{2/3}}\right)}{d^3}+\frac{e x^{2/3}}{d^2}-\frac{x^{4/3}}{2 d}\right)-\frac{1}{3} x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\right)$$

input
```
Int[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

output
```
(-3*(-1/3*(x^2*(a + b*Log[c*(d + e/x^(2/3))^n])) + (b*e*n*((e*x^(2/3))/d^2
 - x^(4/3)/(2*d) - (e^2*Log[d + e/x^(2/3)])/d^3 + (e^2*Log[x^(-2/3)])/d^3)
)/3))/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c{\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}^{n}\right)\right)dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{2\,bd^3x^2\log\left(c\right)+bd^2enx^{\frac{4}{3}}+2\,ad^3x^2-4\,bd^3n\log\left(x^{\frac{1}{3}}\right)-2\,bde^2nx^{\frac{2}{3}}+2\,(bd^3 \cdots}{4\,d^3}$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

output
```
1/4*(2*b*d^3*x^2*log(c) + b*d^2*e*n*x^(4/3) + 2*a*d^3*x^2 - 4*b*d^3*n*log(
x^(1/3)) - 2*b*d*e^2*n*x^(2/3) + 2*(b*d^3 + b*e^3)*n*log(d*x^(2/3) + e) +
2*(b*d^3*n*x^2 - b*d^3*n)*log((d*x + e*x^(1/3))/x))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.67

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{4}\,ben\left(\frac{2\,e^2\log\left(dx^{\frac{2}{3}}+e\right)}{d^3}+\frac{dx^{\frac{4}{3}}-2\,ex^{\frac{2}{3}}}{d^2}\right)+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

output
```
1/4*b*e*n*(2*e^2*log(d*x^(2/3) + e)/d^3 + (d*x^(4/3) - 2*e*x^(2/3))/d^2) +
 1/2*b*x^2*log(c*(d + e/x^(2/3))^n) + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{4}\left(2\,x^2\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+e\left(\frac{2\,e^2\log\left(\left|dx^{\frac{2}{3}}+e\right|\right)}{d^3}+\frac{dx^{\frac{4}{3}}-2\,ex^{\frac{2}{3}}}{d^2}\right)\right)bn+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

output
```
1/2*b*x^2*log(c) + 1/4*(2*x^2*log(d + e/x^(2/3)) + e*(2*e^2*log(abs(d*x^(2
/3) + e))/d^3 + (d*x^(4/3) - 2*e*x^(2/3))/d^2))*b*n + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 25.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{x^{4/3}\left(\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n}{d^2\,x^{2/3}}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2}+\frac{b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{2/3}}+1\right)}{d^3}$$

input
```
int(x*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

output
```
(x^(4/3)*((b*e*n)/(2*d) - (b*e^2*n)/(d^2*x^(2/3))))/2 + (a*x^2)/2 + (b*x^2
*log(c*(d + e/x^(2/3))^n))/2 + (b*e^3*n*atanh((2*e)/(d*x^(2/3)) + 1))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{-2x^{\frac{2}{3}}bd\,e^2n+x^{\frac{4}{3}}b\,d^2en+4\log\left(x^{\frac{1}{3}}\right)b\,e^3n+2\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^nc}{x^{\frac{2n}{3}}}\right)b\,d^3x^2+2\log$$

$$4d^3$$

input
```
int(x*(a+b*log(c*(d+e/x^(2/3))^n)),x)
```

output
```
( - 2*x**(2/3)*b*d*e**2*n + x**(1/3)*b*d**2*e*n*x + 4*log(x**(1/3))*b*e**3
*n + 2*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b*d**3*x**2 + 2*log(((x**
(2/3)*d + e)**n*c)/x**((2*n)/3))*b*e**3 + 2*a*d**3*x**2)/(4*d**3)
```

3.511 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{2ben\sqrt[3]{x}}{d}+ax-\frac{2be^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

output
```
2*b*e*n*x^(1/3)/d+a*x-2*b*e^(3/2)*n*arctan(d^(1/2)*x^(1/3)/e^(1/2))/d^(3/2
)+b*x*ln(c*(d+e/x^(2/3))^n)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=ax+\frac{2ben\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{e}{dx^{2/3}}\right)}{d}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

input
```
Integrate[a + b*Log[c*(d + e/x^(2/3))^n],x]
```

output
```
a*x + (2*b*e*n*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))])/
d + b*x*Log[c*(d + e/x^(2/3))^n]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx$$

$$\Big\downarrow \ 2009$$

$$ax-\frac{2be^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2ben\sqrt[3]{x}}{d}$$

input
```
Int[a + b*Log[c*(d + e/x^(2/3))^n],x]
```

output
```
(2*b*e*n*x^(1/3))/d + a*x - (2*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e
]])/d^(3/2) + b*x*Log[c*(d + e/x^(2/3))^n]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(51) = 102$.

Time = 1.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.37

method	result
default	$ax+b\left(x\ln\left(c\left(\frac{e+d\,x^{\frac{2}{3}}}{x^{\frac{2}{3}}}\right)^{n}\right)+\frac{2en\left(\frac{e\arctan\left(\frac{d^{2}x}{e\sqrt{de}}\right)}{d\sqrt{de}}+\frac{3x^{\frac{1}{3}}}{d}-\frac{2e\arctan\left(\frac{d\,x^{\frac{1}{3}}}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{e\arctan\left(\frac{\sqrt{3}\,\sqrt{d}\,\sqrt{e}-2d\,x^{\frac{1}{3}}}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{e\arctan\left(2c\right.}{}\right.}{3}\right.$
parts	$ax+b\left(x\ln\left(c\left(\frac{e+d\,x^{\frac{2}{3}}}{x^{\frac{2}{3}}}\right)^{n}\right)+\frac{2en\left(\frac{e\arctan\left(\frac{d^{2}x}{e\sqrt{de}}\right)}{d\sqrt{de}}+\frac{3x^{\frac{1}{3}}}{d}-\frac{2e\arctan\left(\frac{d\,x^{\frac{1}{3}}}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{e\arctan\left(\frac{\sqrt{3}\,\sqrt{d}\,\sqrt{e}-2d\,x^{\frac{1}{3}}}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{e\arctan\left(2c\right.}{}\right.}{3}\right.$

input
```
int(a+b*ln(c*(d+e/x^(2/3))^n),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*ln(c*((e+d*x^(2/3))/x^(2/3))^n)+2/3*e*n*(e/d/(d*e)^(1/2)*arctan(d
^2*x/e/(d*e)^(1/2))+3/d*x^(1/3)-2/d*e/(d*e)^(1/2)*arctan(d*x^(1/3)/(d*e)^(
1/2))+1/d*e/(d*e)^(1/2)*arctan((3^(1/2)*d^(1/2)*e^(1/2)-2*d*x^(1/3))/(d*e)
^(1/2))-1/d*e/(d*e)^(1/2)*arctan((2*d*x^(1/3)+3^(1/2)*d^(1/2)*e^(1/2))/(d*
e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.29

$$\int \left(a + b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) dx = \left[\frac{ben\sqrt{-\frac{e}{d}}\log\left(\frac{d^3x^2+2d^2ex\sqrt{-\frac{e}{d}}-e^3-2\left(d^3x\sqrt{-\frac{e}{d}}-de^2\right)x^{\frac{2}{3}}-2\left(d^2ex+de^2\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{d^3x^2+e^3}\right)+bd\cdots}{\cdots}\right.$$

$$-\frac{2ben\sqrt{\frac{e}{d}}\arctan\left(\frac{dx^{\frac{1}{3}}\sqrt{\frac{e}{d}}}{e}\right)-bdn\log\left(dx^{\frac{2}{3}}+e\right)-bdx\log(c)+2bdn\log\left(x^{\frac{1}{3}}\right)-2benx^{\frac{1}{3}}-adx-(bdn\cdots}{d}$$

input
```
integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="fricas")
```

output
```
[(b*e*n*sqrt(-e/d)*log((d^3*x^2 + 2*d^2*e*x*sqrt(-e/d) - e^3 - 2*(d^3*x*sq
rt(-e/d) - d*e^2)*x^(2/3) - 2*(d^2*e*x + d*e^2*sqrt(-e/d))*x^(1/3))/(d^3*x
^2 + e^3)) + b*d*n*log(d*x^(2/3) + e) + b*d*x*log(c) - 2*b*d*n*log(x^(1/3)
) + 2*b*e*n*x^(1/3) + a*d*x + (b*d*n*x - b*d*n)*log((d*x + e*x^(1/3))/x))/
d, -(2*b*e*n*sqrt(e/d)*arctan(d*x^(1/3)*sqrt(e/d)/e) - b*d*n*log(d*x^(2/3)
 + e) - b*d*x*log(c) + 2*b*d*n*log(x^(1/3)) - 2*b*e*n*x^(1/3) - a*d*x - (b
*d*n*x - b*d*n)*log((d*x + e*x^(1/3))/x))/d]
```

Sympy [A] (verification not implemented)

Time = 13.84 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx = ax + b\left(\frac{2en\left(\frac{3\sqrt[3]{x}}{d}-\frac{3e\,\mathrm{atan}\left(\frac{\sqrt[3]{x}}{\sqrt{\frac{e}{d}}}\right)}{d^2\sqrt{\frac{e}{d}}}\right)}{3}+x\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)$$

input
```
integrate(a+b*ln(c*(d+e/x**(2/3))**n),x)
```

output
```
a*x + b*(2*e*n*(3*x**(1/3)/d - 3*e*atan(x**(1/3)/sqrt(e/d))/(d**2*sqrt(e/d
)))/3 + x*log(c*(d + e/x**(2/3))**n))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)dx = \text{Exception raised: ValueError}$$

input
```
integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx = -\left(\left(2\,e\left(\frac{e\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}d}-\frac{x^{\frac{1}{3}}}{d}\right)-x\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)n-x\log\left(c\right)\right)b+ax$$

input
```
integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="giac")
```

output
```
-((2*e*(e*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d) - x^(1/3)/d) - x*log(d
 + e/x^(2/3)))*n - x*log(c))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 25.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx = a\,x + b\,x\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2\,b\,e\,n\,x^{1/3}}{d}-\frac{2\,b\,e^{3/2}\,n\,\mathrm{atan}\left(\frac{\sqrt{d}\,x^{1/3}}{\sqrt{e}}\right)}{d^{3/2}}$$

input
```
int(a + b*log(c*(d + e/x^(2/3))^n),x)
```

output
```
a*x + b*x*log(c*(d + e/x^(2/3))^n) + (2*b*e*n*x^(1/3))/d - (2*b*e^(3/2)*n*
atan((d^(1/2)*x^(1/3))/e^(1/2)))/d^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{-2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)ben+2x^{\frac{1}{3}}bden+\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)b\,d^2x+a\,d^2x}{d^2}$$

input
```
int(a+b*log(c*(d+e/x^(2/3))^n),x)
```

output
```
( - 2*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*b*e*n + 2*x**(1
/3)*b*d*e*n + log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b*d**2*x + a*d**2*
x)/d**2
```

3.512 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx = -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{3}{2}bn\,\text{PolyLog}\left(2,1+\frac{e}{dx^{2/3}}\right)$$

output
```
-3/2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))-3/2*b*n*polylog(2,1+e/d/
x^(2/3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx = a\log(x) - \frac{3}{2}b\left(\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\log\left(-\frac{e}{dx^{2/3}}\right)+n\,\text{PolyLog}\left(2,\frac{d+\frac{e}{x^{2/3}}}{d}\right)\right)$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
```

output
```
a*Log[x] - (3*b*(Log[c*(d + e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] + n*PolyLo
g[2, (d + e/x^(2/3))/d]))/2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2}\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2841}$$

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-ben\int\frac{\log\left(-\frac{e}{dx^{2/3}}\right)}{d+\frac{e}{x^{2/3}}}d\frac{1}{x^{2/3}}\right)$$

$$\Big\downarrow \text{2752}$$

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
```

output
```
(-3*((a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))] + b*n*PolyLog[
2, 1 + e/(d*x^(2/3))]))/2
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))/x,x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*((d*x + e*x^(1/3))/x)^n) + a)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(44) = 88.

Time = 0.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.31

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx=-\frac{3}{2}\left(2\log\left(\frac{dx^{\frac{2}{3}}}{e}+1\right)\log\left(x^{\frac{1}{3}}\right)+\mathrm{Li}_2\left(-\frac{dx^{\frac{2}{3}}}{e}\right)\right)bn+\frac{2\,ben\log\left(x\right)^2+6\,bdnx^{\frac{2}{3}}\log\left(x\right)+6\,be\log\left(\left(dx^{\frac{2}{3}}+e\right)^n\right)\log\left(x\right)-12\,be\log\left(x\right)\log\left(x^{\frac{1}{3}n}\right)-9\,bdnx^{\frac{2}{3}}+6\,(}{6\,e}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="maxima")
```

output
```
-3/2*(2*log(d*x^(2/3)/e + 1)*log(x^(1/3)) + dilog(-d*x^(2/3)/e))*b*n + 1/6
*(2*b*e*n*log(x)^2 + 6*b*d*n*x^(2/3)*log(x) + 6*b*e*log((d*x^(2/3) + e)^n)
*log(x) - 12*b*e*log(x)*log(x^(1/3*n)) - 9*b*d*n*x^(2/3) + 6*(b*e*log(c) +
 a*e)*log(x) - 3*(2*b*d*n*x*log(x) - 3*b*d*n*x)/x^(1/3))/e
```

Giac [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx=\int \frac{a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))/x, x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx=\left(\int \frac{\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)}{x}dx\right)b+\log(x)\,a$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))/x,x)
```

output
```
int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))/x,x)*b + log(x)*a
```

3.513 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = \frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}} - \frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

output
```
2/3*b*n/x-2*b*d*n/e/x^(1/3)-2*b*d^(3/2)*n*arctan(d^(1/2)*x^(1/3)/e^(1/2))/
e^(3/2)-(a+b*ln(c*(d+e/x^(2/3))^n))/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = -\frac{a}{x}+\frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}} + \frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
```

output
```
-(a/x) + (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) + (2*b*d^(3/2)*n*ArcTan[Sqr
t[e]/(Sqrt[d]*x^(1/3))])/e^(3/2) - (b*Log[c*(d + e/x^(2/3))^n])/x
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2905, 795, 864, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}dx$$

$$\Big\downarrow \text{ 2905}$$

$$-\frac{2}{3}ben\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{8/3}}dx-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

$$\Big\downarrow \text{ 795}$$

$$-\frac{2}{3}ben\int \frac{1}{\left(x^{2/3}d+e\right)x^2}dx-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

$$\Big\downarrow \text{ 864}$$

$$-2ben\int \frac{1}{\left(x^{2/3}d+e\right)x^{4/3}}d\sqrt[3]{x}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

$$\Big\downarrow \text{ 264}$$

$$-2ben\left(-\frac{d\int \frac{1}{\left(x^{2/3}d+e\right)x^{2/3}}d\sqrt[3]{x}}{e}-\frac{1}{3ex}\right)-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

$$\Big\downarrow \text{ 264}$$

$$-2ben\left(-\frac{d\left(-\frac{d\int \frac{1}{x^{2/3}d+e}d\sqrt[3]{x}}{e}-\frac{1}{e\sqrt[3]{x}}\right)}{e}-\frac{1}{3ex}\right)-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-2ben\left(-\frac{d\left(-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{1}{e\sqrt[3]{x}}\right)}{e}-\frac{1}{3ex}\right)$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
```

output
```
-2*b*e*n*(-1/3*1/(e*x) - (d*(-(1/(e*x^(1/3))) - (Sqrt[d]*ArcTan[(Sqrt[d]*x
^(1/3))/Sqrt[e]])/e^(3/2)))/e) - (a + b*Log[c*(d + e/x^(2/3))^n])/x
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))/x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.05

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)}{x^{2}}\,dx = \left[\frac{3\,bdnx\sqrt{-\frac{d}{e}}\log\left(\frac{d^{3}x^{2}+2\,de^{2}x\sqrt{-\frac{d}{e}}-e^{3}-2\left(d^{2}ex\sqrt{-\frac{d}{e}}-de^{2}\right)x^{\frac{2}{3}}-2\left(d^{2}ex+e^{3}\sqrt{-\frac{d}{e}}\right)x^{\frac{1}{3}}}{d^{3}x^{2}+e^{3}}\right)}{3\,ex}\right.$$

$$\left.-\frac{6\,bdnx\sqrt{\frac{d}{e}}\arctan\left(x^{\frac{1}{3}}\sqrt{\frac{d}{e}}\right)+3\,ben\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)+6\,bdnx^{\frac{2}{3}}-2\,ben+3\,be\log\left(c\right)+3\,ae}{3\,ex}\right]$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="fricas")
```

output
```
[1/3*(3*b*d*n*x*sqrt(-d/e)*log((d^3*x^2 + 2*d*e^2*x*sqrt(-d/e) - e^3 - 2*(
d^2*e*x*sqrt(-d/e) - d*e^2)*x^(2/3) - 2*(d^2*e*x + e^3*sqrt(-d/e))*x^(1/3)
)/(d^3*x^2 + e^3)) - 3*b*e*n*log((d*x + e*x^(1/3))/x) - 6*b*d*n*x^(2/3) +
2*b*e*n - 3*b*e*log(c) - 3*a*e)/(e*x), -1/3*(6*b*d*n*x*sqrt(d/e)*arctan(x^
(1/3)*sqrt(d/e)) + 3*b*e*n*log((d*x + e*x^(1/3))/x) + 6*b*d*n*x^(2/3) - 2*
b*e*n + 3*b*e*log(c) + 3*a*e)/(e*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x^2}\, dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = -\frac{1}{3}\left(2\,e\left(\frac{3\,d^2\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}e^2}+\frac{3\,dx^{\frac{2}{3}}-e}{e^2x}\right)+\frac{3\,\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}{x}\right)bn-\frac{b\log\left(c\right)}{x}-\frac{a}{x}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="giac")
```

output
```
-1/3*(2*e*(3*d^2*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^2) + (3*d*x^(2/3
) - e)/(e^2*x)) + 3*log(d + e/x^(2/3))/x)*b*n - b*log(c)/x - a/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = \int \frac{a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))/x^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = \frac{-6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)bdnx-6x^{\frac{2}{3}}bden-3\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)b\,e^2-3a\,e^2-}{3e^2x}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x)
```

output
```
( - 6*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*b*d*n*x - 6*x**
(2/3)*b*d*e*n - 3*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b*e**2 - 3*a*e
**2 + 2*b*e**2*n)/(3*e**2*x)
```

3.514 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx=\frac{bn}{6x^2}-\frac{bdn}{4ex^{4/3}}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}$$

output
```
1/6*b*n/x^2-1/4*b*d*n/e/x^(4/3)+1/2*b*d^2*n/e^2/x^(2/3)-1/2*b*d^3*n*ln(d+e
/x^(2/3))/e^3-1/2*(a+b*ln(c*(d+e/x^(2/3))^n))/x^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx=-\frac{a}{2x^2}+\frac{bn}{6x^2}-\frac{bdn}{4ex^{4/3}}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

output
```
-1/2*a/x^2 + (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2
/3)) - (b*d^3*n*Log[d + e/x^(2/3)])/(2*e^3) - (b*Log[c*(d + e/x^(2/3))^n])
/(2*x^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2904, 2842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2}\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^{4/3}}d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2842}$$

$$-\frac{3}{2}\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^2}-\frac{1}{3}ben\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^2}d\frac{1}{x^{2/3}}\right)$$

$$\Big\downarrow \text{49}$$

$$-\frac{3}{2}\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^2}-\frac{1}{3}ben\int \left(-\frac{d^3}{e^3\left(d+\frac{e}{x^{2/3}}\right)}+\frac{d^2}{e^3}-\frac{d}{e^2x^{2/3}}+\frac{1}{ex^{4/3}}\right)d\frac{1}{x^{2/3}}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^2}-\frac{1}{3}ben\left(-\frac{d^3\log\left(d+\frac{e}{x^{2/3}}\right)}{e^4}+\frac{d^2}{e^3x^{2/3}}-\frac{d}{2e^2x^{4/3}}+\frac{1}{3ex^2}\right)\right)$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

output
```
(-3*(-1/3*(b*e*n*(1/(3*e*x^2) - d/(2*e^2*x^(4/3)) + d^2/(e^3*x^(2/3)) - (d
^3*Log[d + e/x^(2/3)])/e^4)) + (a + b*Log[c*(d + e/x^(2/3))^n])/(3*x^2)))/
2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2842
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Simp[b*e*(n/(g*(q + 1)))   Int[(f + g*x)^(q + 1)/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] &&
NeQ[q, -1]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = \frac{6\,bd^2enx^{\frac{4}{3}} - 3\,bde^2nx^{\frac{2}{3}} + 2\,be^3n - 6\,be^3\log\left(c\right) - 6\,ae^3 - 6\,(bd^3nx^2 + be^3n}{12\,e^3x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="fricas")
```

output
```
1/12*(6*b*d^2*e*n*x^(4/3) - 3*b*d*e^2*n*x^(2/3) + 2*b*e^3*n - 6*b*e^3*log(
c) - 6*a*e^3 - 6*(b*d^3*n*x^2 + b*e^3*n)*log((d*x + e*x^(1/3))/x))/(e^3*x^
2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = -\frac{1}{12}\,ben\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4} - \frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4} - \frac{6\,d^2x^{\frac{4}{3}} - 3\,dex^{\frac{2}{3}} + 2\,e^2}{e^3x^2}\right) - \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="maxima")
```

output
```
-1/12*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^
2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/2*b*log(c*(d + e/x^(2/3)
)^n)/x^2 - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = \frac{1}{12}\left(e\left(\frac{12\,d^3\log\left(x^{\frac{1}{3}}\right)}{e^4} - \frac{6\,d^3\log\left(\left|dx^{\frac{2}{3}}+e\right|\right)}{e^4} - \frac{11\,d^3x^2 - 6\,d^2ex^{\frac{4}{3}} + 3\,d\right.\right.}{e^4x^2}$$
$$-\frac{b\log\left(c\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="giac")
```

output
```
1/12*(e*(12*d^3*log(x^(1/3))/e^4 - 6*d^3*log(abs(d*x^(2/3) + e))/e^4 - (11
*d^3*x^2 - 6*d^2*e*x^(4/3) + 3*d*e^2*x^(2/3) - 2*e^3)/(e^4*x^2)) - 6*log(d
 + e/x^(2/3))/x^2)*b*n - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

Mupad [B] (verification not implemented)

Time = 25.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = \frac{b\,n}{6\,x^2} - \frac{a}{2\,x^2} - \frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{2\,x^2} - \frac{b\,d\,n}{4\,e\,x^{4/3}} - \frac{b\,d^3\,n\,\ln\left(d+\frac{e}{x^{2/3}}\right)}{2\,e^3} + \frac{b\,d^2\,n}{2\,e^2\,x^{2/3}}$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))/x^3,x)
```

output
```
(b*n)/(6*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(2/3))^n))/(2*x^2) - (b*d*n)
/(4*e*x^(4/3)) - (b*d^3*n*log(d + e/x^(2/3)))/(2*e^3) + (b*d^2*n)/(2*e^2*x
^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = \frac{-3x^{\frac{2}{3}}bd\,e^2n+6x^{\frac{4}{3}}b\,d^2en-6\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^nc}{x^{\frac{2n}{3}}}\right)b\,d^3x^2-6\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^nc}{x^{\frac{2n}{3}}}\right)b}{12e^3x^2}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x)
```

output
```
( - 3*x**(2/3)*b*d*e**2*n + 6*x**(1/3)*b*d**2*e*n*x - 6*log(((x**(2/3)*d +
 e)**n*c)/x**((2*n)/3))*b*d**3*x**2 - 6*log(((x**(2/3)*d + e)**n*c)/x**((2
*n)/3))*b*e**3 - 6*a*e**3 + 2*b*e**3*n)/(12*e**3*x**2)
```

3.515 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx=\frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}+\frac{2bd^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

output
```
2/27*b*n/x^3-2/21*b*d*n/e/x^(7/3)+2/15*b*d^2*n/e^2/x^(5/3)-2/9*b*d^3*n/e^3
/x+2/3*b*d^4*n/e^4/x^(1/3)+2/3*b*d^(9/2)*n*arctan(d^(1/2)*x^(1/3)/e^(1/2))
/e^(9/2)-1/3*(a+b*ln(c*(d+e/x^(2/3))^n))/x^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{2}{9}ben\left(-\frac{1}{3ex^3}+\frac{3d}{7e^2x^{7/3}}-\frac{3d^2}{5e^3x^{5/3}}+\frac{d^3}{e^4x}-\frac{3d^4}{e^5\sqrt[3]{x}}+\frac{3d^{9/2}\arctan\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{e^{11/2}}\right)-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

output
```
-1/3*a/x^3 - (2*b*e*n*(-1/3*1/(e*x^3) + (3*d)/(7*e^2*x^(7/3)) - (3*d^2)/(5
*e^3*x^(5/3)) + d^3/(e^4*x) - (3*d^4)/(e^5*x^(1/3)) + (3*d^(9/2)*ArcTan[Sq
rt[e]/(Sqrt[d]*x^(1/3))])/e^(11/2)))/9 - (b*Log[c*(d + e/x^(2/3))^n])/(3*x
^3)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2905, 795, 864, 264, 264, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx$$

$$\Big\downarrow \; 2905$$

$$-\frac{2}{9}ben\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{14/3}}dx-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \; 795$$

$$-\frac{2}{9}ben\int \frac{1}{\left(x^{2/3}d+e\right)x^4}dx-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{864}$$

$$-\frac{2}{3}ben\int \frac{1}{\left(x^{2/3}d+e\right)x^{10/3}}d\sqrt[3]{x}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2}{3}ben\left(-\frac{d\int \frac{1}{\left(x^{2/3}d+e\right)x^{8/3}}d\sqrt[3]{x}}{e}-\frac{1}{9ex^3}\right)-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2}{3}ben\left(-\frac{d\left(-\frac{d\int \frac{1}{\left(x^{2/3}d+e\right)x^2}d\sqrt[3]{x}}{e}-\frac{1}{7ex^{7/3}}\right)}{e}-\frac{1}{9ex^3}\right)-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2}{3}ben\left(-\frac{d\left(-\frac{d\left(-\frac{d\int \frac{1}{\left(x^{2/3}d+e\right)x^{4/3}}d\sqrt[3]{x}}{e}-\frac{1}{5ex^{5/3}}\right)}{e}-\frac{1}{7ex^{7/3}}\right)}{e}-\frac{1}{9ex^3}\right)-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2}{3}ben\left(-\frac{d\left(-\frac{d\left(-\frac{d\left(-\frac{d\int\frac{1}{\left(x^{2/3}d+e\right)x^{2/3}}d\sqrt[3]{x}}{e}-\frac{1}{3ex}\right)}{e}-\frac{1}{5ex^{5/3}}\right)}{e}-\frac{1}{7ex^{7/3}}\right)}{e}-\frac{1}{9ex^3}\right)-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

$\downarrow$ 264

$$-\frac{2}{3}ben\left(-\frac{d\left(-\frac{d\left(-\frac{d\left(-\frac{d\left(-\frac{d\int \frac{1}{x^{2/3}d+e}d\sqrt[3]{x}}{e}-\frac{1}{e\sqrt[3]{x}}\right)}{e}-\frac{1}{3ex}\right)}{e}-\frac{1}{5ex^{5/3}}\right)}{e}-\frac{1}{7ex^{7/3}}\right)}{e}-\frac{1}{9ex^3}\right)-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

$\big\downarrow$ 218

$$-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-$$

$$\frac{2}{3}ben\left(-\frac{d\left(-\frac{d\left(-\frac{d\left(-\frac{d\left(-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{1}{e\sqrt[3]{x}}\right)}{e}-\frac{1}{3ex}\right)}{e}-\frac{1}{5ex^{5/3}}\right)}{e}-\frac{1}{7ex^{7/3}}\right)}{e}-\frac{1}{9ex^3}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

output
```
(-2*b*e*n*(-1/9*1/(e*x^3) - (d*(-1/7*1/(e*x^(7/3)) - (d*(-1/5*1/(e*x^(5/3)
) - (d*(-1/3*1/(e*x) - (d*(-(1/(e*x^(1/3))) - (Sqrt[d]*ArcTan[(Sqrt[d]*x^(
1/3))/Sqrt[e]])/e^(3/2)))/e))/e))/e))/e))/3 - (a + b*Log[c*(d + e/x^(2/3))
^n])/(3*x^3)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2905
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Simp[b*e*n*(p/(f*(m + 1)))   Int[x^(n - 1)*((f*x)^(m + 1)/(d +
 e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.57

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)}{x^{4}}\,dx = \left[\frac{315\,bd^{4}nx^{3}\sqrt{-\frac{d}{e}}\log\left(\frac{d^{3}x^{2}-2\,de^{2}x\sqrt{-\frac{d}{e}}-e^{3}+2\left(d^{2}ex\sqrt{-\frac{d}{e}}+de^{2}\right)x^{\frac{2}{3}}-2\left(d^{2}ex-e^{3}\sqrt{-\frac{d}{e}}\right.}{d^{3}x^{2}+e^{3}}\right.}{}\right.$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="fricas")
```

output
```
[1/945*(315*b*d^4*n*x^3*sqrt(-d/e)*log((d^3*x^2 - 2*d*e^2*x*sqrt(-d/e) - e
^3 + 2*(d^2*e*x*sqrt(-d/e) + d*e^2)*x^(2/3) - 2*(d^2*e*x - e^3*sqrt(-d/e))
*x^(1/3))/(d^3*x^2 + e^3)) - 210*b*d^3*e*n*x^2 + 126*b*d^2*e^2*n*x^(4/3) -
 315*b*e^4*n*log((d*x + e*x^(1/3))/x) + 70*b*e^4*n - 315*b*e^4*log(c) - 31
5*a*e^4 + 90*(7*b*d^4*n*x^2 - b*d*e^3*n)*x^(2/3))/(e^4*x^3), 1/945*(630*b*
d^4*n*x^3*sqrt(d/e)*arctan(x^(1/3)*sqrt(d/e)) - 210*b*d^3*e*n*x^2 + 126*b*
d^2*e^2*n*x^(4/3) - 315*b*e^4*n*log((d*x + e*x^(1/3))/x) + 70*b*e^4*n - 31
5*b*e^4*log(c) - 315*a*e^4 + 90*(7*b*d^4*n*x^2 - b*d*e^3*n)*x^(2/3))/(e^4*
x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx=\frac{1}{945}\left(2\,e\left(\frac{315\,d^5\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}e^5}+\frac{315\,d^4x^{\frac{8}{3}}-105\,d^3ex^2+63\,d^2e^2x^{\frac{4}{3}}-45\right.}{e^5x^3}\right.\right.$$
$$-\frac{b\log\left(c\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="giac")
```

output
```
1/945*(2*e*(315*d^5*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^5) + (315*d^4
*x^(8/3) - 105*d^3*e*x^2 + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^(2/3) + 35*e^4)
/(e^5*x^3)) - 315*log(d + e/x^(2/3))/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x
^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx=\int \frac{a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))/x^4,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx=\frac{630\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)b\,d^4n\,x^3+630x^{\frac{8}{3}}b\,d^4en-90x^{\frac{2}{3}}bd\,e^4n+126x^{\frac{4}{3}}b\,d^2}{945e^5x}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x)
```

output
```
(630*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*b*d**4*n*x**3 +
630*x**(2/3)*b*d**4*e*n*x**2 - 90*x**(2/3)*b*d*e**4*n + 126*x**(1/3)*b*d**
2*e**3*n*x - 315*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b*e**5 - 315*a*
e**5 - 210*b*d**3*e**2*n*x**2 + 70*b*e**5*n)/(945*e**5*x**3)
```

3.516 $\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 412

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=-\frac{77b^2e^5n^2x^{2/3}}{120d^5}+\frac{47b^2e^4n^2x^{4/3}}{240d^4}-\frac{3b^2e^3n^2x^2}{40d^3}+\frac{b^2e^2n^2x^{8/3}}{40d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{120d^6}+\frac{be^5n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}-\frac{be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4d^4}+\frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{6d^3}-\frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2}+\frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d}+\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{2d^6}$$

output
```
-77/120*b^2*e^5*n^2*x^(2/3)/d^5+47/240*b^2*e^4*n^2*x^(4/3)/d^4-3/40*b^2*e^
3*n^2*x^2/d^3+1/40*b^2*e^2*n^2*x^(8/3)/d^2+77/120*b^2*e^6*n^2*ln(d+e/x^(2/
3))/d^6+1/2*b*e^5*n*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6-
1/4*b*e^4*n*x^(4/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^4+1/6*b*e^3*n*x^2*(a+b*l
n(c*(d+e/x^(2/3))^n))/d^3-1/8*b*e^2*n*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/
d^2+1/10*b*e*n*x^(10/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d+1/2*b*e^6*n*ln(1-d/(
d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+1/4*x^4*(a+b*ln(c*(d+e/x^(2/
3))^n))^2+137/180*b^2*e^6*n^2*ln(x)/d^6-1/2*b^2*e^6*n^2*polylog(2,d/(d+e/x
^(2/3)))/d^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 830 vs. $2(412) = 824$.

Time = 0.75 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.01

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

output
```
(x^4*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/4 + (b*e*n*(360*a*d*e^4*x^(2/3) -
 462*b*d*e^4*n*x^(2/3) - 180*a*d^2*e^3*x^(4/3) + 141*b*d^2*e^3*n*x^(4/3) +
 120*a*d^3*e^2*x^2 - 54*b*d^3*e^2*n*x^2 - 90*a*d^4*e*x^(8/3) + 18*b*d^4*e*
n*x^(8/3) + 72*a*d^5*x^(10/3) + 822*b*e^5*n*Log[d + e/x^(2/3)] + 360*b*d*e
^4*x^(2/3)*Log[c*(d + e/x^(2/3))^n] - 180*b*d^2*e^3*x^(4/3)*Log[c*(d + e/x
^(2/3))^n] + 120*b*d^3*e^2*x^2*Log[c*(d + e/x^(2/3))^n] - 90*b*d^4*e*x^(8/
3)*Log[c*(d + e/x^(2/3))^n] + 72*b*d^5*x^(10/3)*Log[c*(d + e/x^(2/3))^n] -
 360*a*e^5*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 360*b*e^5*Log[c*(d + e/x^(2/3
))^n]*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 180*b*e^5*n*Log[Sqrt[e] - Sqrt[-d]
*x^(1/3)]^2 - 360*a*e^5*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 360*b*e^5*Log[c*
(d + e/x^(2/3))^n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 180*b*e^5*n*Log[Sqrt[
e] + Sqrt[-d]*x^(1/3)]^2 + 360*b*e^5*n*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log
[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b*e^5*n*Log[Sqrt[e] - Sqrt[-d
]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 720*b*e^5*n*Log[Sqrt[
e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] - 720*b*e^5*n*Lo
g[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 548*b*e^5*
n*Log[x] - 720*b*e^5*n*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 360*b*
e^5*n*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b*e^5*n*PolyL
og[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 720*b*e^5*n*PolyLog[2, 1 + (Sq
rt[-d]*x^(1/3))/Sqrt[e]]))/(720*d^6)
```

Rubi [A] (warning: unable to verify)

Time = 3.86 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.38, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.042$, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$$

$$\Big\downarrow \text{ 2904}$$

$$-\frac{3}{2}\int x^{14/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{ 2845}$$

$$-\frac{3}{2}\left(\frac{1}{3}ben\int \frac{x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d+\frac{e}{x^{2/3}}}d\frac{1}{x^{2/3}}-\frac{1}{6}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$\Big\downarrow$ 2858

$$-\frac{3}{2}\left(\frac{1}{3}bn\int x^{14/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{6}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$\Big\downarrow$ 27

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\int \frac{x^{14/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^6}d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{6}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$\Big\downarrow$ 2789

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\int \frac{x^4\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^6}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int -\frac{x^4\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^5}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right)-\frac{1}{6}x^4\left(a+b\log\left(c\left(d+\right.\right.\right.\right.$$

$\Big\downarrow$ 2756

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int -\frac{x^4}{e^5}d\left(d+\frac{e}{x^{2/3}}\right)-\frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{5e^5}}{d}+\frac{\int -\frac{x^4\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^5}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right)-\frac{1}{6}x^4\left(a\right.\right.$$

$\Big\downarrow$ 54

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{-\frac{1}{5}bn\int \left(-\frac{x^{10/3}}{de^5}+\frac{x^{8/3}}{d^2e^4}-\frac{x^2}{d^3e^3}+\frac{x^{4/3}}{d^4e^2}-\frac{x^{2/3}}{d^5e}+\frac{x^{2/3}}{d^5}\right)d\left(d+\frac{e}{x^{2/3}}\right)-\frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{5e^5}}{d}+\frac{\int -\frac{x^4}{}}{}\right.\right.$$

$\Big\downarrow$ 2009

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\int -\frac{x^4\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^5}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{-\frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{5e^5}-\frac{1}{5}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^5}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^5}-\frac{x^2}{d^4}\right.}{d}\right.\right.$$

$\Big\downarrow$ 2789

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\int -\frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^5}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{-\frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{5e^5}-\frac{1}{5}b\right.$$

$\downarrow$ 2756

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\int \frac{x^{10/3}}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{-\frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right.\right.}{5e^5}\right.$$

$\downarrow$ 54

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\int \left(\frac{x^{8/3}}{de^4}-\frac{x^2}{d^2e^3}+\frac{x^{4/3}}{d^3e^2}-\frac{x^{2/3}}{d^4e}+\frac{x^{2/3}}{d^4}\right)d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}\right.$$

$\downarrow$ 2009

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\int \frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}-\frac{x^{2/3}}{d^3e}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^2}{3de^3}\right.}{d}}{d}\right.$$

$\downarrow$ 2789

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int \frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\frac{e}{x^{2}}\right.}{d^4}\right.}{d}}{d}\right.$$

$\downarrow$ 2756

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{-\frac{1}{3}bn\int -\frac{x^{8/3}}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{3e^3}}{d}}{d}}{d}+\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\cdots}{\cdots}\right.\right.$$

$\downarrow$ 54

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{-\frac{1}{3}bn\int\left(-\frac{x^2}{de^3}+\frac{x^{4/3}}{d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{x^{2/3}}{d^3}\right)d\left(d+\frac{e}{x^{2/3}}\right)-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{3e^3}}{d}+\frac{\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}}{d}+\frac{\frac{x^{8/3}\left(a+\cdots\right.}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 2009

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^3}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^3}-\frac{x^{2/3}}{d^2e}+\frac{x^{4/3}}{2de^2}\right)}{d}}{d}}{d}+\frac{\frac{x^{8/3}\cdots}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 2789

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^3}-\cdots\right.}{d}}{d}}{d}\cdots\right.\right.$$

$\downarrow$ 2756

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\int\frac{x^2}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{\int \frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\cdots\right.}{d^3}\cdots\right.}{d}}{d}\cdots\right.\right.$$

$\downarrow$ 54

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\int\left(\frac{x^{4/3}}{de^2}-\frac{x^{2/3}}{d^2e}+\frac{x^{2/3}}{d^2}\right)d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{3e^3}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^2}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^2}-\frac{x^{2/3}}{de}\right)}{d}}{d}+\frac{-\frac{x^2\left(a+b\log\left(cx^{-\cdots}\right)\right)}{3e^3}\cdots}{\cdots}}{d}\cdots\right.\right.$$

$$\Big\downarrow \ 2789$$

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\int\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int-\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^2}\cdots\right.}{d}}{d}\cdots\right.\right.$$

$$\Big\downarrow \ 2751$$

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{-\frac{bn\int-\frac{x^{2/3}}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{de}}{d}+\frac{\int-\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n\cdots}\right)\right)}{2e^2}\cdots}{\cdots}}{d}\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \ 16$$

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{de}}{d}}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\cdots}{d}\cdots}{\cdots}\right.\right.$$

$\Big\downarrow$ 2779

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{bn\int x^{2/3}\log\left(1-dx^{2/3}\right)d\left(d+\frac{e}{x^{2/3}}\right)}{d}-\frac{\log\left(1-dx^{2/3}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{de}}{d}}{d}+\cdots}{d}\cdots}{\cdots}\right.\right.$$

$\Big\downarrow$ 2838

$$-\frac{3}{2}\left(\frac{1}{3}be^6n\left(\frac{\frac{\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^2}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^2}-\frac{x^{2/3}}{de}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{de}}{d}+\cdots}{d}\cdots}{\cdots}\right.\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

output
```
(-3*(-1/6*(x^4*(a + b*Log[c*(d + e/x^(2/3))^n])^2) + (b*e^6*n*((-1/5*(b*n*
(-(x^(2/3)/(d^4*e)) + x^(4/3)/(2*d^3*e^2) - x^2/(3*d^2*e^3) + x^(8/3)/(4*d
*e^4) + Log[d + e/x^(2/3)]/d^5 - Log[-(e/x^(2/3))]/d^5)) - (x^(10/3)*(a +
b*Log[c/x^((2*n)/3)]))/(5*e^5))/d + ((-1/4*(b*n*(-(x^(2/3)/(d^3*e)) + x^(4
/3)/(2*d^2*e^2) - x^2/(3*d*e^3) + Log[d + e/x^(2/3)]/d^4 - Log[-(e/x^(2/3)
)]/d^4)) + (x^(8/3)*(a + b*Log[c/x^((2*n)/3)]))/(4*e^4))/d + ((-1/3*(b*n*(
-(x^(2/3)/(d^2*e)) + x^(4/3)/(2*d*e^2) + Log[d + e/x^(2/3)]/d^3 - Log[-(e/
x^(2/3))]/d^3)) - (x^2*(a + b*Log[c/x^((2*n)/3)]))/(3*e^3))/d + ((-1/2*(b*
n*(-(x^(2/3)/(d*e)) + Log[d + e/x^(2/3)]/d^2 - Log[-(e/x^(2/3))]/d^2)) + (
x^(4/3)*(a + b*Log[c/x^((2*n)/3)]))/(2*e^2))/d + (((b*n*Log[-(e/x^(2/3))])
/d - ((d + e/x^(2/3))*x^(2/3)*(a + b*Log[c/x^((2*n)/3)]))/(d*e))/d + (-((L
og[1 - d*x^(2/3)]*(a + b*Log[c/x^((2*n)/3)]))/d) + (b*n*PolyLog[2, d*x^(2/
3)])/d)/d)/d)/d)/d)/d))/3))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^2dx$$

input
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Fricas [F]

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^3*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x^3*log(c*((d*x
+ e*x^(1/3))/x)^n) + a^2*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx = \int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*log((d*x^(2/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
 2*a*b*d*log(c) + a^2*d)*x^4 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)
*x^(10/3) + 12*(b^2*d*x^4 + b^2*e*x^(10/3))*log(x^(1/3*n))^2 - (b^2*d*n*x^
4 - 6*(b^2*d*log(c) + a*b*d)*x^4 - 6*(b^2*e*log(c) + a*b*e)*x^(10/3) + 12*
(b^2*d*x^4 + b^2*e*x^(10/3))*log(x^(1/3*n)))*log((d*x^(2/3) + e)^n) - 12*(
(b^2*d*log(c) + a*b*d)*x^4 + (b^2*e*log(c) + a*b*e)*x^(10/3))*log(x^(1/3*n
)))/(d*x + e*x^(1/3)), x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx = \int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \int x^3 \left(a + b\,\ln\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$$

input
```
int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

output
```
int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

Reduce [F]

$$\int x^3 \left(a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \frac{-30x^{\frac{8}{3}}\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right) b^2 d^4 e^2 n - 30x^{\frac{8}{3}} ab\, d^4 e^2 n + 120x^{\frac{2}{3}} abd\, e^5 n + 6x^{\frac{8}{3}} b^2 d^4 e \cdots}{\cdots}$$

input
```
int(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x)
```

output

```
( - 30*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**4*e**2*n
*x**2 - 30*x**(2/3)*a*b*d**4*e**2*n*x**2 + 120*x**(2/3)*a*b*d*e**5*n + 6*x
**(2/3)*b**2*d**4*e**2*n**2*x**2 - 154*x**(2/3)*b**2*d*e**5*n**2 + 24*x**(
1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**5*e*n*x**3 - 60*x**
(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**2*e**4*n*x + 24*x*
*(1/3)*a*b*d**5*e*n*x**3 - 60*x**(1/3)*a*b*d**2*e**4*n*x + 47*x**(1/3)*b**
2*d**2*e**4*n**2*x + 80*int((x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n
)/3)))/(x**(2/3)*d + e),x)*b**2*d**2*e**5*n - 240*log(x**(1/3))*a*b*e**6*n
 + 308*log(x**(1/3))*b**2*e**6*n**2 + 60*log(((x**(2/3)*d + e)**n*c)/x**((
2*n)/3))**2*b**2*d**6*x**4 + 120*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))
*a*b*d**6*x**4 - 120*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b*e**6 +
40*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**3*e**3*n*x**2 + 154*l
og(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*e**6*n + 60*a**2*d**6*x**4 +
 40*a*b*d**3*e**3*n*x**2 - 18*b**2*d**3*e**3*n**2*x**2)/(240*d**6)
```

3.517 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 239

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx &= \frac{b^2e^2n^2x^{2/3}}{2d^2}\\
&-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}\\
&+\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d}\\
&-\frac{be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}\\
&+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{b^2e^3n^2\log(x)}{d^3}+\frac{b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{d^3}\end{aligned}$$

output

```
1/2*b^2*e^2*n^2*x^(2/3)/d^2-1/2*b^2*e^3*n^2*ln(d+e/x^(2/3))/d^3-b*e^2*n*(d
+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+1/2*b*e*n*x^(4/3)*(a+b
*ln(c*(d+e/x^(2/3))^n))/d-b*e^3*n*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(
2/3))^n))/d^3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2-b^2*e^3*n^2*ln(x)/d^3+
b^2*e^3*n^2*polylog(2,d/(d+e/x^(2/3)))/d^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 542 vs. $2(239) = 478$.

Time = 0.47 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.27

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2$$
$$-\frac{ben\left(6dex^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-3d^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-6e^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right.\right.\right.}{}$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

output
```
(x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/2 - (b*e*n*(6*d*e*x^(2/3)*(a + b*
Log[c*(d + e/x^(2/3))^n]) - 3*d^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n])
 - 6*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]
- 6*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] +
 2*b*e^2*n*(3*Log[d + e/x^(2/3)] + 2*Log[x]) + b*e*n*(-3*d*x^(2/3) + 3*e*L
og[d + e/x^(2/3)] + 2*e*Log[x]) + 3*b*e^2*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3
)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e
])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^
(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]) + 3*
b*e^2*n*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]
+ 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))
/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog
[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/(6*d^3)
```

Rubi [A] (warning: unable to verify)

Time = 1.62 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2904, 2845, 2858, 25, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$-\frac{3}{2}\int x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{ 2845}$$

$$-\frac{3}{2}\left(\frac{2}{3}ben\int\frac{x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d+\frac{e}{x^{2/3}}}d\frac{1}{x^{2/3}}-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{ 2858}$$

$$-\frac{3}{2}\left(\frac{2}{3}bn\int x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{3}{2}\left(-\frac{2}{3}bn\int -x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{ 2789}$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right)-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 2756}$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\int\frac{x^2}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right)-\frac{1}{3}x^2\left(a+bl\right.\right.$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\int\left(\frac{x^{4/3}}{de^2}-\frac{x^{2/3}}{d^2e}+\frac{x^{2/3}}{d^2}\right)d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right.\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\int \frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^2}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^2}-\frac{x^{2/3}}{de}\right.}{d}\right.\right.$$

$$\Big\downarrow \ 2789$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{\int \frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\right.}{}\right.\right.$$

$$\Big\downarrow \ 2751$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{-\frac{bn\int -\frac{x^{2/3}}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{de}}{d}+\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\right.}{2e}}{}\right.\right.$$

$$\Big\downarrow \ 16$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{de}}{d}}{d}+\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right.\right.}{2e^2}}{}\right.\right.$$

$$\Big\downarrow \ 2779$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{\frac{bn\int x^{2/3}\log\left(1-dx^{2/3}\right)d\left(d+\frac{e}{x^{2/3}}\right)}{d}-\frac{\log\left(1-dx^{2/3}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right.\right.}{de}}{d}}{d}\right.\right.$$

$$\Big\downarrow \ 2838$$

$$-\frac{3}{2}\left(-\frac{2}{3}be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2e^2}-\frac{1}{2}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^2}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^2}-\frac{x^{2/3}}{de}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\right.}{de}}{d}\right.\right.$$

input
```
Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

output
```
(-3*(-1/3*(x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2) - (2*b*e^3*n*((-1/2*(b*
n*(-(x^(2/3)/(d*e)) + Log[d + e/x^(2/3)]/d^2 - Log[-(e/x^(2/3))]/d^2)) + (
x^(4/3)*(a + b*Log[c/x^((2*n)/3)]))/(2*e^2))/d + (((b*n*Log[-(e/x^(2/3))])
/d - ((d + e/x^(2/3))*x^(2/3)*(a + b*Log[c/x^((2*n)/3)]))/(d*e))/d + (-((L
og[1 - d*x^(2/3)]*(a + b*Log[c/x^((2*n)/3)]))/d) + (b*n*PolyLog[2, d*x^(2/
3)])/d)/d)/d))/3))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*
x^(1/3))/x)^n) + a^2*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*log((d*x^(2/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
 2*a*b*d*log(c) + a^2*d)*x^2 + 12*(b^2*d*x^2 + b^2*e*x^(4/3))*log(x^(1/3*n
))^2 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(4/3) - 2*(b^2*d*n*x^
2 - 3*(b^2*d*log(c) + a*b*d)*x^2 - 3*(b^2*e*log(c) + a*b*e)*x^(4/3) + 6*(b
^2*d*x^2 + b^2*e*x^(4/3))*log(x^(1/3*n)))*log((d*x^(2/3) + e)^n) - 12*((b^
2*d*log(c) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(4/3))*log(x^(1/3*n)))/
(d*x + e*x^(1/3)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\frac{-6x^{\frac{2}{3}}abd\,e^2n+3x^{\frac{2}{3}}b^2d\,e^2n^2+3x^{\frac{4}{3}}\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)b^2d^2en+3x^{\frac{4}{3}}ab\,d^2en-\cdots}{\cdots}$$

input
```
int(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x)
```

output
```
( - 6*x**(2/3)*a*b*d*e**2*n + 3*x**(2/3)*b**2*d*e**2*n**2 + 3*x**(1/3)*log
(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**2*e*n*x + 3*x**(1/3)*a*b*d*
*2*e*n*x - 4*int((x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3)))/(x**
(2/3)*d + e),x)*b**2*d**2*e**2*n + 12*log(x**(1/3))*a*b*e**3*n - 6*log(x**
(1/3))*b**2*e**3*n**2 + 3*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**
2*d**3*x**2 + 6*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b*d**3*x**2 +
6*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b*e**3 - 3*log(((x**(2/3)*d
+ e)**n*c)/x**((2*n)/3))*b**2*e**3*n + 3*a**2*d**3*x**2)/(6*d**3)
```

3.518 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = -\frac{3}{2}\Big(a$$
$$+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\Big)^2\log\left(-\frac{e}{dx^{2/3}}\right)-3bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\text{PolyLog}\left(2,1+\frac{e}{dx^{2/3}}\right)+3b^2n^2\,\text{Pol}$$

output
```
-3/2*(a+b*ln(c*(d+e/x^(2/3))^n))^2*ln(-e/d/x^(2/3))-3*b*n*(a+b*ln(c*(d+e/x
^(2/3))^n))*polylog(2,1+e/d/x^(2/3))+3*b^2*n^2*polylog(3,1+e/d/x^(2/3))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 1701, normalized size of antiderivative = 17.91

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```

output
```
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*Log[x] + 2*b*n
*(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*((Log[d + e/x^(
2/3)] - Log[1 + e/(d*x^(2/3))])*Log[x] + (3*PolyLog[2, -(e/(d*x^(2/3)))])/
2) + 3*b^2*n^2*(Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)]^2*Log[((-I)*Sqrt[d]*
x^(1/3))/Sqrt[e]] + 2*Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)]*Log[(I*Sqrt[e]
)/Sqrt[d] + x^(1/3)]*Log[((-I)*Sqrt[d]*x^(1/3))/Sqrt[e]] + Log[1 - (I*Sqrt
[d]*x^(1/3))/Sqrt[e]]*(-2*Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)] + Log[1 -
(I*Sqrt[d]*x^(1/3))/Sqrt[e]])*(Log[((-I)*Sqrt[d]*x^(1/3))/Sqrt[e]] - Log[(
I*Sqrt[d]*x^(1/3))/Sqrt[e]]) + Log[(I*Sqrt[e])/Sqrt[d] + x^(1/3)]^2*Log[(I
*Sqrt[d]*x^(1/3))/Sqrt[e]] + 2*Log[(Sqrt[e] - I*Sqrt[d]*x^(1/3))/(Sqrt[e]
+ I*Sqrt[d]*x^(1/3))]*Log[1 - (I*Sqrt[d]*x^(1/3))/Sqrt[e]]*(-Log[((-I)*Sqr
t[d]*x^(1/3))/Sqrt[e]] + Log[(I*Sqrt[d]*x^(1/3))/Sqrt[e]]) + Log[(Sqrt[e]
- I*Sqrt[d]*x^(1/3))/(Sqrt[e] + I*Sqrt[d]*x^(1/3))]^2*(Log[(2*Sqrt[e])/(Sq
rt[e] + I*Sqrt[d]*x^(1/3))] + Log[((-I)*Sqrt[d]*x^(1/3))/Sqrt[e]] - Log[(2
*x^(1/3))/(((-I)*Sqrt[e])/Sqrt[d] + x^(1/3))]) + ((-Log[d + e/x^(2/3)] + L
og[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)] + Log[(I*Sqrt[e])/Sqrt[d] + x^(1/3)]
- (2*Log[x])/3)^2*Log[x])/3 + (4*Log[x]^3)/81 + 2*Log[(Sqrt[e] - I*Sqrt[d]
*x^(1/3))/(Sqrt[e] + I*Sqrt[d]*x^(1/3))]*(-PolyLog[2, (I*Sqrt[e] + Sqrt[d]
*x^(1/3))/(I*Sqrt[e] - Sqrt[d]*x^(1/3))] + PolyLog[2, (I*Sqrt[e] + Sqrt[d]
*x^(1/3))/((-I)*Sqrt[e] + Sqrt[d]*x^(1/3))]) + 2*Log[(I*Sqrt[e])/Sqrt[d...
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2}\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2843}$$

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-2ben\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)}{d+\frac{e}{x^{2/3}}}d\frac{1}{x^{2/3}}\right)$$

$\Big\downarrow$ 2881

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-2bn\int x^{2/3}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)d\left(d+\frac{e}{x^{2/3}}\right.\right.$$

$\Big\downarrow$ 2821

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-2bn\left(bn\int x^{2/3}\operatorname{PolyLog}\left(2,\frac{d+\frac{e}{x^{2/3}}}{d}\right)d\left(d+\frac{e}{x^{2/3}}\right)-\operatorname{PolyI}\right.\right.$$

$\Big\downarrow$ 7143

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-2bn\left(bn\operatorname{PolyLog}\left(3,\frac{d+\frac{e}{x^{2/3}}}{d}\right)-\operatorname{PolyLog}\left(2,\frac{d+\frac{e}{x^{2/3}}}{d}\right)\left(a\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```

output
```
(-3*((a + b*Log[c*(d + e/x^(2/3))^n])^2*Log[-(e/(d*x^(2/3)))] - 2*b*n*(-((
a + b*Log[c/x^((2*n)/3)])*PolyLog[2, (d + e/x^(2/3))/d]) + b*n*PolyLog[3,
(d + e/x^(2/3))/d])))/2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(
1/3))/x)^n) + a^2)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="maxima")
```

output
```
b^2*log((d*x^(2/3) + e)^n)^2*log(x) - integrate(-1/3*(12*(b^2*d*x + b^2*e*
x^(1/3))*log(x^(1/3*n))^2 + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x
- 2*(2*b^2*d*n*x*log(x) - 3*(b^2*d*log(c) + a*b*d)*x + 6*(b^2*d*x + b^2*e*
x^(1/3))*log(x^(1/3*n)) - 3*(b^2*e*log(c) + a*b*e)*x^(1/3))*log((d*x^(2/3)
 + e)^n) - 12*((b^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*x^(1/3))*
log(x^(1/3*n)) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(1/3))/(d*x
^2 + e*x^(4/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^2/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))^2/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = \left(\int \frac{\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)^2}{x}dx\right)b^2 + 2\left(\int \frac{\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)}{x}dx\right)ab + \log(x)\,a^2$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x)
```

output
```
int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2/x,x)*b**2 + 2*int(log(((x
**(2/3)*d + e)**n*c)/x**((2*n)/3))/x,x)*a*b + log(x)*a**2
```

3.519 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 276

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3}-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} -\frac{3b^2d^2n^2}{e^2x^{2/3}}+\frac{b^2d^3n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}+\frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} -\frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3} +\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} -\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2}$$

output

```
3/4*b^2*d*n^2*(d+e/x^(2/3))^2/e^3-1/9*b^2*n^2*(d+e/x^(2/3))^3/e^3-3*b^2*d^
2*n^2/e^2/x^(2/3)+1/2*b^2*d^3*n^2*ln(d+e/x^(2/3))^2/e^3+3*b*d^2*n*(d+e/x^(
2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-3/2*b*d*n*(d+e/x^(2/3))^2*(a+b*ln(c*
(d+e/x^(2/3))^n))/e^3+1/3*b*n*(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/
e^3-b*d^3*n*ln(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-1/2*(a+b*ln(c*
(d+e/x^(2/3))^n))^2/x^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.63 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.50

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

output
```
(-18*e^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + b*n*(9*b*d*n*x^(2/3)*(e*(e -
 2*d*x^(2/3)) + 2*d^2*x^(4/3)*Log[d + e/x^(2/3)]) - 2*b*n*(e*(2*e^2 - 3*d*
e*x^(2/3) + 6*d^2*x^(4/3)) - 6*d^3*x^2*Log[d + e/x^(2/3)]) + 12*e^3*(a + b
*Log[c*(d + e/x^(2/3))^n]) - 18*d*e^2*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))
^n]) + 36*d^2*x^(4/3)*(e*(a - b*n) + b*(e + d*x^(2/3))*Log[c*(d + e/x^(2/3
))^n]) - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d
]*x^(1/3)] - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqr
t[-d]*x^(1/3)] - 36*d^3*x^2*((a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x
^(2/3)))] + b*n*PolyLog[2, 1 + e/(d*x^(2/3))]) + 18*b*d^3*n*x^2*(Log[Sqrt[
e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt
[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog
[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3)
)/(2*Sqrt[e])]) + 18*b*d^3*n*x^2*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqr
t[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4
*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3)
)/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/(36*e^3*x^
2)
```

Rubi [A] (warning: unable to verify)

Time = 0.86 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2904, 2845, 2858, 25, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx$$

$$\Big\downarrow \; 2904$$

$$-\frac{3}{2}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{4/3}}d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \; 2845$$

$$-\frac{3}{2}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3x^2}-\frac{2}{3}ben\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{\left(d+\frac{e}{x^{2/3}}\right)x^2}d\frac{1}{x^{2/3}}\right)$$

$$\Big\downarrow \; 2858$$

$$-\frac{3}{2}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3x^2}-\frac{2}{3}bn\int \frac{a+b\log\left(cx^{-2n/3}\right)}{x^{4/3}}d\left(d+\frac{e}{x^{2/3}}\right)\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{3}{2}\left(\frac{2}{3}bn\int -\frac{a+b\log\left(cx^{-2n/3}\right)}{x^{4/3}}d\left(d+\frac{e}{x^{2/3}}\right)+\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3x^2}\right)$$

$$\Big\downarrow \; 27$$

$$-\frac{3}{2}\left(\frac{2bn\int -\frac{e^3\left(a+b\log\left(cx^{-2n/3}\right)\right)}{x^{4/3}}d\left(d+\frac{e}{x^{2/3}}\right)}{3e^3}+\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3x^2}\right)$$

$$\Big\downarrow \; 2772$$

$$-\frac{3}{2}\left(\frac{2bn\left(-bn\int\left(x^{2/3}\log\left(d+\frac{e}{x^{2/3}}\right)d^3-3d^2+\frac{3}{2}\left(d+\frac{e}{x^{2/3}}\right)d-\frac{1}{3x^{4/3}}\right)d\left(d+\frac{e}{x^{2/3}}\right)+d^3\log\left(d+\frac{e}{x^{2/3}}\right)(a+b\right.}{3e^3}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{3}{2}\left(\frac{2bn\left(d^3\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)-3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)+\frac{3d\left(a+b\log\left(cx^{-2n/3}\right)\right)}{2x^{4/3}}-\right.}{3e^3}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

output
```
(-3*((a + b*Log[c*(d + e/x^(2/3))^n])^2/(3*x^2) + (2*b*n*(-(b*n*(-3*d^2*(d
 + e/x^(2/3)) - 1/(9*x^2) + (3*d)/(4*x^(4/3)) + (d^3*Log[d + e/x^(2/3)]^2)
/2)) - 3*d^2*(d + e/x^(2/3))*(a + b*Log[c/x^((2*n)/3)]) - (a + b*Log[c/x^(
(2*n)/3)])/(3*x^2) + (3*d*(a + b*Log[c/x^((2*n)/3)]))/(2*x^(4/3)) + d^3*Lo
g[d + e/x^(2/3)]*(a + b*Log[c/x^((2*n)/3)])))/(3*e^3)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln \left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx =$$
$$-\frac{4\,b^2e^3n^2+18\,b^2e^3\log\left(c\right)^2-12\,abe^3n+18\,a^2e^3+18\left(b^2d^3n^2x^2+b^2e^3n^2\right)\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^2-12\left(b^2e^3n-3\,c\right.}{}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="fricas")
```

output
```
-1/36*(4*b^2*e^3*n^2 + 18*b^2*e^3*log(c)^2 - 12*a*b*e^3*n + 18*a^2*e^3 + 1
8*(b^2*d^3*n^2*x^2 + b^2*e^3*n^2)*log((d*x + e*x^(1/3))/x)^2 - 12*(b^2*e^3
*n - 3*a*b*e^3)*log(c) - 6*(6*b^2*d^2*e*n^2*x^(4/3) - 3*b^2*d*e^2*n^2*x^(2
/3) + 2*b^2*e^3*n^2 - 6*a*b*e^3*n + (11*b^2*d^3*n^2 - 6*a*b*d^3*n)*x^2 - 6
*(b^2*d^3*n*x^2 + b^2*e^3*n)*log(c))*log((d*x + e*x^(1/3))/x) - 3*(5*b^2*d
*e^2*n^2 - 6*b^2*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(2/3) - 6*(6*b^2*d^2*e*
n*x*log(c) - (11*b^2*d^2*e*n^2 - 6*a*b*d^2*e*n)*x)*x^(1/3))/(e^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx =$$
$$-\frac{1}{6}aben\left(\frac{6d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6d^2x^{\frac{4}{3}}-3dex^{\frac{2}{3}}+2e^2}{e^3x^2}\right)$$
$$-\frac{1}{36}\left(6en\left(\frac{6d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6d^2x^{\frac{4}{3}}-3dex^{\frac{2}{3}}+2e^2}{e^3x^2}\right)\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)-\frac{(18d\cdots}{\cdots}\right.$$
$$-\frac{b^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)^2}{2x^2}-\frac{ab\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{x^2}-\frac{a^2}{2x^2}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="maxima")
```

output
```
-1/6*a*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d
^2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/36*(6*e*n*(6*d^3*log(d*
x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3)
 + 2*e^2)/(e^3*x^2))*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*log(d*x^(2/3)
+ e)^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*
e^2*x^(2/3) - 4*e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e)
)*n^2/(e^3*x^2))*b^2 - 1/2*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - a*b*log(c*
(d + e/x^(2/3))^n)/x^2 - 1/2*a^2/x^2
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 25.55 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx=\frac{\frac{d\left(\frac{3a^2}{2}-a\,b\,n+\frac{b^2n^2}{3}\right)}{2\,e}-\frac{d\,(3\,a^2-b^2\,n^2)}{4\,e}}{x^{4/3}}$$

$$-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2\left(\frac{b^2}{2\,x^2}+\frac{b^2\,d^3}{2\,e^3}\right)-\frac{\frac{a^2}{2}-\frac{a\,b\,n}{3}+\frac{b^2\,n^2}{9}}{x^2}-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\left(\frac{b\,(3\,a-b\,n)}{3\,x^2}-\frac{\frac{b\,d\,(3\,a-b\,n)}{2\,e}}{x^{4/3}}\right.$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^3,x)
```

output
```
((d*((3*a^2)/2 + (b^2*n^2)/3 - a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(4*e)
)/x^(4/3) - log(c*(d + e/x^(2/3))^n)^2*(b^2/(2*x^2) + (b^2*d^3)/(2*e^3)) -
 (a^2/2 + (b^2*n^2)/9 - (a*b*n)/3)/x^2 - log(c*(d + e/x^(2/3))^n)*((b*(3*a
 - b*n))/(3*x^2) - ((b*d*(3*a - b*n))/(2*e) - (3*a*b*d)/(2*e))/x^(4/3) + (
d*((b*d*(3*a - b*n))/e - (3*a*b*d)/e))/(e*x^(2/3))) - ((d*((d*((3*a^2)/2 +
 (b^2*n^2)/3 - a*b*n))/e - (d*(3*a^2 - b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2)
/e^2)/x^(2/3) + (log(d + e/x^(2/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(6*e^3
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.19

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = \frac{-18x^{\frac{2}{3}}\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)b^2d\,e^2n-18x^{\frac{2}{3}}abd\,e^2n+15x^{\frac{2}{3}}b^2d\,e^2n^2+36x^{\frac{4}{3}}\mathrm{l}}{}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x)
```

output
```
( - 18*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d*e**2*n -
18*x**(2/3)*a*b*d*e**2*n + 15*x**(2/3)*b**2*d*e**2*n**2 + 36*x**(1/3)*log(
((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**2*e*n*x + 36*x**(1/3)*a*b*d*
*2*e*n*x - 66*x**(1/3)*b**2*d**2*e*n**2*x - 18*log(((x**(2/3)*d + e)**n*c)
/x**((2*n)/3))**2*b**2*d**3*x**2 - 18*log(((x**(2/3)*d + e)**n*c)/x**((2*n
)/3))**2*b**2*e**3 - 36*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b*d**3
*x**2 - 36*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b*e**3 + 66*log(((x
**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**3*n*x**2 + 12*log(((x**(2/3)*d
+ e)**n*c)/x**((2*n)/3))*b**2*e**3*n - 18*a**2*e**3 + 12*a*b*e**3*n - 4*b*
*2*e**3*n**2)/(36*e**3*x**2)
```

3.520 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 482

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6} + \frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6} + \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^6}{72e^6} + \frac{3b^2d^5n^2}{e^5x^{2/3}} - \frac{b^2d^6n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{4e^6} - \frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6} + \frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^6} - \frac{10bd^3n\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^6} + \frac{15bd^2n\left(d+\frac{e}{x^{2/3}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8e^6} - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{12e^6} + \frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4}$$

output

```
-15/8*b^2*d^4*n^2*(d+e/x^(2/3))^2/e^6+10/9*b^2*d^3*n^2*(d+e/x^(2/3))^3/e^6
-15/32*b^2*d^2*n^2*(d+e/x^(2/3))^4/e^6+3/25*b^2*d*n^2*(d+e/x^(2/3))^5/e^6-
1/72*b^2*n^2*(d+e/x^(2/3))^6/e^6+3*b^2*d^5*n^2/e^5/x^(2/3)-1/4*b^2*d^6*n^2
*ln(d+e/x^(2/3))^2/e^6-3*b*d^5*n*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))
/e^6+15/4*b*d^4*n*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-10/3*b*d
^3*n*(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6+15/8*b*d^2*n*(d+e/x^(
2/3))^4*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-3/5*b*d*n*(d+e/x^(2/3))^5*(a+b*ln(
c*(d+e/x^(2/3))^n))/e^6+1/12*b*n*(d+e/x^(2/3))^6*(a+b*ln(c*(d+e/x^(2/3))^n
))/e^6+1/2*b*d^6*n*ln(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-1/4*(a+
b*ln(c*(d+e/x^(2/3))^n))^2/x^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.91 (sec) , antiderivative size = 988, normalized size of antiderivative = 2.05

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

output
```
-1/7200*(1800*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(-600*a*e^6 + 100*
b*e^6*n + 720*a*d*e^5*x^(2/3) - 264*b*d*e^5*n*x^(2/3) - 900*a*d^2*e^4*x^(4
/3) + 555*b*d^2*e^4*n*x^(4/3) + 1200*a*d^3*e^3*x^2 - 1140*b*d^3*e^3*n*x^2
- 1800*a*d^4*e^2*x^(8/3) + 2610*b*d^4*e^2*n*x^(8/3) + 3600*a*d^5*e*x^(10/3
) - 8820*b*d^5*e*n*x^(10/3) + 8820*b*d^6*n*x^4*Log[d + e/x^(2/3)] - 600*b*
e^6*Log[c*(d + e/x^(2/3))^n] + 720*b*d*e^5*x^(2/3)*Log[c*(d + e/x^(2/3))^n
] - 900*b*d^2*e^4*x^(4/3)*Log[c*(d + e/x^(2/3))^n] + 1200*b*d^3*e^3*x^2*Lo
g[c*(d + e/x^(2/3))^n] - 1800*b*d^4*e^2*x^(8/3)*Log[c*(d + e/x^(2/3))^n] +
 3600*b*d^5*e*x^(10/3)*Log[c*(d + e/x^(2/3))^n] - 3600*a*d^6*x^4*Log[Sqrt[
e] - Sqrt[-d]*x^(1/3)] - 3600*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n]*Log[Sqrt[
e] - Sqrt[-d]*x^(1/3)] + 1800*b*d^6*n*x^4*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^
2 - 3600*a*d^6*x^4*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 3600*b*d^6*x^4*Log[c*
(d + e/x^(2/3))^n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 1800*b*d^6*n*x^4*Log[
Sqrt[e] + Sqrt[-d]*x^(1/3)]^2 + 3600*b*d^6*n*x^4*Log[Sqrt[e] + Sqrt[-d]*x^
(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 3600*b*d^6*n*x^4*Log[Sq
rt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 3600*b
*d^6*x^4*Log[c*(d + e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] - 7200*b*d^6*n*x^4
*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] - 7200
*b*d^6*n*x^4*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e
]] + 2400*a*d^6*x^4*Log[x] - 3600*b*d^6*n*x^4*PolyLog[2, 1 + e/(d*x^(2/...
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2904, 2845, 2858, 27, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{10/3}}d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2845}$$

$$-\frac{3}{2}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{6x^4}-\frac{1}{3}ben\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{\left(d+\frac{e}{x^{2/3}}\right)x^4}d\frac{1}{x^{2/3}}\right)$$

$$\Big\downarrow \text{2858}$$

$$-\frac{3}{2}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{6x^4}-\frac{1}{3}bn\int \frac{a+b\log\left(cx^{-2n/3}\right)}{x^{10/3}}d\left(d+\frac{e}{x^{2/3}}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{2}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{6x^4}-\frac{bn\int \frac{e^6\left(a+b\log\left(cx^{-2n/3}\right)\right)}{x^{10/3}}d\left(d+\frac{e}{x^{2/3}}\right)}{3e^6}\right)$$

$$\Big\downarrow \text{2772}$$

$$-\frac{3}{2}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{6x^4}-\frac{bn\left(-bn\int \left(x^{2/3}\log\left(d+\frac{e}{x^{2/3}}\right)d^6-6d^5+\frac{15}{2}\left(d+\frac{e}{x^{2/3}}\right)d^4-\frac{20d^3}{3x^{4/3}}+\frac{15d^2}{4x^2}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{6x^4}-\frac{bn\left(d^6\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)-6d^5\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-\right.\right.\right.}\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

output
```
(-3*((a + b*Log[c*(d + e/x^(2/3))^n])^2/(6*x^4) - (b*n*(-(b*n*(-6*d^5*(d +
 e/x^(2/3)) + 1/(36*x^4) - (6*d)/(25*x^(10/3)) + (15*d^2)/(16*x^(8/3)) - (
20*d^3)/(9*x^2) + (15*d^4)/(4*x^(4/3)) + (d^6*Log[d + e/x^(2/3)]^2)/2)) -
6*d^5*(d + e/x^(2/3))*(a + b*Log[c/x^((2*n)/3)]) + (a + b*Log[c/x^((2*n)/3
)])/(6*x^4) - (6*d*(a + b*Log[c/x^((2*n)/3)]))/(5*x^(10/3)) + (15*d^2*(a +
 b*Log[c/x^((2*n)/3)]))/(4*x^(8/3)) - (20*d^3*(a + b*Log[c/x^((2*n)/3)]))/
(3*x^2) + (15*d^4*(a + b*Log[c/x^((2*n)/3)]))/(2*x^(4/3)) + d^6*Log[d + e/
x^(2/3)]*(a + b*Log[c/x^((2*n)/3)])))/(3*e^6)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x^{5}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^5,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx =$$

$$-\frac{100\,b^2e^6n^2+1800\,b^2e^6\log\left(c\right)^2-600\,abe^6n+1800\,a^2e^6-60\,(19\,b^2d^3e^3n^2-20\,abd^3e^3n)x^2-1800\,(b^2d^6n$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="fricas")
```

output
```
-1/7200*(100*b^2*e^6*n^2 + 1800*b^2*e^6*log(c)^2 - 600*a*b*e^6*n + 1800*a^
2*e^6 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x^2 - 1800*(b^2*d^6*n^2
*x^4 - b^2*e^6*n^2)*log((d*x + e*x^(1/3))/x)^2 + 600*(2*b^2*d^3*e^3*n*x^2
- b^2*e^6*n + 6*a*b*e^6)*log(c) + 60*(20*b^2*d^3*e^3*n^2*x^2 - 10*b^2*e^6*
n^2 + 60*a*b*e^6*n + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^4 - 60*(b^2*d^6*n
*x^4 - b^2*e^6*n)*log(c) - 6*(5*b^2*d^4*e^2*n^2*x^2 - 2*b^2*d*e^5*n^2)*x^(
2/3) + 15*(4*b^2*d^5*e*n^2*x^3 - b^2*d^2*e^4*n^2*x)*x^(1/3))*log((d*x + e*
x^(1/3))/x) - 6*(44*b^2*d*e^5*n^2 - 120*a*b*d*e^5*n - 15*(29*b^2*d^4*e^2*n
^2 - 20*a*b*d^4*e^2*n)*x^2 + 60*(5*b^2*d^4*e^2*n*x^2 - 2*b^2*d*e^5*n)*log(
c))*x^(2/3) - 15*(12*(49*b^2*d^5*e*n^2 - 20*a*b*d^5*e*n)*x^3 - (37*b^2*d^2
*e^4*n^2 - 60*a*b*d^2*e^4*n)*x - 60*(4*b^2*d^5*e*n*x^3 - b^2*d^2*e^4*n*x)*
log(c))*x^(1/3))/(e^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = \frac{1}{120}aben\left(\frac{60\,d^6\log\left(dx^{\frac{2}{3}}+e\right)}{e^7}-\frac{60\,d^6\log\left(x^{\frac{2}{3}}\right)}{e^7}-\frac{60\,d^5x^{\frac{10}{3}}-30\,d^4ex\right.$$
$$+\frac{1}{7200}\left(60\,en\left(\frac{60\,d^6\log\left(dx^{\frac{2}{3}}+e\right)}{e^7}-\frac{60\,d^6\log\left(x^{\frac{2}{3}}\right)}{e^7}-\frac{60\,d^5x^{\frac{10}{3}}-30\,d^4ex^{\frac{8}{3}}+20\,d^3e^2x^2-15\,d^2e^3x^{\frac{4}{3}}+1}{e^6x^4}\right.\right.$$
$$-\frac{b^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)^2}{4\,x^4}-\frac{ab\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{2\,x^4}-\frac{a^2}{4\,x^4}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="maxima")
```

output
```
1/120*a*b*e*n*(60*d^6*log(d*x^(2/3) + e)/e^7 - 60*d^6*log(x^(2/3))/e^7 - (
60*d^5*x^(10/3) - 30*d^4*e*x^(8/3) + 20*d^3*e^2*x^2 - 15*d^2*e^3*x^(4/3) +
 12*d*e^4*x^(2/3) - 10*e^5)/(e^6*x^4)) + 1/7200*(60*e*n*(60*d^6*log(d*x^(2
/3) + e)/e^7 - 60*d^6*log(x^(2/3))/e^7 - (60*d^5*x^(10/3) - 30*d^4*e*x^(8/
3) + 20*d^3*e^2*x^2 - 15*d^2*e^3*x^(4/3) + 12*d*e^4*x^(2/3) - 10*e^5)/(e^6
*x^4))*log(c*(d + e/x^(2/3))^n) - (1800*d^6*x^4*log(d*x^(2/3) + e)^2 + 800
*d^6*x^4*log(x)^2 - 5880*d^6*x^4*log(x) - 8820*d^5*e*x^(10/3) + 2610*d^4*e
^2*x^(8/3) - 1140*d^3*e^3*x^2 + 555*d^2*e^4*x^(4/3) - 264*d*e^5*x^(2/3) +
100*e^6 - 60*(40*d^6*x^4*log(x) - 147*d^6*x^4)*log(d*x^(2/3) + e))*n^2/(e^
6*x^4))*b^2 - 1/4*b^2*log(c*(d + e/x^(2/3))^n)^2/x^4 - 1/2*a*b*log(c*(d +
e/x^(2/3))^n)/x^4 - 1/4*a^2/x^4
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 27.02 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = \frac{b^2 d^6 \ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2}{4e^6} - \frac{b^2 \ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2}{4x^4} - \frac{b^2 n^2}{72x^4} - \frac{a\,b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^4} - \frac{a^2}{4x^4} + \frac{a\,b\,n}{12x^4} + \frac{b^2 n\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{12x^4} - \frac{49b^2d^6n^2\ln\left(d+\frac{e}{x^{2/3}}\right)}{40e^6} + \frac{19b^2d^3n^2}{120e^3x^2} - \frac{37b^2d^2n^2}{480e^2x^{8/3}} - \frac{29b^2d^4n^2}{80e^4x^{4/3}} + \frac{49b^2d^5n^2}{40e^5x^{2/3}} + \frac{11b^2dn^2}{300e\,x^{10/3}} - \frac{b^2d^3n\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{6e^3x^2} + \frac{b^2d^2n\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{8e^2x^{8/3}} + \frac{b^2d^4n\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{4e^4x^{4/3}} - \frac{b^2d^5n\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2e^5x^{2/3}} - \frac{a\,b\,d\,n}{10e\,x^{10/3}} + \frac{a\,b\,d^6n\ln\left(d+\frac{e}{x^{2/3}}\right)}{2e^6} - \frac{b^2dn\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{10e\,x^{10/3}} - \frac{a\,b\,d^3n}{6e^3x^2} + \frac{a\,b\,d^2n}{8e^2x^{8/3}} + \frac{a\,b\,d^4n}{4e^4x^{4/3}} - \frac{a\,b\,d^5n}{2e^5x^{2/3}}$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^5,x)
```

output
```
(b^2*d^6*log(c*(d + e/x^(2/3))^n)^2)/(4*e^6) - (b^2*log(c*(d + e/x^(2/3))^
n)^2)/(4*x^4) - (b^2*n^2)/(72*x^4) - (a*b*log(c*(d + e/x^(2/3))^n))/(2*x^4
) - a^2/(4*x^4) + (a*b*n)/(12*x^4) + (b^2*n*log(c*(d + e/x^(2/3))^n))/(12*
x^4) - (49*b^2*d^6*n^2*log(d + e/x^(2/3)))/(40*e^6) + (19*b^2*d^3*n^2)/(12
0*e^3*x^2) - (37*b^2*d^2*n^2)/(480*e^2*x^(8/3)) - (29*b^2*d^4*n^2)/(80*e^4
*x^(4/3)) + (49*b^2*d^5*n^2)/(40*e^5*x^(2/3)) + (11*b^2*d*n^2)/(300*e*x^(1
0/3)) - (b^2*d^3*n*log(c*(d + e/x^(2/3))^n))/(6*e^3*x^2) + (b^2*d^2*n*log(
c*(d + e/x^(2/3))^n))/(8*e^2*x^(8/3)) + (b^2*d^4*n*log(c*(d + e/x^(2/3))^n
))/(4*e^4*x^(4/3)) - (b^2*d^5*n*log(c*(d + e/x^(2/3))^n))/(2*e^5*x^(2/3))
 - (a*b*d*n)/(10*e*x^(10/3)) + (a*b*d^6*n*log(d + e/x^(2/3)))/(2*e^6) - (b^
2*d*n*log(c*(d + e/x^(2/3))^n))/(10*e*x^(10/3)) - (a*b*d^3*n)/(6*e^3*x^2)
 + (a*b*d^2*n)/(8*e^2*x^(8/3)) + (a*b*d^4*n)/(4*e^4*x^(4/3)) - (a*b*d^5*n)/
(2*e^5*x^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = \frac{1800x^{\frac{8}{3}}\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)b^2d^4e^2n-720x^{\frac{2}{3}}\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)b^2d\,e^5n+180}{}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x)
```

output
```
(1800*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**4*e**2*n*
x**2 - 720*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d*e**5*
n + 1800*x**(2/3)*a*b*d**4*e**2*n*x**2 - 720*x**(2/3)*a*b*d*e**5*n - 2610*
x**(2/3)*b**2*d**4*e**2*n**2*x**2 + 264*x**(2/3)*b**2*d*e**5*n**2 - 3600*x
**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**5*e*n*x**3 + 900
*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**2*e**4*n*x - 3
600*x**(1/3)*a*b*d**5*e*n*x**3 + 900*x**(1/3)*a*b*d**2*e**4*n*x + 8820*x**
(1/3)*b**2*d**5*e*n**2*x**3 - 555*x**(1/3)*b**2*d**2*e**4*n**2*x + 1800*lo
g(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**2*d**6*x**4 - 1800*log(((x**
(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**2*e**6 + 3600*log(((x**(2/3)*d + e)
**n*c)/x**((2*n)/3))*a*b*d**6*x**4 - 3600*log(((x**(2/3)*d + e)**n*c)/x**(
(2*n)/3))*a*b*e**6 - 8820*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d
**6*n*x**4 - 1200*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**3*e**3
*n*x**2 + 600*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*e**6*n - 1800
*a**2*e**6 - 1200*a*b*d**3*e**3*n*x**2 + 600*a*b*e**6*n + 1140*b**2*d**3*e
**3*n**2*x**2 - 100*b**2*e**6*n**2)/(7200*e**6*x**4)
```

3.521 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 24, antiderivative size = 490

$$\begin{aligned}
\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx &= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} \\
&+ \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2} \\
&- \frac{1408b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{315d^{9/2}} - \frac{4ib^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{3d^{9/2}} \\
&+ \frac{8b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}} \\
&- \frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4} + \frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3} \\
&- \frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2} + \frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d} \\
&+ \frac{4be^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}} \\
&+ \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{4ib^2e^{9/2}n^2\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}}
\end{aligned}$$

output
```
-4/3*a*b*e^4*n*x^(1/3)/d^4+568/315*b^2*e^4*n^2*x^(1/3)/d^4-32/105*b^2*e^3*
n^2*x/d^3+8/105*b^2*e^2*n^2*x^(5/3)/d^2-1408/315*b^2*e^(9/2)*n^2*arctan(d^
(1/2)*x^(1/3)/e^(1/2))/d^(9/2)-4/3*I*b^2*e^(9/2)*n^2*arctan(d^(1/2)*x^(1/3
)/e^(1/2))^2/d^(9/2)+8/3*b^2*e^(9/2)*n^2*arctan(d^(1/2)*x^(1/3)/e^(1/2))*l
n(2-2*e^(1/2)/(e^(1/2)-I*d^(1/2)*x^(1/3)))/d^(9/2)-4/3*b^2*e^4*n*x^(1/3)*l
n(c*(d+e/x^(2/3))^n)/d^4+4/9*b*e^3*n*x*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3-4/1
5*b*e^2*n*x^(5/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^2+4/21*b*e*n*x^(7/3)*(a+b*
ln(c*(d+e/x^(2/3))^n))/d+4/3*b*e^(9/2)*n*arctan(d^(1/2)*x^(1/3)/e^(1/2))*(
a+b*ln(c*(d+e/x^(2/3))^n))/d^(9/2)+1/3*x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2-4
/3*I*b^2*e^(9/2)*n^2*polylog(2,-1+2*e^(1/2)/(e^(1/2)-I*d^(1/2)*x^(1/3)))/d
^(9/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.41 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.50

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

output
```
(x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 - 4*b*e*n*((a*e^3*x^(1/3))/d^4 - (
2*b*e^(7/2)*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/d^(9/2) - (2*b*e*n*x^(5/3
)*Hypergeometric2F1[-5/2, 1, -3/2, -(e/(d*x^(2/3)))])/(35*d^2) + (2*b*e^2*
n*x*Hypergeometric2F1[-3/2, 1, -1/2, -(e/(d*x^(2/3)))])/(15*d^3) - (2*b*e^
3*n*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))])/(3*d^4) + (
b*e^3*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/d^4 - (e^2*x*(a + b*Log[c*(d + e/x
^(2/3))^n]))/(3*d^3) + (e*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(5*d^2
) - (x^(7/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(7*d) + (e^(7/2)*(a + b*Log
[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(2*(-d)^(9/2)) - (
e^(7/2)*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/
(2*(-d)^(9/2)) - (b*e^(7/2)*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e
] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[
(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]
] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]))/(4*(-d)^(9/2)) +
(b*e^(7/2)*n*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1
/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(
1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*Po
lyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(4*(-d)^(9/2))))/3
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2908, 2907, 2005, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2dx$$

$$\Big\downarrow \text{2908}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(\frac{4}{9}ben\int\frac{x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d+\frac{e}{x^{2/3}}}d\sqrt[3]{x}+\frac{1}{9}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2005}$$

$$3\left(\frac{4}{9}ben\int \frac{x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{x^{2/3}d+e}d\sqrt[3]{x}+\frac{1}{9}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2926}$$

$$3\left(\frac{4}{9}ben\int\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)e^4}{d^4\left(x^{2/3}d+e\right)}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)e^3}{d^4}+\frac{x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{9}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{4}{9}ben\left(\frac{e^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{9/2}}+\frac{e^2x\left(a+b\log\right.}{}\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

output
```
3*((x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/9 + (4*b*e*n*(-((a*e^3*x^(1/3)
)/d^4) + (142*b*e^3*n*x^(1/3))/(105*d^4) - (8*b*e^2*n*x)/(35*d^3) + (2*b*e
*n*x^(5/3))/(35*d^2) - (352*b*e^(7/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])
/(105*d^(9/2)) - (I*b*e^(7/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^(9/
2) + (2*b*e^(7/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/
(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(9/2) - (b*e^3*x^(1/3)*Log[c*(d + e/x^(2
/3))^n])/d^4 + (e^2*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*d^3) - (e*x^(5/
3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(5*d^2) + (x^(7/3)*(a + b*Log[c*(d +
e/x^(2/3))^n]))/(7*d) + (e^(7/2)*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*
Log[c*(d + e/x^(2/3))^n]))/d^(9/2) - (I*b*e^(7/2)*n*PolyLog[2, -1 + (2*Sqr
t[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(9/2)))/9)
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int x^2 \left(a + b \ln \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^2 dx$$

input
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Fricas [F]

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2 x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x
+ e*x^(1/3))/x)^n) + a^2*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

output
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

Reduce [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\frac{420\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)ab\,e^4n-1408\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)b^2e^4n^2-84x^{\frac{5}{3}}\log}{}$$

input
```
int(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x)
```

output
```
(420*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*a*b*e**4*n - 140
8*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*b**2*e**4*n**2 - 84
*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**3*e**2*n*x - 8
4*x**(2/3)*a*b*d**3*e**2*n*x + 24*x**(2/3)*b**2*d**3*e**2*n**2*x - 105*x**
(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**2*d*e**4 + 60*x**(1/
3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**4*e*n*x**2 - 420*x**(
1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d*e**4*n + 60*x**(1/3)
*a*b*d**4*e*n*x**2 - 420*x**(1/3)*a*b*d*e**4*n + 568*x**(1/3)*b**2*d*e**4*
n**2 + 35*int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2/x**(2/3),x)*b**
2*d*e**4 + 105*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**2*d**5*x**3
 + 210*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b*d**5*x**3 + 140*log((
(x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d**2*e**3*n*x + 105*a**2*d**5*x*
*3 + 140*a*b*d**2*e**3*n*x - 96*b**2*d**2*e**3*n**2*x)/(315*d**5)
```

3.522 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 309

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}} + \frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d} - \frac{4be^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{4ib^2e^{3/2}n^2\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}$$

output
```
4*a*b*e*n*x^(1/3)/d+8*b^2*e^(3/2)*n^2*arctan(d^(1/2)*x^(1/3)/e^(1/2))/d^(3
/2)+4*I*b^2*e^(3/2)*n^2*arctan(d^(1/2)*x^(1/3)/e^(1/2))^2/d^(3/2)-8*b^2*e^
(3/2)*n^2*arctan(d^(1/2)*x^(1/3)/e^(1/2))*ln(2-2*e^(1/2)/(e^(1/2)-I*d^(1/2
)*x^(1/3)))/d^(3/2)+4*b^2*e*n*x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d-4*b*e^(3/2)*
n*arctan(d^(1/2)*x^(1/3)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(3/2)+x*(a
+b*ln(c*(d+e/x^(2/3))^n))^2+4*I*b^2*e^(3/2)*n^2*polylog(2,-1+2*e^(1/2)/(e^
(1/2)-I*d^(1/2)*x^(1/3)))/d^(3/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.69

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + ben\left(\frac{4a\sqrt[3]{x}}{d}-\frac{8b\sqrt{e}n\arctan\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}+\frac{4b\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d}-\frac{2\sqrt{e}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{\right.}}{(-d)^{3/2}}\right.$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

output
```
x*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + b*e*n*((4*a*x^(1/3))/d - (8*b*Sqrt[
e]*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/d^(3/2) + (4*b*x^(1/3)*Log[c*(d +
e/x^(2/3))^n])/d - (2*Sqrt[e]*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e]
 - Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (2*Sqrt[e]*(a + b*Log[c*(d + e/x^(2/3))
^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (b*Sqrt[e]*n*(Log[Sqrt[
e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt
[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog
[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3)
)/(2*Sqrt[e])]))/(-d)^(3/2) + (b*d*Sqrt[e]*n*(Log[Sqrt[e] + Sqrt[-d]*x^(1/
3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*S
qrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt
[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))
/(-d)^(5/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2901, 2907, 2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2dx$$

$$\Big\downarrow \text{2901}$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(\frac{4}{3}ben\int\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d+\frac{e}{x^{2/3}}}d\sqrt[3]{x}+\frac{1}{3}x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\right)$$

$$\Big\downarrow \text{2921}$$

$$3\left(\frac{4}{3}ben\int\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d}-\frac{e\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d\left(x^{2/3}d+e\right)}\right)d\sqrt[3]{x}+\frac{1}{3}x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{4}{3}ben\left(-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+\frac{a\sqrt[3]{x}}{d}+\frac{ib\sqrt{}}{}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

output
```
3*((x*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/3 + (4*b*e*n*((a*x^(1/3))/d + (2
*b*Sqrt[e]*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/d^(3/2) + (I*b*Sqrt[e]*n*A
rcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^(3/2) - (2*b*Sqrt[e]*n*ArcTan[(Sqrt[
d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d
^(3/2) + (b*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/d - (Sqrt[e]*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/d^(3/2) + (I*b*Sqrt[e
]*n*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(3/2)))/
3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [F]

$$\int \left(a+b\ln\left(c{\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}^{n}\right)\right)^{2}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1
/3))/x)^n) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}\,dx$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

output
```
Integral((a + b*log(c*(d + e/x**(2/3))**n))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx = \int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \frac{-12\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)aben+24\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)b^2en^2+3x^{\frac{1}{3}}\log\left(\frac{\left(x^{\frac{2}{3}}\right.}{}\right.}{}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))^2,x)
```

output
```
( - 12*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*a*b*e*n + 24*s
qrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*b**2*e*n**2 + 3*x**(1/
3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**2*d*e + 12*x**(1/3)*log
(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**2*d*e*n + 12*x**(1/3)*a*b*d*e*n
- int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2/x**(2/3),x)*b**2*d*e +
3*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**2*d**2*x + 6*log(((x**(2
/3)*d + e)**n*c)/x**((2*n)/3))*a*b*d**2*x + 3*a**2*d**2*x)/(3*d**2)
```

3.523 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 361

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{8b^2n^2}{9x}+\frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} + \frac{4ib^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} - \frac{8b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}} + \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} + \frac{4ib^2d^{3/2}n^2\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}$$

output
```
-8/9*b^2*n^2/x+32/3*b^2*d*n^2/e/x^(1/3)+32/3*b^2*d^(3/2)*n^2*arctan(d^(1/2
)*x^(1/3)/e^(1/2))/e^(3/2)+4*I*b^2*d^(3/2)*n^2*arctan(d^(1/2)*x^(1/3)/e^(1
/2))^2/e^(3/2)-8*b^2*d^(3/2)*n^2*arctan(d^(1/2)*x^(1/3)/e^(1/2))*ln(2-2*e^
(1/2)/(e^(1/2)-I*d^(1/2)*x^(1/3)))/e^(3/2)+4/3*b*n*(a+b*ln(c*(d+e/x^(2/3))
^n))/x-4*b*d*n*(a+b*ln(c*(d+e/x^(2/3))^n))/e/x^(1/3)-4*b*d^(3/2)*n*arctan(
d^(1/2)*x^(1/3)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^(3/2)-(a+b*ln(c*(d+
e/x^(2/3))^n))^2/x+4*I*b^2*d^(3/2)*n^2*polylog(2,-1+2*e^(1/2)/(e^(1/2)-I*d
^(1/2)*x^(1/3)))/e^(3/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.69

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx =$$

$$-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{bn\left(36ad\sqrt{e}x^{2/3}-72bd\sqrt{e}nx^{2/3}+72bd^{3/2}nx\arctan\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)+8bn\left(\sqrt{e}(e-3dx^{2/3})+3d^{3/2}x\,\mathrm{arcta}\right.\right.}{}}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```

output
```
-1/9*(9*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(36*a*d*Sqrt[e]*x^(2/3)
- 72*b*d*Sqrt[e]*n*x^(2/3) + 72*b*d^(3/2)*n*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1
/3))] + 8*b*n*(Sqrt[e]*(e - 3*d*x^(2/3)) + 3*d^(3/2)*x*ArcTan[Sqrt[e]/(Sqr
t[d]*x^(1/3))]) + 36*b*d*Sqrt[e]*x^(2/3)*Log[c*(d + e/x^(2/3))^n] - 12*e^(
3/2)*(a + b*Log[c*(d + e/x^(2/3))^n]) + 18*Sqrt[-d]*d*x*(a + b*Log[c*(d +
e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 18*(-d)^(3/2)*x*(a + b*Lo
g[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 9*b*(-d)^(3/2)*n
*x*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*L
og[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]
) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqr
t[-d]*x^(1/3))/(2*Sqrt[e])]) + 9*b*Sqrt[-d]*d*n*x*(Log[Sqrt[e] + Sqrt[-d]*
x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))
/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 +
(Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[
e]])))/e^(3/2))/x
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2908, 2907, 2005, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(-\frac{4}{3}ben\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{\left(d+\frac{e}{x^{2/3}}\right)x^2}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3x}\right)$$

$$\Big\downarrow \text{2005}$$

$$3\left(-\frac{4}{3}ben\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{\left(x^{2/3}d+e\right)x^{4/3}}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3x}\right)$$

$$\Big\downarrow \text{2926}$$

$$3\left(-\frac{4}{3}ben\int \left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)d^2}{e^2\left(x^{2/3}d+e\right)}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)d}{e^2x^{2/3}}+\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{ex^{4/3}}\right)d\sqrt[3]{x}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3x}-\frac{4}{3}ben\left(\frac{d^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{5/2}}+\frac{d\left(a+b\log\left(c\right.\right.}{e^2\ldots}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```

output
```
3*(-1/3*(a + b*Log[c*(d + e/x^(2/3))^n])^2/x - (4*b*e*n*((2*b*n)/(9*e*x) -
 (8*b*d*n)/(3*e^2*x^(1/3)) - (8*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[
e]])/(3*e^(5/2)) - (I*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(
5/2) + (2*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e]
)/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(5/2) - (a + b*Log[c*(d + e/x^(2/3))^n
])/(3*e*x) + (d*(a + b*Log[c*(d + e/x^(2/3))^n]))/(e^2*x^(1/3)) + (d^(3/2)
*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(5/
2) - (I*b*d^(3/2)*n*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/
3))])/e^(5/2)))/3)
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(
1/3))/x)^n) + a^2)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx=\frac{-12\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)abdnx-12x^{\frac{2}{3}}abden-4\left(\int\frac{\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)}{x^{\frac{8}{3}}d+e\,x^2}dx\right.}{}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x)
```

output
```
( - 12*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*a*b*d*n*x - 12
*x**(2/3)*a*b*d*e*n - 4*int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))/(x**
(2/3)*d*x**2 + e*x**2),x)*b**2*e**3*n*x - 3*log(((x**(2/3)*d + e)**n*c)/x*
*((2*n)/3))**2*b**2*e**2 - 6*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b
*e**2 - 3*a**2*e**2 + 4*a*b*e**2*n)/(3*e**2*x)
```

3.524 $\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 24, antiderivative size = 773

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

output

```
71/80*b^3*e^5*n^3*x^(2/3)/d^5-3/20*b^3*e^4*n^3*x^(4/3)/d^4+1/40*b^3*e^3*n^
3*x^2/d^3-71/80*b^3*e^6*n^3*ln(d+e/x^(2/3))/d^6-77/40*b^2*e^5*n^2*(d+e/x^(
2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+47/80*b^2*e^4*n^2*x^(4/3)*(a
+b*ln(c*(d+e/x^(2/3))^n))/d^4-9/40*b^2*e^3*n^2*x^2*(a+b*ln(c*(d+e/x^(2/3))
^n))/d^3+3/40*b^2*e^2*n^2*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^2-77/40*b^
2*e^6*n^2*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+3/4*b*e^5*
n*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^6-3/8*b*e^4*n*x^(4
/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^4+1/4*b*e^3*n*x^2*(a+b*ln(c*(d+e/x^(2/
3))^n))^2/d^3-3/16*b*e^2*n*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^2+3/20*
b*e*n*x^(10/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d+3/4*b*e^6*n*ln(1-d/(d+e/x^(
2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^6+1/4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n
))^3-3/2*b^2*e^6*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))/d^6-15/8
*b^3*e^6*n^3*ln(x)/d^6+77/40*b^3*e^6*n^3*polylog(2,d/(d+e/x^(2/3)))/d^6-3/
2*b^2*e^6*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*polylog(2,d/(d+e/x^(2/3)))/d^6-3
/2*b^3*e^6*n^3*polylog(2,1+e/d/x^(2/3))/d^6-3/2*b^3*e^6*n^3*polylog(3,d/(d
+e/x^(2/3)))/d^6
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.50 (sec) , antiderivative size = 5557, normalized size of antiderivative = 7.19

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Result too large to show}$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 10.89 (sec) , antiderivative size = 1384, normalized size of antiderivative = 1.79, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2904, 2845, 2858, 27, 2789, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2}\int x^{14/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2845}$$

$$-\frac{3}{2}\left(\frac{1}{2}ben\int\frac{x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d+\frac{e}{x^{2/3}}}d\frac{1}{x^{2/3}}-\frac{1}{6}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2858}$$

$$-\frac{3}{2}\left(\frac{1}{2}bn\int x^{14/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{6}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\int \frac{x^{14/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{e^6}d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{6}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2789}$$

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{\int \frac{x^4\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{e^6}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int -\frac{x^4\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{e^5}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right)-\frac{1}{6}x^4\left(a+b\log\left(c\left(d+\right.\right.\right.$$

$$\Big\downarrow \text{2756}$$

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\int -\frac{x^4\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^5}d\left(d+\frac{e}{x^{2/3}}\right)-\frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{5e^5}}{d}+\frac{\int -\frac{x^4\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{e^5}d\left(d+\right.}{d}\right.\right.$$

$$\Big\downarrow \text{2789}$$

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\int -\frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^5}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right)-\frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right.\right.}{5e^5}}{d}\right.\right.$$

$$\Big\downarrow \text{2756}$$

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{4e^4}-\frac{1}{2}bn\int \frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{-\frac{2}{5}}{}\right.\right.$$

$$\Big\downarrow \text{54}$$

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\int\left(\frac{x^{8/3}}{de^4}-\frac{x^2}{d^2e^3}+\frac{x^{4/3}}{d^3e^2}-\frac{x^{2/3}}{d^4e}+\frac{x^{2/3}}{d^4}\right)d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d\right.}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2009

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\int \frac{x^{10/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}-\frac{x^{2/3}}{d^3e}+\frac{x^4}{2d}\right.}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2789

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\frac{\int \frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^4}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(\right.}{}\right.}{d}\right.\right.$$

$\downarrow$ 2756

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{2}{5}bn\left(\frac{\frac{\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{-\frac{1}{3}bn\int -\frac{x^{8/3}}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{3e^3}}{d}}{d}+\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n}\right.\right.}{4e^4}}{}\right.}{d}\right.\right.$$

$\downarrow$ 54

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{4e^4}-\frac{1}{2}bn\left(\frac{-\frac{1}{3}bn\int\left(-\frac{x^2}{de^3}+\frac{x^{4/3}}{d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{x^{2/3}}{d^3}\right)d\left(d+\frac{e}{x^{2/3}}\right)-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{3e^3}}{d}+\frac{\int-\frac{x^{8/3}\left(a+b\log\left(cx\right.\right.}{e^3}}{}\right.}{d}}{}\right.\right.$$

$\big\downarrow$ 2009

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{4e^4}-\frac{1}{2}bn\left(\frac{\int-\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{3e^3}-\frac{1}{3}bn\left(\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^3}-\frac{\log\left(\right.}{}\right.}{d}\right.}{d}}{}\right.\right.$$

$\big\downarrow$ 2789

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{4e^4}-\frac{1}{2}bn\left(\frac{\frac{\int-\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}}{d}+\frac{-\frac{x^2\left(a+b\log\left(c\right.\right.}{3e^3}}{}\right.}{d}}{}\right.\right.$$

$\big\downarrow$ 2756

$$-\frac{3}{2}\Bigg(\frac{1}{2}be^6n\Bigg(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\Bigg(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\Big(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}}{d}}{}$$

$\downarrow$ 54

$$-\frac{3}{2}\Bigg(\frac{1}{2}be^6n\Bigg(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\Bigg(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\Big(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}}{d}}{}$$

$\downarrow$ 2009

$$-\frac{3}{2}\Bigg(\frac{1}{2}be^6n\Bigg(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\Bigg(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\Big(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}}{d}}{}$$

$\downarrow$ 2789

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}\right.}{d}\right.}{}\right.\right.$$

$\downarrow$ 2751

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}\right.}{d}\right.}{}\right.\right.$$

$\downarrow$ 16

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}\right.}{d}\right.}{}\right.\right.$$

$\downarrow$ 2755

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}\right.}{d}\right.}{}\right.\right.$$

$\downarrow$ 2754

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}\right.}{d}\right.}{}\right.\right.$$

$\downarrow$ 2779

$$-\frac{3}{2}\left(\frac{1}{2}be^6n\left(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\left(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\left(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}\right.}{d}\right.}{}\right.\right.$$

$\downarrow$ 2821

$$-\frac{3}{2}\Bigg(\frac{1}{2}be^6n\Bigg(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\Bigg(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\Big(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}}{d}}{}$$

$$\Big\downarrow \; 2838$$

$$-\frac{3}{2}\Bigg(\frac{1}{2}be^6n\Bigg(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\Bigg(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\Big(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}}{d}}{}$$

$$\Big\downarrow \; 7143$$

$$-\frac{3}{2}\Bigg(\frac{1}{2}be^6n\Bigg(\frac{-\frac{\left(a+b\log\left(cx^{-2n/3}\right)\right)^2x^{10/3}}{5e^5}-\frac{2}{5}bn\Bigg(\frac{\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{4e^4}-\frac{1}{4}bn\Big(-\frac{x^2}{3de^3}+\frac{x^{4/3}}{2d^2e^2}-\frac{x^{2/3}}{d^3e}+\frac{\log\left(d+\frac{e}{x^{2/3}}\right)}{d^4}-\frac{\log\left(-\frac{e}{x^{2/3}}\right)}{d^4}}{d}}{}$$

input
```
Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

output
```
(-3*(-1/6*(x^4*(a + b*Log[c*(d + e/x^(2/3))^n])^3) + (b*e^6*n*((-1/5*(x^(1
0/3)*(a + b*Log[c/x^((2*n)/3)])^2)/e^5 - (2*b*n*((-1/4*(b*n*(-(x^(2/3)/(d^
3*e)) + x^(4/3)/(2*d^2*e^2) - x^2/(3*d*e^3) + Log[d + e/x^(2/3)]/d^4 - Log
[-(e/x^(2/3))]/d^4)) + (x^(8/3)*(a + b*Log[c/x^((2*n)/3)]))/(4*e^4))/d + (
(-1/3*(b*n*(-(x^(2/3)/(d^2*e)) + x^(4/3)/(2*d*e^2) + Log[d + e/x^(2/3)]/d^
3 - Log[-(e/x^(2/3))]/d^3)) - (x^2*(a + b*Log[c/x^((2*n)/3)]))/(3*e^3))/d
+ ((-1/2*(b*n*(-(x^(2/3)/(d*e)) + Log[d + e/x^(2/3)]/d^2 - Log[-(e/x^(2/3)
)]/d^2)) + (x^(4/3)*(a + b*Log[c/x^((2*n)/3)]))/(2*e^2))/d + (((b*n*Log[-(
e/x^(2/3))])/d - ((d + e/x^(2/3))*x^(2/3)*(a + b*Log[c/x^((2*n)/3)]))/(d*e
))/d + (-((Log[1 - d*x^(2/3)]*(a + b*Log[c/x^((2*n)/3)]))/d) + (b*n*PolyLo
g[2, d*x^(2/3)])/d)/d)/d)/d)/d))/5)/d + (((x^(8/3)*(a + b*Log[c/x^((2*n)/3
)])^2)/(4*e^4) - (b*n*((-1/3*(b*n*(-(x^(2/3)/(d^2*e)) + x^(4/3)/(2*d*e^2)
+ Log[d + e/x^(2/3)]/d^3 - Log[-(e/x^(2/3))]/d^3)) - (x^2*(a + b*Log[c/x^(
(2*n)/3)]))/(3*e^3))/d + ((-1/2*(b*n*(-(x^(2/3)/(d*e)) + Log[d + e/x^(2/3)
]/d^2 - Log[-(e/x^(2/3))]/d^2)) + (x^(4/3)*(a + b*Log[c/x^((2*n)/3)]))/(2*
e^2))/d + (((b*n*Log[-(e/x^(2/3))])/d - ((d + e/x^(2/3))*x^(2/3)*(a + b*Lo
g[c/x^((2*n)/3)]))/(d*e))/d + (-((Log[1 - d*x^(2/3)]*(a + b*Log[c/x^((2*n)
/3)]))/d) + (b*n*PolyLog[2, d*x^(2/3)])/d)/d)/d)/d))/2)/d + ((-1/3*(x^2*(a
 + b*Log[c/x^((2*n)/3)])^2)/e^3 - (2*b*n*((-1/2*(b*n*(-(x^(2/3)/(d*e)) + L
og[d + e/x^(2/3)]/d^2 - Log[-(e/x^(2/3))]/d^2)) + (x^(4/3)*(a + b*Log[c...
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int x^3 \left(a + b \ln \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^3 dx$$

input
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Fricas [F]

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a\right)^3 x^3 dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x^3*log(c*((d*
x + e*x^(1/3))/x)^n)^2 + 3*a^2*b*x^3*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*
x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3 x^3 dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4*log((d*x^(2/3) + e)^n)^3 - integrate(-1/2*(2*(b^3*d*log(c)^3 +
 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^4 + 2*(b^3*e*log(c)^3 +
3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(10/3) - 16*(b^3*d*x^4 +
b^3*e*x^(10/3))*log(x^(1/3*n))^3 - (b^3*d*n*x^4 - 6*(b^3*d*log(c) + a*b^2*
d)*x^4 - 6*(b^3*e*log(c) + a*b^2*e)*x^(10/3) + 12*(b^3*d*x^4 + b^3*e*x^(10
/3))*log(x^(1/3*n)))*log((d*x^(2/3) + e)^n)^2 + 24*((b^3*d*log(c) + a*b^2*
d)*x^4 + (b^3*e*log(c) + a*b^2*e)*x^(10/3))*log(x^(1/3*n))^2 + 6*((b^3*d*l
og(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^4 + (b^3*e*log(c)^2 + 2*a*b^2*e*lo
g(c) + a^2*b*e)*x^(10/3) + 4*(b^3*d*x^4 + b^3*e*x^(10/3))*log(x^(1/3*n))^2
 - 4*((b^3*d*log(c) + a*b^2*d)*x^4 + (b^3*e*log(c) + a*b^2*e)*x^(10/3))*lo
g(x^(1/3*n)))*log((d*x^(2/3) + e)^n) - 12*((b^3*d*log(c)^2 + 2*a*b^2*d*log
(c) + a^2*b*d)*x^4 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(10/3
))*log(x^(1/3*n)))/(d*x + e*x^(1/3)), x)
```

Giac [F]

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3 x^3 dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int x^3\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

input
```
int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
```

output
```
int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

Reduce [F]

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x)
```

output
```
( - 45*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**3*d**4*e**
2*n*x**2 - 90*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b**2*d*
*4*e**2*n*x**2 + 18*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**
3*d**4*e**2*n**2*x**2 - 45*x**(2/3)*a**2*b*d**4*e**2*n*x**2 + 180*x**(2/3)
*a**2*b*d*e**5*n + 18*x**(2/3)*a*b**2*d**4*e**2*n**2*x**2 - 462*x**(2/3)*a
*b**2*d*e**5*n**2 + 213*x**(2/3)*b**3*d*e**5*n**3 + 36*x**(1/3)*log(((x**(
2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**3*d**5*e*n*x**3 - 90*x**(1/3)*log(((
x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**3*d**2*e**4*n*x + 72*x**(1/3)*lo
g(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b**2*d**5*e*n*x**3 - 180*x**(1/3
)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b**2*d**2*e**4*n*x + 141*x**
(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**3*d**2*e**4*n**2*x + 36
*x**(1/3)*a**2*b*d**5*e*n*x**3 - 90*x**(1/3)*a**2*b*d**2*e**4*n*x + 141*x*
*(1/3)*a*b**2*d**2*e**4*n**2*x - 36*x**(1/3)*b**3*d**2*e**4*n**3*x + 120*i
nt((x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2)/(x**(2/3)*d + e
),x)*b**3*d**2*e**5*n + 240*int((x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**(
(2*n)/3)))/(x**(2/3)*d + e),x)*a*b**2*d**2*e**5*n - 308*int((x**(1/3)*log(
((x**(2/3)*d + e)**n*c)/x**((2*n)/3)))/(x**(2/3)*d + e),x)*b**3*d**2*e**5*
n**2 - 360*log(x**(1/3))*a**2*b*e**6*n + 924*log(x**(1/3))*a*b**2*e**6*n**
2 - 426*log(x**(1/3))*b**3*e**6*n**3 + 60*log(((x**(2/3)*d + e)**n*c)/x**(
(2*n)/3))**3*b**3*d**6*x**4 + 180*log(((x**(2/3)*d + e)**n*c)/x**((2*n)...
```

3.525 $\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 22, antiderivative size = 451

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}$$
$$+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}$$
$$-\frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}+\frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d}$$
$$-\frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}$$
$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)}{d^3}+\frac{b^3e^3n^3\log(x)}{d^3}-\frac{3b^3e^3n}{}$$

output
```
3/2*b^2*e^2*n^2*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+3/2*
b^2*e^3*n^2*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3-3/2*b*e^
2*n*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^3+3/4*b*e*n*x^(4
/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d-3/2*b*e^3*n*ln(1-d/(d+e/x^(2/3)))*(a+b
*ln(c*(d+e/x^(2/3))^n))^2/d^3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3+3*b^2*
e^3*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))/d^3+b^3*e^3*n^3*ln(x)
/d^3-3/2*b^3*e^3*n^3*polylog(2,d/(d+e/x^(2/3)))/d^3+3*b^2*e^3*n^2*(a+b*ln(
c*(d+e/x^(2/3))^n))*polylog(2,d/(d+e/x^(2/3)))/d^3+3*b^3*e^3*n^3*polylog(2
,1+e/d/x^(2/3))/d^3+3*b^3*e^3*n^3*polylog(3,d/(d+e/x^(2/3)))/d^3
```

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 3.31 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.83, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.727$, Rules used = {2904, 2845, 2858, 25, 27, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2}\int x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2845}$$

$$-\frac{3}{2}\left(ben\int \frac{x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d+\frac{e}{x^{2/3}}}d\frac{1}{x^{2/3}}-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2858}$$

$$-\frac{3}{2}\left(bn\int x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{3}{2}\left(-bn\int -x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{2}\left(-be^3n\int -\frac{x^{8/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2789}$$

$$-\frac{3}{2}\left(-be^3n\left(\frac{\int -\frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{e^3}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right)-\frac{1}{3}x^2\left(a+b\log\left(c\left(d+\right.\right.\right.\right.$$

$$\Big\downarrow \text{2756}$$

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{2e^2}-bn\int \frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int \frac{x^2\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right)\right.$$

$$\Big\downarrow \text{2789}$$

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\int \frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e^2}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}\right)}{d}+\right.\right.$$

$$\Big\downarrow \text{2751}$$

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{2e^2}-bn\left(\frac{-\frac{bn\int -\frac{x^{2/3}}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{de}}{d}+\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-2n\right.}\right.}{e}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 16

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-\right.}\right.}{de}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2755

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-\right.}\right.}{de}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2754

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\int -\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)}{e}d\left(d+\frac{e}{x^{2/3}}\right)}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-\right.}\right.}{de}}{d}\right.}{d}\right.\right.$$

$\downarrow$ 2779

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\frac{bn\int x^{2/3}\log\left(1-dx^{2/3}\right)d\left(d+\frac{e}{x^{2/3}}\right)}{d}-\frac{\log\left(1-dx^{2/3}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-}{}\right.}{d}\right.\right.$$

$\downarrow$ 2821

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\frac{bn\int x^{2/3}\log\left(1-dx^{2/3}\right)d\left(d+\frac{e}{x^{2/3}}\right)}{d}-\frac{\log\left(1-dx^{2/3}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{d}}{d}+\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\right.\right.}{d}\right.\right.$$

$\downarrow$ 2838

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{\frac{2bn\left(\operatorname{PolyLog}\left(2,dx^{2/3}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)-bn\int x^{2/3}\operatorname{PolyLog}\left(2,dx^{2/3}\right)d\left(d+\frac{e}{x^{2/3}}\right)\right)}{d}-\frac{\log\left(1-dx^{2/3}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{d}}{d}+\frac{-\frac{2b}{}}{}}{d}\right.\right.$$

$\downarrow$ 7143

$$-\frac{3}{2}\left(-be^3n\left(\frac{\frac{x^{4/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)^2}{2e^2}-bn\left(\frac{\frac{bn\log\left(-\frac{e}{x^{2/3}}\right)}{d}-\frac{x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)}{de}}{d}+\frac{\frac{bn\operatorname{PolyLog}\left(2,dx^{2/3}\right)}{d}-\frac{\log\left(1-dx^{2/3}\right)}{}}{d}\right.\right.}{d}\right.\right.$$

input

```
Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

output

```
(-3*(-1/3*(x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3) - b*e^3*n*(((x^(4/3)*(a
 + b*Log[c/x^((2*n)/3)])^2)/(2*e^2) - b*n*(((b*n*Log[-(e/x^(2/3))])/d - ((
d + e/x^(2/3))*x^(2/3)*(a + b*Log[c/x^((2*n)/3)]))/(d*e))/d + (-((Log[1 -
d*x^(2/3)]*(a + b*Log[c/x^((2*n)/3)]))/d) + (b*n*PolyLog[2, d*x^(2/3)])/d)
/d))/d + ((-(((d + e/x^(2/3))*x^(2/3)*(a + b*Log[c/x^((2*n)/3)])^2)/(d*e))
 - (2*b*n*(-(Log[1 - (d + e/x^(2/3))/d]*(a + b*Log[c/x^((2*n)/3)])) - b*n*
PolyLog[2, (d + e/x^(2/3))/d]))/d)/d + (-((Log[1 - d*x^(2/3)]*(a + b*Log[c
/x^((2*n)/3)])^2)/d) + (2*b*n*((a + b*Log[c/x^((2*n)/3)])*PolyLog[2, d*x^(
2/3)] + b*n*PolyLog[3, d*x^(2/3)]))/d)/d)/d)))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, 
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, 
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, 
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), 
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x] 
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q, 
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2845
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Simp[b*e*n*(p/(g*(q + 1)))   Int[(f + g*x)^(q + 1)
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && In
tegersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x +
e*x^(1/3))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2*log((d*x^(2/3) + e)^n)^3 - integrate((8*(b^3*d*x^2 + b^3*e*x^(
4/3))*log(x^(1/3*n))^3 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*
log(c) + a^3*d)*x^2 + (b^3*d*n*x^2 - 3*(b^3*d*log(c) + a*b^2*d)*x^2 - 3*(b
^3*e*log(c) + a*b^2*e)*x^(4/3) + 6*(b^3*d*x^2 + b^3*e*x^(4/3))*log(x^(1/3*
n)))*log((d*x^(2/3) + e)^n)^2 - 12*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^3*e*
log(c) + a*b^2*e)*x^(4/3))*log(x^(1/3*n))^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*
log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(4/3) - 3*((b^3*d*log(c)^2 + 2*a*b^
2*d*log(c) + a^2*b*d)*x^2 + 4*(b^3*d*x^2 + b^3*e*x^(4/3))*log(x^(1/3*n))^2
 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(4/3) - 4*((b^3*d*log(c
) + a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(4/3))*log(x^(1/3*n)))*log((
d*x^(2/3) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 +
 (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(4/3))*log(x^(1/3*n)))/(d
*x + e*x^(1/3)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

Reduce [F]

$$\int x\Big(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\Big)^3\,dx=\frac{-6x^{\frac{2}{3}}a^2bd\,e^2n+6x^{\frac{2}{3}}a\,b^2d\,e^2n^2+3x^{\frac{4}{3}}\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^nc}{x^{\frac{2n}{3}}}\right)^2b^3d^2en+6x^{\frac{4}{3}}\log\Bigg(\cdots}{\cdots}$$

input
```
int(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x)
```

output

```
( - 6*x**(2/3)*a**2*b*d*e**2*n + 6*x**(2/3)*a*b**2*d*e**2*n**2 + 3*x**(1/3
)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**3*d**2*e*n*x + 6*x**(1/3
)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b**2*d**2*e*n*x + 3*x**(1/3)
*a**2*b*d**2*e*n*x - 4*int((x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)
/3))**2)/(x**(2/3)*d + e),x)*b**3*d**2*e**2*n - 8*int((x**(1/3)*log(((x**(
2/3)*d + e)**n*c)/x**((2*n)/3)))/(x**(2/3)*d + e),x)*a*b**2*d**2*e**2*n +
4*int((x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3)))/(x**(2/3)*d + e
),x)*b**3*d**2*e**2*n**2 + 12*log(x**(1/3))*a**2*b*e**3*n - 12*log(x**(1/3
))*a*b**2*e**3*n**2 + 2*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3*b**3*
d**3*x**2 + 6*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*a*b**2*d**3*x**
2 + 6*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a**2*b*d**3*x**2 + 6*log((
(x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a**2*b*e**3 - 6*log(((x**(2/3)*d + e)
**n*c)/x**((2*n)/3))*a*b**2*e**3*n + 2*a**3*d**3*x**2)/(4*d**3)
```

3.526 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx = -\frac{3}{2}\Big(a$$
$$+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\Big)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\text{PolyLog}\left(2,1+\frac{e}{dx^{2/3}}\right)+9b^2n^2\Big(a$$

output
```
-3/2*(a+b*ln(c*(d+e/x^(2/3))^n))^3*ln(-e/d/x^(2/3))-9/2*b*n*(a+b*ln(c*(d+e
/x^(2/3))^n))^2*polylog(2,1+e/d/x^(2/3))+9*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))
^n))*polylog(3,1+e/d/x^(2/3))-9*b^3*n^3*polylog(4,1+e/d/x^(2/3))
```

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2904, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2}\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2843}$$

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-3ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right)}{d+\frac{e}{x^{2/3}}}d\frac{1}{x^{2/3}}\right)$$

$$\Big\downarrow \text{2881}$$

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-3bn\int x^{2/3}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)^2d\left(d+\frac{e}{x^{2/3}}\right.\right.$$

$$\Big\downarrow \text{2821}$$

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-3bn\left(2bn\int x^{2/3}\left(a+b\log\left(cx^{-2n/3}\right)\right)\text{PolyLog}\left(2,\frac{d+\frac{e}{x^{2/3}}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{2830}$$

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+\frac{e}{x^{2/3}}}{d}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)-b\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$-\frac{3}{2}\left(\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-3bn\left(2bn\left(\text{PolyLog}\left(3,\frac{d+\frac{e}{x^{2/3}}}{d}\right)\left(a+b\log\left(cx^{-2n/3}\right)\right)-b\right.\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
```

output
```
(-3*((a + b*Log[c*(d + e/x^(2/3))^n])^3*Log[-(e/(d*x^(2/3)))] - 3*b*n*(-((
a + b*Log[c/x^((2*n)/3)])^2*PolyLog[2, (d + e/x^(2/3))/d]) + 2*b*n*((a + b
*Log[c/x^((2*n)/3)])*PolyLog[3, (d + e/x^(2/3))/d] - b*n*PolyLog[4, (d + e
/x^(2/3))/d]))))/2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x
^(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="maxima")
```

output
```
b^3*log((d*x^(2/3) + e)^n)^3*log(x) - integrate((8*(b^3*d*x + b^3*e*x^(1/3
))*log(x^(1/3*n))^3 + (2*b^3*d*n*x*log(x) - 3*(b^3*d*log(c) + a*b^2*d)*x +
 6*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3*n)) - 3*(b^3*e*log(c) + a*b^2*e)*x
^(1/3))*log((d*x^(2/3) + e)^n)^2 - 12*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e
*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n))^2 - (b^3*d*log(c)^3 + 3*a*b^2*d
*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 3*(4*(b^3*d*x + b^3*e*x^(1/3))*l
og(x^(1/3*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 4*((b^
3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n))
 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(1/3))*log((d*x^(2/3) +
 e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c
)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(1/3))*log(x^(1/3*n)) - (b^3*e*log(c)^
3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(1/3))/(d*x^2 + e*x^(
4/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^3/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))^3/x, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx=\left(\int \frac{\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)^3}{x}dx\right)b^3+3\left(\int \frac{\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)^2}{x}dx\right)a\,b^2+3\left(\int \frac{\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)}{x}dx\right)a^2b+\log(x)\,a^3$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x)
```

output
```
int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3/x,x)*b**3 + 3*int(log(((x
**(2/3)*d + e)**n*c)/x**((2*n)/3))**2/x,x)*a*b**2 + 3*int(log(((x**(2/3)*d
 + e)**n*c)/x**((2*n)/3))/x,x)*a**2*b + log(x)*a**3
```

3.527 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 449

$$\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx =& -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3}+\frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} \\
&-\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9b^3d^2n^3}{e^2x^{2/3}}-\frac{9b^3d^2n^2\left(d+\frac{e}{x^{2/3}}\right)\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{e^3} \\
&+\frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^3} \\
&-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} \\
&+\frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} \\
&-\frac{9bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4e^3} \\
&+\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} \\
&-\frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} \\
&+\frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} \\
&-\frac{\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3}
\end{aligned}$$

output
```
-9/8*b^3*d*n^3*(d+e/x^(2/3))^2/e^3+1/9*b^3*n^3*(d+e/x^(2/3))^3/e^3-9*a*b^2
*d^2*n^2/e^2/x^(2/3)+9*b^3*d^2*n^3/e^2/x^(2/3)-9*b^3*d^2*n^2*(d+e/x^(2/3))
*ln(c*(d+e/x^(2/3))^n)/e^3+9/4*b^2*d*n^2*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^
(2/3))^n))/e^3-1/3*b^2*n^2*(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3
+9/2*b*d^2*n*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3-9/4*b*d*n*(d+
e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3+1/2*b*n*(d+e/x^(2/3))^3*(a+
b*ln(c*(d+e/x^(2/3))^n))^2/e^3-3/2*d^2*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3
))^n))^3/e^3+3/2*d*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3/e^3-1/2*(
d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3/e^3
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = \frac{-36a^3e^3+36a^2be^3n-24ab^2e^3n^2+8b^3e^3n^3-54a^2bde^2nx^{2/3}+90ab^2d}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```

output
```
(-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*
b*d*e^2*n*x^(2/3) + 90*a*b^2*d*e^2*n^2*x^(2/3) - 57*b^3*d*e^2*n^3*x^(2/3)
+ 108*a^2*b*d^2*e*n*x^(4/3) - 396*a*b^2*d^2*e*n^2*x^(4/3) + 510*b^3*d^2*e*
n^3*x^(4/3) + 72*b^3*d^3*n^3*x^2*Log[d + e/x^(2/3)]^3 - 36*b^3*e^3*Log[c*(
d + e/x^(2/3))^n]^3 - 108*a^2*b*d^3*n*x^2*Log[e + d*x^(2/3)] + 396*a*b^2*d
^3*n^2*x^2*Log[e + d*x^(2/3)] - 510*b^3*d^3*n^3*x^2*Log[e + d*x^(2/3)] + 1
2*b^2*d^3*n^2*x^2*Log[d + e/x^(2/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2
/3))^n])*(3*Log[e + d*x^(2/3)] - 2*Log[x]) + 72*a^2*b*d^3*n*x^2*Log[x] - 2
64*a*b^2*d^3*n^2*x^2*Log[x] + 340*b^3*d^3*n^3*x^2*Log[x] - 18*b^2*d^3*n^2*
x^2*Log[d + e/x^(2/3)]^2*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2/3))^n] + 6*
b*n*Log[e + d*x^(2/3)] - 4*b*n*Log[x]) + 18*b^2*Log[c*(d + e/x^(2/3))^n]^2
*(e*(-6*a*e^2 + 2*b*e^2*n - 3*b*d*e*n*x^(2/3) + 6*b*d^2*n*x^(4/3)) - 6*b*d
^3*n*x^2*Log[e + d*x^(2/3)] + 4*b*d^3*n*x^2*Log[x]) - 6*b*Log[c*(d + e/x^(
2/3))^n]*(18*a^2*e^3 - 6*a*b*e*n*(2*e^2 - 3*d*e*x^(2/3) + 6*d^2*x^(4/3)) +
 b^2*e*n^2*(4*e^2 - 15*d*e*x^(2/3) + 66*d^2*x^(4/3)) + 6*b*d^3*n*(6*a - 11
*b*n)*x^2*Log[e + d*x^(2/3)] + 4*b*d^3*n*(-6*a + 11*b*n)*x^2*Log[x]))/(72*
e^3*x^2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$-\frac{3}{2}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^{4/3}}d\frac{1}{x^{2/3}}$$

$$\Big\downarrow \text{2848}$$

$$-\frac{3}{2}\int \left(\frac{\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{e^2}-\frac{2d\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{e^2}+\frac{d^2\left(a+b\log\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}\left(\frac{2b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9e^3}-\frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3}+\frac{6ab^2d^2}{e^2x^{2/}}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```

output
```
(-3*((3*b^3*d*n^3*(d + e/x^(2/3))^2)/(4*e^3) - (2*b^3*n^3*(d + e/x^(2/3))^
3)/(27*e^3) + (6*a*b^2*d^2*n^2)/(e^2*x^(2/3)) - (6*b^3*d^2*n^3)/(e^2*x^(2/
3)) + (6*b^3*d^2*n^2*(d + e/x^(2/3))*Log[c*(d + e/x^(2/3))^n])/e^3 - (3*b^
2*d*n^2*(d + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*e^3) + (2*b
^2*n^2*(d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(9*e^3) - (3*b*
d^2*n*(d + e/x^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/e^3 + (3*b*d*n*(
d + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (b*n*(d + e
/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*e^3) + (d^2*(d + e/x^(2
/3))*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/e^3 - (d*(d + e/x^(2/3))^2*(a + b
*Log[c*(d + e/x^(2/3))^n])^3)/e^3 + ((d + e/x^(2/3))^3*(a + b*Log[c*(d + e
/x^(2/3))^n])^3)/(3*e^3)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.61

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="fricas")
```

output
```
1/72*(8*b^3*e^3*n^3 - 36*b^3*e^3*log(c)^3 - 24*a*b^2*e^3*n^2 + 36*a^2*b*e^
3*n - 36*a^3*e^3 - 36*(b^3*d^3*n^3*x^2 + b^3*e^3*n^3)*log((d*x + e*x^(1/3)
)/x)^3 + 36*(b^3*e^3*n - 3*a*b^2*e^3)*log(c)^2 + 18*(6*b^3*d^2*e*n^3*x^(4/
3) - 3*b^3*d*e^2*n^3*x^(2/3) + 2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + (11*b^3*d
^3*n^3 - 6*a*b^2*d^3*n^2)*x^2 - 6*(b^3*d^3*n^2*x^2 + b^3*e^3*n^2)*log(c))*
log((d*x + e*x^(1/3))/x)^2 - 12*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e
^3)*log(c) - 6*(4*b^3*e^3*n^3 - 12*a*b^2*e^3*n^2 + 18*a^2*b*e^3*n + (85*b^
3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n)*x^2 + 18*(b^3*d^3*n*x^2 + b
^3*e^3*n)*log(c)^2 - 6*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + (11*b^3*d^3*n^2 -
6*a*b^2*d^3*n)*x^2)*log(c) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) -
 6*a*b^2*d*e^2*n^2)*x^(2/3) - 6*(6*b^3*d^2*e*n^2*x*log(c) - (11*b^3*d^2*e*
n^3 - 6*a*b^2*d^2*e*n^2)*x)*x^(1/3))*log((d*x + e*x^(1/3))/x) - 3*(19*b^3*
d*e^2*n^3 + 18*b^3*d*e^2*n*log(c)^2 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*
n - 6*(5*b^3*d*e^2*n^2 - 6*a*b^2*d*e^2*n)*log(c))*x^(2/3) + 6*(18*b^3*d^2*
e*n*x*log(c)^2 - 6*(11*b^3*d^2*e*n^2 - 6*a*b^2*d^2*e*n)*x*log(c) + (85*b^3
*d^2*e*n^3 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n)*x)*x^(1/3))/(e^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.52

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

output
```
-1/4*a^2*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6
*d^2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/12*(6*e*n*(6*d^3*log(
d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/
3) + 2*e^2)/(e^3*x^2))*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*log(d*x^(2/3
) + e)^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*
d*e^2*x^(2/3) - 4*e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) +
e))*n^2/(e^3*x^2))*a*b^2 - 1/216*(54*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6
*d^3*log(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2))
*log(c*(d + e/x^(2/3))^n)^2 + e*n*((108*d^3*x^2*log(d*x^(2/3) + e)^3 - 32*
d^3*x^2*log(x)^3 + 264*d^3*x^2*log(x)^2 - 1020*d^3*x^2*log(x) - 1530*d^2*e
*x^(4/3) + 171*d*e^2*x^(2/3) - 24*e^3 - 54*(4*d^3*x^2*log(x) - 11*d^3*x^2)
*log(d*x^(2/3) + e)^2 + 18*(8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) + 85*d^
3*x^2)*log(d*x^(2/3) + e))*n^2/(e^4*x^2) - 18*(18*d^3*x^2*log(d*x^(2/3) +
e)^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^
2*x^(2/3) - 4*e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*
n*log(c*(d + e/x^(2/3))^n)/(e^4*x^2)))*b^3 - 1/2*b^3*log(c*(d + e/x^(2/3))
^n)^3/x^2 - 3/2*a*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - 3/2*a^2*b*log(c*(d
+ e/x^(2/3))^n)/x^2 - 1/2*a^3/x^2
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 25.63 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx=\frac{\frac{d\left(\frac{3a^3}{2}-\frac{3a^2bn}{2}+ab^2n^2-\frac{b^3n^3}{3}\right)}{2e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{8e}}{x^{4/3}}$$

$$-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^3\left(\frac{b^3}{2x^2}+\frac{b^3d^3}{2e^3}\right)-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2\left(\frac{b^2(3a-bn)}{2x^2}-\frac{\frac{3b^2d(3a-bn)}{2e}-\frac{9ab^2d}{2e}}{2x^{4/3}}+\frac{d\,(6a\right.$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^3,x)
```

output
```
((d*((3*a^3)/2 - (b^3*n^3)/3 + a*b^2*n^2 - (3*a^2*b*n)/2))/(2*e) - (d*(6*a
^3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(8*e))/x^(4/3) - log(c*(d + e/x^(2/3))^n)^3
*(b^3/(2*x^2) + (b^3*d^3)/(2*e^3)) - log(c*(d + e/x^(2/3))^n)^2*((b^2*(3*a
 - b*n))/(2*x^2) - ((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e))/(2*x^
(4/3)) + (d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(4*e^3) + (d*((6*b^2*d*(3*a - b*
n))/e - (18*a*b^2*d)/e))/(4*e*x^(2/3))) - ((d*((d*((3*a^3)/2 - (b^3*n^3)/3
 + a*b^2*n^2 - (3*a^2*b*n)/2))/e - (d*(6*a^3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(
4*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/(2*e^2))/x^(2/3) - (a^3/2 - (b^3*n
^3)/9 + (a*b^2*n^2)/3 - (a^2*b*n)/2)/x^2 - (log(c*(d + e/x^(2/3))^n)*(((d*
(2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^2 - b^2*n^2)))/e + 1
2*b^3*d^2*n^2)/(2*e*x^(2/3)) - (2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*
b*d*e*(3*a^2 - b^2*n^2))/(4*e*x^(4/3)) + (b*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n
))/(3*x^2)))/(2*e) - (log(d + e/x^(2/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^
2 + 18*a^2*b*d^3*n))/(12*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x)
```

output

```
( - 54*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**3*d*e**2*n
 - 108*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b**2*d*e**2*n
+ 90*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**3*d*e**2*n**2 -
 54*x**(2/3)*a**2*b*d*e**2*n + 90*x**(2/3)*a*b**2*d*e**2*n**2 - 57*x**(2/3
)*b**3*d*e**2*n**3 + 108*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3)
)**2*b**3*d**2*e*n*x + 216*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/
3))*a*b**2*d**2*e*n*x - 396*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)
/3))*b**3*d**2*e*n**2*x + 108*x**(1/3)*a**2*b*d**2*e*n*x - 396*x**(1/3)*a*
b**2*d**2*e*n**2*x + 510*x**(1/3)*b**3*d**2*e*n**3*x - 36*log(((x**(2/3)*d
 + e)**n*c)/x**((2*n)/3))**3*b**3*d**3*x**2 - 36*log(((x**(2/3)*d + e)**n*
c)/x**((2*n)/3))**3*b**3*e**3 - 108*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/
3))**2*a*b**2*d**3*x**2 - 108*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2
*a*b**2*e**3 + 198*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**3*d**3*
n*x**2 + 36*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**3*e**3*n - 108
*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a**2*b*d**3*x**2 - 108*log(((x*
*(2/3)*d + e)**n*c)/x**((2*n)/3))*a**2*b*e**3 + 396*log(((x**(2/3)*d + e)*
*n*c)/x**((2*n)/3))*a*b**2*d**3*n*x**2 + 72*log(((x**(2/3)*d + e)**n*c)/x*
*((2*n)/3))*a*b**2*e**3*n - 510*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*
b**3*d**3*n**2*x**2 - 24*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**3*e*
*3*n**2 - 36*a**3*e**3 + 36*a**2*b*e**3*n - 24*a*b**2*e**3*n**2 + 8*b**...
```

3.528 $\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4}+\frac{16b^3e^3n^3x}{105d^3}$$
$$+\frac{1376b^3e^{9/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{105d^{9/2}}+\frac{1408ib^3e^{9/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{105d^{9/2}}$$
$$-\frac{2816b^3e^{9/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{105d^{9/2}}$$
$$+\frac{568b^3e^4n^2\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{105d^4}-\frac{32b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{35d^3}$$
$$+\frac{8b^2e^2n^2x^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{35d^2}$$
$$-\frac{1408b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{105d^{9/2}}$$
$$-\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4}+\frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3d^3}$$
$$-\frac{2be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{5d^2}+\frac{2benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7d}$$
$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{1408ib^3e^{9/2}n^3\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{105d^{9/2}}+\frac{2be^5n\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\right.\right.\right.}{(e+dx^{2/3})}\right.}{3d^4}$$

output
```
568/105*a*b^2*e^4*n^2*x^(1/3)/d^4-16/7*b^3*e^4*n^3*x^(1/3)/d^4+16/105*b^3*
e^3*n^3*x/d^3+1376/105*b^3*e^(9/2)*n^3*arctan(d^(1/2)*x^(1/3)/e^(1/2))/d^(
9/2)+1408/105*I*b^3*e^(9/2)*n^3*arctan(d^(1/2)*x^(1/3)/e^(1/2))^2/d^(9/2)-
2816/105*b^3*e^(9/2)*n^3*arctan(d^(1/2)*x^(1/3)/e^(1/2))*ln(2-2*e^(1/2)/(e
^(1/2)-I*d^(1/2)*x^(1/3)))/d^(9/2)+568/105*b^3*e^4*n^2*x^(1/3)*ln(c*(d+e/x
^(2/3))^n)/d^4-32/35*b^2*e^3*n^2*x*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+8/35*b^
2*e^2*n^2*x^(5/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^2-1408/105*b^2*e^(9/2)*n^2
*arctan(d^(1/2)*x^(1/3)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(9/2)-2*b*e
^4*n*x^(1/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^4+2/3*b*e^3*n*x*(a+b*ln(c*(d+
e/x^(2/3))^n))^2/d^3-2/5*b*e^2*n*x^(5/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^2
+2/7*b*e*n*x^(7/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d+1/3*x^3*(a+b*ln(c*(d+e/
x^(2/3))^n))^3+1408/105*I*b^3*e^(9/2)*n^3*polylog(2,-1+2*e^(1/2)/(e^(1/2)-
I*d^(1/2)*x^(1/3)))/d^(9/2)+2/3*b*e^5*n*Defer(Int)((a+b*ln(c*(d+e/x^(2/3))
^n))^2/(e+d*x^(2/3))/x^(2/3),x)/d^4
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5975 vs. 2(646) = 1292.

Time = 29.29 (sec) , antiderivative size = 5975, normalized size of antiderivative = 248.96

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Result too large to show}$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

output
```
Result too large to show
```

Rubi [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2907, 2005, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 \, dx$$

$$\Big\downarrow \text{2908}$$

$$3 \int x^{8/3} \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3 \left(\frac{2}{3} b e n \int \frac{x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d+\frac{e}{x^{2/3}}} d\sqrt[3]{x}+\frac{1}{9} x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2005}$$

$$3 \left(\frac{2}{3} b e n \int \frac{x^{8/3} \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3} d+e} d\sqrt[3]{x}+\frac{1}{9} x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2926}$$

$$3 \left(\frac{2}{3} b e n \int \left(\frac{\left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 e^4}{d^4 \left(x^{2/3} d+e\right)}-\frac{\left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 e^3}{d^4}+\frac{x^{2/3} \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right.\right.}{d^3}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3 \left(\frac{1}{9} x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{2}{3} b e n \left(\frac{e^4 \int \frac{\left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3} d+e} d\sqrt[3]{x}}{d^4}-\frac{704 b e^{7/2} n \arctan \left(\frac{\sqrt{d} \sqrt[3]{x}}{\sqrt{e}}\right) \left(a\right.}{105 d^{9/}}\right.\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2 \left(a + b \ln \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^3 dx$$

input
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.88

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a\right)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x^2*log(c*((d*
x + e*x^(1/3))/x)^n)^2 + 3*a^2*b*x^2*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*
x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int x^2\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
```

output
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 935, normalized size of antiderivative = 38.96

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x)
```

output

```
(630*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*a**2*b*e**4*n -
4224*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*a*b**2*e**4*n**2
 + 4128*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*b**3*e**4*n**
3 - 126*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**3*d**3*e*
*2*n*x - 252*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b**2*d**
3*e**2*n*x + 72*x**(2/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*b**3*d*
*3*e**2*n**2*x - 126*x**(2/3)*a**2*b*d**3*e**2*n*x + 72*x**(2/3)*a*b**2*d*
*3*e**2*n**2*x - 105*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3
*b**3*d*e**4 + 90*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b*
*3*d**4*e*n*x**2 - 630*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*
*2*b**3*d*e**4*n + 180*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*
a*b**2*d**4*e*n*x**2 - 1260*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)
/3))*a*b**2*d*e**4*n + 1704*x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)
/3))*b**3*d*e**4*n**2 + 90*x**(1/3)*a**2*b*d**4*e*n*x**2 - 630*x**(1/3)*a*
*2*b*d*e**4*n + 1704*x**(1/3)*a*b**2*d*e**4*n**2 - 720*x**(1/3)*b**3*d*e**
4*n**3 + 35*int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3/(x**(2/3)*e +
 x**(1/3)*d*x),x)*b**3*d*e**5 + 420*int(log(((x**(2/3)*d + e)**n*c)/x**((2
*n)/3))/(x**(2/3)*e + x**(1/3)*d*x),x)*a*b**2*d*e**5*n - 1408*int(log(((x*
*(2/3)*d + e)**n*c)/x**((2*n)/3))/(x**(2/3)*e + x**(1/3)*d*x),x)*b**3*d*e*
*5*n**2 + 35*int((x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3...
```

3.529 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = -\frac{24ib^3e^{3/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}}$$
$$+\frac{48b^3e^{3/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}$$
$$+\frac{24b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}$$
$$+\frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}$$
$$+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{24ib^3e^{3/2}n^3\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}-\frac{2be^2n\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right.\right.}{\left(e+dx^{2/3}\right)x^{2/3}}\right.}{d}$$

output
```
-24*I*b^3*e^(3/2)*n^3*arctan(d^(1/2)*x^(1/3)/e^(1/2))^2/d^(3/2)+48*b^3*e^(
3/2)*n^3*arctan(d^(1/2)*x^(1/3)/e^(1/2))*ln(2-2*e^(1/2)/(e^(1/2)-I*d^(1/2)
*x^(1/3)))/d^(3/2)+24*b^2*e^(3/2)*n^2*arctan(d^(1/2)*x^(1/3)/e^(1/2))*(a+b
*ln(c*(d+e/x^(2/3))^n))/d^(3/2)+6*b*e*n*x^(1/3)*(a+b*ln(c*(d+e/x^(2/3))^n)
)^2/d+x*(a+b*ln(c*(d+e/x^(2/3))^n))^3-24*I*b^3*e^(3/2)*n^3*polylog(2,-1+2*
e^(1/2)/(e^(1/2)-I*d^(1/2)*x^(1/3)))/d^(3/2)-2*b*e^2*n*Defer(Int)((a+b*ln(
c*(d+e/x^(2/3))^n))^2/(e+d*x^(2/3))/x^(2/3),x)/d
```

Mathematica [N/A]

Not integrable

Time = 6.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2901, 2907, 2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

$$\Big\downarrow \text{ 2901}$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(2ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d+\frac{e}{x^{2/3}}}d\sqrt[3]{x}+\frac{1}{3}x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\right)$$

$$\Big\downarrow \text{2921}$$

$$3\left(2ben\int \left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}-\frac{e\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d\left(x^{2/3}d+e\right)}\right)d\sqrt[3]{x}+\frac{1}{3}x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{3}x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+2ben\left(-\frac{e\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3}d+e}d\sqrt[3]{x}}{d}+\frac{4b\sqrt{e}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\right.}{d^{3/2}}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

output
```
integral(b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int \left(b\log \left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int \left(a+b\,\ln \left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^3,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 451, normalized size of antiderivative = 22.55

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3} dx = \frac{-18\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)a^{2}ben+72\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)a\,b^{2}e\,n^{2}+3x^{\frac{1}{3}}\log\left(\cdots\right.}{}$$

input

```
int((a+b*log(c*(d+e/x^(2/3))^n))^3,x)
```

output

```
( - 18*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*a**2*b*e*n + 7
2*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*a*b**2*e*n**2 + 3*x
**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3*b**3*d*e + 18*x**(1/3
)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*b**3*d*e*n + 36*x**(1/3)*lo
g(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a*b**2*d*e*n + 18*x**(1/3)*a**2*b*
d*e*n - int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3/(x**(2/3)*e + x**
(1/3)*d*x),x)*b**3*d*e**2 - 12*int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3
))/(x**(2/3)*e + x**(1/3)*d*x),x)*a*b**2*d*e**2*n + 24*int(log(((x**(2/3)*
d + e)**n*c)/x**((2*n)/3))/(x**(2/3)*e + x**(1/3)*d*x),x)*b**3*d*e**2*n**2
 - int((x**(1/3)*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3)/(x**(1/3)*e
 + d*x),x)*b**3*d**2*e + 3*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3*b*
*3*d**2*x + 9*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*a*b**2*d**2*x +
 9*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a**2*b*d**2*x + 3*a**3*d**2*x
)/(3*d**2)
```

3.530 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx = \frac{16b^3n^3}{9x}-\frac{208b^3dn^3}{3e\sqrt[3]{x}}-\frac{208b^3d^{3/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}}-\frac{32ib^3d^{3/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}}+\frac{64b^3d^{3/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x}+\frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}}+\frac{32b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}+\frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}-\frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}-\frac{32ib^3d^{3/2}n^3\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{2bd^2n\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{\left(e+dx^{2/3}\right)x^{2/3}},x\right)}{e}$$

output
```
16/9*b^3*n^3/x-208/3*b^3*d*n^3/e/x^(1/3)-208/3*b^3*d^(3/2)*n^3*arctan(d^(1
/2)*x^(1/3)/e^(1/2))/e^(3/2)-32*I*b^3*d^(3/2)*n^3*arctan(d^(1/2)*x^(1/3)/e
^(1/2))^2/e^(3/2)+64*b^3*d^(3/2)*n^3*arctan(d^(1/2)*x^(1/3)/e^(1/2))*ln(2-
2*e^(1/2)/(e^(1/2)-I*d^(1/2)*x^(1/3)))/e^(3/2)-8/3*b^2*n^2*(a+b*ln(c*(d+e/
x^(2/3))^n))/x+32*b^2*d*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e/x^(1/3)+32*b^2*d
^(3/2)*n^2*arctan(d^(1/2)*x^(1/3)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^(
3/2)+2*b*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/x-6*b*d*n*(a+b*ln(c*(d+e/x^(2/3))
^n))^2/e/x^(1/3)-(a+b*ln(c*(d+e/x^(2/3))^n))^3/x-32*I*b^3*d^(3/2)*n^3*poly
log(2,-1+2*e^(1/2)/(e^(1/2)-I*d^(1/2)*x^(1/3)))/e^(3/2)-2*b*d^2*n*Defer(In
t)((a+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2/3))/x^(2/3),x)/e
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5502 vs. 2(483) = 966.

Time = 19.56 (sec) , antiderivative size = 5502, normalized size of antiderivative = 229.25

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\text{Result too large to show}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

output
```
Result too large to show
```

Rubi [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2907, 2005, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2907}$$

$$3\left(-2ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{\left(d+\frac{e}{x^{2/3}}\right)x^2}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x}\right)$$

$$\Big\downarrow \; 2005$$

$$3\left(-2ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{\left(x^{2/3}d+e\right)x^{4/3}}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x}\right)$$

$$\Big\downarrow \; 2926$$

$$3\left(-2ben\int \left(-\frac{d\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e^2x^{2/3}}+\frac{d^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e^2\left(x^{2/3}d+e\right)}+\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{ex^{4/3}}\right.\right.$$

$$\Big\downarrow \; 2009$$

$$3\left(-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x}-2ben\left(\frac{d^2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3}d+e}d\sqrt[3]{x}}{e^2}-\frac{16bd^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\right.}{3e^{5/2}}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.50

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x
^(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 245, normalized size of antiderivative = 10.21

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx = \frac{-6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)a^2bdnx-6x^{\frac{2}{3}}a^2bden-2\left(\int\frac{\log\left(\frac{\left(x^{\frac{2}{3}}d+e\right)^n c}{x^{\frac{2n}{3}}}\right)^2}{x^{\frac{8}{3}}d+e\,x^2}dx\right.}{}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x)
```

output
```
( - 6*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*a**2*b*d*n*x -
6*x**(2/3)*a**2*b*d*e*n - 2*int(log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))*
*2/(x**(2/3)*d*x**2 + e*x**2),x)*b**3*e**3*n*x - 4*int(log(((x**(2/3)*d +
e)**n*c)/x**((2*n)/3))/(x**(2/3)*d*x**2 + e*x**2),x)*a*b**2*e**3*n*x - log
(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3*b**3*e**2 - 3*log(((x**(2/3)*d +
 e)**n*c)/x**((2*n)/3))**2*a*b**2*e**2 - 3*log(((x**(2/3)*d + e)**n*c)/x**
((2*n)/3))*a**2*b*e**2 - a**3*e**2 + 2*a**2*b*e**2*n)/(e**2*x)
```

3.531 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

output
```
16/729*b^3*n^3/x^3-3088/27783*b^3*d*n^3/e/x^(7/3)+221344/496125*b^3*d^2*n^
3/e^2/x^(5/3)-637984/297675*b^3*d^3*n^3/e^3/x+3475504/99225*b^3*d^4*n^3/e^
4/x^(1/3)+3475504/99225*b^3*d^(9/2)*n^3*arctan(d^(1/2)*x^(1/3)/e^(1/2))/e^
(9/2)+4504/315*I*b^3*d^(9/2)*n^3*polylog(2,-1+2*e^(1/2)/(e^(1/2)-I*d^(1/2)
*x^(1/3)))/e^(9/2)-9008/315*b^3*d^(9/2)*n^3*arctan(d^(1/2)*x^(1/3)/e^(1/2)
)*ln(2-2*e^(1/2)/(e^(1/2)-I*d^(1/2)*x^(1/3)))/e^(9/2)-8/81*b^2*n^2*(a+b*ln
(c*(d+e/x^(2/3))^n))/x^3+128/441*b^2*d*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e/x
^(7/3)-1144/1575*b^2*d^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^2/x^(5/3)+1984/
945*b^2*d^3*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3/x-4504/315*b^2*d^4*n^2*(a+
b*ln(c*(d+e/x^(2/3))^n))/e^4/x^(1/3)-4504/315*b^2*d^(9/2)*n^2*arctan(d^(1/
2)*x^(1/3)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^(9/2)+2/9*b*n*(a+b*ln(c*
(d+e/x^(2/3))^n))^2/x^3-2/7*b*d*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e/x^(7/3)+
2/5*b*d^2*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^2/x^(5/3)-2/3*b*d^3*n*(a+b*ln(
c*(d+e/x^(2/3))^n))^2/e^3/x+2*b*d^4*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^4/x^
(1/3)-1/3*(a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3+4504/315*I*b^3*d^(9/2)*n^3*arc
tan(d^(1/2)*x^(1/3)/e^(1/2))^2/e^(9/2)+2/3*b*d^5*n*Defer(Int)((a+b*ln(c*(d
+e/x^(2/3))^n))^2/(e+d*x^(2/3))/x^(2/3),x)/e^4
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6328 vs. 2(784) = 1568.

Time = 27.19 (sec) , antiderivative size = 6328, normalized size of antiderivative = 263.67

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\text{Result too large to show}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```

output
```
Result too large to show
```

Rubi [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2907, 2005, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}dx$$

$$\Big\downarrow \text{ 2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2907}$$

$$3\left(-\frac{2}{3}ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{\left(d+\frac{e}{x^{2/3}}\right)x^4}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{9x^3}\right)$$

$$\Big\downarrow \text{2005}$$

$$3\left(-\frac{2}{3}ben\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{\left(x^{2/3}d+e\right)x^{10/3}}d\sqrt[3]{x}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{9x^3}\right)$$

$$\Big\downarrow \text{2926}$$

$$3\left(-\frac{2}{3}ben\int \left(-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 d^5}{e^5\left(x^{2/3}d+e\right)}+\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 d^4}{e^5x^{2/3}}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right.\right.}{e^4x^{4/3}}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{9x^3}-\frac{2}{3}ben\left(-\frac{d^5\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^{2/3}d+e}d\sqrt[3]{x}}{e^5}+\frac{2252bd^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(\right.}{315e^{11/}}\right.\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2907
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Simp[b*e*n*p*(q/(f^n*(m + 1)))   Int[(f*x)^(m + n)*((a
+ b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d
, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.50

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="fricas")
```

output
```
integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x
^(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 22.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^4,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^4, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 302, normalized size of antiderivative = 12.58

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx = \frac{630\sqrt{e}\,\sqrt{d}\,atan\left(\frac{x^{\frac{1}{3}}d}{\sqrt{e}\,\sqrt{d}}\right)a^2b\,d^4n\,x^3+630x^{\frac{8}{3}}a^2b\,d^4en-90x^{\frac{2}{3}}a^2bd\,e^4n+}{}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x)
```

output
```
(630*sqrt(e)*sqrt(d)*atan((x**(1/3)*d)/(sqrt(e)*sqrt(d)))*a**2*b*d**4*n*x*
*3 + 630*x**(2/3)*a**2*b*d**4*e*n*x**2 - 90*x**(2/3)*a**2*b*d*e**4*n + 126
*x**(1/3)*a**2*b*d**2*e**3*n*x - 210*int(log(((x**(2/3)*d + e)**n*c)/x**((
2*n)/3))**2/(x**(2/3)*d*x**4 + e*x**4),x)*b**3*e**6*n*x**3 - 420*int(log((
(x**(2/3)*d + e)**n*c)/x**((2*n)/3))/(x**(2/3)*d*x**4 + e*x**4),x)*a*b**2*
e**6*n*x**3 - 105*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**3*b**3*e**5 -
 315*log(((x**(2/3)*d + e)**n*c)/x**((2*n)/3))**2*a*b**2*e**5 - 315*log(((
x**(2/3)*d + e)**n*c)/x**((2*n)/3))*a**2*b*e**5 - 105*a**3*e**5 - 210*a**2
*b*d**3*e**2*n*x**2 + 70*a**2*b*e**5*n)/(315*e**5*x**3)
```

3.532 $\int x^3 \left(a + b \log \left(c\left(d + e\sqrt{x}\right)\right)\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 730

$$\int x^3 \left(a + b \log \left(c\left(d + e\sqrt{x}\right)\right)\right)^p \, dx = \text{Too large to display}$$

output
```
2^(-2-3*p)*GAMMA(p+1,(-8*a-8*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1
/2))))^p/c^8/e^8/exp(8*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(p
+1,(-7*a-7*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(7^p)/c^7
/e^8/exp(7*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)+7*d^2*GAMMA(p+1,(-6*a-6
*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(6^p)/c^6/e^8/exp(6
*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)-14*d^3*GAMMA(p+1,(-5*a-5*b*ln(c*(
d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(5^p)/c^5/e^8/exp(5*a/b)/((-
(a+b*ln(c*(d+e*x^(1/2))))/b)^p)+35*2^(-1-2*p)*d^4*GAMMA(p+1,(-4*a-4*b*ln(c
*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c^4/e^8/exp(4*a/b)/((-(a+b
*ln(c*(d+e*x^(1/2))))/b)^p)-14*d^5*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e*x^(1/2)))
)/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(3^p)/c^3/e^8/exp(3*a/b)/((-(a+b*ln(c*(d+
e*x^(1/2))))/b)^p)+7*d^6*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*
ln(c*(d+e*x^(1/2))))^p/(2^p)/c^2/e^8/exp(2*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))
))/b)^p)-2*d^7*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1
/2))))^p/c/e^8/exp(a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)
```

Mathematica [F]

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

output
```
Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p, x]
```

Rubi [A] (verified)

Time = 2.87 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int x^{7/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \text{ 2848}$$

$$2\int \left(\frac{\left(d+e\sqrt{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{e^7}-\frac{7d\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{e^7}+\frac{21d^2\left(d+e\sqrt{x}\right)^5\left(a+\right.}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{8^{-p-1}e^{-\frac{8a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^8e^8}-\frac{d7^{-p}e^{-\frac{7a}{b}}\left(a\right.}{}\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

output
```
2*((8^(-1 - p)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*
Log[c*(d + e*Sqrt[x])])^p)/(c^8*e^8*E^((8*a)/b)*(-((a + b*Log[c*(d + e*Sqr
t[x])])/b))^p) - (d*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e*Sqrt[x])])^p)/(7^p*c^7*e^8*E^((7*a)/b)*(-((a + b*Log[c*(
d + e*Sqrt[x])])/b))^p) + (7*2^(-1 - p)*d^2*Gamma[1 + p, (-6*(a + b*Log[c*
(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^6*e^8*E^((6
*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (7*d^3*Gamma[1 + p, (-5*
(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*
c^5*e^8*E^((5*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (35*4^(-1 -
 p)*d^4*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(
d + e*Sqrt[x])])^p)/(c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])
/b))^p) - (7*d^5*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (7*2^(-1 - p)*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d
+ e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c^2*e^8*E^((2*a)/b)*
(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (d^7*Gamma[1 + p, -((a + b*Log[
c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^8*E^(a/b)*(
-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3 \left(a + b \ln \left(c \left(d + e\sqrt{x}\right)\right)\right)^p dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Fricas [F]

$$\int x^3 \left(a + b \log \left(c \left(d + e\sqrt{x}\right)\right)\right)^p \, dx = \int \left(b \log \left(\left(e\sqrt{x} + d\right)c\right) + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log \left(c \left(d + e\sqrt{x}\right)\right)\right)^p \, dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int x^3\,\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^p\,dx$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(1/2))))^p,x)
```

output
```
int(x^3*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

Reduce [F]

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \, dx = \text{too large to display}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x)
```

output
```
(840*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**7*e*p**2 + 840*sqrt(x)
*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**7*e*p + 280*sqrt(x)*(log(sqrt(x)*c
*e + c*d)*b + a)**p*b*d**5*e**3*p**2*x + 280*sqrt(x)*(log(sqrt(x)*c*e + c*
d)*b + a)**p*b*d**5*e**3*p*x + 168*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)*
*p*b*d**3*e**5*p**2*x**2 + 168*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*b
*d**3*e**5*p*x**2 + 120*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d*e**7
*p**2*x**3 + 120*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d*e**7*p*x**3
 - 840*(log(sqrt(x)*c*e + c*d)*b + a)**p*log(sqrt(x)*c*e + c*d)*b*d**8*p -
 840*(log(sqrt(x)*c*e + c*d)*b + a)**p*a*d**8*p + 840*(log(sqrt(x)*c*e + c
*d)*b + a)**p*a*e**8*p*x**4 + 840*(log(sqrt(x)*c*e + c*d)*b + a)**p*a*e**8
*x**4 - 420*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**6*e**2*p**2*x - 420*(lo
g(sqrt(x)*c*e + c*d)*b + a)**p*b*d**6*e**2*p*x - 210*(log(sqrt(x)*c*e + c*
d)*b + a)**p*b*d**4*e**4*p**2*x**2 - 210*(log(sqrt(x)*c*e + c*d)*b + a)**p
*b*d**4*e**4*p*x**2 - 140*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**2*e**6*p*
*2*x**3 - 140*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**2*e**6*p*x**3 - 3360*
int((log(sqrt(x)*c*e + c*d)*b + a)**p/(8*sqrt(x)*log(sqrt(x)*c*e + c*d)*a*
b*e + sqrt(x)*log(sqrt(x)*c*e + c*d)*b**2*e*p + 8*sqrt(x)*a**2*e + sqrt(x)
*a*b*e*p + 8*log(sqrt(x)*c*e + c*d)*a*b*d + log(sqrt(x)*c*e + c*d)*b**2*d*
p + 8*a**2*d + a*b*d*p),x)*a*b**2*d**7*e**2*p**3 - 3360*int((log(sqrt(x)*c
*e + c*d)*b + a)**p/(8*sqrt(x)*log(sqrt(x)*c*e + c*d)*a*b*e + sqrt(x)*l...
```

3.533 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 551

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6(a+b\log(c(d+e\sqrt{x})))}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log(c(d+e\sqrt{x}))}{b}\right)^{-p}}{c^6e^6}$$
$$-\frac{2\,5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5(a+b\log(c(d+e\sqrt{x})))}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log(c(d+e\sqrt{x}))}{b}\right)^{-p}}{c^5e^6}$$
$$+\frac{5\,4^{-p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4(a+b\log(c(d+e\sqrt{x})))}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log(c(d+e\sqrt{x}))}{b}\right)^{-p}}{c^4e^6}$$
$$-\frac{20\,3^{-1-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3(a+b\log(c(d+e\sqrt{x})))}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log(c(d+e\sqrt{x}))}{b}\right)^{-p}}{c^3e^6}$$
$$+\frac{5\,2^{-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2(a+b\log(c(d+e\sqrt{x})))}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log(c(d+e\sqrt{x}))}{b}\right)^{-p}}{c^2e^6}$$
$$-\frac{2d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log(c(d+e\sqrt{x}))}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log(c(d+e\sqrt{x}))}{b}\right)^{-p}}{ce^6}$$

output
```
3^(-1-p)*GAMMA(p+1,(-6*a-6*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2
))))^p/(2^p)/c^6/e^6/exp(6*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAM
MA(p+1,(-5*a-5*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(5^p)
/c^5/e^6/exp(5*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)+5*d^2*GAMMA(p+1,(-4
*a-4*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(4^p)/c^4/e^6/e
xp(4*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)-20*3^(-1-p)*d^3*GAMMA(p+1,(-3
*a-3*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c^3/e^6/exp(3*a
/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)+5*d^4*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e
*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^p)/c^2/e^6/exp(2*a/b)/((-(a+
b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d^5*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/2))))/b)
*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^6/exp(a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b
)^p)
```

Mathematica [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

output
```
Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p, x]
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int x^{5/2}\left(a+b\log\left(c(d+e\sqrt{x})\right)\right)^p d\sqrt{x}$$

$$\Big\downarrow \text{2848}$$

$$2\int\left(\frac{\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c(d+e\sqrt{x})\right)\right)^p}{e^5}-\frac{5d\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c(d+e\sqrt{x})\right)\right)^p}{e^5}+\frac{10d^2\left(d+e\sqrt{x}\right)^3\left(a+\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{6^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c(d+e\sqrt{x})\right)\right)^p\left(-\frac{a+b\log(c(d+e\sqrt{x}))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6(a+b\log(c(d+e\sqrt{x})))}{b}\right)}{c^6e^6}-\frac{d5^{-p}e^{-\frac{5a}{b}}\left(a-\right.}{}\right.$$

input

```
Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

output

```
2*((6^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*
Log[c*(d + e*Sqrt[x])])^p)/(c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*Sqr
t[x])])/b))^p) - (d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(
d + e*Sqrt[x])])/b))^p) + (5*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log[
c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c^4*e^6*E^((4*a
)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (10*3^(-1 - p)*d^3*Gamma[1
 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])]
)^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (5*2^
(-1 - p)*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[
x])])/b))^p) - (d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2 \left(a + b \ln \left(c \left(d + e\sqrt{x}\right)\right)\right)^p dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Fricas [F]

$$\int x^2 \left(a + b \log \left(c \left(d + e\sqrt{x}\right)\right)\right)^p \, dx = \int \left(b \log \left(\left(e\sqrt{x} + d\right)c\right) + a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^2, x)
```

Giac [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^p\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/2))))^p,x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

Reduce [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\text{too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x)
```

output

```
(120*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**5*e*p**2 + 120*sqrt(x)
*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**5*e*p + 40*sqrt(x)*(log(sqrt(x)*c*
e + c*d)*b + a)**p*b*d**3*e**3*p**2*x + 40*sqrt(x)*(log(sqrt(x)*c*e + c*d)
*b + a)**p*b*d**3*e**3*p*x + 24*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*
b*d*e**5*p**2*x**2 + 24*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d*e**5
*p*x**2 - 120*(log(sqrt(x)*c*e + c*d)*b + a)**p*log(sqrt(x)*c*e + c*d)*b*d
**6*p - 120*(log(sqrt(x)*c*e + c*d)*b + a)**p*a*d**6*p + 120*(log(sqrt(x)*
c*e + c*d)*b + a)**p*a*e**6*p*x**3 + 120*(log(sqrt(x)*c*e + c*d)*b + a)**p
*a*e**6*x**3 - 60*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**4*e**2*p**2*x - 6
0*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**4*e**2*p*x - 30*(log(sqrt(x)*c*e
+ c*d)*b + a)**p*b*d**2*e**4*p**2*x**2 - 30*(log(sqrt(x)*c*e + c*d)*b + a)
**p*b*d**2*e**4*p*x**2 - 360*int((log(sqrt(x)*c*e + c*d)*b + a)**p/(6*sqrt
(x)*log(sqrt(x)*c*e + c*d)*a*b*e + sqrt(x)*log(sqrt(x)*c*e + c*d)*b**2*e*p
 + 6*sqrt(x)*a**2*e + sqrt(x)*a*b*e*p + 6*log(sqrt(x)*c*e + c*d)*a*b*d + l
og(sqrt(x)*c*e + c*d)*b**2*d*p + 6*a**2*d + a*b*d*p),x)*a*b**2*d**5*e**2*p
**3 - 360*int((log(sqrt(x)*c*e + c*d)*b + a)**p/(6*sqrt(x)*log(sqrt(x)*c*e
 + c*d)*a*b*e + sqrt(x)*log(sqrt(x)*c*e + c*d)*b**2*e*p + 6*sqrt(x)*a**2*e
 + sqrt(x)*a*b*e*p + 6*log(sqrt(x)*c*e + c*d)*a*b*d + log(sqrt(x)*c*e + c*
d)*b**2*d*p + 6*a**2*d + a*b*d*p),x)*a*b**2*d**5*e**2*p**2 - 60*int((log(s
qrt(x)*c*e + c*d)*b + a)**p/(6*sqrt(x)*log(sqrt(x)*c*e + c*d)*a*b*e + s...
```

3.534 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 360

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx &= \frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^4e^4} \\ &\quad - \frac{2\,3^{-p}de^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^3e^4} \\ &\quad + \frac{3\,2^{-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^4} \\ &\quad - \frac{2d^3e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^4}\end{aligned}$$

output
```
2^(-1-2*p)*GAMMA(p+1,(-4*a-4*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1
/2))))^p/c^4/e^4/exp(4*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(p
+1,(-3*a-3*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(3^p)/c^3
/e^4/exp(3*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)+3*d^2*GAMMA(p+1,(-2*a-2
*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^p)/c^2/e^4/exp(2
*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d^3*GAMMA(p+1,-(a+b*ln(c*(d+e*x
^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^4/exp(a/b)/((-(a+b*ln(c*(d+e*
x^(1/2))))/b)^p)
```

Mathematica [A] (verified)

Time = 6.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.64

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \frac{2^{-1-2p}3^{-p}e^{-\frac{4a}{b}}\left(3^p\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-2^{1+p}cde^{a/b}\left(2^{1+p}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)+3^pc\right.\right.}{}$$

input
```
Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

output
```
(2^(-1 - 2*p)*(3^p*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 2
^(1 + p)*c*d*E^(a/b)*(2^(1 + p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[
x])]))/b] + 3^p*c*d*E^(a/b)*(-3*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[
x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[
x])])/b)])))*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^4*e^4*E^((4*a)/b)*(-
((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{2904}$$

$$2\int x^{3/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \text{2848}$$

$$2\int\left(\frac{\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{e^3}-\frac{3d\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{e^3}+\frac{3d^2\left(d+e\sqrt{x}\right)\left(a+b\right.}{e\right.}\right.$$

$\downarrow$ 2009

$$2\left(\frac{4^{-p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^4}-\frac{d3^{-p}e^{-\frac{3a}{b}}\left(a\right.}{}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

output
```
2*((4^(-1 - p)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*
Log[c*(d + e*Sqrt[x])])^p)/(c^4*e^4*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqr
t[x])])/b))^p) - (d*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^3*e^4*E^((3*a)/b)*(-((a + b*Log[c*(
d + e*Sqrt[x])])/b))^p) + (3*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*
(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c^2*e^4*E^((2*a)/
b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (d^3*Gamma[1 + p, -((a + b*L
og[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^4*E^(a/b
)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\, dx = \int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x\, dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\, dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/2))))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \text{too large to display}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/2))))^p,x)
```

output
```
(12*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**3*e*p**2 + 12*sqrt(x)*(
log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**3*e*p + 4*sqrt(x)*(log(sqrt(x)*c*e +
 c*d)*b + a)**p*b*d*e**3*p**2*x + 4*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)
**p*b*d*e**3*p*x - 12*(log(sqrt(x)*c*e + c*d)*b + a)**p*log(sqrt(x)*c*e +
c*d)*b*d**4*p - 12*(log(sqrt(x)*c*e + c*d)*b + a)**p*a*d**4*p + 12*(log(sq
rt(x)*c*e + c*d)*b + a)**p*a*e**4*p*x**2 + 12*(log(sqrt(x)*c*e + c*d)*b +
a)**p*a*e**4*x**2 - 6*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**2*e**2*p**2*x
 - 6*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d**2*e**2*p*x - 24*int((log(sqrt(
x)*c*e + c*d)*b + a)**p/(4*sqrt(x)*log(sqrt(x)*c*e + c*d)*a*b*e + sqrt(x)*
log(sqrt(x)*c*e + c*d)*b**2*e*p + 4*sqrt(x)*a**2*e + sqrt(x)*a*b*e*p + 4*l
og(sqrt(x)*c*e + c*d)*a*b*d + log(sqrt(x)*c*e + c*d)*b**2*d*p + 4*a**2*d +
 a*b*d*p),x)*a*b**2*d**3*e**2*p**3 - 24*int((log(sqrt(x)*c*e + c*d)*b + a)
**p/(4*sqrt(x)*log(sqrt(x)*c*e + c*d)*a*b*e + sqrt(x)*log(sqrt(x)*c*e + c*
d)*b**2*e*p + 4*sqrt(x)*a**2*e + sqrt(x)*a*b*e*p + 4*log(sqrt(x)*c*e + c*d
)*a*b*d + log(sqrt(x)*c*e + c*d)*b**2*d*p + 4*a**2*d + a*b*d*p),x)*a*b**2*
d**3*e**2*p**2 - 6*int((log(sqrt(x)*c*e + c*d)*b + a)**p/(4*sqrt(x)*log(sq
rt(x)*c*e + c*d)*a*b*e + sqrt(x)*log(sqrt(x)*c*e + c*d)*b**2*e*p + 4*sqrt(
x)*a**2*e + sqrt(x)*a*b*e*p + 4*log(sqrt(x)*c*e + c*d)*a*b*d + log(sqrt(x)
*c*e + c*d)*b**2*d*p + 4*a**2*d + a*b*d*p),x)*b**3*d**3*e**2*p**4 - 6*int(
(log(sqrt(x)*c*e + c*d)*b + a)**p/(4*sqrt(x)*log(sqrt(x)*c*e + c*d)*a*b...
```

3.535 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 174

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^2} - \frac{2de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^2}$$

output
```
GAMMA(p+1,(-2*a-2*b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2
^p)/c^2/e^2/exp(2*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(p+1,-(
a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^2/exp(a/b)/((-
(a+b*ln(c*(d+e*x^(1/2))))/b)^p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$
$$=\frac{2^{-p}e^{-\frac{2a}{b}}\left(\Gamma\left(1+p,-\frac{2(a+b\log(c(d+e\sqrt{x})))}{b}\right)-2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log(c(d+e\sqrt{x}))}{b}\right)\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right.\right.\right.}{c^2e^2}$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

output
```
((Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 2^(1 + p)*c*d*E^(a
/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)])*(a + b*Log[c*(d + e
*Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/
b))^p)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2901, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{ 2901}$$

$$2\int \sqrt{x}\left(a+b\log \left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \text{ 2848}$$

$$2\int \left(\frac{\left(d+e\sqrt{x}\right)\left(a+b\log \left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{e}-\frac{d\left(a+b\log \left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{e}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^{2}e^{2}}-\frac{de^{-\frac{a}{b}}\left(a+b\log\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

output
```
2*((2^(-1 - p)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*
Log[c*(d + e*Sqrt[x])])^p)/(c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqr
t[x])])/b))^p) - (d*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*sqrt(x) + c*d) + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))))**p,x)
```

output
```
Integral((a + b*log(c*(d + e*sqrt(x))))**p, x)
```

Maxima [F]

$$\int \left(a + b\log\left(c(d + e\sqrt{x})\right)\right)^p \, dx = \int \left(b\log\left((e\sqrt{x} + d)c\right) + a\right)^p \, dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p, x)
```

Giac [F]

$$\int \left(a + b\log\left(c(d + e\sqrt{x})\right)\right)^p \, dx = \int \left(b\log\left((e\sqrt{x} + d)c\right) + a\right)^p \, dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + b\log\left(c(d + e\sqrt{x})\right)\right)^p \, dx = \int \left(a + b\,\ln\left(c\,\left(d + e\,\sqrt{x}\right)\right)\right)^p \, dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))))^p,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))))^p, x)
```

Reduce [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)\right)\right)^p \, dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))))^p,x)
```

output
```
(2*sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*b*d*e*p**2 + 2*sqrt(x)*(log(s
qrt(x)*c*e + c*d)*b + a)**p*b*d*e*p - 2*(log(sqrt(x)*c*e + c*d)*b + a)**p*
log(sqrt(x)*c*e + c*d)*b*d**2*p - 2*(log(sqrt(x)*c*e + c*d)*b + a)**p*a*d*
*2*p + 2*(log(sqrt(x)*c*e + c*d)*b + a)**p*a*e**2*p*x + 2*(log(sqrt(x)*c*e
 + c*d)*b + a)**p*a*e**2*x - 2*int((log(sqrt(x)*c*e + c*d)*b + a)**p/(2*sq
rt(x)*log(sqrt(x)*c*e + c*d)*a*b*e + sqrt(x)*log(sqrt(x)*c*e + c*d)*b**2*e
*p + 2*sqrt(x)*a**2*e + sqrt(x)*a*b*e*p + 2*log(sqrt(x)*c*e + c*d)*a*b*d +
 log(sqrt(x)*c*e + c*d)*b**2*d*p + 2*a**2*d + a*b*d*p),x)*a*b**2*d*e**2*p*
*3 - 2*int((log(sqrt(x)*c*e + c*d)*b + a)**p/(2*sqrt(x)*log(sqrt(x)*c*e +
c*d)*a*b*e + sqrt(x)*log(sqrt(x)*c*e + c*d)*b**2*e*p + 2*sqrt(x)*a**2*e +
sqrt(x)*a*b*e*p + 2*log(sqrt(x)*c*e + c*d)*a*b*d + log(sqrt(x)*c*e + c*d)*
b**2*d*p + 2*a**2*d + a*b*d*p),x)*a*b**2*d*e**2*p**2 - int((log(sqrt(x)*c*
e + c*d)*b + a)**p/(2*sqrt(x)*log(sqrt(x)*c*e + c*d)*a*b*e + sqrt(x)*log(s
qrt(x)*c*e + c*d)*b**2*e*p + 2*sqrt(x)*a**2*e + sqrt(x)*a*b*e*p + 2*log(sq
rt(x)*c*e + c*d)*a*b*d + log(sqrt(x)*c*e + c*d)*b**2*d*p + 2*a**2*d + a*b*
d*p),x)*b**3*d*e**2*p**4 - int((log(sqrt(x)*c*e + c*d)*b + a)**p/(2*sqrt(x
)*log(sqrt(x)*c*e + c*d)*a*b*e + sqrt(x)*log(sqrt(x)*c*e + c*d)*b**2*e*p +
 2*sqrt(x)*a**2*e + sqrt(x)*a*b*e*p + 2*log(sqrt(x)*c*e + c*d)*a*b*d + log
(sqrt(x)*c*e + c*d)*b**2*d*p + 2*a**2*d + a*b*d*p),x)*b**3*d*e**2*p**3 + 2
*int((sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p*log(sqrt(x)*c*e + c*d))...
```

3.536 $\int \frac{(a+b\log(c(d+e\sqrt{x})))^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

$$\Big\downarrow \ 2908$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \ 2867$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{\sqrt{x}}d\sqrt{x}$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*sqrt(x) + c*d) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))))^p/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 224, normalized size of antiderivative = 10.18

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx = \frac{2\left(\log\left(\sqrt{x}\,ce+cd\right)b+a\right)^p\log\left(\sqrt{x}\,ce+cd\right)b+2\left(\log\left(\sqrt{x}\,ce+cd\right)b+a\right)^p a+\left(\int\frac{\left(\log\left(\sqrt{x}\,ce+cd\right)b+a\right)^p}{-e^2x^2+d^2x}dx\right)b}{b\,(p}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))))^p/x,x)
```

output
```
(2*(log(sqrt(x)*c*e + c*d)*b + a)**p*log(sqrt(x)*c*e + c*d)*b + 2*(log(sqr
t(x)*c*e + c*d)*b + a)**p*a + int((log(sqrt(x)*c*e + c*d)*b + a)**p/(d**2*
x - e**2*x**2),x)*b*d**2*p + int((log(sqrt(x)*c*e + c*d)*b + a)**p/(d**2*x
 - e**2*x**2),x)*b*d**2 - int((sqrt(x)*(log(sqrt(x)*c*e + c*d)*b + a)**p)/
(d**2*x - e**2*x**2),x)*b*d*e*p - int((sqrt(x)*(log(sqrt(x)*c*e + c*d)*b +
 a)**p)/(d**2*x - e**2*x**2),x)*b*d*e)/(b*(p + 1))
```

3.537 $\int \frac{(a+b\log(c(d+e\sqrt{x})))^p}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\, dx = \operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\, dx = \int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\, dx$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$
$$\Big\downarrow \ 2908$$
$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^{3/2}}d\sqrt{x}$$
$$\Big\downarrow \ 2867$$
$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^{3/2}}d\sqrt{x}$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*sqrt(x) + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}}{x^{2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))))^p/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 8.55

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx = \frac{-2\left(\log\left(\sqrt{x}\,ce+cd\right)b+a\right)^p-\left(\int \frac{\left(\log\left(\sqrt{x}\,ce+cd\right)b+a\right)^p}{\log\left(\sqrt{x}\,ce+cd\right)b\,d^2x-\log\left(\sqrt{x}\,ce+cd\right)b\,e^2x^2+a\,d^2x-a\,e^2x^2}dx\right)b\,e^2px+\left(\int \frac{}{\log\left(\sqrt{x}\,ce+c\right.}\right.}{2x}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x)
```

output
```
( - 2*(log(sqrt(x)*c*e + c*d)*b + a)**p - int((log(sqrt(x)*c*e + c*d)*b +
a)**p/(log(sqrt(x)*c*e + c*d)*b*d**2*x - log(sqrt(x)*c*e + c*d)*b*e**2*x**
2 + a*d**2*x - a*e**2*x**2),x)*b*e**2*p*x + int((sqrt(x)*(log(sqrt(x)*c*e
+ c*d)*b + a)**p)/(log(sqrt(x)*c*e + c*d)*b*d**2*x**2 - log(sqrt(x)*c*e +
c*d)*b*e**2*x**3 + a*d**2*x**2 - a*e**2*x**3),x)*b*d*e*p*x)/(2*x)
```

3.538 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 907

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = \text{Too large to display}$$

output
```
GAMMA(p+1,(-4*a-4*b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^
p/(2^(2*p+2))/c^4/e^8/exp(4*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p
+1)*d*(d+e*x^(1/2))^7*GAMMA(p+1,1/2*(-7*a-7*b*ln(c*(d+e*x^(1/2))^2))/b)*(a
+b*ln(c*(d+e*x^(1/2))^2))^p/(7^p)/e^8/exp(7/2*a/b)/(c*(d+e*x^(1/2))^2)^(7/
2)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)+7*d^2*GAMMA(p+1,(-3*a-3*b*ln(c*(d+
e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(3^p)/c^3/e^8/exp(3*a/b)/(
(-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)-7*2^(p+1)*d^3*(d+e*x^(1/2))^5*GAMMA(p+
1,1/2*(-5*a-5*b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(5
^p)/e^8/exp(5/2*a/b)/(c*(d+e*x^(1/2))^2)^(5/2)/((-(a+b*ln(c*(d+e*x^(1/2))^
2))/b)^p)+35*2^(-1-p)*d^4*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e*x^(1/2))^2))/b)*(a
+b*ln(c*(d+e*x^(1/2))^2))^p/c^2/e^8/exp(2*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))^
2))/b)^p)-7*2^(p+1)*d^5*(d+e*x^(1/2))^3*GAMMA(p+1,1/2*(-3*a-3*b*ln(c*(d+e*
x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(3^p)/e^8/exp(3/2*a/b)/(c*(d
+e*x^(1/2))^2)^(3/2)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)+7*d^6*GAMMA(p+1,
-(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^8/exp(a/
b)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p+1)*d^7*(d+e*x^(1/2))*GAMMA(p+
1,-1/2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/e^8/ex
p(1/2*a/b)/(c*(d+e*x^(1/2))^2)^(1/2)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)
```

Mathematica [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

output
```
Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Rubi [A] (verified)

Time = 3.04 (sec) , antiderivative size = 896, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

$$\Big\downarrow \text{ 2904}$$

$$2\int x^{7/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \text{ 2848}$$

$$2\int\left(\frac{\left(d+e\sqrt{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{e^7}-\frac{7d\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{e^7}+\frac{21d^2\left(d+e\sqrt{x}\right)^5}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{2^{-2p-3}e^{-\frac{4a}{b}}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^4e^8}-\frac{\left(\frac{2}{7}\right)^p}{}\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

output
```
2*((2^(-3 - 2*p)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a
+ b*Log[c*(d + e*Sqrt[x])^2])^p)/(c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])^2])/b))^p) - ((2/7)^p*d*(d + e*Sqrt[x])^7*Gamma[1 + p, (-7*(a
+ b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/
(e^8*E^((7*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(7/2)*(-((a + b*Log[c*(d + e*Sq
rt[x])^2])/b))^p) + (7*d^2*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(2*3^p*c^3*e^8*E^((3*a)/b)*(-((
a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*(2/5)^p*d^3*(d + e*Sqrt[x])^5*
Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d
+ e*Sqrt[x])^2])^p)/(e^8*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a
 + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (35*2^(-2 - p)*d^4*Gamma[1 + p, (-
2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)
/(c^2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*(2/3
)^p*d^5*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]
))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^8*E^((3*a)/(2*b))*(c*(d +
 e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (7*d^6*G
amma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqr
t[x])^2])^p)/(2*c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) -
 (2^p*d^7*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*Sqrt[x])^2
])/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^8*E^(a/(2*b))*Sqrt[c*(d + ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Fricas [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\,\ln\left(c\left(d+e\,\sqrt{x}\right)^2\right)\right)^p\,dx$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
```

output
```
int(x^3*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

Reduce [F]

$$\int x^3 \left(a + b\log\left(c\left(d + e\sqrt{x}\right)^2\right)\right)^p dx = \text{too large to display}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x)
```

output
```
(420*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d**7*e*
p**2 + 420*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d
**7*e*p + 140*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*
b*d**5*e**3*p**2*x + 140*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)
*b + a)**p*b*d**5*e**3*p*x + 84*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*
e**2*x)*b + a)**p*b*d**3*e**5*p**2*x**2 + 84*sqrt(x)*(log(2*sqrt(x)*c*d*e
+ c*d**2 + c*e**2*x)*b + a)**p*b*d**3*e**5*p*x**2 + 60*sqrt(x)*(log(2*sqrt
(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d*e**7*p**2*x**3 + 60*sqrt(x)*(
log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d*e**7*p*x**3 - 210*(
log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*log(2*sqrt(x)*c*d*e + c
*d**2 + c*e**2*x)*b*d**8*p - 210*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)
*b + a)**p*a*d**8*p + 210*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)
**p*a*e**8*p*x**4 + 210*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**
p*a*e**8*x**4 - 210*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*
d**6*e**2*p**2*x - 210*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p
*b*d**6*e**2*p*x - 105*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p
*b*d**4*e**4*p**2*x**2 - 105*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b +
 a)**p*b*d**4*e**4*p*x**2 - 70*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b
 + a)**p*b*d**2*e**6*p**2*x**3 - 70*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2
*x)*b + a)**p*b*d**2*e**6*p*x**3 - 1680*int((log(2*sqrt(x)*c*d*e + c*d*...
```

3.539 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 677

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

$$=\frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^3e^6}$$

$$-\frac{2^{1+p}5^{-p}de^{-\frac{5a}{2b}}\left(d+e\sqrt{x}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c(d+\right.}{b}\right.}{e^6\left(c\left(d+e\sqrt{x}\right)^2\right)^{5/2}}$$

$$+\frac{5\;2^{-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^2e^6}$$

$$-\frac{5\;2^{2+p}3^{-1-p}d^3e^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^6\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$

$$+\frac{5d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^6}$$

$$-\frac{2^{1+p}d^5e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right.}{e^6\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

output
```
3^(-1-p)*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1
/2))^2))^p/c^3/e^6/exp(3*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p+1)
*d*(d+e*x^(1/2))^5*GAMMA(p+1,1/2*(-5*a-5*b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*
ln(c*(d+e*x^(1/2))^2))^p/(5^p)/e^6/exp(5/2*a/b)/(c*(d+e*x^(1/2))^2)^(5/2)/
((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)+5*d^2*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e*x
^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(2^p)/c^2/e^6/exp(2*a/b)/((-(
a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)-5*2^(2+p)*3^(-1-p)*d^3*(d+e*x^(1/2))^3*GA
MMA(p+1,1/2*(-3*a-3*b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2)
)^p/e^6/exp(3/2*a/b)/(c*(d+e*x^(1/2))^2)^(3/2)/((-(a+b*ln(c*(d+e*x^(1/2))^
2))/b)^p)+5*d^4*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x
^(1/2))^2))^p/c/e^6/exp(a/b)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p+1)*
d^5*(d+e*x^(1/2))*GAMMA(p+1,-1/2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*
(d+e*x^(1/2))^2))^p/e^6/exp(1/2*a/b)/(c*(d+e*x^(1/2))^2)^(1/2)/((-(a+b*ln(
c*(d+e*x^(1/2))^2))/b)^p)
```

Mathematica [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

output
```
Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

$$\Big\downarrow \text{2904}$$

$$2\int x^{5/2}\left(a+b\log\left(c(d+e\sqrt{x})^2\right)\right)^p d\sqrt{x}$$

$$\Big\downarrow \text{2848}$$

$$2\int \left(\frac{(d+e\sqrt{x})^5\left(a+b\log\left(c(d+e\sqrt{x})^2\right)\right)^p}{e^5}-\frac{5d(d+e\sqrt{x})^4\left(a+b\log\left(c(d+e\sqrt{x})^2\right)\right)^p}{e^5}+\frac{10d^2(d+e\sqrt{x})^3}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c(d+e\sqrt{x})^2\right)\right)^p\left(-\frac{a+b\log\left(c(d+e\sqrt{x})^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c(d+e\sqrt{x})^2\right)\right)}{b}\right)}{2c^3e^6}+\frac{5d^22^{-}}{}\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

output
```
2*((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a +
b*Log[c*(d + e*Sqrt[x])^2])^p)/(2*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])^2])/b))^p) - ((2/5)^p*d*(d + e*Sqrt[x])^5*Gamma[1 + p, (-5*(a
+ b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/
(e^6*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e*Sq
rt[x])^2])/b))^p) + (5*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e
*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c^2*e^6*E^((2*a)/b)
*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (5*2^(1 + p)*3^(-1 - p)*d^3*
(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)
]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*Sqrt[
x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (5*d^4*Gamma[1 +
 p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x])^2]
)^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^p*d^
5*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*Sqrt[x])^2])/b]*(a
 + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^
2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Fricas [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a + b \log \left(c(d + e\sqrt{x})^2\right)\right)^p dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \left(a + b \log \left(c(d + e\sqrt{x})^2\right)\right)^p dx = \int \left(b \log \left((e\sqrt{x} + d)^2 c\right) + a\right)^p x^2 dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^2, x)
```

Giac [F]

$$\int x^2 \left(a + b \log \left(c(d + e\sqrt{x})^2\right)\right)^p dx = \int \left(b \log \left((e\sqrt{x} + d)^2 c\right) + a\right)^p x^2 dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)^p\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

Reduce [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\text{too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x)
```

output

```
(60*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d**5*e*p
**2 + 60*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d**
5*e*p + 20*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d
**3*e**3*p**2*x + 20*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b +
 a)**p*b*d**3*e**3*p*x + 12*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2
*x)*b + a)**p*b*d*e**5*p**2*x**2 + 12*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**
2 + c*e**2*x)*b + a)**p*b*d*e**5*p*x**2 - 30*(log(2*sqrt(x)*c*d*e + c*d**2
 + c*e**2*x)*b + a)**p*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b*d**6*p -
 30*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*a*d**6*p + 30*(log
(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*a*e**6*p*x**3 + 30*(log(2*
sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*a*e**6*x**3 - 30*(log(2*sqrt(
x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d**4*e**2*p**2*x - 30*(log(2*sqr
t(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d**4*e**2*p*x - 15*(log(2*sqrt
(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d**2*e**4*p**2*x**2 - 15*(log(2
*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d**2*e**4*p*x**2 - 180*int
((log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p/(3*sqrt(x)*log(2*sqrt
(x)*c*d*e + c*d**2 + c*e**2*x)*a*b*e + sqrt(x)*log(2*sqrt(x)*c*d*e + c*d**
2 + c*e**2*x)*b**2*e*p + 3*sqrt(x)*a**2*e + sqrt(x)*a*b*e*p + 3*log(2*sqrt
(x)*c*d*e + c*d**2 + c*e**2*x)*a*b*d + log(2*sqrt(x)*c*d*e + c*d**2 + c*e*
*2*x)*b**2*d*p + 3*a**2*d + a*b*d*p),x)*a*b**2*d**5*e**2*p**3 - 180*int...
```

3.540 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 445

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

$$=\frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^2e^4}$$

$$-\frac{2^{1+p}3^{-p}de^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c(d+\right.}{b}\right.}{e^4\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$

$$+\frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^4}$$

$$-\frac{2^{1+p}d^3e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right.}{e^4\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

output
```
2^(-1-p)*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1
/2))^2))^p/c^2/e^4/exp(2*a/b)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p+1)
*d*(d+e*x^(1/2))^3*GAMMA(p+1,1/2*(-3*a-3*b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*
ln(c*(d+e*x^(1/2))^2))^p/(3^p)/e^4/exp(3/2*a/b)/(c*(d+e*x^(1/2))^2)^(3/2)/
((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)+3*d^2*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/
2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^4/exp(a/b)/((-(a+b*ln(c*(d+e*
x^(1/2))^2))/b)^p)-2^(p+1)*d^3*(d+e*x^(1/2))*GAMMA(p+1,-1/2*(a+b*ln(c*(d+e
*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/e^4/exp(1/2*a/b)/(c*(d+e*x^
(1/2))^2)^(1/2)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)
```

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

$$\Big\downarrow \text{2904}$$

$$2\int x^{3/2}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \text{2848}$$

$$2\int \left(\frac{(d+e\sqrt{x})^3\left(a+b\log\left(c(d+e\sqrt{x})^2\right)\right)^p}{e^3}-\frac{3d(d+e\sqrt{x})^2\left(a+b\log\left(c(d+e\sqrt{x})^2\right)\right)^p}{e^3}+\frac{3d^2(d+e\sqrt{x})\left(\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{2^{-p-2}e^{-\frac{2a}{b}}\left(a+b\log\left(c(d+e\sqrt{x})^2\right)\right)^p\left(-\frac{a+b\log\left(c(d+e\sqrt{x})^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c(d+e\sqrt{x})^2\right)\right)}{b}\right)}{c^2e^4}-\frac{d^32^pe}{}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

output
```
2*((2^(-2 - p)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a +
b*Log[c*(d + e*Sqrt[x])^2])^p)/(c^2*e^4*E^((2*a)/b)*(-((a + b*Log[c*(d + e
*Sqrt[x])^2])/b))^p) - ((2/3)^p*d*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a +
b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e
^4*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b
)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(2*c*e^4*E^(a/b)*(-((a + b*Log[c*(d
 + e*Sqrt[x])^2])/b))^p) - (2^p*d^3*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a +
 b*Log[c*(d + e*Sqrt[x])^2])/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^4*E
^(a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b
))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^{2}c\right)+a\right)^{p}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x\left(a+b\,\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = \text{too large to display}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x)
```

output
```
(6*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d**3*e*p*
*2 + 6*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d**3*
e*p + 2*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d*e*
*3*p**2*x + 2*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*
b*d*e**3*p*x - 3*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*log(2
*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b*d**4*p - 3*(log(2*sqrt(x)*c*d*e + c*
d**2 + c*e**2*x)*b + a)**p*a*d**4*p + 3*(log(2*sqrt(x)*c*d*e + c*d**2 + c*
e**2*x)*b + a)**p*a*e**4*p*x**2 + 3*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2
*x)*b + a)**p*a*e**4*x**2 - 3*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b
+ a)**p*b*d**2*e**2*p**2*x - 3*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b
 + a)**p*b*d**2*e**2*p*x - 12*int((log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x
)*b + a)**p/(2*sqrt(x)*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*a*b*e + sq
rt(x)*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b**2*e*p + 2*sqrt(x)*a**2*e
 + sqrt(x)*a*b*e*p + 2*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*a*b*d + lo
g(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b**2*d*p + 2*a**2*d + a*b*d*p),x)*a
*b**2*d**3*e**2*p**3 - 12*int((log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b
+ a)**p/(2*sqrt(x)*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*a*b*e + sqrt(x
)*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b**2*e*p + 2*sqrt(x)*a**2*e + s
qrt(x)*a*b*e*p + 2*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*a*b*d + log(2*
sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b**2*d*p + 2*a**2*d + a*b*d*p),x)*a*...
```

3.541 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 213

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx = \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^2} - \frac{2^{1+p}de^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{e^2\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

output
```
GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/
e^2/exp(a/b)/((-(a+b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p+1)*d*(d+e*x^(1/2))*
GAMMA(p+1,-1/2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^
p/e^2/exp(1/2*a/b)/(c*(d+e*x^(1/2))^2)^(1/2)/((-(a+b*ln(c*(d+e*x^(1/2))^2)
)/b)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$
$$=\frac{e^{-\frac{a}{b}}\left(\sqrt{c\left(d+e\sqrt{x}\right)^2}\,\Gamma\left(1+p,-\frac{a+b\log\left(c(d+e\sqrt{x})^2\right)}{b}\right)-2^{1+p}cde^{\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c(d+e\sqrt{x})^2\right)}{2b}\right.}{ce^2\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

output
```
((Sqrt[c*(d + e*Sqrt[x])^2]*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2]
)/b)] - 2^(1 + p)*c*d*E^(a/(2*b))*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a + b
*Log[c*(d + e*Sqrt[x])^2])/b])*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^2*
E^(a/b)*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^
p)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2901, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

$$\Big\downarrow \; 2901$$

$$2\int \sqrt{x}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \; 2848$$

$$2\int \left(\frac{\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{e}-\frac{d\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{e}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{2ce^2}-\frac{d2^pe^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

output
```
2*((Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d +
e*Sqrt[x])^2])^p)/(2*c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))
^p) - (2^p*d*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*Sqrt[x]
)^2])/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d +
e*Sqrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int \left(b\log \left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int \left(b\log \left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int {\left(a+b\,\ln \left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)}^p\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^2))^p,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\text{too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^2))^p,x)
```

output
```
(2*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d*e*p**2
+ 2*sqrt(x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*b*d*e*p -
(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*log(2*sqrt(x)*c*d*e +
c*d**2 + c*e**2*x)*b*d**2*p - (log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b
+ a)**p*a*d**2*p + (log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*a*e
**2*p*x + (log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*a*e**2*x - 2
*int((log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p/(sqrt(x)*log(2*sq
rt(x)*c*d*e + c*d**2 + c*e**2*x)*a*b*e + sqrt(x)*log(2*sqrt(x)*c*d*e + c*d
**2 + c*e**2*x)*b**2*e*p + sqrt(x)*a**2*e + sqrt(x)*a*b*e*p + log(2*sqrt(x
)*c*d*e + c*d**2 + c*e**2*x)*a*b*d + log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2
*x)*b**2*d*p + a**2*d + a*b*d*p),x)*a*b**2*d*e**2*p**3 - 2*int((log(2*sqrt
(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p/(sqrt(x)*log(2*sqrt(x)*c*d*e + c*
d**2 + c*e**2*x)*a*b*e + sqrt(x)*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*
b**2*e*p + sqrt(x)*a**2*e + sqrt(x)*a*b*e*p + log(2*sqrt(x)*c*d*e + c*d**2
 + c*e**2*x)*a*b*d + log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b**2*d*p + a
**2*d + a*b*d*p),x)*a*b**2*d*e**2*p**2 - 2*int((log(2*sqrt(x)*c*d*e + c*d*
*2 + c*e**2*x)*b + a)**p/(sqrt(x)*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)
*a*b*e + sqrt(x)*log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b**2*e*p + sqrt(
x)*a**2*e + sqrt(x)*a*b*e*p + log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*a*b
*d + log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b**2*d*p + a**2*d + a*b*d...
```

3.542 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

$$\Big\downarrow \text{2908}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{2867}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{\sqrt{x}}d\sqrt{x}$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^2))^p/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^2))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 292, normalized size of antiderivative = 12.17

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$
$$=\frac{\left(\log\left(2\sqrt{x}\,cde+c\,d^2+c\,e^2x\right)b+a\right)^p\log\left(2\sqrt{x}\,cde+c\,d^2+c\,e^2x\right)b+\left(\log\left(2\sqrt{x}\,cde+c\,d^2+c\,e^2x\right)b+a\right)}{}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x)
```

output
```
((log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p*log(2*sqrt(x)*c*d*e +
 c*d**2 + c*e**2*x)*b + (log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**
p*a + int((log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p/(d**2*x - e*
*2*x**2),x)*b*d**2*p + int((log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a
)**p/(d**2*x - e**2*x**2),x)*b*d**2 - int((sqrt(x)*(log(2*sqrt(x)*c*d*e +
c*d**2 + c*e**2*x)*b + a)**p)/(d**2*x - e**2*x**2),x)*b*d*e*p - int((sqrt(
x)*(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p)/(d**2*x - e**2*x**
2),x)*b*d*e)/(b*(p + 1))
```

3.543 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}dx$$

$$\Big\downarrow \text{2908}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{2867}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^{3/2}}d\sqrt{x}$$

input
```
Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{2}c\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/2))^2))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/2))^2))^p/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 10.71

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$
$$=\frac{-\left(\log\left(2\sqrt{x}\,cde+c\,d^2+c\,e^2x\right)b+a\right)^p-\left(\int \frac{\left(\log\left(2\sqrt{x}\,cde+c\,d^2+c\,e^2x\right)b+a\right)^p}{\log\left(2\sqrt{x}\,cde+c\,d^2+c\,e^2x\right)b\,d^2x-\log\left(2\sqrt{x}\,cde+c\,d^2+c\,e^2x\right)b\,e^2x^2+a\,d^2x-a\,e^2x^2}}{x}$$

input
```
int((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

output
```
( - (log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p - int((log(2*sqrt(
x)*c*d*e + c*d**2 + c*e**2*x)*b + a)**p/(log(2*sqrt(x)*c*d*e + c*d**2 + c*
e**2*x)*b*d**2*x - log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b*e**2*x**2 +
a*d**2*x - a*e**2*x**2),x)*b*e**2*p*x + int((sqrt(x)*(log(2*sqrt(x)*c*d*e
+ c*d**2 + c*e**2*x)*b + a)**p)/(log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*
b*d**2*x**2 - log(2*sqrt(x)*c*d*e + c*d**2 + c*e**2*x)*b*e**2*x**3 + a*d**
2*x**2 - a*e**2*x**3),x)*b*d*e*p*x)/x
```

3.544 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\operatorname{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

output
```
Defer(Int)(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$
$$\Big\downarrow \text{ 2908}$$
$$2\int x^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,d\sqrt{x}$$
$$\Big\downarrow \text{ 2910}$$
$$2\int x^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,d\sqrt{x}$$

input
```
Int[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/2))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 25.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int x\,{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(1/2))))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(1/2))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx=\int\left(\log\left(\frac{\sqrt{x}\,cd+ce}{\sqrt{x}}\right)b+a\right)^p xdx$$

input
```
int(x*(a+b*log(c*(d+e/x^(1/2))))^p,x)
```

output
```
int((log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*x,x)
```

3.545 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2901, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx$$

$$\Big\downarrow \text{ 2901}$$

$$2\int \sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p d\sqrt{x}$$

$$\Big\downarrow \text{ 2910}$$

$$2\int \sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p d\sqrt{x}$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 25.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx = \int \left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))))^p,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int \left(\log\left(\frac{\sqrt{x}\,cd+ce}{\sqrt{x}}\right)b+a\right)^p dx$$

input
```
int((a+b*log(c*(d+e/x^(1/2))))^p,x)
```

output
```
int((log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p,x)
```

3.546 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$
$$\Big\downarrow \text{2908}$$
$$2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{\sqrt{x}}d\sqrt{x}$$
$$\Big\downarrow \text{2910}$$
$$2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{\sqrt{x}}d\sqrt{x}$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))))^p/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(\log\left(\frac{\sqrt{x}\,cd+ce}{\sqrt{x}}\right)b+a\right)^p}{x}dx$$

input
```
int((a+b*log(c*(d+e/x^(1/2))))^p/x,x)
```

output
```
int((log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p/x,x)
```

3.547 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 175

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = -\frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^2} + \frac{2de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{ce^2}$$

output
```
-GAMMA(p+1,(-2*a-2*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(
2^p)/c^2/e^2/exp(2*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GAMMA(p+1,-
(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c/e^2/exp(a/b)/((
-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$$
$$=\frac{2^{-p}e^{-\frac{2a}{b}}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\right)\left(a+b\log\left(c\left(d\right.\right.\right.}{c^2e^2}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
```

output
```
((-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(
a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)])*(a + b*Log[c*(d +
e/Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])
/b))^p)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$$
$$\Big\downarrow \text{2904}$$
$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{\sqrt{x}}d\frac{1}{\sqrt{x}}$$
$$\Big\downarrow \text{2848}$$

$$-2\int \left(\frac{\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{e}-\frac{d\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{e}\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^2e^2}-\frac{de^{-\frac{a}{b}}\left(a\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
```

output
```
-2*((2^(-1 - p)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b
*Log[c*(d + e/Sqrt[x])])^p)/(c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])])/b))^p) - (d*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a
+ b*Log[c*(d + e/Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[
x])])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = \text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x)
```

output
```
( - 2*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d*e*p**2 - 2*s
qrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d*e*p + 2*(log((sqrt(
x)*c*d + c*e)/sqrt(x))*b + a)**p*log((sqrt(x)*c*d + c*e)/sqrt(x))*b*d**2*p
*x + 2*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*a*d**2*p*x - 2*(log((sq
rt(x)*c*d + c*e)/sqrt(x))*b + a)**p*a*e**2*p - 2*(log((sqrt(x)*c*d + c*e)/
sqrt(x))*b + a)**p*a*e**2 - 2*int((log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)
**p/(2*sqrt(x)*log((sqrt(x)*c*d + c*e)/sqrt(x))*a*b*e*x + sqrt(x)*log((sqr
t(x)*c*d + c*e)/sqrt(x))*b**2*e*p*x + 2*sqrt(x)*a**2*e*x + sqrt(x)*a*b*e*p
*x + 2*log((sqrt(x)*c*d + c*e)/sqrt(x))*a*b*d*x**2 + log((sqrt(x)*c*d + c*
e)/sqrt(x))*b**2*d*p*x**2 + 2*a**2*d*x**2 + a*b*d*p*x**2),x)*a*b**2*d*e**2
*p**3*x - 2*int((log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p/(2*sqrt(x)*log
((sqrt(x)*c*d + c*e)/sqrt(x))*a*b*e*x + sqrt(x)*log((sqrt(x)*c*d + c*e)/sq
rt(x))*b**2*e*p*x + 2*sqrt(x)*a**2*e*x + sqrt(x)*a*b*e*p*x + 2*log((sqrt(x
)*c*d + c*e)/sqrt(x))*a*b*d*x**2 + log((sqrt(x)*c*d + c*e)/sqrt(x))*b**2*d
*p*x**2 + 2*a**2*d*x**2 + a*b*d*p*x**2),x)*a*b**2*d*e**2*p**2*x - int((log
((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p/(2*sqrt(x)*log((sqrt(x)*c*d + c*e)
/sqrt(x))*a*b*e*x + sqrt(x)*log((sqrt(x)*c*d + c*e)/sqrt(x))*b**2*e*p*x +
2*sqrt(x)*a**2*e*x + sqrt(x)*a*b*e*p*x + 2*log((sqrt(x)*c*d + c*e)/sqrt(x)
)*a*b*d*x**2 + log((sqrt(x)*c*d + c*e)/sqrt(x))*b**2*d*p*x**2 + 2*a**2*d*x
**2 + a*b*d*p*x**2),x)*b**3*d*e**2*p**4*x - int((log((sqrt(x)*c*d + c*e...
```

3.548 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 552

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = -\frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^6e^6}$$
$$+\frac{2\,5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^5e^6}$$
$$-\frac{5\,4^{-p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^4e^6}$$
$$+\frac{20\,3^{-1-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^3e^6}$$
$$-\frac{5\,2^{-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^6}$$
$$+\frac{2d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{ce^6}$$

output
```
-3^(-1-p)*GAMMA(p+1,(-6*a-6*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/
2))))^p/(2^p)/c^6/e^6/exp(6*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GA
MMA(p+1,(-5*a-5*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(5^p
)/c^5/e^6/exp(5*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)-5*d^2*GAMMA(p+1,(-
4*a-4*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(4^p)/c^4/e^6/
exp(4*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)+20*3^(-1-p)*d^3*GAMMA(p+1,(-
3*a-3*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^3/e^6/exp(3*
a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)-5*d^4*GAMMA(p+1,(-2*a-2*b*ln(c*(d+
e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(2^p)/c^2/e^6/exp(2*a/b)/((-(a
+b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d^5*GAMMA(p+1,-(a+b*ln(c*(d+e/x^(1/2))))/b
)*(a+b*ln(c*(d+e/x^(1/2))))^p/c/e^6/exp(a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/
b)^p)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.59

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = \frac{3^{-1-p}20^{-p}e^{-\frac{6a}{b}}\left(-10^p\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(2^{1+2p}3^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```

output
```
(3^(-1 - p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]) +
 c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/S
qrt[x])]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[
c*(d + e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e/Sqrt[x])]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p,
(-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p
, -((a + b*Log[c*(d + e/Sqrt[x])])/b)])))))*(a + b*Log[c*(d + e/Sqrt[x])])
^p)/(20^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^{5/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2848}$$

$$-2\int \left(\frac{\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{e^5}-\frac{5d\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{e^5}+\frac{10d^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{6^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^6e^6}-\frac{d5^{-p}e^{-}}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```

output
```
-2*((6^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b
*Log[c*(d + e/Sqrt[x])])^p)/(c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])])/b))^p) - (d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(
a + b*Log[c*(d + e/Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*
(d + e/Sqrt[x])])/b))^p) + (5*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log
[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (10*3^(-1 - p)*d^3*Gamma[
1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])
])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (5*2
^(-1 - p)*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*L
og[c*(d + e/Sqrt[x])])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt
[x])])/b))^p) - (d^5*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a
+ b*Log[c*(d + e/Sqrt[x])])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[
x])])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))))^p/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))))^p/x^4,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = \text{too large to display}$$

input

```
int((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x)
```

output

```
( - 120*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**5*e*p**2*
x**2 - 120*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**5*e*p*
x**2 - 40*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**3*e**3*
p**2*x - 40*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**3*e**
3*p*x - 24*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d*e**5*p*
*2 - 24*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d*e**5*p + 1
20*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*log((sqrt(x)*c*d + c*e)/sqr
t(x))*b*d**6*p*x**3 + 120*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*a*d*
*6*p*x**3 - 120*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*a*e**6*p - 120
*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*a*e**6 + 60*(log((sqrt(x)*c*d
 + c*e)/sqrt(x))*b + a)**p*b*d**4*e**2*p**2*x**2 + 60*(log((sqrt(x)*c*d +
c*e)/sqrt(x))*b + a)**p*b*d**4*e**2*p*x**2 + 30*(log((sqrt(x)*c*d + c*e)/s
qrt(x))*b + a)**p*b*d**2*e**4*p**2*x + 30*(log((sqrt(x)*c*d + c*e)/sqrt(x)
)*b + a)**p*b*d**2*e**4*p*x + 90*int((log((sqrt(x)*c*d + c*e)/sqrt(x))*b +
 a)**p/(6*sqrt(x)*log((sqrt(x)*c*d + c*e)/sqrt(x))*a*b*d*x**3 + sqrt(x)*lo
g((sqrt(x)*c*d + c*e)/sqrt(x))*b**2*d*p*x**3 + 6*sqrt(x)*a**2*d*x**3 + sqr
t(x)*a*b*d*p*x**3 + 6*log((sqrt(x)*c*d + c*e)/sqrt(x))*a*b*e*x**3 + log((s
qrt(x)*c*d + c*e)/sqrt(x))*b**2*e*p*x**3 + 6*a**2*e*x**3 + a*b*e*p*x**3),x
)*a*b**2*d**2*e**5*p**3*x**3 + 90*int((log((sqrt(x)*c*d + c*e)/sqrt(x))*b
+ a)**p/(6*sqrt(x)*log((sqrt(x)*c*d + c*e)/sqrt(x))*a*b*d*x**3 + sqrt(x...
```

3.549 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 926

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = \text{Too large to display}$$

output
```
-5^(-1-p)*GAMMA(p+1,(-10*a-10*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(
1/2))))^p/(2^p)/c^10/e^10/exp(10*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)+2
*d*GAMMA(p+1,(-9*a-9*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p
/(9^p)/c^9/e^10/exp(9*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)-9*d^2*GAMMA(
p+1,(-8*a-8*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(8^p)/c^
8/e^10/exp(8*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)+24*d^3*GAMMA(p+1,(-7*
a-7*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(7^p)/c^7/e^10/e
xp(7*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)-7*6^(1-p)*d^4*GAMMA(p+1,(-6*a
-6*b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^6/e^10/exp(6*a/
b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)+252*5^(-1-p)*d^5*GAMMA(p+1,(-5*a-5*b
*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^5/e^10/exp(5*a/b)/(
(-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)-21*2^(1-2*p)*d^6*GAMMA(p+1,(-4*a-4*b*ln(
c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^4/e^10/exp(4*a/b)/((-(a
+b*ln(c*(d+e/x^(1/2))))/b)^p)+8*3^(1-p)*d^7*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e/
x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^3/e^10/exp(3*a/b)/((-(a+b*ln(c
*(d+e/x^(1/2))))/b)^p)-9*d^8*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e/x^(1/2))))/b)*(
a+b*ln(c*(d+e/x^(1/2))))^p/(2^p)/c^2/e^10/exp(2*a/b)/((-(a+b*ln(c*(d+e/x^(
1/2))))/b)^p)+2*d^9*GAMMA(p+1,-(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e
/x^(1/2))))^p/c/e^10/exp(a/b)/((-(a+b*ln(c*(d+e/x^(1/2))))/b)^p)
```

Mathematica [A] (verified)

Time = 5.95 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.57

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx$$

$$= \frac{5^{-1-p}504^{-p}e^{-\frac{10a}{b}}\left(-252^p\Gamma\left(1+p,-\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(2^{1+3p}5^{1+p}7^p\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\right.\right.\right.}{b}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

output
```
(5^(-1 - p)*(-(252^p*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b])
 + c*d*E^(a/b)*(2^(1 + 3*p)*5^(1 + p)*7^p*Gamma[1 + p, (-9*(a + b*Log[c*(d
 + e/Sqrt[x])]))/b] + c*d*E^(a/b)*(-(7^p*45^(1 + p)*Gamma[1 + p, (-8*(a +
b*Log[c*(d + e/Sqrt[x])]))/b]) + 2^p*c*d*E^(a/b)*(2^(3 + 2*p)*3^(1 + 2*p)*
5^(1 + p)*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 7^p*c*d*E^
(a/b)*(-7*30^(1 + p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b] +
 c*d*E^(a/b)*(7*36^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])])
)/b] + 3^p*5^(1 + p)*c*d*E^(a/b)*(-14*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Lo
g[c*(d + e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(3*2^(3 + p)*Gamma[1 + p, (-3*
(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 3^p*c*d*E^(a/b)*(-9*Gamma[1 + p, (-2*
(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -(
(a + b*Log[c*(d + e/Sqrt[x])])/b)]))))))))))*(a + b*Log[c*(d + e/Sqrt[x])])
^p)/(504^p*c^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rubi [A] (verified)

Time = 3.34 (sec) , antiderivative size = 929, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}dx$$

$$\Big\downarrow \text{ 2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^{9/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 2848}$$

$$-2\int \left(\frac{\left(d+\frac{e}{\sqrt{x}}\right)^9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{e^9}-\frac{9d\left(d+\frac{e}{\sqrt{x}}\right)^8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{e^9}+\frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)^7\left(a\right.}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-2\left(\frac{10^{-p-1}e^{-\frac{10a}{b}}\Gamma\left(p+1,-\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^{10}e^{10}}-\frac{9^{-p}d}{\quad}\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

output
```
-2*((10^(-1 - p)*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e/Sqrt[x])])^p)/(c^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d +
 e/Sqrt[x])])/b))^p) - (d*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/Sqrt[x])]))
/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(9^p*c^9*e^10*E^((9*a)/b)*(-((a + b*
Log[c*(d + e/Sqrt[x])])/b))^p) + (9*2^(-1 - 3*p)*d^2*Gamma[1 + p, (-8*(a +
 b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^8*e^10
*E^((8*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (12*d^3*Gamma[1 +
p, (-7*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p
)/(7^p*c^7*e^10*E^((7*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (7*
3^(1 - p)*d^4*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*L
og[c*(d + e/Sqrt[x])])^p)/(2^p*c^6*e^10*E^((6*a)/b)*(-((a + b*Log[c*(d + e
/Sqrt[x])])/b))^p) - (126*5^(-1 - p)*d^5*Gamma[1 + p, (-5*(a + b*Log[c*(d
+ e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^5*e^10*E^((5*a)/b)
*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (21*d^6*Gamma[1 + p, (-4*(a +
b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(4^p*c^4*e
^10*E^((4*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (4*3^(1 - p)*d^
7*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(c^3*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^
p) + (9*2^(-1 - p)*d^8*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]
*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^2*e^10*E^((2*a)/b)*(-((a + b*Log[...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x^{6}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))))^p/x^6,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))))^p/x^6,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x^{6}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}}{x^{6}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^6}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^6}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))))^p/x^6,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))))^p/x^6, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx=\text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x)
```

output
```
( - 5040*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**9*e*p**2
*x**4 - 5040*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**9*e*
p*x**4 - 1680*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**7*e
**3*p**2*x**3 - 1680*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b
*d**7*e**3*p*x**3 - 1008*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)*
*p*b*d**5*e**5*p**2*x**2 - 1008*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x))*
b + a)**p*b*d**5*e**5*p*x**2 - 720*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(x
))*b + a)**p*b*d**3*e**7*p**2*x - 720*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqr
t(x))*b + a)**p*b*d**3*e**7*p*x - 560*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqr
t(x))*b + a)**p*b*d*e**9*p**2 - 560*sqrt(x)*(log((sqrt(x)*c*d + c*e)/sqrt(
x))*b + a)**p*b*d*e**9*p + 5040*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**
p*log((sqrt(x)*c*d + c*e)/sqrt(x))*b*d**10*p*x**5 + 5040*(log((sqrt(x)*c*d
 + c*e)/sqrt(x))*b + a)**p*a*d**10*p*x**5 - 5040*(log((sqrt(x)*c*d + c*e)/
sqrt(x))*b + a)**p*a*e**10*p - 5040*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b +
a)**p*a*e**10 + 2520*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**8*e*
*2*p**2*x**4 + 2520*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**8*e**
2*p*x**4 + 1260*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**6*e**4*p*
*2*x**3 + 1260*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**6*e**4*p*x
**3 + 840*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**4*e**6*p**2*x**
2 + 840*(log((sqrt(x)*c*d + c*e)/sqrt(x))*b + a)**p*b*d**4*e**6*p*x**2 ...
```

3.550 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p,x\right)$$

output
```
Defer(Int)(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

$$\Big\downarrow \text{2908}$$

$$2\int x^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,d\sqrt{x}$$

$$\Big\downarrow \text{2910}$$

$$2\int x^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,d\sqrt{x}$$

input
```
Int[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx = \int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx = \text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 25.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(1/2))^2))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(1/2))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int\left(\log\left(\frac{2\sqrt{x}\,cde+c\,d^2x+c\,e^2}{x}\right)b+a\right)^p xdx$$

input
```
int(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x)
```

output
```
int((log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*x,x)
```

3.551 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx = \text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p, x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx = \int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2901, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

$$\Big\downarrow \text{2901}$$

$$2\int \sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p d\sqrt{x}$$

$$\Big\downarrow \text{2910}$$

$$2\int \sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p d\sqrt{x}$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)+a\right)^{p}dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 25.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx=\int \left(\log\left(\frac{2\sqrt{x}\,cde+c\,d^2x+c\,e^2}{x}\right)b+a\right)^p dx$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^2))^p,x)
```

output
```
int((log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p,x)
```

3.552 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx = \text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}dx$$

$$\Big\downarrow \text{2908}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{2910}$$

$$2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{\sqrt{x}}d\sqrt{x}$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(\log\left(\frac{2\sqrt{x}\,cde+c\,d^2x+c\,e^2}{x}\right)b+a\right)^p}{x}dx$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x)
```

output
```
int((log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p/x,x)
```

3.553 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 216

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = \frac{2^{1+p}de^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}} - \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{ce^2}$$

output
```
2^(p+1)*d*(d+e/x^(1/2))*GAMMA(p+1,1/2*(-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b
*ln(c*(d+e/x^(1/2))^2))^p/e^2/exp(1/2*a/b)/(c*(d+e/x^(1/2))^2)^(1/2)/((-(a
+b*ln(c*(d+e/x^(1/2))^2))/b)^p)-GAMMA(p+1,-(a+b*ln(c*(d+e/x^(1/2))^2))/b)*
(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/e^2/exp(a/b)/((-(a+b*ln(c*(d+e/x^(1/2))^2)
)/b)^p)
```

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2, x]
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{\sqrt{x}}\,d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2848}$$

$$-2\int \left(\frac{\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{e}-\frac{d\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{e}\right)d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)}{2ce^2}-\frac{d2^pe^{-\frac{a}{2b}}\left(d+\right.}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
```

output
```
-2*((Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d +
 e/Sqrt[x])^2])^p)/(2*c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b)
)^p) - (2^p*d*(d + e/Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/Sqrt[x
])^2])/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d +
 e/Sqrt[x])^2]*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = \text{too large to display}$$

input

```
int((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x)
```

output

```
( - 2*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d*
e*p**2 - 2*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p
*b*d*e*p + (log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*log((2*
sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b*d**2*p*x + (log((2*sqrt(x)*c*d*e +
 c*d**2*x + c*e**2)/x)*b + a)**p*a*d**2*p*x - (log((2*sqrt(x)*c*d*e + c*d*
*2*x + c*e**2)/x)*b + a)**p*a*e**2*p - (log((2*sqrt(x)*c*d*e + c*d**2*x +
c*e**2)/x)*b + a)**p*a*e**2 - 2*int((log((2*sqrt(x)*c*d*e + c*d**2*x + c*e
**2)/x)*b + a)**p/(sqrt(x)*log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*a*
b*e*x + sqrt(x)*log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b**2*e*p*x +
sqrt(x)*a**2*e*x + sqrt(x)*a*b*e*p*x + log((2*sqrt(x)*c*d*e + c*d**2*x + c
*e**2)/x)*a*b*d*x**2 + log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b**2*d
*p*x**2 + a**2*d*x**2 + a*b*d*p*x**2),x)*a*b**2*d*e**2*p**3*x - 2*int((log
((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p/(sqrt(x)*log((2*sqrt(x
)*c*d*e + c*d**2*x + c*e**2)/x)*a*b*e*x + sqrt(x)*log((2*sqrt(x)*c*d*e + c
*d**2*x + c*e**2)/x)*b**2*e*p*x + sqrt(x)*a**2*e*x + sqrt(x)*a*b*e*p*x + l
og((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*a*b*d*x**2 + log((2*sqrt(x)*c*
d*e + c*d**2*x + c*e**2)/x)*b**2*d*p*x**2 + a**2*d*x**2 + a*b*d*p*x**2),x)
*a*b**2*d*e**2*p**2*x - 2*int((log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x
)*b + a)**p/(sqrt(x)*log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*a*b*e*x
+ sqrt(x)*log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b**2*e*p*x + sqr...
```

3.554 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 676

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\text{Too large to display}$$

output

```
-3^(-1-p)*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(
1/2))^2))^p/c^3/e^6/exp(3*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(p+1
)*d*(d+e/x^(1/2))^5*GAMMA(p+1,1/2*(-5*a-5*b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b
*ln(c*(d+e/x^(1/2))^2))^p/(5^p)/e^6/exp(5/2*a/b)/(c*(d+e/x^(1/2))^2)^(5/2)
/((-(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)-5*d^2*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e/
x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/(2^p)/c^2/e^6/exp(2*a/b)/((-
(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)+5*2^(2+p)*3^(-1-p)*d^3*(d+e/x^(1/2))^3*G
AMMA(p+1,1/2*(-3*a-3*b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2
))^p/e^6/exp(3/2*a/b)/(c*(d+e/x^(1/2))^2)^(3/2)/((-(a+b*ln(c*(d+e/x^(1/2))
^2))/b)^p)-5*d^4*GAMMA(p+1,-(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/
x^(1/2))^2))^p/c/e^6/exp(a/b)/((-(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(p+1)
*d^5*(d+e/x^(1/2))*GAMMA(p+1,-1/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c
*(d+e/x^(1/2))^2))^p/e^6/exp(1/2*a/b)/(c*(d+e/x^(1/2))^2)^(1/2)/((-(a+b*ln
(c*(d+e/x^(1/2))^2))/b)^p)
```

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4, x]
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^{5/2}}\,d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2848}$$

$$-2\int \left(\frac{\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{e^5}-\frac{5d\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{e^5}+\frac{10d^2\left(d+\frac{e}{\sqrt{x}}\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)}{2c^3e^6}+-\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
```

output
```
-2*((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a +
 b*Log[c*(d + e/Sqrt[x])^2])^p)/(2*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d
+ e/Sqrt[x])^2])/b))^p) - ((2/5)^p*d*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a
 + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)
/(e^6*E^((5*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/S
qrt[x])^2])/b))^p) + (5*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d +
e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c^2*e^6*E^((2*a)/b
)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (5*2^(1 + p)*3^(-1 - p)*d^3
*(d + e/Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b
)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e/Sqrt
[x])^2)^(3/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (5*d^4*Gamma[1
+ p, -((a + b*Log[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2
])^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (2^p*d
^5*(d + e/Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/Sqrt[x])^2])/b]*(
a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])
^2]*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)\right)^{p}}{x^{4}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{2}\right)+a\right)^{p}}{x^{4}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^4,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x)
```

output

```
( - 60*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d
**5*e*p**2*x**2 - 60*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)
*b + a)**p*b*d**5*e*p*x**2 - 20*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x +
 c*e**2)/x)*b + a)**p*b*d**3*e**3*p**2*x - 20*sqrt(x)*(log((2*sqrt(x)*c*d*
e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d**3*e**3*p*x - 12*sqrt(x)*(log((2*s
qrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d*e**5*p**2 - 12*sqrt(x)*
(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d*e**5*p + 30*(l
og((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*log((2*sqrt(x)*c*d*e
 + c*d**2*x + c*e**2)/x)*b*d**6*p*x**3 + 30*(log((2*sqrt(x)*c*d*e + c*d**2
*x + c*e**2)/x)*b + a)**p*a*d**6*p*x**3 - 30*(log((2*sqrt(x)*c*d*e + c*d**
2*x + c*e**2)/x)*b + a)**p*a*e**6*p - 30*(log((2*sqrt(x)*c*d*e + c*d**2*x
+ c*e**2)/x)*b + a)**p*a*e**6 + 30*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e*
*2)/x)*b + a)**p*b*d**4*e**2*p**2*x**2 + 30*(log((2*sqrt(x)*c*d*e + c*d**2
*x + c*e**2)/x)*b + a)**p*b*d**4*e**2*p*x**2 + 15*(log((2*sqrt(x)*c*d*e +
c*d**2*x + c*e**2)/x)*b + a)**p*b*d**2*e**4*p**2*x + 15*(log((2*sqrt(x)*c*
d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d**2*e**4*p*x + 45*int((log((2*sqr
t(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p/(3*sqrt(x)*log((2*sqrt(x)*c*d
*e + c*d**2*x + c*e**2)/x)*a*b*d*x**3 + sqrt(x)*log((2*sqrt(x)*c*d*e + c*d
**2*x + c*e**2)/x)*b**2*d*p*x**3 + 3*sqrt(x)*a**2*d*x**3 + sqrt(x)*a*b*d*p
*x**3 + 3*log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*a*b*e*x**3 + log...
```

3.555 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1141

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx = \text{Too large to display}$$

output
```
-5^(-1-p)*GAMMA(p+1,(-5*a-5*b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(
1/2))^2))^p/c^5/e^10/exp(5*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(p+
1)*d*(d+e/x^(1/2))^9*GAMMA(p+1,1/2*(-9*a-9*b*ln(c*(d+e/x^(1/2))^2))/b)*(a+
b*ln(c*(d+e/x^(1/2))^2))^p/(9^p)/e^10/exp(9/2*a/b)/(c*(d+e/x^(1/2))^2)^(9/
2)/((-(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)-9*d^2*GAMMA(p+1,(-4*a-4*b*ln(c*(d+
e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/(4^p)/c^4/e^10/exp(4*a/b)/
((-(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)+3*2^(3+p)*d^3*(d+e/x^(1/2))^7*GAMMA(p
+1,1/2*(-7*a-7*b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/(
7^p)/e^10/exp(7/2*a/b)/(c*(d+e/x^(1/2))^2)^(7/2)/((-(a+b*ln(c*(d+e/x^(1/2)
)^2))/b)^p)-14*3^(1-p)*d^4*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e/x^(1/2))^2))/b)*(
a+b*ln(c*(d+e/x^(1/2))^2))^p/c^3/e^10/exp(3*a/b)/((-(a+b*ln(c*(d+e/x^(1/2)
)^2))/b)^p)+63*2^(2+p)*5^(-1-p)*d^5*(d+e/x^(1/2))^5*GAMMA(p+1,1/2*(-5*a-5*
b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/e^10/exp(5/2*a/b
)/(c*(d+e/x^(1/2))^2)^(5/2)/((-(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)-21*2^(1-p
)*d^6*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2)
)^2))^p/c^2/e^10/exp(2*a/b)/((-(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(3+p)*3
^(1-p)*d^7*(d+e/x^(1/2))^3*GAMMA(p+1,1/2*(-3*a-3*b*ln(c*(d+e/x^(1/2))^2))/
b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/e^10/exp(3/2*a/b)/(c*(d+e/x^(1/2))^2)^(3/
2)/((-(a+b*ln(c*(d+e/x^(1/2))^2))/b)^p)-9*d^8*GAMMA(p+1,-(a+b*ln(c*(d+e/x^
(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/e^10/exp(a/b)/((-(a+b*ln(...
```

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6, x]
```

Rubi [A] (verified)

Time = 3.63 (sec) , antiderivative size = 1143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}dx$$

$$\Big\downarrow \text{2904}$$

$$-2\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^{9/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{2848}$$

$$-2\int \left(\frac{\left(d+\frac{e}{\sqrt{x}}\right)^9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{e^9}-\frac{9d\left(d+\frac{e}{\sqrt{x}}\right)^8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{e^9}+\frac{36d^2\left(d+\frac{e}{\sqrt{x}}\right)}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{5^{-p-1}e^{-\frac{5a}{b}}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{2c^5e^{10}}-\right.$$

input
```
Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]
```

output

```
-2*((5^(-1 - p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a +
 b*Log[c*(d + e/Sqrt[x])^2])^p)/(2*c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e/Sqrt[x])^2])/b))^p) - ((2/9)^p*d*(d + e/Sqrt[x])^9*Gamma[1 + p, (-9*(
a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p
)/(e^10*E^((9*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(9/2)*(-((a + b*Log[c*(d + e
/Sqrt[x])^2])/b))^p) + (9*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(
d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c^4*e^10*E^((4
*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (3*2^(2 + p)*d^3*(d +
e/Sqrt[x])^7*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a
+ b*Log[c*(d + e/Sqrt[x])^2])^p)/(7^p*e^10*E^((7*a)/(2*b))*(c*(d + e/Sqrt[
x])^2)^(7/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (7*3^(1 - p)*d^4
*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e
/Sqrt[x])^2])^p)/(c^3*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])
/b))^p) - (63*2^(1 + p)*5^(-1 - p)*d^5*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*
(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^
p)/(e^10*E^((5*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d +
e/Sqrt[x])^2])/b))^p) + (21*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[
x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(2^p*c^2*e^10*E^((2*a)/b)*
(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (2^(2 + p)*3^(1 - p)*d^7*(d +
 e/Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]...
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^6,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^6,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^6}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^6}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^6}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^6,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^6, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx = \text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x)
```

output
```
( - 2520*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b
*d**9*e*p**2*x**4 - 2520*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2
)/x)*b + a)**p*b*d**9*e*p*x**4 - 840*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**
2*x + c*e**2)/x)*b + a)**p*b*d**7*e**3*p**2*x**3 - 840*sqrt(x)*(log((2*sqr
t(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d**7*e**3*p*x**3 - 504*sqrt
(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d**5*e**5*p*
*2*x**2 - 504*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)
**p*b*d**5*e**5*p*x**2 - 360*sqrt(x)*(log((2*sqrt(x)*c*d*e + c*d**2*x + c*
e**2)/x)*b + a)**p*b*d**3*e**7*p**2*x - 360*sqrt(x)*(log((2*sqrt(x)*c*d*e
+ c*d**2*x + c*e**2)/x)*b + a)**p*b*d**3*e**7*p*x - 280*sqrt(x)*(log((2*sq
rt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d*e**9*p**2 - 280*sqrt(x)*
(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d*e**9*p + 1260*
(log((2*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*log((2*sqrt(x)*c*d
*e + c*d**2*x + c*e**2)/x)*b*d**10*p*x**5 + 1260*(log((2*sqrt(x)*c*d*e + c
*d**2*x + c*e**2)/x)*b + a)**p*a*d**10*p*x**5 - 1260*(log((2*sqrt(x)*c*d*e
 + c*d**2*x + c*e**2)/x)*b + a)**p*a*e**10*p - 1260*(log((2*sqrt(x)*c*d*e
+ c*d**2*x + c*e**2)/x)*b + a)**p*a*e**10 + 1260*(log((2*sqrt(x)*c*d*e + c
*d**2*x + c*e**2)/x)*b + a)**p*b*d**8*e**2*p**2*x**4 + 1260*(log((2*sqrt(x
)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d**8*e**2*p*x**4 + 630*(log((2
*sqrt(x)*c*d*e + c*d**2*x + c*e**2)/x)*b + a)**p*b*d**6*e**4*p**2*x**3 ...
```

3.556 $\int x^3 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)\right)\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 1121

$$\int x^3 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)\right)\right)^p \, dx = \text{Too large to display}$$

output
```
4^(-1-p)*GAMMA(p+1,(-12*a-12*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1
/3))))^p/(3^p)/c^12/e^12/exp(12*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-3*
d*GAMMA(p+1,(-11*a-11*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^
p/(11^p)/c^11/e^12/exp(11*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+33*2^(-1
-p)*d^2*GAMMA(p+1,(-10*a-10*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/
3))))^p/(5^p)/c^10/e^12/exp(10*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-55*
d^3*GAMMA(p+1,(-9*a-9*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^
p/(9^p)/c^9/e^12/exp(9*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+495*2^(-2-3
*p)*d^4*GAMMA(p+1,(-8*a-8*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3)
)))^p/c^8/e^12/exp(8*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-198*d^5*GAMMA
(p+1,(-7*a-7*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(7^p)/c
^7/e^12/exp(7*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+77*3^(1-p)*d^6*GAMMA
(p+1,(-6*a-6*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(2^p)/c
^6/e^12/exp(6*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-198*d^7*GAMMA(p+1,(-
5*a-5*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^12
/exp(5*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+495*4^(-1-p)*d^8*GAMMA(p+1,
(-4*a-4*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^4/e^12/exp
(4*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-55*d^9*GAMMA(p+1,(-3*a-3*b*ln(c
*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^3/e^12/exp(3*a/b)/
((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+33*2^(-1-p)*d^10*GAMMA(p+1,(-2*a-2*b...
```

Mathematica [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

output
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p, x]
```

Rubi [A] (verified)

Time = 4.04 (sec) , antiderivative size = 1115, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{11/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int\left(\frac{\left(d+e\sqrt[3]{x}\right)^{11}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{e^{11}}-\frac{11d\left(d+e\sqrt[3]{x}\right)^{10}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{e^{11}}+\frac{55d^2\left(d+e\sqrt[3]{x}\right)^9}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{12^{-p-1}e^{-\frac{12a}{b}}\Gamma\left(p+1,-\frac{12\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^{12}e^{12}}-\frac{11^{-}}{}\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

output
```
3*((12^(-1 - p)*Gamma[1 + p, (-12*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
b*Log[c*(d + e*x^(1/3))])^p)/(c^12*e^12*E^((12*a)/b)*(-((a + b*Log[c*(d +
e*x^(1/3))])/b))^p) - (d*Gamma[1 + p, (-11*(a + b*Log[c*(d + e*x^(1/3))]))
/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(11^p*c^11*e^12*E^((11*a)/b)*(-((a +
 b*Log[c*(d + e*x^(1/3))])/b))^p) + (11*2^(-1 - p)*d^2*Gamma[1 + p, (-10*(
a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c
^10*e^12*E^((10*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (55*3^(-1
 - 2*p)*d^3*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(c^9*e^12*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/
3))])/b))^p) + (165*2^(-2 - 3*p)*d^4*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*
x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^8*e^12*E^((8*a)/b)*(-(
(a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (66*d^5*Gamma[1 + p, (-7*(a + b*Lo
g[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^12*
E^((7*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (77*d^6*Gamma[1 + p
, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)
/(6^p*c^6*e^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (66*
d^7*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d +
e*x^(1/3))])^p)/(5^p*c^5*e^12*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))]
)/b))^p) + (165*4^(-1 - p)*d^8*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3
))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^12*E^((4*a)/b)*(-((a ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^{3}\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Fricas [F]

$$\int x^{3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x^{3}\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3(a+b\log(c(d+e\sqrt[3]{x})))^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3(a+b\log(c(d+e\sqrt[3]{x})))^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\log(c(d+e\sqrt[3]{x})))^p\,dx=\int x^3\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(1/3))))^p,x)
```

output
```
int(x^3*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

Reduce [F]

$$\int x^3 \left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)\right)\right)^p \, dx = \text{too large to display}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x)
```

output
```
( - 41580*x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**10*e**2*p**2 -
41580*x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**10*e**2*p + 16632*x
**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**7*e**5*p**2*x + 16632*x**(
2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**7*e**5*p*x - 10395*x**(2/3)*(
log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**4*e**8*p**2*x**2 - 10395*x**(2/3)*(
log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**4*e**8*p*x**2 + 7560*x**(2/3)*(log(
x**(1/3)*c*e + c*d)*b + a)**p*b*d*e**11*p**2*x**3 + 7560*x**(2/3)*(log(x**
(1/3)*c*e + c*d)*b + a)**p*b*d*e**11*p*x**3 + 83160*x**(1/3)*(log(x**(1/3)
*c*e + c*d)*b + a)**p*b*d**11*e*p**2 + 83160*x**(1/3)*(log(x**(1/3)*c*e +
c*d)*b + a)**p*b*d**11*e*p - 20790*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a
)**p*b*d**8*e**4*p**2*x - 20790*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**
p*b*d**8*e**4*p*x + 11880*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d*
*5*e**7*p**2*x**2 + 11880*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d*
*5*e**7*p*x**2 - 8316*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**2*e
**10*p**2*x**3 - 8316*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**2*e
**10*p*x**3 - 83160*(log(x**(1/3)*c*e + c*d)*b + a)**p*log(x**(1/3)*c*e +
c*d)*b*d**12*p - 83160*(log(x**(1/3)*c*e + c*d)*b + a)**p*a*d**12*p + 8316
0*(log(x**(1/3)*c*e + c*d)*b + a)**p*a*e**12*p*x**4 + 83160*(log(x**(1/3)*
c*e + c*d)*b + a)**p*a*e**12*x**4 + 27720*(log(x**(1/3)*c*e + c*d)*b + a)*
*p*b*d**9*e**3*p**2*x + 27720*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**9...
```

3.557 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 831

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = \text{Too large to display}$$

output
```
3^(-1-2*p)*GAMMA(p+1,(-9*a-9*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1
/3))))^p/c^9/e^9/exp(9*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(p
+1,(-8*a-8*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(8^p)/c^8
/e^9/exp(8*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+12*d^2*GAMMA(p+1,(-7*a-
7*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(7^p)/c^7/e^9/exp(
7*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-7*2^(2-p)*d^3*GAMMA(p+1,(-6*a-6*
b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^6/e^9/exp(6*
a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+42*d^4*GAMMA(p+1,(-5*a-5*b*ln(c*(d
+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^9/exp(5*a/b)/((-(
a+b*ln(c*(d+e*x^(1/3))))/b)^p)-21*2^(1-2*p)*d^5*GAMMA(p+1,(-4*a-4*b*ln(c*(
d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^4/e^9/exp(4*a/b)/((-(a+b*l
n(c*(d+e*x^(1/3))))/b)^p)+28*d^6*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e*x^(1/3))))/
b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^3/e^9/exp(3*a/b)/((-(a+b*ln(c*(d+e*
x^(1/3))))/b)^p)-3*2^(2-p)*d^7*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e*x^(1/3))))/b)
*(a+b*ln(c*(d+e*x^(1/3))))^p/c^2/e^9/exp(2*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))
))/b)^p)+3*d^8*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1
/3))))^p/c/e^9/exp(a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)
```

Mathematica [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

output
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p, x]
```

Rubi [A] (verified)

Time = 3.03 (sec) , antiderivative size = 836, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int\left(\frac{\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{e^8}-\frac{8d\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{e^8}+\frac{28d^2\left(d+e\sqrt[3]{x}\right)^6\left(a\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{9^{-p-1}e^{-\frac{9a}{b}}\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^9e^9}-\frac{8^{-p}de^{-}}{}\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

output
```
3*((9^(-1 - p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*
Log[c*(d + e*x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(
1/3))])/b))^p) - (d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(
d + e*x^(1/3))])/b))^p) + (4*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1
/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((
a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (7*2^(2 - p)*3^(-1 - p)*d^3*Gamma[1
 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))]
)^p)/(c^6*e^9*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (14*d
^4*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e
*x^(1/3))])^p)/(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/
b))^p) - (7*2^(1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]
))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log
[c*(d + e*x^(1/3))])/b))^p) + (28*3^(-1 - p)*d^6*Gamma[1 + p, (-3*(a + b*L
og[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^3*e^9*E^((
3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (2^(2 - p)*d^7*Gamma[1
+ p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])
^p)/(c^2*e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (d^8*G
amma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/
3))])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^{2} \left(a +b \ln \left(c \left(d +e \,x^{\frac{1}{3}}\right)\right)\right)^{p}d x$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Fricas [F]

$$\int x^2 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)\right)\right)^p \, dx=\int { {\left(b \log \left({\left(e x^{\frac{1}{3}} + d\right)} c\right) + a\right)}^{p} x^{2} \,d x }$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)\right)\right)^p \, dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^2, x)
```

Giac [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int x^2\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/3))))^p,x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

Reduce [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\text{too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x)
```

output
```
(1260*x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**7*e**2*p**2 + 1260*
x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**7*e**2*p - 504*x**(2/3)*(
log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**4*e**5*p**2*x - 504*x**(2/3)*(log(x
**(1/3)*c*e + c*d)*b + a)**p*b*d**4*e**5*p*x + 315*x**(2/3)*(log(x**(1/3)*
c*e + c*d)*b + a)**p*b*d*e**8*p**2*x**2 + 315*x**(2/3)*(log(x**(1/3)*c*e +
 c*d)*b + a)**p*b*d*e**8*p*x**2 - 2520*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b
 + a)**p*b*d**8*e*p**2 - 2520*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*
b*d**8*e*p + 630*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**5*e**4*p
**2*x + 630*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**5*e**4*p*x -
360*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**2*e**7*p**2*x**2 - 36
0*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**2*e**7*p*x**2 + 2520*(l
og(x**(1/3)*c*e + c*d)*b + a)**p*log(x**(1/3)*c*e + c*d)*b*d**9*p + 2520*(
log(x**(1/3)*c*e + c*d)*b + a)**p*a*d**9*p + 2520*(log(x**(1/3)*c*e + c*d)
*b + a)**p*a*e**9*p*x**3 + 2520*(log(x**(1/3)*c*e + c*d)*b + a)**p*a*e**9*
x**3 - 840*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**6*e**3*p**2*x - 840*(lo
g(x**(1/3)*c*e + c*d)*b + a)**p*b*d**6*e**3*p*x + 420*(log(x**(1/3)*c*e +
c*d)*b + a)**p*b*d**3*e**6*p**2*x**2 + 420*(log(x**(1/3)*c*e + c*d)*b + a)
**p*b*d**3*e**6*p*x**2 + 7560*int((log(x**(1/3)*c*e + c*d)*b + a)**p/(9*x*
*(2/3)*log(x**(1/3)*c*e + c*d)*a*b*e + x**(2/3)*log(x**(1/3)*c*e + c*d)*b*
*2*e*p + 9*x**(2/3)*a**2*e + x**(2/3)*a*b*e*p + 9*x**(1/3)*log(x**(1/3)...
```

3.558 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 553

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$
$$=\frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^6e^6}$$
$$-\frac{3\ 5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^5e^6}$$
$$+\frac{15\ 2^{-1-2p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right.}{c^4e^6}$$
$$-\frac{10\ 3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^6}$$
$$+\frac{15\ 2^{-1-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)}{c^2e^6}$$
$$-\frac{3d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{ce^6}$$

output

```
2^(-1-p)*GAMMA(p+1,(-6*a-6*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3
))))^p/(3^p)/c^6/e^6/exp(6*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAM
MA(p+1,(-5*a-5*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)
/c^5/e^6/exp(5*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+15*2^(-1-2*p)*d^2*G
AMMA(p+1,(-4*a-4*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^4
/e^6/exp(4*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-10*d^3*GAMMA(p+1,(-3*a-
3*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^3/e^6/exp(
3*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+15*2^(-1-p)*d^4*GAMMA(p+1,(-2*a-
2*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^2/e^6/exp(2*a/b)
/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d^5*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/3
))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^6/exp(a/b)/((-(a+b*ln(c*(d+e*x^(1/
3))))/b)^p)
```

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e*x^(1/3))])^p, x]
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int\left(\frac{\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{e^5}-\frac{5d\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{e^5}+\frac{10d^2\left(d+e\sqrt[3]{x}\right)^3\left(a\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{6^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^6e^6}-\frac{d5^{-p}e^{-}}{}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

output
```
3*((6^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*
Log[c*(d + e*x^(1/3))])^p)/(c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(
1/3))])/b))^p) - (d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(
d + e*x^(1/3))])/b))^p) + (5*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log[
c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^6*E^((4*a
)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (10*3^(-1 - p)*d^3*Gamma[1
 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))]
)^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (5*2^
(-1 - p)*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/
3))])/b))^p) - (d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/3))))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\text{too large to display}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/3))))^p,x)
```

output
```
( - 90*x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**4*e**2*p**2 - 90*x
**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**4*e**2*p + 36*x**(2/3)*(lo
g(x**(1/3)*c*e + c*d)*b + a)**p*b*d*e**5*p**2*x + 36*x**(2/3)*(log(x**(1/3
)*c*e + c*d)*b + a)**p*b*d*e**5*p*x + 180*x**(1/3)*(log(x**(1/3)*c*e + c*d
)*b + a)**p*b*d**5*e*p**2 + 180*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**
p*b*d**5*e*p - 45*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**2*e**4*
p**2*x - 45*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**2*e**4*p*x -
180*(log(x**(1/3)*c*e + c*d)*b + a)**p*log(x**(1/3)*c*e + c*d)*b*d**6*p -
180*(log(x**(1/3)*c*e + c*d)*b + a)**p*a*d**6*p + 180*(log(x**(1/3)*c*e +
c*d)*b + a)**p*a*e**6*p*x**2 + 180*(log(x**(1/3)*c*e + c*d)*b + a)**p*a*e*
*6*x**2 + 60*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d**3*e**3*p**2*x + 60*(l
og(x**(1/3)*c*e + c*d)*b + a)**p*b*d**3*e**3*p*x - 360*int((log(x**(1/3)*c
*e + c*d)*b + a)**p/(6*x**(2/3)*log(x**(1/3)*c*e + c*d)*a*b*e + x**(2/3)*l
og(x**(1/3)*c*e + c*d)*b**2*e*p + 6*x**(2/3)*a**2*e + x**(2/3)*a*b*e*p + 6
*x**(1/3)*log(x**(1/3)*c*e + c*d)*a*b*d + x**(1/3)*log(x**(1/3)*c*e + c*d)
*b**2*d*p + 6*x**(1/3)*a**2*d + x**(1/3)*a*b*d*p),x)*a*b**2*d**5*e**2*p**3
 - 360*int((log(x**(1/3)*c*e + c*d)*b + a)**p/(6*x**(2/3)*log(x**(1/3)*c*e
 + c*d)*a*b*e + x**(2/3)*log(x**(1/3)*c*e + c*d)*b**2*e*p + 6*x**(2/3)*a**
2*e + x**(2/3)*a*b*e*p + 6*x**(1/3)*log(x**(1/3)*c*e + c*d)*a*b*d + x**(1/
3)*log(x**(1/3)*c*e + c*d)*b**2*d*p + 6*x**(1/3)*a**2*d + x**(1/3)*a*b*...
```

3.559 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 266

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^3}$$
$$-\frac{3\,2^{-p}de^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^2e^3}$$
$$+\frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{ce^3}$$

output

```
GAMMA(p+1,(-3*a-3*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3
^p)/c^3/e^3/exp(3*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(p+1,(-
2*a-2*b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(2^p)/c^2/e^3/
exp(2*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))))/b)^p)+3*d^2*GAMMA(p+1,-(a+b*ln(c*(
d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^3/exp(a/b)/((-(a+b*ln(c*
(d+e*x^(1/3))))/b)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.65

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

$$=\frac{6^{-p}e^{-\frac{3a}{b}}\left(2^p\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+2\right.\right.}{c^3e^3}$$

input

```
Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

output

```
((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 3^(1 + p)*c*d*
E^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^p*c*d*E^
(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]))*(a + b*Log[c*(d
+ e*x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))
])/b))^p)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2901, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{2901}$$

$$3\int x^{2/3}\left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int \left(\frac{\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{e^2}-\frac{2d\left(d+e\sqrt[3]{x}\right)\left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{e^2}+\frac{d^2\left(a+b\log \left(c\left(d+e\right.\right.\right.}{e^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^3e^3}-\frac{d2^{-p}e^{-}}{}\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

output
```
3*((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*
Log[c*(d + e*x^(1/3))])^p)/(c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(
1/3))])/b))^p) - (d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(
d + e*x^(1/3))])/b))^p) + (d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))
])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(1/3) + c*d) + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))))**p,x)
```

output
```
Integral((a + b*log(c*(d + e*x**(1/3))))**p, x)
```

Maxima [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int \left(b\log \left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int \left(b\log \left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int \left(a+b\,\ln \left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))))^p,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))))^p, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\text{too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))))^p,x)
```

output
```
(3*x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d*e**2*p**2 + 3*x**(2/3)*
(log(x**(1/3)*c*e + c*d)*b + a)**p*b*d*e**2*p - 6*x**(1/3)*(log(x**(1/3)*c
*e + c*d)*b + a)**p*b*d**2*e*p**2 - 6*x**(1/3)*(log(x**(1/3)*c*e + c*d)*b
+ a)**p*b*d**2*e*p + 6*(log(x**(1/3)*c*e + c*d)*b + a)**p*log(x**(1/3)*c*e
 + c*d)*b*d**3*p + 6*(log(x**(1/3)*c*e + c*d)*b + a)**p*a*d**3*p + 6*(log(
x**(1/3)*c*e + c*d)*b + a)**p*a*e**3*p*x + 6*(log(x**(1/3)*c*e + c*d)*b +
a)**p*a*e**3*x + 6*int((log(x**(1/3)*c*e + c*d)*b + a)**p/(3*x**(2/3)*log(
x**(1/3)*c*e + c*d)*a*b*e + x**(2/3)*log(x**(1/3)*c*e + c*d)*b**2*e*p + 3*
x**(2/3)*a**2*e + x**(2/3)*a*b*e*p + 3*x**(1/3)*log(x**(1/3)*c*e + c*d)*a*
b*d + x**(1/3)*log(x**(1/3)*c*e + c*d)*b**2*d*p + 3*x**(1/3)*a**2*d + x**(
1/3)*a*b*d*p),x)*a*b**2*d**2*e**2*p**3 + 6*int((log(x**(1/3)*c*e + c*d)*b
+ a)**p/(3*x**(2/3)*log(x**(1/3)*c*e + c*d)*a*b*e + x**(2/3)*log(x**(1/3)*
c*e + c*d)*b**2*e*p + 3*x**(2/3)*a**2*e + x**(2/3)*a*b*e*p + 3*x**(1/3)*lo
g(x**(1/3)*c*e + c*d)*a*b*d + x**(1/3)*log(x**(1/3)*c*e + c*d)*b**2*d*p +
3*x**(1/3)*a**2*d + x**(1/3)*a*b*d*p),x)*a*b**2*d**2*e**2*p**2 + 2*int((lo
g(x**(1/3)*c*e + c*d)*b + a)**p/(3*x**(2/3)*log(x**(1/3)*c*e + c*d)*a*b*e
+ x**(2/3)*log(x**(1/3)*c*e + c*d)*b**2*e*p + 3*x**(2/3)*a**2*e + x**(2/3)
*a*b*e*p + 3*x**(1/3)*log(x**(1/3)*c*e + c*d)*a*b*d + x**(1/3)*log(x**(1/3
)*c*e + c*d)*b**2*d*p + 3*x**(1/3)*a**2*d + x**(1/3)*a*b*d*p),x)*b**3*d**2
*e**2*p**4 + 2*int((log(x**(1/3)*c*e + c*d)*b + a)**p/(3*x**(2/3)*log(x...
```

3.560 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

$$\Big\downarrow \ 2908$$

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 2867$$

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\log(c(d+e\sqrt[3]{x})))^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(1/3) + c*d) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+e\sqrt[3]{x})))^p}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))))^p/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 5.91

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$
$$= \frac{3\left(\log\left(x^{\frac{1}{3}}ce+cd\right)b+a\right)^p\log\left(x^{\frac{1}{3}}ce+cd\right)b+3\left(\log\left(x^{\frac{1}{3}}ce+cd\right)b+a\right)^p a+\left(\int\frac{\left(\log\left(x^{\frac{1}{3}}ce+cd\right)b+a\right)^p}{x^{\frac{4}{3}}e+dx}dx\right) b}{b\,(p+1)}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))))^p/x,x)
```

output
```
(3*(log(x**(1/3)*c*e + c*d)*b + a)**p*log(x**(1/3)*c*e + c*d)*b + 3*(log(x
**(1/3)*c*e + c*d)*b + a)**p*a + int((log(x**(1/3)*c*e + c*d)*b + a)**p/(x
**(1/3)*e*x + d*x),x)*b*d*p + int((log(x**(1/3)*c*e + c*d)*b + a)**p/(x**(
1/3)*e*x + d*x),x)*b*d)/(b*(p + 1))
```

3.561 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}dx$$
$$\Big\downarrow \text{2908}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{2867}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\log(c(d+e\sqrt[3]{x})))^p}{x^2}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(1/3) + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+e\sqrt[3]{x})))^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))))^p/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 532, normalized size of antiderivative = 24.18

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x)
```

output
```
( - 6*x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*a*d**2 + 6*x**(1/3)*(log
(x**(1/3)*c*e + c*d)*b + a)**p*b*e**2*p*x - 3*(log(x**(1/3)*c*e + c*d)*b +
 a)**p*b*d*e*p*x - 2*x**(2/3)*int(((log(x**(1/3)*c*e + c*d)*b + a)**p*log(
x**(1/3)*c*e + c*d))/(x**(2/3)*log(x**(1/3)*c*e + c*d)*b*d*x + x**(2/3)*a*
d*x + log(x**(1/3)*c*e + c*d)*b*e*x**2 + a*e*x**2),x)*b**2*d**2*e*p*x + 2*
x**(2/3)*int((x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p*log(x**(1/3)*c*e
 + c*d))/(x**(2/3)*log(x**(1/3)*c*e + c*d)*b*d*x + x**(2/3)*a*d*x + log(x*
*(1/3)*c*e + c*d)*b*e*x**2 + a*e*x**2),x)*b**2*e**3*p*x + 2*x**(2/3)*int((
x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p)/(x**(2/3)*log(x**(1/3)*c*e +
c*d)*b*d*x + x**(2/3)*a*d*x + log(x**(1/3)*c*e + c*d)*b*e*x**2 + a*e*x**2)
,x)*a*b*e**3*p*x - 2*x**(2/3)*int((x**(2/3)*(log(x**(1/3)*c*e + c*d)*b + a
)**p)/(x**(2/3)*log(x**(1/3)*c*e + c*d)*b*d*x + x**(2/3)*a*d*x + log(x**(1
/3)*c*e + c*d)*b*e*x**2 + a*e*x**2),x)*b**2*e**3*p**2*x + x**(2/3)*int((x*
*(1/3)*(log(x**(1/3)*c*e + c*d)*b + a)**p)/(x**(2/3)*log(x**(1/3)*c*e + c*
d)*b*d*x + x**(2/3)*a*d*x + log(x**(1/3)*c*e + c*d)*b*e*x**2 + a*e*x**2),x
)*b**2*d*e**2*p**2*x)/(6*x**(2/3)*a*d**2*x)
```

3.562 $\int x^3 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^2\right)\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 1363

$$\int x^3 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^2\right)\right)^p \, dx = \text{Too large to display}$$

output
```
2^(-2-p)*GAMMA(p+1,(-6*a-6*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1
/3))^2))^p/(3^p)/c^6/e^12/exp(6*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)-
3*(2/11)^p*d*(d+e*x^(1/3))^11*GAMMA(p+1,1/2*(-11*a-11*b*ln(c*(d+e*x^(1/3))
^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(11/2*a/b)/(c*(d+e*x^(1/3))^
2)^(11/2)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)+33/2*d^2*GAMMA(p+1,(-5*a-5*
b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(5^p)/c^5/e^12/e
xp(5*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)-55*(2/9)^p*d^3*(d+e*x^(1/3)
)^9*GAMMA(p+1,1/2*(-9*a-9*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/
3))^2))^p/e^12/exp(9/2*a/b)/(c*(d+e*x^(1/3))^2)^(9/2)/((-(a+b*ln(c*(d+e*x^
(1/3))^2))/b)^p)+495*d^4*GAMMA(p+1,(-4*a-4*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+
b*ln(c*(d+e*x^(1/3))^2))^p/(2^(2*p+2))/c^4/e^12/exp(4*a/b)/((-(a+b*ln(c*(d
+e*x^(1/3))^2))/b)^p)-99*2^(p+1)*d^5*(d+e*x^(1/3))^7*GAMMA(p+1,1/2*(-7*a-7
*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(7^p)/e^12/exp(
7/2*a/b)/(c*(d+e*x^(1/3))^2)^(7/2)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)+77
*3^(1-p)*d^6*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*
x^(1/3))^2))^p/c^3/e^12/exp(3*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)-99
*2^(p+1)*d^7*(d+e*x^(1/3))^5*GAMMA(p+1,1/2*(-5*a-5*b*ln(c*(d+e*x^(1/3))^2)
)/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(5^p)/e^12/exp(5/2*a/b)/(c*(d+e*x^(1/3)
)^2)^(5/2)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)+495*2^(-2-p)*d^8*GAMMA(p+1
,(-2*a-2*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^2/...
```

Mathematica [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

output
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Rubi [A] (verified)

Time = 4.41 (sec) , antiderivative size = 1375, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{11/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int \left(\frac{\left(d+e\sqrt[3]{x}\right)^{11}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{e^{11}}-\frac{11d\left(d+e\sqrt[3]{x}\right)^{10}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{e^{11}}+\frac{55d^2\left(d+e\sqrt[3]{\right.}}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{2^{-p-2}3^{-p-1}e^{-\frac{6a}{b}}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)}{c^6e^{12}}\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

output
```
3*((2^(-2 - p)*3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))^2]
))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^6*e^12*E^((6*a)/b)*(-((a + b*
Log[c*(d + e*x^(1/3))^2])/b))^p) - ((2/11)^p*d*(d + e*x^(1/3))^11*Gamma[1
+ p, (-11*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(
1/3))^2])^p)/(e^12*E^((11*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(11/2)*(-((a + b
*Log[c*(d + e*x^(1/3))^2])/b))^p) + (11*d^2*Gamma[1 + p, (-5*(a + b*Log[c*
(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*5^p*c^5*e^1
2*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*2^p*3^(-1 -
 2*p)*d^3*(d + e*x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))^
2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^((9*a)/(2*b))*(c*(
d + e*x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (165*
d^4*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d
+ e*x^(1/3))^2])^p)/(2^(2*(1 + p))*c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))^2])/b))^p) - (33*2^(1 + p)*d^5*(d + e*x^(1/3))^7*Gamma[1 + p
, (-7*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(7^p*e^12*E^((7*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (77*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d
 + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*c^3*e^12*E^
((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (33*2^(1 + p)*d^7*(
d + e*x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^2\right)\right)^p dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Fricas [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)^2\right)\right)^p\,dx$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
```

output
```
int(x^3*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

Reduce [F]

$$\int x^3 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p dx = \text{too large to display}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x)
```

output
```
( - 41580*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a
)**p*b*d**10*e**2*p**2 - 41580*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*
c*d*e + c*d**2)*b + a)**p*b*d**10*e**2*p + 16632*x**(2/3)*(log(x**(2/3)*c*
e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**7*e**5*p**2*x + 16632*x**
(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**7*e
**5*p*x - 10395*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)
*b + a)**p*b*d**4*e**8*p**2*x**2 - 10395*x**(2/3)*(log(x**(2/3)*c*e**2 + 2
*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**4*e**8*p*x**2 + 7560*x**(2/3)*(lo
g(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d*e**11*p**2*x*
*3 + 7560*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a
)**p*b*d*e**11*p*x**3 + 83160*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c
*d*e + c*d**2)*b + a)**p*b*d**11*e*p**2 + 83160*x**(1/3)*(log(x**(2/3)*c*e
**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**11*e*p - 20790*x**(1/3)*(l
og(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**8*e**4*p**2
*x - 20790*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b +
a)**p*b*d**8*e**4*p*x + 11880*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c
*d*e + c*d**2)*b + a)**p*b*d**5*e**7*p**2*x**2 + 11880*x**(1/3)*(log(x**(2
/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**5*e**7*p*x**2 - 831
6*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d
**2*e**10*p**2*x**3 - 8316*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c...
```

3.563 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1035

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = \text{Too large to display}$$

output
```
2^p*3^(-1-2*p)*(d+e*x^(1/3))^9*GAMMA(p+1,1/2*(-9*a-9*b*ln(c*(d+e*x^(1/3))^
2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^9/exp(9/2*a/b)/(c*(d+e*x^(1/3))^2)^
(9/2)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*d*GAMMA(p+1,(-4*a-4*b*ln(c*(d
+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(4^p)/c^4/e^9/exp(4*a/b)/
((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)+3*2^(2+p)*d^2*(d+e*x^(1/3))^7*GAMMA(p
+1,1/2*(-7*a-7*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(
7^p)/e^9/exp(7/2*a/b)/(c*(d+e*x^(1/3))^2)^(7/2)/((-(a+b*ln(c*(d+e*x^(1/3))
^2))/b)^p)-28*d^3*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*
(d+e*x^(1/3))^2))^p/(3^p)/c^3/e^9/exp(3*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))^2)
)/b)^p)+21*2^(p+1)*d^4*(d+e*x^(1/3))^5*GAMMA(p+1,1/2*(-5*a-5*b*ln(c*(d+e*x
^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(5^p)/e^9/exp(5/2*a/b)/(c*(d+
e*x^(1/3))^2)^(5/2)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)-21*2^(1-p)*d^5*GA
MMA(p+1,(-2*a-2*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/
c^2/e^9/exp(2*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)+7*2^(2+p)*d^6*(d+e
*x^(1/3))^3*GAMMA(p+1,1/2*(-3*a-3*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d
+e*x^(1/3))^2))^p/(3^p)/e^9/exp(3/2*a/b)/(c*(d+e*x^(1/3))^2)^(3/2)/((-(a+b
*ln(c*(d+e*x^(1/3))^2))/b)^p)-12*d^7*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/3))^2)
)/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^9/exp(a/b)/((-(a+b*ln(c*(d+e*x^(1/3
))^2))/b)^p)+3*2^p*d^8*(d+e*x^(1/3))*GAMMA(p+1,-1/2*(a+b*ln(c*(d+e*x^(1/3)
)^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^9/exp(1/2*a/b)/(c*(d+e*x^(1/3)...
```

Mathematica [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

output
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Rubi [A] (verified)

Time = 3.35 (sec) , antiderivative size = 1039, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \log \left(c(d+e\sqrt[3]{x})^2\right)\right)^p dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{8/3} \left(a+b \log \left(c(d+e\sqrt[3]{x})^2\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int \left(\frac{(d+e\sqrt[3]{x})^8 \left(a+b \log \left(c(d+e\sqrt[3]{x})^2\right)\right)^p}{e^8}-\frac{8d(d+e\sqrt[3]{x})^7 \left(a+b \log \left(c(d+e\sqrt[3]{x})^2\right)\right)^p}{e^8}+\frac{28d^2(d+e\sqrt[3]{x})\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{2^p 9^{-p-1} e^{-\frac{9a}{2b}} (d+e\sqrt[3]{x})^9 \Gamma\left(p+1,-\frac{9\left(a+b \log \left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right) \left(a+b \log \left(c(d+e\sqrt[3]{x})^2\right)\right)^p \left(-\frac{a+b \log \left(c\left(d+e\sqrt[3]{}\right.\right.}{b}\right.}{e^9 \left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}}\right.$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

output
```
3*((2^p*9^(-1 - p)*(d + e*x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*
x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*
b))*(c*(d + e*x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
 - (d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(
d + e*x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1
/3))^2])/b))^p) + (2^(2 + p)*d^2*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^
p*e^9*E^((7*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x
^(1/3))^2])/b))^p) - (28*3^(-1 - p)*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d +
 e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^3*e^9*E^((3*a)/
b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (7*2^(1 + p)*d^4*(d + e*x^
(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*
Log[c*(d + e*x^(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2
)^(5/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (7*2^(1 - p)*d^5*Gamm
a[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1
/3))^2])^p)/(c^2*e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p
) + (7*2^(2 + p)*3^(-1 - p)*d^6*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*
Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9
*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3
))^2])/b))^p) - (4*d^7*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^2\right)\right)^p dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Fricas [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^2, x)
```

Giac [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x^2\left(a+b\ln\left(c\left(d+e\,x^{1/3}\right)^2\right)\right)^p\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

Reduce [F]

$$\int x^2 \left(a + b \log \left(c \left(d + e \sqrt[3]{x}\right)^2\right)\right)^p \, dx = \text{too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x)
```

output
```
(1260*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p
*b*d**7*e**2*p**2 + 1260*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e
+ c*d**2)*b + a)**p*b*d**7*e**2*p - 504*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*
x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**4*e**5*p**2*x - 504*x**(2/3)*(log(
x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**4*e**5*p*x + 3
15*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*
d*e**8*p**2*x**2 + 315*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e +
c*d**2)*b + a)**p*b*d*e**8*p*x**2 - 2520*x**(1/3)*(log(x**(2/3)*c*e**2 + 2
*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**8*e*p**2 - 2520*x**(1/3)*(log(x**
(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**8*e*p + 630*x**(1
/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**5*e**
4*p**2*x + 630*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*
b + a)**p*b*d**5*e**4*p*x - 360*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)
*c*d*e + c*d**2)*b + a)**p*b*d**2*e**7*p**2*x**2 - 360*x**(1/3)*(log(x**(2
/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**2*e**7*p*x**2 + 126
0*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*log(x**(2/3)
*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b*d**9*p + 1260*(log(x**(2/3)*c*e**2
+ 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*a*d**9*p + 1260*(log(x**(2/3)*c*e**
2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*a*e**9*p*x**3 + 1260*(log(x**(2/3
)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*a*e**9*x**3 - 840*(log(...
```

3.564 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 673

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\text{Too large to display}$$

output
```
1/2*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^
2))^p/(3^p)/c^3/e^6/exp(3*a/b)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*(2/5
)^p*d*(d+e*x^(1/3))^5*GAMMA(p+1,1/2*(-5*a-5*b*ln(c*(d+e*x^(1/3))^2))/b)*(a
+b*ln(c*(d+e*x^(1/3))^2))^p/e^6/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/((-
(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)+15*2^(-1-p)*d^2*GAMMA(p+1,(-2*a-2*b*ln(c
*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^2/e^6/exp(2*a/b)/((-
(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)-5*2^(p+1)*d^3*(d+e*x^(1/3))^3*GAMMA(p+1,
1/2*(-3*a-3*b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(3^p
)/e^6/exp(3/2*a/b)/(c*(d+e*x^(1/3))^2)^(3/2)/((-(a+b*ln(c*(d+e*x^(1/3))^2)
)/b)^p)+15/2*d^4*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*
x^(1/3))^2))^p/c/e^6/exp(a/b)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*2^p*d
^5*(d+e*x^(1/3))*GAMMA(p+1,-1/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(
d+e*x^(1/3))^2))^p/e^6/exp(1/2*a/b)/(c*(d+e*x^(1/3))^2)^(1/2)/((-(a+b*ln(c
*(d+e*x^(1/3))^2))/b)^p)
```

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

$$\Big\downarrow \text{2904}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int\left(\frac{\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{e^5}-\frac{5d\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{e^5}+\frac{10d^2\left(d+e\sqrt[3]{x}\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)}{2c^3e^6}+\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

output
```
3*((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a +
b*Log[c*(d + e*x^(1/3))^2])^p)/(2*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d +
 e*x^(1/3))^2])/b))^p) - ((2/5)^p*d*(d + e*x^(1/3))^5*Gamma[1 + p, (-5*(a
+ b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/
(e^6*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^
(1/3))^2])/b))^p) + (5*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e
*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)
*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (5*2^(1 + p)*3^(-1 - p)*d^3*
(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)
]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*x^(1/
3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (5*d^4*Gamma[1 +
 p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2]
)^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (2^p*d^
5*(d + e*x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/3))^2])/b]*(a
 + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^
2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x\,{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)}^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\text{too large to display}$$

input
```
int(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x)
```

output
```
( - 90*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**
p*b*d**4*e**2*p**2 - 90*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e +
 c*d**2)*b + a)**p*b*d**4*e**2*p + 36*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x*
*(1/3)*c*d*e + c*d**2)*b + a)**p*b*d*e**5*p**2*x + 36*x**(2/3)*(log(x**(2/
3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d*e**5*p*x + 180*x**(1/
3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**5*e*p*
*2 + 180*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)
**p*b*d**5*e*p - 45*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d
**2)*b + a)**p*b*d**2*e**4*p**2*x - 45*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x
**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**2*e**4*p*x - 90*(log(x**(2/3)*c*e**
2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*log(x**(2/3)*c*e**2 + 2*x**(1/3)*
c*d*e + c*d**2)*b*d**6*p - 90*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*
d**2)*b + a)**p*a*d**6*p + 90*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*
d**2)*b + a)**p*a*e**6*p*x**2 + 90*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e
 + c*d**2)*b + a)**p*a*e**6*x**2 + 60*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*
d*e + c*d**2)*b + a)**p*b*d**3*e**3*p**2*x + 60*(log(x**(2/3)*c*e**2 + 2*x
**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**3*e**3*p*x - 360*int((log(x**(2/3)*
c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p/(3*x**(2/3)*log(x**(2/3)*c*e
**2 + 2*x**(1/3)*c*d*e + c*d**2)*a*b*e + x**(2/3)*log(x**(2/3)*c*e**2 + 2*
x**(1/3)*c*d*e + c*d**2)*b**2*e*p + 3*x**(2/3)*a**2*e + x**(2/3)*a*b*e*...
```

3.565 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 338

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

$$= \frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

$$- \frac{3de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}}{ce^3}$$

$$+ \frac{3\,2^p d^2 e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}$$

output
```
(2/3)^p*(d+e*x^(1/3))^3*GAMMA(p+1,1/2*(-3*a-3*b*ln(c*(d+e*x^(1/3))^2))/b)*
(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^3/exp(3/2*a/b)/(c*(d+e*x^(1/3))^2)^(3/2)/(
(-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*d*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(1/3))
^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^3/exp(a/b)/((-(a+b*ln(c*(d+e*x^(
1/3))^2))/b)^p)+3*2^p*d^2*(d+e*x^(1/3))*GAMMA(p+1,-1/2*(a+b*ln(c*(d+e*x^(1
/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^3/exp(1/2*a/b)/(c*(d+e*x^(1/3)
)^2)^(1/2)/((-(a+b*ln(c*(d+e*x^(1/3))^2))/b)^p)
```

Mathematica [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2901, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

$$\Big\downarrow \text{2901}$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2848}$$

$$3\int \left(\frac{(d+e\sqrt[3]{x})^2 \left(a+b\log \left(c(d+e\sqrt[3]{x})^2\right)\right)^p}{e^2}-\frac{2d(d+e\sqrt[3]{x}) \left(a+b\log \left(c(d+e\sqrt[3]{x})^2\right)\right)^p}{e^2}+\frac{d^2\left(a+b\log \left(c(\right.\right.}{e^2}\right.$$

$\downarrow$ 2009

$$3\left(\frac{d^2 2^p e^{-\frac{a}{2b}}(d+e\sqrt[3]{x}) \left(a+b\log \left(c(d+e\sqrt[3]{x})^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right.}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

output
```
3*((2^p*3^(-1 - p)*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*
x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*
b))*(c*(d + e*x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
 - (d*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d
+ e*x^(1/3))^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))
^p) + (2^p*d^2*(d + e*x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/
3))^2])/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d
+ e*x^(1/3))^2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int \left(b\log \left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int \left(b\log \left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int {\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)}^p\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^2))^p,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\text{too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^2))^p,x)
```

output
```
(3*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*b*
d*e**2*p**2 + 3*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)
*b + a)**p*b*d*e**2*p - 6*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e
 + c*d**2)*b + a)**p*b*d**2*e*p**2 - 6*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x
**(1/3)*c*d*e + c*d**2)*b + a)**p*b*d**2*e*p + 3*(log(x**(2/3)*c*e**2 + 2*
x**(1/3)*c*d*e + c*d**2)*b + a)**p*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e
+ c*d**2)*b*d**3*p + 3*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b
 + a)**p*a*d**3*p + 3*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b
+ a)**p*a*e**3*p*x + 3*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b
 + a)**p*a*e**3*x + 12*int((log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**
2)*b + a)**p/(3*x**(2/3)*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*
a*b*e + 2*x**(2/3)*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b**2*e
*p + 3*x**(2/3)*a**2*e + 2*x**(2/3)*a*b*e*p + 3*x**(1/3)*log(x**(2/3)*c*e*
*2 + 2*x**(1/3)*c*d*e + c*d**2)*a*b*d + 2*x**(1/3)*log(x**(2/3)*c*e**2 + 2
*x**(1/3)*c*d*e + c*d**2)*b**2*d*p + 3*x**(1/3)*a**2*d + 2*x**(1/3)*a*b*d*
p),x)*a*b**2*d**2*e**2*p**3 + 12*int((log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d
*e + c*d**2)*b + a)**p/(3*x**(2/3)*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e
+ c*d**2)*a*b*e + 2*x**(2/3)*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d*
*2)*b**2*e*p + 3*x**(2/3)*a**2*e + 2*x**(2/3)*a*b*e*p + 3*x**(1/3)*log(x**
(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*a*b*d + 2*x**(1/3)*log(x**(2/...
```

3.566 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}dx$$

$$\Big\downarrow \text{ 2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2867}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^2))^p/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^2))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 8.04

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$
$$=\frac{3\left(\log\left(x^{\frac{2}{3}}c\,e^2+2x^{\frac{1}{3}}cde+c\,d^2\right)b+a\right)^p\log\left(x^{\frac{2}{3}}c\,e^2+2x^{\frac{1}{3}}cde+c\,d^2\right)b+3\left(\log\left(x^{\frac{2}{3}}c\,e^2+2x^{\frac{1}{3}}cde+c\,d^2\right)\right.}{2b\,(p+1)}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x)
```

output
```
(3*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p*log(x**(2/3
)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + 3*(log(x**(2/3)*c*e**2 + 2*x**(1
/3)*c*d*e + c*d**2)*b + a)**p*a + 2*int((log(x**(2/3)*c*e**2 + 2*x**(1/3)*
c*d*e + c*d**2)*b + a)**p/(x**(1/3)*e*x + d*x),x)*b*d*p + 2*int((log(x**(2
/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p/(x**(1/3)*e*x + d*x),x)*
b*d)/(2*b*(p + 1))
```

3.567 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2867}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}dx$$
$$\Big\downarrow \text{2908}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{2867}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2867
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(AFx_), x_Sy
mbol] :> Unintegrable[AFx*(a + b*Log[c*(d + e*x)^n])^p, x] /; FreeQ[{a, b,
c, d, e, n, p}, x] && AlgebraicFunctionQ[AFx, x, True]
```

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(1/3))^2))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(1/3))^2))^p/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 773, normalized size of antiderivative = 32.21

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

output

```
( - 3*x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p
*a*d**2 + 6*x**(1/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b +
 a)**p*b*e**2*p*x - 3*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b
+ a)**p*b*d*e*p*x - 2*x**(2/3)*int(((log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*
e + c*d**2)*b + a)**p*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2))/(x
**(2/3)*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b*d*x + x**(2/3)*
a*d*x + log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b*e*x**2 + a*e*x*
*2),x)*b**2*d**2*e*p*x + 2*x**(2/3)*int((x**(2/3)*(log(x**(2/3)*c*e**2 + 2
*x**(1/3)*c*d*e + c*d**2)*b + a)**p*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e
 + c*d**2))/(x**(2/3)*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b*d
*x + x**(2/3)*a*d*x + log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b*e
*x**2 + a*e*x**2),x)*b**2*e**3*p*x + 2*x**(2/3)*int((x**(2/3)*(log(x**(2/3
)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p)/(x**(2/3)*log(x**(2/3)*c*
e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b*d*x + x**(2/3)*a*d*x + log(x**(2/3)*c*
e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b*e*x**2 + a*e*x**2),x)*a*b*e**3*p*x - 4
*x**(2/3)*int((x**(2/3)*(log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*
b + a)**p)/(x**(2/3)*log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b*d*
x + x**(2/3)*a*d*x + log(x**(2/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b*e*
x**2 + a*e*x**2),x)*b**2*e**3*p**2*x + 2*x**(2/3)*int((x**(1/3)*(log(x**(2
/3)*c*e**2 + 2*x**(1/3)*c*d*e + c*d**2)*b + a)**p)/(x**(2/3)*log(x**(2/...
```

3.568 $\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 557

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx = \frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+\cdots}{\cdots}\right.}{c^6e^6}$$
$$-\frac{3\,5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2c^5e^6}$$
$$+\frac{15\,2^{-2(1+p)}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^4e^6}$$
$$-\frac{5\,3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^3e^6}$$
$$+\frac{15\,2^{-2-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^2e^6}$$
$$-\frac{3d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2ce^6}$$

output
```
2^(-2-p)*GAMMA(p+1,(-6*a-6*b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3
))))^p/(3^p)/c^6/e^6/exp(6*a/b)/((-(a+b*ln(c*(d+e*x^(2/3))))/b)^p)-3/2*d*G
AMMA(p+1,(-5*a-5*b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(5^
p)/c^5/e^6/exp(5*a/b)/((-(a+b*ln(c*(d+e*x^(2/3))))/b)^p)+15*d^2*GAMMA(p+1,
(-4*a-4*b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(2^(2*p+2))/
c^4/e^6/exp(4*a/b)/((-(a+b*ln(c*(d+e*x^(2/3))))/b)^p)-5*d^3*GAMMA(p+1,(-3*
a-3*b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(3^p)/c^3/e^6/ex
p(3*a/b)/((-(a+b*ln(c*(d+e*x^(2/3))))/b)^p)+15*2^(-2-p)*d^4*GAMMA(p+1,(-2*
a-2*b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c^2/e^6/exp(2*a/
b)/((-(a+b*ln(c*(d+e*x^(2/3))))/b)^p)-3/2*d^5*GAMMA(p+1,-(a+b*ln(c*(d+e*x^
(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c/e^6/exp(a/b)/((-(a+b*ln(c*(d+e*x
^(2/3))))/b)^p)
```

Mathematica [F]

$$\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

output
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int x^{10/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p dx^{2/3}$$

$\Big\downarrow$ 2848

$$\frac{3}{2}\int\left(\frac{\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c(d+ex^{2/3})\right)\right)^p}{e^5}-\frac{5d\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c(d+ex^{2/3})\right)\right)^p}{e^5}+\frac{10d^2\left(d+ex^{2/3}\right)\right.$$

$\Big\downarrow$ 2009

$$\frac{3}{2}\left(\frac{6^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c(d+ex^{2/3})\right)\right)^p\left(-\frac{a+b\log(c(d+ex^{2/3}))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6(a+b\log(c(d+ex^{2/3})))}{b}\right)}{c^6e^6}-\frac{d5^{-p}e^{-\frac{5a}{b}}}{\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

output
```
(3*((6^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b
*Log[c*(d + e*x^(2/3))])^p)/(c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^
(2/3))])/b))^p) - (d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(
a + b*Log[c*(d + e*x^(2/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*
(d + e*x^(2/3))])/b))^p) + (5*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log
[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) - (10*3^(-1 - p)*d^3*Gamma[
1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))
])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) + (5*2
^(-1 - p)*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*L
og[c*(d + e*x^(2/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(2
/3))])/b))^p) - (d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])/b)]*(a
+ b*Log[c*(d + e*x^(2/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/
3))])/b))^p)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3 \left(a + b \ln \left(c \left(d + e\, x^{\frac{2}{3}}\right)\right)\right)^p dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Fricas [F]

$$\int x^3 (a + b \log (c(d + ex^{2/3})))^p \, dx = \int \left(b \log \left(\left(ex^{\frac{2}{3}} + d\right)c\right) + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p \, dx = \int x^3 \, \left(a+b\, \ln \left(c\, \left(d+e\, x^{2/3}\right)\right)\right)^p \, dx$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(2/3))))^p,x)
```

output
```
int(x^3*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

Reduce [F]

$$\int x^3 \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p \, dx = \text{too large to display}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x)
```

output
```
(180*x**(2/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p*b*d**5*e*p**2 + 180*x**(2
/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p*b*d**5*e*p - 45*x**(2/3)*(log(x**(2
/3)*c*e + c*d)*b + a)**p*b*d**2*e**4*p**2*x**2 - 45*x**(2/3)*(log(x**(2/3)
*c*e + c*d)*b + a)**p*b*d**2*e**4*p*x**2 - 90*x**(1/3)*(log(x**(2/3)*c*e +
 c*d)*b + a)**p*b*d**4*e**2*p**2*x - 90*x**(1/3)*(log(x**(2/3)*c*e + c*d)*
b + a)**p*b*d**4*e**2*p*x + 36*x**(1/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p
*b*d*e**5*p**2*x**3 + 36*x**(1/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p*b*d*e
**5*p*x**3 - 180*(log(x**(2/3)*c*e + c*d)*b + a)**p*log(x**(2/3)*c*e + c*d
)*b*d**6*p - 180*(log(x**(2/3)*c*e + c*d)*b + a)**p*a*d**6*p + 180*(log(x*
*(2/3)*c*e + c*d)*b + a)**p*a*e**6*p*x**4 + 180*(log(x**(2/3)*c*e + c*d)*b
 + a)**p*a*e**6*x**4 + 60*(log(x**(2/3)*c*e + c*d)*b + a)**p*b*d**3*e**3*p
**2*x**2 + 60*(log(x**(2/3)*c*e + c*d)*b + a)**p*b*d**3*e**3*p*x**2 - 144*
int(((log(x**(2/3)*c*e + c*d)*b + a)**p*x**3)/(6*x**(2/3)*log(x**(2/3)*c*e
 + c*d)*a*b*e + x**(2/3)*log(x**(2/3)*c*e + c*d)*b**2*e*p + 6*x**(2/3)*a**
2*e + x**(2/3)*a*b*e*p + 6*log(x**(2/3)*c*e + c*d)*a*b*d + log(x**(2/3)*c*
e + c*d)*b**2*d*p + 6*a**2*d + a*b*d*p),x)*a*b**2*d*e**6*p**3 - 144*int(((
log(x**(2/3)*c*e + c*d)*b + a)**p*x**3)/(6*x**(2/3)*log(x**(2/3)*c*e + c*d
)*a*b*e + x**(2/3)*log(x**(2/3)*c*e + c*d)*b**2*e*p + 6*x**(2/3)*a**2*e +
x**(2/3)*a*b*e*p + 6*log(x**(2/3)*c*e + c*d)*a*b*d + log(x**(2/3)*c*e + c*
d)*b**2*d*p + 6*a**2*d + a*b*d*p),x)*a*b**2*d*e**6*p**2 - 24*int(((log(...
```

3.569 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 273

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log(c}{}\right.}{2c^3e^3}$$
$$-\frac{3\,2^{-1-p}de^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^2e^3}$$
$$+\frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2ce^3}$$

output
```
1/2*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^
p/(3^p)/c^3/e^3/exp(3*a/b)/((-(a+b*ln(c*(d+e*x^(2/3))))/b)^p)-3*2^(-1-p)*d
*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c
^2/e^3/exp(2*a/b)/((-(a+b*ln(c*(d+e*x^(2/3))))/b)^p)+3/2*d^2*GAMMA(p+1,-(a
+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c/e^3/exp(a/b)/((-(
a+b*ln(c*(d+e*x^(2/3))))/b)^p)
```

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \ 2904$$

$$\frac{3}{2}\int x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx^{2/3}$$

$$\Big\downarrow \ 2848$$

$$\frac{3}{2}\int\left(\frac{\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{e^2}-\frac{2d\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{e^2}+\frac{d^2\left(a+b\log\left(c\left(d\right.\right.\right.}{e^2}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{3}{2}\left(\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^3e^3}-\frac{d2^{-p}e^{-\frac{2a}{b}}}{}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

output
```
(3*((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b
*Log[c*(d + e*x^(2/3))])^p)/(c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^
(2/3))])/b))^p) - (d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(
a + b*Log[c*(d + e*x^(2/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*
(d + e*x^(2/3))])/b))^p) + (d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3)
)])/b)]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(
d + e*x^(2/3))])/b))^p)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e*x^(2/3))))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\text{too large to display}$$

input
```
int(x*(a+b*log(c*(d+e*x^(2/3))))^p,x)
```

output

```
( - 6*x**(2/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p*b*d**2*e*p**2 - 6*x**(2/
3)*(log(x**(2/3)*c*e + c*d)*b + a)**p*b*d**2*e*p + 3*x**(1/3)*(log(x**(2/3
)*c*e + c*d)*b + a)**p*b*d*e**2*p**2*x + 3*x**(1/3)*(log(x**(2/3)*c*e + c*
d)*b + a)**p*b*d*e**2*p*x + 6*(log(x**(2/3)*c*e + c*d)*b + a)**p*log(x**(2
/3)*c*e + c*d)*b*d**3*p + 6*(log(x**(2/3)*c*e + c*d)*b + a)**p*a*d**3*p +
6*(log(x**(2/3)*c*e + c*d)*b + a)**p*a*e**3*p*x**2 + 6*(log(x**(2/3)*c*e +
 c*d)*b + a)**p*a*e**3*x**2 - 6*int(((log(x**(2/3)*c*e + c*d)*b + a)**p*x)
/(3*x**(2/3)*log(x**(2/3)*c*e + c*d)*a*b*e + x**(2/3)*log(x**(2/3)*c*e + c
*d)*b**2*e*p + 3*x**(2/3)*a**2*e + x**(2/3)*a*b*e*p + 3*log(x**(2/3)*c*e +
 c*d)*a*b*d + log(x**(2/3)*c*e + c*d)*b**2*d*p + 3*a**2*d + a*b*d*p),x)*a*
b**2*d*e**3*p**3 - 6*int(((log(x**(2/3)*c*e + c*d)*b + a)**p*x)/(3*x**(2/3
)*log(x**(2/3)*c*e + c*d)*a*b*e + x**(2/3)*log(x**(2/3)*c*e + c*d)*b**2*e*
p + 3*x**(2/3)*a**2*e + x**(2/3)*a*b*e*p + 3*log(x**(2/3)*c*e + c*d)*a*b*d
 + log(x**(2/3)*c*e + c*d)*b**2*d*p + 3*a**2*d + a*b*d*p),x)*a*b**2*d*e**3
*p**2 - 2*int(((log(x**(2/3)*c*e + c*d)*b + a)**p*x)/(3*x**(2/3)*log(x**(2
/3)*c*e + c*d)*a*b*e + x**(2/3)*log(x**(2/3)*c*e + c*d)*b**2*e*p + 3*x**(2
/3)*a**2*e + x**(2/3)*a*b*e*p + 3*log(x**(2/3)*c*e + c*d)*a*b*d + log(x**(
2/3)*c*e + c*d)*b**2*d*p + 3*a**2*d + a*b*d*p),x)*b**3*d*e**3*p**4 - 2*int
(((log(x**(2/3)*c*e + c*d)*b + a)**p*x)/(3*x**(2/3)*log(x**(2/3)*c*e + c*d
)*a*b*e + x**(2/3)*log(x**(2/3)*c*e + c*d)*b**2*e*p + 3*x**(2/3)*a**2*e...
```

3.570 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}dx$$
$$\Big\downarrow \text{2908}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{2910}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^{p}}{x}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^{p}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))))^p/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 6.05

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\frac{3\left(\log\left(x^{\frac{2}{3}}ce+cd\right)b+a\right)^p\log\left(x^{\frac{2}{3}}ce+cd\right)b+3\left(\log\left(x^{\frac{2}{3}}ce+cd\right)b+a\right.}{2b\,(p}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))))^p/x,x)
```

output
```
(3*(log(x**(2/3)*c*e + c*d)*b + a)**p*log(x**(2/3)*c*e + c*d)*b + 3*(log(x
**(2/3)*c*e + c*d)*b + a)**p*a + 2*int((log(x**(2/3)*c*e + c*d)*b + a)**p/
(x**(2/3)*e*x + d*x),x)*b*d*p + 2*int((log(x**(2/3)*c*e + c*d)*b + a)**p/(
x**(2/3)*e*x + d*x),x)*b*d)/(2*b*(p + 1))
```

3.571 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$
$$\Big\downarrow \text{ 2908}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^{7/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{ 2910}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^{7/3}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^p}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}}{x^{3}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^p}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))))^p/x^3,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))))^p/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 533, normalized size of antiderivative = 24.23

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x)
```

output
```
(6*x**(2/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p*b*e**2*p*x - 6*x**(1/3)*(lo
g(x**(2/3)*c*e + c*d)*b + a)**p*a*d**2 - 3*(log(x**(2/3)*c*e + c*d)*b + a)
**p*b*d*e*p*x - 4*x**(1/3)*int(((log(x**(2/3)*c*e + c*d)*b + a)**p*log(x**
(2/3)*c*e + c*d))/(x**(1/3)*log(x**(2/3)*c*e + c*d)*b*d*x**2 + x**(1/3)*a*
d*x**2 + log(x**(2/3)*c*e + c*d)*b*e*x**3 + a*e*x**3),x)*b**2*d**2*e*p*x**
2 + 2*x**(1/3)*int((x**(2/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p)/(x**(1/3)
*log(x**(2/3)*c*e + c*d)*b*d*x**2 + x**(1/3)*a*d*x**2 + log(x**(2/3)*c*e +
 c*d)*b*e*x**3 + a*e*x**3),x)*b**2*d*e**2*p**2*x**2 + 4*x**(1/3)*int((x**(
1/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p*log(x**(2/3)*c*e + c*d))/(x**(1/3)
*log(x**(2/3)*c*e + c*d)*b*d*x + x**(1/3)*a*d*x + log(x**(2/3)*c*e + c*d)*
b*e*x**2 + a*e*x**2),x)*b**2*e**3*p*x**2 + 4*x**(1/3)*int((x**(1/3)*(log(x
**(2/3)*c*e + c*d)*b + a)**p)/(x**(1/3)*log(x**(2/3)*c*e + c*d)*b*d*x + x*
*(1/3)*a*d*x + log(x**(2/3)*c*e + c*d)*b*e*x**2 + a*e*x**2),x)*a*b*e**3*p*
x**2 - 4*x**(1/3)*int((x**(1/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p)/(x**(1
/3)*log(x**(2/3)*c*e + c*d)*b*d*x + x**(1/3)*a*d*x + log(x**(2/3)*c*e + c*
d)*b*e*x**2 + a*e*x**2),x)*b**2*e**3*p**2*x**2)/(12*x**(1/3)*a*d**2*x**2)
```

3.572 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\text{Int}\left(x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p,x\right)$$

output
```
Defer(Int)(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

output
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{2908}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,d\sqrt[3]{x}$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^2 \left(a + b \ln \left(c \left(d + e\, x^{\frac{2}{3}}\right)\right)\right)^p dx$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int x^2 (a + b\log (c(d + ex^{2/3})))^p \, dx = \int \left(b\log \left(\left(ex^{\frac{2}{3}} + d\right)c\right) + a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (a + b\log (c(d + ex^{2/3})))^p \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))))^p,x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 1475, normalized size of antiderivative = 67.05

$$\int x^2 \left(a + b \log \left(c \left(d + e x^{2/3}\right)\right)\right)^p \, dx = \text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x)
```

output
```
(105*(log(x**(2/3)*c*e + c*d)*b + a)**p*e**3*x**3 + 1260*int((log(x**(2/3)
*c*e + c*d)*b + a)**p/(9*x**(2/3)*log(x**(2/3)*c*e + c*d)*a*b*e + 2*x**(2/
3)*log(x**(2/3)*c*e + c*d)*b**2*e*p + 9*x**(2/3)*a**2*e + 2*x**(2/3)*a*b*e
*p + 9*log(x**(2/3)*c*e + c*d)*a*b*d + 2*log(x**(2/3)*c*e + c*d)*b**2*d*p
+ 9*a**2*d + 2*a*b*d*p),x)*b**2*d**4*p**2 - 630*int(((log(x**(2/3)*c*e + c
*d)*b + a)**p*x**3)/(9*x**(1/3)*log(x**(2/3)*c*e + c*d)*a*b*d + 2*x**(1/3)
*log(x**(2/3)*c*e + c*d)*b**2*d*p + 9*x**(1/3)*a**2*d + 2*x**(1/3)*a*b*d*p
 + 9*log(x**(2/3)*c*e + c*d)*a*b*e*x + 2*log(x**(2/3)*c*e + c*d)*b**2*e*p*
x + 9*a**2*e*x + 2*a*b*e*p*x),x)*a*b*e**4*p - 140*int(((log(x**(2/3)*c*e +
 c*d)*b + a)**p*x**3)/(9*x**(1/3)*log(x**(2/3)*c*e + c*d)*a*b*d + 2*x**(1/
3)*log(x**(2/3)*c*e + c*d)*b**2*d*p + 9*x**(1/3)*a**2*d + 2*x**(1/3)*a*b*d
*p + 9*log(x**(2/3)*c*e + c*d)*a*b*e*x + 2*log(x**(2/3)*c*e + c*d)*b**2*e*
p*x + 9*a**2*e*x + 2*a*b*e*p*x),x)*b**2*e**4*p**2 - 252*int(((log(x**(2/3)
*c*e + c*d)*b + a)**p*x**2)/(9*x**(2/3)*log(x**(2/3)*c*e + c*d)*a*b*d + 2*
x**(2/3)*log(x**(2/3)*c*e + c*d)*b**2*d*p + 9*x**(2/3)*a**2*d + 2*x**(2/3)
*a*b*d*p + 9*x**(1/3)*log(x**(2/3)*c*e + c*d)*a*b*e*x + 2*x**(1/3)*log(x**
(2/3)*c*e + c*d)*b**2*e*p*x + 9*x**(1/3)*a**2*e*x + 2*x**(1/3)*a*b*e*p*x),
x)*b**2*d**2*e**2*p**2 - 180*int(((log(x**(2/3)*c*e + c*d)*b + a)**p*x**2)
/(9*x**(2/3)*log(x**(2/3)*c*e + c*d)*a*b*e + 2*x**(2/3)*log(x**(2/3)*c*e +
 c*d)*b**2*e*p + 9*x**(2/3)*a**2*e + 2*x**(2/3)*a*b*e*p + 9*log(x**(2/3...
```

3.573 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\text{Int}\left(\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p,x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2901, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$
$$\Big\downarrow \text{2901}$$
$$3\int x^{2/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,d\sqrt[3]{x}$$
$$\Big\downarrow \text{2910}$$
$$3\int x^{2/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(a+b\ln \left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^{p}\,dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(2/3) + c*d) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 25.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int {\left(a+b\,\ln \left(c\,\left(d+e\,x^{2/3}\right)\right)\right)}^p\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))))^p,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 578, normalized size of antiderivative = 32.11

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))))^p,x)
```

output
```
(3*(log(x**(2/3)*c*e + c*d)*b + a)**p*x - 12*int((log(x**(2/3)*c*e + c*d)*
b + a)**p/(3*x**(2/3)*log(x**(2/3)*c*e + c*d)*a*b*e + 2*x**(2/3)*log(x**(2
/3)*c*e + c*d)*b**2*e*p + 3*x**(2/3)*a**2*e + 2*x**(2/3)*a*b*e*p + 3*log(x
**(2/3)*c*e + c*d)*a*b*d + 2*log(x**(2/3)*c*e + c*d)*b**2*d*p + 3*a**2*d +
 2*a*b*d*p),x)*b**2*d*p**2 - 6*int(((log(x**(2/3)*c*e + c*d)*b + a)**p*x)/
(3*x**(1/3)*log(x**(2/3)*c*e + c*d)*a*b*d + 2*x**(1/3)*log(x**(2/3)*c*e +
c*d)*b**2*d*p + 3*x**(1/3)*a**2*d + 2*x**(1/3)*a*b*d*p + 3*log(x**(2/3)*c*
e + c*d)*a*b*e*x + 2*log(x**(2/3)*c*e + c*d)*b**2*e*p*x + 3*a**2*e*x + 2*a
*b*e*p*x),x)*a*b*e*p - 4*int(((log(x**(2/3)*c*e + c*d)*b + a)**p*x)/(3*x**
(1/3)*log(x**(2/3)*c*e + c*d)*a*b*d + 2*x**(1/3)*log(x**(2/3)*c*e + c*d)*b
**2*d*p + 3*x**(1/3)*a**2*d + 2*x**(1/3)*a*b*d*p + 3*log(x**(2/3)*c*e + c*
d)*a*b*e*x + 2*log(x**(2/3)*c*e + c*d)*b**2*e*p*x + 3*a**2*e*x + 2*a*b*e*p
*x),x)*b**2*e*p**2 + 12*int((x**(1/3)*(log(x**(2/3)*c*e + c*d)*b + a)**p)/
(3*x**(1/3)*log(x**(2/3)*c*e + c*d)*a*b*d + 2*x**(1/3)*log(x**(2/3)*c*e +
c*d)*b**2*d*p + 3*x**(1/3)*a**2*d + 2*x**(1/3)*a*b*d*p + 3*log(x**(2/3)*c*
e + c*d)*a*b*e*x + 2*log(x**(2/3)*c*e + c*d)*b**2*e*p*x + 3*a**2*e*x + 2*a
*b*e*p*x),x)*b**2*d*p**2)/3
```

3.574 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^{p}}{x^{2}}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^{p}}{x^{2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))))^p/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.55

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx = \frac{-3\left(\log\left(x^{\frac{2}{3}}ce+cd\right)b+a\right)^p+2\left(\int\frac{\left(\log\left(x^{\frac{2}{3}}ce+cd\right)b+a\right)^p}{x^{\frac{4}{3}}\log\left(x^{\frac{2}{3}}ce+cd\right)bd+x^{\frac{4}{3}}ad+\log\left(x^{\frac{2}{3}}ce+cd\right)b\cdots}\cdots}{3x}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x)
```

output
```
( - 3*(log(x**(2/3)*c*e + c*d)*b + a)**p + 2*int((log(x**(2/3)*c*e + c*d)*
b + a)**p/(x**(1/3)*log(x**(2/3)*c*e + c*d)*b*d*x + x**(1/3)*a*d*x + log(x
**(2/3)*c*e + c*d)*b*e*x**2 + a*e*x**2),x)*b*e*p*x)/(3*x)
```

3.575 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 678

$$\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx =$$

$$-\frac{3\ 2^{-1+p}d^5e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right.\right.}{b}\right.}{e^6\sqrt{c\left(d+ex^{2/3}\right)^2}}$$

$$+\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{4c^3e^6}$$

$$-\frac{3\ 2^{-1+p}5^{-p}de^{-\frac{5a}{2b}}\left(d+ex^{2/3}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(\right.}{}\right.}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{5/2}}$$

$$+\frac{15\ 2^{-2-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{c^2e^6}$$

$$-\frac{5\left(\frac{2}{3}\right)^p d^3e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}$$

$$+\frac{15d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{4ce^6}$$

output
```
-3*2^(-1+p)*d^5*(d+e*x^(2/3))*GAMMA(p+1,1/2*(-a-b*ln(c*(d+e*x^(2/3))^2))/b
)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/e^6/exp(1/2*a/b)/(c*(d+e*x^(2/3))^2)^(1/2)
/((-(a+b*ln(c*(d+e*x^(2/3))^2))/b)^p)+1/4*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e*x^
(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/(3^p)/c^3/e^6/exp(3*a/b)/((-(a
+b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3*2^(-1+p)*d*(d+e*x^(2/3))^5*GAMMA(p+1,1/2
*(-5*a-5*b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/(5^p)/e
^6/exp(5/2*a/b)/(c*(d+e*x^(2/3))^2)^(5/2)/((-(a+b*ln(c*(d+e*x^(2/3))^2))/b
)^p)+15*2^(-2-p)*d^2*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln
(c*(d+e*x^(2/3))^2))^p/c^2/e^6/exp(2*a/b)/((-(a+b*ln(c*(d+e*x^(2/3))^2))/b
)^p)-5*(2/3)^p*d^3*(d+e*x^(2/3))^3*GAMMA(p+1,1/2*(-3*a-3*b*ln(c*(d+e*x^(2/
3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/e^6/exp(3/2*a/b)/(c*(d+e*x^(2/3))
^2)^(3/2)/((-(a+b*ln(c*(d+e*x^(2/3))^2))/b)^p)+15/4*d^4*GAMMA(p+1,-(a+b*ln
(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c/e^6/exp(a/b)/((-(a
+b*ln(c*(d+e*x^(2/3))^2))/b)^p)
```

Mathematica [F]

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

output
```
Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

$$\Big\downarrow \; 2904$$

$$\frac{3}{2}\int x^{10/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx^{2/3}$$

$$\Big\downarrow \; 2848$$

$$\frac{3}{2}\int\left(\frac{\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{e^5}-\frac{5d\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{e^5}+\frac{10d^2\left(d+ex\right.}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{3}{2}\left(\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)}{2c^3e^6}+\frac{5d}{}\right.$$

input
```
Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

output
```
(3*((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/b]*(a +
 b*Log[c*(d + e*x^(2/3))^2])^p)/(2*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d
+ e*x^(2/3))^2])/b))^p) - ((2/5)^p*d*(d + e*x^(2/3))^5*Gamma[1 + p, (-5*(a
 + b*Log[c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)
/(e^6*E^((5*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x
^(2/3))^2])/b))^p) + (5*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d +
e*x^(2/3))^2]))/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(c^2*e^6*E^((2*a)/b
)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (5*2^(1 + p)*3^(-1 - p)*d^3
*(d + e*x^(2/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/(2*b
)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*x^(2
/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) + (5*d^4*Gamma[1
+ p, -((a + b*Log[c*(d + e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x^(2/3))^2
])^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (2^p*d
^5*(d + e*x^(2/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(2/3))^2])/b]*(
a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(2/3))
^2]*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x^3 \left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^2\right)\right)^p dx$$

input
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Fricas [F]

$$\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^px^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log\left(c\left(d + ex^{2/3}\right)^2\right)\right)^p \, dx = \int x^3 \left(a + b \ln\left(c\left(d + e\,x^{2/3}\right)^2\right)\right)^p \, dx$$

input
```
int(x^3*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
```

output
```
int(x^3*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

Reduce [F]

$$\int x^3 \left(a + b \log\left(c\left(d + ex^{2/3}\right)^2\right)\right)^p \, dx = \text{too large to display}$$

input
```
int(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x)
```

output
```
(180*x**(2/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**
p*b*d**5*e*p**2 + 180*x**(2/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x +
 c*d**2)*b + a)**p*b*d**5*e*p - 45*x**(2/3)*(log(2*x**(2/3)*c*d*e + x**(1/
3)*c*e**2*x + c*d**2)*b + a)**p*b*d**2*e**4*p**2*x**2 - 45*x**(2/3)*(log(2
*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*b*d**2*e**4*p*x**2
 - 90*x**(1/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)*
*p*b*d**4*e**2*p**2*x - 90*x**(1/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**
2*x + c*d**2)*b + a)**p*b*d**4*e**2*p*x + 36*x**(1/3)*(log(2*x**(2/3)*c*d*
e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*b*d*e**5*p**2*x**3 + 36*x**(1/3)
*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*b*d*e**5*p*
x**3 - 90*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*lo
g(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b*d**6*p - 90*(log(2*x**(
2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*a*d**6*p + 90*(log(2*x*
*(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*a*e**6*p*x**4 + 90*(l
og(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*a*e**6*x**4 +
60*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*b*d**3*e*
*3*p**2*x**2 + 60*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b +
a)**p*b*d**3*e**3*p*x**2 - 144*int(((log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**
2*x + c*d**2)*b + a)**p*x**3)/(3*x**(2/3)*log(2*x**(2/3)*c*d*e + x**(1/3)*
c*e**2*x + c*d**2)*a*b*e + x**(2/3)*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e...
```

3.576 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 350

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\frac{3\,2^{-1+p}d^2e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\right.\right.\right.}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}$$

$$+\frac{2^{-1+p}3^{-p}e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{e^3\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}$$

$$-\frac{3de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{2ce^3}$$

output
```
3*2^(-1+p)*d^2*(d+e*x^(2/3))*GAMMA(p+1,1/2*(-a-b*ln(c*(d+e*x^(2/3))^2))/b)
*(a+b*ln(c*(d+e*x^(2/3))^2))^p/e^3/exp(1/2*a/b)/(c*(d+e*x^(2/3))^2)^(1/2)/
((-(a+b*ln(c*(d+e*x^(2/3))^2))/b)^p)+2^(-1+p)*(d+e*x^(2/3))^3*GAMMA(p+1,1/
2*(-3*a-3*b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/(3^p)/
e^3/exp(3/2*a/b)/(c*(d+e*x^(2/3))^2)^(3/2)/((-(a+b*ln(c*(d+e*x^(2/3))^2))/
b)^p)-3/2*d*GAMMA(p+1,-(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/
3))^2))^p/c/e^3/exp(a/b)/((-(a+b*ln(c*(d+e*x^(2/3))^2))/b)^p)
```

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

$$\Big\downarrow \text{2904}$$

$$\frac{3}{2}\int x^{4/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx^{2/3}$$

$$\Big\downarrow \text{2848}$$

$$\frac{3}{2}\int\left(\frac{\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{e^2}-\frac{2d\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{e^2}+\frac{d^2\left(a+b\log\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\left(\frac{d^2 2^p e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right.}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}\right.$$

input
```
Int[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

output
```
(3*((2^p*3^(-1 - p)*(d + e*x^(2/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e
*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^3*E^((3*a)/(2
*b))*(c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p
) - (d*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))^2])/b)]*(a + b*Log[c*(d
 + e*x^(2/3))^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b)
)^p) + (2^p*d^2*(d + e*x^(2/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(2
/3))^2])/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d
 + e*x^(2/3))^2]*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int x \left(a+b \ln \left(c \left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^px\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^2\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

Reduce [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\text{too large to display}$$

input
```
int(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x)
```

output

```
( - 6*x**(2/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)*
*p*b*d**2*e*p**2 - 6*x**(2/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x +
c*d**2)*b + a)**p*b*d**2*e*p + 3*x**(1/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)
*c*e**2*x + c*d**2)*b + a)**p*b*d*e**2*p**2*x + 3*x**(1/3)*(log(2*x**(2/3)
*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*b*d*e**2*p*x + 3*(log(2*x**
(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*log(2*x**(2/3)*c*d*e +
 x**(1/3)*c*e**2*x + c*d**2)*b*d**3*p + 3*(log(2*x**(2/3)*c*d*e + x**(1/3)
*c*e**2*x + c*d**2)*b + a)**p*a*d**3*p + 3*(log(2*x**(2/3)*c*d*e + x**(1/3
)*c*e**2*x + c*d**2)*b + a)**p*a*e**3*p*x**2 + 3*(log(2*x**(2/3)*c*d*e + x
**(1/3)*c*e**2*x + c*d**2)*b + a)**p*a*e**3*x**2 - 12*int(((log(2*x**(2/3)
*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*x)/(3*x**(2/3)*log(2*x**(2/
3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*e + 2*x**(2/3)*log(2*x**(2/3)*c
*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*e*p + 3*x**(2/3)*a**2*e + 2*x**(2/
3)*a*b*e*p + 3*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*d +
2*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*d*p + 3*a**2*d +
 2*a*b*d*p),x)*a*b**2*d*e**3*p**3 - 12*int(((log(2*x**(2/3)*c*d*e + x**(1/
3)*c*e**2*x + c*d**2)*b + a)**p*x)/(3*x**(2/3)*log(2*x**(2/3)*c*d*e + x**(
1/3)*c*e**2*x + c*d**2)*a*b*e + 2*x**(2/3)*log(2*x**(2/3)*c*d*e + x**(1/3)
*c*e**2*x + c*d**2)*b**2*e*p + 3*x**(2/3)*a**2*e + 2*x**(2/3)*a*b*e*p + 3*
log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*d + 2*log(2*x**(...
```

3.577 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}dx$$
$$\Big\downarrow \text{2908}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{2910}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln \left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^2))^p/x,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^2))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 8.04

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\frac{3\left(\log\left(2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}c\,e^2+c\,d^2\right)b+a\right)^p\log\left(2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}c\,e^2+c\,d^2\right)}{}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x)
```

output
```
(3*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*log(2*x**
(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + 3*(log(2*x**(2/3)*c*d*e + x*
*(1/3)*c*e**2*x + c*d**2)*b + a)**p*a + 4*int((log(2*x**(2/3)*c*d*e + x**(
1/3)*c*e**2*x + c*d**2)*b + a)**p/(x**(2/3)*e*x + d*x),x)*b*d*p + 4*int((l
og(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p/(x**(2/3)*e*x
+ d*x),x)*b*d)/(4*b*(p + 1))
```

3.578 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}dx$$
$$\Big\downarrow \text{2908}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^{7/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{2910}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^{7/3}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{2}\right)\right)^{p}}{x^{3}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x^{3}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^3, x)
```

Giac [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^3,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 773, normalized size of antiderivative = 32.21

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

output
```
(6*x**(2/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*
b*e**2*p*x - 3*x**(1/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2
)*b + a)**p*a*d**2 - 3*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)
*b + a)**p*b*d*e*p*x - 4*x**(1/3)*int(((log(2*x**(2/3)*c*d*e + x**(1/3)*c*
e**2*x + c*d**2)*b + a)**p*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d*
*2))/(x**(1/3)*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b*d*x**2
 + x**(1/3)*a*d*x**2 + log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*
b*e*x**3 + a*e*x**3),x)*b**2*d**2*e*p*x**2 + 4*x**(1/3)*int((x**(2/3)*(log
(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p)/(x**(1/3)*log(2
*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b*d*x**2 + x**(1/3)*a*d*x**2
 + log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b*e*x**3 + a*e*x**3)
,x)*b**2*d*e**2*p**2*x**2 + 4*x**(1/3)*int((x**(1/3)*(log(2*x**(2/3)*c*d*e
 + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*log(2*x**(2/3)*c*d*e + x**(1/3)*c
*e**2*x + c*d**2))/(x**(1/3)*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*
d**2)*b*d*x + x**(1/3)*a*d*x + log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x +
c*d**2)*b*e*x**2 + a*e*x**2),x)*b**2*e**3*p*x**2 + 4*x**(1/3)*int((x**(1/3
)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p)/(x**(1/3)
*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b*d*x + x**(1/3)*a*d*x
 + log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b*e*x**2 + a*e*x**2)
,x)*a*b*e**3*p*x**2 - 8*x**(1/3)*int((x**(1/3)*(log(2*x**(2/3)*c*d*e + ...
```

3.579 $\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p \,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p \,dx = \text{Int}\left(x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p,x\right)$$

output
```
Defer(Int)(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p \,dx = \int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p \,dx$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

output
```
Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx$$
$$\Big\downarrow \text{ 2908}$$
$$3 \int x^{8/3} \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p d\sqrt[3]{x}$$
$$\Big\downarrow \text{ 2910}$$
$$3 \int x^{8/3} \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{2} \left(a +b \ln \left(c \left(d +e \,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}d x$$

input
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx=\int { {\left(b \log \left({\left(e x^{\frac{2}{3}} + d\right)}^{2} c\right) + a\right)}^{p} x^{2} \,d x}$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^px^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^px^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
```

output
```
int(x^2*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 2087, normalized size of antiderivative = 86.96

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\text{Too large to display}$$

input
```
int(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x)
```

output
```
(105*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*e**3*x*
*3 + 5040*int((log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**
p/(9*x**(2/3)*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*e + 4
*x**(2/3)*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*e*p + 9*
x**(2/3)*a**2*e + 4*x**(2/3)*a*b*e*p + 9*log(2*x**(2/3)*c*d*e + x**(1/3)*c
*e**2*x + c*d**2)*a*b*d + 4*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d
**2)*b**2*d*p + 9*a**2*d + 4*a*b*d*p),x)*b**2*d**4*p**2 - 1260*int(((log(2
*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*x**3)/(9*x**(1/3)*
log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*d + 4*x**(1/3)*log(
2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*d*p + 9*x**(1/3)*a**2*
d + 4*x**(1/3)*a*b*d*p + 9*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d*
*2)*a*b*e*x + 4*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*e*
p*x + 9*a**2*e*x + 4*a*b*e*p*x),x)*a*b*e**4*p - 560*int(((log(2*x**(2/3)*c
*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*x**3)/(9*x**(1/3)*log(2*x**(2
/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*d + 4*x**(1/3)*log(2*x**(2/3)*
c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*d*p + 9*x**(1/3)*a**2*d + 4*x**(1
/3)*a*b*d*p + 9*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*e*x
 + 4*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*e*p*x + 9*a**
2*e*x + 4*a*b*e*p*x),x)*b**2*e**4*p**2 - 1008*int(((log(2*x**(2/3)*c*d*e +
 x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*x**2)/(9*x**(2/3)*log(2*x**(2/3)...
```

3.580 $\int \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx=\text{Int}\left(\left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p,x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx=\int \left(a+b \log \left(c \left(d+e x^{2/3}\right)^2\right)\right)^p \, dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2901, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx$$

$$\Big\downarrow \ 2901$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \ 2910$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^{2}\right)\right)^{p}\,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^{2}\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 25.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^2))^p,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 830, normalized size of antiderivative = 41.50

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\text{Too large to display}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^2))^p,x)
```

output
```
(3*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*x - 48*in
t((log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p/(3*x**(2/3
)*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*e + 4*x**(2/3)*lo
g(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*e*p + 3*x**(2/3)*a**
2*e + 4*x**(2/3)*a*b*e*p + 3*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*
d**2)*a*b*d + 4*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*d*
p + 3*a**2*d + 4*a*b*d*p),x)*b**2*d*p**2 - 12*int(((log(2*x**(2/3)*c*d*e +
 x**(1/3)*c*e**2*x + c*d**2)*b + a)**p*x)/(3*x**(1/3)*log(2*x**(2/3)*c*d*e
 + x**(1/3)*c*e**2*x + c*d**2)*a*b*d + 4*x**(1/3)*log(2*x**(2/3)*c*d*e + x
**(1/3)*c*e**2*x + c*d**2)*b**2*d*p + 3*x**(1/3)*a**2*d + 4*x**(1/3)*a*b*d
*p + 3*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*e*x + 4*log(
2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b**2*e*p*x + 3*a**2*e*x + 4
*a*b*e*p*x),x)*a*b*e*p - 16*int(((log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x
 + c*d**2)*b + a)**p*x)/(3*x**(1/3)*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2
*x + c*d**2)*a*b*d + 4*x**(1/3)*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x +
 c*d**2)*b**2*d*p + 3*x**(1/3)*a**2*d + 4*x**(1/3)*a*b*d*p + 3*log(2*x**(2
/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*a*b*e*x + 4*log(2*x**(2/3)*c*d*e +
 x**(1/3)*c*e**2*x + c*d**2)*b**2*e*p*x + 3*a**2*e*x + 4*a*b*e*p*x),x)*b**
2*e*p**2 + 48*int((x**(1/3)*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*
d**2)*b + a)**p)/(3*x**(1/3)*log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x +...
```

3.581 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 6.17

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\frac{-3\left(\log\left(2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}c\,e^2+c\,d^2\right)b+a\right)^p+4\left(\int \frac{(\mathrm{lo}}{x^{\frac{4}{3}}\log\left(2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}c\,e^2+\right.}\right.}{3x}$$

input
```
int((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

output
```
( - 3*(log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p + 4*in
t((log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b + a)**p/(x**(1/3)*
log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b*d*x + x**(1/3)*a*d*x
 + log(2*x**(2/3)*c*d*e + x**(1/3)*c*e**2*x + c*d**2)*b*e*x**2 + a*e*x**2),
x)*b*e*p*x)/(3*x)
```

3.582 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p,x\right)$$

output
```
Defer(Int)(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p dx$$

$$\Big\downarrow \text{2908}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^{p}\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/3))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 25.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(1/3))))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(1/3))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int\left(\log\left(\frac{x^{\frac{1}{3}}cd+ce}{x^{\frac{1}{3}}}\right)b+a\right)^p x\,dx$$

input
```
int(x*(a+b*log(c*(d+e/x^(1/3))))^p,x)
```

output
```
int((log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*x,x)
```

3.583 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\operatorname{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p,x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2901, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p dx$$

$$\Big\downarrow \text{ 2901}$$

$$3\int x^{2/3}\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2910}$$

$$3\int x^{2/3}\left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^{p}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 25.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p dx=\int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))))^p,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int \left(\log \left(\frac{x^{\frac{1}{3}}cd+ce}{x^{\frac{1}{3}}}\right)b+a\right)^p dx$$

input
```
int((a+b*log(c*(d+e/x^(1/3))))^p,x)
```

output
```
int((log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p,x)
```

3.584 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^{p}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))))^p/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(\log\left(\frac{x^{\frac{1}{3}}cd+ce}{x^{\frac{1}{3}}}\right)b+a\right)^p}{x}dx$$

input
```
int((a+b*log(c*(d+e/x^(1/3))))^p/x,x)
```

output
```
int((log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p/x,x)
```

3.585 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 267

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^3e^3}$$
$$+\frac{3\ 2^{-p}de^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^2e^3}$$
$$-\frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{ce^3}$$

output
```
-GAMMA(p+1,(-3*a-3*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(
3^p)/c^3/e^3/exp(3*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(p+1,(
-2*a-2*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(2^p)/c^2/e^3
/exp(2*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)-3*d^2*GAMMA(p+1,-(a+b*ln(c*
(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c/e^3/exp(a/b)/((-(a+b*ln(c
*(d+e/x^(1/3))))/b)^p)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx =$$

$$-\frac{6^{-p}e^{-\frac{3a}{b}}\left(2^p\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

output
```
-(((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*
d*E^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^p*c*d*
E^(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]))*(a + b*Log[c*(
d + e/x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3
))])/b))^p))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}dx$$

$\Big\downarrow$ 2904

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^{2/3}}d\frac{1}{\sqrt[3]{x}}$$

$\Big\downarrow$ 2848

$$-3\int \left(\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{e^2}-\frac{2d\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{e^2}+\frac{d^2\left(a+b\mathrm{l}\right.}{}\right.$$

$\Big\downarrow$ 2009

$$-3\left(\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^3e^3}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

output
```
-3*((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b
*Log[c*(d + e/x^(1/3))])^p)/(c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^
(1/3))])/b))^p) - (d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*
(d + e/x^(1/3))])/b))^p) + (d^2*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3)
)])/b)]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(
d + e/x^(1/3))])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^{p}}{x^{2}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx=\text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x)
```

output

```
(6*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**2*e*p**2 +
6*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**2*e*p - 3*x*
*(1/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d*e**2*p**2 - 3*x**
(1/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d*e**2*p - 6*(log((x
**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*log((x**(1/3)*c*d + c*e)/x**(1/3))*
b*d**3*p*x - 6*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*a*d**3*p*x -
6*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*a*e**3*p - 6*(log((x**(1/3
)*c*d + c*e)/x**(1/3))*b + a)**p*a*e**3 + 6*int((log((x**(1/3)*c*d + c*e)/
x**(1/3))*b + a)**p/(3*x**(2/3)*log((x**(1/3)*c*d + c*e)/x**(1/3))*a*b*d*x
 + x**(2/3)*log((x**(1/3)*c*d + c*e)/x**(1/3))*b**2*d*p*x + 3*x**(2/3)*a**
2*d*x + x**(2/3)*a*b*d*p*x + 3*x**(1/3)*log((x**(1/3)*c*d + c*e)/x**(1/3))
*a*b*e*x + x**(1/3)*log((x**(1/3)*c*d + c*e)/x**(1/3))*b**2*e*p*x + 3*x**(
1/3)*a**2*e*x + x**(1/3)*a*b*e*p*x),x)*a*b**2*d**2*e**2*p**3*x + 6*int((lo
g((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p/(3*x**(2/3)*log((x**(1/3)*c*d +
 c*e)/x**(1/3))*a*b*d*x + x**(2/3)*log((x**(1/3)*c*d + c*e)/x**(1/3))*b**2
*d*p*x + 3*x**(2/3)*a**2*d*x + x**(2/3)*a*b*d*p*x + 3*x**(1/3)*log((x**(1/
3)*c*d + c*e)/x**(1/3))*a*b*e*x + x**(1/3)*log((x**(1/3)*c*d + c*e)/x**(1/
3))*b**2*e*p*x + 3*x**(1/3)*a**2*e*x + x**(1/3)*a*b*e*p*x),x)*a*b**2*d**2*
e**2*p**2*x + 2*int((log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p/(3*x**(2
/3)*log((x**(1/3)*c*d + c*e)/x**(1/3))*a*b*d*x + x**(2/3)*log((x**(1/3)...
```

3.586 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 554

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx =$$

$$-\frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^6e^6}$$

$$+\frac{3\ 5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^5e^6}$$

$$-\frac{15\ 2^{-1-2p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^4e^6}$$

$$+\frac{10\ 3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^3e^6}$$

$$-\frac{15\ 2^{-1-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^2e^6}$$

$$+\frac{3d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{ce^6}$$

output
```
-2^(-1-p)*GAMMA(p+1,(-6*a-6*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/
3))))^p/(3^p)/c^6/e^6/exp(6*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GA
MMA(p+1,(-5*a-5*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(5^p
)/c^5/e^6/exp(5*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)-15*2^(-1-2*p)*d^2*
GAMMA(p+1,(-4*a-4*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^
4/e^6/exp(4*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)+10*d^3*GAMMA(p+1,(-3*a
-3*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p)/c^3/e^6/exp
(3*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)-15*2^(-1-p)*d^4*GAMMA(p+1,(-2*a
-2*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^2/e^6/exp(2*a/b
)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d^5*GAMMA(p+1,-(a+b*ln(c*(d+e/x^(1/
3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c/e^6/exp(a/b)/((-(a+b*ln(c*(d+e/x^(1
/3))))/b)^p)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.59

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx$$

$$= \frac{2^{-1-2p}15^{-p}e^{-\frac{6a}{b}}\left(-10^p\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(2^{1+2p}3^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d\right.\right.\right.}{b}\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```

output
```
(2^(-1 - 2*p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b])
 + c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e
/x^(1/3))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Lo
g[c*(d + e/x^(1/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*
(a + b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p
, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 +
 p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]))))))*(a + b*Log[c*(d + e/x^(1/3))
])^p)/(15^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^{5/3}}d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2848}$$

$$-3\int \left(\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{e^5}-\frac{5d\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{e^5}+\frac{10d^2\left(d+\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-3\left(\frac{6^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^6e^6}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```

output
```
-3*((6^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b
*Log[c*(d + e/x^(1/3))])^p)/(c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^
(1/3))])/b))^p) - (d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*
(d + e/x^(1/3))])/b))^p) + (5*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log
[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (10*3^(-1 - p)*d^3*Gamma[
1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))
])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (5*2
^(-1 - p)*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*L
og[c*(d + e/x^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p) - (d^5*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(a
+ b*Log[c*(d + e/x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/
3))])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^p}{x^3}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))))^p/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))))^p/x^3,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = \text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x)
```

output
```
( - 180*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**5*e*p*
*2*x - 180*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**5*e
*p*x + 45*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**2*e*
*4*p**2 + 45*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**2
*e**4*p + 90*x**(1/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**4
*e**2*p**2*x + 90*x**(1/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b
*d**4*e**2*p*x - 36*x**(1/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p
*b*d*e**5*p**2 - 36*x**(1/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p
*b*d*e**5*p + 180*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*log((x**(1
/3)*c*d + c*e)/x**(1/3))*b*d**6*p*x**2 + 180*(log((x**(1/3)*c*d + c*e)/x**
(1/3))*b + a)**p*a*d**6*p*x**2 - 180*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b
 + a)**p*a*e**6*p - 180*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*a*e*
*6 - 60*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**3*e**3*p**2*x -
 60*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**3*e**3*p*x + 90*int
((log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p/(6*x**(2/3)*log((x**(1/3)*c
*d + c*e)/x**(1/3))*a*b*d*x**2 + x**(2/3)*log((x**(1/3)*c*d + c*e)/x**(1/3
))*b**2*d*p*x**2 + 6*x**(2/3)*a**2*d*x**2 + x**(2/3)*a*b*d*p*x**2 + 6*x**(
1/3)*log((x**(1/3)*c*d + c*e)/x**(1/3))*a*b*e*x**2 + x**(1/3)*log((x**(1/3
)*c*d + c*e)/x**(1/3))*b**2*e*p*x**2 + 6*x**(1/3)*a**2*e*x**2 + x**(1/3)*a
*b*e*p*x**2),x)*a*b**2*d**2*e**5*p**3*x**2 + 90*int((log((x**(1/3)*c*d ...
```

3.587 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 832

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx = \text{Too large to display}$$

output

```
-3^(-1-2*p)*GAMMA(p+1,(-9*a-9*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(
1/3))))^p/c^9/e^9/exp(9*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(
p+1,(-8*a-8*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(8^p)/c^
8/e^9/exp(8*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)-12*d^2*GAMMA(p+1,(-7*a
-7*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(7^p)/c^7/e^9/exp
(7*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)+7*2^(2-p)*d^3*GAMMA(p+1,(-6*a-6
*b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p)/c^6/e^9/exp(6
*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)-42*d^4*GAMMA(p+1,(-5*a-5*b*ln(c*(
d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(5^p)/c^5/e^9/exp(5*a/b)/((-
(a+b*ln(c*(d+e/x^(1/3))))/b)^p)+21*2^(1-2*p)*d^5*GAMMA(p+1,(-4*a-4*b*ln(c*
(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^4/e^9/exp(4*a/b)/((-(a+b*
ln(c*(d+e/x^(1/3))))/b)^p)-28*d^6*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e/x^(1/3))))
/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p)/c^3/e^9/exp(3*a/b)/((-(a+b*ln(c*(d+e
/x^(1/3))))/b)^p)+3*2^(2-p)*d^7*GAMMA(p+1,(-2*a-2*b*ln(c*(d+e/x^(1/3))))/b
)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^2/e^9/exp(2*a/b)/((-(a+b*ln(c*(d+e/x^(1/3)
)))/b)^p)-3*d^8*GAMMA(p+1,-(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(
1/3))))^p/c/e^9/exp(a/b)/((-(a+b*ln(c*(d+e/x^(1/3))))/b)^p)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.60

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx =$$

$$-\frac{3^{-1-2p}280^{-p}e^{-\frac{9a}{b}}\left(280^p\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)-9^{1+p}35^pcde^{a/b}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+\right.\right.\right.}{b}\right.\right.}{}$$

input

```
Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```

output
```
-((3^(-1 - 2*p)*(280^p*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]
 - 9^(1 + p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3)
)]))/b] + 2^(2 + 3*p)*5^p*9^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-7*(
a + b*Log[c*(d + e/x^(1/3))]))/b] - 5^p*84^(1 + p)*c^3*d^3*E^((3*a)/b)*Gam
ma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + 3*p)*63^(1 + p)*
c^4*d^4*E^((4*a)/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b] -
5^p*126^(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x
^(1/3))]))/b] + 2^(2 + 3*p)*5^p*21^(1 + p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p
, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b] - 35^p*36^(1 + p)*c^7*d^7*E^((7*a
)/b)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 9^(1 + p)*280^p
*c^8*d^8*E^((8*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)])*(a
 + b*Log[c*(d + e/x^(1/3))])^p)/(280^p*c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p))
```

Rubi [A] (verified)

Time = 2.92 (sec) , antiderivative size = 836, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^{8/3}}d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2848}$$

$$-3\int \left(\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{e^8}-\frac{8d\left(d+\frac{e}{\sqrt[3]{x}}\right)^7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{e^8}+\frac{28d^2\left(d+\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-3\left(\frac{9^{-p-1}e^{-\frac{9a}{b}}\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^9e^9}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```

output
```
-3*((9^(-1 - p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b
*Log[c*(d + e/x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^
(1/3))])/b))^p) - (d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*
(d + e/x^(1/3))])/b))^p) + (4*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/x^(
1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-(
(a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (7*2^(2 - p)*3^(-1 - p)*d^3*Gamma[
1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))
])^p)/(c^6*e^9*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (14*
d^4*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d +
e/x^(1/3))])^p)/(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])
/b))^p) - (7*2^(1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))
]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Lo
g[c*(d + e/x^(1/3))])/b))^p) + (28*3^(-1 - p)*d^6*Gamma[1 + p, (-3*(a + b*
Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^3*e^9*E^(
(3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (2^(2 - p)*d^7*Gamma[1
 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))]
)^p)/(c^2*e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (d^8*
Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*(d + e/x^(1
/3))])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))))^p/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^{p}}{x^{4}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x^{4}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))))^p/x^4,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx=\text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x)
```

output
```
(2520*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**8*e*p**2
*x**2 + 2520*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**8
*e*p*x**2 - 630*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d
**5*e**4*p**2*x - 630*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)*
*p*b*d**5*e**4*p*x + 360*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b +
a)**p*b*d**2*e**7*p**2 + 360*x**(2/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3))*
b + a)**p*b*d**2*e**7*p - 1260*x**(1/3)*(log((x**(1/3)*c*d + c*e)/x**(1/3)
)*b + a)**p*b*d**7*e**2*p**2*x**2 - 1260*x**(1/3)*(log((x**(1/3)*c*d + c*e
)/x**(1/3))*b + a)**p*b*d**7*e**2*p*x**2 + 504*x**(1/3)*(log((x**(1/3)*c*d
 + c*e)/x**(1/3))*b + a)**p*b*d**4*e**5*p**2*x + 504*x**(1/3)*(log((x**(1/
3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d**4*e**5*p*x - 315*x**(1/3)*(log((x**
(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d*e**8*p**2 - 315*x**(1/3)*(log((x*
*(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*b*d*e**8*p - 2520*(log((x**(1/3)*c*d
 + c*e)/x**(1/3))*b + a)**p*log((x**(1/3)*c*d + c*e)/x**(1/3))*b*d**9*p*x*
*3 - 2520*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*a*d**9*p*x**3 - 25
20*(log((x**(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*a*e**9*p - 2520*(log((x**
(1/3)*c*d + c*e)/x**(1/3))*b + a)**p*a*e**9 + 840*(log((x**(1/3)*c*d + c*e
)/x**(1/3))*b + a)**p*b*d**6*e**3*p**2*x**2 + 840*(log((x**(1/3)*c*d + c*e
)/x**(1/3))*b + a)**p*b*d**6*e**3*p*x**2 - 420*(log((x**(1/3)*c*d + c*e)/x
**(1/3))*b + a)**p*b*d**3*e**6*p**2*x - 420*(log((x**(1/3)*c*d + c*e)/x...
```

3.588 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

output
```
Defer(Int)(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx$$

$$\Big\downarrow \text{2908}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{2}\right)\right)^{p}dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p*x, x
)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{2}\right)\right)^{p}dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 25.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^2\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(1/3))^2))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(1/3))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int\left(\log\left(\frac{x^{\frac{2}{3}}c\,d^2+2x^{\frac{1}{3}}cde+c\,e^2}{x^{\frac{2}{3}}}\right)b+a\right)^p xdx$$

input
```
int(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x)
```

output
```
int((log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p
*x,x)
```

3.589 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\operatorname{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2901, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx$$

$$\Big\downarrow \text{2901}$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)\right)^{p} dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{2}\right)\right)^{p} dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p} dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{2}\right)\right)^{p} dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 25.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx = \int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)^2\right)\right)^p dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int \left(\log \left(\frac{x^{\frac{2}{3}}c\,d^2+2x^{\frac{1}{3}}cde+c\,e^2}{x^{\frac{2}{3}}}\right)b+a\right)^p dx$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^2))^p,x)
```

output
```
int((log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p
,x)
```

3.590 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{2}\right)\right)^{p}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x, x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)^2\right) + a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(\log\left(\frac{x^{\frac{2}{3}}c\,d^2+2x^{\frac{1}{3}}cde+c\,e^2}{x^{\frac{2}{3}}}\right)b+a\right)^p}{x}dx$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x)
```

output
```
int((log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p
/x,x)
```

3.591 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 342

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = -\frac{3\,2^p d^2 e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(\right.\right.}{}\right.}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}} - \frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^3\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}} + \frac{3de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}}{ce^3}$$

output

```
-3*2^p*d^2*(d+e/x^(1/3))*GAMMA(p+1,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+
b*ln(c*(d+e/x^(1/3))^2))^p/e^3/exp(1/2*a/b)/(c*(d+e/x^(1/3))^2)^(1/2)/((-(
a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)-(2/3)^p*(d+e/x^(1/3))^3*GAMMA(p+1,1/2*(-3
*a-3*b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^3/exp(3/2
*a/b)/(c*(d+e/x^(1/3))^2)^(3/2)/((-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*d*G
AMMA(p+1,-(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e
^3/exp(a/b)/((-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)
```

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2, x]
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^{2/3}}\,d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2848}$$

$$-3\int \left(\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{e^2}-\frac{2d\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{e^2}+\frac{d^2\left(a+\right.}{}\right.$$

$\downarrow$ 2009

$$-3\left(\frac{d^2 2^p e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\right.\right.}{2b}\right.}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
```

output
```
-3*((2^p*3^(-1 - p)*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e
/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^((3*a)/(2
*b))*(c*(d + e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p
) - (d*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d
 + e/x^(1/3))^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b)
)^p) + (2^p*d^2*(d + e/x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/x^(1
/3))^2])/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d
 + e/x^(1/3))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{2}\right)\right)^{p}}{x^{2}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^2,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = \text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x)
```

output
```
(6*x**(2/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b
 + a)**p*b*d**2*e*p**2 + 6*x**(2/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d
*e + c*e**2)/x**(2/3))*b + a)**p*b*d**2*e*p - 3*x**(1/3)*(log((x**(2/3)*c*
d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d*e**2*p**2 - 3*x*
*(1/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)
**p*b*d*e**2*p - 3*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(
2/3))*b + a)**p*log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3)
)*b*d**3*p*x - 3*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/
3))*b + a)**p*a*d**3*p*x - 3*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*
e**2)/x**(2/3))*b + a)**p*a*e**3*p - 3*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*
c*d*e + c*e**2)/x**(2/3))*b + a)**p*a*e**3 + 12*int((log((x**(2/3)*c*d**2
+ 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p/(3*x**(2/3)*log((x**(2/3)
*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*a*b*d*x + 2*x**(2/3)*log((x
**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b**2*d*p*x + 3*x**(2
/3)*a**2*d*x + 2*x**(2/3)*a*b*d*p*x + 3*x**(1/3)*log((x**(2/3)*c*d**2 + 2*
x**(1/3)*c*d*e + c*e**2)/x**(2/3))*a*b*e*x + 2*x**(1/3)*log((x**(2/3)*c*d*
*2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b**2*e*p*x + 3*x**(1/3)*a**2*e*x
 + 2*x**(1/3)*a*b*e*p*x),x)*a*b**2*d**2*e**2*p**3*x + 12*int((log((x**(2/3
)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p/(3*x**(2/3)*log(
(x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*a*b*d*x + 2*x**...
```

3.592 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 673

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx = \text{Too large to display}$$

output

```
-1/2*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))
^2))^p/(3^p)/c^3/e^6/exp(3*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*(2/
5)^p*d*(d+e/x^(1/3))^5*GAMMA(p+1,1/2*(-5*a-5*b*ln(c*(d+e/x^(1/3))^2))/b)*(
a+b*ln(c*(d+e/x^(1/3))^2))^p/e^6/exp(5/2*a/b)/(c*(d+e/x^(1/3))^2)^(5/2)/((
-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)-15*2^(-1-p)*d^2*GAMMA(p+1,(-2*a-2*b*ln(
c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c^2/e^6/exp(2*a/b)/((
-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)+5*2^(p+1)*d^3*(d+e/x^(1/3))^3*GAMMA(p+1
,1/2*(-3*a-3*b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(3^
p)/e^6/exp(3/2*a/b)/(c*(d+e/x^(1/3))^2)^(3/2)/((-(a+b*ln(c*(d+e/x^(1/3))^2
))/b)^p)-15/2*d^4*GAMMA(p+1,-(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e
/x^(1/3))^2))^p/c/e^6/exp(a/b)/((-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*2^p*
d^5*(d+e/x^(1/3))*GAMMA(p+1,-1/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*
(d+e/x^(1/3))^2))^p/e^6/exp(1/2*a/b)/(c*(d+e/x^(1/3))^2)^(1/2)/((-(a+b*ln(
c*(d+e/x^(1/3))^2))/b)^p)
```

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3, x]
```

Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^{5/3}}\,d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2848}$$

$$-3\int \left(\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{e^5}-\frac{5d\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{e^5}+\frac{10d^2\left(\right.}{}\right.$$

$\downarrow$ 2009

$$-3\left(\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right.}{b}\right.}{2c^3e^6}\right.$$

input

```
Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
```

output

```
-3*((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a +
 b*Log[c*(d + e/x^(1/3))^2])^p)/(2*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d
+ e/x^(1/3))^2])/b))^p) - ((2/5)^p*d*(d + e/x^(1/3))^5*Gamma[1 + p, (-5*(a
 + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)
/(e^6*E^((5*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e/x
^(1/3))^2])/b))^p) + (5*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d +
e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b
)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (5*2^(1 + p)*3^(-1 - p)*d^3
*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b
)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e/x^(1
/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (5*d^4*Gamma[1
+ p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/3))^2
])^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (2^p*d
^5*(d + e/x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/x^(1/3))^2])/b]*(
a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3))
^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)\right)^{p}}{x^{3}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^3,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{2}\right)\right)^{p}}{x^{3}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x^{3}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^3,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x^3}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^3,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx=\text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x)
```

output
```
( - 180*x**(2/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/
3))*b + a)**p*b*d**5*e*p**2*x - 180*x**(2/3)*(log((x**(2/3)*c*d**2 + 2*x**
(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d**5*e*p*x + 45*x**(2/3)*(log(
(x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d**2*e
**4*p**2 + 45*x**(2/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/
x**(2/3))*b + a)**p*b*d**2*e**4*p + 90*x**(1/3)*(log((x**(2/3)*c*d**2 + 2*
x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d**4*e**2*p**2*x + 90*x**(1
/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p
*b*d**4*e**2*p*x - 36*x**(1/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e +
c*e**2)/x**(2/3))*b + a)**p*b*d*e**5*p**2 - 36*x**(1/3)*(log((x**(2/3)*c*d
**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d*e**5*p + 90*(log(
(x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*log((x**
(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b*d**6*p*x**2 + 90*(lo
g((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*a*d**6
*p*x**2 - 90*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*
b + a)**p*a*e**6*p - 90*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)
/x**(2/3))*b + a)**p*a*e**6 - 60*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e
+ c*e**2)/x**(2/3))*b + a)**p*b*d**3*e**3*p**2*x - 60*(log((x**(2/3)*c*d**
2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d**3*e**3*p*x + 90*in
t((log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**...
```

3.593 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1036

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx = \text{Too large to display}$$

output
```
-2^p*3^(-1-2*p)*(d+e/x^(1/3))^9*GAMMA(p+1,1/2*(-9*a-9*b*ln(c*(d+e/x^(1/3))
^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^9/exp(9/2*a/b)/(c*(d+e/x^(1/3))^2)
^(9/2)/((-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*d*GAMMA(p+1,(-4*a-4*b*ln(c*(
d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(4^p)/c^4/e^9/exp(4*a/b)
/((-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)-3*2^(2+p)*d^2*(d+e/x^(1/3))^7*GAMMA(
p+1,1/2*(-7*a-7*b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/
(7^p)/e^9/exp(7/2*a/b)/(c*(d+e/x^(1/3))^2)^(7/2)/((-(a+b*ln(c*(d+e/x^(1/3)
)^2))/b)^p)+28*d^3*GAMMA(p+1,(-3*a-3*b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c
*(d+e/x^(1/3))^2))^p/(3^p)/c^3/e^9/exp(3*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))^2
))/b)^p)-21*2^(p+1)*d^4*(d+e/x^(1/3))^5*GAMMA(p+1,1/2*(-5*a-5*b*ln(c*(d+e/
x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(5^p)/e^9/exp(5/2*a/b)/(c*(d
+e/x^(1/3))^2)^(5/2)/((-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)+21*2^(1-p)*d^5*G
AMMA(p+1,(-2*a-2*b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p
/c^2/e^9/exp(2*a/b)/((-(a+b*ln(c*(d+e/x^(1/3))^2))/b)^p)-7*2^(2+p)*d^6*(d+
e/x^(1/3))^3*GAMMA(p+1,1/2*(-3*a-3*b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(
d+e/x^(1/3))^2))^p/(3^p)/e^9/exp(3/2*a/b)/(c*(d+e/x^(1/3))^2)^(3/2)/((-(a+
b*ln(c*(d+e/x^(1/3))^2))/b)^p)+12*d^7*GAMMA(p+1,-(a+b*ln(c*(d+e/x^(1/3))^2
))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e^9/exp(a/b)/((-(a+b*ln(c*(d+e/x^(1/
3))^2))/b)^p)-3*2^p*d^8*(d+e/x^(1/3))*GAMMA(p+1,-1/2*(a+b*ln(c*(d+e/x^(1/3
))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^9/exp(1/2*a/b)/(c*(d+e/x^(1/3...
```

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4, x]
```

Rubi [A] (verified)

Time = 3.39 (sec) , antiderivative size = 1039, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2904, 2848, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}dx$$

$$\Big\downarrow \text{2904}$$

$$-3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^{8/3}}d\frac{1}{\sqrt[3]{x}}$$

$$\Big\downarrow \text{2848}$$

$$-3\int \left(\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{e^8}-\frac{8d\left(d+\frac{e}{\sqrt[3]{x}}\right)^7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{e^8}+\frac{28d^2\left(\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-3\left(\frac{2^p9^{-p-1}e^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^9\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{9/2}}\right.$$

input
```
Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
```

output
```
-3*((2^p*9^(-1 - p)*(d + e/x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e
/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^((9*a)/(2
*b))*(c*(d + e/x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p
) - (d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*
(d + e/x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(
1/3))^2])/b))^p) + (2^(2 + p)*d^2*(d + e/x^(1/3))^7*Gamma[1 + p, (-7*(a +
b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(7
^p*e^9*E^((7*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e/
x^(1/3))^2])/b))^p) - (28*3^(-1 - p)*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d
+ e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c^3*e^9*E^((3*a)
/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (7*2^(1 + p)*d^4*(d + e/x
^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b
*Log[c*(d + e/x^(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e/x^(1/3))^
2)^(5/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (7*2^(1 - p)*d^5*Gam
ma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e/x^(
1/3))^2])^p)/(c^2*e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^
p) + (7*2^(2 + p)*3^(-1 - p)*d^6*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b
*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^
9*E^((3*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/
3))^2])/b))^p) - (4*d^7*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2848
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)\right)^{p}}{x^{4}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^4,x)
```

output
```
int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{2}\right)\right)^{p}}{x^{4}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{2}\right)+a\right)^{p}}{x^{4}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^4,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{2}\right)\right)^{p}}{x^{4}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x^4}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^4,x)
```

output
```
int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx = \text{too large to display}$$

input
```
int((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x)
```

output
```
(2520*x**(2/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3)
)*b + a)**p*b*d**8*e*p**2*x**2 + 2520*x**(2/3)*(log((x**(2/3)*c*d**2 + 2*x
**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d**8*e*p*x**2 - 630*x**(2/3)
*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*
d**5*e**4*p**2*x - 630*x**(2/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e +
 c*e**2)/x**(2/3))*b + a)**p*b*d**5*e**4*p*x + 360*x**(2/3)*(log((x**(2/3)
*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d**2*e**7*p**2
+ 360*x**(2/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3)
)*b + a)**p*b*d**2*e**7*p - 1260*x**(1/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/
3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d**7*e**2*p**2*x**2 - 1260*x**(1/
3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*
b*d**7*e**2*p*x**2 + 504*x**(1/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e
 + c*e**2)/x**(2/3))*b + a)**p*b*d**4*e**5*p**2*x + 504*x**(1/3)*(log((x**
(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d**4*e**5*
p*x - 315*x**(1/3)*(log((x**(2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(
2/3))*b + a)**p*b*d*e**8*p**2 - 315*x**(1/3)*(log((x**(2/3)*c*d**2 + 2*x**
(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*b*d*e**8*p - 1260*(log((x**(2/3)
*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*log((x**(2/3)*c*d
**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b*d**9*p*x**3 - 1260*(log((x**(
2/3)*c*d**2 + 2*x**(1/3)*c*d*e + c*e**2)/x**(2/3))*b + a)**p*a*d**9*p*x...
```

3.594 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\text{Int}\left(x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

output
```
Defer(Int)(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

output
```
Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

$$\Big\downarrow \text{2908}$$

$$3\int x^{11/3} \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int x^{11/3} \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,d\sqrt[3]{x}$$

input
```
Int[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p dx$$

input
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p \, dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)\right) + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p \, dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)\right) + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p \, dx = \int x^3 \left(a + b \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$$

input
```
int(x^3*(a + b*log(c*(d + e/x^(2/3))))^p,x)
```

output
```
int(x^3*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(\log\left(\frac{x^{\frac{2}{3}}cd+ce}{x^{\frac{2}{3}}}\right)b+a\right)^p x^3dx$$

input
```
int(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x)
```

output
```
int((log((x**(2/3)*c*d + c*e)/x**(2/3))*b + a)**p*x**3,x)
```

3.595 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p dx = \text{Int}\left(x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p, x\right)$$

output
```
Defer(Int)(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p dx = \int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p dx$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

output
```
Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p dx$$

$$\Big\downarrow \text{2908}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{2} \left(a +b \ln \left(c \left(d +\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^{p}d x$$

input
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx=\int \left(b \log \left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p} x^{2} \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x^2\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))))^p,x)
```

output
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \,dx=\int \left(\log\left(\frac{x^{\frac{2}{3}}cd+ce}{x^{\frac{2}{3}}}\right)b+a\right)^p x^2dx$$

input
```
int(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x)
```

output
```
int((log((x**(2/3)*c*d + c*e)/x**(2/3))*b + a)**p*x**2,x)
```

3.596 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

output
```
Defer(Int)(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$$
$$\Big\downarrow \text{ 2908}$$
$$3 \int x^{5/3} \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p d\sqrt[3]{x}$$
$$\Big\downarrow \text{ 2910}$$
$$3 \int x^{5/3} \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^{p}\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 25.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(2/3))))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(\log\left(\frac{x^{\frac{2}{3}}cd+ce}{x^{\frac{2}{3}}}\right)b+a\right)^p x dx$$

input
```
int(x*(a+b*log(c*(d+e/x^(2/3))))^p,x)
```

output
```
int((log((x**(2/3)*c*d + c*e)/x**(2/3))*b + a)**p*x,x)
```

3.597 $\int \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx=\text{Int}\left(\left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx=\int \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2901, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$
$$\Big\downarrow \text{2901}$$
$$3\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,d\sqrt[3]{x}$$
$$\Big\downarrow \text{2910}$$
$$3\int x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(a+b\ln \left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^{p}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(b\log \left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 25.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))))^p,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(\log\left(\frac{x^{\frac{2}{3}}cd+ce}{x^{\frac{2}{3}}}\right)b+a\right)^p dx$$

input
```
int((a+b*log(c*(d+e/x^(2/3))))^p,x)
```

output
```
int((log((x**(2/3)*c*d + c*e)/x**(2/3))*b + a)**p,x)
```

3.598 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

$$\Big\downarrow \ 2908$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{\sqrt[3]{x}}\,d\sqrt[3]{x}$$

$$\Big\downarrow \ 2910$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{\sqrt[3]{x}}\,d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^{p}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))))^p/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(\log\left(\frac{x^{\frac{2}{3}}cd+ce}{x^{\frac{2}{3}}}\right)b+a\right)^p}{x}dx$$

input
```
int((a+b*log(c*(d+e/x^(2/3))))^p/x,x)
```

output
```
int((log((x**(2/3)*c*d + c*e)/x**(2/3))*b + a)**p/x,x)
```

3.599 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

$$\Big\downarrow \text{2908}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))))^p/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 5.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx=\frac{-3\left(\log\left(\frac{x^{\frac{2}{3}}cd+ce}{x^{\frac{2}{3}}}\right)b+a\right)^p-2\left(\int \frac{\left(\log\left(\frac{x^{\frac{2}{3}}cd+ce}{x^{\frac{2}{3}}}\right)b+a\right)^p}{x^{\frac{8}{3}}\log\left(\frac{x^{\frac{2}{3}}cd+ce}{x^{\frac{2}{3}}}\right)bd+x^{\frac{8}{3}}ad+\log\left(\frac{x^{\frac{2}{3}}cd+ce}{x^{\frac{2}{3}}}\right)be\,x^2}\right.}{3x}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x)
```

output
```
( - 3*(log((x**(2/3)*c*d + c*e)/x**(2/3))*b + a)**p - 2*int((log((x**(2/3)
*c*d + c*e)/x**(2/3))*b + a)**p/(x**(2/3)*log((x**(2/3)*c*d + c*e)/x**(2/3
))*b*d*x**2 + x**(2/3)*a*d*x**2 + log((x**(2/3)*c*d + c*e)/x**(2/3))*b*e*x
**2 + a*e*x**2),x)*b*e*p*x)/(3*x)
```

3.600 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \text{Int}\left(x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p, x\right)$$

output
```
Defer(Int)(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

input
```
Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

output
```
Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

$$\Big\downarrow \text{2908}$$

$$3\int x^{11/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int x^{11/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^3 \left(a + b\ln\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p dx$$

input
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

output
```
int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int x^3 \left(a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \int \left(b\log\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^2\right) + a\right)^p x^3 \, dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*
x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^3, x)
```

Giac [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x^3\,\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^2\right)\right)^p\,dx$$

input
```
int(x^3*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
```

output
```
int(x^3*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(\log\left(\frac{2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}c\,d^2+c\,e^2}{x^{\frac{4}{3}}}\right)b+a\right)^p x^3dx$$

input
```
int(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x)
```

output
```
int((log((2*x**(2/3)*c*d*e + x**(1/3)*c*d**2*x + c*e**2)/(x**(1/3)*x))*b +
 a)**p*x**3,x)
```

3.601 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\text{Int}\left(x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

output
```
Defer(Int)(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

output
```
Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$
$$\Big\downarrow \text{2908}$$
$$3\int x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p d\sqrt[3]{x}$$
$$\Big\downarrow \text{2910}$$
$$3\int x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p dx$$

input
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

output
```
int(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^2\, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*
x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

input
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
```

output
```
int(x^2*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(\log\left(\frac{2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}c\,d^2+c\,e^2}{x^{\frac{4}{3}}}\right)b+a\right)^p x^2dx$$

input
```
int(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x)
```

output
```
int((log((2*x**(2/3)*c*d*e + x**(1/3)*c*d**2*x + c*e**2)/(x**(1/3)*x))*b +
 a)**p*x**2,x)
```

3.602 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

output
```
Defer(Int)(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

input
```
Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

output
```
Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

$$\Big\downarrow \text{2908}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \text{2910}$$

$$3\int x^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)\right)^{p}dx$$

input
```
int(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

output
```
int(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p}\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p}x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*
x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x\,dx$$

input
```
integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 25.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

input
```
int(x*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
```

output
```
int(x*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(\log\left(\frac{2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}c\,d^2+c\,e^2}{x^{\frac{4}{3}}}\right)b+a\right)^p x dx$$

input
```
int(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x)
```

output
```
int((log((2*x**(2/3)*c*d*e + x**(1/3)*c*d**2*x + c*e**2)/(x**(1/3)*x))*b +
 a)**p*x,x)
```

3.603 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p, x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Mathematica [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2901, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

$$\Big\downarrow \; 2901$$

$$3\int x^{2/3}\left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

$$\Big\downarrow \; 2910$$

$$3\int x^{2/3}\left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2901
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*Log[c*
(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
 x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)\right)^{p}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 25.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int {\left(a+b\,\ln \left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^2\right)\right)}^p \,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^2))^p,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int \left(\log \left(\frac{2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}c\,d^{2}+c\,e^{2}}{x^{\frac{4}{3}}}\right)b+a\right)^{p}dx$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^2))^p,x)
```

output
```
int((log((2*x**(2/3)*c*d*e + x**(1/3)*c*d**2*x + c*e**2)/(x**(1/3)*x))*b +
 a)**p,x)
```

3.604 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}dx$$
$$\Big\downarrow \text{ 2908}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{ 2910}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{\sqrt[3]{x}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)\right)^{p}}{x}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p/
x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 25.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^2))^p/x,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^2))^p/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(\log\left(\frac{2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}c\,d^2+c\,e^2}{x^{\frac{4}{3}}}\right)b+a\right)^p}{x}dx$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x)
```

output
```
int((log((2*x**(2/3)*c*d*e + x**(1/3)*c*d**2*x + c*e**2)/(x**(1/3)*x))*b +
 a)**p/x,x)
```

3.605 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2},x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2908, 2910}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}dx$$
$$\Big\downarrow \text{2908}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{2910}$$
$$3\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^{4/3}}d\sqrt[3]{x}$$

input
```
Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2908
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, p, q}, x] && FractionQ[n]
```

rule 2910
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*
(x_))^(m_.), x_Symbol] :> Unintegrable[(f*x)^m*(a + b*Log[c*(d + e*x^n)^p])
^q, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)\right)^{p}}{x^{2}}dx$$

input
```
int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

output
```
int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{2}\right)\right)^{p}}{x^{2}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{2}\right)+a\right)^{p}}{x^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p/
x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 25.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

input
```
int((a + b*log(c*(d + e/x^(2/3))^2))^p/x^2,x)
```

output
```
int((a + b*log(c*(d + e/x^(2/3))^2))^p/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 6.83

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\frac{-3\left(\log\left(\frac{2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}cd^2+ce^2}{x^{\frac{4}{3}}}\right)b+a\right)^p-4\left(\int \frac{\left(\log\left(\frac{2x^{\frac{2}{3}}cd}{}\right.\right.}{x^{\frac{8}{3}}\log\left(\frac{2x^{\frac{2}{3}}cde+x^{\frac{4}{3}}cd^2+ce^2}{x^{\frac{4}{3}}}\right)bd+a}\right.}{3x}$$

input
```
int((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

output
```
( - 3*(log((2*x**(2/3)*c*d*e + x**(1/3)*c*d**2*x + c*e**2)/(x**(1/3)*x))*b
 + a)**p - 4*int((log((2*x**(2/3)*c*d*e + x**(1/3)*c*d**2*x + c*e**2)/(x**
(1/3)*x))*b + a)**p/(x**(2/3)*log((2*x**(2/3)*c*d*e + x**(1/3)*c*d**2*x +
c*e**2)/(x**(1/3)*x))*b*d*x**2 + x**(2/3)*a*d*x**2 + log((2*x**(2/3)*c*d*e
 + x**(1/3)*c*d**2*x + c*e**2)/(x**(1/3)*x))*b*e*x**2 + a*e*x**2),x)*b*e*p
*x)/(3*x)
```

3.606 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 474

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \frac{2af\sqrt{hx}}{h}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}-\frac{2\sqrt{2}b\sqrt[4]{d}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}-\frac{2\sqrt{2}bd^{3/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}+\frac{2\sqrt{2}b\sqrt[4]{d}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}+\frac{2\sqrt{2}bd^{3/4}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}+\frac{2\sqrt{2}b\sqrt[4]{d}fp\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{e}\sqrt{h}}-\frac{2\sqrt{2}bd^{3/4}gp\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{3e^{3/4}\sqrt{h}}+\frac{2bf\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h}+\frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2}$$

output
```
2*a*f*(h*x)^(1/2)/h-8*b*f*p*(h*x)^(1/2)/h-8/9*b*g*p*(h*x)^(3/2)/h^2-2*2^(1
/2)*b*d^(1/4)*f*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^
(1/4)/h^(1/2)-2/3*2^(1/2)*b*d^(3/4)*g*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/
2)/d^(1/4)/h^(1/2))/e^(3/4)/h^(1/2)+2*2^(1/2)*b*d^(1/4)*f*p*arctan(1+2^(1/
2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1/4)/h^(1/2)+2/3*2^(1/2)*b*d^(3
/4)*g*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(3/4)/h^(1
/2)+2*2^(1/2)*b*d^(1/4)*f*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^
(1/2)/(d^(1/2)+e^(1/2)*x))/e^(1/4)/h^(1/2)-2/3*2^(1/2)*b*d^(3/4)*g*p*arcta
nh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/e^(3/4
)/h^(1/2)+2*b*f*(h*x)^(1/2)*ln(c*(e*x^2+d)^p)/h+2/3*g*(h*x)^(3/2)*(a+b*ln(
c*(e*x^2+d)^p))/h^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.79

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \frac{2\sqrt{x}\left(af\sqrt{x}-4bfp\sqrt{x}+\frac{1}{3}agx^{3/2}-\frac{4}{9}bgpx^{3/2}-\frac{\sqrt{2}b\sqrt[4]{d}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}+\frac{\sqrt{2}b\sqrt[4]{d}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}\right.}{}$$

input
```
Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

output
```
(2*Sqrt[x]*(a*f*Sqrt[x] - 4*b*f*p*Sqrt[x] + (a*g*x^(3/2))/3 - (4*b*g*p*x^(
3/2))/9 - (Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1
/4)])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x]
)/d^(1/4)])/e^(1/4) - (2*b*(-d)^(3/4)*g*p*ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1
/4)])/(3*e^(3/4)) + (2*b*(-d)^(3/4)*g*p*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)^(1/
4)])/(3*e^(3/4)) - (b*d^(1/4)*f*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sq
rt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + (b*d^(1/4)*f*p*Log[Sqrt[d] + Sqrt[
2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + b*f*Sqrt[x]*L
og[c*(d + e*x^2)^p] + (b*g*x^(3/2)*Log[c*(d + e*x^2)^p])/3))/Sqrt[h*x]
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2917, 27, 2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}dx$$

$$\Big\downarrow \text{2917}$$

$$\frac{2\int \frac{(fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h}d\sqrt{hx}}{h}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int (fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{2921}$$

$$\frac{2\int \left(fh\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)+ghx\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)\right)d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{1}{3}g(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)+afh\sqrt{hx}-\frac{\sqrt{2}bd^{3/4}gh^{3/2}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}}+\frac{\sqrt{2}bd^{3/4}gh^{3/2}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.}{3e^{3/4}}\cdots\right.}{\cdots}$$

input
```
Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

output
```
(2*(a*f*h*Sqrt[h*x] - 4*b*f*h*p*Sqrt[h*x] - (4*b*g*p*(h*x)^(3/2))/9 - (Sqr
t[2]*b*d^(1/4)*f*h^(3/2)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/e^(1/4) - (Sqrt[2]*b*d^(3/4)*g*h^(3/2)*p*ArcTan[1 - (Sqrt[2]*e
^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)) + (Sqrt[2]*b*d^(1/4)*f*h
^(3/2)*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/e^(1/4
) + (Sqrt[2]*b*d^(3/4)*g*h^(3/2)*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/
(d^(1/4)*Sqrt[h])])/(3*e^(3/4)) + b*f*h*Sqrt[h*x]*Log[c*(d + e*x^2)^p] + (
g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/3 - (b*d^(1/4)*f*h^(3/2)*p*Log
[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sq
rt[2]*e^(1/4)) + (b*d^(3/4)*g*h^(3/2)*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(3*Sqrt[2]*e^(3/4)) + (b*d^(1/4)*f
*h^(3/2)*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*S
qrt[h*x]])/(Sqrt[2]*e^(1/4)) - (b*d^(3/4)*g*h^(3/2)*p*Log[Sqrt[d]*h + Sqrt
[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(3*Sqrt[2]*e^(3/4)))
)/h^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{\sqrt{hx}}dx$$

input
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

output
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1196 vs. $2(338) = 676$.

Time = 0.12 (sec) , antiderivative size = 1196, normalized size of antiderivative = 2.52

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="frica
s")
```

output
```
-2/9*(3*h*sqrt(-(6*b^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^
2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4
- b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*
b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) + 3*(9*b^2*e^2*f^3 - b^2*d
*e*f*g^2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18
*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))) - 3*h*sqrt(-(6*b
^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^
3*g^4)*p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(
h*x)*p^3 - 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 +
b^4*d^3*g^4)*p^4/(e^3*h^2)) + 3*(9*b^2*e^2*f^3 - b^2*d*e*f*g^2)*h*p^2)*sqr
t(-(6*b^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 +
 b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))) - 3*h*sqrt(-(6*b^2*d*f*g*p^2 - e*h*s
qrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)
))/(e*h))*log(-32*(81*b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(e^2*g
*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^
3*h^2)) - 3*(9*b^2*e^2*f^3 - b^2*d*e*f*g^2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2
- e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e
^3*h^2)))/(e*h))) + 3*h*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4*d*e^2*f
^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log(-32*(8
1*b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 - 32*(e^2*g*h^2*sqrt(-(81*b^...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(338) = 676$.

Time = 0.13 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.59

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="maxim
a")
```

output
```
2/3*b*g*x^2*log((e*x^2 + d)^p*c)/sqrt(h*x) + 2/3*a*g*x^2/sqrt(h*x) + 2*sqr
t(h*x)*b*f*log((e*x^2 + d)^p*c)/h - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^4*log(
sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2
)^(3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqr
t(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)*h^3*log(-(sq
rt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*
x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1
/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h
^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4)
- 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)
^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)))
*d/e)*b*e*f*p/h^3 + 2*sqrt(h*x)*a*f/h - 1/9*(3*d*h^4*(sqrt(2)*log(sqrt(e)*
h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*
e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/
4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt
(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqr
t(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x
)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqr
t(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e
))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + ...
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.17

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="giac"
)
```

output
```
1/9*(6*sqrt(h*x)*b*g*x*log(c) + 9*(e*(2*sqrt(2)*(d*e^3*h^2)^(1/4)*arctan(1
/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^2 +
2*sqrt(2)*(d*e^3*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) -
 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^2 + sqrt(2)*(d*e^3*h^2)^(1/4)*log(h*x + s
qrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^2 - sqrt(2)*(d*e^3*h^2
)^(1/4)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^2 -
 8*sqrt(h*x)/e) + 2*sqrt(h*x)*log(e*x^2 + d))*b*f*p + 6*sqrt(h*x)*a*g*x +
18*sqrt(h*x)*b*f*log(c) + (6*sqrt(h*x)*h*x*log(e*x^2 + d) - (8*sqrt(h*x)*h
*x/e - 6*sqrt(2)*(d*e^3*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(
1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^4 - 6*sqrt(2)*(d*e^3*h^2)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/
e^4 + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h
*x) + sqrt(d*h^2/e))/e^4 - 3*sqrt(2)*(d*e^3*h^2)^(3/4)*log(h*x - sqrt(2)*(
d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^4)*e)*b*g*p/h + 18*sqrt(h*x)*a
*f)/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{\sqrt{h\,x}}\,dx$$

input
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2),x)
```

output
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.76

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx$$

$$=\frac{\sqrt{h}\left(-6e^{\frac{1}{4}}d^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{e}}{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}}\right)bgp-18e^{\frac{3}{4}}d^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{e}}{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}}\right)bfp+6e^{\frac{1}{4}}d^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{e\right.}{}\right.}{}$$

input
```
int((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

output
```
(sqrt(h)*( - 6*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*g*p - 18*e**(3/4)*d**(1
/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)
*d**(1/4)*sqrt(2)))*b*f*p + 6*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**
(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*g*p + 18
*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqr
t(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*f*p + 6*e**(1/4)*d**(3/4)*sqrt(2)*log
( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*g*p - 3*e**
(1/4)*d**(3/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*g - 18*e**(3/4)*d**(1/4)*s
qrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*f
*p + 9*e**(3/4)*d**(1/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*f + 18*sqrt(x)*l
og((d + e*x**2)**p*c)*b*e*f + 6*sqrt(x)*log((d + e*x**2)**p*c)*b*e*g*x + 1
8*sqrt(x)*a*e*f + 6*sqrt(x)*a*e*g*x - 72*sqrt(x)*b*e*f*p - 8*sqrt(x)*b*e*g
*p*x))/(9*e*h)
```

3.607 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 449

$$\begin{aligned}
\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx &= \frac{2ag\sqrt{hx}}{h^2}-\frac{8bgp\sqrt{hx}}{h^2} \\
&-\frac{2\sqrt{2}b\sqrt[4]{e}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}-\frac{2\sqrt{2}b\sqrt[4]{d}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}} \\
&+\frac{2\sqrt{2}b\sqrt[4]{e}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}+\frac{2\sqrt{2}b\sqrt[4]{d}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}} \\
&-\frac{2\sqrt{2}b\sqrt[4]{e}fp\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{d}h^{3/2}}+\frac{2\sqrt{2}b\sqrt[4]{d}gp\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{e}h^{3/2}} \\
&+\frac{2bg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2}-\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}
\end{aligned}$$

output
```
2*a*g*(h*x)^(1/2)/h^2-8*b*g*p*(h*x)^(1/2)/h^2-2*2^(1/2)*b*e^(1/4)*f*p*arct
an(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(1/4)/h^(3/2)-2*2^(1/2
)*b*d^(1/4)*g*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1
/4)/h^(3/2)+2*2^(1/2)*b*e^(1/4)*f*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d
^(1/4)/h^(1/2))/d^(1/4)/h^(3/2)+2*2^(1/2)*b*d^(1/4)*g*p*arctan(1+2^(1/2)*e
^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1/4)/h^(3/2)-2*2^(1/2)*b*e^(1/4)*f*
p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))
/d^(1/4)/h^(3/2)+2*2^(1/2)*b*d^(1/4)*g*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(
h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/e^(1/4)/h^(3/2)+2*b*g*(h*x)^(1/2)*
ln(c*(e*x^2+d)^p)/h^2-2*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.74

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2x^{3/2}\left(ag\sqrt{x}-4bgp\sqrt{x}-\frac{\sqrt{2}b\sqrt[4]{d}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}+\frac{\sqrt{2}b\sqrt[4]{d}\cdots}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

output
```
(2*x^(3/2)*(a*g*Sqrt[x] - 4*b*g*p*Sqrt[x] - (Sqrt[2]*b*d^(1/4)*g*p*ArcTan[
1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*g*p*A
rcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (2*b*e^(1/4)*f*p*(
ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5
/4)]))/(-d)^(1/4) - (b*d^(1/4)*g*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*S
qrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + (b*d^(1/4)*g*p*Log[Sqrt[d] + Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + b*g*Sqrt[x]*
Log[c*(d + e*x^2)^p] - (f*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[x]))/(h*x)^(3
/2)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}dx$$

$$\Big\downarrow \text{2917}$$

$$\frac{2\int \frac{(fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^2x}d\sqrt{hx}}{h}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{(fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{hx}d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{2926}$$

$$\frac{2\int \left(g\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)+\frac{f\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{x}\right)d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{fh\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}+ag\sqrt{hx}-\frac{\sqrt{2}b\sqrt[4]{e}f\sqrt{h}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}}+\frac{\sqrt{2}b\sqrt[4]{e}f\sqrt{h}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{d}}-\frac{\sqrt{2}b\sqrt[4]{\ldots}}{\ldots}\right.$$

input
```
Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

output
```
(2*(a*g*Sqrt[h*x] - 4*b*g*p*Sqrt[h*x] - (Sqrt[2]*b*e^(1/4)*f*Sqrt[h]*p*Arc
Tan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/d^(1/4) - (Sqrt[2]
*b*d^(1/4)*g*Sqrt[h]*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqr
t[h])])/e^(1/4) + (Sqrt[2]*b*e^(1/4)*f*Sqrt[h]*p*ArcTan[1 + (Sqrt[2]*e^(1/
4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/d^(1/4) + (Sqrt[2]*b*d^(1/4)*g*Sqrt[h]*p
*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/e^(1/4) + b*g*
Sqrt[h*x]*Log[c*(d + e*x^2)^p] - (f*h*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h
*x] + (b*e^(1/4)*f*Sqrt[h]*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)
*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sqrt[2]*d^(1/4)) - (b*d^(1/4)*g*Sqrt[h]*p*Lo
g[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(S
qrt[2]*e^(1/4)) - (b*e^(1/4)*f*Sqrt[h]*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqr
t[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sqrt[2]*d^(1/4)) + (b*d^(1/4)*g*
Sqrt[h]*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sq
rt[h*x]])/(Sqrt[2]*e^(1/4))))/h^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{3}{2}}}dx$$

input
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

output
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(325) = 650$.

Time = 0.12 (sec) , antiderivative size = 1162, normalized size of antiderivative = 2.59

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="frica
s")
```

output
```
2*(h^2*x*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2
 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^3*e^2*f^4 - b^3*d^2*g^4)*s
qrt(h*x)*p^3 + 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*
d^2*g^4)*p^4/(d*e*h^6)) - (b^2*d*e*f^2*g - b^2*d^2*g^3)*h^2*p^2)*sqrt(-(2*
b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^
4/(d*e*h^6)))/h^3)) - h^2*x*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4
- 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^3*e^2*f
^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 - 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 - 2*b^4
*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) - (b^2*d*e*f^2*g - b^2*d^2*g^3)
*h^2*p^2)*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^
2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)) - h^2*x*sqrt(-(2*b^2*f*g*p^2 - h^3*
sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)
*log(-32*(b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(d*e*f*h^5*sqrt(-(
b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) + (b^2*d*e*f
^2*g - b^2*d^2*g^3)*h^2*p^2)*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*e^2*f^4
 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)) + h^2*x*sqrt(-(2
*b^2*f*g*p^2 - h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p
^4/(d*e*h^6)))/h^3)*log(-32*(b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 - 32
*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e
*h^6)) + (b^2*d*e*f^2*g - b^2*d^2*g^3)*h^2*p^2)*sqrt(-(2*b^2*f*g*p^2 - ...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(325) = 650$.

Time = 0.13 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.63

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="maxim
a")
```

output
```
-b*e*f*p*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4
) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)
*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sq
rt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/
4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h
^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e
)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4
)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(
2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)
*sqrt(e)))/h + 2*b*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(3/2) + 2*a*g*x^2/(h*x
)^(3/2) - 2*b*f*log((e*x^2 + d)^p*c)/(sqrt(h*x)*h) - (8*sqrt(h*x)*h^2/e -
(sqrt(2)*h^4*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + s
qrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*
(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqr
t(2)*h^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^
(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(
d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqr
t(d)) + sqrt(2)*h^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^
2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h)
+ sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*...
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.00

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx =$$

$$-\frac{2\left(\frac{bfp}{\sqrt{hx}}-\frac{\sqrt{hx}bgp}{h}\right)\log\left(eh^2x^2+dh^2\right)-\frac{2\left(bfp\log\left(h^2\right)-bf\log(c)-af\right)}{\sqrt{hx}}+\frac{2\left(bgp\log\left(h^2\right)+4\,bgp-bg\log(c)-ag\right)\sqrt{hx}}{h}-\frac{2\left(\sqrt{2}(d\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="giac"
)
```

output
```
-(2*(b*f*p/sqrt(h*x) - sqrt(h*x)*b*g*p/h)*log(e*h^2*x^2 + d*h^2) - 2*(b*f*
p*log(h^2) - b*f*log(c) - a*f)/sqrt(h*x) + 2*(b*g*p*log(h^2) + 4*b*g*p - b
*g*log(c) - a*g)*sqrt(h*x)/h - 2*(sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g*h*p +
sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/
4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h^2) - 2*(sqrt(2)*(d*e^3*h^2)^(1
/4)*b*d*e*g*h*p + sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h^2) - (sqrt(
2)*(d*e^3*h^2)^(1/4)*b*d*e*g*h*p - sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*log(h*
x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d*e^2*h^2) + (sqrt
(2)*(d*e^3*h^2)^(1/4)*b*d*e*g*h*p - sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*log(h
*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d*e^2*h^2))/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{3/2}}\,dx$$

input
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2),x)
```

output
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{\sqrt{h}\left(-2\sqrt{x}\,e^{\frac{5}{4}}d^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{e}}{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}}\right)bfp-2\sqrt{x}\,e^{\frac{3}{4}}d^{\frac{5}{4}}\sqrt{2}\right.}{}$$

input
```
int((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

output
```
(sqrt(h)*( - 2*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f*p - 2*sqrt(
x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x)*s
qrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d*g*p + 2*sqrt(x)*e**(1/4)*d**(3/4)
*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d*
*(1/4)*sqrt(2)))*b*e*f*p + 2*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1
/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d
*g*p + 2*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4
)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*e*f*p - sqrt(x)*e**(1/4)*d**(3/4)*sqrt(
2)*log((d + e*x**2)**p*c)*b*e*f - 2*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log(
 - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*d*g*p + sqrt
(x)*e**(3/4)*d**(1/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*d*g - 2*log((d + e*
x**2)**p*c)*b*d*e*f + 2*log((d + e*x**2)**p*c)*b*d*e*g*x - 2*a*d*e*f + 2*a
*d*e*g*x - 8*b*d*e*g*p*x))/(sqrt(x)*d*e*h**2)
```

3.608 $\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 431

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = -\frac{2\sqrt{2}be^{3/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} - \frac{2\sqrt{2}b\sqrt[4]{e}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} + \frac{2\sqrt{2}be^{3/4}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} + \frac{2\sqrt{2}be^{3/4}fp\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{3d^{3/4}h^{5/2}} - \frac{2\sqrt{2}b\sqrt[4]{e}gp\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{d}h^{5/2}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}}$$

output
```
-2/3*2^(1/2)*b*e^(3/4)*f*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^
(1/2))/d^(3/4)/h^(5/2)-2*2^(1/2)*b*e^(1/4)*g*p*arctan(1-2^(1/2)*e^(1/4)*(h
*x)^(1/2)/d^(1/4)/h^(1/2))/d^(1/4)/h^(5/2)+2/3*2^(1/2)*b*e^(3/4)*f*p*arcta
n(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(3/4)/h^(5/2)+2*2^(1/2)
*b*e^(1/4)*g*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(1/
4)/h^(5/2)+2/3*2^(1/2)*b*e^(3/4)*f*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)
^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(3/4)/h^(5/2)-2*2^(1/2)*b*e^(1/4)*g*
p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))
/d^(1/4)/h^(5/2)-2/3*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(3/2)-2*g*(a+b*ln(c
*(e*x^2+d)^p))/h^2/(h*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.63

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2x^{5/2}\left(\frac{2b\sqrt[4]{e}gp\left(\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+\operatorname{arctanh}\left(\frac{d\sqrt[4]{e}\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}} - \frac{be^{3/4}fp\left(2\operatorname{arc}\right.}{}\right.}{}$$

input
```
Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

output
```
(2*x^(5/2)*((2*b*e^(1/4)*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTa
nh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*e^(3/4)*f*p*(2*ArcTan
[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sq
rt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*
x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(3*Sqrt[
2]*d^(3/4)) - (f*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g*(a + b*Log
[c*(d + e*x^2)^p]))/Sqrt[x]))/(h*x)^(5/2)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}dx$$

$$\Big\downarrow \text{2917}$$

$$\frac{2\int \frac{(fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^3x^2}d\sqrt{hx}}{h}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{(fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^2x^2}d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{2926}$$

$$\frac{2\int \left(\frac{g\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{hx}+\frac{f\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{hx^2}\right)d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{fh\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3(hx)^{3/2}}-\frac{g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}-\frac{\sqrt{2}be^{3/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}\sqrt{h}}+\frac{\sqrt{2}be^{3/4}fp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{3d^{3/4}\sqrt{h}} \right.$$

input
```
Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

output
```
(2*(-1/3*(Sqrt[2]*b*e^(3/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^
(1/4)*Sqrt[h])])/(d^(3/4)*Sqrt[h]) - (Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 - (Sq
rt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*Sqrt[h]) + (Sqrt[2]*
b*e^(3/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(
3*d^(3/4)*Sqrt[h]) + (Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sq
rt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*Sqrt[h]) - (f*h*(a + b*Log[c*(d + e*
x^2)^p]))/(3*(h*x)^(3/2)) - (g*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x] - (
b*e^(3/4)*f*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h
]*Sqrt[h*x]])/(3*Sqrt[2]*d^(3/4)*Sqrt[h]) + (b*e^(1/4)*g*p*Log[Sqrt[d]*h +
 Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sqrt[2]*d^(1/4
)*Sqrt[h]) + (b*e^(3/4)*f*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*
e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(3*Sqrt[2]*d^(3/4)*Sqrt[h]) - (b*e^(1/4)*g*p*L
og[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(
Sqrt[2]*d^(1/4)*Sqrt[h])))/h^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{5}{2}}}dx$$

input
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

output
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(303) = 606$.

Time = 0.13 (sec) , antiderivative size = 1236, normalized size of antiderivative = 2.87

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="frica
s")
```

output
```
-2/3*(h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d
*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^
3*f^4 - 81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 + 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f
^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) + (b^2*d*e^2
*f^3 - 9*b^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b
^4*e^3*f^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*
h^5))) - h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^
4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3
*e^3*f^4 - 81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 - 32*(3*d^3*g*h^8*sqrt(-(b^4*e^
3*f^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) + (b^2*d*
e^2*f^3 - 9*b^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(
-(b^4*e^3*f^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/
(d*h^5))) - h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 18
*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(
b^3*e^3*f^4 - 81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 + 32*(3*d^3*g*h^8*sqrt(-(b^4
*e^3*f^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) - (b^2
*d*e^2*f^3 - 9*b^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sq
rt(-(b^4*e^3*f^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)
))/(d*h^5))) + h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 -
 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(303) = 606$.

Time = 0.13 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.64

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="maxim
a")
```

output
```
-b*e*g*p*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4
) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)
*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sq
rt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/
4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h
^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e
)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4
)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(
2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)
*sqrt(e)))/h^2 - 2*b*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(5/2) + 1/3*(sqrt(2)
*h^2*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h
)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^
(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)*h*l
og(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*
sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/
4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)) + sq
rt(2)*h*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(
1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d
*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt
(d)))*b*e*f*p/h^3 - 2*a*g*x^2/(h*x)^(5/2) - 2/3*b*f*log((e*x^2 + d)^p*c...
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx =$$

$$-\frac{\frac{2(3bghpx+bfhp)\log(eh^2x^2+dh^2)}{\sqrt{hx}h^2x}-\frac{2(3bghpx\log(h^2)+bfhp\log(h^2)-3bghx\log(c)-3aghx-bfh\log(c)-afh)}{\sqrt{hx}h^2x}-\frac{2\left(\sqrt{2}(de^3h^2)^{\frac{1}{4}}be^2fhp\right.}{}}{}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="giac"
)
```

output
```
-1/3*(2*(3*b*g*h*p*x + b*f*h*p)*log(e*h^2*x^2 + d*h^2)/(sqrt(h*x)*h^2*x) -
 2*(3*b*g*h*p*x*log(h^2) + b*f*h*p*log(h^2) - 3*b*g*h*x*log(c) - 3*a*g*h*x
 - b*f*h*log(c) - a*f*h)/(sqrt(h*x)*h^2*x) - 2*(sqrt(2)*(d*e^3*h^2)^(1/4)*
b*e^2*f*h*p + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*arctan(1/2*sqrt(2)*(sqrt(
2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h^3) - 2*(sqrt(2
)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d
*e^2*h^3) - (sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p - 3*sqrt(2)*(d*e^3*h^2)
^(3/4)*b*g*p)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))
/(d*e^2*h^3) + (sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p - 3*sqrt(2)*(d*e^3*h
^2)^(3/4)*b*g*p)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/
e))/(d*e^2*h^3))/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{5/2}}\,dx$$

input
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2),x)
```

output
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{\sqrt{h}\left(-6\sqrt{x}\,e^{\frac{1}{4}}d^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{e}}{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}}\right)bgpx-2\sqrt{x}\,e^{\frac{3}{4}}d^{\frac{1}{4}}\sqrt{}\right.}{}$$

input
```
int((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

output
```
(sqrt(h)*( - 6*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*g*p*x - 2*sqrt(
x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x)*s
qrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*f*p*x + 6*sqrt(x)*e**(1/4)*d**(3/4)
*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d*
*(1/4)*sqrt(2)))*b*g*p*x + 2*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1
/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*f
*p*x + 6*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4
)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*g*p*x - 3*sqrt(x)*e**(1/4)*d**(3/4)*sqr
t(2)*log((d + e*x**2)**p*c)*b*g*x - 2*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*lo
g( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*f*p*x + sq
rt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*f*x - 2*log((d +
e*x**2)**p*c)*b*d*f - 6*log((d + e*x**2)**p*c)*b*d*g*x - 2*a*d*f - 6*a*d*g
*x))/(3*sqrt(x)*d*h**3*x)
```

3.609 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 460

$$
\begin{aligned}
\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx &= -\frac{8befp}{5dh^3\sqrt{hx}} \\
&+\frac{2\sqrt{2}be^{5/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}-\frac{2\sqrt{2}be^{3/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} \\
&-\frac{2\sqrt{2}be^{5/4}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}+\frac{2\sqrt{2}be^{3/4}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} \\
&+\frac{2\sqrt{2}be^{5/4}fp\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{5d^{5/4}h^{7/2}}+\frac{2\sqrt{2}be^{3/4}gp\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{3d^{3/4}h^{7/2}} \\
&-\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}}-\frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}}
\end{aligned}
$$

output
```
-8/5*b*e*f*p/d/h^3/(h*x)^(1/2)+2/5*2^(1/2)*b*e^(5/4)*f*p*arctan(1-2^(1/2)*
e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(5/4)/h^(7/2)-2/3*2^(1/2)*b*e^(3/4)
*g*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(3/4)/h^(7/2)
-2/5*2^(1/2)*b*e^(5/4)*f*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^
(1/2))/d^(5/4)/h^(7/2)+2/3*2^(1/2)*b*e^(3/4)*g*p*arctan(1+2^(1/2)*e^(1/4)*
(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(3/4)/h^(7/2)+2/5*2^(1/2)*b*e^(5/4)*f*p*arc
tanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(5
/4)/h^(7/2)+2/3*2^(1/2)*b*e^(3/4)*g*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x
)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(3/4)/h^(7/2)-2/5*f*(a+b*ln(c*(e*x^
2+d)^p))/h/(h*x)^(5/2)-2/3*g*(a+b*ln(c*(e*x^2+d)^p))/h^2/(h*x)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.51

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{x\left(-24befpx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\frac{ex^2}{d}\right)-5\sqrt{2}b\sqrt[4]{c}\right.}{}$$

input
```
Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

output
```
(x*(-24*b*e*f*p*x^2*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] - 5*Sqrt
[2]*b*d^(1/4)*e^(3/4)*g*p*x^(5/2)*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/
d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] -
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]) - 6*d*f*(a + b*Log[c*(d + e*x^2)^p]) - 1
0*d*g*x*(a + b*Log[c*(d + e*x^2)^p])))/(15*d*(h*x)^(7/2))
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}dx$$

$$\Big\downarrow \text{ 2917}$$

$$\frac{2\int \frac{(fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^4x^3}d\sqrt{hx}}{h}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\int \frac{(fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^3x^3}d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{ 2926}$$

$$\frac{2\int \left(\frac{g\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^2x^2}+\frac{f\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^2x^3}\right)d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{fh\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5(hx)^{5/2}}-\frac{g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3(hx)^{3/2}}+\frac{\sqrt{2}be^{5/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{3/2}}-\frac{\sqrt{2}be^{5/4}fp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{5d^{5/4}h^{3/2}}\right.$$

input
```
Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

output
```
(2*((-4*b*e*f*p)/(5*d*h*Sqrt[h*x]) + (Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 - (Sq
rt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(3/2)) - (Sqrt[2
]*b*e^(3/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])
/(3*d^(3/4)*h^(3/2)) - (Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*
Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(3/2)) + (Sqrt[2]*b*e^(3/4)*g*
p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^
(3/2)) - (f*h*(a + b*Log[c*(d + e*x^2)^p]))/(5*(h*x)^(5/2)) - (g*(a + b*Lo
g[c*(d + e*x^2)^p]))/(3*(h*x)^(3/2)) - (b*e^(5/4)*f*p*Log[Sqrt[d]*h + Sqrt
[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(5*Sqrt[2]*d^(5/4)*h
^(3/2)) - (b*e^(3/4)*g*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(
1/4)*Sqrt[h]*Sqrt[h*x]])/(3*Sqrt[2]*d^(3/4)*h^(3/2)) + (b*e^(5/4)*f*p*Log[
Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(5*S
qrt[2]*d^(5/4)*h^(3/2)) + (b*e^(3/4)*g*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqr
t[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(3*Sqrt[2]*d^(3/4)*h^(3/2))))/h^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{7}{2}}}dx$$

input
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

output
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. $2(320) = 640$.

Time = 0.16 (sec) , antiderivative size = 1348, normalized size of antiderivative = 2.93

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="frica
s")
```

output
```
2/15*(d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^
2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*
log(-32*(81*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 + 32*(3*d^4*f
*h^11*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)
*p^4/(d^5*h^14)) - 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqr
t((d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3
*g^4)*p^4/(d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt((
d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^
4)*p^4/(d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*log(-32*(81*b^3*e^4*f^
4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 - 32*(3*d^4*f*h^11*sqrt(-(81*b^4*e^
5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 5*(
9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b
^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14))
+ 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4
*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) -
30*b^2*e^2*f*g*p^2)/(d^2*h^7))*log(-32*(81*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g
^4)*sqrt(h*x)*p^3 + 32*(3*d^4*f*h^11*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4
*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 5*(9*b^2*d^2*e^2*f^2*g -
 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4
*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. 2(320) = 640.

Time = 0.14 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.57

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="maxim
a")
```

output
```
1/5*b*e*f*p*(e*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*
e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - s
qrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)
) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)
*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2
)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*
sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2
)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) +
 sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt
(e)*h)*sqrt(e)))/d - 8/(sqrt(h*x)*d))/h^3 - 2/3*b*g*x^2*log((e*x^2 + d)^p*
c)/(h*x)^(7/2) + 1/3*(sqrt(2)*h^2*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*
sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(s
qrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)
^(3/4)*e^(1/4)) + sqrt(2)*h*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(
2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqr
t(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqr
t(d)*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h
) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sq
rt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(
sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)))*b*e*g*p/h^4 - 2/3*a*g*x^2/(h*x)^(7/2...
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2\left(5\sqrt{2}(de^3h^2)^{\frac{1}{4}}bdeghp-3\sqrt{2}(de^3h^2)^{\frac{3}{4}}bfp\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}+2\sqrt{hx}\right)}{2\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}}\right)}{d^2eh} +$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="giac"
)
```

output
```
1/15*(2*(5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g*h*p - 3*sqrt(2)*(d*e^3*h^2)^(
3/4)*b*f*p)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*
h^2/e)^(1/4))/(d^2*e*h) + 2*(5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g*h*p - 3*s
qrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/
4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d^2*e*h) + (5*sqrt(2)*(d*e^3*h^2)^(1/4
)*b*d*e*g*h*p + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*log(h*x + sqrt(2)*(d*h^
2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h) - (5*sqrt(2)*(d*e^3*h^2)^(
1/4)*b*d*e*g*h*p + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*log(h*x - sqrt(2)*(d
*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h) - 2*(5*b*g*h^3*p*x + 3*
b*f*h^3*p)*log(e*h^2*x^2 + d*h^2)/(sqrt(h*x)*h^2*x^2) - 2*(12*b*e*f*h^3*p*
x^2 - 5*b*d*g*h^3*p*x*log(h^2) - 3*b*d*f*h^3*p*log(h^2) + 5*b*d*g*h^3*x*lo
g(c) + 5*a*d*g*h^3*x + 3*b*d*f*h^3*log(c) + 3*a*d*f*h^3)/(sqrt(h*x)*d*h^2*
x^2))/h^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,(e\,x^2+d)^p\right)\right)}{(h\,x)^{7/2}}\,dx$$

input
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2),x)
```

output
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.87

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{\sqrt{h}\left(6\sqrt{x}\,e^{\frac{5}{4}}d^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{e}}{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}}\right)bfp\,x^2-10\sqrt{x}\,e^{\frac{3}{4}}d^{\frac{5}{4}}\sqrt{}\right.}{}$$

input
```
int((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

output
```
(sqrt(h)*(6*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt
(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f*p*x**2 - 10*sq
rt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d*g*p*x**2 - 6*sqrt(x)*e**(1/4)*
d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**
(1/4)*d**(1/4)*sqrt(2)))*b*e*f*p*x**2 + 10*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(
2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)
*sqrt(2)))*b*d*g*p*x**2 - 6*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log( - sqrt(
x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*e*f*p*x**2 + 3*sqrt(
x)*e**(1/4)*d**(3/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*e*f*x**2 - 10*sqrt(x
)*e**(3/4)*d**(1/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqr
t(d) + sqrt(e)*x)*b*d*g*p*x**2 + 5*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log((
d + e*x**2)**p*c)*b*d*g*x**2 - 6*log((d + e*x**2)**p*c)*b*d**2*f - 10*log(
(d + e*x**2)**p*c)*b*d**2*g*x - 6*a*d**2*f - 10*a*d**2*g*x - 24*b*d*e*f*p*
x**2))/(15*sqrt(x)*d**2*h**4*x**2)
```

3.610 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 481

$$\begin{aligned}
\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx &= -\frac{8befp}{21dh^3(hx)^{3/2}}-\frac{8begp}{5dh^4\sqrt{hx}}\\
&+\frac{2\sqrt{2}be^{7/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}+\frac{2\sqrt{2}be^{5/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}\\
&-\frac{2\sqrt{2}be^{7/4}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}-\frac{2\sqrt{2}be^{5/4}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}\\
&-\frac{2\sqrt{2}be^{7/4}fp\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{7d^{7/4}h^{9/2}}+\frac{2\sqrt{2}be^{5/4}gp\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{5d^{5/4}h^{9/2}}\\
&-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7h(hx)^{7/2}}-\frac{2g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^2(hx)^{5/2}}
\end{aligned}$$

output
```
-8/21*b*e*f*p/d/h^3/(h*x)^(3/2)-8/5*b*e*g*p/d/h^4/(h*x)^(1/2)+2/7*2^(1/2)*
b*e^(7/4)*f*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(7/4
)/h^(9/2)+2/5*2^(1/2)*b*e^(5/4)*g*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d
^(1/4)/h^(1/2))/d^(5/4)/h^(9/2)-2/7*2^(1/2)*b*e^(7/4)*f*p*arctan(1+2^(1/2)
*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(7/4)/h^(9/2)-2/5*2^(1/2)*b*e^(5/4
)*g*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(5/4)/h^(9/2
)-2/7*2^(1/2)*b*e^(7/4)*f*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^
(1/2)/(d^(1/2)+e^(1/2)*x))/d^(7/4)/h^(9/2)+2/5*2^(1/2)*b*e^(5/4)*g*p*arcta
nh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(5/4
)/h^(9/2)-2/7*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(7/2)-2/5*g*(a+b*ln(c*(e*x
^2+d)^p))/h^2/(h*x)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.21

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = -\frac{2\sqrt{hx}\left(20befpx^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\frac{ex^2}{d}\right)+84begpx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\frac{ex^2}{d}\right)\right.}{105dh^5x^4}$$

input
```
Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

output
```
(-2*Sqrt[h*x]*(20*b*e*f*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)
] + 84*b*e*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] + 3*d*(5*
f + 7*g*x)*(a + b*Log[c*(d + e*x^2)^p])))/(105*d*h^5*x^4)
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}dx$$

$$\Big\downarrow \text{ 2917}$$

$$\frac{2\int \frac{(fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^5x^4}d\sqrt{hx}}{h}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\int \frac{(fh+gxh)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^4x^4}d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{ 2926}$$

$$\frac{2\int \left(\frac{g\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^3x^3}+\frac{f\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^3x^4}\right)d\sqrt{hx}}{h^2}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{fh\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{7(hx)^{7/2}}-\frac{g\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5(hx)^{5/2}}+\frac{\sqrt{2}be^{7/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{5/2}}-\frac{\sqrt{2}be^{7/4}fp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{7d^{7/4}h^{5/2}}\right.$$

input
```
Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

output
```
(2*((-4*b*e*f*p)/(21*d*h*(h*x)^(3/2)) - (4*b*e*g*p)/(5*d*h^2*Sqrt[h*x]) +
(Sqrt[2]*b*e^(7/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqr
t[h])])/(7*d^(7/4)*h^(5/2)) + (Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 - (Sqrt[2]*e
^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(5/2)) - (Sqrt[2]*b*e^(
7/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(
7/4)*h^(5/2)) - (Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*
x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(5/2)) - (f*h*(a + b*Log[c*(d + e*x^2
)^p]))/(7*(h*x)^(7/2)) - (g*(a + b*Log[c*(d + e*x^2)^p]))/(5*(h*x)^(5/2))
+ (b*e^(7/4)*f*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqr
t[h]*Sqrt[h*x]])/(7*Sqrt[2]*d^(7/4)*h^(5/2)) - (b*e^(5/4)*g*p*Log[Sqrt[d]*
h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(5*Sqrt[2]*d
^(5/4)*h^(5/2)) - (b*e^(7/4)*f*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(
1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(7*Sqrt[2]*d^(7/4)*h^(5/2)) + (b*e^(5/4)*
g*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x
]])/(5*Sqrt[2]*d^(5/4)*h^(5/2))))/h^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{9}{2}}}dx$$

input
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

output
```
int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. 2(337) = 674.

Time = 0.15 (sec) , antiderivative size = 1369, normalized size of antiderivative = 2.85

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="frica
s")
```

output
```
2/105*(3*d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*
f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3
*h^9))*log(-32*(625*b^3*e^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 + 32
*(7*d^6*g*h^14*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*
d^2*e^5*g^4)*p^4/(d^7*h^18)) + 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^
2)*h^5*p^2)*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2
 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9)))
 - 3*d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*
g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9
))*log(-32*(625*b^3*e^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 - 32*(7*
d^6*g*h^14*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*
e^5*g^4)*p^4/(d^7*h^18)) + 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h
^5*p^2)*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2
401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - 3
*d*h^5*x^4*sqrt((d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 +
 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*lo
g(-32*(625*b^3*e^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 + 32*(7*d^6*g
*h^14*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g
^4)*p^4/(d^7*h^18)) - 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^
2)*sqrt((d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(337) = 674.

Time = 0.14 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.54

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="maxim
a")
```

output
```
-1/21*b*e*f*p*(3*(sqrt(2)*e^(3/4)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*
sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*e^(3/4)*log(sqrt(e)
*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4)
+ sqrt(2)*e*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)
*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2
)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*
sqrt(d)*h) + sqrt(2)*e*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d
*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*
h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*
sqrt(e)*h)*sqrt(d)*h))/d + 8/((h*x)^(3/2)*d))/h^3 + 1/5*b*e*g*p*(e*(sqrt(2
)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(
(d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*s
qrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt
(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)
*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4
) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log
(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sq
rt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)
*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/d - 8
/(sqrt(h*x)*d))/h^4 - 2/5*b*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(9/2) - 2/...
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx =$$

$$-\frac{\frac{6\,(7\,bghpx+5\,bfhp)\log\left(eh^2x^2+dh^2\right)}{\sqrt{hx}h^4x^3}+\frac{6\left(5\,\sqrt{2}(de^3h^2)^{\frac{1}{4}}be^2fhp+7\,\sqrt{2}(de^3h^2)^{\frac{3}{4}}bgp\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}+2\,\sqrt{hx}\right)}{2\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}}\right)}{d^2eh^5}+\frac{6\left(5\,\sqrt{2}(de^3h\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="giac"
)
```

output
```
-1/105*(6*(7*b*g*h*p*x + 5*b*f*h*p)*log(e*h^2*x^2 + d*h^2)/(sqrt(h*x)*h^4*
x^3) + 6*(5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p + 7*sqrt(2)*(d*e^3*h^2)^
(3/4)*b*g*p)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d
*h^2/e)^(1/4))/(d^2*e*h^5) + 6*(5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p +
7*sqrt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^
(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d^2*e*h^5) + 3*(5*sqrt(2)*(d*e^3*h^
2)^(1/4)*b*e^2*f*h*p - 7*sqrt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*log(h*x + sqrt(2
)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h^5) - 3*(5*sqrt(2)*(d
*e^3*h^2)^(1/4)*b*e^2*f*h*p - 7*sqrt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*log(h*x -
 sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h^5) + 2*(84*b*
e*g*h^3*p*x^3 + 20*b*e*f*h^3*p*x^2 - 21*b*d*g*h^3*p*x*log(h^2) - 15*b*d*f*
h^3*p*log(h^2) + 21*b*d*g*h^3*x*log(c) + 21*a*d*g*h^3*x + 15*b*d*f*h^3*log
(c) + 15*a*d*f*h^3)/(sqrt(h*x)*d*h^6*x^3))/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{9/2}}\,dx$$

input
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2),x)
```

output
```
int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \frac{\sqrt{h}\left(42\sqrt{x}\,e^{\frac{5}{4}}d^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{e}}{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}}\right)bgp\,x^3+30\sqrt{x}\,e^{\frac{7}{4}}d^{\frac{1}{4}}}{}$$

input
```
int((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

output
```
(sqrt(h)*(42*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*g*p*x**3 + 30*s
qrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f*p*x**3 - 42*sqrt(x)*e**(1/4
)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e
**(1/4)*d**(1/4)*sqrt(2)))*b*e*g*p*x**3 - 30*sqrt(x)*e**(3/4)*d**(1/4)*sqr
t(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/
4)*sqrt(2)))*b*e*f*p*x**3 - 42*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log( - sq
rt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*e*g*p*x**3 + 21*s
qrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*e*g*x**3 + 30*sq
rt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) +
 sqrt(d) + sqrt(e)*x)*b*e*f*p*x**3 - 15*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*
log((d + e*x**2)**p*c)*b*e*f*x**3 - 30*log((d + e*x**2)**p*c)*b*d**2*f - 4
2*log((d + e*x**2)**p*c)*b*d**2*g*x - 30*a*d**2*f - 42*a*d**2*g*x - 40*b*d
*e*f*p*x**2 - 168*b*d*e*g*p*x**3))/(105*sqrt(x)*d**2*h**5*x**3)
```

3.611 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 760

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \frac{2af^2\sqrt{hx}}{h}-\frac{8bf^2p\sqrt{hx}}{h}+\frac{8bdg^2p\sqrt{hx}}{5eh}-\frac{16bfgp(hx)^{3/2}}{9h^2}-\frac{8bg^2p(hx)^{5/2}}{25h^3}-\frac{2\sqrt{2}b\sqrt[4]{d}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}-\frac{4\sqrt{2}bd^{3/4}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}+\frac{2\sqrt{2}bd^{5/4}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5e^{5/4}\sqrt{h}}+\frac{2\sqrt{2}b\sqrt[4]{d}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}+\frac{4\sqrt{2}bd^{3/4}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}-\frac{2\sqrt{2}bd^{5/4}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5e^{5/4}\sqrt{h}}+\frac{2\sqrt{2}b\sqrt[4]{d}f^2p\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{e}\sqrt{h}}-\frac{4\sqrt{2}bd^{3/4}fgp\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{3e^{3/4}\sqrt{h}}-\frac{2\sqrt{2}bd^{5/4}g^2p\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{5e^{5/4}\sqrt{h}}+\frac{2bf^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h}+\frac{4fg(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2}+\frac{2g^2(hx)^{5/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^3}$$

output
```
2*a*f^2*(h*x)^(1/2)/h-8*b*f^2*p*(h*x)^(1/2)/h+8/5*b*d*g^2*p*(h*x)^(1/2)/e/
h-16/9*b*f*g*p*(h*x)^(3/2)/h^2-8/25*b*g^2*p*(h*x)^(5/2)/h^3-2*2^(1/2)*b*d^
(1/4)*f^2*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1/4)/
h^(1/2)-4/3*2^(1/2)*b*d^(3/4)*f*g*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d
^(1/4)/h^(1/2))/e^(3/4)/h^(1/2)+2/5*2^(1/2)*b*d^(5/4)*g^2*p*arctan(1-2^(1/
2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(5/4)/h^(1/2)+2*2^(1/2)*b*d^(1/4
)*f^2*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1/4)/h^(1
/2)+4/3*2^(1/2)*b*d^(3/4)*f*g*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/
4)/h^(1/2))/e^(3/4)/h^(1/2)-2/5*2^(1/2)*b*d^(5/4)*g^2*p*arctan(1+2^(1/2)*e
^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(5/4)/h^(1/2)+2*2^(1/2)*b*d^(1/4)*f^
2*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x
))/e^(1/4)/h^(1/2)-4/3*2^(1/2)*b*d^(3/4)*f*g*p*arctanh(2^(1/2)*d^(1/4)*e^(
1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/e^(3/4)/h^(1/2)-2/5*2^(1/2)*
b*d^(5/4)*g^2*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/
2)+e^(1/2)*x))/e^(5/4)/h^(1/2)+2*b*f^2*(h*x)^(1/2)*ln(c*(e*x^2+d)^p)/h+4/3
*f*g*(h*x)^(3/2)*(a+b*ln(c*(e*x^2+d)^p))/h^2+2/5*g^2*(h*x)^(5/2)*(a+b*ln(c
*(e*x^2+d)^p))/h^3
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 628, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx$$

$$= \frac{2\sqrt{x}\left(af^2\sqrt{x}-4bf^2p\sqrt{x}+\frac{2}{3}afgx^{3/2}-\frac{8}{9}bfgpx^{3/2}-\frac{\sqrt{2}b\sqrt[4]{d}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}+\frac{\sqrt{2}b\sqrt[4]{d}f^2p\arctan\left(1+\cdots\right.}{\sqrt[4]{e}}\right.}{}$$

input
```
Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

output
```
(2*Sqrt[x]*(a*f^2*Sqrt[x] - 4*b*f^2*p*Sqrt[x] + (2*a*f*g*x^(3/2))/3 - (8*b
*f*g*p*x^(3/2))/9 - (Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*S
qrt[x])/d^(1/4)])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e
^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) - (4*b*(-d)^(3/4)*f*g*p*ArcTan[(e^(1/4)*
Sqrt[x])/(-d)^(1/4)])/(3*e^(3/4)) + (4*b*(-d)^(3/4)*f*g*p*ArcTanh[(e^(1/4)
*Sqrt[x])/(-d)^(1/4)])/(3*e^(3/4)) - (b*d^(1/4)*f^2*p*Log[Sqrt[d] - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + (b*d^(1/4)*f^2
*p*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^
(1/4)) - (b*g^2*p*(-40*d*e^(1/4)*Sqrt[x] + 8*e^(5/4)*x^(5/2) - 10*Sqrt[2]*
d^(5/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + 10*Sqrt[2]*d^(5/4)
*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 5*Sqrt[2]*d^(5/4)*Log[Sqr
t[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] + 5*Sqrt[2]*d^(5/4)*Lo
g[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(50*e^(5/4)) +
b*f^2*Sqrt[x]*Log[c*(d + e*x^2)^p] + (2*b*f*g*x^(3/2)*Log[c*(d + e*x^2)^p]
)/3 + (g^2*x^(5/2)*(a + b*Log[c*(d + e*x^2)^p]))/5))/Sqrt[h*x]
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2917, 27, 2921, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}dx$$

$$\Big\downarrow \ 2917$$

$$\frac{2\int \frac{(fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^2}d\sqrt{hx}}{h}$$

$$\Big\downarrow \ 27$$

$$\frac{2\int (fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)d\sqrt{hx}}{h^3}$$

$$\Big\downarrow \ 2921$$

$$\frac{2\int \left(f^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)h^2+g^2x^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)h^2+2fgx\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)h^2\right)d\sqrt{h}}{h^3}$$

$\downarrow$ 2009

$$2\left(-\frac{\sqrt{2}b\sqrt[4]{d}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)h^{5/2}}{\sqrt[4]{e}}+\frac{\sqrt{2}bd^{5/4}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)h^{5/2}}{5e^{5/4}}-\frac{2\sqrt{2}bd^{3/4}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)h^{5/2}}{3e^{3/4}}\right.$$

input
```
Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

output
```
(2*(a*f^2*h^2*Sqrt[h*x] - 4*b*f^2*h^2*p*Sqrt[h*x] + (4*b*d*g^2*h^2*p*Sqrt[
h*x])/(5*e) - (8*b*f*g*h*p*(h*x)^(3/2))/9 - (4*b*g^2*p*(h*x)^(5/2))/25 - (
Sqrt[2]*b*d^(1/4)*f^2*h^(5/2)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^
(1/4)*Sqrt[h])])/e^(1/4) - (2*Sqrt[2]*b*d^(3/4)*f*g*h^(5/2)*p*ArcTan[1 - (
Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)) + (Sqrt[2]*b*d^
(5/4)*g^2*h^(5/2)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h
])])/(5*e^(5/4)) + (Sqrt[2]*b*d^(1/4)*f^2*h^(5/2)*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/e^(1/4) + (2*Sqrt[2]*b*d^(3/4)*f*g*h^
(5/2)*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3
/4)) - (Sqrt[2]*b*d^(5/4)*g^2*h^(5/2)*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h
*x])/(d^(1/4)*Sqrt[h])])/(5*e^(5/4)) + b*f^2*h^2*Sqrt[h*x]*Log[c*(d + e*x^
2)^p] + (2*f*g*h*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/3 + (g^2*(h*x)^
(5/2)*(a + b*Log[c*(d + e*x^2)^p]))/5 - (b*d^(1/4)*f^2*h^(5/2)*p*Log[Sqrt[
d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sqrt[2]*
e^(1/4)) + (Sqrt[2]*b*d^(3/4)*f*g*h^(5/2)*p*Log[Sqrt[d]*h + Sqrt[e]*h*x -
Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(3*e^(3/4)) + (b*d^(5/4)*g^2*h
^(5/2)*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqr
t[h*x]])/(5*Sqrt[2]*e^(5/4)) + (b*d^(1/4)*f^2*h^(5/2)*p*Log[Sqrt[d]*h + Sq
rt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sqrt[2]*e^(1/4))
- (Sqrt[2]*b*d^(3/4)*f*g*h^(5/2)*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2921
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Maple [F]

$$\int \frac{(gx+f)^{2} \left(a+b\ln \left(c(e\,x^{2}+d)^{p}\right)\right)}{\sqrt{hx}}dx$$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

output
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2178 vs. $2(548) = 1096$.

Time = 0.14 (sec) , antiderivative size = 2178, normalized size of antiderivative = 2.87

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="fri
cas")
```

output
```
2/225*(15*e*h*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*
f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*
g^8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*l
og(16*(50625*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*
g^4 - 1620*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(10*e^4*
f*g*h^2*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4
*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2))
 + 3*(1125*b^2*e^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4
- 9*b^2*d^3*e*g^6)*h*p^2)*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*
b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 +
 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2
)/(e^2*h))) - 15*e*h*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d
^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b
^4*d^5*g^8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^
2*h))*log(16*(50625*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e
^2*f^4*g^4 - 1620*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(
10*e^4*f*g*h^2*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40
150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^
5*h^2)) + 3*(1125*b^2*e^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f
^2*g^4 - 9*b^2*d^3*e*g^6)*h*p^2)*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(548) = 1096$.

Time = 0.14 (sec) , antiderivative size = 1165, normalized size of antiderivative = 1.53

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="max
ima")
```

output
```
2/5*b*g^2*x^3*log((e*x^2 + d)^p*c)/sqrt(h*x) + 2/5*a*g^2*x^3/sqrt(h*x) + 4
/3*b*f*g*x^2*log((e*x^2 + d)^p*c)/sqrt(h*x) + 4/3*a*f*g*x^2/sqrt(h*x) + 2*
sqrt(h*x)*b*f^2*log((e*x^2 + d)^p*c)/h - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^4
*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((
d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4
)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)*h^3*log
(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sq
rt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)
*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)) + sqrt
(2)*h^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(
1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d
*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt
(d))*d/e)*b*e*f^2*p/h^3 + 2*sqrt(h*x)*a*f^2/h - 2/9*(3*d*h^4*(sqrt(2)*log
(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^
2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*s
qrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt
(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2
*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sq
rt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt...
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.18

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="gia
c")
```

output
```
1/225*(90*sqrt(h*x)*b*g^2*x^2*log(c) + 90*sqrt(h*x)*a*g^2*x^2 + 300*sqrt(h
*x)*b*f*g*x*log(c) + 225*(e*(2*sqrt(2)*(d*e^3*h^2)^(1/4)*arctan(1/2*sqrt(2
)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^2 + 2*sqrt(2)
*(d*e^3*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h
*x))/(d*h^2/e)^(1/4))/e^2 + sqrt(2)*(d*e^3*h^2)^(1/4)*log(h*x + sqrt(2)*(d
*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^2 - sqrt(2)*(d*e^3*h^2)^(1/4)*l
og(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^2 - 8*sqrt(h
*x)/e) + 2*sqrt(h*x)*log(e*x^2 + d))*b*f^2*p + 300*sqrt(h*x)*a*f*g*x + 450
*sqrt(h*x)*b*f^2*log(c) + 50*(6*sqrt(h*x)*h*x*log(e*x^2 + d) - (8*sqrt(h*x
)*h*x/e - 6*sqrt(2)*(d*e^3*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e
)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^4 - 6*sqrt(2)*(d*e^3*h^2)^(3/4)*
arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4
))/e^4 + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqr
t(h*x) + sqrt(d*h^2/e))/e^4 - 3*sqrt(2)*(d*e^3*h^2)^(3/4)*log(h*x - sqrt(2
)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^4)*e)*b*f*g*p/h + 450*sqrt(
h*x)*a*f^2 + 9*(10*sqrt(h*x)*h^2*x^2*log(e*x^2 + d) - (10*sqrt(2)*(d*e^3*h
^2)^(1/4)*d*h^2*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))
/(d*h^2/e)^(1/4))/e^3 + 10*sqrt(2)*(d*e^3*h^2)^(1/4)*d*h^2*arctan(-1/2*sqr
t(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^3 + 5*sqrt
(2)*(d*e^3*h^2)^(1/4)*d*h^2*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \int \frac{(f+g\,x)^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{\sqrt{h\,x}}\,dx$$

input
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2),x)
```

output
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 606, normalized size of antiderivative = 0.80

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

output
```
(sqrt(h)*( - 300*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2)
 - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f*g*p + 90*e**(3/4)
*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(e))/(e*
*(1/4)*d**(1/4)*sqrt(2)))*b*d*g**2*p - 450*e**(3/4)*d**(1/4)*sqrt(2)*atan(
(e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)
))*b*e*f**2*p + 300*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt
(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f*g*p - 90*e**(3
/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/
(e**(1/4)*d**(1/4)*sqrt(2)))*b*d*g**2*p + 450*e**(3/4)*d**(1/4)*sqrt(2)*at
an((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt
(2)))*b*e*f**2*p + 300*e**(1/4)*d**(3/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d
**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*e*f*g*p - 150*e**(1/4)*d**(3/4)*s
qrt(2)*log((d + e*x**2)**p*c)*b*e*f*g + 90*e**(3/4)*d**(1/4)*sqrt(2)*log(
- sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*d*g**2*p - 45
0*e**(3/4)*d**(1/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqr
t(d) + sqrt(e)*x)*b*e*f**2*p - 45*e**(3/4)*d**(1/4)*sqrt(2)*log((d + e*x**
2)**p*c)*b*d*g**2 + 225*e**(3/4)*d**(1/4)*sqrt(2)*log((d + e*x**2)**p*c)*b
*e*f**2 + 450*sqrt(x)*log((d + e*x**2)**p*c)*b*e**2*f**2 + 300*sqrt(x)*log
((d + e*x**2)**p*c)*b*e**2*f*g*x + 90*sqrt(x)*log((d + e*x**2)**p*c)*b*e**
2*g**2*x**2 + 450*sqrt(x)*a*e**2*f**2 + 300*sqrt(x)*a*e**2*f*g*x + 90*s...
```

3.612 $\int \frac{(f+gx)^2 \left(a+b \log \left(c \left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 709

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{4afg\sqrt{hx}}{h^2}-\frac{16bfgp\sqrt{hx}}{h^2}-\frac{8bg^2p(hx)^{3/2}}{9h^3}-\frac{2\sqrt{2}b\sqrt[4]{e}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}-\frac{4\sqrt{2}b\sqrt[4]{d}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}-\frac{2\sqrt{2}bd^{3/4}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}}+\frac{2\sqrt{2}b\sqrt[4]{e}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}+\frac{4\sqrt{2}b\sqrt[4]{d}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}+\frac{2\sqrt{2}bd^{3/4}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}}-\frac{2\sqrt{2}b\sqrt[4]{e}f^2p\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{d}h^{3/2}}+\frac{4\sqrt{2}b\sqrt[4]{d}fgp\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{e}h^{3/2}}-\frac{2\sqrt{2}bd^{3/4}g^2p\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{3e^{3/4}h^{3/2}}+\frac{4bfg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2}-\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}+\frac{2g^2(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3}$$

output
```
4*a*f*g*(h*x)^(1/2)/h^2-16*b*f*g*p*(h*x)^(1/2)/h^2-8/9*b*g^2*p*(h*x)^(3/2)
/h^3-2*2^(1/2)*b*e^(1/4)*f^2*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4
)/h^(1/2))/d^(1/4)/h^(3/2)-4*2^(1/2)*b*d^(1/4)*f*g*p*arctan(1-2^(1/2)*e^(1
/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1/4)/h^(3/2)-2/3*2^(1/2)*b*d^(3/4)*g^2
*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(3/4)/h^(3/2)+2
*2^(1/2)*b*e^(1/4)*f^2*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1
/2))/d^(1/4)/h^(3/2)+4*2^(1/2)*b*d^(1/4)*f*g*p*arctan(1+2^(1/2)*e^(1/4)*(h
*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1/4)/h^(3/2)+2/3*2^(1/2)*b*d^(3/4)*g^2*p*arc
tan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(3/4)/h^(3/2)-2*2^(1/
2)*b*e^(1/4)*f^2*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^
(1/2)+e^(1/2)*x))/d^(1/4)/h^(3/2)+4*2^(1/2)*b*d^(1/4)*f*g*p*arctanh(2^(1/2
)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/e^(1/4)/h^(3/2)
-2/3*2^(1/2)*b*d^(3/4)*g^2*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h
^(1/2)/(d^(1/2)+e^(1/2)*x))/e^(3/4)/h^(3/2)+4*b*f*g*(h*x)^(1/2)*ln(c*(e*x^
2+d)^p)/h^2-2*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(1/2)+2/3*g^2*(h*x)^(3/2
)*(a+b*ln(c*(e*x^2+d)^p))/h^3
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.67

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2x^{3/2}\left(2afg\sqrt{x}-8bfgp\sqrt{x}+\frac{1}{3}ag^2x^{3/2}-\frac{4}{9}bg^2px^{3/2}-\frac{2\sqrt{2}b\sqrt[4]{}}{}\right.}{}$$

input
```
Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

output
```
(2*x^(3/2)*(2*a*f*g*Sqrt[x] - 8*b*f*g*p*Sqrt[x] + (a*g^2*x^(3/2))/3 - (4*b
*g^2*p*x^(3/2))/9 - (2*Sqrt[2]*b*d^(1/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)
*Sqrt[x])/d^(1/4)])/e^(1/4) + (2*Sqrt[2]*b*d^(1/4)*f*g*p*ArcTan[1 + (Sqrt[
2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) - (2*b*(-d)^(3/4)*g^2*p*ArcTan[(e^(1
/4)*Sqrt[x])/(-d)^(1/4)])/(3*e^(3/4)) + (2*b*(-d)^(3/4)*g^2*p*ArcTanh[(e^(
1/4)*Sqrt[x])/(-d)^(1/4)])/(3*e^(3/4)) + (2*b*e^(1/4)*f^2*p*(ArcTan[(e^(1/
4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(
1/4) - (Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt
[x] + Sqrt[e]*x])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d] + Sqrt[2]
*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/e^(1/4) + 2*b*f*g*Sqrt[x]*Log[c*(d
+ e*x^2)^p] + (b*g^2*x^(3/2)*Log[c*(d + e*x^2)^p])/3 - (f^2*(a + b*Log[c*(
d + e*x^2)^p]))/Sqrt[x]))/(h*x)^(3/2)
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}dx$$

$$\Big\downarrow \text{2917}$$

$$\frac{2\int \frac{(fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^3x}d\sqrt{hx}}{h}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{(fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{hx}d\sqrt{hx}}{h^3}$$

$$\Big\downarrow \text{2926}$$

$$\frac{2\int \left(\frac{h\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{x}+2gh\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f+g^2hx\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)\right)d\sqrt{hx}}{h^3}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{\sqrt{2}b\sqrt[4]{e}h^{3/2}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{\sqrt[4]{d}}+\frac{\sqrt{2}b\sqrt[4]{e}h^{3/2}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{\sqrt[4]{d}}-\frac{h^2(a+b\log(c(ex^2+d)^p))f^2}{\sqrt{hx}}+\frac{b\sqrt[4]{e}h^{3/2}p\,l\ldots}{}\right.$$

input
```
Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

output
```
(2*(2*a*f*g*h*Sqrt[h*x] - 8*b*f*g*h*p*Sqrt[h*x] - (4*b*g^2*p*(h*x)^(3/2))/
9 - (Sqrt[2]*b*e^(1/4)*f^2*h^(3/2)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x]
)/(d^(1/4)*Sqrt[h])])/d^(1/4) - (2*Sqrt[2]*b*d^(1/4)*f*g*h^(3/2)*p*ArcTan[
1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/e^(1/4) - (Sqrt[2]*b*d
^(3/4)*g^2*h^(3/2)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(3*e^(3/4)) + (Sqrt[2]*b*e^(1/4)*f^2*h^(3/2)*p*ArcTan[1 + (Sqrt[2]*e
^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/d^(1/4) + (2*Sqrt[2]*b*d^(1/4)*f*g*h
^(3/2)*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/e^(1/4
) + (Sqrt[2]*b*d^(3/4)*g^2*h^(3/2)*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x]
)/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)) + 2*b*f*g*h*Sqrt[h*x]*Log[c*(d + e*x^2)^
p] - (f^2*h^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x] + (g^2*(h*x)^(3/2)*(
a + b*Log[c*(d + e*x^2)^p]))/3 + (b*e^(1/4)*f^2*h^(3/2)*p*Log[Sqrt[d]*h +
Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sqrt[2]*d^(1/4)
) - (Sqrt[2]*b*d^(1/4)*f*g*h^(3/2)*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]
*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/e^(1/4) + (b*d^(3/4)*g^2*h^(3/2)*p*Lo
g[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(3
*Sqrt[2]*e^(3/4)) - (b*e^(1/4)*f^2*h^(3/2)*p*Log[Sqrt[d]*h + Sqrt[e]*h*x +
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sqrt[2]*d^(1/4)) + (Sqrt[2]*
b*d^(1/4)*f*g*h^(3/2)*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h]*Sqrt[h*x]])/e^(1/4) - (b*d^(3/4)*g^2*h^(3/2)*p*Log[Sqrt[d]*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{(gx+f)^2 \left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{3}{2}}}dx$$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

output
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2118 vs. $2(513) = 1026$.

Time = 0.17 (sec) , antiderivative size = 2118, normalized size of antiderivative = 2.99

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="fri
cas")
```

output
```
-2/9*(3*h^2*x*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 +
 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*
h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*b^3*e^4*
f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3*e*f^2*
g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 + 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h^5*sqr
t(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*
b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 6*(9*b^2*d*e^3*f^5*g -
 30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt(-(e*h^3*sqrt(-(81
*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^
3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*
g^3)*p^2)/(e*h^3))) - 3*h^2*x*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4
*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*
g^8)*p^4/(d*e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log
(32*(81*b^3*e^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 1
2*b^3*d^3*e*f^2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 - 32*((3*d*e^3*f^2 + d^2*
e^2*g^2)*h^5*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e
^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 6*(9*b
^2*d*e^3*f^5*g - 30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt(-
(e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4
*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 12*(3*b^2...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1119 vs. 2(513) = 1026.

Time = 0.14 (sec) , antiderivative size = 1119, normalized size of antiderivative = 1.58

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="max
ima")
```

output
```
2/3*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(3/2) - b*e*f^2*p*(sqrt(2)*log(sq
rt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^
(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)
*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt
(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e)
)/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sq
rt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(
2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*
sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4)
 + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/h + 2/3*a*g^2
*x^3/(h*x)^(3/2) + 4*b*f*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(3/2) + 4*a*f*g*
x^2/(h*x)^(3/2) - 2*b*f^2*log((e*x^2 + d)^p*c)/(sqrt(h*x)*h) - 2*(8*sqrt(h
*x)*h^2/e - (sqrt(2)*h^4*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)
*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*
x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^
(1/4)) + sqrt(2)*h^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h
^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h)
 - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sq
rt(e)*h)*sqrt(d)) + sqrt(2)*h^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - s
qrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt...
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.81

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{6 \left(\frac{\sqrt{hx}bg^2px}{h} - \frac{3\,bf^2p}{\sqrt{hx}} + \frac{6\,\sqrt{hx}bfgp}{h}\right)\log\left(eh^2x^2+dh^2\right) - \frac{2\,(3\,bg^2p\log}{}}{}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="gia
c")
```

output
```
1/9*(6*(sqrt(h*x)*b*g^2*p*x/h - 3*b*f^2*p/sqrt(h*x) + 6*sqrt(h*x)*b*f*g*p/
h)*log(e*h^2*x^2 + d*h^2) - 2*(3*b*g^2*p*log(h^2) + 4*b*g^2*p - 3*b*g^2*lo
g(c) - 3*a*g^2)*sqrt(h*x)*x/h + 6*(6*sqrt(2)*b*d*e^2*f*g*h*p + 3*sqrt(2)*s
qrt(d*e)*b*e^2*f^2*h*p + sqrt(2)*sqrt(d*e)*b*d*e*g^2*h*p)*arctan(1/2*sqrt(
2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^3*h^2)^(3
/4) + 6*(6*sqrt(2)*b*d*e^2*f*g*h*p + 3*sqrt(2)*sqrt(d*e)*b*e^2*f^2*h*p + s
qrt(2)*sqrt(d*e)*b*d*e*g^2*h*p)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/
4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^3*h^2)^(3/4) + 3*(6*sqrt(2)*b*d*e^
2*f*g*h*p + 3*sqrt(2)*sqrt(d*e)*b*e^2*f^2*h*p + sqrt(2)*sqrt(d*e)*b*d*e*g^
2*h*p)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d*e^3
*h^2)^(3/4) - 3*(6*sqrt(2)*b*d*e^2*f*g*h*p - 3*sqrt(2)*sqrt(d*e)*b*e^2*f^2
*h*p - sqrt(2)*sqrt(d*e)*b*d*e*g^2*h*p)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*
sqrt(h*x) + sqrt(d*h^2/e))/(d*e^3*h^2)^(3/4) + 18*(b*f^2*p*log(h^2) - b*f^
2*log(c) - a*f^2)/sqrt(h*x) - 36*(b*f*g*p*log(h^2) + 4*b*f*g*p - b*f*g*log
(c) - a*f*g)*sqrt(h*x)/h)/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \int \frac{{\left(f+g\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{3/2}}\,dx$$

input
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2),x)
```

output
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 590, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

output
```
(sqrt(h)*( - 6*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d*g**2*p - 18*s
qrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f**2*p - 36*sqrt(x)*e**(3/4)*
d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(e))/(e**
(1/4)*d**(1/4)*sqrt(2)))*b*d*f*g*p + 6*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*a
tan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqr
t(2)))*b*d*g**2*p + 18*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d*
*(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f**2*
p + 36*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) +
 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d*f*g*p + 6*sqrt(x)*e**
(1/4)*d**(3/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d)
+ sqrt(e)*x)*b*d*g**2*p + 18*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log( - sqrt
(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*e*f**2*p - 3*sqrt(x
)*e**(1/4)*d**(3/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*d*g**2 - 9*sqrt(x)*e*
*(1/4)*d**(3/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*e*f**2 - 36*sqrt(x)*e**(3
/4)*d**(1/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) +
sqrt(e)*x)*b*d*f*g*p + 18*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log((d + e*x**
2)**p*c)*b*d*f*g - 18*log((d + e*x**2)**p*c)*b*d*e*f**2 + 36*log((d + e*x*
*2)**p*c)*b*d*e*f*g*x + 6*log((d + e*x**2)**p*c)*b*d*e*g**2*x**2 - 18*a...
```

3.613 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 692

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}be^{3/4}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} - \frac{4\sqrt{2}b\sqrt[4]{e}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} - \frac{2\sqrt{2}b\sqrt[4]{d}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{5/2}} + \frac{2\sqrt{2}be^{3/4}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} + \frac{4\sqrt{2}b\sqrt[4]{e}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} + \frac{2\sqrt{2}b\sqrt[4]{d}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{5/2}} + \frac{2\sqrt{2}be^{3/4}f^2p\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{3d^{3/4}h^{5/2}} - \frac{4\sqrt{2}b\sqrt[4]{e}fgp\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{d}h^{5/2}} + \frac{2\sqrt{2}b\sqrt[4]{d}g^2p\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{e}h^{5/2}} + \frac{2bg^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^3} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}}$$

output
```
2*a*g^2*(h*x)^(1/2)/h^3-8*b*g^2*p*(h*x)^(1/2)/h^3-2/3*2^(1/2)*b*e^(3/4)*f^
2*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(3/4)/h^(5/2)-
4*2^(1/2)*b*e^(1/4)*f*g*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(
1/2))/d^(1/4)/h^(5/2)-2*2^(1/2)*b*d^(1/4)*g^2*p*arctan(1-2^(1/2)*e^(1/4)*(
h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1/4)/h^(5/2)+2/3*2^(1/2)*b*e^(3/4)*f^2*p*ar
ctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(3/4)/h^(5/2)+4*2^(1
/2)*b*e^(1/4)*f*g*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/
d^(1/4)/h^(5/2)+2*2^(1/2)*b*d^(1/4)*g^2*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(
1/2)/d^(1/4)/h^(1/2))/e^(1/4)/h^(5/2)+2/3*2^(1/2)*b*e^(3/4)*f^2*p*arctanh(
2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(3/4)/h
^(5/2)-4*2^(1/2)*b*e^(1/4)*f*g*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/
2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(1/4)/h^(5/2)+2*2^(1/2)*b*d^(1/4)*g^2*p*
arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/e
^(1/4)/h^(5/2)+2*b*g^2*(h*x)^(1/2)*ln(c*(e*x^2+d)^p)/h^3-2/3*f^2*(a+b*ln(c
*(e*x^2+d)^p))/h/(h*x)^(3/2)-4*f*g*(a+b*ln(c*(e*x^2+d)^p))/h^2/(h*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.76

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2x^{5/2}\left(ag^2\sqrt{x}-4bg^2p\sqrt{x}-\frac{\sqrt{2}b\sqrt[4]{d}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}+\frac{\sqrt{2}b}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

output
```
(2*x^(5/2)*(a*g^2*Sqrt[x] - 4*b*g^2*p*Sqrt[x] - (Sqrt[2]*b*d^(1/4)*g^2*p*A
rcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (Sqrt[2]*b*d^(1/4)
*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (4*b*e^(1/
4)*f*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x
])/(-d)^(5/4)]))/(-d)^(1/4) - (b*d^(1/4)*g^2*p*Log[Sqrt[d] - Sqrt[2]*d^(1/
4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) - (b*e^(3/4)*f^2*p*(2*A
rcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/
4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqr
t[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(3*
Sqrt[2]*d^(3/4)) + (b*d^(1/4)*g^2*p*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*
Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + b*g^2*Sqrt[x]*Log[c*(d + e*x^2)^
p] - (f^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (2*f*g*(a + b*Log[c*
(d + e*x^2)^p]))/Sqrt[x]))/(h*x)^(5/2)
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}dx$$

$$\Big\downarrow \text{2917}$$

$$\frac{2\int \frac{(fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^4x^2}d\sqrt{hx}}{h}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{(fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^2x^2}d\sqrt{hx}}{h^3}$$

$$\Big\downarrow \text{2926}$$

$$\frac{2\int \left(\frac{\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{x^2}+\frac{2g\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f}{x}+g^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)\right)d\sqrt{hx}}{h^3}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{\sqrt{2}be^{3/4}\sqrt{h}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{3d^{3/4}}+\frac{\sqrt{2}be^{3/4}\sqrt{h}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{3d^{3/4}}-\frac{h^2(a+b\log(c(ex^2+d)^p))f^2}{3(hx)^{3/2}}-\frac{be^{3/4}\sqrt{h}p\log\left(\right.}{}\right.$$

input
```
Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

output
```
(2*(a*g^2*Sqrt[h*x] - 4*b*g^2*p*Sqrt[h*x] - (Sqrt[2]*b*e^(3/4)*f^2*Sqrt[h]
*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4))
- (2*Sqrt[2]*b*e^(1/4)*f*g*Sqrt[h]*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x]
)/(d^(1/4)*Sqrt[h])])/d^(1/4) - (Sqrt[2]*b*d^(1/4)*g^2*Sqrt[h]*p*ArcTan[1
- (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/e^(1/4) + (Sqrt[2]*b*e^(
3/4)*f^2*Sqrt[h]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(3*d^(3/4)) + (2*Sqrt[2]*b*e^(1/4)*f*g*Sqrt[h]*p*ArcTan[1 + (Sqrt[2]*e
^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/d^(1/4) + (Sqrt[2]*b*d^(1/4)*g^2*Sqr
t[h]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/e^(1/4)
+ b*g^2*Sqrt[h*x]*Log[c*(d + e*x^2)^p] - (f^2*h^2*(a + b*Log[c*(d + e*x^2)
^p]))/(3*(h*x)^(3/2)) - (2*f*g*h*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x] -
 (b*e^(3/4)*f^2*Sqrt[h]*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^
(1/4)*Sqrt[h]*Sqrt[h*x]])/(3*Sqrt[2]*d^(3/4)) + (Sqrt[2]*b*e^(1/4)*f*g*Sqr
t[h]*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[
h*x]])/d^(1/4) - (b*d^(1/4)*g^2*Sqrt[h]*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sqrt[2]*e^(1/4)) + (b*e^(3/4)*f
^2*Sqrt[h]*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]
*Sqrt[h*x]])/(3*Sqrt[2]*d^(3/4)) - (Sqrt[2]*b*e^(1/4)*f*g*Sqrt[h]*p*Log[Sq
rt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/d^(1/4
) + (b*d^(1/4)*g^2*Sqrt[h]*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{(gx+f)^2 \left(a+b\ln \left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{5}{2}}}dx$$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

output
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2112 vs. $2(500) = 1000$.

Time = 0.19 (sec) , antiderivative size = 2112, normalized size of antiderivative = 3.05

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="fri
cas")
```

output

```
2/3*(h^3*x^2*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*
b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h
^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(16*(b^3*e^4*f^8
 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*d^3*e*f^2*g^6
 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(6*d^3*e*f*g*h^8*sqrt(-(b^4*e^4*f^8
- 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 +
 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + (b^2*d*e^3*f^6 - 27*b^2*d^2*e^2*f^4*g
^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(-(d*h^5*sqrt(-(b
^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*
e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*
f*g^3)*p^2)/(d*h^5))) - h^3*x^2*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d
*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^
4*g^8)*p^4/(d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*
log(16*(b^3*e^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 10
8*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(6*d^3*e*f*g*h^8*
sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*
b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + (b^2*d*e^3*f^6 - 2
7*b^2*d^2*e^2*f^4*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sq
rt(-(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4
*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + 12*(...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)
```

output

```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. $2(500) = 1000$.

Time = 0.14 (sec) , antiderivative size = 1102, normalized size of antiderivative = 1.59

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="max
ima")
```

output
```
2*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(5/2) - 2*b*e*f*g*p*(sqrt(2)*log(sq
rt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^
(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)
*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt
(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e)
)/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sq
rt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(
2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*
sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4)
 + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/h^2 + 2*a*g^2
*x^3/(h*x)^(5/2) - 4*b*f*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(5/2) + 1/3*(sqr
t(2)*h^2*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(
d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(sqrt(e)*h*x - sqrt(2)*(d*h
^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)
*h*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4)
- 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)
^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d))
+ sqrt(2)*h*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)
*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2
)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*...
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 631, normalized size of antiderivative = 0.91

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="gia
c")
```

output
```
1/3*(2*(3*sqrt(h*x)*b*g^2*p/h^2 - (6*b*f*g*h*p*x + b*f^2*h*p)/(sqrt(h*x)*h
^2*x))*log(e*h^2*x^2 + d*h^2) - 6*(b*g^2*p*log(h^2) + 4*b*g^2*p - b*g^2*lo
g(c) - a*g^2)*sqrt(h*x)/h^2 + 2*(6*b*f*g*h*p*x*log(h^2) + b*f^2*h*p*log(h^
2) - 6*b*f*g*h*x*log(c) - 6*a*f*g*h*x - b*f^2*h*log(c) - a*f^2*h)/(sqrt(h*
x)*h^2*x) + 2*(sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p + 3*sqrt(2)*(d*e^3*
h^2)^(1/4)*b*d*e*g^2*h*p + 6*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*arctan(1/2
*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h
^3) + 2*(sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p + 3*sqrt(2)*(d*e^3*h^2)^(
1/4)*b*d*e*g^2*h*p + 6*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h^3) +
 (sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p + 3*sqrt(2)*(d*e^3*h^2)^(1/4)*b*
d*e*g^2*h*p - 6*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*log(h*x + sqrt(2)*(d*h^
2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d*e^2*h^3) - (sqrt(2)*(d*e^3*h^2)^(
1/4)*b*e^2*f^2*h*p + 3*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g^2*h*p - 6*sqrt(2)
*(d*e^3*h^2)^(3/4)*b*f*g*p)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) +
sqrt(d*h^2/e))/(d*e^2*h^3))/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \int \frac{(f+g\,x)^2\,\left(a+b\,\ln\left(c\,{(e\,x^2+d)}^p\right)\right)}{(h\,x)^{5/2}}\,dx$$

input
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2),x)
```

output
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 595, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

output
```
(sqrt(h)*( - 12*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f*g*p*x - 6*
sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt
(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d*g**2*p*x - 2*sqrt(x)*e**(3/4
)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(e))/(e
**(1/4)*d**(1/4)*sqrt(2)))*b*e*f**2*p*x + 12*sqrt(x)*e**(1/4)*d**(3/4)*sqr
t(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/
4)*sqrt(2)))*b*e*f*g*p*x + 6*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1
/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d
*g**2*p*x + 2*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sq
rt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f**2*p*x + 12*
sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2)
 + sqrt(d) + sqrt(e)*x)*b*e*f*g*p*x - 6*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*
log((d + e*x**2)**p*c)*b*e*f*g*x - 6*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log
( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*d*g**2*p*x
- 2*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqr
t(2) + sqrt(d) + sqrt(e)*x)*b*e*f**2*p*x + 3*sqrt(x)*e**(3/4)*d**(1/4)*sqr
t(2)*log((d + e*x**2)**p*c)*b*d*g**2*x + sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)
*log((d + e*x**2)**p*c)*b*e*f**2*x - 2*log((d + e*x**2)**p*c)*b*d*e*f**2 -
 12*log((d + e*x**2)**p*c)*b*d*e*f*g*x + 6*log((d + e*x**2)**p*c)*b*d*e...
```

3.614 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 693

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx =$$
$$-\frac{8bef^2p}{5dh^3\sqrt{hx}}+\frac{2\sqrt{2}be^{5/4}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}$$
$$-\frac{4\sqrt{2}be^{3/4}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}$$
$$-\frac{2\sqrt{2}b\sqrt[4]{e}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}}$$
$$-\frac{2\sqrt{2}be^{5/4}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}$$
$$+\frac{4\sqrt{2}be^{3/4}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}$$
$$+\frac{2\sqrt{2}b\sqrt[4]{e}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}}$$
$$+\frac{2\sqrt{2}be^{5/4}f^2p\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{5d^{5/4}h^{7/2}}$$
$$+\frac{4\sqrt{2}be^{3/4}fgp\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{3d^{3/4}h^{7/2}}$$
$$-\frac{2\sqrt{2}b\sqrt[4]{e}g^2p\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{\sqrt[4]{d}h^{7/2}}-\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}}$$
$$-\frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}}-\frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^3\sqrt{hx}}$$

output
```
-8/5*b*e*f^2*p/d/h^3/(h*x)^(1/2)+2/5*2^(1/2)*b*e^(5/4)*f^2*p*arctan(1-2^(1
/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(5/4)/h^(7/2)-4/3*2^(1/2)*b*e^(
3/4)*f*g*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(3/4)/h
^(7/2)-2*2^(1/2)*b*e^(1/4)*g^2*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))/d^(1/4)/h^(7/2)-2/5*2^(1/2)*b*e^(5/4)*f^2*p*arctan(1+2^(1/2)*
e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(5/4)/h^(7/2)+4/3*2^(1/2)*b*e^(3/4)
*f*g*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(3/4)/h^(7/
2)+2*2^(1/2)*b*e^(1/4)*g^2*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/
h^(1/2))/d^(1/4)/h^(7/2)+2/5*2^(1/2)*b*e^(5/4)*f^2*p*arctanh(2^(1/2)*d^(1/
4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(5/4)/h^(7/2)+4/3*2^
(1/2)*b*e^(3/4)*f*g*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/
(d^(1/2)+e^(1/2)*x))/d^(3/4)/h^(7/2)-2*2^(1/2)*b*e^(1/4)*g^2*p*arctanh(2^(
1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(1/4)/h^(7
/2)-2/5*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(5/2)-4/3*f*g*(a+b*ln(c*(e*x^2
+d)^p))/h^2/(h*x)^(3/2)-2*g^2*(a+b*ln(c*(e*x^2+d)^p))/h^3/(h*x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.08 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.49

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2x^{7/2}\left(\frac{2b\sqrt[4]{e}g^2p\left(\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+\operatorname{arctanh}\left(\frac{d\sqrt[4]{e}\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}} - \frac{4bef^2p\,\text{Hype}\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

output
```
(2*x^(7/2)*((2*b*e^(1/4)*g^2*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + Arc
Tanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (4*b*e*f^2*p*Hypergeom
etric2F1[-1/4, 1, 3/4, -((e*x^2)/d)])/(5*d*Sqrt[x]) - (Sqrt[2]*b*e^(3/4)*f
*g*p*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt
[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt
[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]
*x]))/(3*d^(3/4)) - (f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*x^(5/2)) - (2*f*
g*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g^2*(a + b*Log[c*(d + e*x^2
)^p]))/Sqrt[x]))/(h*x)^(7/2)
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}} \, dx$$

$$\Big\downarrow \; 2917$$

$$\frac{2\int \frac{(fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^5x^3}d\sqrt{hx}}{h}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{(fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^3x^3}d\sqrt{hx}}{h^3}$$

$$\Big\downarrow \; 2926$$

$$\frac{2\int \left(\frac{\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{hx^3}+\frac{2g\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f}{hx^2}+\frac{g^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{hx}\right)d\sqrt{hx}}{h^3}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{\sqrt{2}be^{5/4}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{5d^{5/4}\sqrt{h}}-\frac{\sqrt{2}be^{5/4}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{5d^{5/4}\sqrt{h}}-\frac{h^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{5(hx)^{5/2}}-\frac{be^{5/4}p\log\left(\sqrt{e}xh+\sqrt{d}h\right.}{5\sqrt{2}\ldots}\right.$$

input
```
Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

output
```
(2*((-4*b*e*f^2*p)/(5*d*Sqrt[h*x]) + (Sqrt[2]*b*e^(5/4)*f^2*p*ArcTan[1 - (
Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*Sqrt[h]) - (2*Sq
rt[2]*b*e^(3/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt
[h])])/(3*d^(3/4)*Sqrt[h]) - (Sqrt[2]*b*e^(1/4)*g^2*p*ArcTan[1 - (Sqrt[2]*
e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*Sqrt[h]) - (Sqrt[2]*b*e^(5
/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*Sqrt[h]) + (2*Sqrt[2]*b*e^(3/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sq
rt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*Sqrt[h]) + (Sqrt[2]*b*e^(1/4)*g^2*
p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*Sqrt
[h]) - (f^2*h^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*(h*x)^(5/2)) - (2*f*g*h*(
a + b*Log[c*(d + e*x^2)^p]))/(3*(h*x)^(3/2)) - (g^2*(a + b*Log[c*(d + e*x^
2)^p]))/Sqrt[h*x] - (b*e^(5/4)*f^2*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]
*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(5*Sqrt[2]*d^(5/4)*Sqrt[h]) - (Sqrt[2
]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sq
rt[h]*Sqrt[h*x]])/(3*d^(3/4)*Sqrt[h]) + (b*e^(1/4)*g^2*p*Log[Sqrt[d]*h + S
qrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(Sqrt[2]*d^(1/4)*
Sqrt[h]) + (b*e^(5/4)*f^2*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*
e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(5*Sqrt[2]*d^(5/4)*Sqrt[h]) + (Sqrt[2]*b*e^(3/
4)*f*g*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqr
t[h*x]])/(3*d^(3/4)*Sqrt[h]) - (b*e^(1/4)*g^2*p*Log[Sqrt[d]*h + Sqrt[e]...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{(gx+f)^2 \left(a+b\ln \left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{7}{2}}}dx$$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

output
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2205 vs. $2(493) = 986$.

Time $= 0.26$ (sec) , antiderivative size $= 2205$, normalized size of antiderivative $= 3.18$

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="fri
cas")
```

output
```
-2/15*(d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*
g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^
4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*
h^7))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g^2 + 2150*b^3*d^2*e^3*f
^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e*g^8)*sqrt(h*x)*p^3 +
32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*
f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^
4*d^4*e*g^8)*p^4/(d^5*h^14)) - 10*(9*b^2*d^2*e^3*f^5*g - 190*b^2*d^3*e^2*f
^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8
 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*
f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2
*d*e*f*g^3)*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f
^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^
2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b
^2*d*e*f*g^3)*p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*
g^2 + 2150*b^3*d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4
*e*g^8)*sqrt(h*x)*p^3 - 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e
^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^
3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 10*(9*b^2*d^2*e^3*f
^5*g - 190*b^2*d^3*e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt((d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(493) = 986$.

Time = 0.14 (sec) , antiderivative size = 1088, normalized size of antiderivative = 1.57

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="max
ima")
```

output
```
-2*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(7/2) + 1/5*b*e*f^2*p*(e*(sqrt(2)*
log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d
*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqr
t(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2
)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*s
qrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4)
+ 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-
(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt
(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e
^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/d - 8/(
sqrt(h*x)*d))/h^3 - b*e*g^2*p*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(
1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(
sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2
)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2
)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt
(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt
(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) -
 sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(
d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqr
t(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/h^3 - 2*a*g^2*x^3/(h*x)^(7/2) - 4/3*b*f...
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 675, normalized size of antiderivative = 0.97

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="gia
c")
```

output
```
-1/15*(2*(15*b*g^2*h^3*p*x^2 + 10*b*f*g*h^3*p*x + 3*b*f^2*h^3*p)*log(e*h^2
*x^2 + d*h^2)/(sqrt(h*x)*h^2*x^2) - 2*(10*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e^
2*f*g*h*p - 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*e*f^2*p + 15*sqrt(2)*(d*e^3*h^2)
^(3/4)*b*d*g^2*p)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x
))/(d*h^2/e)^(1/4))/(d^2*e^2*h) - 2*(10*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e^2*
f*g*h*p - 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*e*f^2*p + 15*sqrt(2)*(d*e^3*h^2)^(
3/4)*b*d*g^2*p)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x)
)/(d*h^2/e)^(1/4))/(d^2*e^2*h) - (10*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e^2*f*g
*h*p + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*e*f^2*p - 15*sqrt(2)*(d*e^3*h^2)^(3/4
)*b*d*g^2*p)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/
(d^2*e^2*h) + (10*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e^2*f*g*h*p + 3*sqrt(2)*(d
*e^3*h^2)^(3/4)*b*e*f^2*p - 15*sqrt(2)*(d*e^3*h^2)^(3/4)*b*d*g^2*p)*log(h*
x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e^2*h) - 2*(15
*b*d*g^2*h^3*p*x^2*log(h^2) - 12*b*e*f^2*h^3*p*x^2 + 10*b*d*f*g*h^3*p*x*lo
g(h^2) - 15*b*d*g^2*h^3*x^2*log(c) - 15*a*d*g^2*h^3*x^2 + 3*b*d*f^2*h^3*p*
log(h^2) - 10*b*d*f*g*h^3*x*log(c) - 10*a*d*f*g*h^3*x - 3*b*d*f^2*h^3*log(
c) - 3*a*d*f^2*h^3)/(sqrt(h*x)*d*h^2*x^2))/h^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \int \frac{(f+g\,x)^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{(h\,x)}^{7/2}}\,dx$$

input
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2),x)
```

output
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx=\text{Too large to display}$$

input
```
int((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

output
```
(sqrt(h)*( - 30*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*
sqrt(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d*g**2*p*x**2
+ 6*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f**2*p*x**2 - 20*sqrt(x)
*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x)*sqr
t(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d*f*g*p*x**2 + 30*sqrt(x)*e**(1/4)*d*
*(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1
/4)*d**(1/4)*sqrt(2)))*b*d*g**2*p*x**2 - 6*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(
2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)
*sqrt(2)))*b*e*f**2*p*x**2 + 20*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e*
*(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*
b*d*f*g*p*x**2 + 30*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log( - sqrt(x)*e**(1
/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*d*g**2*p*x**2 - 6*sqrt(x)*e*
*(1/4)*d**(3/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d)
 + sqrt(e)*x)*b*e*f**2*p*x**2 - 15*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log((
d + e*x**2)**p*c)*b*d*g**2*x**2 + 3*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log(
(d + e*x**2)**p*c)*b*e*f**2*x**2 - 20*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*lo
g( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*d*f*g*p*x*
*2 + 10*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*d*f*g*x
**2 - 6*log((d + e*x**2)**p*c)*b*d**2*f**2 - 20*log((d + e*x**2)**p*c)*...
```

3.615 $\int \frac{(f+gx)^2 \left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 723

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} + \frac{2\sqrt{2}be^{7/4}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} + \frac{4\sqrt{2}be^{5/4}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}} - \frac{2\sqrt{2}be^{3/4}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}} - \frac{2\sqrt{2}be^{7/4}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} - \frac{4\sqrt{2}be^{5/4}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}} + \frac{2\sqrt{2}be^{3/4}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}} - \frac{2\sqrt{2}be^{7/4}f^2p\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{7d^{7/4}h^{9/2}} + \frac{4\sqrt{2}be^{5/4}fgp\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{5d^{5/4}h^{9/2}} + \frac{2\sqrt{2}be^{3/4}g^2p\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}}{\sqrt{h}\left(\sqrt{d}+\sqrt{e}x\right)}\right)}{3d^{3/4}h^{9/2}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3(hx)^{3/2}}$$

output
```
-8/21*b*e*f^2*p/d/h^3/(h*x)^(3/2)-16/5*b*e*f*g*p/d/h^4/(h*x)^(1/2)+2/7*2^(
1/2)*b*e^(7/4)*f^2*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))
/d^(7/4)/h^(9/2)+4/5*2^(1/2)*b*e^(5/4)*f*g*p*arctan(1-2^(1/2)*e^(1/4)*(h*x
)^(1/2)/d^(1/4)/h^(1/2))/d^(5/4)/h^(9/2)-2/3*2^(1/2)*b*e^(3/4)*g^2*p*arcta
n(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(3/4)/h^(9/2)-2/7*2^(1/
2)*b*e^(7/4)*f^2*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d
^(7/4)/h^(9/2)-4/5*2^(1/2)*b*e^(5/4)*f*g*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^
(1/2)/d^(1/4)/h^(1/2))/d^(5/4)/h^(9/2)+2/3*2^(1/2)*b*e^(3/4)*g^2*p*arctan(
1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/d^(3/4)/h^(9/2)-2/7*2^(1/2)
*b*e^(7/4)*f^2*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1
/2)+e^(1/2)*x))/d^(7/4)/h^(9/2)+4/5*2^(1/2)*b*e^(5/4)*f*g*p*arctanh(2^(1/2
)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(5/4)/h^(9/2)
+2/3*2^(1/2)*b*e^(3/4)*g^2*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h
^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(3/4)/h^(9/2)-2/7*f^2*(a+b*ln(c*(e*x^2+d)^p)
)/h/(h*x)^(7/2)-4/5*f*g*(a+b*ln(c*(e*x^2+d)^p))/h^2/(h*x)^(5/2)-2/3*g^2*(a
+b*ln(c*(e*x^2+d)^p))/h^3/(h*x)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.41

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \frac{x\left(-40bef^2px^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\frac{ex^2}{d}\right)-336be\ldots\right.}{}$$

input
```
Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

output
```
(x*(-40*b*e*f^2*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)] - 336*
b*e*f*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] - 35*Sqrt[2]*b
*d^(1/4)*e^(3/4)*g^2*p*x^(7/2)*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(
1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)
*e^(1/4)*Sqrt[x] + Sqrt[e]*x]) - 30*d*f^2*(a + b*Log[c*(d + e*x^2)^p]) - 8
4*d*f*g*x*(a + b*Log[c*(d + e*x^2)^p]) - 70*d*g^2*x^2*(a + b*Log[c*(d + e*
x^2)^p])))/(105*d*(h*x)^(9/2))
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 947, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}} \, dx$$

$$\Big\downarrow \text{2917}$$

$$\frac{2\int \frac{(fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^6x^4}d\sqrt{hx}}{h}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{(fh+gxh)^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^4x^4}d\sqrt{hx}}{h^3}$$

$$\Big\downarrow \text{2926}$$

$$\frac{2\int \left(\frac{\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{h^2x^4}+\frac{2g\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f}{h^2x^3}+\frac{g^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{h^2x^2}\right)d\sqrt{hx}}{h^3}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\sqrt{2}be^{7/4}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{7d^{7/4}h^{3/2}}-\frac{\sqrt{2}be^{7/4}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{7d^{7/4}h^{3/2}}-\frac{h^2\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)f^2}{7(hx)^{7/2}}+\frac{be^{7/4}p\log\left(\sqrt{e}xh+\sqrt{d}h\right.}{7\sqrt{2}d}\right.$$

input
```
Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

output
```
(2*((-4*b*e*f^2*p)/(21*d*(h*x)^(3/2)) - (8*b*e*f*g*p)/(5*d*h*Sqrt[h*x]) +
(Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(7*d^(7/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(5/4)*f*g*p*ArcTan[1 - (Sqr
t[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(3/2)) - (Sqrt[2]
*b*e^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])]
)/(3*d^(3/4)*h^(3/2)) - (Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/
4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7/4)*h^(3/2)) - (2*Sqrt[2]*b*e^(5/
4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(
5/4)*h^(3/2)) + (Sqrt[2]*b*e^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(3/2)) - (f^2*h^2*(a + b*Log[c*(d +
 e*x^2)^p]))/(7*(h*x)^(7/2)) - (2*f*g*h*(a + b*Log[c*(d + e*x^2)^p]))/(5*(
h*x)^(5/2)) - (g^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*(h*x)^(3/2)) + (b*e^(7
/4)*f^2*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sq
rt[h*x]])/(7*Sqrt[2]*d^(7/4)*h^(3/2)) - (Sqrt[2]*b*e^(5/4)*f*g*p*Log[Sqrt[
d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(5*d^(5/4
)*h^(3/2)) - (b*e^(3/4)*g^2*p*Log[Sqrt[d]*h + Sqrt[e]*h*x - Sqrt[2]*d^(1/4
)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(3*Sqrt[2]*d^(3/4)*h^(3/2)) - (b*e^(7/4)*f^2
*p*Log[Sqrt[d]*h + Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]
])/(7*Sqrt[2]*d^(7/4)*h^(3/2)) + (Sqrt[2]*b*e^(5/4)*f*g*p*Log[Sqrt[d]*h +
Sqrt[e]*h*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h]*Sqrt[h*x]])/(5*d^(5/4)*h^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{(gx+f)^2 \left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{9}{2}}}dx$$

input
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

output
```
int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2283 vs. $2(511) = 1022$.

Time $= 0.22$ (sec) , antiderivative size $= 2283$, normalized size of antiderivative $= 3.16$

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="fri
cas")
```

output
```
-2/105*(d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*
e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 +
1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*f^3*g - 7*b^2*d*
e^2*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*b^3*e^6*f^8 - 472500*b^3*d*e^5*f^
6*g^2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^2*g^6 + 150062
5*b^3*d^4*e^2*g^8)*sqrt(h*x)*p^3 + 16*(42*d^6*f*g*h^14*sqrt(-(50625*b^4*e^
7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*
b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 5*(675*b^
2*d^2*e^4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^2*g^4 - 85
75*b^2*d^5*e*g^6)*h^5*p^2)*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 12663
00*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f
^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*f^3*g -
 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))) - d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(506
25*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 -
 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) +
420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*b^
3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 25725
00*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*x)*p^3 - 16*(42*d
^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846
*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1110 vs. 2(511) = 1022.

Time = 0.14 (sec) , antiderivative size = 1110, normalized size of antiderivative = 1.54

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="max
ima")
```

output
```
-1/21*b*e*f^2*p*(3*(sqrt(2)*e^(3/4)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4
)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*e^(3/4)*log(sqrt(
e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4
) + sqrt(2)*e*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/
4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt
(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h
)*sqrt(d)*h) + sqrt(2)*e*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*
(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e
)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d
)*sqrt(e)*h)*sqrt(d)*h))/d + 8/((h*x)^(3/2)*d))/h^3 - 2/3*b*g^2*x^3*log((e
*x^2 + d)^p*c)/(h*x)^(9/2) + 2/5*b*e*f*g*p*(e*(sqrt(2)*log(sqrt(e)*h*x + s
qrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)
) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sq
rt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqr
t(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sq
rt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(
e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt
(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqr
t(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x
)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/d - 8/(sqrt(h*x)*d))/h^...
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 688, normalized size of antiderivative = 0.95

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="gia
c")
```

output
```
-1/105*(2*(15*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p - 35*sqrt(2)*(d*e^3*
h^2)^(1/4)*b*d*e*g^2*h*p + 42*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*arctan(1/
2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d^2*e*
h^5) + 2*(15*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p - 35*sqrt(2)*(d*e^3*h
^2)^(1/4)*b*d*e*g^2*h*p + 42*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d^2*e*
h^5) + (15*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p - 35*sqrt(2)*(d*e^3*h^2
)^(1/4)*b*d*e*g^2*h*p - 42*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*log(h*x + sq
rt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h^5) - (15*sqrt(2)
*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p - 35*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g^2*
h*p - 42*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*log(h*x - sqrt(2)*(d*h^2/e)^(1
/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h^5) + 2*(35*b*g^2*h^2*p*x^2 + 42*b*
f*g*h^2*p*x + 15*b*f^2*h^2*p)*log(e*h^2*x^2 + d*h^2)/(sqrt(h*x)*h^5*x^3) +
 2*(168*b*e*f*g*h^3*p*x^3 - 35*b*d*g^2*h^3*p*x^2*log(h^2) + 20*b*e*f^2*h^3
*p*x^2 - 42*b*d*f*g*h^3*p*x*log(h^2) + 35*b*d*g^2*h^3*x^2*log(c) + 35*a*d*
g^2*h^3*x^2 - 15*b*d*f^2*h^3*p*log(h^2) + 42*b*d*f*g*h^3*x*log(c) + 42*a*d
*f*g*h^3*x + 15*b*d*f^2*h^3*log(c) + 15*a*d*f^2*h^3)/(sqrt(h*x)*d*h^6*x^3)
)/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \int \frac{{\left(f+g\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{9/2}}\,dx$$

input
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2),x)
```

output
```
int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 634, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

output
```
(sqrt(h)*(84*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqr
t(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f*g*p*x**3 - 70
*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqr
t(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*d*g**2*p*x**3 + 30*sqrt(x)*e*
*(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(e
))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*e*f**2*p*x**3 - 84*sqrt(x)*e**(1/4)*d**(
3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4
)*d**(1/4)*sqrt(2)))*b*e*f*g*p*x**3 + 70*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)
*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*s
qrt(2)))*b*d*g**2*p*x**3 - 30*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(
1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*
e*f**2*p*x**3 - 84*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log( - sqrt(x)*e**(1/
4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*e*f*g*p*x**3 + 42*sqrt(x)*e**
(1/4)*d**(3/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*e*f*g*x**3 - 70*sqrt(x)*e*
*(3/4)*d**(1/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d)
 + sqrt(e)*x)*b*d*g**2*p*x**3 + 30*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log(
- sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*e*f**2*p*x**3
 + 35*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*d*g**2*x*
*3 - 15*sqrt(x)*e**(3/4)*d**(1/4)*sqrt(2)*log((d + e*x**2)**p*c)*b*e*f**2*
x**3 - 30*log((d + e*x**2)**p*c)*b*d**2*f**2 - 84*log((d + e*x**2)**p*c...
```

3.616 $\int \frac{\sqrt{hx}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f+gx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1601

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx = \text{Too large to display}$$

output
```
2*a*(h*x)^(1/2)/g-8*b*p*(h*x)^(1/2)/g-2*2^(1/2)*b*d^(1/4)*h^(1/2)*p*arctan
(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1/4)/g+2*2^(1/2)*b*d^(1
/4)*h^(1/2)*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2))/e^(1/4
)/g+2*2^(1/2)*b*d^(1/4)*h^(1/2)*p*arctanh(2^(1/2)*d^(1/4)*e^(1/4)*(h*x)^(1
/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/e^(1/4)/g+2*b*(h*x)^(1/2)*ln(c*(e*x^2+d)^
p)/g-2*f^(1/2)*h^(1/2)*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*(a+b*ln
(c*(e*x^2+d)^p))/g^(3/2)-8*b*f^(1/2)*h^(1/2)*p*arctan(g^(1/2)*(h*x)^(1/2)/
f^(1/2)/h^(1/2))*ln(2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/
2)))/g^(3/2)+2*b*f^(1/2)*h^(1/2)*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1
/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/
2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(
1/2)-I*g^(1/2)*(h*x)^(1/2)))/g^(3/2)+2*b*f^(1/2)*h^(1/2)*p*arctan(g^(1/2)*
(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)-e^(
1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-
I*g^(1/2)*(h*x)^(1/2)))/g^(3/2)+2*b*f^(1/2)*h^(1/2)*p*arctan(g^(1/2)*(h*x)
^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2
)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^
(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/g^(3/2)+2*b*f^(1/2)*h^(1/2
)*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)
^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/...
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 1506, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]
```

output
```
(Sqrt[h*x]*(2*a*Sqrt[g]*Sqrt[x] - 8*b*Sqrt[g]*p*Sqrt[x] - (2*Sqrt[2]*b*d^(
1/4)*Sqrt[g]*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (2
*Sqrt[2]*b*d^(1/4)*Sqrt[g]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)]
)/e^(1/4) - (Sqrt[2]*b*d^(1/4)*Sqrt[g]*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(
1/4)*Sqrt[x] + Sqrt[e]*x])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*Sqrt[g]*p*Log[Sqrt
[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/e^(1/4) + 2*b*Sqrt[g]*
Sqrt[x]*Log[c*(d + e*x^2)^p] + Sqrt[-f]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a
 + b*Log[c*(d + e*x^2)^p]) - Sqrt[-f]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a +
 b*Log[c*(d + e*x^2)^p]) - b*Sqrt[-f]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4
)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqr
t[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*
Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqr
t[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sq
rt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/
4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[
g]*Sqrt[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*S
qrt[-f] - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*S
qrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*
(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] +
 PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + ...
```

Rubi [A] (verified)

Time = 4.75 (sec) , antiderivative size = 1677, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f+gx}dx$$

$$\Big\downarrow \ 2917$$

$$\frac{2\int \frac{h^2x\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{fh+gxh}d\sqrt{hx}}{h}$$

$$\Big\downarrow \ 27$$

$$2\int \frac{hx\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{fh+gxh}d\sqrt{hx}$$

$$\Big\downarrow \; 2926$$

$$2\int \left(\frac{a+b\log\left(c\left(ex^2+d\right)^p\right)}{g}-\frac{fh\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{g(fh+gxh)}\right)d\sqrt{hx}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{\sqrt{hx}a}{g}-\frac{\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}g}+\frac{\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{e}g}+\frac{b\sqrt{hx}\log\left(c\left(ex^2+\right.\right.}{g}\right.$$

input
```
Int[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]
```

output
```
2*((a*Sqrt[h*x])/g - (4*b*p*Sqrt[h*x])/g - (Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Ar
cTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*g) + (Sq
rt[2]*b*d^(1/4)*Sqrt[h]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*
Sqrt[h])])/(e^(1/4)*g) + (b*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/g - (Sqrt[f]*S
qrt[h]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x
^2)^p]))/g^(3/2) - (4*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt
[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*
x])])/g^(3/2) + (b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*S
qrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqr
t[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[
f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (b*Sqrt[f]*Sqrt[h]*p*ArcTan
[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4
)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*
(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/g^(3/2) + (b*Sqrt[f]*Sqrt[h]*p*
ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[
h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h
] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/
g^(3/2) + (b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h]
)]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^
(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{\sqrt{hx}\,\left(a+b\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\right)}{gx+f}dx$$

input
```
int((h*x)^(1/2)*(a+b*ln(c*(e*x^2+d)^p))/(g*x+f),x)
```

output
```
int((h*x)^(1/2)*(a+b*ln(c*(e*x^2+d)^p))/(g*x+f),x)
```

Fricas [F]

$$\int \frac{\sqrt{hx}(a + b\log(c(d + ex^2)^p))}{f + gx} dx = \int \frac{\sqrt{hx}(b\log((ex^2 + d)^p c) + a)}{gx + f} dx$$

input
```
integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{hx}(a + b\log(c(d + ex^2)^p))}{f + gx} dx = \text{Timed out}$$

input
```
integrate((h*x)**(1/2)*(a+b*ln(c*(e*x**2+d)**p))/(g*x+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{hx}(a + b\log(c(d + ex^2)^p))}{f + gx} dx = \int \frac{\sqrt{hx}(b\log((ex^2 + d)^p c) + a)}{gx + f} dx$$

input
```
integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="maxim
a")
```

output
```
b*integrate((sqrt(h)*sqrt(x)*log((e*x^2 + d)^p) + sqrt(h)*sqrt(x)*log(c))/
(g*x + f), x) - 2*(f*h^2*arctan(sqrt(h*x)*g/sqrt(f*g*h))/(sqrt(f*g*h)*g) -
 sqrt(h*x)*h/g)*a/h
```

Giac [F]

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx=\int \frac{\sqrt{hx}\left(b\log\left((ex^2+d)^p c\right)+a\right)}{gx+f}\,dx$$

input
```
integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="giac"
)
```

output
```
integrate(sqrt(h*x)*(b*log((e*x^2 + d)^p*c) + a)/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx=\int \frac{\sqrt{h\,x}\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{f+g\,x}\,dx$$

input
```
int(((h*x)^(1/2)*(a + b*log(c*(d + e*x^2)^p)))/(f + g*x),x)
```

output
```
int(((h*x)^(1/2)*(a + b*log(c*(d + e*x^2)^p)))/(f + g*x), x)
```

Reduce [F]

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx$$
$$=\frac{\sqrt{h}\left(-2e^{\frac{3}{4}}d^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{e}}{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}}\right)bgp+2e^{\frac{3}{4}}d^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\sqrt{e}}{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}}\right)bgp-2\sqrt{g}\sqrt{f}\,atan\left(\frac{\sqrt{x}}{\sqrt{g}\sqrt{}}\right.}{}$$

input
```
int((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x)
```

output
```
(sqrt(h)*( - 2*e**(3/4)*d**(1/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*g*p + 2*e**(3/4)*d**(1/
4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(e))/(e**(1/4)*
d**(1/4)*sqrt(2)))*b*g*p - 2*sqrt(g)*sqrt(f)*atan((sqrt(x)*g)/(sqrt(g)*sqr
t(f)))*a*e - e**(3/4)*d**(1/4)*sqrt(2)*log( - sqrt(x)*e**(1/4)*d**(1/4)*sq
rt(2) + sqrt(d) + sqrt(e)*x)*b*g*p + e**(3/4)*d**(1/4)*sqrt(2)*log(sqrt(x)
*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*g*p + 2*sqrt(x)*a*e*g
- 4*int(sqrt(x)/(d*f*x + d*g*x**2 + e*f*x**3 + e*g*x**4),x)*b*d*e*f*g*p -
4*int(sqrt(x)/(d*f + d*g*x + e*f*x**2 + e*g*x**3),x)*b*d*e*g**2*p + int((s
qrt(x)*log((d + e*x**2)**p*c)*x**2)/(d*f + d*g*x + e*f*x**2 + e*g*x**3),x)
*b*e**2*g**2 + int((sqrt(x)*log((d + e*x**2)**p*c))/(d*f + d*g*x + e*f*x**
2 + e*g*x**3),x)*b*d*e*g**2))/(e*g**2)
```

3.617 $\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{hx}(f+gx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1361

$$\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{hx}(f+gx)}\,dx = \text{Too large to display}$$

output
```
2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))/f^(1
/2)/g^(1/2)/h^(1/2)+8*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2
*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/
h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(
1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/
2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^
(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h
^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*
e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)
))/f^(1/2)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2
))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^(1/2)
)/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/
2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h
*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4
)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g
^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)-4*I*b*p*polylog(2,1-2*f^(1/2)
*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+
I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)
*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f
^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*po...
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 1297, normalized size of antiderivative = 0.95

$$\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{hx}(f+gx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```

output
```
(Sqrt[x]*(a*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4)
 - e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[
-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x])
)/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x
]] - b*p*Log[(Sqrt[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f]
+ I*(-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g
]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]
*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - a*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p
*Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/
4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4
) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqr
t[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x
]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*S
qrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt
[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*Log[Sqrt[
-f] - Sqrt[g]*Sqrt[x]]*Log[c*(d + e*x^2)^p] - b*Log[Sqrt[-f] + Sqrt[g]*Sqr
t[x]]*Log[c*(d + e*x^2)^p] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*S
qrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (e^(1/4
)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])]
 - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[...
```

Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 1261, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2917, 27, 2920, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{hx}(f+gx)}dx$$

$$\Big\downarrow \ 2917$$

$$\frac{2\int \frac{h\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{fh+gxh}d\sqrt{hx}}{h}$$

$$\Big\downarrow \ 27$$

$$2\int \frac{a+b\log\left(c\left(ex^2+d\right)^p\right)}{fh+gxh}d\sqrt{hx}$$

$\Big\downarrow$ 2920

$$2\left(\frac{\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}-\frac{4bep\int \frac{h^{3/2}(hx)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)}{\sqrt{f}\sqrt{g}(ex^2h^2+dh^2)}d\sqrt{hx}}{h^2}\right)$$

$\Big\downarrow$ 27

$$2\left(\frac{\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}-\frac{4bep\int \frac{(hx)^{3/2}\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)}{ex^2h^2+dh^2}d\sqrt{hx}}{\sqrt{f}\sqrt{g}\sqrt{h}}\right)$$

$\Big\downarrow$ 7276

$$2\left(\frac{\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}-\frac{4bep\int \left(\frac{\sqrt{hx}\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)}{2(ehx-\sqrt{-d}\sqrt{e}h)}+\frac{\sqrt{hx}\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)}{2(exh+\sqrt{-d}\sqrt{e}h)}\right)d\sqrt{hx}}{\sqrt{f}\sqrt{g}\sqrt{h}}\right)$$

$\Big\downarrow$ 2009

$$2\left(\frac{\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}-\frac{4bep\left(-\frac{\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{e}+\frac{\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```

output
```
2*((ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)
^p]))/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (4*b*e*p*(-((ArcTan[(Sqrt[g]*Sqrt[h*x])/
(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sq
rt[h*x])])/e) + (ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt
[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4
)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[
g]*Sqrt[h*x]))])/(4*e) + (ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Lo
g[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4
)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])
/(4*e) + (ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqr
t[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[
g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt
[h*x]))])/(4*e) + (ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sq
rt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f
] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(4*e) +
 ((I/2)*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sq
rt[h*x])])/e - ((I/8)*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4
)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)
*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/e - ((I/8)*Po
lyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2920
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Simp[b*e*n*p   Int[u*(x^(n - 1)/(d + e*x^n)), x
], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{a+b\ln\left(c{\left(e\,x^{2}+d\right)}^{p}\right)}{\sqrt{hx}\,\left(gx+f\right)}dx$$

input
```
int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x)
```

output
```
int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx = \int \frac{b\log\left((ex^2+d)^pc\right)+a}{(gx+f)\sqrt{hx}}\,dx$$

input
```
integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*h*x^2 + f*h*x
), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex^2)^p\right)}{\sqrt{hx}(f + gx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2)/(g*x+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + b\log\left(c(d + ex^2)^p\right)}{\sqrt{hx}(f + gx)}\,dx = \int \frac{b\log\left((ex^2 + d)^p c\right) + a}{(gx + f)\sqrt{hx}}\,dx$$

input
```
integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="maxim
a")
```

output
```
b*integrate((sqrt(h)*log((e*x^2 + d)^p) + sqrt(h)*log(c))/(g*h*x^(3/2) + f
*h*sqrt(x)), x) + 2*a*arctan(sqrt(h*x)*g/sqrt(f*g*h))/sqrt(f*g*h)
```

Giac [F]

$$\int \frac{a + b\log\left(c(d + ex^2)^p\right)}{\sqrt{hx}(f + gx)}\,dx = \int \frac{b\log\left((ex^2 + d)^p c\right) + a}{(gx + f)\sqrt{hx}}\,dx$$

input
```
integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="giac"
)
```

output
```
integrate((b*log((e*x^2 + d)^p*c) + a)/((g*x + f)*sqrt(h*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx=\int \frac{a+b\,\ln\left(c\,(e\,x^2+d)^p\right)}{(f+g\,x)\,\sqrt{h\,x}}\,dx$$

input
```
int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(1/2)),x)
```

output
```
int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx=\frac{\sqrt{h}\left(2\sqrt{g}\,\sqrt{f}\,atan\left(\frac{\sqrt{x}\,g}{\sqrt{g}\,\sqrt{f}}\right)a+\left(\int \frac{\log\left((e\,x^2+d)^p c\right)}{\sqrt{x}\,f+\sqrt{x}\,gx}dx\right)bfg\right)}{fgh}$$

input
```
int((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x)
```

output
```
(sqrt(h)*(2*sqrt(g)*sqrt(f)*atan((sqrt(x)*g)/(sqrt(g)*sqrt(f)))*a + int(lo
g((d + e*x**2)**p*c)/(sqrt(x)*f + sqrt(x)*g*x),x)*b*f*g))/(f*g*h)
```

3.618 $\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{(hx)^{3/2}(f+gx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1580

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \text{Too large to display}$$

output
```
-2*2^(1/2)*b*e^(1/4)*p*arctan(1-2^(1/2)*e^(1/4)*(h*x)^(1/2)/d^(1/4)/h^(1/2
))/d^(1/4)/f/h^(3/2)+2*2^(1/2)*b*e^(1/4)*p*arctan(1+2^(1/2)*e^(1/4)*(h*x)^
(1/2)/d^(1/4)/h^(1/2))/d^(1/4)/f/h^(3/2)-2*2^(1/2)*b*e^(1/4)*p*arctanh(2^(
1/2)*d^(1/4)*e^(1/4)*(h*x)^(1/2)/h^(1/2)/(d^(1/2)+e^(1/2)*x))/d^(1/4)/f/h^
(3/2)-2*(a+b*ln(c*(e*x^2+d)^p))/f/h/(h*x)^(1/2)-2*g^(1/2)*arctan(g^(1/2)*(
h*x)^(1/2)/f^(1/2)/h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))/f^(3/2)/h^(3/2)-8*b*g^
(1/2)*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*h^(1/2)/(
f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(3/2)/h^(3/2)+2*b*g^(1/2)*p*arct
an(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)
^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^
(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(3/2)/h^
(3/2)+2*b*g^(1/2)*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(-2*f^(1
/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-
d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(3/2)/h^(3/2)
+2*b*g^(1/2)*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^
(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1
/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)
^(1/2)))/f^(3/2)/h^(3/2)+2*b*g^(1/2)*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/
h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*
e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1...
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 1336, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```

output
```
(x^(3/2)*((4*b*e^(1/4)*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(
d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (2*(a + b*Log[c*(d + e*x^2)^
p]))/Sqrt[x] + (f*Sqrt[g]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a + b*Log[c*(d
+ e*x^2)^p]))/(-f)^(3/2) + (Sqrt[g]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a + b
*Log[c*(d + e*x^2)^p]))/Sqrt[-f] + (b*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^(1/4)
- e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-
f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*
e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] +
Log[(Sqrt[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^
(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4)
 + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f]
 - Sqrt[g]*Sqrt[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e
^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - S
qrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (
e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqr
t[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-
f] + (-d)^(1/4)*Sqrt[g])]))/Sqrt[-f] + (b*f*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^
(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqr
t[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/
(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[...
```

Rubi [A] (verified)

Time = 4.18 (sec) , antiderivative size = 1658, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2917, 27, 2926, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{(hx)^{3/2}(f+gx)}dx$$

$$\Big\downarrow \text{2917}$$

$$\frac{2\int \frac{a+b\log\left(c\left(ex^2+d\right)^p\right)}{x(fh+gxh)}d\sqrt{hx}}{h}$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{a+b\log\left(c\left(ex^2+d\right)^p\right)}{hx(fh+gxh)}d\sqrt{hx}$$

$\Big\downarrow$ 2926

$$2\int \left(\frac{a+b\log\left(c\left(ex^2+d\right)^p\right)}{fh^2x}-\frac{g\left(a+b\log\left(c\left(ex^2+d\right)^p\right)\right)}{fh(fh+gxh)}\right)d\sqrt{hx}$$

$\Big\downarrow$ 2009

$$2\left(-\frac{\sqrt{2}b\sqrt[4]{e}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}}+\frac{\sqrt{2}b\sqrt[4]{e}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{d}fh^{3/2}}-\frac{\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\right.\right.}{f^{3/2}h^{3/2}}\right.$$

input

```
Int[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```

output

```
2*(-((Sqrt[2]*b*e^(1/4)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*
Sqrt[h])])/(d^(1/4)*f*h^(3/2))) + (Sqrt[2]*b*e^(1/4)*p*ArcTan[1 + (Sqrt[2]
*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*f*h^(3/2)) - (a + b*Log[c
*(d + e*x^2)^p])/(f*h*Sqrt[h*x]) - (Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sq
rt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p]))/(f^(3/2)*h^(3/2)) - (4*b*Sqr
t[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h
])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/2)) + (b*Sqrt[g
]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*S
qrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqr
t[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])
)])/(f^(3/2)*h^(3/2)) + (b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*S
qrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))
/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sq
rt[h*x]))])/(f^(3/2)*h^(3/2)) + (b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(
1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h]
)*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (b*Sqrt[g
]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*(
(-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*
Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) +...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2917
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Simp[k/h   Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*
(d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

rule 2926
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(e\,x^2+d)^p\right)}{(hx)^{\frac{3}{2}}\,(gx+f)}dx$$

input
```
int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x)
```

output
```
int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x)
```

Fricas [F]

$$\int \frac{a+b\log \left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \int \frac{b\log \left((ex^2+d)^pc\right)+a}{(gx+f)\,(hx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="frica
s")
```

output
```
integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*h^2*x^3 + f*h
^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2)/(g*x+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\log \left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \int \frac{b\log \left((ex^2+d)^pc\right)+a}{(gx+f)\,(hx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="maxim
a")
```

output
```
b*integrate((sqrt(h)*log((e*x^2 + d)^p) + sqrt(h)*log(c))/(g*h^2*x^(5/2) +
 f*h^2*x^(3/2)), x) - 2*a*(g*arctan(sqrt(h*x)*g/sqrt(f*g*h))/(sqrt(f*g*h)*
f) + 1/(sqrt(h*x)*f))/h
```

Giac [F]

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \int \frac{b\log\left((ex^2+d)^pc\right)+a}{(gx+f)\,(hx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="giac"
)
```

output
```
integrate((b*log((e*x^2 + d)^p*c) + a)/((g*x + f)*(h*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \int \frac{a+b\,\ln\left(c\,(e\,x^2+d)^p\right)}{(f+g\,x)\,(h\,x)^{3/2}}\,dx$$

input
```
int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(3/2)),x)
```

output
```
int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \frac{\sqrt{h}\left(-2\sqrt{x}\,e^{\frac{1}{4}}d^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{e}}{e^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{2}}\right)bfp+2\sqrt{x}\,e^{\frac{1}{4}}d^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{e^{\frac{1}{4}}d^{\frac{1}{4}}}{e}\right.}{}$$

input
```
int((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x)
```

output
```
(sqrt(h)*( - 2*sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*f*p + 2*sqrt(x)
*e**(1/4)*d**(3/4)*sqrt(2)*atan((e**(1/4)*d**(1/4)*sqrt(2) + 2*sqrt(x)*sqr
t(e))/(e**(1/4)*d**(1/4)*sqrt(2)))*b*f*p - 2*sqrt(x)*sqrt(g)*sqrt(f)*atan(
(sqrt(x)*g)/(sqrt(g)*sqrt(f)))*a*d + sqrt(x)*e**(1/4)*d**(3/4)*sqrt(2)*log
( - sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(d) + sqrt(e)*x)*b*f*p - sqrt(
x)*e**(1/4)*d**(3/4)*sqrt(2)*log(sqrt(x)*e**(1/4)*d**(1/4)*sqrt(2) + sqrt(
d) + sqrt(e)*x)*b*f*p - 4*sqrt(x)*int(sqrt(x)/(d*f*x + d*g*x**2 + e*f*x**3
 + e*g*x**4),x)*b*d**2*f*g*p - 4*sqrt(x)*int(sqrt(x)/(d*f + d*g*x + e*f*x*
*2 + e*g*x**3),x)*b*d**2*g**2*p + sqrt(x)*int((sqrt(x)*log((d + e*x**2)**p
*c)*x**2)/(d*f + d*g*x + e*f*x**2 + e*g*x**3),x)*b*d*e*g**2 + sqrt(x)*int(
(sqrt(x)*log((d + e*x**2)**p*c))/(d*f + d*g*x + e*f*x**2 + e*g*x**3),x)*b*
d**2*g**2 - 2*log((d + e*x**2)**p*c)*b*d*f - 2*log((d + e*x**2)**p*c)*b*d*
g*x - 2*a*d*f + 8*b*d*g*p*x))/(sqrt(x)*d*f**2*h**2)
```

3.619 $\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx = -\frac{\log(fx^p)\,\text{PolyLog}(2,-ex^m)}{m} + \frac{p\,\text{PolyLog}(3,-ex^m)}{m^2}$$

output
```
-ln(f*x^p)*polylog(2,-e*x^m)/m+p*polylog(3,-e*x^m)/m^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx = -\frac{\log(fx^p)\,\text{PolyLog}(2,-ex^m)}{m} + \frac{p\,\text{PolyLog}(3,-ex^m)}{m^2}$$

input
```
Integrate[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
```

output
```
-((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(ex^m+1)\log(fx^p)}{x}\,dx$$

$$\Big\downarrow \text{2821}$$

$$\frac{p\int \frac{\text{PolyLog}(2,-ex^m)}{x}dx}{m}-\frac{\text{PolyLog}(2,-ex^m)\log(fx^p)}{m}$$

$$\Big\downarrow \text{7143}$$

$$\frac{p\,\text{PolyLog}(3,-ex^m)}{m^2}-\frac{\text{PolyLog}(2,-ex^m)\log(fx^p)}{m}$$

input
```
Int[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
```

output
```
-((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.63 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.48

method	result
risch	$-\frac{p\ln(x)\operatorname{polylog}(2,-e\,x^m)}{m}+\frac{p\operatorname{polylog}(3,-e\,x^m)}{m^2}-\frac{(\ln(x^p)-p\ln(x))\operatorname{dilog}(1+e\,x^m)}{m}-\frac{\left(\frac{i\pi\operatorname{csgn}(ix^p)\operatorname{csgn}(if\,x^p)^2}{2}-\frac{i\pi\operatorname{csgn}(ix\right.}{}$

input
```
int(ln(f*x^p)*ln(1+e*x^m)/x,x,method=_RETURNVERBOSE)
```

output
```
-p/m*ln(x)*polylog(2,-e*x^m)+p*polylog(3,-e*x^m)/m^2-1/m*(ln(x^p)-p*ln(x))
*dilog(1+e*x^m)-(1/2*I*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-1/2*I*Pi*csgn(I*x^p)
*csgn(I*f*x^p)*csgn(I*f)-1/2*I*Pi*csgn(I*f*x^p)^3+1/2*I*Pi*csgn(I*f*x^p)^2
*csgn(I*f)+ln(f))/m*dilog(1+e*x^m)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx = -\frac{(mp\log(x)+m\log(f))\mathrm{Li}_2(-ex^m)-p\mathrm{polylog}(3,-ex^m)}{m^2}$$

input
```
integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="fricas")
```

output
```
-((m*p*log(x) + m*log(f))*dilog(-e*x^m) - p*polylog(3, -e*x^m))/m^2
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(ln(f*x**p)*ln(1+e*x**m)/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx = \int \frac{\log(ex^m+1)\log(fx^p)}{x}\,dx$$

input
```
integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="maxima")
```

output
```
-1/2*(p*log(x)^2 - 2*log(f)*log(x) - 2*log(x)*log(x^p))*log(e*x^m + 1) - i
ntegrate(1/2*(2*e*m*x^m*log(x)*log(x^p) - (e*m*p*log(x)^2 - 2*e*m*log(f)*l
og(x))*x^m)/(e*x*x^m + x), x)
```

Giac [F]

$$\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx = \int \frac{\log(ex^m+1)\log(fx^p)}{x}\,dx$$

input
```
integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="giac")
```

output
```
integrate(log(e*x^m + 1)*log(f*x^p)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (fx^p)\log (1+ex^m)}{x}\,dx=\int \frac{\ln \left(f\,x^p\right)\,\ln \left(e\,x^m+1\right)}{x} \,dx$$

input
```
int((log(f*x^p)*log(e*x^m + 1))/x,x)
```

output
```
int((log(f*x^p)*log(e*x^m + 1))/x, x)
```

Reduce [F]

$$\int \frac{\log (fx^p)\log (1+ex^m)}{x}\,dx=\int \frac{\log(x^m e+1)\log(x^p f)}{x}dx$$

input
```
int(log(f*x^p)*log(1+e*x^m)/x,x)
```

output
```
int((log(x**m*e + 1)*log(x**p*f))/x,x)
```

3.620 $\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{2p\log(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2} - \frac{2p^2\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{em^3}$$

output
```
ln(f*x^p)^2*ln(1+e*x^m/d)/e/m+2*p*ln(f*x^p)*polylog(2,-e*x^m/d)/e/m^2-2*p^
2*polylog(3,-e*x^m/d)/e/m^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(75) = 150.

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.93

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx$$
$$= \frac{p^2\log^3(x) + 3p\log^2(x)\left(-p\log(x)+\log(fx^p)\right) + 3\log(x)\left(-p\log(x)+\log(fx^p)\right)^2 - \frac{3(-p\log(x)+\log(fx^p))^2(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

output
```
(p^2*Log[x]^3 + 3*p*Log[x]^2*(-(p*Log[x]) + Log[f*x^p]) + 3*Log[x]*(-(p*Lo
g[x]) + Log[f*x^p])^2 - (3*(-(p*Log[x]) + Log[f*x^p])^2*(Log[x^m] - Log[d*
m*(d + e*x^m)]))/m - (6*p*(-(p*Log[x]) + Log[f*x^p])*((m^2*Log[x]^2)/2 + (
-(m*Log[x]) + Log[-((e*x^m)/d)])*Log[d + e*x^m] + PolyLog[2, 1 + (e*x^m)/d
]))/m^2 + (3*p^2*(m^2*Log[x]^2*Log[1 + d/(e*x^m)] - 2*m*Log[x]*PolyLog[2,
-(d/(e*x^m))] - 2*PolyLog[3, -(d/(e*x^m))]))/m^3)/(3*e)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2775, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-1}\log^2(fx^p)}{d+ex^m}dx$$

$$\Big\downarrow \text{2775}$$

$$\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2p\int \frac{\log(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{x}dx}{em}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2p\left(\frac{p\int \frac{\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{x}dx}{m}-\frac{\log(fx^p)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2p\left(\frac{p\,\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{\log(fx^p)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}$$

input
```
Int[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

output
```
(Log[f*x^p]^2*Log[1 + (e*x^m)/d])/(e*m) - (2*p*(-((Log[f*x^p]*PolyLog[2, -
((e*x^m)/d)])/m) + (p*PolyLog[3, -((e*x^m)/d)])/m^2))/(e*m)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.61 (sec) , antiderivative size = 496, normalized size of antiderivative = 6.61

method	result
risch	$\frac{\ln(d+e\,x^m)\ln(x)^2p^2}{me}-\frac{2\ln(d+e\,x^m)\ln(x)\ln(x^p)p}{me}+\frac{\ln(d+e\,x^m)\ln(x^p)^2}{me}+\frac{p^2\ln(x)^2\ln\left(1+\frac{e\,x^m}{d}\right)}{me}+\frac{2p^2\ln(x)\operatorname{polylog}\left(2,-\right.}{m^2e}$

input
```
int(x^(m-1)*ln(f*x^p)^2/(d+e*x^m),x,method=_RETURNVERBOSE)
```

output
```
1/m*ln(d+e*x^m)/e*ln(x)^2*p^2-2/m*ln(d+e*x^m)/e*ln(x)*ln(x^p)*p+1/m*ln(d+e
*x^m)/e*ln(x^p)^2+1/m*p^2/e*ln(x)^2*ln(1+e*x^m/d)+2/m^2*p^2/e*ln(x)*polylo
g(2,-e*x^m/d)-2*p^2*polylog(3,-e*x^m/d)/e/m^3-2/m^2*p^2*dilog((d+e*x^m)/d)
/e*ln(x)+2/m^2*p*dilog((d+e*x^m)/d)/e*ln(x^p)-2/m*p^2*ln(x)^2*ln((d+e*x^m)
/d)/e+2/m*p*ln(x)*ln((d+e*x^m)/d)/e*ln(x^p)+(I*Pi*csgn(I*x^p)*csgn(I*f*x^p
)^2-I*Pi*csgn(I*x^p)*csgn(I*f*x^p)*csgn(I*f)-I*Pi*csgn(I*f*x^p)^3+I*Pi*csg
n(I*f*x^p)^2*csgn(I*f)+2*ln(f))*(1/m*(ln(x^p)-p*ln(x))*ln(d+e*x^m)/e+1/m^2
*p*dilog((d+e*x^m)/d)/e+1/m*p*ln(x)*ln((d+e*x^m)/d)/e)+1/4*(I*Pi*csgn(I*x^
p)*csgn(I*f*x^p)^2-I*Pi*csgn(I*x^p)*csgn(I*f*x^p)*csgn(I*f)-I*Pi*csgn(I*f*
x^p)^3+I*Pi*csgn(I*f*x^p)^2*csgn(I*f)+2*ln(f))^2/m*ln(d+e*x^m)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \frac{m^2\log(ex^m+d)\log(f)^2 - 2\,p^2\text{polylog}\left(3,-\frac{ex^m}{d}\right) + 2\left(mp^2\log(x)+mp\log(f)\right)\text{Li}_2\left(-\frac{ex^m+d}{d}+1\right) + \left(m^2\right.}{em^3}$$

input
```
integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="fricas")
```

output
```
(m^2*log(e*x^m + d)*log(f)^2 - 2*p^2*polylog(3, -e*x^m/d) + 2*(m*p^2*log(x
) + m*p*log(f))*dilog(-(e*x^m + d)/d + 1) + (m^2*p^2*log(x)^2 + 2*m^2*p*lo
g(f)*log(x))*log((e*x^m + d)/d))/(e*m^3)
```

Sympy [F]

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \int \frac{x^{m-1}\log(fx^p)^2}{d+ex^m}\,dx$$

input
```
integrate(x**(-1+m)*ln(f*x**p)**2/(d+e*x**m),x)
```

output
```
Integral(x**(m - 1)*log(f*x**p)**2/(d + e*x**m), x)
```

Maxima [F]

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \int \frac{x^{m-1}\log(fx^p)^2}{ex^m+d}\,dx$$

input
```
integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="maxima")
```

output
```
integrate(x^(m - 1)*log(f*x^p)^2/(e*x^m + d), x)
```

Giac [F]

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \int \frac{x^{m-1}\log(fx^p)^2}{ex^m+d}\,dx$$

input
```
integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*log(f*x^p)^2/(e*x^m + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \int \frac{x^{m-1}\ln(f\,x^p)^2}{d+e\,x^m}\,dx$$

input
```
int((x^(m - 1)*log(f*x^p)^2)/(d + e*x^m),x)
```

output
```
int((x^(m - 1)*log(f*x^p)^2)/(d + e*x^m), x)
```

Reduce [F]

$$\int \frac{x^{-1+m} \log^2 (f x^p)}{d + e x^m} \, dx = \int \frac{x^m \log(x^p f)^2}{x^m e x + d x} dx$$

input
```
int(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x)
```

output
```
int((x**m*log(x**p*f)**2)/(x**m*e*x + d*x),x)
```

3.621 $\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 161

$$\int \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p} - \frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p} - \frac{bn\log^3(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} + \frac{3bnp\log^2(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2} - \frac{6bnp^2\log(fx^p)\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3} + \frac{6bnp^3\operatorname{PolyLog}\left(5,-\frac{ex^m}{d}\right)}{m^4}$$

output
```
1/4*ln(f*x^p)^4*(a+b*ln(c*(d+e*x^m)^n))/p-1/4*b*n*ln(f*x^p)^4*ln(1+e*x^m/d
)/p-b*n*ln(f*x^p)^3*polylog(2,-e*x^m/d)/m+3*b*n*p*ln(f*x^p)^2*polylog(3,-e
*x^m/d)/m^2-6*b*n*p^2*ln(f*x^p)*polylog(4,-e*x^m/d)/m^3+6*b*n*p^3*polylog(
5,-e*x^m/d)/m^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 659 vs. $2(161) = 322$.

Time = 0.34 (sec) , antiderivative size = 659, normalized size of antiderivative = 4.09

$$\int \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

output
```
(-3*b*m*n*p^3*Log[x]^5)/10 + (3*b*m*n*p^2*Log[x]^4*Log[f*x^p])/4 - (b*m*n*
p*Log[x]^3*Log[f*x^p]^2)/2 + (a*Log[f*x^p]^4)/(4*p) - (3*b*n*p^3*Log[x]^4*
Log[1 + d/(e*x^m)])/4 + 2*b*n*p^2*Log[x]^3*Log[f*x^p]*Log[1 + d/(e*x^m)] -
 (3*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[1 + d/(e*x^m)])/2 + b*n*p^3*Log[x]^4*L
og[d + e*x^m] - (b*n*p^3*Log[x]^3*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m - 3*
b*n*p^2*Log[x]^3*Log[f*x^p]*Log[d + e*x^m] + (3*b*n*p^2*Log[x]^2*Log[-((e*
x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m + 3*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[
d + e*x^m] - (3*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]^2*Log[d + e*x^m]
)/m - b*n*Log[x]*Log[f*x^p]^3*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[
f*x^p]^3*Log[d + e*x^m])/m - (b*p^3*Log[x]^4*Log[c*(d + e*x^m)^n])/4 + b*p
^2*Log[x]^3*Log[f*x^p]*Log[c*(d + e*x^m)^n] - (3*b*p*Log[x]^2*Log[f*x^p]^2
*Log[c*(d + e*x^m)^n])/2 + b*Log[x]*Log[f*x^p]^3*Log[c*(d + e*x^m)^n] + (b
*n*p*Log[x]*(p^2*Log[x]^2 - 3*p*Log[x]*Log[f*x^p] + 3*Log[f*x^p]^2)*PolyLo
g[2, -(d/(e*x^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])^3*PolyLog[2, 1 + (e*x
^m)/d])/m + (3*b*n*p*Log[f*x^p]^2*PolyLog[3, -(d/(e*x^m))])/m^2 + (6*b*n*p
^2*Log[f*x^p]*PolyLog[4, -(d/(e*x^m))])/m^3 + (6*b*n*p^3*PolyLog[5, -(d/(e
*x^m))])/m^4
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2931, 2775, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$$

$$\Big\downarrow \text{2931}$$

$$\frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p}-\frac{bemn\int \frac{x^{m-1}\log^4(fx^p)}{ex^m+d}dx}{4p}$$

$$\Big\downarrow \text{2775}$$

$$\frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p}-\frac{bemn\left(\frac{\log^4(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{4p\int \frac{\log^3(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{x}dx}{em}\right)}{4p}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p}-\frac{bemn\left(\frac{\log^4(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{4p\left(\frac{3p\int \frac{\log^2(fx^p)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{x}dx}{m}-\frac{\log^3(fx^p)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}\right)}{4p}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p}-\frac{bemn\left(\frac{\log^4(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{4p\left(\frac{3p\left(\frac{\log^2(fx^p)\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m}-\frac{2p\int \frac{\log(fx^p)\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{x}dx}{m}\right)}{m}-\frac{\log^3(fx^p)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}\right)}{4p}$$

$$\Big\downarrow \text{2830}$$

$$\frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p} - \frac{bemn\left(\frac{\log^4(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em} - \frac{4p\left(\frac{3p\left(\frac{\log^2(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m} - \frac{2p\left(\frac{\log(fx^p)\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m} - \frac{p\int\frac{\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{x}dx}{m}\right)}{m}\right)}{m} - \frac{\log^3(f}{}}{em}\right)}{4p}$$

↓ 7143

$$\frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p} - \frac{bemn\left(\frac{\log^4(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em} - \frac{4p\left(\frac{3p\left(\frac{\log^2(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m} - \frac{2p\left(\frac{\log(fx^p)\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m} - \frac{p\operatorname{PolyLog}\left(5,-\frac{ex^m}{d}\right)}{m^2}\right)}{m}\right)}{m} - \frac{\log^3(fx^p)}{}}{em}\right)}{4p}$$

input
```
Int[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

output
```
(Log[f*x^p]^4*(a + b*Log[c*(d + e*x^m)^n]))/(4*p) - (b*e*m*n*((Log[f*x^p]^
4*Log[1 + (e*x^m)/d])/(e*m) - (4*p*(-((Log[f*x^p]^3*PolyLog[2, -((e*x^m)/d
)])/m) + (3*p*((Log[f*x^p]^2*PolyLog[3, -((e*x^m)/d)])/m - (2*p*((Log[f*x^
p]*PolyLog[4, -((e*x^m)/d)])/m - (p*PolyLog[5, -((e*x^m)/d)])/m^2))/m))/m)
)/(e*m)))/(4*p)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2931
```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Simp[b*e*n*(p/(q*(m + 1)))   Int[x^(n - 1
)*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p, q}, x] && NeQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln (f x^p)^3 (a+b\ln (c(d+e x^m)^n))}{x}dx$$

input
```
int(ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

output
```
int(ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(156) = 312$.

Time = 0.14 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.59

$$\int \frac{\log^3 (fx^p) (a+b\log (c(d+ex^m)^n))}{x} dx = \frac{24\,bnp^3\text{polylog}\left(5,-\frac{ex^m}{d}\right)+4\,(bm^4\log (c)+am^4)\log (f)^3\log (x)+6\,(bm^4p\log (c)+am^4p)\log (f)^2\log (}{}$$

input
```
integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

output
```
1/4*(24*b*n*p^3*polylog(5, -e*x^m/d) + 4*(b*m^4*log(c) + a*m^4)*log(f)^3*l
og(x) + 6*(b*m^4*p*log(c) + a*m^4*p)*log(f)^2*log(x)^2 + 4*(b*m^4*p^2*log(
c) + a*m^4*p^2)*log(f)*log(x)^3 + (b*m^4*p^3*log(c) + a*m^4*p^3)*log(x)^4
- 4*(b*m^3*n*p^3*log(x)^3 + 3*b*m^3*n*p^2*log(f)*log(x)^2 + 3*b*m^3*n*p*lo
g(f)^2*log(x) + b*m^3*n*log(f)^3)*dilog(-(e*x^m + d)/d + 1) + (b*m^4*n*p^3
*log(x)^4 + 4*b*m^4*n*p^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2
+ 4*b*m^4*n*log(f)^3*log(x))*log(e*x^m + d) - (b*m^4*n*p^3*log(x)^4 + 4*b*
m^4*n*p^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2 + 4*b*m^4*n*log(
f)^3*log(x))*log((e*x^m + d)/d) - 24*(b*m*n*p^3*log(x) + b*m*n*p^2*log(f))
*polylog(4, -e*x^m/d) + 12*(b*m^2*n*p^3*log(x)^2 + 2*b*m^2*n*p^2*log(f)*lo
g(x) + b*m^2*n*p*log(f)^2)*polylog(3, -e*x^m/d))/m^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(ln(f*x**p)**3*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{(b\log((ex^m+d)^nc)+a)\log(fx^p)^3}{x}\,dx$$

input
```
integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

output
```
-1/4*(b*p^3*log(x)^4 - 4*b*p^2*log(f)*log(x)^3 + 6*b*p*log(f)^2*log(x)^2 -
 4*b*log(f)^3*log(x) - 4*b*log(x)*log(x^p)^3 + 6*(b*p*log(x)^2 - 2*b*log(f
)*log(x))*log(x^p)^2 - 4*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log
(f)^2*log(x))*log(x^p))*log((e*x^m + d)^n) - integrate(-1/4*(4*b*d*log(c)*
log(f)^3 + 4*a*d*log(f)^3 + 4*(b*d*log(c) + a*d - (b*e*m*n*log(x) - b*e*lo
g(c) - a*e)*x^m)*log(x^p)^3 + 6*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*e
*m*n*p*log(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log(f) + 2*a*e*lo
g(f))*x^m)*log(x^p)^2 + (b*e*m*n*p^3*log(x)^4 - 4*b*e*m*n*p^2*log(f)*log(x
)^3 + 6*b*e*m*n*p*log(f)^2*log(x)^2 - 4*b*e*m*n*log(f)^3*log(x) + 4*b*e*lo
g(c)*log(f)^3 + 4*a*e*log(f)^3)*x^m + 4*(3*b*d*log(c)*log(f)^2 + 3*a*d*log
(f)^2 - (b*e*m*n*p^2*log(x)^3 - 3*b*e*m*n*p*log(f)*log(x)^2 + 3*b*e*m*n*lo
g(f)^2*log(x) - 3*b*e*log(c)*log(f)^2 - 3*a*e*log(f)^2)*x^m)*log(x^p))/(e*
x*x^m + d*x), x)
```

Giac [F]

$$\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{(b\log((ex^m+d)^n c)+a)\log(fx^p)^3}{x}\,dx$$

input
```
integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{\ln(f\,x^p)^3\,(a+b\,\ln(c\,(d+e\,x^m)^n))}{x}\,dx$$

input
```
int((log(f*x^p)^3*(a + b*log(c*(d + e*x^m)^n)))/x,x)
```

output
```
int((log(f*x^p)^3*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

Reduce [F]

$$\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{4\left(\int \frac{\log((x^m e+d)^n c)\log(x^p f)^3}{x}dx\right)bp+\log(x^p f)^4 a}{4p}$$

input
```
int(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x)
```

output
```
(4*int((log((x**m*e + d)**n*c)*log(x**p*f)**3)/x,x)*b*p + log(x**p*f)**4*a
)/(4*p)
```

3.622 $\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{3p} - \frac{bn\log^2(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} + \frac{2bnp\log(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2} - \frac{2bnp^2\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}$$

output
```
1/3*ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/p-1/3*b*n*ln(f*x^p)^3*ln(1+e*x^m/d
)/p-b*n*ln(f*x^p)^2*polylog(2,-e*x^m/d)/m+2*b*n*p*ln(f*x^p)*polylog(3,-e*x
^m/d)/m^2-2*b*n*p^2*polylog(4,-e*x^m/d)/m^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 456 vs. $2(132) = 264$.

Time = 0.28 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.45

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$$
$$= \frac{1}{4}bmnp^2\log^4(x) - \frac{1}{3}bmnp\log^3(x)\log(fx^p) + \frac{a\log^3(fx^p)}{3p} + \frac{2}{3}bnp^2\log^3(x)\log\left(1+\frac{dx^{-m}}{e}\right) - bnp\log^2(x)\log(fx^p)\log\left(1+\frac{dx^{-m}}{e}\right) - bnp^2\log^3(x)\log(d+ex^m) + \frac{bnp^2\log^2(x)\log\left(-\frac{ex^m}{d}\right)\log(d+ex^m)}{m} + 2bnp\log^2(x)\log(fx^p)\log(d+ex^m) - \frac{2bnp\log(x)\log\left(-\frac{ex^m}{d}\right)\log(fx^p)\log(d+ex^m)}{m} - bn\log(x)\log^2(fx^p)\log(d+ex^m) + \frac{bn\log\left(-\frac{ex^m}{d}\right)\log^2(fx^p)\log(d+ex^m)}{m} + \frac{1}{3}bp^2\log^3(x)\log(c(d+ex^m)^n) - bp\log^2(x)\log(fx^p)\log(c(d+ex^m)^n) + b\log(x)\log^2(fx^p)\log(c(d+ex^m)^n) - \frac{bnp\log(x)(p\log(x)-2\log(fx^p))\,\text{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m} + \frac{bn(-p\log(x)+\log(fx^p))^2\,\text{PolyLog}\left(2,1+\frac{ex^m}{d}\right)}{m} + \frac{2bnp\log(fx^p)\,\text{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2} + \frac{2bnp^2\,\text{PolyLog}\left(4,-\frac{dx^{-m}}{e}\right)}{m^3}$$

input
```
Integrate[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

output
```
(b*m*n*p^2*Log[x]^4)/4 - (b*m*n*p*Log[x]^3*Log[f*x^p])/3 + (a*Log[f*x^p]^3
)/(3*p) + (2*b*n*p^2*Log[x]^3*Log[1 + d/(e*x^m)])/3 - b*n*p*Log[x]^2*Log[f
*x^p]*Log[1 + d/(e*x^m)] - b*n*p^2*Log[x]^3*Log[d + e*x^m] + (b*n*p^2*Log[
x]^2*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m + 2*b*n*p*Log[x]^2*Log[f*x^p]*Log
[d + e*x^m] - (2*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])
/m - b*n*Log[x]*Log[f*x^p]^2*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[f
*x^p]^2*Log[d + e*x^m])/m + (b*p^2*Log[x]^3*Log[c*(d + e*x^m)^n])/3 - b*p*
Log[x]^2*Log[f*x^p]*Log[c*(d + e*x^m)^n] + b*Log[x]*Log[f*x^p]^2*Log[c*(d
+ e*x^m)^n] - (b*n*p*Log[x]*(p*Log[x] - 2*Log[f*x^p])*PolyLog[2, -(d/(e*x^
m))])/m + (b*n*(-(p*Log[x]) + Log[f*x^p])^2*PolyLog[2, 1 + (e*x^m)/d])/m +
 (2*b*n*p*Log[f*x^p]*PolyLog[3, -(d/(e*x^m))])/m^2 + (2*b*n*p^2*PolyLog[4,
 -(d/(e*x^m))])/m^3
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2931, 2775, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}dx$$

$$\Big\downarrow \text{2931}$$

$$\frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p}-\frac{bemn\int \frac{x^{m-1}\log^3(fx^p)}{ex^m+d}dx}{3p}$$

$$\Big\downarrow \text{2775}$$

$$\frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p}-\frac{bemn\left(\frac{\log^3(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{3p\int \frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{x}dx}{em}\right)}{3p}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p}-\frac{bemn\left(\frac{\log^3(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{3p\left(\frac{2p\int\frac{\log(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{x}dx}{m}-\frac{\log^2(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}\right)}{3p}$$

$$\Big\downarrow \; 2830$$

$$\frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p}-\frac{bemn\left(\frac{\log^3(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{3p\left(\frac{2p\left(\frac{\log(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m}-\frac{p\int\frac{\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{x}dx}{m}\right)}{m}-\frac{\log^2(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}\right)}{3p}$$

$$\Big\downarrow \; 7143$$

$$\frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p}-\frac{bemn\left(\frac{\log^3(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{3p\left(\frac{2p\left(\frac{\log(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m}-\frac{p\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^2}\right)}{m}-\frac{\log^2(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}\right)}{3p}$$

input
```
Int[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

output
```
(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/(3*p) - (b*e*m*n*((Log[f*x^p]^
3*Log[1 + (e*x^m)/d])/(e*m) - (3*p*(-((Log[f*x^p]^2*PolyLog[2, -((e*x^m)/d
)])/m) + (2*p*((Log[f*x^p]*PolyLog[3, -((e*x^m)/d)])/m - (p*PolyLog[4, -((
e*x^m)/d)])/m^2))/m))/(e*m)))/(3*p)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2931
```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Simp[b*e*n*(p/(q*(m + 1)))   Int[x^(n - 1
)*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p, q}, x] && NeQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln (f\,x^{p})^{2} \left(a+b\ln \left(c(d+e\,x^{m})^{n}\right)\right)}{x}dx$$

input
```
int(ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

output
```
int(ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(127) = 254.

Time = 0.13 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.13

$$\int \frac{\log^2 (fx^p) (a+b\log (c(d+ex^m)^n))}{x}\,dx = -\frac{6\,bnp^2\mathrm{polylog}\left(4,-\frac{ex^m}{d}\right)-3\left(bm^3\log(c)+am^3\right)\log(f)^2\log(x)-3\left(bm^3p\log(c)+am^3p\right)\log(f)\log(\ldots$$

input
```
integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

output
```
-1/3*(6*b*n*p^2*polylog(4, -e*x^m/d) - 3*(b*m^3*log(c) + a*m^3)*log(f)^2*l
og(x) - 3*(b*m^3*p*log(c) + a*m^3*p)*log(f)*log(x)^2 - (b*m^3*p^2*log(c) +
 a*m^3*p^2)*log(x)^3 + 3*(b*m^2*n*p^2*log(x)^2 + 2*b*m^2*n*p*log(f)*log(x)
 + b*m^2*n*log(f)^2)*dilog(-(e*x^m + d)/d + 1) - (b*m^3*n*p^2*log(x)^3 + 3
*b*m^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*log(x))*log(e*x^m + d) + (
b*m^3*n*p^2*log(x)^3 + 3*b*m^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*lo
g(x))*log((e*x^m + d)/d) - 6*(b*m*n*p^2*log(x) + b*m*n*p*log(f))*polylog(3
, -e*x^m/d))/m^3
```

Sympy [F]

$$\int \frac{\log^2(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \int \frac{\left(a+b\log\left(c(d+ex^m)^n\right)\right)\log\left(fx^p\right)^2}{x}\,dx$$

input
```
integrate(ln(f*x**p)**2*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x**m)**n))*log(f*x**p)**2/x, x)
```

Maxima [F]

$$\int \frac{\log^2(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \int \frac{\left(b\log\left((ex^m+d)^n c\right)+a\right)\log\left(fx^p\right)^2}{x}\,dx$$

input
```
integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

output
```
1/3*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log(f)^2*log(x) + 3*b*lo
g(x)*log(x^p)^2 - 3*(b*p*log(x)^2 - 2*b*log(f)*log(x))*log(x^p))*log((e*x^
m + d)^n) - integrate(-1/3*(3*b*d*log(c)*log(f)^2 + 3*a*d*log(f)^2 + 3*(b*
d*log(c) + a*d - (b*e*m*n*log(x) - b*e*log(c) - a*e)*x^m)*log(x^p)^2 - (b*
e*m*n*p^2*log(x)^3 - 3*b*e*m*n*p*log(f)*log(x)^2 + 3*b*e*m*n*log(f)^2*log(
x) - 3*b*e*log(c)*log(f)^2 - 3*a*e*log(f)^2)*x^m + 3*(2*b*d*log(c)*log(f)
+ 2*a*d*log(f) + (b*e*m*n*p*log(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log
(c)*log(f) + 2*a*e*log(f))*x^m)*log(x^p))/(e*x*x^m + d*x), x)
```

Giac [F]

$$\int \frac{\log^2(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \int \frac{\left(b\log\left((ex^m+d)^n c\right)+a\right)\log\left(fx^p\right)^2}{x}\,dx$$

input
```
integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx=\int \frac{\ln(f\,x^p)^2\,(a+b\,\ln(c\,(d+e\,x^m)^n))}{x}\,dx$$

input
```
int((log(f*x^p)^2*(a + b*log(c*(d + e*x^m)^n)))/x,x)
```

output
```
int((log(f*x^p)^2*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

Reduce [F]

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx=\frac{3\left(\int \frac{\log((x^me+d)^nc)\log(x^pf)^2}{x}dx\right)bp+\log(x^pf)^3a}{3p}$$

input
```
int(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x)
```

output
```
(3*int((log((x**m*e + d)**n*c)*log(x**p*f)**2)/x,x)*b*p + log(x**p*f)**3*a
)/(3*p)
```

3.623 $\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{2p} - \frac{bn\log(fx^p)\,\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} + \frac{bnp\,\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}$$

output
```
1/2*ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/p-1/2*b*n*ln(f*x^p)^2*ln(1+e*x^m/d
)/p-b*n*ln(f*x^p)*polylog(2,-e*x^m/d)/m+b*n*p*polylog(3,-e*x^m/d)/m^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 265 vs. $2(102) = 204$.

Time = 0.25 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.60

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$$
$$= -\frac{1}{6}bmnp\log^3(x)+\frac{a\log^2(fx^p)}{2p}-\frac{1}{2}bnp\log^2(x)\log\left(1+\frac{dx^{-m}}{e}\right)$$
$$+bnp\log^2(x)\log(d+ex^m)-\frac{bnp\log(x)\log\left(-\frac{ex^m}{d}\right)\log(d+ex^m)}{m}$$
$$-bn\log(x)\log(fx^p)\log(d+ex^m)$$
$$+\frac{bn\log\left(-\frac{ex^m}{d}\right)\log(fx^p)\log(d+ex^m)}{m}-\frac{1}{2}bp\log^2(x)\log(c(d+ex^m)^n)$$
$$+b\log(x)\log(fx^p)\log(c(d+ex^m)^n)+\frac{bnp\log(x)\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m}$$
$$-\frac{bn(p\log(x)-\log(fx^p))\operatorname{PolyLog}\left(2,1+\frac{ex^m}{d}\right)}{m}+\frac{bnp\operatorname{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2}$$

input
```
Integrate[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

output
```
-1/6*(b*m*n*p*Log[x]^3) + (a*Log[f*x^p]^2)/(2*p) - (b*n*p*Log[x]^2*Log[1 +
 d/(e*x^m)])/2 + b*n*p*Log[x]^2*Log[d + e*x^m] - (b*n*p*Log[x]*Log[-((e*x^
m)/d)]*Log[d + e*x^m])/m - b*n*Log[x]*Log[f*x^p]*Log[d + e*x^m] + (b*n*Log
[-((e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m - (b*p*Log[x]^2*Log[c*(d + e*x
^m)^n])/2 + b*Log[x]*Log[f*x^p]*Log[c*(d + e*x^m)^n] + (b*n*p*Log[x]*PolyL
og[2, -(d/(e*x^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])*PolyLog[2, 1 + (e*x^
m)/d])/m + (b*n*p*PolyLog[3, -(d/(e*x^m))])/m^2
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2931, 2775, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}dx$$

$$\Big\downarrow \text{2931}$$

$$\frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p}-\frac{bemn\int \frac{x^{m-1}\log^2(fx^p)}{ex^m+d}dx}{2p}$$

$$\Big\downarrow \text{2775}$$

$$\frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p}-\frac{bemn\left(\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2p\int \frac{\log(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{x}dx}{em}\right)}{2p}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p}-\frac{bemn\left(\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2p\left(\frac{p\int \frac{\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{x}dx}{m}-\frac{\log(fx^p)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}\right)}{2p}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p}-\frac{bemn\left(\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2p\left(\frac{p\,\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}-\frac{\log(fx^p)\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m}\right)}{em}\right)}{2p}$$

input
```
Int[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

output
```
(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/(2*p) - (b*e*m*n*((Log[f*x^p]^
2*Log[1 + (e*x^m)/d])/(e*m) - (2*p*(-((Log[f*x^p]*PolyLog[2, -((e*x^m)/d)]
)/m) + (p*PolyLog[3, -((e*x^m)/d)])/m^2))/(e*m)))/(2*p)
```

Defintions of rubi rules used

rule 2775
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Simp[b*f^m*n*(p/(e*r))   Int[Log[1 + e*(x^r/d)]*((a +
b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &
& EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2931
```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Simp[b*e*n*(p/(q*(m + 1)))   Int[x^(n - 1
)*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p, q}, x] && NeQ[m, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\ln (f\,x^{p}) \left(a+b\ln \left(c(d+e\,x^{m})^{n}\right)\right)}{x}dx$$

input
```
int(ln(f*x^p)*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

output
```
int(ln(f*x^p)*(a+b*ln(c*(d+e*x^m)^n))/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.58

$$\int \frac{\log (fx^p) (a+b\log (c(d+ex^m)^n))}{x}\,dx = \frac{2\,bnp\mathrm{polylog}\left(3,-\frac{ex^m}{d}\right)+2\,(bm^2\log(c)+am^2)\log(f)\log(x)+(bm^2p\log(c)+am^2p)\log(x)^2-2\,(bmn$$

input
```
integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

output
```
1/2*(2*b*n*p*polylog(3, -e*x^m/d) + 2*(b*m^2*log(c) + a*m^2)*log(f)*log(x)
 + (b*m^2*p*log(c) + a*m^2*p)*log(x)^2 - 2*(b*m*n*p*log(x) + b*m*n*log(f))
*dilog(-(e*x^m + d)/d + 1) + (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x)
)*log(e*x^m + d) - (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x))*log((e*x
^m + d)/d))/m^2
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log (fx^p) (a+b\log (c(d+ex^m)^n))}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(ln(f*x**p)*(a+b*ln(c*(d+e*x**m)**n))/x,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{(b\log((ex^m+d)^nc)+a)\log(fx^p)}{x}\,dx$$

input
```
integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

output
```
-1/2*(b*p*log(x)^2 - 2*b*log(f)*log(x) - 2*b*log(x)*log(x^p))*log((e*x^m +
 d)^n) - integrate(-1/2*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*e*m*n*p*l
og(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log(f) + 2*a*e*log(f))*x^
m + 2*(b*d*log(c) + a*d - (b*e*m*n*log(x) - b*e*log(c) - a*e)*x^m)*log(x^p
))/(e*x*x^m + d*x), x)
```

Giac [F]

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{(b\log((ex^m+d)^nc)+a)\log(fx^p)}{x}\,dx$$

input
```
integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{\ln(f\,x^p)\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)\right)}{x}\,dx$$

input
```
int((log(f*x^p)*(a + b*log(c*(d + e*x^m)^n)))/x,x)
```

output
```
int((log(f*x^p)*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

Reduce [F]

$$\int \frac{\log (fx^p) (a+b\log (c(d+ex^m)^n))}{x}\,dx = \frac{2\left(\int \frac{\log((x^m e+d)^n c)\log(x^p f)}{x}dx\right)bp+\log(x^p f)^2 a}{2p}$$

input
```
int(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x)
```

output
```
(2*int((log((x**m*e + d)**n*c)*log(x**p*f))/x,x)*b*p + log(x**p*f)**2*a)/(
2*p)
```

3.624 $\int \frac{a+b\log(c(d+ex^m)^n)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = \frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{m} + \frac{bn\,\text{PolyLog}\left(2,1+\frac{ex^m}{d}\right)}{m}$$

output
```
ln(-e*x^m/d)*(a+b*ln(c*(d+e*x^m)^n))/m+b*n*polylog(2,1+e*x^m/d)/m
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = a\log(x) + \frac{b\left(\log\left(-\frac{ex^m}{d}\right)\log\left(c(d+ex^m)^n\right)+n\,\text{PolyLog}\left(2,\frac{d+ex^m}{d}\right)\right)}{m}$$

input
```
Integrate[(a + b*Log[c*(d + e*x^m)^n])/x,x]
```

output
```
a*Log[x] + (b*(Log[-((e*x^m)/d)]*Log[c*(d + e*x^m)^n] + n*PolyLog[2, (d +
e*x^m)/d]))/m
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx$$

$$\Big\downarrow \ \textbf{2904}$$

$$\frac{\int x^{-m}(a+b\log\left(c(ex^m+d)^n\right))\,dx^m}{m}$$

$$\Big\downarrow \ \textbf{2841}$$

$$\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)-ben\int \frac{\log\left(-\frac{ex^m}{d}\right)}{ex^m+d}dx^m}{m}$$

$$\Big\downarrow \ \textbf{2752}$$

$$\frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)+bn\,\text{PolyLog}\left(2,\frac{ex^m}{d}+1\right)}{m}$$

input
```
Int[(a + b*Log[c*(d + e*x^m)^n])/x,x]
```

output
```
(Log[-((e*x^m)/d)]*(a + b*Log[c*(d + e*x^m)^n]) + b*n*PolyLog[2, 1 + (e*x^
m)/d])/m
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.78 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.67

method	result
risch	$b\ln \left(x \right) \ln \left(\left(d +e \,x^{m}\right)^{n}\right)+\left(\frac{i b \pi \,\operatorname{csgn}\left(i \left(d +e \,x^{m}\right)^{n}\right) \operatorname{csgn}\left(i c \left(d +e \,x^{m}\right)^{n}\right)^{2}}{2}-\frac{i b \pi \,\operatorname{csgn}\left(i \left(d +e \,x^{m}\right)^{n}\right) \operatorname{csgn}\left(i c \left(d +e \,x^{m}\right)^{n}\right) \operatorname{csg}}{2}\right.$

input
```
int((a+b*ln(c*(d+e*x^m)^n))/x,x,method=_RETURNVERBOSE)
```

output
```
b*ln(x)*ln((d+e*x^m)^n)+(1/2*I*b*Pi*csgn(I*(d+e*x^m)^n)*csgn(I*c*(d+e*x^m)
^n)^2-1/2*I*b*Pi*csgn(I*(d+e*x^m)^n)*csgn(I*c*(d+e*x^m)^n)*csgn(I*c)-1/2*I
*b*Pi*csgn(I*c*(d+e*x^m)^n)^3+1/2*I*b*Pi*csgn(I*c*(d+e*x^m)^n)^2*csgn(I*c)
+b*ln(c)+a)*ln(x)-b/m*n*dilog((d+e*x^m)/d)-b*n*ln(x)*ln((d+e*x^m)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = \frac{bmn\log\left(ex^m+d\right)\log\left(x\right)-bmn\log\left(x\right)\log\left(\frac{ex^m+d}{d}\right)-bn\mathrm{Li}_2\left(-\frac{ex^m+d}{d}+1\right)+\left(bm\log\left(c\right)+am\right)\log\left(x\right)}{m}$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

output
```
(b*m*n*log(e*x^m + d)*log(x) - b*m*n*log(x)*log((e*x^m + d)/d) - b*n*dilog
(-(e*x^m + d)/d + 1) + (b*m*log(c) + a*m)*log(x))/m
```

Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = \int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**m)**n))/x,x)
```

output
```
Integral((a + b*log(c*(d + e*x**m)**n))/x, x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = \int \frac{b\log\left((ex^m+d)^n c\right)+a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

output
```
1/2*(2*d*m*n*integrate(log(x)/(e*x*x^m + d*x), x) - m*n*log(x)^2 + 2*log((
e*x^m + d)^n)*log(x) + 2*log(c)*log(x))*b + a*log(x)
```

Giac [F]

$$\int \frac{a + b\log\left(c(d + ex^m)^n\right)}{x}\,dx = \int \frac{b\log\left((ex^m + d)^n c\right) + a}{x}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^m + d)^n*c) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex^m)^n\right)}{x}\,dx = \int \frac{a + b\,\ln\left(c\,(d + e\,x^m)^n\right)}{x}\,dx$$

input
```
int((a + b*log(c*(d + e*x^m)^n))/x,x)
```

output
```
int((a + b*log(c*(d + e*x^m)^n))/x, x)
```

Reduce [F]

$$\int \frac{a + b\log\left(c(d + ex^m)^n\right)}{x}\,dx = \frac{2\left(\int \frac{\log((x^m e+d)^n c)}{x^m ex+dx}dx\right) bdmn + \log((x^m e + d)^n\, c)^2 b + 2\log(x)\, amn}{2mn}$$

input
```
int((a+b*log(c*(d+e*x^m)^n))/x,x)
```

output
```
(2*int(log((x**m*e + d)**n*c)/(x**m*e*x + d*x),x)*b*d*m*n + log((x**m*e +
d)**n*c)**2*b + 2*log(x)*a*m*n)/(2*m*n)
```

3.625 $\int \frac{a+b\log(c(d+ex^m)^n)}{x\log(fx^p)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx=\frac{a\log\left(\log\left(fx^p\right)\right)}{p}+b\text{Int}\left(\frac{\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)},x\right)$$

output
```
a*ln(ln(f*x^p))/p+b*Defer(Int)(ln(c*(d+e*x^m)^n)/x/ln(f*x^p),x)
```

Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx=\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]), x]
```

Rubi [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a}{x\log\left(fx^p\right)}+\frac{b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b\int \frac{\log\left(c(ex^m+d)^n\right)}{x\log\left(fx^p\right)}dx+\frac{a\log\left(\log\left(fx^p\right)\right)}{p}$$

input
```
Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln\left(c(d+e\,x^{m})^{n}\right)}{x\ln\left(f\,x^{p}\right)}dx$$

input
```
int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p),x)
```

output
```
int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^{m})^{n}\right)}{x\log\left(fx^{p}\right)}\,dx=\int \frac{b\log\left((ex^{m}+d)^{n}c\right)+a}{x\log\left(fx^{p}\right)}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="fricas")
```

output
```
integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)), x)
```

Sympy [N/A]

Not integrable

Time = 46.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c(d+ex^{m})^{n}\right)}{x\log\left(fx^{p}\right)}\,dx=\int \frac{a+b\log\left(c(d+ex^{m})^{n}\right)}{x\log\left(fx^{p}\right)}\,dx$$

input
```
integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p),x)
```

output
```
Integral((a + b*log(c*(d + e*x**m)**n))/(x*log(f*x**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{a + b\log\left(c(d + ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx = \int \frac{b\log\left((ex^m + d)^n c\right) + a}{x\log\left(fx^p\right)}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="maxima")
```

output
```
b*integrate((log((e*x^m + d)^n) + log(c))/(x*log(f) + x*log(x^p)), x) + a*
log(log(f*x^p))/p
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a + b\log\left(c(d + ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx = \int \frac{b\log\left((ex^m + d)^n c\right) + a}{x\log\left(fx^p\right)}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="giac")
```

output
```
integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)), x)
```

Mupad [N/A]

Not integrable

Time = 25.94 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,\ln\left(f\,x^p\right)}\,dx$$

input
```
int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)),x)
```

output
```
int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx=\frac{\left(\int \frac{\log\left(\left(x^m e+d\right)^n c\right)}{\log\left(x^p f\right)x}dx\right)bp+\log\left(\log\left(x^p f\right)\right)a}{p}$$

input
```
int((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x)
```

output
```
(int(log((x**m*e + d)**n*c)/(log(x**p*f)*x),x)*b*p + log(log(x**p*f))*a)/p
```

3.626 $\int \frac{a+b\log(c(d+ex^m)^n)}{x\log^2(fx^p)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx = -\frac{a+b\log\left(c(d+ex^m)^n\right)}{p\log\left(fx^p\right)} + \frac{bemn\text{Int}\left(\frac{x^{-1+m}}{(d+ex^m)\log(fx^p)},x\right)}{p}$$

output
```
-(a+b*ln(c*(d+e*x^m)^n))/p/ln(f*x^p)+b*e*m*n*Defer(Int)(x^(-1+m)/(d+e*x^m)
/ln(f*x^p),x)/p
```

Mathematica [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx = \int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2931, 2796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx$$

$$\Big\downarrow \; 2931$$

$$\frac{bemn\int \frac{x^{m-1}}{(ex^m+d)\log(fx^p)}dx}{p}-\frac{a+b\log\left(c(d+ex^m)^n\right)}{p\log\left(fx^p\right)}$$

$$\Big\downarrow \; 2796$$

$$\frac{bemn\int \frac{x^{m-1}}{(ex^m+d)\log(fx^p)}dx}{p}-\frac{a+b\log\left(c(d+ex^m)^n\right)}{p\log\left(fx^p\right)}$$

input
```
Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2796
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^r)^q*(a
 + b*Log[c*x^n])^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x]
```

rule 2931
```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Simp[b*e*n*(p/(q*(m + 1)))   Int[x^(n - 1
)*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p, q}, x] && NeQ[m, -1]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln\left(c(d+e\,x^{m})^{n}\right)}{x\ln\left(f\,x^{p}\right)^{2}}dx$$

input
```
int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^2,x)
```

output
```
int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^{m})^{n}\right)}{x\log^{2}\left(fx^{p}\right)}\,dx=\int \frac{b\log\left((ex^{m}+d)^{n}c\right)+a}{x\log\left(fx^{p}\right)^{2}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="fricas")
```

output
```
integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx = \int \frac{b\log\left((ex^m+d)^n c\right)+a}{x\log\left(fx^p\right)^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="maxima")
```

output
```
(e*m*n*integrate(x^m/(e*p*x*x^m*log(f) + d*p*x*log(f) + (e*p*x*x^m + d*p*x
)*log(x^p)), x) - (log((e*x^m + d)^n) + log(c))/(p*log(f) + p*log(x^p)))*b
 - a/(p*log(f*x^p))
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx = \int \frac{b\log\left((ex^m+d)^n c\right)+a}{x\log\left(fx^p\right)^2}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

Mupad [N/A]

Not integrable

Time = 26.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2(fx^p)}\,dx=\int \frac{a+b\,\ln\left(c\left(d+e\,x^m\right)^n\right)}{x\ln\left(f\,x^p\right)^2}\,dx$$

input
```
int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^2),x)
```

output
```
int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.18

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2(fx^p)}\,dx = \frac{-\left(\int \frac{1}{x^m\log(x^pf)ex+\log(x^pf)dx}dx\right)\log(x^pf)\,bdmnp+\log(\log(x^pf))\log(x^pf)\,bmn-\log((x^me+d)^n\,c)\,bp-a}{\log(x^pf)\,p^2}$$

input
```
int((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x)
```

output
```
( - int(1/(x**m*log(x**p*f)*e*x + log(x**p*f)*d*x),x)*log(x**p*f)*b*d*m*n*
p + log(log(x**p*f))*log(x**p*f)*b*m*n - log((x**m*e + d)**n*c)*b*p - a*p)
/(log(x**p*f)*p**2)
```

3.627 $\int \frac{a+b\log(c(d+ex^m)^n)}{x\log^3(fx^p)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3(fx^p)}\,dx=-\frac{a+b\log\left(c(d+ex^m)^n\right)}{2p\log^2(fx^p)}+\frac{bemn\text{Int}\left(\frac{x^{-1+m}}{(d+ex^m)\log^2(fx^p)},x\right)}{2p}$$

output
```
-1/2*(a+b*ln(c*(d+e*x^m)^n))/p/ln(f*x^p)^2+1/2*b*e*m*n*Defer(Int)(x^(-1+m)
/(d+e*x^m)/ln(f*x^p)^2,x)/p
```

Mathematica [N/A]

Not integrable

Time = 13.93 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3(fx^p)}\,dx=\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3(fx^p)}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
```

output
```
Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3), x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2931, 2796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}dx$$

$$\Big\downarrow \text{ 2931}$$

$$\frac{bemn\int \frac{x^{m-1}}{(ex^m+d)\log^2(fx^p)}dx}{2p}-\frac{a+b\log\left(c(d+ex^m)^n\right)}{2p\log^2\left(fx^p\right)}$$

$$\Big\downarrow \text{ 2796}$$

$$\frac{bemn\int \frac{x^{m-1}}{(ex^m+d)\log^2(fx^p)}dx}{2p}-\frac{a+b\log\left(c(d+ex^m)^n\right)}{2p\log^2\left(fx^p\right)}$$

input
```
Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2796
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Unintegrable[(f*x)^m*(d + e*x^r)^q*(a
 + b*Log[c*x^n])^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x]
```

rule 2931
```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Simp[b*e*n*(p/(q*(m + 1)))   Int[x^(n - 1
)*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p, q}, x] && NeQ[m, -1]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln \left(c(d+e\,x^{m})^{n}\right)}{x\ln \left(f\,x^{p}\right)^{3}}dx$$

input
```
int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^3,x)
```

output
```
int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log \left(c(d+ex^{m})^{n}\right)}{x\log^{3}\left(fx^{p}\right)}\,dx=\int \frac{b\log \left((ex^{m}+d)^{n}c\right)+a}{x\log \left(fx^{p}\right)^{3}}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="fricas")
```

output
```
integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 238, normalized size of antiderivative = 8.50

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx = \int \frac{b\log\left((ex^m+d)^n c\right)+a}{x\log\left(fx^p\right)^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="maxima")
```

output
```
1/2*(2*d*e*m^2*n*integrate(1/2*x^m/(e^2*p^2*x*x^(2*m)*log(f) + 2*d*e*p^2*x
*x^m*log(f) + d^2*p^2*x*log(f) + (e^2*p^2*x*x^(2*m) + 2*d*e*p^2*x*x^m + d^
2*p^2*x)*log(x^p)), x) - (e*m*n*x^m*log(x^p) + d*p*log(c) + (e*m*n*log(f)
+ e*p*log(c))*x^m + (e*p*x^m + d*p)*log((e*x^m + d)^n))/(e*p^2*x^m*log(f)^
2 + d*p^2*log(f)^2 + (e*p^2*x^m + d*p^2)*log(x^p)^2 + 2*(e*p^2*x^m*log(f)
+ d*p^2*log(f))*log(x^p)))*b - 1/2*a/(p*log(f*x^p)^2)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log^3 (fx^p)}\,dx=\int \frac{b\log \left((ex^m+d)^n c\right)+a}{x\log \left(fx^p\right)^3}\,dx$$

input
```
integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="giac")
```

output
```
integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

Mupad [N/A]

Not integrable

Time = 26.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log^3 (fx^p)}\,dx=\int \frac{a+b\,\ln \left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,{\ln \left(f\,x^p\right)}^3}\,dx$$

input
```
int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^3),x)
```

output
```
int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.82

$$\int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log^3 (fx^p)}\,dx=\frac{\left(\int \frac{x^m}{x^m\log(x^p f)^2 ex+\log(x^p f)^2 dx}dx\right)\log(x^p f)^2 bemn-\log\left((x^m e+d)^n c\right)b-a}{2\log\left(x^p f\right)^2 p}$$

input
```
int((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x)
```

output
```
(int(x**m/(x**m*log(x**p*f)**2*e*x + log(x**p*f)**2*d*x),x)*log(x**p*f)**2
*b*e*m*n - log((x**m*e + d)**n*c)*b - a)/(2*log(x**p*f)**2*p)
```

3.628 $\int \frac{\log(1+x^2)}{\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{\log(1+x^2)}{\sqrt{1+x^2}} \, dx = \text{arcsinh}(x) \log\left(1+x^2\right) + \log^2\left(x+\sqrt{1+x^2}\right) - 2\log\left(x+\sqrt{1+x^2}\right)\log\left(1+\left(x+\sqrt{1+x^2}\right)^2\right) - \text{PolyLog}\left(2,-\left(x+\sqrt{1+x^2}\right)^2\right)$$

output
```
arcsinh(x)*ln(x^2+1)+ln(x+(x^2+1)^(1/2))^2-2*ln(x+(x^2+1)^(1/2))*ln(1+(x+(
x^2+1)^(1/2))^2)-polylog(2,-(x+(x^2+1)^(1/2))^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.55

$$\int \frac{\log(1+x^2)}{\sqrt{1+x^2}} \, dx = -\frac{i\sqrt{-x^2}\left(\arcsin\left(\sqrt{1+x^2}\right)^2 - 2i\arcsin\left(\sqrt{1+x^2}\right)\log\left(1-e^{-2i\arcsin\left(\sqrt{1+x^2}\right)}\right) + \log\left(1+x^2\right)\log\left(\sqrt{-x\right.\right.}{x}$$

input
```
Integrate[Log[1 + x^2]/Sqrt[1 + x^2],x]
```

output
```
((-I)*Sqrt[-x^2]*(ArcSin[Sqrt[1 + x^2]]^2 - (2*I)*ArcSin[Sqrt[1 + x^2]]*Lo
g[1 - E^((-2*I)*ArcSin[Sqrt[1 + x^2]])] + Log[1 + x^2]*Log[Sqrt[-x^2] + I*
Sqrt[1 + x^2]] + PolyLog[2, E^((-2*I)*ArcSin[Sqrt[1 + x^2]])]))/x
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x^2+1)}{\sqrt{x^2+1}}\,dx$$

$$\Big\downarrow \; 2923$$

$$\int \frac{\log(x^2+1)}{\sqrt{x^2+1}}dx$$

input
```
Int[Log[1 + x^2]/Sqrt[1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(x^2+1)}{\sqrt{x^2+1}}dx$$

input
```
int(ln(x^2+1)/(x^2+1)^(1/2),x)
```

output
```
int(ln(x^2+1)/(x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log(1+x^2)}{\sqrt{1+x^2}}\,dx = \int \frac{\log(x^2+1)}{\sqrt{x^2+1}}\,dx$$

input
```
integrate(log(x^2+1)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(log(x^2 + 1)/sqrt(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log(1+x^2)}{\sqrt{1+x^2}}\,dx = \int \frac{\log(x^2+1)}{\sqrt{x^2+1}}\,dx$$

input
```
integrate(ln(x**2+1)/(x**2+1)**(1/2),x)
```

output
```
Integral(log(x**2 + 1)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\log(1+x^2)}{\sqrt{1+x^2}}\,dx = \int \frac{\log(x^2+1)}{\sqrt{x^2+1}}\,dx$$

input
```
integrate(log(x^2+1)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(x^2 + 1)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log(1+x^2)}{\sqrt{1+x^2}}\,dx = \int \frac{\log(x^2+1)}{\sqrt{x^2+1}}\,dx$$

input
```
integrate(log(x^2+1)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(x^2 + 1)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1+x^2)}{\sqrt{1+x^2}}\,dx = \int \frac{\ln(x^2+1)}{\sqrt{x^2+1}}\,dx$$

input
```
int(log(x^2 + 1)/(x^2 + 1)^(1/2),x)
```

output
```
int(log(x^2 + 1)/(x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log (1+x^2)}{\sqrt{1+x^2}}\,dx=\int \frac{\log(x^2+1)}{\sqrt{x^2+1}}dx$$

input
```
int(log(x^2+1)/(x^2+1)^(1/2),x)
```

output
```
int(log(x**2 + 1)/sqrt(x**2 + 1),x)
```

3.629 $\int \frac{\log(-1-x^2)}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 75

$$\int \frac{\log(-1-x^2)}{\sqrt{1+x^2}}\,dx = \operatorname{arcsinh}(x)\log(-1-x^2) + \log^2\left(x+\sqrt{1+x^2}\right) - 2\log\left(x+\sqrt{1+x^2}\right)\log\left(1+\left(x+\sqrt{1+x^2}\right)^2\right) - \operatorname{PolyLog}\left(2,-\left(x+\sqrt{1+x^2}\right)^2\right)$$

output
```
arcsinh(x)*ln(-x^2-1)+ln(x+(x^2+1)^(1/2))^2-2*ln(x+(x^2+1)^(1/2))*ln(1+(x+
(x^2+1)^(1/2))^2)-polylog(2,-(x+(x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.80

$$\int \frac{\log(-1-x^2)}{\sqrt{1+x^2}}\,dx = -\frac{\sqrt{-x^2}\sqrt{-1-x^2}\left(-\operatorname{arcsinh}\left(\sqrt{-1-x^2}\right)^2 - 2\operatorname{arcsinh}\left(\sqrt{-1-x^2}\right)\log\left(1-e^{-2\operatorname{arcsinh}\left(\sqrt{-1-x^2}\right)}\right) + \log\left(\right.}{x\sqrt{1+x^2}}$$

input
```
Integrate[Log[-1 - x^2]/Sqrt[1 + x^2],x]
```

output
```
-((Sqrt[-x^2]*Sqrt[-1 - x^2]*(-ArcSinh[Sqrt[-1 - x^2]]^2 - 2*ArcSinh[Sqrt[
-1 - x^2]]*Log[1 - E^(-2*ArcSinh[Sqrt[-1 - x^2]])] + Log[-1 - x^2]*Log[Sqr
t[-x^2] + Sqrt[-1 - x^2]] + PolyLog[2, E^(-2*ArcSinh[Sqrt[-1 - x^2]])]))/(
x*Sqrt[1 + x^2]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(-x^2-1\right)}{\sqrt{x^2+1}} \, dx$$
$$\Big\downarrow \; 2923$$
$$\int \frac{\log \left(-x^2-1\right)}{\sqrt{x^2+1}}dx$$

input
```
Int[Log[-1 - x^2]/Sqrt[1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln \left(-x^{2}-1\right)}{\sqrt{x^{2}+1}}dx$$

input
```
int(ln(-x^2-1)/(x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2-1)/(x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log \left(-1-x^{2}\right)}{\sqrt{1+x^{2}}}\,dx = \int \frac{\log \left(-x^{2}-1\right)}{\sqrt{x^{2}+1}}\,dx$$

input
```
integrate(log(-x^2-1)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(log(-x^2 - 1)/sqrt(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log \left(-1-x^{2}\right)}{\sqrt{1+x^{2}}}\,dx = \int \frac{\log \left(-x^{2}-1\right)}{\sqrt{x^{2}+1}}\,dx$$

input
```
integrate(ln(-x**2-1)/(x**2+1)**(1/2),x)
```

output
```
Integral(log(-x**2 - 1)/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\log(-1-x^2)}{\sqrt{1+x^2}}\,dx = \int \frac{\log(-x^2-1)}{\sqrt{x^2+1}}\,dx$$

input
```
integrate(log(-x^2-1)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 - 1)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log(-1-x^2)}{\sqrt{1+x^2}}\,dx = \int \frac{\log(-x^2-1)}{\sqrt{x^2+1}}\,dx$$

input
```
integrate(log(-x^2-1)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 - 1)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(-1-x^2)}{\sqrt{1+x^2}}\,dx = \int \frac{\ln(-x^2-1)}{\sqrt{x^2+1}}\,dx$$

input
```
int(log(- x^2 - 1)/(x^2 + 1)^(1/2),x)
```

output
```
int(log(- x^2 - 1)/(x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(-1-x^2)}{\sqrt{1+x^2}}\,dx = \int \frac{\log(-x^2-1)}{\sqrt{x^2+1}}dx$$

input
```
int(log(-x^2-1)/(x^2+1)^(1/2),x)
```

output
```
int(log( - x**2 - 1)/sqrt(x**2 + 1),x)
```

3.630 $\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \arcsin(x)\log\left(1-x^2\right) - i\log^2\left(ix+\sqrt{1-x^2}\right) + 2i\log\left(ix+\sqrt{1-x^2}\right)\log\left(1+\left(ix+\sqrt{1-x^2}\right)^2\right) + i\,\mathrm{PolyLog}\left(2,-\left(ix+\sqrt{1-x^2}\right)^2\right)$$

output
```
arcsin(x)*ln(-x^2+1)-I*ln(I*x+(-x^2+1)^(1/2))^2+2*I*ln(I*x+(-x^2+1)^(1/2))
*ln(1+(I*x+(-x^2+1)^(1/2))^2)+I*polylog(2,-(I*x+(-x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.79

$$\begin{aligned}\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx &= -2i\pi\arcsin(x)+i\arcsin(x)^2-4\pi\log\left(1+e^{-i\arcsin(x)}\right)\\ &\quad-\pi\log\left(1-ie^{i\arcsin(x)}\right)-2\arcsin(x)\log\left(1-ie^{i\arcsin(x)}\right)\\ &\quad+\pi\log\left(1+ie^{i\arcsin(x)}\right)-2\arcsin(x)\log\left(1+ie^{i\arcsin(x)}\right)\\ &\quad+\arcsin(x)\log\left(1-x^2\right)+4\pi\log\left(\cos\left(\frac{\arcsin(x)}{2}\right)\right)\\ &\quad-\pi\log\left(-\cos\left(\frac{1}{4}(\pi+2\arcsin(x))\right)\right)\\ &\quad+\pi\log\left(\sin\left(\frac{1}{4}(\pi+2\arcsin(x))\right)\right)\\ &\quad+2i\,\mathrm{PolyLog}\left(2,-ie^{i\arcsin(x)}\right)+2i\,\mathrm{PolyLog}\left(2,ie^{i\arcsin(x)}\right)\end{aligned}$$

input
```
Integrate[Log[1 - x^2]/Sqrt[1 - x^2],x]
```

output
```
(-2*I)*Pi*ArcSin[x] + I*ArcSin[x]^2 - 4*Pi*Log[1 + E^((-I)*ArcSin[x])] - P
i*Log[1 - I*E^(I*ArcSin[x])] - 2*ArcSin[x]*Log[1 - I*E^(I*ArcSin[x])] + Pi
*Log[1 + I*E^(I*ArcSin[x])] - 2*ArcSin[x]*Log[1 + I*E^(I*ArcSin[x])] + Arc
Sin[x]*Log[1 - x^2] + 4*Pi*Log[Cos[ArcSin[x]/2]] - Pi*Log[-Cos[(Pi + 2*Arc
Sin[x])/4]] + Pi*Log[Sin[(Pi + 2*ArcSin[x])/4]] + (2*I)*PolyLog[2, (-I)*E^
(I*ArcSin[x])] + (2*I)*PolyLog[2, I*E^(I*ArcSin[x])]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \; 2923$$

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx$$

input
```
Int[Log[1 - x^2]/Sqrt[1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{\sqrt{-x^2+1}}dx$$

input
```
int(ln(-x^2+1)/(-x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/(-x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*log(-x^2 + 1)/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(1-x^2)}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/(-x**2+1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\ln(1-x^2)}{\sqrt{1-x^2}}\,dx$$

input
```
int(log(1 - x^2)/(1 - x^2)^(1/2),x)
```

output
```
int(log(1 - x^2)/(1 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}}dx$$

input
```
int(log(-x^2+1)/(-x^2+1)^(1/2),x)
```

output
```
int(log( - x**2 + 1)/sqrt( - x**2 + 1),x)
```

3.631 $\int \frac{\log(-1+x^2)}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{\log(-1+x^2)}{\sqrt{1-x^2}}\,dx = \arcsin(x)\log\left(-1+x^2\right) - i\log^2\left(ix+\sqrt{1-x^2}\right) + 2i\log\left(ix+\sqrt{1-x^2}\right)\log\left(1+\left(ix+\sqrt{1-x^2}\right)^2\right) + i\,\text{PolyLog}\left(2,-\left(ix+\sqrt{1-x^2}\right)^2\right)$$

output
```
arcsin(x)*ln(x^2-1)-I*ln(I*x+(-x^2+1)^(1/2))^2+2*I*ln(I*x+(-x^2+1)^(1/2))*
ln(1+(I*x+(-x^2+1)^(1/2))^2)+I*polylog(2,-(I*x+(-x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \frac{\log(-1+x^2)}{\sqrt{1-x^2}}\,dx = \frac{\sqrt{x^2}\sqrt{-1+x^2}\left(-\text{arcsinh}\left(\sqrt{-1+x^2}\right)^2 - 2\text{arcsinh}\left(\sqrt{-1+x^2}\right)\log\left(1-e^{-2\text{arcsinh}\left(\sqrt{-1+x^2}\right)}\right) + \log\left(-1 \right.\right.}{x\sqrt{1-x^2}}$$

input
```
Integrate[Log[-1 + x^2]/Sqrt[1 - x^2],x]
```

output
```
(Sqrt[x^2]*Sqrt[-1 + x^2]*(-ArcSinh[Sqrt[-1 + x^2]]^2 - 2*ArcSinh[Sqrt[-1
+ x^2]]*Log[1 - E^(-2*ArcSinh[Sqrt[-1 + x^2]])] + Log[-1 + x^2]*Log[Sqrt[x
^2] + Sqrt[-1 + x^2]] + PolyLog[2, E^(-2*ArcSinh[Sqrt[-1 + x^2]])]))/(x*Sq
rt[1 - x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (x^2-1)}{\sqrt{1-x^2}} \, dx$$

$$\Big\downarrow \text{ 2923}$$

$$\int \frac{\log (x^2-1)}{\sqrt{1-x^2}}dx$$

input
```
Int[Log[-1 + x^2]/Sqrt[1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln \left(x^2-1\right)}{\sqrt{-x^2+1}}dx$$

input
```
int(ln(x^2-1)/(-x^2+1)^(1/2),x)
```

output
```
int(ln(x^2-1)/(-x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log \left(-1+x^2\right)}{\sqrt{1-x^2}}\, dx = \int \frac{\log \left(x^2-1\right)}{\sqrt{-x^2+1}}\, dx$$

input
```
integrate(log(x^2-1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*log(x^2 - 1)/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\log \left(-1+x^2\right)}{\sqrt{1-x^2}}\, dx = \int \frac{\log \left(x^2-1\right)}{\sqrt{-\left(x-1\right)\left(x+1\right)}}\, dx$$

input
```
integrate(ln(x**2-1)/(-x**2+1)**(1/2),x)
```

output
```
Integral(log(x**2 - 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\log(-1+x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(x^2-1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(log(x^2-1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(x^2 - 1)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log(-1+x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(x^2-1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(log(x^2-1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(x^2 - 1)/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(-1+x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\ln(x^2-1)}{\sqrt{1-x^2}}\,dx$$

input
```
int(log(x^2 - 1)/(1 - x^2)^(1/2),x)
```

output
```
int(log(x^2 - 1)/(1 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(-1+x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(x^2-1)}{\sqrt{-x^2+1}}dx$$

input
```
int(log(x^2-1)/(-x^2+1)^(1/2),x)
```

output
```
int(log(x**2 - 1)/sqrt( - x**2 + 1),x)
```

3.632 $\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\begin{aligned}\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx &= \log\left(1-x^2\right)\log\left(x+\sqrt{-1+x^2}\right)+\log^2\left(x+\sqrt{-1+x^2}\right)\\ &\quad -2\log\left(x+\sqrt{-1+x^2}\right)\log\left(1-\left(x+\sqrt{-1+x^2}\right)^2\right)\\ &\quad -\operatorname{PolyLog}\left(2,\left(x+\sqrt{-1+x^2}\right)^2\right)\end{aligned}$$

output
```
ln(-x^2+1)*ln(x+(x^2-1)^(1/2))+ln(x+(x^2-1)^(1/2))^2-2*ln(x+(x^2-1)^(1/2))
*ln(1-(x+(x^2-1)^(1/2))^2)-polylog(2,(x+(x^2-1)^(1/2))^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx$$

$$= \frac{i\sqrt{1-x^2}\left(\arcsin\left(\sqrt{1-x^2}\right)^2-2i\arcsin\left(\sqrt{1-x^2}\right)\log\left(1-e^{-2i\arcsin\left(\sqrt{1-x^2}\right)}\right)+\log\left(1-x^2\right)\log\left(\sqrt{x^2}\right.\right.}{\sqrt{1-\frac{1}{x^2}}\,x}$$

input
```
Integrate[Log[1 - x^2]/Sqrt[-1 + x^2],x]
```

output
```
(I*Sqrt[1 - x^2]*(ArcSin[Sqrt[1 - x^2]]^2 - (2*I)*ArcSin[Sqrt[1 - x^2]]*Lo
g[1 - E^((-2*I)*ArcSin[Sqrt[1 - x^2]])] + Log[1 - x^2]*Log[Sqrt[x^2] + I*S
qrt[1 - x^2]] + PolyLog[2, E^((-2*I)*ArcSin[Sqrt[1 - x^2]])]))/(Sqrt[1 - x
^(-2)]*x)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{\sqrt{x^2-1}}\,dx$$

$$\Big\downarrow \text{ 2923}$$

$$\int \frac{\log(1-x^2)}{\sqrt{x^2-1}}dx$$

input
```
Int[Log[1 - x^2]/Sqrt[-1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{\sqrt{x^2-1}}dx$$

input
```
int(ln(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/(x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(1-x^2)}{\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/(x**2-1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/sqrt((x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Giac [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\ln(1-x^2)}{\sqrt{x^2-1}}\,dx$$

input
```
int(log(1 - x^2)/(x^2 - 1)^(1/2),x)
```

output
```
int(log(1 - x^2)/(x^2 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}}dx$$

input
```
int(log(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
int(log( - x**2 + 1)/sqrt(x**2 - 1),x)
```

3.633 $\int \frac{\log(-1+x^2)}{\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\begin{aligned}\int \frac{\log(-1+x^2)}{\sqrt{-1+x^2}}\,dx &= \log(-1+x^2)\log\left(x+\sqrt{-1+x^2}\right)+\log^2\left(x+\sqrt{-1+x^2}\right)\\ &\quad -2\log\left(x+\sqrt{-1+x^2}\right)\log\left(1-\left(x+\sqrt{-1+x^2}\right)^2\right)\\ &\quad -\operatorname{PolyLog}\left(2,\left(x+\sqrt{-1+x^2}\right)^2\right)\end{aligned}$$

output
```
ln(x^2-1)*ln(x+(x^2-1)^(1/2))+ln(x+(x^2-1)^(1/2))^2-2*ln(x+(x^2-1)^(1/2))*
ln(1-(x+(x^2-1)^(1/2))^2)-polylog(2,(x+(x^2-1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int \frac{\log(-1+x^2)}{\sqrt{-1+x^2}}\,dx = \frac{\sqrt{-1+x^2}\left(-\operatorname{arcsinh}\left(\sqrt{-1+x^2}\right)^2-2\operatorname{arcsinh}\left(\sqrt{-1+x^2}\right)\log\left(1-e^{-2\operatorname{arcsinh}\left(\sqrt{-1+x^2}\right)}\right)+\log(-1+x^2)\right.}{x\sqrt{\frac{-1+x^2}{x^2}}}$$

input
```
Integrate[Log[-1 + x^2]/Sqrt[-1 + x^2],x]
```

output
```
(Sqrt[-1 + x^2]*(-ArcSinh[Sqrt[-1 + x^2]]^2 - 2*ArcSinh[Sqrt[-1 + x^2]]*Lo
g[1 - E^(-2*ArcSinh[Sqrt[-1 + x^2]])] + Log[-1 + x^2]*Log[Sqrt[x^2] + Sqrt
[-1 + x^2]] + PolyLog[2, E^(-2*ArcSinh[Sqrt[-1 + x^2]])]))/(x*Sqrt[(-1 + x
^2)/x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x^2-1)}{\sqrt{x^2-1}}\,dx$$
$$\Big\downarrow \ 2923$$
$$\int \frac{\log(x^2-1)}{\sqrt{x^2-1}}dx$$

input
```
Int[Log[-1 + x^2]/Sqrt[-1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(x^2-1)}{\sqrt{x^2-1}}dx$$

input
```
int(ln(x^2-1)/(x^2-1)^(1/2),x)
```

output
```
int(ln(x^2-1)/(x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log(-1+x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(x^2-1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(log(x^2-1)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(log(x^2 - 1)/sqrt(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\log(-1+x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(x^2-1)}{\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(x**2-1)/(x**2-1)**(1/2),x)
```

output
```
Integral(log(x**2 - 1)/sqrt((x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\log(-1+x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(x^2-1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(log(x^2-1)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(x^2 - 1)/sqrt(x^2 - 1), x)
```

Giac [F]

$$\int \frac{\log(-1+x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(x^2-1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(log(x^2-1)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(x^2 - 1)/sqrt(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(-1+x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\ln(x^2-1)}{\sqrt{x^2-1}}\,dx$$

input
```
int(log(x^2 - 1)/(x^2 - 1)^(1/2),x)
```

output
```
int(log(x^2 - 1)/(x^2 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(-1+x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(x^2-1)}{\sqrt{x^2-1}}dx$$

input
```
int(log(x^2-1)/(x^2-1)^(1/2),x)
```

output
```
int(log(x**2 - 1)/sqrt(x**2 - 1),x)
```

3.634 $\int \frac{\log(1+x^2)}{\sqrt{-1-x^2}} dx$

Optimal result

Integrand size = 18, antiderivative size = 154

$$\int \frac{\log (1+x^2)}{\sqrt{-1-x^2}} dx = \frac{\sqrt{1+x^2}\text{arcsinh}(x) \log (1+x^2)}{\sqrt{-1-x^2}} + \frac{\sqrt{1+x^2} \log^2 \left(x+\sqrt{1+x^2}\right)}{\sqrt{-1-x^2}} - \frac{2\sqrt{1+x^2} \log \left(x+\sqrt{1+x^2}\right) \log \left(1+\left(x+\sqrt{1+x^2}\right)^2\right)}{\sqrt{-1-x^2}} - \frac{\sqrt{1+x^2}\,\text{PolyLog}\left(2,-\left(x+\sqrt{1+x^2}\right)^2\right)}{\sqrt{-1-x^2}}$$

output
```
(x^2+1)^(1/2)*arcsinh(x)*ln(x^2+1)/(-x^2-1)^(1/2)+(x^2+1)^(1/2)*ln(x+(x^2+
1)^(1/2))^2/(-x^2-1)^(1/2)-2*(x^2+1)^(1/2)*ln(x+(x^2+1)^(1/2))*ln(1+(x+(x^
2+1)^(1/2))^2)/(-x^2-1)^(1/2)-(x^2+1)^(1/2)*polylog(2,-(x+(x^2+1)^(1/2))^2
)/(-x^2-1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

$$\int \frac{\log(1+x^2)}{\sqrt{-1-x^2}}\,dx =$$
$$-\frac{i\sqrt{1+\frac{1}{x^2}}x\left(\arcsin\left(\sqrt{1+x^2}\right)^2-2i\arcsin\left(\sqrt{1+x^2}\right)\log\left(1-e^{-2i\arcsin\left(\sqrt{1+x^2}\right)}\right)+\log(1+x^2)\log\left(\sqrt{\right.\right.}{\sqrt{1+x^2}}$$

input
```
Integrate[Log[1 + x^2]/Sqrt[-1 - x^2],x]
```

output
```
((-I)*Sqrt[1 + x^(-2)]*x*(ArcSin[Sqrt[1 + x^2]]^2 - (2*I)*ArcSin[Sqrt[1 +
x^2]]*Log[1 - E^((-2*I)*ArcSin[Sqrt[1 + x^2]])] + Log[1 + x^2]*Log[Sqrt[-x
^2] + I*Sqrt[1 + x^2]] + PolyLog[2, E^((-2*I)*ArcSin[Sqrt[1 + x^2]])]))/Sq
rt[1 + x^2]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(x^2+1)}{\sqrt{-x^2-1}}\,dx$$
$$\Big\downarrow \; 2923$$
$$\int \frac{\log(x^2+1)}{\sqrt{-x^2-1}}dx$$

input
```
Int[Log[1 + x^2]/Sqrt[-1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln \left(x^2+1\right)}{\sqrt{-x^2-1}}dx$$

input
```
int(ln(x^2+1)/(-x^2-1)^(1/2),x)
```

output
```
int(ln(x^2+1)/(-x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log \left(1+x^2\right)}{\sqrt{-1-x^2}}\, dx = \int \frac{\log \left(x^2+1\right)}{\sqrt{-x^2-1}}\, dx$$

input
```
integrate(log(x^2+1)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 - 1)*log(x^2 + 1)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log \left(1+x^2\right)}{\sqrt{-1-x^2}}\, dx = \int \frac{\log \left(x^2+1\right)}{\sqrt{-x^2-1}}\, dx$$

input
```
integrate(ln(x**2+1)/(-x**2-1)**(1/2),x)
```

output
```
Integral(log(x**2 + 1)/sqrt(-x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\log(1+x^2)}{\sqrt{-1-x^2}}\,dx = \int \frac{\log(x^2+1)}{\sqrt{-x^2-1}}\,dx$$

input
```
integrate(log(x^2+1)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(x^2 + 1)/sqrt(-x^2 - 1), x)
```

Giac [F]

$$\int \frac{\log(1+x^2)}{\sqrt{-1-x^2}}\,dx = \int \frac{\log(x^2+1)}{\sqrt{-x^2-1}}\,dx$$

input
```
integrate(log(x^2+1)/(-x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(x^2 + 1)/sqrt(-x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1+x^2)}{\sqrt{-1-x^2}}\,dx = \int \frac{\ln(x^2+1)}{\sqrt{-x^2-1}}\,dx$$

input
```
int(log(x^2 + 1)/(- x^2 - 1)^(1/2),x)
```

output
```
int(log(x^2 + 1)/(- x^2 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log (1 + x^2)}{\sqrt{-1 - x^2}} \, dx = -\left(\int \frac{\log(x^2 + 1)}{\sqrt{x^2 + 1}} dx \right) i$$

input
```
int(log(x^2+1)/(-x^2-1)^(1/2),x)
```

output
```
- int(log(x**2 + 1)/sqrt(x**2 + 1),x)*i
```

3.635 $\int \frac{\log(-1-x^2)}{\sqrt{-1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{\log(-1-x^2)}{\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\operatorname{arcsinh}(x)\log(-1-x^2)}{\sqrt{-1-x^2}} + \frac{\sqrt{1+x^2}\log^2\left(x+\sqrt{1+x^2}\right)}{\sqrt{-1-x^2}} - \frac{2\sqrt{1+x^2}\log\left(x+\sqrt{1+x^2}\right)\log\left(1+\left(x+\sqrt{1+x^2}\right)^2\right)}{\sqrt{-1-x^2}} - \frac{\sqrt{1+x^2}\operatorname{PolyLog}\left(2,-\left(x+\sqrt{1+x^2}\right)^2\right)}{\sqrt{-1-x^2}}$$

output
```
(x^2+1)^(1/2)*arcsinh(x)*ln(-x^2-1)/(-x^2-1)^(1/2)+(x^2+1)^(1/2)*ln(x+(x^2
+1)^(1/2))^2/(-x^2-1)^(1/2)-2*(x^2+1)^(1/2)*ln(x+(x^2+1)^(1/2))*ln(1+(x+(x
^2+1)^(1/2))^2)/(-x^2-1)^(1/2)-(x^2+1)^(1/2)*polylog(2,-(x+(x^2+1)^(1/2))^
2)/(-x^2-1)^(1/2)
```

Mathematica [F]

$$\int \frac{\log(-1-x^2)}{\sqrt{-1-x^2}}\,dx = \int \frac{\log(-1-x^2)}{\sqrt{-1-x^2}}\,dx$$

input
```
Integrate[Log[-1 - x^2]/Sqrt[-1 - x^2],x]
```

output
```
Integrate[Log[-1 - x^2]/Sqrt[-1 - x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(-x^2-1)}{\sqrt{-x^2-1}}\,dx$$
$$\Big\downarrow \ 2923$$
$$\int \frac{\log(-x^2-1)}{\sqrt{-x^2-1}}dx$$

input
```
Int[Log[-1 - x^2]/Sqrt[-1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln\left(-x^2-1\right)}{\sqrt{-x^2-1}}dx$$

input
```
int(ln(-x^2-1)/(-x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2-1)/(-x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log\left(-1-x^2\right)}{\sqrt{-1-x^2}}\,dx = \int \frac{\log\left(-x^2-1\right)}{\sqrt{-x^2-1}}\,dx$$

input
```
integrate(log(-x^2-1)/(-x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 - 1)*log(-x^2 - 1)/(x^2 + 1), x)
```

Sympy [F]

$$\int \frac{\log\left(-1-x^2\right)}{\sqrt{-1-x^2}}\,dx = \int \frac{\log\left(-x^2-1\right)}{\sqrt{-x^2-1}}\,dx$$

input
```
integrate(ln(-x**2-1)/(-x**2-1)**(1/2),x)
```

output
```
Integral(log(-x**2 - 1)/sqrt(-x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\log(-1-x^2)}{\sqrt{-1-x^2}}\,dx = \int \frac{\log(-x^2-1)}{\sqrt{-x^2-1}}\,dx$$

input
```
integrate(log(-x^2-1)/(-x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 - 1)/sqrt(-x^2 - 1), x)
```

Giac [F]

$$\int \frac{\log(-1-x^2)}{\sqrt{-1-x^2}}\,dx = \int \frac{\log(-x^2-1)}{\sqrt{-x^2-1}}\,dx$$

input
```
integrate(log(-x^2-1)/(-x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 - 1)/sqrt(-x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(-1-x^2)}{\sqrt{-1-x^2}}\,dx = \int \frac{\ln(-x^2-1)}{\sqrt{-x^2-1}}\,dx$$

input
```
int(log(- x^2 - 1)/(- x^2 - 1)^(1/2),x)
```

output
```
int(log(- x^2 - 1)/(- x^2 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(-1-x^2)}{\sqrt{-1-x^2}}\,dx = -\left(\int \frac{\log(-x^2-1)}{\sqrt{x^2+1}}dx\right)i$$

input
```
int(log(-x^2-1)/(-x^2-1)^(1/2),x)
```

output
```
- int(log( - x**2 - 1)/sqrt(x**2 + 1),x)*i
```

3.636 $\int \frac{x^7 \log(1-x^2)}{\sqrt{1-x^2}}\, dx$

Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{1-x^2}}\, dx = 2\sqrt{1-x^2} - \frac{2}{3}(1-x^2)^{3/2} + \frac{6}{25}(1-x^2)^{5/2} - \frac{2}{49}(1-x^2)^{7/2} - \sqrt{1-x^2}\log(1-x^2) + (1-x^2)^{3/2}\log(1-x^2) - \frac{3}{5}(1-x^2)^{5/2}\log(1-x^2) + \frac{1}{7}(1-x^2)^{7/2}\log(1-x^2)$$

output
```
2*(-x^2+1)^(1/2)-2/3*(-x^2+1)^(3/2)+6/25*(-x^2+1)^(5/2)-2/49*(-x^2+1)^(7/2
)-(-x^2+1)^(1/2)*ln(-x^2+1)+(-x^2+1)^(3/2)*ln(-x^2+1)-3/5*(-x^2+1)^(5/2)*l
n(-x^2+1)+1/7*(-x^2+1)^(7/2)*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.42

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{1-x^2}}\, dx = \frac{\sqrt{1-x^2}(2(2816+568x^2+216x^4+75x^6)-105(16+8x^2+6x^4+5x^6)\log(1-x^2))}{3675}$$

input
```
Integrate[(x^7*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(2*(2816 + 568*x^2 + 216*x^4 + 75*x^6) - 105*(16 + 8*x^2 +
6*x^4 + 5*x^6)*Log[1 - x^2]))/3675
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{1-x^2}} dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^7 \log(1-x^2)}{\sqrt{1-x^2}} dx$$

input
```
Int[(x^7*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

method	result
risch	$\frac{(5x^6+6x^4+8x^2+16)(x^2-1)\ln(-x^2+1)}{35\sqrt{-x^2+1}} - \frac{2(75x^6+216x^4+568x^2+2816)(x^2-1)}{3675\sqrt{-x^2+1}}$
orering	$\frac{(x-1)(1+x)(975x^8+1401x^6+2736x^4+14600x^2-19712)\ln(-x^2+1)}{3675x^2\sqrt{-x^2+1}} - \frac{(75x^6+216x^4+568x^2+2816)(x-1)^2(1+x)^2\left(\frac{7x^6\ln(-x^2}{\sqrt{-x^2+}}\right.}{3675x^8}$

input
```
int(x^7*ln(-x^2+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*x^6+6*x^4+8*x^2+16)*(x^2-1)/(-x^2+1)^(1/2)*ln(-x^2+1)-2/3675*(75*x
^6+216*x^4+568*x^2+2816)*(x^2-1)/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{1}{3675}\left(150\,x^6+432\,x^4+1136\,x^2-105\left(5\,x^6+6\,x^4+8\,x^2+16\right)\log\left(-x^2+1\right)+5632\right)\sqrt{-x^2+1}$$

input
```
integrate(x^7*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3675*(150*x^6 + 432*x^4 + 1136*x^2 - 105*(5*x^6 + 6*x^4 + 8*x^2 + 16)*lo
g(-x^2 + 1) + 5632)*sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 130.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \sqrt{1-x^2}\left(-\frac{x^6}{7}-\frac{6x^4}{35}-\frac{8x^2}{35}-\frac{16}{35}\right)\log\left(1-x^2\right) + 2\sqrt{1-x^2}\left(\frac{x^6}{49}+\frac{72x^4}{1225}+\frac{568x^2}{3675}+\frac{2816}{3675}\right)$$

input
```
integrate(x**7*ln(-x**2+1)/(-x**2+1)**(1/2),x)
```

output
```
sqrt(1 - x**2)*(-x**6/7 - 6*x**4/35 - 8*x**2/35 - 16/35)*log(1 - x**2) + 2
*sqrt(1 - x**2)*(x**6/49 + 72*x**4/1225 + 568*x**2/3675 + 2816/3675)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{2}{49}\sqrt{-x^2+1}x^6 + \frac{144}{1225}\sqrt{-x^2+1}x^4 + \frac{1136}{3675}\sqrt{-x^2+1}x^2 - \frac{1}{35}\left(5\sqrt{-x^2+1}x^6 + 6\sqrt{-x^2+1}x^4 + 8\sqrt{-x^2+1}x^2 + 16\sqrt{-x^2+1}\right)\log\left(-x^2+1\right) + \frac{5632}{3675}\sqrt{-x^2+1}$$

input
```
integrate(x^7*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
2/49*sqrt(-x^2 + 1)*x^6 + 144/1225*sqrt(-x^2 + 1)*x^4 + 1136/3675*sqrt(-x^
2 + 1)*x^2 - 1/35*(5*sqrt(-x^2 + 1)*x^6 + 6*sqrt(-x^2 + 1)*x^4 + 8*sqrt(-x
^2 + 1)*x^2 + 16*sqrt(-x^2 + 1))*log(-x^2 + 1) + 5632/3675*sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{2}{49}\left(x^2-1\right)^3\sqrt{-x^2+1} + \frac{6}{25}\left(x^2-1\right)^2\sqrt{-x^2+1} - \frac{1}{35}\left(5\left(x^2-1\right)^3\sqrt{-x^2+1} + 21\left(x^2-1\right)^2\sqrt{-x^2+1} - 35\left(-x^2+1\right)^{\frac{3}{2}} + 35\sqrt{-x^2+1}\right)\log\left(-x^2+1\right) - \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}} + 2\sqrt{-x^2+1}$$

input
```
integrate(x^7*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
2/49*(x^2 - 1)^3*sqrt(-x^2 + 1) + 6/25*(x^2 - 1)^2*sqrt(-x^2 + 1) - 1/35*(
5*(x^2 - 1)^3*sqrt(-x^2 + 1) + 21*(x^2 - 1)^2*sqrt(-x^2 + 1) - 35*(-x^2 +
1)^(3/2) + 35*sqrt(-x^2 + 1))*log(-x^2 + 1) - 2/3*(-x^2 + 1)^(3/2) + 2*sqr
t(-x^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \log (1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{x^7\,\ln (1-x^2)}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^7*log(1 - x^2))/(1 - x^2)^(1/2),x)
```

output
```
int((x^7*log(1 - x^2))/(1 - x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int \frac{x^7 \log (1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{\sqrt{-x^2+1}\,(-525\log(-x^2+1)\,x^6-630\log(-x^2+1)\,x^4-840\log(-x^2+1)\,x^2-1680\log(-x^2+1)+1}{3675}$$

input
```
int(x^7*log(-x^2+1)/(-x^2+1)^(1/2),x)
```

output
```
(sqrt( - x**2 + 1)*( - 525*log( - x**2 + 1)*x**6 - 630*log( - x**2 + 1)*x*
*4 - 840*log( - x**2 + 1)*x**2 - 1680*log( - x**2 + 1) + 150*x**6 + 432*x*
*4 + 1136*x**2 + 5632))/3675
```

3.637 $\int \frac{x^5 \log(1-x^2)}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{1-x^2}} \, dx = 2\sqrt{1-x^2} - \frac{4}{9}(1-x^2)^{3/2} + \frac{2}{25}(1-x^2)^{5/2} - \sqrt{1-x^2}\log(1-x^2) + \frac{2}{3}(1-x^2)^{3/2}\log(1-x^2) - \frac{1}{5}(1-x^2)^{5/2}\log(1-x^2)$$

output
```
2*(-x^2+1)^(1/2)-4/9*(-x^2+1)^(3/2)+2/25*(-x^2+1)^(5/2)-(-x^2+1)^(1/2)*ln(
-x^2+1)+2/3*(-x^2+1)^(3/2)*ln(-x^2+1)-1/5*(-x^2+1)^(5/2)*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.44

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{1-x^2}} \, dx = \frac{1}{225}\sqrt{1-x^2}\left(368 + 64x^2 + 18x^4 - 15(8 + 4x^2 + 3x^4)\log(1-x^2)\right)$$

input
```
Integrate[(x^5*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(368 + 64*x^2 + 18*x^4 - 15*(8 + 4*x^2 + 3*x^4)*Log[1 - x^2
]))/225
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \log \left(1-x^2\right)}{\sqrt{1-x^2}} \, dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^5 \log \left(1-x^2\right)}{\sqrt{1-x^2}}dx$$

input
```
Int[(x^5*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

method	result
risch	$\frac{\left(3 x^{4}+4 x^{2}+8\right) \left(x^{2}-1\right) \ln \left(-x^{2}+1\right)}{15 \sqrt{-x^{2}+1}}-\frac{2 \left(9 x^{4}+32 x^{2}+184\right) \left(x^{2}-1\right)}{225 \sqrt{-x^{2}+1}}$
orering	$\frac{\left(x-1\right) \left(1+x\right) \left(81 x^{6}+143 x^{4}+696 x^{2}-920\right) \ln \left(-x^{2}+1\right)}{225 x^{2} \sqrt{-x^{2}+1}}-\frac{\left(9 x^{4}+32 x^{2}+184\right) \left(x-1\right)^{2} \left(1+x\right)^{2} \left(\frac{5 x^{4} \ln \left(-x^{2}+1\right)}{\sqrt{-x^{2}+1}}-\frac{2 x^{6}}{\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{x^{6} \ln \left(-x \right.}{\left(-x^{2}+\right.}$

input
```
int(x^5*ln(-x^2+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*x^4+4*x^2+8)*(x^2-1)/(-x^2+1)^(1/2)*ln(-x^2+1)-2/225*(9*x^4+32*x^2
+184)*(x^2-1)/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.41

$$\int \frac{x^5 \log (1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{1}{225}\left(18\,x^4 + 64\,x^2 - 15\left(3\,x^4 + 4\,x^2 + 8\right)\log\left(-x^2 + 1\right) + 368\right)\sqrt{-x^2 + 1}$$

input
```
integrate(x^5*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/225*(18*x^4 + 64*x^2 - 15*(3*x^4 + 4*x^2 + 8)*log(-x^2 + 1) + 368)*sqrt(
-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 37.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{x^5 \log (1-x^2)}{\sqrt{1-x^2}}\,dx = \sqrt{1-x^2}\left(-\frac{x^4}{5} - \frac{4x^2}{15} - \frac{8}{15}\right)\log\left(1-x^2\right) + 2\sqrt{1-x^2}\left(\frac{x^4}{25} + \frac{32x^2}{225} + \frac{184}{225}\right)$$

input
```
integrate(x**5*ln(-x**2+1)/(-x**2+1)**(1/2),x)
```

output
```
sqrt(1 - x**2)*(-x**4/5 - 4*x**2/15 - 8/15)*log(1 - x**2) + 2*sqrt(1 - x**
2)*(x**4/25 + 32*x**2/225 + 184/225)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{2}{25}\sqrt{-x^2+1}x^4 + \frac{64}{225}\sqrt{-x^2+1}x^2 - \frac{1}{15}\left(3\sqrt{-x^2+1}x^4 + 4\sqrt{-x^2+1}x^2 + 8\sqrt{-x^2+1}\right)\log\left(-x^2+1\right) + \frac{368}{225}\sqrt{-x^2+1}$$

input
```
integrate(x^5*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
2/25*sqrt(-x^2 + 1)*x^4 + 64/225*sqrt(-x^2 + 1)*x^2 - 1/15*(3*sqrt(-x^2 +
1)*x^4 + 4*sqrt(-x^2 + 1)*x^2 + 8*sqrt(-x^2 + 1))*log(-x^2 + 1) + 368/225*
sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.83

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{2}{25}\left(x^2-1\right)^2\sqrt{-x^2+1} - \frac{1}{15}\left(3\left(x^2-1\right)^2\sqrt{-x^2+1} - 10\left(-x^2+1\right)^{\frac{3}{2}} + 15\sqrt{-x^2+1}\right)\log\left(-x^2+1\right) - \frac{4}{9}\left(-x^2+1\right)^{\frac{3}{2}} + 2\sqrt{-x^2+1}$$

input
```
integrate(x^5*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
2/25*(x^2 - 1)^2*sqrt(-x^2 + 1) - 1/15*(3*(x^2 - 1)^2*sqrt(-x^2 + 1) - 10*
(-x^2 + 1)^(3/2) + 15*sqrt(-x^2 + 1))*log(-x^2 + 1) - 4/9*(-x^2 + 1)^(3/2)
 + 2*sqrt(-x^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{x^5\,\ln(1-x^2)}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^5*log(1 - x^2))/(1 - x^2)^(1/2),x)
```

output
```
int((x^5*log(1 - x^2))/(1 - x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{\sqrt{-x^2+1}\,(-45\log(-x^2+1)\,x^4-60\log(-x^2+1)\,x^2-120\log(-x^2+1)+18x^4+64x^2+368)}{225}$$

input
```
int(x^5*log(-x^2+1)/(-x^2+1)^(1/2),x)
```

output
```
(sqrt( - x**2 + 1)*( - 45*log( - x**2 + 1)*x**4 - 60*log( - x**2 + 1)*x**2
 - 120*log( - x**2 + 1) + 18*x**4 + 64*x**2 + 368))/225
```

3.638 $\int \frac{x^3 \log(1-x^2)}{\sqrt{1-x^2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{1-x^2}} dx = 2\sqrt{1-x^2} - \frac{2}{9}(1-x^2)^{3/2} - \sqrt{1-x^2}\log(1-x^2) + \frac{1}{3}(1-x^2)^{3/2}\log(1-x^2)$$

output
```
2*(-x^2+1)^(1/2)-2/9*(-x^2+1)^(3/2)-(-x^2+1)^(1/2)*ln(-x^2+1)+1/3*(-x^2+1)
^(3/2)*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.52

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{1-x^2}} dx = \frac{1}{9}\sqrt{1-x^2}(2(8+x^2) - 3(2+x^2)\log(1-x^2))$$

input
```
Integrate[(x^3*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(2*(8 + x^2) - 3*(2 + x^2)*Log[1 - x^2]))/9
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log (1-x^2)}{\sqrt{1-x^2}} \, dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{x^3 \log (1-x^2)}{\sqrt{1-x^2}}dx$$

input
```
Int[(x^3*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{(x^2+2)(x^2-1)\ln(-x^2+1)}{3\sqrt{-x^2+1}}-\frac{2(x^2+8)(x^2-1)}{9\sqrt{-x^2+1}}$	52
orering	$\frac{(x-1)(1+x)(5x^4+19x^2-24)\ln(-x^2+1)}{9x^2\sqrt{-x^2+1}}-\frac{(x^2+8)(x-1)^2(1+x)^2\left(\frac{3x^2\ln(-x^2+1)}{\sqrt{-x^2+1}}-\frac{2x^4}{(-x^2+1)^{\frac{3}{2}}}+\frac{x^4\ln(-x^2+1)}{(-x^2+1)^{\frac{3}{2}}}\right)}{9x^4}$	120

input
```
int(x^3*ln(-x^2+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+2)*(x^2-1)/(-x^2+1)^(1/2)*ln(-x^2+1)-2/9*(x^2+8)*(x^2-1)/(-x^2+1)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{1}{9}\left(2\,x^2 - 3\left(x^2+2\right)\log\left(-x^2+1\right) + 16\right)\sqrt{-x^2+1}$$

input
```
integrate(x^3*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/9*(2*x^2 - 3*(x^2 + 2)*log(-x^2 + 1) + 16)*sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \sqrt{1-x^2}\left(-\frac{x^2}{3}-\frac{2}{3}\right)\log\left(1-x^2\right) + 2\sqrt{1-x^2}\left(\frac{x^2}{9}+\frac{8}{9}\right)$$

input
```
integrate(x**3*ln(-x**2+1)/(-x**2+1)**(1/2),x)
```

output
```
sqrt(1 - x**2)*(-x**2/3 - 2/3)*log(1 - x**2) + 2*sqrt(1 - x**2)*(x**2/9 +
8/9)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{2}{9}\sqrt{-x^2+1}x^2 - \frac{1}{3}\left(\sqrt{-x^2+1}x^2 + 2\sqrt{-x^2+1}\right)\log\left(-x^2+1\right) + \frac{16}{9}\sqrt{-x^2+1}$$

input
```
integrate(x^3*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
2/9*sqrt(-x^2 + 1)*x^2 - 1/3*(sqrt(-x^2 + 1)*x^2 + 2*sqrt(-x^2 + 1))*log(-
x^2 + 1) + 16/9*sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{1}{3}\left(\left(-x^2+1\right)^{\frac{3}{2}} - 3\sqrt{-x^2+1}\right)\log\left(-x^2+1\right) - \frac{2}{9}\left(-x^2+1\right)^{\frac{3}{2}} + 2\sqrt{-x^2+1}$$

input
```
integrate(x^3*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/3*((-x^2 + 1)^(3/2) - 3*sqrt(-x^2 + 1))*log(-x^2 + 1) - 2/9*(-x^2 + 1)^(
3/2) + 2*sqrt(-x^2 + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{x^3 \ln(1-x^2)}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^3*log(1 - x^2))/(1 - x^2)^(1/2),x)
```

output
```
int((x^3*log(1 - x^2))/(1 - x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.55

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \frac{\sqrt{-x^2+1}\,(-3\log(-x^2+1)\,x^2-6\log(-x^2+1)+2x^2+16)}{9}$$

input
```
int(x^3*log(-x^2+1)/(-x^2+1)^(1/2),x)
```

output
```
(sqrt( - x**2 + 1)*( - 3*log( - x**2 + 1)*x**2 - 6*log( - x**2 + 1) + 2*x*
*2 + 16))/9
```

3.639 $\int \frac{x \log(1-x^2)}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{x \log(1-x^2)}{\sqrt{1-x^2}} \, dx = 2\sqrt{1-x^2} - \sqrt{1-x^2} \log(1-x^2)$$

output
```
2*(-x^2+1)^(1/2)-(-x^2+1)^(1/2)*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{x \log(1-x^2)}{\sqrt{1-x^2}} \, dx = -\sqrt{1-x^2}\left(-2+\log(1-x^2)\right)$$

input
```
Integrate[(x*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
-(Sqrt[1 - x^2]*(-2 + Log[1 - x^2]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log(1-x^2)}{\sqrt{1-x^2}} \, dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{x \log(1-x^2)}{\sqrt{1-x^2}} dx$$

input
```
Int[(x*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$2\sqrt{-x^2+1}-\sqrt{-x^2+1}\,\ln(-x^2+1)$	32
default	$2\sqrt{-x^2+1}-\sqrt{-x^2+1}\,\ln(-x^2+1)$	32
risch	$\frac{\ln(-x^2+1)(x^2-1)}{\sqrt{-x^2+1}}-\frac{2(x^2-1)}{\sqrt{-x^2+1}}$	41
orering	$\frac{(x^2-1)(x-1)(1+x)\ln(-x^2+1)}{x^2\sqrt{-x^2+1}}-\frac{(x-1)^2(1+x)^2\left(\frac{\ln(-x^2+1)}{\sqrt{-x^2+1}}-\frac{2x^2}{(-x^2+1)^{\frac{3}{2}}}+\frac{x^2\ln(-x^2+1)}{(-x^2+1)^{\frac{3}{2}}}\right)}{x^2}$	103

input
```
int(x*ln(-x^2+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-x^2+1)^(1/2)-(-x^2+1)^(1/2)*ln(-x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{x \log(1-x^2)}{\sqrt{1-x^2}} \, dx = -\sqrt{-x^2+1}\left(\log\left(-x^2+1\right)-2\right)$$

input
```
integrate(x*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 1)*(log(-x^2 + 1) - 2)
```

Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{x \log(1-x^2)}{\sqrt{1-x^2}} \, dx = -\sqrt{1-x^2}\log\left(1-x^2\right)+2\sqrt{1-x^2}$$

input
```
integrate(x*ln(-x**2+1)/(-x**2+1)**(1/2),x)
```

output
```
-sqrt(1 - x**2)*log(1 - x**2) + 2*sqrt(1 - x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x\log(1-x^2)}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}\log\left(-x^2+1\right)+2\sqrt{-x^2+1}$$

input
```
integrate(x*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1)*log(-x^2 + 1) + 2*sqrt(-x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x\log(1-x^2)}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1}\log\left(-x^2+1\right)+2\sqrt{-x^2+1}$$

input
```
integrate(x*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1)*log(-x^2 + 1) + 2*sqrt(-x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 25.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{x\log(1-x^2)}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}\,\left(\ln\left(1-x^2\right)-2\right)$$

input
```
int((x*log(1 - x^2))/(1 - x^2)^(1/2),x)
```

output
```
-(1 - x^2)^(1/2)*(log(1 - x^2) - 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{x \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \sqrt{-x^2+1}\left(-\log\left(-x^2+1\right)+2\right)$$

input

```
int(x*log(-x^2+1)/(-x^2+1)^(1/2),x)
```

output

```
sqrt( - x**2 + 1)*( - log( - x**2 + 1) + 2)
```

3.640 $\int \frac{\log(1-x^2)}{x\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\log(1-x^2)}{x\sqrt{1-x^2}}\,dx = -\text{arctanh}\left(\sqrt{1-x^2}\right)\log(1-x^2) - \text{PolyLog}\left(2,-\sqrt{1-x^2}\right) + \text{PolyLog}\left(2,\sqrt{1-x^2}\right)$$

output
```
-arctanh((-x^2+1)^(1/2))*ln(-x^2+1)-polylog(2,-(-x^2+1)^(1/2))+polylog(2,(
-x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.60

$$\int \frac{\log(1-x^2)}{x\sqrt{1-x^2}}\,dx = \frac{1}{2}\log(1-x^2)\log\left(1-\sqrt{1-x^2}\right) - \frac{1}{2}\log(1-x^2)\log\left(1+\sqrt{1-x^2}\right) - \text{PolyLog}\left(2,-\sqrt{1-x^2}\right) + \text{PolyLog}\left(2,\sqrt{1-x^2}\right)$$

input
```
Integrate[Log[1 - x^2]/(x*Sqrt[1 - x^2]),x]
```

output
```
(Log[1 - x^2]*Log[1 - Sqrt[1 - x^2]])/2 - (Log[1 - x^2]*Log[1 + Sqrt[1 - x
^2]])/2 - PolyLog[2, -Sqrt[1 - x^2]] + PolyLog[2, Sqrt[1 - x^2]]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x\sqrt{1-x^2}}\,dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{\log(1-x^2)}{x\sqrt{1-x^2}}dx$$

input
```
Int[Log[1 - x^2]/(x*Sqrt[1 - x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x\sqrt{-x^2+1}}dx$$

input
```
int(ln(-x^2+1)/x/(-x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x/(-x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log (1-x^2)}{x\sqrt{1-x^2}}\,dx=\int \frac{\log (-x^2+1)}{\sqrt{-x^2+1}x}\,dx$$

input
```
integrate(log(-x^2+1)/x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*log(-x^2 + 1)/(x^3 - x), x)
```

Sympy [F]

$$\int \frac{\log (1-x^2)}{x\sqrt{1-x^2}}\,dx=\int \frac{\log (1-x^2)}{x\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x/(-x**2+1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x*sqrt(-(x - 1)*(x + 1))), x)
```

Maxima [F]

$$\int \frac{\log (1-x^2)}{x\sqrt{1-x^2}}\,dx=\int \frac{\log (-x^2+1)}{\sqrt{-x^2+1}x}\,dx$$

input
```
integrate(log(-x^2+1)/x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(-x^2 + 1)*x), x)
```

Giac [F]

$$\int \frac{\log(1-x^2)}{x\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}x}\,dx$$

input
```
integrate(log(-x^2+1)/x/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(-x^2 + 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x\sqrt{1-x^2}}\,dx = \int \frac{\ln(1-x^2)}{x\sqrt{1-x^2}}\,dx$$

input
```
int(log(1 - x^2)/(x*(1 - x^2)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x*(1 - x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{x\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}\,x}dx$$

input
```
int(log(-x^2+1)/x/(-x^2+1)^(1/2),x)
```

output
```
int(log( - x**2 + 1)/(sqrt( - x**2 + 1)*x),x)
```

3.641 $\int \frac{\log(1-x^2)}{x^3\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int \frac{\log(1-x^2)}{x^3\sqrt{1-x^2}}\,dx = \operatorname{arctanh}\left(\sqrt{1-x^2}\right) - \frac{\sqrt{1-x^2}\log(1-x^2)}{2x^2} - \frac{1}{2}\operatorname{arctanh}\left(\sqrt{1-x^2}\right)\log(1-x^2) - \frac{1}{2}\operatorname{PolyLog}\left(2,-\sqrt{1-x^2}\right) + \frac{1}{2}\operatorname{PolyLog}\left(2,\sqrt{1-x^2}\right)$$

output
```
arctanh((-x^2+1)^(1/2))-1/2*(-x^2+1)^(1/2)*ln(-x^2+1)/x^2-1/2*arctanh((-x^
2+1)^(1/2))*ln(-x^2+1)-1/2*polylog(2,-(-x^2+1)^(1/2))+1/2*polylog(2,(-x^2+
1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.45

$$\int \frac{\log(1-x^2)}{x^3\sqrt{1-x^2}}\,dx = \frac{1}{4}\left(-\frac{2\sqrt{1-x^2}\log(1-x^2)}{x^2} - 2\log\left(1-\sqrt{1-x^2}\right) + \log(1-x^2)\log\left(1-\sqrt{1-x^2}\right) + 2\log\left(1+\sqrt{1-x^2}\right) - \log(1-x^2)\log\left(1+\sqrt{1-x^2}\right) - 2\,\text{PolyLog}\left(2,-\sqrt{1-x^2}\right) + 2\,\text{PolyLog}\left(2,\sqrt{1-x^2}\right)\right)$$

input
```
Integrate[Log[1 - x^2]/(x^3*Sqrt[1 - x^2]),x]
```

output
```
((-2*Sqrt[1 - x^2]*Log[1 - x^2])/x^2 - 2*Log[1 - Sqrt[1 - x^2]] + Log[1 -
x^2]*Log[1 - Sqrt[1 - x^2]] + 2*Log[1 + Sqrt[1 - x^2]] - Log[1 - x^2]*Log[
1 + Sqrt[1 - x^2]] - 2*PolyLog[2, -Sqrt[1 - x^2]] + 2*PolyLog[2, Sqrt[1 -
x^2]])/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^3\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \ 2929$$

$$\int \frac{\log(1-x^2)}{x^3\sqrt{1-x^2}}dx$$

input
```
Int[Log[1 - x^2]/(x^3*Sqrt[1 - x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^3\sqrt{-x^2+1}}dx$$

input
```
int(ln(-x^2+1)/x^3/(-x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^3/(-x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log(1-x^2)}{x^3\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}x^3}\,dx$$

input
```
integrate(log(-x^2+1)/x^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*log(-x^2 + 1)/(x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^3\sqrt{1-x^2}}\,dx = \int \frac{\log(1-x^2)}{x^3\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**3/(-x**2+1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**3*sqrt(-(x - 1)*(x + 1))), x)
```

Maxima [F]

$$\int \frac{\log (1-x^2)}{x^3\sqrt{1-x^2}}\,dx = \int \frac{\log (-x^2+1)}{\sqrt{-x^2+1}x^3}\,dx$$

input
```
integrate(log(-x^2+1)/x^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(-x^2 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{\log (1-x^2)}{x^3\sqrt{1-x^2}}\,dx = \int \frac{\log (-x^2+1)}{\sqrt{-x^2+1}x^3}\,dx$$

input
```
integrate(log(-x^2+1)/x^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(-x^2 + 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (1-x^2)}{x^3\sqrt{1-x^2}}\,dx = \int \frac{\ln (1-x^2)}{x^3\,\sqrt{1-x^2}}\,dx$$

input
```
int(log(1 - x^2)/(x^3*(1 - x^2)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^3*(1 - x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{x^3\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}\,x^3}dx$$

input

```
int(log(-x^2+1)/x^3/(-x^2+1)^(1/2),x)
```

output

```
int(log( - x**2 + 1)/(sqrt( - x**2 + 1)*x**3),x)
```

3.642 $\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx = \frac{\sqrt{1-x^2}}{4x^2} + \operatorname{arctanh}\left(\sqrt{1-x^2}\right) - \frac{\sqrt{1-x^2}\log(1-x^2)}{4x^4} - \frac{3\sqrt{1-x^2}\log(1-x^2)}{8x^2} - \frac{3}{8}\operatorname{arctanh}\left(\sqrt{1-x^2}\right)\log(1-x^2) - \frac{3}{8}\operatorname{PolyLog}\left(2,-\sqrt{1-x^2}\right) + \frac{3}{8}\operatorname{PolyLog}\left(2,\sqrt{1-x^2}\right)$$

output
```
1/4*(-x^2+1)^(1/2)/x^2+arctanh((-x^2+1)^(1/2))-1/4*(-x^2+1)^(1/2)*ln(-x^2+
1)/x^4-3/8*(-x^2+1)^(1/2)*ln(-x^2+1)/x^2-3/8*arctanh((-x^2+1)^(1/2))*ln(-x
^2+1)-3/8*polylog(2,-(-x^2+1)^(1/2))+3/8*polylog(2,(-x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.29

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx = \frac{1}{16}\left(\frac{4\sqrt{1-x^2}}{x^2} - \frac{4\sqrt{1-x^2}\log(1-x^2)}{x^4} - \frac{6\sqrt{1-x^2}\log(1-x^2)}{x^2} - 8\log\left(1-\sqrt{1-x^2}\right) + 3\log(1-x^2)\log\left(1-\sqrt{1-x^2}\right) + 8\log\left(1+\sqrt{1-x^2}\right) - 3\log(1-x^2)\log\left(1+\sqrt{1-x^2}\right) - 6\,\text{PolyLog}\left(2,-\sqrt{1-x^2}\right) + 6\,\text{PolyLog}\left(2,\sqrt{1-x^2}\right)\right)$$

input
```
Integrate[Log[1 - x^2]/(x^5*Sqrt[1 - x^2]),x]
```

output
```
((4*Sqrt[1 - x^2])/x^2 - (4*Sqrt[1 - x^2]*Log[1 - x^2])/x^4 - (6*Sqrt[1 -
x^2]*Log[1 - x^2])/x^2 - 8*Log[1 - Sqrt[1 - x^2]] + 3*Log[1 - x^2]*Log[1 -
 Sqrt[1 - x^2]] + 8*Log[1 + Sqrt[1 - x^2]] - 3*Log[1 - x^2]*Log[1 + Sqrt[1
 - x^2]] - 6*PolyLog[2, -Sqrt[1 - x^2]] + 6*PolyLog[2, Sqrt[1 - x^2]])/16
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}dx$$

input
```
Int[Log[1 - x^2]/(x^5*Sqrt[1 - x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^5\sqrt{-x^2+1}}dx$$

input
```
int(ln(-x^2+1)/x^5/(-x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^5/(-x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}x^5}\,dx$$

input
```
integrate(log(-x^2+1)/x^5/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*log(-x^2 + 1)/(x^7 - x^5), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx = \int \frac{\log(1-x^2)}{x^5\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**5/(-x**2+1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**5*sqrt(-(x - 1)*(x + 1))), x)
```

Maxima [F]

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}x^5}\,dx$$

input
```
integrate(log(-x^2+1)/x^5/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(-x^2 + 1)*x^5), x)
```

Giac [F]

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}x^5}\,dx$$

input
```
integrate(log(-x^2+1)/x^5/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(-x^2 + 1)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^5\,\sqrt{1-x^2}}\,dx$$

input
```
int(log(1 - x^2)/(x^5*(1 - x^2)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^5*(1 - x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{x^5\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}\,x^5}dx$$

input
```
int(log(-x^2+1)/x^5/(-x^2+1)^(1/2),x)
```

output
```
int(log( - x**2 + 1)/(sqrt( - x**2 + 1)*x**5),x)
```

3.643 $\int \frac{x^2 \log(1-x^2)}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 374

$$\begin{aligned}\int \frac{x^2\log(1-x^2)}{\sqrt{1-x^2}}\,dx &= \frac{\sqrt{-1+x^2}}{8\sqrt{1-x^2}\left(x+\sqrt{-1+x^2}\right)^2} - \frac{\sqrt{-1+x^2}\left(x+\sqrt{-1+x^2}\right)^2}{8\sqrt{1-x^2}} \\ &\quad - \frac{\sqrt{-1+x^2}\log(1-x^2)}{8\sqrt{1-x^2}\left(x+\sqrt{-1+x^2}\right)^2} \\ &\quad + \frac{\sqrt{-1+x^2}\left(x+\sqrt{-1+x^2}\right)^2\log(1-x^2)}{8\sqrt{1-x^2}} \\ &\quad - \frac{\sqrt{-1+x^2}\log\left(x+\sqrt{-1+x^2}\right)}{2\sqrt{1-x^2}} \\ &\quad + \frac{\sqrt{-1+x^2}\log(1-x^2)\log\left(x+\sqrt{-1+x^2}\right)}{2\sqrt{1-x^2}} \\ &\quad + \frac{\sqrt{-1+x^2}\log^2\left(x+\sqrt{-1+x^2}\right)}{2\sqrt{1-x^2}} \\ &\quad - \frac{\sqrt{-1+x^2}\log\left(x+\sqrt{-1+x^2}\right)\log\left(1-\left(x+\sqrt{-1+x^2}\right)^2\right)}{\sqrt{1-x^2}} \\ &\quad - \frac{\sqrt{-1+x^2}\,\mathrm{PolyLog}\left(2,\left(x+\sqrt{-1+x^2}\right)^2\right)}{2\sqrt{1-x^2}}\end{aligned}$$

output
```
1/8*(x^2-1)^(1/2)/(-x^2+1)^(1/2)/(x+(x^2-1)^(1/2))^2-1/8*(x^2-1)^(1/2)*(x+
(x^2-1)^(1/2))^2/(-x^2+1)^(1/2)-1/8*(x^2-1)^(1/2)*ln(-x^2+1)/(-x^2+1)^(1/2
)/(x+(x^2-1)^(1/2))^2+1/8*(x^2-1)^(1/2)*(x+(x^2-1)^(1/2))^2*ln(-x^2+1)/(-x
^2+1)^(1/2)-1/2*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))/(-x^2+1)^(1/2)+1/2*(x^2-
1)^(1/2)*ln(-x^2+1)*ln(x+(x^2-1)^(1/2))/(-x^2+1)^(1/2)+1/2*(x^2-1)^(1/2)*l
n(x+(x^2-1)^(1/2))^2/(-x^2+1)^(1/2)-(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))*ln(1
-(x+(x^2-1)^(1/2))^2)/(-x^2+1)^(1/2)-1/2*(x^2-1)^(1/2)*polylog(2,(x+(x^2-1
)^(1/2))^2)/(-x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.52

$$\int \frac{x^2 \log (1-x^2)}{\sqrt{1-x^2}} \, dx = \frac{x^2\sqrt{1-x^2} + i\sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)^2 + \sqrt{x^2}\arcsin\left(\sqrt{1-x^2}\right)\left(1+2\log\left(1-e^{-2i\arcsin\left(\sqrt{1-x^2}\right)}\right)\right) - x^2}{2}$$

input
```
Integrate[(x^2*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
(x^2*Sqrt[1 - x^2] + I*Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2 + Sqrt[x^2]*ArcSi
n[Sqrt[1 - x^2]]*(1 + 2*Log[1 - E^((-2*I)*ArcSin[Sqrt[1 - x^2]])]) - x^2*S
qrt[1 - x^2]*Log[1 - x^2] + I*Sqrt[x^2]*Log[1 - x^2]*Log[Sqrt[x^2] + I*Sqr
t[1 - x^2]] + I*Sqrt[x^2]*PolyLog[2, E^((-2*I)*ArcSin[Sqrt[1 - x^2]])])/(2
*x)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log (1-x^2)}{\sqrt{1-x^2}} \, dx$$

$$\downarrow 2929$$

$$\int \frac{x^2 \log (1-x^2)}{\sqrt{1-x^2}}dx$$

input
```
Int[(x^2*Log[1 - x^2])/Sqrt[1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^2 \ln (-x^2+1)}{\sqrt{-x^2+1}}dx$$

input
```
int(x^2*ln(-x^2+1)/(-x^2+1)^(1/2),x)
```

output
```
int(x^2*ln(-x^2+1)/(-x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2 \log (1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \log (-x^2+1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^2*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*x^2*log(-x^2 + 1)/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \log(1-x^2)}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate(x**2*ln(-x**2+1)/(-x**2+1)**(1/2),x)
```

output
```
Integral(x**2*log(1 - x**2)/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \log(-x^2+1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^2*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*log(-x^2 + 1)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{x^2 \log(-x^2+1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^2*log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*log(-x^2 + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log (1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{x^2\,\ln (1-x^2)}{\sqrt{1-x^2}}\,dx$$

input
```
int((x^2*log(1 - x^2))/(1 - x^2)^(1/2),x)
```

output
```
int((x^2*log(1 - x^2))/(1 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \log (1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)\,x^2}{\sqrt{-x^2+1}}dx$$

input
```
int(x^2*log(-x^2+1)/(-x^2+1)^(1/2),x)
```

output
```
int((log( - x**2 + 1)*x**2)/sqrt( - x**2 + 1),x)
```

3.644 $\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \arcsin(x)\log\left(1-x^2\right) - i\log^2\left(ix+\sqrt{1-x^2}\right) + 2i\log\left(ix+\sqrt{1-x^2}\right)\log\left(1+\left(ix+\sqrt{1-x^2}\right)^2\right) + i\,\mathrm{PolyLog}\left(2,-\left(ix+\sqrt{1-x^2}\right)^2\right)$$

output
```
arcsin(x)*ln(-x^2+1)-I*ln(I*x+(-x^2+1)^(1/2))^2+2*I*ln(I*x+(-x^2+1)^(1/2))
*ln(1+(I*x+(-x^2+1)^(1/2))^2)+I*polylog(2,-(I*x+(-x^2+1)^(1/2))^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.79

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = -2i\pi\arcsin(x)+i\arcsin(x)^2-4\pi\log\left(1+e^{-i\arcsin(x)}\right) - \pi\log\left(1-ie^{i\arcsin(x)}\right)-2\arcsin(x)\log\left(1-ie^{i\arcsin(x)}\right) + \pi\log\left(1+ie^{i\arcsin(x)}\right)-2\arcsin(x)\log\left(1+ie^{i\arcsin(x)}\right) + \arcsin(x)\log\left(1-x^2\right)+4\pi\log\left(\cos\left(\frac{\arcsin(x)}{2}\right)\right) - \pi\log\left(-\cos\left(\frac{1}{4}(\pi+2\arcsin(x))\right)\right) + \pi\log\left(\sin\left(\frac{1}{4}(\pi+2\arcsin(x))\right)\right) + 2i\,\text{PolyLog}\left(2,-ie^{i\arcsin(x)}\right)+2i\,\text{PolyLog}\left(2,ie^{i\arcsin(x)}\right)$$

input
```
Integrate[Log[1 - x^2]/Sqrt[1 - x^2],x]
```

output
```
(-2*I)*Pi*ArcSin[x] + I*ArcSin[x]^2 - 4*Pi*Log[1 + E^((-I)*ArcSin[x])] - P
i*Log[1 - I*E^(I*ArcSin[x])] - 2*ArcSin[x]*Log[1 - I*E^(I*ArcSin[x])] + Pi
*Log[1 + I*E^(I*ArcSin[x])] - 2*ArcSin[x]*Log[1 + I*E^(I*ArcSin[x])] + Arc
Sin[x]*Log[1 - x^2] + 4*Pi*Log[Cos[ArcSin[x]/2]] - Pi*Log[-Cos[(Pi + 2*Arc
Sin[x])/4]] + Pi*Log[Sin[(Pi + 2*ArcSin[x])/4]] + (2*I)*PolyLog[2, (-I)*E^
(I*ArcSin[x])] + (2*I)*PolyLog[2, I*E^(I*ArcSin[x])]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \; 2923$$

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx$$

input
```
Int[Log[1 - x^2]/Sqrt[1 - x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln \left(-x^{2}+1\right)}{\sqrt{-x^{2}+1}}dx$$

input
```
int(ln(-x^2+1)/(-x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/(-x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log (1-x^2)}{\sqrt{1-x^2}}\,dx=\int \frac{\log \left(-x^{2}+1\right)}{\sqrt{-x^{2}+1}}\,dx$$

input
```
integrate(log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*log(-x^2 + 1)/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(1-x^2)}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/(-x**2+1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(log(-x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\ln(1-x^2)}{\sqrt{1-x^2}}\,dx$$

input
```
int(log(1 - x^2)/(1 - x^2)^(1/2),x)
```

output
```
int(log(1 - x^2)/(1 - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{\sqrt{1-x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{-x^2+1}}dx$$

input
```
int(log(-x^2+1)/(-x^2+1)^(1/2),x)
```

output
```
int(log( - x**2 + 1)/sqrt( - x**2 + 1),x)
```

3.645 $\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx = -2\arcsin(x) - \frac{\sqrt{1-x^2}\log(1-x^2)}{x}$$

output
```
-2*arcsin(x)-(-x^2+1)^(1/2)*ln(-x^2+1)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx = -2\arcsin(x) - \frac{\sqrt{1-x^2}\log(1-x^2)}{x}$$

input
```
Integrate[Log[1 - x^2]/(x^2*Sqrt[1 - x^2]),x]
```

output
```
-2*ArcSin[x] - (Sqrt[1 - x^2]*Log[1 - x^2])/x
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}dx$$

input
```
Int[Log[1 - x^2]/(x^2*Sqrt[1 - x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^2\sqrt{-x^2+1}}dx$$

input
```
int(ln(-x^2+1)/x^2/(-x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^2/(-x^2+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx = \frac{4\,x\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}\log(-x^2+1)}{x}$$

input
```
integrate(log(-x^2+1)/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
(4*x*arctan((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*log(-x^2 + 1))/x
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx = \int \frac{\log(1-x^2)}{x^2\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**2/(-x**2+1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**2*sqrt(-(x - 1)*(x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{-x^2+1}\log(-x^2+1)}{x}-2\arcsin(x)$$

input
```
integrate(log(-x^2+1)/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1)*log(-x^2 + 1)/x - 2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx = \frac{1}{2}\left(\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1}-1}{x}\right)\log\left(-x^2+1\right) - 2\arcsin(x)$$

input
```
integrate(log(-x^2+1)/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)*log(-x^2 + 1) - 2*ar
csin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^2\sqrt{1-x^2}}\,dx$$

input
```
int(log(1 - x^2)/(x^2*(1 - x^2)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^2*(1 - x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{\log(1-x^2)}{x^2\sqrt{1-x^2}}\,dx = \frac{-2asin(x)\,x - \sqrt{-x^2+1}\log\left(\frac{\tan\left(\frac{asin(x)}{2}\right)^4 - 2\tan\left(\frac{asin(x)}{2}\right)^2+1}{\tan\left(\frac{asin(x)}{2}\right)^4 + 2\tan\left(\frac{asin(x)}{2}\right)^2+1}\right)}{x}$$

input
```
int(log(-x^2+1)/x^2/(-x^2+1)^(1/2),x)
```

output
```
( - 2*asin(x)*x - sqrt( - x**2 + 1)*log((tan(asin(x)/2)**4 - 2*tan(asin(x)
/2)**2 + 1)/(tan(asin(x)/2)**4 + 2*tan(asin(x)/2)**2 + 1)))/x
```

3.646 $\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}\,dx = \frac{2\sqrt{1-x^2}}{3x} - \frac{4\arcsin(x)}{3} - \frac{\sqrt{1-x^2}\log(1-x^2)}{3x^3} - \frac{2\sqrt{1-x^2}\log(1-x^2)}{3x}$$

output
```
2/3*(-x^2+1)^(1/2)/x-4/3*arcsin(x)-1/3*(-x^2+1)^(1/2)*ln(-x^2+1)/x^3-2/3*(
-x^2+1)^(1/2)*ln(-x^2+1)/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}\,dx = \frac{1}{3}\left(-4\arcsin(x) + \frac{\sqrt{1-x^2}(2x^2-(1+2x^2)\log(1-x^2))}{x^3}\right)$$

input
```
Integrate[Log[1 - x^2]/(x^4*Sqrt[1 - x^2]),x]
```

output
```
(-4*ArcSin[x] + (Sqrt[1 - x^2]*(2*x^2 - (1 + 2*x^2)*Log[1 - x^2]))/x^3)/3
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}dx$$

input
```
Int[Log[1 - x^2]/(x^4*Sqrt[1 - x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^4\sqrt{-x^2+1}}dx$$

input
```
int(ln(-x^2+1)/x^4/(-x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^4/(-x^2+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}\,dx = \frac{8\,x^3\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+(2\,x^2-(2\,x^2+1)\log(-x^2+1))\sqrt{-x^2+1}}{3\,x^3}$$

input
```
integrate(log(-x^2+1)/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(8*x^3*arctan((sqrt(-x^2 + 1) - 1)/x) + (2*x^2 - (2*x^2 + 1)*log(-x^2
+ 1))*sqrt(-x^2 + 1))/x^3
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}\,dx = \int \frac{\log(1-x^2)}{x^4\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**4/(-x**2+1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**4*sqrt(-(x - 1)*(x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}\,dx = -\frac{1}{3}\left(\frac{2\sqrt{-x^2+1}}{x}+\frac{\sqrt{-x^2+1}}{x^3}\right)\log(-x^2+1) + \frac{2\sqrt{-x^2+1}}{3\,x} - \frac{4}{3}\arcsin(x)$$

input
```
integrate(log(-x^2+1)/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*log(-x^2 + 1) + 2/3*sqrt(-x
^2 + 1)/x - 4/3*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.55

$$\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}\,dx = \frac{1}{24}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3} - \frac{9\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\log\left(-x^2+1\right) - \frac{x}{3\left(\sqrt{-x^2+1}-1\right)} + \frac{\sqrt{-x^2+1}-1}{3\,x} - \frac{4}{3}\arcsin(x)$$

input
```
integrate(log(-x^2+1)/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/24*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(s
qrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*log(-x^2 + 1) - 1/3*x/(
sqrt(-x^2 + 1) - 1) + 1/3*(sqrt(-x^2 + 1) - 1)/x - 4/3*arcsin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^4\sqrt{1-x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^4\,\sqrt{1-x^2}}\,dx$$

input
```
int(log(1 - x^2)/(x^4*(1 - x^2)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^4*(1 - x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

$$\int \frac{\log (1-x^2)}{x^4\sqrt{1-x^2}}\,dx$$
$$= \frac{-4asin(x)\,x^3 - 2\sqrt{-x^2+1}\log\left(\frac{\tan\left(\frac{asin(x)}{2}\right)^4-2\tan\left(\frac{asin(x)}{2}\right)^2+1}{\tan\left(\frac{asin(x)}{2}\right)^4+2\tan\left(\frac{asin(x)}{2}\right)^2+1}\right)x^2 - \sqrt{-x^2+1}\log\left(\frac{\tan\left(\frac{asin(x)}{2}\right)^4-2\tan\left(\frac{asin(x)}{2}\right)^2+1}{\tan\left(\frac{asin(x)}{2}\right)^4+2\tan\left(\frac{asin(x)}{2}\right)^2+1}\right)}{3x^3}$$

input
```
int(log(-x^2+1)/x^4/(-x^2+1)^(1/2),x)
```

output
```
( - 4*asin(x)*x**3 - 2*sqrt( - x**2 + 1)*log((tan(asin(x)/2)**4 - 2*tan(as
in(x)/2)**2 + 1)/(tan(asin(x)/2)**4 + 2*tan(asin(x)/2)**2 + 1))*x**2 - sqr
t( - x**2 + 1)*log((tan(asin(x)/2)**4 - 2*tan(asin(x)/2)**2 + 1)/(tan(asin
(x)/2)**4 + 2*tan(asin(x)/2)**2 + 1)) + 2*sqrt( - x**2 + 1)*x**2)/(3*x**3)
```

3.647 $\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx = \frac{2\sqrt{1-x^2}}{15x^3}+\frac{4\sqrt{1-x^2}}{5x}-\frac{16\arcsin(x)}{15}-\frac{\sqrt{1-x^2}\log(1-x^2)}{5x^5}-\frac{4\sqrt{1-x^2}\log(1-x^2)}{15x^3}-\frac{8\sqrt{1-x^2}\log(1-x^2)}{15x}$$

output
```
2/15*(-x^2+1)^(1/2)/x^3+4/5*(-x^2+1)^(1/2)/x-16/15*arcsin(x)-1/5*(-x^2+1)^
(1/2)*ln(-x^2+1)/x^5-4/15*(-x^2+1)^(1/2)*ln(-x^2+1)/x^3-8/15*(-x^2+1)^(1/2
)*ln(-x^2+1)/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx = \frac{1}{15}\left(-16\arcsin(x)+\frac{\sqrt{1-x^2}(2(x^2+6x^4)-(3+4x^2+8x^4)\log(1-x^2))}{x^5}\right)$$

input
```
Integrate[Log[1 - x^2]/(x^6*Sqrt[1 - x^2]),x]
```

output
```
(-16*ArcSin[x] + (Sqrt[1 - x^2]*(2*(x^2 + 6*x^4) - (3 + 4*x^2 + 8*x^4)*Log
[1 - x^2]))/x^5)/15
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx$$
$$\Big\downarrow \text{2929}$$
$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}dx$$

input
```
Int[Log[1 - x^2]/(x^6*Sqrt[1 - x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^6\sqrt{-x^2+1}}dx$$

input
```
int(ln(-x^2+1)/x^6/(-x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^6/(-x^2+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx = \frac{32\,x^5\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + (12\,x^4 + 2\,x^2 - (8\,x^4 + 4\,x^2 + 3)\log(-x^2+1))\sqrt{-x^2+1}}{15\,x^5}$$

input
```
integrate(log(-x^2+1)/x^6/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(32*x^5*arctan((sqrt(-x^2 + 1) - 1)/x) + (12*x^4 + 2*x^2 - (8*x^4 + 4
*x^2 + 3)*log(-x^2 + 1))*sqrt(-x^2 + 1))/x^5
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx = \int \frac{\log(1-x^2)}{x^6\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**6/(-x**2+1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**6*sqrt(-(x - 1)*(x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx = -\frac{1}{15}\left(\frac{8\sqrt{-x^2+1}}{x}+\frac{4\sqrt{-x^2+1}}{x^3}+\frac{3\sqrt{-x^2+1}}{x^5}\right)\log\left(-x^2+1\right) + \frac{4\sqrt{-x^2+1}}{5\,x}+\frac{2\sqrt{-x^2+1}}{15\,x^3}-\frac{16}{15}\arcsin\left(x\right)$$

input
```
integrate(log(-x^2+1)/x^6/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/15*(8*sqrt(-x^2 + 1)/x + 4*sqrt(-x^2 + 1)/x^3 + 3*sqrt(-x^2 + 1)/x^5)*l
og(-x^2 + 1) + 4/5*sqrt(-x^2 + 1)/x + 2/15*sqrt(-x^2 + 1)/x^3 - 16/15*arcs
in(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.61

$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx = \frac{1}{480}\left(\frac{x^5\left(\frac{25\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{150\left(\sqrt{-x^2+1}-1\right)^4}{x^4}+3\right)}{\left(\sqrt{-x^2+1}-1\right)^5}-\frac{150\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{25\left(\sqrt{-x^2+1}-1\right)^3}{x^3}-\frac{3\left(\sqrt{\cdots}\right.}{\cdots}\right. \left.+1\right) - \frac{x^3\left(\frac{25\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{60\left(\sqrt{-x^2+1}-1\right)^3}+\frac{5\left(\sqrt{-x^2+1}-1\right)}{12\,x}+\frac{\left(\sqrt{-x^2+1}-1\right)^3}{60\,x^3}-\frac{16}{15}\arcsin\left(x\right)$$

input
```
integrate(log(-x^2+1)/x^6/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/480*(x^5*(25*(sqrt(-x^2 + 1) - 1)^2/x^2 + 150*(sqrt(-x^2 + 1) - 1)^4/x^4
 + 3)/(sqrt(-x^2 + 1) - 1)^5 - 150*(sqrt(-x^2 + 1) - 1)/x - 25*(sqrt(-x^2
+ 1) - 1)^3/x^3 - 3*(sqrt(-x^2 + 1) - 1)^5/x^5)*log(-x^2 + 1) - 1/60*x^3*(
25*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 + 5/12*(sqrt(-x^
2 + 1) - 1)/x + 1/60*(sqrt(-x^2 + 1) - 1)^3/x^3 - 16/15*arcsin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^6\,\sqrt{1-x^2}}\,dx$$

input
```
int(log(1 - x^2)/(x^6*(1 - x^2)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^6*(1 - x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.61

$$\int \frac{\log(1-x^2)}{x^6\sqrt{1-x^2}}\,dx = \frac{-16asin(x)\,x^5 - 8\sqrt{-x^2+1}\log\left(\frac{\tan\left(\frac{asin(x)}{2}\right)^4 - 2\tan\left(\frac{asin(x)}{2}\right)^2+1}{\tan\left(\frac{asin(x)}{2}\right)^4+2\tan\left(\frac{asin(x)}{2}\right)^2+1}\right)x^4 - 4\sqrt{-x^2+1}\log\left(\frac{\tan\left(\frac{asin(x)}{2}\right)^4-2\tan\left(\frac{asi}{}\right.}{\tan\left(\frac{asin(x)}{2}\right)^4+2\tan\left(\frac{asi}{}\right.}\right.}{15x^5}$$

input
```
int(log(-x^2+1)/x^6/(-x^2+1)^(1/2),x)
```

output
```
( - 16*asin(x)*x**5 - 8*sqrt( - x**2 + 1)*log((tan(asin(x)/2)**4 - 2*tan(a
sin(x)/2)**2 + 1)/(tan(asin(x)/2)**4 + 2*tan(asin(x)/2)**2 + 1))*x**4 - 4*
sqrt( - x**2 + 1)*log((tan(asin(x)/2)**4 - 2*tan(asin(x)/2)**2 + 1)/(tan(a
sin(x)/2)**4 + 2*tan(asin(x)/2)**2 + 1))*x**2 - 3*sqrt( - x**2 + 1)*log((t
an(asin(x)/2)**4 - 2*tan(asin(x)/2)**2 + 1)/(tan(asin(x)/2)**4 + 2*tan(asi
n(x)/2)**2 + 1)) + 12*sqrt( - x**2 + 1)*x**4 + 2*sqrt( - x**2 + 1)*x**2)/(
15*x**5)
```

3.648 $\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 165

$$\begin{aligned}\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx &= \frac{2\sqrt{1-x^2}}{35x^5}+\frac{4\sqrt{1-x^2}}{21x^3}+\frac{88\sqrt{1-x^2}}{105x}-\frac{32\arcsin(x)}{35}\\ &\quad-\frac{\sqrt{1-x^2}\log(1-x^2)}{7x^7}-\frac{6\sqrt{1-x^2}\log(1-x^2)}{35x^5}\\ &\quad-\frac{8\sqrt{1-x^2}\log(1-x^2)}{35x^3}-\frac{16\sqrt{1-x^2}\log(1-x^2)}{35x}\end{aligned}$$

output
```
2/35*(-x^2+1)^(1/2)/x^5+4/21*(-x^2+1)^(1/2)/x^3+88/105*(-x^2+1)^(1/2)/x-32
/35*arcsin(x)-1/7*(-x^2+1)^(1/2)*ln(-x^2+1)/x^7-6/35*(-x^2+1)^(1/2)*ln(-x^
2+1)/x^5-8/35*(-x^2+1)^(1/2)*ln(-x^2+1)/x^3-16/35*(-x^2+1)^(1/2)*ln(-x^2+1
)/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx = \frac{1}{105}\left(-96\arcsin(x) + \frac{\sqrt{1-x^2}\left(6x^2+20x^4+88x^6-3(5+6x^2+8x^4+16x^6)\log(1-x^2)\right)}{x^7}\right)$$

input
```
Integrate[Log[1 - x^2]/(x^8*Sqrt[1 - x^2]),x]
```

output
```
(-96*ArcSin[x] + (Sqrt[1 - x^2]*(6*x^2 + 20*x^4 + 88*x^6 - 3*(5 + 6*x^2 +
8*x^4 + 16*x^6)*Log[1 - x^2]))/x^7)/105
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx$$

input
```
Int[Log[1 - x^2]/(x^8*Sqrt[1 - x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^8\sqrt{-x^2+1}}dx$$

input
```
int(ln(-x^2+1)/x^8/(-x^2+1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^8/(-x^2+1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.48

$$\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx = \frac{192\,x^7\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + (88\,x^6+20\,x^4+6\,x^2-3\,(16\,x^6+8\,x^4+6\,x^2+5)\log(-x^2+1))\sqrt{-x^2+1}}{105\,x^7}$$

input
```
integrate(log(-x^2+1)/x^8/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(192*x^7*arctan((sqrt(-x^2 + 1) - 1)/x) + (88*x^6 + 20*x^4 + 6*x^2 -
 3*(16*x^6 + 8*x^4 + 6*x^2 + 5)*log(-x^2 + 1))*sqrt(-x^2 + 1))/x^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx = \text{Timed out}$$

input
```
integrate(ln(-x**2+1)/x**8/(-x**2+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.69

$$\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx = -\frac{1}{35}\left(\frac{16\sqrt{-x^2+1}}{x}+\frac{8\sqrt{-x^2+1}}{x^3}+\frac{6\sqrt{-x^2+1}}{x^5}+\frac{5\sqrt{-x^2+1}}{x^7}\right)\log\left(-x^2+1\right) + \frac{88\sqrt{-x^2+1}}{105\,x}+\frac{4\sqrt{-x^2+1}}{21\,x^3}+\frac{2\sqrt{-x^2+1}}{35\,x^5}-\frac{32}{35}\arcsin(x)$$

input
```
integrate(log(-x^2+1)/x^8/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/35*(16*sqrt(-x^2 + 1)/x + 8*sqrt(-x^2 + 1)/x^3 + 6*sqrt(-x^2 + 1)/x^5 +
 5*sqrt(-x^2 + 1)/x^7)*log(-x^2 + 1) + 88/105*sqrt(-x^2 + 1)/x + 4/21*sqrt
(-x^2 + 1)/x^3 + 2/35*sqrt(-x^2 + 1)/x^5 - 32/35*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.62

$$\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{4480}\left(\frac{x^7\left(\frac{49\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{245\left(\sqrt{-x^2+1}-1\right)^4}{x^4}+\frac{1225\left(\sqrt{-x^2+1}-1\right)^6}{x^6}+5\right)}{\left(\sqrt{-x^2+1}-1\right)^7} - \frac{1225\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{245\left(\sqrt{\right.}}{}\right.$$
$$+1) - \frac{x^5\left(\frac{49\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{750\left(\sqrt{-x^2+1}-1\right)^4}{x^4}+3\right)}{1680\left(\sqrt{-x^2+1}-1\right)^5}$$
$$+ \frac{25\left(\sqrt{-x^2+1}-1\right)}{56\,x} + \frac{7\left(\sqrt{-x^2+1}-1\right)^3}{240\,x^3} + \frac{\left(\sqrt{-x^2+1}-1\right)^5}{560\,x^5} - \frac{32}{35}\arcsin(x)$$

input
```
integrate(log(-x^2+1)/x^8/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/4480*(x^7*(49*(sqrt(-x^2 + 1) - 1)^2/x^2 + 245*(sqrt(-x^2 + 1) - 1)^4/x^
4 + 1225*(sqrt(-x^2 + 1) - 1)^6/x^6 + 5)/(sqrt(-x^2 + 1) - 1)^7 - 1225*(sq
rt(-x^2 + 1) - 1)/x - 245*(sqrt(-x^2 + 1) - 1)^3/x^3 - 49*(sqrt(-x^2 + 1)
- 1)^5/x^5 - 5*(sqrt(-x^2 + 1) - 1)^7/x^7)*log(-x^2 + 1) - 1/1680*x^5*(49*
(sqrt(-x^2 + 1) - 1)^2/x^2 + 750*(sqrt(-x^2 + 1) - 1)^4/x^4 + 3)/(sqrt(-x^
2 + 1) - 1)^5 + 25/56*(sqrt(-x^2 + 1) - 1)/x + 7/240*(sqrt(-x^2 + 1) - 1)^
3/x^3 + 1/560*(sqrt(-x^2 + 1) - 1)^5/x^5 - 32/35*arcsin(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^8\,\sqrt{1-x^2}}\,dx$$

input
```
int(log(1 - x^2)/(x^8*(1 - x^2)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^8*(1 - x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.58

$$\int \frac{\log(1-x^2)}{x^8\sqrt{1-x^2}}\,dx$$
$$= \frac{-96 asin(x)\,x^7 - 48\sqrt{-x^2+1}\log\left(\frac{\tan\left(\frac{asin(x)}{2}\right)^4-2\tan\left(\frac{asin(x)}{2}\right)^2+1}{\tan\left(\frac{asin(x)}{2}\right)^4+2\tan\left(\frac{asin(x)}{2}\right)^2+1}\right)x^6 - 24\sqrt{-x^2+1}\log\left(\frac{\tan\left(\frac{asin(x)}{2}\right)^4-2\tan\left(\right.}{\tan\left(\frac{asin(x)}{2}\right)^4+2\tan\left(\right.}\right.}{}$$

input
```
int(log(-x^2+1)/x^8/(-x^2+1)^(1/2),x)
```

output
```
( - 96*asin(x)*x**7 - 48*sqrt( - x**2 + 1)*log((tan(asin(x)/2)**4 - 2*tan(
asin(x)/2)**2 + 1)/(tan(asin(x)/2)**4 + 2*tan(asin(x)/2)**2 + 1))*x**6 - 2
4*sqrt( - x**2 + 1)*log((tan(asin(x)/2)**4 - 2*tan(asin(x)/2)**2 + 1)/(tan
(asin(x)/2)**4 + 2*tan(asin(x)/2)**2 + 1))*x**4 - 18*sqrt( - x**2 + 1)*log
((tan(asin(x)/2)**4 - 2*tan(asin(x)/2)**2 + 1)/(tan(asin(x)/2)**4 + 2*tan(
asin(x)/2)**2 + 1))*x**2 - 15*sqrt( - x**2 + 1)*log((tan(asin(x)/2)**4 - 2
*tan(asin(x)/2)**2 + 1)/(tan(asin(x)/2)**4 + 2*tan(asin(x)/2)**2 + 1)) + 8
8*sqrt( - x**2 + 1)*x**6 + 20*sqrt( - x**2 + 1)*x**4 + 6*sqrt( - x**2 + 1)
*x**2)/(105*x**7)
```

3.649 $\int \frac{x^7 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = -2\sqrt{-1+x^2}-\frac{2}{3}\left(-1+x^2\right)^{3/2}-\frac{6}{25}\left(-1+x^2\right)^{5/2}-\frac{2}{49}\left(-1+x^2\right)^{7/2} +\sqrt{-1+x^2}\log\left(1-x^2\right)+\left(-1+x^2\right)^{3/2}\log\left(1-x^2\right)+\frac{3}{5}\left(-1+x^2\right)^{5/2}\log\left(1-x^2\right)+\frac{1}{7}\left(-1+x^2\right)^{7/2}\log\left(1-x^2\right)$$

output
```
-2*(x^2-1)^(1/2)-2/3*(x^2-1)^(3/2)-6/25*(x^2-1)^(5/2)-2/49*(x^2-1)^(7/2)+(
x^2-1)^(1/2)*ln(-x^2+1)+(x^2-1)^(3/2)*ln(-x^2+1)+3/5*(x^2-1)^(5/2)*ln(-x^2
+1)+1/7*(x^2-1)^(7/2)*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.47

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \frac{\sqrt{-1+x^2}\left(-2(2816+568x^2+216x^4+75x^6)+105(16+8x^2+6x^4+5x^6)\log\left(1-x^2\right)\right)}{3675}$$

input
```
Integrate[(x^7*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
(Sqrt[-1 + x^2]*(-2*(2816 + 568*x^2 + 216*x^4 + 75*x^6) + 105*(16 + 8*x^2
+ 6*x^4 + 5*x^6)*Log[1 - x^2]))/3675
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{x^2-1}} dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^7 \log(1-x^2)}{\sqrt{x^2-1}} dx$$

input
```
Int[(x^7*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.48

method	result
risch	$\frac{(5x^6+6x^4+8x^2+16)\sqrt{x^2-1}\,\ln(-x^2+1)}{35} - \frac{2(75x^6+216x^4+568x^2+2816)\sqrt{x^2-1}}{3675}$
orering	$\frac{(x-1)(1+x)(975x^8+1401x^6+2736x^4+14600x^2-19712)\ln(-x^2+1)}{3675x^2\sqrt{x^2-1}} - \frac{(75x^6+216x^4+568x^2+2816)(x-1)^2(1+x)^2\left(\frac{7x^6\ln(-x^2}{\sqrt{x^2-1}}\right.}{3675x^8}$

input
```
int(x^7*ln(-x^2+1)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*x^6+6*x^4+8*x^2+16)*(x^2-1)^(1/2)*ln(-x^2+1)-2/3675*(75*x^6+216*x^
4+568*x^2+2816)*(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.41

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = -\frac{1}{3675}\left(150\,x^6 + 432\,x^4 + 1136\,x^2 - 105\left(5\,x^6 + 6\,x^4 + 8\,x^2 + 16\right)\log\left(-x^2+1\right) + 5632\right)\sqrt{x^2-1}$$

input
```
integrate(x^7*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3675*(150*x^6 + 432*x^4 + 1136*x^2 - 105*(5*x^6 + 6*x^4 + 8*x^2 + 16)*l
og(-x^2 + 1) + 5632)*sqrt(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.08

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \frac{x^6\sqrt{x^2-1}\log(1-x^2)}{7} - \frac{2x^6\sqrt{x^2-1}}{49} + \frac{6x^4\sqrt{x^2-1}\log(1-x^2)}{35} - \frac{144x^4\sqrt{x^2-1}}{1225} + \frac{8x^2\sqrt{x^2-1}\log(1-x^2)}{35} - \frac{1136x^2\sqrt{x^2-1}}{3675} + \frac{16\sqrt{x^2-1}\log(1-x^2)}{35} - \frac{5632\sqrt{x^2-1}}{3675}$$

input
```
integrate(x**7*ln(-x**2+1)/(x**2-1)**(1/2),x)
```

output
```
x**6*sqrt(x**2 - 1)*log(1 - x**2)/7 - 2*x**6*sqrt(x**2 - 1)/49 + 6*x**4*sq
rt(x**2 - 1)*log(1 - x**2)/35 - 144*x**4*sqrt(x**2 - 1)/1225 + 8*x**2*sqrt
(x**2 - 1)*log(1 - x**2)/35 - 1136*x**2*sqrt(x**2 - 1)/3675 + 16*sqrt(x**2
 - 1)*log(1 - x**2)/35 - 5632*sqrt(x**2 - 1)/3675
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = -\frac{2}{49}\left(x^2-1\right)^{\frac{3}{2}}x^4 - \frac{194}{1225}\left(x^2-1\right)^{\frac{3}{2}}x^2 + \frac{1}{35}\left(5\sqrt{x^2-1}x^6 + 6\sqrt{x^2-1}x^4 + 8\sqrt{x^2-1}x^2 + 16\sqrt{x^2-1}\right)\log\left(-x^2+1\right) - \frac{1718}{3675}\left(x^2-1\right)^{\frac{3}{2}} - 2\sqrt{x^2-1}$$

input
```
integrate(x^7*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
-2/49*(x^2 - 1)^(3/2)*x^4 - 194/1225*(x^2 - 1)^(3/2)*x^2 + 1/35*(5*sqrt(x^
2 - 1)*x^6 + 6*sqrt(x^2 - 1)*x^4 + 8*sqrt(x^2 - 1)*x^2 + 16*sqrt(x^2 - 1))
*log(-x^2 + 1) - 1718/3675*(x^2 - 1)^(3/2) - 2*sqrt(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.65

$$\int \frac{x^7 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = -\frac{2}{49}\left(x^2-1\right)^{\frac{7}{2}} - \frac{6}{25}\left(x^2-1\right)^{\frac{5}{2}} + \frac{1}{35}\left(5\left(x^2-1\right)^{\frac{7}{2}} + 21\left(x^2-1\right)^{\frac{5}{2}} + 35\left(x^2-1\right)^{\frac{3}{2}} + 35\sqrt{x^2-1}\right)\log\left(-x^2+1\right) - \frac{2}{3}\left(x^2-1\right)^{\frac{3}{2}} - 2\sqrt{x^2-1}$$

input
```
integrate(x^7*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
-2/49*(x^2 - 1)^(7/2) - 6/25*(x^2 - 1)^(5/2) + 1/35*(5*(x^2 - 1)^(7/2) + 2
1*(x^2 - 1)^(5/2) + 35*(x^2 - 1)^(3/2) + 35*sqrt(x^2 - 1))*log(-x^2 + 1) -
 2/3*(x^2 - 1)^(3/2) - 2*sqrt(x^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \log (1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^7 \ln (1-x^2)}{\sqrt{x^2-1}}\,dx$$

input
```
int((x^7*log(1 - x^2))/(x^2 - 1)^(1/2),x)
```

output
```
int((x^7*log(1 - x^2))/(x^2 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.57

$$\int \frac{x^7 \log (1-x^2)}{\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\,(525\log(-x^2+1)\,x^6+630\log(-x^2+1)\,x^4+840\log(-x^2+1)\,x^2+1680\log(-x^2+1)-150x}{3675}$$

input
```
int(x^7*log(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
(sqrt(x**2 - 1)*(525*log( - x**2 + 1)*x**6 + 630*log( - x**2 + 1)*x**4 + 8
40*log( - x**2 + 1)*x**2 + 1680*log( - x**2 + 1) - 150*x**6 - 432*x**4 - 1
136*x**2 - 5632))/3675
```

3.650 $\int \frac{x^5 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx = -2\sqrt{-1+x^2} - \frac{4}{9}\left(-1+x^2\right)^{3/2} - \frac{2}{25}\left(-1+x^2\right)^{5/2} + \sqrt{-1+x^2}\log\left(1-x^2\right) + \frac{2}{3}\left(-1+x^2\right)^{3/2}\log\left(1-x^2\right) + \frac{1}{5}\left(-1+x^2\right)^{5/2}\log\left(1-x^2\right)$$

output
```
-2*(x^2-1)^(1/2)-4/9*(x^2-1)^(3/2)-2/25*(x^2-1)^(5/2)+(x^2-1)^(1/2)*ln(-x^
2+1)+2/3*(x^2-1)^(3/2)*ln(-x^2+1)+1/5*(x^2-1)^(5/2)*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.51

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx = \frac{1}{225}\sqrt{-1+x^2}\left(-2\left(184+32x^2+9x^4\right) + 15\left(8+4x^2+3x^4\right)\log\left(1-x^2\right)\right)$$

input
```
Integrate[(x^5*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
(Sqrt[-1 + x^2]*(-2*(184 + 32*x^2 + 9*x^4) + 15*(8 + 4*x^2 + 3*x^4)*Log[1
- x^2]))/225
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{x^2-1}} dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^5 \log(1-x^2)}{\sqrt{x^2-1}} dx$$

input
```
Int[(x^5*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

method	result
risch	$\frac{(3x^4+4x^2+8)\sqrt{x^2-1}\,\ln(-x^2+1)}{15}-\frac{2(9x^4+32x^2+184)\sqrt{x^2-1}}{225}$
orering	$\frac{(x-1)(1+x)(81x^6+143x^4+696x^2-920)\ln(-x^2+1)}{225x^2\sqrt{x^2-1}}-\frac{(9x^4+32x^2+184)(x-1)^2(1+x)^2\left(\frac{5x^4\ln(-x^2+1)}{\sqrt{x^2-1}}-\frac{2x^6}{(-x^2+1)\sqrt{x^2-1}}-\frac{x^6 \mathrm{l}}{(}\right.}{225x^6}$

input
```
int(x^5*ln(-x^2+1)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*x^4+4*x^2+8)*(x^2-1)^(1/2)*ln(-x^2+1)-2/225*(9*x^4+32*x^2+184)*(x^
2-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.44

$$\int \frac{x^5\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = -\frac{1}{225}\left(18\,x^4+64\,x^2-15\left(3\,x^4+4\,x^2+8\right)\log\left(-x^2+1\right)+368\right)\sqrt{x^2-1}$$

input
```
integrate(x^5*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/225*(18*x^4 + 64*x^2 - 15*(3*x^4 + 4*x^2 + 8)*log(-x^2 + 1) + 368)*sqrt
(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \frac{x^5\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \frac{x^4\sqrt{x^2-1}\log(1-x^2)}{5}-\frac{2x^4\sqrt{x^2-1}}{25}+\frac{4x^2\sqrt{x^2-1}\log(1-x^2)}{15} - \frac{64x^2\sqrt{x^2-1}}{225}+\frac{8\sqrt{x^2-1}\log(1-x^2)}{15}-\frac{368\sqrt{x^2-1}}{225}$$

input
```
integrate(x**5*ln(-x**2+1)/(x**2-1)**(1/2),x)
```

output
```
x**4*sqrt(x**2 - 1)*log(1 - x**2)/5 - 2*x**4*sqrt(x**2 - 1)/25 + 4*x**2*sq
rt(x**2 - 1)*log(1 - x**2)/15 - 64*x**2*sqrt(x**2 - 1)/225 + 8*sqrt(x**2 -
 1)*log(1 - x**2)/15 - 368*sqrt(x**2 - 1)/225
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx = -\frac{2}{25}\left(x^2-1\right)^{\frac{3}{2}} x^2 + \frac{1}{15}\left(3\sqrt{x^2-1}x^4 + 4\sqrt{x^2-1}x^2 + 8\sqrt{x^2-1}\right)\log\left(-x^2+1\right) - \frac{82}{225}\left(x^2-1\right)^{\frac{3}{2}} - 2\sqrt{x^2-1}$$

input
```
integrate(x^5*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
-2/25*(x^2 - 1)^(3/2)*x^2 + 1/15*(3*sqrt(x^2 - 1)*x^4 + 4*sqrt(x^2 - 1)*x^
2 + 8*sqrt(x^2 - 1))*log(-x^2 + 1) - 82/225*(x^2 - 1)^(3/2) - 2*sqrt(x^2 -
 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{x^5 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx = -\frac{2}{25}\left(x^2-1\right)^{\frac{5}{2}} + \frac{1}{15}\left(3\left(x^2-1\right)^{\frac{5}{2}} + 10\left(x^2-1\right)^{\frac{3}{2}} + 15\sqrt{x^2-1}\right)\log\left(-x^2+1\right) - \frac{4}{9}\left(x^2-1\right)^{\frac{3}{2}} - 2\sqrt{x^2-1}$$

input
```
integrate(x^5*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
-2/25*(x^2 - 1)^(5/2) + 1/15*(3*(x^2 - 1)^(5/2) + 10*(x^2 - 1)^(3/2) + 15*
sqrt(x^2 - 1))*log(-x^2 + 1) - 4/9*(x^2 - 1)^(3/2) - 2*sqrt(x^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log (1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^5 \ln (1-x^2)}{\sqrt{x^2-1}}\,dx$$

input
```
int((x^5*log(1 - x^2))/(x^2 - 1)^(1/2),x)
```

output
```
int((x^5*log(1 - x^2))/(x^2 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int \frac{x^5 \log (1-x^2)}{\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\,(45\log(-x^2+1)\,x^4+60\log(-x^2+1)\,x^2+120\log(-x^2+1)-18x^4-64x^2-368)}{225}$$

input
```
int(x^5*log(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
(sqrt(x**2 - 1)*(45*log( - x**2 + 1)*x**4 + 60*log( - x**2 + 1)*x**2 + 120
*log( - x**2 + 1) - 18*x**4 - 64*x**2 - 368))/225
```

3.651 $\int \frac{x^3 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx = -2\sqrt{-1+x^2} - \frac{2}{9}\left(-1+x^2\right)^{3/2} + \sqrt{-1+x^2}\log\left(1-x^2\right) + \frac{1}{3}\left(-1+x^2\right)^{3/2}\log\left(1-x^2\right)$$

output
```
-2*(x^2-1)^(1/2)-2/9*(x^2-1)^(3/2)+(x^2-1)^(1/2)*ln(-x^2+1)+1/3*(x^2-1)^(3
/2)*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx = \frac{1}{9}\sqrt{-1+x^2}\left(-2\left(8+x^2\right)+3\left(2+x^2\right)\log\left(1-x^2\right)\right)$$

input
```
Integrate[(x^3*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
(Sqrt[-1 + x^2]*(-2*(8 + x^2) + 3*(2 + x^2)*Log[1 - x^2]))/9
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \log (1-x^2)}{\sqrt{x^2-1}} \, dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^3 \log (1-x^2)}{\sqrt{x^2-1}} dx$$

input
```
Int[(x^3*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{(x^2+2)\sqrt{x^2-1}\,\ln(-x^2+1)}{3}-\frac{2(x^2+8)\sqrt{x^2-1}}{9}$	38
orering	$\frac{(x-1)(1+x)(5x^4+19x^2-24)\ln(-x^2+1)}{9x^2\sqrt{x^2-1}}-\frac{(x^2+8)(x-1)^2(1+x)^2\left(\frac{3x^2\ln(-x^2+1)}{\sqrt{x^2-1}}-\frac{2x^4}{(-x^2+1)\sqrt{x^2-1}}-\frac{x^4\ln(-x^2+1)}{(x^2-1)^{\frac{3}{2}}}\right)}{9x^4}$	122

input
```
int(x^3*ln(-x^2+1)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+2)*(x^2-1)^(1/2)*ln(-x^2+1)-2/9*(x^2+8)*(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.48

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx = -\frac{1}{9}\left(2\,x^2 - 3\left(x^2+2\right)\log\left(-x^2+1\right) + 16\right)\sqrt{x^2-1}$$

input
```
integrate(x^3*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(2*x^2 - 3*(x^2 + 2)*log(-x^2 + 1) + 16)*sqrt(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{-1+x^2}}\, dx = \frac{x^2\sqrt{x^2-1}\log(1-x^2)}{3} - \frac{2x^2\sqrt{x^2-1}}{9} + \frac{2\sqrt{x^2-1}\log(1-x^2)}{3} - \frac{16\sqrt{x^2-1}}{9}$$

input
```
integrate(x**3*ln(-x**2+1)/(x**2-1)**(1/2),x)
```

output
```
x**2*sqrt(x**2 - 1)*log(1 - x**2)/3 - 2*x**2*sqrt(x**2 - 1)/9 + 2*sqrt(x**
2 - 1)*log(1 - x**2)/3 - 16*sqrt(x**2 - 1)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \frac{1}{3}\left(\sqrt{x^2-1}x^2 + 2\sqrt{x^2-1}\right)\log\left(-x^2+1\right) - \frac{2}{9}\left(x^2-1\right)^{\frac{3}{2}} - 2\sqrt{x^2-1}$$

input
```
integrate(x^3*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(sqrt(x^2 - 1)*x^2 + 2*sqrt(x^2 - 1))*log(-x^2 + 1) - 2/9*(x^2 - 1)^(3
/2) - 2*sqrt(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{x^3 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \frac{1}{3}\left(\left(x^2-1\right)^{\frac{3}{2}} + 3\sqrt{x^2-1}\right)\log\left(-x^2+1\right) - \frac{2}{9}\left(x^2-1\right)^{\frac{3}{2}} - 2\sqrt{x^2-1}$$

input
```
integrate(x^3*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
1/3*((x^2 - 1)^(3/2) + 3*sqrt(x^2 - 1))*log(-x^2 + 1) - 2/9*(x^2 - 1)^(3/2
) - 2*sqrt(x^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^3\ln(1-x^2)}{\sqrt{x^2-1}}\,dx$$

input
```
int((x^3*log(1 - x^2))/(x^2 - 1)^(1/2),x)
```

output
```
int((x^3*log(1 - x^2))/(x^2 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

$$\int \frac{x^3\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\,(3\log(-x^2+1)\,x^2+6\log(-x^2+1)-2x^2-16)}{9}$$

input
```
int(x^3*log(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
(sqrt(x**2 - 1)*(3*log( - x**2 + 1)*x**2 + 6*log( - x**2 + 1) - 2*x**2 - 1
6))/9
```

3.652 $\int \frac{x \log(1-x^2)}{\sqrt{-1+x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{x \log(1-x^2)}{\sqrt{-1+x^2}} \, dx = -2\sqrt{-1+x^2} + \sqrt{-1+x^2} \log(1-x^2)$$

output
```
-2*(x^2-1)^(1/2)+(x^2-1)^(1/2)*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x \log(1-x^2)}{\sqrt{-1+x^2}} \, dx = \sqrt{-1+x^2}\left(-2+\log(1-x^2)\right)$$

input
```
Integrate[(x*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
Sqrt[-1 + x^2]*(-2 + Log[1 - x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \log (1-x^2)}{\sqrt{x^2-1}} \, dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x \log (1-x^2)}{\sqrt{x^2-1}} dx$$

input
```
Int[(x*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result	size
risch	$-2\sqrt{x^2-1}+\sqrt{x^2-1}\,\ln\left(-x^2+1\right)$	27
orering	$\frac{\sqrt{x^2-1}\,(x-1)(1+x)\ln(-x^2+1)}{x^2}-\frac{(x-1)^2(1+x)^2\left(\frac{\ln\left(-x^2+1\right)}{\sqrt{x^2-1}}-\frac{2x^2}{\left(-x^2+1\right)\sqrt{x^2-1}}-\frac{x^2\ln\left(-x^2+1\right)}{\left(x^2-1\right)^{\frac{3}{2}}}\right)}{x^2}$	100

input
```
int(x*ln(-x^2+1)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(x^2-1)^(1/2)+(x^2-1)^(1/2)*ln(-x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\left(\log\left(-x^2+1\right)-2\right)$$

input
```
integrate(x*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 - 1)*(log(-x^2 + 1) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\log\left(1-x^2\right)-2\sqrt{x^2-1}$$

input
```
integrate(x*ln(-x**2+1)/(x**2-1)**(1/2),x)
```

output
```
sqrt(x**2 - 1)*log(1 - x**2) - 2*sqrt(x**2 - 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{x\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\log\left(-x^2+1\right)-2\sqrt{x^2-1}$$

input
```
integrate(x*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1)*log(-x^2 + 1) - 2*sqrt(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{x\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\log(-x^2+1) - 2\sqrt{x^2-1}$$

input
```
integrate(x*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 - 1)*log(-x^2 + 1) - 2*sqrt(x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \frac{x\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\left(\ln(1-x^2)-2\right)$$

input
```
int((x*log(1 - x^2))/(x^2 - 1)^(1/2),x)
```

output
```
(x^2 - 1)^(1/2)*(log(1 - x^2) - 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \frac{x\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \sqrt{x^2-1}\left(\log(-x^2+1)-2\right)$$

input
```
int(x*log(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
sqrt(x**2 - 1)*(log( - x**2 + 1) - 2)
```

3.653 $\int \frac{\log(1-x^2)}{x\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\log(1-x^2)}{x\sqrt{-1+x^2}}\,dx = \arctan\left(\sqrt{-1+x^2}\right)\log\left(1-x^2\right) - i\,\text{PolyLog}\left(2,-i\sqrt{-1+x^2}\right) + i\,\text{PolyLog}\left(2,i\sqrt{-1+x^2}\right)$$

output
```
arctan((x^2-1)^(1/2))*ln(-x^2+1)-I*polylog(2,-I*(x^2-1)^(1/2))+I*polylog(2
,I*(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.81

$$\int \frac{\log(1-x^2)}{x\sqrt{-1+x^2}}\,dx = -\frac{1}{2}i\log\left(1-x^2\right)\log\left(-i\left(i-\sqrt{-1+x^2}\right)\right) + \frac{1}{2}i\log\left(1-x^2\right)\log\left(-i\left(i+\sqrt{-1+x^2}\right)\right) - i\,\text{PolyLog}\left(2,-i\sqrt{-1+x^2}\right) + i\,\text{PolyLog}\left(2,i\sqrt{-1+x^2}\right)$$

input
```
Integrate[Log[1 - x^2]/(x*Sqrt[-1 + x^2]),x]
```

output
```
(-1/2*I)*Log[1 - x^2]*Log[(-I)*(I - Sqrt[-1 + x^2])] + (I/2)*Log[1 - x^2]*
Log[(-I)*(I + Sqrt[-1 + x^2])] - I*PolyLog[2, (-I)*Sqrt[-1 + x^2]] + I*Pol
yLog[2, I*Sqrt[-1 + x^2]]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(1-x^2\right)}{x \sqrt{x^2-1}} \, dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{\log \left(1-x^2\right)}{x \sqrt{x^2-1}}dx$$

input
```
Int[Log[1 - x^2]/(x*Sqrt[-1 + x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x\sqrt{x^2-1}}dx$$

input
```
int(ln(-x^2+1)/x/(x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x/(x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log(1-x^2)}{x\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}x}\,dx$$

input
```
integrate(log(-x^2+1)/x/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 - 1)*log(-x^2 + 1)/(x^3 - x), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x\sqrt{-1+x^2}}\,dx = \int \frac{\log(1-x^2)}{x\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x/(x**2-1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x*sqrt((x - 1)*(x + 1))), x)
```

Maxima [F]

$$\int \frac{\log(1-x^2)}{x\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}x}\,dx$$

input
```
integrate(log(-x^2+1)/x/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(x^2 - 1)*x), x)
```

Giac [F]

$$\int \frac{\log(1-x^2)}{x\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}x}\,dx$$

input
```
integrate(log(-x^2+1)/x/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(x^2 - 1)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x\sqrt{-1+x^2}}\,dx = \int \frac{\ln(1-x^2)}{x\sqrt{x^2-1}}\,dx$$

input
```
int(log(1 - x^2)/(x*(x^2 - 1)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x*(x^2 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\log (1-x^2)}{x\sqrt{-1+x^2}}\, dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}\, x} dx$$

input
```
int(log(-x^2+1)/x/(x^2-1)^(1/2),x)
```

output
```
int(log( - x**2 + 1)/(sqrt(x**2 - 1)*x),x)
```

3.654 $\int \frac{\log(1-x^2)}{x^3\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\begin{aligned}\int \frac{\log(1-x^2)}{x^3\sqrt{-1+x^2}}\,dx = &-\arctan\left(\sqrt{-1+x^2}\right)+\frac{\sqrt{-1+x^2}\log(1-x^2)}{2x^2}\\&+\frac{1}{2}\arctan\left(\sqrt{-1+x^2}\right)\log(1-x^2)\\&-\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-i\sqrt{-1+x^2}\right)+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,i\sqrt{-1+x^2}\right)\end{aligned}$$

output
```
-arctan((x^2-1)^(1/2))+1/2*(x^2-1)^(1/2)*ln(-x^2+1)/x^2+1/2*arctan((x^2-1)
^(1/2))*ln(-x^2+1)-1/2*I*polylog(2,-I*(x^2-1)^(1/2))+1/2*I*polylog(2,I*(x^
2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.57

$$\int \frac{\log(1-x^2)}{x^3\sqrt{-1+x^2}}\,dx = \frac{1}{4}\left(\frac{2\sqrt{-1+x^2}\log(1-x^2)}{x^2} + 2i\log\left(i-\sqrt{-1+x^2}\right) + i\log(1-x^2)\log\left(1-i\sqrt{-1+x^2}\right) - i\log(1-x^2)\log\left(1+i\sqrt{-1+x^2}\right) - 2i\log\left(i+\sqrt{-1+x^2}\right) - 2i\,\mathrm{PolyLog}\left(2,-i\sqrt{-1+x^2}\right) + 2i\,\mathrm{PolyLog}\left(2,i\sqrt{-1+x^2}\right)\right)$$

input
```
Integrate[Log[1 - x^2]/(x^3*Sqrt[-1 + x^2]),x]
```

output
```
((2*Sqrt[-1 + x^2]*Log[1 - x^2])/x^2 + (2*I)*Log[I - Sqrt[-1 + x^2]] + I*L
og[1 - x^2]*Log[1 - I*Sqrt[-1 + x^2]] - I*Log[1 - x^2]*Log[1 + I*Sqrt[-1 +
 x^2]] - (2*I)*Log[I + Sqrt[-1 + x^2]] - (2*I)*PolyLog[2, (-I)*Sqrt[-1 + x
^2]] + (2*I)*PolyLog[2, I*Sqrt[-1 + x^2]])/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^3\sqrt{x^2-1}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{\log(1-x^2)}{x^3\sqrt{x^2-1}}dx$$

input
```
Int[Log[1 - x^2]/(x^3*Sqrt[-1 + x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^3\sqrt{x^2-1}}dx$$

input
```
int(ln(-x^2+1)/x^3/(x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^3/(x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log(1-x^2)}{x^3\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}x^3}\,dx$$

input
```
integrate(log(-x^2+1)/x^3/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 - 1)*log(-x^2 + 1)/(x^5 - x^3), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^3\sqrt{-1+x^2}}\,dx = \int \frac{\log(1-x^2)}{x^3\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**3/(x**2-1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**3*sqrt((x - 1)*(x + 1))), x)
```

Maxima [F]

$$\int \frac{\log(1-x^2)}{x^3\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}x^3}\,dx$$

input
```
integrate(log(-x^2+1)/x^3/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(x^2 - 1)*x^3), x)
```

Giac [F]

$$\int \frac{\log(1-x^2)}{x^3\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}x^3}\,dx$$

input
```
integrate(log(-x^2+1)/x^3/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(x^2 - 1)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^3\sqrt{-1+x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^3\sqrt{x^2-1}}\,dx$$

input
```
int(log(1 - x^2)/(x^3*(x^2 - 1)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^3*(x^2 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{x^3\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}\,x^3}dx$$

input
```
int(log(-x^2+1)/x^3/(x^2-1)^(1/2),x)
```

output
```
int(log( - x**2 + 1)/(sqrt(x**2 - 1)*x**3),x)
```

3.655 $\int \frac{\log(1-x^2)}{x^5\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\begin{aligned}\int \frac{\log(1-x^2)}{x^5\sqrt{-1+x^2}}\,dx = &-\frac{\sqrt{-1+x^2}}{4x^2}-\arctan\left(\sqrt{-1+x^2}\right)+\frac{\sqrt{-1+x^2}\log(1-x^2)}{4x^4}\\ &+\frac{3\sqrt{-1+x^2}\log(1-x^2)}{8x^2}+\frac{3}{8}\arctan\left(\sqrt{-1+x^2}\right)\log(1-x^2)\\ &-\frac{3}{8}i\,\mathrm{PolyLog}\left(2,-i\sqrt{-1+x^2}\right)+\frac{3}{8}i\,\mathrm{PolyLog}\left(2,i\sqrt{-1+x^2}\right)\end{aligned}$$

output

```
-1/4*(x^2-1)^(1/2)/x^2-arctan((x^2-1)^(1/2))+1/4*(x^2-1)^(1/2)*ln(-x^2+1)/
x^4+3/8*(x^2-1)^(1/2)*ln(-x^2+1)/x^2+3/8*arctan((x^2-1)^(1/2))*ln(-x^2+1)-
3/8*I*polylog(2,-I*(x^2-1)^(1/2))+3/8*I*polylog(2,I*(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.38

$$\int \frac{\log(1-x^2)}{x^5\sqrt{-1+x^2}}\,dx = \frac{1}{16}\left(-\frac{4\sqrt{-1+x^2}}{x^2}+\frac{4\sqrt{-1+x^2}\log(1-x^2)}{x^4}+\frac{6\sqrt{-1+x^2}\log(1-x^2)}{x^2}+8i\log\left(i-\sqrt{-1+x^2}\right)+3i\log(1-x^2)\log\left(1-i\sqrt{-1+x^2}\right)-3i\log(1-x^2)\log\left(1+i\sqrt{-1+x^2}\right)-8i\log\left(i+\sqrt{-1+x^2}\right)-6i\,\text{PolyLog}\left(2,-i\sqrt{-1+x^2}\right)+6i\,\text{PolyLog}\left(2,i\sqrt{-1+x^2}\right)\right)$$

input
```
Integrate[Log[1 - x^2]/(x^5*Sqrt[-1 + x^2]),x]
```

output
```
((-4*Sqrt[-1 + x^2])/x^2 + (4*Sqrt[-1 + x^2]*Log[1 - x^2])/x^4 + (6*Sqrt[-
1 + x^2]*Log[1 - x^2])/x^2 + (8*I)*Log[I - Sqrt[-1 + x^2]] + (3*I)*Log[1 -
 x^2]*Log[1 - I*Sqrt[-1 + x^2]] - (3*I)*Log[1 - x^2]*Log[1 + I*Sqrt[-1 + x
^2]] - (8*I)*Log[I + Sqrt[-1 + x^2]] - (6*I)*PolyLog[2, (-I)*Sqrt[-1 + x^2
]] + (6*I)*PolyLog[2, I*Sqrt[-1 + x^2]])/16
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^5\sqrt{x^2-1}}\,dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{\log(1-x^2)}{x^5\sqrt{x^2-1}}\,dx$$

input
```
Int[Log[1 - x^2]/(x^5*Sqrt[-1 + x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln \left(-x^{2}+1\right)}{x^{5} \sqrt{x^{2}-1}}dx$$

input
```
int(ln(-x^2+1)/x^5/(x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^5/(x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log (1-x^2)}{x^5\sqrt{-1+x^2}}\, dx = \int \frac{\log \left(-x^{2}+1\right)}{\sqrt{x^{2}-1}x^{5}}\, dx$$

input
```
integrate(log(-x^2+1)/x^5/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 - 1)*log(-x^2 + 1)/(x^7 - x^5), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^5\sqrt{-1+x^2}}\,dx = \int \frac{\log(1-x^2)}{x^5\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**5/(x**2-1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**5*sqrt((x - 1)*(x + 1))), x)
```

Maxima [F]

$$\int \frac{\log(1-x^2)}{x^5\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}x^5}\,dx$$

input
```
integrate(log(-x^2+1)/x^5/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(x^2 - 1)*x^5), x)
```

Giac [F]

$$\int \frac{\log(1-x^2)}{x^5\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}x^5}\,dx$$

input
```
integrate(log(-x^2+1)/x^5/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/(sqrt(x^2 - 1)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^5\sqrt{-1+x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^5\,\sqrt{x^2-1}}\,dx$$

input
```
int(log(1 - x^2)/(x^5*(x^2 - 1)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^5*(x^2 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{x^5\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}\,x^5}dx$$

input
```
int(log(-x^2+1)/x^5/(x^2-1)^(1/2),x)
```

output
```
int(log( - x**2 + 1)/(sqrt(x**2 - 1)*x***5),x)
```

3.656 $\int \frac{x^4 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 280

$$\begin{aligned}\int \frac{x^4 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx &= \frac{1}{128\left(x+\sqrt{-1+x^2}\right)^4}+\frac{5}{32\left(x+\sqrt{-1+x^2}\right)^2}\\
&\quad-\frac{5}{32}\left(x+\sqrt{-1+x^2}\right)^2-\frac{1}{128}\left(x+\sqrt{-1+x^2}\right)^4\\
&\quad-\frac{\log(1-x^2)}{64\left(x+\sqrt{-1+x^2}\right)^4}-\frac{\log(1-x^2)}{8\left(x+\sqrt{-1+x^2}\right)^2}\\
&\quad+\frac{1}{8}\left(x+\sqrt{-1+x^2}\right)^2\log(1-x^2)\\
&\quad+\frac{1}{64}\left(x+\sqrt{-1+x^2}\right)^4\log(1-x^2)-\frac{9}{16}\log\left(x+\sqrt{-1+x^2}\right)\\
&\quad+\frac{3}{8}\log(1-x^2)\log\left(x+\sqrt{-1+x^2}\right)+\frac{3}{8}\log^2\left(x+\sqrt{-1+x^2}\right)\\
&\quad-\frac{3}{4}\log\left(x+\sqrt{-1+x^2}\right)\log\left(1-\left(x+\sqrt{-1+x^2}\right)^2\right)\\
&\quad-\frac{3}{8}\text{PolyLog}\left(2,\left(x+\sqrt{-1+x^2}\right)^2\right)\end{aligned}$$

output
```
1/128/(x+(x^2-1)^(1/2))^4+5/32/(x+(x^2-1)^(1/2))^2-5/32*(x+(x^2-1)^(1/2))^
2-1/128*(x+(x^2-1)^(1/2))^4-1/64*ln(-x^2+1)/(x+(x^2-1)^(1/2))^4-1/8*ln(-x^
2+1)/(x+(x^2-1)^(1/2))^2+1/8*(x+(x^2-1)^(1/2))^2*ln(-x^2+1)+1/64*(x+(x^2-1
)^(1/2))^4*ln(-x^2+1)-9/16*ln(x+(x^2-1)^(1/2))+3/8*ln(-x^2+1)*ln(x+(x^2-1)
^(1/2))+3/8*ln(x+(x^2-1)^(1/2))^2-3/4*ln(x+(x^2-1)^(1/2))*ln(1-(x+(x^2-1)^
(1/2))^2)-3/8*polylog(2,(x+(x^2-1)^(1/2))^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.85

$$\int \frac{x^4 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = -\frac{x\sqrt{1-x^2}\left(-2(x^2)^{3/2}\sqrt{1-x^2}-9\sqrt{x^2-x^4}-6i\arcsin\left(\sqrt{1-x^2}\right)^2-3\arcsin\left(\sqrt{1-x^2}\right)\left(3+4\log\left(\right.\right.\right.}{}$$

input
```
Integrate[(x^4*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
-1/16*(x*Sqrt[1 - x^2]*(-2*(x^2)^(3/2)*Sqrt[1 - x^2] - 9*Sqrt[x^2 - x^4] -
 (6*I)*ArcSin[Sqrt[1 - x^2]]^2 - 3*ArcSin[Sqrt[1 - x^2]]*(3 + 4*Log[1 - E^
((-2*I)*ArcSin[Sqrt[1 - x^2]])]) + 4*(x^2)^(3/2)*Sqrt[1 - x^2]*Log[1 - x^2
] + 6*Sqrt[x^2 - x^4]*Log[1 - x^2] - (6*I)*Log[1 - x^2]*Log[Sqrt[x^2] + I*
Sqrt[1 - x^2]] - (6*I)*PolyLog[2, E^((-2*I)*ArcSin[Sqrt[1 - x^2]])]))/(Sqr
t[x^2]*Sqrt[-1 + x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \log(1-x^2)}{\sqrt{x^2-1}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{x^4 \log (1-x^2)}{\sqrt{x^2-1}}dx$$

input
```
Int[(x^4*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^4 \ln (-x^2+1)}{\sqrt{x^2-1}}dx$$

input
```
int(x^4*ln(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
int(x^4*ln(-x^2+1)/(x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4 \log (1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^4 \log (-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(x^4*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^4*log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^4 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^4 \log(1-x^2)}{\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(x**4*ln(-x**2+1)/(x**2-1)**(1/2),x)
```

output
```
Integral(x**4*log(1 - x**2)/sqrt((x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^4 \log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(x^4*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4*log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Giac [F]

$$\int \frac{x^4 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^4 \log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(x^4*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4*log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^4 \ln(1-x^2)}{\sqrt{x^2-1}}\,dx$$

input
```
int((x^4*log(1 - x^2))/(x^2 - 1)^(1/2),x)
```

output
```
int((x^4*log(1 - x^2))/(x^2 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)\,x^4}{\sqrt{x^2-1}}dx$$

input
```
int(x^4*log(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
int((log( - x**2 + 1)*x**4)/sqrt(x**2 - 1),x)
```

3.657 $\int \frac{x^2 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 194

$$\begin{aligned}\int \frac{x^2 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx &= \frac{1}{8\left(x+\sqrt{-1+x^2}\right)^2}-\frac{1}{8}\left(x+\sqrt{-1+x^2}\right)^2-\frac{\log(1-x^2)}{8\left(x+\sqrt{-1+x^2}\right)^2}\\&\quad+\frac{1}{8}\left(x+\sqrt{-1+x^2}\right)^2\log(1-x^2)-\frac{1}{2}\log\left(x+\sqrt{-1+x^2}\right)\\&\quad+\frac{1}{2}\log(1-x^2)\log\left(x+\sqrt{-1+x^2}\right)+\frac{1}{2}\log^2\left(x+\sqrt{-1+x^2}\right)\\&\quad-\log\left(x+\sqrt{-1+x^2}\right)\log\left(1-\left(x+\sqrt{-1+x^2}\right)^2\right)\\&\quad-\frac{1}{2}\operatorname{PolyLog}\left(2,\left(x+\sqrt{-1+x^2}\right)^2\right)\end{aligned}$$

output
```
1/8/(x+(x^2-1)^(1/2))^2-1/8*(x+(x^2-1)^(1/2))^2-1/8*ln(-x^2+1)/(x+(x^2-1)^
(1/2))^2+1/8*(x+(x^2-1)^(1/2))^2*ln(-x^2+1)-1/2*ln(x+(x^2-1)^(1/2))+1/2*ln
(-x^2+1)*ln(x+(x^2-1)^(1/2))+1/2*ln(x+(x^2-1)^(1/2))^2-ln(x+(x^2-1)^(1/2))
*ln(1-(x+(x^2-1)^(1/2))^2)-1/2*polylog(2,(x+(x^2-1)^(1/2))^2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.97

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{-1+x^2}} \, dx = -\frac{x\sqrt{1-x^2}\left(-\sqrt{x^2-x^4} - i\arcsin\left(\sqrt{1-x^2}\right)^2 - \arcsin\left(\sqrt{1-x^2}\right)\left(1+2\log\left(1-e^{-2i\arcsin\left(\sqrt{1-x^2}\right)}\right)\right)\right.}{2\sqrt{x^2}\sqrt{}}$$

input
```
Integrate[(x^2*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
-1/2*(x*Sqrt[1 - x^2]*(-Sqrt[x^2 - x^4] - I*ArcSin[Sqrt[1 - x^2]]^2 - ArcS
in[Sqrt[1 - x^2]]*(1 + 2*Log[1 - E^((-2*I)*ArcSin[Sqrt[1 - x^2]])]) + Sqrt
[x^2 - x^4]*Log[1 - x^2] - I*Log[1 - x^2]*Log[Sqrt[x^2] + I*Sqrt[1 - x^2]]
 - I*PolyLog[2, E^((-2*I)*ArcSin[Sqrt[1 - x^2]])]))/(Sqrt[x^2]*Sqrt[-1 + x
^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{x^2-1}} \, dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{x^2-1}} dx$$

input
```
Int[(x^2*Log[1 - x^2])/Sqrt[-1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^2 \ln(-x^2+1)}{\sqrt{x^2-1}}dx$$

input
```
int(x^2*ln(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
int(x^2*ln(-x^2+1)/(x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^2 \log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(x^2*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2*log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^2 \log(1-x^2)}{\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(x**2*ln(-x**2+1)/(x**2-1)**(1/2),x)
```

output
```
Integral(x**2*log(1 - x**2)/sqrt((x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^2 \log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(x^2*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Giac [F]

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^2 \log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(x^2*log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{x^2 \ln(1-x^2)}{\sqrt{x^2-1}}\,dx$$

input
```
int((x^2*log(1 - x^2))/(x^2 - 1)^(1/2),x)
```

output
```
int((x^2*log(1 - x^2))/(x^2 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \log (1 - x^2)}{\sqrt{-1 + x^2}} \, dx = \int \frac{\log(-x^2 + 1) \, x^2}{\sqrt{x^2 - 1}} dx$$

input
```
int(x^2*log(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
int((log( - x**2 + 1)*x**2)/sqrt(x**2 - 1),x)
```

3.658 $\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \log(1-x^2)\log\left(x+\sqrt{-1+x^2}\right)+\log^2\left(x+\sqrt{-1+x^2}\right) - 2\log\left(x+\sqrt{-1+x^2}\right)\log\left(1-\left(x+\sqrt{-1+x^2}\right)^2\right) - \text{PolyLog}\left(2,\left(x+\sqrt{-1+x^2}\right)^2\right)$$

output
```
ln(-x^2+1)*ln(x+(x^2-1)^(1/2))+ln(x+(x^2-1)^(1/2))^2-2*ln(x+(x^2-1)^(1/2))
*ln(1-(x+(x^2-1)^(1/2))^2)-polylog(2,(x+(x^2-1)^(1/2))^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \frac{i\sqrt{1-x^2}\left(\arcsin\left(\sqrt{1-x^2}\right)^2-2i\arcsin\left(\sqrt{1-x^2}\right)\log\left(1-e^{-2i\arcsin\left(\sqrt{1-x^2}\right)}\right)+\log(1-x^2)\log\left(\sqrt{x^2}\right.\right.}{\sqrt{1-\frac{1}{x^2}}\,x}$$

input
```
Integrate[Log[1 - x^2]/Sqrt[-1 + x^2],x]
```

output
```
(I*Sqrt[1 - x^2]*(ArcSin[Sqrt[1 - x^2]]^2 - (2*I)*ArcSin[Sqrt[1 - x^2]]*Lo
g[1 - E^((-2*I)*ArcSin[Sqrt[1 - x^2]])] + Log[1 - x^2]*Log[Sqrt[x^2] + I*S
qrt[1 - x^2]] + PolyLog[2, E^((-2*I)*ArcSin[Sqrt[1 - x^2]])]))/(Sqrt[1 - x
^(-2)]*x)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{\sqrt{x^2-1}}\,dx$$
$$\Big\downarrow \ 2923$$
$$\int \frac{\log(1-x^2)}{\sqrt{x^2-1}}dx$$

input
```
Int[Log[1 - x^2]/Sqrt[-1 + x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{\sqrt{x^2-1}}dx$$

input
```
int(ln(-x^2+1)/(x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/(x^2-1)^(1/2),x)
```

Fricas [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(1-x^2)}{\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/(x**2-1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/sqrt((x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Giac [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}}\,dx$$

input
```
integrate(log(-x^2+1)/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log(-x^2 + 1)/sqrt(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\ln(1-x^2)}{\sqrt{x^2-1}}\,dx$$

input
```
int(log(1 - x^2)/(x^2 - 1)^(1/2),x)
```

output
```
int(log(1 - x^2)/(x^2 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{\log(1-x^2)}{\sqrt{-1+x^2}}\,dx = \int \frac{\log(-x^2+1)}{\sqrt{x^2-1}}dx$$

input

```
int(log(-x^2+1)/(x^2-1)^(1/2),x)
```

output

```
int(log( - x**2 + 1)/sqrt(x**2 - 1),x)
```

3.659 $\int \frac{\log(1-x^2)}{x^2\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{\log(1-x^2)}{x^2\sqrt{-1+x^2}}\,dx = -2\text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)+\frac{\sqrt{-1+x^2}\log(1-x^2)}{x}$$

output
```
-2*arctanh(x/(x^2-1)^(1/2))+(x^2-1)^(1/2)*ln(-x^2+1)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\log(1-x^2)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{-1+x^2}\log(1-x^2)}{x}-2\log\left(x+\sqrt{-1+x^2}\right)$$

input
```
Integrate[Log[1 - x^2]/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
(Sqrt[-1 + x^2]*Log[1 - x^2])/x - 2*Log[x + Sqrt[-1 + x^2]]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^2\sqrt{x^2-1}}\,dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{\log(1-x^2)}{x^2\sqrt{x^2-1}}dx$$

input
```
Int[Log[1 - x^2]/(x^2*Sqrt[-1 + x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^2\sqrt{x^2-1}}dx$$

input
```
int(ln(-x^2+1)/x^2/(x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^2/(x^2-1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\log (1-x^2)}{x^2\sqrt{-1+x^2}}\,dx = \frac{2\,x\log \left(-x+\sqrt{x^2-1}\right)+\sqrt{x^2-1}\log \left(-x^2+1\right)}{x}$$

input
```
integrate(log(-x^2+1)/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
(2*x*log(-x + sqrt(x^2 - 1)) + sqrt(x^2 - 1)*log(-x^2 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{\log (1-x^2)}{x^2\sqrt{-1+x^2}}\,dx = \left(\begin{cases} \frac{\sqrt{x^2-1}}{x} & \text{for } x>-1\wedge x<1 \end{cases}\right)\log \left(1-x^2\right) - 2\left(\begin{cases} \text{NaN} & \text{for } x<-1 \\ \log \left(x+\sqrt{x^2-1}\right) & \text{for } x<1 \\ \text{NaN} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(ln(-x**2+1)/x**2/(x**2-1)**(1/2),x)
```

output
```
Piecewise((sqrt(x**2 - 1)/x, (x > -1) & (x < 1)))*log(1 - x**2) - 2*Piecew
ise((nan, x < -1), (log(x + sqrt(x**2 - 1)), x < 1), (nan, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\log (1-x^2)}{x^2\sqrt{-1+x^2}}\,dx = \frac{\sqrt{x^2-1}\log \left(-x^2+1\right)}{x} + 2\,\log \left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate(log(-x^2+1)/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1)*log(-x^2 + 1)/x + 2*log(-x + sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{\log(1-x^2)}{x^2\sqrt{-1+x^2}}\,dx = \frac{2\log(-x^2+1)}{\left(x-\sqrt{x^2-1}\right)^2+1} - \log(|x+1|) - \log(|x-1|) + 2\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate(log(-x^2+1)/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
2*log(-x^2 + 1)/((x - sqrt(x^2 - 1))^2 + 1) - log(abs(x + 1)) - log(abs(x
- 1)) + 2*log(abs(-x + sqrt(x^2 - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^2\sqrt{-1+x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^2\sqrt{x^2-1}}\,dx$$

input
```
int(log(1 - x^2)/(x^2*(x^2 - 1)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^2*(x^2 - 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{\log(1-x^2)}{x^2\sqrt{-1+x^2}}\,dx$$
$$= \frac{\sqrt{x^2-1}\log(-x^2+1)+\log(-x^2+1)\,x-2\log\left(\sqrt{x^2-1}+x-1\right)x-2\log\left(\sqrt{x^2-1}+x+1\right)x}{x}$$

input
```
int(log(-x^2+1)/x^2/(x^2-1)^(1/2),x)
```

output
```
(sqrt(x**2 - 1)*log( - x**2 + 1) + log( - x**2 + 1)*x - 2*log(sqrt(x**2 -
1) + x - 1)*x - 2*log(sqrt(x**2 - 1) + x + 1)*x)/x
```

3.660 $\int \frac{\log(1-x^2)}{x^4\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\log(1-x^2)}{x^4\sqrt{-1+x^2}}\,dx = -\frac{2\sqrt{-1+x^2}}{3x} - \frac{4}{3}\text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right) + \frac{\sqrt{-1+x^2}\log(1-x^2)}{3x^3} + \frac{2\sqrt{-1+x^2}\log(1-x^2)}{3x}$$

output
```
-2/3*(x^2-1)^(1/2)/x-4/3*arctanh(x/(x^2-1)^(1/2))+1/3*(x^2-1)^(1/2)*ln(-x^
2+1)/x^3+2/3*(x^2-1)^(1/2)*ln(-x^2+1)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{\log(1-x^2)}{x^4\sqrt{-1+x^2}}\,dx = -\frac{2\sqrt{-1+x^2}}{3x} + \frac{\sqrt{-1+x^2}(1+2x^2)\log(1-x^2)}{3x^3} - \frac{4}{3}\log\left(x+\sqrt{-1+x^2}\right)$$

input
```
Integrate[Log[1 - x^2]/(x^4*Sqrt[-1 + x^2]),x]
```

output
```
(-2*Sqrt[-1 + x^2])/(3*x) + (Sqrt[-1 + x^2]*(1 + 2*x^2)*Log[1 - x^2])/(3*x
^3) - (4*Log[x + Sqrt[-1 + x^2]])/3
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^4\sqrt{x^2-1}}\,dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{\log(1-x^2)}{x^4\sqrt{x^2-1}}dx$$

input
```
Int[Log[1 - x^2]/(x^4*Sqrt[-1 + x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^4\sqrt{x^2-1}}dx$$

input
```
int(ln(-x^2+1)/x^4/(x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^4/(x^2-1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{\log(1-x^2)}{x^4\sqrt{-1+x^2}}\,dx = \frac{4\,x^3\log\left(-x+\sqrt{x^2-1}\right)-2\,x^3-\left(2\,x^2-\left(2\,x^2+1\right)\log\left(-x^2+1\right)\right)\sqrt{x^2-1}}{3\,x^3}$$

input
```
integrate(log(-x^2+1)/x^4/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(4*x^3*log(-x + sqrt(x^2 - 1)) - 2*x^3 - (2*x^2 - (2*x^2 + 1)*log(-x^2
 + 1))*sqrt(x^2 - 1))/x^3
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^4\sqrt{-1+x^2}}\,dx = \int \frac{\log(1-x^2)}{x^4\sqrt{(x-1)\,(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**4/(x**2-1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**4*sqrt((x - 1)*(x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{\log(1-x^2)}{x^4\sqrt{-1+x^2}}\,dx = \frac{1}{3}\left(\frac{2\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2-1}}{x^3}\right)\log\left(-x^2+1\right) - \frac{2\sqrt{x+1}\sqrt{x-1}}{3x} + \frac{4}{3}\log\left(-\frac{1}{3}x + \frac{1}{3}\sqrt{x^2-1}\right)$$

input
```
integrate(log(-x^2+1)/x^4/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(2*sqrt(x^2 - 1)/x + sqrt(x^2 - 1)/x^3)*log(-x^2 + 1) - 2/3*sqrt(x + 1
)*sqrt(x - 1)/x + 4/3*log(-1/3*x + 1/3*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{\log(1-x^2)}{x^4\sqrt{-1+x^2}}\,dx = -\frac{4}{3\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)} + \frac{4\left(3\left(x-\sqrt{x^2-1}\right)^2+1\right)\log\left(-x^2+1\right)}{3\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^3} + \frac{2}{3}\log\left(\left(x-\sqrt{x^2-1}\right)^2\right) - \frac{2}{3}\log\left(|x+1|\right) - \frac{2}{3}\log\left(|x-1|\right)$$

input
```
integrate(log(-x^2+1)/x^4/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
-4/3/((x - sqrt(x^2 - 1))^2 + 1) + 4/3*(3*(x - sqrt(x^2 - 1))^2 + 1)*log(-
x^2 + 1)/((x - sqrt(x^2 - 1))^2 + 1)^3 + 2/3*log((x - sqrt(x^2 - 1))^2) -
2/3*log(abs(x + 1)) - 2/3*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^4\sqrt{-1+x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^4\,\sqrt{x^2-1}}\,dx$$

input
```
int(log(1 - x^2)/(x^4*(x^2 - 1)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^4*(x^2 - 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23

$$\int \frac{\log(1-x^2)}{x^4\sqrt{-1+x^2}}\,dx = \frac{6\sqrt{x^2-1}\log(-x^2+1)\,x^2+3\sqrt{x^2-1}\log(-x^2+1)-6\sqrt{x^2-1}\,x^2+6\log(-x^2+1)\,x^3-12\log\left(\sqrt{x^2-\right.}{9x^3}$$

input
```
int(log(-x^2+1)/x^4/(x^2-1)^(1/2),x)
```

output
```
(6*sqrt(x**2 - 1)*log( - x**2 + 1)*x**2 + 3*sqrt(x**2 - 1)*log( - x**2 + 1
) - 6*sqrt(x**2 - 1)*x**2 + 6*log( - x**2 + 1)*x**3 - 12*log(sqrt(x**2 - 1
) + x - 1)*x**3 - 12*log(sqrt(x**2 - 1) + x + 1)*x**3 + 2*x**3)/(9*x**3)
```

3.661 $\int \frac{\log(1-x^2)}{x^6\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{\log(1-x^2)}{x^6\sqrt{-1+x^2}}\,dx = -\frac{2\sqrt{-1+x^2}}{15x^3}-\frac{4\sqrt{-1+x^2}}{5x}-\frac{16}{15}\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)+\frac{\sqrt{-1+x^2}\log(1-x^2)}{5x^5}+\frac{4\sqrt{-1+x^2}\log(1-x^2)}{15x^3}+\frac{8\sqrt{-1+x^2}\log(1-x^2)}{15x}$$

output
```
-2/15*(x^2-1)^(1/2)/x^3-4/5*(x^2-1)^(1/2)/x-16/15*arctanh(x/(x^2-1)^(1/2))
+1/5*(x^2-1)^(1/2)*ln(-x^2+1)/x^5+4/15*(x^2-1)^(1/2)*ln(-x^2+1)/x^3+8/15*(
x^2-1)^(1/2)*ln(-x^2+1)/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{\log(1-x^2)}{x^6\sqrt{-1+x^2}}\,dx = \left(-\frac{2}{15x^3}-\frac{4}{5x}\right)\sqrt{-1+x^2}+\frac{\sqrt{-1+x^2}(3+4x^2+8x^4)\log(1-x^2)}{15x^5}-\frac{16}{15}\log\left(x+\sqrt{-1+x^2}\right)$$

input
```
Integrate[Log[1 - x^2]/(x^6*Sqrt[-1 + x^2]),x]
```

output
```
(-2/(15*x^3) - 4/(5*x))*Sqrt[-1 + x^2] + (Sqrt[-1 + x^2]*(3 + 4*x^2 + 8*x^
4)*Log[1 - x^2])/(15*x^5) - (16*Log[x + Sqrt[-1 + x^2]])/15
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^6\sqrt{x^2-1}}\,dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{\log(1-x^2)}{x^6\sqrt{x^2-1}}dx$$

input
```
Int[Log[1 - x^2]/(x^6*Sqrt[-1 + x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^6\sqrt{x^2-1}}dx$$

input
```
int(ln(-x^2+1)/x^6/(x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^6/(x^2-1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{\log(1-x^2)}{x^6\sqrt{-1+x^2}}\,dx = \frac{16\,x^5\log\left(-x+\sqrt{x^2-1}\right)-12\,x^5-\left(12\,x^4+2\,x^2-\left(8\,x^4+4\,x^2+3\right)\log\left(-x^2+1\right)\right)\sqrt{x^2-1}}{15\,x^5}$$

input
```
integrate(log(-x^2+1)/x^6/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(16*x^5*log(-x + sqrt(x^2 - 1)) - 12*x^5 - (12*x^4 + 2*x^2 - (8*x^4 +
 4*x^2 + 3)*log(-x^2 + 1))*sqrt(x^2 - 1))/x^5
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^6\sqrt{-1+x^2}}\,dx = \int \frac{\log(1-x^2)}{x^6\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**6/(x**2-1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**6*sqrt((x - 1)*(x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{\log(1-x^2)}{x^6\sqrt{-1+x^2}}\,dx = \frac{1}{15}\left(\frac{8\sqrt{x^2-1}}{x}+\frac{4\sqrt{x^2-1}}{x^3}+\frac{3\sqrt{x^2-1}}{x^5}\right)\log(-x^2+1) - \frac{8\sqrt{x+1}\sqrt{x-1}}{15\,x} - \frac{2\,(2\,x^2+1)\sqrt{x+1}\sqrt{x-1}}{15\,x^3} + \frac{16}{15}\log\left(-\frac{1}{15}\,x+\frac{1}{15}\sqrt{x^2-1}\right)$$

input
```
integrate(log(-x^2+1)/x^6/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(8*sqrt(x^2 - 1)/x + 4*sqrt(x^2 - 1)/x^3 + 3*sqrt(x^2 - 1)/x^5)*log(-
x^2 + 1) - 8/15*sqrt(x + 1)*sqrt(x - 1)/x - 2/15*(2*x^2 + 1)*sqrt(x + 1)*s
qrt(x - 1)/x^3 + 16/15*log(-1/15*x + 1/15*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17

$$\int \frac{\log(1-x^2)}{x^6\sqrt{-1+x^2}}\,dx = -\frac{8\left(2\left(x-\sqrt{x^2-1}\right)^4+7\left(x-\sqrt{x^2-1}\right)^2+3\right)}{15\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^3} + \frac{16\left(10\left(x-\sqrt{x^2-1}\right)^4+5\left(x-\sqrt{x^2-1}\right)^2+1\right)\log(-x^2+1)}{15\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^5} + \frac{8}{15}\log\left(\left(x-\sqrt{x^2-1}\right)^2\right) - \frac{8}{15}\log(|x+1|) - \frac{8}{15}\log(|x-1|)$$

input
```
integrate(log(-x^2+1)/x^6/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
-8/15*(2*(x - sqrt(x^2 - 1))^4 + 7*(x - sqrt(x^2 - 1))^2 + 3)/((x - sqrt(x
^2 - 1))^2 + 1)^3 + 16/15*(10*(x - sqrt(x^2 - 1))^4 + 5*(x - sqrt(x^2 - 1)
)^2 + 1)*log(-x^2 + 1)/((x - sqrt(x^2 - 1))^2 + 1)^5 + 8/15*log((x - sqrt(
x^2 - 1))^2) - 8/15*log(abs(x + 1)) - 8/15*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1-x^2)}{x^6\sqrt{-1+x^2}}\,dx = \int \frac{\ln(1-x^2)}{x^6\,\sqrt{x^2-1}}\,dx$$

input
```
int(log(1 - x^2)/(x^6*(x^2 - 1)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^6*(x^2 - 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\log(1-x^2)}{x^6\sqrt{-1+x^2}}\,dx = \frac{40\sqrt{x^2-1}\log(-x^2+1)\,x^4+20\sqrt{x^2-1}\log(-x^2+1)\,x^2+15\sqrt{x^2-1}\log(-x^2+1)-60\sqrt{x^2-1}\,x^4-}{75}$$

input
```
int(log(-x^2+1)/x^6/(x^2-1)^(1/2),x)
```

output
```
(40*sqrt(x**2 - 1)*log( - x**2 + 1)*x**4 + 20*sqrt(x**2 - 1)*log( - x**2 +
 1)*x**2 + 15*sqrt(x**2 - 1)*log( - x**2 + 1) - 60*sqrt(x**2 - 1)*x**4 - 1
0*sqrt(x**2 - 1)*x**2 + 40*log( - x**2 + 1)*x**5 - 80*log(sqrt(x**2 - 1) +
 x - 1)*x**5 - 80*log(sqrt(x**2 - 1) + x + 1)*x**5 + 44*x**5)/(75*x**5)
```

3.662 $\int \frac{\log(1-x^2)}{x^8\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}\int \frac{\log(1-x^2)}{x^8\sqrt{-1+x^2}}\,dx = &-\frac{2\sqrt{-1+x^2}}{35x^5}-\frac{4\sqrt{-1+x^2}}{21x^3}\\&-\frac{88\sqrt{-1+x^2}}{105x}-\frac{32}{35}\operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)\\&+\frac{\sqrt{-1+x^2}\log(1-x^2)}{7x^7}+\frac{6\sqrt{-1+x^2}\log(1-x^2)}{35x^5}\\&+\frac{8\sqrt{-1+x^2}\log(1-x^2)}{35x^3}+\frac{16\sqrt{-1+x^2}\log(1-x^2)}{35x}\end{aligned}$$

output
```
-2/35*(x^2-1)^(1/2)/x^5-4/21*(x^2-1)^(1/2)/x^3-88/105*(x^2-1)^(1/2)/x-32/3
5*arctanh(x/(x^2-1)^(1/2))+1/7*(x^2-1)^(1/2)*ln(-x^2+1)/x^7+6/35*(x^2-1)^(
1/2)*ln(-x^2+1)/x^5+8/35*(x^2-1)^(1/2)*ln(-x^2+1)/x^3+16/35*(x^2-1)^(1/2)*
ln(-x^2+1)/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.54

$$\int \frac{\log(1-x^2)}{x^8\sqrt{-1+x^2}}\,dx$$
$$= \frac{-2x^2\sqrt{-1+x^2}(3+10x^2+44x^4)+3\sqrt{-1+x^2}(5+6x^2+8x^4+16x^6)\log(1-x^2)-96x^7\log\left(x+\sqrt{-\right.}{105x^7}$$

input
```
Integrate[Log[1 - x^2]/(x^8*Sqrt[-1 + x^2]),x]
```

output
```
(-2*x^2*Sqrt[-1 + x^2]*(3 + 10*x^2 + 44*x^4) + 3*Sqrt[-1 + x^2]*(5 + 6*x^2
 + 8*x^4 + 16*x^6)*Log[1 - x^2] - 96*x^7*Log[x + Sqrt[-1 + x^2]])/(105*x^7
)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log(1-x^2)}{x^8\sqrt{x^2-1}}\,dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{\log(1-x^2)}{x^8\sqrt{x^2-1}}dx$$

input
```
Int[Log[1 - x^2]/(x^8*Sqrt[-1 + x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{\ln(-x^2+1)}{x^8\sqrt{x^2-1}}dx$$

input
```
int(ln(-x^2+1)/x^8/(x^2-1)^(1/2),x)
```

output
```
int(ln(-x^2+1)/x^8/(x^2-1)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int \frac{\log(1-x^2)}{x^8\sqrt{-1+x^2}}\,dx = \frac{96\,x^7\log\left(-x+\sqrt{x^2-1}\right)-88\,x^7-\left(88\,x^6+20\,x^4+6\,x^2-3\,(16\,x^6+8\,x^4+6\,x^2+5)\log\left(-x^2+1\right)\right)\sqrt{x^2-1}}{105\,x^7}$$

input
```
integrate(log(-x^2+1)/x^8/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(96*x^7*log(-x + sqrt(x^2 - 1)) - 88*x^7 - (88*x^6 + 20*x^4 + 6*x^2
- 3*(16*x^6 + 8*x^4 + 6*x^2 + 5)*log(-x^2 + 1))*sqrt(x^2 - 1))/x^7
```

Sympy [F]

$$\int \frac{\log(1-x^2)}{x^8\sqrt{-1+x^2}}\,dx = \int \frac{\log(1-x^2)}{x^8\sqrt{(x-1)(x+1)}}\,dx$$

input
```
integrate(ln(-x**2+1)/x**8/(x**2-1)**(1/2),x)
```

output
```
Integral(log(1 - x**2)/(x**8*sqrt((x - 1)*(x + 1))), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.87

$$\int \frac{\log(1-x^2)}{x^8\sqrt{-1+x^2}}\,dx = \frac{1}{35}\left(\frac{16\sqrt{x^2-1}}{x}+\frac{8\sqrt{x^2-1}}{x^3}+\frac{6\sqrt{x^2-1}}{x^5}+\frac{5\sqrt{x^2-1}}{x^7}\right)\log(-x^2+1) - \frac{16\sqrt{x+1}\sqrt{x-1}}{35x} - \frac{4(2x^2+1)\sqrt{x+1}\sqrt{x-1}}{35x^3} - \frac{2(8x^4+4x^2+3)\sqrt{x+1}\sqrt{x-1}}{105x^5} + \frac{32}{35}\log\left(-\frac{1}{35}x+\frac{1}{35}\sqrt{x^2-1}\right)$$

input
```
integrate(log(-x^2+1)/x^8/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
1/35*(16*sqrt(x^2 - 1)/x + 8*sqrt(x^2 - 1)/x^3 + 6*sqrt(x^2 - 1)/x^5 + 5*s
qrt(x^2 - 1)/x^7)*log(-x^2 + 1) - 16/35*sqrt(x + 1)*sqrt(x - 1)/x - 4/35*(
2*x^2 + 1)*sqrt(x + 1)*sqrt(x - 1)/x^3 - 2/105*(8*x^4 + 4*x^2 + 3)*sqrt(x
+ 1)*sqrt(x - 1)/x^5 + 32/35*log(-1/35*x + 1/35*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16

$$\int \frac{\log (1-x^2)}{x^8\sqrt{-1+x^2}}\,dx = -\frac{16\left(6\left(x-\sqrt{x^2-1}\right)^8+33\left(x-\sqrt{x^2-1}\right)^6+77\left(x-\sqrt{x^2-1}\right)^4+49\left(x-\sqrt{x^2-1}\right)^2+11\right)}{105\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^5} + \frac{32\left(35\left(x-\sqrt{x^2-1}\right)^6+21\left(x-\sqrt{x^2-1}\right)^4+7\left(x-\sqrt{x^2-1}\right)^2+1\right)\log(-x^2+1)}{35\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^7} + \frac{16}{35}\log\left(\left(x-\sqrt{x^2-1}\right)^2\right) - \frac{16}{35}\log(|x+1|) - \frac{16}{35}\log(|x-1|)$$

input
```
integrate(log(-x^2+1)/x^8/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
-16/105*(6*(x - sqrt(x^2 - 1))^8 + 33*(x - sqrt(x^2 - 1))^6 + 77*(x - sqrt
(x^2 - 1))^4 + 49*(x - sqrt(x^2 - 1))^2 + 11)/((x - sqrt(x^2 - 1))^2 + 1)^
5 + 32/35*(35*(x - sqrt(x^2 - 1))^6 + 21*(x - sqrt(x^2 - 1))^4 + 7*(x - sq
rt(x^2 - 1))^2 + 1)*log(-x^2 + 1)/((x - sqrt(x^2 - 1))^2 + 1)^7 + 16/35*lo
g((x - sqrt(x^2 - 1))^2) - 16/35*log(abs(x + 1)) - 16/35*log(abs(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (1-x^2)}{x^8\sqrt{-1+x^2}}\,dx = \int \frac{\ln (1-x^2)}{x^8\,\sqrt{x^2-1}}\,dx$$

input
```
int(log(1 - x^2)/(x^8*(x^2 - 1)^(1/2)),x)
```

output
```
int(log(1 - x^2)/(x^8*(x^2 - 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int \frac{\log(1-x^2)}{x^8\sqrt{-1+x^2}}\,dx$$
$$= \frac{336\sqrt{x^2-1}\log(-x^2+1)\,x^6 + 168\sqrt{x^2-1}\log(-x^2+1)\,x^4 + 126\sqrt{x^2-1}\log(-x^2+1)\,x^2 + 105\sqrt{x^2-}}{}$$

input
```
int(log(-x^2+1)/x^8/(x^2-1)^(1/2),x)
```

output
```
(336*sqrt(x**2 - 1)*log( - x**2 + 1)*x**6 + 168*sqrt(x**2 - 1)*log( - x**2
 + 1)*x**4 + 126*sqrt(x**2 - 1)*log( - x**2 + 1)*x**2 + 105*sqrt(x**2 - 1)
*log( - x**2 + 1) - 616*sqrt(x**2 - 1)*x**6 - 140*sqrt(x**2 - 1)*x**4 - 42
*sqrt(x**2 - 1)*x**2 + 336*log( - x**2 + 1)*x**7 - 672*log(sqrt(x**2 - 1)
+ x - 1)*x**7 - 672*log(sqrt(x**2 - 1) + x + 1)*x**7 + 520*x**7)/(735*x**7
)
```

3.663 $\int \frac{x^7\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 223

$$\int \frac{x^7(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = \frac{128bp\sqrt{4+gx^2}}{g^4} - \frac{32bp(4+gx^2)^{3/2}}{3g^4} + \frac{24bp(4+gx^2)^{5/2}}{25g^4} - \frac{2bp(4+gx^2)^{7/2}}{49g^4} - \frac{64\sqrt{4+gx^2}(a+b\log(c(4d+dgx^2)^p))}{g^4} + \frac{16(4+gx^2)^{3/2}(a+b\log(c(4d+dgx^2)^p))}{g^4} - \frac{12(4+gx^2)^{5/2}(a+b\log(c(4d+dgx^2)^p))}{5g^4} + \frac{(4+gx^2)^{7/2}(a+b\log(c(4d+dgx^2)^p))}{7g^4}$$

output
```
128*b*p*(g*x^2+4)^(1/2)/g^4-32/3*b*p*(g*x^2+4)^(3/2)/g^4+24/25*b*p*(g*x^2+
4)^(5/2)/g^4-2/49*b*p*(g*x^2+4)^(7/2)/g^4-64*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*
g*x^2+4*d)^p))/g^4+16*(g*x^2+4)^(3/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/g^4-12/5
*(g*x^2+4)^(5/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/g^4+1/7*(g*x^2+4)^(7/2)*(a+b*
ln(c*(d*g*x^2+4*d)^p))/g^4
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52

$$\int \frac{x^7\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx$$
$$=\frac{\sqrt{4+gx^2}\left(2bp(180224-9088gx^2+864g^2x^4-75g^3x^6)+105a(-1024+128gx^2-24g^2x^4+5g^3x^6)+1\right.}{3675g^4}$$

input
```
Integrate[(x^7*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
(Sqrt[4 + g*x^2]*(2*b*p*(180224 - 9088*g*x^2 + 864*g^2*x^4 - 75*g^3*x^6) +
 105*a*(-1024 + 128*g*x^2 - 24*g^2*x^4 + 5*g^3*x^6) + 105*b*(-1024 + 128*g
*x^2 - 24*g^2*x^4 + 5*g^3*x^6)*Log[c*(d*(4 + g*x^2))^p]))/(3675*g^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(a+b\log\left(c\left(dgx^2+4d\right)^p\right)\right)}{\sqrt{gx^2+4}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{x^7\left(a+b\log\left(c\left(dgx^2+4d\right)^p\right)\right)}{\sqrt{gx^2+4}}dx$$

input
```
Int[(x^7*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^7 \left(a+b\ln\left(c(dg\,x^2+4d)^p\right)\right)}{\sqrt{g\,x^2+4}}dx$$

input
```
int(x^7*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
int(x^7*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.69

$$\int \frac{x^7 \left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx =$$
$$-\frac{(75\,(2\,bg^3p-7\,ag^3)x^6-72\,(24\,bg^2p-35\,ag^2)x^4+128\,(142\,bgp-105\,ag)x^2-360448\,bp-105\,(5\,bg^3p}{}$$

input
```
integrate(x^7*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/3675*(75*(2*b*g^3*p - 7*a*g^3)*x^6 - 72*(24*b*g^2*p - 35*a*g^2)*x^4 + 1
28*(142*b*g*p - 105*a*g)*x^2 - 360448*b*p - 105*(5*b*g^3*p*x^6 - 24*b*g^2*
p*x^4 + 128*b*g*p*x^2 - 1024*b*p)*log(d*g*x^2 + 4*d) - 105*(5*b*g^3*x^6 -
24*b*g^2*x^4 + 128*b*g*x^2 - 1024*b)*log(c) + 107520*a)*sqrt(g*x^2 + 4)/g^
4
```

Sympy [A] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.49

$$\int \frac{x^7(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx$$
$$= \begin{cases} \frac{ax^6\sqrt{gx^2+4}}{7g} - \frac{24ax^4\sqrt{gx^2+4}}{35g^2} + \frac{128ax^2\sqrt{gx^2+4}}{35g^3} - \frac{1024a\sqrt{gx^2+4}}{35g^4} - \frac{2bpx^6\sqrt{gx^2+4}}{49g} + \frac{bx^6\sqrt{gx^2+4}\log(c(dgx^2+4d)^p)}{7g} + \frac{576bp}{1} \\ \frac{x^8\left(\frac{a}{2}+\frac{b\log(c(4d)^p)}{2}\right)}{8} \end{cases}$$

input
```
integrate(x**7*(a+b*ln(c*(d*g*x**2+4*d)**p))/(g*x**2+4)**(1/2),x)
```

output
```
Piecewise((a*x**6*sqrt(g*x**2 + 4)/(7*g) - 24*a*x**4*sqrt(g*x**2 + 4)/(35*
g**2) + 128*a*x**2*sqrt(g*x**2 + 4)/(35*g**3) - 1024*a*sqrt(g*x**2 + 4)/(3
5*g**4) - 2*b*p*x**6*sqrt(g*x**2 + 4)/(49*g) + b*x**6*sqrt(g*x**2 + 4)*log
(c*(d*g*x**2 + 4*d)**p)/(7*g) + 576*b*p*x**4*sqrt(g*x**2 + 4)/(1225*g**2)
- 24*b*x**4*sqrt(g*x**2 + 4)*log(c*(d*g*x**2 + 4*d)**p)/(35*g**2) - 18176*
b*p*x**2*sqrt(g*x**2 + 4)/(3675*g**3) + 128*b*x**2*sqrt(g*x**2 + 4)*log(c*
(d*g*x**2 + 4*d)**p)/(35*g**3) + 360448*b*p*sqrt(g*x**2 + 4)/(3675*g**4) -
 1024*b*sqrt(g*x**2 + 4)*log(c*(d*g*x**2 + 4*d)**p)/(35*g**4), Ne(g, 0)),
(x**8*(a/2 + b*log(c*(4*d)**p)/2)/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.92

$$\int \frac{x^7(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx$$
$$= \frac{1}{35}\left(\frac{5\sqrt{gx^2+4}x^6}{g} - \frac{24\sqrt{gx^2+4}x^4}{g^2} + \frac{128\sqrt{gx^2+4}x^2}{g^3} - \frac{1024\sqrt{gx^2+4}}{g^4}\right)b\log\left((dgx^2+4d)^pc\right)$$
$$+ \frac{1}{35}\left(\frac{5\sqrt{gx^2+4}x^6}{g} - \frac{24\sqrt{gx^2+4}x^4}{g^2} + \frac{128\sqrt{gx^2+4}x^2}{g^3} - \frac{1024\sqrt{gx^2+4}}{g^4}\right)a$$
$$- \frac{2\left(75(gx^2+4)^{\frac{7}{2}} - 1764(gx^2+4)^{\frac{5}{2}} + 19600(gx^2+4)^{\frac{3}{2}} - 235200\sqrt{gx^2+4}\right)bp}{3675\,g^4}$$

input
```
integrate(x^7*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
1/35*(5*sqrt(g*x^2 + 4)*x^6/g - 24*sqrt(g*x^2 + 4)*x^4/g^2 + 128*sqrt(g*x^
2 + 4)*x^2/g^3 - 1024*sqrt(g*x^2 + 4)/g^4)*b*log((d*g*x^2 + 4*d)^p*c) + 1/
35*(5*sqrt(g*x^2 + 4)*x^6/g - 24*sqrt(g*x^2 + 4)*x^4/g^2 + 128*sqrt(g*x^2
+ 4)*x^2/g^3 - 1024*sqrt(g*x^2 + 4)/g^4)*a - 2/3675*(75*(g*x^2 + 4)^(7/2)
- 1764*(g*x^2 + 4)^(5/2) + 19600*(g*x^2 + 4)^(3/2) - 235200*sqrt(g*x^2 + 4
))*b*p/g^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.01

$$\int \frac{x^7(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = \frac{bp\left(\frac{105\left(5\,(gx^2+4)^{\frac{7}{2}}-84\,(gx^2+4)^{\frac{5}{2}}+560\,(gx^2+4)^{\frac{3}{2}}-2240\,\sqrt{gx^2+4}\right)\log(dgx^2+4\,d)}{g^3}-\frac{2\left(75\,(gx^2+4)^{\frac{7}{2}}-1764\,(gx^2+4)^{\frac{5}{2}}+19600\,(gx^2+4\right.}{g^3}\right.}{}$$

input
```
integrate(x^7*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
1/3675*(b*p*(105*(5*(g*x^2 + 4)^(7/2) - 84*(g*x^2 + 4)^(5/2) + 560*(g*x^2
+ 4)^(3/2) - 2240*sqrt(g*x^2 + 4))*log(d*g*x^2 + 4*d)/g^3 - 2*(75*(g*x^2 +
 4)^(7/2) - 1764*(g*x^2 + 4)^(5/2) + 19600*(g*x^2 + 4)^(3/2) - 235200*sqrt
(g*x^2 + 4))/g^3) + 105*(5*(g*x^2 + 4)^(7/2) - 84*(g*x^2 + 4)^(5/2) + 560*
(g*x^2 + 4)^(3/2) - 2240*sqrt(g*x^2 + 4))*b*log(c)/g^3 + 105*(5*(g*x^2 + 4
)^(7/2) - 84*(g*x^2 + 4)^(5/2) + 560*(g*x^2 + 4)^(3/2) - 2240*sqrt(g*x^2 +
 4))*a/g^3)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \left(a+b\log \left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx=\int \frac{x^7\,\left(a+b\,\ln \left(c\,{\left(d\,g\,x^2+4\,d\right)}^p\right)\right)}{\sqrt{g\,x^2+4}}\,dx$$

input
```
int((x^7*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2),x)
```

output
```
int((x^7*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.43

$$\int \frac{x^7 \left(a+b\log \left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = \frac{\sqrt{g\,x^2+4}\left(525\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^p c}{\left(2\sqrt{g\,x^2+4}+2\sqrt{g}\,x\right)^{2p}}\right)b\,g^3x^6-2520\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^p c}{\left(2\sqrt{g\,x^2+4}+2\sqrt{g}\,x\right)^{2p}}\right)b\,g^2x^4+\cdots}{\cdots}$$

input
```
int(x^7*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
(sqrt(g*x**2 + 4)*(525*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)
**(2*p)*4**p*c)/(2*sqrt(g*x**2 + 4) + 2*sqrt(g)*x)**(2*p))*b*g**3*x**6 - 2
520*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**(2*p)*4**p*c)/(2*
sqrt(g*x**2 + 4) + 2*sqrt(g)*x)**(2*p))*b*g**2*x**4 + 13440*log((d**p*(sqr
t(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**(2*p)*4**p*c)/(2*sqrt(g*x**2 + 4) +
 2*sqrt(g)*x)**(2*p))*b*g*x**2 - 107520*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4
)*x + g*x**2 + 4)**(2*p)*4**p*c)/(2*sqrt(g*x**2 + 4) + 2*sqrt(g)*x)**(2*p)
)*b + 525*a*g**3*x**6 - 2520*a*g**2*x**4 + 13440*a*g*x**2 - 107520*a - 150
*b*g**3*p*x**6 + 1728*b*g**2*p*x**4 - 18176*b*g*p*x**2 + 360448*b*p))/(367
5*g**4)
```

3.664 $\int \frac{x^5\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 168

$$\begin{aligned}\int \frac{x^5\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = &-\frac{32bp\sqrt{4+gx^2}}{g^3}+\frac{16bp\left(4+gx^2\right)^{3/2}}{9g^3}\\ &-\frac{2bp\left(4+gx^2\right)^{5/2}}{25g^3}\\ &+\frac{16\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{g^3}\\ &-\frac{8\left(4+gx^2\right)^{3/2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{3g^3}\\ &+\frac{\left(4+gx^2\right)^{5/2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{5g^3}\end{aligned}$$

output
```
-32*b*p*(g*x^2+4)^(1/2)/g^3+16/9*b*p*(g*x^2+4)^(3/2)/g^3-2/25*b*p*(g*x^2+4
)^(5/2)/g^3+16*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/g^3-8/3*(g*x^2+
4)^(3/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/g^3+1/5*(g*x^2+4)^(5/2)*(a+b*ln(c*(d*
g*x^2+4*d)^p))/g^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.54

$$\int \frac{x^5(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = \frac{\sqrt{4+gx^2}(15a(128-16gx^2+3g^2x^4)-2bp(2944-128gx^2+9g^2x^4)+15b(128-16gx^2+3g^2x^4)\log(c$$

$$225g^3$$

input
```
Integrate[(x^5*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
(Sqrt[4 + g*x^2]*(15*a*(128 - 16*g*x^2 + 3*g^2*x^4) - 2*b*p*(2944 - 128*g*
x^2 + 9*g^2*x^4) + 15*b*(128 - 16*g*x^2 + 3*g^2*x^4)*Log[c*(d*(4 + g*x^2))
^p]))/(225*g^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\log(c(dgx^2+4d)^p))}{\sqrt{gx^2+4}}\,dx$$

$$\Big\downarrow \text{2929}$$

$$\int \frac{x^5(a+b\log(c(dgx^2+4d)^p))}{\sqrt{gx^2+4}}dx$$

input
```
Int[(x^5*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^5\left(a+b\ln\left(c(dg\,x^2+4d)^p\right)\right)}{\sqrt{g\,x^2+4}}dx$$

input
```
int(x^5*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
int(x^5*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.69

$$\int \frac{x^5\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = -\frac{(9\,(2\,bg^2p-5\,ag^2)x^4-16\,(16\,bgp-15\,ag)x^2+5888\,bp-15\,(3\,bg^2px^4-16\,bgpx^2+128\,bp)\log\left(dgx^2\right.}{225\,g^3}$$

input
```
integrate(x^5*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/225*(9*(2*b*g^2*p - 5*a*g^2)*x^4 - 16*(16*b*g*p - 15*a*g)*x^2 + 5888*b*
p - 15*(3*b*g^2*p*x^4 - 16*b*g*p*x^2 + 128*b*p)*log(d*g*x^2 + 4*d) - 15*(3
*b*g^2*x^4 - 16*b*g*x^2 + 128*b)*log(c) - 1920*a)*sqrt(g*x^2 + 4)/g^3
```

Sympy [A] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.49

$$\int \frac{x^5\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx$$
$$=\begin{cases}\frac{ax^4\sqrt{gx^2+4}}{5g}-\frac{16ax^2\sqrt{gx^2+4}}{15g^2}+\frac{128a\sqrt{gx^2+4}}{15g^3}-\frac{2bpx^4\sqrt{gx^2+4}}{25g}+\frac{bx^4\sqrt{gx^2+4}\log\left(c(dgx^2+4d)^p\right)}{5g}+\frac{256bpx^2\sqrt{gx^2+4}}{225g^2}-\frac{16bx^2}{} \\ \frac{x^6\left(\frac{a}{2}+\frac{b\log\left(c(4d)^p\right)}{2}\right)}{6}\end{cases}$$

input
```
integrate(x**5*(a+b*ln(c*(d*g*x**2+4*d)**p))/(g*x**2+4)**(1/2),x)
```

output
```
Piecewise((a*x**4*sqrt(g*x**2 + 4)/(5*g) - 16*a*x**2*sqrt(g*x**2 + 4)/(15*
g**2) + 128*a*sqrt(g*x**2 + 4)/(15*g**3) - 2*b*p*x**4*sqrt(g*x**2 + 4)/(25
*g) + b*x**4*sqrt(g*x**2 + 4)*log(c*(d*g*x**2 + 4*d)**p)/(5*g) + 256*b*p*x
**2*sqrt(g*x**2 + 4)/(225*g**2) - 16*b*x**2*sqrt(g*x**2 + 4)*log(c*(d*g*x*
*2 + 4*d)**p)/(15*g**2) - 5888*b*p*sqrt(g*x**2 + 4)/(225*g**3) + 128*b*sqr
t(g*x**2 + 4)*log(c*(d*g*x**2 + 4*d)**p)/(15*g**3), Ne(g, 0)), (x**6*(a/2
+ b*log(c*(4*d)**p)/2)/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.96

$$\int \frac{x^5\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx$$
$$=\frac{1}{15}\left(\frac{3\sqrt{gx^2+4}x^4}{g}-\frac{16\sqrt{gx^2+4}x^2}{g^2}+\frac{128\sqrt{gx^2+4}}{g^3}\right)b\log\left(\left(dgx^2+4\,d\right)^pc\right)$$
$$+\frac{1}{15}\left(\frac{3\sqrt{gx^2+4}x^4}{g}-\frac{16\sqrt{gx^2+4}x^2}{g^2}+\frac{128\sqrt{gx^2+4}}{g^3}\right)a$$
$$-\frac{2\left(9\left(gx^2+4\right)^{\frac{5}{2}}-200\left(gx^2+4\right)^{\frac{3}{2}}+3600\sqrt{gx^2+4}\right)bp}{225\,g^3}$$

input
```
integrate(x^5*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
1/15*(3*sqrt(g*x^2 + 4)*x^4/g - 16*sqrt(g*x^2 + 4)*x^2/g^2 + 128*sqrt(g*x^
2 + 4)/g^3)*b*log((d*g*x^2 + 4*d)^p*c) + 1/15*(3*sqrt(g*x^2 + 4)*x^4/g - 1
6*sqrt(g*x^2 + 4)*x^2/g^2 + 128*sqrt(g*x^2 + 4)/g^3)*a - 2/225*(9*(g*x^2 +
 4)^(5/2) - 200*(g*x^2 + 4)^(3/2) + 3600*sqrt(g*x^2 + 4))*b*p/g^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04

$$\int \frac{x^5(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = -\frac{\frac{\left(18\,(gx^2+4)^{\frac{5}{2}}-15\left(3\,(gx^2+4)^{\frac{5}{2}}-40\,(gx^2+4)^{\frac{3}{2}}+240\,\sqrt{gx^2+4}\right)\log(dgx^2+4\,d)-400\,(gx^2+4)^{\frac{3}{2}}+7200\,\sqrt{gx^2+4}\right)bp}{g^2}-\frac{15\left(3\,(gx^2+4)^{\frac{5}{2}}-\cdots\right.}{}}{225\,g}$$

input
```
integrate(x^5*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
-1/225*((18*(g*x^2 + 4)^(5/2) - 15*(3*(g*x^2 + 4)^(5/2) - 40*(g*x^2 + 4)^(
3/2) + 240*sqrt(g*x^2 + 4))*log(d*g*x^2 + 4*d) - 400*(g*x^2 + 4)^(3/2) + 7
200*sqrt(g*x^2 + 4))*b*p/g^2 - 15*(3*(g*x^2 + 4)^(5/2) - 40*(g*x^2 + 4)^(3
/2) + 240*sqrt(g*x^2 + 4))*b*log(c)/g^2 - 15*(3*(g*x^2 + 4)^(5/2) - 40*(g*
x^2 + 4)^(3/2) + 240*sqrt(g*x^2 + 4))*a/g^2)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = \int \frac{x^5\,\left(a+b\,\ln\left(c\,{\left(d\,g\,x^2+4\,d\right)}^p\right)\right)}{\sqrt{g\,x^2+4}}\,dx$$

input
```
int((x^5*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2),x)
```

output
```
int((x^5*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

$$\int \frac{x^5(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx$$

$$= \frac{\sqrt{g\,x^2+4}\left(45\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^pc}{\left(2\sqrt{g\,x^2+4}+2\sqrt{g}\,x\right)^{2p}}\right)b\,g^2x^4-240\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^pc}{\left(2\sqrt{g\,x^2+4}+2\sqrt{g}\,x\right)^{2p}}\right)bg\,x^2+19}{22}$$

input
```
int(x^5*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
(sqrt(g*x**2 + 4)*(45*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)*
*(2*p)*4**p*c)/(2*sqrt(g*x**2 + 4) + 2*sqrt(g)*x)**(2*p))*b*g**2*x**4 - 24
0*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**(2*p)*4**p*c)/(2*sq
rt(g*x**2 + 4) + 2*sqrt(g)*x)**(2*p))*b*g*x**2 + 1920*log((d**p*(sqrt(g)*s
qrt(g*x**2 + 4)*x + g*x**2 + 4)**(2*p)*4**p*c)/(2*sqrt(g*x**2 + 4) + 2*sqr
t(g)*x)**(2*p))*b + 45*a*g**2*x**4 - 240*a*g*x**2 + 1920*a - 18*b*g**2*p*x
**4 + 256*b*g*p*x**2 - 5888*b*p))/(225*g**3)
```

3.665 $\int \frac{x^3\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 111

$$\int \frac{x^3\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = \frac{8bp\sqrt{4+gx^2}}{g^2}-\frac{2bp(4+gx^2)^{3/2}}{9g^2} - \frac{4\sqrt{4+gx^2}\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{g^2} + \frac{(4+gx^2)^{3/2}\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{3g^2}$$

output
```
8*b*p*(g*x^2+4)^(1/2)/g^2-2/9*b*p*(g*x^2+4)^(3/2)/g^2-4*(g*x^2+4)^(1/2)*(a
+b*ln(c*(d*g*x^2+4*d)^p))/g^2+1/3*(g*x^2+4)^(3/2)*(a+b*ln(c*(d*g*x^2+4*d)^
p))/g^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int \frac{x^3(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = \frac{\sqrt{4+gx^2}(-2bp(-32+gx^2)+3a(-8+gx^2)+3b(-8+gx^2)\log(c(d(4+gx^2))^p))}{9g^2}$$

input
```
Integrate[(x^3*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
(Sqrt[4 + g*x^2]*(-2*b*p*(-32 + g*x^2) + 3*a*(-8 + g*x^2) + 3*b*(-8 + g*x^
2)*Log[c*(d*(4 + g*x^2))^p]))/(9*g^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\log(c(dgx^2+4d)^p))}{\sqrt{gx^2+4}}\,dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{x^3(a+b\log(c(dgx^2+4d)^p))}{\sqrt{gx^2+4}}dx$$

input
```
Int[(x^3*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^3\left(a+b\ln\left(c(dg\,x^2+4d)^p\right)\right)}{\sqrt{g\,x^2+4}}dx$$

input
```
int(x^3*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
int(x^3*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int \frac{x^3\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = -\frac{((2\,bgp-3\,ag)x^2-64\,bp-3\,(bgpx^2-8\,bp)\log\left(dgx^2+4\,d\right)-3\,(bgx^2-8\,b)\log\left(c\right)+24\,a)\sqrt{gx^2+4}}{9\,g^2}$$

input
```
integrate(x^3*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/9*((2*b*g*p - 3*a*g)*x^2 - 64*b*p - 3*(b*g*p*x^2 - 8*b*p)*log(d*g*x^2 +
 4*d) - 3*(b*g*x^2 - 8*b)*log(c) + 24*a)*sqrt(g*x^2 + 4)/g^2
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.50

$$\int \frac{x^3(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = \begin{cases} \frac{ax^2\sqrt{gx^2+4}}{3g} - \frac{8a\sqrt{gx^2+4}}{3g^2} - \frac{2bpx^2\sqrt{gx^2+4}}{9g} + \frac{bx^2\sqrt{gx^2+4}\log(c(dgx^2+4d)^p)}{3g} + \frac{64bp\sqrt{gx^2+4}}{9g^2} - \frac{8b\sqrt{gx^2+4}\log(c(dgx^2+4d)^p)}{3g^2} \\ \frac{x^4\left(\frac{a}{2}+\frac{b\log(c(4d)^p)}{2}\right)}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*ln(c*(d*g*x**2+4*d)**p))/(g*x**2+4)**(1/2),x)
```

output
```
Piecewise((a*x**2*sqrt(g*x**2 + 4)/(3*g) - 8*a*sqrt(g*x**2 + 4)/(3*g**2) -
 2*b*p*x**2*sqrt(g*x**2 + 4)/(9*g) + b*x**2*sqrt(g*x**2 + 4)*log(c*(d*g*x*
*2 + 4*d)**p)/(3*g) + 64*b*p*sqrt(g*x**2 + 4)/(9*g**2) - 8*b*sqrt(g*x**2 +
 4)*log(c*(d*g*x**2 + 4*d)**p)/(3*g**2), Ne(g, 0)), (x**4*(a/2 + b*log(c*(
4*d)**p)/2)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{x^3(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = \frac{1}{3}\left(\frac{\sqrt{gx^2+4}x^2}{g} - \frac{8\sqrt{gx^2+4}}{g^2}\right)b\log\left((dgx^2+4d)^p c\right) + \frac{1}{3}\left(\frac{\sqrt{gx^2+4}x^2}{g} - \frac{8\sqrt{gx^2+4}}{g^2}\right)a - \frac{2\left((gx^2+4)^{\frac{3}{2}} - 36\sqrt{gx^2+4}\right)bp}{9g^2}$$

input
```
integrate(x^3*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
1/3*(sqrt(g*x^2 + 4)*x^2/g - 8*sqrt(g*x^2 + 4)/g^2)*b*log((d*g*x^2 + 4*d)^
p*c) + 1/3*(sqrt(g*x^2 + 4)*x^2/g - 8*sqrt(g*x^2 + 4)/g^2)*a - 2/9*((g*x^2
 + 4)^(3/2) - 36*sqrt(g*x^2 + 4))*b*p/g^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13

$$\int \frac{x^3\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx$$
$$= \frac{\frac{\left(3\left((gx^2+4)^{\frac{3}{2}}-12\sqrt{gx^2+4}\right)\log\left(dgx^2+4d\right)-2\,(gx^2+4)^{\frac{3}{2}}+72\sqrt{gx^2+4}\right)bp}{g}+\frac{3\left((gx^2+4)^{\frac{3}{2}}-12\sqrt{gx^2+4}\right)b\log(c)}{g}+\frac{3\left((gx^2+4)^{\frac{3}{2}}-12\right.}{g}}{9\,g}$$

input
```
integrate(x^3*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
1/9*((3*((g*x^2 + 4)^(3/2) - 12*sqrt(g*x^2 + 4))*log(d*g*x^2 + 4*d) - 2*(g
*x^2 + 4)^(3/2) + 72*sqrt(g*x^2 + 4))*b*p/g + 3*((g*x^2 + 4)^(3/2) - 12*sq
rt(g*x^2 + 4))*b*log(c)/g + 3*((g*x^2 + 4)^(3/2) - 12*sqrt(g*x^2 + 4))*a/g
)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx=\int \frac{x^3\,\left(a+b\,\ln\left(c\,{\left(d\,g\,x^2+4\,d\right)}^p\right)\right)}{\sqrt{g\,x^2+4}}\,dx$$

input
```
int((x^3*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2),x)
```

output
```
int((x^3*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.39

$$\int \frac{x^3 (a + b\log (c(4d + dgx^2)^p))}{\sqrt{4 + gx^2}}\, dx = \frac{\sqrt{g\,x^2+4}\left(3\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p} 4^p c}{\left(2\sqrt{g\,x^2+4}+2\sqrt{g}\,x\right)^{2p}}\right) bg\,x^2 - 24\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p} 4^p c}{\left(2\sqrt{g\,x^2+4}+2\sqrt{g}\,x\right)^{2p}}\right) b + 3ag\,x^2 - 2}{9g^2}$$

input
```
int(x^3*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
(sqrt(g*x**2 + 4)*(3*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**
(2*p)*4**p*c)/(2*sqrt(g*x**2 + 4) + 2*sqrt(g)*x)**(2*p))*b*g*x**2 - 24*log
((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**(2*p)*4**p*c)/(2*sqrt(g*
x**2 + 4) + 2*sqrt(g)*x)**(2*p))*b + 3*a*g*x**2 - 24*a - 2*b*g*p*x**2 + 64
*b*p))/(9*g**2)
```

3.666 $\int \frac{x\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 53

$$\int \frac{x\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx=-\frac{2bp\sqrt{4+gx^2}}{g}+\frac{\sqrt{4+gx^2}\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{g}$$

output
```
-2*b*p*(g*x^2+4)^(1/2)/g+(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/g
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx=\frac{\sqrt{4+gx^2}\left(a-2bp+b\log\left(c(d(4+gx^2))^p\right)\right)}{g}$$

input
```
Integrate[(x*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
(Sqrt[4 + g*x^2]*(a - 2*b*p + b*Log[c*(d*(4 + g*x^2))^p]))/g
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\log\left(c\left(dgx^2+4d\right)^p\right)\right)}{\sqrt{gx^2+4}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{x\left(a+b\log\left(c\left(dgx^2+4d\right)^p\right)\right)}{\sqrt{gx^2+4}}dx$$

input
```
Int[(x*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x\left(a+b\ln\left(c\left(dg\,x^2+4d\right)^p\right)\right)}{\sqrt{g\,x^2+4}}dx$$

input
```
int(x*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
int(x*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = \frac{\sqrt{gx^2+4}\,(bp\log\left(dgx^2+4\,d\right)-2\,bp+b\log\left(c\right)+a)}{g}$$

input
```
integrate(x*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="fri
cas")
```

output
```
sqrt(g*x^2 + 4)*(b*p*log(d*g*x^2 + 4*d) - 2*b*p + b*log(c) + a)/g
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{x\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = \begin{cases} \frac{a\sqrt{gx^2+4}}{g} - \frac{2bp\sqrt{gx^2+4}}{g} + \frac{b\sqrt{gx^2+4}\log\left(c\left(dgx^2+4d\right)^p\right)}{g} & \text{for } g \neq 0 \\ \frac{x^2\left(\frac{a}{2}+\frac{b\log\left(c(4d)^p\right)}{2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*ln(c*(d*g*x**2+4*d)**p))/(g*x**2+4)**(1/2),x)
```

output
```
Piecewise((a*sqrt(g*x**2 + 4)/g - 2*b*p*sqrt(g*x**2 + 4)/g + b*sqrt(g*x**2
 + 4)*log(c*(d*g*x**2 + 4*d)**p)/g, Ne(g, 0)), (x**2*(a/2 + b*log(c*(4*d)*
*p)/2)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \frac{x\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = -\frac{2\sqrt{gx^2+4}bp}{g} + \frac{\sqrt{gx^2+4}b\log\left((dgx^2+4\,d)^p c\right)}{g} + \frac{\sqrt{gx^2+4}a}{g}$$

input
```
integrate(x*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="max
ima")
```

output
```
-2*sqrt(g*x^2 + 4)*b*p/g + sqrt(g*x^2 + 4)*b*log((d*g*x^2 + 4*d)^p*c)/g +
sqrt(g*x^2 + 4)*a/g
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \frac{x\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = \frac{\sqrt{gx^2+4}bp\log(d) + \left(\sqrt{gx^2+4}\log(gx^2+4) - 2\sqrt{gx^2+4}\right)bp + \sqrt{gx^2+4}b\log(c) + \sqrt{gx^2+4}a}{g}$$

input
```
integrate(x*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="gia
c")
```

output
```
(sqrt(g*x^2 + 4)*b*p*log(d) + (sqrt(g*x^2 + 4)*log(g*x^2 + 4) - 2*sqrt(g*x
^2 + 4))*b*p + sqrt(g*x^2 + 4)*b*log(c) + sqrt(g*x^2 + 4)*a)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx=\int \frac{x\,\left(a+b\,\ln\left(c\,{\left(d\,g\,x^2+4\,d\right)}^p\right)\right)}{\sqrt{g\,x^2+4}}\,dx$$

input
```
int((x*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2),x)
```

output
```
int((x*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{x\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = \frac{\sqrt{g\,x^2+4}\,\left(\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^p c}{\left(2\sqrt{g\,x^2+4}+2\sqrt{g}\,x\right)^{2p}}\right)b+a-2bp\right)}{g}$$

input
```
int(x*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
(sqrt(g*x**2 + 4)*(log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**(2
*p)*4**p*c)/(2*sqrt(g*x**2 + 4) + 2*sqrt(g)*x)**(2*p))*b + a - 2*b*p))/g
```

3.667 $\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 86

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x\sqrt{4+gx^2}}\,dx = -\frac{1}{2}\text{arctanh}\left(\frac{1}{2}\sqrt{4+gx^2}\right)\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right) - \frac{1}{2}bp\,\text{PolyLog}\left(2,-\frac{1}{2}\sqrt{4+gx^2}\right) + \frac{1}{2}bp\,\text{PolyLog}\left(2,\frac{1}{2}\sqrt{4+gx^2}\right)$$

output

```
-1/2*arctanh(1/2*(g*x^2+4)^(1/2))*(a+b*ln(c*(d*g*x^2+4*d)^p))-1/2*b*p*poly
log(2,-1/2*(g*x^2+4)^(1/2))+1/2*b*p*polylog(2,1/2*(g*x^2+4)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x\sqrt{4+gx^2}}\,dx = \frac{1}{4}\left(\left(a+b\log\left(c\left(d\left(4+gx^2\right)\right)^p\right)\right)\left(\log\left(2-\sqrt{4+gx^2}\right)-\log\left(2+\sqrt{4+gx^2}\right)\right)-2bp\,\text{PolyLog}\left(2,-\frac{1}{2}\sqrt{4+gx^2}\right)+2bp\,\text{PolyLog}\left(2,\frac{1}{2}\sqrt{4+gx^2}\right)\right)$$

input
```
Integrate[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x*Sqrt[4 + g*x^2]),x]
```

output
```
((a + b*Log[c*(d*(4 + g*x^2))^p])*(Log[2 - Sqrt[4 + g*x^2]] - Log[2 + Sqrt
[4 + g*x^2]]) - 2*b*p*PolyLog[2, -1/2*Sqrt[4 + g*x^2]] + 2*b*p*PolyLog[2,
Sqrt[4 + g*x^2]/2])/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x\sqrt{gx^2+4}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x\sqrt{gx^2+4}}dx$$

input
```
Int[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x*Sqrt[4 + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(dg\,x^{2}+4d)^{p}\right)}{x\sqrt{g\,x^{2}+4}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x/(g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x/(g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x\sqrt{4+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+4\,d)^pc\right)+a}{\sqrt{gx^2+4}x}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x/(g*x^2+4)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((sqrt(g*x^2 + 4)*b*log((d*g*x^2 + 4*d)^p*c) + sqrt(g*x^2 + 4)*a)/
(g*x^3 + 4*x), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x\sqrt{4+gx^2}}\,dx=\int \frac{a+b\log\left(c(dgx^2+4d)^p\right)}{x\sqrt{gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+4*d)**p))/x/(g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*g*x**2 + 4*d)**p))/(x*sqrt(g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x\sqrt{4+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+4\,d)^p c\right)+a}{\sqrt{gx^2+4}x}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x/(g*x^2+4)^(1/2),x, algorithm="max
ima")
```

output
```
-1/2*a*arcsinh(2/(sqrt(g)*abs(x))) + b*integrate((p*log(g*x^2 + 4) + p*log
(d) + log(c))/(sqrt(g*x^2 + 4)*x), x)
```

Giac [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x\sqrt{4+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+4\,d)^p c\right)+a}{\sqrt{gx^2+4}x}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x/(g*x^2+4)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((b*log((d*g*x^2 + 4*d)^p*c) + a)/(sqrt(g*x^2 + 4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x\sqrt{4+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,(d\,g\,x^2+4\,d)^p\right)}{x\,\sqrt{g\,x^2+4}}\,dx$$

input
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x*(g*x^2 + 4)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x*(g*x^2 + 4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x\sqrt{4+gx^2}}\,dx=\left(\int \frac{\log\left((dg\,x^2+4d)^p\,c\right)}{\sqrt{g\,x^2+4}\,x}dx\right)b+\frac{\log\left(\frac{\sqrt{g\,x^2+4}}{2}+\frac{\sqrt{g}\,x}{2}-1\right)a}{2}-\frac{\log\left(\frac{\sqrt{g\,x^2+4}}{2}+\frac{\sqrt{g}\,x}{2}+1\right)a}{2}$$

input
```
int((a+b*log(c*(d*g*x^2+4*d)^p))/x/(g*x^2+4)^(1/2),x)
```

output
```
(2*int(log((d*g*x**2 + 4*d)**p*c)/(sqrt(g*x**2 + 4)*x),x)*b + log((sqrt(g*
x**2 + 4) + sqrt(g)*x - 2)/2)*a - log((sqrt(g*x**2 + 4) + sqrt(g)*x + 2)/2
)*a)/2
```

3.668 $\int \frac{a+b\log\left(c\left(4d+dgx^2\right)^p\right)}{x^3\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 149

$$\begin{aligned}\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^3\sqrt{4+gx^2}}\,dx = & -\frac{1}{8}bgp\,\text{arctanh}\left(\frac{1}{2}\sqrt{4+gx^2}\right) \\ & -\frac{\sqrt{4+gx^2}\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{8x^2} \\ & +\frac{1}{16}g\,\text{arctanh}\left(\frac{1}{2}\sqrt{4+gx^2}\right)\left(a\right. \\ & \left.+b\log\left(c\left(4d+dgx^2\right)^p\right)\right) \\ & +\frac{1}{16}bgp\,\text{PolyLog}\left(2,-\frac{1}{2}\sqrt{4+gx^2}\right) \\ & -\frac{1}{16}bgp\,\text{PolyLog}\left(2,\frac{1}{2}\sqrt{4+gx^2}\right)\end{aligned}$$

output
```
-1/8*b*g*p*arctanh(1/2*(g*x^2+4)^(1/2))-1/8*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g
*x^2+4*d)^p))/x^2+1/16*g*arctanh(1/2*(g*x^2+4)^(1/2))*(a+b*ln(c*(d*g*x^2+4
*d)^p))+1/16*b*g*p*polylog(2,-1/2*(g*x^2+4)^(1/2))-1/16*b*g*p*polylog(2,1/
2*(g*x^2+4)^(1/2))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.74

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^3\sqrt{4+gx^2}}\,dx =$$
$$-\frac{4a\sqrt{4+gx^2}+4b\sqrt{4+gx^2}\log\left(c(d(4+gx^2))^p\right)+agx^2\log\left(2-\sqrt{4+gx^2}\right)-2bgpx^2\log\left(2-\sqrt{4+g\ldots\right.}}{\ldots}$$

input
```
Integrate[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^3*Sqrt[4 + g*x^2]),x]
```

output
```
-1/32*(4*a*Sqrt[4 + g*x^2] + 4*b*Sqrt[4 + g*x^2]*Log[c*(d*(4 + g*x^2))^p]
+ a*g*x^2*Log[2 - Sqrt[4 + g*x^2]] - 2*b*g*p*x^2*Log[2 - Sqrt[4 + g*x^2]]
+ b*g*x^2*Log[c*(d*(4 + g*x^2))^p]*Log[2 - Sqrt[4 + g*x^2]] - a*g*x^2*Log[
2 + Sqrt[4 + g*x^2]] + 2*b*g*p*x^2*Log[2 + Sqrt[4 + g*x^2]] - b*g*x^2*Log[
c*(d*(4 + g*x^2))^p]*Log[2 + Sqrt[4 + g*x^2]] - 2*b*g*p*x^2*PolyLog[2, -1/
2*Sqrt[4 + g*x^2]] + 2*b*g*p*x^2*PolyLog[2, Sqrt[4 + g*x^2]/2])/x^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^3\sqrt{gx^2+4}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^3\sqrt{gx^2+4}}dx$$

input
```
Int[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^3*Sqrt[4 + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a + b\ln\left(c(dg\,x^2 + 4d)^p\right)}{x^3\sqrt{g\,x^2 + 4}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^3/(g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^3/(g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{a + b\log\left(c(4d + dgx^2)^p\right)}{x^3\sqrt{4 + gx^2}}\,dx = \int \frac{b\log\left((dgx^2 + 4\,d)^p c\right) + a}{\sqrt{gx^2 + 4}x^3}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^3/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((sqrt(g*x^2 + 4)*b*log((d*g*x^2 + 4*d)^p*c) + sqrt(g*x^2 + 4)*a)/
(g*x^5 + 4*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^3\sqrt{4+gx^2}}\,dx = \int \frac{a+b\log\left(c(dgx^2+4d)^p\right)}{x^3\sqrt{gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+4*d)**p))/x**3/(g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*g*x**2 + 4*d)**p))/(x**3*sqrt(g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^3\sqrt{4+gx^2}}\,dx = \int \frac{b\log\left((dgx^2+4\,d)^p c\right)+a}{\sqrt{gx^2+4}x^3}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^3/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
1/16*(g*arcsinh(2/(sqrt(g)*abs(x))) - 2*sqrt(g*x^2 + 4)/x^2)*a + b*integra
te((p*log(g*x^2 + 4) + p*log(d) + log(c))/(sqrt(g*x^2 + 4)*x^3), x)
```

Giac [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^3\sqrt{4+gx^2}}\,dx = \int \frac{b\log\left((dgx^2+4\,d)^p c\right)+a}{\sqrt{gx^2+4}x^3}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^3/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*log((d*g*x^2 + 4*d)^p*c) + a)/(sqrt(g*x^2 + 4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^3\sqrt{4+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+4\,d\right)}^p\right)}{x^3\,\sqrt{g\,x^2+4}}\,dx$$

input
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^3*(g*x^2 + 4)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^3*(g*x^2 + 4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^3\sqrt{4+gx^2}}\,dx$$
$$=\frac{-2\sqrt{g\,x^2+4}\,a+16\left(\int \frac{\log\left(\left(dg\,x^2+4d\right)^p c\right)}{\sqrt{g\,x^2+4}\,x^3}dx\right)b\,x^2-\log\left(\frac{\sqrt{g\,x^2+4}}{2}+\frac{\sqrt{g}\,x}{2}-1\right)ag\,x^2+\log\left(\frac{\sqrt{g\,x^2+4}}{2}+\frac{\sqrt{g}\,x}{2}+\right.}{16x^2}$$

input
```
int((a+b*log(c*(d*g*x^2+4*d)^p))/x^3/(g*x^2+4)^(1/2),x)
```

output
```
( - 2*sqrt(g*x**2 + 4)*a + 16*int(log((d*g*x**2 + 4*d)**p*c)/(sqrt(g*x**2
+ 4)*x**3),x)*b*x**2 - log((sqrt(g*x**2 + 4) + sqrt(g)*x - 2)/2)*a*g*x**2
+ log((sqrt(g*x**2 + 4) + sqrt(g)*x + 2)/2)*a*g*x**2)/(16*x**2)
```

3.669 $\int \frac{a+b\log\left(c\left(4d+dgx^2\right)^p\right)}{x^5\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 216

$$\begin{aligned}\int \frac{a+b\log\left(c\left(4d+dgx^2\right)^p\right)}{x^5\sqrt{4+gx^2}}\,dx &= -\frac{bgp\sqrt{4+gx^2}}{64x^2}+\frac{1}{32}bg^2p\operatorname{arctanh}\left(\frac{1}{2}\sqrt{4+gx^2}\right)\\ &\quad-\frac{\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{16x^4}\\ &\quad+\frac{3g\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{128x^2}\\ &\quad-\frac{3}{256}g^2\operatorname{arctanh}\left(\frac{1}{2}\sqrt{4+gx^2}\right)\left(a\right.\\ &\quad\left.+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)\\ &\quad-\frac{3}{256}bg^2p\operatorname{PolyLog}\left(2,-\frac{1}{2}\sqrt{4+gx^2}\right)\\ &\quad+\frac{3}{256}bg^2p\operatorname{PolyLog}\left(2,\frac{1}{2}\sqrt{4+gx^2}\right)\end{aligned}$$

output

```
-1/64*b*g*p*(g*x^2+4)^(1/2)/x^2+1/32*b*g^2*p*arctanh(1/2*(g*x^2+4)^(1/2))-
1/16*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/x^4+3/128*g*(g*x^2+4)^(1/
2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/x^2-3/256*g^2*arctanh(1/2*(g*x^2+4)^(1/2))*
(a+b*ln(c*(d*g*x^2+4*d)^p))-3/256*b*g^2*p*polylog(2,-1/2*(g*x^2+4)^(1/2))+
3/256*b*g^2*p*polylog(2,1/2*(g*x^2+4)^(1/2))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.61

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^5\sqrt{4+gx^2}}\,dx = -\frac{32a\sqrt{4+gx^2}-12agx^2\sqrt{4+gx^2}+8bgpx^2\sqrt{4+gx^2}+32b\sqrt{4+gx^2}\log\left(c(d(4+gx^2))^p\right)-12bgx^2\sqrt{4}}{}$$

input
```
Integrate[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^5*Sqrt[4 + g*x^2]),x]
```

output
```
-1/512*(32*a*Sqrt[4 + g*x^2] - 12*a*g*x^2*Sqrt[4 + g*x^2] + 8*b*g*p*x^2*Sq
rt[4 + g*x^2] + 32*b*Sqrt[4 + g*x^2]*Log[c*(d*(4 + g*x^2))^p] - 12*b*g*x^2
*Sqrt[4 + g*x^2]*Log[c*(d*(4 + g*x^2))^p] - 3*a*g^2*x^4*Log[2 - Sqrt[4 + g
*x^2]] + 8*b*g^2*p*x^4*Log[2 - Sqrt[4 + g*x^2]] - 3*b*g^2*x^4*Log[c*(d*(4
+ g*x^2))^p]*Log[2 - Sqrt[4 + g*x^2]] + 3*a*g^2*x^4*Log[2 + Sqrt[4 + g*x^2
]] - 8*b*g^2*p*x^4*Log[2 + Sqrt[4 + g*x^2]] + 3*b*g^2*x^4*Log[c*(d*(4 + g*
x^2))^p]*Log[2 + Sqrt[4 + g*x^2]] + 6*b*g^2*p*x^4*PolyLog[2, -1/2*Sqrt[4 +
 g*x^2]] - 6*b*g^2*p*x^4*PolyLog[2, Sqrt[4 + g*x^2]/2])/x^4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^5\sqrt{gx^2+4}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^5\sqrt{gx^2+4}}dx$$

input
```
Int[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^5*Sqrt[4 + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(dg\,x^2+4d)^p\right)}{x^5\sqrt{g\,x^2+4}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^5/(g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^5/(g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^5\sqrt{4+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+4\,d)^pc\right)+a}{\sqrt{gx^2+4}x^5}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^5/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((sqrt(g*x^2 + 4)*b*log((d*g*x^2 + 4*d)^p*c) + sqrt(g*x^2 + 4)*a)/
(g*x^7 + 4*x^5), x)
```

Sympy [F]

$$\int \frac{a + b\log\left(c(4d + dgx^2)^p\right)}{x^5\sqrt{4 + gx^2}}\,dx = \int \frac{a + b\log\left(c(dgx^2 + 4d)^p\right)}{x^5\sqrt{gx^2 + 4}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+4*d)**p))/x**5/(g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*g*x**2 + 4*d)**p))/(x**5*sqrt(g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a + b\log\left(c(4d + dgx^2)^p\right)}{x^5\sqrt{4 + gx^2}}\,dx = \int \frac{b\log\left((dgx^2 + 4\,d)^p c\right) + a}{\sqrt{gx^2 + 4}x^5}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^5/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
-1/256*(3*g^2*arcsinh(2/(sqrt(g)*abs(x))) - 6*sqrt(g*x^2 + 4)*g/x^2 + 16*s
qrt(g*x^2 + 4)/x^4)*a + b*integrate((p*log(g*x^2 + 4) + p*log(d) + log(c))
/(sqrt(g*x^2 + 4)*x^5), x)
```

Giac [F]

$$\int \frac{a + b\log\left(c(4d + dgx^2)^p\right)}{x^5\sqrt{4 + gx^2}}\,dx = \int \frac{b\log\left((dgx^2 + 4\,d)^p c\right) + a}{\sqrt{gx^2 + 4}x^5}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^5/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*log((d*g*x^2 + 4*d)^p*c) + a)/(sqrt(g*x^2 + 4)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^5\sqrt{4+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,(d\,g\,x^2+4\,d)^p\right)}{x^5\,\sqrt{g\,x^2+4}}\,dx$$

input
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^5*(g*x^2 + 4)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^5*(g*x^2 + 4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^5\sqrt{4+gx^2}}\,dx$$
$$=\frac{6\sqrt{g\,x^2+4}\,ag\,x^2-16\sqrt{g\,x^2+4}\,a+256\left(\int \frac{\log\left(\left(dg\,x^2+4d\right)^p c\right)}{\sqrt{g\,x^2+4}\,x^5}dx\right)b\,x^4+3\log\left(\frac{\sqrt{g\,x^2+4}}{2}+\frac{\sqrt{g}\,x}{2}-1\right)a\,g^2x^4-}{256x^4}$$

input
```
int((a+b*log(c*(d*g*x^2+4*d)^p))/x^5/(g*x^2+4)^(1/2),x)
```

output
```
(6*sqrt(g*x**2 + 4)*a*g*x**2 - 16*sqrt(g*x**2 + 4)*a + 256*int(log((d*g*x*
*2 + 4*d)**p*c)/(sqrt(g*x**2 + 4)*x**5),x)*b*x**4 + 3*log((sqrt(g*x**2 + 4
) + sqrt(g)*x - 2)/2)*a*g**2*x**4 - 3*log((sqrt(g*x**2 + 4) + sqrt(g)*x +
2)/2)*a*g**2*x**4)/(256*x**4)
```

3.670 $\int \frac{x^2\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 383

$$\begin{aligned}
&\int \frac{x^2\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx \\
&= \frac{2bp}{g^{3/2}\left(\sqrt{g}x+\sqrt{4+gx^2}\right)^2}+\frac{a\left(\sqrt{g}x+\sqrt{4+gx^2}\right)^2}{8g^{3/2}}-\frac{bp\left(\sqrt{g}x+\sqrt{4+gx^2}\right)^2}{8g^{3/2}} \\
&\quad+\frac{b\left(\sqrt{g}x+\sqrt{4+gx^2}\right)^2\log\left(c(d(4+gx^2))^p\right)}{8g^{3/2}}-\frac{2\left(a+b\log\left(c(d(4+gx^2))^p\right)\right)}{g^{3/2}\left(\sqrt{g}x+\sqrt{4+gx^2}\right)^2} \\
&\quad+\frac{2bp\log\left(\sqrt{g}x+\sqrt{4+gx^2}\right)}{g^{3/2}}-\frac{2\left(a+b\log\left(c(d(4+gx^2))^p\right)\right)\log\left(\sqrt{g}x+\sqrt{4+gx^2}\right)}{g^{3/2}} \\
&\quad-\frac{2bp\log^2\left(\sqrt{g}x+\sqrt{4+gx^2}\right)}{g^{3/2}} \\
&\quad+\frac{4bp\log\left(\sqrt{g}x+\sqrt{4+gx^2}\right)\log\left(1+\frac{1}{4}\left(\sqrt{g}x+\sqrt{4+gx^2}\right)^2\right)}{g^{3/2}} \\
&\quad+\frac{2bp\,\text{PolyLog}\left(2,-\frac{1}{4}\left(\sqrt{g}x+\sqrt{4+gx^2}\right)^2\right)}{g^{3/2}}
\end{aligned}$$

output
```
2*b*p/g^(3/2)/(g^(1/2)*x+(g*x^2+4)^(1/2))^2+1/8*a*(g^(1/2)*x+(g*x^2+4)^(1/
2))^2/g^(3/2)-1/8*b*p*(g^(1/2)*x+(g*x^2+4)^(1/2))^2/g^(3/2)+1/8*b*(g^(1/2)
*x+(g*x^2+4)^(1/2))^2*ln(c*(d*(g*x^2+4))^p)/g^(3/2)-2*(a+b*ln(c*(d*(g*x^2+
4))^p))/g^(3/2)/(g^(1/2)*x+(g*x^2+4)^(1/2))^2+2*b*p*ln(g^(1/2)*x+(g*x^2+4)
^(1/2))/g^(3/2)-2*(a+b*ln(c*(d*(g*x^2+4))^p))*ln(g^(1/2)*x+(g*x^2+4)^(1/2)
)/g^(3/2)-2*b*p*ln(g^(1/2)*x+(g*x^2+4)^(1/2))^2/g^(3/2)+4*b*p*ln(g^(1/2)*x
+(g*x^2+4)^(1/2))*ln(1+1/4*(g^(1/2)*x+(g*x^2+4)^(1/2))^2)/g^(3/2)+2*b*p*po
lylog(2,-1/4*(g^(1/2)*x+(g*x^2+4)^(1/2))^2)/g^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.75 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.43

$$\int \frac{x^2\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}}\,dx = \frac{1}{2}\left(-\frac{4bpx}{g\sqrt{4+gx^2}}-\frac{bpx^3}{\sqrt{4+gx^2}}+\frac{ax\sqrt{4+gx^2}}{g}-\frac{4a\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}{g^{3/2}}+\frac{4bp\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}{g^{3/2}}+\frac{8ibp\pi\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}{g^{3/2}}+\frac{4bp\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)^2}{g^{3/2}}-\frac{4ibp\pi\log\left(1-ie^{-\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}\right)}{g^{3/2}}+\frac{8bp\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)\log\left(1-ie^{-\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}\right)}{g^{3/2}}+\frac{4ibp\pi\log\left(1+ie^{-\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}\right)}{g^{3/2}}+\frac{8bp\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)\log\left(1+ie^{-\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}\right)}{g^{3/2}}-\frac{16ibp\pi\log\left(1+e^{\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}\right)}{g^{3/2}}+\frac{bx\sqrt{4+gx^2}\log\left(c(d(4+gx^2))^p\right)}{g}-\frac{4b\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)\log\left(c(d(4+gx^2))^p\right)}{g^{3/2}}-\frac{4ibp\pi\log\left(-\cos\left(\frac{1}{4}\left(\pi+2i\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)\right)\right)\right)}{g^{3/2}}+\frac{16ibp\pi\log\left(\cosh\left(\frac{1}{2}\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)\right)\right)}{g^{3/2}}+\frac{4ibp\pi\log\left(\sin\left(\frac{1}{4}\left(\pi+2i\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)\right)\right)\right)}{g^{3/2}}-\frac{8bp\,\text{PolyLog}\left(2,-ie^{-\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}\right)}{g^{3/2}}-\frac{8bp\,\text{PolyLog}\left(2,ie^{-\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}\right)}{g^{3/2}}\right)$$

input
```
Integrate[(x^2*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
((-4*b*p*x)/(g*Sqrt[4 + g*x^2]) - (b*p*x^3)/Sqrt[4 + g*x^2] + (a*x*Sqrt[4
+ g*x^2])/g - (4*a*ArcSinh[(Sqrt[g]*x)/2])/g^(3/2) + (4*b*p*ArcSinh[(Sqrt[
g]*x)/2])/g^(3/2) + ((8*I)*b*p*Pi*ArcSinh[(Sqrt[g]*x)/2])/g^(3/2) + (4*b*p
*ArcSinh[(Sqrt[g]*x)/2]^2)/g^(3/2) - ((4*I)*b*p*Pi*Log[1 - I/E^ArcSinh[(Sq
rt[g]*x)/2]])/g^(3/2) + (8*b*p*ArcSinh[(Sqrt[g]*x)/2]*Log[1 - I/E^ArcSinh[
(Sqrt[g]*x)/2]])/g^(3/2) + ((4*I)*b*p*Pi*Log[1 + I/E^ArcSinh[(Sqrt[g]*x)/2
]])/g^(3/2) + (8*b*p*ArcSinh[(Sqrt[g]*x)/2]*Log[1 + I/E^ArcSinh[(Sqrt[g]*x
)/2]])/g^(3/2) - ((16*I)*b*p*Pi*Log[1 + E^ArcSinh[(Sqrt[g]*x)/2]])/g^(3/2)
 + (b*x*Sqrt[4 + g*x^2]*Log[c*(d*(4 + g*x^2))^p])/g - (4*b*ArcSinh[(Sqrt[g
]*x)/2]*Log[c*(d*(4 + g*x^2))^p])/g^(3/2) - ((4*I)*b*p*Pi*Log[-Cos[(Pi + (
2*I)*ArcSinh[(Sqrt[g]*x)/2])/4]])/g^(3/2) + ((16*I)*b*p*Pi*Log[Cosh[ArcSin
h[(Sqrt[g]*x)/2]/2]])/g^(3/2) + ((4*I)*b*p*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[
(Sqrt[g]*x)/2])/4]])/g^(3/2) - (8*b*p*PolyLog[2, (-I)/E^ArcSinh[(Sqrt[g]*x
)/2]])/g^(3/2) - (8*b*p*PolyLog[2, I/E^ArcSinh[(Sqrt[g]*x)/2]])/g^(3/2))/2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\log\left(c\left(dgx^2+4d\right)^p\right)\right)}{\sqrt{gx^2+4}}dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^2\left(a+b\log\left(c\left(dgx^2+4d\right)^p\right)\right)}{\sqrt{gx^2+4}}dx$$

input
```
Int[(x^2*(a + b*Log[c*(4*d + d*g*x^2)^p]))/Sqrt[4 + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^{2} \left(a+b \ln \left(c {\left(d g \,x^{2}+4 d \right)}^{p}\right)\right)}{\sqrt{g \,x^{2}+4}}d x$$

input
```
int(x^2*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
int(x^2*(a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2 \left(a+b \log \left(c(4d+dgx^2)^p\right)\right)}{\sqrt{4+gx^2}} \, dx=\int \frac{{\left(b \log \left({\left(d g x^{2} + 4 \, d\right)}^{p} c\right) + a\right)} x^{2}}{\sqrt{g x^{2} + 4}} \,d x$$

input
```
integrate(x^2*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((sqrt(g*x^2 + 4)*b*x^2*log((d*g*x^2 + 4*d)^p*c) + sqrt(g*x^2 + 4)
*a*x^2)/(g*x^2 + 4), x)
```

Sympy [F]

$$\int \frac{x^2 \left(a + b \log \left(c(4d + dgx^2)^p\right)\right)}{\sqrt{4 + gx^2}} \, dx = \int \frac{x^2 \left(a + b \log \left(c(dgx^2 + 4d)^p\right)\right)}{\sqrt{gx^2 + 4}} \, dx$$

input
```
integrate(x**2*(a+b*ln(c*(d*g*x**2+4*d)**p))/(g*x**2+4)**(1/2),x)
```

output
```
Integral(x**2*(a + b*log(c*(d*g*x**2 + 4*d)**p))/sqrt(g*x**2 + 4), x)
```

Maxima [F]

$$\int \frac{x^2 \left(a + b \log \left(c(4d + dgx^2)^p\right)\right)}{\sqrt{4 + gx^2}} \, dx = \int \frac{\left(b \log \left((dgx^2 + 4\,d)^p c\right) + a\right)x^2}{\sqrt{gx^2 + 4}} \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
1/2*a*(sqrt(g*x^2 + 4)*x/g - 4*arcsinh(1/2*sqrt(g)*x)/g^(3/2)) + b*integra
te((p*x^2*log(g*x^2 + 4) + (p*log(d) + log(c))*x^2)/sqrt(g*x^2 + 4), x)
```

Giac [F]

$$\int \frac{x^2 \left(a + b \log \left(c(4d + dgx^2)^p\right)\right)}{\sqrt{4 + gx^2}} \, dx = \int \frac{\left(b \log \left((dgx^2 + 4\,d)^p c\right) + a\right)x^2}{\sqrt{gx^2 + 4}} \, dx$$

input
```
integrate(x^2*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*log((d*g*x^2 + 4*d)^p*c) + a)*x^2/sqrt(g*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d\,g\,x^2+4\,d)^p))}{\sqrt{g\,x^2+4}}\,dx$$

input
```
int((x^2*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2),x)
```

output
```
int((x^2*(a + b*log(c*(4*d + d*g*x^2)^p)))/(g*x^2 + 4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\log(c(4d+dgx^2)^p))}{\sqrt{4+gx^2}}\,dx = \frac{\sqrt{g\,x^2+4}\,agx-4\sqrt{g}\log\left(\frac{\sqrt{g\,x^2+4}}{2}+\frac{\sqrt{g}\,x}{2}\right)a+2\left(\int \frac{\log((dg\,x^2+4d)^p\,c)\,x^2}{\sqrt{g\,x^2+4}}dx\right)b\,g^2}{2g^2}$$

input
```
int(x^2*(a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
(sqrt(g*x**2 + 4)*a*g*x - 4*sqrt(g)*log((sqrt(g*x**2 + 4) + sqrt(g)*x)/2)*
a + 2*int((log((d*g*x**2 + 4*d)**p*c)*x**2)/sqrt(g*x**2 + 4),x)*b*g**2)/(2
*g**2)
```

3.671 $\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 174

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{\sqrt{4+gx^2}}\,dx = \frac{\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{\sqrt{g}} + \frac{bp\log^2\left(\frac{\sqrt{g}x}{2}+\sqrt{1+\frac{gx^2}{4}}\right)}{\sqrt{g}} - \frac{2bp\log\left(\frac{\sqrt{g}x}{2}+\sqrt{1+\frac{gx^2}{4}}\right)\log\left(1+\left(\frac{\sqrt{g}x}{2}+\sqrt{1+\frac{gx^2}{4}}\right)^2\right)}{\sqrt{g}} - \frac{bp\,\text{PolyLog}\left(2,-\frac{1}{4}\left(\sqrt{g}x+\sqrt{4+gx^2}\right)^2\right)}{\sqrt{g}}$$

output

```
arcsinh(1/2*g^(1/2)*x)*(a+b*ln(c*(d*g*x^2+4*d)^p))/g^(1/2)+b*p*ln(1/2*g^(1
/2)*x+1/2*(g*x^2+4)^(1/2))^2/g^(1/2)-2*b*p*ln(1/2*g^(1/2)*x+1/2*(g*x^2+4)^
(1/2))*ln(1+(1/2*g^(1/2)*x+1/2*(g*x^2+4)^(1/2))^2)/g^(1/2)-b*p*polylog(2,-
1/4*(g^(1/2)*x+(g*x^2+4)^(1/2))^2)/g^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.14

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{\sqrt{4+gx^2}}\,dx$$
$$= \frac{a\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)-2ibp\pi\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)-bp\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)^2+ibp\pi\log\left(1-ie^{-\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)}\right)-2bp\text{arcsinh}}{}$$

input
```
Integrate[(a + b*Log[c*(4*d + d*g*x^2)^p])/Sqrt[4 + g*x^2],x]
```

output
```
(a*ArcSinh[(Sqrt[g]*x)/2] - (2*I)*b*p*Pi*ArcSinh[(Sqrt[g]*x)/2] - b*p*ArcS
inh[(Sqrt[g]*x)/2]^2 + I*b*p*Pi*Log[1 - I/E^ArcSinh[(Sqrt[g]*x)/2]] - 2*b*
p*ArcSinh[(Sqrt[g]*x)/2]*Log[1 - I/E^ArcSinh[(Sqrt[g]*x)/2]] - I*b*p*Pi*Lo
g[1 + I/E^ArcSinh[(Sqrt[g]*x)/2]] - 2*b*p*ArcSinh[(Sqrt[g]*x)/2]*Log[1 + I
/E^ArcSinh[(Sqrt[g]*x)/2]] + (4*I)*b*p*Pi*Log[1 + E^ArcSinh[(Sqrt[g]*x)/2]
] + b*ArcSinh[(Sqrt[g]*x)/2]*Log[c*(d*(4 + g*x^2))^p] + I*b*p*Pi*Log[-Cos[
(Pi + (2*I)*ArcSinh[(Sqrt[g]*x)/2])/4]] - (4*I)*b*p*Pi*Log[Cosh[ArcSinh[(S
qrt[g]*x)/2]/2]] - I*b*p*Pi*Log[Sin[(Pi + (2*I)*ArcSinh[(Sqrt[g]*x)/2])/4]
] + 2*b*p*PolyLog[2, (-I)/E^ArcSinh[(Sqrt[g]*x)/2]] + 2*b*p*PolyLog[2, I/E
^ArcSinh[(Sqrt[g]*x)/2]])/Sqrt[g]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{\sqrt{gx^2+4}}\,dx$$
$$\Big\downarrow \text{ 2923}$$
$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{\sqrt{gx^2+4}}dx$$

input
```
Int[(a + b*Log[c*(4*d + d*g*x^2)^p])/Sqrt[4 + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a + b\ln\left(c(dg\,x^2 + 4d)^p\right)}{\sqrt{g\,x^2 + 4}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{a + b\log\left(c(4d + dgx^2)^p\right)}{\sqrt{4 + gx^2}}\,dx = \int \frac{b\log\left((dgx^2 + 4\,d)^p c\right) + a}{\sqrt{gx^2 + 4}}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(g*x^2 + 4)*b*log((d*g*x^2 + 4*d)^p*c) + sqrt(g*x^2 + 4)*a)/
(g*x^2 + 4), x)
```

Sympy [F]

$$\int \frac{a+b\log \left(c(4d+dgx^2)^p\right)}{\sqrt{4+gx^2}}\,dx=\int \frac{a+b\log \left(c(dgx^2+4d)^p\right)}{\sqrt{gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+4*d)**p))/(g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*g*x**2 + 4*d)**p))/sqrt(g*x**2 + 4), x)
```

Maxima [F]

$$\int \frac{a+b\log \left(c(4d+dgx^2)^p\right)}{\sqrt{4+gx^2}}\,dx=\int \frac{b\log \left((dgx^2+4\,d)^pc\right)+a}{\sqrt{gx^2+4}}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="maxim
a")
```

output
```
b*integrate((p*log(g*x^2 + 4) + p*log(d) + log(c))/sqrt(g*x^2 + 4), x) + a
*arcsinh(1/2*sqrt(g)*x)/sqrt(g)
```

Giac [F]

$$\int \frac{a+b\log \left(c(4d+dgx^2)^p\right)}{\sqrt{4+gx^2}}\,dx=\int \frac{b\log \left((dgx^2+4\,d)^pc\right)+a}{\sqrt{gx^2+4}}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*log((d*g*x^2 + 4*d)^p*c) + a)/sqrt(g*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{\sqrt{4+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+4\,d\right)}^p\right)}{\sqrt{g\,x^2+4}}\,dx$$

input
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(g*x^2 + 4)^(1/2),x)
```

output
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(g*x^2 + 4)^(1/2), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{\sqrt{4+gx^2}}\,dx=\frac{\sqrt{g}\log\left(\frac{\sqrt{g\,x^2+4}}{2}+\frac{\sqrt{g}\,x}{2}\right)a+\left(\int \frac{\log\left(\left(dg\,x^2+4d\right)^p c\right)}{\sqrt{g\,x^2+4}}dx\right)bg}{g}$$

input
```
int((a+b*log(c*(d*g*x^2+4*d)^p))/(g*x^2+4)^(1/2),x)
```

output
```
(sqrt(g)*log((sqrt(g*x**2 + 4) + sqrt(g)*x)/2)*a + int(log((d*g*x**2 + 4*d
)**p*c)/sqrt(g*x**2 + 4),x)*b*g)/g
```

3.672 $\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^2\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 60

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^2\sqrt{4+gx^2}}\,dx=\frac{1}{2}b\sqrt{g}p\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)-\frac{\sqrt{4+gx^2}\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{4x}$$

output
```
1/2*b*g^(1/2)*p*arcsinh(1/2*g^(1/2)*x)-1/4*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*
x^2+4*d)^p))/x
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^2\sqrt{4+gx^2}}\,dx=\frac{1}{2}b\sqrt{g}p\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)-\frac{\sqrt{4+gx^2}\left(a+b\log\left(c(d(4+gx^2))^p\right)\right)}{4x}$$

input
```
Integrate[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^2*Sqrt[4 + g*x^2]),x]
```

output
```
(b*Sqrt[g]*p*ArcSinh[(Sqrt[g]*x)/2])/2 - (Sqrt[4 + g*x^2]*(a + b*Log[c*(d*
(4 + g*x^2))^p]))/(4*x)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^2\sqrt{gx^2+4}}dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^2\sqrt{gx^2+4}}dx$$

input
```
Int[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^2*Sqrt[4 + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a + b\ln\left(c(dg\,x^2 + 4d)^p\right)}{x^2\sqrt{g\,x^2 + 4}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^2/(g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^2/(g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.37

$$\int \frac{a + b\log\left(c(4d + dgx^2)^p\right)}{x^2\sqrt{4 + gx^2}}\,dx = \left[\frac{2\,b\sqrt{g}px\log\left(-\sqrt{g}x - \sqrt{gx^2 + 4}\right) - \sqrt{gx^2 + 4}(bp\log\left(dgx^2 + 4\,d\right) + b\log\left(c\right) + a)}{4\,x}, -\frac{4\,b\sqrt{-g}px\arctan\left(\frac{\sqrt{gx^2+4}\sqrt{-g}-2\sqrt{-g}}{gx}\right) + \sqrt{gx^2 + 4}(bp\log\left(dgx^2 + 4\,d\right) + b\log\left(c\right) + a)}{4\,x}\right]$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^2/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
[1/4*(2*b*sqrt(g)*p*x*log(-sqrt(g)*x - sqrt(g*x^2 + 4)) - sqrt(g*x^2 + 4)*
(b*p*log(d*g*x^2 + 4*d) + b*log(c) + a))/x, -1/4*(4*b*sqrt(-g)*p*x*arctan(
(sqrt(g*x^2 + 4)*sqrt(-g) - 2*sqrt(-g))/(g*x)) + sqrt(g*x^2 + 4)*(b*p*log(
d*g*x^2 + 4*d) + b*log(c) + a))/x]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^2\sqrt{4+gx^2}}\,dx=\int \frac{a+b\log\left(c(dgx^2+4d)^p\right)}{x^2\sqrt{gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+4*d)**p))/x**2/(g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*g*x**2 + 4*d)**p))/(x**2*sqrt(g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^2\sqrt{4+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+4\,d)^pc\right)+a}{\sqrt{gx^2+4}x^2}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^2/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
1/4*(4*g^2*p*integrate(1/2*x^2/(g*x^2 + 4)^(3/2), x) - ((g*p*log(d) - 2*g*
p + g*log(c))*x^2 + (g*p*x^2 + 4*p)*log(g*x^2 + 4) + 4*p*log(d) + 4*log(c)
)/(sqrt(g*x^2 + 4)*x))*b - 1/4*sqrt(g*x^2 + 4)*a/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^2\sqrt{4+gx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^2/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^2\sqrt{4+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,(d\,g\,x^2+4\,d)^p\right)}{x^2\,\sqrt{g\,x^2+4}}\,dx$$

input
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^2*(g*x^2 + 4)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^2*(g*x^2 + 4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.08

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^2\sqrt{4+gx^2}}\,dx = \frac{-\sqrt{g\,x^2+4}\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^pc}{\left(\sqrt{g\,x^2+4}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b-\sqrt{g\,x^2+4}\,a+2\sqrt{g}\log\left(\frac{\sqrt{g}\,\sqrt{g\,x^2+4}\,x}{2}+\frac{g\,x^2}{2}+2\right)bpx-}{4x}$$

input
```
int((a+b*log(c*(d*g*x^2+4*d)^p))/x^2/(g*x^2+4)^(1/2),x)
```

output
```
( - sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**
(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b - sqrt(g
*x**2 + 4)*a + 2*sqrt(g)*log((sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)/2)*
b*p*x - sqrt(g)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**(2*p)
*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*x - sqrt(g)*a
*x)/(4*x)
```

3.673 $\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^4\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 119

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^4\sqrt{4+gx^2}}\,dx = -\frac{bgp\sqrt{4+gx^2}}{24x}-\frac{1}{12}bg^{3/2}p\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)-\frac{\sqrt{4+gx^2}\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{12x^3}+\frac{g\sqrt{4+gx^2}\left(a+b\log\left(c(4d+dgx^2)^p\right)\right)}{24x}$$

output
```
-1/24*b*g*p*(g*x^2+4)^(1/2)/x-1/12*b*g^(3/2)*p*arcsinh(1/2*g^(1/2)*x)-1/12
*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/x^3+1/24*g*(g*x^2+4)^(1/2)*(a
+b*ln(c*(d*g*x^2+4*d)^p))/x
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^4\sqrt{4+gx^2}}\,dx = \frac{1}{24}\left(-2bg^{3/2}p\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right) + \frac{\sqrt{4+gx^2}(-2a+agx^2-bgpx^2+b(-2+gx^2)\log\left(c(d(4+gx^2))^p\right))}{x^3}\right)$$

input
```
Integrate[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^4*Sqrt[4 + g*x^2]),x]
```

output
```
(-2*b*g^(3/2)*p*ArcSinh[(Sqrt[g]*x)/2] + (Sqrt[4 + g*x^2]*(-2*a + a*g*x^2
- b*g*p*x^2 + b*(-2 + g*x^2)*Log[c*(d*(4 + g*x^2))^p]))/x^3)/24
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^4\sqrt{gx^2+4}}\,dx$$
$$\Big\downarrow \text{2929}$$
$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^4\sqrt{gx^2+4}}dx$$

input
```
Int[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^4*Sqrt[4 + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(dg\,x^{2}+4d)^{p}\right)}{x^{4}\sqrt{g\,x^{2}+4}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^4/(g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^4/(g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.84

$$\int \frac{a+b\log\left(c(4d+dgx^{2})^{p}\right)}{x^{4}\sqrt{4+gx^{2}}}\,dx$$
$$= \left[\frac{2\,bg^{\frac{3}{2}}px^{3}\log\left(\sqrt{g}x-\sqrt{gx^{2}+4}\right)-((bgp-ag)x^{2}-(bgpx^{2}-2\,bp)\log\left(dgx^{2}+4\,d\right)-(bgx^{2}-2\,b)\log\left(c\right.}{24\,x^{3}}\right.$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^4/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
[1/24*(2*b*g^(3/2)*p*x^3*log(sqrt(g)*x - sqrt(g*x^2 + 4)) - ((b*g*p - a*g)
*x^2 - (b*g*p*x^2 - 2*b*p)*log(d*g*x^2 + 4*d) - (b*g*x^2 - 2*b)*log(c) + 2
*a)*sqrt(g*x^2 + 4))/x^3, 1/24*(4*b*sqrt(-g)*g*p*x^3*arctan((sqrt(g*x^2 +
4)*sqrt(-g) - 2*sqrt(-g))/(g*x)) - ((b*g*p - a*g)*x^2 - (b*g*p*x^2 - 2*b*p
)*log(d*g*x^2 + 4*d) - (b*g*x^2 - 2*b)*log(c) + 2*a)*sqrt(g*x^2 + 4))/x^3]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^4\sqrt{4+gx^2}}\,dx=\int \frac{a+b\log\left(c(dgx^2+4d)^p\right)}{x^4\sqrt{gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+4*d)**p))/x**4/(g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*g*x**2 + 4*d)**p))/(x**4*sqrt(g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^4\sqrt{4+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+4\,d)^pc\right)+a}{\sqrt{gx^2+4}x^4}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^4/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
-1/24*(24*g^3*p*integrate(1/12*x^2/(g*x^2 + 4)^(3/2), x) - ((g^2*p*log(d)
- 3*g^2*p + g^2*log(c))*x^4 + 2*(g*p*log(d) - 2*g*p + g*log(c))*x^2 + (g^2
*p*x^4 + 2*g*p*x^2 - 8*p)*log(g*x^2 + 4) - 8*p*log(d) - 8*log(c))/(sqrt(g*
x^2 + 4)*x^3))*b + 1/24*a*(sqrt(g*x^2 + 4)*g/x - 2*sqrt(g*x^2 + 4)/x^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^4\sqrt{4+gx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^4/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(4d+dgx^2)^p\right)}{x^4\sqrt{4+gx^2}}\,dx=\int \frac{a+b\,\ln \left(c\,{\left(d\,g\,x^2+4\,d\right)}^p\right)}{x^4\,\sqrt{g\,x^2+4}}\,dx$$

input
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^4*(g*x^2 + 4)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^4*(g*x^2 + 4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.58

$$\int \frac{a+b\log \left(c(4d+dgx^2)^p\right)}{x^4\sqrt{4+gx^2}}\,dx$$
$$=\frac{3\sqrt{g\,x^2+4}\,\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^p c}{\left(\sqrt{g\,x^2+4}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b g\,x^2-6\sqrt{g\,x^2+4}\,\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^p c}{\left(\sqrt{g\,x^2+4}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b+3\sqrt{}}{}$$

input
```
int((a+b*log(c*(d*g*x^2+4*d)^p))/x^4/(g*x^2+4)^(1/2),x)
```

output

```
(3*sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**(
2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g*x**2 -
6*sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**(2
*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b + 3*sqrt(g
*x**2 + 4)*a*g*x**2 - 6*sqrt(g*x**2 + 4)*a - 3*sqrt(g*x**2 + 4)*b*g*p*x**2
 - 6*sqrt(g)*log((sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)/2)*b*g*p*x**3 +
 3*sqrt(g)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**(2*p)*4**p
*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g*x**3 - 3*sqrt(g)
*a*g*x**3 + sqrt(g)*b*g*p*x**3)/(72*x**3)
```

3.674 $\int \frac{a+b\log\left(c\left(4d+dgx^2\right)^p\right)}{x^6\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 182

$$\begin{aligned}\int \frac{a+b\log\left(c\left(4d+dgx^2\right)^p\right)}{x^6\sqrt{4+gx^2}}\,dx = &-\frac{bgp\sqrt{4+gx^2}}{120x^3}+\frac{bg^2p\sqrt{4+gx^2}}{80x}\\ &+\frac{1}{60}bg^{5/2}p\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)\\ &-\frac{\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{20x^5}\\ &+\frac{g\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{60x^3}\\ &-\frac{g^2\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{120x}\end{aligned}$$

output
```
-1/120*b*g*p*(g*x^2+4)^(1/2)/x^3+1/80*b*g^2*p*(g*x^2+4)^(1/2)/x+1/60*b*g^(
5/2)*p*arcsinh(1/2*g^(1/2)*x)-1/20*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+4*d)
^p))/x^5+1/60*g*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/x^3-1/120*g^2*
(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/x
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.59

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^6\sqrt{4+gx^2}}\,dx = \frac{1}{240}\left(4bg^{5/2}p\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)-\frac{\sqrt{4+gx^2}\left(bgpx^2(2-3gx^2)+2a(6-2gx^2+g^2x^4)+2b(6-2gx^2+g^2x^4)\log\left(c(d(4+gx^2))^p\right)\right)}{x^5}\right)$$

input
```
Integrate[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^6*Sqrt[4 + g*x^2]),x]
```

output
```
(4*b*g^(5/2)*p*ArcSinh[(Sqrt[g]*x)/2] - (Sqrt[4 + g*x^2]*(b*g*p*x^2*(2 - 3
*g*x^2) + 2*a*(6 - 2*g*x^2 + g^2*x^4) + 2*b*(6 - 2*g*x^2 + g^2*x^4)*Log[c*
(d*(4 + g*x^2))^p]))/x^5)/240
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^6\sqrt{gx^2+4}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^6\sqrt{gx^2+4}}dx$$

input
```
Int[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^6*Sqrt[4 + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(dg\,x^2+4d)^p\right)}{x^6\sqrt{g\,x^2+4}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^6/(g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^6/(g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.63

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^6\sqrt{4+gx^2}}\,dx$$
$$= \left[\frac{4\,bg^{\frac{5}{2}}px^5\log\left(-\sqrt{g}x-\sqrt{gx^2+4}\right)+((3\,bg^2p-2\,ag^2)x^4-2\,(bgp-2\,ag)x^2-2\,(bg^2px^4-2\,bgpx^2+6\,b}{240\,x^5}\right.$$
$$-\frac{8\,b\sqrt{-g}g^2px^5\arctan\left(\frac{\sqrt{gx^2+4}\sqrt{-g}-2\sqrt{-g}}{gx}\right)-((3\,bg^2p-2\,ag^2)x^4-2\,(bgp-2\,ag)x^2-2\,(bg^2px^4-2\,bgp}{240\,x^5}$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^6/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
[1/240*(4*b*g^(5/2)*p*x^5*log(-sqrt(g)*x - sqrt(g*x^2 + 4)) + ((3*b*g^2*p
- 2*a*g^2)*x^4 - 2*(b*g*p - 2*a*g)*x^2 - 2*(b*g^2*p*x^4 - 2*b*g*p*x^2 + 6*
b*p)*log(d*g*x^2 + 4*d) - 2*(b*g^2*x^4 - 2*b*g*x^2 + 6*b)*log(c) - 12*a)*s
qrt(g*x^2 + 4))/x^5, -1/240*(8*b*sqrt(-g)*g^2*p*x^5*arctan((sqrt(g*x^2 + 4
)*sqrt(-g) - 2*sqrt(-g))/(g*x)) - ((3*b*g^2*p - 2*a*g^2)*x^4 - 2*(b*g*p -
2*a*g)*x^2 - 2*(b*g^2*p*x^4 - 2*b*g*p*x^2 + 6*b*p)*log(d*g*x^2 + 4*d) - 2*
(b*g^2*x^4 - 2*b*g*x^2 + 6*b)*log(c) - 12*a)*sqrt(g*x^2 + 4))/x^5]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^6\sqrt{4+gx^2}}\,dx=\int \frac{a+b\log\left(c(dgx^2+4d)^p\right)}{x^6\sqrt{gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+4*d)**p))/x**6/(g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*g*x**2 + 4*d)**p))/(x**6*sqrt(g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^6\sqrt{4+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+4\,d)^pc\right)+a}{\sqrt{gx^2+4}x^6}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^6/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
-1/120*(sqrt(g*x^2 + 4)*g^2/x - 2*sqrt(g*x^2 + 4)*g/x^3 + 6*sqrt(g*x^2 + 4
)/x^5)*a + 1/240*(240*g^4*p*integrate(1/60*x^2/(g*x^2 + 4)^(3/2), x) - ((2
*g^3*p*log(d) - 7*g^3*p + 2*g^3*log(c))*x^6 + 2*(2*g^2*p*log(d) - 5*g^2*p
+ 2*g^2*log(c))*x^4 - 4*(g*p*log(d) - 2*g*p + g*log(c))*x^2 + 2*(g^3*p*x^6
 + 2*g^2*p*x^4 - 2*g*p*x^2 + 24*p)*log(g*x^2 + 4) + 48*p*log(d) + 48*log(c
))/(sqrt(g*x^2 + 4)*x^5))*b
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^6\sqrt{4+gx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^6/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^6\sqrt{4+gx^2}}\,dx = \int \frac{a+b\,\ln\left(c\,(d\,g\,x^2+4\,d)^p\right)}{x^6\,\sqrt{g\,x^2+4}}\,dx$$

input
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^6*(g*x^2 + 4)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^6*(g*x^2 + 4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.34

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^6\sqrt{4+gx^2}}\,dx$$

$$= \frac{-10\sqrt{g\,x^2+4}\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^p c}{\left(\sqrt{g\,x^2+4}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b\,g^2x^4+20\sqrt{g\,x^2+4}\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^p c}{\left(\sqrt{g\,x^2+4}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)bg\cdots}{\cdots}$$

input
```
int((a+b*log(c*(d*g*x^2+4*d)^p))/x^6/(g*x^2+4)^(1/2),x)
```

output
```
( - 10*sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4
)**(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g**2*
x**4 + 20*sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2
+ 4)**(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g*
x**2 - 60*sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2
+ 4)**(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b -
10*sqrt(g*x**2 + 4)*a*g**2*x**4 + 20*sqrt(g*x**2 + 4)*a*g*x**2 - 60*sqrt(g
*x**2 + 4)*a + 15*sqrt(g*x**2 + 4)*b*g**2*p*x**4 - 10*sqrt(g*x**2 + 4)*b*g
*p*x**2 + 20*sqrt(g)*log((sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)/2)*b*g*
*2*p*x**5 - 10*sqrt(g)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)
**(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g**2*x
**5 + 10*sqrt(g)*a*g**2*x**5 - 11*sqrt(g)*b*g**2*p*x**5)/(1200*x**5)
```

3.675 $\int \frac{a+b\log\left(c\left(4d+dgx^2\right)^p\right)}{x^8\sqrt{4+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 245

$$\begin{aligned}\int \frac{a+b\log\left(c\left(4d+dgx^2\right)^p\right)}{x^8\sqrt{4+gx^2}}\,dx = &-\frac{bgp\sqrt{4+gx^2}}{280x^5}+\frac{bg^2p\sqrt{4+gx^2}}{336x^3} \\ &-\frac{11bg^3p\sqrt{4+gx^2}}{3360x}-\frac{1}{280}bg^{7/2}p\operatorname{arcsinh}\left(\frac{\sqrt{g}x}{2}\right) \\ &-\frac{\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{28x^7} \\ &+\frac{3g\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{280x^5} \\ &-\frac{g^2\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{280x^3} \\ &+\frac{g^3\sqrt{4+gx^2}\left(a+b\log\left(c\left(4d+dgx^2\right)^p\right)\right)}{560x}\end{aligned}$$

output
```
-1/280*b*g*p*(g*x^2+4)^(1/2)/x^5+1/336*b*g^2*p*(g*x^2+4)^(1/2)/x^3-11/3360
*b*g^3*p*(g*x^2+4)^(1/2)/x-1/280*b*g^(7/2)*p*arcsinh(1/2*g^(1/2)*x)-1/28*(
g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/x^7+3/280*g*(g*x^2+4)^(1/2)*(a+
b*ln(c*(d*g*x^2+4*d)^p))/x^5-1/280*g^2*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+
4*d)^p))/x^3+1/560*g^3*(g*x^2+4)^(1/2)*(a+b*ln(c*(d*g*x^2+4*d)^p))/x
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.53

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^8\sqrt{4+gx^2}}\,dx$$
$$=\frac{-12bg^{7/2}p\,\text{arcsinh}\left(\frac{\sqrt{g}x}{2}\right)+\frac{\sqrt{4+gx^2}(bgpx^2(-12+10gx^2-11g^2x^4)+6a(-20+6gx^2-2g^2x^4+g^3x^6)+6b(-20+6gx^2-2g^2x^4+g^3x^6)\,\text{lo}}{x^7}}{3360}$$

input
```
Integrate[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^8*Sqrt[4 + g*x^2]),x]
```

output
```
(-12*b*g^(7/2)*p*ArcSinh[(Sqrt[g]*x)/2] + (Sqrt[4 + g*x^2]*(b*g*p*x^2*(-12
 + 10*g*x^2 - 11*g^2*x^4) + 6*a*(-20 + 6*g*x^2 - 2*g^2*x^4 + g^3*x^6) + 6*
b*(-20 + 6*g*x^2 - 2*g^2*x^4 + g^3*x^6)*Log[c*(d*(4 + g*x^2))^p]))/x^7)/33
60
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^8\sqrt{gx^2+4}}\,dx$$
$$\Big\downarrow \text{2929}$$
$$\int \frac{a+b\log\left(c\left(dgx^2+4d\right)^p\right)}{x^8\sqrt{gx^2+4}}\,dx$$

input
```
Int[(a + b*Log[c*(4*d + d*g*x^2)^p])/(x^8*Sqrt[4 + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(dg\,x^2+4d)^p\right)}{x^8\sqrt{g\,x^2+4}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^8/(g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+4*d)^p))/x^8/(g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.52

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^8\sqrt{4+gx^2}}\,dx$$
$$= \left[\frac{12\,bg^{\frac{7}{2}}px^7\log\left(\sqrt{g}x-\sqrt{gx^2+4}\right)-((11\,bg^3p-6\,ag^3)x^6-2\,(5\,bg^2p-6\,ag^2)x^4+12\,(bgp-3\,ag)x^2-\right.$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^8/(g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
[1/3360*(12*b*g^(7/2)*p*x^7*log(sqrt(g)*x - sqrt(g*x^2 + 4)) - ((11*b*g^3*
p - 6*a*g^3)*x^6 - 2*(5*b*g^2*p - 6*a*g^2)*x^4 + 12*(b*g*p - 3*a*g)*x^2 -
6*(b*g^3*p*x^6 - 2*b*g^2*p*x^4 + 6*b*g*p*x^2 - 20*b*p)*log(d*g*x^2 + 4*d)
- 6*(b*g^3*x^6 - 2*b*g^2*x^4 + 6*b*g*x^2 - 20*b)*log(c) + 120*a)*sqrt(g*x^
2 + 4))/x^7, 1/3360*(24*b*sqrt(-g)*g^3*p*x^7*arctan((sqrt(g*x^2 + 4)*sqrt(
-g) - 2*sqrt(-g))/(g*x)) - ((11*b*g^3*p - 6*a*g^3)*x^6 - 2*(5*b*g^2*p - 6*
a*g^2)*x^4 + 12*(b*g*p - 3*a*g)*x^2 - 6*(b*g^3*p*x^6 - 2*b*g^2*p*x^4 + 6*b
*g*p*x^2 - 20*b*p)*log(d*g*x^2 + 4*d) - 6*(b*g^3*x^6 - 2*b*g^2*x^4 + 6*b*g
*x^2 - 20*b)*log(c) + 120*a)*sqrt(g*x^2 + 4))/x^7]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^8\sqrt{4+gx^2}}\,dx=\int \frac{a+b\log\left(c(dgx^2+4d)^p\right)}{x^8\sqrt{gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+4*d)**p))/x**8/(g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*g*x**2 + 4*d)**p))/(x**8*sqrt(g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^8\sqrt{4+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+4\,d)^pc\right)+a}{\sqrt{gx^2+4}x^8}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^8/(g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
1/560*(sqrt(g*x^2 + 4)*g^3/x - 2*sqrt(g*x^2 + 4)*g^2/x^3 + 6*sqrt(g*x^2 +
4)*g/x^5 - 20*sqrt(g*x^2 + 4)/x^7)*a - 1/3360*(3360*g^5*p*integrate(1/280*
x^2/(g*x^2 + 4)^(3/2), x) - ((6*g^4*p*log(d) - 23*g^4*p + 6*g^4*log(c))*x^
8 + 2*(6*g^3*p*log(d) - 17*g^3*p + 6*g^3*log(c))*x^6 - 4*(3*g^2*p*log(d) -
 7*g^2*p + 3*g^2*log(c))*x^4 + 24*(g*p*log(d) - 2*g*p + g*log(c))*x^2 + 6*
(g^4*p*x^8 + 2*g^3*p*x^6 - 2*g^2*p*x^4 + 4*g*p*x^2 - 80*p)*log(g*x^2 + 4)
- 480*p*log(d) - 480*log(c))/(sqrt(g*x^2 + 4)*x^7))*b
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^8\sqrt{4+gx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(d*g*x^2+4*d)^p))/x^8/(g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^8\sqrt{4+gx^2}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+4\,d\right)}^p\right)}{x^8\,\sqrt{g\,x^2+4}}\,dx$$

input
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^8*(g*x^2 + 4)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d + d*g*x^2)^p))/(x^8*(g*x^2 + 4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.19

$$\int \frac{a+b\log\left(c(4d+dgx^2)^p\right)}{x^8\sqrt{4+gx^2}}\,dx$$
$$= \frac{42\sqrt{g\,x^2+4}\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^pc}{\left(\sqrt{g\,x^2+4}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b\,g^3x^6-84\sqrt{g\,x^2+4}\log\left(\frac{d^p\left(\sqrt{g}\,\sqrt{g\,x^2+4}\,x+g\,x^2+4\right)^{2p}4^pc}{\left(\sqrt{g\,x^2+4}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b\,g^2}{}$$

input
```
int((a+b*log(c*(d*g*x^2+4*d)^p))/x^8/(g*x^2+4)^(1/2),x)
```

output
```
(42*sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)**
(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g**3*x**
6 - 84*sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4
)**(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g**2*
x**4 + 252*sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2
 + 4)**(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g
*x**2 - 840*sqrt(g*x**2 + 4)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**
2 + 4)**(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b
+ 42*sqrt(g*x**2 + 4)*a*g**3*x**6 - 84*sqrt(g*x**2 + 4)*a*g**2*x**4 + 252*
sqrt(g*x**2 + 4)*a*g*x**2 - 840*sqrt(g*x**2 + 4)*a - 77*sqrt(g*x**2 + 4)*b
*g**3*p*x**6 + 70*sqrt(g*x**2 + 4)*b*g**2*p*x**4 - 84*sqrt(g*x**2 + 4)*b*g
*p*x**2 - 84*sqrt(g)*log((sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)/2)*b*g*
*3*p*x**7 + 42*sqrt(g)*log((d**p*(sqrt(g)*sqrt(g*x**2 + 4)*x + g*x**2 + 4)
**(2*p)*4**p*c)/((sqrt(g*x**2 + 4) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g**3*x
**7 - 42*sqrt(g)*a*g**3*x**7 + 65*sqrt(g)*b*g**3*p*x**7)/(23520*x**7)
```

3.676 $\int \frac{x^7\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 235

$$\int \frac{x^7 (a+b\log (c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}\,dx = \frac{128bp\sqrt{4-gx^2}}{g^4} - \frac{32bp(4-gx^2)^{3/2}}{3g^4} + \frac{24bp(4-gx^2)^{5/2}}{25g^4} - \frac{2bp(4-gx^2)^{7/2}}{49g^4} - \frac{64\sqrt{4-gx^2}(a+b\log (c(4d-dgx^2)^p))}{g^4} + \frac{16(4-gx^2)^{3/2} (a+b\log (c(4d-dgx^2)^p))}{g^4} - \frac{12(4-gx^2)^{5/2} (a+b\log (c(4d-dgx^2)^p))}{5g^4} + \frac{(4-gx^2)^{7/2} (a+b\log (c(4d-dgx^2)^p))}{7g^4}$$

output
```
128*b*p*(-g*x^2+4)^(1/2)/g^4-32/3*b*p*(-g*x^2+4)^(3/2)/g^4+24/25*b*p*(-g*x
^2+4)^(5/2)/g^4-2/49*b*p*(-g*x^2+4)^(7/2)/g^4-64*(-g*x^2+4)^(1/2)*(a+b*ln(
c*(-d*g*x^2+4*d)^p))/g^4+16*(-g*x^2+4)^(3/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/
g^4-12/5*(-g*x^2+4)^(5/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/g^4+1/7*(-g*x^2+4)^
(7/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/g^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.50

$$\int \frac{x^7 \left(a+b\log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = \frac{\sqrt{4-gx^2}\left(-105a(1024+128gx^2+24g^2x^4+5g^3x^6)+2bp(180224+9088gx^2+864g^2x^4+75g^3x^6)-10\right.}{3675g^4}$$

input
```
Integrate[(x^7*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output
```
(Sqrt[4 - g*x^2]*(-105*a*(1024 + 128*g*x^2 + 24*g^2*x^4 + 5*g^3*x^6) + 2*b
*p*(180224 + 9088*g*x^2 + 864*g^2*x^4 + 75*g^3*x^6) - 105*b*(1024 + 128*g*
x^2 + 24*g^2*x^4 + 5*g^3*x^6)*Log[c*(d*(4 - g*x^2))^p]))/(3675*g^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(a+b\log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{x^7 \left(a+b\log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}dx$$

input
```
Int[(x^7*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^7 \left(a+b\ln\left(c(-dg\,x^2+4d)^p\right)\right)}{\sqrt{-g\,x^2+4}}dx$$

input
```
int(x^7*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
int(x^7*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \frac{x^7 \left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = \frac{(75\,(2\,bg^3p-7\,ag^3)x^6+72\,(24\,bg^2p-35\,ag^2)x^4+128\,(142\,bgp-105\,ag)x^2+360448\,bp-105\,(5\,bg^3px^6$$

input
```
integrate(x^7*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
1/3675*(75*(2*b*g^3*p - 7*a*g^3)*x^6 + 72*(24*b*g^2*p - 35*a*g^2)*x^4 + 12
8*(142*b*g*p - 105*a*g)*x^2 + 360448*b*p - 105*(5*b*g^3*p*x^6 + 24*b*g^2*p
*x^4 + 128*b*g*p*x^2 + 1024*b*p)*log(-d*g*x^2 + 4*d) - 105*(5*b*g^3*x^6 +
24*b*g^2*x^4 + 128*b*g*x^2 + 1024*b)*log(c) - 107520*a)*sqrt(-g*x^2 + 4)/g
^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(a+b*ln(c*(-d*g*x**2+4*d)**p))/(-g*x**2+4)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int \frac{x^7(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}\,dx = -\frac{1}{35}\left(\frac{5\sqrt{-gx^2+4}x^6}{g}+\frac{24\sqrt{-gx^2+4}x^4}{g^2}+\frac{128\sqrt{-gx^2+4}x^2}{g^3}+\frac{1024\sqrt{-gx^2+4}}{g^4}\right)b\log\left((-dgx^2+4\right. - \frac{1}{35}\left(\frac{5\sqrt{-gx^2+4}x^6}{g}+\frac{24\sqrt{-gx^2+4}x^4}{g^2}+\frac{128\sqrt{-gx^2+4}x^2}{g^3}+\frac{1024\sqrt{-gx^2+4}}{g^4}\right)a - \frac{2\left(75(-gx^2+4)^{\frac{7}{2}}-1764(-gx^2+4)^{\frac{5}{2}}+19600(-gx^2+4)^{\frac{3}{2}}-235200\sqrt{-gx^2+4}\right)bp}{3675g^4}$$

input
```
integrate(x^7*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/35*(5*sqrt(-g*x^2 + 4)*x^6/g + 24*sqrt(-g*x^2 + 4)*x^4/g^2 + 128*sqrt(-
g*x^2 + 4)*x^2/g^3 + 1024*sqrt(-g*x^2 + 4)/g^4)*b*log((-d*g*x^2 + 4*d)^p*c
) - 1/35*(5*sqrt(-g*x^2 + 4)*x^6/g + 24*sqrt(-g*x^2 + 4)*x^4/g^2 + 128*sqr
t(-g*x^2 + 4)*x^2/g^3 + 1024*sqrt(-g*x^2 + 4)/g^4)*a - 2/3675*(75*(-g*x^2
+ 4)^(7/2) - 1764*(-g*x^2 + 4)^(5/2) + 19600*(-g*x^2 + 4)^(3/2) - 235200*s
qrt(-g*x^2 + 4))*b*p/g^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.34

$$\int \frac{x^7 \left(a+b\log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx =$$

$$-\frac{bp\left(\frac{105\left(5\,(gx^2-4)^3\sqrt{-gx^2+4}+84\,(gx^2-4)^2\sqrt{-gx^2+4}-560\,(-gx^2+4)^{\frac{3}{2}}+2240\,\sqrt{-gx^2+4}\right)\log(-dgx^2+4\,d)}{g^3}-\frac{2\left(75\,(gx^2-4)^3\sqrt{-\dots}\right.}{}\right.}{}$$

input
```
integrate(x^7*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
-1/3675*(b*p*(105*(5*(g*x^2 - 4)^3*sqrt(-g*x^2 + 4) + 84*(g*x^2 - 4)^2*sqr
t(-g*x^2 + 4) - 560*(-g*x^2 + 4)^(3/2) + 2240*sqrt(-g*x^2 + 4))*log(-d*g*x
^2 + 4*d)/g^3 - 2*(75*(g*x^2 - 4)^3*sqrt(-g*x^2 + 4) + 1764*(g*x^2 - 4)^2*
sqrt(-g*x^2 + 4) - 19600*(-g*x^2 + 4)^(3/2) + 235200*sqrt(-g*x^2 + 4))/g^3
) + 105*(5*(g*x^2 - 4)^3*sqrt(-g*x^2 + 4) + 84*(g*x^2 - 4)^2*sqrt(-g*x^2 +
 4) - 560*(-g*x^2 + 4)^(3/2) + 2240*sqrt(-g*x^2 + 4))*b*log(c)/g^3 + 105*(
5*(g*x^2 - 4)^3*sqrt(-g*x^2 + 4) + 84*(g*x^2 - 4)^2*sqrt(-g*x^2 + 4) - 560
*(-g*x^2 + 4)^(3/2) + 2240*sqrt(-g*x^2 + 4))*a/g^3)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \left(a+b\log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = \int \frac{x^7\,\left(a+b\,\ln\left(c\,{\left(4\,d-d\,g\,x^2\right)}^p\right)\right)}{\sqrt{4-g\,x^2}}\,dx$$

input
```
int((x^7*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2),x)
```

output
```
int((x^7*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.88

$$\int \frac{x^7(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}\,dx$$
$$= \frac{\sqrt{-g\,x^2+4}\left(-525\log\left(\frac{d^p(-g\,x^2+4)^p 4^p c}{2^{2p}}\right)b\,g^3x^6-2520\log\left(\frac{d^p(-g\,x^2+4)^p 4^p c}{2^{2p}}\right)b\,g^2x^4-13440\log\left(\frac{d^p(-g\,x^2+4)^p}{2^{2p}}\right.\right.}{}$$

input
```
int(x^7*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
(sqrt( - g*x**2 + 4)*( - 525*log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p)
)*b*g**3*x**6 - 2520*log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p))*b*g**2
*x**4 - 13440*log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p))*b*g*x**2 - 10
7520*log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p))*b - 525*a*g**3*x**6 -
2520*a*g**2*x**4 - 13440*a*g*x**2 - 107520*a + 150*b*g**3*p*x**6 + 1728*b*
g**2*p*x**4 + 18176*b*g*p*x**2 + 360448*b*p))/(3675*g**4)
```

3.677 $\int \frac{x^5\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 177

$$\int \frac{x^5\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = \frac{32bp\sqrt{4-gx^2}}{g^3}-\frac{16bp\left(4-gx^2\right)^{3/2}}{9g^3}+\frac{2bp\left(4-gx^2\right)^{5/2}}{25g^3}-\frac{16\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{g^3}+\frac{8\left(4-gx^2\right)^{3/2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{3g^3}-\frac{\left(4-gx^2\right)^{5/2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{5g^3}$$

output
```
32*b*p*(-g*x^2+4)^(1/2)/g^3-16/9*b*p*(-g*x^2+4)^(3/2)/g^3+2/25*b*p*(-g*x^2
+4)^(5/2)/g^3-16*(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/g^3+8/3*(-g
*x^2+4)^(3/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/g^3-1/5*(-g*x^2+4)^(5/2)*(a+b*l
n(c*(-d*g*x^2+4*d)^p))/g^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

$$\int \frac{x^5 \left(a+b \log \left(c \left(4 d-d g x^2\right)^p\right)\right)}{\sqrt{4-g x^2}} \, dx = \frac{\sqrt{4-gx^2}\left(-15a(128+16gx^2+3g^2x^4)+2bp(2944+128gx^2+9g^2x^4)-15b(128+16gx^2+3g^2x^4)\log\right.}{225g^3}$$

input
```
Integrate[(x^5*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output
```
(Sqrt[4 - g*x^2]*(-15*a*(128 + 16*g*x^2 + 3*g^2*x^4) + 2*b*p*(2944 + 128*g
*x^2 + 9*g^2*x^4) - 15*b*(128 + 16*g*x^2 + 3*g^2*x^4)*Log[c*(d*(4 - g*x^2)
)^p]))/(225*g^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(a+b \log \left(c \left(4 d-d g x^2\right)^p\right)\right)}{\sqrt{4-g x^2}} \, dx$$

$$\Big\downarrow \text{2929}$$

$$\int \frac{x^5 \left(a+b \log \left(c \left(4 d-d g x^2\right)^p\right)\right)}{\sqrt{4-g x^2}}dx$$

input
```
Int[(x^5*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^5\left(a+b\ln\left(c(-dg\,x^2+4d)^p\right)\right)}{\sqrt{-g\,x^2+4}}dx$$

input
```
int(x^5*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
int(x^5*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.67

$$\int \frac{x^5\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$$
$$= \frac{(9\,(2\,bg^2p-5\,ag^2)x^4+16\,(16\,bgp-15\,ag)x^2+5888\,bp-15\,(3\,bg^2px^4+16\,bgpx^2+128\,bp)\log\left(-dgx^2\right.}{225\,g^3}$$

input
```
integrate(x^5*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
1/225*(9*(2*b*g^2*p - 5*a*g^2)*x^4 + 16*(16*b*g*p - 15*a*g)*x^2 + 5888*b*p
 - 15*(3*b*g^2*p*x^4 + 16*b*g*p*x^2 + 128*b*p)*log(-d*g*x^2 + 4*d) - 15*(3
*b*g^2*x^4 + 16*b*g*x^2 + 128*b)*log(c) - 1920*a)*sqrt(-g*x^2 + 4)/g^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \left(a+b\log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*ln(c*(-d*g*x**2+4*d)**p))/(-g*x**2+4)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

$$\int \frac{x^5 \left(a+b\log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = -\frac{1}{15}\left(\frac{3\sqrt{-gx^2+4}x^4}{g}+\frac{16\sqrt{-gx^2+4}x^2}{g^2}+\frac{128\sqrt{-gx^2+4}}{g^3}\right)b\log\left(\left(-dgx^2+4\,d\right)^p c\right) - \frac{1}{15}\left(\frac{3\sqrt{-gx^2+4}x^4}{g}+\frac{16\sqrt{-gx^2+4}x^2}{g^2}+\frac{128\sqrt{-gx^2+4}}{g^3}\right)a + \frac{2\left(9\left(-gx^2+4\right)^{\frac{5}{2}}-200\left(-gx^2+4\right)^{\frac{3}{2}}+3600\sqrt{-gx^2+4}\right)bp}{225\,g^3}$$

input
```
integrate(x^5*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/15*(3*sqrt(-g*x^2 + 4)*x^4/g + 16*sqrt(-g*x^2 + 4)*x^2/g^2 + 128*sqrt(-
g*x^2 + 4)/g^3)*b*log((-d*g*x^2 + 4*d)^p*c) - 1/15*(3*sqrt(-g*x^2 + 4)*x^4
/g + 16*sqrt(-g*x^2 + 4)*x^2/g^2 + 128*sqrt(-g*x^2 + 4)/g^3)*a + 2/225*(9*
(-g*x^2 + 4)^(5/2) - 200*(-g*x^2 + 4)^(3/2) + 3600*sqrt(-g*x^2 + 4))*b*p/g
^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.30

$$\int \frac{x^5(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}\,dx =$$

$$-\frac{bp\left(\frac{15\left(3\,(gx^2-4)^2\sqrt{-gx^2+4}-40\,(-gx^2+4)^{\frac{3}{2}}+240\,\sqrt{-gx^2+4}\right)\log(-dgx^2+4\,d)}{g^2}-\frac{2\left(9\,(gx^2-4)^2\sqrt{-gx^2+4}-200\,(-gx^2+4)^{\frac{3}{2}}+360\right.}{g^2}}{}$$

input
```
integrate(x^5*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
-1/225*(b*p*(15*(3*(g*x^2 - 4)^2*sqrt(-g*x^2 + 4) - 40*(-g*x^2 + 4)^(3/2)
+ 240*sqrt(-g*x^2 + 4))*log(-d*g*x^2 + 4*d)/g^2 - 2*(9*(g*x^2 - 4)^2*sqrt(
-g*x^2 + 4) - 200*(-g*x^2 + 4)^(3/2) + 3600*sqrt(-g*x^2 + 4))/g^2) + 15*(3
*(g*x^2 - 4)^2*sqrt(-g*x^2 + 4) - 40*(-g*x^2 + 4)^(3/2) + 240*sqrt(-g*x^2
+ 4))*b*log(c)/g^2 + 15*(3*(g*x^2 - 4)^2*sqrt(-g*x^2 + 4) - 40*(-g*x^2 + 4
)^(3/2) + 240*sqrt(-g*x^2 + 4))*a/g^2)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}\,dx = \int \frac{x^5\,\left(a+b\,\ln\left(c\,{\left(4\,d-d\,g\,x^2\right)}^p\right)\right)}{\sqrt{4-g\,x^2}}\,dx$$

input
```
int((x^5*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86

$$\int \frac{x^5 \left(a+b\log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$$
$$= \frac{\sqrt{-g\,x^2+4}\,\left(-45\log\left(\frac{d^p\left(-g\,x^2+4\right)^p 4^p c}{2^{2p}}\right) b\,g^2x^4-240\log\left(\frac{d^p\left(-g\,x^2+4\right)^p 4^p c}{2^{2p}}\right) bg\,x^2-1920\log\left(\frac{d^p\left(-g\,x^2+4\right)^p 4^p c}{2^{2p}}\right)\right.}{225g^3}$$

input
```
int(x^5*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
(sqrt( - g*x**2 + 4)*( - 45*log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p))
*b*g**2*x**4 - 240*log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p))*b*g*x**2
 - 1920*log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p))*b - 45*a*g**2*x**4
- 240*a*g*x**2 - 1920*a + 18*b*g**2*p*x**4 + 256*b*g*p*x**2 + 5888*b*p))/(
225*g**3)
```

3.678 $\int \frac{x^3\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 117

$$\int \frac{x^3\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx=\frac{8bp\sqrt{4-gx^2}}{g^2}-\frac{2bp(4-gx^2)^{3/2}}{9g^2}-\frac{4\sqrt{4-gx^2}\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{g^2}+\frac{(4-gx^2)^{3/2}\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{3g^2}$$

output
```
8*b*p*(-g*x^2+4)^(1/2)/g^2-2/9*b*p*(-g*x^2+4)^(3/2)/g^2-4*(-g*x^2+4)^(1/2)
*(a+b*ln(c*(-d*g*x^2+4*d)^p))/g^2+1/3*(-g*x^2+4)^(3/2)*(a+b*ln(c*(-d*g*x^2
+4*d)^p))/g^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

$$\int \frac{x^3 \left(a+b \log \left(c \left(4 d-d g x^2\right)^p\right)\right)}{\sqrt{4-g x^2}} \, dx = \frac{\sqrt{4-g x^2} \left(-3 a \left(8+g x^2\right)+2 b p \left(32+g x^2\right)-3 b \left(8+g x^2\right) \log \left(c \left(d \left(4-g x^2\right)\right)^p\right)\right)}{9 g^2}$$

input

```
Integrate[(x^3*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output

```
(Sqrt[4 - g*x^2]*(-3*a*(8 + g*x^2) + 2*b*p*(32 + g*x^2) - 3*b*(8 + g*x^2)*
Log[c*(d*(4 - g*x^2))^p]))/(9*g^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b \log \left(c \left(4 d-d g x^2\right)^p\right)\right)}{\sqrt{4-g x^2}} \, dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{x^3 \left(a+b \log \left(c \left(4 d-d g x^2\right)^p\right)\right)}{\sqrt{4-g x^2}}dx$$

input

```
Int[(x^3*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^3 \left(a + b \ln \left(c(-dg\,x^2 + 4d)^p\right)\right)}{\sqrt{-g\,x^2 + 4}}dx$$

input
```
int(x^3*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
int(x^3*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \frac{x^3 \left(a + b \log \left(c(4d - dgx^2)^p\right)\right)}{\sqrt{4 - gx^2}}\,dx = \frac{((2\,bgp - 3\,ag)x^2 + 64\,bp - 3\,(bgpx^2 + 8\,bp) \log \left(-dgx^2 + 4\,d\right) - 3\,(bgx^2 + 8\,b) \log \left(c\right) - 24\,a)\sqrt{-gx^2 + 4}}{9\,g^2}$$

input
```
integrate(x^3*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
1/9*((2*b*g*p - 3*a*g)*x^2 + 64*b*p - 3*(b*g*p*x^2 + 8*b*p)*log(-d*g*x^2 +
 4*d) - 3*(b*g*x^2 + 8*b)*log(c) - 24*a)*sqrt(-g*x^2 + 4)/g^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*ln(c*(-d*g*x**2+4*d)**p))/(-g*x**2+4)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\begin{aligned}&\int \frac{x^3(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}\,dx \\ &= -\frac{1}{3}\left(\frac{\sqrt{-gx^2+4}x^2}{g}+\frac{8\sqrt{-gx^2+4}}{g^2}\right)b\log\left((-dgx^2+4\,d)^pc\right) \\ &\quad -\frac{1}{3}\left(\frac{\sqrt{-gx^2+4}x^2}{g}+\frac{8\sqrt{-gx^2+4}}{g^2}\right)a-\frac{2\left((-gx^2+4)^{\frac{3}{2}}-36\sqrt{-gx^2+4}\right)bp}{9\,g^2}\end{aligned}$$

input
```
integrate(x^3*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/3*(sqrt(-g*x^2 + 4)*x^2/g + 8*sqrt(-g*x^2 + 4)/g^2)*b*log((-d*g*x^2 + 4
*d)^p*c) - 1/3*(sqrt(-g*x^2 + 4)*x^2/g + 8*sqrt(-g*x^2 + 4)/g^2)*a - 2/9*(
(-g*x^2 + 4)^(3/2) - 36*sqrt(-g*x^2 + 4))*b*p/g^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int \frac{x^3\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$$
$$= \frac{\frac{\left(3\left((-gx^2+4)^{\frac{3}{2}}-12\sqrt{-gx^2+4}\right)\log(-dgx^2+4d)-2(-gx^2+4)^{\frac{3}{2}}+72\sqrt{-gx^2+4}\right)bp}{g}+\frac{3\left((-gx^2+4)^{\frac{3}{2}}-12\sqrt{-gx^2+4}\right)b\log(c)}{g}+\frac{3\left((-\right.}{}}{9g}$$

input
```
integrate(x^3*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
1/9*((3*((-g*x^2 + 4)^(3/2) - 12*sqrt(-g*x^2 + 4))*log(-d*g*x^2 + 4*d) - 2
*(-g*x^2 + 4)^(3/2) + 72*sqrt(-g*x^2 + 4))*b*p/g + 3*((-g*x^2 + 4)^(3/2) -
 12*sqrt(-g*x^2 + 4))*b*log(c)/g + 3*((-g*x^2 + 4)^(3/2) - 12*sqrt(-g*x^2
+ 4))*a/g)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = \int \frac{x^3\left(a+b\ln\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$$

input
```
int((x^3*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{x^3(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}\,dx$$
$$=\frac{\sqrt{-g\,x^2+4}\left(-3\log\left(\frac{d^p(-g\,x^2+4)^p 4^p c}{2^{2p}}\right)bg\,x^2-24\log\left(\frac{d^p(-g\,x^2+4)^p 4^p c}{2^{2p}}\right)b-3ag\,x^2-24a+2bgp\,x^2+64bp\right)}{9g^2}$$

input
```
int(x^3*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
(sqrt( - g*x**2 + 4)*( - 3*log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p))*
b*g*x**2 - 24*log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p))*b - 3*a*g*x**
2 - 24*a + 2*b*g*p*x**2 + 64*b*p))/(9*g**2)
```

3.679 $\int \frac{x\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 57

$$\int \frac{x\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = \frac{2bp\sqrt{4-gx^2}}{g}-\frac{\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{g}$$

output
```
2*b*p*(-g*x^2+4)^(1/2)/g-(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/g
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

$$\int \frac{x\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = -\frac{\sqrt{4-gx^2}\left(a-2bp+b\log\left(c\left(d\left(4-gx^2\right)\right)^p\right)\right)}{g}$$

input
```
Integrate[(x*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output
```
-((Sqrt[4 - g*x^2]*(a - 2*b*p + b*Log[c*(d*(4 - g*x^2))^p]))/g)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$$
$$\Big\downarrow \text{2929}$$
$$\int \frac{x\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}dx$$

input
```
Int[(x*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x\left(a+b\ln\left(c\left(-dg\,x^{2}+4d\right)^{p}\right)\right)}{\sqrt{-g\,x^{2}+4}}dx$$

input
```
int(x*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
int(x*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

$$\int \frac{x\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = -\frac{\sqrt{-gx^2+4}(bp\log\left(-dgx^2+4\,d\right)-2\,bp+b\log\left(c\right)+a)}{g}$$

input
```
integrate(x*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
-sqrt(-g*x^2 + 4)*(b*p*log(-d*g*x^2 + 4*d) - 2*b*p + b*log(c) + a)/g
```

Sympy [F]

$$\int \frac{x\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = \int \frac{x\left(a+b\log\left(c(-dgx^2+4d)^p\right)\right)}{\sqrt{-gx^2+4}}\,dx$$

input
```
integrate(x*(a+b*ln(c*(-d*g*x**2+4*d)**p))/(-g*x**2+4)**(1/2),x)
```

output
```
Integral(x*(a + b*log(c*(-d*g*x**2 + 4*d)**p))/sqrt(-g*x**2 + 4), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{x\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx = \frac{2\sqrt{-gx^2+4}bp}{g} - \frac{\sqrt{-gx^2+4}b\log\left((-dgx^2+4\,d)^pc\right)}{g} - \frac{\sqrt{-gx^2+4}a}{g}$$

input
```
integrate(x*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
2*sqrt(-g*x^2 + 4)*b*p/g - sqrt(-g*x^2 + 4)*b*log((-d*g*x^2 + 4*d)^p*c)/g
- sqrt(-g*x^2 + 4)*a/g
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int \frac{x(a + b\log(c(4d - dgx^2)^p))}{\sqrt{4 - gx^2}}\,dx = -\frac{\sqrt{-gx^2+4}bp\log(d) + \left(\sqrt{-gx^2+4}\log(-gx^2+4) - 2\sqrt{-gx^2+4}\right)bp + \sqrt{-gx^2+4}b\log(c) + \sqrt{-g}}{g}$$

input
```
integrate(x*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
-(sqrt(-g*x^2 + 4)*b*p*log(d) + (sqrt(-g*x^2 + 4)*log(-g*x^2 + 4) - 2*sqrt
(-g*x^2 + 4))*b*p + sqrt(-g*x^2 + 4)*b*log(c) + sqrt(-g*x^2 + 4)*a)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\log(c(4d - dgx^2)^p))}{\sqrt{4 - gx^2}}\,dx = \int \frac{x\,\left(a + b\,\ln\left(c\,{\left(4\,d - d\,g\,x^2\right)}^p\right)\right)}{\sqrt{4 - g\,x^2}}\,dx$$

input
```
int((x*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2),x)
```

output
```
int((x*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{x\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx=\frac{\sqrt{-g\,x^2+4}\,\left(-\log\left(\frac{d^p\left(-g\,x^2+4\right)^p 4^p c}{2^{2p}}\right)b-a+2bp\right)}{g}$$

input
```
int(x*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
(sqrt( - g*x**2 + 4)*( - log((d**p*( - g*x**2 + 4)**p*4**p*c)/2**(2*p))*b
- a + 2*b*p))/g
```

3.680 $\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 90

$$\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x\sqrt{4-gx^2}}\,dx = -\frac{1}{2}\text{arctanh}\left(\frac{1}{2}\sqrt{4-gx^2}\right)\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right) - \frac{1}{2}bp\,\text{PolyLog}\left(2,-\frac{1}{2}\sqrt{4-gx^2}\right) + \frac{1}{2}bp\,\text{PolyLog}\left(2,\frac{1}{2}\sqrt{4-gx^2}\right)$$

output
```
-1/2*arctanh(1/2*(-g*x^2+4)^(1/2))*(a+b*ln(c*(-d*g*x^2+4*d)^p))-1/2*b*p*po
lylog(2,-1/2*(-g*x^2+4)^(1/2))+1/2*b*p*polylog(2,1/2*(-g*x^2+4)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x\sqrt{4-gx^2}}\,dx = \frac{1}{4}\Bigg(\left(a+b\log\left(c\left(-d\left(-4+gx^2\right)\right)^p\right)\right)\left(\log\left(2-\sqrt{4-gx^2}\right)-\log\left(2+\sqrt{4-gx^2}\right)\right)-2bp\,\text{PolyLog}\left(2,-\frac{1}{2}\sqrt{4-gx^2}\right)+2bp\,\text{PolyLog}\left(2,\frac{1}{2}\sqrt{4-gx^2}\right)\Bigg)$$

input
```
Integrate[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x*Sqrt[4 - g*x^2]),x]
```

output
```
((a + b*Log[c*(-(d*(-4 + g*x^2)))^p])*(Log[2 - Sqrt[4 - g*x^2]] - Log[2 +
Sqrt[4 - g*x^2]]) - 2*b*p*PolyLog[2, -1/2*Sqrt[4 - g*x^2]] + 2*b*p*PolyLog
[2, Sqrt[4 - g*x^2]/2])/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x\sqrt{4-gx^2}}\,dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x\sqrt{4-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x*Sqrt[4 - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(-dg\,x^2+4d)^p\right)}{x\sqrt{-g\,x^2+4}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x/(-g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x/(-g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x\sqrt{4-gx^2}}\,dx=\int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}x}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x/(-g*x^2+4)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-(sqrt(-g*x^2 + 4)*b*log((-d*g*x^2 + 4*d)^p*c) + sqrt(-g*x^2 + 4)
*a)/(g*x^3 - 4*x), x)
```

Sympy [F]

$$\int \frac{a + b\log\left(c(4d - dgx^2)^p\right)}{x\sqrt{4 - gx^2}}\,dx = \int \frac{a + b\log\left(c(-dgx^2 + 4d)^p\right)}{x\sqrt{-gx^2 + 4}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+4*d)**p))/x/(-g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(-d*g*x**2 + 4*d)**p))/(x*sqrt(-g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a + b\log\left(c(4d - dgx^2)^p\right)}{x\sqrt{4 - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + 4\,d)^p c\right) + a}{\sqrt{-gx^2 + 4}x}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x/(-g*x^2+4)^(1/2),x, algorithm="m
axima")
```

output
```
b*integrate((p*log(d) + log((-g*x^2 + 4)^p) + log(c))/(sqrt(-g*x^2 + 4)*x)
, x) - 1/2*a*log(4*sqrt(-g*x^2 + 4)/abs(x) + 8/abs(x))
```

Giac [F]

$$\int \frac{a + b\log\left(c(4d - dgx^2)^p\right)}{x\sqrt{4 - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + 4\,d)^p c\right) + a}{\sqrt{-gx^2 + 4}x}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x/(-g*x^2+4)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*log((-d*g*x^2 + 4*d)^p*c) + a)/(sqrt(-g*x^2 + 4)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x\sqrt{4-gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(4\,d-d\,g\,x^2\right)}^p\right)}{x\,\sqrt{4-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x*(4 - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x*(4 - g*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x\sqrt{4-gx^2}}\,dx=\left(\int \frac{\log\left(\left(-dg\,x^2+4d\right)^p c\right)}{\sqrt{-g\,x^2+4}\,x}dx\right)b+\frac{\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)\right)a}{2}$$

input
```
int((a+b*log(c*(-d*g*x^2+4*d)^p))/x/(-g*x^2+4)^(1/2),x)
```

output
```
(2*int(log(( - d*g*x**2 + 4*d)**p*c)/(sqrt( - g*x**2 + 4)*x),x)*b + log(ta
n(asin((sqrt(g)*x)/2)/2))*a)/2
```

3.681 $\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^3\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 156

$$\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^3\sqrt{4-gx^2}}\,dx = \frac{1}{8}bgp\text{arctanh}\left(\frac{1}{2}\sqrt{4-gx^2}\right) - \frac{\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{8x^2} - \frac{1}{16}g\text{arctanh}\left(\frac{1}{2}\sqrt{4-gx^2}\right)\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right) - \frac{1}{16}bgp\,\text{PolyLog}\left(2,-\frac{1}{2}\sqrt{4-gx^2}\right) + \frac{1}{16}bgp\,\text{PolyLog}\left(2,\frac{1}{2}\sqrt{4-gx^2}\right)$$

output

```
1/8*b*g*p*arctanh(1/2*(-g*x^2+4)^(1/2))-1/8*(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d
*g*x^2+4*d)^p))/x^2-1/16*g*arctanh(1/2*(-g*x^2+4)^(1/2))*(a+b*ln(c*(-d*g*x
^2+4*d)^p))-1/16*b*g*p*polylog(2,-1/2*(-g*x^2+4)^(1/2))+1/16*b*g*p*polylog
(2,1/2*(-g*x^2+4)^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.75

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^3\sqrt{4-gx^2}}\,dx =$$
$$-\frac{4a\sqrt{4-gx^2}+4b\sqrt{4-gx^2}\log\left(c(-d(-4+gx^2))^p\right)-agx^2\log\left(2-\sqrt{4-gx^2}\right)+2bgpx^2\log\left(2-\sqrt{4}\right.}{}$$

input
```
Integrate[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^3*Sqrt[4 - g*x^2]),x]
```

output
```
-1/32*(4*a*Sqrt[4 - g*x^2] + 4*b*Sqrt[4 - g*x^2]*Log[c*(-(d*(-4 + g*x^2)))
^p] - a*g*x^2*Log[2 - Sqrt[4 - g*x^2]] + 2*b*g*p*x^2*Log[2 - Sqrt[4 - g*x^
2]] - b*g*x^2*Log[c*(-(d*(-4 + g*x^2)))^p]*Log[2 - Sqrt[4 - g*x^2]] + a*g*
x^2*Log[2 + Sqrt[4 - g*x^2]] - 2*b*g*p*x^2*Log[2 + Sqrt[4 - g*x^2]] + b*g*
x^2*Log[c*(-(d*(-4 + g*x^2)))^p]*Log[2 + Sqrt[4 - g*x^2]] + 2*b*g*p*x^2*Po
lyLog[2, -1/2*Sqrt[4 - g*x^2]] - 2*b*g*p*x^2*PolyLog[2, Sqrt[4 - g*x^2]/2]
)/x^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^3\sqrt{4-gx^2}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^3\sqrt{4-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^3*Sqrt[4 - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a + b\ln\left(c(-dg\,x^2 + 4d)^p\right)}{x^3\sqrt{-g\,x^2 + 4}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^3/(-g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^3/(-g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{a + b\log\left(c(4d - dgx^2)^p\right)}{x^3\sqrt{4 - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + 4\,d)^p c\right) + a}{\sqrt{-gx^2 + 4}x^3}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^3/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-(sqrt(-g*x^2 + 4)*b*log((-d*g*x^2 + 4*d)^p*c) + sqrt(-g*x^2 + 4)
*a)/(g*x^5 - 4*x^3), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^3\sqrt{4-gx^2}}\,dx=\int \frac{a+b\log\left(c(-dgx^2+4d)^p\right)}{x^3\sqrt{-gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+4*d)**p))/x**3/(-g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(-d*g*x**2 + 4*d)**p))/(x**3*sqrt(-g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^3\sqrt{4-gx^2}}\,dx=\int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}x^3}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^3/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/16*(g*log(4*sqrt(-g*x^2 + 4)/abs(x) + 8/abs(x)) + 2*sqrt(-g*x^2 + 4)/x^
2)*a + b*integrate((p*log(d) + log((-g*x^2 + 4)^p) + log(c))/(sqrt(-g*x^2
+ 4)*x^3), x)
```

Giac [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^3\sqrt{4-gx^2}}\,dx=\int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}x^3}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^3/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((b*log((-d*g*x^2 + 4*d)^p*c) + a)/(sqrt(-g*x^2 + 4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^3\sqrt{4-gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(4\,d-d\,g\,x^2\right)}^p\right)}{x^3\,\sqrt{4-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^3*(4 - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^3*(4 - g*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^3\sqrt{4-gx^2}}\,dx = \frac{-2\sqrt{-g\,x^2+4}\,a+16\left(\int \frac{\log\left(\left(-dg\,x^2+4d\right)^p c\right)}{\sqrt{-g\,x^2+4}\,x^3}dx\right)b\,x^2+\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)\right)ag\,x^2}{16x^2}$$

input
```
int((a+b*log(c*(-d*g*x^2+4*d)^p))/x^3/(-g*x^2+4)^(1/2),x)
```

output
```
( - 2*sqrt( - g*x**2 + 4)*a + 16*int(log(( - d*g*x**2 + 4*d)**p*c)/(sqrt(
- g*x**2 + 4)*x**3),x)*b*x**2 + log(tan(asin((sqrt(g)*x)/2)/2))*a*g*x**2)/
(16*x**2)
```

3.682 $\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^5\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 226

$$\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^5\sqrt{4-gx^2}}\,dx = \frac{bgp\sqrt{4-gx^2}}{64x^2}+\frac{1}{32}bg^2p\,\text{arctanh}\left(\frac{1}{2}\sqrt{4-gx^2}\right) - \frac{\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{16x^4} - \frac{3g\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{128x^2} - \frac{3}{256}g^2\,\text{arctanh}\left(\frac{1}{2}\sqrt{4-gx^2}\right)\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right) - \frac{3}{256}bg^2p\,\text{PolyLog}\left(2,-\frac{1}{2}\sqrt{4-gx^2}\right) + \frac{3}{256}bg^2p\,\text{PolyLog}\left(2,\frac{1}{2}\sqrt{4-gx^2}\right)$$

output
```
1/64*b*g*p*(-g*x^2+4)^(1/2)/x^2+1/32*b*g^2*p*arctanh(1/2*(-g*x^2+4)^(1/2))
-1/16*(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/x^4-3/128*g*(-g*x^2+4)
^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/x^2-3/256*g^2*arctanh(1/2*(-g*x^2+4)^(
1/2))*(a+b*ln(c*(-d*g*x^2+4*d)^p))-3/256*b*g^2*p*polylog(2,-1/2*(-g*x^2+4)
^(1/2))+3/256*b*g^2*p*polylog(2,1/2*(-g*x^2+4)^(1/2))
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.61

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^5\sqrt{4-gx^2}}\,dx = -\frac{32a\sqrt{4-gx^2}+12agx^2\sqrt{4-gx^2}-8bgpx^2\sqrt{4-gx^2}+32b\sqrt{4-gx^2}\log\left(c(-d(-4+gx^2))^p\right)+12bgx}{}$$

input
```
Integrate[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^5*Sqrt[4 - g*x^2]),x]
```

output
```
-1/512*(32*a*Sqrt[4 - g*x^2] + 12*a*g*x^2*Sqrt[4 - g*x^2] - 8*b*g*p*x^2*Sq
rt[4 - g*x^2] + 32*b*Sqrt[4 - g*x^2]*Log[c*(-(d*(-4 + g*x^2)))^p] + 12*b*g
*x^2*Sqrt[4 - g*x^2]*Log[c*(-(d*(-4 + g*x^2)))^p] - 3*a*g^2*x^4*Log[2 - Sq
rt[4 - g*x^2]] + 8*b*g^2*p*x^4*Log[2 - Sqrt[4 - g*x^2]] - 3*b*g^2*x^4*Log[
c*(-(d*(-4 + g*x^2)))^p]*Log[2 - Sqrt[4 - g*x^2]] + 3*a*g^2*x^4*Log[2 + Sq
rt[4 - g*x^2]] - 8*b*g^2*p*x^4*Log[2 + Sqrt[4 - g*x^2]] + 3*b*g^2*x^4*Log[
c*(-(d*(-4 + g*x^2)))^p]*Log[2 + Sqrt[4 - g*x^2]] + 6*b*g^2*p*x^4*PolyLog[
2, -1/2*Sqrt[4 - g*x^2]] - 6*b*g^2*p*x^4*PolyLog[2, Sqrt[4 - g*x^2]/2])/x^
4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^5\sqrt{4-gx^2}}\,dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^5\sqrt{4-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^5*Sqrt[4 - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a + b\ln\left(c(-dg\,x^2 + 4d)^p\right)}{x^5\sqrt{-g\,x^2 + 4}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^5/(-g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^5/(-g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{a + b\log\left(c(4d - dgx^2)^p\right)}{x^5\sqrt{4 - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + 4\,d)^p c\right) + a}{\sqrt{-gx^2 + 4}x^5}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^5/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-(sqrt(-g*x^2 + 4)*b*log((-d*g*x^2 + 4*d)^p*c) + sqrt(-g*x^2 + 4)
*a)/(g*x^7 - 4*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^5\sqrt{4-gx^2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+4*d)**p))/x**5/(-g*x**2+4)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^5\sqrt{4-gx^2}}\,dx = \int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}x^5}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^5/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/256*(3*g^2*log(4*sqrt(-g*x^2 + 4)/abs(x) + 8/abs(x)) + 6*sqrt(-g*x^2 +
4)*g/x^2 + 16*sqrt(-g*x^2 + 4)/x^4)*a + b*integrate((p*log(d) + log((-g*x^
2 + 4)^p) + log(c))/(sqrt(-g*x^2 + 4)*x^5), x)
```

Giac [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^5\sqrt{4-gx^2}}\,dx = \int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}x^5}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^5/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((b*log((-d*g*x^2 + 4*d)^p*c) + a)/(sqrt(-g*x^2 + 4)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^5\sqrt{4-gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(4\,d-d\,g\,x^2\right)}^p\right)}{x^5\,\sqrt{4-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^5*(4 - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^5*(4 - g*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^5\sqrt{4-gx^2}}\,dx = \frac{-6\sqrt{-g\,x^2+4}\,ag\,x^2-16\sqrt{-g\,x^2+4}\,a+256\left(\int \frac{\log\left(\left(-dg\,x^2+4d\right)^p c\right)}{\sqrt{-g\,x^2+4}\,x^5}dx\right)b\,x^4+3\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)\right)a\,g}{256x^4}$$

input
```
int((a+b*log(c*(-d*g*x^2+4*d)^p))/x^5/(-g*x^2+4)^(1/2),x)
```

output
```
( - 6*sqrt( - g*x**2 + 4)*a*g*x**2 - 16*sqrt( - g*x**2 + 4)*a + 256*int(lo
g(( - d*g*x**2 + 4*d)**p*c)/(sqrt( - g*x**2 + 4)*x**5),x)*b*x**4 + 3*log(t
an(asin((sqrt(g)*x)/2)/2))*a*g**2*x**4)/(256*x**4)
```

3.683 $\int \frac{x^2\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 568

$$\int \frac{x^2\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx$$
$$= \frac{2bp\sqrt{-\frac{4}{g}+x^2}}{g^2\sqrt{4-gx^2}\left(x+\sqrt{-\frac{4}{g}+x^2}\right)^2}+\frac{a\sqrt{-\frac{4}{g}+x^2}\left(x+\sqrt{-\frac{4}{g}+x^2}\right)^2}{8\sqrt{4-gx^2}}$$
$$-\frac{bp\sqrt{-\frac{4}{g}+x^2}\left(x+\sqrt{-\frac{4}{g}+x^2}\right)^2}{8\sqrt{4-gx^2}}$$
$$+\frac{b\sqrt{-\frac{4}{g}+x^2}\left(x+\sqrt{-\frac{4}{g}+x^2}\right)^2\log\left(c(4d-dgx^2)^p\right)}{8\sqrt{4-gx^2}}$$
$$-\frac{2\sqrt{-\frac{4}{g}+x^2}\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{g^2\sqrt{4-gx^2}\left(x+\sqrt{-\frac{4}{g}+x^2}\right)^2}-\frac{2bp\sqrt{-\frac{4}{g}+x^2}\log\left(x+\sqrt{-\frac{4}{g}+x^2}\right)}{g\sqrt{4-gx^2}}$$
$$+\frac{2\sqrt{-\frac{4}{g}+x^2}\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)\log\left(x+\sqrt{-\frac{4}{g}+x^2}\right)}{g\sqrt{4-gx^2}}$$
$$+\frac{2bp\sqrt{-\frac{4}{g}+x^2}\log^2\left(x+\sqrt{-\frac{4}{g}+x^2}\right)}{g\sqrt{4-gx^2}}$$
$$-\frac{4bp\sqrt{-\frac{4}{g}+x^2}\log\left(x+\sqrt{-\frac{4}{g}+x^2}\right)\log\left(1-\frac{1}{4}g\left(x+\sqrt{-\frac{4}{g}+x^2}\right)^2\right)}{g\sqrt{4-gx^2}}$$
$$-\frac{2bp\sqrt{-\frac{4}{g}+x^2}\,\mathrm{PolyLog}\left(2,\frac{1}{4}g\left(x+\sqrt{-\frac{4}{g}+x^2}\right)^2\right)}{g\sqrt{4-gx^2}}$$

output
```
2*b*p*(-4/g+x^2)^(1/2)/g^2/(-g*x^2+4)^(1/2)/(x+(-4/g+x^2)^(1/2))^2+1/8*a*(
-4/g+x^2)^(1/2)*(x+(-4/g+x^2)^(1/2))^2/(-g*x^2+4)^(1/2)-1/8*b*p*(-4/g+x^2)
^(1/2)*(x+(-4/g+x^2)^(1/2))^2/(-g*x^2+4)^(1/2)+1/8*b*(-4/g+x^2)^(1/2)*(x+(
-4/g+x^2)^(1/2))^2*ln(c*(-d*g*x^2+4*d)^p)/(-g*x^2+4)^(1/2)-2*(-4/g+x^2)^(1
/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/g^2/(-g*x^2+4)^(1/2)/(x+(-4/g+x^2)^(1/2))
^2-2*b*p*(-4/g+x^2)^(1/2)*ln(x+(-4/g+x^2)^(1/2))/g/(-g*x^2+4)^(1/2)+2*(-4/
g+x^2)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))*ln(x+(-4/g+x^2)^(1/2))/g/(-g*x^2
+4)^(1/2)+2*b*p*(-4/g+x^2)^(1/2)*ln(x+(-4/g+x^2)^(1/2))^2/g/(-g*x^2+4)^(1/
2)-4*b*p*(-4/g+x^2)^(1/2)*ln(x+(-4/g+x^2)^(1/2))*ln(1-1/4*g*(x+(-4/g+x^2)^
(1/2))^2)/g/(-g*x^2+4)^(1/2)-2*b*p*(-4/g+x^2)^(1/2)*polylog(2,1/4*g*(x+(-4
/g+x^2)^(1/2))^2)/g/(-g*x^2+4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.84

$$\int \frac{x^2\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx =$$

$$-\frac{a\sqrt{g}x\sqrt{4-gx^2}-b\sqrt{g}px\sqrt{4-gx^2}-4a\arcsin\left(\frac{\sqrt{g}x}{2}\right)+4bp\arcsin\left(\frac{\sqrt{g}x}{2}\right)+8ibp\pi\arcsin\left(\frac{\sqrt{g}x}{2}\right)-4ib}{}$$

input
```
Integrate[(x^2*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output
```
-1/2*(a*Sqrt[g]*x*Sqrt[4 - g*x^2] - b*Sqrt[g]*p*x*Sqrt[4 - g*x^2] - 4*a*Ar
cSin[(Sqrt[g]*x)/2] + 4*b*p*ArcSin[(Sqrt[g]*x)/2] + (8*I)*b*p*Pi*ArcSin[(S
qrt[g]*x)/2] - (4*I)*b*p*ArcSin[(Sqrt[g]*x)/2]^2 + 16*b*p*Pi*Log[1 + E^((-
I)*ArcSin[(Sqrt[g]*x)/2])] + 4*b*p*Pi*Log[1 - I*E^(I*ArcSin[(Sqrt[g]*x)/2]
)] + 8*b*p*ArcSin[(Sqrt[g]*x)/2]*Log[1 - I*E^(I*ArcSin[(Sqrt[g]*x)/2])] -
4*b*p*Pi*Log[1 + I*E^(I*ArcSin[(Sqrt[g]*x)/2])] + 8*b*p*ArcSin[(Sqrt[g]*x)
/2]*Log[1 + I*E^(I*ArcSin[(Sqrt[g]*x)/2])] + b*Sqrt[g]*x*Sqrt[4 - g*x^2]*L
og[c*(d*(4 - g*x^2))^p] - 4*b*ArcSin[(Sqrt[g]*x)/2]*Log[c*(d*(4 - g*x^2))^
p] - 16*b*p*Pi*Log[Cos[ArcSin[(Sqrt[g]*x)/2]/2]] + 4*b*p*Pi*Log[-Cos[(Pi +
 2*ArcSin[(Sqrt[g]*x)/2])/4]] - 4*b*p*Pi*Log[Sin[(Pi + 2*ArcSin[(Sqrt[g]*x
)/2])/4]] - (8*I)*b*p*PolyLog[2, (-I)*E^(I*ArcSin[(Sqrt[g]*x)/2])] - (8*I)
*b*p*PolyLog[2, I*E^(I*ArcSin[(Sqrt[g]*x)/2])])/g^(3/2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^2(a+b\log(c(4d-dgx^2)^p))}{\sqrt{4-gx^2}}dx$$

input
```
Int[(x^2*(a + b*Log[c*(4*d - d*g*x^2)^p]))/Sqrt[4 - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^2(a+b\ln(c(-dg\,x^2+4d)^p))}{\sqrt{-g\,x^2+4}}dx$$

input
```
int(x^2*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
int(x^2*(a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2 \left(a+b \log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\, dx = \int \frac{\left(b \log \left(\left(-dgx^2+4\, d\right)^p c\right)+a\right)x^2}{\sqrt{-gx^2+4}}\, dx$$

input
```
integrate(x^2*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-(sqrt(-g*x^2 + 4)*b*x^2*log((-d*g*x^2 + 4*d)^p*c) + sqrt(-g*x^2
+ 4)*a*x^2)/(g*x^2 - 4), x)
```

Sympy [F]

$$\int \frac{x^2 \left(a+b \log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\, dx = \int \frac{x^2 \left(a+b \log \left(c(-dgx^2+4d)^p\right)\right)}{\sqrt{-gx^2+4}}\, dx$$

input
```
integrate(x**2*(a+b*ln(c*(-d*g*x**2+4*d)**p))/(-g*x**2+4)**(1/2),x)
```

output
```
Integral(x**2*(a + b*log(c*(-d*g*x**2 + 4*d)**p))/sqrt(-g*x**2 + 4), x)
```

Maxima [F]

$$\int \frac{x^2 \left(a+b \log \left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\, dx = \int \frac{\left(b \log \left(\left(-dgx^2+4\, d\right)^p c\right)+a\right)x^2}{\sqrt{-gx^2+4}}\, dx$$

input
```
integrate(x^2*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/2*a*(sqrt(-g*x^2 + 4)*x/g - 4*arcsin(1/2*sqrt(g)*x)/g^(3/2)) + b*integr
ate(((p*log(d) + log(c))*x^2 + x^2*log((-g*x^2 + 4)^p))/sqrt(-g*x^2 + 4),
x)
```

Giac [F]

$$\int \frac{x^2\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx=\int \frac{\left(b\log\left((-dgx^2+4\,d)^pc\right)+a\right)x^2}{\sqrt{-gx^2+4}}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((b*log((-d*g*x^2 + 4*d)^p*c) + a)*x^2/sqrt(-g*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx=\int \frac{x^2\,\left(a+b\,\ln\left(c\,{\left(4\,d-d\,g\,x^2\right)}^p\right)\right)}{\sqrt{4-g\,x^2}}\,dx$$

input
```
int((x^2*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*log(c*(4*d - d*g*x^2)^p)))/(4 - g*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{4-gx^2}}\,dx=\frac{4\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{2}\right)a-\sqrt{-g\,x^2+4}\,agx+2\left(\int\frac{\log\left((-dg\,x^2+4d)^pc\right)x^2}{\sqrt{-g\,x^2+4}}dx\right)b\,g^2}{2g^2}$$

input
```
int(x^2*(a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
(4*sqrt(g)*asin((sqrt(g)*x)/2)*a - sqrt( - g*x**2 + 4)*a*g*x + 2*int((log(
( - d*g*x**2 + 4*d)**p*c)*x**2)/sqrt( - g*x**2 + 4),x)*b*g**2)/(2*g**2)
```

3.684 $\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 192

$$\begin{aligned}\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}\,dx &= \frac{\arcsin\left(\frac{\sqrt{g}x}{2}\right)\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{\sqrt{g}} - \frac{ibp\log^2\left(\frac{1}{2}i\sqrt{g}x+\sqrt{1-\frac{gx^2}{4}}\right)}{\sqrt{g}}\\ &+ \frac{2ibp\log\left(\frac{1}{2}i\sqrt{g}x+\sqrt{1-\frac{gx^2}{4}}\right)\log\left(1+\left(\frac{1}{2}i\sqrt{g}x+\sqrt{1-\frac{gx^2}{4}}\right)^2\right)}{\sqrt{g}}\\ &+ \frac{ibp\,\text{PolyLog}\left(2,-\frac{1}{4}\left(i\sqrt{g}x+\sqrt{4-gx^2}\right)^2\right)}{\sqrt{g}}\end{aligned}$$

output
```
arcsin(1/2*g^(1/2)*x)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/g^(1/2)-I*b*p*ln(1/2*I*
g^(1/2)*x+1/2*(-g*x^2+4)^(1/2))^2/g^(1/2)+2*I*b*p*ln(1/2*I*g^(1/2)*x+1/2*(
-g*x^2+4)^(1/2))*ln(1+(1/2*I*g^(1/2)*x+1/2*(-g*x^2+4)^(1/2))^2)/g^(1/2)+I*
b*p*polylog(2,-1/4*(I*g^(1/2)*x+(-g*x^2+4)^(1/2))^2)/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.97

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}\,dx$$
$$=\frac{a\arcsin\left(\frac{\sqrt{g}x}{2}\right)-2ibp\pi\arcsin\left(\frac{\sqrt{g}x}{2}\right)+ibp\arcsin\left(\frac{\sqrt{g}x}{2}\right)^2-4bp\pi\log\left(1+e^{-i\arcsin\left(\frac{\sqrt{g}x}{2}\right)}\right)-bp\pi\log\left(1\right.}{}$$

input
```
Integrate[(a + b*Log[c*(4*d - d*g*x^2)^p])/Sqrt[4 - g*x^2],x]
```

output
```
(a*ArcSin[(Sqrt[g]*x)/2] - (2*I)*b*p*Pi*ArcSin[(Sqrt[g]*x)/2] + I*b*p*ArcS
in[(Sqrt[g]*x)/2]^2 - 4*b*p*Pi*Log[1 + E^((-I)*ArcSin[(Sqrt[g]*x)/2])] - b
*p*Pi*Log[1 - I*E^(I*ArcSin[(Sqrt[g]*x)/2])] - 2*b*p*ArcSin[(Sqrt[g]*x)/2]
*Log[1 - I*E^(I*ArcSin[(Sqrt[g]*x)/2])] + b*p*Pi*Log[1 + I*E^(I*ArcSin[(Sq
rt[g]*x)/2])] - 2*b*p*ArcSin[(Sqrt[g]*x)/2]*Log[1 + I*E^(I*ArcSin[(Sqrt[g]
*x)/2])] + b*ArcSin[(Sqrt[g]*x)/2]*Log[c*(d*(4 - g*x^2))^p] + 4*b*p*Pi*Log
[Cos[ArcSin[(Sqrt[g]*x)/2]/2]] - b*p*Pi*Log[-Cos[(Pi + 2*ArcSin[(Sqrt[g]*x
)/2])/4]] + b*p*Pi*Log[Sin[(Pi + 2*ArcSin[(Sqrt[g]*x)/2])/4]] + (2*I)*b*p*
PolyLog[2, (-I)*E^(I*ArcSin[(Sqrt[g]*x)/2])] + (2*I)*b*p*PolyLog[2, I*E^(I
*ArcSin[(Sqrt[g]*x)/2])])/Sqrt[g]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}\,dx$$
$$\Big\downarrow \text{2923}$$
$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(4*d - d*g*x^2)^p])/Sqrt[4 - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(-dg\,x^2+4d)^p\right)}{\sqrt{-g\,x^2+4}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}\,dx = \int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(-(sqrt(-g*x^2 + 4)*b*log((-d*g*x^2 + 4*d)^p*c) + sqrt(-g*x^2 + 4)
*a)/(g*x^2 - 4), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}\,dx = \int \frac{a+b\log\left(c(-dgx^2+4d)^p\right)}{\sqrt{-gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+4*d)**p))/(-g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(-d*g*x**2 + 4*d)**p))/sqrt(-g*x**2 + 4), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}\,dx = \int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm="max
ima")
```

output
```
b*integrate((p*log(d) + log((-g*x^2 + 4)^p) + log(c))/sqrt(-g*x^2 + 4), x)
 + a*arcsin(1/2*sqrt(g)*x)/sqrt(g)
```

Giac [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}\,dx = \int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((b*log((-d*g*x^2 + 4*d)^p*c) + a)/sqrt(-g*x^2 + 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(4\,d-d\,g\,x^2\right)}^p\right)}{\sqrt{4-g\,x^2}} \,dx$$

input
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(4 - g*x^2)^(1/2),x)
```

output
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(4 - g*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{\sqrt{4-gx^2}}\,dx = \frac{\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{2}\right)a+\left(\int \frac{\log\left((-dg\,x^2+4d)^p c\right)}{\sqrt{-g\,x^2+4}}dx\right)bg}{g}$$

input
```
int((a+b*log(c*(-d*g*x^2+4*d)^p))/(-g*x^2+4)^(1/2),x)
```

output
```
(sqrt(g)*asin((sqrt(g)*x)/2)*a + int(log(( - d*g*x**2 + 4*d)**p*c)/sqrt( -
 g*x**2 + 4),x)*b*g)/g
```

3.685 $\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^2\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 62

$$\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^2\sqrt{4-gx^2}}\,dx=-\frac{1}{2}b\sqrt{g}p\arcsin\left(\frac{\sqrt{g}x}{2}\right)-\frac{\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{4x}$$

output
```
-1/2*b*g^(1/2)*p*arcsin(1/2*g^(1/2)*x)-1/4*(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*
g*x^2+4*d)^p))/x
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^2\sqrt{4-gx^2}}\,dx=-\frac{1}{2}b\sqrt{g}p\arcsin\left(\frac{\sqrt{g}x}{2}\right)-\frac{\sqrt{4-gx^2}\left(a+b\log\left(c\left(d\left(4-gx^2\right)\right)^p\right)\right)}{4x}$$

input
```
Integrate[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^2*Sqrt[4 - g*x^2]),x]
```

output
```
-1/2*(b*Sqrt[g]*p*ArcSin[(Sqrt[g]*x)/2]) - (Sqrt[4 - g*x^2]*(a + b*Log[c*(
d*(4 - g*x^2))^p]))/(4*x)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^2\sqrt{4-gx^2}}\,dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^2\sqrt{4-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^2*Sqrt[4 - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(-dg\,x^2+4d)^p\right)}{x^2\sqrt{-g\,x^2+4}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^2/(-g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^2/(-g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.24

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^2\sqrt{4-gx^2}}\,dx = \left[\frac{2\,b\sqrt{-g}px\log\left(-\sqrt{-g}x-\sqrt{-gx^2+4}\right)-\sqrt{-gx^2+4}(bp\log\left(-dgx^2+4\,d\right)+b\log\left(c\right)+a)}{4\,x}, \frac{4\,b\sqrt{g}px\,\mathrm{a}}{}\right.$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^2/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/4*(2*b*sqrt(-g)*p*x*log(-sqrt(-g)*x - sqrt(-g*x^2 + 4)) - sqrt(-g*x^2 +
 4)*(b*p*log(-d*g*x^2 + 4*d) + b*log(c) + a))/x, 1/4*(4*b*sqrt(g)*p*x*arct
an((sqrt(-g*x^2 + 4) - 2)/(sqrt(g)*x)) - sqrt(-g*x^2 + 4)*(b*p*log(-d*g*x^
2 + 4*d) + b*log(c) + a))/x]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^2\sqrt{4-gx^2}}\,dx = \int \frac{a+b\log\left(c(-dgx^2+4d)^p\right)}{x^2\sqrt{-gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+4*d)**p))/x**2/(-g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(-d*g*x**2 + 4*d)**p))/(x**2*sqrt(-g*x**2 + 4)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^2\sqrt{4-gx^2}}\,dx = -\frac{1}{2}\,b\sqrt{g}p\arcsin\left(\frac{1}{2}\,\sqrt{g}x\right) - \frac{\sqrt{-gx^2+4}b\log\left((-dgx^2+4\,d)^pc\right)}{4\,x} - \frac{\sqrt{-gx^2+4}a}{4\,x}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^2/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/2*b*sqrt(g)*p*arcsin(1/2*sqrt(g)*x) - 1/4*sqrt(-g*x^2 + 4)*b*log((-d*g*
x^2 + 4*d)^p*c)/x - 1/4*sqrt(-g*x^2 + 4)*a/x
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^2\sqrt{4-gx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^2/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^2\sqrt{4-gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(4\,d-d\,g\,x^2\right)}^p\right)}{x^2\,\sqrt{4-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^2*(4 - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^2*(4 - g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^2\sqrt{4-gx^2}}\,dx = \frac{-2\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{2}\right)bpx-\sqrt{-g\,x^2+4}\log\left(\frac{d^p\left(-\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)^2+1\right)^{2p}4^p c}{\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)^2+1\right)^{2p}}\right)b-\sqrt{-g\,x^2+4}\,a}{4x}$$

input
```
int((a+b*log(c*(-d*g*x^2+4*d)^p))/x^2/(-g*x^2+4)^(1/2),x)
```

output
```
( - 2*sqrt(g)*asin((sqrt(g)*x)/2)*b*p*x - sqrt( - g*x**2 + 4)*log((d**p*(
- tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((sqrt(g)*x)/
2)/2)**2 + 1)**(2*p))*b - sqrt( - g*x**2 + 4)*a)/(4*x)
```

3.686 $\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^4\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 124

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}\,dx = \frac{bgp\sqrt{4-gx^2}}{24x}-\frac{1}{12}bg^{3/2}p\arcsin\left(\frac{\sqrt{g}x}{2}\right) -\frac{\sqrt{4-gx^2}\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{12x^3} -\frac{g\sqrt{4-gx^2}\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{24x}$$

output
```
1/24*b*g*p*(-g*x^2+4)^(1/2)/x-1/12*b*g^(3/2)*p*arcsin(1/2*g^(1/2)*x)-1/12*
(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/x^3-1/24*g*(-g*x^2+4)^(1/2)*
(a+b*ln(c*(-d*g*x^2+4*d)^p))/x
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}\,dx = \frac{1}{24}\left(-2bg^{3/2}p\arcsin\left(\frac{\sqrt{g}x}{2}\right) - \frac{\sqrt{4-gx^2}\left(2a+agx^2-bgpx^2+b(2+gx^2)\log\left(c(d(4-gx^2))^p\right)\right)}{x^3}\right)$$

input
```
Integrate[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^4*Sqrt[4 - g*x^2]),x]
```

output
```
(-2*b*g^(3/2)*p*ArcSin[(Sqrt[g]*x)/2] - (Sqrt[4 - g*x^2]*(2*a + a*g*x^2 -
b*g*p*x^2 + b*(2 + g*x^2)*Log[c*(d*(4 - g*x^2))^p]))/x^3)/24
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}\,dx$$
$$\Big\downarrow \text{2929}$$
$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^4*Sqrt[4 - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(-dg\,x^2+4d)^p\right)}{x^4\sqrt{-g\,x^2+4}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^4/(-g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^4/(-g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.73

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}\,dx = \left[\frac{2\,b\sqrt{-g}gpx^3\log\left(-\sqrt{-g}x-\sqrt{-gx^2+4}\right)+((bgp-ag)x^2-(bgpx^2+2\,bp)\log\left(-dgx^2+4\,d\right)-(bgx^2}{24\,x^3}\right.$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^4/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/24*(2*b*sqrt(-g)*g*p*x^3*log(-sqrt(-g)*x - sqrt(-g*x^2 + 4)) + ((b*g*p
- a*g)*x^2 - (b*g*p*x^2 + 2*b*p)*log(-d*g*x^2 + 4*d) - (b*g*x^2 + 2*b)*log
(c) - 2*a)*sqrt(-g*x^2 + 4))/x^3, 1/24*(4*b*g^(3/2)*p*x^3*arctan((sqrt(-g*
x^2 + 4) - 2)/(sqrt(g)*x)) + ((b*g*p - a*g)*x^2 - (b*g*p*x^2 + 2*b*p)*log(
-d*g*x^2 + 4*d) - (b*g*x^2 + 2*b)*log(c) - 2*a)*sqrt(-g*x^2 + 4))/x^3]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}\,dx = \int \frac{a+b\log\left(c(-dgx^2+4d)^p\right)}{x^4\sqrt{-gx^2+4}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+4*d)**p))/x**4/(-g*x**2+4)**(1/2),x)
```

output
```
Integral((a + b*log(c*(-d*g*x**2 + 4*d)**p))/(x**4*sqrt(-g*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}\,dx = \int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}x^4}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^4/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/24*a*(sqrt(-g*x^2 + 4)*g/x + 2*sqrt(-g*x^2 + 4)/x^3) + 1/24*b*((g^2*x^4
 - 2*g*x^2 - 8)*log((-g*x^2 + 4)^p)/(sqrt(-g*x^2 + 4)*x^3) - 2*integrate((
g^2*p*x^4 + 2*g*p*x^2 - 12*p*log(d) - 12*log(c))/(sqrt(-g*x^2 + 4)*x^4), x
))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^4/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(4\,d-d\,g\,x^2\right)}^p\right)}{x^4\,\sqrt{4-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^4*(4 - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^4*(4 - g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.52

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^4\sqrt{4-gx^2}}\,dx = \frac{-2\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{2}\right)bgp\,x^3-\sqrt{-g\,x^2+4}\log\left(\frac{d^p\left(-\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)^2+1\right)^{2p}4^p c}{\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)^2+1\right)^{2p}}\right)bg\,x^2-2\sqrt{-g\,x^2+4}\log\left(\cdots\right.}{24x^3}$$

input
```
int((a+b*log(c*(-d*g*x^2+4*d)^p))/x^4/(-g*x^2+4)^(1/2),x)
```

output
```
( - 2*sqrt(g)*asin((sqrt(g)*x)/2)*b*g*p*x**3 - sqrt( - g*x**2 + 4)*log((d*
*p*( - tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((sqrt(g
)*x)/2)/2)**2 + 1)**(2*p))*b*g*x**2 - 2*sqrt( - g*x**2 + 4)*log((d**p*( -
tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((sqrt(g)*x)/2)
/2)**2 + 1)**(2*p))*b - sqrt( - g*x**2 + 4)*a*g*x**2 - 2*sqrt( - g*x**2 +
4)*a + sqrt( - g*x**2 + 4)*b*g*p*x**2)/(24*x**3)
```

3.687 $\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^6\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 190

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^6\sqrt{4-gx^2}}\,dx = \frac{bgp\sqrt{4-gx^2}}{120x^3}+\frac{bg^2p\sqrt{4-gx^2}}{80x} - \frac{1}{60}bg^{5/2}p\arcsin\left(\frac{\sqrt{g}x}{2}\right) - \frac{\sqrt{4-gx^2}\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{20x^5} - \frac{g\sqrt{4-gx^2}\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{60x^3} - \frac{g^2\sqrt{4-gx^2}\left(a+b\log\left(c(4d-dgx^2)^p\right)\right)}{120x}$$

output
```
1/120*b*g*p*(-g*x^2+4)^(1/2)/x^3+1/80*b*g^2*p*(-g*x^2+4)^(1/2)/x-1/60*b*g^
(5/2)*p*arcsin(1/2*g^(1/2)*x)-1/20*(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*
d)^p))/x^5-1/60*g*(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/x^3-1/120*
g^2*(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/x
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.57

$$\int \frac{a+b\log \left(c(4d-dgx^2)^p\right)}{x^6\sqrt{4-gx^2}}\,dx = \frac{1}{240}\left(-4bg^{5/2}p\arcsin\left(\frac{\sqrt{g}x}{2}\right) + \frac{\sqrt{4-gx^2}\left(bgpx^2(2+3gx^2)-2a(6+2gx^2+g^2x^4)-2b(6+2gx^2+g^2x^4)\log\left(c(d(4-gx^2))^p\right)\right)}{x^5}\right)$$

input
```
Integrate[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^6*Sqrt[4 - g*x^2]),x]
```

output
```
(-4*b*g^(5/2)*p*ArcSin[(Sqrt[g]*x)/2] + (Sqrt[4 - g*x^2]*(b*g*p*x^2*(2 + 3
*g*x^2) - 2*a*(6 + 2*g*x^2 + g^2*x^4) - 2*b*(6 + 2*g*x^2 + g^2*x^4)*Log[c*
(d*(4 - g*x^2))^p]))/x^5)/240
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log \left(c(4d-dgx^2)^p\right)}{x^6\sqrt{4-gx^2}}\,dx$$

$$\Big\downarrow \text{2929}$$

$$\int \frac{a+b\log \left(c(4d-dgx^2)^p\right)}{x^6\sqrt{4-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^6*Sqrt[4 - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(-dg\,x^2+4d)^p\right)}{x^6\sqrt{-g\,x^2+4}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^6/(-g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^6/(-g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.54

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^6\sqrt{4-gx^2}}\,dx$$
$$= \left[\frac{4\,b\sqrt{-g}g^2px^5\log\left(-\sqrt{-g}x-\sqrt{-gx^2+4}\right)+\left((3\,bg^2p-2\,ag^2)x^4+2\,(bgp-2\,ag)x^2-2\,(bg^2px^4+2\,bg\right.}{240\,x^5}\right.$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^6/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/240*(4*b*sqrt(-g)*g^2*p*x^5*log(-sqrt(-g)*x - sqrt(-g*x^2 + 4)) + ((3*b
*g^2*p - 2*a*g^2)*x^4 + 2*(b*g*p - 2*a*g)*x^2 - 2*(b*g^2*p*x^4 + 2*b*g*p*x
^2 + 6*b*p)*log(-d*g*x^2 + 4*d) - 2*(b*g^2*x^4 + 2*b*g*x^2 + 6*b)*log(c) -
 12*a)*sqrt(-g*x^2 + 4))/x^5, 1/240*(8*b*g^(5/2)*p*x^5*arctan((sqrt(-g*x^2
 + 4) - 2)/(sqrt(g)*x)) + ((3*b*g^2*p - 2*a*g^2)*x^4 + 2*(b*g*p - 2*a*g)*x
^2 - 2*(b*g^2*p*x^4 + 2*b*g*p*x^2 + 6*b*p)*log(-d*g*x^2 + 4*d) - 2*(b*g^2*
x^4 + 2*b*g*x^2 + 6*b)*log(c) - 12*a)*sqrt(-g*x^2 + 4))/x^5]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^6\sqrt{4-gx^2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+4*d)**p))/x**6/(-g*x**2+4)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^6\sqrt{4-gx^2}}\,dx=\int \frac{b\log\left((-dgx^2+4\,d)^pc\right)+a}{\sqrt{-gx^2+4}x^6}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^6/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/120*(sqrt(-g*x^2 + 4)*g^2/x + 2*sqrt(-g*x^2 + 4)*g/x^3 + 6*sqrt(-g*x^2
+ 4)/x^5)*a + 1/120*b*((g^3*x^6 - 2*g^2*x^4 - 2*g*x^2 - 24)*log((-g*x^2 +
4)^p)/(sqrt(-g*x^2 + 4)*x^5) - 2*integrate((g^3*p*x^6 + 2*g^2*p*x^4 + 6*g*
p*x^2 - 60*p*log(d) - 60*log(c))/(sqrt(-g*x^2 + 4)*x^6), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^6\sqrt{4-gx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^6/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(4d - dgx^2)^p\right)}{x^6\sqrt{4 - gx^2}}\,dx = \int \frac{a + b\,\ln\left(c\,{\left(4\,d - d\,g\,x^2\right)}^p\right)}{x^6\,\sqrt{4 - g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^6*(4 - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^6*(4 - g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.54

$$\int \frac{a + b\log\left(c(4d - dgx^2)^p\right)}{x^6\sqrt{4 - gx^2}}\,dx = \frac{-4\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{2}\right) b\,g^2 p\,x^5 - 2\sqrt{-g\,x^2+4}\log\left(\frac{d^p\left(-\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)^2+1\right)^{2p} 4^p c}{\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)^2+1\right)^{2p}}\right) b\,g^2 x^4 - 4\sqrt{-g\,x^2+4}\log}{}$$

input
```
int((a+b*log(c*(-d*g*x^2+4*d)^p))/x^6/(-g*x^2+4)^(1/2),x)
```

output
```
( - 4*sqrt(g)*asin((sqrt(g)*x)/2)*b*g**2*p*x**5 - 2*sqrt( - g*x**2 + 4)*lo
g((d**p*( - tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((s
qrt(g)*x)/2)/2)**2 + 1)**(2*p))*b*g**2*x**4 - 4*sqrt( - g*x**2 + 4)*log((d
**p*( - tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((sqrt(
g)*x)/2)/2)**2 + 1)**(2*p))*b*g*x**2 - 12*sqrt( - g*x**2 + 4)*log((d**p*(
- tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((sqrt(g)*x)/
2)/2)**2 + 1)**(2*p))*b - 2*sqrt( - g*x**2 + 4)*a*g**2*x**4 - 4*sqrt( - g*
x**2 + 4)*a*g*x**2 - 12*sqrt( - g*x**2 + 4)*a + 3*sqrt( - g*x**2 + 4)*b*g*
*2*p*x**4 + 2*sqrt( - g*x**2 + 4)*b*g*p*x**2)/(240*x**5)
```

3.688 $\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^8\sqrt{4-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 256

$$\begin{aligned}\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^8\sqrt{4-gx^2}}\,dx = {} & \frac{bgp\sqrt{4-gx^2}}{280x^5}+\frac{bg^2p\sqrt{4-gx^2}}{336x^3} \\ & +\frac{11bg^3p\sqrt{4-gx^2}}{3360x}-\frac{1}{280}bg^{7/2}p\arcsin\left(\frac{\sqrt{g}x}{2}\right) \\ & -\frac{\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{28x^7} \\ & -\frac{3g\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{280x^5} \\ & -\frac{g^2\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{280x^3} \\ & -\frac{g^3\sqrt{4-gx^2}\left(a+b\log\left(c\left(4d-dgx^2\right)^p\right)\right)}{560x}\end{aligned}$$

output

```
1/280*b*g*p*(-g*x^2+4)^(1/2)/x^5+1/336*b*g^2*p*(-g*x^2+4)^(1/2)/x^3+11/336
0*b*g^3*p*(-g*x^2+4)^(1/2)/x-1/280*b*g^(7/2)*p*arcsin(1/2*g^(1/2)*x)-1/28*
(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p))/x^7-3/280*g*(-g*x^2+4)^(1/2)
*(a+b*ln(c*(-d*g*x^2+4*d)^p))/x^5-1/280*g^2*(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d
*g*x^2+4*d)^p))/x^3-1/560*g^3*(-g*x^2+4)^(1/2)*(a+b*ln(c*(-d*g*x^2+4*d)^p)
)/x
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.52

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^8\sqrt{4-gx^2}}\,dx$$
$$= \frac{-12bg^{7/2}p\arcsin\left(\frac{\sqrt{g}x}{2}\right)+\frac{\sqrt{4-gx^2}\left(bgpx^2(12+10gx^2+11g^2x^4)-6a(20+6gx^2+2g^2x^4+g^3x^6)-6b(20+6gx^2+2g^2x^4+g^3x^6)\log(c(d(\right.}{x^7}}{3360}$$

input
```
Integrate[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^8*Sqrt[4 - g*x^2]),x]
```

output
```
(-12*b*g^(7/2)*p*ArcSin[(Sqrt[g]*x)/2] + (Sqrt[4 - g*x^2]*(b*g*p*x^2*(12 +
 10*g*x^2 + 11*g^2*x^4) - 6*a*(20 + 6*g*x^2 + 2*g^2*x^4 + g^3*x^6) - 6*b*(
20 + 6*g*x^2 + 2*g^2*x^4 + g^3*x^6)*Log[c*(d*(4 - g*x^2))^p]))/x^7)/3360
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^8\sqrt{4-gx^2}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{a+b\log\left(c\left(4d-dgx^2\right)^p\right)}{x^8\sqrt{4-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(4*d - d*g*x^2)^p])/(x^8*Sqrt[4 - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(-dg\,x^2+4d)^p\right)}{x^8\sqrt{-g\,x^2+4}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^8/(-g*x^2+4)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+4*d)^p))/x^8/(-g*x^2+4)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.44

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^8\sqrt{4-gx^2}}\,dx$$
$$= \left[\frac{12\,b\sqrt{-g}g^3px^7\log\left(-\sqrt{-g}x-\sqrt{-gx^2+4}\right)+((11\,bg^3p-6\,ag^3)x^6+2\,(5\,bg^2p-6\,ag^2)x^4+12\,(bgp-\right.$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^8/(-g*x^2+4)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/3360*(12*b*sqrt(-g)*g^3*p*x^7*log(-sqrt(-g)*x - sqrt(-g*x^2 + 4)) + ((1
1*b*g^3*p - 6*a*g^3)*x^6 + 2*(5*b*g^2*p - 6*a*g^2)*x^4 + 12*(b*g*p - 3*a*g
)*x^2 - 6*(b*g^3*p*x^6 + 2*b*g^2*p*x^4 + 6*b*g*p*x^2 + 20*b*p)*log(-d*g*x^
2 + 4*d) - 6*(b*g^3*x^6 + 2*b*g^2*x^4 + 6*b*g*x^2 + 20*b)*log(c) - 120*a)*
sqrt(-g*x^2 + 4))/x^7, 1/3360*(24*b*g^(7/2)*p*x^7*arctan((sqrt(-g*x^2 + 4)
 - 2)/(sqrt(g)*x)) + ((11*b*g^3*p - 6*a*g^3)*x^6 + 2*(5*b*g^2*p - 6*a*g^2)
*x^4 + 12*(b*g*p - 3*a*g)*x^2 - 6*(b*g^3*p*x^6 + 2*b*g^2*p*x^4 + 6*b*g*p*x
^2 + 20*b*p)*log(-d*g*x^2 + 4*d) - 6*(b*g^3*x^6 + 2*b*g^2*x^4 + 6*b*g*x^2
+ 20*b)*log(c) - 120*a)*sqrt(-g*x^2 + 4))/x^7]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(4d - dgx^2)^p\right)}{x^8\sqrt{4 - gx^2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+4*d)**p))/x**8/(-g*x**2+4)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + b\log\left(c(4d - dgx^2)^p\right)}{x^8\sqrt{4 - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + 4\,d)^p c\right) + a}{\sqrt{-gx^2 + 4}x^8}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^8/(-g*x^2+4)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/560*(sqrt(-g*x^2 + 4)*g^3/x + 2*sqrt(-g*x^2 + 4)*g^2/x^3 + 6*sqrt(-g*x^
2 + 4)*g/x^5 + 20*sqrt(-g*x^2 + 4)/x^7)*a + 1/560*b*((g^4*x^8 - 2*g^3*x^6
- 2*g^2*x^4 - 4*g*x^2 - 80)*log((-g*x^2 + 4)^p)/(sqrt(-g*x^2 + 4)*x^7) - 2
*integrate((g^4*p*x^8 + 2*g^3*p*x^6 + 6*g^2*p*x^4 + 20*g*p*x^2 - 280*p*log
(d) - 280*log(c))/(sqrt(-g*x^2 + 4)*x^8), x))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^8\sqrt{4-gx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+4*d)^p))/x^8/(-g*x^2+4)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^8\sqrt{4-gx^2}}\,dx = \int \frac{a+b\,\ln\left(c\,(4\,d-d\,g\,x^2)^p\right)}{x^8\,\sqrt{4-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^8*(4 - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(4*d - d*g*x^2)^p))/(x^8*(4 - g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.54

$$\int \frac{a+b\log\left(c(4d-dgx^2)^p\right)}{x^8\sqrt{4-gx^2}}\,dx$$

$$= \frac{-12\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{2}\right)b\,g^3p\,x^7-6\sqrt{-g\,x^2+4}\log\left(\frac{d^p\left(-\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)^2+1\right)^{2p}4^pc}{\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{2}\right)}{2}\right)^2+1\right)^{2p}}\right)b\,g^3x^6-12\sqrt{-g\,x^2+4}\,\mathrm{l}}{}$$

input
```
int((a+b*log(c*(-d*g*x^2+4*d)^p))/x^8/(-g*x^2+4)^(1/2),x)
```

output
```
( - 12*sqrt(g)*asin((sqrt(g)*x)/2)*b*g**3*p*x**7 - 6*sqrt( - g*x**2 + 4)*l
og((d**p*( - tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((
sqrt(g)*x)/2)/2)**2 + 1)**(2*p))*b*g**3*x**6 - 12*sqrt( - g*x**2 + 4)*log(
(d**p*( - tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((sqr
t(g)*x)/2)/2)**2 + 1)**(2*p))*b*g**2*x**4 - 36*sqrt( - g*x**2 + 4)*log((d*
*p*( - tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((sqrt(g
)*x)/2)/2)**2 + 1)**(2*p))*b*g*x**2 - 120*sqrt( - g*x**2 + 4)*log((d**p*(
- tan(asin((sqrt(g)*x)/2)/2)**2 + 1)**(2*p)*4**p*c)/(tan(asin((sqrt(g)*x)/
2)/2)**2 + 1)**(2*p))*b - 6*sqrt( - g*x**2 + 4)*a*g**3*x**6 - 12*sqrt( - g
*x**2 + 4)*a*g**2*x**4 - 36*sqrt( - g*x**2 + 4)*a*g*x**2 - 120*sqrt( - g*x
**2 + 4)*a + 11*sqrt( - g*x**2 + 4)*b*g**3*p*x**6 + 10*sqrt( - g*x**2 + 4)
*b*g**2*p*x**4 + 12*sqrt( - g*x**2 + 4)*b*g*p*x**2)/(3360*x**7)
```

3.689 $\int \frac{x^7\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 236

$$\int \frac{x^7\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = \frac{2bf^3p\sqrt{f+gx^2}}{g^4}-\frac{2bf^2p\left(f+gx^2\right)^{3/2}}{3g^4}+\frac{6bfp\left(f+gx^2\right)^{5/2}}{25g^4}-\frac{2bp\left(f+gx^2\right)^{7/2}}{49g^4}-\frac{f^3\sqrt{f+gx^2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{g^4}+\frac{f^2\left(f+gx^2\right)^{3/2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{g^4}-\frac{3f\left(f+gx^2\right)^{5/2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{5g^4}+\frac{\left(f+gx^2\right)^{7/2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{7g^4}$$

output
```
2*b*f^3*p*(g*x^2+f)^(1/2)/g^4-2/3*b*f^2*p*(g*x^2+f)^(3/2)/g^4+6/25*b*f*p*(
g*x^2+f)^(5/2)/g^4-2/49*b*p*(g*x^2+f)^(7/2)/g^4-f^3*(g*x^2+f)^(1/2)*(a+b*l
n(c*(d*g*x^2+d*f)^p))/g^4+f^2*(g*x^2+f)^(3/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/
g^4-3/5*f*(g*x^2+f)^(5/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/g^4+1/7*(g*x^2+f)^(7
/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/g^4
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.59

$$\int \frac{x^7\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = -\frac{\sqrt{f+gx^2}\left(105a(16f^3-8f^2gx^2+6fg^2x^4-5g^3x^6)+2bp(-2816f^3+568f^2gx^2-216fg^2x^4+75g^3x^6\right.}{3675g^4}$$

input
```
Integrate[(x^7*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
-1/3675*(Sqrt[f + g*x^2]*(105*a*(16*f^3 - 8*f^2*g*x^2 + 6*f*g^2*x^4 - 5*g^
3*x^6) + 2*b*p*(-2816*f^3 + 568*f^2*g*x^2 - 216*f*g^2*x^4 + 75*g^3*x^6) +
105*b*(16*f^3 - 8*f^2*g*x^2 + 6*f*g^2*x^4 - 5*g^3*x^6)*Log[c*(d*(f + g*x^2
))^p]))/g^4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{x^7\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}dx$$

input
```
Int[(x^7*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^7\left(a+b\ln\left(c\left(dg\,x^2+df\right)^p\right)\right)}{\sqrt{g\,x^2+f}}dx$$

input
```
int(x^7*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
int(x^7*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.77

$$\int \frac{x^7\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = -\frac{(75\,(2\,bg^3p-7\,ag^3)x^6-5632\,bf^3p-18\,(24\,bf\,g^2p-35\,af\,g^2)x^4+1680\,af^3+8\,(142\,bf^2gp-105\,af^2g}{}$$

input
```
integrate(x^7*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/3675*(75*(2*b*g^3*p - 7*a*g^3)*x^6 - 5632*b*f^3*p - 18*(24*b*f*g^2*p -
35*a*f*g^2)*x^4 + 1680*a*f^3 + 8*(142*b*f^2*g*p - 105*a*f^2*g)*x^2 - 105*(
5*b*g^3*p*x^6 - 6*b*f*g^2*p*x^4 + 8*b*f^2*g*p*x^2 - 16*b*f^3*p)*log(d*g*x^
2 + d*f) - 105*(5*b*g^3*x^6 - 6*b*f*g^2*x^4 + 8*b*f^2*g*x^2 - 16*b*f^3)*lo
g(c))*sqrt(g*x^2 + f)/g^4
```

Sympy [A] (verification not implemented)

Time = 16.29 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.53

$$\int \frac{x^7 \left(a + b \log \left(c(df + dgx^2)^p\right)\right)}{\sqrt{f + gx^2}} \, dx$$
$$= \begin{cases} -\frac{16af^3\sqrt{f+gx^2}}{35g^4} + \frac{8af^2x^2\sqrt{f+gx^2}}{35g^3} - \frac{6afx^4\sqrt{f+gx^2}}{35g^2} + \frac{ax^6\sqrt{f+gx^2}}{7g} + \frac{5632bf^3p\sqrt{f+gx^2}}{3675g^4} - \frac{16bf^3\sqrt{f+gx^2}\log\left(c(df+dgx^2)^p\right)}{35g^4} \\ \frac{x^8(a+b\log\left(c(df)^p\right))}{8\sqrt{f}} \end{cases}$$

input
```
integrate(x**7*(a+b*ln(c*(d*g*x**2+d*f)**p))/(g*x**2+f)**(1/2),x)
```

output
```
Piecewise((-16*a*f**3*sqrt(f + g*x**2)/(35*g**4) + 8*a*f**2*x**2*sqrt(f +
g*x**2)/(35*g**3) - 6*a*f*x**4*sqrt(f + g*x**2)/(35*g**2) + a*x**6*sqrt(f
+ g*x**2)/(7*g) + 5632*b*f**3*p*sqrt(f + g*x**2)/(3675*g**4) - 16*b*f**3*s
qrt(f + g*x**2)*log(c*(d*f + d*g*x**2)**p)/(35*g**4) - 1136*b*f**2*p*x**2*
sqrt(f + g*x**2)/(3675*g**3) + 8*b*f**2*x**2*sqrt(f + g*x**2)*log(c*(d*f +
 d*g*x**2)**p)/(35*g**3) + 144*b*f*p*x**4*sqrt(f + g*x**2)/(1225*g**2) - 6
*b*f*x**4*sqrt(f + g*x**2)*log(c*(d*f + d*g*x**2)**p)/(35*g**2) - 2*b*p*x*
*6*sqrt(f + g*x**2)/(49*g) + b*x**6*sqrt(f + g*x**2)*log(c*(d*f + d*g*x**2
)**p)/(7*g), Ne(g, 0)), (x**8*(a + b*log(c*(d*f)**p))/(8*sqrt(f)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.96

$$\int \frac{x^7 \left(a + b \log \left(c(df + dgx^2)^p\right)\right)}{\sqrt{f + gx^2}} \, dx$$
$$= \frac{1}{35} \left(\frac{5\sqrt{gx^2+f}x^6}{g} - \frac{6\sqrt{gx^2+f}fx^4}{g^2} + \frac{8\sqrt{gx^2+f}f^2x^2}{g^3} - \frac{16\sqrt{gx^2+f}f^3}{g^4}\right) b \log\left(\left(dgx^2+df\right)^p c\right)$$
$$+ \frac{1}{35} \left(\frac{5\sqrt{gx^2+f}x^6}{g} - \frac{6\sqrt{gx^2+f}fx^4}{g^2} + \frac{8\sqrt{gx^2+f}f^2x^2}{g^3} - \frac{16\sqrt{gx^2+f}f^3}{g^4}\right) a$$
$$- \frac{2\left(75\left(gx^2+f\right)^{\frac{7}{2}} - 441\left(gx^2+f\right)^{\frac{5}{2}}f + 1225\left(gx^2+f\right)^{\frac{3}{2}}f^2 - 3675\sqrt{gx^2+f}f^3\right)bp}{3675\,g^4}$$

input
```
integrate(x^7*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
1/35*(5*sqrt(g*x^2 + f)*x^6/g - 6*sqrt(g*x^2 + f)*f*x^4/g^2 + 8*sqrt(g*x^2
 + f)*f^2*x^2/g^3 - 16*sqrt(g*x^2 + f)*f^3/g^4)*b*log((d*g*x^2 + d*f)^p*c)
 + 1/35*(5*sqrt(g*x^2 + f)*x^6/g - 6*sqrt(g*x^2 + f)*f*x^4/g^2 + 8*sqrt(g*
x^2 + f)*f^2*x^2/g^3 - 16*sqrt(g*x^2 + f)*f^3/g^4)*a - 2/3675*(75*(g*x^2 +
 f)^(7/2) - 441*(g*x^2 + f)^(5/2)*f + 1225*(g*x^2 + f)^(3/2)*f^2 - 3675*sq
rt(g*x^2 + f)*f^3)*b*p/g^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.07

$$\int \frac{x^7 \left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$
$$= \frac{bp\left(\frac{105\left(5\left(gx^2+f\right)^{\frac{7}{2}}-21\left(gx^2+f\right)^{\frac{5}{2}}f+35\left(gx^2+f\right)^{\frac{3}{2}}f^2-35\sqrt{gx^2+f}\,f^3\right)\log(dgx^2+df)}{g^3}-\frac{2\left(75\left(gx^2+f\right)^{\frac{7}{2}}-441\left(gx^2+f\right)^{\frac{5}{2}}f+1225\left(gx^2\right.\right.}{g^3}\right.}{}$$

input
```
integrate(x^7*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
1/3675*(b*p*(105*(5*(g*x^2 + f)^(7/2) - 21*(g*x^2 + f)^(5/2)*f + 35*(g*x^2
 + f)^(3/2)*f^2 - 35*sqrt(g*x^2 + f)*f^3)*log(d*g*x^2 + d*f)/g^3 - 2*(75*(
g*x^2 + f)^(7/2) - 441*(g*x^2 + f)^(5/2)*f + 1225*(g*x^2 + f)^(3/2)*f^2 -
3675*sqrt(g*x^2 + f)*f^3)/g^3) + 105*(5*(g*x^2 + f)^(7/2) - 21*(g*x^2 + f)
^(5/2)*f + 35*(g*x^2 + f)^(3/2)*f^2 - 35*sqrt(g*x^2 + f)*f^3)*b*log(c)/g^3
 + 105*(5*(g*x^2 + f)^(7/2) - 21*(g*x^2 + f)^(5/2)*f + 35*(g*x^2 + f)^(3/2
)*f^2 - 35*sqrt(g*x^2 + f)*f^3)*a/g^3)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{x^7\,\left(a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)\right)}{\sqrt{g\,x^2+f}}\,dx$$

input
```
int((x^7*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2),x)
```

output
```
int((x^7*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.47

$$\int \frac{x^7\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$
$$=\frac{\sqrt{g\,x^2+f}\,\left(-1680\log\left(\frac{f^p d^p\left(\sqrt{g}\,\sqrt{g\,x^2+f}\,x+f+g\,x^2\right)^{2p}c}{\left(\sqrt{f}\,\sqrt{g\,x^2+f}+\sqrt{g}\,\sqrt{f}\,x\right)^{2p}}\right)b\,f^3+840\log\left(\frac{f^p d^p\left(\sqrt{g}\,\sqrt{g\,x^2+f}\,x+f+g\,x^2\right)^{2p}c}{\left(\sqrt{f}\,\sqrt{g\,x^2+f}+\sqrt{g}\,\sqrt{f}\,x\right)^{2p}}\right)b\,f^2g\,x^2$$

input
```
int(x^7*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f + g*x**2)*( - 1680*log((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f
+ g*x**2)**(2*p)*c)/(sqrt(f)*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))
*b*f**3 + 840*log((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f + g*x**2)**(2
*p)*c)/(sqrt(f)*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))*b*f**2*g*x**
2 - 630*log((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f + g*x**2)**(2*p)*c)
/(sqrt(f)*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))*b*f*g**2*x**4 + 52
5*log((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f + g*x**2)**(2*p)*c)/(sqrt
(f)*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))*b*g**3*x**6 - 1680*a*f**
3 + 840*a*f**2*g*x**2 - 630*a*f*g**2*x**4 + 525*a*g**3*x**6 + 5632*b*f**3*
p - 1136*b*f**2*g*p*x**2 + 432*b*f*g**2*p*x**4 - 150*b*g**3*p*x**6))/(3675
*g**4)
```

3.690 $\int \frac{x^5\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 175

$$\begin{aligned}\int \frac{x^5\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = &-\frac{2bf^2p\sqrt{f+gx^2}}{g^3}+\frac{4bfp(f+gx^2)^{3/2}}{9g^3}\\ &-\frac{2bp(f+gx^2)^{5/2}}{25g^3}\\ &+\frac{f^2\sqrt{f+gx^2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{g^3}\\ &-\frac{2f(f+gx^2)^{3/2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{3g^3}\\ &+\frac{(f+gx^2)^{5/2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{5g^3}\end{aligned}$$

output

```
-2*b*f^2*p*(g*x^2+f)^(1/2)/g^3+4/9*b*f*p*(g*x^2+f)^(3/2)/g^3-2/25*b*p*(g*x
^2+f)^(5/2)/g^3+f^2*(g*x^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/g^3-2/3*f*
(g*x^2+f)^(3/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/g^3+1/5*(g*x^2+f)^(5/2)*(a+b*l
n(c*(d*g*x^2+d*f)^p))/g^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

$$\int \frac{x^5 \left(a+b \log \left(c \left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}} \, dx = \frac{\sqrt{f+gx^2} \left(15a(8f^2-4fgx^2+3g^2x^4)-2bp(184f^2-32fgx^2+9g^2x^4)+15b(8f^2-4fgx^2+3g^2x^4) \log\right.}{225g^3}$$

input
```
Integrate[(x^5*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
(Sqrt[f + g*x^2]*(15*a*(8*f^2 - 4*f*g*x^2 + 3*g^2*x^4) - 2*b*p*(184*f^2 -
32*f*g*x^2 + 9*g^2*x^4) + 15*b*(8*f^2 - 4*f*g*x^2 + 3*g^2*x^4)*Log[c*(d*(f
 + g*x^2))^p]))/(225*g^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(a+b \log \left(c \left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}} \, dx$$

$$\Big\downarrow \text{2929}$$

$$\int \frac{x^5 \left(a+b \log \left(c \left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}dx$$

input
```
Int[(x^5*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^5 \left(a+b\ln \left(c(dg\,x^2+df)^p\right)\right)}{\sqrt{g\,x^2+f}}dx$$

input
```
int(x^5*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
int(x^5*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.75

$$\int \frac{x^5 \left(a+b\log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = -\frac{(9\,(2\,bg^2p-5\,ag^2)x^4+368\,bf^2p-120\,af^2-4\,(16\,bfgp-15\,afg)x^2-15\,(3\,bg^2px^4-4\,bfgpx^2+8\,bf}{225\,g^3}$$

input
```
integrate(x^5*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/225*(9*(2*b*g^2*p - 5*a*g^2)*x^4 + 368*b*f^2*p - 120*a*f^2 - 4*(16*b*f*
g*p - 15*a*f*g)*x^2 - 15*(3*b*g^2*p*x^4 - 4*b*f*g*p*x^2 + 8*b*f^2*p)*log(d
*g*x^2 + d*f) - 15*(3*b*g^2*x^4 - 4*b*f*g*x^2 + 8*b*f^2)*log(c))*sqrt(g*x^
2 + f)/g^3
```

Sympy [A] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.53

$$\int \frac{x^5\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = \begin{cases} \frac{8af^2\sqrt{f+gx^2}}{15g^3} - \frac{4afx^2\sqrt{f+gx^2}}{15g^2} + \frac{ax^4\sqrt{f+gx^2}}{5g} - \frac{368bf^2p\sqrt{f+gx^2}}{225g^3} + \frac{8bf^2\sqrt{f+gx^2}\log\left(c(df+dgx^2)^p\right)}{15g^3} + \frac{64bfpx^2\sqrt{f+gx^2}}{225g^2} - \\ \frac{x^6(a+b\log(c(df)^p))}{6\sqrt{f}} \end{cases}$$

input
```
integrate(x**5*(a+b*ln(c*(d*g*x**2+d*f)**p))/(g*x**2+f)**(1/2),x)
```

output
```
Piecewise((8*a*f**2*sqrt(f + g*x**2)/(15*g**3) - 4*a*f*x**2*sqrt(f + g*x**
2)/(15*g**2) + a*x**4*sqrt(f + g*x**2)/(5*g) - 368*b*f**2*p*sqrt(f + g*x**
2)/(225*g**3) + 8*b*f**2*sqrt(f + g*x**2)*log(c*(d*f + d*g*x**2)**p)/(15*g
**3) + 64*b*f*p*x**2*sqrt(f + g*x**2)/(225*g**2) - 4*b*f*x**2*sqrt(f + g*x
**2)*log(c*(d*f + d*g*x**2)**p)/(15*g**2) - 2*b*p*x**4*sqrt(f + g*x**2)/(2
5*g) + b*x**4*sqrt(f + g*x**2)*log(c*(d*f + d*g*x**2)**p)/(5*g), Ne(g, 0))
, (x**6*(a + b*log(c*(d*f)**p))/(6*sqrt(f)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^5\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx &= \frac{1}{15}\left(\frac{3\sqrt{gx^2+f}x^4}{g} - \frac{4\sqrt{gx^2+f}fx^2}{g^2} + \frac{8\sqrt{gx^2+f}f^2}{g^3}\right)b\log\left(\left(dgx^2+df\right)^pc\right) \\ &\quad + \frac{1}{15}\left(\frac{3\sqrt{gx^2+f}x^4}{g} - \frac{4\sqrt{gx^2+f}fx^2}{g^2} + \frac{8\sqrt{gx^2+f}f^2}{g^3}\right)a \\ &\quad - \frac{2\left(9\left(gx^2+f\right)^{\frac{5}{2}} - 50\left(gx^2+f\right)^{\frac{3}{2}}f + 225\sqrt{gx^2+f}f^2\right)bp}{225\,g^3}\end{aligned}$$

input
```
integrate(x^5*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
1/15*(3*sqrt(g*x^2 + f)*x^4/g - 4*sqrt(g*x^2 + f)*f*x^2/g^2 + 8*sqrt(g*x^2
 + f)*f^2/g^3)*b*log((d*g*x^2 + d*f)^p*c) + 1/15*(3*sqrt(g*x^2 + f)*x^4/g
- 4*sqrt(g*x^2 + f)*f*x^2/g^2 + 8*sqrt(g*x^2 + f)*f^2/g^3)*a - 2/225*(9*(g
*x^2 + f)^(5/2) - 50*(g*x^2 + f)^(3/2)*f + 225*sqrt(g*x^2 + f)*f^2)*b*p/g^
3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09

$$\int \frac{x^5\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = -\frac{\frac{\left(18\left(gx^2+f\right)^{\frac{5}{2}}-100\left(gx^2+f\right)^{\frac{3}{2}}f+450\sqrt{gx^2+f}\,f^2-15\left(3\left(gx^2+f\right)^{\frac{5}{2}}-10\left(gx^2+f\right)^{\frac{3}{2}}f+15\sqrt{gx^2+f}\,f^2\right)\log\left(dgx^2+df\right)\right)bp}{g^2}-\frac{15\left(3\left(gx^2\right.\right.}{}}{225\,g}$$

input
```
integrate(x^5*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
-1/225*((18*(g*x^2 + f)^(5/2) - 100*(g*x^2 + f)^(3/2)*f + 450*sqrt(g*x^2 +
 f)*f^2 - 15*(3*(g*x^2 + f)^(5/2) - 10*(g*x^2 + f)^(3/2)*f + 15*sqrt(g*x^2
 + f)*f^2)*log(d*g*x^2 + d*f))*b*p/g^2 - 15*(3*(g*x^2 + f)^(5/2) - 10*(g*x
^2 + f)^(3/2)*f + 15*sqrt(g*x^2 + f)*f^2)*b*log(c)/g^2 - 15*(3*(g*x^2 + f)
^(5/2) - 10*(g*x^2 + f)^(3/2)*f + 15*sqrt(g*x^2 + f)*f^2)*a/g^2)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = \int \frac{x^5\,\left(a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)\right)}{\sqrt{g\,x^2+f}}\,dx$$

input
```
int((x^5*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.45

$$\int \frac{x^5(a+b\log(c(df+dgx^2)^p))}{\sqrt{f+gx^2}}\,dx$$
$$= \frac{\sqrt{g\,x^2+f}\left(120\log\left(\frac{f^p d^p\left(\sqrt{g}\,\sqrt{g\,x^2+f}\,x+f+g\,x^2\right)^{2p}c}{\left(\sqrt{f}\,\sqrt{g\,x^2+f}+\sqrt{g}\,\sqrt{f}\,x\right)^{2p}}\right)b\,f^2-60\log\left(\frac{f^p d^p\left(\sqrt{g}\,\sqrt{g\,x^2+f}\,x+f+g\,x^2\right)^{2p}c}{\left(\sqrt{f}\,\sqrt{g\,x^2+f}+\sqrt{g}\,\sqrt{f}\,x\right)^{2p}}\right)bfg\,x^2+45\right.}{}$$

input
```
int(x^5*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f + g*x**2)*(120*log((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f + g*
x**2)**(2*p)*c)/(sqrt(f)*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))*b*f
**2 - 60*log((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f + g*x**2)**(2*p)*c
)/(sqrt(f)*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))*b*f*g*x**2 + 45*l
og((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f + g*x**2)**(2*p)*c)/(sqrt(f)
*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))*b*g**2*x**4 + 120*a*f**2 -
60*a*f*g*x**2 + 45*a*g**2*x**4 - 368*b*f**2*p + 64*b*f*g*p*x**2 - 18*b*g**
2*p*x**4))/(225*g**3)
```

3.691 $\int \frac{x^3\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 113

$$\int \frac{x^3(a+b\log(c(df+dgx^2)^p))}{\sqrt{f+gx^2}}\,dx = \frac{2bfp\sqrt{f+gx^2}}{g^2}-\frac{2bp(f+gx^2)^{3/2}}{9g^2}-\frac{f\sqrt{f+gx^2}(a+b\log(c(df+dgx^2)^p))}{g^2}+\frac{(f+gx^2)^{3/2}(a+b\log(c(df+dgx^2)^p))}{3g^2}$$

output
```
2*b*f*p*(g*x^2+f)^(1/2)/g^2-2/9*b*p*(g*x^2+f)^(3/2)/g^2-f*(g*x^2+f)^(1/2)*
(a+b*ln(c*(d*g*x^2+d*f)^p))/g^2+1/3*(g*x^2+f)^(3/2)*(a+b*ln(c*(d*g*x^2+d*f
)^p))/g^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.61

$$\int \frac{x^3\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$
$$=\frac{\sqrt{f+gx^2}\left(-6af+16bfp+3agx^2-2bgpx^2+b(-6f+3gx^2)\log\left(c(d(f+gx^2))^p\right)\right)}{9g^2}$$

input
```
Integrate[(x^3*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
(Sqrt[f + g*x^2]*(-6*a*f + 16*b*f*p + 3*a*g*x^2 - 2*b*g*p*x^2 + b*(-6*f +
3*g*x^2)*Log[c*(d*(f + g*x^2))^p]))/(9*g^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^3\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}dx$$

input
```
Int[(x^3*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^3 \left(a + b \ln \left(c \left(d g\, x^2 + d f\right)^p\right)\right)}{\sqrt{g\, x^2 + f}}dx$$

input
```
int(x^3*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
int(x^3*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \frac{x^3 \left(a + b \log \left(c \left(df + dgx^2\right)^p\right)\right)}{\sqrt{f + gx^2}}\, dx$$
$$= \frac{\left(16\, bfp - (2\, bgp - 3\, ag)x^2 - 6\, af + 3\, (bgpx^2 - 2\, bfp) \log \left(dgx^2 + df\right) + 3\, (bgx^2 - 2\, bf) \log (c)\right)\sqrt{gx^2 +}}{9\, g^2}$$

input
```
integrate(x^3*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
1/9*(16*b*f*p - (2*b*g*p - 3*a*g)*x^2 - 6*a*f + 3*(b*g*p*x^2 - 2*b*f*p)*lo
g(d*g*x^2 + d*f) + 3*(b*g*x^2 - 2*b*f)*log(c))*sqrt(g*x^2 + f)/g^2
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.53

$$\int \frac{x^3 \left(a+b \log \left(c (df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$
$$= \begin{cases} -\frac{2af\sqrt{f+gx^2}}{3g^2} + \frac{ax^2\sqrt{f+gx^2}}{3g} + \frac{16bfp\sqrt{f+gx^2}}{9g^2} - \frac{2bf\sqrt{f+gx^2}\log\left(c(df+dgx^2)^p\right)}{3g^2} - \frac{2bpx^2\sqrt{f+gx^2}}{9g} + \frac{bx^2\sqrt{f+gx^2}\log(c(df+dg}{3g} \\ \frac{x^4(a+b\log(c(df)^p))}{4\sqrt{f}} \end{cases}$$

input
```
integrate(x**3*(a+b*ln(c*(d*g*x**2+d*f)**p))/(g*x**2+f)**(1/2),x)
```

output
```
Piecewise((-2*a*f*sqrt(f + g*x**2)/(3*g**2) + a*x**2*sqrt(f + g*x**2)/(3*g
) + 16*b*f*p*sqrt(f + g*x**2)/(9*g**2) - 2*b*f*sqrt(f + g*x**2)*log(c*(d*f
 + d*g*x**2)**p)/(3*g**2) - 2*b*p*x**2*sqrt(f + g*x**2)/(9*g) + b*x**2*sqr
t(f + g*x**2)*log(c*(d*f + d*g*x**2)**p)/(3*g), Ne(g, 0)), (x**4*(a + b*lo
g(c*(d*f)**p))/(4*sqrt(f)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{x^3 \left(a+b \log \left(c (df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$
$$= \frac{1}{3}\left(\frac{\sqrt{gx^2+f}x^2}{g} - \frac{2\sqrt{gx^2+f}f}{g^2}\right) b \log\left((dgx^2+df)^p c\right) + \frac{1}{3}\left(\frac{\sqrt{gx^2+f}x^2}{g} - \frac{2\sqrt{gx^2+f}f}{g^2}\right) a - \frac{2\left((gx^2+f)^{\frac{3}{2}} - 9\sqrt{gx^2+f}f\right) bp}{9g^2}$$

input
```
integrate(x^3*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
1/3*(sqrt(g*x^2 + f)*x^2/g - 2*sqrt(g*x^2 + f)*f/g^2)*b*log((d*g*x^2 + d*f
)^p*c) + 1/3*(sqrt(g*x^2 + f)*x^2/g - 2*sqrt(g*x^2 + f)*f/g^2)*a - 2/9*((g
*x^2 + f)^(3/2) - 9*sqrt(g*x^2 + f)*f)*b*p/g^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{x^3\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$
$$=\frac{\frac{\left(3\left((gx^2+f)^{\frac{3}{2}}-3\sqrt{gx^2+f}f\right)\log(dgx^2+df)-2(gx^2+f)^{\frac{3}{2}}+18\sqrt{gx^2+f}f\right)bp}{g}+\frac{3\left((gx^2+f)^{\frac{3}{2}}-3\sqrt{gx^2+f}f\right)b\log(c)}{g}+\frac{3\left((gx^2+f)^{\frac{3}{2}}-3\right.}{g}}{9g}$$

input
```
integrate(x^3*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
1/9*((3*((g*x^2 + f)^(3/2) - 3*sqrt(g*x^2 + f)*f)*log(d*g*x^2 + d*f) - 2*(
g*x^2 + f)^(3/2) + 18*sqrt(g*x^2 + f)*f)*b*p/g + 3*((g*x^2 + f)^(3/2) - 3*
sqrt(g*x^2 + f)*f)*b*log(c)/g + 3*((g*x^2 + f)^(3/2) - 3*sqrt(g*x^2 + f)*f
)*a/g)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{x^3\,\left(a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)\right)}{\sqrt{g\,x^2+f}}\,dx$$

input
```
int((x^3*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.42

$$\int \frac{x^3(a+b\log(c(df+dgx^2)^p))}{\sqrt{f+gx^2}}\,dx = \frac{\sqrt{g\,x^2+f}\left(-6\log\left(\frac{f^p d^p\left(\sqrt{g}\,\sqrt{g\,x^2+f}\,x+f+g\,x^2\right)^{2p}c}{\left(\sqrt{f}\,\sqrt{g\,x^2+f}+\sqrt{g}\,\sqrt{f}\,x\right)^{2p}}\right)bf+3\log\left(\frac{f^p d^p\left(\sqrt{g}\,\sqrt{g\,x^2+f}\,x+f+g\,x^2\right)^{2p}c}{\left(\sqrt{f}\,\sqrt{g\,x^2+f}+\sqrt{g}\,\sqrt{f}\,x\right)^{2p}}\right)bg\,x^2-6af+\right.}{9g^2}$$

input
```
int(x^3*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f + g*x**2)*( - 6*log((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f + g
*x**2)**(2*p)*c)/(sqrt(f)*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))*b*
f + 3*log((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f + g*x**2)**(2*p)*c)/(
sqrt(f)*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))*b*g*x**2 - 6*a*f + 3
*a*g*x**2 + 16*b*f*p - 2*b*g*p*x**2))/(9*g**2)
```

3.692 $\int \frac{x\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 53

$$\int \frac{x\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=-\frac{2bp\sqrt{f+gx^2}}{g}+\frac{\sqrt{f+gx^2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{g}$$

output
```
-2*b*p*(g*x^2+f)^(1/2)/g+(g*x^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/g
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=\frac{\sqrt{f+gx^2}\left(a-2bp+b\log\left(c\left(d\left(f+gx^2\right)\right)^p\right)\right)}{g}$$

input
```
Integrate[(x*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
(Sqrt[f + g*x^2]*(a - 2*b*p + b*Log[c*(d*(f + g*x^2))^p]))/g
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\log(c(df+dgx^2)^p))}{\sqrt{f+gx^2}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{x(a+b\log(c(df+dgx^2)^p))}{\sqrt{f+gx^2}}dx$$

input
```
Int[(x*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x(a+b\ln(c(dg\,x^2+df)^p))}{\sqrt{g\,x^2+f}}dx$$

input
```
int(x*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
int(x*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x(a+b\log(c(df+dgx^2)^p))}{\sqrt{f+gx^2}}\,dx = \frac{\sqrt{gx^2+f}(bp\log(dgx^2+df)-2\,bp+b\log(c)+a)}{g}$$

input
```
integrate(x*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="fri
cas")
```

output
```
sqrt(g*x^2 + f)*(b*p*log(d*g*x^2 + d*f) - 2*b*p + b*log(c) + a)/g
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{x(a+b\log(c(df+dgx^2)^p))}{\sqrt{f+gx^2}}\,dx = \begin{cases} \frac{a\sqrt{f+gx^2}}{g} - \frac{2bp\sqrt{f+gx^2}}{g} + \frac{b\sqrt{f+gx^2}\log(c(df+dgx^2)^p)}{g} & \text{for } g \neq 0 \\ \frac{x^2(a+b\log(c(df)^p))}{2\sqrt{f}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*ln(c*(d*g*x**2+d*f)**p))/(g*x**2+f)**(1/2),x)
```

output
```
Piecewise((a*sqrt(f + g*x**2)/g - 2*b*p*sqrt(f + g*x**2)/g + b*sqrt(f + g*
x**2)*log(c*(d*f + d*g*x**2)**p)/g, Ne(g, 0)), (x**2*(a + b*log(c*(d*f)**p
))/(2*sqrt(f)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \frac{x\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = -\frac{2\sqrt{gx^2+f}bp}{g} + \frac{\sqrt{gx^2+f}b\log\left((dgx^2+df)^p c\right)}{g} + \frac{\sqrt{gx^2+f}a}{g}$$

input
```
integrate(x*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="max
ima")
```

output
```
-2*sqrt(g*x^2 + f)*b*p/g + sqrt(g*x^2 + f)*b*log((d*g*x^2 + d*f)^p*c)/g +
sqrt(g*x^2 + f)*a/g
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{x\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = \frac{\left(\sqrt{gx^2+f}\log\left(dgx^2+df\right)-2\sqrt{gx^2+f}\right)bp+\sqrt{gx^2+f}b\log\left(c\right)+\sqrt{gx^2+f}a}{g}$$

input
```
integrate(x*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="gia
c")
```

output
```
((sqrt(g*x^2 + f)*log(d*g*x^2 + d*f) - 2*sqrt(g*x^2 + f))*b*p + sqrt(g*x^2
 + f)*b*log(c) + sqrt(g*x^2 + f)*a)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{x\,\left(a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)\right)}{\sqrt{g\,x^2+f}}\,dx$$

input
```
int((x*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2),x)
```

output
```
int((x*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \frac{x\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = \frac{\sqrt{g\,x^2+f}\,\left(\log\left(\frac{f^p d^p \left(\sqrt{g}\,\sqrt{g\,x^2+f}\,x+f+g\,x^2\right)^{2p} c}{\left(\sqrt{f}\,\sqrt{g\,x^2+f}+\sqrt{g}\,\sqrt{f}\,x\right)^{2p}}\right) b+a-2bp\right)}{g}$$

input
```
int(x*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f + g*x**2)*(log((f**p*d**p*(sqrt(g)*sqrt(f + g*x**2)*x + f + g*x**2
)**(2*p)*c)/(sqrt(f)*sqrt(f + g*x**2) + sqrt(g)*sqrt(f)*x)**(2*p))*b + a -
 2*b*p))/g
```

3.693 $\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 101

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x\sqrt{f+gx^2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f}}-\frac{bp\operatorname{PolyLog}\left(2,-\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{bp\operatorname{PolyLog}\left(2,\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)}{\sqrt{f}}$$

output
```
-arctanh((g*x^2+f)^(1/2)/f^(1/2))*(a+b*ln(c*(d*g*x^2+d*f)^p))/f^(1/2)-b*p*
polylog(2,-(g*x^2+f)^(1/2)/f^(1/2))/f^(1/2)+b*p*polylog(2,(g*x^2+f)^(1/2)/
f^(1/2))/f^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x\sqrt{f+gx^2}}\,dx$$
$$= \frac{\left(a+b\log\left(c(d(f+gx^2))^p\right)\right)\left(\log\left(1-\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)-\log\left(1+\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)\right)-2bp\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)+2}{2\sqrt{f}}$$

input
```
Integrate[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x*Sqrt[f + g*x^2]),x]
```

output
```
((a + b*Log[c*(d*(f + g*x^2))^p])*(Log[1 - Sqrt[f + g*x^2]/Sqrt[f]] - Log[
1 + Sqrt[f + g*x^2]/Sqrt[f]]) - 2*b*p*PolyLog[2, -(Sqrt[f + g*x^2]/Sqrt[f]
)] + 2*b*p*PolyLog[2, Sqrt[f + g*x^2]/Sqrt[f]])/(2*Sqrt[f])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x\sqrt{f+gx^2}}\,dx$$
$$\Big\downarrow \text{2929}$$
$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x\sqrt{f+gx^2}}\,dx$$

input
```
Int[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x*Sqrt[f + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln \left(c(dg\,x^2+df)^p\right)}{x\sqrt{g\,x^2+f}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x/(g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x/(g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log \left(c(df+dgx^2)^p\right)}{x\sqrt{f+gx^2}}\,dx=\int \frac{b\log \left((dgx^2+df)^pc\right)+a}{\sqrt{gx^2+f}x}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x/(g*x^2+f)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((sqrt(g*x^2 + f)*b*log((d*g*x^2 + d*f)^p*c) + sqrt(g*x^2 + f)*a)/
(g*x^3 + f*x), x)
```

Sympy [F]

$$\int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x\sqrt{f + gx^2}}\,dx = \int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x\sqrt{f + gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+d*f)**p))/x/(g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f + d*g*x**2)**p))/(x*sqrt(f + g*x**2)), x)
```

Maxima [F]

$$\int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x\sqrt{f + gx^2}}\,dx = \int \frac{b\log\left((dgx^2 + df)^p c\right) + a}{\sqrt{gx^2 + f}x}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x/(g*x^2+f)^(1/2),x, algorithm="max
ima")
```

output
```
b*integrate((p*log(g*x^2 + f) + p*log(d) + log(c))/(sqrt(g*x^2 + f)*x), x)
 - a*arcsinh(f/(sqrt(f*g)*abs(x)))/sqrt(f)
```

Giac [F]

$$\int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x\sqrt{f + gx^2}}\,dx = \int \frac{b\log\left((dgx^2 + df)^p c\right) + a}{\sqrt{gx^2 + f}x}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x/(g*x^2+f)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((b*log((d*g*x^2 + d*f)^p*c) + a)/(sqrt(g*x^2 + f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x\sqrt{f+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)}{x\,\sqrt{g\,x^2+f}}\,dx$$

input
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x*(f + g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x*(f + g*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x\sqrt{f+gx^2}}\,dx$$
$$=\frac{\sqrt{f}\,\log\left(\frac{\sqrt{g\,x^2+f}-\sqrt{f}+\sqrt{g}\,x}{\sqrt{f}}\right)a-\sqrt{f}\,\log\left(\frac{\sqrt{g\,x^2+f}+\sqrt{f}+\sqrt{g}\,x}{\sqrt{f}}\right)a+\left(\int \frac{\log\left(\left(dg\,x^2+df\right)^p c\right)}{\sqrt{g\,x^2+f}\,x}dx\right)bf}{f}$$

input
```
int((a+b*log(c*(d*g*x^2+d*f)^p))/x/(g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f)*log((sqrt(f + g*x**2) - sqrt(f) + sqrt(g)*x)/sqrt(f))*a - sqrt(f)
*log((sqrt(f + g*x**2) + sqrt(f) + sqrt(g)*x)/sqrt(f))*a + int(log((d*f +
d*g*x**2)**p*c)/(sqrt(f + g*x**2)*x),x)*b*f)/f
```

3.694 $\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^3\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 179

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^3\sqrt{f+gx^2}}\,dx=-\frac{bgp\operatorname{arctanh}\left(\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)}{f^{3/2}}-\frac{\sqrt{f+gx^2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{2fx^2}+\frac{g\operatorname{arctanh}\left(\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{2f^{3/2}}+\frac{bgp\operatorname{PolyLog}\left(2,-\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)}{2f^{3/2}}-\frac{bgp\operatorname{PolyLog}\left(2,\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)}{2f^{3/2}}$$

output
```
-b*g*p*arctanh((g*x^2+f)^(1/2)/f^(1/2))/f^(3/2)-1/2*(g*x^2+f)^(1/2)*(a+b*l
n(c*(d*g*x^2+d*f)^p))/f/x^2+1/2*g*arctanh((g*x^2+f)^(1/2)/f^(1/2))*(a+b*ln
(c*(d*g*x^2+d*f)^p))/f^(3/2)+1/2*b*g*p*polylog(2,-(g*x^2+f)^(1/2)/f^(1/2))
/f^(3/2)-1/2*b*g*p*polylog(2,(g*x^2+f)^(1/2)/f^(1/2))/f^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.73

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^3\sqrt{f+gx^2}}\,dx =$$
$$-\frac{2a\sqrt{f}\sqrt{f+gx^2}+2b\sqrt{f}\sqrt{f+gx^2}\log\left(c(d(f+gx^2))^p\right)-2bgpx^2\log\left(\sqrt{f}-\sqrt{f+gx^2}\right)+2bgpx^2\log(}{}$$

input
```
Integrate[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^3*Sqrt[f + g*x^2]),x]
```

output
```
-1/4*(2*a*Sqrt[f]*Sqrt[f + g*x^2] + 2*b*Sqrt[f]*Sqrt[f + g*x^2]*Log[c*(d*(
f + g*x^2))^p] - 2*b*g*p*x^2*Log[Sqrt[f] - Sqrt[f + g*x^2]] + 2*b*g*p*x^2*
Log[Sqrt[f] + Sqrt[f + g*x^2]] + a*g*x^2*Log[1 - Sqrt[f + g*x^2]/Sqrt[f]]
+ b*g*x^2*Log[c*(d*(f + g*x^2))^p]*Log[1 - Sqrt[f + g*x^2]/Sqrt[f]] - a*g*
x^2*Log[1 + Sqrt[f + g*x^2]/Sqrt[f]] - b*g*x^2*Log[c*(d*(f + g*x^2))^p]*Lo
g[1 + Sqrt[f + g*x^2]/Sqrt[f]] - 2*b*g*p*x^2*PolyLog[2, -(Sqrt[f + g*x^2]/
Sqrt[f])] + 2*b*g*p*x^2*PolyLog[2, Sqrt[f + g*x^2]/Sqrt[f]])/(f^(3/2)*x^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^3\sqrt{f+gx^2}}\,dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^3\sqrt{f+gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^3*Sqrt[f + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(dg\,x^{2}+df)^{p}\right)}{x^{3}\sqrt{g\,x^{2}+f}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^3/(g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^3/(g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^3\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+df)^pc\right)+a}{\sqrt{gx^2+f}x^3}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^3/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((sqrt(g*x^2 + f)*b*log((d*g*x^2 + d*f)^p*c) + sqrt(g*x^2 + f)*a)/
(g*x^5 + f*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x^3\sqrt{f + gx^2}}\,dx = \int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x^3\sqrt{f + gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+d*f)**p))/x**3/(g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f + d*g*x**2)**p))/(x**3*sqrt(f + g*x**2)), x)
```

Maxima [F]

$$\int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x^3\sqrt{f + gx^2}}\,dx = \int \frac{b\log\left((dgx^2 + df)^p c\right) + a}{\sqrt{gx^2 + f}x^3}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^3/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
1/2*a*(g*arcsinh(f/(sqrt(f*g)*abs(x)))/f^(3/2) - sqrt(g*x^2 + f)/(f*x^2))
+ b*integrate((p*log(g*x^2 + f) + p*log(d) + log(c))/(sqrt(g*x^2 + f)*x^3)
, x)
```

Giac [F]

$$\int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x^3\sqrt{f + gx^2}}\,dx = \int \frac{b\log\left((dgx^2 + df)^p c\right) + a}{\sqrt{gx^2 + f}x^3}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^3/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*log((d*g*x^2 + d*f)^p*c) + a)/(sqrt(g*x^2 + f)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^3\sqrt{f+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)}{x^3\,\sqrt{g\,x^2+f}}\,dx$$

input
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^3*(f + g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^3*(f + g*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^3\sqrt{f+gx^2}}\,dx = \frac{-\sqrt{g\,x^2+f}\,af-\sqrt{f}\,\log\left(\frac{\sqrt{g\,x^2+f}-\sqrt{f}+\sqrt{g}\,x}{\sqrt{f}}\right)ag\,x^2+\sqrt{f}\,\log\left(\frac{\sqrt{g\,x^2+f}+\sqrt{f}+\sqrt{g}\,x}{\sqrt{f}}\right)ag\,x^2+2\left(\int \frac{\log\left(\left(dg\,x^2+df\right)^p}{\sqrt{g\,x^2+f}\,x^3}\right.}{2f^2x^2}$$

input
```
int((a+b*log(c*(d*g*x^2+d*f)^p))/x^3/(g*x^2+f)^(1/2),x)
```

output
```
( - sqrt(f + g*x**2)*a*f - sqrt(f)*log((sqrt(f + g*x**2) - sqrt(f) + sqrt(
g)*x)/sqrt(f))*a*g*x**2 + sqrt(f)*log((sqrt(f + g*x**2) + sqrt(f) + sqrt(g
)*x)/sqrt(f))*a*g*x**2 + 2*int(log((d*f + d*g*x**2)**p*c)/(sqrt(f + g*x**2
)*x**3),x)*b*f**2*x**2)/(2*f**2*x**2)
```

3.695 $\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^5\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 251

$$\begin{aligned}\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^5\sqrt{f+gx^2}}\,dx &= -\frac{bgp\sqrt{f+gx^2}}{4f^2x^2}+\frac{bg^2p\text{arctanh}\left(\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)}{f^{5/2}} \\ &\quad -\frac{\sqrt{f+gx^2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{4fx^4} \\ &\quad +\frac{3g\sqrt{f+gx^2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{8f^2x^2} \\ &\quad -\frac{3g^2\text{arctanh}\left(\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{8f^{5/2}} \\ &\quad -\frac{3bg^2p\,\text{PolyLog}\left(2,-\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)}{8f^{5/2}} \\ &\quad +\frac{3bg^2p\,\text{PolyLog}\left(2,\frac{\sqrt{f+gx^2}}{\sqrt{f}}\right)}{8f^{5/2}}\end{aligned}$$

output
```
-1/4*b*g*p*(g*x^2+f)^(1/2)/f^2/x^2+b*g^2*p*arctanh((g*x^2+f)^(1/2)/f^(1/2)
)/f^(5/2)-1/4*(g*x^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/f/x^4+3/8*g*(g*x
^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/f^2/x^2-3/8*g^2*arctanh((g*x^2+f)^
(1/2)/f^(1/2))*(a+b*ln(c*(d*g*x^2+d*f)^p))/f^(5/2)-3/8*b*g^2*p*polylog(2,-
(g*x^2+f)^(1/2)/f^(1/2))/f^(5/2)+3/8*b*g^2*p*polylog(2,(g*x^2+f)^(1/2)/f^(
1/2))/f^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.64

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}\,dx = \frac{-4af^{3/2}\sqrt{f+gx^2}+6a\sqrt{f}gx^2\sqrt{f+gx^2}-4b\sqrt{f}gpx^2\sqrt{f+gx^2}-4bf^{3/2}\sqrt{f+gx^2}\log\left(c(d(f+gx^2))^p\right)}{}$$

input
```
Integrate[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^5*Sqrt[f + g*x^2]),x]
```

output
```
(-4*a*f^(3/2)*Sqrt[f + g*x^2] + 6*a*Sqrt[f]*g*x^2*Sqrt[f + g*x^2] - 4*b*Sq
rt[f]*g*p*x^2*Sqrt[f + g*x^2] - 4*b*f^(3/2)*Sqrt[f + g*x^2]*Log[c*(d*(f +
g*x^2))^p] + 6*b*Sqrt[f]*g*x^2*Sqrt[f + g*x^2]*Log[c*(d*(f + g*x^2))^p] -
8*b*g^2*p*x^4*Log[Sqrt[f] - Sqrt[f + g*x^2]] + 8*b*g^2*p*x^4*Log[Sqrt[f] +
 Sqrt[f + g*x^2]] + 3*a*g^2*x^4*Log[1 - Sqrt[f + g*x^2]/Sqrt[f]] + 3*b*g^2
*x^4*Log[c*(d*(f + g*x^2))^p]*Log[1 - Sqrt[f + g*x^2]/Sqrt[f]] - 3*a*g^2*x
^4*Log[1 + Sqrt[f + g*x^2]/Sqrt[f]] - 3*b*g^2*x^4*Log[c*(d*(f + g*x^2))^p]
*Log[1 + Sqrt[f + g*x^2]/Sqrt[f]] - 6*b*g^2*p*x^4*PolyLog[2, -(Sqrt[f + g*
x^2]/Sqrt[f])] + 6*b*g^2*p*x^4*PolyLog[2, Sqrt[f + g*x^2]/Sqrt[f]])/(16*f^
(5/2)*x^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}\,dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^5*Sqrt[f + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(dg\,x^2+df)^p\right)}{x^5\sqrt{g\,x^2+f}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^5/(g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^5/(g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+df)^p c\right)+a}{\sqrt{gx^2+f}x^5}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^5/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((sqrt(g*x^2 + f)*b*log((d*g*x^2 + d*f)^p*c) + sqrt(g*x^2 + f)*a)/
(g*x^7 + f*x^5), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}\,dx=\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+d*f)**p))/x**5/(g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f + d*g*x**2)**p))/(x**5*sqrt(f + g*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+df)^p c\right)+a}{\sqrt{gx^2+f}x^5}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^5/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
-1/8*a*(3*g^2*arcsinh(f/(sqrt(f*g)*abs(x)))/f^(5/2) - 3*sqrt(g*x^2 + f)*g/
(f^2*x^2) + 2*sqrt(g*x^2 + f)/(f*x^4)) + b*integrate((p*log(g*x^2 + f) + p
*log(d) + log(c))/(sqrt(g*x^2 + f)*x^5), x)
```

Giac [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+df)^pc\right)+a}{\sqrt{gx^2+f}x^5}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^5/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*log((d*g*x^2 + d*f)^p*c) + a)/(sqrt(g*x^2 + f)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)}{x^5\,\sqrt{g\,x^2+f}}\,dx$$

input
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^5*(f + g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^5*(f + g*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^5\sqrt{f+gx^2}}\,dx$$
$$=\frac{-2\sqrt{g\,x^2+f}\,a\,f^2+3\sqrt{g\,x^2+f}\,a f g\,x^2+3\sqrt{f}\,\log\left(\frac{\sqrt{g\,x^2+f}-\sqrt{f}+\sqrt{g}\,x}{\sqrt{f}}\right)a\,g^2x^4-3\sqrt{f}\,\log\left(\frac{\sqrt{g\,x^2+f}+\sqrt{f}+\sqrt{g}\,x}{\sqrt{f}}\right.}{8f^3x^4}$$

input
```
int((a+b*log(c*(d*g*x^2+d*f)^p))/x^5/(g*x^2+f)^(1/2),x)
```

output

```
( - 2*sqrt(f + g*x**2)*a*f**2 + 3*sqrt(f + g*x**2)*a*f*g*x**2 + 3*sqrt(f)*
log((sqrt(f + g*x**2) - sqrt(f) + sqrt(g)*x)/sqrt(f))*a*g**2*x**4 - 3*sqrt
(f)*log((sqrt(f + g*x**2) + sqrt(f) + sqrt(g)*x)/sqrt(f))*a*g**2*x**4 + 8*
int(log((d*f + d*g*x**2)**p*c)/(sqrt(f + g*x**2)*x**5),x)*b*f**3*x**4)/(8*
f**3*x**4)
```

3.696 $\int \frac{x^2\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 406

$$
\begin{aligned}
\int \frac{x^2\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx
&= \frac{bf^2p}{8g^{3/2}\left(\sqrt{g}x+\sqrt{f+gx^2}\right)^2}+\frac{a\left(\sqrt{g}x+\sqrt{f+gx^2}\right)^2}{8g^{3/2}} \\
&\quad-\frac{bp\left(\sqrt{g}x+\sqrt{f+gx^2}\right)^2}{8g^{3/2}}+\frac{b\left(\sqrt{g}x+\sqrt{f+gx^2}\right)^2\log\left(c(d(f+gx^2))^p\right)}{8g^{3/2}} \\
&\quad-\frac{f^2\left(a+b\log\left(c(d(f+gx^2))^p\right)\right)}{8g^{3/2}\left(\sqrt{g}x+\sqrt{f+gx^2}\right)^2}+\frac{bfp\log\left(\sqrt{g}x+\sqrt{f+gx^2}\right)}{2g^{3/2}} \\
&\quad-\frac{f\left(a+b\log\left(c(d(f+gx^2))^p\right)\right)\log\left(\sqrt{g}x+\sqrt{f+gx^2}\right)}{2g^{3/2}} \\
&\quad-\frac{bfp\log^2\left(\sqrt{g}x+\sqrt{f+gx^2}\right)}{2g^{3/2}} \\
&\quad+\frac{bfp\log\left(\sqrt{g}x+\sqrt{f+gx^2}\right)\log\left(1+\frac{\left(\sqrt{g}x+\sqrt{f+gx^2}\right)^2}{f}\right)}{g^{3/2}} \\
&\quad+\frac{bfp\,\mathrm{PolyLog}\left(2,-\frac{\left(\sqrt{g}x+\sqrt{f+gx^2}\right)^2}{f}\right)}{2g^{3/2}}
\end{aligned}
$$

output
```
1/8*b*f^2*p/g^(3/2)/(g^(1/2)*x+(g*x^2+f)^(1/2))^2+1/8*a*(g^(1/2)*x+(g*x^2+
f)^(1/2))^2/g^(3/2)-1/8*b*p*(g^(1/2)*x+(g*x^2+f)^(1/2))^2/g^(3/2)+1/8*b*(g
^(1/2)*x+(g*x^2+f)^(1/2))^2*ln(c*(d*(g*x^2+f))^p)/g^(3/2)-1/8*f^2*(a+b*ln(
c*(d*(g*x^2+f))^p))/g^(3/2)/(g^(1/2)*x+(g*x^2+f)^(1/2))^2+1/2*b*f*p*ln(g^(
1/2)*x+(g*x^2+f)^(1/2))/g^(3/2)-1/2*f*(a+b*ln(c*(d*(g*x^2+f))^p))*ln(g^(1/
2)*x+(g*x^2+f)^(1/2))/g^(3/2)-1/2*b*f*p*ln(g^(1/2)*x+(g*x^2+f)^(1/2))^2/g^
(3/2)+b*f*p*ln(g^(1/2)*x+(g*x^2+f)^(1/2))*ln(1+(g^(1/2)*x+(g*x^2+f)^(1/2))
^2/f)/g^(3/2)+1/2*b*f*p*polylog(2,-(g^(1/2)*x+(g*x^2+f)^(1/2))^2/f)/g^(3/2
)
```

Mathematica [F]

$$\int \frac{x^2 \left(a+b\log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{x^2 \left(a+b\log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$

input
```
Integrate[(x^2*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
Integrate[(x^2*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b\log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^2 \left(a+b\log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}dx$$

input
```
Int[(x^2*(a + b*Log[c*(d*f + d*g*x^2)^p]))/Sqrt[f + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^2\left(a+b\ln\left(c(dg\,x^2+df)^p\right)\right)}{\sqrt{g\,x^2+f}}dx$$

input
```
int(x^2*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
int(x^2*(a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{\left(b\log\left((dgx^2+df)^pc\right)+a\right)x^2}{\sqrt{gx^2+f}}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((sqrt(g*x^2 + f)*b*x^2*log((d*g*x^2 + d*f)^p*c) + sqrt(g*x^2 + f)
*a*x^2)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{x^2 \left(a+b \log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{x^2 \left(a+b \log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx$$

input
```
integrate(x**2*(a+b*ln(c*(d*g*x**2+d*f)**p))/(g*x**2+f)**(1/2),x)
```

output
```
Integral(x**2*(a + b*log(c*(d*f + d*g*x**2)**p))/sqrt(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{x^2 \left(a+b \log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{\left(b \log \left((dgx^2+df)^p c\right)+a\right)x^2}{\sqrt{gx^2+f}}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
1/2*a*(sqrt(g*x^2 + f)*x/g - f*arcsinh(g*x/sqrt(f*g))/g^(3/2)) + b*integra
te((p*x^2*log(g*x^2 + f) + (p*log(d) + log(c))*x^2)/sqrt(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{x^2 \left(a+b \log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{\left(b \log \left((dgx^2+df)^p c\right)+a\right)x^2}{\sqrt{gx^2+f}}\,dx$$

input
```
integrate(x^2*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*log((d*g*x^2 + d*f)^p*c) + a)*x^2/sqrt(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \left(a+b\log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = \int \frac{x^2\,\left(a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)\right)}{\sqrt{g\,x^2+f}}\,dx$$

input
```
int((x^2*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*log(c*(d*f + d*g*x^2)^p)))/(f + g*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2 \left(a+b\log \left(c(df+dgx^2)^p\right)\right)}{\sqrt{f+gx^2}}\,dx = \frac{\sqrt{g\,x^2+f}\,agx-\sqrt{g}\log\left(\frac{\sqrt{g\,x^2+f}+\sqrt{g}\,x}{\sqrt{f}}\right)af+2\left(\int \frac{\log\left(\left(dg\,x^2+df\right)^p c\right)x^2}{\sqrt{g\,x^2+f}}dx\right)b\,g^2}{2g^2}$$

input
```
int(x^2*(a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f + g*x**2)*a*g*x - sqrt(g)*log((sqrt(f + g*x**2) + sqrt(g)*x)/sqrt(
f))*a*f + 2*int((log((d*f + d*g*x**2)**p*c)*x**2)/sqrt(f + g*x**2),x)*b*g*
*2)/(2*g**2)
```

3.697 $\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 169

$$\begin{aligned}\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{\sqrt{f+gx^2}}\,dx &= \frac{\left(a+b\log\left(c(d(f+gx^2))^p\right)\right)\log\left(\sqrt{g}x+\sqrt{f+gx^2}\right)}{\sqrt{g}}+\frac{bp\log^2\left(\sqrt{g}x+\sqrt{f+gx^2}\right)}{\sqrt{g}}\\ &-\frac{2bp\log\left(\sqrt{g}x+\sqrt{f+gx^2}\right)\log\left(1+\frac{\left(\sqrt{g}x+\sqrt{f+gx^2}\right)^2}{f}\right)}{\sqrt{g}}\\ &-\frac{bp\,\mathrm{PolyLog}\left(2,-\frac{\left(\sqrt{g}x+\sqrt{f+gx^2}\right)^2}{f}\right)}{\sqrt{g}}\end{aligned}$$

output
```
(a+b*ln(c*(d*(g*x^2+f))^p))*ln(g^(1/2)*x+(g*x^2+f)^(1/2))/g^(1/2)+b*p*ln(g
^(1/2)*x+(g*x^2+f)^(1/2))^2/g^(1/2)-2*b*p*ln(g^(1/2)*x+(g*x^2+f)^(1/2))*ln
(1+(g^(1/2)*x+(g*x^2+f)^(1/2))^2/f)/g^(1/2)-b*p*polylog(2,-(g^(1/2)*x+(g*x
^2+f)^(1/2))^2/f)/g^(1/2)
```

Mathematica [F]

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{\sqrt{f+gx^2}}\,dx$$

input
```
Integrate[(a + b*Log[c*(d*f + d*g*x^2)^p])/Sqrt[f + g*x^2],x]
```

output
```
Integrate[(a + b*Log[c*(d*f + d*g*x^2)^p])/Sqrt[f + g*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{\sqrt{f+gx^2}}\,dx$$

$$\Big\downarrow \text{ 2923}$$

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{\sqrt{f+gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f + d*g*x^2)^p])/Sqrt[f + g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln \left(c{\left(dg\,x^{2}+df\right)}^{p}\right)}{\sqrt{g\,x^{2}+f}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log \left(c(df+dgx^2)^p\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{b\log \left((dgx^2+df)^pc\right)+a}{\sqrt{gx^2+f}}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="frica
s")
```

output
```
integral((sqrt(g*x^2 + f)*b*log((d*g*x^2 + d*f)^p*c) + sqrt(g*x^2 + f)*a)/
(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{\sqrt{f+gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+d*f)**p))/(g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f + d*g*x**2)**p))/sqrt(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+df)^pc\right)+a}{\sqrt{gx^2+f}}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="maxim
a")
```

output
```
b*integrate((p*log(g*x^2 + f) + p*log(d) + log(c))/sqrt(g*x^2 + f), x) + a
*arcsinh(g*x/sqrt(f*g))/sqrt(g)
```

Giac [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+df)^pc\right)+a}{\sqrt{gx^2+f}}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((b*log((d*g*x^2 + d*f)^p*c) + a)/sqrt(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{\sqrt{f+gx^2}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)}{\sqrt{g\,x^2+f}}\,dx$$

input
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(f + g*x^2)^(1/2),x)
```

output
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(f + g*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{\sqrt{f+gx^2}}\,dx = \frac{\sqrt{g}\,\log\left(\frac{\sqrt{g\,x^2+f}+\sqrt{g}\,x}{\sqrt{f}}\right)a+\left(\int \frac{\log\left(\left(dg\,x^2+df\right)^p c\right)}{\sqrt{g\,x^2+f}}dx\right)bg}{g}$$

input
```
int((a+b*log(c*(d*g*x^2+d*f)^p))/(g*x^2+f)^(1/2),x)
```

output
```
(sqrt(g)*log((sqrt(f + g*x**2) + sqrt(g)*x)/sqrt(f))*a + int(log((d*f + d*
g*x**2)**p*c)/sqrt(f + g*x**2),x)*b*g)/g
```

3.698 $\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^2\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 70

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^2\sqrt{f+gx^2}}\,dx=\frac{2b\sqrt{g}p\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f+gx^2}}\right)}{f}-\frac{\sqrt{f+gx^2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{fx}$$

output
```
2*b*g^(1/2)*p*arctanh(g^(1/2)*x/(g*x^2+f)^(1/2))/f-(g*x^2+f)^(1/2)*(a+b*ln
(c*(d*g*x^2+d*f)^p))/f/x
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^2\sqrt{f+gx^2}}\,dx=-\frac{a\sqrt{f+gx^2}+b\sqrt{f+gx^2}\log\left(c(d(f+gx^2))^p\right)-2b\sqrt{g}px\log\left(gx+\sqrt{g}\sqrt{f+gx^2}\right)}{fx}$$

input
```
Integrate[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^2*Sqrt[f + g*x^2]),x]
```

output
```
-((a*Sqrt[f + g*x^2] + b*Sqrt[f + g*x^2]*Log[c*(d*(f + g*x^2))^p] - 2*b*Sq
rt[g]*p*x*Log[g*x + Sqrt[g]*Sqrt[f + g*x^2]])/(f*x))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^2\sqrt{f+gx^2}}\,dx$$

$$\Big\downarrow \ 2929$$

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^2\sqrt{f+gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^2*Sqrt[f + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(dg\,x^2+df)^p\right)}{x^2\sqrt{g\,x^2+f}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^2/(g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^2/(g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.99

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^2\sqrt{f+gx^2}}\,dx = \left[\frac{b\sqrt{g}px\log\left(-2\,gx^2-2\,\sqrt{gx^2+f}\sqrt{g}x-f\right)-\sqrt{gx^2+f}(bp\log\left(dgx^2+df\right)+b\log\left(c\right)+a)}{fx}, -\frac{2\,b\sqrt{-g}px\arctan\left(\frac{\sqrt{-g}x}{\sqrt{gx^2+f}}\right)+\sqrt{gx^2+f}(bp\log\left(dgx^2+df\right)+b\log\left(c\right)+a)}{fx}\right]$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^2/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
[(b*sqrt(g)*p*x*log(-2*g*x^2 - 2*sqrt(g*x^2 + f)*sqrt(g)*x - f) - sqrt(g*x
^2 + f)*(b*p*log(d*g*x^2 + d*f) + b*log(c) + a))/(f*x), -(2*b*sqrt(-g)*p*x
*arctan(sqrt(-g)*x/sqrt(g*x^2 + f)) + sqrt(g*x^2 + f)*(b*p*log(d*g*x^2 + d
*f) + b*log(c) + a))/(f*x)]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^2\sqrt{f+gx^2}}\,dx=\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^2\sqrt{f+gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+d*f)**p))/x**2/(g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f + d*g*x**2)**p))/(x**2*sqrt(f + g*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^2\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+df)^pc\right)+a}{\sqrt{gx^2+f}x^2}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^2/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
(2*g^2*p*integrate(x^2/((f*g*x^2 + f^2)*sqrt(g*x^2 + f)), x) - ((g*p*log(d
) - 2*g*p + g*log(c))*x^2 + f*p*log(d) + (g*p*x^2 + f*p)*log(g*x^2 + f) +
f*log(c))/(sqrt(g*x^2 + f)*f*x))*b - sqrt(g*x^2 + f)*a/(f*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(62) = 124$.

Time = 0.46 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.17

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^2\sqrt{f+gx^2}}\,dx$$
$$=bp\left(\frac{2\sqrt{g}\log\left(dgx^2+df\right)}{\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f}+\frac{\sqrt{g}\log\left(|gx^2+f|\right)}{f}-\frac{2\sqrt{g}\log\left(\left|-\sqrt{g}x+\sqrt{gx^2+f}\right|\right)}{f}\right)$$
$$+\frac{2b\sqrt{g}\log\left(c\right)}{\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f}+\frac{2a\sqrt{g}}{\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f}$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^2/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
b*p*(2*sqrt(g)*log(d*g*x^2 + d*f)/((sqrt(g)*x - sqrt(g*x^2 + f))^2 - f) +
sqrt(g)*log(abs(g*x^2 + f))/f - 2*sqrt(g)*log(abs(-sqrt(g)*x + sqrt(g*x^2
+ f)))/f) + 2*b*sqrt(g)*log(c)/((sqrt(g)*x - sqrt(g*x^2 + f))^2 - f) + 2*a
*sqrt(g)/((sqrt(g)*x - sqrt(g*x^2 + f))^2 - f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^2\sqrt{f+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)}{x^2\,\sqrt{g\,x^2+f}}\,dx$$

input
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^2*(f + g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^2*(f + g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.79

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^2\sqrt{f+gx^2}}\,dx$$
$$=\frac{-\sqrt{g\,x^2+f}\,\log\left(\frac{d^p\left(2\sqrt{g}\,\sqrt{g\,x^2+f}\,x+2f+2g\,x^2\right)^{2p}c}{\left(\sqrt{g\,x^2+f}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b-\sqrt{g\,x^2+f}\,a+2\sqrt{g}\log\left(\frac{2\sqrt{g}\,\sqrt{g\,x^2+f}\,x+2f+2g\,x^2}{f}\right)bpx-}{fx}$$

input
```
int((a+b*log(c*(d*g*x^2+d*f)^p))/x^2/(g*x^2+f)^(1/2),x)
```

output
```
( - sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*g*x
**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b - sqrt(
f + g*x**2)*a + 2*sqrt(g)*log((2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*g*x*
*2)/f)*b*p*x - sqrt(g)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*g
*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*x - s
qrt(g)*a*x)/(f*x)
```

3.699 $\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^4\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 139

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^4\sqrt{f+gx^2}}\,dx = -\frac{2bgp\sqrt{f+gx^2}}{3f^2x}-\frac{4bg^{3/2}p\,\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f+gx^2}}\right)}{3f^2} - \frac{\sqrt{f+gx^2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{3fx^3} + \frac{2g\sqrt{f+gx^2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{3f^2x}$$

output
```
-2/3*b*g*p*(g*x^2+f)^(1/2)/f^2/x-4/3*b*g^(3/2)*p*arctanh(g^(1/2)*x/(g*x^2+
f)^(1/2))/f^2-1/3*(g*x^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/f/x^3+2/3*g*
(g*x^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/f^2/x
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^4\sqrt{f+gx^2}}\,dx =$$
$$-\frac{\sqrt{f+gx^2}(af-2agx^2+2bgpx^2)+b(f-2gx^2)\sqrt{f+gx^2}\log\left(c(d(f+gx^2))^p\right)+4bg^{3/2}px^3\log\left(gx+\right.}{3f^2x^3}$$

input
```
Integrate[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^4*Sqrt[f + g*x^2]),x]
```

output
```
-1/3*(Sqrt[f + g*x^2]*(a*f - 2*a*g*x^2 + 2*b*g*p*x^2) + b*(f - 2*g*x^2)*Sq
rt[f + g*x^2]*Log[c*(d*(f + g*x^2))^p] + 4*b*g^(3/2)*p*x^3*Log[g*x + Sqrt[
g]*Sqrt[f + g*x^2]])/(f^2*x^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^4\sqrt{f+gx^2}}\,dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^4\sqrt{f+gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^4*Sqrt[f + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a + b\ln\left(c(dg\,x^2 + df)^p\right)}{x^4\sqrt{g\,x^2 + f}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^4/(g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^4/(g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.65

$$\int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x^4\sqrt{f + gx^2}}\,dx$$
$$= \left[\frac{2\,bg^{\frac{3}{2}}px^3\log\left(-2\,gx^2 + 2\,\sqrt{gx^2 + f}\sqrt{g}x - f\right) - (2\,(bgp - ag)x^2 + af - (2\,bgpx^2 - bfp)\log\left(dgx^2 + df\right.}{3\,f^2x^3}\right.$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^4/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
[1/3*(2*b*g^(3/2)*p*x^3*log(-2*g*x^2 + 2*sqrt(g*x^2 + f)*sqrt(g)*x - f) -
(2*(b*g*p - a*g)*x^2 + a*f - (2*b*g*p*x^2 - b*f*p)*log(d*g*x^2 + d*f) - (2
*b*g*x^2 - b*f)*log(c))*sqrt(g*x^2 + f))/(f^2*x^3), 1/3*(4*b*sqrt(-g)*g*p*
x^3*arctan(sqrt(-g)*x/sqrt(g*x^2 + f)) - (2*(b*g*p - a*g)*x^2 + a*f - (2*b
*g*p*x^2 - b*f*p)*log(d*g*x^2 + d*f) - (2*b*g*x^2 - b*f)*log(c))*sqrt(g*x^
2 + f))/(f^2*x^3)]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^4\sqrt{f+gx^2}}\,dx = \int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^4\sqrt{f+gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+d*f)**p))/x**4/(g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f + d*g*x**2)**p))/(x**4*sqrt(f + g*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^4\sqrt{f+gx^2}}\,dx = \int \frac{b\log\left((dgx^2+df)^pc\right)+a}{\sqrt{gx^2+f}x^4}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^4/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
-1/3*(12*g^3*p*integrate(1/3*x^2/((f^2*g*x^2 + f^3)*sqrt(g*x^2 + f)), x) -
 (2*(g^2*p*log(d) - 3*g^2*p + g^2*log(c))*x^4 - f^2*p*log(d) + (f*g*p*log(
d) - 2*f*g*p + f*g*log(c))*x^2 - f^2*log(c) + (2*g^2*p*x^4 + f*g*p*x^2 - f
^2*p)*log(g*x^2 + f))/(sqrt(g*x^2 + f)*f^2*x^3))*b + 1/3*a*(2*sqrt(g*x^2 +
 f)*g/(f^2*x) - sqrt(g*x^2 + f)/(f*x^3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(119) = 238$.

Time = 0.70 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.89

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^4\sqrt{f+gx^2}}\,dx$$
$$=\frac{2}{3}bp\left(\frac{2\left(3\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f\right)g^{\frac{3}{2}}\log\left(dgx^2+df\right)}{\left(\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f\right)^3}+\frac{g^{\frac{3}{2}}\log\left(\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2\right)}{f^2}-\frac{g^{\frac{3}{2}}\log\left(|gx^2\right.}{f^2}\right.$$
$$+\frac{4\left(3\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f\right)bg^{\frac{3}{2}}\log\left(c\right)}{3\left(\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f\right)^3}+\frac{4\left(3\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f\right)ag^{\frac{3}{2}}}{3\left(\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f\right)^3}$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^4/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
2/3*b*p*(2*(3*(sqrt(g)*x - sqrt(g*x^2 + f))^2 - f)*g^(3/2)*log(d*g*x^2 + d
*f)/((sqrt(g)*x - sqrt(g*x^2 + f))^2 - f)^3 + g^(3/2)*log((sqrt(g)*x - sqr
t(g*x^2 + f))^2)/f^2 - g^(3/2)*log(abs(g*x^2 + f))/f^2 + 2*g^(3/2)/(((sqrt
(g)*x - sqrt(g*x^2 + f))^2 - f)*f)) + 4/3*(3*(sqrt(g)*x - sqrt(g*x^2 + f))
^2 - f)*b*g^(3/2)*log(c)/((sqrt(g)*x - sqrt(g*x^2 + f))^2 - f)^3 + 4/3*(3*
(sqrt(g)*x - sqrt(g*x^2 + f))^2 - f)*a*g^(3/2)/((sqrt(g)*x - sqrt(g*x^2 +
f))^2 - f)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^4\sqrt{f+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)}{x^4\,\sqrt{g\,x^2+f}}\,dx$$

input
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^4*(f + g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^4*(f + g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.32

$$\int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x^4\sqrt{f + gx^2}}\,dx$$

$$= \frac{-3\sqrt{g\,x^2+f}\log\left(\frac{d^p\left(2\sqrt{g}\,\sqrt{g\,x^2+f}\,x+2f+2g\,x^2\right)^{2p}c}{\left(\sqrt{g\,x^2+f}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)bf + 6\sqrt{g\,x^2+f}\log\left(\frac{d^p\left(2\sqrt{g}\,\sqrt{g\,x^2+f}\,x+2f+2g\,x^2\right)^{2p}c}{\left(\sqrt{g\,x^2+f}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)bg\,x^2}{}$$

input
```
int((a+b*log(c*(d*g*x^2+d*f)^p))/x^4/(g*x^2+f)^(1/2),x)
```

output
```
( - 3*sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*g
*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*f + 6
*sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*g*x**2
)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g*x**2 - 3
*sqrt(f + g*x**2)*a*f + 6*sqrt(f + g*x**2)*a*g*x**2 - 6*sqrt(f + g*x**2)*b
*g*p*x**2 - 12*sqrt(g)*log((2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*g*x**2)
/f)*b*g*p*x**3 + 6*sqrt(g)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f +
 2*g*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g
*x**3 - 6*sqrt(g)*a*g*x**3 + 2*sqrt(g)*b*g*p*x**3)/(9*f**2*x**3)
```

3.700 $\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^6\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 208

$$\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^6\sqrt{f+gx^2}}\,dx = -\frac{2bgp\sqrt{f+gx^2}}{15f^2x^3}+\frac{4bg^2p\sqrt{f+gx^2}}{5f^3x} + \frac{16bg^{5/2}p\,\text{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f+gx^2}}\right)}{15f^3} - \frac{\sqrt{f+gx^2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{5fx^5} + \frac{4g\sqrt{f+gx^2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{15f^2x^3} - \frac{8g^2\sqrt{f+gx^2}\left(a+b\log\left(c\left(df+dgx^2\right)^p\right)\right)}{15f^3x}$$

output
```
-2/15*b*g*p*(g*x^2+f)^(1/2)/f^2/x^3+4/5*b*g^2*p*(g*x^2+f)^(1/2)/f^3/x+16/1
5*b*g^(5/2)*p*arctanh(g^(1/2)*x/(g*x^2+f)^(1/2))/f^3-1/5*(g*x^2+f)^(1/2)*(
a+b*ln(c*(d*g*x^2+d*f)^p))/f/x^5+4/15*g*(g*x^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2
+d*f)^p))/f^2/x^3-8/15*g^2*(g*x^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/f^3
/x
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.71

$$\int \frac{a+b\log \left(c\left(df+dgx^2\right)^p\right)}{x^6\sqrt{f+gx^2}}\,dx$$
$$= \frac{\sqrt{f+gx^2}\left(2bgpx^2(-f+6gx^2)+a(-3f^2+4fgx^2-8g^2x^4)\right)+b\sqrt{f+gx^2}(-3f^2+4fgx^2-8g^2x^4)\log\left(\right.}{15f^3x^5}$$

input
```
Integrate[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^6*Sqrt[f + g*x^2]),x]
```

output
```
(Sqrt[f + g*x^2]*(2*b*g*p*x^2*(-f + 6*g*x^2) + a*(-3*f^2 + 4*f*g*x^2 - 8*g
^2*x^4)) + b*Sqrt[f + g*x^2]*(-3*f^2 + 4*f*g*x^2 - 8*g^2*x^4)*Log[c*(d*(f
+ g*x^2))^p] + 16*b*g^(5/2)*p*x^5*Log[g*x + Sqrt[g]*Sqrt[f + g*x^2]])/(15*
f^3*x^5)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log \left(c\left(df+dgx^2\right)^p\right)}{x^6\sqrt{f+gx^2}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{a+b\log \left(c\left(df+dgx^2\right)^p\right)}{x^6\sqrt{f+gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^6*Sqrt[f + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c{\left(dg\,x^{2}+df\right)}^{p}\right)}{x^{6}\sqrt{g\,x^{2}+f}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^6/(g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^6/(g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.58

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^6\sqrt{f+gx^2}}\,dx = \left[\frac{8\,bg^{\frac{5}{2}}px^5\log\left(-2\,gx^2-2\,\sqrt{gx^2+f}\sqrt{g}x-f\right)+(4\,(3\,bg^2p-2\,ag^2)x^4-3\,af^2-2\,(bfgp-2\,afg)x^2-(}{15\,f^3x^5}\right.$$
$$-\frac{16\,b\sqrt{-g}g^2px^5\arctan\left(\frac{\sqrt{-g}x}{\sqrt{gx^2+f}}\right)-(4\,(3\,bg^2p-2\,ag^2)x^4-3\,af^2-2\,(bfgp-2\,afg)x^2-(8\,bg^2px^4-}{15\,f^3x^5}$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^6/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
[1/15*(8*b*g^(5/2)*p*x^5*log(-2*g*x^2 - 2*sqrt(g*x^2 + f)*sqrt(g)*x - f) +
 (4*(3*b*g^2*p - 2*a*g^2)*x^4 - 3*a*f^2 - 2*(b*f*g*p - 2*a*f*g)*x^2 - (8*b
*g^2*p*x^4 - 4*b*f*g*p*x^2 + 3*b*f^2*p)*log(d*g*x^2 + d*f) - (8*b*g^2*x^4
- 4*b*f*g*x^2 + 3*b*f^2)*log(c))*sqrt(g*x^2 + f))/(f^3*x^5), -1/15*(16*b*s
qrt(-g)*g^2*p*x^5*arctan(sqrt(-g)*x/sqrt(g*x^2 + f)) - (4*(3*b*g^2*p - 2*a
*g^2)*x^4 - 3*a*f^2 - 2*(b*f*g*p - 2*a*f*g)*x^2 - (8*b*g^2*p*x^4 - 4*b*f*g
*p*x^2 + 3*b*f^2*p)*log(d*g*x^2 + d*f) - (8*b*g^2*x^4 - 4*b*f*g*x^2 + 3*b*
f^2)*log(c))*sqrt(g*x^2 + f))/(f^3*x^5)]
```

Sympy [F]

$$\int \frac{a + b \log \left(c(df + dgx^2)^p\right)}{x^6\sqrt{f + gx^2}}\, dx = \int \frac{a + b \log \left(c(df + dgx^2)^p\right)}{x^6\sqrt{f + gx^2}}\, dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+d*f)**p))/x**6/(g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f + d*g*x**2)**p))/(x**6*sqrt(f + g*x**2)), x)
```

Maxima [F]

$$\int \frac{a + b \log \left(c(df + dgx^2)^p\right)}{x^6\sqrt{f + gx^2}}\, dx = \int \frac{b \log \left((dgx^2 + df)^p c\right) + a}{\sqrt{gx^2 + f}x^6}\, dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^6/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
1/15*(240*g^4*p*integrate(1/15*x^2/((f^3*g*x^2 + f^4)*sqrt(g*x^2 + f)), x)
 - (4*(2*g^3*p*log(d) - 7*g^3*p + 2*g^3*log(c))*x^6 + 2*(2*f*g^2*p*log(d)
- 5*f*g^2*p + 2*f*g^2*log(c))*x^4 + 3*f^3*p*log(d) + 3*f^3*log(c) - (f^2*g
*p*log(d) - 2*f^2*g*p + f^2*g*log(c))*x^2 + (8*g^3*p*x^6 + 4*f*g^2*p*x^4 -
 f^2*g*p*x^2 + 3*f^3*p)*log(g*x^2 + f))/(sqrt(g*x^2 + f)*f^3*x^5))*b - 1/1
5*a*(8*sqrt(g*x^2 + f)*g^2/(f^3*x) - 4*sqrt(g*x^2 + f)*g/(f^2*x^3) + 3*sqr
t(g*x^2 + f)/(f*x^5))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(180) = 360$.

Time = 0.98 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.84

$$\int \frac{a + b\log\left(c(df + dgx^2)^p\right)}{x^6\sqrt{f + gx^2}}\,dx =$$

$$-\frac{8}{15}\,bp\left(\frac{g^{\frac{5}{2}}\log\left(\left(\sqrt{g}x - \sqrt{gx^2+f}\right)^2\right)}{f^3} - \frac{g^{\frac{5}{2}}\log\left(|gx^2+f|\right)}{f^3} - \frac{2\left(10\left(\sqrt{g}x - \sqrt{gx^2+f}\right)^4 - 5\left(\sqrt{g}x - \right.\right.}{\left(\left(\sqrt{g}x - \sqrt{gx^2\right.\right.}\right.$$

$$+ \frac{16\left(10\left(\sqrt{g}x - \sqrt{gx^2+f}\right)^4 - 5\left(\sqrt{g}x - \sqrt{gx^2+f}\right)^2 f + f^2\right)bg^{\frac{5}{2}}\log(c)}{15\left(\left(\sqrt{g}x - \sqrt{gx^2+f}\right)^2 - f\right)^5}$$

$$+ \frac{16\left(10\left(\sqrt{g}x - \sqrt{gx^2+f}\right)^4 - 5\left(\sqrt{g}x - \sqrt{gx^2+f}\right)^2 f + f^2\right)ag^{\frac{5}{2}}}{15\left(\left(\sqrt{g}x - \sqrt{gx^2+f}\right)^2 - f\right)^5}$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^6/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
-8/15*b*p*(g^(5/2)*log((sqrt(g)*x - sqrt(g*x^2 + f))^2)/f^3 - g^(5/2)*log(
abs(g*x^2 + f))/f^3 - 2*(10*(sqrt(g)*x - sqrt(g*x^2 + f))^4 - 5*(sqrt(g)*x
 - sqrt(g*x^2 + f))^2*f + f^2)*g^(5/2)*log(d*g*x^2 + d*f)/((sqrt(g)*x - sq
rt(g*x^2 + f))^2 - f)^5 + (2*(sqrt(g)*x - sqrt(g*x^2 + f))^4*g^(5/2) - 7*(
sqrt(g)*x - sqrt(g*x^2 + f))^2*f*g^(5/2) + 3*f^2*g^(5/2))/(((sqrt(g)*x - s
qrt(g*x^2 + f))^2 - f)^3*f^2)) + 16/15*(10*(sqrt(g)*x - sqrt(g*x^2 + f))^4
 - 5*(sqrt(g)*x - sqrt(g*x^2 + f))^2*f + f^2)*b*g^(5/2)*log(c)/((sqrt(g)*x
 - sqrt(g*x^2 + f))^2 - f)^5 + 16/15*(10*(sqrt(g)*x - sqrt(g*x^2 + f))^4 -
 5*(sqrt(g)*x - sqrt(g*x^2 + f))^2*f + f^2)*a*g^(5/2)/((sqrt(g)*x - sqrt(g
*x^2 + f))^2 - f)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^6\sqrt{f+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)}{x^6\,\sqrt{g\,x^2+f}}\,dx$$

input
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^6*(f + g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^6*(f + g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.15

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^6\sqrt{f+gx^2}}\,dx$$
$$=\frac{-15\sqrt{g\,x^2+f}\,\log\left(\frac{d^p\left(2\sqrt{g}\,\sqrt{g\,x^2+f}\,x+2f+2g\,x^2\right)^{2p}c}{\left(\sqrt{g\,x^2+f}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b\,f^2+20\sqrt{g\,x^2+f}\,\log\left(\frac{d^p\left(2\sqrt{g}\,\sqrt{g\,x^2+f}\,x+2f+2g\,x^2\right)^{2p}c}{\left(\sqrt{g\,x^2+f}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b}{}$$

input
```
int((a+b*log(c*(d*g*x^2+d*f)^p))/x^6/(g*x^2+f)^(1/2),x)
```

output
```
( - 15*sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*
g*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*f**2
 + 20*sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*g
*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*f*g*x
**2 - 40*sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f +
2*g*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g*
*2*x**4 - 15*sqrt(f + g*x**2)*a*f**2 + 20*sqrt(f + g*x**2)*a*f*g*x**2 - 40
*sqrt(f + g*x**2)*a*g**2*x**4 - 10*sqrt(f + g*x**2)*b*f*g*p*x**2 + 60*sqrt
(f + g*x**2)*b*g**2*p*x**4 + 80*sqrt(g)*log((2*sqrt(g)*sqrt(f + g*x**2)*x
+ 2*f + 2*g*x**2)/f)*b*g**2*p*x**5 - 40*sqrt(g)*log((d**p*(2*sqrt(g)*sqrt(
f + g*x**2)*x + 2*f + 2*g*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)*
*(2*p)*2**(2*p)))*b*g**2*x**5 + 40*sqrt(g)*a*g**2*x**5 - 44*sqrt(g)*b*g**2
*p*x**5)/(75*f**3*x**5)
```

3.701 $\int \frac{a+b\log\left(c\left(df+dgx^2\right)^p\right)}{x^8\sqrt{f+gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 277

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx = -\frac{2bgp\sqrt{f+gx^2}}{35f^2x^5}+\frac{4bg^2p\sqrt{f+gx^2}}{21f^3x^3} -\frac{88bg^3p\sqrt{f+gx^2}}{105f^4x}-\frac{32bg^{7/2}p\operatorname{arctanh}\left(\frac{\sqrt{g}x}{\sqrt{f+gx^2}}\right)}{35f^4} -\frac{\sqrt{f+gx^2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{7fx^7} +\frac{6g\sqrt{f+gx^2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{35f^2x^5} -\frac{8g^2\sqrt{f+gx^2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{35f^3x^3} +\frac{16g^3\sqrt{f+gx^2}\left(a+b\log\left(c(df+dgx^2)^p\right)\right)}{35f^4x}$$

output
```
-2/35*b*g*p*(g*x^2+f)^(1/2)/f^2/x^5+4/21*b*g^2*p*(g*x^2+f)^(1/2)/f^3/x^3-8
8/105*b*g^3*p*(g*x^2+f)^(1/2)/f^4/x-32/35*b*g^(7/2)*p*arctanh(g^(1/2)*x/(g
*x^2+f)^(1/2))/f^4-1/7*(g*x^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/f/x^7+6
/35*g*(g*x^2+f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/f^2/x^5-8/35*g^2*(g*x^2+
f)^(1/2)*(a+b*ln(c*(d*g*x^2+d*f)^p))/f^3/x^3+16/35*g^3*(g*x^2+f)^(1/2)*(a+
b*ln(c*(d*g*x^2+d*f)^p))/f^4/x
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.66

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx = \frac{\sqrt{f+gx^2}(-2bgpx^2(3f^2-10fgx^2+44g^2x^4)-3a(5f^3-6f^2gx^2+8fg^2x^4-16g^3x^6))-3b\sqrt{f+gx^2}(5}{105f^4x^7}$$

input
```
Integrate[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^8*Sqrt[f + g*x^2]),x]
```

output
```
(Sqrt[f + g*x^2]*(-2*b*g*p*x^2*(3*f^2 - 10*f*g*x^2 + 44*g^2*x^4) - 3*a*(5*
f^3 - 6*f^2*g*x^2 + 8*f*g^2*x^4 - 16*g^3*x^6)) - 3*b*Sqrt[f + g*x^2]*(5*f^
3 - 6*f^2*g*x^2 + 8*f*g^2*x^4 - 16*g^3*x^6)*Log[c*(d*(f + g*x^2))^p] - 96*
b*g^(7/2)*p*x^7*Log[g*x + Sqrt[g]*Sqrt[f + g*x^2]])/(105*f^4*x^7)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f + d*g*x^2)^p])/(x^8*Sqrt[f + g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(dg\,x^2+df)^p\right)}{x^8\sqrt{g\,x^2+f}}dx$$

input
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^8/(g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(d*g*x^2+d*f)^p))/x^8/(g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.55

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx = \left[\frac{48\,bg^{\frac{7}{2}}px^7\log\left(-2\,gx^2+2\,\sqrt{gx^2+f}\sqrt{g}x-f\right)-(8\,(11\,bg^3p-6\,ag^3)x^6-4\,(5\,bfg^2p-6\,afg^2)x^4+15\right.$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^8/(g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
[1/105*(48*b*g^(7/2)*p*x^7*log(-2*g*x^2 + 2*sqrt(g*x^2 + f)*sqrt(g)*x - f)
 - (8*(11*b*g^3*p - 6*a*g^3)*x^6 - 4*(5*b*f*g^2*p - 6*a*f*g^2)*x^4 + 15*a*
f^3 + 6*(b*f^2*g*p - 3*a*f^2*g)*x^2 - 3*(16*b*g^3*p*x^6 - 8*b*f*g^2*p*x^4
+ 6*b*f^2*g*p*x^2 - 5*b*f^3*p)*log(d*g*x^2 + d*f) - 3*(16*b*g^3*x^6 - 8*b*
f*g^2*x^4 + 6*b*f^2*g*x^2 - 5*b*f^3)*log(c))*sqrt(g*x^2 + f))/(f^4*x^7), 1
/105*(96*b*sqrt(-g)*g^3*p*x^7*arctan(sqrt(-g)*x/sqrt(g*x^2 + f)) - (8*(11*
b*g^3*p - 6*a*g^3)*x^6 - 4*(5*b*f*g^2*p - 6*a*f*g^2)*x^4 + 15*a*f^3 + 6*(b
*f^2*g*p - 3*a*f^2*g)*x^2 - 3*(16*b*g^3*p*x^6 - 8*b*f*g^2*p*x^4 + 6*b*f^2*
g*p*x^2 - 5*b*f^3*p)*log(d*g*x^2 + d*f) - 3*(16*b*g^3*x^6 - 8*b*f*g^2*x^4
+ 6*b*f^2*g*x^2 - 5*b*f^3)*log(c))*sqrt(g*x^2 + f))/(f^4*x^7)]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx=\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(d*g*x**2+d*f)**p))/x**8/(g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f + d*g*x**2)**p))/(x**8*sqrt(f + g*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx=\int \frac{b\log\left((dgx^2+df)^pc\right)+a}{\sqrt{gx^2+f}x^8}\,dx$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^8/(g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
-1/105*(3360*g^5*p*integrate(1/35*x^2/((f^4*g*x^2 + f^5)*sqrt(g*x^2 + f)),
 x) - (8*(6*g^4*p*log(d) - 23*g^4*p + 6*g^4*log(c))*x^8 + 4*(6*f*g^3*p*log
(d) - 17*f*g^3*p + 6*f*g^3*log(c))*x^6 - 15*f^4*p*log(d) - 2*(3*f^2*g^2*p*
log(d) - 7*f^2*g^2*p + 3*f^2*g^2*log(c))*x^4 - 15*f^4*log(c) + 3*(f^3*g*p*
log(d) - 2*f^3*g*p + f^3*g*log(c))*x^2 + 3*(16*g^4*p*x^8 + 8*f*g^3*p*x^6 -
 2*f^2*g^2*p*x^4 + f^3*g*p*x^2 - 5*f^4*p)*log(g*x^2 + f))/(sqrt(g*x^2 + f)
*f^4*x^7))*b + 1/35*a*(16*sqrt(g*x^2 + f)*g^3/(f^4*x) - 8*sqrt(g*x^2 + f)*
g^2/(f^3*x^3) + 6*sqrt(g*x^2 + f)*g/(f^2*x^5) - 5*sqrt(g*x^2 + f)/(f*x^7))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(241) = 482$.

Time = 1.24 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.86

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx$$
$$= \frac{16}{105}\,bp\left(\frac{3\,g^{\frac{7}{2}}\log\left(\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2\right)}{f^4} - \frac{3\,g^{\frac{7}{2}}\log\left(|gx^2+f|\right)}{f^4} + \frac{6\left(35\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^6 - 21\left(\sqrt{g}x\right.\right.}{}\right.$$
$$+\frac{32\left(35\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^6-21\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^4f+7\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2f^2-f^3\right)bg^{\frac{7}{2}}\log(c)}{35\left(\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f\right)^7}$$
$$+\frac{32\left(35\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^6-21\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^4f+7\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2f^2-f^3\right)ag^{\frac{7}{2}}}{35\left(\left(\sqrt{g}x-\sqrt{gx^2+f}\right)^2-f\right)^7}$$

input
```
integrate((a+b*log(c*(d*g*x^2+d*f)^p))/x^8/(g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
16/105*b*p*(3*g^(7/2)*log((sqrt(g)*x - sqrt(g*x^2 + f))^2)/f^4 - 3*g^(7/2)
*log(abs(g*x^2 + f))/f^4 + 6*(35*(sqrt(g)*x - sqrt(g*x^2 + f))^6 - 21*(sqr
t(g)*x - sqrt(g*x^2 + f))^4*f + 7*(sqrt(g)*x - sqrt(g*x^2 + f))^2*f^2 - f^
3)*g^(7/2)*log(d*g*x^2 + d*f)/((sqrt(g)*x - sqrt(g*x^2 + f))^2 - f)^7 + (6
*(sqrt(g)*x - sqrt(g*x^2 + f))^8*g^(7/2) - 33*(sqrt(g)*x - sqrt(g*x^2 + f)
)^6*f*g^(7/2) + 77*(sqrt(g)*x - sqrt(g*x^2 + f))^4*f^2*g^(7/2) - 49*(sqrt(
g)*x - sqrt(g*x^2 + f))^2*f^3*g^(7/2) + 11*f^4*g^(7/2))/(((sqrt(g)*x - sqr
t(g*x^2 + f))^2 - f)^5*f^3)) + 32/35*(35*(sqrt(g)*x - sqrt(g*x^2 + f))^6 -
 21*(sqrt(g)*x - sqrt(g*x^2 + f))^4*f + 7*(sqrt(g)*x - sqrt(g*x^2 + f))^2*
f^2 - f^3)*b*g^(7/2)*log(c)/((sqrt(g)*x - sqrt(g*x^2 + f))^2 - f)^7 + 32/3
5*(35*(sqrt(g)*x - sqrt(g*x^2 + f))^6 - 21*(sqrt(g)*x - sqrt(g*x^2 + f))^4
*f + 7*(sqrt(g)*x - sqrt(g*x^2 + f))^2*f^2 - f^3)*a*g^(7/2)/((sqrt(g)*x -
sqrt(g*x^2 + f))^2 - f)^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,g\,x^2+d\,f\right)}^p\right)}{x^8\,\sqrt{g\,x^2+f}}\,dx$$

input
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^8*(f + g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f + d*g*x^2)^p))/(x^8*(f + g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.05

$$\int \frac{a+b\log\left(c(df+dgx^2)^p\right)}{x^8\sqrt{f+gx^2}}\,dx$$
$$=\frac{-105\sqrt{g\,x^2+f}\,\log\left(\frac{d^p\left(2\sqrt{g}\,\sqrt{g\,x^2+f}\,x+2f+2g\,x^2\right)^{2p}c}{\left(\sqrt{g\,x^2+f}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)b\,f^3+126\sqrt{g\,x^2+f}\,\log\left(\frac{d^p\left(2\sqrt{g}\,\sqrt{g\,x^2+f}\,x+2f+2g\,x^2\right)^{2p}c}{\left(\sqrt{g\,x^2+f}+\sqrt{g}\,x\right)^{2p}2^{2p}}\right)}{}$$

input
```
int((a+b*log(c*(d*g*x^2+d*f)^p))/x^8/(g*x^2+f)^(1/2),x)
```

output

```
( - 105*sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2
*g*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*f**
3 + 126*sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2
*g*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*f**
2*g*x**2 - 168*sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x +
2*f + 2*g*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p))
)*b*f*g**2*x**4 + 336*sqrt(f + g*x**2)*log((d**p*(2*sqrt(g)*sqrt(f + g*x**
2)*x + 2*f + 2*g*x**2)**(2*p)*c)/((sqrt(f + g*x**2) + sqrt(g)*x)**(2*p)*2*
*(2*p)))*b*g**3*x**6 - 105*sqrt(f + g*x**2)*a*f**3 + 126*sqrt(f + g*x**2)*
a*f**2*g*x**2 - 168*sqrt(f + g*x**2)*a*f*g**2*x**4 + 336*sqrt(f + g*x**2)*
a*g**3*x**6 - 42*sqrt(f + g*x**2)*b*f**2*g*p*x**2 + 140*sqrt(f + g*x**2)*b
*f*g**2*p*x**4 - 616*sqrt(f + g*x**2)*b*g**3*p*x**6 - 672*sqrt(g)*log((2*s
qrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*g*x**2)/f)*b*g**3*p*x**7 + 336*sqrt(g)
*log((d**p*(2*sqrt(g)*sqrt(f + g*x**2)*x + 2*f + 2*g*x**2)**(2*p)*c)/((sqr
t(f + g*x**2) + sqrt(g)*x)**(2*p)*2**(2*p)))*b*g**3*x**7 - 336*sqrt(g)*a*g
**3*x**7 + 520*sqrt(g)*b*g**3*p*x**7)/(735*f**4*x**7)
```

3.702 $\int \frac{x^7\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 248

$$\int \frac{x^7\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx=\frac{2bf^3p\sqrt{f-gx^2}}{g^4}-\frac{2bf^2p(f-gx^2)^{3/2}}{3g^4}+\frac{6bfp(f-gx^2)^{5/2}}{25g^4}-\frac{2bp(f-gx^2)^{7/2}}{49g^4}-\frac{f^3\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{g^4}+\frac{f^2(f-gx^2)^{3/2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{g^4}-\frac{3f(f-gx^2)^{5/2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{5g^4}+\frac{(f-gx^2)^{7/2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{7g^4}$$

output

```
2*b*f^3*p*(-g*x^2+f)^(1/2)/g^4-2/3*b*f^2*p*(-g*x^2+f)^(3/2)/g^4+6/25*b*f*p
*(-g*x^2+f)^(5/2)/g^4-2/49*b*p*(-g*x^2+f)^(7/2)/g^4-f^3*(-g*x^2+f)^(1/2)*(
a+b*ln(c*(-d*g*x^2+d*f)^p))/g^4+f^2*(-g*x^2+f)^(3/2)*(a+b*ln(c*(-d*g*x^2+d
*f)^p))/g^4-3/5*f*(-g*x^2+f)^(5/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/g^4+1/7*(-
g*x^2+f)^(7/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/g^4
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.57

$$\int \frac{x^7\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$=\frac{\sqrt{f-gx^2}\left(-105a(16f^3+8f^2gx^2+6fg^2x^4+5g^3x^6)+2bp(2816f^3+568f^2gx^2+216fg^2x^4+75g^3x^6)\right.-}{3675g^4}$$

input
```
Integrate[(x^7*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
(Sqrt[f - g*x^2]*(-105*a*(16*f^3 + 8*f^2*g*x^2 + 6*f*g^2*x^4 + 5*g^3*x^6)
+ 2*b*p*(2816*f^3 + 568*f^2*g*x^2 + 216*f*g^2*x^4 + 75*g^3*x^6) - 105*b*(1
6*f^3 + 8*f^2*g*x^2 + 6*f*g^2*x^4 + 5*g^3*x^6)*Log[c*(d*(f - g*x^2))^p]))/
(3675*g^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x^7\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}dx$$

input
```
Int[(x^7*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^7 \left(a+b\ln \left(c(-dg\,x^2+df)^p\right)\right)}{\sqrt{-g\,x^2+f}}dx$$

input
```
int(x^7*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
int(x^7*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.74

$$\int \frac{x^7 \left(a+b\log \left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = \frac{(75\,(2\,bg^3p-7\,ag^3)x^6+5632\,bf^3p+18\,(24\,bfg^2p-35\,afg^2)x^4-1680\,af^3+8\,(142\,bf^2gp-105\,af^2g)x$$

input
```
integrate(x^7*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
1/3675*(75*(2*b*g^3*p - 7*a*g^3)*x^6 + 5632*b*f^3*p + 18*(24*b*f*g^2*p - 3
5*a*f*g^2)*x^4 - 1680*a*f^3 + 8*(142*b*f^2*g*p - 105*a*f^2*g)*x^2 - 105*(5
*b*g^3*p*x^6 + 6*b*f*g^2*p*x^4 + 8*b*f^2*g*p*x^2 + 16*b*f^3*p)*log(-d*g*x^
2 + d*f) - 105*(5*b*g^3*x^6 + 6*b*f*g^2*x^4 + 8*b*f^2*g*x^2 + 16*b*f^3)*lo
g(c))*sqrt(-g*x^2 + f)/g^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x**7*(a+b*ln(c*(-d*g*x**2+d*f)**p))/(-g*x**2+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.97

$$\int \frac{x^7\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx= -\frac{1}{35}\left(\frac{5\sqrt{-gx^2+f}x^6}{g}+\frac{6\sqrt{-gx^2+f}fx^4}{g^2}+\frac{8\sqrt{-gx^2+f}f^2x^2}{g^3}+\frac{16\sqrt{-gx^2+f}f^3}{g^4}\right)b\log\left(\left(-dgx^2+\right.\right. -\frac{1}{35}\left(\frac{5\sqrt{-gx^2+f}x^6}{g}+\frac{6\sqrt{-gx^2+f}fx^4}{g^2}+\frac{8\sqrt{-gx^2+f}f^2x^2}{g^3}+\frac{16\sqrt{-gx^2+f}f^3}{g^4}\right)a -\frac{2\left(75\left(-gx^2+f\right)^{\frac{7}{2}}-441\left(-gx^2+f\right)^{\frac{5}{2}}f+1225\left(-gx^2+f\right)^{\frac{3}{2}}f^2-3675\sqrt{-gx^2+f}f^3\right)bp}{3675\,g^4}$$

input
```
integrate(x^7*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/35*(5*sqrt(-g*x^2 + f)*x^6/g + 6*sqrt(-g*x^2 + f)*f*x^4/g^2 + 8*sqrt(-g
*x^2 + f)*f^2*x^2/g^3 + 16*sqrt(-g*x^2 + f)*f^3/g^4)*b*log((-d*g*x^2 + d*f
)^p*c) - 1/35*(5*sqrt(-g*x^2 + f)*x^6/g + 6*sqrt(-g*x^2 + f)*f*x^4/g^2 + 8
*sqrt(-g*x^2 + f)*f^2*x^2/g^3 + 16*sqrt(-g*x^2 + f)*f^3/g^4)*a - 2/3675*(7
5*(-g*x^2 + f)^(7/2) - 441*(-g*x^2 + f)^(5/2)*f + 1225*(-g*x^2 + f)^(3/2)*
f^2 - 3675*sqrt(-g*x^2 + f)*f^3)*b*p/g^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.44

$$\int \frac{x^7 (a+b\log (c(df-dgx^2)^p))}{\sqrt{f-gx^2}}\,dx = -\frac{bp\left(\frac{105\left(5\left(gx^2-f\right)^3\sqrt{-gx^2+f}+21\left(gx^2-f\right)^2\sqrt{-gx^2+f}f-35\left(-gx^2+f\right)^{\frac{3}{2}}f^2+35\sqrt{-gx^2+f}f^3\right)\log(-dgx^2+df)}{g^3}-\frac{2\left(75\left(gx^2-f\right)^3 \cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(x^7*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
-1/3675*(b*p*(105*(5*(g*x^2 - f)^3*sqrt(-g*x^2 + f) + 21*(g*x^2 - f)^2*sqr
t(-g*x^2 + f)*f - 35*(-g*x^2 + f)^(3/2)*f^2 + 35*sqrt(-g*x^2 + f)*f^3)*log
(-d*g*x^2 + d*f)/g^3 - 2*(75*(g*x^2 - f)^3*sqrt(-g*x^2 + f) + 441*(g*x^2 -
 f)^2*sqrt(-g*x^2 + f)*f - 1225*(-g*x^2 + f)^(3/2)*f^2 + 3675*sqrt(-g*x^2
+ f)*f^3)/g^3) + 105*(5*(g*x^2 - f)^3*sqrt(-g*x^2 + f) + 21*(g*x^2 - f)^2*
sqrt(-g*x^2 + f)*f - 35*(-g*x^2 + f)^(3/2)*f^2 + 35*sqrt(-g*x^2 + f)*f^3)*
b*log(c)/g^3 + 105*(5*(g*x^2 - f)^3*sqrt(-g*x^2 + f) + 21*(g*x^2 - f)^2*sq
rt(-g*x^2 + f)*f - 35*(-g*x^2 + f)^(3/2)*f^2 + 35*sqrt(-g*x^2 + f)*f^3)*a/
g^3)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 (a+b\log (c(df-dgx^2)^p))}{\sqrt{f-gx^2}}\,dx = \int \frac{x^7\,\left(a+b\,\ln \left(c\,{\left(d\,f-d\,g\,x^2\right)}^p\right)\right)}{\sqrt{f-g\,x^2}}\,dx$$

input
```
int((x^7*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2),x)
```

output
```
int((x^7*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.76

$$\int \frac{x^7\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$= \frac{\sqrt{-g\,x^2+f}\,\left(-1680\log\left(d^p(-g\,x^2+f)^p\,c\right)b\,f^3-840\log\left(d^p(-g\,x^2+f)^p\,c\right)b\,f^2g\,x^2-630\log\left(d^p(-g\,x^2\right.\right.}{}$$

input
```
int(x^7*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f - g*x**2)*( - 1680*log(d**p*(f - g*x**2)**p*c)*b*f**3 - 840*log(d*
*p*(f - g*x**2)**p*c)*b*f**2*g*x**2 - 630*log(d**p*(f - g*x**2)**p*c)*b*f*
g**2*x**4 - 525*log(d**p*(f - g*x**2)**p*c)*b*g**3*x**6 - 1680*a*f**3 - 84
0*a*f**2*g*x**2 - 630*a*f*g**2*x**4 - 525*a*g**3*x**6 + 5632*b*f**3*p + 11
36*b*f**2*g*p*x**2 + 432*b*f*g**2*p*x**4 + 150*b*g**3*p*x**6))/(3675*g**4)
```

3.703 $\int \frac{x^5\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 185

$$\int \frac{x^5\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = \frac{2bf^2p\sqrt{f-gx^2}}{g^3}-\frac{4bfp\left(f-gx^2\right)^{3/2}}{9g^3}+\frac{2bp\left(f-gx^2\right)^{5/2}}{25g^3}-\frac{f^2\sqrt{f-gx^2}\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{g^3}+\frac{2f\left(f-gx^2\right)^{3/2}\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{3g^3}-\frac{\left(f-gx^2\right)^{5/2}\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{5g^3}$$

output
```
2*b*f^2*p*(-g*x^2+f)^(1/2)/g^3-4/9*b*f*p*(-g*x^2+f)^(3/2)/g^3+2/25*b*p*(-g
*x^2+f)^(5/2)/g^3-f^2*(-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/g^3+2/
3*f*(-g*x^2+f)^(3/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/g^3-1/5*(-g*x^2+f)^(5/2)
*(a+b*ln(c*(-d*g*x^2+d*f)^p))/g^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.58

$$\int \frac{x^5 \left(a+b\log \left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = \frac{\sqrt{f-gx^2}\left(-15a(8f^2+4fgx^2+3g^2x^4)+2bp(184f^2+32fgx^2+9g^2x^4)-15b(8f^2+4fgx^2+3g^2x^4)\log\left(c\left(d\left(f-gx^2\right)\right)^p\right)\right)}{225g^3}$$

input
```
Integrate[(x^5*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
(Sqrt[f - g*x^2]*(-15*a*(8*f^2 + 4*f*g*x^2 + 3*g^2*x^4) + 2*b*p*(184*f^2 +
 32*f*g*x^2 + 9*g^2*x^4) - 15*b*(8*f^2 + 4*f*g*x^2 + 3*g^2*x^4)*Log[c*(d*(
f - g*x^2))^p]))/(225*g^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(a+b\log \left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{x^5 \left(a+b\log \left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}dx$$

input
```
Int[(x^5*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^5 \left(a+b\ln \left(c{\left(-dg\,x^2+df\right)}^p\right)\right)}{\sqrt{-g\,x^2+f}}dx$$

input
```
int(x^5*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
int(x^5*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.72

$$\int \frac{x^5 \left(a+b\log \left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$= \frac{(9\,(2\,bg^2p-5\,ag^2)x^4+368\,bf^2p-120\,af^2+4\,(16\,bfgp-15\,afg)x^2-15\,(3\,bg^2px^4+4\,bfgpx^2+8\,bf^2p}{225\,g^3}$$

input
```
integrate(x^5*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
1/225*(9*(2*b*g^2*p - 5*a*g^2)*x^4 + 368*b*f^2*p - 120*a*f^2 + 4*(16*b*f*g
*p - 15*a*f*g)*x^2 - 15*(3*b*g^2*p*x^4 + 4*b*f*g*p*x^2 + 8*b*f^2*p)*log(-d
*g*x^2 + d*f) - 15*(3*b*g^2*x^4 + 4*b*f*g*x^2 + 8*b*f^2)*log(c))*sqrt(-g*x
^2 + f)/g^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 (a+b\log (c(df-dgx^2)^p))}{\sqrt{f-gx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*ln(c*(-d*g*x**2+d*f)**p))/(-g*x**2+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^5 (a+b\log (c(df-dgx^2)^p))}{\sqrt{f-gx^2}}\,dx = & -\frac{1}{15}\left(\frac{3\sqrt{-gx^2+f}x^4}{g}+\frac{4\sqrt{-gx^2+f}fx^2}{g^2}+\frac{8\sqrt{-gx^2+f}f^2}{g^3}\right)b\log\left((-dgx^2+df)^pc\right)\\ & -\frac{1}{15}\left(\frac{3\sqrt{-gx^2+f}x^4}{g}+\frac{4\sqrt{-gx^2+f}fx^2}{g^2}+\frac{8\sqrt{-gx^2+f}f^2}{g^3}\right)a\\ & +\frac{2\left(9(-gx^2+f)^{\frac{5}{2}}-50(-gx^2+f)^{\frac{3}{2}}f+225\sqrt{-gx^2+f}f^2\right)bp}{225\,g^3}\end{aligned}$$

input
```
integrate(x^5*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/15*(3*sqrt(-g*x^2 + f)*x^4/g + 4*sqrt(-g*x^2 + f)*f*x^2/g^2 + 8*sqrt(-g
*x^2 + f)*f^2/g^3)*b*log((-d*g*x^2 + d*f)^p*c) - 1/15*(3*sqrt(-g*x^2 + f)*
x^4/g + 4*sqrt(-g*x^2 + f)*f*x^2/g^2 + 8*sqrt(-g*x^2 + f)*f^2/g^3)*a + 2/2
25*(9*(-g*x^2 + f)^(5/2) - 50*(-g*x^2 + f)^(3/2)*f + 225*sqrt(-g*x^2 + f)*
f^2)*b*p/g^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.37

$$\int \frac{x^5(a + b\log(c(df - dgx^2)^p))}{\sqrt{f - gx^2}}\,dx =$$

$$-\frac{bp\left(\frac{15\left(3\,(gx^2-f)^2\sqrt{-gx^2+f}-10\,(-gx^2+f)^{\frac{3}{2}}f+15\,\sqrt{-gx^2+f}\,f^2\right)\log(-dgx^2+df)}{g^2} - \frac{2\left(9\,(gx^2-f)^2\sqrt{-gx^2+f}-50\,(-gx^2+f)^{\frac{3}{2}}f+\right.}{g^2}}{}$$

input
```
integrate(x^5*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
-1/225*(b*p*(15*(3*(g*x^2 - f)^2*sqrt(-g*x^2 + f) - 10*(-g*x^2 + f)^(3/2)*
f + 15*sqrt(-g*x^2 + f)*f^2)*log(-d*g*x^2 + d*f)/g^2 - 2*(9*(g*x^2 - f)^2*
sqrt(-g*x^2 + f) - 50*(-g*x^2 + f)^(3/2)*f + 225*sqrt(-g*x^2 + f)*f^2)/g^2
) + 15*(3*(g*x^2 - f)^2*sqrt(-g*x^2 + f) - 10*(-g*x^2 + f)^(3/2)*f + 15*sq
rt(-g*x^2 + f)*f^2)*b*log(c)/g^2 + 15*(3*(g*x^2 - f)^2*sqrt(-g*x^2 + f) -
10*(-g*x^2 + f)^(3/2)*f + 15*sqrt(-g*x^2 + f)*f^2)*a/g^2)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\log(c(df - dgx^2)^p))}{\sqrt{f - gx^2}}\,dx = \int \frac{x^5\,(a + b\,\ln(c\,(d\,f - d\,g\,x^2)^p))}{\sqrt{f - g\,x^2}}\,dx$$

input
```
int((x^5*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.73

$$\int \frac{x^5\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$=\frac{\sqrt{-g\,x^2+f}\,\left(-120\log\left(d^p(-g\,x^2+f)^p\,c\right)b\,f^2-60\log\left(d^p(-g\,x^2+f)^p\,c\right)bfg\,x^2-45\log\left(d^p(-g\,x^2+f)\right.\right.}{225g^3}$$

input
```
int(x^5*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f - g*x**2)*( - 120*log(d**p*(f - g*x**2)**p*c)*b*f**2 - 60*log(d**p
*(f - g*x**2)**p*c)*b*f*g*x**2 - 45*log(d**p*(f - g*x**2)**p*c)*b*g**2*x**
4 - 120*a*f**2 - 60*a*f*g*x**2 - 45*a*g**2*x**4 + 368*b*f**2*p + 64*b*f*g*
p*x**2 + 18*b*g**2*p*x**4))/(225*g**3)
```

3.704 $\int \frac{x^3\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 119

$$\int \frac{x^3(a+b\log(c(df-dgx^2)^p))}{\sqrt{f-gx^2}}\,dx = \frac{2bfp\sqrt{f-gx^2}}{g^2}-\frac{2bp(f-gx^2)^{3/2}}{9g^2} - \frac{f\sqrt{f-gx^2}(a+b\log(c(df-dgx^2)^p))}{g^2} + \frac{(f-gx^2)^{3/2}\,(a+b\log(c(df-dgx^2)^p))}{3g^2}$$

output
```
2*b*f*p*(-g*x^2+f)^(1/2)/g^2-2/9*b*p*(-g*x^2+f)^(3/2)/g^2-f*(-g*x^2+f)^(1/
2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/g^2+1/3*(-g*x^2+f)^(3/2)*(a+b*ln(c*(-d*g*x
^2+d*f)^p))/g^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.61

$$\int \frac{x^3\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = \frac{\sqrt{f-gx^2}\left(-3a(2f+gx^2)+2bp(8f+gx^2)-3b(2f+gx^2)\log\left(c(d(f-gx^2))^p\right)\right)}{9g^2}$$

input
```
Integrate[(x^3*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
(Sqrt[f - g*x^2]*(-3*a*(2*f + g*x^2) + 2*b*p*(8*f + g*x^2) - 3*b*(2*f + g*
x^2)*Log[c*(d*(f - g*x^2))^p]))/(9*g^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{x^3\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}dx$$

input
```
Int[(x^3*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^3 \left(a+b\ln \left(c(-dg\,x^2+df)^p\right)\right)}{\sqrt{-g\,x^2+f}}dx$$

input
```
int(x^3*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
int(x^3*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{x^3 \left(a+b\log \left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = \frac{(16\,bfp+(2\,bgp-3\,ag)x^2-6\,af-3\,(bgpx^2+2\,bfp)\log \left(-dgx^2+df\right)-3\,(bgx^2+2\,bf)\log \left(c\right))\sqrt{-gx^2}}{9\,g^2}$$

input
```
integrate(x^3*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
1/9*(16*b*f*p + (2*b*g*p - 3*a*g)*x^2 - 6*a*f - 3*(b*g*p*x^2 + 2*b*f*p)*lo
g(-d*g*x^2 + d*f) - 3*(b*g*x^2 + 2*b*f)*log(c))*sqrt(-g*x^2 + f)/g^2
```

Sympy [F]

$$\int \frac{x^3\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx=\int \frac{x^3\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$

input
```
integrate(x**3*(a+b*ln(c*(-d*g*x**2+d*f)**p))/(-g*x**2+f)**(1/2),x)
```

output
```
Integral(x**3*(a + b*log(c*(d*f - d*g*x**2)**p))/sqrt(f - g*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \frac{x^3\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx\\&=-\frac{1}{3}\left(\frac{\sqrt{-gx^2+f}x^2}{g}+\frac{2\sqrt{-gx^2+f}f}{g^2}\right)b\log\left(\left(-dgx^2+df\right)^pc\right)\\&\quad-\frac{1}{3}\left(\frac{\sqrt{-gx^2+f}x^2}{g}+\frac{2\sqrt{-gx^2+f}f}{g^2}\right)a-\frac{2\left(\left(-gx^2+f\right)^{\frac{3}{2}}-9\sqrt{-gx^2+f}f\right)bp}{9\,g^2}\end{aligned}$$

input
```
integrate(x^3*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/3*(sqrt(-g*x^2 + f)*x^2/g + 2*sqrt(-g*x^2 + f)*f/g^2)*b*log((-d*g*x^2 +
 d*f)^p*c) - 1/3*(sqrt(-g*x^2 + f)*x^2/g + 2*sqrt(-g*x^2 + f)*f/g^2)*a - 2
/9*((-g*x^2 + f)^(3/2) - 9*sqrt(-g*x^2 + f)*f)*b*p/g^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.16

$$\int \frac{x^3\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$=\frac{\frac{\left(3\left((-gx^2+f)^{\frac{3}{2}}-3\sqrt{-gx^2+f}f\right)\log(-dgx^2+df)-2(-gx^2+f)^{\frac{3}{2}}+18\sqrt{-gx^2+f}f\right)bp}{g}+\frac{3\left((-gx^2+f)^{\frac{3}{2}}-3\sqrt{-gx^2+f}f\right)b\log(c)}{g}+\frac{3(}{}}{9g}$$

input
```
integrate(x^3*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
1/9*((3*((-g*x^2 + f)^(3/2) - 3*sqrt(-g*x^2 + f)*f)*log(-d*g*x^2 + d*f) -
2*(-g*x^2 + f)^(3/2) + 18*sqrt(-g*x^2 + f)*f)*b*p/g + 3*((-g*x^2 + f)^(3/2
) - 3*sqrt(-g*x^2 + f)*f)*b*log(c)/g + 3*((-g*x^2 + f)^(3/2) - 3*sqrt(-g*x
^2 + f)*f)*a/g)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx=\int \frac{x^3\,\left(a+b\,\ln\left(c\,{\left(d\,f-d\,g\,x^2\right)}^p\right)\right)}{\sqrt{f-g\,x^2}}\,dx$$

input
```
int((x^3*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{x^3 \left(a + b \log \left(c(df - dgx^2)^p\right)\right)}{\sqrt{f - gx^2}}\, dx$$
$$= \frac{\sqrt{-g\,x^2 + f}\,\left(-6\log\left(d^p(-g\,x^2 + f)^p\,c\right) bf - 3\log\left(d^p(-g\,x^2 + f)^p\,c\right) bg\,x^2 - 6af - 3ag\,x^2 + 16bfp + 2b}{9g^2}$$

input
```
int(x^3*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f - g*x**2)*( - 6*log(d**p*(f - g*x**2)**p*c)*b*f - 3*log(d**p*(f -
g*x**2)**p*c)*b*g*x**2 - 6*a*f - 3*a*g*x**2 + 16*b*f*p + 2*b*g*p*x**2))/(9
*g**2)
```

3.705 $\int \frac{x\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 57

$$\int \frac{x\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = \frac{2bp\sqrt{f-gx^2}}{g}-\frac{\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{g}$$

output
```
2*b*p*(-g*x^2+f)^(1/2)/g-(-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/g
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

$$\int \frac{x\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = -\frac{\sqrt{f-gx^2}\left(a-2bp+b\log\left(c(d(f-gx^2))^p\right)\right)}{g}$$

input
```
Integrate[(x*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
-((Sqrt[f - g*x^2]*(a - 2*b*p + b*Log[c*(d*(f - g*x^2))^p]))/g)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$\Big\downarrow \text{ 2929}$$
$$\int \frac{x\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f-gx^2}}dx$$

input
```
Int[(x*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x\left(a+b\ln\left(c\left(-dg\,x^2+df\right)^p\right)\right)}{\sqrt{-g\,x^2+f}}dx$$

input
```
int(x*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
int(x*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

$$\int \frac{x\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = -\frac{\sqrt{-gx^2+f}(bp\log\left(-dgx^2+df\right)-2\,bp+b\log\left(c\right)+a)}{g}$$

input
```
integrate(x*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
-sqrt(-g*x^2 + f)*(b*p*log(-d*g*x^2 + d*f) - 2*b*p + b*log(c) + a)/g
```

Sympy [F]

$$\int \frac{x\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = \int \frac{x\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$

input
```
integrate(x*(a+b*ln(c*(-d*g*x**2+d*f)**p))/(-g*x**2+f)**(1/2),x)
```

output
```
Integral(x*(a + b*log(c*(d*f - d*g*x**2)**p))/sqrt(f - g*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{x\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx = \frac{2\sqrt{-gx^2+f}bp}{g} - \frac{\sqrt{-gx^2+f}b\log\left((-dgx^2+df)^pc\right)}{g} - \frac{\sqrt{-gx^2+f}a}{g}$$

input
```
integrate(x*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
2*sqrt(-g*x^2 + f)*b*p/g - sqrt(-g*x^2 + f)*b*log((-d*g*x^2 + d*f)^p*c)/g
- sqrt(-g*x^2 + f)*a/g
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{x(a+b\log (c(df-dgx^2)^p))}{\sqrt{f-gx^2}}\,dx = -\frac{\left(\sqrt{-gx^2+f}\log \left(-dgx^2+df\right)-2\sqrt{-gx^2+f}\right)bp+\sqrt{-gx^2+f}b\log (c)+\sqrt{-gx^2+f}a}{g}$$

input
```
integrate(x*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
-((sqrt(-g*x^2 + f)*log(-d*g*x^2 + d*f) - 2*sqrt(-g*x^2 + f))*b*p + sqrt(-
g*x^2 + f)*b*log(c) + sqrt(-g*x^2 + f)*a)/g
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\log (c(df-dgx^2)^p))}{\sqrt{f-gx^2}}\,dx = \int \frac{x\,\left(a+b\,\ln \left(c\,{\left(d\,f-d\,g\,x^2\right)}^p\right)\right)}{\sqrt{f-g\,x^2}}\,dx$$

input
```
int((x*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2),x)
```

output
```
int((x*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

$$\int \frac{x\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx=\frac{\sqrt{-g\,x^2+f}\,\left(-\log\left(d^p\left(-g\,x^2+f\right)^p c\right)b-a+2bp\right)}{g}$$

input
```
int(x*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f - g*x**2)*( - log(d**p*(f - g*x**2)**p*c)*b - a + 2*b*p))/g
```

3.706 $\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 105

$$\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x\sqrt{f-gx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{\sqrt{f}} - \frac{bp\operatorname{PolyLog}\left(2,-\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{bp\operatorname{PolyLog}\left(2,\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)}{\sqrt{f}}$$

output

```
-arctanh((-g*x^2+f)^(1/2)/f^(1/2))*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f^(1/2)-b*
p*polylog(2,-(-g*x^2+f)^(1/2)/f^(1/2))/f^(1/2)+b*p*polylog(2,(-g*x^2+f)^(1
/2)/f^(1/2))/f^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x\sqrt{f-gx^2}}\,dx$$
$$=\frac{\left(a+b\log\left(c(d(f-gx^2))^p\right)\right)\left(\log\left(1-\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)-\log\left(1+\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)\right)-2bp\,\text{PolyLog}\left(2,-\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)+2}{2\sqrt{f}}$$

input
```
Integrate[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x*Sqrt[f - g*x^2]),x]
```

output
```
((a + b*Log[c*(d*(f - g*x^2))^p])*(Log[1 - Sqrt[f - g*x^2]/Sqrt[f]] - Log[
1 + Sqrt[f - g*x^2]/Sqrt[f]]) - 2*b*p*PolyLog[2, -(Sqrt[f - g*x^2]/Sqrt[f]
)] + 2*b*p*PolyLog[2, Sqrt[f - g*x^2]/Sqrt[f]])/(2*Sqrt[f])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x\sqrt{f-gx^2}}\,dx$$
$$\Big\downarrow \text{2929}$$
$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x\sqrt{f-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x*Sqrt[f - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(-dg\,x^{2}+df)^{p}\right)}{x\sqrt{-g\,x^{2}+f}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x/(-g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x/(-g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(df-dgx^{2})^{p}\right)}{x\sqrt{f-gx^{2}}}\,dx=\int \frac{b\log\left((-dgx^{2}+df)^{p}c\right)+a}{\sqrt{-gx^{2}+f}x}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x/(-g*x^2+f)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-(sqrt(-g*x^2 + f)*b*log((-d*g*x^2 + d*f)^p*c) + sqrt(-g*x^2 + f)
*a)/(g*x^3 - f*x), x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x\sqrt{f-gx^2}}\,dx = \int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x\sqrt{f-gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+d*f)**p))/x/(-g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f - d*g*x**2)**p))/(x*sqrt(f - g*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x\sqrt{f-gx^2}}\,dx = \int \frac{b\log\left((-dgx^2+df)^p c\right)+a}{\sqrt{-gx^2+f}x}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x/(-g*x^2+f)^(1/2),x, algorithm="m
axima")
```

output
```
b*integrate((p*log(d) + log((-g*x^2 + f)^p) + log(c))/(sqrt(-g*x^2 + f)*x)
, x) - a*log(2*sqrt(-g*x^2 + f)*sqrt(f)/abs(x) + 2*f/abs(x))/sqrt(f)
```

Giac [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x\sqrt{f-gx^2}}\,dx = \int \frac{b\log\left((-dgx^2+df)^p c\right)+a}{\sqrt{-gx^2+f}x}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x/(-g*x^2+f)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*log((-d*g*x^2 + d*f)^p*c) + a)/(sqrt(-g*x^2 + f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x\sqrt{f-gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,f-d\,g\,x^2\right)}^p\right)}{x\,\sqrt{f-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x*(f - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x*(f - g*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x\sqrt{f-gx^2}}\,dx=\frac{\sqrt{f}\,\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)\right)a+\left(\int \frac{\log\left(\left(-dg\,x^2+df\right)^p c\right)}{\sqrt{-g\,x^2+f}\,x}dx\right)bf}{f}$$

input
```
int((a+b*log(c*(-d*g*x^2+d*f)^p))/x/(-g*x^2+f)^(1/2),x)
```

output
```
(sqrt(f)*log(tan(asin((sqrt(g)*x)/sqrt(f))/2))*a + int(log((d*f - d*g*x**2
)**p*c)/(sqrt(f - g*x**2)*x),x)*b*f)/f
```

3.707 $\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^3\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 185

$$\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^3\sqrt{f-gx^2}}\,dx = \frac{bgp\,\text{arctanh}\left(\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)}{f^{3/2}} - \frac{\sqrt{f-gx^2}\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{2fx^2} - \frac{g\,\text{arctanh}\left(\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{2f^{3/2}} - \frac{bgp\,\text{PolyLog}\left(2,-\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)}{2f^{3/2}} + \frac{bgp\,\text{PolyLog}\left(2,\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)}{2f^{3/2}}$$

output
```
b*g*p*arctanh((-g*x^2+f)^(1/2)/f^(1/2))/f^(3/2)-1/2*(-g*x^2+f)^(1/2)*(a+b*
ln(c*(-d*g*x^2+d*f)^p))/f/x^2-1/2*g*arctanh((-g*x^2+f)^(1/2)/f^(1/2))*(a+b
*ln(c*(-d*g*x^2+d*f)^p))/f^(3/2)-1/2*b*g*p*polylog(2,-(-g*x^2+f)^(1/2)/f^(
1/2))/f^(3/2)+1/2*b*g*p*polylog(2,(-g*x^2+f)^(1/2)/f^(1/2))/f^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.75

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^3\sqrt{f-gx^2}}\,dx$$
$$= \frac{-2a\sqrt{f}\sqrt{f-gx^2}-2b\sqrt{f}\sqrt{f-gx^2}\log\left(c(d(f-gx^2))^p\right)-2bgpx^2\log\left(\sqrt{f}-\sqrt{f-gx^2}\right)+2bgpx^2\log\left(\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^3*Sqrt[f - g*x^2]),x]
```

output
```
(-2*a*Sqrt[f]*Sqrt[f - g*x^2] - 2*b*Sqrt[f]*Sqrt[f - g*x^2]*Log[c*(d*(f -
g*x^2))^p] - 2*b*g*p*x^2*Log[Sqrt[f] - Sqrt[f - g*x^2]] + 2*b*g*p*x^2*Log[
Sqrt[f] + Sqrt[f - g*x^2]] + a*g*x^2*Log[1 - Sqrt[f - g*x^2]/Sqrt[f]] + b*
g*x^2*Log[c*(d*(f - g*x^2))^p]*Log[1 - Sqrt[f - g*x^2]/Sqrt[f]] - a*g*x^2*
Log[1 + Sqrt[f - g*x^2]/Sqrt[f]] - b*g*x^2*Log[c*(d*(f - g*x^2))^p]*Log[1
+ Sqrt[f - g*x^2]/Sqrt[f]] - 2*b*g*p*x^2*PolyLog[2, -(Sqrt[f - g*x^2]/Sqrt
[f])] + 2*b*g*p*x^2*PolyLog[2, Sqrt[f - g*x^2]/Sqrt[f]])/(4*f^(3/2)*x^2)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^3\sqrt{f-gx^2}}\,dx$$
$$\Big\downarrow \; 2929$$
$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^3\sqrt{f-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^3*Sqrt[f - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln \left(c(-dg\,x^{2}+df)^{p}\right)}{x^{3}\sqrt{-g\,x^{2}+f}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^3/(-g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^3/(-g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{x^3\sqrt{f-gx^2}}\,dx=\int \frac{b\log \left((-dgx^2+df)^pc\right)+a}{\sqrt{-gx^2+f}x^3}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^3/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-(sqrt(-g*x^2 + f)*b*log((-d*g*x^2 + d*f)^p*c) + sqrt(-g*x^2 + f)
*a)/(g*x^5 - f*x^3), x)
```

Sympy [F]

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^3\sqrt{f - gx^2}}\,dx = \int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^3\sqrt{f - gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+d*f)**p))/x**3/(-g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f - d*g*x**2)**p))/(x**3*sqrt(f - g*x**2)), x)
```

Maxima [F]

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^3\sqrt{f - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + df)^p c\right) + a}{\sqrt{-gx^2 + f}x^3}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^3/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/2*a*(g*log(2*sqrt(-g*x^2 + f)*sqrt(f)/abs(x) + 2*f/abs(x))/f^(3/2) + sq
rt(-g*x^2 + f)/(f*x^2)) + b*integrate((p*log(d) + log((-g*x^2 + f)^p) + lo
g(c))/(sqrt(-g*x^2 + f)*x^3), x)
```

Giac [F]

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^3\sqrt{f - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + df)^p c\right) + a}{\sqrt{-gx^2 + f}x^3}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^3/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((b*log((-d*g*x^2 + d*f)^p*c) + a)/(sqrt(-g*x^2 + f)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^3\sqrt{f-gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,(d\,f-d\,g\,x^2)^p\right)}{x^3\,\sqrt{f-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^3*(f - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^3*(f - g*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^3\sqrt{f-gx^2}}\,dx$$
$$=\frac{-\sqrt{-g\,x^2+f}\,af+\sqrt{f}\,\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)\right)ag\,x^2+2\left(\int\frac{\log\left(\left(-dg\,x^2+df\right)^p c\right)}{\sqrt{-g\,x^2+f}\,x^3}dx\right)b\,f^2x^2}{2f^2x^2}$$

input
```
int((a+b*log(c*(-d*g*x^2+d*f)^p))/x^3/(-g*x^2+f)^(1/2),x)
```

output
```
( - sqrt(f - g*x**2)*a*f + sqrt(f)*log(tan(asin((sqrt(g)*x)/sqrt(f))/2))*a
*g*x**2 + 2*int(log((d*f - d*g*x**2)**p*c)/(sqrt(f - g*x**2)*x**3),x)*b*f*
*2*x**2)/(2*f**2*x**2)
```

3.708 $\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^5\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 261

$$\begin{aligned}\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^5\sqrt{f-gx^2}}\,dx &= \frac{bgp\sqrt{f-gx^2}}{4f^2x^2}+\frac{bg^2p\,\text{arctanh}\left(\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)}{f^{5/2}} \\ &\quad -\frac{\sqrt{f-gx^2}\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{4fx^4} \\ &\quad -\frac{3g\sqrt{f-gx^2}\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{8f^2x^2} \\ &\quad -\frac{3g^2\text{arctanh}\left(\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{8f^{5/2}} \\ &\quad -\frac{3bg^2p\,\text{PolyLog}\left(2,-\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)}{8f^{5/2}} \\ &\quad +\frac{3bg^2p\,\text{PolyLog}\left(2,\frac{\sqrt{f-gx^2}}{\sqrt{f}}\right)}{8f^{5/2}}\end{aligned}$$

output
```
1/4*b*g*p*(-g*x^2+f)^(1/2)/f^2/x^2+b*g^2*p*arctanh((-g*x^2+f)^(1/2)/f^(1/2
))/f^(5/2)-1/4*(-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f/x^4-3/8*g*(
-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f^2/x^2-3/8*g^2*arctanh((-g*x
^2+f)^(1/2)/f^(1/2))*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f^(5/2)-3/8*b*g^2*p*poly
log(2,-(-g*x^2+f)^(1/2)/f^(1/2))/f^(5/2)+3/8*b*g^2*p*polylog(2,(-g*x^2+f)^
(1/2)/f^(1/2))/f^(5/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.64

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^5\sqrt{f-gx^2}}\,dx =$$
$$-\frac{4af^{3/2}\sqrt{f-gx^2}+6a\sqrt{f}gx^2\sqrt{f-gx^2}-4b\sqrt{f}gpx^2\sqrt{f-gx^2}+4bf^{3/2}\sqrt{f-gx^2}\log\left(c(d(f-gx^2))^p\right)}{}$$

input
```
Integrate[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^5*Sqrt[f - g*x^2]),x]
```

output
```
-1/16*(4*a*f^(3/2)*Sqrt[f - g*x^2] + 6*a*Sqrt[f]*g*x^2*Sqrt[f - g*x^2] - 4
*b*Sqrt[f]*g*p*x^2*Sqrt[f - g*x^2] + 4*b*f^(3/2)*Sqrt[f - g*x^2]*Log[c*(d*
(f - g*x^2))^p] + 6*b*Sqrt[f]*g*x^2*Sqrt[f - g*x^2]*Log[c*(d*(f - g*x^2))^
p] + 8*b*g^2*p*x^4*Log[Sqrt[f] - Sqrt[f - g*x^2]] - 8*b*g^2*p*x^4*Log[Sqrt
[f] + Sqrt[f - g*x^2]] - 3*a*g^2*x^4*Log[1 - Sqrt[f - g*x^2]/Sqrt[f]] - 3*
b*g^2*x^4*Log[c*(d*(f - g*x^2))^p]*Log[1 - Sqrt[f - g*x^2]/Sqrt[f]] + 3*a*
g^2*x^4*Log[1 + Sqrt[f - g*x^2]/Sqrt[f]] + 3*b*g^2*x^4*Log[c*(d*(f - g*x^2
))^p]*Log[1 + Sqrt[f - g*x^2]/Sqrt[f]] + 6*b*g^2*p*x^4*PolyLog[2, -(Sqrt[f
 - g*x^2]/Sqrt[f])] - 6*b*g^2*p*x^4*PolyLog[2, Sqrt[f - g*x^2]/Sqrt[f]])/(
f^(5/2)*x^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^5\sqrt{f-gx^2}}\,dx$$
$$\Big\downarrow \text{2929}$$
$$\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^5\sqrt{f-gx^2}}\,dx$$

input
```
Int[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^5*Sqrt[f - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(-dg\,x^2+df\right)^p\right)}{x^5\sqrt{-g\,x^2+f}}\,dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^5/(-g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^5/(-g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^5\sqrt{f-gx^2}}\,dx=\int \frac{b\log\left((-dgx^2+df)^pc\right)+a}{\sqrt{-gx^2+f}x^5}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^5/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-(sqrt(-g*x^2 + f)*b*log((-d*g*x^2 + d*f)^p*c) + sqrt(-g*x^2 + f)
*a)/(g*x^7 - f*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^5\sqrt{f-gx^2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+d*f)**p))/x**5/(-g*x**2+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^5\sqrt{f-gx^2}}\,dx=\int \frac{b\log\left((-dgx^2+df)^pc\right)+a}{\sqrt{-gx^2+f}x^5}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^5/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/8*a*(3*g^2*log(2*sqrt(-g*x^2 + f)*sqrt(f)/abs(x) + 2*f/abs(x))/f^(5/2)
+ 3*sqrt(-g*x^2 + f)*g/(f^2*x^2) + 2*sqrt(-g*x^2 + f)/(f*x^4)) + b*integra
te((p*log(d) + log((-g*x^2 + f)^p) + log(c))/(sqrt(-g*x^2 + f)*x^5), x)
```

Giac [F]

$$\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{x^5\sqrt{f-gx^2}}\,dx=\int \frac{b\log \left((-dgx^2+df)^pc\right)+a}{\sqrt{-gx^2+f}x^5}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^5/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((b*log((-d*g*x^2 + d*f)^p*c) + a)/(sqrt(-g*x^2 + f)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{x^5\sqrt{f-gx^2}}\,dx=\int \frac{a+b\,\ln \left(c\,{\left(d\,f-d\,g\,x^2\right)}^p\right)}{x^5\,\sqrt{f-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^5*(f - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^5*(f - g*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{x^5\sqrt{f-gx^2}}\,dx$$
$$=\frac{-2\sqrt{-g\,x^2+f}\,a\,f^2-3\sqrt{-g\,x^2+f}\,afg\,x^2+3\sqrt{f}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)\right)a\,g^2x^4+8\left(\int\frac{\log((-dg\,x^2+df)^pc)}{\sqrt{-g\,x^2+f}\,x^5}\right.}{8f^3x^4}$$

input
```
int((a+b*log(c*(-d*g*x^2+d*f)^p))/x^5/(-g*x^2+f)^(1/2),x)
```

output
```
( - 2*sqrt(f - g*x**2)*a*f**2 - 3*sqrt(f - g*x**2)*a*f*g*x**2 + 3*sqrt(f)*
log(tan(asin((sqrt(g)*x)/sqrt(f))/2))*a*g**2*x**4 + 8*int(log((d*f - d*g*x
**2)**p*c)/(sqrt(f - g*x**2)*x**5),x)*b*f**3*x**4)/(8*f**3*x**4)
```

3.709 $\int \frac{x^2\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 613

$$\int \frac{x^2\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$= \frac{bf^2p\sqrt{-\frac{f}{g}+x^2}}{8g^2\sqrt{f-gx^2}\left(x+\sqrt{-\frac{f}{g}+x^2}\right)^2} + \frac{a\sqrt{-\frac{f}{g}+x^2}\left(x+\sqrt{-\frac{f}{g}+x^2}\right)^2}{8\sqrt{f-gx^2}}$$
$$- \frac{bp\sqrt{-\frac{f}{g}+x^2}\left(x+\sqrt{-\frac{f}{g}+x^2}\right)^2}{8\sqrt{f-gx^2}}$$
$$+ \frac{b\sqrt{-\frac{f}{g}+x^2}\left(x+\sqrt{-\frac{f}{g}+x^2}\right)^2\log\left(c(df-dgx^2)^p\right)}{8\sqrt{f-gx^2}}$$
$$- \frac{f^2\sqrt{-\frac{f}{g}+x^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{8g^2\sqrt{f-gx^2}\left(x+\sqrt{-\frac{f}{g}+x^2}\right)^2} - \frac{bfp\sqrt{-\frac{f}{g}+x^2}\log\left(x+\sqrt{-\frac{f}{g}+x^2}\right)}{2g\sqrt{f-gx^2}}$$
$$+ \frac{f\sqrt{-\frac{f}{g}+x^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)\log\left(x+\sqrt{-\frac{f}{g}+x^2}\right)}{2g\sqrt{f-gx^2}}$$
$$+ \frac{bfp\sqrt{-\frac{f}{g}+x^2}\log^2\left(x+\sqrt{-\frac{f}{g}+x^2}\right)}{2g\sqrt{f-gx^2}}$$
$$- \frac{bfp\sqrt{-\frac{f}{g}+x^2}\log\left(x+\sqrt{-\frac{f}{g}+x^2}\right)\log\left(1-\frac{g\left(x+\sqrt{-\frac{f}{g}+x^2}\right)^2}{f}\right)}{g\sqrt{f-gx^2}}$$
$$- \frac{bfp\sqrt{-\frac{f}{g}+x^2}\,\mathrm{PolyLog}\left(2,\frac{g\left(x+\sqrt{-\frac{f}{g}+x^2}\right)^2}{f}\right)}{2g\sqrt{f-gx^2}}$$

output
```
1/8*b*f^2*p*(-f/g+x^2)^(1/2)/g^2/(-g*x^2+f)^(1/2)/(x+(-f/g+x^2)^(1/2))^2+1
/8*a*(-f/g+x^2)^(1/2)*(x+(-f/g+x^2)^(1/2))^2/(-g*x^2+f)^(1/2)-1/8*b*p*(-f/
g+x^2)^(1/2)*(x+(-f/g+x^2)^(1/2))^2/(-g*x^2+f)^(1/2)+1/8*b*(-f/g+x^2)^(1/2
)*(x+(-f/g+x^2)^(1/2))^2*ln(c*(-d*g*x^2+d*f)^p)/(-g*x^2+f)^(1/2)-1/8*f^2*(
-f/g+x^2)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/g^2/(-g*x^2+f)^(1/2)/(x+(-f/g
+x^2)^(1/2))^2-1/2*b*f*p*(-f/g+x^2)^(1/2)*ln(x+(-f/g+x^2)^(1/2))/g/(-g*x^2
+f)^(1/2)+1/2*f*(-f/g+x^2)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))*ln(x+(-f/g+x
^2)^(1/2))/g/(-g*x^2+f)^(1/2)+1/2*b*f*p*(-f/g+x^2)^(1/2)*ln(x+(-f/g+x^2)^(
1/2))^2/g/(-g*x^2+f)^(1/2)-b*f*p*(-f/g+x^2)^(1/2)*ln(x+(-f/g+x^2)^(1/2))*l
n(1-g*(x+(-f/g+x^2)^(1/2))^2/f)/g/(-g*x^2+f)^(1/2)-1/2*b*f*p*(-f/g+x^2)^(1
/2)*polylog(2,g*(x+(-f/g+x^2)^(1/2))^2/f)/g/(-g*x^2+f)^(1/2)
```

Mathematica [F]

$$\int \frac{x^2\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx=\int \frac{x^2\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$

input
```
Integrate[(x^2*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
Integrate[(x^2*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{x^2\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{\sqrt{f-gx^2}}dx$$

input
```
Int[(x^2*(a + b*Log[c*(d*f - d*g*x^2)^p]))/Sqrt[f - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{x^2 \left(a + b \ln \left(c(-dg\, x^2 + df)^p\right)\right)}{\sqrt{-g\, x^2 + f}}dx$$

input
```
int(x^2*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
int(x^2*(a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2 \left(a + b \log \left(c(df - dgx^2)^p\right)\right)}{\sqrt{f - gx^2}}\, dx = \int \frac{\left(b \log \left((-dgx^2 + df)^p c\right) + a\right)x^2}{\sqrt{-gx^2 + f}}\, dx$$

input
```
integrate(x^2*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
integral(-(sqrt(-g*x^2 + f)*b*x^2*log((-d*g*x^2 + d*f)^p*c) + sqrt(-g*x^2
+ f)*a*x^2)/(g*x^2 - f), x)
```

Sympy [F]

$$\int \frac{x^2 \left(a+b \log \left(c \left(d f-d g x^2\right)^p\right)\right)}{\sqrt{f-g x^2}} \, dx=\int \frac{x^2 \left(a+b \log \left(c \left(d f-d g x^2\right)^p\right)\right)}{\sqrt{f-g x^2}} \, dx$$

input
```
integrate(x**2*(a+b*ln(c*(-d*g*x**2+d*f)**p))/(-g*x**2+f)**(1/2),x)
```

output
```
Integral(x**2*(a + b*log(c*(d*f - d*g*x**2)**p))/sqrt(f - g*x**2), x)
```

Maxima [F]

$$\int \frac{x^2 \left(a+b \log \left(c \left(d f-d g x^2\right)^p\right)\right)}{\sqrt{f-g x^2}} \, dx=\int \frac{\left(b \log \left(\left(-d g x^2+d f\right)^p c\right)+a\right) x^2}{\sqrt{-g x^2+f}} \, dx$$

input
```
integrate(x^2*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/2*a*(sqrt(-g*x^2 + f)*x/g - f*arcsin(g*x/sqrt(f*g))/g^(3/2)) + b*integr
ate(((p*log(d) + log(c))*x^2 + x^2*log((-g*x^2 + f)^p))/sqrt(-g*x^2 + f),
x)
```

Giac [F]

$$\int \frac{x^2 \left(a+b \log \left(c \left(d f-d g x^2\right)^p\right)\right)}{\sqrt{f-g x^2}} \, dx=\int \frac{\left(b \log \left(\left(-d g x^2+d f\right)^p c\right)+a\right) x^2}{\sqrt{-g x^2+f}} \, dx$$

input
```
integrate(x^2*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((b*log((-d*g*x^2 + d*f)^p*c) + a)*x^2/sqrt(-g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\log(c(df-dgx^2)^p))}{\sqrt{f-gx^2}}\,dx = \int \frac{x^2\,(a+b\,\ln(c\,(d\,f-d\,g\,x^2)^p))}{\sqrt{f-g\,x^2}}\,dx$$

input
```
int((x^2*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*log(c*(d*f - d*g*x^2)^p)))/(f - g*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(a+b\log(c(df-dgx^2)^p))}{\sqrt{f-gx^2}}\,dx = \frac{\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)af-\sqrt{-g\,x^2+f}\,agx+2\left(\int \frac{\log((-dg\,x^2+df)^p c)x^2}{\sqrt{-g\,x^2+f}}dx\right)b\,g^2}{2g^2}$$

input
```
int(x^2*(a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
(sqrt(g)*asin((sqrt(g)*x)/sqrt(f))*a*f - sqrt(f - g*x**2)*a*g*x + 2*int((l
og((d*f - d*g*x**2)**p*c)*x**2)/sqrt(f - g*x**2),x)*b*g**2)/(2*g**2)
```

3.710 $\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 242

$$\begin{aligned}\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{\sqrt{f-gx^2}}\,dx &= \frac{\sqrt{-\frac{f}{g}+x^2}\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)\log\left(x+\sqrt{-\frac{f}{g}+x^2}\right)}{\sqrt{f-gx^2}} \\ &\quad+\frac{bp\sqrt{-\frac{f}{g}+x^2}\log^2\left(x+\sqrt{-\frac{f}{g}+x^2}\right)}{\sqrt{f-gx^2}} \\ &\quad-\frac{2bp\sqrt{-\frac{f}{g}+x^2}\log\left(x+\sqrt{-\frac{f}{g}+x^2}\right)\log\left(1-\frac{g\left(x+\sqrt{-\frac{f}{g}+x^2}\right)^2}{f}\right)}{\sqrt{f-gx^2}} \\ &\quad-\frac{bp\sqrt{-\frac{f}{g}+x^2}\,\mathrm{PolyLog}\left(2,\frac{g\left(x+\sqrt{-\frac{f}{g}+x^2}\right)^2}{f}\right)}{\sqrt{f-gx^2}}\end{aligned}$$

output
```
(-f/g+x^2)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))*ln(x+(-f/g+x^2)^(1/2))/(-g*x
^2+f)^(1/2)+b*p*(-f/g+x^2)^(1/2)*ln(x+(-f/g+x^2)^(1/2))^2/(-g*x^2+f)^(1/2)
-2*b*p*(-f/g+x^2)^(1/2)*ln(x+(-f/g+x^2)^(1/2))*ln(1-g*(x+(-f/g+x^2)^(1/2))
^2/f)/(-g*x^2+f)^(1/2)-b*p*(-f/g+x^2)^(1/2)*polylog(2,g*(x+(-f/g+x^2)^(1/2
))^2/f)/(-g*x^2+f)^(1/2)
```

Mathematica [F]

$$\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{\sqrt{f-gx^2}}\,dx=\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{\sqrt{f-gx^2}}\,dx$$

input
```
Integrate[(a + b*Log[c*(d*f - d*g*x^2)^p])/Sqrt[f - g*x^2],x]
```

output
```
Integrate[(a + b*Log[c*(d*f - d*g*x^2)^p])/Sqrt[f - g*x^2], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{\sqrt{f-gx^2}}\,dx$$
$$\Big\downarrow \text{ 2923}$$
$$\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{\sqrt{f-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f - d*g*x^2)^p])/Sqrt[f - g*x^2],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2923
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(f + g*x^s)^r*(a + b*Log
[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(-dg\,x^{2}+df)^{p}\right)}{\sqrt{-g\,x^{2}+f}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c(df-dgx^{2})^{p}\right)}{\sqrt{f-gx^{2}}}\,dx=\int \frac{b\log\left((-dgx^{2}+df)^{p}c\right)+a}{\sqrt{-gx^{2}+f}}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(-(sqrt(-g*x^2 + f)*b*log((-d*g*x^2 + d*f)^p*c) + sqrt(-g*x^2 + f)
*a)/(g*x^2 - f), x)
```

Sympy [F]

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{\sqrt{f - gx^2}}\,dx = \int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{\sqrt{f - gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+d*f)**p))/(-g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f - d*g*x**2)**p))/sqrt(f - g*x**2), x)
```

Maxima [F]

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{\sqrt{f - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + df)^p c\right) + a}{\sqrt{-gx^2 + f}}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm="max
ima")
```

output
```
b*integrate((p*log(d) + log((-g*x^2 + f)^p) + log(c))/sqrt(-g*x^2 + f), x)
 + a*arcsin(g*x/sqrt(f*g))/sqrt(g)
```

Giac [F]

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{\sqrt{f - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + df)^p c\right) + a}{\sqrt{-gx^2 + f}}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((b*log((-d*g*x^2 + d*f)^p*c) + a)/sqrt(-g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{\sqrt{f-gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,f-d\,g\,x^2\right)}^p\right)}{\sqrt{f-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(f - g*x^2)^(1/2),x)
```

output
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(f - g*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{\sqrt{f-gx^2}}\,dx=\frac{\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)a+\left(\int \frac{\log\left(\left(-dg\,x^2+df\right)^p c\right)}{\sqrt{-g\,x^2+f}}dx\right)bg}{g}$$

input
```
int((a+b*log(c*(-d*g*x^2+d*f)^p))/(-g*x^2+f)^(1/2),x)
```

output
```
(sqrt(g)*asin((sqrt(g)*x)/sqrt(f))*a + int(log((d*f - d*g*x**2)**p*c)/sqrt
(f - g*x**2),x)*b*g)/g
```

3.711 $\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^2\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 73

$$\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^2\sqrt{f-gx^2}}\,dx=-\frac{2b\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f-gx^2}}\right)}{f}-\frac{\sqrt{f-gx^2}\left(a+b\log\left(c\left(df-dgx^2\right)^p\right)\right)}{fx}$$

output
```
-2*b*g^(1/2)*p*arctan(g^(1/2)*x/(-g*x^2+f)^(1/2))/f-(-g*x^2+f)^(1/2)*(a+b*
ln(c*(-d*g*x^2+d*f)^p))/f/x
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^2\sqrt{f-gx^2}}\,dx=-\frac{2b\sqrt{g}px\arctan\left(\frac{\sqrt{g}x}{\sqrt{f-gx^2}}\right)+\sqrt{f-gx^2}\left(a+b\log\left(c\left(d\left(f-gx^2\right)\right)^p\right)\right)}{fx}$$

input
```
Integrate[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^2*Sqrt[f - g*x^2]),x]
```

output
```
-((2*b*Sqrt[g]*p*x*ArcTan[(Sqrt[g]*x)/Sqrt[f - g*x^2]] + Sqrt[f - g*x^2]*(
a + b*Log[c*(d*(f - g*x^2))^p]))/(f*x))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^2\sqrt{f-gx^2}}dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{a+b\log\left(c\left(df-dgx^2\right)^p\right)}{x^2\sqrt{f-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^2*Sqrt[f - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln \left(c(-dg\,x^2+df)^p\right)}{x^2\sqrt{-g\,x^2+f}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^2/(-g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^2/(-g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.14

$$\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{x^2\sqrt{f-gx^2}}\,dx = \left[\frac{b\sqrt{-g}px\log \left(2\,gx^2-2\,\sqrt{-gx^2+f}\sqrt{-g}x-f\right)-\sqrt{-gx^2+f}(bp\log \left(-dgx^2+df\right)+b\log \left(c\right)+a)}{fx}, \frac{2\,b}{}\right.$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^2/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
[(b*sqrt(-g)*p*x*log(2*g*x^2 - 2*sqrt(-g*x^2 + f)*sqrt(-g)*x - f) - sqrt(-
g*x^2 + f)*(b*p*log(-d*g*x^2 + d*f) + b*log(c) + a))/(f*x), (2*b*sqrt(g)*p
*x*arctan(sqrt(-g*x^2 + f)*sqrt(g)*x/(g*x^2 - f)) - sqrt(-g*x^2 + f)*(b*p*
log(-d*g*x^2 + d*f) + b*log(c) + a))/(f*x)]
```

Sympy [F]

$$\int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{x^2\sqrt{f-gx^2}}\,dx = \int \frac{a+b\log \left(c(df-dgx^2)^p\right)}{x^2\sqrt{f-gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+d*f)**p))/x**2/(-g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f - d*g*x**2)**p))/(x**2*sqrt(f - g*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^2\sqrt{f - gx^2}}\,dx = -\frac{2\,b\sqrt{g}p\arcsin\left(\frac{x}{\sqrt{\frac{f}{g}}}\right)}{f} - \frac{\sqrt{-gx^2 + f}b\log\left((-dgx^2 + df)^p c\right)}{fx} - \frac{\sqrt{-gx^2 + f}a}{fx}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^2/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-2*b*sqrt(g)*p*arcsin(x/sqrt(f/g))/f - sqrt(-g*x^2 + f)*b*log((-d*g*x^2 +
d*f)^p*c)/(f*x) - sqrt(-g*x^2 + f)*a/(f*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(65) = 130.

Time = 0.47 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.44

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^2\sqrt{f - gx^2}}\,dx = bp\left(\frac{2\sqrt{-g}\log\left(-dgx^2 + df\right)}{\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 - f} + \frac{\sqrt{-g}\log\left(|gx^2 - f|\right)}{f} + \frac{2\,g\log\left(\left|-\sqrt{-g}x + \sqrt{-gx^2 + f}\right|\right)}{f\sqrt{-g}}\right) + \frac{2\,b\sqrt{-g}\log\left(c\right)}{\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 - f} + \frac{2\,a\sqrt{-g}}{\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 - f}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^2/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
b*p*(2*sqrt(-g)*log(-d*g*x^2 + d*f)/((sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f
) + sqrt(-g)*log(abs(g*x^2 - f))/f + 2*g*log(abs(-sqrt(-g)*x + sqrt(-g*x^2
 + f)))/(f*sqrt(-g))) + 2*b*sqrt(-g)*log(c)/((sqrt(-g)*x - sqrt(-g*x^2 + f
))^2 - f) + 2*a*sqrt(-g)/((sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^2\sqrt{f-gx^2}}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d\,f-d\,g\,x^2\right)}^p\right)}{x^2\,\sqrt{f-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^2*(f - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^2*(f - g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^2\sqrt{f-gx^2}}\,dx$$
$$=\frac{-2\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)bpx-\sqrt{-g\,x^2+f}\,\log\left(\frac{f^p d^p\left(-\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)^2+1\right)^{2p}c}{\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)^2+1\right)^{2p}}\right)b-\sqrt{-g\,x^2+f}\,a}{fx}$$

input
```
int((a+b*log(c*(-d*g*x^2+d*f)^p))/x^2/(-g*x^2+f)^(1/2),x)
```

output
```
( - 2*sqrt(g)*asin((sqrt(g)*x)/sqrt(f))*b*p*x - sqrt(f - g*x**2)*log((f**p
*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p)*c)/(tan(asin((sq
rt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b - sqrt(f - g*x**2)*a)/(f*x)
```

3.712 $\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx = \frac{2bgp\sqrt{f-gx^2}}{3f^2x}-\frac{4bg^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f-gx^2}}\right)}{3f^2}-\frac{\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{3fx^3}-\frac{2g\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{3f^2x}$$

output
```
2/3*b*g*p*(-g*x^2+f)^(1/2)/f^2/x-4/3*b*g^(3/2)*p*arctan(g^(1/2)*x/(-g*x^2+
f)^(1/2))/f^2-1/3*(-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f/x^3-2/3*
g*(-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f^2/x
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx = -\frac{4bg^{3/2}px^3\arctan\left(\frac{\sqrt{g}x}{\sqrt{f-gx^2}}\right)+\sqrt{f-gx^2}\left(af+2agx^2-2bgpx^2+b(f+2gx^2)\log\left(c(d(f-gx^2))^p\right)\right)}{3f^2x^3}$$

input
```
Integrate[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^4*Sqrt[f - g*x^2]),x]
```

output
```
-1/3*(4*b*g^(3/2)*p*x^3*ArcTan[(Sqrt[g]*x)/Sqrt[f - g*x^2]] + Sqrt[f - g*x
^2]*(a*f + 2*a*g*x^2 - 2*b*g*p*x^2 + b*(f + 2*g*x^2)*Log[c*(d*(f - g*x^2))
^p]))/(f^2*x^3)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx$$

$$\Big\downarrow \; 2929$$

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^4*Sqrt[f - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c(-dg\,x^2+df)^p\right)}{x^4\sqrt{-g\,x^2+f}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^4/(-g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^4/(-g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.67

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx$$

$$= \left[\frac{2\,b\sqrt{-g}gpx^3\log\left(2\,gx^2-2\,\sqrt{-gx^2+f}\,\sqrt{-g}x-f\right)+\left(2\,(bgp-ag)x^2-af-(2\,bgpx^2+bfp)\log\left(-dg\right.\right.}{3\,f^2x^3}\right.$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^4/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/3*(2*b*sqrt(-g)*g*p*x^3*log(2*g*x^2 - 2*sqrt(-g*x^2 + f)*sqrt(-g)*x - f
) + (2*(b*g*p - a*g)*x^2 - a*f - (2*b*g*p*x^2 + b*f*p)*log(-d*g*x^2 + d*f)
 - (2*b*g*x^2 + b*f)*log(c))*sqrt(-g*x^2 + f))/(f^2*x^3), 1/3*(4*b*g^(3/2)
*p*x^3*arctan(sqrt(-g*x^2 + f)*sqrt(g)*x/(g*x^2 - f)) + (2*(b*g*p - a*g)*x
^2 - a*f - (2*b*g*p*x^2 + b*f*p)*log(-d*g*x^2 + d*f) - (2*b*g*x^2 + b*f)*l
og(c))*sqrt(-g*x^2 + f))/(f^2*x^3)]
```

Sympy [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx = \int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+d*f)**p))/x**4/(-g*x**2+f)**(1/2),x)
```

output
```
Integral((a + b*log(c*(d*f - d*g*x**2)**p))/(x**4*sqrt(f - g*x**2)), x)
```

Maxima [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx = \int \frac{b\log\left((-dgx^2+df)^p c\right)+a}{\sqrt{-gx^2+f}x^4}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^4/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/3*b*(integrate((4*g^2*p*x^4 + 2*f*g*p*x^2 - 3*f^2*p*log(d) - 3*f^2*log(
c))/(sqrt(-g*x^2 + f)*x^4), x)/f^2 - (2*g^2*x^4 - f*g*x^2 - f^2)*log((-g*x
^2 + f)^p)/(sqrt(-g*x^2 + f)*f^2*x^3)) - 1/3*a*(2*sqrt(-g*x^2 + f)*g/(f^2*
x) + sqrt(-g*x^2 + f)/(f*x^3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(125) = 250$.

Time = 0.74 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.12

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx = -\frac{2}{3}bp\left(\frac{2\left(3\left(\sqrt{-g}x-\sqrt{-gx^2+f}\right)^2-f\right)\sqrt{-g}g\log\left(-dgx^2+df\right)}{\left(\left(\sqrt{-g}x-\sqrt{-gx^2+f}\right)^2-f\right)^3}+\frac{\sqrt{-g}g\log\left(\left(\sqrt{-g}x-\sqrt{-gx^2+\right.\right.}{f^2}\right. - \frac{4\left(3\left(\sqrt{-g}x-\sqrt{-gx^2+f}\right)^2-f\right)b\sqrt{-g}g\log(c)}{3\left(\left(\sqrt{-g}x-\sqrt{-gx^2+f}\right)^2-f\right)^3} - \frac{4\left(3\left(\sqrt{-g}x-\sqrt{-gx^2+f}\right)^2-f\right)a\sqrt{-g}g}{3\left(\left(\sqrt{-g}x-\sqrt{-gx^2+f}\right)^2-f\right)^3}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^4/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
-2/3*b*p*(2*(3*(sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)*sqrt(-g)*g*log(-d*g*
x^2 + d*f)/((sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)^3 + sqrt(-g)*g*log((sqr
t(-g)*x - sqrt(-g*x^2 + f))^2)/f^2 - sqrt(-g)*g*log(abs(g*x^2 - f))/f^2 +
2*sqrt(-g)*g/(((sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)*f)) - 4/3*(3*(sqrt(-
g)*x - sqrt(-g*x^2 + f))^2 - f)*b*sqrt(-g)*g*log(c)/((sqrt(-g)*x - sqrt(-g
*x^2 + f))^2 - f)^3 - 4/3*(3*(sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)*a*sqrt
(-g)*g/((sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d\,f-d\,g\,x^2\right)}^p\right)}{x^4\,\sqrt{f-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^4*(f - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^4*(f - g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.44

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^4\sqrt{f-gx^2}}\,dx$$

$$= \frac{-4\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)bgp\,x^3-\sqrt{-g\,x^2+f}\log\left(\frac{f^pd^p\left(-\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)^2+1\right)^{2p}c}{\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)^2+1\right)^{2p}}\right)bf-2\sqrt{-g\,x^2+f}\log\left(\frac{f^pd^p}{\cdots}\right.}{3f^2x^3}$$

input
```
int((a+b*log(c*(-d*g*x^2+d*f)^p))/x^4/(-g*x^2+f)^(1/2),x)
```

output
```
( - 4*sqrt(g)*asin((sqrt(g)*x)/sqrt(f))*b*g*p*x**3 - sqrt(f - g*x**2)*log(
(f**p*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p)*c)/(tan(asi
n((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b*f - 2*sqrt(f - g*x**2)*log((f*
*p*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p)*c)/(tan(asin((
sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b*g*x**2 - sqrt(f - g*x**2)*a*f - 2
*sqrt(f - g*x**2)*a*g*x**2 + 2*sqrt(f - g*x**2)*b*g*p*x**2)/(3*f**2*x**3)
```

3.713 $\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 217

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}\,dx = \frac{2bgp\sqrt{f-gx^2}}{15f^2x^3}+\frac{4bg^2p\sqrt{f-gx^2}}{5f^3x} - \frac{16bg^{5/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f-gx^2}}\right)}{15f^3} - \frac{\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{5fx^5} - \frac{4g\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{15f^2x^3} - \frac{8g^2\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{15f^3x}$$

output
```
2/15*b*g*p*(-g*x^2+f)^(1/2)/f^2/x^3+4/5*b*g^2*p*(-g*x^2+f)^(1/2)/f^3/x-16/
15*b*g^(5/2)*p*arctan(g^(1/2)*x/(-g*x^2+f)^(1/2))/f^3-1/5*(-g*x^2+f)^(1/2)
*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f/x^5-4/15*g*(-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*
g*x^2+d*f)^p))/f^2/x^3-8/15*g^2*(-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^
p))/f^3/x
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}\,dx = -\frac{16bg^{5/2}px^5\arctan\left(\frac{\sqrt{g}x}{\sqrt{f-gx^2}}\right)+\sqrt{f-gx^2}\left(-2bgpx^2(f+6gx^2)+a(3f^2+4fgx^2+8g^2x^4)+b(3f^2+4\right.}{15f^3x^5}$$

input
```
Integrate[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^6*Sqrt[f - g*x^2]),x]
```

output
```
-1/15*(16*b*g^(5/2)*p*x^5*ArcTan[(Sqrt[g]*x)/Sqrt[f - g*x^2]] + Sqrt[f - g
*x^2]*(-2*b*g*p*x^2*(f + 6*g*x^2) + a*(3*f^2 + 4*f*g*x^2 + 8*g^2*x^4) + b*
(3*f^2 + 4*f*g*x^2 + 8*g^2*x^4)*Log[c*(d*(f - g*x^2))^p]))/(f^3*x^5)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}\,dx$$

$$\Big\downarrow \text{ 2929}$$

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^6*Sqrt[f - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(-dg\,x^{2}+df\right)^{p}\right)}{x^{6}\sqrt{-g\,x^{2}+f}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^6/(-g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^6/(-g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.59

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}\,dx$$
$$= \left[\frac{8\,b\sqrt{-g}g^2px^5\log\left(2\,gx^2-2\,\sqrt{-gx^2+f}\sqrt{-g}x-f\right)+\left(4\,(3\,bg^2p-2\,ag^2)x^4-3\,af^2+2\,(bfgp-2\,afg\right.}{15}\right.$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^6/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/15*(8*b*sqrt(-g)*g^2*p*x^5*log(2*g*x^2 - 2*sqrt(-g*x^2 + f)*sqrt(-g)*x
- f) + (4*(3*b*g^2*p - 2*a*g^2)*x^4 - 3*a*f^2 + 2*(b*f*g*p - 2*a*f*g)*x^2
- (8*b*g^2*p*x^4 + 4*b*f*g*p*x^2 + 3*b*f^2*p)*log(-d*g*x^2 + d*f) - (8*b*g
^2*x^4 + 4*b*f*g*x^2 + 3*b*f^2)*log(c))*sqrt(-g*x^2 + f))/(f^3*x^5), 1/15*
(16*b*g^(5/2)*p*x^5*arctan(sqrt(-g*x^2 + f)*sqrt(g)*x/(g*x^2 - f)) + (4*(3
*b*g^2*p - 2*a*g^2)*x^4 - 3*a*f^2 + 2*(b*f*g*p - 2*a*f*g)*x^2 - (8*b*g^2*p
*x^4 + 4*b*f*g*p*x^2 + 3*b*f^2*p)*log(-d*g*x^2 + d*f) - (8*b*g^2*x^4 + 4*b
*f*g*x^2 + 3*b*f^2)*log(c))*sqrt(-g*x^2 + f))/(f^3*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+d*f)**p))/x**6/(-g*x**2+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}\,dx=\int \frac{b\log\left((-dgx^2+df)^pc\right)+a}{\sqrt{-gx^2+f}\,x^6}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^6/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/15*a*(8*sqrt(-g*x^2 + f)*g^2/(f^3*x) + 4*sqrt(-g*x^2 + f)*g/(f^2*x^3) +
 3*sqrt(-g*x^2 + f)/(f*x^5)) - 1/15*b*(integrate((16*g^3*p*x^6 + 8*f*g^2*p
*x^4 + 6*f^2*g*p*x^2 - 15*f^3*p*log(d) - 15*f^3*log(c))/(sqrt(-g*x^2 + f)*
x^6), x)/f^3 - (8*g^3*x^6 - 4*f*g^2*x^4 - f^2*g*x^2 - 3*f^3)*log((-g*x^2 +
 f)^p)/(sqrt(-g*x^2 + f)*f^3*x^5))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(189) = 378$.

Time = 1.03 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.14

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^6\sqrt{f - gx^2}}\,dx = -\frac{8}{15}bp\left(\frac{\sqrt{-g}g^2\log\left(\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2\right)}{f^3} - \frac{\sqrt{-g}g^2\log\left(|gx^2 - f|\right)}{f^3} - \frac{2\left(10\left(\sqrt{-g}x - \sqrt{-gx^2 + \right.\right.}{}\right.$$

$$+ \frac{16\left(10\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^4 - 5\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 f + f^2\right)b\sqrt{-g}g^2\log(c)}{15\left(\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 - f\right)^5}$$

$$+ \frac{16\left(10\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^4 - 5\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 f + f^2\right)a\sqrt{-g}g^2}{15\left(\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 - f\right)^5}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^6/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
-8/15*b*p*(sqrt(-g)*g^2*log((sqrt(-g)*x - sqrt(-g*x^2 + f))^2)/f^3 - sqrt(
-g)*g^2*log(abs(g*x^2 - f))/f^3 - 2*(10*(sqrt(-g)*x - sqrt(-g*x^2 + f))^4
 - 5*(sqrt(-g)*x - sqrt(-g*x^2 + f))^2*f + f^2)*sqrt(-g)*g^2*log(-d*g*x^2 +
 d*f)/((sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)^5 + (2*(sqrt(-g)*x - sqrt(-g
*x^2 + f))^4*sqrt(-g)*g^2 - 7*(sqrt(-g)*x - sqrt(-g*x^2 + f))^2*f*sqrt(-g)
*g^2 + 3*f^2*sqrt(-g)*g^2)/(((sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)^3*f^2)
) + 16/15*(10*(sqrt(-g)*x - sqrt(-g*x^2 + f))^4 - 5*(sqrt(-g)*x - sqrt(-g*
x^2 + f))^2*f + f^2)*b*sqrt(-g)*g^2*log(c)/((sqrt(-g)*x - sqrt(-g*x^2 + f)
)^2 - f)^5 + 16/15*(10*(sqrt(-g)*x - sqrt(-g*x^2 + f))^4 - 5*(sqrt(-g)*x -
 sqrt(-g*x^2 + f))^2*f + f^2)*a*sqrt(-g)*g^2/((sqrt(-g)*x - sqrt(-g*x^2 +
f))^2 - f)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}\,dx = \int \frac{a+b\,\ln\left(c\left(d\,f-d\,g\,x^2\right)^p\right)}{x^6\,\sqrt{f-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^6*(f - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^6*(f - g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.50

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^6\sqrt{f-gx^2}}\,dx = \frac{-16\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right) b\,g^2 p\,x^5 - 3\sqrt{-g\,x^2+f}\,\log\left(\frac{f^p d^p \left(-\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)^2+1\right)^{2p} c}{\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)^2+1\right)^{2p}}\right) b\,f^2 - 4\sqrt{-g\,x^2+f}\,\log}{}$$

input
```
int((a+b*log(c*(-d*g*x^2+d*f)^p))/x^6/(-g*x^2+f)^(1/2),x)
```

output
```
( - 16*sqrt(g)*asin((sqrt(g)*x)/sqrt(f))*b*g**2*p*x**5 - 3*sqrt(f - g*x**2
)*log((f**p*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p)*c)/(t
an(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b*f**2 - 4*sqrt(f - g*x**2
)*log((f**p*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p)*c)/(t
an(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b*f*g*x**2 - 8*sqrt(f - g*
x**2)*log((f**p*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p)*c
)/(tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b*g**2*x**4 - 3*sqrt(f
 - g*x**2)*a*f**2 - 4*sqrt(f - g*x**2)*a*f*g*x**2 - 8*sqrt(f - g*x**2)*a*g
**2*x**4 + 2*sqrt(f - g*x**2)*b*f*g*p*x**2 + 12*sqrt(f - g*x**2)*b*g**2*p*
x**4)/(15*f**3*x**5)
```

3.714 $\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^8\sqrt{f-gx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 289

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^8\sqrt{f-gx^2}}\,dx = \frac{2bgp\sqrt{f-gx^2}}{35f^2x^5}+\frac{4bg^2p\sqrt{f-gx^2}}{21f^3x^3}+\frac{88bg^3p\sqrt{f-gx^2}}{105f^4x}-\frac{32bg^{7/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f-gx^2}}\right)}{35f^4}-\frac{\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{7fx^7}-\frac{6g\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{35f^2x^5}-\frac{8g^2\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{35f^3x^3}-\frac{16g^3\sqrt{f-gx^2}\left(a+b\log\left(c(df-dgx^2)^p\right)\right)}{35f^4x}$$

output

```
2/35*b*g*p*(-g*x^2+f)^(1/2)/f^2/x^5+4/21*b*g^2*p*(-g*x^2+f)^(1/2)/f^3/x^3+
88/105*b*g^3*p*(-g*x^2+f)^(1/2)/f^4/x-32/35*b*g^(7/2)*p*arctan(g^(1/2)*x/(
-g*x^2+f)^(1/2))/f^4-1/7*(-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f/x
^7-6/35*g*(-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f^2/x^5-8/35*g^2*(
-g*x^2+f)^(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f^3/x^3-16/35*g^3*(-g*x^2+f)^
(1/2)*(a+b*ln(c*(-d*g*x^2+d*f)^p))/f^4/x
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.59

$$\int \frac{a+b\log \left(c\left(df-dgx^2\right)^p\right)}{x^8\sqrt{f-gx^2}}\,dx$$
$$=\frac{-96bg^{7/2}px^7\arctan\left(\frac{\sqrt{g}x}{\sqrt{f-gx^2}}\right)+\sqrt{f-gx^2}\left(2bgpx^2\left(3f^2+10fgx^2+44g^2x^4\right)-3a\left(5f^3+6f^2gx^2+8fg^2\right.\right.}{105f^4x^7}$$

input
```
Integrate[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^8*Sqrt[f - g*x^2]),x]
```

output
```
(-96*b*g^(7/2)*p*x^7*ArcTan[(Sqrt[g]*x)/Sqrt[f - g*x^2]] + Sqrt[f - g*x^2]
*(2*b*g*p*x^2*(3*f^2 + 10*f*g*x^2 + 44*g^2*x^4) - 3*a*(5*f^3 + 6*f^2*g*x^2
 + 8*f*g^2*x^4 + 16*g^3*x^6) - 3*b*(5*f^3 + 6*f^2*g*x^2 + 8*f*g^2*x^4 + 16
*g^3*x^6)*Log[c*(d*(f - g*x^2))^p]))/(105*f^4*x^7)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\log \left(c\left(df-dgx^2\right)^p\right)}{x^8\sqrt{f-gx^2}}\,dx$$
$$\Big\downarrow \ 2929$$
$$\int \frac{a+b\log \left(c\left(df-dgx^2\right)^p\right)}{x^8\sqrt{f-gx^2}}dx$$

input
```
Int[(a + b*Log[c*(d*f - d*g*x^2)^p])/(x^8*Sqrt[f - g*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2929
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*
(x_))^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Unintegrable[(h*x)
^m*(f + g*x^s)^r*(a + b*Log[c*(d + e*x^n)^p])^q, x] /; FreeQ[{a, b, c, d, e
, f, g, h, m, n, p, q, r, s}, x]
```

Maple [F]

$$\int \frac{a+b\ln \left(c(-dg\,x^{2}+df)^{p}\right)}{x^{8}\sqrt{-g\,x^{2}+f}}dx$$

input
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^8/(-g*x^2+f)^(1/2),x)
```

output
```
int((a+b*ln(c*(-d*g*x^2+d*f)^p))/x^8/(-g*x^2+f)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.54

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^8\sqrt{f-gx^2}}\,dx$$
$$=\left[\frac{48\,b\sqrt{-g}g^3px^7\log\left(2\,gx^2-2\,\sqrt{-gx^2+f}\sqrt{-g}x-f\right)+(8\,(11\,bg^3p-6\,ag^3)x^6+4\,(5\,bfg^2p-6\,afg^2)x^{}}{}\right.$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^8/(-g*x^2+f)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/105*(48*b*sqrt(-g)*g^3*p*x^7*log(2*g*x^2 - 2*sqrt(-g*x^2 + f)*sqrt(-g)*
x - f) + (8*(11*b*g^3*p - 6*a*g^3)*x^6 + 4*(5*b*f*g^2*p - 6*a*f*g^2)*x^4 -
 15*a*f^3 + 6*(b*f^2*g*p - 3*a*f^2*g)*x^2 - 3*(16*b*g^3*p*x^6 + 8*b*f*g^2*
p*x^4 + 6*b*f^2*g*p*x^2 + 5*b*f^3*p)*log(-d*g*x^2 + d*f) - 3*(16*b*g^3*x^6
 + 8*b*f*g^2*x^4 + 6*b*f^2*g*x^2 + 5*b*f^3)*log(c))*sqrt(-g*x^2 + f))/(f^4
*x^7), 1/105*(96*b*g^(7/2)*p*x^7*arctan(sqrt(-g*x^2 + f)*sqrt(g)*x/(g*x^2
- f)) + (8*(11*b*g^3*p - 6*a*g^3)*x^6 + 4*(5*b*f*g^2*p - 6*a*f*g^2)*x^4 -
15*a*f^3 + 6*(b*f^2*g*p - 3*a*f^2*g)*x^2 - 3*(16*b*g^3*p*x^6 + 8*b*f*g^2*p
*x^4 + 6*b*f^2*g*p*x^2 + 5*b*f^3*p)*log(-d*g*x^2 + d*f) - 3*(16*b*g^3*x^6
+ 8*b*f*g^2*x^4 + 6*b*f^2*g*x^2 + 5*b*f^3)*log(c))*sqrt(-g*x^2 + f))/(f^4*
x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^8\sqrt{f - gx^2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*ln(c*(-d*g*x**2+d*f)**p))/x**8/(-g*x**2+f)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^8\sqrt{f - gx^2}}\,dx = \int \frac{b\log\left((-dgx^2 + df)^p c\right) + a}{\sqrt{-gx^2 + f}x^8}\,dx$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^8/(-g*x^2+f)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/35*a*(16*sqrt(-g*x^2 + f)*g^3/(f^4*x) + 8*sqrt(-g*x^2 + f)*g^2/(f^3*x^3
) + 6*sqrt(-g*x^2 + f)*g/(f^2*x^5) + 5*sqrt(-g*x^2 + f)/(f*x^7)) - 1/35*b*
(integrate((32*g^4*p*x^8 + 16*f*g^3*p*x^6 + 12*f^2*g^2*p*x^4 + 10*f^3*g*p*
x^2 - 35*f^4*p*log(d) - 35*f^4*log(c))/(sqrt(-g*x^2 + f)*x^8), x)/f^4 - (1
6*g^4*x^8 - 8*f*g^3*x^6 - 2*f^2*g^2*x^4 - f^3*g*x^2 - 5*f^4)*log((-g*x^2 +
 f)^p)/(sqrt(-g*x^2 + f)*f^4*x^7))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(253) = 506$.

Time = 1.19 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.16

$$\int \frac{a + b\log\left(c(df - dgx^2)^p\right)}{x^8\sqrt{f - gx^2}}\,dx =$$

$$-\frac{16}{105}\,bp\left(\frac{3\sqrt{-g}g^3\log\left(\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2\right)}{f^4} - \frac{3\sqrt{-g}g^3\log\left(|gx^2 - f|\right)}{f^4} + \frac{6\left(35\left(\sqrt{-g}x - \sqrt{-gx\right.\right.}{}\right.$$

$$-\frac{32\left(35\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^6 - 21\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^4 f + 7\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 f^2 - f^3\right)b}{35\left(\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 - f\right)^7}$$

$$-\frac{32\left(35\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^6 - 21\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^4 f + 7\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 f^2 - f^3\right)a}{35\left(\left(\sqrt{-g}x - \sqrt{-gx^2 + f}\right)^2 - f\right)^7}$$

input
```
integrate((a+b*log(c*(-d*g*x^2+d*f)^p))/x^8/(-g*x^2+f)^(1/2),x, algorithm=
"giac")
```

output
```
-16/105*b*p*(3*sqrt(-g)*g^3*log((sqrt(-g)*x - sqrt(-g*x^2 + f))^2)/f^4 - 3
*sqrt(-g)*g^3*log(abs(g*x^2 - f))/f^4 + 6*(35*(sqrt(-g)*x - sqrt(-g*x^2 +
f))^6 - 21*(sqrt(-g)*x - sqrt(-g*x^2 + f))^4*f + 7*(sqrt(-g)*x - sqrt(-g*x
^2 + f))^2*f^2 - f^3)*sqrt(-g)*g^3*log(-d*g*x^2 + d*f)/((sqrt(-g)*x - sqrt
(-g*x^2 + f))^2 - f)^7 + (6*(sqrt(-g)*x - sqrt(-g*x^2 + f))^8*sqrt(-g)*g^3
 - 33*(sqrt(-g)*x - sqrt(-g*x^2 + f))^6*f*sqrt(-g)*g^3 + 77*(sqrt(-g)*x -
sqrt(-g*x^2 + f))^4*f^2*sqrt(-g)*g^3 - 49*(sqrt(-g)*x - sqrt(-g*x^2 + f))^
2*f^3*sqrt(-g)*g^3 + 11*f^4*sqrt(-g)*g^3)/(((sqrt(-g)*x - sqrt(-g*x^2 + f)
)^2 - f)^5*f^3)) - 32/35*(35*(sqrt(-g)*x - sqrt(-g*x^2 + f))^6 - 21*(sqrt(
-g)*x - sqrt(-g*x^2 + f))^4*f + 7*(sqrt(-g)*x - sqrt(-g*x^2 + f))^2*f^2 -
f^3)*b*sqrt(-g)*g^3*log(c)/((sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)^7 - 32/
35*(35*(sqrt(-g)*x - sqrt(-g*x^2 + f))^6 - 21*(sqrt(-g)*x - sqrt(-g*x^2 +
f))^4*f + 7*(sqrt(-g)*x - sqrt(-g*x^2 + f))^2*f^2 - f^3)*a*sqrt(-g)*g^3/((
sqrt(-g)*x - sqrt(-g*x^2 + f))^2 - f)^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^8\sqrt{f-gx^2}}\,dx = \int \frac{a+b\,\ln\left(c\,(d\,f-d\,g\,x^2)^p\right)}{x^8\,\sqrt{f-g\,x^2}}\,dx$$

input
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^8*(f - g*x^2)^(1/2)),x)
```

output
```
int((a + b*log(c*(d*f - d*g*x^2)^p))/(x^8*(f - g*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.53

$$\int \frac{a+b\log\left(c(df-dgx^2)^p\right)}{x^8\sqrt{f-gx^2}}\,dx = \frac{-96\sqrt{g}\,asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right) b\,g^3 p\,x^7 - 15\sqrt{-g\,x^2+f}\,\log\left(\frac{f^p d^p\left(-\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)^2+1\right)^{2p} c}{\left(\tan\left(\frac{asin\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{2}\right)^2+1\right)^{2p}}\right) b\,f^3 - 18\sqrt{-g\,x^2+f}\,\mathrm{lo}}{}$$

input
```
int((a+b*log(c*(-d*g*x^2+d*f)^p))/x^8/(-g*x^2+f)^(1/2),x)
```

output
```
( - 96*sqrt(g)*asin((sqrt(g)*x)/sqrt(f))*b*g**3*p*x**7 - 15*sqrt(f - g*x**
2)*log((f**p*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p)*c)/(
tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b*f**3 - 18*sqrt(f - g*x*
*2)*log((f**p*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p)*c)/
(tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b*f**2*g*x**2 - 24*sqrt(
f - g*x**2)*log((f**p*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(
2*p)*c)/(tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b*f*g**2*x**4 -
48*sqrt(f - g*x**2)*log((f**p*d**p*( - tan(asin((sqrt(g)*x)/sqrt(f))/2)**2
 + 1)**(2*p)*c)/(tan(asin((sqrt(g)*x)/sqrt(f))/2)**2 + 1)**(2*p))*b*g**3*x
**6 - 15*sqrt(f - g*x**2)*a*f**3 - 18*sqrt(f - g*x**2)*a*f**2*g*x**2 - 24*
sqrt(f - g*x**2)*a*f*g**2*x**4 - 48*sqrt(f - g*x**2)*a*g**3*x**6 + 6*sqrt(
f - g*x**2)*b*f**2*g*p*x**2 + 20*sqrt(f - g*x**2)*b*f*g**2*p*x**4 + 88*sqr
t(f - g*x**2)*b*g**3*p*x**6)/(105*f**4*x**7)
```

3.715 $\int \log \left(c(d+e(f+gx)^p)^q\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \log \left(c(d+e(f+gx)^p)^q\right)\,dx = -\frac{epq(f+gx)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{p},2+\frac{1}{p},-\frac{e(f+gx)^p}{d}\right)}{dg(1+p)} + \frac{(f+gx)\log\left(c(d+e(f+gx)^p)^q\right)}{g}$$

output
```
-e*p*q*(g*x+f)^(p+1)*hypergeom([1, 1+1/p],[2+1/p],-e*(g*x+f)^p/d)/d/g/(p+1
)+(g*x+f)*ln(c*(d+e*(g*x+f)^p)^q)/g
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \log \left(c(d+e(f+gx)^p)^q\right)\,dx = -pqx + \frac{pq(f+gx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{p},1+\frac{1}{p},-\frac{e(f+gx)^p}{d}\right)}{g} + \frac{(f+gx)\log\left(c(d+e(f+gx)^p)^q\right)}{g}$$

input
```
Integrate[Log[c*(d + e*(f + g*x)^p)^q],x]
```

output
```
-(p*q*x) + (p*q*(f + g*x)*Hypergeometric2F1[1, p^(-1), 1 + p^(-1), -((e*(f
 + g*x)^p)/d)])/g + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q])/g
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2933, 2898, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c(d+e(f+gx)^p)^q\right) \, dx$$

$$\Big\downarrow \text{ 2933}$$

$$\frac{\int \log \left(c(e(f+gx)^p+d)^q\right)d(f+gx)}{g}$$

$$\Big\downarrow \text{ 2898}$$

$$\frac{(f+gx)\log \left(c(d+e(f+gx)^p)^q\right)-epq\int \frac{(f+gx)^p}{e(f+gx)^p+d}d(f+gx)}{g}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(f+gx)\log \left(c(d+e(f+gx)^p)^q\right)-\frac{epq(f+gx)^{p+1}\text{Hypergeometric2F1}\left(1,1+\frac{1}{p},2+\frac{1}{p},-\frac{e(f+gx)^p}{d}\right)}{d(p+1)}}{g}$$

input
```
Int[Log[c*(d + e*(f + g*x)^p)^q],x]
```

output
```
(-((e*p*q*(f + g*x)^(1 + p)*Hypergeometric2F1[1, 1 + p^(-1), 2 + p^(-1), -
((e*(f + g*x)^p)/d)])/(d*(1 + p))) + (f + g*x)*Log[c*(d + e*(f + g*x)^p)^q
])/g
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 2933
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
 x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
 && (EqQ[q, 1] || IntegerQ[n])
```

Maple [F]

$$\int \ln \left(c(d+e(gx+f)^{p})^{q}\right) dx$$

input
```
int(ln(c*(d+e*(g*x+f)^p)^q),x)
```

output
```
int(ln(c*(d+e*(g*x+f)^p)^q),x)
```

Fricas [F]

$$\int \log \left(c(d+e(f+gx)^{p})^{q}\right) dx = \int \log \left(((gx+f)^{p}e+d)^{q}c\right) dx$$

input
```
integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="fricas")
```

output
```
integral(log(((g*x + f)^p*e + d)^q*c), x)
```

Sympy [F]

$$\int \log \left(c(d+e(f+gx)^p)^q\right)\,dx=\int \log \left(c(d+e(f+gx)^p)^q\right)dx$$

input
```
integrate(ln(c*(d+e*(g*x+f)**p)**q),x)
```

output
```
Integral(log(c*(d + e*(f + g*x)**p)**q), x)
```

Maxima [F]

$$\int \log \left(c(d+e(f+gx)^p)^q\right)\,dx=\int \log \left(((gx+f)^pe+d)^qc\right)\,dx$$

input
```
integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="maxima")
```

output
```
d*g*p*q*integrate(x/(d*g*x + (e*g*x + e*f)*(g*x + f)^p + d*f), x) + (f*p*q
*log(g*x + f) + g*x*log(((g*x + f)^p*e + d)^q) - (g*p*q - g*log(c))*x)/g
```

Giac [F]

$$\int \log \left(c(d+e(f+gx)^p)^q\right)\,dx=\int \log \left(((gx+f)^pe+d)^qc\right)\,dx$$

input
```
integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="giac")
```

output
```
integrate(log(((g*x + f)^p*e + d)^q*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(c(d+e(f+gx)^p)^q\right) \, dx = \int \ln \left(c \left(d+e \left(f+g \, x\right)^p\right)^q\right) \, dx$$

input
```
int(log(c*(d + e*(f + g*x)^p)^q),x)
```

output
```
int(log(c*(d + e*(f + g*x)^p)^q), x)
```

Reduce [F]

$$\int \log \left(c(d+e(f+gx)^p)^q\right) \, dx = \frac{\left(\int \frac{x}{(gx+f)^p ef+(gx+f)^p egx+df+dgx}dx\right) d\, g^2 pq + \log(gx+f)\, fpq + \log(((gx+f)^p\, e+d)^q\, c)\, gx - gpqx}{g}$$

input
```
int(log(c*(d+e*(g*x+f)^p)^q),x)
```

output
```
(int(x/((f + g*x)**p*e*f + (f + g*x)**p*e*g*x + d*f + d*g*x),x)*d*g**2*p*q
 + log(f + g*x)*f*p*q + log(((f + g*x)**p*e + d)**q*c)*g*x - g*p*q*x)/g
```

3.716 $\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 169

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx = -3qx-\frac{\sqrt{3}\sqrt[3]{d}q\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}(f+gx)}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}g}+\frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g}-\frac{\sqrt[3]{d}q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}g}+\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g}$$

output
```
-3*q*x-3^(1/2)*d^(1/3)*q*arctan(1/3*(d^(1/3)-2*e^(1/3)*(g*x+f))*3^(1/2)/d^
(1/3))/e^(1/3)/g+d^(1/3)*q*ln(d^(1/3)+e^(1/3)*(g*x+f))/e^(1/3)/g-1/2*d^(1/
3)*q*ln(d^(2/3)-d^(1/3)*e^(1/3)*(g*x+f)+e^(2/3)*(g*x+f)^2)/e^(1/3)/g+(g*x+
f)*ln(c*(d+e*(g*x+f)^3)^q)/g
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.87

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx=-3qx$$
$$+\frac{\sqrt[3]{d}q\left(2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{d}+2\sqrt[3]{e}(f+gx)}{\sqrt{3}\sqrt[3]{d}}\right)+2\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^2\right.}{2\sqrt[3]{e}g}$$
$$+\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g}$$

input
```
Integrate[Log[c*(d + e*(f + g*x)^3)^q],x]
```

output
```
-3*q*x + (d^(1/3)*q*(2*Sqrt[3]*ArcTan[(-d^(1/3) + 2*e^(1/3)*(f + g*x))/(Sq
rt[3]*d^(1/3))] + 2*Log[d^(1/3) + e^(1/3)*(f + g*x)] - Log[d^(2/3) - d^(1/
3)*e^(1/3)*(f + g*x) + e^(2/3)*(f + g*x)^2]))/(2*e^(1/3)*g) + ((f + g*x)*L
og[c*(d + e*(f + g*x)^3)^q])/g
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {2933, 2898, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx$$
$$\Big\downarrow \text{2933}$$
$$\frac{\int \log \left(c\left(e(f+gx)^3+d\right)^q\right)d(f+gx)}{g}$$
$$\Big\downarrow \text{2898}$$
$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\int \frac{(f+gx)^3}{e(f+gx)^3+d}d(f+gx)}{g}$$

$$\Big\downarrow \; 843$$

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\left(\frac{f+gx}{e}-\frac{d\int\frac{1}{e(f+gx)^3+d}d(f+gx)}{e}\right)}{g}$$

$$\Big\downarrow \; 750$$

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\left(\frac{f+gx}{e}-\frac{d\left(\frac{\int\frac{2\sqrt[3]{d}-\sqrt[3]{e}(f+gx)}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}}d(f+gx)}{3d^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{e}(f+gx)+\sqrt[3]{d}}d(f+gx)}{3d^{2/3}}\right)}{e}\right)}{g}$$

$$\Big\downarrow \; 16$$

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\left(\frac{f+gx}{e}-\frac{d\left(\frac{\int\frac{2\sqrt[3]{d}-\sqrt[3]{e}(f+gx)}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}}d(f+gx)}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{3d^{2/3}\sqrt[3]{e}}\right)}{e}\right)}{g}$$

$$\Big\downarrow \; 1142$$

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\left(\frac{f+gx}{e}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}}d(f+gx)-\int-\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{\cdots}\right.}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{\cdots}}}{3d^{2/3}}\right.}{e}\right.}{g}$$

$\downarrow$ 25

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\left(\frac{f+gx}{e}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}}d(f+gx)+\int\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{\cdots}\right.}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{\cdots}}}{3d^{2/3}}\right.}{e}\right.}{g}$$

$\downarrow$ 27

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\left(\frac{f+gx}{e}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{d}\int\frac{1}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}}d(f+gx)+\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{\cdots}}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\cdots}}{3d^{2/3}}\right.}{e}\right.}{g}$$

$\downarrow$ 1082

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\left(\frac{f+gx}{e}-\frac{d\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}(f+gx)}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}}d(f+gx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{e}(f+gx)}{\sqrt[3]{d}}\right)^2-3}}{\sqrt[3]{e}}}{3d^{2/3}}}{e}\right.}{g}$$

$\downarrow$ 217

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\left(\frac{f+gx}{e}-\frac{d\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{d}-2\sqrt[3]{e}(f+gx)}{e^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}}d(f+gx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}(f+gx)}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3d^{2/3}}}{e}\right.}{g}$$

$\downarrow$ 1103

$$\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)-3eq\left(\frac{f+gx}{e}-\frac{d\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}(f+gx)}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}}}{3d^{2/3}}+\right.}{e}\right.}{g}$$

input
```
Int[Log[c*(d + e*(f + g*x)^3)^q],x]
```

output
```
(-3*e*q*((f + g*x)/e - (d*(Log[d^(1/3) + e^(1/3)*(f + g*x)]/(3*d^(2/3)*e^(
1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*e^(1/3)*(f + g*x))/d^(1/3))/Sqrt[3]])/e
^(1/3)) - Log[d^(2/3) - d^(1/3)*e^(1/3)*(f + g*x) + e^(2/3)*(f + g*x)^2]/(
2*e^(1/3)))/(3*d^(2/3))))/e) + (f + g*x)*Log[c*(d + e*(f + g*x)^3)^q])/g
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 2933
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
 x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
 && (EqQ[q, 1] || IntegerQ[n])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

method	result
parts	$\ln\left(c\left(d+e(gx+f)^{3}\right)^{q}\right)x-3geq\left(\frac{x}{ge}+\frac{\sum_{\_R=\mathrm{RootOf}\left(g^{3}e\_Z^{3}+3ef\,g^{2}\_Z^{2}+3e\,f^{2}g\_Z+e\,f^{3}+d\right)}\frac{\left(-\_R^{2}ef\,g^{2}-2\_R\right.}{g^{2}\_R^{2}}}{3e^{2}g^{2}}\right.$
default	$x\ln\left(c(e\,g^{3}x^{3}+3ef\,g^{2}x^{2}+3e\,f^{2}gx+e\,f^{3}+d)^{q}\right)-3geq\left(\frac{x}{ge}+\frac{\sum_{\_R=\mathrm{RootOf}\left(g^{3}e\_Z^{3}+3ef\,g^{2}\_Z^{2}+3e\,f^{2}g\_\right.}}{}\right.$
risch	$x\ln\left(\left(d+e(gx+f)^{3}\right)^{q}\right)+\frac{i c \mathrm{csgn}\left(ic\left(d+e(gx+f)^{3}\right)^{q}\right)^{2}\mathrm{csgn}\left(i\left(d+e(gx+f)^{3}\right)^{q}\right)\pi x}{2}-\frac{i\pi x\ \mathrm{csgn}\left(i\left(d+e(gx+f)^{3}\right)^{q}\right)\mathrm{c}}{}$

input
```
int(ln(c*(d+e*(g*x+f)^3)^q),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(d+e*(g*x+f)^3)^q)*x-3*g*e*q*(1/g/e*x+1/3/e^2/g^2*sum((-_R^2*e*f*g^2-
2*_R*e*f^2*g-e*f^3-d)/(_R^2*g^2+2*_R*f*g+f^2)*ln(x-_R),_R=RootOf(_Z^3*e*g^
3+3*_Z^2*e*f*g^2+3*_Z*e*f^2*g+e*f^3+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 1134, normalized size of antiderivative = 6.71

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right) \, dx = \text{Too large to display}$$

input
```
integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="fricas")
```

output
```
1/4*(4*g*q*x*log(e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d) - 12
*g*q*x - 2*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3
)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g*log(q*x - 1/2*(-1/2*f^3*q^3/
g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3
) + 1) + f*q/g) + 4*g*x*log(c) + (((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) +
 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 6*f
*q + sqrt(3)*g*sqrt(-(((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*
q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^
3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*
sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2))*log(2*g*q*x + 1/2*((-1/2*
f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(
I*sqrt(3) + 1) - 2*f*q/g)*g + 3*f*q + 1/2*sqrt(3)*g*sqrt(-(((-1/2*f^3*q^3/
g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3
) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(
e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2
*q^2)/g^2)) + (((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d
*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 6*f*q - sqrt(3)*g*sqrt
(-(((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3
))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d*q
^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2...
```

Sympy [F(-1)]

Timed out.

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right) dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e*(g*x+f)**3)**q),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right) dx = \int \log \left(\left((gx+f)^3e+d\right)^q c\right) dx$$

input
```
integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="maxima")
```

output
```
-(3*q - log(c))*x + 3*q*integrate((e*f*g^2*x^2 + 2*e*f^2*g*x + e*f^3 + d)/
(e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d), x) + x*log((e*g^3*x
^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d)^q)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.54

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right) dx = qx\log \left(eg^3x^3+3\,efg^2x^2+3\,ef^2gx+ef^3+d\right) - (3\,q-\log(c))x + \frac{fq\log\left(|eg^3x^3+3\,efg^2x^2+3\,ef^2gx+ef^3+d|\right)}{g} + \frac{2\,\sqrt{3}(de^2g^6q^3)^{\frac{1}{3}}\arctan\left(-\frac{egx+ef+(de^2)^{\frac{1}{3}}}{\sqrt{3}egx+\sqrt{3}ef-\sqrt{3}(de^2)^{\frac{1}{3}}}\right) - (de^2g^6q^3)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}egx+\sqrt{3}ef-\sqrt{3}(de^2)^{\frac{1}{3}}\right)^2\right.}{2\,eg^3}$$

input
```
integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="giac")
```

output
```
q*x*log(e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d) - (3*q - log(
c))*x + f*q*log(abs(e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d))/
g + 1/2*(2*sqrt(3)*(d*e^2*g^6*q^3)^(1/3)*arctan(-(e*g*x + e*f + (d*e^2)^(1
/3))/(sqrt(3)*e*g*x + sqrt(3)*e*f - sqrt(3)*(d*e^2)^(1/3))) - (d*e^2*g^6*q
^3)^(1/3)*log(4*(sqrt(3)*e*g*x + sqrt(3)*e*f - sqrt(3)*(d*e^2)^(1/3))^2 +
4*(e*g*x + e*f + (d*e^2)^(1/3))^2) + 2*(d*e^2*g^6*q^3)^(1/3)*log(abs(e*g*x
 + e*f + (d*e^2)^(1/3))))/(e*g^3)
```

Mupad [B] (verification not implemented)

Time = 25.87 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx = x\,\ln\left(c\,\left(d+e\,{\left(f+g\,x\right)}^3\right)^q\right) - \left(\sum_{k=1}^{3}\ln\left(d\,e^2\,g^5\,\left(\mathrm{root}\left(b^3\,e\,g^3+3\,b^2\,e\,f\,g^2\,q+3\,b\,e\,f^2\,g\,q^2+e\,f^3\,q^3+d\,q^3,b,k\right)\,g+f\,q\right)\,\left(\mathrm{root}\left(b^3\,e\,g^3+3\,b^2 \right.\right.\right.$$
$$\left.\left.+3\,b^2\,e\,f\,g^2\,q+3\,b\,e\,f^2\,g\,q^2+e\,f^3\,q^3+d\,q^3,b,k\right)\right) - 3\,q\,x$$

input
```
int(log(c*(d + e*(f + g*x)^3)^q),x)
```

output
```
x*log(c*(d + e*(f + g*x)^3)^q) - symsum(log(9*d*e^2*g^5*(root(b^3*e*g^3 +
3*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^3, b, k)*g + f*q)*(roo
t(b^3*e*g^3 + 3*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^3, b, k)
 - q*x))*root(b^3*e*g^3 + 3*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 +
d*q^3, b, k), k, 1, 3) - 3*q*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.21

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx$$
$$= \frac{-2d^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}f-2e^{\frac{1}{3}}gx}{d^{\frac{1}{3}}\sqrt{3}}\right)q+3d^{\frac{1}{3}}\log\left(d^{\frac{1}{3}}+e^{\frac{1}{3}}f+e^{\frac{1}{3}}gx\right)q-d^{\frac{1}{3}}\log\left(\left(e\,g^3x^3+3ef\,g^2x^2+3e\,f^2gx\right.\right.}{}$$

input

```
int(log(c*(d+e*(g*x+f)^3)^q),x)
```

output

```
( - 2*d**(1/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*f - 2*e**(1/3)*g*x)/(d*
*(1/3)*sqrt(3)))*q + 3*d**(1/3)*log(d**(1/3) + e**(1/3)*f + e**(1/3)*g*x)*
q - d**(1/3)*log((d + e*f**3 + 3*e*f**2*g*x + 3*e*f*g**2*x**2 + e*g**3*x**
3)**q*c) + 2*e**(1/3)*log((d + e*f**3 + 3*e*f**2*g*x + 3*e*f*g**2*x**2 + e
*g**3*x**3)**q*c)*f + 2*e**(1/3)*log((d + e*f**3 + 3*e*f**2*g*x + 3*e*f*g*
*2*x**2 + e*g**3*x**3)**q*c)*g*x - 6*e**(1/3)*g*q*x)/(2*e**(1/3)*g)
```

3.717 $\int \log \left(c\left(d+e(f+gx)^2\right)^q\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right) \, dx = -2qx+\frac{2\sqrt{d}q\arctan\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g}+\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}$$

output
```
-2*q*x+2*d^(1/2)*q*arctan(e^(1/2)*(g*x+f)/d^(1/2))/e^(1/2)/g+(g*x+f)*ln(c*
(d+e*(g*x+f)^2)^q)/g
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right) \, dx = -2qx+\frac{2\sqrt{d}q\arctan\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g}+\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}$$

input
```
Integrate[Log[c*(d + e*(f + g*x)^2)^q],x]
```

output
```
-2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + (
(f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2933, 2898, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right) \, dx$$

$$\Big\downarrow \text{2933}$$

$$\frac{\int \log \left(c\left(e(f+gx)^2+d\right)^q\right)d(f+gx)}{g}$$

$$\Big\downarrow \text{2898}$$

$$\frac{(f+gx)\log \left(c\left(d+e(f+gx)^2\right)^q\right)-2eq\int \frac{(f+gx)^2}{e(f+gx)^2+d}d(f+gx)}{g}$$

$$\Big\downarrow \text{262}$$

$$\frac{(f+gx)\log \left(c\left(d+e(f+gx)^2\right)^q\right)-2eq\left(\frac{f+gx}{e}-\frac{d\int \frac{1}{e(f+gx)^2+d}d(f+gx)}{e}\right)}{g}$$

$$\Big\downarrow \text{218}$$

$$\frac{(f+gx)\log \left(c\left(d+e(f+gx)^2\right)^q\right)-2eq\left(\frac{f+gx}{e}-\frac{\sqrt{d}\arctan \left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{e^{3/2}}\right)}{g}$$

input
```
Int[Log[c*(d + e*(f + g*x)^2)^q],x]
```

output
```
(-2*e*q*((f + g*x)/e - (Sqrt[d]*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/e^(3/
2)) + (f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 2933
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
 x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
 && (EqQ[q, 1] || IntegerQ[n])
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

method	result
parts	$\ln\left(c\left(d+(gx+f)^2 e\right)^q\right)x-2qeg\left(\frac{x}{ge}+\frac{-\frac{f\ln\left(e\,g^2x^2+2efgx+f^2e+d\right)}{2g}-\frac{d\arctan\left(\frac{2e\,g^2x+2efg}{2g\sqrt{de}}\right)}{g\sqrt{de}}}{eg}\right)$
default	$x\ln\left(c(e\,g^2x^2+2efgx+f^2e+d)^q\right)-2qeg\left(\frac{x}{ge}+\frac{-\frac{f\ln\left(e\,g^2x^2+2efgx+f^2e+d\right)}{2g}-\frac{d\arctan\left(\frac{2e\,g^2x+2efg}{2g\sqrt{de}}\right)}{g\sqrt{de}}}{eg}\right)$
risch	$x\ln\left(\left(d+(gx+f)^2e\right)^q\right)+\frac{i\mathrm{csgn}\left(ic\left(d+(gx+f)^2e\right)^q\right)^2\mathrm{csgn}\left(i\left(d+(gx+f)^2e\right)^q\right)\pi x}{2}-\frac{i\pi x\ \mathrm{csgn}\left(i\left(d+(gx+f)^2e\right)^q\right)\ \mathrm{c}}{}$

input
```
int(ln(c*(d+(g*x+f)^2*e)^q),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(d+(g*x+f)^2*e)^q)*x-2*q*e*g*(1/g/e*x+1/e/g*(-1/2*f/g*ln(e*g^2*x^2+2*
e*f*g*x+e*f^2+d)-d/g/(d*e)^(1/2)*arctan(1/2*(2*e*g^2*x+2*e*f*g)/g/(d*e)^(1
/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.27

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx$$

$$= \left[-\frac{2\,gqx - gx\log\left(c\right) - q\sqrt{-\frac{d}{e}}\log\left(\frac{eg^2x^2+2\,efgx+ef^2+2\,(egx+ef)\sqrt{-\frac{d}{e}}-d}{eg^2x^2+2\,efgx+ef^2+d}\right) - (gqx+fq)\log\left(eg^2x^2+2\,efgx\right.}{g}\right.$$

$$\left. -\frac{2\,gqx - gx\log\left(c\right) - 2\,q\sqrt{\frac{d}{e}}\arctan\left(\frac{(egx+ef)\sqrt{\frac{d}{e}}}{d}\right) - (gqx+fq)\log\left(eg^2x^2+2\,efgx+ef^2+d\right)}{g}\right]$$

input
```
integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="fricas")
```

output
```
[-(2*g*q*x - g*x*log(c) - q*sqrt(-d/e)*log((e*g^2*x^2 + 2*e*f*g*x + e*f^2
+ 2*(e*g*x + e*f)*sqrt(-d/e) - d)/(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d)) - (
g*q*x + f*q)*log(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d))/g, -(2*g*q*x - g*x*lo
g(c) - 2*q*sqrt(d/e)*arctan((e*g*x + e*f)*sqrt(d/e)/d) - (g*q*x + f*q)*log
(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d))/g]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(58) = 116$.

Time = 119.94 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.73

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right) dx = \begin{cases} x \log \left(0^q c\right) \\ x \log \left(cd^q\right) \\ x \log \left(c\left(d+ef^2\right)^q\right) \\ \frac{f \log \left(c\left(ef^2+2efgx+eg^2x^2\right)^q\right)}{g} - 2qx + x \log \left(c\left(ef^2+2efgx+eg^2x^2\right)^q\right) \\ \frac{2dq \log \left(\frac{f}{g}+x-\frac{\sqrt{-de}}{eg}\right)}{g\sqrt{-de}} - \frac{d \log \left(c\left(d+ef^2+2efgx+eg^2x^2\right)^q\right)}{g\sqrt{-de}} + \frac{f \log \left(c\left(d+ef^2+2efgx+eg^2x^2\right)^q\right)}{g} - 2qx + x \log \left(c\left(d+ef^2+\right.\right. \end{cases}$$

input
```
integrate(ln(c*(d+e*(g*x+f)**2)**q),x)
```

output
```
Piecewise((x*log(0**q*c), Eq(d, 0) & Eq(e, 0) & Eq(g, 0)), (x*log(c*d**q),
 Eq(e, 0)), (x*log(c*(d + e*f**2)**q), Eq(g, 0)), (f*log(c*(e*f**2 + 2*e*f
*g*x + e*g**2*x**2)**q)/g - 2*q*x + x*log(c*(e*f**2 + 2*e*f*g*x + e*g**2*x
**2)**q), Eq(d, 0)), (2*d*q*log(f/g + x - sqrt(-d*e)/(e*g))/(g*sqrt(-d*e))
 - d*log(c*(d + e*f**2 + 2*e*f*g*x + e*g**2*x**2)**q)/(g*sqrt(-d*e)) + f*l
og(c*(d + e*f**2 + 2*e*f*g*x + e*g**2*x**2)**q)/g - 2*q*x + x*log(c*(d + e
*f**2 + 2*e*f*g*x + e*g**2*x**2)**q), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right) dx = \text{Exception raised: ValueError}$$

input
```
integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx = qx\log \left(eg^2x^2+2\,efgx+ef^2+d\right)-(2\,q-\log \left(c\right))x+\frac{fq\log \left(eg^2x^2+2\,efgx+ef^2+d\right)}{g}+\frac{2\,dq\arctan \left(\frac{egx+ef}{\sqrt{de}}\right)}{\sqrt{de}g}$$

input
```
integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="giac")
```

output
```
q*x*log(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d) - (2*q - log(c))*x + f*q*log(e*
g^2*x^2 + 2*e*f*g*x + e*f^2 + d)/g + 2*d*q*arctan((e*g*x + e*f)/sqrt(d*e))
/(sqrt(d*e)*g)
```

Mupad [B] (verification not implemented)

Time = 25.67 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx = x\,\ln \left(c\,{\left(d+e\,{\left(f+g\,x\right)}^2\right)}^q\right)-2\,q\,x+\frac{f\,q\,\ln \left(e\,f^2+2\,e\,f\,g\,x+e\,g^2\,x^2+d\right)}{g}+\frac{2\,\sqrt{d}\,q\,\mathrm{atan}\left(\frac{\sqrt{e}\,f}{\sqrt{d}}+\frac{\sqrt{e}\,g\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g}$$

input
```
int(log(c*(d + e*(f + g*x)^2)^q),x)
```

output
```
x*log(c*(d + e*(f + g*x)^2)^q) - 2*q*x + (f*q*log(d + e*f^2 + e*g^2*x^2 +
2*e*f*g*x))/g + (2*d^(1/2)*q*atan((e^(1/2)*f)/d^(1/2) + (e^(1/2)*g*x)/d^(1
/2)))/(e^(1/2)*g)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right) \, dx$$
$$= \frac{2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{egx+ef}{\sqrt{e}\,\sqrt{d}}\right) q+\log\left(\left(e\,g^2x^2+2efgx+e\,f^2+d\right)^q c\right) ef+\log\left(\left(e\,g^2x^2+2efgx+e\,f^2+d\right)^q c\right)}{eg}$$

input
```
int(log(c*(d+e*(g*x+f)^2)^q),x)
```

output
```
(2*sqrt(e)*sqrt(d)*atan((e*f + e*g*x)/(sqrt(e)*sqrt(d)))*q + log((d + e*f*
*2 + 2*e*f*g*x + e*g**2*x**2)**q*c)*e*f + log((d + e*f**2 + 2*e*f*g*x + e*
g**2*x**2)**q*c)*e*g*x - 2*e*g*q*x)/(e*g)
```

3.718 $\int \log \left(c(d+e(f+gx))^q\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \log \left(c(d+e(f+gx))^q\right) \, dx = -qx+\frac{(d+ef+egx)\log \left(c(d+ef+egx)^q\right)}{eg}$$

output
```
-q*x+(e*g*x+e*f+d)*ln(c*(e*g*x+e*f+d)^q)/e/g
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \log \left(c(d+e(f+gx))^q\right) \, dx = -qx+\frac{dq\log(d+ef+egx)}{eg}+\frac{(f+gx)\log \left(c(d+e(f+gx))^q\right)}{g}$$

input
```
Integrate[Log[c*(d + e*(f + g*x))^q],x]
```

output
```
-(q*x) + (d*q*Log[d + e*f + e*g*x])/(e*g) + ((f + g*x)*Log[c*(d + e*(f + g
*x))^q])/g
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2894, 2836, 2732}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c(d+e(f+gx))^q\right) \, dx$$

$$\Big\downarrow \text{ 2894}$$

$$\int \log \left(c(d+ef+egx)^q\right)dx$$

$$\Big\downarrow \text{ 2836}$$

$$\frac{\int \log \left(c(d+ef+egx)^q\right)d(d+ef+egx)}{eg}$$

$$\Big\downarrow \text{ 2732}$$

$$\frac{(d+ef+egx)\log \left(c(d+ef+egx)^q\right)-q(d+ef+egx)}{eg}$$

input
```
Int[Log[c*(d + e*(f + g*x))^q],x]
```

output
```
(-(q*(d + e*f + e*g*x)) + (d + e*f + e*g*x)*Log[c*(d + e*f + e*g*x)^q])/(e
*g)
```

Defintions of rubi rules used

rule 2732
```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

rule 2836
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Simp[1/e   Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{
a, b, c, d, e, n, p}, x]
```

rule 2894
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

method	result
norman	$x \ln\left(c\,e^{q \ln(d+(gx+f)e)}\right)+\frac{q(ef+d)\ln(d+(gx+f)e)}{eg}-qx$
default	$x \ln\left(c(egx+ef+d)^{q}\right)-qeg\left(\frac{x}{ge}+\frac{(-ef-d)\ln(egx+ef+d)}{e^{2}g^{2}}\right)$
parts	$\ln\left(c(d+(gx+f)\,e)^{q}\right)x-qeg\left(\frac{x}{ge}+\frac{(-ef-d)\ln(egx+ef+d)}{e^{2}g^{2}}\right)$
parallelrisch	$\frac{2\ln(egx+ef+d)efq+x\ln(c(egx+ef+d)^{q})eg-geqx+2\ln(egx+ef+d)dq-\ln(c(egx+ef+d)^{q})ef+efq-d\ln(c(egx+ef+d)^{q})+}{eg}$
risch	$x\ln\left((egx+ef+d)^{q}\right)+\frac{i\pi x\ \mathrm{csgn}(i(egx+ef+d)^{q})\ \mathrm{csgn}(ic(egx+ef+d)^{q})^{2}}{2}-\frac{i\pi x\ \mathrm{csgn}(i(egx+ef+d)^{q})\ \mathrm{csgn}(ic(eg}{2}$

input
```
int(ln(c*(d+(g*x+f)*e)^q),x,method=_RETURNVERBOSE)
```

output
```
x*ln(c*exp(q*ln(d+(g*x+f)*e)))+q*(e*f+d)/e/g*ln(d+(g*x+f)*e)-q*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \log\left(c(d+e(f+gx))^{q}\right)\,dx = -\frac{egqx-egx\log(c)-(egqx+(ef+d)q)\log(egx+ef+d)}{eg}$$

input
```
integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="fricas")
```

output
```
-(e*g*q*x - e*g*x*log(c) - (e*g*q*x + (e*f + d)*q)*log(e*g*x + e*f + d))/(
e*g)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.29

$$\int \log\left(c(d+e(f+gx))^q\right)\,dx = \begin{cases} x\log\left(cd^q\right) & \text{for } e=0\wedge(e=0\vee g=0)\\ x\log\left(c(d+ef)^q\right) & \text{for } g=0\\ \frac{d\log\left(c(d+ef+egx)^q\right)}{eg}+\frac{f\log\left(c(d+ef+egx)^q\right)}{g}-qx+x\log\left(c(d+ef+egx)^q\right) & \text{otherwise}\end{cases}$$

input
```
integrate(ln(c*(d+e*(g*x+f))**q),x)
```

output
```
Piecewise((x*log(c*d**q), Eq(e, 0) & (Eq(e, 0) | Eq(g, 0))), (x*log(c*(d +
 e*f)**q), Eq(g, 0)), (d*log(c*(d + e*f + e*g*x)**q)/(e*g) + f*log(c*(d +
e*f + e*g*x)**q)/g - q*x + x*log(c*(d + e*f + e*g*x)**q), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \log\left(c(d+e(f+gx))^q\right)\,dx = -egq\left(\frac{x}{eg}-\frac{(ef+d)\log(egx+ef+d)}{e^2g^2}\right)+x\log\left(((gx+f)e+d)^q c\right)$$

input
```
integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="maxima")
```

output
```
-e*g*q*(x/(e*g) - (e*f + d)*log(e*g*x + e*f + d)/(e^2*g^2)) + x*log(((g*x
+ f)*e + d)^q*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \log \left(c(d+e(f+gx))^q\right) \, dx = \frac{(egx+ef+d)q \log (egx+ef+d)}{eg} - \frac{(egx+ef+d)q}{eg} + \frac{(egx+ef+d) \log (c)}{eg}$$

input
```
integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="giac")
```

output
```
(e*g*x + e*f + d)*q*log(e*g*x + e*f + d)/(e*g) - (e*g*x + e*f + d)*q/(e*g)
 + (e*g*x + e*f + d)*log(c)/(e*g)
```

Mupad [B] (verification not implemented)

Time = 25.70 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \log \left(c(d+e(f+gx))^q\right) \, dx = x \ln \left(c \left(d+e \left(f+g x\right)\right)^q\right) - q x + \frac{\ln \left(d+e f+e g x\right) \left(d q+e f q\right)}{e g}$$

input
```
int(log(c*(d + e*(f + g*x))^q),x)
```

output
```
x*log(c*(d + e*(f + g*x))^q) - q*x + (log(d + e*f + e*g*x)*(d*q + e*f*q))/
(e*g)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int \log \left(c(d+e(f+gx))^q\right) \, dx$$
$$= \frac{\log((egx+ef+d)^q c)\, d+\log((egx+ef+d)^q c)\, ef+\log((egx+ef+d)^q c)\, egx-egqx}{eg}$$

input
```
int(log(c*(d+e*(g*x+f))^q),x)
```

output
```
(log((d + e*f + e*g*x)**q*c)*d + log((d + e*f + e*g*x)**q*c)*e*f + log((d
+ e*f + e*g*x)**q*c)*e*g*x - e*g*q*x)/(e*g)
```

3.719 $\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)\,dx=\frac{(f+gx)\log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq\log(e+d(f+gx))}{dg}$$

output
```
(g*x+f)*ln(c*(d+e/(g*x+f))^q)/g+e*q*ln(e+d*(g*x+f))/d/g
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)\,dx =\frac{-dfq\log(f+gx)+(e+df)q\log(e+df+dgx)+dgx\log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{dg}$$

input
```
Integrate[Log[c*(d + e/(f + g*x))^q],x]
```

output
```
(-(d*f*q*Log[f + g*x]) + (e + d*f)*q*Log[e + d*f + d*g*x] + d*g*x*Log[c*(d
 + e/(f + g*x))^q])/(d*g)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2933, 2898, 795, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right) \, dx$$

$$\Big\downarrow \text{2933}$$

$$\frac{\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)d(f+gx)}{g}$$

$$\Big\downarrow \text{2898}$$

$$\frac{eq\int \frac{1}{(f+gx)\left(d+\frac{e}{f+gx}\right)}d(f+gx)+(f+gx)\log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}$$

$$\Big\downarrow \text{795}$$

$$\frac{eq\int \frac{1}{e+d(f+gx)}d(f+gx)+(f+gx)\log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}$$

$$\Big\downarrow \text{16}$$

$$\frac{(f+gx)\log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)+\frac{eq\log (d(f+gx)+e)}{d}}{g}$$

input
```
Int[Log[c*(d + e/(f + g*x))^q],x]
```

output
```
((f + g*x)*Log[c*(d + e/(f + g*x))^q] + (e*q*Log[e + d*(f + g*x)])/d)/g
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 2933
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
 x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
 && (EqQ[q, 1] || IntegerQ[n])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

method	result	size
parts	$\ln\left(c\left(d+\frac{e}{gx+f}\right)^{q}\right)x+qeg\left(-\frac{f\ln(gx+f)}{g^{2}e}+\frac{(df+e)\ln(xdg+df+e)}{e\,g^{2}d}\right)$	65
default	$x\ln\left(c\left(\frac{xdg+df+e}{gx+f}\right)^{q}\right)+qeg\left(-\frac{f\ln(gx+f)}{g^{2}e}+\frac{(df+e)\ln(xdg+df+e)}{e\,g^{2}d}\right)$	71
parallelrisch	$-\frac{-x\ln\left(c\left(\frac{xdg+df+e}{gx+f}\right)^{q}\right)d\,g^{2}q-\ln(gx+f)eg\,q^{2}-\ln\left(c\left(\frac{xdg+df+e}{gx+f}\right)^{q}\right)dfgq-\ln\left(c\left(\frac{xdg+df+e}{gx+f}\right)^{q}\right)egq}{d\,g^{2}q}$	111

input
```
int(ln(c*(d+e/(g*x+f))^q),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(d+e/(g*x+f))^q)*x+q*e*g*(-f/g^2/e*ln(g*x+f)+(d*f+e)/e/g^2/d*ln(d*g*x
+d*f+e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right) dx = \frac{dgqx\log\left(\frac{dgx+df+e}{gx+f}\right) - dfq\log(gx+f) + dgx\log(c) + (df+e)q\log(dgx+df+e)}{dg}$$

input
```
integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="fricas")
```

output
```
(d*g*q*x*log((d*g*x + d*f + e)/(g*x + f)) - d*f*q*log(g*x + f) + d*g*x*log
(c) + (d*f + e)*q*log(d*g*x + d*f + e))/(d*g)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(36) = 72.

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right) dx = \begin{cases} x\log\left(c\left(\frac{e}{f}\right)^q\right) & \text{for } d=0 \wedge g=0 \\ \frac{f\log\left(c\left(\frac{e}{f+gx}\right)^q\right)}{g} + qx + x\log\left(c\left(\frac{e}{f+gx}\right)^q\right) & \text{for } d=0 \\ x\log\left(c\left(d+\frac{e}{f}\right)^q\right) & \text{for } g=0 \\ \frac{f\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g} + x\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right) + \frac{eq\log(df+dgx+e)}{dg} & \text{otherwise} \end{cases}$$

input
```
integrate(ln(c*(d+e/(g*x+f))**q),x)
```

output
```
Piecewise((x*log(c*(e/f)**q), Eq(d, 0) & Eq(g, 0)), (f*log(c*(e/(f + g*x))
**q)/g + q*x + x*log(c*(e/(f + g*x))**q), Eq(d, 0)), (x*log(c*(d + e/f)**q
), Eq(g, 0)), (f*log(c*(d + e/(f + g*x))**q)/g + x*log(c*(d + e/(f + g*x))
**q) + e*q*log(d*f + d*g*x + e)/(d*g), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)\,dx = -egq\left(\frac{f\log\left(gx+f\right)}{eg^2}-\frac{(df+e)\log\left(dgx+df+e\right)}{deg^2}\right) + x\log\left(c\left(d+\frac{e}{gx+f}\right)^q\right)$$

input
```
integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="maxima")
```

output
```
-e*g*q*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*g^2))
 + x*log(c*(d + e/(g*x + f))^q)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(45) = 90$.

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.80

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right)\,dx = \left(\frac{e^2q\log\left(\frac{dgx+df+e}{gx+f}\right)}{dg^2-\frac{(dgx+df+e)g^2}{gx+f}}+\frac{e^2\log\left(c\right)}{dg^2-\frac{(dgx+df+e)g^2}{gx+f}}+\frac{e^2q\log\left(-d+\frac{dgx+df+e}{gx+f}\right)}{dg^2}-\frac{e^2q\log\left(\frac{dgx+df+e}{gx+f}\right)}{dg^2}\right)\left(\frac{dfg}{e^2}-(d\right.$$

input
```
integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="giac")
```

output
```
(e^2*q*log((d*g*x + d*f + e)/(g*x + f))/(d*g^2 - (d*g*x + d*f + e)*g^2/(g*
x + f)) + e^2*log(c)/(d*g^2 - (d*g*x + d*f + e)*g^2/(g*x + f)) + e^2*q*log
(-d + (d*g*x + d*f + e)/(g*x + f))/(d*g^2) - e^2*q*log((d*g*x + d*f + e)/(
g*x + f))/(d*g^2))*(d*f*g/e^2 - (d*f + e)*g/e^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right) dx = x \ln \left(c\left(d+\frac{e}{f+g\,x}\right)^q\right) - \frac{f\,q\,\ln(f+g\,x)}{g} + \frac{f\,q\,\ln(e+d\,f+d\,g\,x)}{g} + \frac{e\,q\,\ln(e+d\,f+d\,g\,x)}{d\,g}$$

input
```
int(log(c*(d + e/(f + g*x))^q),x)
```

output
```
x*log(c*(d + e/(f + g*x))^q) - (f*q*log(f + g*x))/g + (f*q*log(e + d*f + d
*g*x))/g + (e*q*log(e + d*f + d*g*x))/(d*g)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right) dx = \frac{\log(gx+f)\,eq + \log\left(\frac{(dgx+df+e)^q c}{(gx+f)^q}\right) df + \log\left(\frac{(dgx+df+e)^q c}{(gx+f)^q}\right) dgx + \log\left(\frac{(dgx+df+e)^q c}{(gx+f)^q}\right) e}{dg}$$

input
```
int(log(c*(d+e/(g*x+f))^q),x)
```

output
```
(log(f + g*x)*e*q + log(((d*f + d*g*x + e)**q*c)/(f + g*x)**q)*d*f + log((
(d*f + d*g*x + e)**q*c)/(f + g*x)**q)*d*g*x + log(((d*f + d*g*x + e)**q*c)
/(f + g*x)**q)*e)/(d*g)
```

3.720 $\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx=\frac{2\sqrt{e}q\arctan\left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}g}+\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

output
```
2*e^(1/2)*q*arctan(d^(1/2)*(g*x+f)/e^(1/2))/d^(1/2)/g+(g*x+f)*ln(c*(d+e/(g
*x+f)^2)^q)/g
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx = \frac{\frac{2\sqrt{e}q\arctan\left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}}-2fq\log(f+gx)+gx\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)+fq\log\left(e+d(f+gx)^2\right)}{g}$$

input
```
Integrate[Log[c*(d + e/(f + g*x)^2)^q],x]
```

output
```
((2*Sqrt[e]*q*ArcTan[(Sqrt[d]*(f + g*x))/Sqrt[e]])/Sqrt[d] - 2*f*q*Log[f +
 g*x] + g*x*Log[c*(d + e/(f + g*x)^2)^q] + f*q*Log[e + d*(f + g*x)^2])/g
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2933, 2898, 795, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right) dx$$

$$\Big\downarrow \text{ 2933}$$

$$\frac{\int \log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right) d(f+gx)}{g}$$

$$\Big\downarrow \text{ 2898}$$

$$\frac{2eq\int \frac{1}{(f+gx)^2\left(d+\frac{e}{(f+gx)^2}\right)}d(f+gx)+(f+gx)\log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

$$\Big\downarrow \text{ 795}$$

$$\frac{2eq\int \frac{1}{d(f+gx)^2+e}d(f+gx)+(f+gx)\log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{2\sqrt{e}q\arctan\left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}}+(f+gx)\log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

input
```
Int[Log[c*(d + e/(f + g*x)^2)^q],x]
```

output
```
((2*Sqrt[e]*q*ArcTan[(Sqrt[d]*(f + g*x))/Sqrt[e]])/Sqrt[d] + (f + g*x)*Log
[c*(d + e/(f + g*x)^2)^q])/g
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 2933
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
 x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
 && (EqQ[q, 1] || IntegerQ[n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.

Time = 1.88 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.88

method	result	siz
parts	$\ln\left(c\left(d+\frac{e}{(gx+f)^2}\right)^q\right)x+2qeg\left(\frac{\frac{f\ln\left(d\,g^2x^2+2dfgx+f^2d+e\right)}{2g}+\frac{e\arctan\left(\frac{2d\,g^2x+2dfg}{2g\sqrt{de}}\right)}{g\sqrt{de}}}{eg}-\frac{f\ln(gx+f)}{g^2e}\right)$	11
default	$x\ln\left(c\left(\frac{d\,g^2x^2+2dfgx+f^2d+e}{(gx+f)^2}\right)^q\right)+2qeg\left(\frac{\frac{f\ln\left(d\,g^2x^2+2dfgx+f^2d+e\right)}{2g}+\frac{e\arctan\left(\frac{2d\,g^2x+2dfg}{2g\sqrt{de}}\right)}{g\sqrt{de}}}{eg}-\frac{f\ln(gx+f)}{g^2e}\right)$	12

input
```
int(ln(c*(d+e/(g*x+f)^2)^q),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(d+e/(g*x+f)^2)^q)*x+2*q*e*g*(1/e/g*(1/2*f/g*ln(d*g^2*x^2+2*d*f*g*x+d
*f^2+e)+e/g/(d*e)^(1/2)*arctan(1/2*(2*d*g^2*x+2*d*f*g)/g/(d*e)^(1/2)))-f/g
^2/e*ln(g*x+f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(51) = 102$.

Time = 0.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 4.86

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx$$
$$= \left[\frac{gqx\log\left(\frac{dg^2x^2+2\,dfgx+df^2+e}{g^2x^2+2\,fgx+f^2}\right)+fq\log\left(dg^2x^2+2\,dfgx+df^2+e\right)-2\,fq\log\left(gx+f\right)+gx\log\left(c\right)+q\sqrt{\cdot}}{g}\right.$$

input
```
integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="fricas")
```

output
```
[(g*q*x*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2))
 + f*q*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*l
og(c) + q*sqrt(-e/d)*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + 2*(d*g*x + d*f)*
sqrt(-e/d) - e)/(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)))/g, (g*q*x*log((d*g^2
*x^2 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2)) + f*q*log(d*g^2*x
^2 + 2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*log(c) + 2*q*sqrt(e
/d)*arctan((d*g*x + d*f)*sqrt(e/d)/e))/g]
```

Sympy [F(-1)]

Timed out.

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx=\text{Timed out}$$

input
```
integrate(ln(c*(d+e/(g*x+f)**2)**q),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(51) = 102$.

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.36

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx = deg^4q\left(\frac{f\log\left(dg^2x^2+2\,dfgx+df^2+e\right)}{deg^5}-\frac{2\,f\log\left(|gx+f|\right)}{deg^5}+\frac{2\,\arctan\left(\frac{dgx+df}{\sqrt{de}}\right)}{\sqrt{de}dg^5}\right) + qx\log\left(dg^2x^2+2\,dfgx+df^2+e\right)-qx\log\left(g^2x^2+2\,fgx+f^2\right)+x\log\left(c\right)$$

input
```
integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="giac")
```

output
```
d*e*g^4*q*(f*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(d*e*g^5) - 2*f*log(ab
s(g*x + f))/(d*e*g^5) + 2*arctan((d*g*x + d*f)/sqrt(d*e))/(sqrt(d*e)*d*g^5
)) + q*x*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - q*x*log(g^2*x^2 + 2*f*g*
x + f^2) + x*log(c)
```

Mupad [B] (verification not implemented)

Time = 26.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.76

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx$$
$$= x\,\ln\left(c\left(d+\frac{e}{(f+g\,x)^2}\right)^q\right)-\frac{2\,f\,q\,\ln(f+g\,x)}{g}$$
$$+\frac{\ln\left(e\sqrt{-d\,e}-3\,d\,f^2\sqrt{-d\,e}+4\,d\,e\,f+d\,e\,g\,x-3\,d\,f\,g\,x\sqrt{-d\,e}\right)\,\left(q\sqrt{-d\,e}+d\,f\,q\right)}{d\,g}$$
$$-\frac{\ln\left(3\,d\,f^2\sqrt{-d\,e}-e\sqrt{-d\,e}+4\,d\,e\,f+d\,e\,g\,x+3\,d\,f\,g\,x\sqrt{-d\,e}\right)\,\left(q\sqrt{-d\,e}-d\,f\,q\right)}{d\,g}$$

input
```
int(log(c*(d + e/(f + g*x)^2)^q),x)
```

output
```
x*log(c*(d + e/(f + g*x)^2)^q) - (2*f*q*log(f + g*x))/g + (log(e*(-d*e)^(1
/2) - 3*d*f^2*(-d*e)^(1/2) + 4*d*e*f + d*e*g*x - 3*d*f*g*x*(-d*e)^(1/2))*(
q*(-d*e)^(1/2) + d*f*q))/(d*g) - (log(3*d*f^2*(-d*e)^(1/2) - e*(-d*e)^(1/2
) + 4*d*e*f + d*e*g*x + 3*d*f*g*x*(-d*e)^(1/2))*(q*(-d*e)^(1/2) - d*f*q))/
(d*g)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.24

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)dx$$
$$=\frac{2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{dgx+df}{\sqrt{e}\,\sqrt{d}}\right)q+\log\left(\frac{(d\,g^2x^2+2dfgx+d\,f^2+e)^q c}{(g^2x^2+2fgx+f^2)^q}\right)df+\log\left(\frac{(d\,g^2x^2+2dfgx+d\,f^2+e)^q c}{(g^2x^2+2fgx+f^2)^q}\right)dgx}{dg}$$

input
```
int(log(c*(d+e/(g*x+f)^2)^q),x)
```

output
```
(2*sqrt(e)*sqrt(d)*atan((d*f + d*g*x)/(sqrt(e)*sqrt(d)))*q + log(((d*f**2
+ 2*d*f*g*x + d*g**2*x**2 + e)**q*c)/(f**2 + 2*f*g*x + g**2*x**2)**q)*d*f
+ log(((d*f**2 + 2*d*f*g*x + d*g**2*x**2 + e)**q*c)/(f**2 + 2*f*g*x + g**2
*x**2)**q)*d*g*x)/(d*g)
```

3.721 $\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 165

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx = -\frac{\sqrt{3}\sqrt[3]{e}q\arctan\left(\frac{\sqrt[3]{e}-2\sqrt[3]{d}(f+gx)}{\sqrt{3}\sqrt[3]{e}}\right)}{\sqrt[3]{d}g} + \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(f+gx)\right)}{\sqrt[3]{d}g} - \frac{\sqrt[3]{e}q\log\left(e^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}(f+gx)^2\right)}{2\sqrt[3]{d}g}$$

output
```
-3^(1/2)*e^(1/3)*q*arctan(1/3*(e^(1/3)-2*d^(1/3)*(g*x+f))*3^(1/2)/e^(1/3))
/d^(1/3)/g+(g*x+f)*ln(c*(d+e/(g*x+f)^3)^q)/g+e^(1/3)*q*ln(e^(1/3)+d^(1/3)*
(g*x+f))/d^(1/3)/g-1/2*e^(1/3)*q*ln(e^(2/3)-d^(1/3)*e^(1/3)*(g*x+f)+d^(2/3
)*(g*x+f)^2)/d^(1/3)/g
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx = -\frac{3eq\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{e}{d(f+gx)^3}\right)}{2dg(f+gx)^2} + \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}$$

input
```
Integrate[Log[c*(d + e/(f + g*x)^3)^q],x]
```

output
```
(-3*e*q*Hypergeometric2F1[2/3, 1, 5/3, -(e/(d*(f + g*x)^3))])/(2*d*g*(f +
g*x)^2) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {2933, 2898, 795, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx$$

$$\Big\downarrow \text{2933}$$

$$\frac{\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)d(f+gx)}{g}$$

$$\Big\downarrow \text{2898}$$

$$\frac{3eq\int \frac{1}{(f+gx)^3\left(d+\frac{e}{(f+gx)^3}\right)}d(f+gx)+(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}$$

$$\Big\downarrow \; 795$$

$$\frac{3eq\int \frac{1}{d(f+gx)^3+e}d(f+gx)+(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}$$

$$\Big\downarrow \; 750$$

$$\frac{3eq\left(\frac{\int \frac{2\sqrt[3]{e}-\sqrt[3]{d}(f+gx)}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)}{3e^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}(f+gx)+\sqrt[3]{e}}d(f+gx)}{3e^{2/3}}\right)+(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}$$

$$\Big\downarrow \; 16$$

$$\frac{3eq\left(\frac{\int \frac{2\sqrt[3]{e}-\sqrt[3]{d}(f+gx)}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)}{3e^{2/3}}+\frac{\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{3\sqrt[3]{d}e^{2/3}}\right)+(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}$$

$$\Big\downarrow \; 1142$$

$$\frac{3eq\left(\frac{\frac{3}{2}\sqrt[3]{e}\int \frac{1}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{e}-2\sqrt[3]{d}(f+gx)\right)}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)}{2\sqrt[3]{d}}}{3e^{2/3}}+\frac{\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{3\sqrt[3]{d}e^{2/3}}\right)+}{g}$$

$$\Big\downarrow \; 25$$

$$\frac{3eq\left(\frac{\frac{3}{2}\sqrt[3]{e}\int \frac{1}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{e}-2\sqrt[3]{d}(f+gx)\right)}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)}{2\sqrt[3]{d}}}{3e^{2/3}}+\frac{\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{3\sqrt[3]{d}e^{2/3}}\right)+(}{g}$$

$$\Big\downarrow \; 27$$

$$\frac{3eq\left(\frac{\frac{3}{2}\sqrt[3]{e}\int\frac{1}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)+\frac{1}{2}\int\frac{\sqrt[3]{e}-2\sqrt[3]{d}(f+gx)}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)}{3e^{2/3}}+\frac{\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{3\sqrt[3]{d}e^{2/3}}\right)+}{g}$$

$\big\downarrow$ 1082

$$\frac{3eq\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{e}-2\sqrt[3]{d}(f+gx)}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{d}(f+gx)}{\sqrt[3]{e}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}(f+gx)}{\sqrt[3]{e}}\right)}{\sqrt[3]{d}}}{3e^{2/3}}+\frac{\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{3\sqrt[3]{d}e^{2/3}}\right)+(f}{g}$$

$\big\downarrow$ 217

$$\frac{3eq\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{e}-2\sqrt[3]{d}(f+gx)}{d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}}d(f+gx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}(f+gx)}{\sqrt[3]{e}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3e^{2/3}}+\frac{\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{3\sqrt[3]{d}e^{2/3}}\right)+(f+gx)\log\left(c\left(d\right.\right.}{g}$$

$\big\downarrow$ 1103

$$\frac{3eq\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}(f+gx)}{\sqrt[3]{e}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}\right)}{2\sqrt[3]{d}}}{3e^{2/3}}+\frac{\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{3\sqrt[3]{d}e^{2/3}}\right)+(f+gx)\log\left(c\left(d+\right.\right.}{g}$$

input
```
Int[Log[c*(d + e/(f + g*x)^3)^q],x]
```

output
```
((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q] + 3*e*q*(Log[e^(1/3) + d^(1/3)*(f
+ g*x)]/(3*d^(1/3)*e^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(f + g*x)
)/e^(1/3))/Sqrt[3]])/d^(1/3)) - Log[e^(2/3) - d^(1/3)*e^(1/3)*(f + g*x) +
d^(2/3)*(f + g*x)^2]/(2*d^(1/3)))/(3*e^(2/3))))/g
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2898
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Simp[e*n*p   Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
 e, n, p}, x]
```

rule 2933
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
 x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
 && (EqQ[q, 1] || IntegerQ[n])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84

method	result
parts	$\ln\left(c\left(d+\frac{e}{(gx+f)^3}\right)^q\right)x+3qeg\left(-\frac{f\ln(gx+f)}{g^2e}+\frac{\sum_{\_R=\mathrm{RootOf}\left(dg^3\_Z^3+3dfg^2\_Z^2+3df^2g\_Z+df^3+e\right)}\frac{\left(\_R^2dfg^2+\right.}{g^2}}{3dg^2e}\right.$
default	$x\ln\left(c\left(\frac{dg^3x^3+3dfg^2x^2+3df^2gx+df^3+e}{(gx+f)^3}\right)^q\right)+3qeg\left(-\frac{f\ln(gx+f)}{g^2e}+\frac{\sum_{\_R=\mathrm{RootOf}\left(dg^3\_Z^3+3dfg^2\_Z^2+3df^2g\_Z+d\right.}}{}\right.$

input
```
int(ln(c*(d+e/(g*x+f)^3)^q),x,method=_RETURNVERBOSE)
```

output
```
ln(c*(d+e/(g*x+f)^3)^q)*x+3*q*e*g*(-f/g^2/e*ln(g*x+f)+1/3/d/g^2*sum((_R^2*
d*f*g^2+2*_R*d*f^2*g+d*f^3+e)/(_R^2*g^2+2*_R*f*g+f^2)*ln(x-_R),_R=RootOf(_
Z^3*d*g^3+3*_Z^2*d*f*g^2+3*_Z*d*f^2*g+d*f^3+e))/e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 1169, normalized size of antiderivative = 7.08

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)dx = \text{Too large to display}$$

input
```
integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="fricas")
```

output
```
1/4*(4*g*q*x*log((d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)/(g^
3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)) - 12*f*q*log(g*x + f) - 2*((-1/2*f
^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I
*sqrt(3) + 1) - 2*f*q/g)*g*log(q*x - 1/2*(-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*
g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) + f*q/g) + 4
*g*x*log(c) + (((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e
*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 6*f*q + sqrt(3)*g*sqrt
(-(((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3
))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q
^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*
q/g)*f*g*q + 4*f^2*q^2)/g^2))*log(2*g*q*x + 1/2*((-1/2*f^3*q^3/g^3 + 1/2*e
*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*
f*q/g)*g + 3*f*q + 1/2*sqrt(3)*g*sqrt(-(((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*
g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2
*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/
(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2)) + (((-1
/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3
)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 6*f*q - sqrt(3)*g*sqrt(-(((-1/2*f^3*q^3/g
^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3)
 + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2...
```

Sympy [F(-1)]

Timed out.

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx = \text{Timed out}$$

input
```
integrate(ln(c*(d+e/(g*x+f)**3)**q),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx = \int \log \left(c\left(d+\frac{e}{(gx+f)^3}\right)^q\right) dx$$

input
```
integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="maxima")
```

output
```
3*q*integrate((d*f*g^2*x^2 + 2*d*f^2*g*x + d*f^3 + e)/(d*g^3*x^3 + 3*d*f*g
^2*x^2 + 3*d*f^2*g*x + d*f^3 + e), x) - (3*f*q*log(g*x + f) - g*x*log((d*g
^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)^q) + 3*g*x*log((g*x + f)
^q) - g*x*log(c))/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(132) = 264$.

Time = 0.46 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.93

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx$$
$$= \frac{1}{2} deg^5 q \left(\frac{2 f \log \left(|dg^3x^3 + 3 dfg^2x^2 + 3 df^2gx + df^3 + e|\right)}{deg^6} - \frac{6 f \log \left(|gx + f|\right)}{deg^6} + \frac{2 \sqrt{3}(d^5e^4g^{21})^{\frac{1}{3}} \arctan}{\right.$$
$$+ qx \log \left(dg^3x^3 + 3 dfg^2x^2 + 3 df^2gx + df^3 + e\right)$$
$$- qx \log \left(g^3x^3 + 3 fg^2x^2 + 3 f^2gx + f^3\right) + x \log (c)$$

input
```
integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="giac")
```

output
```
1/2*d*e*g^5*q*(2*f*log(abs(d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3
 + e))/(d*e*g^6) - 6*f*log(abs(g*x + f))/(d*e*g^6) + (2*sqrt(3)*(d^5*e^4*g
^21)^(1/3)*arctan(-(d*g*x + d*f + (d^2*e)^(1/3))/(sqrt(3)*d*g*x + sqrt(3)*
d*f - sqrt(3)*(d^2*e)^(1/3))) - (d^5*e^4*g^21)^(1/3)*log(4*(sqrt(3)*d*g*x
+ sqrt(3)*d*f - sqrt(3)*(d^2*e)^(1/3))^2 + 4*(d*g*x + d*f + (d^2*e)^(1/3))
^2) + 2*(d^5*e^4*g^21)^(1/3)*log(abs(d*g*x + d*f + (d^2*e)^(1/3))))/(d^3*e
^2*g^13)) + q*x*log(d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e) -
 q*x*log(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3) + x*log(c)
```

Mupad [B] (verification not implemented)

Time = 26.12 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.02

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx = x\,\ln\left(c\left(d+\frac{e}{(f+g\,x)^3}\right)^q\right)$$
$$-\left(\sum_{k=1}^{3}\ln\left(-d^2\,e^2\,g^{11}\,\left(3\,e\,q^3\,x+\mathrm{root}\left(d\,g^3\,z^3+3\,d\,f\,g^2\,q\,z^2+3\,d\,f^2\,g\,q^2\,z+d\,f^3\,q^3+e\,q^3,z,k\right)\,e\,q^2+\mathrm{root}\left(d\right.\right.\right.\right.$$
$$\left.+3\,d\,f\,g^2\,q\,z^2+3\,d\,f^2\,g\,q^2\,z+d\,f^3\,q^3+e\,q^3,z,k\right)\right)-\frac{3\,f\,q\,\ln\left(f+g\,x\right)}{g}$$

input
```
int(log(c*(d + e/(f + g*x)^3)^q),x)
```

output
```
x*log(c*(d + e/(f + g*x)^3)^q) - symsum(log(-9*d^2*e^2*g^11*(3*e*q^3*x + r
oot(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z,
k)*e*q^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^
3 + e*q^3, z, k)^3*d*f*g^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*
g*q^2*z + d*f^3*q^3 + e*q^3, z, k)*d*f^3*q^2 + 4*root(d*g^3*z^3 + 3*d*f*g^
2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)^3*d*g^3*x + 8*root(d*
g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)^2*d
*f^2*g*q + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^
3 + e*q^3, z, k)*d*f^2*g*q^2*x + 8*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*
f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)^2*d*f*g^2*q*x))*root(d*g^3*z^3 + 3*
d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k), k, 1, 3) - (3*
f*q*log(f + g*x))/g
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.84

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)dx$$

$$= \frac{2e^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2d^{\frac{1}{3}}f+2d^{\frac{1}{3}}gx-e^{\frac{1}{3}}}{e^{\frac{1}{3}}\sqrt{3}}\right)q+2d^{\frac{1}{3}}\log\left(\frac{(d\,g^3x^3+3df\,g^2x^2+3d\,f^2gx+d\,f^3+e)^q c}{(g^3x^3+3f\,g^2x^2+3f^2gx+f^3)^q}\right)f+2d^{\frac{1}{3}}\log\left(\frac{(d\,g^3x^3+3df\,g^2x^2+3d\,f}{(g^3x^3+3f\,g^2x^2+3}\right.}{2}$$

input
```
int(log(c*(d+e/(g*x+f)^3)^q),x)
```

output
```
(2*e**(1/3)*sqrt(3)*atan((2*d**(1/3)*f + 2*d**(1/3)*g*x - e**(1/3))/(e**(1
/3)*sqrt(3)))*q + 2*d**(1/3)*log(((d*f**3 + 3*d*f**2*g*x + 3*d*f*g**2*x**2
 + d*g**3*x**3 + e)**q*c)/(f**3 + 3*f**2*g*x + 3*f*g**2*x**2 + g**3*x**3)*
*q)*f + 2*d**(1/3)*log(((d*f**3 + 3*d*f**2*g*x + 3*d*f*g**2*x**2 + d*g**3*
x**3 + e)**q*c)/(f**3 + 3*f**2*g*x + 3*f*g**2*x**2 + g**3*x**3)**q)*g*x +
3*e**(1/3)*log(d**(1/3)*f + d**(1/3)*g*x + e**(1/3))*q - 3*e**(1/3)*log(f
+ g*x)*q - e**(1/3)*log(((d*f**3 + 3*d*f**2*g*x + 3*d*f*g**2*x**2 + d*g**3
*x**3 + e)**q*c)/(f**3 + 3*f**2*g*x + 3*f*g**2*x**2 + g**3*x**3)**q))/(2*d
**(1/3)*g)
```

3.722 $\int \left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx=\text{Int}\left(\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n,x\right)$$

output
```
Defer(Int)((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx=\int \left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$$

input
```
Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]
```

output
```
Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2934}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$$

$$\Big\downarrow \text{2934}$$

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$$

input
```
Int[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2934
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Unintegrable[(a + b*Log[c*(d + e*(f + g*x)^n)^p])^q,
 x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^n dx$$

input
```
int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

output
```
int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^n\,dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="fricas")
```

output
```
integral((b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a)^n, x)
```

Sympy [N/A]

Not integrable

Time = 15.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx$$

input
```
integrate((a+b*ln(c*(d+e/(g*x+f))**p))**n,x)
```

output
```
Integral((a + b*log(c*(d + e/(f + g*x))**p))**n, x)
```

Maxima [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^n dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="maxima")
```

output
```
integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^n dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

Mupad [N/A]

Not integrable

Time = 26.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx = \int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^n dx$$

input
```
int((a + b*log(c*(d + e/(f + g*x))^p))^n,x)
```

output
```
int((a + b*log(c*(d + e/(f + g*x))^p))^n, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 251, normalized size of antiderivative = 11.41

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx = \left(\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)b+a\right)^n x + \left(\left(\int \frac{\left(\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)b+a\right.}{\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)bd f^2+2\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)bdf gx+\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)bd g^2x^2+\log\left(\frac{(dgx+df+e)}{(gx+f)^p}\right.}\right.\right.$$

input
```
int((a+b*log(c*(d+e/(g*x+f))^p))^n,x)
```

output
```
(log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b + a)**n*x + int(((log(((d*f
+ d*g*x + e)**p*c)/(f + g*x)**p)*b + a)**n*x)/(log(((d*f + d*g*x + e)**p*c
)/(f + g*x)**p)*b*d*f**2 + 2*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*
d*f*g*x + log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*d*g**2*x**2 + log((
(d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*e*f + log(((d*f + d*g*x + e)**p*c)
/(f + g*x)**p)*b*e*g*x + a*d*f**2 + 2*a*d*f*g*x + a*d*g**2*x**2 + a*e*f +
a*e*g*x),x)*b*e*g*n*p
```

3.723 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx = -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} - \frac{12b^2ep^2\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\text{PolyLog}\left(2,1+\frac{e}{d(f+gx)}\right)}{dg} + \frac{24b^3ep^3\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\text{PolyLog}\left(3,1+\frac{e}{d(f+gx)}\right)}{dg} - \frac{24b^4ep^4\,\text{PolyLog}\left(4,1+\frac{e}{d(f+gx)}\right)}{dg}$$

output
```
-4*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))^3/d/g+(e+d*(g*x+f))*
(a+b*ln(c*(d+e/(g*x+f))^p))^4/d/g-12*b^2*e*p^2*(a+b*ln(c*(d+e/(g*x+f))^p))
^2*polylog(2,1+e/d/(g*x+f))/d/g+24*b^3*e*p^3*(a+b*ln(c*(d+e/(g*x+f))^p))*p
olylog(3,1+e/d/(g*x+f))/d/g-24*b^4*e*p^4*polylog(4,1+e/d/(g*x+f))/d/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 743 vs. $2(221) = 442$.

Time = 1.47 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.36

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```

output
```
(-4*b*p*(d*f*Log[f + g*x] - (e + d*f)*Log[e + d*f + d*g*x] - d*g*x*Log[(e
+ d*f + d*g*x)/(f + g*x)])*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/
(f + g*x))^p])^3 + d*g*x*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f
 + g*x))^p])^4 - 6*b^2*p^2*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/
(f + g*x))^p])^2*(d*f*Log[-(e/(d*f + d*g*x))]^2 + 2*d*f*Log[-(e/(d*f + d*g
*x))]*Log[(e + d*f + d*g*x)/e] - 2*d*f*Log[-(e/(d*f + d*g*x))]*Log[(e + d*
f + d*g*x)/(f + g*x)] - 2*(e + d*f)*Log[e + d*f + d*g*x]*Log[(e + d*f + d*
g*x)/(f + g*x)] - d*g*x*Log[(e + d*f + d*g*x)/(f + g*x)]^2 - 2*d*f*PolyLog
[2, -((d*(f + g*x))/e)] - (e + d*f)*((2*Log[-((d*(f + g*x))/e)] - Log[e +
d*f + d*g*x])*Log[e + d*f + d*g*x] + 2*PolyLog[2, (e + d*f + d*g*x)/e])) +
 4*b^3*p^3*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(
Log[d + e/(f + g*x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*f + d*g*x)*L
og[d + e/(f + g*x)]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 + e/(d*f + d*
g*x)] + 6*e*PolyLog[3, 1 + e/(d*f + d*g*x)]) - b^4*p^4*(4*e*Log[-(e/(d*f +
 d*g*x))]*Log[d + e/(f + g*x)]^3 - e*Log[d + e/(f + g*x)]^4 - d*f*Log[d +
e/(f + g*x)]^4 - d*g*x*Log[d + e/(f + g*x)]^4 + 12*e*Log[d + e/(f + g*x)]^
2*PolyLog[2, 1 + e/(d*f + d*g*x)] - 24*e*Log[d + e/(f + g*x)]*PolyLog[3, 1
 + e/(d*f + d*g*x)] + 24*e*PolyLog[4, 1 + e/(d*f + d*g*x)]))/(d*g)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2933, 2899, 2904, 2843, 2881, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4dx$$

$\Big\downarrow$ 2933

$$\frac{\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4d(f+gx)}{g}$$

$\Big\downarrow$ 2899

$$\frac{\frac{4bep\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{f+gx}d(f+gx)}{d}+\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{d}}{g}$$

$\Big\downarrow$ 2904

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{d}-\frac{4bep\int (f+gx)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3d\frac{1}{f+gx}}{d}}{g}$$

$\Big\downarrow$ 2843

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{d}-\frac{4bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3-3bep\int \frac{\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right.}{d+\frac{e}{f+gx}}\right.}{d}}{g}$$

$\Big\downarrow$ 2881

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{d}-\frac{4bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3-3bp\int (f+gx)\log\left(\frac{d-f-gx}{d}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+}\right.\right.\right.\right.}{d}}{g}$$

$\Big\downarrow$ 2821

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{d}-\frac{4bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3-3bp\left(2bp\int (f+gx)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\right.\right.}{d}}{g}$$

$\Big\downarrow$ 2830

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{d}-\frac{4bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3-3bp\left(2bp\left(\text{PolyLog}\left(3,\frac{d+\frac{e}{f+gx}}{d}\right)\left(a+b\log\left(c\left(\right.\right.\right.\right.\right.\right.}{}}{g}$$

$\Big\downarrow$ 7143

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{d} - \frac{4bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3-3bp\left(2bp\left(\text{PolyLog}\left(3,\frac{d+\frac{e}{f+gx}}{d}\right)\left(a+b\log\left(c\left(\cdots\right.\right.\right.\right.\right.\right.}{\cdots}}{g}$$

input
```
Int[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```

output
```
(((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^4)/d - (4*b*e*p*(Lo
g[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^3 - 3*b*p*(-((a +
 b*Log[c*(d + e/(f + g*x))^p])^2*PolyLog[2, (d + e/(f + g*x))/d]) + 2*b*p*
((a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[3, (d + e/(f + g*x))/d] - b*p*
PolyLog[4, (d + e/(f + g*x))/d]))))/d)/g
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2899
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Simp[b*e*p*(q/d)   I
nt[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p}
, x] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2933
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
 x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
 && (EqQ[q, 1] || IntegerQ[n])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{4}dx$$

input
```
int((a+b*ln(c*(d+e/(g*x+f))^p))^4,x)
```

output
```
int((a+b*ln(c*(d+e/(g*x+f))^p))^4,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{4} dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{4} dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="fricas")
```

output
```
integral(b^4*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^4 + 4*a*b^3*log(c*((d*
g*x + d*f + e)/(g*x + f))^p)^3 + 6*a^2*b^2*log(c*((d*g*x + d*f + e)/(g*x +
 f))^p)^2 + 4*a^3*b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a^4, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{4} dx = \int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{4} dx$$

input
```
integrate((a+b*ln(c*(d+e/(g*x+f))**p))**4,x)
```

output
```
Integral((a + b*log(c*(d + e/(f + g*x))**p))**4, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{4} dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{4} dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="maxima")
```

output
```
-4*a^3*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d
*e*g^2)) + 4*a^3*b*x*log(c*(d + e/(g*x + f))^p) + a^4*x + (b^4*d*g*x*log((
d*g*x + d*f + e)^p)^4 - 4*(b^4*d*f*p*log(g*x + f) + b^4*d*g*x*log((g*x + f
)^p) - (d*f*p + e*p)*b^4*log(d*g*x + d*f + e) - (b^4*d*g*log(c) + a*b^3*d*
g)*x)*log((d*g*x + d*f + e)^p)^3)/(d*g) + integrate(((d*f + e)*b^4*log(c)^
4 + 4*(d*f + e)*a*b^3*log(c)^3 + 6*(d*f + e)*a^2*b^2*log(c)^2 + (b^4*d*g*x
 + (d*f + e)*b^4)*log((g*x + f)^p)^4 - 4*((d*f + e)*b^4*log(c) + (d*f + e)
*a*b^3 + (b^4*d*g*log(c) + a*b^3*d*g)*x)*log((g*x + f)^p)^3 + 6*(2*b^4*d*f
*p^2*log(g*x + f) + (d*f + e)*b^4*log(c)^2 - 2*(d*f*p^2 + e*p^2)*b^4*log(d
*g*x + d*f + e) + 2*(d*f + e)*a*b^3*log(c) + (d*f + e)*a^2*b^2 + (b^4*d*g*
x + (d*f + e)*b^4)*log((g*x + f)^p)^2 + (a^2*b^2*d*g - 2*(d*g*p - d*g*log(
c))*a*b^3 - (2*d*g*p*log(c) - d*g*log(c)^2)*b^4)*x - 2*((d*f + e)*b^4*log(
c) + (d*f + e)*a*b^3 + (a*b^3*d*g - (d*g*p - d*g*log(c))*b^4)*x)*log((g*x
+ f)^p))*log((d*g*x + d*f + e)^p)^2 + 6*((d*f + e)*b^4*log(c)^2 + 2*(d*f +
 e)*a*b^3*log(c) + (d*f + e)*a^2*b^2 + (b^4*d*g*log(c)^2 + 2*a*b^3*d*g*log
(c) + a^2*b^2*d*g)*x)*log((g*x + f)^p)^2 + (b^4*d*g*log(c)^4 + 4*a*b^3*d*g
*log(c)^3 + 6*a^2*b^2*d*g*log(c)^2)*x + 4*((d*f + e)*b^4*log(c)^3 + 3*(d*f
 + e)*a*b^3*log(c)^2 + 3*(d*f + e)*a^2*b^2*log(c) - (b^4*d*g*x + (d*f + e)
*b^4)*log((g*x + f)^p)^3 + 3*((d*f + e)*b^4*log(c) + (d*f + e)*a*b^3 + (b^
4*d*g*log(c) + a*b^3*d*g)*x)*log((g*x + f)^p)^2 + (b^4*d*g*log(c)^3 + 3...
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4\,dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^4\,dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/(g*x + f))^p) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx=\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^4 dx$$

input
```
int((a + b*log(c*(d + e/(f + g*x))^p))^4,x)
```

output
```
int((a + b*log(c*(d + e/(f + g*x))^p))^4, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx$$
$$=\frac{4\left(\int \frac{\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)^3 x}{d g^2x^2+2dfgx+d f^2+egx+ef}dx\right) b^4de\,g^2p+12\left(\int \frac{\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)^2 x}{d g^2x^2+2dfgx+d f^2+egx+ef}dx\right) a\,b^3de\,g^2p+12\left(\int \frac{\log\left(\frac{(d}{}\right.}{d g^2x^2+2df}\right.}{}$$

input
```
int((a+b*log(c*(d+e/(g*x+f))^p))^4,x)
```

output
```
(4*int((log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)**3*x)/(d*f**2 + 2*d*f*g
*x + d*g**2*x**2 + e*f + e*g*x),x)*b**4*d*e*g**2*p + 12*int((log(((d*f + d
*g*x + e)**p*c)/(f + g*x)**p)**2*x)/(d*f**2 + 2*d*f*g*x + d*g**2*x**2 + e*
f + e*g*x),x)*a*b**3*d*e*g**2*p + 12*int((log(((d*f + d*g*x + e)**p*c)/(f
+ g*x)**p)*x)/(d*f**2 + 2*d*f*g*x + d*g**2*x**2 + e*f + e*g*x),x)*a**2*b**
2*d*e*g**2*p + 4*log(f + g*x)*a**3*b*e*p + log(((d*f + d*g*x + e)**p*c)/(f
 + g*x)**p)**4*b**4*d*g*x + 4*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)**
3*a*b**3*d*g*x + 6*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)**2*a**2*b**2
*d*g*x + 4*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a**3*b*d*f + 4*log((
(d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a**3*b*d*g*x + 4*log(((d*f + d*g*x +
 e)**p*c)/(f + g*x)**p)*a**3*b*e + a**4*d*g*x)/(d*g)
```

3.724 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx$

Optimal result

Integrand size = 22, antiderivative size = 168

$$
\begin{aligned}
\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx
&= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} \\
&\quad+\frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} \\
&\quad-\frac{6b^2ep^2\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e}{d(f+gx)}\right)}{dg} \\
&\quad+\frac{6b^3ep^3\operatorname{PolyLog}\left(3,1+\frac{e}{d(f+gx)}\right)}{dg}
\end{aligned}
$$

output

```
-3*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))^2/d/g+(e+d*(g*x+f))*
(a+b*ln(c*(d+e/(g*x+f))^p))^3/d/g-6*b^2*e*p^2*(a+b*ln(c*(d+e/(g*x+f))^p))*
polylog(2,1+e/d/(g*x+f))/d/g+6*b^3*e*p^3*polylog(3,1+e/d/(g*x+f))/d/g
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 441 vs. $2(168) = 336$.

Time = 0.75 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.62

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx$$
$$= \frac{3bdp(f+gx)\log\left(d+\frac{e}{f+gx}\right)\left(a-bp\log\left(d+\frac{e}{f+gx}\right)+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2+d(f+gx)\left(a-bp\log\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```

output
```
(3*b*d*p*(f + g*x)*Log[d + e/(f + g*x)]*(a - b*p*Log[d + e/(f + g*x)] + b*
Log[c*(d + e/(f + g*x))^p])^2 + d*(f + g*x)*(a - b*p*Log[d + e/(f + g*x)]
+ b*Log[c*(d + e/(f + g*x))^p])^3 + 3*b*e*p*(a - b*p*Log[d + e/(f + g*x)]
+ b*Log[c*(d + e/(f + g*x))^p])^2*Log[e + d*(f + g*x)] + 3*b^2*p^2*(a - b*
p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(d*(f + g*x)*Log[d
+ e/(f + g*x)]^2 + e*(Log[e/d + f + g*x]^2 + 2*(Log[f + g*x] - Log[e/d + f
 + g*x] + Log[d + e/(f + g*x)])*Log[e + d*(f + g*x)] - 2*(Log[f + g*x]*Log
[1 + (d*(f + g*x))/e] + PolyLog[2, -((d*(f + g*x))/e)]))) + b^3*p^3*(Log[d
 + e/(f + g*x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*f + d*g*x)*Log[d
+ e/(f + g*x)]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 + e/(d*f + d*g*x)]
 + 6*e*PolyLog[3, 1 + e/(d*f + d*g*x)]))/(d*g)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2933, 2899, 2904, 2843, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx$$
$$\Big\downarrow \text{2933}$$

$$\frac{\int \left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3d(f+gx)}{g}$$

$\Big\downarrow$ 2899

$$\frac{\frac{3bep\int \frac{\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{f+gx}d(f+gx)}{d}+\frac{(d(f+gx)+e)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{d}}{g}$$

$\Big\downarrow$ 2904

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{d}-\frac{3bep\int (f+gx)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2d\frac{1}{f+gx}}{d}}{g}$$

$\Big\downarrow$ 2843

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{d}-\frac{3bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2-2bep\int \frac{\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right.}{d+\frac{e}{f+gx}}}{d}}{g}$$

$\Big\downarrow$ 2881

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{d}-\frac{3bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2-2bp\int (f+gx)\log\left(\frac{d-f-gx}{d}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+}\right.\right.\right.\right.}{d}}{g}$$

$\Big\downarrow$ 2821

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{d}-\frac{3bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2-2bp\left(bp\int (f+gx)\operatorname{PolyLog}\left(2,\frac{d+\frac{e}{f+gx}}{d}\right)d\left(d+\right.\right.\right.}{d}}{g}$$

$\Big\downarrow$ 7143

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{d}-\frac{3bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log \left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2-2bp\left(bp\operatorname{PolyLog}\left(3,\frac{d+\frac{e}{f+gx}}{d}\right)-\operatorname{PolyLog}\left(2,\frac{d}{-}\right.\right.\right.}{d}}{g}$$

input
```
Int[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```

output
```
(((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/d - (3*b*e*p*(Lo
g[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^2 - 2*b*p*(-((a +
 b*Log[c*(d + e/(f + g*x))^p])*PolyLog[2, (d + e/(f + g*x))/d]) + b*p*Poly
Log[3, (d + e/(f + g*x))/d])))/d)/g
```

Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2899
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Simp[b*e*p*(q/d)   I
nt[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p}
, x] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2933
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
 x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
 && (EqQ[q, 1] || IntegerQ[n])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{3}dx$$

input
```
int((a+b*ln(c*(d+e/(g*x+f))^p))^3,x)
```

output
```
int((a+b*ln(c*(d+e/(g*x+f))^p))^3,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{3}dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{3}dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="fricas")
```

output
```
integral(b^3*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^3 + 3*a*b^2*log(c*((d*
g*x + d*f + e)/(g*x + f))^p)^2 + 3*a^2*b*log(c*((d*g*x + d*f + e)/(g*x + f
))^p) + a^3, x)
```

Sympy [F]

$$\int \left(a + b\log\left(c\left(d + \frac{e}{f + gx}\right)^{p}\right)\right)^{3} dx = \int \left(a + b\log\left(c\left(d + \frac{e}{f + gx}\right)^{p}\right)\right)^{3} dx$$

input
```
integrate((a+b*ln(c*(d+e/(g*x+f))**p))**3,x)
```

output
```
Integral((a + b*log(c*(d + e/(f + g*x))**p))**3, x)
```

Maxima [F]

$$\int \left(a + b\log\left(c\left(d + \frac{e}{f + gx}\right)^{p}\right)\right)^{3} dx = \int \left(b\log\left(c\left(d + \frac{e}{gx + f}\right)^{p}\right) + a\right)^{3} dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="maxima")
```

output
```
-3*a^2*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d
*e*g^2)) + 3*a^2*b*x*log(c*(d + e/(g*x + f))^p) + a^3*x + (b^3*d*g*x*log((
d*g*x + d*f + e)^p)^3 - 3*(b^3*d*f*p*log(g*x + f) + b^3*d*g*x*log((g*x + f
)^p) - (d*f*p + e*p)*b^3*log(d*g*x + d*f + e) - (b^3*d*g*log(c) + a*b^2*d*
g)*x)*log((d*g*x + d*f + e)^p)^2)/(d*g) + integrate(((d*f + e)*b^3*log(c)^
3 + 3*(d*f + e)*a*b^2*log(c)^2 - (b^3*d*g*x + (d*f + e)*b^3)*log((g*x + f)
^p)^3 + 3*((d*f + e)*b^3*log(c) + (d*f + e)*a*b^2 + (b^3*d*g*log(c) + a*b^
2*d*g)*x)*log((g*x + f)^p)^2 + (b^3*d*g*log(c)^3 + 3*a*b^2*d*g*log(c)^2)*x
 + 3*(2*b^3*d*f*p^2*log(g*x + f) + (d*f + e)*b^3*log(c)^2 - 2*(d*f*p^2 + e
*p^2)*b^3*log(d*g*x + d*f + e) + 2*(d*f + e)*a*b^2*log(c) + (b^3*d*g*x + (
d*f + e)*b^3)*log((g*x + f)^p)^2 - (2*(d*g*p - d*g*log(c))*a*b^2 + (2*d*g*
p*log(c) - d*g*log(c)^2)*b^3)*x - 2*((d*f + e)*b^3*log(c) + (d*f + e)*a*b^
2 + (a*b^2*d*g - (d*g*p - d*g*log(c))*b^3)*x)*log((g*x + f)^p))*log((d*g*x
 + d*f + e)^p) - 3*((d*f + e)*b^3*log(c)^2 + 2*(d*f + e)*a*b^2*log(c) + (b
^3*d*g*log(c)^2 + 2*a*b^2*d*g*log(c))*x)*log((g*x + f)^p))/(d*g*x + d*f +
e), x)
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^3 dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/(g*x + f))^p) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^3 dx$$

input
```
int((a + b*log(c*(d + e/(f + g*x))^p))^3,x)
```

output
```
int((a + b*log(c*(d + e/(f + g*x))^p))^3, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \frac{3\left(\int \frac{\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)^2 x}{d g^2x^2+2dfgx+d f^2+egx+ef}dx\right) b^3de\, g^2p+6\left(\int \frac{\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right) x}{d g^2x^2+2dfgx+d f^2+egx+ef}dx\right) a\, b^2de\, g^2p+3\log(gx+f)\, a^2}{}$$

input
```
int((a+b*log(c*(d+e/(g*x+f))^p))^3,x)
```

output
```
(3*int((log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)**2*x)/(d*f**2 + 2*d*f*g
*x + d*g**2*x**2 + e*f + e*g*x),x)*b**3*d*e*g**2*p + 6*int((log(((d*f + d*
g*x + e)**p*c)/(f + g*x)**p)*x)/(d*f**2 + 2*d*f*g*x + d*g**2*x**2 + e*f +
e*g*x),x)*a*b**2*d*e*g**2*p + 3*log(f + g*x)*a**2*b*e*p + log(((d*f + d*g*
x + e)**p*c)/(f + g*x)**p)**3*b**3*d*g*x + 3*log(((d*f + d*g*x + e)**p*c)/
(f + g*x)**p)**2*a*b**2*d*g*x + 3*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**
p)*a**2*b*d*f + 3*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a**2*b*d*g*x
+ 3*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a**2*b*e + a**3*d*g*x)/(d*g
)
```

3.725 $\int \left(a+b \log \left(c \left(d+\frac{e}{f+g x}\right)^p\right)\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \left(a+b \log \left(c \left(d+\frac{e}{f+g x}\right)^p\right)\right)^2 \, dx = -\frac{2 b e p \log \left(-\frac{e}{d (f+g x)}\right) \left(a+b \log \left(c \left(d+\frac{e}{f+g x}\right)^p\right)\right)}{d g} + \frac{(e+d (f+g x)) \left(a+b \log \left(c \left(d+\frac{e}{f+g x}\right)^p\right)\right)^2}{d g} - \frac{2 b^2 e p^2 \operatorname{PolyLog}\left(2,1+\frac{e}{d (f+g x)}\right)}{d g}$$

output

```
-2*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))/d/g+(e+d*(g*x+f))*(a
+b*ln(c*(d+e/(g*x+f))^p))^2/d/g-2*b^2*e*p^2*polylog(2,1+e/d/(g*x+f))/d/g
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.90

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx = x\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 - \frac{bp\left(2df\log(f+gx)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)-2(e+df)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\log(e+d(f\right.}{}$$

input
```
Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

output
```
x*(a + b*Log[c*(d + e/(f + g*x))^p])^2 - (b*p*(2*d*f*Log[f + g*x]*(a + b*L
og[c*(d + e/(f + g*x))^p]) - 2*(e + d*f)*(a + b*Log[c*(d + e/(f + g*x))^p]
)*Log[e + d*(f + g*x)] + b*d*f*p*(Log[f + g*x]*(Log[f + g*x] - 2*Log[(e +
d*f + d*g*x)/e]) - 2*PolyLog[2, -((d*(f + g*x))/e)]) - b*(e + d*f)*p*((2*L
og[-((d*(f + g*x))/e)] - Log[e + d*f + d*g*x])*Log[e + d*f + d*g*x] + 2*Po
lyLog[2, (e + d*f + d*g*x)/e])))/(d*g)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2933, 2899, 2904, 2841, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx$$

$$\Big\downarrow \text{ 2933}$$

$$\frac{\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 d(f+gx)}{g}$$

$$\Big\downarrow \text{ 2899}$$

$$\frac{\frac{2bep\int \frac{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{f+gx}d(f+gx)}{d}+\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{d}}{g}$$

$$\Big\downarrow \text{ 2904}$$

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{d}-\frac{2bep\int (f+gx)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)d\frac{1}{f+gx}}{d}}{g}$$

$$\Big\downarrow \text{ 2841}$$

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{d}-\frac{2bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)-bep\int \frac{\log\left(-\frac{e}{d(f+gx)}\right)}{d+\frac{e}{f+gx}}d\frac{1}{f+gx}\right)}{d}}{g}$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{d}-\frac{2bep\left(\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)+bp\,\text{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\right)}{d}}{g}$$

input
```
Int[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

output
```
(((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/d - (2*b*e*p*(Lo
g[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p]) + b*p*PolyLog[2,
1 + e/(d*(f + g*x))]))/d)/g
```

Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2899
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Simp[b*e*p*(q/d)   I
nt[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p}
, x] && IGtQ[q, 0]
```

rule 2904
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*L
og[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
 x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &
&  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

rule 2933
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
 x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
 && (EqQ[q, 1] || IntegerQ[n])
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{2}dx$$

input
```
int((a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

output
```
int((a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{2}dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
```

output
```
integral(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d*g*
x + d*f + e)/(g*x + f))^p) + a^2, x)
```

Sympy [F]

$$\int\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}dx=\int\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}dx$$

input
```
integrate((a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
```

output
```
Integral((a + b*log(c*(d + e/(f + g*x))**p))**2, x)
```

Maxima [F]

$$\int\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}dx=\int\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{2}dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
```

output
```
-2*a*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e
*g^2)) + 2*a*b*x*log(c*(d + e/(g*x + f))^p) + a^2*x + b^2*((d*g*x*log((d*g
*x + d*f + e)^p)^2 + d*g*x*log((g*x + f)^p)^2 - (d*f*p^2 + e*p^2)*log(d*g*
x + d*f + e)^2 + 2*(d*f*p^2 + e*p^2)*log(d*g*x + d*f + e)*log(g*x + f) - 2
*(d*f*p*log(g*x + f) + d*g*x*log((g*x + f)^p) - d*g*x*log(c) - (d*f*p + e*
p)*log(d*g*x + d*f + e))*log((d*g*x + d*f + e)^p) + 2*(d*f*p*log(g*x + f)
- d*g*x*log(c) - (d*f*p + e*p)*log(d*g*x + d*f + e))*log((g*x + f)^p))/(d*
g) - integrate(-(d*g^2*x^2*log(c)^2 + (d*f^2 + e*f)*log(c)^2 + (2*e*g*p*lo
g(c) + (2*d*f*g + e*g)*log(c)^2)*x - 2*(d*f^2*p^2 + 2*e*f*p^2 + (d*f*g*p^2
 + e*g*p^2)*x)*log(g*x + f))/(d*g^2*x^2 + d*f^2 + e*f + (2*d*f*g + e*g)*x)
, x))
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\,dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^2\,dx$$

input
```
integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/(g*x + f))^p) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\,dx=\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^2\,dx$$

input
```
int((a + b*log(c*(d + e/(f + g*x))^p))^2,x)
```

output
```
int((a + b*log(c*(d + e/(f + g*x))^p))^2, x)
```

Reduce [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\,dx$$
$$=\frac{2\left(\int \frac{\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)x}{d\,g^2x^2+2dfgx+d\,f^2+egx+ef}dx\right)b^2de\,g^2p+2\log(gx+f)\,abep+\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)^2 b^2dgx+2\log\left(\frac{(dgx+df}{(gx+}\right.}{dg}$$

input
```
int((a+b*log(c*(d+e/(g*x+f))^p))^2,x)
```

output

```
(2*int((log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*x)/(d*f**2 + 2*d*f*g*x
+ d*g**2*x**2 + e*f + e*g*x),x)*b**2*d*e*g**2*p + 2*log(f + g*x)*a*b*e*p +
 log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)**2*b**2*d*g*x + 2*log(((d*f +
d*g*x + e)**p*c)/(f + g*x)**p)*a*b*d*f + 2*log(((d*f + d*g*x + e)**p*c)/(f
 + g*x)**p)*a*b*d*g*x + 2*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a*b*e
 + a**2*d*g*x)/(d*g)
```

3.726 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\,dx=ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(e+d(f+gx))}{dg}$$

output
```
a*x+b*(g*x+f)*ln(c*(d+e/(g*x+f))^p)/g+b*e*p*ln(e+d*(g*x+f))/d/g
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\,dx=ax-begp\left(\frac{f\log(f+gx)}{eg^2}-\frac{(e+df)\log(e+df+dgx)}{deg^2}\right)+bx\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)$$

input
```
Integrate[a + b*Log[c*(d + e/(f + g*x))^p],x]
```

output
```
a*x - b*e*g*p*((f*Log[f + g*x])/(e*g^2) - ((e + d*f)*Log[e + d*f + d*g*x])
/(d*e*g^2)) + b*x*Log[c*(d + e/(f + g*x))^p]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(d(f+gx)+e)}{dg}$$

input
```
Int[a + b*Log[c*(d + e/(f + g*x))^p],x]
```

output
```
a*x + (b*(f + g*x)*Log[c*(d + e/(f + g*x))^p])/g + (b*e*p*Log[e + d*(f + g
*x)])/(d*g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.54

method	result	size
default	$ax+b\left(x\ln\left(c\left(\frac{xdg+df+e}{gx+f}\right)^{p}\right)+peg\left(-\frac{f\ln(gx+f)}{g^{2}e}+\frac{(df+e)\ln(xdg+df+e)}{e\,g^{2}d}\right)\right)$	77
parts	$ax+b\left(x\ln\left(c\left(\frac{xdg+df+e}{gx+f}\right)^{p}\right)+peg\left(-\frac{f\ln(gx+f)}{g^{2}e}+\frac{(df+e)\ln(xdg+df+e)}{e\,g^{2}d}\right)\right)$	77
parallelrisch	$-\frac{b\left(-x\ln\left(c\left(\frac{xdg+df+e}{gx+f}\right)^{p}\right)d\,g^{2}p-\ln(gx+f)eg\,p^{2}-\ln\left(c\left(\frac{xdg+df+e}{gx+f}\right)^{p}\right)dfgp-\ln\left(c\left(\frac{xdg+df+e}{gx+f}\right)^{p}\right)egp\right)}{d\,g^{2}p}+ax$	116

input
```
int(a+b*ln(c*(d+e/(g*x+f))^p),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*ln(c*((d*g*x+d*f+e)/(g*x+f))^p)+p*e*g*(-f/g^2/e*ln(g*x+f)+(d*f+e)
/e/g^2/d*ln(d*g*x+d*f+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

$$\int\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)dx$$
$$=\frac{bdgpx\log\left(\frac{dgx+df+e}{gx+f}\right)-bdfp\log(gx+f)+bdgx\log(c)+adgx+(bdf+be)p\log(dgx+df+e)}{dg}$$

input
```
integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="fricas")
```

output
```
(b*d*g*p*x*log((d*g*x + d*f + e)/(g*x + f)) - b*d*f*p*log(g*x + f) + b*d*g
*x*log(c) + a*d*g*x + (b*d*f + b*e)*p*log(d*g*x + d*f + e))/(d*g)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.14

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx = ax + b\left(\begin{cases} x\log\left(c\left(\frac{e}{f}\right)^p\right) & \text{for}\ d=0\wedge g=0 \\ \frac{f\log\left(c\left(\frac{e}{f+gx}\right)^p\right)}{g}+px+x\log\left(c\left(\frac{e}{f+gx}\right)^p\right) & \text{for}\ d=0 \\ x\log\left(c\left(d+\frac{e}{f}\right)^p\right) & \text{for}\ g=0 \\ \frac{f\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+x\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)+\frac{ep\log(df+dgx+e)}{dg} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*ln(c*(d+e/(g*x+f))**p),x)
```

output
```
a*x + b*Piecewise((x*log(c*(e/f)**p), Eq(d, 0) & Eq(g, 0)), (f*log(c*(e/(f
 + g*x))**p)/g + p*x + x*log(c*(e/(f + g*x))**p), Eq(d, 0)), (x*log(c*(d +
 e/f)**p), Eq(g, 0)), (f*log(c*(d + e/(f + g*x))**p)/g + x*log(c*(d + e/(f
 + g*x))**p) + e*p*log(d*f + d*g*x + e)/(d*g), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx = -begp\left(\frac{f\log(gx+f)}{eg^2}-\frac{(df+e)\log(dgx+df+e)}{deg^2}\right) + bx\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+ax$$

input
```
integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="maxima")
```

output
```
-b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*g^2
)) + b*x*log(c*(d + e/(g*x + f))^p) + a*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.52

$$\int \left(a + b\log\left(c\left(d + \frac{e}{f + gx}\right)^p\right)\right)dx$$
$$= \left(\frac{e^2 p \log\left(\frac{dgx+df+e}{gx+f}\right)}{dg^2 - \frac{(dgx+df+e)g^2}{gx+f}} + \frac{e^2\log(c)}{dg^2 - \frac{(dgx+df+e)g^2}{gx+f}} + \frac{e^2 p\log\left(-d + \frac{dgx+df+e}{gx+f}\right)}{dg^2} - \frac{e^2 p\log\left(\frac{dgx+df+e}{gx+f}\right)}{dg^2}\right) b\left(\frac{dfg}{e^2} - \frac{(\right.$$
$$+ ax$$

input
```
integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="giac")
```

output
```
(e^2*p*log((d*g*x + d*f + e)/(g*x + f))/(d*g^2 - (d*g*x + d*f + e)*g^2/(g*
x + f)) + e^2*log(c)/(d*g^2 - (d*g*x + d*f + e)*g^2/(g*x + f)) + e^2*p*log
(-d + (d*g*x + d*f + e)/(g*x + f))/(d*g^2) - e^2*p*log((d*g*x + d*f + e)/(
g*x + f))/(d*g^2))*b*(d*f*g/e^2 - (d*f + e)*g/e^2) + a*x
```

Mupad [B] (verification not implemented)

Time = 25.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \left(a + b\log\left(c\left(d + \frac{e}{f + gx}\right)^p\right)\right)dx = a\,x + b\,x\,\ln\left(c\left(d + \frac{e}{f + g\,x}\right)^p\right) - \frac{b\,f\,p\,\ln(f + g\,x)}{g} + \frac{b\,p\,\ln(e + d\,f + d\,g\,x)\,(e + d\,f)}{d\,g}$$

input
```
int(a + b*log(c*(d + e/(f + g*x))^p),x)
```

output
```
a*x + b*x*log(c*(d + e/(f + g*x))^p) - (b*f*p*log(f + g*x))/g + (b*p*log(e
 + d*f + d*g*x)*(e + d*f))/(d*g)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.08

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\,dx$$
$$=\frac{\log(gx+f)\,bep+\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)bdf+\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)bdgx+\log\left(\frac{(dgx+df+e)^p c}{(gx+f)^p}\right)be+adgx}{dg}$$

input
```
int(a+b*log(c*(d+e/(g*x+f))^p),x)
```

output
```
(log(f + g*x)*b*e*p + log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*d*f + l
og(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*d*g*x + log(((d*f + d*g*x + e)
**p*c)/(f + g*x)**p)*b*e + a*d*g*x)/(d*g)
```

3.727 $\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\text{Int}\left(\frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)},x\right)$$

output
```
Defer(Int)(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]
```

output
```
Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2934}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}dx$$
$$\Big\downarrow \text{ 2934}$$
$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}dx$$

input
```
Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2934
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Unintegrable[(a + b*Log[c*(d + e*(f + g*x)^n)^p])^q,
 x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)}dx$$

input
```
int(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

output
```
int(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)}\,dx=\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a}\,dx$$

input
```
integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="fricas")
```

output
```
integral(1/(b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p)),x)
```

output
```
Integral(1/(a + b*log(c*(d + e/(f + g*x))**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a}\,dx$$

input
```
integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="maxima")
```

output
```
integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a}\,dx$$

input
```
integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="giac")
```

output
```
integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

Mupad [N/A]

Not integrable

Time = 26.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{a+b\,\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^p\right)}\,dx$$

input
```
int(1/(a + b*log(c*(d + e/(f + g*x))^p)),x)
```

output
```
int(1/(a + b*log(c*(d + e/(f + g*x))^p)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 661, normalized size of antiderivative = 30.05

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*log(c*(d+e/(g*x+f))^p)),x)
```

output
```
(int(x**2/(log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*d*f**2 + 2*log(((d
*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*d*f*g*x + log(((d*f + d*g*x + e)**p*
c)/(f + g*x)**p)*b*d*g**2*x**2 + log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p
)*b*e*f + log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*e*g*x + a*d*f**2 +
2*a*d*f*g*x + a*d*g**2*x**2 + a*e*f + a*e*g*x),x)*b*d*e*g**3*p + 2*int(x/(
log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*d*f**2 + 2*log(((d*f + d*g*x
+ e)**p*c)/(f + g*x)**p)*b*d*f*g*x + log(((d*f + d*g*x + e)**p*c)/(f + g*x
)**p)*b*d*g**2*x**2 + log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*e*f + l
og(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*e*g*x + a*d*f**2 + 2*a*d*f*g*x
 + a*d*g**2*x**2 + a*e*f + a*e*g*x),x)*b*d*e*f*g**2*p + int(x/(log(((d*f +
 d*g*x + e)**p*c)/(f + g*x)**p)*b*d*f**2 + 2*log(((d*f + d*g*x + e)**p*c)/
(f + g*x)**p)*b*d*f*g*x + log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*d*g
**2*x**2 + log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*b*e*f + log(((d*f +
d*g*x + e)**p*c)/(f + g*x)**p)*b*e*g*x + a*d*f**2 + 2*a*d*f*g*x + a*d*g**2
*x**2 + a*e*f + a*e*g*x),x)*b*e**2*g**2*p - log(log(((d*f + d*g*x + e)**p*
c)/(f + g*x)**p)*b + a)*d*f**2 - log(log(((d*f + d*g*x + e)**p*c)/(f + g*x
)**p)*b + a)*e*f)/(b*e*g*p)
```

3.728 $\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2},x\right)$$

output
```
Defer(Int)(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

input
```
Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
```

output
```
Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2934\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}dx$$
$$\Big\downarrow \text{2934}$$
$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}dx$$

input
```
Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2934
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Unintegrable[(a + b*Log[c*(d + e*(f + g*x)^n)^p])^q,
 x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{2}}dx$$

input
```
int(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

output
```
int(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}}\,dx=\int \frac{1}{\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{2}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d
*g*x + d*f + e)/(g*x + f))^p) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}}\,dx$$

input
```
integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
```

output
```
Integral((a + b*log(c*(d + e/(f + g*x))**p))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 6.73

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}}\,dx=\int \frac{1}{\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{2}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
```

output
```
(d*g^2*x^2 + d*f^2 + e*f + (2*d*f*g + e*g)*x)/(b^2*e*g*p*log((d*g*x + d*f
+ e)^p) - b^2*e*g*p*log((g*x + f)^p) + b^2*e*g*p*log(c) + a*b*e*g*p) - int
egrate((2*d*g*x + 2*d*f + e)/(b^2*e*p*log((d*g*x + d*f + e)^p) - b^2*e*p*l
og((g*x + f)^p) + b^2*e*p*log(c) + a*b*e*p), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}}\,dx=\int \frac{1}{\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{2}}\,dx$$

input
```
integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")
```

output
```
integrate((b*log(c*(d + e/(g*x + f))^p) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 25.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^2}\,dx$$

input
```
int(1/(a + b*log(c*(d + e/(f + g*x))^p))^2,x)
```

output
```
int(1/(a + b*log(c*(d + e/(f + g*x))^p))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 2449, normalized size of antiderivative = 111.32

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x)
```

output

```
(int(x**2/(log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)**2*b**2*d*f**2 + 2*l
og(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)**2*b**2*d*f*g*x + log(((d*f + d*
g*x + e)**p*c)/(f + g*x)**p)**2*b**2*d*g**2*x**2 + log(((d*f + d*g*x + e)*
*p*c)/(f + g*x)**p)**2*b**2*e*f + log(((d*f + d*g*x + e)**p*c)/(f + g*x)**
p)**2*b**2*e*g*x + 2*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a*b*d*f**2
 + 4*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a*b*d*f*g*x + 2*log(((d*f
+ d*g*x + e)**p*c)/(f + g*x)**p)*a*b*d*g**2*x**2 + 2*log(((d*f + d*g*x + e
)**p*c)/(f + g*x)**p)*a*b*e*f + 2*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**
p)*a*b*e*g*x + a**2*d*f**2 + 2*a**2*d*f*g*x + a**2*d*g**2*x**2 + a**2*e*f
+ a**2*e*g*x),x)*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a*b*d*e*g**3*p
 + int(x**2/(log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)**2*b**2*d*f**2 + 2
*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)**2*b**2*d*f*g*x + log(((d*f +
d*g*x + e)**p*c)/(f + g*x)**p)**2*b**2*d*g**2*x**2 + log(((d*f + d*g*x + e
)**p*c)/(f + g*x)**p)**2*b**2*e*f + log(((d*f + d*g*x + e)**p*c)/(f + g*x)
**p)**2*b**2*e*g*x + 2*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a*b*d*f*
*2 + 4*log(((d*f + d*g*x + e)**p*c)/(f + g*x)**p)*a*b*d*f*g*x + 2*log(((d*
f + d*g*x + e)**p*c)/(f + g*x)**p)*a*b*d*g**2*x**2 + 2*log(((d*f + d*g*x +
 e)**p*c)/(f + g*x)**p)*a*b*e*f + 2*log(((d*f + d*g*x + e)**p*c)/(f + g*x)
**p)*a*b*e*g*x + a**2*d*f**2 + 2*a**2*d*f*g*x + a**2*d*g**2*x**2 + a**2*e*
f + a**2*e*g*x),x)*a**2*d*e*g**3*p + 2*int(x/(log(((d*f + d*g*x + e)**p...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                      Small rewrite of logic in main function to make it*)
(*                      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file